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ARE OPTION-IMPLIED FORECASTS OF EXCHANGE RATE
VOLATILITY EXCESSIVELY VARIABLE?

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ABSTRACT

Market participants' forecasts of future exchange rate volatility can be recovered from option contracts on foreign currencies. Such implicit volatility forecasts for four currencies are used to test rational expectations jointly with the applicability of the standard Black-Scholes formula. First, we examine the null hypothesis that the market-anticipated one-month-ahead standard deviation is an unbiased estimator of the subsequent realized standard deviation. The parametric regression method rejects this hypothesis overwhelmingly: the implicit forecasts are themselves excessively variable. Simulations indicate that the rejection is not caused by non-normality of the error term. Second, we use a nonparametric method to test a weaker version of market rationality: the market can correctly forecast the direction of the change in exchange rate volatility. This time, the weaker version of rationality is confirmed. Third, we investigate how market forecasts are formed. We find some evidence that market participants put heavy weight on lagged volatility when forecasting future volatility. Finally, results from the Alternating Conditional Expectations algorithm provide further support for the central finding that when the market predicts a large deviation of volatility from its mean, it could do better by moderating its forecast.

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"It would be wise to cut expectations in half."

Anonymous, from a fortune cookie in a Chinese restaurant

Section 1 Introduction

While there are by now hundreds of tests of rational expectations in financial markets, most do not specify what the alternative hypothesis is. Bilson (1981) and Froot (1989) have specified an alternative hypothesis: the proposition that investors' forecasts have a tendency to be excessively variable. In other words, when their forecast is high it is on average too high and when it is low it is on average too low; investors would do better by placing more weight on the long run average value of the price they are trying to forecast. As unattractive as a rejection of rational expectations is to economists on *a priori* grounds, there is evidence to support it.¹

In the foreign exchange market, the evidence concerns investors' estimates of the expected future spot rate. Tryon (1979), Longworth (1981), Bilson (1981) and others measure the expected future change in the spot rate by the discount in the forward exchange market, and find that the estimates are biased in the direction of excessive variability.² Frankel and Froot (1987)

¹ Froot (1989) and Frankel and Stock (1987) describe how this evidence relates to the variance bounds tests of Shiller (1981) and LeRoy and Porter (1981).

² The problem with these results as a test of rational expectations is the possibility of a risk premium. In other words, in determining the forward rate, risk-averse investors may add a risk premium to their forecasts of future spot rates, which would cause the forward rate to be a biased forecast of the future spot rate even if the investors' forecast is unbiased. Other tests that find bias in the forward rate include Fama (1984) and Hsieh (1984).

and Froot and Frankel (1989) measure the expected future change in the spot rate by survey data, and find the same bias.³

In this paper, we use options data to examine the analogous excess-variability hypothesis for the case of the second moment of exchange rates.⁴ That is, we test that investors could do better in their forecasts of the variance by putting more weight on the long-run average value. Options data provide a relatively clean test because the options pricing formula is derived from a no-arbitrage argument and investors' forecasts of volatility are the only quantity not directly observed by an econometrician. It must be acknowledged that, here too, it is a joint hypothesis that we are testing: there could be a failure of one of the assumptions of the Black-Scholes formula, such as the assumption that the spot rate follows a diffusion process. But even if a rejection of the null hypothesis were due to a failure of an auxiliary assumption, it would still be an important finding, as it would still mean that it is dangerous to rely on options prices' implicit volatilities as forecasts of future variances.

It would be disturbing to conclude that the market's expectations are completely foolish. In the second stage of the test, we look at a weaker version of rationality: the market can correctly predict the direction of the change in exchange rate

³ A problem with these results is that many readers are skeptical of survey data because, among other reasons, investors may strategically misrepresent themselves in a survey. In contrast, the forecasts of future volatility implicit in options prices are used by traders in executing actual trading.

⁴ Stein (1989) examines the term structure of the option-implied volatility in stocks, and finds that the short-term stock volatility tends to overreact relative to the long-term volatility.

volatility. A nonparametric method is used to examine the weaker criterion of rationality. Here, the market expectations are found to be informative about subsequent events.

There exists a relatively small empirical literature testing the efficiency of foreign exchange options prices. Using a simulation method, Borensztein and Dooley (1987) found bias in the foreign exchange option data, namely, that there are deviations from the Black-Scholes values for call options that are deep "out of the money." (In contrast, the option data used in this paper are entirely those that are closest to being at the money.) The direction of the bias was an apparent overvaluation of the option, implying an overestimation of the likelihood of change in the value of the dollar. Similarly, Bodurtha and Courtadon (1987) find a (small) bias toward apparent overpricing of call and put options on foreign exchange, which is equivalent to apparent overestimation of volatility. Bates (1990) argues that an asymmetric jump-diffusion process is more appropriate than the standard processes assumed for the spot rate. He, like Borensztein and Dooley, examines data from the mid-1980s and believes that the apparent failure of the Black-Scholes formula to fit out-of-the-money options is attributable to market perceptions in each period of a certain probability that the dollar will plunge in value. Melino and Turnbull (1991) allow for the spot rate process to differ from the standard log-normal, but again find that the options prices overestimate volatility.

Among other papers using data on foreign exchange options, Lyons (1988) has estimated option-implied volatilities for three currencies and used them in a test of time-varying risk premium in the forward discount, though this was not a test of whether the

implied volatility is an unbiased estimator of the future realized volatility. Wei (1991) uses implied exchange rate volatility to examine the effects of anticipated volatility on bid-ask spreads in foreign exchange markets; an increase in anticipated volatility is found to widen the spreads.

The balance of this paper is organized as follows: Section 2 describes the data source and selection criteria, as well as the formula used in extracting market-anticipated volatility. Based on these estimates, Section 3 examines, through a parametric regression, if the implied standard deviation is an unbiased estimator of the realized one. It also uses simulations to determine how sensitive the size of the test in a small sample is to non-normal distributions of the error term. Section 4 tests a weaker version of rationality with a nonparametric method. Section 5 looks closely at the formation of the market anticipation and possible patterns of the expectational errors. Section 6 uses the technique of Alternating Conditional Expectations to test further the proposition that emerged from Section 3, namely that market participants would do better if their forecasts put more weight on the long-run average volatility. Finally, section 7 reviews the findings.

Section 2 Estimating the market anticipated exchange volatility

2.1 Data Description

Currency option trading started at the end of 1982 on the Philadelphia Exchange. In 1985, the Chicago Board of Exchange also began to trade currency options. The data used in this study are

of options on the Philadelphia Exchange from February of 1983 to January of 1990. Early trading was very thin. The first few months of data might give unreliable estimates of the standard deviations, and so are discarded. The source of the option and spot exchange rate data is various issues of the Wall Street Journal. The data are described as follows.

(1) The four most heavily traded currency options: British Pound, West German Mark, Japanese Yen and Swiss Franc, all relative to the US Dollar.

(2) Call options (American style) that are closest to being at the money. These options are most heavily traded and thus yield more reliable estimates of implied volatility.

(3) Contracts signed on the third Wednesday of each month.

(4) If possible, contracts that matured in the following month. Otherwise, contracts with the next nearest maturity.

(5) The closing quote for the spot exchange rates on the same day the option contract is signed and on the same Exchange.

The interest rates are: the 3-month Treasury Bill rate for the United States, and call money rates for the other four countries. The source is OECD Main Economic Indicators. Daily exchange rates are used to calculate the realized standard deviations in the spot exchange rates. They are the closing bid rates in the London market. The data are from Data Resource Inc.'s financial database.

2.2 The Valuation Model

One model commonly used by the market in calculating European currency option value is the Garman-Kohlhagen (1983) formula, which is equivalent to a version of Black-Scholes formula for options on

a stock which pays a continuous stream of dividends (with the foreign interest rate essentially substituted for the dividend rate). Underlying the Garman-Kohlhagen formula is the assumption regarding the stochastic process governing the asset on which the option is written. Let the spot exchange rate be $S(t)$, which gives units of domestic currency per unit of foreign currency. $S(t)$ is assumed to follow a log normal process.

$$dS(t)/S(t) = udt + \sigma dw(t) \quad (1)$$

where u is its instantaneous mean rate of change, σ is the standard deviation of the instantaneous rate of change, and $w(t)$ is a standard Wiener process. By Ito's lemma, $\ln S(t) - \ln S(0) = (u - 0.5\sigma^2)t + \sigma w(t)$. In other words, for all $j > 0$,

$$\begin{aligned} E\{ \ln S(t+j) - \ln S(t) / S(t) \} &= (u - 0.5\sigma^2) j \\ \text{Var}\{ \ln S(t+j) - \ln S(t) / S(t) \} &= \sigma^2 j \end{aligned} \quad (2)$$

Given the process for $S(t)$ in (1), the Garman-Kohlhagen value of a European call option on foreign exchange is given by the following expression:

$$\begin{aligned} C[S(t), K, T, i, i^*, \sigma] \\ = S(t) \exp[-i^*(T-t)] N(d + \sigma(T-t)^{0.5}) - K \exp[-i(T-t)] N(d) \end{aligned} \quad (3)$$

with the following definitions:

$C(.)$ ----- the value of the call option
 $S(t)$ ----- current spot exchange rate

K ----- strike price
 T ----- maturity date
 i ----- instantaneous domestic nominal interest rate
 i* ----- instantaneous foreign nominal interest rate
 σ ----- market anticipated standard deviation from time t
 to maturity T, the same σ as in (1).

$$d = \frac{\ln[S(t)/K] + (i - i^* - 0.5 \sigma^2) \cdot (T-t)}{\sigma (T-t)^{0.5}}$$

 N(.) ----- the value of the cumulative normal distribution
 function.

One property of (3) that will be frequently used in the next section is that the call value is an increasing function of σ . The more volatile the spot exchange rate is, the more likely is the spot rate to exceed the strike price during the specified time interval, and hence the more valuable is the option to buy foreign currency at that price.

We use American options in our study, because European option trading started in 1985 and thus has too short a history for time series analysis. Unfortunately, there is no closed-form solution such as (3) for the American options. However, as long as early exercise is not heavy, the value of American call and European call are known to be very close. Some random checking of the values of European and American calls with the same striking price and maturity date reveals that they are indeed quite close. Buttler (1989) and Adams and Wyatt (1989) have independently shown through simulations that when the domestic interest rate is higher than the foreign interest rate, the prices of a European and an American call are identical. Furthermore, the difference between the values of the two types of call options is small when the domestic interest

rate is equal to or slightly lower than the foreign interest rate.⁵ Throughout our sample, the US interest rate was higher than the Japanese, German and Swiss interest rates (though it was not higher than the British interest rate). Therefore, we do not expect the errors resulting from use of the European formula to be serious. But the appropriateness of the European option feature, and other aspects of the Garman-Kohlhagen formula, should be considered to be part of the joint null hypothesis. In any case, future research could consider using some numerical approximation method such as the binomial model to improve the estimates of the implied volatility.

There are two other potential sources of error in applying the Garman-Kohlhagen (GK) formula to price currency options. Both have to do with the assumption regarding the spot exchange rate process. With either one of these two complications, currency options can no longer be priced by a no-arbitrage argument. Indeed, there can be no generally agreed-upon method to price options in such cases, since it would have to depend critically on the particular specification of the jump process or the volatility process.

First, volatility is assumed to be non-random, but could in principle also follow a stochastic process. Wiggins (1987) and Hull and White (1987) are among those who have analyzed the effect of stochastic volatility on proper pricing of options. The effect is to introduce a risk premium into the option-pricing formula. Melino and Turnbull (1990) argue that allowing for this risk premium to be non-zero (though constant) sharply reduces the magnitude of apparent errors in pricing foreign exchange options (though there is no evident guarantee that this premium is in fact related to risk in the proper way, as opposed to being merely a free parameter that helps to fit the data).

Hull and White (1987) and Ng (1991) show that under certain special conditions (volatility instantaneously uncorrelated with aggregate consumption), the risk premium becomes zero. In that

⁵ See also Shastri and Tandon (1986) and Jorion and Stoughton (1987).

case, a version of the Black-Scholes formula with the average volatility over the interval of the option substituted for the known volatility becomes appropriate. One can interpret the null hypothesis considered in this paper to be the joint hypothesis consisting of this uncorrelatedness assumption regarding the spot rate process together with the proposition that the forecast of average volatility for each interval is unbiased.

The second potential source of error is that, while the GK formula assumes the exchange rate process to be a lognormal or diffusion process, it could in reality be a mixture of diffusion and jump processes (Jorion, 1988). As mentioned earlier, Borensztein and Dooley (1987) and Bates (1990) attribute the finding of apparent overvaluation in out-of-the-money options to the existence of a jump process, and specifically to the perceived possibility in the mid-1980s of a future large depreciation of the dollar. It has also been argued that a jump process is particularly plausible when one takes into account frequent government intervention in foreign exchange markets (Ball and Roma, 1990).

The present paper differs from earlier studies of bias in that the options examined are at the money (or close to it), which we suspect makes the possibility of a jump process less important as a potential source of apparent bias. The assumption that the spot rate follows a diffusion process, nevertheless, is part of the joint null hypothesis that we test.

2.3 Estimation of the implied volatility and comparison with the realized volatility

Given $S(t)$, K , T , i and i^* , equation (3) defines an implicit function in the market estimate of the standard deviation σ .

We can rewrite (3) as

$$g(\sigma) = C(S, K, T, i, i^*, \sigma) - C = 0. \quad (4)$$

Notice that σ appears (with different powers) in the numerator and the denominator of d , which in turn is an argument of the cumulative normal function. Because (4) is a non-linear equation, the Gauss-Newton iteration method is used to solve for σ .

The corresponding monthly series of the realized standard deviation is computed from daily exchange rates. Let $\text{Log } S(j)$ be the log of the spot rate on day j . Then the realized standard deviation (rsd) for a given month is defined as the standard deviation of changes in the logarithm of the daily spot rate, $\text{Log } S(j+1) - \text{Log } S(j)$, from the third Wednesday of the month to the third Wednesday of the following month. Notice that this definition is consistent with the assumption regarding the stochastic process of the spot rate, as given in (2). Plots of the option-implied volatility against ex post realized volatility for each of the four currencies are presented in Figure 1.

The means of the four implied volatility are: British pound, 0.00592; Deutsche Mark, 0.00618; Japanese yen, 0.00520; and Swiss franc, 0.00616. In comparison, the means of the realized volatility are: British pound, 0.00686; Deutsche Mark; 0.00667, Japanese yen, 0.00568; and Swiss franc, 0.00700. Notice that the four implied volatility are lower than the corresponding realized ones. This suggests that at-the-money options are not "overpriced" compared to deep-in-the-money or deep-out-of-the-money options.

One shortcoming of the data is that the call prices, spot exchange rates and interest rates are not exactly time-synchronized. In future, we will use time-stamped transaction data to improve in this regard as well as to expand the sample size.

Section 3: A parametric test of strong rationality.

The hypothesis tested in this section is that the market-anticipated standard deviation is an unbiased estimator of the subsequent realized one. We require that the market not only get the direction of change correctly, but also get the magnitude

of change correctly on average. In section 4, we will test a weaker notion of rationality, which only requires that investors are on average on the right side of the market.

3.1 Tests of the unbiasedness of the isd based on linear regressions

To test formally the hypothesis that isd is an unbiased predictor of the future rsd, consider the following regression:

$$\text{rsd}_{t+1} = a + \beta \text{isd}_t + e_{t+1} \quad (5)$$

where rsd_{t+1} is the realized standard deviation from t to $t+1$, and isd_t is the forecast of the standard deviation from t to $t+1$ that is implicit in the option price observed in the market at time t . The null hypothesis is $H_0: \beta = 1$. One might also include the restriction $a=0$ as part of the null hypothesis, but we decided not to, so as to focus on excess variability as the alternative hypothesis. This makes the test more conservative, in the sense that it is more difficult to reject H_0 than otherwise.

The results of regression (5) for the four exchange rates are reported in Panel A of Table 1. Notice that for each of the four currencies, the null $\beta = 1$ is rejected at the 5% level. In fact, the β 's in two of the four equations are not significantly different from zero at the 5% level. The point estimates are all smaller than one, suggesting that market participants tend to overpredict the magnitude of volatility. They would be wise in their forecasts to put more weight on the long-run average volatility. The quote given at the beginning of the paper seems appropriate: "it would be wise to cut expectations in half."

If there is heteroskedasticity in the data, the standard errors can be underestimated. This could cause the null hypothesis to be incorrectly rejected even when it is true. To examine this possibility, White's test of heteroskedasticity is performed and the results are reported in Panel A of Table 1. The hypothesis of no heteroskedasticity is rejected for the German Mark at the five percent level, but not for the other three currencies. Standard errors can also be reestimated using White's method of correcting

for unknown heteroskedasticity. The results are in the square brackets in Panel A of Table 1. The standard errors are generally larger than without the correction. But even with this correction, the null hypothesis that the slope parameter is one can be rejected for all four currencies at the five percent level.

One may also wonder about the possibility of a structural break in the sample. We split the sample into two equal-sized subsamples, and use a Chow test to examine for structural break. The results are reported in the last column of Panel A, Table 1. Except for the British Pound, there is no evidence of a structural break at the five percent level.

The first moments of exchange rate changes are known to be highly correlated across currencies, and it is likely that the second moments are as well. To take advantage of the possible cross-currency correlations, we use the method of seemingly unrelated regression (SUR) to estimate the four equations as a system. Because the SUR method takes into account cross-equation restrictions on the covariance matrix of the error terms, it should yield more efficient estimator of the β 's. The results using SUR are reported in Panel B of Table 1. Again, the null hypothesis of unbiasedness is rejected for all four currencies at the 5% level.

[Table-1 about here]

Before concluding that isd is a biased estimator of the rsd , we will note two types of problems that could conceivably invalidate the test. The first problem is the possibility of incorrect size of the test. This problem will be more carefully examined in the next subsection. The second problem is related to observations derived from option contracts with overlapping time to maturity.

Before the end of 1985 options were available only at four maturity dates: March, June, September and December. Monthly series of isd necessarily contain overlapping time periods. Overlapping time periods in isd would cause serial correlation of the prediction errors for isd , and thus $e_{t,1}$ in (5) would not be

white noise. To overcome this problem, we have redone the SUR on a sub-sample that excludes the observations with overlapping horizons.⁶ The results are reported in Panel C of Table 1. From Panels B and C of Table 1, we see that after correcting for overlapping observations, the SUR estimates of β 's are still statistically different from one, which confirms the finding in Panels A and B of the same table.

One might worry about possible non-stationarity in the data that could invalidate the tests. We performed a Dickey-Fuller test for unit roots on the *rsd* series, and found that the null hypothesis of a unit root is rejected for the four currencies. When the SUR estimation is redone for variables in first-differenced form, the β estimates are still statistically different from one. To save space, these results are not reported here.

OLS estimation equation-by-equation and SUR estimation, with or without correcting overlapping observations, all point to the same conclusion: that the null ($H_0: \beta=1$) is overwhelmingly rejected. This says that the implied standard deviation is a biased predictor of the realized standard deviation in the exchange market.

3.2 Non-normality and size of the test in small samples: Some simulation results

The point estimation and hypothesis testing reported above depend on assumptions made about the regression model. In particular, these include an assumption that the error term in the equation (5) has a normal distribution.⁷ As the standard deviation

⁶. We could apply Hansen and Hodrick's (1980) method of moments to correct for problems with overlapping observations. However, since the fraction of the data that come from overlapping contracts is small, we lose only a few observations by simply dropping those observations.

⁷ We know that the level of exchange rates is characterized by leptokurtosis and moderate skewness, particularly in high frequency data (see, for example, Boothe and Glassman, 1987). It is possible that the standard deviation series of exchange rates (or its

can never be less than zero, its true distribution cannot literally be normal. But then the question naturally arises whether our estimates and inference are still valid.

First, we know that in large samples, non-normality does not matter: The slope estimator will be consistent and asymptotically normal, as long as the regressor is uncorrelated with the error term, which is true under the null hypothesis of rational expectations. Second, even in small samples, the estimator is still unbiased. However, the distribution of the estimator in a small sample is not guaranteed to be normal. Given that the null of unbiasedness is rejected in the previous subsection, a relevant question for us is how sensitive the size of the test is to non-normal distributions of the error term. This is the subject of investigation in this subsection.

One way of checking sensitivity to the normality assumption is to run the regressions with all variables in logarithms instead of in levels. The log-normal distribution has the advantage that it implies that the standard deviation can never be negative.⁸ The results are presented in Table 2. Two features are evident. First, the slope estimates are all statistically different from one at the 5% level for all four currencies. This is true whether the OLS or SUR method is used. The point estimates here are generally smaller than the corresponding ones in Table 1. Second, the intercept estimates are all statistically different from zero.

[Again, the White's test of heteroskedasticity is performed and heteroskedasticity-consistent standard error are estimated. The

logarithmic transformation) also has a non-normal distribution.

⁸ It is possible that the logarithmic transformation could actually introduce bias into the hypothesis testing. If the implied standard deviation in levels is an unbiased estimator of the realized standard deviation in levels, Jensen's inequality implies that the logarithm of the *isd* is not an unbiased estimator of the logarithm of the *rsd*. But the Black-Scholes formula does not give us any reason to choose the level of the standard deviation over the log. (Under the assumptions of the formula, the standard deviation is known with certainty, so that there is no issue.)

White's test indicates that there is problem of heteroskedasticity for British pound and Japanese yen. But even when correct for the heteroskedasticity using the White's method, the results of the hypothesis testing regarding the slope parameter are unaffected.]

[Table 2 about here]

Let us now formally examine the moment properties of the residuals from OLS regressions, presented in Table 3. Panel A reports the properties of OLS residuals from regressions with variables in levels. The skewness of the residuals for the British Pound, German Mark, Japanese Yen and Swiss Franc is, respectively, 1.0967, 0.564, 1.584, and 0.9317. In a chi-square test, all four are statistically different from the skewness of a normal distribution at the five percent level. The kurtosis of the residuals for the four currencies is, respectively, 5.053, 3.237, 6.281, and 3.936. Based on a chi-square test at the 5% level, the kurtosis parameters for the British Pound, Japanese yen and Swiss Franc are also statistically different from those of a normal distribution. In individual Jarque-Bera tests, the null of normality is rejected for three of the four currencies at the five percent level. Panel B reports similar statistics for residuals from OLS regressions with variables expressed in logarithms. Here, the kurtosis parameters are no longer statistically different from those of a normal distribution. The Jarque-Bera tests also fail to reject the null hypothesis of normality at the five percent level. This suggests that the logarithmic transformation indeed makes the residuals much closer to normality. However, the four skewness parameters are still statistically different from those of a normal distribution. In other words the deviation from normality, although smaller, does not vanish completely.

[Table 3 about here]

Having established that the errors are not normal, we proceed

to investigate whether we reject the null of unbiasedness too much in our sample. We generate 1000 samples according to the null hypothesis and count the number of rejections using a t-statistic and its conventional critical value. Specifically, we specify the error term, z , as follows:

$$z = h [e^w - e^{\mu + 0.5g}]$$

where w is a normal variate with mean μ and variance g . The advantage of this specification is that the moments of z can be easily computed with the aid of the moment generating function of a normal variate. For example, z can be shown to have the following properties: (let $r = e^g$)

- (a) $E(z) = 0$
- (b) $\text{Var}(z) = h^2 e^{2\mu} r(r-1)$
- (c) $E(z^3) = h^3 e^{3\mu} r^{3/2} [r^3 - 3r + 2]$
- (d) $E(z^4) = h^4 e^{4\mu} [r^8 - 4r^5 + 6r^3 - 3r^2]$
- (e) $\text{Skewness}(z) = E(z^3)/[E(z^2)]^{3/2} = [r^3 - 3r + 2]/[r - 1]^{3/2}$
- (f) $\text{Kurtosis}(z) = E(z^4)/[E(z^2)]^2 = r^4 + 2r^3 + 3r^2 - 3$

In principle, for any arbitrary distribution of the error term we can, by appropriately choosing μ , g and h , use z to match any given three moments of the distribution (in addition to the zero mean). Sometimes this can not be done perfectly. For example, the skewness and kurtosis coefficients involve only one parameter. However, this will not be a serious problem for us as will be clear later. Table 4 shows how skewness and kurtosis change as g (or r) increases. It is clear both are monotonically increasing functions of the parameter. If we choose (μ, g, h) to be $(0, 0.07, 0.01265)$, we can approximate reasonably well the average magnitude of sample variance, skewness and kurtosis of the OLS residuals as reported in Table 3.

[Table 4 about here]

We do 14 different permutations of the triple of parameters (we restrict μ to be zero). For each chosen triple of the parameter, we generate one thousand samples of size 85 under the null that the slope coefficient is one (and intercept is 0.03), and compute the percentage of times we reject the null with a conventional t test at five percent significance level. The results are reported in Table 5.

[Table 5 about here]

First of all, we vary g in the neighborhood of 0.07 while keeping h fixed at 0.01265. As g varies from 0.04 to 1, the skewness of the error term, z , varies from a low of 0.614 to a high of 6.18, the kurtosis from 3.678 to 113.94. Column (6) reports the "true" size of the test in the simulation, Column (7) the average point estimate of the slope. The point estimate is very close to one. The percentage of rejection is somewhere between 5.2 percent and 6.6 percent. Therefore, the true size of the test in the small sample due to non-normality is higher than 5 percent, but not by much. This is true even in the case of very high skewness and kurtosis for the error term.

Next, we vary the value of h in the neighborhood of 0.01265, keeping g at 0.07. As h changes from 0.01165 to 0.110, the variance of the error term changes from $10.55 \cdot 10^{-6}$ to $940.96 \cdot 10^{-6}$. Again we can see that the average point estimate is quite close to one and the percentage of times rejecting the null is not much different from 5 percent.

To summarize, our simulations are designed to examine whether deviations from normality can cause too many rejections of the t test in small samples. Within the family of non-normal distributions chosen here, the answer is no. The true size of the test is only slightly different from 5 percent, even when we vary the variance, skewness and kurtosis of the error distribution to a considerably large magnitude relative to their actual magnitude in the sample. Therefore, the rejection of the unbiasedness hypothesis

in the previous subsection seems to be robust to the distributional assumption.⁹

Section 4. Weak rationality and a nonparametric test

It would be disturbing to conclude that market participants are so foolish that they make forecasts that are completely irrelevant to subsequent realized events. Therefore, we continue our study here by looking at a weaker version of rationality. This version of rationality says that market participants are forward-looking and can predict the direction of the change in exchange rate volatility. It is weaker than the version of rational expectations tested in section 3.1, because it looks only at the direction of the change as opposed to the magnitude of the change.

In Tables 1 and 2, most of the slope estimates, though less than one, were statistically significantly greater than zero. So we already know that the market's anticipated volatility is informative in one sense: it can help predict the realized volatility relative to its sample mean. Here we test whether the option-implied volatility is informative about the direction of change in the realized standard deviation relative to its last period's value. We take as our null hypothesis that the option-implied standard deviation is useless as a forecast of the future realized change in the standard deviation. Included in the null hypothesis is the possibility that the *isd* fails to beat the random walk as a description of the standard deviation, which predicts that the expected change is always zero.

The test is non-parametric in nature and thus is robust to distributional assumptions of the exchange rate process. In addition, it permits non-stationarity of conditional probabilities over time. It was developed by Henriksson and Merton (1981) in the context of mutual fund performance evaluation, and adapted by

⁹ Of course, the simulation results are valid only up to the family of distributions that has been considered.

Havenner and Modjtahedi (1988) and Lai (1990) to examine the usefulness of foreign exchange forecasts.

Let p_1 be the conditional probability of making a successful forecast when the realized standard deviation in the subsequent period decreases or does not change, and p_2 the conditional probability of making a successful forecast when the realized standard deviation increases. I.e., $p_1 = \text{Prob}[\Delta \text{isd} \leq 0 \mid \Delta \text{rsd} \leq 0]$, and $p_2 = \text{Prob}[\Delta \text{isd} > 0 \mid \Delta \text{rsd} > 0]$. Henriksson and Merton (1981) show that a sufficient statistic for evaluating weak rationality is $p_1 + p_2$. A necessary and sufficient condition for the market's forecast to have no value is that $p_1 + p_2 = 1$. For example, a forecaster always using a random walk process would have $p_1 = 1$, $p_2 = 0$ and $p_1 + p_2 = 1$.

Let n_1 be the number of times $\Delta \text{rsd} \leq 0$ and n_2 number of times $\Delta \text{rsd} > 0$. $N = n_1 + n_2$ is the total sample size. Let m_1 be the number of successful forecasts in the sample when $\Delta \text{rsd} \leq 0$, m_2 number of unsuccessful forecasts when $\Delta \text{rsd} > 0$. $M = m_1 + m_2$ is the total number of times $\Delta \text{isd} \leq 0$ in the entire sample. By definition, $p_1 = E(m_1/n_1)$ and $p_2 = 1 - E(m_2/n_2)$.

Under the null hypothesis that the market forecasts are useless, $H_0: p_1 + p_2 = 1$, we have $E(m_1/n_1) = E(m_2/n_2) = p_1$. Henriksson and Merton show that under the null, the probability distribution for m_1 -- the number of correct forecasts, given that $\Delta \text{rsd} \leq 0$ -- has the form of a hypergeometric distribution and is independent of both p_1 and p_2 . I.e.,

$$\text{Pr}(m_1 = x \mid n_1, n_2, M) = \frac{\binom{n_1}{x} \binom{n_2}{M-x}}{\binom{N}{M}}$$

This is the basis of the test. However, the computation of the test statistic is difficult for even moderate numbers of m_1 , n_1 and n_2 , since factorial and gamma function are cumbersome to calculate. Fortunately, a normal distribution approximation is available for the hypergeometric distribution. The approximation is very accurate even for small samples, as long as n_1 is roughly equal to $N/2$. The

parameters used for this normal approximation are the mean and variance for the hypergeometric distribution, which can be written as follows: $E(m_1) = Mm_1/N$ and $var(m_1) = m_1 n \ln 2 (N-M) / [N^2 (N-1)]$.

Table 6 presents results of applying this nonparametric test to our data. In the actual calculations, we take out all the data points that involve contracts of overlapping maturities (as explained in footnote 4). As a result, we have 64 data points for each of the four currencies. We then look at both a one-tail test (corresponding to an alternative: $p_1 + p_2 > 1$) and a two-tail test (corresponding to an alternative: $p_1 + p_2 \neq 1$). From Table 6, it can be seen that the null hypothesis of useless forecasts is rejected for the four currencies, individually and jointly, and for both one-tail and two-tail tests, at five percent significance level. For the British Pound for example, out of 64 sample points, $Ar_{sd} \leq 0$ 30 times. Out of these 30 down times, the market got 27 correct forecasts. The point estimate of $p_1 + p_2 = 1.282$. Corresponding to a critical value at the 5 percent, the required m_1 to reject the null is 24.65 for a one-tail test and 25.06 for a two-tail test, when a normal approximation is used to compute the test statistic. Since the actual value of m_1 is 27, the null hypothesis that isd is useless is rejected at the five percent level for both tests. Similarly, the point estimates of $p_1 + p_2$ for the German Mark, Japanese Yen and Swiss Franc are 1.375, 1.356 and 1.291, respectively. They are all statistically different from one at the five percent level for both the one-tail and two-tail tests. If we pool the four currencies together, the null hypothesis of isd being useless is also rejected at the five percent level for both the one-tail and the two-tail tests.

[Table 6 about here]

Given the overwhelming rejections of the null hypothesis, we might ask again how sensitive the probability of type-one error is to the small size of the sample when normal approximations are employed. In other words, does the test reject too much when the

market forecasts are actually useless? Luckily, Henriksson and Merton have conducted some simulations to check the true size of the test in small samples when normal approximation is employed to compute the statistic. They found that the size of the test is not much influenced by normal approximation in a sample as small as 50 data points, provided that n_1 is close to $N/2$. Since for all four of our currencies, n_1 is quite close to half of the total sample size, we can safely conclude that the rejections of the null are not caused by unusually large type-one errors.

In conclusion, even though the previous section rejects the strong version of rationality by parametric regression method, this section finds that a weaker version of rationality is supported by the data. While the market ex ante anticipation of exchange rate volatility may not be an unbiased estimate of the magnitude of the realized volatility, it does rationally forecast the direction of change in the realized volatility. The market forecast unambiguously outperforms some naive forecast rules, such as a random walk rule, as far as direction of change is concerned.

Section 5. The formation of the isd and pattern of the expectational errors

One naturally wonders how market participants form their estimate of the exchange volatility to be used in currency options. Specifically, we investigate three questions in this section. The first question is whether investors learn from their forecast errors in the past and whether they effectively have adaptive expectations. Second, we examine if the expectational errors are serially correlated. Third, we examine the possibility that the nature of the forecasting errors is best described by saying that market participants put too much weight on recently observed volatility, and not enough on the long-run average volatility.

5.1 The possibility of learning and adaptive expectations

Let us define the difference between the realized and market forecasted standard deviations, $rsd_{t,1} - isd_t$, as the expectation error at time t . To see if market participants learn from their mistakes, we regress the implied standard deviation on last period's prediction error (and last period's isd). The result is reported in Table 7. (This regression uses the SUR method on non-overlapping observations.) The point estimates for the adaptive expectations parameter, which is the coefficient on last period's prediction error, range from 0.2266 for the Swiss Franc to 0.3035 for the British Pound. They are all positive and all statistically different from zero. This says that when last period's isd_{t-1} overpredicts rsd_t by 1 unit, market participants on average adjust this period's prediction isd_t downward by about 0.27 units.

The point estimates of the coefficient for last period's isd are similar in magnitude to the coefficient on the contemporaneous prediction error.¹⁰ This suggests that a proper specification might simply have market participants form their forecasts of future volatility based only on recent observed volatility. We will pursue this idea in section 5.3.

[Table 7 about here]

5.2 Serial correlation in the expectational errors

One implication of the rational expectations hypothesis is that the expectational errors should be serially uncorrelated. A finding of positive serial correlation would imply that the adaptive expectations parameter estimated in the preceding section is lower than is optimal. We formally test this implication now with the

¹⁰ They are statistically significantly greater than zero, ranging from 0.2647 for the Swiss Franc to 0.5019 for the Pound. This means that if last period's forecast, isd_{t-1} , is high, then this period's forecast, isd_t , tends to be high, and vice versa.

Portmanteau Q-test. The results of the test for lags 1,3,5,7,9 and 11 are reported in Table 8.

One can see that for the Pound, Yen and Swiss Franc, there is evidence that the expectational errors are serially correlated. For example, with $P=1$, the null hypothesis of no serial correlation is rejected for the Swiss Franc at 5% level. With $P=9$ and 11, the null hypothesis is rejected for both the Pound and Yen at the 10 % level. The expectational errors for the German mark, in contrast, appear to be serially uncorrelated for all lags. To summarize, there is some evidence that the expectational errors are serially correlated, but the evidence is not strong.

[Table 8 about here]

5.3 Do investors put too much weight on recent volatility?

In section 3 we saw that -- whatever information investors base their forecasts of future volatility on -- they appear to put too much weight on that information and not enough on the long-run average level of volatility. But we said nothing there about what information investors use. There has been a great deal of empirical research, most of it using ARCH and GARCH techniques, showing that lagged volatility is a key variable, both in describing the actual process that generates volatility and in describing the model to which investors appear from their economic behavior to subscribe.¹¹ Textbooks instruct would-be options traders to use historical volatility to price options. A natural question, then, is whether the nature of investors' systematic forecast errors can be described as putting greater weight on lagged volatility than is optimal.

Table 9 reports regressions against the previous two lags of the realized standard deviation. For each currency, the dependent variable in the first equation tested is the *isd*. In each case, the

¹¹ For example, Garman and Klass (1980) [and Hsieh and Manaster (1988)].

lagged rsd is indeed a statistically significant determinant of the isd (and in most cases the twice-lagged rsd is as well). The dependent variable in the second test for each currency is the realized rsd over the coming period. In each case, the lagged rsd is again a statistically significant determinant.

In three out of four cases, the coefficient in the isd regression is greater than the coefficient in the rsd regression. In other words, the lagged volatility is not quite as important a determinant of the future volatility as market participants think it is. Is this difference statistically significant? To answer this question, the third test for each currency regresses the difference between the rsd and isd against the lagged volatilities. None of the negative coefficients is statistically different from zero. Thus, although the point estimates suggest that market participants may be putting too much weight on the lagged volatility, the significance levels are not high enough to allow us to conclude with any confidence that this is indeed the nature of the mistake that they are making.

[Table 9 about here]

6. Another technique for testing the relationship between the expectational errors and the level of the expectations

In this section, we continue to examine the existence of the systematic pattern in the expectational errors that was uncovered in section 3. We use the Alternating Conditional Expectations (ACE) algorithm to provide some clues to the search. The ACE algorithm is a non-parametric procedure, designed to find the optimal transformations (possibly non-linear) of the dependent variable and a set of explanatory variables so as to minimize the mean squared errors in regressions.

To ease interpretations, we restrict the transformations on the realized standard deviations (the dependent variable), to be linear. Under the null hypothesis that the isd is an unbiased estimator of

the *rsd*, one expects the required ACE transformation of the *isd* (the explanatory variable) to be linear, too. The results of the ACE procedure are reported in Figures 2-5. Figure 2 is the ACE result for the British Pound. The top graph is a scatter plot of the ACE-transformed *rsd* series against the original *rsd* series. It is linear by construction. The bottom graph is a scatter plot of the ACE-transformed *isd* series against the original series. Figures 3, 4 and 5 are similar pairs of scatter plots for the German Mark, Japanese Yen and Swiss Franc. The scale of all the transformed series is normalized to have zero mean.

From the bottom graphs in each figure, we can see the type of transformation of the implied standard deviation required in order to maximize the correlation between them and the corresponding realized standard deviation series. Two characteristics of the *isd* series are suggested by these plots. First, for all of the four currencies, the required transformation of *isd* is not linear, as would be the case under the unbiasedness hypothesis. Second, with the exception of the Yen, there is a common pattern to the required transformation of the *isd* series. The transformation of the two tails is different from that of the middle range. In general, when the *isd* gets big relative to its mean (i.e., on the right tails), ACE assigns a smaller weight relative to the weights assigned to *isd*'s around the mean. When the *isd* gets small relative to its mean (i.e., on the left tails), ACE raises the weight relative to the weights assigned to *isd*'s around the mean. This is true for the Yen too, except that the changes in weights in this case are more dramatic than for the other currencies. Based on these results, we hypothesize that the market expectations are overly excited on the two tails. When the market predicts a high volatility next month, it tends to overpredict (i.e., it predicts too high a volatility relative to the true one). When it predicts a low volatility in the following month, the market also tends to overpredict (i.e., it predicts too low a volatility relative to the true one).

This suggests that the market does overpredict the deviations of *isd* from its mean either when it predicts a high or a low

volatility. To echo the quote from the beginning of the paper, "it would be wise to cut expectations" a bit for the market.

Section 7: Overview and conclusions

This paper investigates the rationality of the market in forming its *ex ante* anticipation of the one-month-ahead exchange rate volatility for four currencies from February of 1983 to January of 1990. The market *ex ante* anticipation of exchange volatility is inferred from call option contracts on foreign currencies. We first examined a strong version of the rational expectations question: Is the implied standard deviation an unbiased predictor of the future realized standard deviation? We found that strong rationality is overwhelmingly rejected. This is true both in OLS and in SUR estimations, with or without correction for overlapping observations. Because there exists evidence of non-normality in the residuals of the OLS regressions, we then used a simulation method to determine whether the rejection of the strong rationality could be caused by incorrect size of the test in small samples. We found that the size of the test is not altered very much even if we consider distributions with much higher variance, skewness and kurtosis than observed from the OLS residuals. Thus, the rejection of strong rationality seems to be robust to the distributional assumption..

Some caution in the interpretation of the rejection of unbiasedness is in order before concluding that expectations are not rational. It is always possible that the market anticipation of volatility is incorrectly estimated. One candidate explanation is that, contrary to the assumption underlying the Black-Scholes and Garman-Kohlhagen formulas, the spot exchange rate process contains significant jumps. It could be a mixture of diffusion and jump processes, as proposed by Jorion (1988), Ball and Roma (1990) and Bates (1990). If the market participants take this into account in forming their anticipations, the anticipated volatility inferred by the Garman-Kohlhagen formula may be incorrect as an estimate of

investors' forecasts.¹² The magnitude of this effect on the point estimates and hypothesis-testing needs to be examined more carefully in future research. But even if the observed bias is due to the failure of one of the Garman-Kohlhagen assumptions rather than investors' mistakes, an important finding stands: the implicit volatilities extracted from options prices in the standard way are not optimal forecasts of future volatilities.

A weaker version of rationality was also examined: the market can rationally forecast the direction of change, regardless of the magnitude of the change. A nonparametric test developed by Henriksson and Merton was employed. The test is robust to the form of the distribution of the standard deviation process and can permit certain forms of nonstationarity. Here, the null hypothesis of the market anticipation being useless is overwhelmingly rejected. The market anticipation satisfies the weaker rationality condition.

As to the question of how the market forms its anticipations, we find some evidence of learning behavior and adaptive expectations. When the expectational error series are examined by the Portmanteau Q test, some evidence is found for the presence of serial correlation, suggesting that the investors may not adapt quickly enough. On the other hand there is also weak evidence suggesting that they may put excessive weight on the most recent volatility.

When an ACE algorithm is performed on the isd and rsd series, it seems that the market is overexcited on both tails of the isd series. That is to say, when the market predicts a high volatility, it tends to predict too high a value; when it predicts a low volatility, it tends to predict too low a value. The central finding of the paper is that expectational errors, $rsd_{t-1} - isd_t$, are

¹² Another possible interpretation, however, is that, because the Garman-Kohlhagen formula is known to be widely used in the markets, the extracted volatilities are what the market has in mind, regardless whether the assumptions underlying the derivation of the formula are correct, and that therefore our results do after all suggest bias in investors' formation of expectations.

negatively related to the level of the expectations, isd_t . Market participants could improve their forecasts of future volatility by putting more weight on the long-run average.

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Table 1: Testing the unbiasedness of the ISD, 1983:2-1990:1

$$rsd_{t+1} = \alpha + \beta isd_t + \varepsilon_{t+1}$$

Currency	α	β	adj.R ²	DW	White's test	Chow's test
A. OLS estimation, whole sample (N = 85)						
BP	0.002369 (0.000777)* {0.000972}*	0.7592 (0.1247)*# {0.1618}*	0.30	1.74	3.538 {0.170}	6.0098* {0.004}
DM	0.002717 (0.000962)* {0.000935}*	0.6407 (0.1509)*# {0.1609}*	0.17	1.62	5.884 {0.053}	0.2391 {0.788}
JY	0.005130 (0.000993)* {0.001246}*	0.1058 (0.1850)# {0.2286}	-0.008	1.61	3.102 {0.212}	1.851 {0.164}
SF	0.005241 (0.000999)* {0.001237}*	0.2869 (0.1564)# {0.2010}	0.027	1.33	2.790 {0.248}	0.630 {0.535}
B. SUR estimation, whole sample (N = 85)						
BP	0.003726* (0.000557)	0.5299*# (0.0845)	0.31	1.53		
DM	0.005048* (0.000522)	0.2633*# (0.0749)	0.18	1.47		
JY	0.005185* (0.000787)	0.0954 # (0.1441)	0.003	1.61		
SF	0.005770* (0.000506)	0.2011*# (0.0703)	0.039	1.31		
C. SUR estimation, excluding data from contracts with overlapping maturities (N = 64)						
BP	0.00497* (0.00077)	0.3049*# (0.1281)	0.053	1.57		
DM	0.00553* (0.00062)	0.1859 # (0.0954)	0.096	1.53		
JY	0.00503* (0.00080)	0.1806 # (0.1506)	0.016	1.78		
SF	0.00582* (0.00064)	0.2167*# (0.0970)	0.029	1.64		

Notes:

- (1) Standard errors are in parentheses.
- (2) Heteroskedasticity-consistent standard errors are in square brackets.
- (3) P-values are in curly brackets.
- (4) * denotes that the estimate is statistically different from zero at 5% level.
- (5) # denotes that the estimate is statistically different from one at 5% level.

Table 2: Testing the unbiasedness of the ISD, 1983:2-1990:1

$$\log(\text{rsd}_{t+1}) = \alpha + \beta \log(\text{isd}_t) + \varepsilon_{t+1}$$

Currency	α	β	adj.R ²	DW	White's test	Chow's test
A. OLS estimation, whole sample (N = 85)						
BP	-2.555 (0.5635)* [0.7549]*	0.4811 (0.1085)*# [0.1465]*	0.18	1.51	9.431 {0.009}*	6.506* {0.002}*
DM	-2.247 (0.7170)* [0.6752]*	0.5515 (0.1399)*# [0.1306]*	0.15	1.57	1.2641 {0.531}	0.226 {0.798}
JY	-4.9322 (0.7756)* [1.0768]*	0.0575 (0.1464)# [0.2049]	-0.01	1.61	8.2812 {0.016}*	2.128 {0.126}
SF	-3.4850 (0.7072)* [0.7657]*	0.2992 (0.1378)*# [0.1487]	0.04	1.28	0.8786 {0.644}	0.8139 {0.447}
B. SUR estimation, whole sample (N = 85)						
BP	-3.0693* (0.4284)	0.3818*# (0.0824)	0.19	1.51		
DM	-3.7551* (0.3750)	0.2569*# (0.0729)	0.16	1.45		
JY	-4.6656* (0.6101)	0.1079*# (0.1150)	0.002	1.62		
SF	-3.8327* (0.3365)	0.2314*# (0.0652)	0.054	1.26		
C. SUR estimation, excluding data from contracts with overlapping maturities (N = 64)						
BP	-3.7569* (0.5558)	0.2488*# (0.1063)	0.039	1.53		
DM	-3.8597* (0.4703)	0.2346*# (0.0907)	0.12	1.56		
JY	-4.4147* (0.6296)	0.1452 # (0.1178)	0.008	1.80		
SF	-3.7658* (0.4220)	0.2388*# (0.0815)	0.051	1.54		

Notes:

- (1) Standard errors are in parentheses.
- (2) Heteroskedasticity-consistent standard errors are in square brackets.
- (3) P-values are in curly brackets.
- (4) * denotes that the estimate is statistically different from zero at 5% level.
- (5) # denotes that the estimate is statistically different from one at 5% level.

Table 3: Properties of the OLS residuals and tests of normality

A. Residuals from the OLS regressions in levels: $rsd_{t+1} = \alpha + \beta isd_t + \varepsilon_{t+1}$				
	BP	DM	JY	SF
$u_2 \cdot 10^6$	4.926	4.612	4.900	5.821
$u_3 \cdot 10^9$	11.990	5.584	17.179	13.086
$u_4 \cdot 10^{12}$	122.584	68.851	150.779	133.399
skewness	1.097*	0.564*	1.584*	0.932*
kurtosis	5.053*	3.237	6.281*	3.936*
Jarque-Bera	31.967*	4.701	73.669*	15.403*

B. Residuals from the OLS regressions in logarithm: $\log(rs d_{t+1}) = \alpha + \beta \log(isd_t) + \varepsilon_{t+1}$				
	BP	DM	JY	SF
$u_2 \cdot 10^2$	10.333	10.202	12.778	10.766
$u_3 \cdot 10^3$	7.296	-9.470	12.449	5.618
$u_4 \cdot 10^4$	429.220	338.090	573.116	327.915
skewness	0.220*	-0.291*	0.273*	0.159*
kurtosis	4.020	3.248	3.510	2.829
Jarque-Bera	4.368	1.414	1.974	0.462

Notes:

- (1) u_j denotes the j^{th} central moment. Skewness = $u_3/\sqrt{u_2^3}$. Kurtosis = u_4/u_2^2 .
- (2) Jarque-Bera statistic = $T\{[S^2/6] + [(K-3)^2/24]\}$, where T is the sample size, S the skewness, and K the kurtosis. Under the null hypothesis of normality, the Jarque-Bera statistic has a Chi-square distribution with 2 degrees of freedom. The critical value at 5% level is 5.991. Under the null hypothesis of normality, $T[S^2/6]$ and $T[(K-3)^2/24]$, individually, has a Chi-square distribution with 1 degree of freedom. The critical values of S and K to reject the null at 5% level, are 0.05207 and 4.0414, respectively.
- (3) * denotes "different from that under normality at 5% level."

Table 4: Skewness and kurtosis of z as a function of γ
(with h being positive)

γ	$\exp(\gamma)$	Skewness	Kurtosis
0.0001	1.000100	0.030001	3.001600
0.01	1.010050	0.301759	3.162323
0.05	1.051271	0.690903	3.860583
0.1	1.105170	1.007008	4.855750
0.3	1.349858	1.981403	10.70567
0.5	1.648721	2.938798	21.50727
0.7	2.013752	4.041258	41.94258
0.04	1.040810	0.614294	3.678365
0.07	1.072508	0.827344	4.241307
0.1	1.105170	1.007008	4.855750
0.13	1.138828	1.169517	5.526779
0.4	1.491824	2.448824	15.26988
0.7	2.013752	4.041258	41.94258
1	2.718281	6.184877	113.9363

Notes:

- (1) Let w be a normal variate with mean u and variance γ . z is defined as follows:
 $z = h[\exp(w) - \exp(u + 0.5\gamma)]$.
- (2) If h is negative, then all the skewness would also be negative (with the same magnitude as in the table).

Table 5: Simulation for size of the t-test under non-normality
(sample size = 85, repetition size per simulation = 1000)

parameters		—implied properties of z—			t-test	point estimate
$h \cdot 10^3$ (1)	γ (2)	var(z) (3)	skewness (4)	kurtosis (5)	true size (6)	average β (7)
12.65	0.04	6.7972	0.614294	3.678365	0.066	0.9986
12.65	0.07	12.444	0.827344	4.241307	0.056	1.0028
12.65	0.1	18.600	1.007008	4.855750	0.057	1.0100
12.65	0.13	25.300	1.169517	5.526779	0.052	1.0035
12.65	0.4	117.41	2.448824	15.26988	0.058	1.0049
12.65	0.7	326.68	4.041258	41.94258	0.060	0.9640
12.65	1	747.43	6.184877	113.9363	0.058	1.0430
11.65	0.07	10.555	0.827344	4.241307	0.064	0.9963
12.65	0.07	12.444	0.827344	4.241307	0.058	0.9931
13.65	0.07	14.489	0.827344	4.241307	0.056	0.9913
14.65	0.07	16.690	0.827344	4.241307	0.063	0.9965
20	0.07	31.106	0.827344	4.241307	0.058	0.9714
50	0.07	194.41	0.827344	4.241307	0.052	1.0501
80	0.07	497.70	0.827344	4.241307	0.048	0.9341
110	0.07	940.96	0.827344	4.241307	0.049	0.8987
-12.65	0.04	6.7972	-0.614294	3.678365	0.061	0.9977
-12.65	0.07	12.444	-0.827344	4.241307	0.050	1.0045
-12.65	0.1	18.600	-1.007008	4.855750	0.068	1.0043
-12.65	0.13	25.300	-1.169517	5.526779	0.049	1.0121
-12.65	0.4	117.41	-2.448824	15.26988	0.070	0.9798
-12.65	0.7	326.68	-4.041258	41.94258	0.049	1.0417
-12.65	1	747.43	-6.184877	113.9363	0.048	1.0033

Note:

See the notes for Table 4 for the definitions of h , γ and z .

Table 6: Test of the weaker version of rationality
(Data from overlapping contracts excluded)
 $H_0: p_1 + p_2 = 1$. (isd is useless as predictor of rsd)

Currency	N	n_1	n_2	M	m_1	m_2	m_{1u}	$p_1 + p_2$	m_1 needed to reject H_0	
									1-tail test	2-tail test
BP	64	30	34	48	27	21	30	1.282	24.65*	25.06*
DM	64	32	32	44	28	16	32	1.375	24.15*	24.56*
JY	64	31	33	44	27	17	31	1.356	23.72*	24.188
SF	64	29	35	45	25	20	29	1.291	22.64*	23.07*
Total	256	122	134	181	107	74	122	1.325	90.87*	91.75*

Notes:

- (1) Point estimate of $(p_1 + p_2) = (m_1/n_1) + (n_2 - m_2)/n_2$.
- (2) $m_{1L} = \max[0, m - n_2]$, $m_{1u} = \min[n_1, M]$. $m_{1L} \leq m_1 \leq m_{1u}$.
- (3) * denotes that the null hypothesis that the market's forecast is useless is rejected at 5% level.

Table 7: Factors that affect ISD, 1983:2 - 1990:1

$$\text{isd}_t = \theta_0 + \theta_1(\text{rsd}_t - \text{isd}_{t-1}) + \theta_2 \text{isd}_{t-1} + \text{error}$$

SUR estimation, excluding data from overlapping contracts.

Currency	θ_0	θ_1	θ_2	adj.R ²	DW
BP	0.00245* (0.00041)	0.3035* (0.0525)	0.5019* (0.638)	0.52	1.84
DM	0.00312* (0.00046)	0.2946* (0.0530)	0.4264* (0.0720)	0.40	2.10
JY	0.00328* (0.00055)	0.2409* (0.0506)	0.2898* (0.0969)	0.19	2.21
SF	0.00406* (0.00056)	0.2266* (0.0558)	0.2647* (0.0827)	0.20	2.02

Notes:

- (1) The standard errors are in the parentheses.
- (2) * denotes that the estimate in question is statistically different from zero at 5% level.

Table 8: The Portmanteau Q test of the expectational errors, 1983:2-1990:1

$$H_0: \text{rsd}_{t+1} - \text{isd}_t \text{ is serially uncorrelated}$$

Lag P	Q-BP	Q-DM	Q-JY	Q-SF	Critical value to reject H_0	
					$\alpha = 0.05$	$\alpha = 0.10$
1	0.34	1.69	2.46	6.58*	3.841	2.71
3	5.05	1.85	6.97#	7.52#	7.815	6.25
5	6.93	3.40	8.85	7.68	11.07	9.24
7	8.78	8.78	14.8*	12.3#	14.07	12.02
9	16.0#	10.3	17.3*	12.6	18.31	15.99
11	17.6#	11.4	17.9#	14.2	19.68	17.28

Notes:

- (1) The standard errors are in the parentheses.
- (2) * denotes the null hypothesis of no serial correlation is rejected at 5% level.
- (3) # denotes the null hypothesis of no serial correlation is rejected at 10% level.

Table 9: Do Investors Respond Too Much to Lagged Volatility?
(1983:4 - 1990:2)

Dependent Variable	Independent Variable			\bar{R}^2	DW Durbin's h	S.E.E.
	c	log rsd(-1)	log rsd(-2)			
logBPLSD	-1.547* (0.433)	0.5135* (0.0819)	0.2071* (0.0825)	0.474	1.39	0.240
logBPRSD	-2.221* (0.581)	0.3656* (0.1098)	0.1945# (0.1106)	0.216	1.97 (-0.004)	0.322
logBPRSD-logBPLSD	-0.6747* (0.6673)	-0.1479 (0.1262)	-0.0125 (0.1270)	-0.0015	1.64	0.370
logDMISD	-2.6238* (0.3722)	0.3849* (0.0637)	0.1096# (0.0637)	0.381	1.52	0.192
logDMRSD	-3.351* (0.6488)	0.3321* (0.1111)	0.00602 (0.1110)	0.091	2.01 (0.344)	0.335
logDMRSD-logDMISD	-0.7269 (0.6319)	-0.0528 (0.1082)	-0.1035 (0.1082)	-0.0042	1.76	0.326
logJYISD	-3.856* (0.59009)	0.2470* (0.0751)	0.0301 (0.0755)	0.108	1.71	0.243
logJYRSD	-3.761* (0.7414)	0.1783# (0.1112)	0.1034 (0.1118)	0.026	2.05	0.359
logJYRSD-logJYISD	0.0946 (0.9041)	-0.0687 (0.1356)	0.0733 (0.1363)	-0.019	(-2.16) 1.67	0.438
logSFISD	-3.148* (0.459)	0.1793* (0.0833)	0.2165* (0.0831)	0.171	1.57	0.236
logSFRSD	-2.882* (0.6123)	0.3937* (0.0319)	0.0319 (0.1110)	0.145	2.01	0.315
logSFRSD-logSFISD	0.2665 (0.7195)	0.2143# (0.1306)	-1.1845 (0.1304)	0.017	(0.031) 1.74	0.370

Figure 1: Option-implied standard deviations vs. realized ones

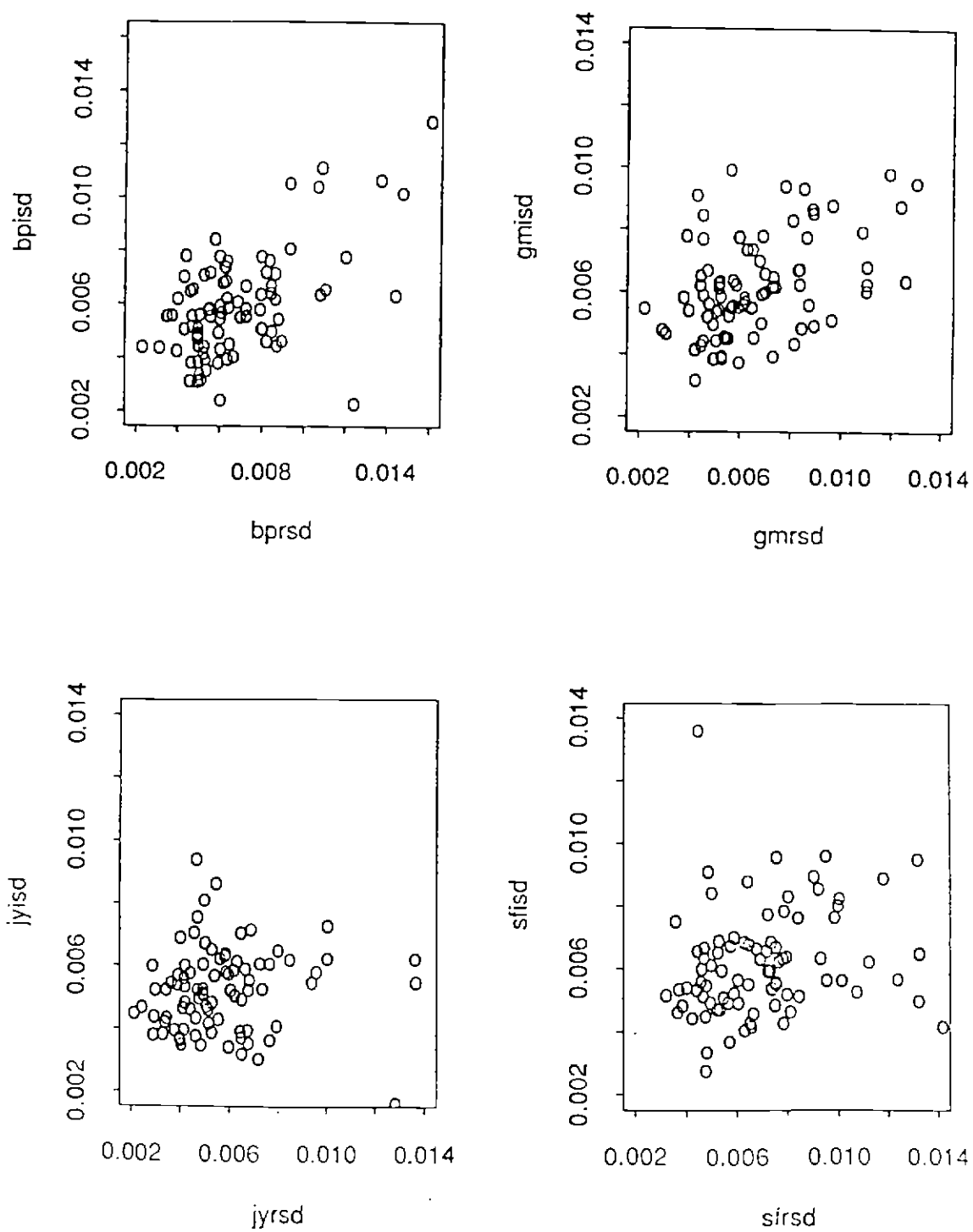
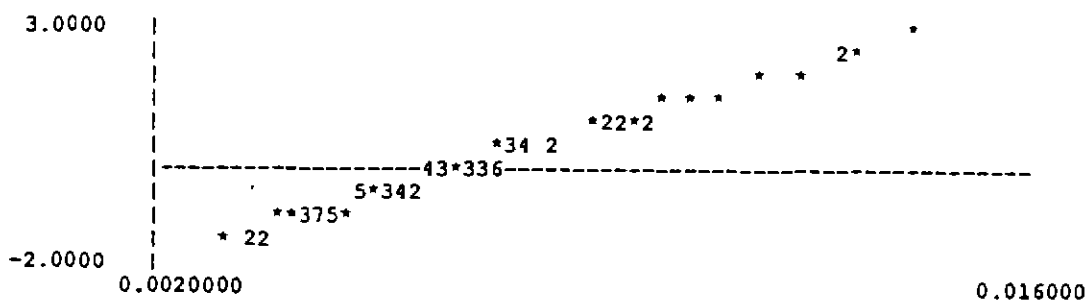


Figure 2: ACE results: British pound

SD for each independent variable:
2.50122e-01

X variable: sfrsd
Y variable: tsfr



X variable: sfisd1
Y variable: tsfi

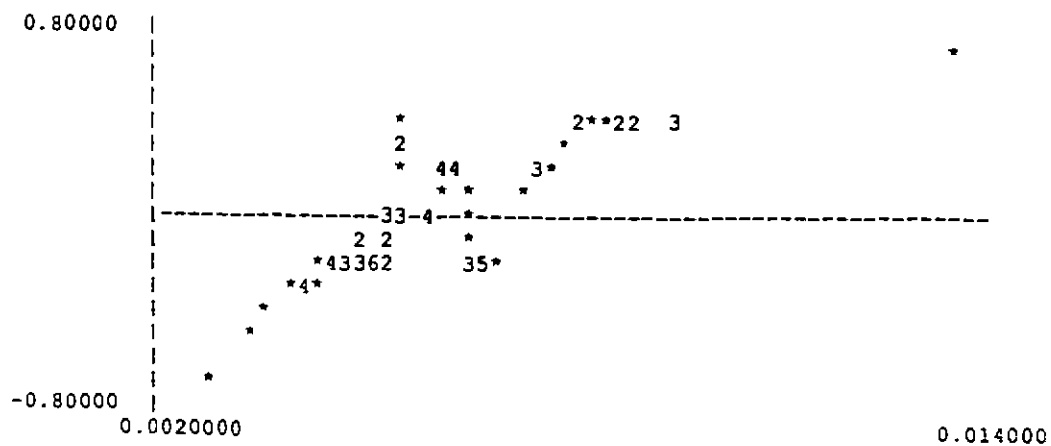
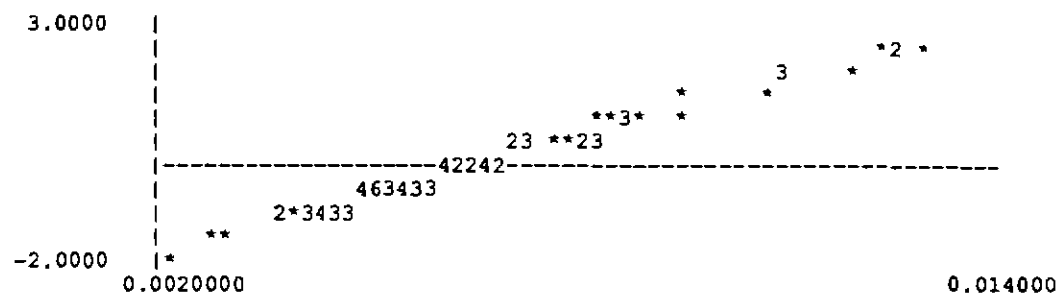


Figure 3: ACE results: German mark

SD for each independent variable:
4.25844e-01

X variable: gmrsl
Y variable: tgm



X variable: gmrsl
Y variable: tgm

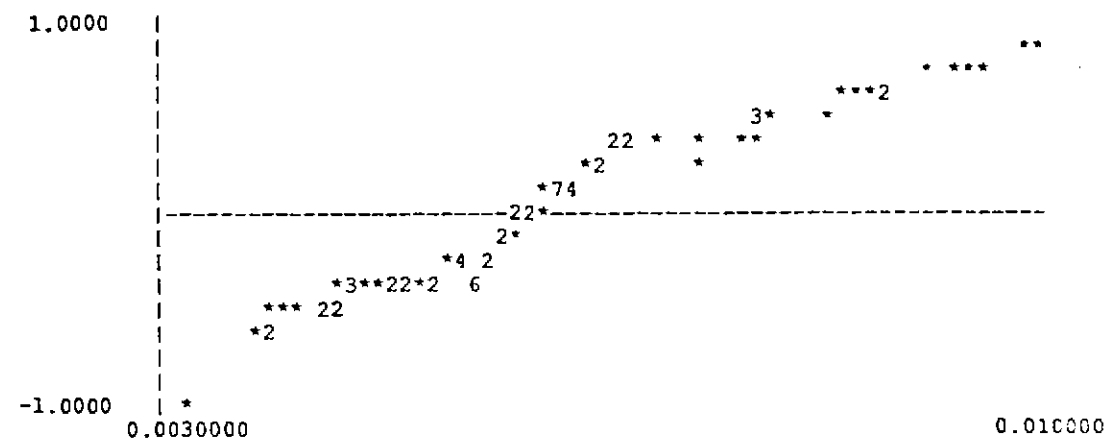
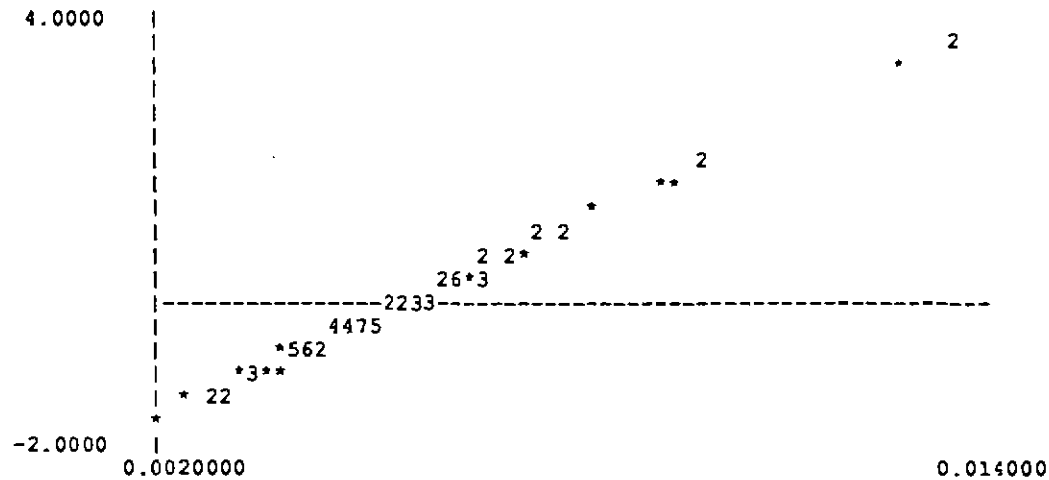


Figure 4: ACE results: Japanese yen

SD for each independent variable:
4.58392e-01

X variable: jyrds
Y variable: tjyr



X variable: jyisd1
Y variable: tjyi

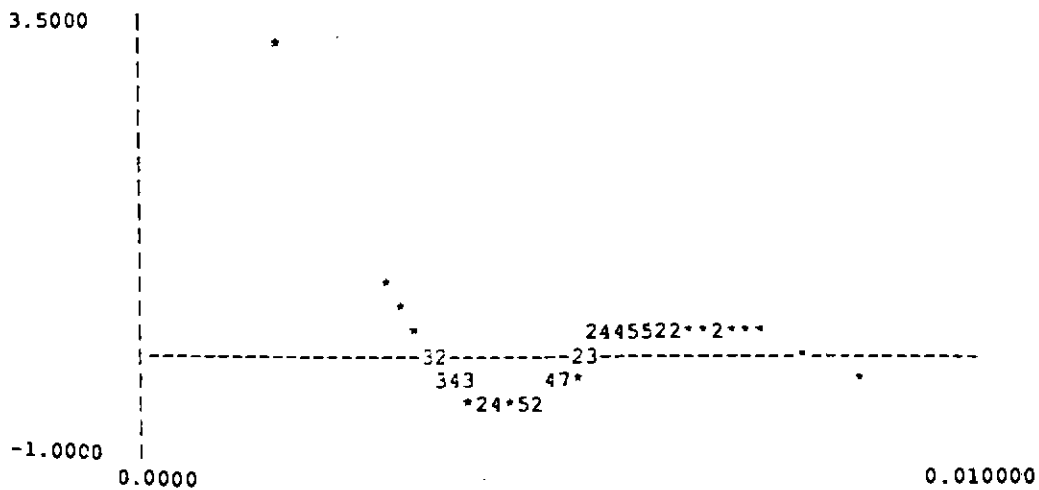
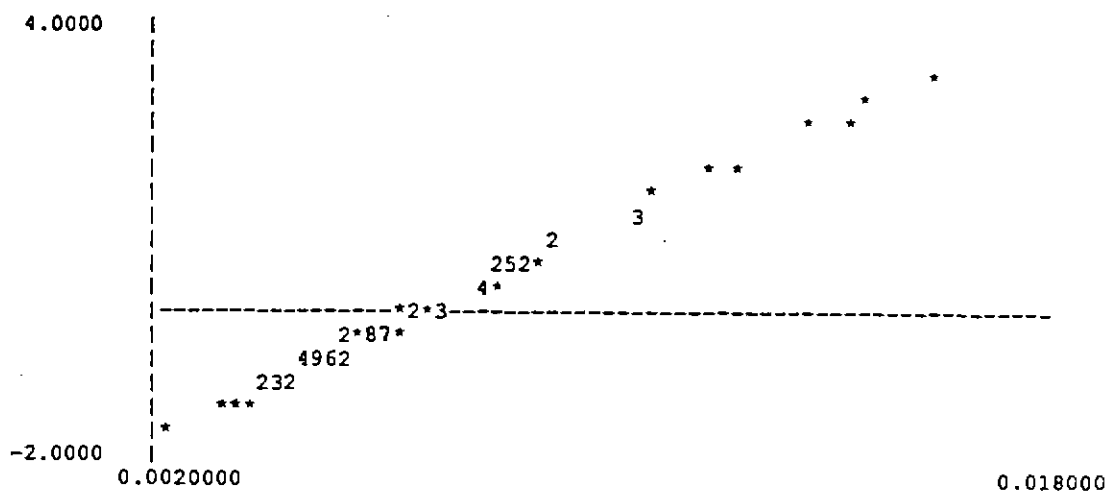


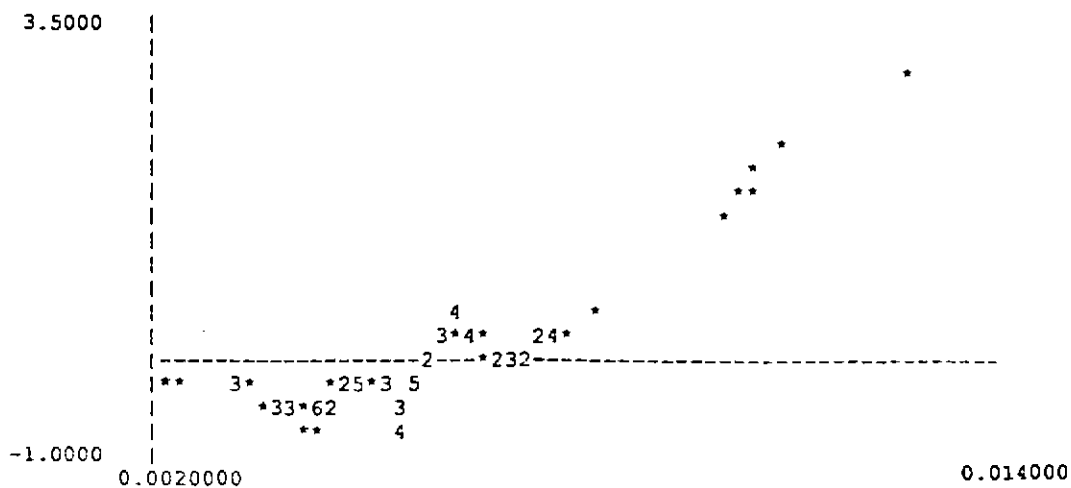
Figure 5: ACE results: Swiss franc

SD for each independent variable:
6.74165e-01

X variable: bprsd
Y variable: tbpr



X variable: bpsd1
Y variable: tbpi



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