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EARNINGS, DIVIDEND POLICY, AND PRESENT VALUE RELATIONS: BUILDING BLOCKS OF DIVIDEND POLICY INVARIANT CASH FLOWS

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ABSTRACT

In a Modigliani-Miller world, price equals the risk-adjusted present value of future dividends and dividend policy is irrelevant for asset pricing. This paper searches for cash flows with two characteristics: asset prices can be calculated from their present values and they are invariant with respect to dividend policy. Residual income measures with these features are identified under two assumptions: dividend policy does not alter risk premiums and income earned from investments associated with dividend policy includes capital gains and losses. These results hold for otherwise arbitrary risk premiums in the general no-arbitrage approach to the valuation of uncertain income streams.

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There is a peculiar schizophrenia underlying the present value relation. On the one hand, the value of equity is the risk-adjusted present value of expected future dividends. On the other hand, value additivity implies that dividend policy is irrelevant. Hence, the conventional practice of assuming a particular dividend policy and computing its present value is fraught with hazard. Under the null hypothesis of efficiency, there is substantial reason to believe that any assumed dividend policy is misspecified since managers have no obvious incentive to adopt or maintain a consistent dividend policy.[1]

For many years, there was comparatively little interest in academia in the computation of present values. The efficient markets hypothesis, which has dominated research over the last quarter century, presumes that asset prices equal their underlying intrinsic values. This focused attention on the behavior of asset prices and particular risk-adjustment procedures, retarding the development of improved procedures for calculating intrinsic values.

It is fair to say that most academic research has been concerned with risk-adjustment procedures for holding period returns. I want to focus instead on procedures for measuring cash flows more fundamental than dividends in a particular sense—like dividends, prices are their risk-adjusted present values but, unlike dividends, they are invariant with respect to changes in dividend policy. Accomplishment of this task would facilitate the calculation of measures of intrinsic value that are not price dependent, simplifying the testing of the efficient markets hypothesis or the exploitation of its predictions for valuation purposes. It does so by

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[1]There is now an enormous literature that examines present value calculations for evidence of excess volatility under particular assumptions about dividend policy and expected returns. See, for example, Campbell and Shiller(1987,1988a,b), Grossman and Shiller(1981), Kleidon(1986), Leroy and Porter(1981), Mankiw, Romer, and Shapiro(1985), Marsh and Merton(1986,1987), Mattey and Meese(1986), Shiller(1979,1981,1984), and West(1987,1988).

providing an explicit link between accounting measures of capital (i.e., book value) and rates of return (i.e., return on equity) and their economic counterparts.

There are three building blocks of dividend policy invariant cash flows which comprise the body of the paper. The next section sets out the general no-arbitrage valuation environment used throughout the paper and the class of accounting relations that can be accommodated by the analysis. The main result is the translation of the no-arbitrage pricing relation for future dividend payments into a corresponding procedure for valuing arbitrary stocks and flows. The second section analyzes the role of dividend policy in the dividend policy invariance of residual income measures based on economic income. It shows that one must generally define financial policy to be the net issuance of contingent claims by firms plus the synthetic securities implicitly created by particular dividend payout strategies. The third section provides a corresponding analysis of accounting-based residual income measures, emphasizing the role of capital gains and losses in their dividend policy invariance. A brief appendix contains the proof of the first proposition.

1. The Arithmetic of Stocks, Flows, and Present Value Relations

The first building block is the specification of both the valuation environment and the class of accounting relations to be considered. Generality is clearly desirable in both dimensions; I do not wish to restrict attention to particular equilibrium valuation or even efficient markets models or to tie the analysis to a particular definition of accrued or realized income. Accordingly, I will consider present value relations that require only no-arbitrage in frictionless markets and any accounting numbers that obey simple stock/flow relations.

The valuation environment assumed throughout the paper is the general no-arbitrage approach to the valuation of uncertain income streams. This is restrictive in some dimensions—it assumes the absence of frictions like taxes, transactions costs, and constraints on short sales as well as value-relevant asymmetries in the information available to investors.[2] Nevertheless, it satisfies the requirements listed above since it does not require market efficiency, particular risk/return models, or complete markets. Following Rubinstein(1976) and Ross(1978), the general no-arbitrage pricing relation is:

$$P_t = \sum_{j=1}^{\infty} \frac{E[d_{t+j} y_{t,j}/I_t]}{(1+\rho_{t,j})^j};\ y_{t,j} > 0;\ E[y_{t,j}/I_t]=1\ \forall\ t,j > 0 \qquad (1)$$

where P_t is the value of a claim to the income stream d_{t+j}, d_{t+j} represents income received at time t+j, $\rho_{t,j}$ is the discount factor at time t for a riskless j period pure discount bond,[3] $y_{t,j}$ is the normalized pricing kernel (generally equal to investors' intertemporal marginal utility functionals for time t+j multiplied by the price of a riskless j period pure discount bond),[4] and the expectation operator $E[\bullet/I_t]$ reflects probability beliefs held conditional on information available at time t. There is no requirement that expectations are rational—probability beliefs need only satisfy the general mathematical properties of an expectation.

The valuation equation (1) is compatible with two extreme views of the

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[2]More precisely, the latter statement requires investors to perceive a deterministic relation between states of the world and cash flows at time t. This generally permits moral hazard and adverse selection problems (but not unresolved information asymmetries) to affect cash flows. Frictions generally make discount factors and pricing kernels investor-specific although proportional transactions costs can usually be accommodated. General conditions for pricing relations like (1) to hold in the presence of frictions are not known.

[3]If there is no nominally riskless asset, the no-arbitrage present value relation is:

$$P_t = \sum_{j=1}^{\infty} E[d_{t+j} Y_{t,j}/I_t];\ Y_{t,j} > 0$$

where $E[d_{t+j} Y_{t,j}/I_t]$ is the discount factor for a j-period zero beta asset.

[4]The pricing kernel is not necessarily unique in incomplete markets. That is, constraints are placed on Arrow-Debreu prices in incomplete markets but the assumption of no-arbitrage alone is generally insufficient to uniquely identify them in this case.

nature of financial markets. This is an efficient markets model when investors have rational expectations given appropriate specification of their information sets since market price equals the objective intrinsic or fundamental value. However, it is also consistent with a wide variety of inefficient markets models. For example, investor expectations need not be rational so that price need not equal objective intrinsic value and some investors could even irrationally ignore the presence of arbitrage opportunities from their perspective so that risk premiums (i.e., the pricing kernels $y_{t,j}$) are not those that would occur if all investors were rational.[5]

Note also that the central tenets of modern financial theory are embodied in this general present value relation. The role of modern portfolio theory is implicit in the risk adjustment used to generate the certainty equivalents of the cash flows d_{t+j}. Modigliani-Miller propositions follow from the notion of value additivity embedded in (1)—the value of a claim to the income stream d_{t+j} is the sum of the values of claims to any arbitrary decomposition of this stream. This observation on the linearity of expectations (or, equivalently, on the linearity of price systems) implies that the value of the firm is independent of its capital structure and the value of an equity claim is independent of the dividend policy of the firm.[6] Modigliani-Miller reasoning will play a critical role in the analysis that follows.

The conversion of the valuation relation (1) into an operational device for calculating intrinsic value requires knowledge of all of its factors. Much of modern financial research has been devoted to the identification of the appropriate risk

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[5]If horizons were finite, equation (1) permits an additional source of irrationality—the possibility of the terminal price (and, hence, the current price) not reflecting intrinsic value due to irrationality of future investors. Obviously, the no-arbitrage pricing relation is not compatible with all sorts of market inefficiency.

[6]This means that beliefs are assumed to be invariant with respect to financial and dividend policy. This further restricts the kinds of market inefficiency compatible with the analysis.

adjustment procedures (i.e., identification of $y_{t,j}$) when expectations are rational. I want to focus instead on the cash flows d_{t+j}. An implication of the general present value relation is that security values are independent of all zero net present value alterations of the stream of cash flows. When the security under consideration is a common stock, this property is usually referred to as the Modigliani-Miller dividend policy irrelevance proposition.

Dividend policy irrelevance greatly complicates the calculation of present values. Legal, institutional, and moral hazard considerations suggest that dividend payouts will not generally follow the convenient stochastic processes typically assumed by econometricians. This would occur, for example, in the plausible case where managers were committed to a stable policy of dividend payments with two features—retained income must be positive when paid-in capital exceeds book value and retained earnings are bounded below by zero because of an eventual 100% payout policy. Moreover, managers might find it convenient to consider occasional alterations of dividend policy. Conventional assumptions like the stationarity of dividend growth rates or detrended dividend payments leads to incorrect present value calculations in these circumstances. Put differently, present value calculations are not invariant with respect to dividend policy assumptions even though prices are invariant with respect to dividend policy.

Therefore, it is desirable to measure cash flows more fundamental than dividends. In particular, I will seek to identify cash flows with two characteristics: (1) like dividends, they can be used to compute asset values and (2) their conditional expectations are invariant with respect to dividend policy. Mathematically, a random variable z_{t+j} has these characteristics when:

$$P_t = Z_t + \sum_{j=1}^{\infty} \frac{E[z_{t+j} y_{t,j} / I_t]}{(1+\rho_{t,j})^j} \tag{2}$$

for some variable Z_t known at time t where Z_t and $E[z_{t+j}y_{t,j}/I_t]$ do not vary with possible (and feasible) future dividend policies. I will call cash flows like z_{t+j} dividend policy invariant.

This definition of dividend policy invariance has obvious intuitive appeal. It permits the application of conventional time series methods to random variables like $z_t y_{t,j}$ to determine conditional expectations and to facilitate the calculation of present values. Otherwise, present value calculations require knowledge of how the conditional expectations of $d_{t+j}y_{t,j}$ vary with dividend policy as well as the nature of the firm's actual (or potential) dividend policy unless the analyst can measure and forecast economic income.[7]

I will begin the search for such cash flows by delineating the general class of stock/flow relations that can be accommodated in the analysis and restating the present value relation in terms of these stocks and flows. In particular, I will confine attention to accumulation equations of the form:

$$x_t - d_t = X_t - X_{t-1} \tag{3}$$

The result presented below is for arbitrary choices of the flow variables x_t and the stock variables X_t, so long as X_t does not grow too quickly.

Stocks and flows like these arise from the capital maintenance notions of income that underlie modern accounting in which assets are often carried at historical cost or book value. For example, if x_t is earnings or net income calculated on an accrual basis and there are no new stock issues, then X_t is the book value of owners' equity. Similarly, if x_t is the change in cash (i.e., net income calculated on a realized basis), then X_t is the book value of all noncash assets.

The general present value relation may be rewritten in terms of these stock

[7]Economic income is dividend policy invariant by definition for reasons discussed later.

and flows. This translation is recorded as Proposition 1 and may be found in Ohlson(1989a) for the case of risk neutrality and a constant riskless interest rate.

Proposition 1: Let security values satisfy the general no-arbitrage pricing relation (1) and let x_t and X_t be any numbers satisfying the accumulation equation:

$$x_{t+j} - d_{t+j} = X_{t+j} - X_{t+j-1}; \qquad X_t \text{ given}$$

such that $\lim_{T\to\infty} \frac{E[X_{t+T}y_{t,T}/I_t]}{(1+\rho_{t,T})^T} = 0$. Then:

$$P_t = X_t + \sum_{j=1}^{\infty} \frac{E[x_{t+j}-f_{t,j}X_{t+j-1}/I_t]}{(1+\rho_{t,j})^j} + \sum_{j=1}^{\infty} \frac{\text{cov}[x_{t+j},y_{t,j}/I_t]}{(1+\rho_{t,j})^j}$$

$$+ \sum_{j=1}^{\infty} \frac{\text{cov}[X_{t+j-1},y_{t,j}-(1+f_{t,j})y_{t,j-1}/I_t]}{(1+\rho_{t,j})^j} \tag{4}$$

where $f_{t,j}$ is the forward rate for period t+j, i.e., $1+f_{t,j} = \frac{(1+\rho_{t,j})^j}{(1+\rho_{t,j-1})^{j-1}}$.

Proof: The proof is contained in the Appendix.

The three terms in the summation merit special comment. The latter two terms simply shows that stock/flow relations like (3) yield complicated expressions for risk premiums although they are obviously not changed by this translation.[8] The first term is called residual income when x_t is earnings, X_t is the book value of owners' equity, and the forward rate equals the cost-of-capital (typically assumed to be constant). Residual income has a long history in managerial compensation and performance measurement and plays an important role in the analysis that follows.

It is also worth emphasizing what is not said by Proposition 1. Proposition 1 is a statement about arbitrary stocks and flows. As such, it cannot point to particular

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[8]That is, $\text{cov}[d_{t+j},y_{t,j}/I_t] = \text{cov}[x_{t+j},y_{t,j}/I_t] - \text{cov}[X_{t+j},y_{t,j}/I_t] + \text{cov}[X_{t+j-1},y_{t,j}/I_t]$.

stock/flow relations as being especially useful for valuation purposes. Economic reasoning is needed to identify dividend policy invariant cash flows and the means by which they can be estimated from accounting measurements.

2. Dividend Policy, Risk Premiums, and the Dividend Policy Invariance of Residual Economic Income

The next building block of cash flows that are invariant with respect to changes in dividend policy requires the definition of dividend policy. Following Modigliani and Miller(1958) and Miller and Modigliani(1961), it is useful to distinguish three kinds of policies undertaken by the firm: investment policy, financial policy, and dividend policy. Investment policy governs the firm's investment in physical (and, for some companies, financial) assets. Financial policy refers to the issuance of contingent claims to finance the desired level of investment that divide the stream of income generated by investment policy. Dividend policy reflects alterations of the stream of payments to equity claimants.

As is customary in Modigliani-Miller analyses, we want to isolate the effects of dividend policy on equity prices. Accordingly, I will assume that both investment and financial policy are exogenous and will focus on the effects of dividend policy. This means that I will take income from investment and the payments to other claimants as given. In these circumstances, dividend policy involves zero net present value alterations of the dividend stream.

It is difficult to distinguish investment, financial, and dividend policy even if there were no accounting measurement problems. One simply cannot tell from a balance sheet whether retained earnings represent investment, financial, or dividend policy. Earnings retention might reflect financial policy since internal finance is one way to finance future investment. Alternatively, it might represent dividend policy as the firm translates foregone current dividend payments into future payouts. Similarly, one cannot disentangle these possibilities by looking at changes in firm assets—the firm will invest retained earnings in assets in both cases. Finally, one cannot distinguish these effects at the time of physical investment so long as the firm is retaining earnings and engaging in both financial and dividend policy related transactions at that time. Only specialized knowledge about the firm can tell an analyst whether particular financial assets are or were earmarked for future investments or dividend payments.

These ambiguities arise because of our valuation methods: we replace the intricate operations of a business firm with a disembodied replicating portfolio of contingent claims. Changes in dividend policy alter the menu of contingent claims that replicate the risk and return characteristics of the dividend stream. Financial policy involves the actual menu of contingent claims used to finance investment. Hence, dividend and financial policy changes are indistinguishable without more detailed knowledge of the firm's inner workings. This knowledge is absent when we model the firm as a black box generating cash flows.

Consequently, it is not clear how to identify the cash flows and net assets associated with these conceptually distinct elements of corporate policy even in the absence of accounting measurement problems. This is readily seen by considering the relevant flows assuming no measurement problems. Accordingly, let π_t denote the economic profits from investment policy net of the changes in the value of the firm's physical assets (i.e., economic depreciation). Similarly, let ψ_t denote the net income (or expenditure) on financial policy inclusive of both earnings retention for future investments and capital gains and losses on the financial liabilities and assets of the firm. The inclusion of these capital gains and losses is not essential and will be discussed further in the next section.

How does dividend policy fit into this picture? The simple dividend policy of paying out net economic profits (i.e., $\pi_t+\psi_t$, a capital levy on the equity claimants when net economic profits are negative) creates no additional cash flows or assets to consider. However, any dividend policy in which payouts differ from net economic profits generates additional financial assets or liabilities with their attendant impact on future cash flows. Accordingly, let ζ_t represent the net profit or loss from the investment of prior differences between net economic profits and dividend payments inclusive of capital gains and losses. The inclusion of capital gains and losses in ζ_t plays a substantive role in the analysis, as will be seen in the next section. For later reference, let Z_t denote the market value of the portion of the firm's assets associated with dividend policy.

In these circumstances, let x_t be total economic earnings inclusive of capital gains and losses which has three components:

$$x_t = \pi_t + \psi_t + \zeta_t \tag{5}$$

This decomposition implies that $\pi_t+\psi_t$ is unaffected by dividend policy and that ζ_t is the dividend policy dependent component. The invariance of net economic profits to dividend policy plays a central role in the analysis.

Now consider the role of dividend policy in this setting. Decompose dividends into net economic profits (i.e., dividend payouts in the absence of dividend policy) and a residual:

$$d_t = \pi_t + \psi_t + \upsilon_t \tag{6}$$

Since dividend policy involves only zero net present value alterations of the dividend stream, the market value of the assets associated with dividend policy is also the risk-adjusted present value of the residual υ_t:

$$Z_t = \sum_{j=1}^{\infty} \frac{E\left[\upsilon_{t+j} y_{t,j} / I_t\right]}{(1+\rho_{t,j})^j} \tag{7}$$

which implies that equity prices satisfy:

$$P_t = Z_t + \sum_{j=1}^{\infty} \frac{E\left[\left(\pi_{t+j}+\psi_{t+j}\right)y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j} \tag{8}$$

Alternatively, the stock/flow relation (3) may be rewritten as:[9]

$$\zeta_t - \upsilon_t = X_t - X_{t-1} \tag{9}$$

by taking the difference of (5) and (6). Application of Proposition 1 to (9) yields:

$$\sum_{j=1}^{\infty} \frac{E\left[\upsilon_{t+j}y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j} = X_t + \sum_{j=1}^{\infty} \frac{E\left[\zeta_{t+j}-f_{t,j}X_{t+j-1}/I_t\right]}{(1+\rho_{t,j})^j} + \sum_{j=1}^{\infty} \frac{\mathrm{cov}\left[\zeta_{t+j}, y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j}$$
$$+ \sum_{j=1}^{\infty} \frac{\mathrm{cov}\left[X_{t+j-1}, y_{t,j}-(1+f_{t,j})y_{t,j-1}/I_t\right]}{(1+\rho_{t,j})^j} \tag{10}$$

The weakness of the present value constraint:

$$Z_t = X_t + \sum_{j=1}^{\infty} \frac{E\left[\zeta_{t+j}-f_{t,j}X_{t+j-1}/I_t\right]}{(1+\rho_{t,j})^j} + \sum_{j=1}^{\infty} \frac{\mathrm{cov}\left[\zeta_{t+j}, y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j}$$
$$+ \sum_{j=1}^{\infty} \frac{\mathrm{cov}\left[X_{t+j-1}, y_{t,j}-(1+f_{t,j})y_{t,j-1}/I_t\right]}{(1+\rho_{t,j})^j} \tag{11}$$

explains why it is difficult to construct dividend policy invariant flows. The conditional expectations of $\zeta_{t+j}-f_{t,j}X_{t+j-1}$ and the conditional covariances involving ζ_{t+j}, X_{t+j-1}, $y_{t,j}$, and $y_{t,j-1}$ for each j are generally nonzero and will typically vary with dividend policy even though their present values are equal to Z_t. Clearly, the individual components of the risk premium play an important role in dividend

[9]Note also that X_t represents the market value of the net assets of stockholders since economic income includes all unrealized capital gains and losses.

policy invariance. Cash flows will not possess the invariance property so long as intertemporal shifts in the risk premium components cause expected risk-adjusted cash flows to systematically change with dividend policy.

This issue arises because the effects of zero net present value changes in the dividend stream can be like those of investment and financial policy—they can alter the equity risk premium (i.e., the present value of the covariance terms in the pricing relations (1) and (4)). As noted earlier, firms could retain additional earnings in the future, invest the proceeds in contingent claims, and pay out the income from these investments as dividends later on. An obvious example is the future repurchase of some of the firm's outstanding contingent claims such as its debt. Such policies will not alter current equity prices but will typically change the division into risk premiums and expected present values of dividends.[10] They will also generally change the risk premiums associated with each period t+j (i.e., $\text{cov}[d_{t+j}, y_{t,j}/I_t]$).

Accordingly, the first major assumption or definition made here is that all policy changes that alter equity risk premiums are changes in either investment or financial policy. This is consistent with the exogeneity of income from investment and the payments to other claimants—in the example above, the purchase of additional contingent claims alters the net flow of payments to other claimants. Hence, this definition restricts changes in dividend policy to be those zero net present value alterations of the dividend stream that leave equity risk premiums unaffected.[11] In terms of the preceding analysis, this restriction implies that:

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[10]These statements apply to current earnings retention for *cum*-dividend current prices.

[11]This definition places unusual burdens on the analyst in practical applications. The investigator must identify investment and financial policy and thus the exogenous income stream. If the firm changes dividend policy in a way that alters the risk premium, the analyst must decompose this change into investment and

$$0 = \sum_{j=1}^{\infty} \frac{\operatorname{cov}\left[\upsilon_{t+j}, y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j}$$

$$= \sum_{j=1}^{\infty} \frac{\operatorname{cov}\left[\zeta_{t+j}, y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j} + \sum_{j=1}^{\infty} \frac{\operatorname{cov}\left[X_{t+j-1}, y_{t,j}-(1+f_{t,j})y_{t,j-1}/I_t\right]}{(1+\rho_{t,j})^j} \tag{12}$$

and that $\operatorname{cov}[\upsilon_{t+j}, y_{t,j}/I_t]$ and $\operatorname{cov}[\zeta_{t+j}, y_{t,j}/I_t]+\operatorname{cov}[X_{t+j-1}, y_{t,j}-(1+f_{t,j})y_{t,j-1}/I_t]$ are not changed by dividend policy.[12]

The need for this assumption arises because changes in financial and dividend policy are generally indistinguishable when the firm is viewed as a black box generating cash flows. Changes in dividend policy generally alter the portfolio of contingent claims that replicate the risk and return characteristics of the dividend stream. Hence, they also constitute a synthetic security that replicates this portfolio. In this sense, any change in dividend policy that alters equity risk premiums can be viewed as changing the net financial assets of the firm inclusive of synthetic securities. The difficulty of differentiating the actual from synthetic changes in financial policy associated with dividend policy changes is eliminated by confining dividend policy to zero net present value, zero risk premium alterations of the dividend stream.[13] This is also a definition of financial policy.

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financial policy components. The present analysis then applies to the residual zero net present value, zero risk premium dividend stream alteration.

[12]This confines the investments associated with dividend policy to zero beta assets like discount and fixed coupon bonds held to maturity and rolled over into zero beta assets. It is related to the Modigliani-Miller equivalent risk class formalism.

[13]Since:

$$P_t = \sum_{j=1}^{\infty} \frac{E\left[d_{t+j}q_{t,j}/I_t\right]}{(1+\rho_{t,j})^j}; \qquad q_{t,j} = 1+\operatorname{cov}\left[\frac{d_{t+j}}{E(d_{t+j}/I_t)}, y_{t,j}/I_t\right]$$

Taken together, the characterization of prices in (8) and of dividend policy in (11) and (12) implies that the residual income variable $x_{t+j}-f_{t,j}X_{t+j-1}$ is dividend policy invariant, a fact recorded here as Proposition 2.

Proposition 2: Suppose that security prices satisfy the general no-arbitrage pricing relation (1). Let x_t be total economic earnings as defined in (5) satisfying the accumulation equation:

$$x_{t+j} - d_{t+j} = X_{t+j} - X_{t+j-1}; \qquad X_t \text{ given}$$

such that $\lim_{T\to\infty} \frac{E[X_{t+T}y_{t,T}/I_t]}{(1+\rho_{t,T})^T} = 0$. Suppose that equity risk premiums are unaffected by dividend policy (i.e., $Cov[d_{t+j}, y_{t,j}/I_t] = Cov[(\pi_{t+j}+\psi_{t+j}, y_{t,j}/I_t])$. Then the residual income variable $x_{t+j}-f_{t,j}X_{t+j-1}$ is dividend policy invariant in that:

$$E[(x_{t+j}-f_{t,j}X_{t+j-1})y_{t,j}/I_t] = E[(\pi_{t+j}+\psi_{t+j})y_{t,j}/I_t]$$

for all choices of dividend policy and prices satisfy:

$$P_t = X_t + \sum_{j=1}^{\infty} \frac{E\left[(x_{t+j}-f_{t,j}X_{t+j-1})y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j} = Z_t + \sum_{j=1}^{\infty} \frac{E\left[\left(\pi_{t+j}+\psi_{t+j}\right)y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j} \qquad (13)$$

Proof: The results follows directly from the assumptions about dividend policy and the present value relations (8) and (11).

These are the ingredients required to synthesize cash flows that are unaffected by alterations of dividend policy in the absence of accounting measurement problems. Proposition 1 indicates how the general present value relation (1) can be rewritten for general stock/flow relations of the form (3). Confining the definition

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it might appear natural to define dividend policy as a set of restrictions on the covariance terms $q_{t,j}$. Unfortunately, this does not generally lead to dividend policy invariant income measures because the stock/flow relation (3) is additive while $E\left[(x_{t+j}-X_{t+j}+X_{t+j-1})q_{t,j}/I_t\right]$ is multiplicative. It does work in the special case of deterministic discount factors as noted in the next section.

of dividend policy to those zero net present value alterations of the dividend stream that leave equity risk premiums unchanged prevents intertemporal shifts in the risk premium components from causing expected risk-adjusted cash flows to systematically change with dividend policy. This condition is restrictive but essential—without it, it is necessary to risk adjust the flows x_t and the stocks X_t, making their conditional expectations dependent on dividend policy.

Unfortunately, the analysis in this section involved economic income variables inclusive of economic asset depreciation. In practice, it is often difficult to assess changes in asset values in a verifiable and replicable way. Consequently, the present analysis is not terribly useful unless it can be applied to accounting income or cash flow measures. This is the focus of the next section.

3. Dividend Policy Invariance and Accounting Income Measures

The final building block of dividend policy invariant cash flows involves the translation of the preceding analysis from economic to accounting earnings. Economic profits are not accounting profits, generally differing in their treatment of asset depreciation. This section identifies the circumstances in which the dividend policy invariance of accounting measures of residual income arises.

Accordingly, consider two broad accounting income measures: earnings (i.e., accrued income inclusive of accounting depreciation) and the net change in cash (i.e., the sum of the cash flows from operating, investment, and financing activities).[14] The question at hand is whether the inclusion of capital gains and

[14]Other income concepts could be handled as well so long as their associated stocks are treated in an internally consistent manner. For example, earnings could be replaced by earnings available for common (i.e., earnings adjusted for nonrecurring items, unjustified income or contingency recognition, noncomparable inventory and depreciation measures, consolidation of subsidiaries, affiliates, and unrecorded assets and liabilities, and provision for income taxes). See Cottle, Murray, and Block(1988), Chapters 10 through 17, for a detailed guide to the construction of earnings available for common (i.e., true operating earnings).

losses in net economic profits $\pi_{t+j}+\psi_{t+j}$ and income associated with dividend policy ζ_t materially affects the analysis.

In terms of the earlier analysis, accounting income measures take the form:

$$x_t = \pi_t + \psi_t + \zeta_t + \varepsilon_{\pi t} + \varepsilon_{\psi t} + \varepsilon_{\zeta t} \tag{14}$$

where the 'measurement errors' $\varepsilon_{\pi t}$, $\varepsilon_{\psi t}$, and $\varepsilon_{\zeta t}$ reflect the difference between the components of the economic and accounting income measures. Cash flow income measures differ in that they make no allowance for net asset depreciation while earnings are generally adjusted for measured but not economic depreciation. Accordingly, investment is calculated net of economic depreciation for economic income, net of accounting depreciation for earnings, and gross of depreciation on a cash flow basis. Not surprisingly, these 'measurement error' components have different effects on the construction of dividend policy invariant cash flows.

There are two present value relations that help identify the issues associated with the distinction between accounting and economic income measures. The first arises from the application of the stock/flow relation (3) to income as defined in (12), yielding a pricing relation of the form (4). The second is the pricing relation (8) for economic income and the market value of assets associated with dividend policy. The difference of these two pricing relations implies:

$$0 = X_t - Z_t + \sum_{j=1}^{\infty} \frac{E\left[\zeta_{t+j}+\varepsilon_{\pi t+j}+\varepsilon_{\psi t+j}+\varepsilon_{\zeta t+j}-f_{t,j}X_{t+j-1}/I_t\right]}{(1+\rho_{t,j})^j}$$

$$+\sum_{j=1}^{\infty} \frac{\operatorname{cov}\left[\zeta_{t+j}+\varepsilon_{\pi t+j}+\varepsilon_{\psi t+j}+\varepsilon_{\zeta t+j}, y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j} + \sum_{j=1}^{\infty} \frac{\operatorname{cov}\left[X_{t+j-1}, y_{t,j}-(1+f_{t,j})y_{t,j-1}/I_t\right]}{(1+\rho_{t,j})^j} \tag{15}$$

which implies that:

$$0 = X_t - Z_t + \sum_{j=1}^{\infty} \frac{E\left[\left(\varepsilon_{\pi t+j}+\varepsilon_{\psi t+j}\right)y_{t+j}/I_t\right]+E\left[\left(X_{t+j-1}-Z_{t+j-1}\right)\left(y_{t,j}-(1+f_{t,j})y_{t,j-1}\right)/I_t\right]}{(1+\rho_{t,j})^j}$$

$$+\sum_{j=1}^{\infty} \frac{E\left[\zeta_{t+j}+\varepsilon_{\zeta t+j}-f_{t,j}Z_{t+j-1}/I_t\right]}{(1+\rho_{t,j})^j}+\sum_{j=1}^{\infty} \frac{\operatorname{cov}\left[\zeta_{t+j}+\varepsilon_{\zeta t+j},y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j} \tag{16}$$

$$+\sum_{j=1}^{\infty} \frac{\operatorname{cov}\left[Z_{t+j-1},y_{t,j}-(1+f_{t,j})y_{t,j-1}/I_t\right]}{(1+\rho_{t,j})^j}$$

where Z_t is the market value of assets associated with dividend policy and X_t is the stock variable associated with the income variable (12) (which differs from the corresponding stock for economic income defined in Section 2).

Consider first the 'measurement error' components $\varepsilon_{\pi t}$ and $\varepsilon_{\psi t}$ associated with net economic profits $\pi_t+\psi_t$ (i.e., the first line of (16) above) for cash flow income measures. Cash flows measure income gross of depreciation—depreciation implicitly figures in the present value of future income which is reduced by future sales or disposal of depreciated capital or by future declines in the productivity of capital (measured gross of depreciation). As such, the present values are unaffected but the timing of expected cash flows and risk premiums are changed. This is reflected in the difference between the book value of noncash assets X_t and the market value of dividend policy related assets Z_t, the second present value term in the first line of (16), and the risk-adjusted present value of the 'measurement error' components $\varepsilon_{\pi t}$ and $\varepsilon_{\psi t}$.

Similarly, earnings introduce the distinction between measured and economic depreciation and the need to account for this difference in present value computations. This too results in a difference between the market and book values of owners' equity and in the altered timing of expected cash flows and risk premiums. These terms show up in (16) as the risk-adjusted present value of the 'measurement error' components $\varepsilon_{\pi t}$ and $\varepsilon_{\psi t}$ (which differ from those associated with cash income) and in the current and present value differences between the market value based 'book' value of owners equity in (11) and the corresponding accounting measure of book value implicit in (16).

These differences between accounting and economic income measures are unimportant for the purpose of identifying dividend policy invariant cash flows. The reason is simple—neither difference between accounting and economic income is related to changes in dividend policy. Accordingly, these distinctions affect the details of present value calculations but introduce no dependence of accounting income measures on dividend policy.

By contrast, the dividend policy dependent component of income found in the last two lines of (16) requires the other major assumption of the analysis—ζ_t represents the net profit or loss from investments associated with dividend policy inclusive of capital gains and losses.[15] This could literally mean that all relevant capital gains and losses are fully realized each period. Alternatively, this corresponds to the assumption that all such investments are marked to market each period for accounting purposes. In terms of (16), this involves the assumption that $\varepsilon_{\zeta t+j}$ is equal to zero for all j (although $E[\varepsilon_{\zeta t+j} y_{t,j}/I_t] = 0$ is all that is needed to obtain the results that follow).

The need for this assumption is straightforward. If the financial assets associated with dividend policy are not marked to market, the measurement error

[15]This assumption implicitly requires broad measures of accounting income. It would be less plausible for narrow definitions of the accounting income variable since some portion of ζ_t might then be inadvertently omitted.

in accounting income includes unrealized capital gains and losses on these assets. While this would not generate dividend policy dependence for financial assets such as zero coupon and fixed income securities held to maturity, the unrealized capital gains and losses of other zero beta assets will generally depend on dividend policy.

This assumption renders the residual income variable $x_{t+j}-f_{t,j}X_{t+j-1}$ based on accounting income dividend policy invariant, as noted in Proposition 3.

Proposition 3: Suppose that security prices satisfy the general no-arbitrage pricing relation (1). Let x_t be some broad accounting income measure like net cash flow or earnings satisfying the accumulation equation:

$$x_{t+j} - d_{t+j} = X_{t+j} - X_{t+j-1}; \qquad X_t \text{ given}$$

such that $\lim_{T\to\infty} \dfrac{E[X_{t+T}y_{t,T}/I_t]}{(1+\rho_{t,T})^T} = 0$. Suppose that equity risk premiums are unaffected by dividend policy (i.e., $Cov[d_{t+j},y_{t,j}/I_t]=Cov[(\pi_{t+j}+\psi_{t+j},y_{t,j}/I_t])$ and that dividend policy related income includes capital gains and losses. Then the residual income variable $x_{t+j}-f_{t,j}X_{t+j-1}$ is dividend policy invariant in that:

$$E[(x_{t+j}-f_{t,j}X_{t+j-1})y_{t,j}/I_t] = E[(\pi_{t+j}+\psi_{t+j})y_{t,j}/I_t]$$

for all choices of dividend policy and prices satisfy:

$$P_1 = X_t + \sum_{j=1}^{\infty} \frac{E\left[(x_{t+j}-f_{t,j}X_{t+j-1})y_{1,j}/I_t\right]}{(1+\rho_{t,j})^j}$$

$$= Z_t + \sum_{j=1}^{\infty} \frac{E\left[\left(\pi_{t+j}+\psi_{t+j}+\varepsilon_{\pi 1+j}+\varepsilon_{\psi t+j}\right)y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j} \tag{17}$$

Proof: This follows directly from Proposition 2 and the preceding discussion.

Hence, there are three building blocks of dividend policy invariant cash flows. The first is the general present value relation (4) for stock/flow relations of the form

(3). The second is the restriction of dividend policy to zero net present value, zero risk premium alterations of the dividend stream. The final building block is the assumption that income associated with dividend policy reported in broad accounting income measures includes any capital gains and losses. Proposition 3 also provides a framework for distinguishing accounting from economic rates of return and capital stock measures.

The two assumptions—dividend policy does not change risk premiums and any dividend policy related income includes marking relevant assets to market for accounting purposes—are necessary for the dividend policy invariance of residual income. The risk premium restriction eliminates the need to risk-adjust the dividend policy related components of income which, in turn, purges their conditional expectations of dependence on dividend policy. Without restrictions on the accounting treatment of dividend policy related income, the residual income variable $x_{t+j}-f_{t,j}X_{t+j-1}$ would include the unrealized gains and losses associated with dividend policy, which clearly can depend on dividend policy. The first assumption restricts dividend policy substantially and the second is a counterfactual proposition about the general accounting treatment of capital gains and losses.

A more positive view is that the analysis does cover a broad range of dividend policies under plausible restrictions. For example, suppose retained earnings reflect savings by the firm designed to eliminate potential future borrowing constraints (or other constraints on the issuance of future contingent claims) much as household savings plays this role in some versions of the permanent income hypothesis of consumption. It is reasonable to suppose that such precautionary savings would be placed in safe assets because of their self-insurance nature. Discount and coupon bonds held to maturity would be safe in this sense and satisfy the requirements of Proposition 3. On this view, accounting residual income measures are invariant to a large class of dividend policy changes.

In addition, these observations have obvious significance for present value calculations given actual practice. It is commonplace to assume that the present value relation holds for expected future dividends discounted by deterministic discount factors of the form:

$$P_t = \sum_{j=1}^{\infty} \frac{E[d_{t+j}/I_t]}{(1+r_{t,j})^j} \tag{18}$$

with $r_{t,j}$ a constant independent of dividend policy. Examples include constant expected returns ($r_{t,j} = r$), constant expected excess returns ($\frac{(1+r_{t,j})^j}{(1+\rho_{t,j})^j} = (1+\delta)^j$ for some constant δ), or constant expected excess returns with a flat term structure of interest rates ($\rho_{t,j} = \rho_t \; \forall \; j$). These specializations of the pricing kernel $y_{t,j}$ make residual income variables of the form $x_{t+j}-\phi_{t,j}x_{t+j-1}$ dividend policy invariant.[16] In any event, one can certainly make a case for using some residual income variable in place of dividends in present value calculations.

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[16]The variable $\phi_{t,j}$ is defined by:

$$1+\phi_{t,j} = \frac{(1+r_{t,j})^j}{(1+r_{t,j-1})^{j-1}}$$

These restrictions on discount factors implicitly restrict dividend policy since:

$$\frac{1}{(1+r_{t,j})^j} = \frac{1+\mathrm{cov}\left[\frac{d_{t+j}}{E(d_{t+j}/I_t)}, y_{t,j}/I_t\right]}{(1+\rho_{t,j})^j}$$

For example, constant discount factors constrain $1+\mathrm{cov}\left[\frac{d_{t+j}}{E(d_{t+j}/I_t)}, y_{t,j}/I_t\right]$ to be proportional to $(1+\rho_{t,j})^j$, substantially restricting dividend policy. Similarly, if excess risk premiums are constant (i.e., $\frac{(1+r_{t,j})^j}{(1+\rho_{t,j})^j} = (1+\delta)^j$), $1+\mathrm{cov}\left[\frac{d_{t+j}}{E(d_{t+j}/I_t)}, y_{t,j}/I_t\right] = (1+\delta)^j$, again implicitly restricting dividend policy.

4. Conclusion

This paper had a very simple motivation. It is well-known that dividend policy is irrelevant in a Modigliani-Miller world. It is equally well-known that price equals the risk-adjusted present value of expected future dividends in these circumstances. The latter fact would not appear to be especially useful for the purposes of computing present values in the absence of *a priori* knowledge of the nature of (value-irrelevant) dividend policy.

There are three building blocks of dividend policy irrelevant cash flows identified in these pages. The first is the calculation of stocks from the difference between arbitrary flows and dividends and the translation of the risk-adjusted present value relation for future dividends into one for the future values of these stocks and flows. The second is the breakdown of corporate policy into investment, financial, and dividend policy and the requirement that dividend policies that synthesize risky contingent claims are classified as financial policy. The final building block is the assumption that the accounting treatment of income from dividend policy includes all relevant capital gains and losses, permitting the calculation of the present value of the cash flow effects of dividend policy in each future period.

Is there some easily measured cash flow that is useful for computing present values and is invariant with respect to dividend policy? The answer to this question is a qualified yes. The analysis identified two necessary conditions for the dividend policy invariance of residual income measures. The first is that dividend policy does not alter equity risk premiums, which are those that would prevail in a world where firms paid out 100% of economic profit as dividends. The second is that the component of accounting income associated with dividend policy includes its associated capital gains and losses. Under these conditions, residual income measures are invariant with respect to dividend policy.

This is moderately bad news in that it places unusual burdens on the analyst. If changes in dividend policies alter equity risk premiums, the analysis requires the attribution of this change to three components: (1) income from investment policy; (2) the payments to other claimants; and (3) the residual dividend policy component. It also requires that the dividend policy component includes capital gains and losses whereas accounting income measures typically only include realized gains and losses. Hence, the analysis would appear to apply only under very restrictive circumstances.

On the other hand, these are burdens that are usually assumed away in actual practice. It is common to assume that either expected returns or equity risk premiums are constant, thus assuming away the problem of the effects of dividend policy on risk premiums (but not on the flow of dividends). Similarly, the constant risk premium assumption effectively limits investment associated with dividend policy to zero beta assets like bonds held to maturity, on which capital gains and losses are not an issue. It is reasonable to suppose that the capital gains and losses associated with other zero beta assets would not substantially alter the analysis.

It is fair to say that techniques for computing the intrinsic value of equity securities have lagged over the past few decades. This is doubtless a consequence of the efficient markets hypothesis, which shifted attention away from such calculations toward the behavior of prices under the assumption that asset prices were equal to their underlying intrinsic values. This paper is a small step toward facilitating such computations by identifying cash flows that are more fundamental than dividends in their invariance with respect to some forms of dividend policy.

Appendix

Proof of Proposition 1: The proof involves trivial algebraic manipulation of the present value relation in the absence of a nominally riskless asset.

$$P_t = \sum_{j=1}^{\infty} E[d_{t+j} Y_{t,j} / I_t]$$

$$= \sum_{j=1}^{\infty} E\left[(x_{t+j} - X_{t+j} + X_{t+j-1}) Y_{t,j} / I_t\right]$$

$$= X_t + \sum_{j=1}^{\infty} E[x_{t+j} Y_{t,j} / I_t] + E\left[(Y_{t,j} - Y_{t,j-1}) X_{t+j-1} / I_t\right] \quad .$$

since $\lim_{T \to \infty} E[X_{t+T} Y_{t,T} / I_t] = 0$ by assumption. If, in addition, there is a nominally riskless asset, $Y_{t,j} = \frac{y_{t,j}}{(1+\rho_{t,j})^j}$ and $Y_{t,j} - Y_{t,j-1} = \frac{y_{t,j} - (1+f_{t,j}) y_{t,j-1}}{(1+\rho_{t,j})^j}$ so that $E[Y_{t,j} / I_t] = \frac{1}{(1+\rho_{t,j})^j}$ and $E[Y_{t,j} - Y_{t,j-1} / I_t] = \frac{-f_{t,j}}{(1+\rho_{t,j})^j}$ where $f_{t,j}$ denote the period t+j forward rate implicit in the yield curve. Consequently:

$$P_t = X_t + \sum_{j=1}^{\infty} \frac{E[x_{t+j} - f_{t,j} X_{t+j-1} / I_t]}{(1+\rho_{t,j})^j} + \sum_{j=1}^{\infty} \frac{\operatorname{cov}[x_{t+j}, y_{t,j} / I_t]}{(1+\rho_{t,j})^j}$$

$$+ \sum_{j=1}^{\infty} \frac{\operatorname{cov}[X_{t+j-1}, y_{t,j} - (1+f_{t,j}) y_{t,j-1} / I_t]}{(1+\rho_{t,j})^j}$$

The present value relation can also be rewritten in terms of future one period riskless rates since $Y_{t,j} - Y_{t,j-1} = (Y_{t+j-1,1} - 1) Y_{t,j-1}$ and $Y_{t+j-1,1} = \frac{y_{t+j-1,1}}{1+\rho_{t+j-1,1}}$.

Q.E.D.

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