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ABSTRACT

This paper explores the impact of incomplete markets and strong complementarities on the time series properties of aggregate activity. We consider an economy which consists of a large number of industries whose production functions both are nonconvex and exhibit localized technological complementarities. The productivity of each industry at t is determined by the production decisions of technologically similar industries at t - 1. No markets exist to coordinate production decisions. This feature implies that aggregate output dynamics for the model are quite different from those predicted by the associated Arrow-Debreu economy. First, multiple stochastic equilibria exist in aggregate activity. These equilibria are distinguished by differences in the mean and the variance of output. Second, output movements are persistent as aggregate productivity shocks indefinitely affect real activity by shifting the economy across equilibria. As a result, the model can exhibit periods of boom and depression.

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Introduction

Recent developments in theoretical macroeconomics have emphasized the potential for multiple, Pareto-rankable equilibria to exist for economies where various Arrow-Debreu assumptions are violated. Authors such as Diamond [1982] have emphasized how incomplete markets can allow economies to become trapped in Pareto-inferior equilibria; Heller [1986] has obtained similar results due to imperfect competition. These different approaches share the idea that strong complementarities in behavior can lead to multiplicity. Intuitively, when technological or demand spillovers make agents sufficiently interdependent, high and low levels of activity can represent internally consistent equilibria in the absence of complete, competitive markets. Most of these models describe multiple steady states in economies rather than multiple nondegenerate time series paths and consequently cannot address issues of aggregate fluctuations. Further, this literature has not shown how economies can shift across equilibria, inducing periods of boom and depression.

An independent literature has argued that aggregate fluctuations are strongly persistent. Researchers have concluded from a variety of perspectives that aggregate output in advanced industrialized economies contains a unit root. Despite controversy over the exact magnitude of the permanent component, the effects of current events on real activity apparently persist over long horizons.

The purpose of the current paper is to link the new multiplicity results in macroeconomic theory with the evidence on output persistence. We do this by modelling coordination problems in an explicitly stochastic framework. As developed in Durlauf [1990,1991], the microeconomic specification of the economy is expressed as a set of conditional probability measures describing how individual agents behave given the economy's history. An aggregate equilibrium exists when one can find a joint probability measure over all agents which is consistent with these conditional measures; multiplicity occurs when several such measures exist. This approach permits one to directly describe the time series properties of aggregate fluctuations along different equilibrium paths.

Specifically, we examine the capital accumulation problems of a set of infinitely-lived industries. We deviate from standard analyses in two respects. First, each industry faces a nonconvex production technology. Second, industries experience technological complementarities as past high production decisions by each industry increase the current productivity of several industries through dynamic learning-by-doing or other effects. Industries do not coordinate production decisions because of incomplete markets. By describing how output levels and productivity evolve as industries interact over time, the model characterizes the impact of complementarities and incomplete markets on the structure of aggregate fluctuations.

Our basic results are twofold. First, we show that with strong complementarities and incomplete markets, multiple stochastic equilibria exist in aggregate activity. These equilibria are distinguished by differences in both the mean and the variance of output. Second, we illustrate how aggregate output movements will be persistent as aggregate productivity shocks indefinitely affect real activity by shifting the economy across equilibria. Although the current model does not exhibit a unit root, one will emerge if deterministic technical change is introduced.

I. A model of interacting industries

Consider a countable infinity of industries indexed by i.[1] Each industry consists of many small, identical firms. All firms produce a homogeneous good; industries are distinguished by distinct production functions rather than distinct outputs. The homogeneous final good may be consumed by the owners of the firms or converted to a capital good which fully depreciates after one period. Industry i's behavior is proportional to the behavior of a representative firm which chooses a capital stock sequence $\{K_{i,t}\}$ to maximize the present discounted value of profits $\Pi_{i,t}$

[1]Durlauf [1990] derives a general equilibrium version of this economy.

$$\Pi_{i,t} = \mathrm{E}(\sum_{j=0}^{\infty} \beta^j (Y_{i,t+j} - K_{i,t+j}) \mid \mathfrak{F}_t). \tag{1}$$

$Y_{i,t}$ equals the output of the i'th industry's representative firm at t; $\mathfrak{F}_t$ equals all available information at t. Initial endowments $Y_{i,0}$ provide starting capital.

Aggregate behavior is determined by the interactions of many heterogeneous industries employing nonconvex technologies. Production occurs with a one period lag; firms at $t-1$ employ both one of two production techniques and a level of capital to determine output at t. Only one technique may be used at a time. Cooper [1987] and Murphy, Shleifer, and Vishny [1989] exploit similar technologies to analyze multiple equilibria; Milgrom and Roberts [1990] discuss how this type of nonconvexity can arise as firms internally coordinate many complementary activities. The technique-specific production functions produce $Y_{1,i,t}$ and $Y_{2,i,t}$ through

$$Y_{1,i,t} = f_1(K_{i,t-1} - F_i, \zeta_{i,t-1}, \xi_{t-1})$$

$$Y_{2,i,t} = f_2(K_{i,t-1}, \eta_{i,t-1}, \xi_{t-1}). \tag{2}$$

$\zeta_{i,t}$ and $\eta_{i,t}$ are industry-specific productivity shocks; ξ_t is an aggregate productivity shock and F_i is a fixed overhead capital cost. $\zeta_{i,t-1}$, $\eta_{i,t-1}$, and ξ_{t-1} are elements of $\mathfrak{F}_{t-1}$. Recalling that firms within an industry are identical, we define $\omega_{i,t}$ which equals 1 if technique 1 is used by industry i at t, 0 otherwise and $\underset{\sim}{\omega}_t = \{\ldots\omega_{i-1,t}, \omega_{i,t}, \omega_{i+1,t}\ldots\}$ which equals the joint set of techniques employed at t.

We make the following assumptions. First, each technique fulfills standard curvature conditions. Further, we associate technique 1 with high production. Specifically, net capital $NK_{i,t}$, which equals $K_{i,t} - F_i$ for technique 1 and $K_{i,t}$ for technique 2, has a strictly higher marginal (and by implication total) product when used with technique 1 than technique 2. A firm chooses technique 1 if it is willing to pay fixed capital costs in exchange for higher output.

Assumption 1. Restrictions on technique-specific production functions

For all realizations of $\zeta_{i,t}$, $\eta_{i,t}$, ξ_t, *and* NK,

A. $f_1(0,\zeta_{i,t},\xi_t) = f_2(0,\eta_{i,t},\xi_t) = 0.$

B. $\dfrac{\partial f_1(0,\zeta_{i,t},\xi_t)}{\partial NK} = \dfrac{\partial f_2(0,\eta_{i,t},\xi_t)}{\partial NK} = \infty;\ \dfrac{\partial f_1(\infty,\zeta_{i,t},\xi_t)}{\partial NK} = \dfrac{\partial f_2(\infty,\eta_{i,t},\xi_t)}{\partial NK} = 0.$

C. $\dfrac{\partial f_1(NK,\zeta_{i,t},\xi_t)}{\partial NK} > \dfrac{\partial f_2(NK,\eta_{i,t},\xi_t)}{\partial NK}.$

Both techniques are assumed to exhibit technological complementarities, as the history of realized activity determines the parameters of the production function at t. Romer's [1986] model of social increasing returns shares this feature. Our complementarities differ from Romer's in two respects. First, all complementarities are local as the production function of each firm is affected by the production decisions of a finite number of industries. The index i orders industries by similarity in technology; spillovers occur only between similar technologies. David [1988] describes the historical importance of local complementarities in the evolution of technical innovations. Second, our complementarities are explicitly dynamic. Past production decisions affect current productivity, which captures the idea of learning-by-doing.

Specifically, we model the complementarities through the dependence of the productivity shocks $\zeta_{i,t}$ and $\eta_{i,t}$ on the history of technique choices (See Durlauf [1990] for a justification). Complementarities are assumed to be the only source of dependence across shocks. $Prob(x \mid y)$ denotes the conditional probability measure of x given information y; $x(y)$ denotes the random variable associated with this measure. $\Delta_{k,l} = \{i-k \ldots i \ldots i+l\}$ indexes the industries which affect industry i's productivity.

Assumption 2. Conditional probability structure of productivity shocks

A. $Prob(\zeta_{i,t} \mid \mathfrak{F}_{t-1}) = Prob(\zeta_{i,t} \mid \omega_{j,t-1}\ \forall\ j\ \in \Delta_{k,l})$.

B. $Prob(\eta_{i,t} \mid \mathfrak{F}_{t-1}) = Prob(\eta_{i,t} \mid \omega_{j,t-1}\ \forall\ j\ \in\ \Delta_{k,l})$.

C. The random pairs $(\zeta_{i,t} - \zeta_{i,t}(\mathfrak{F}_{t-1}), \eta_{i,t} - \eta_{i,t}(\mathfrak{F}_{t-1}))$ *are mutually independent of each other and of* $\xi_t - \xi_t(\mathfrak{F}_{t-1})\ \forall\ i$.

No markets exist whereby individual firms can coordinate to exploit complementarities. Consequently, no industry may be compensated for choosing technique 1 in order to expand the production sets of other industries; nor, given our conceptualization of industries as aggregates of many small producers, can firms within an industry strategically choose a technique in order to induce higher future productivity through complementarities. Market incompleteness combines with the production nonconvexity to fundamentally affect aggregate dynamics.

II. Local complementarities and multiple equilibria

We initially analyze the economy without aggregate shocks, by setting $\xi_t = 0\ \forall\ t$. From our assumptions, one may show that equilibrium industry technique choices obey conditional probabilities of the form

$$Prob(\omega_{i,t} \mid \mathfrak{F}_{t-1}) = Prob(\omega_{i,t} \mid \omega_{j,t-1}\ \forall\ j\ \in \Delta_{k,l}). \tag{3}$$

Once technique choices are determined, one can solve for the optimal levels of capital and output for each firm. In fact, a sufficient condition for the existence of equilibrium capital and output sequences for all firms is the existence of a joint probability measure over all technique choices which is consistent with the conditional measures (3). Durlauf [1990] verifies that such a joint measure exists for any initial conditions $\underline{\omega}_0$.

We now restrict the conditional probabilities in order to discuss multiplicity and dynamics. Past choices of technique 1 are assumed to improve the current relative productivity of the technique. As a result, technique 1 choices will propagate over time. Further, we assume that $\underline{\omega}_t=\underline{1}$ is a steady state, which means that when all productivity spillovers are active, the effects are so strong that high production is always optimal.

Assumption 3. Impact of past technique choices on current technique probabilities[2]

Let ω and ω' denote two realizations of $\underline{\omega}_{t-1}$. If $\omega_j \geq \omega'_j \ \forall\, j \in \Delta_{k,l}$, then

A. $Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = \omega_j \ \forall\, j \in \Delta_{k,l}) \geq Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = \omega'_j \ \forall\, j \in \Delta_{k,l})$.

B. $Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = 1 \ \forall\, j \in \Delta_{k,l}) = 1$.

Whenever some industry chooses $\omega_{i,t} = 0$, a positive productivity feedback is lost. Different configurations of choices at $t-1$ determine different production sets and conditional technique choice probabilities for each industry. We bound the technique choice probabilities from below and above by $\Theta^{min}_{k,l}$ and $\Theta^{max}_{k,l}$ respectively.

$$\Theta^{min}_{k,l} \leq Prob(\omega_{i,t} = 1 \mid \omega_{j,t-1} = 0 \ for\ some\ j \in \Delta_{k,l}) \leq \Theta^{max}_{k,l} \tag{4}$$

Since $\underline{\omega}_t=\underline{1}$ is an equilibrium, multiple equilibria exist if for some initial conditions, $\underline{\omega}_t=\underline{1}$ fails to emerge as t grows. Notice that even if $\underline{\omega}_0 = \underline{0}$, favorable

[2]This assumption can be reformulated in terms of restrictions on the technique-specific production functions.

productivity shocks will periodically induce industries to produce using technique 1. The choice of technique 1 by one industry, through the complementarities, increases the probability that the technique is subsequently chosen in several industries. With strong spillovers, these effects may build up, allowing $\underset{\sim}{\omega}_t=\underset{\sim}{1}$ to emerge from any initial conditions. The model therefore allows us to analyze the stability of a high aggregate output equilibrium from arbitrary initial conditions.

In fact, the limiting behavior of the economy is determined by the bounds $\Theta_{k,l}^{min}$ and $\Theta_{k,l}^{max}$. If the probability of high production by an industry is sufficiently large for all production histories, then the spillover effects induced by spontaneous technique 1 choices cause the economy to iterate towards high production. Alternatively, if technique 1 probabilities are too low in the absence of active spillovers, spontaneous technique 1 choices will not generate sufficient momentum to achieve the $\underset{\sim}{\omega}_t=\underset{\sim}{1}$ equilibrium. $\Theta_{k,l}^{min}$ and $\Theta_{k,l}^{max}$ bound the degree of complementarity in the economy. Large values of $\Theta_{k,l}^{min}$ imply complementarities are weak as technique 1 is chosen relatively frequently regardless of the past. Conversely, small values of $\Theta_{k,l}^{max}$ imply strong complementarities; the probability of current high production is very sensitive to past technique choices. Theorem 1, proven in Durlauf [1990], shows how long run industry behavior is jointly determined by initial conditions and conditional technique probabilities.

Theorem 1. Conditions for uniqueness versus multiplicity of long run equilibrium

For every nonnull index set $\Delta_{k,l}$, there exist numbers $0 < \underline{\Theta}_{\Delta k,l} < \overline{\Theta}_{\Delta k,l} < 1$ such that

A. If $\Theta_{k,l}^{max} \leq \underline{\Theta}_{\Delta_{k,l}}$, then $\lim_{t \Rightarrow \infty} Prob(\omega_{i,t} = 1 \mid \underset{\sim}{\omega}_0 = \underset{\sim}{0}) < 1$.

If complementarities are sufficiently strong, no industry converges to the high production technique almost surely from economy-wide low production technique initial conditions.

B. If $\Theta_{k,l}^{min} \geq \overline{\Theta}_{\Delta_{k,l}}$, *then* $\lim_{t \Rightarrow \infty} Prob(\omega_{i,t} = 1 \mid \underline{\omega}_0 = \underline{0}) = 1$.

If complementarities are sufficiently weak, each industry converges to the high production technique almost surely from economy-wide low production technique initial conditions.

One can associate $\underline{\omega}_t = \underline{1}$ with the equilibrium which would emerge if all firms chose their production levels cooperatively. If production through technique 1 is sufficiently large for $\underline{\omega}_t = \underline{1}$ versus any other configuration, then $\underline{\omega}_t = \underline{1}$ emerges as the cooperative equilibrium after one period. Consequently, incompleteness of markets lowers the mean and increases the variance of industry and aggregate output along the inefficient equilibrium path, as technique choices fluctuate over time. When industries fail to coordinate, production decisions become dependent on idiosyncratic productivity shocks. Observe that the volatility associated with the inefficient equilibrium is caused by fundamentals. Simulations in Durlauf [1990,1991] show that aggregate output can obey a wide range of AR processes, depending on $\Delta_{k,l}$.

III. Path dependence and aggregate shocks

Now consider the role of the aggregate shocks ξ_t. By affecting many industries simultaneously, these shocks act in a way analogous to changing the initial conditions of the economy. Path dependence occurs as one realization of ξ_t permanently changes the equilibrium in the absence of future offsetting shocks. We assume that sufficiently unfavorable aggregate productivity draws make technique 1 unlikely whereas sufficiently favorable draws ensure the use of the technique.

Assumption 4. Impact of aggregate shocks on technique choice

There exist numbers a and b, with $Prob(\xi_t \leq a)$ ***and*** $Prob(\xi_t \geq b)$ ***both nonzero, such that***

A. $Prob(\omega_{i,t}=1 \mid \xi_t \leq a,\ \omega_{j,t-1}=1\ \forall\, j \in \Delta_{k,l}) \leq \Theta^{min}_{k,l}$.[3]

B. $Prob(\omega_{i,t}=1 \mid \xi_t \geq b,\ \omega_{j,t-1}=0\ \forall\, j \in \Delta_{k,l}) = 1$.

When this assumption holds, aggregate shocks can have an indefinite effect on real activity. Durlauf [1991] verifies

Theorem 2. Path dependence due to aggregate shocks

Let $\xi_t = 0\ \forall\ t > T$ *and* $\Theta^{max}_{k,l} \leq \underline{\Theta}_{\Delta_{k,l}}$. *The economy exhibits path dependence as the realization of* ξ_T *affects the limiting technique choice probabilities for all industries.*

A. $\lim_{t \Rightarrow \infty} Prob(\omega_{i,t}=1 \mid \xi_T \leq a) < 1$.

B. $\lim_{t \Rightarrow \infty} Prob(\omega_{i,t}=1 \mid \xi_T \geq b) = 1$.

This result shows how fluctuations can be persistent. For example, once many sectors simultaneously decline due to an adverse aggregate shock, productivity enhancing complementarities are lost until a subsequent favorable draw restores them. If ξ_t is ergodic, then the economy will cycle between the equilibria.

Several interpretations beyond productivity can be applied to the aggregate shocks. Interpreting ξ_t as a proxy for the financial sector, the model indicates how the breakdown of financial institutions, such as occurred during the Depression, can cause indefinite output loss. Alternatively, Durlauf [1990] shows how ξ_t can represent the cost of production inputs provided by leading sectors such as transportation or steel. In this case, the growth of leading sectors improves the relative profitability of high production,

[3] This bound is defined in Theorem 1.

which can lead to a takeoff in growth as the economy shifts across equilibria.

IV. Summary and conclusions

This paper has explored how economies can exhibit multiple equilibria and output persistence as a consequence of dynamic coordination failure. These features arise when strong technological complementarities interact with incomplete markets. Low production initial conditions prevent an economy from realizing positive technological complementarities. Further, aggregate shocks can generate indefinite movements in total output as productivity feedbacks induced by complementarities emerge or disappear. The model exhibits both persistence of shocks as well as a mechanism for reversals of booms and downturns. One application of these ideas is to explore whether output behavior during the Depression and World War II can be interpreted along these lines.

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