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ACCOUNTING FOR GROWTH WITH NEW INPUTS

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ABSTRACT

In this paper we examine how to account for growth when new inputs are being created. In particular, we obtain a decomposition of growth into that due to a *higher quantity* of existing inputs, and that due to a *greater range* of inputs. This decomposition is first obtained for a single firm, with a CES production function. We then generalize to the GNP function of an economy, and again show how a decomposition of growth in GNP can be obtained. An example is presented of a two-sector economy, where new inputs are endogenously created each period, and a simple aggregate production function exists. Data for this economy are simulated, and the GNP function is estimated using various different measures of the factor inputs.

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1. INTRODUCTION

Since the work of Solow (1957), it has been known that technological change accounts for a significant portion of GNP growth in industrialized economies. This technological change has been measured by either the estimated time trend in regressions of aggregate output on inputs, or by indexes of total factor productivity. Since under either method productivity is measured as a residual, it incorporates *all* factors which influence GNP growth other than the increase in measured inputs. Despite various refinements to the measurement of total factor productivity, we still do not have a convincing explanation for its source (see Jorgenson and Griliches, 1967; Solow, 1988).

Recent literature has suggested a potential source of productivity gains: namely, the creation of new inputs under monopolistic competition. Ethier (1982) has argued that the development of new intermediate inputs leads to greater specialization in the use of resources, and higher productivity; see also Markusen (1989). Subsequent authors, including Romer (1990) and Grossman and Helpman (1991), have examined models where continuous growth is made possible by the creation of new inputs. Romer (1987) has considered the form of the aggregate production function in such an economy, and argues that the contribution of capital to growth may significantly exceed its cost share in output. His results have recently been reexamined by Benhabib and Jovanovic (1991).

In this paper we shall examine how to account for growth when new inputs are being created. In particular, we are interested in obtaining a decomposition of growth into that due to a *higher quantity* of existing inputs, and that due to a *greater range* of inputs. In section 2, we show how this decomposition can be obtained for a single firm, with a CES production function.[1] In section 3 we generalize to the GNP function of an economy, and again show how a decomposition of growth in GNP can be obtained. The properties of the GNP function are characterized in detail.

In section 4 we present an example of a two-sector economy, where new inputs are endogenously created each period, and a simple aggregate production function exists. In section 5 we use this example to estimate real GNP as a function of primary resources. We suppose that the productivity gains due to new inputs are not explicitly measured, but picked up in the time trend of this regression. We find that the time trend is significant, and that the estimated coefficients of the production function are biased. This bias arises due to correlation between the observed primary resources, and the unobserved but endogenous productivity gains. Conclusions are given in section 6.

2. SINGLE FIRM

We shall initially consider a single, competitive firm with constant returns to scale. We shall adopt a CES production function:

$$y = f(x,M) = \left[\sum_{i=1}^{M} a_i x_i^{\theta} \right]^{1/\theta}, \quad 0 < \theta < 1, \tag{1}$$

where x_i is the quantity of input $i=1,\ldots,M$, $x = (x_1,\ldots,x_M)$ denotes the vector of inputs, and y is the output. The elasticity of substitution between the inputs is given by $\sigma = 1/(1-\theta)$. We shall denote the prices of the inputs as $p_i > 0$, and suppose that the quantities $x_i > 0$ solve the cost-minimization problem:

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[1] The results in section 2 are also reported in Feenstra, Markusen and Zeile (1992), who estimate the impact of new inputs on productivity for a sample of Korean industries.

$$\min_{x_i > 0} \sum_{i=1}^{M} p_i x_i \text{ subject to } y = f(x,M). \tag{2}$$

In our first result, we let $M_0 \leq M_1$ denote two ranges of inputs. We shall suppose that the M_1-dimensional vector $x > 0$ solves (2) for prices $p > 0$. We are then interested in relating the outputs $f(x,M_1)$ and $f(x,M_0)$, as well as relating their first derivatives:

<u>Proposition 1</u>.

(a) $f(x,M_1) = f(x,M_0)\lambda^{1/\theta}$, where $\lambda \equiv \left[\sum_{i=1}^{M_1} p_i x_i \Big/ \sum_{i=1}^{M_0} p_i x_i\right]$. (3)

(b) $\partial f(x,M_1)/\partial x_i = [\partial f(x,M_0)/\partial x_i]\lambda^{(1-\theta)/\theta}$, where $1 \leq i \leq M_0$.

The proofs of all Propositions are given in the Appendix. Part (a) shows that the outputs obtained with the ranges of inputs M_0 and M_1 are related by the factor $\lambda^{1/\theta} \geq 1$. This variable is endogenously determined by the cost-minimizing input choices of the firm. It can be easily measured as the ratio of expenditure on the full and restricted set of inputs at the common prices p_i, raised to the power $(1/\theta)$. As θ becomes smaller then λ increases, indicating that the new inputs which are not close substitutes for existing ones lead to a larger increase in output. Part (b) shows that the derivatives of $f(x,M_1)$ exceed those of $f(x,M_0)$, by the factor $\lambda^{(1-\theta)/\theta} \geq 1$. Thus, for given quantities of the inputs, their marginal product will rise as the range of inputs increases.

To show how Proposition 1 can be used to obtain a decomposition of the growth in output, suppose that the vector of inputs $x_t > 0$ is cost-minimizing when prices are $p_t > 0$ and the number of inputs is M_t, $t = 0,1$, with $M_0 \leq M_1$. Then the ratio of outputs is:

$$f(x_1,M_1)/f(x_0,M_0) = [f(x_1,M_1)/f(x_1,M_0)][f(x_1,M_0)/f(x_0,M_0)]$$

$$= \lambda^{1/\theta}\,[f(x_1,M_0)/f(x_0,M_0)]$$

$$= \lambda^{1/\theta}\,Q(x_0,p_0,x_1,p_1;M_0). \quad (4)$$

The first line of (4) is an identity and the second line is obtained from Proposition 1(a), with λ defined as in (3) but with the prices and quantities indexed by period 1.

To obtain the third line, we note that $f(x_1,M_0)/f(x_0,M_0)$ is the ratio of two CES functions with the same number of inputs, where the inputs x_1 and x_0 are cost-minimizing for the prices p_1 and p_0, respectively.[2] Then we can use the formula of Sato (1976) and Vartia (1976) to express $f(x_1,M_0)/f(x_0,M_0)$ as an *exact* quantity index, denoted by $Q(x_0,p_0,x_1,p_1;M_0)$. As the notion suggests, this quantity index can be computed with the available price and quantity data, and does not depend on the unknown parameters a_i in the production function (1). Thus, we have obtained a decomposition of output growth into that due to changes in the input *quantities* (as measured by the index Q), and that due to changes in the range of inputs (as measured by the factor λ).

An alternative decomposition can be obtained by using an *implicit* quantity index. Let $P(x_0,p_0,x_1,p_1;M_0) = c(p_1,M_0)/c(p_0,M_0)$ denote the Sato-Vartia price index for the CES unit-cost function. This price index is computed by ignoring the new inputs in the second period. Then define the implicit quantity index $\tilde{Q}$

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[2] Note that the inputs x_{1i} for $i = 1,\ldots,M_0$ satisfy the cost minimization problem (2) for prices p_{1i} and the output $\tilde{y}_1 \equiv f(x_1,M_0)$.

as $\tilde{Q}(x_0,p_0,x_1,p_1) \equiv (C_1/C_0)/P(x_0,p_0,x_1,p_1;M_0)$, where C_t denotes total costs in period t = 0,1. This index is interpreted as the ratio of total costs in the two periods, divided by the price index which ignores the new inputs. From Feenstra (1991), the ratio of unit-costs can be measured by $c(p_1,M_1)/c(p_0,M_0) = P(x_0,p_0,x_1,p_1;M_0)\lambda^{-(1-\theta)/\theta}$. Then the ratio of outputs equals:

$$f(x_1,M_1)/f(x_0,M_0) = (C_1/C_0)/[c(p_1,M_1)/c(p_0,M_0)] \\ = \tilde{Q}(x_0,p_0,x_1,p_1)\lambda^{(1-\theta)/\theta}, \tag{4'}$$

where the first line is obtained since C_t equals $c(p_t,M_t)f(x_t,M_t)$, and the second line follows from our results above. This decomposition of output growth is preferred to (4), since the bias $\lambda^{(1-\theta)/\theta}$ is less than $\lambda^{1-\theta}$, for $0 < \theta < 1$. In particular, since $(1-\theta)/\theta = 1/(\sigma-1)$, as $\theta \to 1$ and the elasticity of substitution is very large then the term $\lambda^{(1-\theta)/\theta}$ will approach unity. Thus, the quantity index $\tilde{Q}(x_0,p_0,x_1,p_1)$ will equal the ratio of outputs as the new inputs become perfect substitutes for existing inputs.

Our results can be readily extended to the case where some inputs are disappearing, while others are new. Results of this type for the CES unit-cost function are presented in Feenstra (1991). Rather than discuss this extension here, we turn next to the economy-wide GNP function.

3. GNP FUNCTION

We consider an economy where new intermediate inputs are used in the production of final goods. We will show that GNP can be expressed as a function of the primary inputs, and of the variables $\lambda_1,\ldots,\lambda_N$ defined as in (3) for *each* of the final goods industries. Using this function, we will be able to decompose the growth of GNP into that due to a greater quantity of primary inputs, and that due to an expanded range of intermediate inputs.

To develop our notation, suppose there are L primary inputs denoted by ℓ (think of labor and land). Primary factors are used to produce intermediate inputs, and these are combined with additional primary factors to produce the final goods. Let there be N final goods indexed by j, each of which are produced with M_j intermediate inputs indexed by i. The production function for each final good is weakly separable between the intermediate and primary inputs, and is given by,

$$y_j = g_j[f_j(x_j, M_j), v_j], \qquad j=1,\dots,N, \tag{5}$$

where x_j is the M_j-dimensional vector of intermediate inputs used in the final goods industry j; f_j is a CES function as in (1) with parameter θ_j; and v_j is the L-dimensional vector of primary inputs. We assume that g_j is increasing and concave, and will sometimes specify that it is homogeneous of degree one.

We suppose that the intermediate inputs are *customized* to each final good industry, so that the production function for the input i used in the final goods industry j depends on both i and j:

$$x_{ij} = h_{ij}(v_{ij}), \qquad i=1,\dots,M_j, \qquad j=1,\dots,N, \tag{6}$$

where v_{ij} is the L-dimensional vector of primary inputs (excluding possible fixed costs of product development). We also assume that h_{ij} is increasing and concave, with $h_{ij}(0) = 0$, and will sometimes specify that it is homogeneous of degree one. Finally, the resource constraint for the economy is that,

$$\sum_{j=1}^{N} v_j + \sum_{j=1}^{N} \sum_{i=1}^{M_j} v_{ij} = \overline{v}, \tag{7}$$

where $\overline{v}$ is the L-dimensional vector of primary resource devoted to goods production. Note that (7) excludes resources which are devoted to the creation of new inputs (i.e., to cover the fixed costs of product development).

Let the prices of final goods be denoted by q_j, j=1,...,N. Then in a *perfectly competitive* economy with fixed ranges of intermediate inputs M_j, GNP would be obtained as the maximum of $\Sigma_j q_j y_j$ subject to (5), (6) and (7). We will instead consider a *monopolistically competitive* economy, where the range of inputs M_j can change. In the next section we present a fully specified example where M_j is endogenous. Here we shall simply treat M_j as a parameter, but recognize that changes in it affect GNP.

We suppose that final goods are produced competitively, while the intermediate inputs are produced under monopolistic competition, so that marginal cost pricing is not used. Instead, we need to compute the derived demand from the CES functions in (5) and (1), and the associated marginal revenue: letting p_{ij} denote the price of the intermediate input i used in industry j, the marginal revenue is given by $\theta_j p_{ij}$. Then each intermediate goods producer will hire primary factors to the point where the marginal revenue product equals the wage:

$$\theta_j p_{ij}(\partial h_{ij}/\partial v_{ij}) = w, \tag{8}$$

where w is the L-dimensional vector of wages for primary inputs.

To develop the GNP function for the economy, we first rewrite (5) using our results from Proposition 1:

$$y_j = g_j[f_j(x_j, M_{j_0})\lambda_j^{1/\theta_j}, v_j], \qquad j=1,\dots,N, \tag{9}$$

where M_{j0} is an *initial* set of inputs available, and λ_j is defined as in (3) but with the subscript j added to denote the intermediates used by industry j:

$$\lambda_j \equiv \left[\sum_{i=1}^{M_j} p_{ij}x_{ij} / \sum_{i=1}^{M_{j0}} p_{ij}x_{ij} \right]. \tag{3'}$$

We shall be holding M_{j0} constant, and allowing λ_j to vary as a parameter, depending on the range of new inputs available. We next define an *artificial* vector of primary factor endowments as:

$$\tilde{v} \equiv \sum_{j=1}^{N} v_j + \sum_{j=1}^{N} \sum_{i=1}^{M_{j0}} v_{ij} (\lambda_j/\theta_j) . \tag{10}$$

Notice that the final summation in (10) is over the restricted set of inputs M_{j0}, and that the primary inputs v_{ij} are adjusted by the factor (λ_j/θ_j). We will not attempt to justify this artificial measurement of primary factors until after we show how $\tilde{v}$ can be used.

Let $\lambda = (\lambda_1,...,\lambda_N)$ denote the vector of parameters λ_j for each final goods industry. Then consider the following definition of the GNP function:

$$G(q,\tilde{v},\lambda) \equiv \max \sum_{j=1}^{N} q_j y_j \text{ subject to (6), (9) and (10),} \tag{11}$$

where the choice variables in (11) are $x_{ij} > 0$ for $i=1,...,M_{j0}$, $v_j \geq 0$ and $v_{ij} \geq 0$. We substitute (9) for y_j in the objective function of (11), and let $\tilde{p}_{ij}$ and the vector w denote the Lagrange multipliers on the constraints (6) and (10), respectively. Then the first-order conditions for an interior solution of (11) are:

$$q_j(\partial g_j/\partial v_j) = w, \tag{12}$$

$$q_j\left[\frac{\partial g_j}{\partial f_j}\ \frac{\partial f_j(x_j,M_{j0})}{\partial x_{ij}}\right]\lambda_j^{1/\theta_j} = \tilde{p}_{ij}. \tag{13}$$

$$\tilde{p}_{ij}(\partial h_{ij}/\partial v_{ij}) = w(\lambda_j/\theta_j). \tag{14}$$

Condition (12) simply states that the marginal value product of primary resources devoted to final good production equals the wage w. To interpret the next two conditions, define $p_{ij} \equiv \tilde{p}_{ij}/\lambda_j$ as a transformation of the Lagrange multiplier. Then using Proposition 1(b) we can rewrite (13) as:

$$q_j\left[\frac{\partial g_j}{\partial f_j}\ \frac{\partial f_j(x_j,M_j)}{\partial x_{ij}}\right] = p_{ij}\ . \tag{15}$$

This is the familiar condition that the marginal value product of intermediate inputs used in the production of final goods equal their price p_{ij}. Using the same definition of p_{ij} we can rewrite (14) as $\theta_j p_{ij}(\partial h_{ij}/\partial v_{ij}) = w$, which is identical to (8). This condition states that the marginal *revenue* product of primary factors used to produce intermediate inputs equals the wage, as required under monopolistic competition.

Summing up, we have shown that the first-order conditions for (11) can be written as (8), (12), and (15), which are precisely the equilibrium conditions of the economy with monopolistically competitive production of intermediate inputs. We conclude that $G(q,\tilde{v},\lambda)$ will be the value of GNP in this economy, depending on q, $\tilde{v}$, and λ. Several properties of this GNP function are summarized by:

Proposition 2.

(a) $G(q,\tilde{v},\lambda)$ is increasing and concave in $\tilde{v}$, with $G_{\tilde{v}} = w$.

(b) $G(q,\tilde{v},\lambda)$ is increasing and convex in q, with $G_q = y$.

(c) $G(q,\tilde{v},\lambda)$ is homogeneous of degree one in q, and homogeneous of degree one in $\tilde{v}$ if g_j and h_{ij} are homogeneous of degree one.

(d) $G(q,\tilde{v},\lambda)$ is increasing in λ, and $G_{\lambda_j} \geq \lambda_j^{-1} \frac{(1-\theta_j)}{\theta_j} \sum_{i=1}^{M_j} p_{ij}x_{ij}$ with equality when h_{ij} are homogeneous of degree one.

(e) Suppose that $h_{ij}(v_{ij}) = h_j(v_{ij})$ for all i and j, and that h_j is homogeneous of degree one. Then:

$$\tilde{v} = \sum_{j=1}^{N} v_j + \sum_{j=1}^{N} \sum_{i=1}^{M_j} v_{ij}/\theta_j . \tag{16}$$

Parts (a), (b) and (c) show that the GNP function for the monopolistically competitive economy satisfies the conventional properties which also hold in the competitive case. To interpret (d), write this condition (as an equality) in elasticity form as,

$$G_{\lambda_j}(\lambda_j/G) = \frac{(1-\theta_j)}{\theta_j} \sum_{i=1}^{M_j} p_{ij}x_{ij}/G.$$

Thus, the elasticity of GNP with respect to the range of intermediate λ_j equals the *share of these intermediate inputs in total GNP*, multiplied by $(1-\theta_j)/\theta_j$. This latter term enters because the contribution of the new inputs to GNP will depend on how closely they substitute for existing inputs: when $\theta_j = 1$ then new inputs are perfect substitutes for existing ones in (1), and so their presence has no impact on GNP. More generally, since $(1-\theta_j)/\theta_j = 1/(\sigma_j-1)$, the new inputs will have a larger impact when their elasticity of substitution $\sigma_j > 1$ with existing inputs is lower.

Under the conditions in (e),[3] the artificial inputs $\tilde{v}$ are measured in the simpler form, which is interpreted as the *total resources devoted to production including the fixed costs of product development.* To see this, note that (16) is the same as $\tilde{v}$ in (7) except that the resources v_{ij} are weighted by $(1/\theta_j)$. This weighting reflects the resources used *plus* the profits earned in the production of intermediates: letting c_{ij} denote the unit-costs of producing an intermediate x_{ij}, the price is $p_{ij} = c_{ij}/\theta_j$ and so profits are $(p_{ij}-c_{ij})x_{ij} = (1-\theta_j)c_{ij}x_{ij}/\theta_j$. By summing profits and total (variable) costs we then obtain $c_{ij}x_{ij}/\theta_j$, so that the resources used in the industry must be inflated by $1/\theta_j$ to include these profits. We interpret the profits earned per period as equalling the fixed costs of product development, where the total fixed costs are amortized over time. Indeed, it is common to impute the profits earned in an industry as some form of capital when constructing measures of aggregate resources, so that we could use existing measures of resource endowments for $\tilde{v}$.

In the next sections we develop a specific example of the GNP function.

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[3] The first condition in (e) is somewhat unusual, since it states that all the intermediate inputs x_{ij} used in the final goods industry j must be produced with the *same* production function h_j. This condition is more reasonable if we think of the intermediates x_{ij} as just *different varieties* of the same basic input i used in industry j, where all varieties have the same production function. Inputs into this industry which are quite different could be represented by *another* CES function entering into the final good production g_j in (5).

4. A SIMPLE DYNAMIC MONOPOLISTIC-COMPETITION MODEL

We now turn to the question of constructing an explicit dynamic model in which new goods are created endogenously over time. Having constructed such a model, we then simulate a dynamic economy. The model we develop has several attractive features. First, the new goods are created by an endogenous, optimizing process rather than introduced exogenously by some unspecified or ad hoc process. Second, the model is a monopolistic-competition formulation that is similar to our discussion above and ties in well with recent literature (Romer, 1987, 1990; Grossman and Helpman, 1991; Markusen, 1991). Third, real income increases are driven solely by the introduction of new goods and capital accumulation, and there is no technical change in the usual sense, but a traditional growth accounting procedure may generally identify it as such.

The economy consist of two final-goods sectors, with outputs y_1 and y_2. Roughly speaking, lower-case letters correspond to flow variables, while upper-case letters correspond to stocks. Consumers maximize a standard time-separable utility function, where ρ is the rate of time preference:

$$W = \sum_{t=0}^{T} \left(1+\rho_t\right)^{-t} \left(y_{1t}^{\alpha} y_{2t}^{1-\alpha}\right)^{\eta}, \qquad \eta < 1. \tag{17}$$

Sector 2 is competitive, producing output (y_2) with Labor (L), capital (K), and a sector-specific factor (R, which could be land or natural resources):

$$y_{2t} = L_{2t}^{\beta} K_{2t}^{\varepsilon} R_t^{1-\beta-\varepsilon}, \qquad 0 < \beta, \varepsilon < 1. \tag{18}$$

Sector 1 costlessly assembles intermediate inputs x_i as in Ethier (1982):

$$y_{1t} = \left[\sum_{i=1}^{M_t} x_{it}^{\theta} \right]^{1/\theta}, \quad 0 < \theta < 1 . \tag{19}$$

The number of intermediate inputs (M) and their output levels (x_i) are endogenous. The x_i are produced with identical constant-returns, production functions using capital and labor:

$$x_{it} = L_{ixt}^{\gamma} K_{ixt}^{1-\gamma} . \tag{20}$$

In addition, to begin production of a new input, a once-and-for-all fixed input F_i must be created. This fixed input corresponds to the notion of non-depreciating knowledge capital that must be acquired before introducing a new good. F_i is a constant, but substitution between capital and labor is allowed. We specify the factor intensities of x_{it} and F_i as identical, so that factor intensity does not vary with the scale of production, but that assumption is in no way important to the results that follow:

$$F_i = L_{ift}^{\gamma} K_{ift}^{1-\gamma}. \tag{21}$$

For simplicity, one unit of new capital (k) is produced from one unit of labor: $k_t = L_{kt}$. The endowments L_t and R_t may grow exogenously over time. Adding up constraints for labor and capital are as follows:

$$L_t = L_{xt} + L_{ft} + L_{2t} + L_{kt}, \quad L_{xt} \equiv \sum_{i=1}^{M} L_{ixt}, \quad L_{ft} \equiv \sum_{i=1}^{M} L_{ift} . \tag{22a}$$

$$K_t = K_{xt} + K_{ft} + K_{2t}, \quad K_{xt} = \sum_{i=1}^{M} K_{ixt}, \quad K_{ft} \equiv \sum_{i=1}^{M} K_{ift} . \tag{22b}$$

The equation of motion for the capital stock uses a constant depreciation rate (δ):

$$K_t = (1-\delta)K_{t-1} + k_t \tag{23}$$

Let p_{it} denote the price of x_{it} in terms of y_{2t}, and $c(w_t,r_t)$ denote the unit-cost for x_{it} or F_i, where w_t and r_t are the rental prices of labor and capital in terms of y_{2t}. We define, for $\tau \geq t$, real interest rates $i_{\tau t}$ such that $(1 + i_{\tau t})$ is the intertemporal relative price of y_{2t} in terms of $y_{2\tau}$ ($i_{\tau t} = 0$ for $\tau = t$). Consumers maximize (17) subject to an intertemporal budget constraint that requires that the present value at t = 0 of the income stream from factor rents equal the present value of consumption expenditure on y_{1t} and y_{2t}. There is thus an implicit asset market in which consumers can borrow or lend, and in which entering firms can borrow to finance fixed costs. We can think of asset-market clearing as determining the $i_{\tau t}$.

The symmetry of the x_{it} in (19) and (20) implies that any input that is produced is produced in the same amount and sells for the same price. Hence the i subscript will generally be dropped. A firm producing an input x_t views factor prices as parametric, and maximizes profits in terms of the competitive good y_{2t}. A firm entering at time t maximizes the present value of profits:

$$\max \pi_t = \sum_{\tau=1}^{T} \left[(1+i_{\tau t})^{-1}(p_\tau - c(w_\tau,r_\tau))x_\tau \right] - c(w_t,r_t)F.$$

The full solution to the model determines a price vector $(p_t,w_t,r_t,v_t,i_{\tau t})$ for each t and $\tau \geq t$. Suppose that there are many x_t producers, such that we can assume a monopolistically competitive market structure. With demand for an input given from the CES production function (19), the marginal revenue from selling an intermediate input is θp_t. Then equilibrium in the monopolistically competitive sector is given by marginal revenue equals marginal cost, and zero profits due to free entry:

$$p_t\theta = c(w_t,r_t)\,, \quad \pi_t = 0. \tag{24}$$

Define the following aggregate factor supplies as in the previous section:

$$\tilde{L}_t \equiv L_{2t} + L_{xt}/\theta, \qquad \tilde{K}_t \equiv K_{2t} + K_{xt}/\theta. \tag{25}$$

Let U_t be defined as within-period utility, or real GNP. In the Appendix, we show how the equilibrium conditions (24) determine the allocation of labor and capital across sectors, and therefore the output of each good. Real GNP as a function of the factor supplies in (25) can then be obtained as:

<u>Proposition 3</u>.

$$U_t \equiv y_{1t}^{\alpha}y_{2t}^{1-\alpha} = C\tilde{L}_t^{a_1}\tilde{K}_t^{a_2}R_t^{a_3}M_t^{a_4}, \qquad \sum_{i=1}^{3}a_i = 1, \qquad \sum_{i=1}^{4}a_i > 1, \tag{26}$$

where C is a constant and:

$$a_1 = \alpha\gamma + (1-\alpha)\beta,\ a_2 = \alpha(1-\gamma) + (1-\alpha)\varepsilon,\ a_3 = (1-\alpha)(1-\beta-\varepsilon),\ a_4 = \alpha(1-\theta)/\theta. \tag{27}$$

Equation (26) gives single-period real GNP, and several features are of interest. First, there are constant returns to scale in the (correctly measured) primary inputs, as we showed in the general case. Second, if we view M_t as the proxy for accumulated knowledge capital (since M_t is linear in F), then there are increasing returns to scale in all four inputs. Third, since $\alpha(1-\theta)/\theta$

appears as the exponent of M_t in (26), the results are consistent with Proposition 2(d), where $\lambda_t = M_t/M_0$ in our previous notation and α is the share in GNP of sector 1 using the new intermediates inputs.

5. SIMULATION AND ESTIMATION

We solved the two-sector dynamic monopolistic competition model of the previous section numerically, and the output series generated was used as a data set for estimation purposes. We report here a simulation run over twelve time periods, and solved as a simultaneous general-equilibrium system using Rutherford's (1989) non-linear complementarity software, MPS/GE.[4] The model solves for 73 activity levels, the prices of 97 commodities and factors, consumer income, and the values of 16 auxilliary variables used to incorporate the scale economies: 187 non-linear inequalities in 187 unknowns. In the simulation, new intermediate goods were introduced over the first nine time periods, and new capital was produced over the first eleven time periods.[5]

Simulation results were then treated as a time series to estimate a GNP function. Results are presented for seven log-linear regressions in Table 1. The first regression is the true GNP function in (26), with coefficients correctly given by (27) and summing to 1.1.

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[4] Rutherford's non-linear complementarity easily handles corner solutions in which some activities are not active in some time periods. This is vital to the simulation of a model like this with a finite time horizon. There exists a steady-state version of this model in an infinite horizon case, but because there is no variation in the data in the steady state, the regressions to follow cannot be estimated. The model quickly converges to steady-state values, so adding additional time periods does not contribute anything new, but does greatly increase the dimensionality.

[5] The data used for R_t fluctuated slightly around a constant value, while L_t grew at a rate of 2.5% for most periods, but jumped by 25% in periods 8 and 9. Other parameters used are listed in Table 1. With a discount factor of $\delta = 0.25$, each period can be interpreted as several years.

The second regression uses the correctly-measured inputs, but omits the product differentiation variable M_t. Here the contribution of $\tilde{K}_t$ is significantly overestimated, while R_t gets a large but statistically insignificant coefficient. The coefficients on $\tilde{L}_t$ and $\tilde{K}_t$ add up to almost exactly one, indicating an estimation of roughly constant returns to scale to the two statistically-significant inputs, and increasing returns to all three inputs.

The third and fourth regressions add time-trends to proxy technical change. Regression 3 uses time t and so the estimated GNP function is of the form $U = e^{\lambda t}(\ )$, while 4 uses ℓnt and so GNP is of the form $U = t^{\lambda}(\ )$. The time trend in regression 3 is significant neither statistically nor in magnitude, and increasing returns to the primary inputs are estimated.[6] Regression 4 estimates approximate constant returns to the primary inputs (R_t is again insignificant), and a high secular "technical change" is identified. This fourth regression thus suggests no scale economies but rather increasing, concave secular technical change.

The fifth, sixth, and seventh regressions are counterparts to 2, 3, and 4 except that they mismeasure inputs by including in L_t and K_t factors used in the fixed costs of acquiring knowledge capital (i.e., K_t is as given in (22b), while L_t is as given in (22a) minus labor used in capital formation L_{kt}).[7] Thus 5, 6, and 7 ignore the whole concept of the creation of knowledge as an input, as

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[6] The insignificance of t is presumably due to the fact that, with the fixed time horizon, the introduction of new products stops after the ninth time period, making the apparent exogenous technical change process "concave." Thus the convex function $e^{\lambda t}$ is not a good fit for the sample. Conversely, the technical change process t^{λ} is concave for $\lambda < 1$, and that is what is estimated in regressions 4 and 7.

[7] We could also use definitions of $\tilde{L}_t$ and $\tilde{K}_t$ as in (25) but without dividing by θ (i.e., ignoring the distortion). But this would only affect the constant term of the regressions in the present case because, with Cobb-Douglas preferences, the shares of labor devoted to y_{2t} and to x_{it} are constant. Thus, this alternative definition of $\tilde{L}_t$ and $\tilde{K}_t$ yields coefficients identical to those in regressions 1-4.

well as ignoring the product-differentiation variable. Regressions 5 and 6 estimate strong scale economies to the primary inputs; equation 6 shows a weak and insignificant exponential time trend similar to regression 3. Output is estimated to be homogeneous of degree (approximately) 1.3 in factor inputs. Regression 7 estimates slightly decreasing returns to primary inputs (homogeneity of degree 0.92), and a strong, concave time trend, giving qualitatively similar results to regression 4.

The model was simulated using a variety of other parameter values, and the results were qualitatively very similar. For example, we increased γ from 0.4 to 0.6, so that the production of x_{it} was labor intensive relative to y_{2t}. As in the results of Table 1, we found that: (1) the estimated elasticity of output with respect to capital was significantly higher than its true value when the product differentiation variable was omitted, while the estimated elasticity of labor was about equal to or lower than its true value; (2) this result continued to hold when the exponential time trend was added, and that estimated time trend was small and statistically insignificant; (3) the "concave" time trend variable, on the other hand, was significant and lowered the estimated capital elasticity to nearly its true value and output share.

The results of Table 1 are interesting in light of the work of Romer (1987) and Benhabib and Jovanovic (1991). These authors are partly interested in the empirical puzzle that the estimated elasticity of aggregate output with respect to capital exceeds the empirical share of capital in output. Romer offers an externality argument while Benhabib and Jovanovic offer an exogenous technical change argument. Our model is very similar to Romer's and does indeed produce the empirical stylized fact. Using the "correct" inputs $(\tilde{L}_t, \tilde{K}_t, R_t)$ the share of capital in GNP is 0.425, the same as the elasticity estimated in regression 1. Misspecified regressions 2 and 3 in Table 1 produce

TABLE 1.

$\alpha = 0.5,\ \theta = 0.833,\ \gamma = 0.4,\ \beta = 0.5,\ \varepsilon = 0.25,\ \rho = 0.25,\ \delta = 0.1,\ \eta = 0.167$

$\ln U = \ln\left[y_1^{\alpha} y_2^{1-\alpha}\right]$ is dependent variable

1.	$G(\tilde{L}, \tilde{K}, R, M)$	$\ln \tilde{L}_t$	$\ln \tilde{K}_t$	$\ln R_t$	$\ln M_t$	R^2
		0.4500	0.4250	0.1250	0.1000	1.0
2.	$G(\tilde{L}, \tilde{K}, R)$	$\ln \tilde{L}_t$	$\ln \tilde{K}_t$	$\ln R_t$		
		0.4090	0.5810	0.2531		0.9996
		(19.644)	(27.250)	(1.202)		
3.	$G(\tilde{L}, \tilde{K}, R, T)$	$\ln \tilde{L}_t$	$\ln \tilde{K}_t$	$\ln R_t$	t	
		0.4811	0.5687	0.2052	-0.0044	0.9996
		(7.543)	(24.537)	(0.981)	(1.193)	
4.	$G(\tilde{L}, \tilde{K}, R, T)$	$\ln \tilde{L}_t$	$\ln \tilde{K}_t$	$\ln R_t$	$\ln t$	
		0.3408	0.4476	0.2337	0.1444	0.9998
		(13.566)	(10.356)	(1.650)	(3.272)	
5.	$G(L, K, R)$	$\ln L_t$	$\ln K_t$	$\ln R_t$		
		0.4557	0.6915	0.2034		0.9986
		(12.240)	(16.838)	(0.530)		
6.	$G(L, K, R, T)$	$\ln L_t$	$\ln K_t$	$\ln R_t$	t	
		0.2991	0.7101	0.3114	0.0093	0.9990
		(2.969)	(18.243)	(0.879)	(1.650)	
7.	$G(L, K, R, T)$	$\ln L_t$	$\ln K_t$	$\ln R_t$	$\ln t$	
		0.3139	0.4129	0.1914	0.2592	0.9999
		(29.586)	(22.731)	(2.959)	(16.582)	

Notes: t-statistics are in parentheses.

an elasticity estimate for capital that greatly exceeds the share of capital. Using the "incorrect" factor input measures in regressions 5, 6, and 7 the actual share of capital in output varies slightly over the sample between 44% and 41% of output. Thus in regressions 5 and 6, we also have the result that the estimated capital elasticity significantly exceeds the share of capital in output. Using the (concave) time trend t^{λ} as a proxy for exogenous technical change lowers the capital elasticity in cases 4 and 7 to approximately capital's share, suggestive of Benhabib and Jovanovic. In a sense our model makes quite a similar point to their paper: aggregate scale economies and technical change are hard to distinguish. We have shown that both can arise in the estimated regressions from the endogenous creation of new intermediate inputs.

6. CONCLUSIONS

This paper is motivated both by the growth accounting literature, and by the recent work in trade and growth theories using monopolistic competition models. The latter suggests one of many possible contributions to the large residuals or time trends found in the growth-accounting literature, namely, the creation of new intermediate inputs over time. An increased "division of labor" through new intermediates enhances productivity of the economy, but fixed costs limit the number of new inputs developed at any point in time.

We derive and analyze a single-period GNP function for such an economy, and show how we can separate the contributions of the quantity of primary inputs production from the range of intermediates. We then construct and analyze a dynamic monopolistic competition model in which new intermediates are introduced endogenously. The model is simulated, and the time series obtained are used in a number of regression equations, which attempt to account for growth. The omission of the range of intermediates creates biased estimates of the relative contribution of primary factors, and strong time trends. These biases arise due to correlation between the observed primary resources, and the unobserved but endogenous productivity gains due to new inputs.

APPENDIX

Proof of Proposition 1

Proof of (a):

Using (1), the first-order conditions for (2) are

$$p_i - \mu \left[\sum_{i=1}^{M_1} a_i x_i^{\theta} \right]^{(1-\theta)/\theta} a_i x_i^{\theta-1} = 0 \,, \qquad \text{(A1)}$$

where μ is a Lagrange multiplier. Multiply each equation of (A1) by x_i, sum over the full and restricted sets of inputs, divide the resulting equations and raise to the power $1/\theta$ to obtain:

$$\left[\frac{\sum_{i=1}^{M_1} p_i x_i}{\sum_{i=1}^{M_0} p_i x_i} \right]^{1/\theta} = \left[\frac{\sum_{i=1}^{M_1} a_i x_i^{\theta}}{\sum_{i=1}^{M_0} a_i x_i^{\theta}} \right]^{1/\theta} = \frac{f(x,M_1)}{f(x,M_0)} \,. \qquad \text{(A2)}$$

Proof of (b):

The marginal product of an x_i given the full and restricted sets of inputs are:

$$\partial f(x_1,M_1)/\partial x_i = \left[\sum_{i=1}^{M_1} a_i x_i^{\theta} \right]^{(1-\theta)/\theta} a_i x_i^{\theta-1}$$

and, (A3)

$$\partial f(x_1,M_0)/\partial x_i = \left[\sum_{i=1}^{M_0} a_i x_i^{\theta} \right]^{(1-\theta)/\theta} a_i x_i^{\theta-1} \,.$$

Dividing these two equations and using (A2) gives us part (b).

Proof of Proposition 2

Proof of (a), (b), (c):

At constant λ, (a) and (b) are just the standard envelope properties of the optimized GNP function: since the factor inputs are optimal at the initial λ, $G_{\tilde{v}}$ and G_q are the partial derivatives of the optimized value (11). Concavity in $\tilde{v}$ and convexity in q are also conventional properties of the GNP function, for fixed λ. Part (c) is seen from the fact that (11) describes a constant-returns economy, so long as g_j and h_{ij} are homogeneous of degree one.

Proof of (d):

The envelope property applied to (11) implies that the vector of derivatives G_λ are the partial derivatives of the programming problem, where λ_j appears twice: in g_j [equation (9)] and in the resource constraint (10). Attaching the Lagrange multiplier w to the latter, G_{λ_j} is given by:

$$\partial G/\partial\lambda_j = \frac{1}{\theta_j}\,[q_j(\partial g_j/\partial f_j)(\partial f_j/\partial x_{ij})\lambda_j^{(1-\theta_j)/\theta_j} - \sum_{\ell=1}^{L} w_\ell \sum_{i=1}^{M_{j0}} v_{ij\ell}]\,. \qquad (A4)$$

We have $f_j(x_j, M_{j0}) = \sum_{i=1}^{M_{j0}} (\partial f_j/\partial x_{ij})x_{ij}$ since $f_j(x_j, M_{j0})$ is homogeneous of degree one in x_{ij}. Substituting this into the first term on the right of (A4) and using (13):

$$q_j \frac{\partial g_j}{\partial f_j} f_j(x_j, M_{j0})\, \lambda_j^{(1-\theta_j)/\theta_j} = \sum_{i=1}^{M_{j0}} \tilde{p}_{ij} x_{ij}/\lambda_{ij} = \sum_{i=1}^{M_{j0}} p_{ij} x_{ij}\,. \qquad (A5)$$

where we have used the definition of $p_{ij} \equiv \tilde{p}_{ij}/\lambda_j$.

Now turn to the final sum of terms in (A4). Consider the pricing rules (8) and multiply both sides of the ℓ^{th} element by $v_{ij\ell}$. Summing over $\ell=1,\dots,L$ and $i=1,\dots,M_{j_0}$ we have:

$$\theta_j \sum_{i=1}^{M_{j_0}} p_{ij} \sum_{\ell=1}^{L} \frac{\partial h_{ij}}{\partial v_{ij\ell}} v_{ij\ell} = \sum_{\ell=1}^{L} w_\ell \left(\sum_{i=1}^{M_{j_0}} v_{ij\ell} \right). \qquad (A6)$$

Since h_{ij} is concave in v_{ij} with $h_{ij}(0) = 0$, then $\sum_{\ell=1}^{L}(\partial h_i/\partial v_{ij\ell})v_{ij\ell} \leq h_{ij}(v_{ij}) = x_{ij}$. It follows from (A6) that:

$$\sum_{\ell=1}^{L} w_\ell \left(\sum_{i=1}^{M_{j_0}} v_{ij\ell} \right) \leq \theta_j \sum_{i=1}^{M_{j_0}} p_{ij} x_{ij} . \qquad (A7)$$

Substituting (A7) and (A5) into (A4) we obtain our result:

$$\partial G/\partial \lambda_j \geq \frac{(1-\theta_j)}{\theta_j} \sum_{i=1}^{M_{j_0}} p_{ij} x_{ij} = \lambda_j^{-1} \frac{(1-\theta_j)}{\theta_j} \sum_{i=1}^{M_j} p_{ij} x_{ij} , \qquad (A8)$$

by definition of λ_j in (3'). When h_{ij} is homogeneous of degree one then (A7) and (A8) hold as equalities.

Proof of (e):

$h_{ij}(v_{ij}) = h_j(v_{ij})$ is a symmetry assumption that in turn implies $p_{ij} = p_j$, and that the optimal input vectors v_{ij} and v_{kj} are proportional. Referring back to (3'), we see that λ_j takes on the simple form:

$$\lambda_j = \left[\sum_{i=1}^{M_j} x_{ij} / \sum_{i=1}^{M_{j_0}} x_{ij} \right] = h_j \left[\sum_{i=1}^{M_j} v_{ij} \right] / h_j \left[\sum_{i=1}^{M_{j_0}} v_{ij} \right]. \qquad (A9)$$

where the second equality follows since v_{ij} and v_{kj} are proportional and since h_j is homogeneous of degree one. Rearranging (A9):

$$\lambda_j h_j \left[\sum_{i=1}^{M_{j_0}} v_{ij} \right] = h_j \left[\sum_{i=1}^{M_j} v_{ij} \right], \text{ which implies } \lambda_j \sum_{i=1}^{M_{j_0}} v_{ij} = \sum_{i=1}^{M_j} v_{ij} , \qquad \text{(A10)}$$

where the latter implication follows again since v_{ij} and v_{kj} are proportional and h_j is homogeneous of degree one. Then (16) follows by substituting (A10) into definition (10).

Proof of Proposition 3.

Let q_t denote the price of y_{1t} in terms of y_{2t}. The demand for x_t is a derived demand, with the input producers receiving the value of the marginal product from their intermediate input. Multiplying (19) by q_t and differentiating with respect to a single input x, p_t is then given by:

$$p_t = \left[q_t y_{1t}^{1-\theta} \right] x_t^{\theta-1}. \qquad \text{(A11)}$$

Multiply both sides of the first equation in (24) by x_t and using (A11), we obtain $c(w_t,r_t)x_t/\theta = q_t y_{1t}^{1-\theta} x_t^{\theta}$. Now sum over all the intermediate inputs and use (19):

$$\sum_{i=1}^{M} c(w_t,r_t)x_t/\theta = q_t y_{1t}^{1-\theta} \sum_{i=1}^{M} x_{it}^{\theta} = q_t y_{1t} . \qquad \text{(A12)}$$

Since (20) has constant returns to scale, the left-hand side of (A12) is the total (variable) cost of producing all the inputs divided by θ, and from (20) the shares of labor and capital in x_t are γ and $(1-\gamma)$ respectively. Thus, we obtain:

$$w_t L_{xt}/\theta = \gamma q_t y_{1t}, \quad r_t K_{xt}/\theta = (1-\gamma) q_t y_{1t}.$$

Using (18) for the shares of labor (β) and capital (ε) in y_{2t}, and (17) for the shares of good 1 (α) and good 2 ($1-\alpha$) in within-period expenditure ($q_t y_{1t} + y_{2t}$), we then have:

$$\frac{w_t L_{xt}/\theta}{w_t L_{2t}} = \frac{\gamma q_t y_{1t}}{\beta y_{2t}} = \frac{\gamma\alpha}{\beta(1-\alpha)}, \text{ and } \frac{r_t K_{xt}/\theta}{r_t K_{2t}} = \frac{(1-\gamma)\alpha}{\varepsilon(1-\alpha)}. \qquad \text{(A13)}$$

Combining (A13) with the factor supplies defined in (25) we obtain:

$$L_{xt}/\theta = \frac{\gamma\alpha}{\gamma\alpha + \beta(1-\alpha)} \tilde{L}_t, \quad L_{2t} = \frac{\beta(1-\alpha)}{\gamma\alpha + \beta(1-\alpha)} \tilde{L}_t. \qquad \text{(A14a)}$$

$$K_{xt}/\theta = \frac{(1-\gamma)\alpha}{(1-\gamma)\alpha + \varepsilon(1-\alpha)} \tilde{K}_t, \quad K_{2t} = \frac{\varepsilon(1-\alpha)}{(1-\gamma)\alpha+\varepsilon(1-\alpha)} \tilde{K}_t. \qquad \text{(A14b)}$$

Using these results in (19), y_{1t} can then be written as:

$$y_{1t} = \left[x_t^{\theta} M_t\right]^{1/\theta} = (M_t x_t) M_t^{(1-\theta)/\theta} = L_{xt}^{\gamma} K_{xt}^{1-\gamma} M_t^{(1-\theta)/\theta} = A\tilde{L}_t^{\gamma} \tilde{K}_t^{1-\gamma} M_t^{(1-\theta)/\theta}$$

where A is a constant. Similarly, substituting (A14) into (18) we have:

$$y_{2t} = B\tilde{L}_t^{\beta} \tilde{K}_t^{\varepsilon} R_t^{1-\beta-\varepsilon},$$

where B is a constant. Then Proposition 3 follows directly.

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