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ABSTRACT

This paper develops a general framework for analyzing corporate risk management policies. We begin by observing that if external sources of finance are more costly to corporations than internally generated funds, there will typically be a benefit to hedging: hedging adds value to the extent that it helps ensure that a corporation has sufficient internal funds available to take advantage of attractive investment opportunities. We then argue that this simple observation has wide-ranging implications for the design of risk management strategies. We delineate how these strategies should depend on such factors as shocks to investment and financing opportunities. We also discuss exchange-rate hedging strategies for multinationals, as well as strategies involving "nonlinear" instruments like options.

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Risk Management:
Coordinating Corporate Investment and Financing Policies

1. Introduction

Corporations take risk management very seriously – recent surveys find that risk management is ranked by financial executives as one of their most important objectives.[1] Given its real-world prominence, one might guess that the topic of risk management would command a great deal of attention from researchers in finance, and that practitioners would therefore have a well-developed body of wisdom from which to draw in formulating hedging strategies.

Such a guess would, however, be at best only half correct. Finance theory does do a good job of instructing firms on the implementation of hedges. For example, if a refining company decides that it wants to use options to reduce its exposure to oil prices by a certain amount, a Black-Scholes type model can help the company calculate the number of contracts needed. Indeed, there is an extensive literature that covers numerous practical aspects of what might be termed "hedging mechanics", from the computation of hedge ratios to the institutional peculiarities of individual contracts.

Unfortunately, finance theory has had much less clear-cut guidance to offer on the logically prior questions of hedging strategy – why and how much to hedge. Paradoxically, the same arbitrage logic that helps the refining company calculate option deltas also implies that there may be no reason for it to engage in hedging activity in the first place. According to the Modigliani- Miller paradigm, buying and selling oil options contracts cannot alter the company's value, since individual investors in the company's stock can always buy and sell such contracts themselves if they care to adjust their exposure to oil prices.

It is not that there are no stories to explain why firms might wish to hedge. Indeed, a number of potential explanations have been developed recently, by, among others, Stulz (1984); Smith and Stulz (1985); Smith, Smithson and Wilford (1990);

[1] See Rawls and Smithson (1990).

Breeden and Viswanathan (1990); and Lessard (1990). However, it is fair to say that there is not yet a single, accepted framework which can be used to guide hedging strategies. This gap in knowledge is illustrated in the most recent edition of Brealey and Myers'(1991) textbook. Brealey and Myers do devote an entire chapter to the topic of "Hedging Financial Risk", but the chapter focuses almost exclusively on questions relating to hedging implementation. Less than one page is devoted to discussing reasons why firms might hedge.

In this paper, we present a simple, yet very general framework for thinking about corporate risk management strategies. Our model not only explains why firms might find it desirable to hedge under a wide range of circumstances, but perhaps more importantly, also provides quite explicit guidance as to the amount and type of hedging that is optimal. The basic force that drives all our results is a capital market imperfection that makes externally-obtained funds more expensive than those generated internally. Although the inside vs. outside money distinction has received a great deal of attention in recent theoretical and empirical work, its implications for risk management have (surprisingly, in our view) been left largely undeveloped.

Our logic can be understood as follows. If a firm does not hedge, there will be some variability in the cashflows generated by assets in place. Simple accounting implies that this variability in internal cashflow must result in either: a) variability in the amount of money raised externally; or b) variability in the amount of investment. Variability in investment will generally be undesirable, to the extent that there are diminishing marginal returns to investment. (i.e., to the extent that output is a concave function of investment.) If the supply of external finance were perfectly elastic, the optimal ex-post solution would thus be to leave investment plans unaltered in the face of variations in internal cashflow, taking up all the slack by changing the quantity of outside money raised. Unfortunately, this approach no longer works well if the marginal cost of funds goes up with the amount raised externally. Now a shortfall in cash may be met with some increase in outside financing, but also some decrease in investment. Thus variability in cashflows now disturbs both investment and financing plans in a way that is costly to the firm. To the extent that hedging can reduce this variability in cashflows, it can increase the value of the firm.

In our framework, risk management stands squarely at the interface between investment and financing policies – a firm's optimal hedging program depends on both the range of investment opportunities it has available and its ability to raise funds from external sources. In this sense, our framework encompasses as a special case some previous work (see, e.g., Smith and Stulz (1985)) that focuses on bankruptcy costs as a determinant of hedging policies. This work can be thought of as showing how hedging can keep the total expected costs of debt financing in check, but it for the most part does not examine the investment-side consequences of risk management.

More generally, investment considerations play no role in many of the explanations of hedging seen in the literature. One important exception is Lessard (1990).[2] Lessard writes: "...the most compelling arguments for hedging lie in ensuring the firm's ability to meet two critical sets of cash flow commitments. (1) the exercise prices of their operating options reflected in their growth opportunities (for example, the R&D or promotion budgets) and (2) their dividends...The growth options argument hinges on the observation that, in the case of a funding shortfall relative to investment opportunities, raising external capital will be costly."

A similar line of reasoning is advanced by Lewent and Kearney (1990) in explaining Merck's philosophy of risk management. They note that a key factor in deciding whether to hedge is "the potential effect of cash flow volatility on our ability to execute our strategic plan – particularly, to make the investments in R&D that furnish the basis for future growth."

The model that we develop below is very much in the spirit of these verbal

[2] Other exceptions include Froot, Scharfstein and Stein (1989) Smith, Smithson and Wilford (1990), and Stulz (1990), all of whom examine the implications of hedging for investment in the presence of a large debt overhang. These papers are discussed in further detail below.

arguments. However, our principal goal is not simply to formalize the insight that costly external finance can generate a rationale for hedging. Rather, we are most interested in fleshing out the practical implications of this insight – i.e., in specifying how optimal hedging strategies can be determined in a variety of settings.

The plan of the paper is as follows. In Section 2, we briefly sketch several other explanations of corporate risk management that have been offered. In Section 3, we present our model in its most elemental form, and use it to demonstrate the basic rationale for hedging. We then examine a series of practical applications of our framework. In Section 4, we extend the model to show how optimal hedge ratios can be calculated as a function of shocks to investment and financing opportunities. Section 5 considers the question of optimal currency hedging by multinationals that have investment opportunities in more than one country. Section 6 examines "nonlinear" hedging strategies that make use of options and other complex hedging instruments. Section 7 concludes.

2. Other Rationales for Corporate Risk Management

2.1. Managerial motives

Stulz (1984) argues that corporate hedging is an outgrowth of the risk aversion of managers. While outside stockholders' ability to diversify will effectively make them indifferent to the amount of hedging activity undertaken, the same cannot be said for managers, who may hold a relatively large portion of their wealth in the firm's stock. Thus managers can be made strictly better off (without costing outside shareholders anything) by reducing the variance of total firm value.

One weakness of the Stulz theory is that it implicitly relies on the assumption that managers face significant costs when trading in hedging contracts for their own account – otherwise, they would be able to adjust the risks they face without having to involve the firm directly in any hedging activities. At the same time, unless one also introduces transactions costs to hedging at the corporate level, the Stulz theory makes the extreme prediction that firms will hedge as much as possible – that is, until the variance of stock prices is minimized.

A very different managerial theory of hedging, based on asymmetric information, is put forward by Breeden and Viswanathan (1990) and DeMarzo and Duffie (1992). In both of these models, the labor market revises its opinions about the ability of managers based on their firms' performance. This can lead some managers to undertake hedges in an attempt to influence the labor market's perceptions.

2.2. Taxes

Smith and Stulz (1985) argue that if taxes are a convex function of earnings, it will generally be optimal for firms to hedge. The logic is straightforward – convexity implies that a more volatile earnings stream leads to higher expected taxes than a less volatile earnings stream. Convexity in the tax function is quite plausible for some firms, particularly those who face a significant probability of negative earnings and are unable to carry forward 100 percent of their tax losses to subsequent periods.

2.3. Costs of financial distress and debt capacity

For a given level of debt, hedging can reduce the probability that a firm will find itself in a situation where it is unable to repay that debt. Thus if financial distress is costly, and if there is an advantage to having debt in the capital structure (say due to taxes or agency problems associated with "free cash flow") hedging may be used as a means to increase debt capacity. The simplest variant of this argument, put forth by Smith and Stulz (1985), simply assumes that bankruptcy involves some exogenous transactions costs. Stulz (1990) employs a similar argument, assuming that bankruptcy eliminates the possibility of undertaking future investments.

A more sophisticated version of the argument invokes Myers' (1977) "debt overhang" underinvestment effect to endogenize the costs of financial distress. This rationale for hedging (or equivalently, for using debt indexed to exogenous sources of risk) is given by Froot, Scharfstein and Stein (1989) in the context of highly indebted LDCs. The same basic point is made in a corporate finance setting by Smith, Smithson and Wilford (1990), and Stulz (1990). Of all the theories surveyed in this section, the debt overhang rationale for hedging is the closest to that presented here. Like the model of this paper, it draws an explicit link between firms' hedging strategies and their investment behavior.

3. The Basic Paradigm

3.1. A simple model of the benefits to hedging

As stated above, hedging is beneficial if it can allow a firm to avoid unnecessary fluctuations in either investment spending or funds raised from outside investors. To illustrate this point, it is best to begin with a very simple and general framework. Afterwards, we demonstrate how this simple framework corresponds to a well-known optimizing model of costly external finance.

Consider a firm which faces a two-period investment/financing decision. In the first period the firm has an amount of liquid assets, w. At this time the firm must choose its investment expenditures and external financing needs. In the second period, the output from the investment is realized and outside investors are repaid.

On the investment side, let the net present value of investment expenditures be given by:

$$F(I) = f(I) - I, \tag{1}$$

where I is investment, $f(I)$ is the subsequent expected level of output, $f' > 0$ and $f'' < 0$. For notational simplicity we assume the discount rate is equal to zero.

As will become clear, the company prefers to finance investment with internal funds first before turning to external sources. Therefore, the company will raise from outside investors an amount e, so that:

$$I = w + e. \tag{2}$$

Given the discount rate of zero, outside investors require an expected repayment of e in the second period.

We assume, however, that there are additional (deadweight) costs to the firm of external finance, which we denote by C. (Per dollar raised, these funds therefore cost $\frac{C}{e}$ above the riskless rate.) These costs could arise from a number of sources.

First, they could originate in costs of bankruptcy and financial distress, which include direct costs (e.g., legal fees) as well as indirect costs (e.g., decreased product-market competitiveness and underinvestment). Second, such costs could arise from informational asymmetries between managers and outside investors. Or, to the extent that managers are not full residual claimants, there may be agency costs associated with motivating and monitoring managers who resort to certain types of outside finance. Finally, managers may obtain private benefits from limiting their dependence on external investors. Thus even if there are no observable costs to external finance, management may *act* as though external financing has real economic costs.[3]

Regardless of which interpretation one chooses, the deadweight costs should be an increasing function of the amount of external finance. We represent these costs as $C = C(e)$ and note that $C_e \geq 0$.[4]

The issue of hedging arises when first-period wealth, w, is random. To the extent that there are marketable risks that are correlated with w, the firm may attempt to alter the distribution of w by undertaking hedging transactions in period zero. For simplicity, we make the extreme assumption that *all* the fluctuations in w are completely hedgeable, and furthermore that hedging has no effect on the expected level of w.[5] Given this assumption, complete hedging will clearly be beneficial if and only if profits are a concave function of internal wealth.

To explore the impact of hedging on optimal financing and investment decisions, we solve the model backwards, starting with the firm's first-period investment decision. The firm enters the first period with internal resources of w and chooses investment (and thereby the amount of external financing, $e = I - w$) to maximize

[3] On costs of external finance, see e.g., Townsend (1979), Myers and Majluf (1984), Jensen and Meckling (1976), and Myers (1977) among many others.

[4] A more general formulation of these costs would allow them to depend also on the scale of the investment project undertaken, $C = C(I, e)$. This would make it possible for a firm to lower its per-dollar costs of external finance by undertaking larger investment projects. The qualitative nature of our results is unaffected (although the exposition is somewhat complicated) by using this more general formulation. As we discuss below, either formulation can be rationalized in an optimal-contracting framework.

[5] This assumption would follow from risk neutrality on the part of investors. It is straightforward to extend our analysis to the case where systematic risk is priced in equilibrium.

net expected profits:

$$P(w) = \max_I \; F(I) - C(e). \tag{3}$$

The first-order condition for this problem is:

$$F_I = f_I - 1 = C_e, \tag{4}$$

where we have used the fact that, in the second period when w is given, $\frac{de}{dI} = 1$. Equation (4) implies that there is underinvestment – the optimal level of investment, I^*, is below the first-best level, which would set $f_I = 1$.

Moving to period zero, the firm chooses its hedging policy to maximize expected profits. As noted above, random fluctuations in w reduce expected profits if $P(w)$ is a concave function. Using the first-order condition in (4), the second derivative of profits is given by:

$$P_{ww} = f_{II}\left(\frac{dI^*}{dw}\right)^2 - C_{ee}\left(\frac{dI^*}{dw} - 1\right)^2, \tag{5}$$

where f_{II} and C_{ee} are evaluated at $I = I^*$. If this expression is globally negative, then hedging raises average profits. Equation (5) can be rewritten by applying the implicit function theorem to (4) to yield:[6]

$$P_{ww} = f_{II}\frac{dI^*}{dw}. \tag{6}$$

Equation (6) clarifies the sense in which hedging activity is determined by the interaction of investment and financing considerations. If hedging is to be beneficial, two conditions must be satisfied: *i*) marginal returns on investment must be decreasing; and *ii*) the level of internal wealth must have a positive impact on the optimal level of investment. The latter condition is a ubiquitous feature of models

[6] The first-order condition (4) and the implicit function theorem together imply that I^*, satisfies

$$\frac{dI^*}{dw} = \frac{-C_{ee}}{f_{II} - C_{ee}},$$

at $I = I^*$. We assume that the second-order conditions with respect to investment are satisfied, so that the denominator of this expression is always negative.

of external finance in the face of information and/or incentive problems. Furthermore, there is substantial empirical evidence suggesting that corporate investment is indeed sensitive to levels of internal cash flow.[7]

Two simple examples may help to further develop the intuition behind equations (5) and (6). In the first, assume that a company has no access at all to financial markets. In this case, C is always equal to zero in equilibrium, and any variation in w is reflected one-for-one in changes in investment, $\frac{dI^*}{dw} = 1$. Equations (5) and (6) then tell us that $P_{ww} = f_{II}$: the concavity of the profit function comes solely from the concavity of the production technology.

In the second polar example, investment is completely fixed (e.g., the company has only one indivisible investment project with high returns). Now any fluctuations in internal funds translate one-for-one into fluctuations in the amount of external funds that must be raised, $\frac{dI^*}{dw} = 0$. Equation (5) then says that the concavity of the profit function comes exclusively from the convexity of the C function, i.e., $P_{ww} = -C_{ee}$.

Clearly, for intermediate cases – those in which $0 < \frac{dI^*}{dw} < 1$ – the concavity of the profit function will come from both the concavity of the investment technology and the convexity of the financing-cost function. Another way to see this is to substitute out $\frac{dI^*}{dw}$ from equation (5), yielding

$$P_{ww} = \frac{-f_{II}C_{ee}}{f_{II} - C_{ee}}. \tag{7}$$

Equation (7) illustrates again that hedging is driven by an interaction between investment and financing considerations (as represented by f_{II} and C_{ee}, respectively).

Thus far we have used an arbitrary specification for the C function to establish conditions under which hedging is value-increasing. However, it is unclear whether these conditions (i.e., the requirement that $C_{ee} \geq 0$) would emerge naturally if we derived the C function from an optimizing model with rational agents. Next, we examine an important class of such models, and demonstrate that the required convexity in C obtains under a wide range of parameterizations.

[7] See, for example, Fazzari, Hubbard and Petersen (1988), and Hoshi, Kashyap and Scharfstein (1991).

3.2. Hedging in an optimal contracting model

The model we adopt is a variant of the costly-state-verification approach developed by Townsend (1979) and Gale and Hellwig (1985). As we shall see, the prescription that companies should hedge takes the form of a simple and fairly weak restriction on the specification of this CSV model. Moreover, we are able to rewrite the $C(e)$ function explicitly in terms of parameters of the CSV model.

As before, we assume that in the first period a firm can invest an amount I, which yields a gross payoff of $f(I)$ in the second period. Also in the second period, the firm generates *additional* random cash flows of x from its pre-existing assets. The cumulative distribution and density of x are given by $G(x)$ and $g(x)$, respectively.

As in the Townsend and Gale-Hellwig models, we assume that cash flows are costlessly observable to company insiders, but are observable to external creditors only at some cost. In particular, we suppose that the cash flows from the *existing* assets can be observed at a cost c, but that it is infinitely costly to observe the cash flows from the new investment project. As is well known, when $c > 0$, the optimal contract between outside investors and the company will be a standard debt contract. In return for receiving e in the first period, the company is required to repay in the second period a state-invariant amount D. If the company fails to perform, creditors pay the monitoring costs, then observe – and keep for themselves – company profits. States in which monitoring occurs can be interpreted as bankruptcy.

Our formulation of the CSV model is slightly different from that in Townsend and Gale-Hellwig: we suppose that a set of pre-existing assets entirely determines the firm's capacity for external finance, so that this capacity is unaffected by current investment spending. This parallels our setup in section 3.1 above, where we assumed that new investment spending has no independent effect on deadweight costs for a given level of external finance. That is, in both models C can be represented simply as $C(e)$. This assumption simplifies our analysis, but does not affect the basic results.[8]

Under these circumstances, the company chooses investment and outside financing to maximize

$$L \equiv \max_{I,D} \ f(I) + \int_{D}^{\infty} (x - D) g(x) dx, \tag{8}$$

subject to a non-negative profit constraint for outside investors:

$$\int_{-\infty}^{D} (x - c) g(x) dx + \int_{D}^{\infty} D g(x) dx \geq I - w. \tag{9}$$

The first-order conditions for this constrained optimization problem are:

$$\frac{\partial L}{\partial D} = (\lambda - 1)(1 - G(D)) - \lambda c g(D) = 0, \tag{10}$$

$$\frac{\partial L}{\partial I} = f_I - \lambda = 0, \tag{11}$$

where λ is the Lagrange multiplier on constraint (9).

Equations (10) and (11) together imply that the firm sets I^* such that

$$f_I = \frac{1 - G(D)}{1 - G(D) - c g(D)} \geq 1. \tag{12}$$

If there are no deadweight costs ($c = 0$) the firm sets investment efficiently ($f_I = 1$). However, if $c > 0$, then the firm underinvests, setting $f_I > 1$.[9] Underinvestment occurs in this model because an increase in I necessitates an increase in D, which raises the probability of bankruptcy. At the optimum, the firm reduces investment from the first-best level in order to economize on deadweight costs.

In this setup, there is a direct correspondence between expected deadweight costs of external finance and the probability of bankruptcy:

$$C(e) = c G(D), \tag{13}$$

[8] One way to rationalize this assumption would be to suppose that the assets in place are comprised of physical capital that has some value in liquidation, whereas the new investment is in intangible assets (e.g., R&D, market share, etc.) that have no value in liquidation.

[9] This analysis assumes that there exists an optimally-chosen D such that $1 - G(D) - cg(D) > 0$ and that investors' zero-profit constraint (9) is satisfied. Otherwise, there would be no solution to the problem in (8), and no investment would take place.

where equation (9) implicitly defines the function $D = D(e)$.

One can verify that the first-order condition, $f_I = c_e + 1$, derived in section 3.1, is identical to (12) above. From equation (11), it is clear that the expected shadow value of an additional dollar of internal wealth ($L_w = \lambda$) is equal to the marginal return on investment, which is given by f_I.

As before, hedging raises the value of the company if profits are concave in internal wealth, i.e., $L_{ww} = \frac{d\lambda}{dw} = F_{II}\frac{dI^*}{dw} < 0$. (Note that this is the same condition we derived in equation (6) for our reduced-form model.) Totally differentiating equations (9) through (11) and solving for $\frac{dI^*}{dw}$, one can show that a sufficient condition for $\frac{dI^*}{dw} > 0 \; \forall x$ is that the hazard rate $\frac{g(x)}{1-G(x)}$ is strictly increasing in x. This is a fairly weak condition, and is satisfied for the normal, exponential, and uniform distributions, among others.[10] Thus, when $f_{II} < 0$ and the hazard rate of $G(.)$ is increasing, hedging is optimal in this CSV framework.

4. Optimal Hedging with Changing Investment and Financing Opportunities

So far our results create a very simplistic picture of optimal hedging policies – firms with increasing marginal costs of external finance should always fully hedge their cashflows. In this section, we extend our analysis to incorporate randomness in both investment and financing opportunities. As will be seen, these considerations lead to a richer range of solutions to the optimal hedging problem.

4.1. Changing investment opportunities

In the discussion above, we have assumed that a firm's investment opportunities were nonstochastic, and thus independent of the cash flows from its assets in place. In many cases, however, this assumption is unrealistic. For example, a company engaged in oil exploration and development will find that both its current cash flows (i.e., the net revenues from its already-developed fields) and the marginal product of additional investments (i.e., expenditures on further exploration) decline when the price of oil falls. For such a company, hedging against oil-price declines is less

[10] The same restriction on the hazard rate also implies that $C_{ee} > 0$. This can be seen by twice differentiating equation (13), and then by noting that equation (9) implicitly defines $D = D(e)$.

valuable – even without hedging, the supply of internal funds tends to match the demand for funds.

It is straightforward to extend the analysis of the previous section to address the question of the optimal hedge ratio in a world of changing investment opportunities. If we focus for the moment on linear hedging strategies (i.e., forward sales or purchases), the hedging decision can be modelled by writing internal funds as[11]

$$w = w_0(h + (1 - h)\epsilon), \tag{14}$$

where h is the "hedge ratio" chosen by the firm, and ϵ is the primitive source of uncertainty.[12] To keep things simple, we assume that ϵ – the return on the risky asset – is distributed normally, with a mean of 1 and a variance of σ^2.[13]

To model changing investment opportunities, we redefine profits as

$$F(I) = \theta f(I) - I, \tag{15}$$

where $\theta = \alpha(\epsilon - \bar{\epsilon}) + 1$. In this formulation, α is a measure of the correlation between investment opportunities and the risk to be hedged.

In period zero, the firm must choose h to maximize expected profits:

$$\max_h \; E\left[P(w)\right], \tag{16}$$

where the expectation is taken with respect to ϵ. The first-order condition for this problem is

$$E\left[P_w \frac{dw}{dh}\right] = 0. \tag{17}$$

Equation (17) simplifies to

$$E\left[P_w(1 - \epsilon)\right] = 0, \tag{18}$$

[11] In section 6 below, we consider alternative, nonlinear hedging strategies that involve instruments such as options.

[12] To see what (14) implies for actual futures positions and prices, define z_0 as the current futures price and q_1 as the future spot price of the variable in question. The variable ϵ then corresponds to $\epsilon = \frac{q_1}{z_0}$ and a hedging position of h corresponds to selling $h\frac{w_0}{z_0}$ futures contracts.

[13] Assuming that the mean of ϵ is one implies, as before, that the expected level of wealth is unaffected by the amount of hedging.

which can be written as:

$$\text{cov}(P_w, \epsilon) = 0. \tag{19}$$

To simplify the covariance term, we use a second-order Taylor series approximation (which is exact if the asset's return, ϵ, is normally distributed) with respect to h around $\epsilon = 1$.[14] Equation (19) and a little algebra then yield the optimal hedge ratio:

$$h^* = 1 + \alpha \frac{E\left[f_I P_{ww}/\theta f_{II}\right]}{w_0 \bar{P}_{ww}}, \tag{20}$$

where a bar over a variable implies that an expectation has been taken with respect to ϵ, e.g., $\bar{P}_{ww} = E[P_{ww}]$.

The last term in equation (20) takes account of the direct effect of ϵ on output. Clearly, if $\alpha = 0$ (i.e., there is no correlation between investment opportunities and the availability of internal funds), it is optimal to hedge fully (i.e., $h^* = 1$), as in section 3 above. However, if $\alpha > 0$, then the firm will wish to do less hedging.

It should be noted that according to equation (20), h^* need not necessarily be between zero and one. The possibility of $h^* < 0$ arises when investment opportunities are extremely sensitive to the risk variable. In that case it may make sense for a firm to actually *increase* its exposure to the variable in question, so as to have sufficient cash when ϵ is high and very large investments are required. Conversely, optimal hedge ratios greater than one will arise when investment opportunities are negatively correlated with current cashflows. In this case it makes sense to "overhedge," so as to have more cash when ϵ is *low*.[15]

To build some intuition for why companies with different investment opportunities might implement different hedging strategies, consider the following example.

[14] If x and y are normally distributed, and $a()$ and $b()$ are differentiable functions, then $\text{cov}(a(x), b(y)) = E_x[a_x]E_y[b_y]$ $\text{cov}(x, y)$. See Rubinstein (1976) for a proof. Note that if we were to assume that ϵ is log-normally distributed (with the same mean and variance as above), we would arrive at results very similar to those given throughout the paper.

[15] Note that while $h^* < 0$ or $h^* > 1$ may (according to equation (20)) be optimal for the firm, such positions may implicitly leave the firm with negative first-period resources in some states. As a consequence, the capital market may no longer charge default-free prices for futures contracts, because these contracts can now involve credit risk. For example, a firm with initial wealth consisting of nothing but 100 gold bricks may not be able to buy more *on net*, because it has no non-gold collateral. (That firm would have no resources to pay for the additional purchases if the price of gold were to fall to zero.) Similarly, a firm that sells futures contracts for more than the equivalent of 100 gold bricks might be unable to make good on its position when gold prices rise sufficiently. This entire credit-risk issue disappears, however, if we are willing to assume that the investment function satisfies the Inada conditions, i.e., that the marginal product of investment is infinite at $I = 0$. In this case the optimal hedge ratio in equation (20) endogenously ensures that firm resources (and hence investment) are positive in all states.

Suppose there are two companies engaged in natural resource exploration and extraction. Company g is a gold company. It currently owns developed mines which produce 100 units of gold in period one at zero marginal cost. Thus company g's period one cash flows are $100\tilde{p}_g$, where $\tilde{p}_g$ is the random price of gold.

Company g also has the opportunity to invest in additional exploration activities in period one. If it spends an amount I on exploration, it discovers undeveloped lodes containing $f_g(I)$ units of gold. Before the gold can be extracted, however, a further *per unit* development cost of c_g must be paid in period 2. Thus, the net returns to an exploration investment of I are given by $(\tilde{p}_g - c_g)f_g(I) - I$.

Company o is an oil company. It most respects it is very similar to company g. Its period-one cash flows are $100\tilde{p}_o$, and it is assumed that $\tilde{p}_o$ has the same distribution as $\tilde{p}_g$. Thus, both companies face exactly the same risks with regard to the nature of their period-one cashflows.

Company o also can uncover undeveloped reserves containing $f_o(I)$ units of oil by spending an amount I on exploration in period one. Company o's development costs are higher than company g's – it must pay $c_o > c_g$ in period two to develop the new reserves before they can be extracted. Thus, the net returns to an exploration investment of I are given by $(\tilde{p}_o - c_o)f_o(I) - I$. To preserve comparability across the two companies, it is further assumed that $f_o(I) = \frac{\bar{p}-c_g}{\bar{p}-c_o} f_g(I)$, where $\bar{p}$ is the mean of both price distributions. This implies that in the "base case" where commodity prices equal their means, both companies have the same marginal product of capital at any given level of investment.

The key difference between company o and company g is the fact that higher development costs make company o's investment opportunities *more leveraged* with respect to commodity prices. For example, if $c_g = 0$ and $c_o = 50$, the marginal product of capital for the gold company falls by 10 percent when gold prices fall from 100 to 90. However, the marginal product of capital for the oil company falls by *20 percent* when oil prices fall from 100 to 90.

In terminology of the above model, this difference in technology can be represented as a higher value of the parameter α for the oil company. Thus, the two companies should pursue different hedging strategies, with company g hedging more than company o.

4.2. Changing financing opportunities

Up to now, we have assumed that the supply schedule for external finance – given by the $C(e)$ function – is exogenously fixed and insensitive to the risks impacting the firm's cashflows. However, it seems quite possible that negative shocks to a firm's current cashflows might also make it more costly for the firm to raise money from outside investors. If this is the case, it may make sense for the firm to hedge more than it otherwise would. This will allow the firm to fund its investments while making *less* use of external finance in bad times than in good times.[16]

We can formalize this insight by generalizing the C function to be $C(e, \phi)$, where ϕ is given by $\delta(\epsilon - \bar{\epsilon}) + 1$. Such a generalization emerges naturally from the CSV model sketched in section 3.2. Suppose that instead of yielding x, the assets already in place yield ϕx. That is, the eventual proceeds from assets in place are correlated with the risk variable ϵ, and δ measures the strength of this correlation. As long as the distribution of x satisfies the increasing-hazard-rate property, then the $C(e, \phi)$ function that emerges from the CSV setting has the feature that $C_{e\phi} < 0$ (for fixed first-period wealth). This simply means that marginal costs of external finance, C_e, are lower for higher realizations of ϵ.

If we assume for the moment that α – which measures the correlation of *investment* opportunities with ϵ – is zero, we can derive an expression that gives us the pure effect of changing financing opportunities on the hedge ratio. The methodology is the same as before. In particular, the first-order condition in (19) still applies. But now the optimal hedge ratio is given by:

$$h^* = 1 + \delta \frac{\bar{C}_{e\phi}}{w_0 \bar{P}_{ww}}. \tag{21}$$

[16] We thank Tim Luehrman for suggesting this case to us.

Given that $C_{\epsilon\phi} < 0$, the optimal hedge ratio is greater than one, with the effect being greater the more sensitive are assets in place to the risk variable ϵ. Again the intuition is that hedging must now allow the firm to fund its investments and yet conserve on borrowing at those times when external finance is most expensive.[17]

However, even with a nonstochastic production technology (i.e., $\alpha = 0$), it is no longer true that investment is completely insulated from shocks to ϵ. This is purely a consequence of the fact that we are restricting ourselves to linear hedging strategies. Nonstochastic investment would (by the firm's first-order conditions) require that, once the hedge is in place, C_ϵ be independent of ϕ. This generally cannot be accomplished using futures alone. In section 6 below, we argue that if options are available, the firm will indeed wish to construct a hedging strategy that leads to nonstochastic investment.

5. Risk Management for Multinationals

Our framework also has implications for multinational companies' risk management strategies.[18] Multinationals have sales and production opportunities in a number of different countries. In addition, the goods that they produce at any given location may either be targeted for local consumption (i.e, nontradeable goods, such as McDonalds hamburgers) or for worldwide markets (i.e., tradeable goods, such as semiconductors). These factors complicate the hedging problem for multinational corporations.

We begin with a quite general framework which builds on that of the previous sections. Assume that the multinational can invest in two locations, "home" and "abroad," and that profits are given by:

$$P(w) = f^H(I^H) + \theta f^A(I^A) - I^H - \gamma I^A - C(e), \tag{22}$$

where $\theta = \alpha(\epsilon - \bar{\epsilon}) + 1$, $\gamma = \beta(\epsilon - \bar{\epsilon}) + 1$, and the production functions, $f^i(I^i)$,

[17] In this particular case, there is no default risk associated with the futures position that implements the desired hedge ratio. The futures position will only incur large losses in those states where assets in place are extremely valuable. In such states the funds that can be raised against assets in place ensure that the firm will make good on its futures position.

[18] Conversations with Don Lessard were especially helpful in motivating the work in this section. See Adler and Dumas (1983) for an overview of the traditional arguments for hedging exchange-rate risk.

$i = A, H$ are increasing and concave. In this expression, ϵ now represents the home-currency price of the foreign currency, and α and β are parameters (between zero and one) which index the sensitivity of foreign revenues and foreign investment costs to the exchange rate.[19] Implicitly, equation (22) treats the domestic currency as the numeraire.[20]

It is easiest to build an understanding of equation (22) by examining several special cases:

Case 1: *Exchange-rate exposure for both investment costs and revenues from foreign operations*, $\alpha = \beta = 1$. This case might correspond to situations where both the outputs and the investment inputs are nontraded goods.[21] An example might be a McDonald's restaurant in Tokyo, since local factors are required to begin operations.

More generally, even for goods traded on international markets, the law of one price may fail to hold. Indeed, the prices of such traded goods may behave like those of nontraded goods. The literature on "pricing to market" provides substantial empirical support for this view.[22]

Case 2: *Exchange-rate exposure for foreign investment costs but no exchange-rate exposure for either foreign or domestic revenues*, $\alpha = 0$ and $\beta = 1$. This case might correspond to a situation where the output from both plants is sold at the same price on the domestic market.[23] An example might be ball bearings, which can be produced using primarily local factors, but which are sold on a global market.

Case 3: *No exchange-rate exposure for investment costs but exchange-rate exposure for foreign revenues*, $\alpha = 1$ and $\beta = 0$. This case might correspond, as above, to a situation where the outputs are nontraded goods. However, now the investment inputs used in both locations are purchased on a single domestic market

[19] Note that our earlier formulation in section 4 can be interpreted as a degenerate case of equation (22), with $\beta = 0$ and I^H fixed at zero – i.e., no investment in one of the two countries.

[20] In this formulation, the external borrowing facility is also denominated in the home currency. In terms of the CSV model developed in section 3.2, this amounts to assuming that the payoff z on the pre-existing asset is home-currency denominated. Thus, we are suppressing the issues relating to changing financing opportunities raised in section 4.2.

[21] Effectively, this assumes that the foreign-currency price of nontradeable goods is not affected by exchange-rate changes.

[22] See for example Marston (1990) and Froot and Klemperer (1989).

[23] This will be correct provided that this domestic-currency price is constant.

at the same price. An example might be a construction company, like Bechtel, which makes heavy use of construction equipment that is sold on a global market.

In order to finance these different investments, the firm requires external finance of an amount

$$e = I^H + \gamma I^A - w. \tag{23}$$

Maintaining our focus on linear hedging strategies, w continues to be given by equation (14) above. In this formulation, a hedge ratio of one means that period-zero wealth, w_0, is held entirely in the domestic currency. In contrast, a hedge ratio of zero means that wealth is held entirely in the foreign currency.

Using arguments analogous to those developed above, we can solve for the optimal hedge ratio. (See the appendix for a sketch of the derivation.)

$$h^* = 1 + \frac{E\left[(\alpha\gamma - \beta\theta) f_I^A P_{ww} / \theta f_{II}^A\right]}{w_0 \bar{P}_{ww}} - \beta \frac{E\left[I^A P_{ww}\right]}{w_0 \bar{P}_{ww}}, \tag{24}$$

where

$$P_{ww} = \frac{f_{II}^H \theta f_{II}^A C_{ee}}{C_{ee}(\gamma^2 f_{II}^H + \theta f_{II}^A) - \theta f_{II}^H f_{II}^A} < 0. \tag{25}$$

There are two basic components of the optimal hedge ratio in (24). First, there is a slightly more complex version of the "changing-investment-opportunity-set" term, $\frac{E[(\alpha\gamma - \beta\theta) f_I^A P_{ww}/\theta f_{II}^A]}{w_0 \bar{P}_{ww}}$, which effectively captures the *net* exchange-rate exposure of foreign-investment profitability. Second, there is a new "lock-in" term, $\beta \frac{E[I^A P_{ww}]}{w_0 \bar{P}_{ww}}$, which is, loosely speaking, driven by the expected size of the foreign investment relative to internal wealth.

We can understand this lock-in term better by focusing on case 1 above, where $\alpha = \beta = 1$. In this case (or in any case with $\alpha = \beta$), (24) can be simplified considerably – the changing-investment-opportunity-set term disappears completely, and the lock-in term itself becomes easier to interpret. In particular, we demonstrate in the appendix that:

Proposition 1. If $\alpha = \beta$, then the optimal hedging strategy is such that investment in both locations is independent of the exchange rate: $I^H(\epsilon) = \bar{I}^H$; and

$I^A(\epsilon) = \bar{I}^A\ \forall\epsilon$. This hedging strategy is given by $h^* = 1 - \beta\bar{I}^A/w_0$.

To understand the intuition behind the proposition, imagine that the company did not hedge at all but that the actual realization of the exchange rate coincided with its expectation, $\epsilon = \bar{\epsilon}$.[24] One could then solve for the optimal first-period levels of investment. What hedging does is to *assure* that domestic and foreign investment will always be at exactly these levels, regardless of the actual realization of the exchange rate. In other words, hedging *locks in* the ability to carry out a predetermined (as of period zero) investment plan, where that plan is based on the expected future exchange rate.

In case 2, with $\alpha = 0$ and $\beta = 1$, the lock-in term remains. However, it takes on a more complicated form, since I^A and P_{ww} are now random variables, and it is no longer generally true that $E[I^A P_{ww}] = \bar{I}^A \bar{P}_{ww}$. In addition, the hedge ratio is increased by the changing-investment-opportunity-set term, $\frac{-E[f_I^A P_{ww}/f_{II}^A]}{w_0 \bar{P}_{ww}}$. This term implies that it is optimal to hold relatively *more* of the domestic currency than in case 1. The logic is similar to that developed in section 4 above. When the domestic currency depreciates, investments abroad become less attractive due to higher input costs. Thus, less foreign investment is warranted, and there is less need to hold foreign currency as a hedge against such an outcome.

Finally, in case 3, with $\alpha = 1$ and $\beta = 0$, there is no lock-in effect. Because the price of foreign investment is insensitive to the exchange rate, it is unnecessary to hold foreign currency to guarantee a given level of foreign investment. At the same time, it is still worthwhile to hold *some* wealth in the form of foreign currency. This is because the correlation of net investment opportunities with the value of the domestic currency is now *negative* – when the domestic currency depreciates, returns on foreign investment are now *high*.

6. Nonlinear Hedging Strategies

Thus far we have restricted our attention to hedges which employ only forward

[24] Note that with $\bar{\epsilon} = 1$, the expected future spot rate is equal to the forward rate.

or futures contracts. With these instruments, the sensitivity of internal wealth to changes in the risk variable to be hedged is constrained to be a constant. That is, $\frac{dw}{d\epsilon} = (1-h)w_0$, which is independent of the realization of ϵ. While such linear hedges can add value, they generally will not *maximize* value if other, nonlinear instruments, such as options, are available. Options effectively create the possibility for hedge ratios to be "customized" on a state-by-state basis.

To see why a firm might want its hedge ratios to be sensitive to the realization of ϵ, let us return to our oil-company example. We argued that the oil company's investment opportunities become less attractive when the price of oil falls, and that this militated in favor of leaving its cashflows somewhat exposed to these fluctuations. But suppose we use futures to pick a single, state-independent hedge ratio, and that this hedge ratio results in the oil company cutting capital investment expenditures by 2 percent for every 1 percent decline in the price of oil. This might make good sense for small fluctuations in oil prices – perhaps the company's level of investment *should* be cut by 20 percent when oil prices fall by 10 percent. But it may not make equally good sense for the company to completely eliminate its investment spending when oil prices fall by 50 percent.

If this is the case, the oil company may wish to do some of its hedging with options. For example, by adding out-of-the-money puts on oil to its futures hedging position, the company can give itself relatively more protection against large decreases in the price of oil than against small decreases. (Similarly, the company might also write out-of-the-money calls on oil, if a linear hedging strategy results in "too much" cash for very large increases in the price of oil.)

We can develop the general logic for nonlinear hedging strategies using the same basic setup as in section 5. We denote the frequency distribution of the random variable, ϵ, by $p(\epsilon)$. If we assume complete markets, the firm's hedging problem now becomes one of choosing a profile for wealth across states of nature, $w^* = w^*(\epsilon)$, to maximize expected profits:

$$\max_{w(\epsilon)} \int_{\epsilon} P(\epsilon, w(\epsilon)) p(\epsilon) d\epsilon, \tag{26}$$

subject to the "fair-pricing" constraint that hedging cannot change the expected level of wealth,

$$\int_{\epsilon} w(\epsilon)p(\epsilon)d\epsilon = w_0, \tag{27}$$

and to the first-order conditions for domestic and foreign investment (which are given in equations (A1) and (A2) of the appendix).[25]

The first-order condition for the constrained optimization problem in (26) is given by

$$P_w = \lambda, \tag{28}$$

where λ is the Lagrange multiplier on the constraint (27). Equation (28) implicitly defines an optimal level of wealth in every state. Note that because λ is constant across states, the implicit function theorem can be applied to (28), which after some algebra yields an expression for the optimal hedge ratio in each state:

$$\frac{dw^*(\epsilon)}{d\epsilon} = \frac{P_{w\epsilon}}{-P_{ww}} = -(\alpha\gamma - \beta\theta)\frac{f_I^A}{w_0\theta f_{II}^A} + \frac{\beta I^A}{w_0}, \tag{29}$$

where $w^* = w^*(\epsilon)$ describes the optimal level of wealth for every value of ϵ.[26]

One can use (29) to see when the first-best hedge can be attained using only futures contracts. In such cases, it must be that $\frac{dw^*}{d\epsilon}$ is a constant. Thus, we have

Proposition 2. With $\alpha = \beta$, futures contracts alone can provide value-maximizing hedges. In all other cases, options may be required to obtain the value-maximizing hedge.

Futures hedging alone is thus optimal: *i*) in the simple models of section 3 with fixed investment and financing opportunities; and *ii*) in our multinational setup of section 5 whenever there is the complete lock-in described in Proposition 1. In

[25] It is also important to check the whether the candidate solution that emerges from (26) and (27) involves negative wealth in any states. If so, then an additional, non-negativity constraint on internal wealth, $w \geq 0$, $\forall\epsilon$, might also be imposed in the maximization problem, in order to address the concerns about credit risk raised in footnote 16.

[26] The expression on the right-hand side of (29) can be shown to be a function (denoted by l), of both internal wealth and ϵ:

$$\frac{dw^*(\epsilon)}{d\epsilon} = -(\alpha\gamma - \beta\theta)\frac{f_I^A}{w_0\theta f_{II}^A} + \frac{\beta I^A}{w_0} = l(w^*(\epsilon), \epsilon).$$

This expression defines the basic differential equation which the optimal level of wealth must satisfy. (The constraint (27) provides the restriction that ties down the constant of integration.)

contrast, options will be needed for implementing the optimal hedges when either $\alpha \neq \beta$ or when there are state-dependent financing opportunities ($\delta \neq 0$) as in section 4.2. In the latter case, the use of options allows investment to be completely insulated from shocks to financing opportunities.[27]

For those cases in which options are required, equation (29) implicitly yields a recipe for the number of options to be purchased at different strike prices. While the first derivative of wealth, $\frac{dw^*}{d\epsilon}$, gives us the optimal exposure to ϵ, it is the *second* derivative, $\frac{d^2w^*}{d\epsilon^2}$, that describes the "density" of the options position at each strike price in the optimal hedge portfolio. Intuitively, an option at a strike price of $\hat{\epsilon}$ is indispensible for *changing* the degree of exposure at the point where $\epsilon = \hat{\epsilon}$. Thus, for example, if there are regions in which $\frac{d^2w^*}{d\epsilon^2}$ is large and positive, a substantial number of call options with strike prices in that region should be added. In contrast, for regions in which the hedge ratio is constant, $\frac{d^2w^*}{d\epsilon^2} = 0$, no additional options are required.

To see the role for options more concretely, consider the following numerical example. Suppose that there are three equally-probable states of nature, 1, 2 and 3, and that a firm's first-best levels of investment (i.e., that for which $f_I = 1$) are 6, 9 and 15, in each state respectively. Suppose also that at any level of investment below 6, the firm will be unable to compete and will be forced into bankruptcy, and that the firm has no access to external finance. Finally, suppose that internal wealth is initially equal to 10, and that a no-hedging strategy yields 5, 10, and 15 of internal funds available for investment. (See Table 1 below for a schematic.)

If the firm has only futures contracts available to it, it can increase state-one internal wealth only through an equivalent reduction in state-three wealth. Its optimal hedge will therefore be predicated on protecting revenues in the lowest state, and will lead to an internal wealth configuration of something like 6, 10, and 14. This is a better profile than without hedging, but it does not generate first-best levels of investment.

[27] To see this, note that with non-stochastic production technology, $F_I = P_w$, which by (28) is a constant.

Now suppose that options become available. With its futures hedge in place, the firm has excess cash in state two and insufficient cash in state three. The value-maximizing hedging strategy therefore involves buying 1 state-one "put" option (which pays 1 in state one and zero otherwise) and 2 state-three "call" options (each of which pay 1 in state three and zero otherwise). Because each option costs 1/3, their total cost is 1, which exactly eliminates the previously-existing excess cash balance in state two. (See Table 1.) Options are therefore valuable when value-maximizing hedge ratios are not constant.[28]

[28]By put-call parity, one can achieve an equivalent hedge by using only the put (or call) option together with a different quantity of futures, or by using options alone.

7. Conclusion

Although we have explored a number of applications of our basic risk management paradigm, our simplifying assumptions have left several interesting questions unanswered. First, since our model is essentially a static one – there is only a single period during which investment takes place – we have not addressed any of the potentially important intertemporal issues associated with risk management. For example: 1) over what time horizon should one structure hedges?; 2) Given that different hedging instruments (e.g., futures vs. forwards) involve different time patterns of cash inflows and outflows, what are the relative advantages and disadvantages of each?

Second, we have assumed that all the risks impacting a firm's cashflows are marketable and thus can be hedged. However, this will not in general be true. For example, a firm's cashflows will be abnormally low if its new product introduction fails, but there may be no futures market in which this risk can be laid off.

In this is the case, such unmarketable idiosyncratic risks will (in a work with costly external finance) impose real costs on the firm. Capital budgeting procedures should therefore take these costs into account. Consequently, the CAPM (or any other standard asset pricing model) will no longer be universally valid as a capital budgeting tool. In other words, when investment projects impose large idiosyncratic risks that cannot be directly sold off, a second-best risk management strategy will involve reducing the level of investment in these projects below that implied by a CAPM-type discounting procedure.

The *magnitude* of the deviation from traditional capital budgeting principles should depend on the same sorts of factors that we identified above as determinants of the optimal hedging strategy. For example, if the unmarketable idiosyncratic risk on the investment currently being evaluated is closely correlated with the availability of future investment opportunities, then the logic developed in section 4.1 suggests that there is relatively less reason to "hedge" by skimping on this investment. In contrast, if the investment in question is uncorrelated with the availability of future investment opportunities, it should be evaluated more harshly.

8. Appendix

Derivation of Equation (24). First, note that at the moment when the investments are made, ϵ has already been realized. It follows that the first-order condition of (22) with respect to domestic investment is

$$f_I^H = \frac{\theta}{\gamma} f_I^A, \tag{A1}$$

which says that the firm equalizes the marginal-revenue product of an additional unit of domestic currency across investments. Second, note that the marginal return on domestic investment will be always be set equal to the marginal cost of an additional unit (in domestic currency terms) of external finance,

$$f_I^H = C_e + 1. \tag{A2}$$

Together these equations, along with the budget constraint in (23) tie down the optimal choices for domestic and foreign investment, *for given wealth* of w. By applying the implicit function theorem to them, one can determine the sensitivity of optimal investment plans to changes in ϵ, $\frac{dI^H}{d\epsilon}$ and $\frac{dI^A}{d\epsilon}$.

Moving back to the initial period when the hedging decision is made, equation (22) must be maximized with respect to h. The first-order condition for this problem is identical to that given in equations (17) through (19). Applying the formula for covariance given in footnote 15, equation (19) can be rewritten:

$$E\left[C_{ee}\left(\frac{dI^H}{d\epsilon} + \gamma\frac{dI^A}{d\epsilon} + \beta I^A - (1-h)w_0\right)\right] = 0. \tag{A3}$$

Substituting in the expressions for $\frac{dI^H}{d\epsilon}$ and $\frac{dI^A}{d\epsilon}$ derived above and simplifying yields equation (24).

Proof of Proposition 1. We start by hypothesizing that I^H and I^A non-stochastic, and $h = 1 - \beta \bar{I}^A / w_0$. We then verify that this is optimal, i.e., that the first-order conditions for both hedging (equation (24)) and investment (equations (A1) and (A2) above) are satisfied.

First, note that I^H and I^A constant and $h = 1 - \beta \bar{I}^A / w_0$ together imply, from the budget constraint in (23), that $\frac{de}{d\epsilon} = 0$ – external financing is independent of the exchange rate. This implies that C_e is independent of ϵ. But, given the first-order condition in (A2), this in turn implies that it is optimal for I^H to be independent of ϵ. Similarly, when $\alpha = \beta$, the first-order condition in (A1) reduces to $f_I^H = f_I^A$. So if it is optimal for I^H to be constant, then it is optimal for I^A to be constant also.

This establishes that a constant I^H and I^A are optimal, *given* $h = 1 - \beta \bar{I}^A / w_0$. We now must check that this hypothesized hedge ratio is itself optimal. This now follows immediately from (24), once we note that $E[I^A P_{ww}]$ can be simplified to $\bar{I}^A \bar{P}_{ww}$ when I^A is non-stochastic.

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Table 1

Hypothetical hedging strategies and investment spending
(with initial wealth of 10)

state	proba-bility	optimal investment spending	Net Funds Available for Investment: no hedging (1)	futures hedge (2)	payoffs to options position (3)	net hedge with options (2) + (3)− cost
1	1/3	6	5	6	1	$6+1-1=6$
2	1/3	9	10	10	0	$10+0-1=9$
3	1/3	15	15	14	2	$14+2-1=15$
Total cost of options			$-1/3-2/3=-1$			