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ABSTRACT

Micro estimates of the Marshallian elasticity of labor supply are small and typically positive, whereas cross-country and time-series patterns of hours imply a strong negative relationship between wages and hours. I reconcile these two apparently contradictory observations using a single utility specification and taking into account heterogeneity in non-labor income. Micro estimates condition on non-labor income, while macro variation allows capital income to adjust alongside labor income, which strengthens the income effect. A model with heterogeneous households and exogenous capital income yields closed-form expressions in which the distribution of the labor share shapes the gap between the micro and the macro elasticities. A cross-sectional regression of hours on wages that conditions on the labor share recovers the macro elasticity. A dynamic model with heterogeneous households and incomplete asset markets reproduces both elasticities as outcomes when disciplined by joint moments of wages, hours, consumption, and wealth. The income effects that bridge the gap between the two elasticities imply marginal propensities to earn that lie in the range of estimates of micro studies on lottery winners.

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1 Introduction

A central puzzle in labor economics is the divergence between micro and macro estimates of the wage elasticity of labor supply. Chetty, Guren, Manoli, and Weber (2012) highlight that business cycle models require larger Frisch elasticities of labor supply than the elasticities estimated from micro-level datasets. I emphasize another divergence, which has not been formally analyzed by the literature thus far and concerns long-term comparisons of labor supply. Drawing on cross-sectional and quasi-experimental variation, micro studies estimate small and typically positive Marshallian elasticities of labor supply. For example, the meta-analysis of Keane (2011) reports a central value of 0.06. By contrast, on the macro side, Bick, Fuchs-Schündeln, and Lagakos (2018) find that households in higher-income countries work fewer hours. Boppart and Krusell (2020) find that hours per worker have fallen in developed economies over the past century, despite substantial growth in wages. The divergence between micro and macro estimates is striking over long horizons. The average estimate in the two macro studies implies that hours per capita fall from 2,000 to 1,340 over a century. With a 2 percent wage growth, if one used the Marshallian elasticity of 0.06 from the micro studies, one would have concluded that hours actually increased to 2,255.

Reconciling, within a single utility specification, estimates of the Marshallian elasticity from micro-level data ("micro elasticity") with the elasticity implied by the cross-country or time-series observations ("macro elasticity") remains an open question.[1] This paper offers a reconciliation and argues that small and often positive micro elasticities are not inconsistent with negative and larger in magnitude macro elasticities. Micro estimates of the Marshallian elasticity come from cross-sectional comparisons of individuals at a point in time that hold non-labor income constant. Macro variation, by contrast, allows capital income to adjust along with labor income. Once we allow for this adjustment, the income effect on hours is stronger.

Beyond this conceptual point, the paper aims to make a more substantive, quantitative contribution. Are standard utility specifications in models with realistic heterogeneity across households simultaneously consistent with the micro elasticity estimated from cross-sectional variation and the macro elasticity implied by comparisons of hours over time or across countries?

[1] The micro elasticities from the literature that I refer to are estimated along the intensive margin. For the macro elasticities, Bick, Fuchs-Schündeln, and Lagakos (2018) find that cross-country differences in hours per person are shaped by both the intensive and the extensive margin. However, Boppart and Krusell (2020) document that the long-run decline in hours per person comes almost entirely from the intensive margin. Because the divergence in the elasticities that I am interested in occurs primarily along the intensive margin, the models abstract from the extensive margin of labor supply.

If so, are the income effects that bridge the gap between the elasticities consistent with the income effects estimated in micro studies using exogenous variation in household resources? I calibrate preferences and sources of heterogeneity in two models and arrive at affirmative answers to both questions. The divergence between the elasticities occurs because they are distinct objects. The contribution of this paper is to show that this distinction has novel and sharp quantitative implications. The theory yields closed-form expressions of the elasticity gap using a single utility specification and allowing for realistic heterogeneity across households. It shows that the gap is a function of preference parameters and the observed joint distribution of wages, hours, and consumption. The theory produces additional testable implications, such as the magnitude of the marginal propensity to earn, that are consistent with quasi-experimental evidence. It is embedded in general equilibrium, which leaves no degrees of freedom about how non-labor income responds in comparative statics.

The first model features exogenous capital income, as in the textbook treatment of labor supply, with the only difference being that household decisions are part of a general equilibrium. The second model retains general equilibrium, but endogenizes the distribution of capital income. General equilibrium is not necessary for calculating the micro elasticity, but becomes necessary for calculating the macro elasticity, because this is the response of aggregate hours to changes in the aggregate wage. Modeling heterogeneity is essential for my purposes, because both aggregate macro variables and population averages of the Marshallian elasticity might depend on the distribution of the sources of heterogeneity across households.

The model with exogenous capital income is a static, competitive economy with heterogeneous households who supply labor and are endowed with capital income that accrues from a decreasing returns to scale aggregate technology. Households are heterogeneous in their disutility of work, productivity, and the share of aggregate capital income that they receive. The model is cast in general equilibrium by imposing restrictions on the distribution of capital income. While exogenous capital income at the household level may appear unrealistic, the model is particularly revealing for the economic mechanisms underlying the results, because it allows for closed-form expressions for the micro and macro elasticities of labor supply. I show that the elasticity gap depends on the joint distribution of labor and capital income in the population, as summarized by households' labor share. An important result is that the macro elasticity is the same across all households. This result reflects the assumptions of a constant aggregate labor share and separable isoelastic preferences, which imply that households scale their labor supply by a common factor in response to a change in the aggregate wage. As a methodological by-product, I demonstrate that conditioning a cross-sectional regression of hours on wages on the labor share of households recovers exactly the macro elasticity. Thus, replacing non-labor income with the labor share in the regression converts the estimand from micro to macro.

The baseline version of the model uses separable isoelastic preferences as in MaCurdy (1981), which are commonly used in applied research. However, the reconciliation between the micro and macro elasticities does not rest on separable isoelastic preferences. The micro-elasticity formula generalizes to any utility function, with the curvature parameters of the utility function now varying across households at their optimal choices. The generalization of the macro-elasticity formula to include household-specific curvature parameters depends on the distribution of aggregate capital income. The formula extends to any utility function if capital income is distributed in a way that keeps households' labor share constant over time. If households derive a constant share of capital income, it extends to utility functions whose marginal rate of substitution takes a multiplicative-power form or is homogeneous of degree one in consumption.

To quantify the model with exogenous capital income, I prove in closed form the identification of parameters that govern the heterogeneity across households from moments of the joint distribution of wages, hours, and consumption. The model targets explicitly a value of 0.06 for the micro elasticity and a value of -0.17 for the macro elasticity.[2] The gap between these two elasticities is determined by the observed heterogeneity in labor income, the inferred heterogeneity in non-labor income, and calibrated preference parameters.

The second model is an Aiyagari (1994) economy, which does not impose ad-hoc restrictions on the distribution of aggregate capital income. The decreasing returns to scale technology in the model with exogenous capital income is replaced by a constant returns to scale technology with respect to both capital and labor. Capital income accrues endogenously to households who, in addition to labor, choose their optimal savings. Households are heterogeneous in their disutility of work, in idiosyncratic productivity, and additionally in their discount factor, with the latter allowing the model to generate a realistic wealth distribution. I also introduce a proportional tax on income and a lump-sum transfer, which help disentangle the distribution

[2]The sensitivity checks that I perform show that the mechanisms do not rest on this particular value, as they apply to any given gap between the micro and the macro elasticities. The target value of 0.06 comes from the meta-analysis of Keane (2011) and is in the range of estimates reported elsewhere in the literature. Other estimates of the micro-level Marshallian elasticity of labor supply can be found in the classic analysis of Blundell and MaCurdy (1999). For the macro elasticity, I draw a value in the midpoint of Bick, Fuchs-Schündeln, and Lagakos (2018) and Boppart and Krusell (2020), because their values are close to each other.

of consumption from the distributions of labor income and wealth.

The goal of quantifying the model with endogenous capital income is different from that of the model with exogenous capital income, which was calibrated to match the micro and macro elasticities by design. Here, I am interested in evaluating how close the model comes to generating as *outcomes* the estimated micro and macro elasticities from the data. I quantify the model to target the same statistics as the model with exogenous capital income, except for the micro and macro elasticities. In addition, I use statistics from the distribution of wealth. The main result is that the model, disciplined by statistics of the distribution of wages, hours, consumption, and wealth, reproduces both the micro and the macro elasticities.

In both models, the micro elasticity of each household depends on its labor share. Since the labor share is a function of wages, hours, and consumption, second moments of the labor share distribution are not free parameters but are pinned down by second moments of the distribution of wages, hours, and consumption. The difference between the two models is that the model with endogenous capital income restricts the cross-sectional moments of wages, hours, and consumption to be consistent with the savings decisions of households in general equilibrium. Targeting moments of the wealth distribution provides an external validation of this model, because wealth moments discipline the distribution of the micro elasticity.

The income effect from changes in non-labor income is at the core of the mechanism that reconciles the micro and macro elasticities. I compare the strength of the income effects in my models to empirical studies that estimate the marginal propensity to earn in response to lottery winnings. The model with exogenous capital income generates an average marginal propensity to earn close to -0.3 and the model with endogenous capital income generates an average marginal propensity to earn close to -0.5. These estimates are comparable to the range of estimates reported by the literature. Golosov, Graber, Mogstad, and Novgorodsky (2024) conduct an event study for U.S. lottery winners and estimate a marginal propensity to earn of -0.52. These authors' estimate is somewhat larger than the comparable estimate of -0.33 in Cesarini, Lindqvist, Notowidigdo, and Östling (2017), who study lottery winners in Sweden. Given the significant heterogeneity in income effects, the model with endogenous capital income also generates substantial heterogeneity in the Marshallian elasticity of labor supply. The heterogeneity at the bottom of the distribution is comparable to that in the structural model of Attanasio, Levell, Low, and Sánchez-Marcos (2018), estimated on women's data from the United States. Further, the 90th and 50th percentiles of the elasticity in my model match closely the corresponding percentiles in the study of Graber, Håvarstein, Mogstad, Torsvik, and Vestad (2026), who use variation from a tax reform and lottery winnings in Norway to estimate the uncompensated elasticity of labor supply.

The divergence between micro and macro estimates of labor supply elasticities has been studied extensively since Chetty, Guren, Manoli, and Weber (2012) documented that models of the business cycle require Frisch elasticities far in excess of Frisch elasticities estimated from micro-level panel data. The leading reconciliations of this gap operate through the extensive margin of labor supply, human capital accumulation, and various frictions on optimization decisions.[3] The divergence I study is different in nature, because it concerns the Marshallian rather than the Frisch elasticity. The relevant contrast is between cross-sectional comparisons across individuals and the time-series or cross-country comparison of aggregate labor supply, rather than between the individual and the representative agent, and the reconciling force is the income effect that operates through capital income.[4]

The literature on tax policy and cross-country labor supply typically works with compensated elasticities. In the micro-public tradition, Diamond and Saez (2011), Saez, Slemrod, and Giertz (2012), and Piketty, Saez, and Stantcheva (2014) assume away income effects in deriving optimal-tax formulas. In the macro-public tradition, Prescott (2004), Rogerson (2008), and Trabandt and Uhlig (2011) rebate tax revenues as transfers to households, which eliminates income effects from the elasticity that drives steady-state hours and the Laffer curve. The distinction between these two literatures is that the former imports compensated elasticities from quasi-experimental tax-reform studies, whereas the latter either assumes Cobb-Douglas preferences over consumption and leisure, which imply a high compensated elasticity, or calibrates the Frisch elasticity to high values as in the macro tradition. My distinction is different. Both the micro and the macro elasticities are Marshallian and both incorporate an income effect, but the income effect on hours is stronger at the macro level because capital income adjusts with the aggregate wage in general equilibrium. This adjustment is not a degree of freedom for the

[3] One strand emphasizes the extensive margin, which generates a larger aggregate elasticity than the elasticity at the individual level (Hansen, 1985; Rogerson, 1988). A second strand emphasizes human capital accumulation, under which observed wage and hours responses understate the structural elasticity (Imai and Keane, 2004; Rogerson and Wallenius, 2009). A third emphasizes optimization frictions (Chetty, 2012), and a fourth shows that borrowing constraints bias micro estimates of the elasticity downward (Domeij and Flóden, 2006). Keane and Rogerson (2012) argue that, once some of these forces are accounted for, the apparent inconsistency is largely resolved.

[4] The mechanism in Kleven, Kreiner, Larsen, and Søgaard (2025) is complementary, as it quantifies the divergence of Marshallian elasticities between the short run, with an estimated value of 0.2, and the long run, with an estimated value of 0.5, due to dynamic returns to effort.

researcher. By contrast, in the tax policy literature, reasonable arguments can be made about which equilibrium adjustments are required in order to absorb changes in taxes.[5]

Boppart and Krusell (2020) is the closest antecedent to this paper, as they anticipated, informally, the mechanism that I formalize. From a theoretical perspective, their main contribution is to characterize preferences that are consistent with a steady decline in hours along a balanced growth path. What Boppart and Krusell do not attempt to do is an explicit analysis of the cross section, which concerns the micro elasticity, and how it relates to their macro elasticity. They acknowledge that their theory, absent other sources of heterogeneity, would predict a negative correlation between wages and hours in the cross section. In discussing the reasons why their theory cannot be rejected by a cross-sectional regression, Boppart and Krusell mention that the ideal experiment would scale wage and non-wage income together and that heterogeneity in other dimensions could matter for labor supply. The factors that Boppart and Krusell identify as necessary to be controlled for are the same factors that shape the gap between the micro and macro elasticities in my framework. By modeling heterogeneity in capital income explicitly, I show that the cross-sectional Marshallian elasticity and the time-series or cross-country macro elasticity are distinct objects of the same model and that the gap between them is quantifiable.

2 Model with Exogenous Capital Income

I begin with a simple model of labor supply that generates explicit solutions for the micro and macro elasticities of labor supply and can be quantified almost in closed form. Time is discrete and the horizon is infinite, $t = 0, 1, 2....$ The model is driven by changes in aggregate productivity, A_t, over time. There are no dynamics in aggregate variables or household variables, so the comparative statics concern comparisons across static equilibria which are indexed by different aggregate productivity levels. The final consumption good is the numeraire.

2.1 The Model

Firm. A representative firm produces a final consumption good, Y_t, using aggregate labor services, L_t. Production is Cobb-Douglas with labor share $1 - \alpha$, where $\alpha \in (0, 1)$:

$$Y_t = A_t L_t^{1-\alpha}, \qquad L_t = \int \theta(\iota) n_t(\iota) \, \mathrm{d}F(\iota), \tag{1}$$

[5]Whether to rebate tax revenues or not is the essence of the debate between Prescott (2004) and Alesina, Glaeser, and Sacerdote (2005). The former paper rebates revenues back to households, so the compensated elasticity is the relevant one. The latter paper argues that if revenues are not fully rebated, the income effect survives and the relevant elasticity is the lower uncompensated one.

where $n_t(\iota)$ is hours worked and $\theta(\iota)$ is efficiency units of household ι, and aggregate labor services, L_t, sum up all households' efficiency units. The firm takes the aggregate wage, w_t, as given and chooses L_t to maximize profits, $\Pi_t = Y_t - w_t L_t$. The first-order condition yields

$$w_t = (1-\alpha)\frac{Y_t}{L_t}. \tag{2}$$

With a Cobb-Douglas production function, aggregate income is distributed with constant shares to profits, $\Pi_t = \alpha Y_t$, and to labor income, $w_t L_t = (1-\alpha)Y_t$.

Households. There is a continuum of households indexed by $\iota \in [0,1]$ and distributed according to the function $F(\iota)$. Households are heterogeneous in three dimensions: their efficiency units of labor, $\theta(\iota)$, their disutility of work, $\chi(\iota)$, and their non-labor income, $\pi_t(\iota)$. Without loss of generality in the model with exogenous capital income, assume that $\theta(\iota)$ and $\chi(\iota)$ do not vary over time.

Each household ι has MaCurdy (1981) preferences over consumption $c_t(\iota)$ and hours $n_t(\iota)$:

$$U_t(\iota) = \frac{c_t(\iota)^{1-\gamma} - 1}{1-\gamma} - \frac{\chi(\iota)}{1+1/\varepsilon} n_t(\iota)^{1+1/\varepsilon}, \tag{3}$$

where $\gamma > 0$ is the curvature of utility with respect to consumption and $\varepsilon > 0$ is the Frisch elasticity of labor supply. Households take the aggregate wage, w_t, and aggregate profits, Π_t, as given, and choose consumption and labor supply to maximize utility in equation (3) subject to the budget constraint:

$$c_t(\iota) = w_t \theta(\iota) n_t(\iota) + \pi_t(\iota), \tag{4}$$

where non-labor income $\pi_t(\iota)$ is treated as exogenous from the household's perspective. Optimal labor supply satisfies the usual first-order condition:

$$\chi(\iota) n_t(\iota)^{1/\varepsilon} c_t(\iota)^{\gamma} = w_t \theta(\iota). \tag{5}$$

MaCurdy preferences are commonly used in applied research. Under these preferences, I obtain a closed-form expression for the macro elasticity, irrespective of how capital income is distributed across households. As I demonstrate in Section 2.3, the reconciliation between the micro and macro elasticities can be generalized to environments without separable isoelastic preferences. However, the closed-form expression for the macro elasticity requires a specific rule for the allocation of capital income across households.

Allocation of capital income. Next, I discuss the distribution of aggregate profits Π_t across households. I posit a general affine rule with time-invariant primitives $\nu(\iota)$ and $\xi(\iota)$ across households:

$$\pi_t(\iota) = \nu(\iota)\Pi_t + \xi(\iota)w_t\theta(\iota)n_t(\iota). \tag{6}$$

This rule nests two natural cases. We obtain a "fixed-share" rule when $\xi(\iota) = 0$, so that each household always receives a fixed share of aggregate profits. We obtain a "constant-labor-share" rule when $\nu(\iota) = 0$, because each household's non-labor income scales proportionally with its own labor income. A household's labor share is given by

$$\zeta_t(\iota) \equiv \frac{w_t\theta(\iota)n_t(\iota)}{c_t(\iota)}, \tag{7}$$

which becomes constant, $\zeta_t(\iota) = \frac{1}{1+\xi(\iota)}$, under $\nu(\iota) = 0$. This rule allows the labor share to differ across households, but assumes that for an individual household the labor share remains constant over time. As demonstrated below, the fixed-share rule, $\xi(\iota) = 0$, also yields a constant labor share over time for all households under MaCurdy preferences. Given an appropriate choice of $\nu(\iota)$, the two rules lead to equivalent allocations of consumption and labor supply across households. Further, the same result applies to any other rule that generates common labor supply responses across households.

Capital market clearing requires $\Pi_t = \int \pi_t(\iota)\mathrm{d}F(\iota)$ at every t. Using the distribution of aggregate income between profits and labor, $\Pi_t = \alpha Y_t$ and $w_t L_t = (1-\alpha)Y_t$, I recast capital market clearing as a single restriction on the primitives of the allocation rule:

$$\alpha = \alpha\bar{\nu} + (1-\alpha)\tilde{\xi}_t, \qquad \bar{\nu} = \int \nu(\iota)\mathrm{d}F(\iota), \qquad \tilde{\xi}_t \equiv \frac{\int \xi(\iota)\theta(\iota)n_t(\iota)\mathrm{d}F(\iota)}{\int \theta(\iota)n_t(\iota)\mathrm{d}F(\iota)}, \tag{8}$$

where $\bar{\nu}$ is the mean of $\nu(\iota)$ and $\tilde{\xi}_t$ is the labor-income-weighted mean of $\xi(\iota)$. A result of the subsequent analysis is that $\tilde{\xi}_t$ becomes time invariant, so capital market clearing is satisfied at every t once it is satisfied at one t.

Equilibrium. Given an aggregate productivity, A_t, and an allocation rule in equation (6), a competitive equilibrium at date t consists of an aggregate wage w_t, aggregate quantities (Y_t, L_t, Π_t), and household choices $\{n_t(\iota), c_t(\iota)\}$ such that:

1. taking as given the aggregate wage w_t, the firm maximizes profits by setting labor demand according to equation (2);

2. taking as given the aggregate wage w_t, aggregate profits Π_t, and the allocation rule in equation (6), each household ι maximizes utility by setting consumption according to equation (4) and labor supply according to equation (5);

3. the goods market clears, $Y_t = \int c_t(\iota)\mathrm{d}F(\iota)$, the labor market clears, $L_t = \int \theta(\iota)n_t(\iota)\mathrm{d}F(\iota)$, and the capital market clears, $\Pi_t = \int \pi_t(\iota)\mathrm{d}F(\iota)$. The latter condition is equivalent to equation (8).

2.2 Elasticities of Labor Supply

In this section, I derive expressions for the micro and macro elasticities and formalize their relationship. The Marshallian elasticity for household ι is the percent change of labor supply, $n_t(\iota)$, in response to a one percent change in its own wage $W_t(\iota) \equiv w_t\theta(\iota)$, holding non-labor income, $\pi_t(\iota)$, and disutility of work, $\chi(\iota)$, constant. The aggregate wage, w_t, is the same for all households at a point in time and thus, up to preferences and non-labor income, labor supply differs across households because of differences in efficiency units, $\theta(\iota)$. The macro elasticity of labor supply measures the response of aggregate labor, L_t, to a one percent change in the aggregate wage, w_t. The aggregate wage is endogenous and changes only because of changes in aggregate productivity, A_t, over time. Equivalently, one can think of different levels of A_t as capturing cross-country variation in productivity in a given period, in which case t would index different countries.

Proposition 1 (Micro and macro elasticities)**.** At any equilibrium allocation, the Marshallian elasticity of labor supply for household ι is given by

$$\eta_\theta(\iota) \equiv \left.\frac{\partial \ln n_t(\iota)}{\partial \ln(W_t(\iota))}\right|_{\pi_t(\iota),\,\chi(\iota)} = \frac{1-\gamma\zeta_t(\iota)}{1/\varepsilon + \gamma\zeta_t(\iota)}, \tag{9}$$

where $\zeta_t(\iota)$ is the household's labor share in equilibrium, given by equation (7).

The macro elasticity of labor supply is given by

$$\eta_w(\iota) \equiv \frac{\mathrm{d}\ln L_t}{\mathrm{d}\ln w_t} = \frac{1-\gamma}{1/\varepsilon+\gamma} = \frac{\mathrm{d}\ln n_t(\iota)}{\mathrm{d}\ln w_t}, \tag{10}$$

which coincides with the elasticity of each household with respect to a change in the aggregate wage over time. Further, the macro elasticity is independent of the primitives $\nu(\iota)$ and $\xi(\iota)$ that determine the distribution of capital income.

The proof is given in Appendix A.1. The micro and macro elasticities are different objects. The micro elasticity in equation (9) varies across households according to their labor share,

$\zeta_t(\iota)$. Its population mean depends on parameters ε and γ and the distribution of $\zeta_t(\iota)$ across households. By contrast, the macro elasticity in equation (10) is constant across households and independent of the distribution of $\zeta_t(\iota)$. As the two formulas show, for $\gamma > 1$, the macro elasticity is negative, while the average micro elasticity in the population may still be positive.

The micro elasticity is derived from implicit differentiation of the first-order condition (5), as in Keane (2011). The numerator of the elasticity in equation (9) has an ambiguous sign. This reflects the familiar substitution and income effects from changes in the wage. In response to a higher wage, households tend to substitute toward labor, because it becomes relatively more expensive to consume leisure. At the same time, households tend to work less, because leisure is a normal good. The strength of the income effect is heterogeneous across households and parameterized by the term $\gamma\zeta_t(\iota)$. A higher curvature of the utility function with respect to consumption, γ, strengthens the income effect. Households with a higher labor share, $\zeta_t(\iota)$, have a stronger income effect than households with a lower labor share, because a larger share of their consumption is financed through labor income as opposed to non-labor income.

An object that researchers are interested in estimating is the population mean of the micro elasticity, $\bar{\eta}_\theta \equiv \int \eta_\theta(\iota)\mathrm{d}F(\iota)$. Equation (9) shows how the distribution of the labor share affects the mean micro elasticity. The micro elasticity is decreasing and convex in $\zeta_t(\iota)$. Thus, larger dispersion in the labor share implies a larger gap between the mean micro elasticity and the elasticity evaluated at the mean labor share. This formula also shows why cross-sectional regressions of log hours on log wages do not in general recover the mean micro elasticity, even if wages are uncorrelated with non-labor income and preferences. The micro elasticity is heterogeneous across households when they have heterogeneous labor shares. With heterogeneity in elasticities, the OLS coefficient recovers a weighted average of the household elasticities, with weights proportional to the squared deviation of the log wage from its mean.

Following the survey of Keane (2011), various research designs attempt to estimate the population average of the Marshallian elasticity, $\bar{\eta}_\theta$. These can be grouped into three categories. Cross-sectional regressions of log hours on log wages with controls for non-labor income identify $\bar{\eta}_\theta$ when the elasticity is common across households and the controls are adequate (Pencavel, 1986; Blundell and MaCurdy, 1999). Tax-reform and nonlinear-budget-set approaches identify $\bar{\eta}_\theta$ by exploiting variation in the marginal after-tax wage while holding non-labor income fixed (Hausman, 1981; Eissa and Liebman, 1996; Blundell, Duncan, and Meghir, 1998; Graber, Håvarstein, Mogstad, Torsvik, and Vestad, 2026). By contrast, life-cycle variation in wages and hours holding the marginal utility of wealth fixed identifies the Frisch elasticity ε rather than the Marshallian elasticity $\bar{\eta}_\theta$ (MaCurdy, 1981; Browning, Deaton, and Irish, 1985; Altonji, 1986; Pistaferri, 2003). The value of 0.06 targeted in the calibration is the central estimate from Keane's synthesis of estimates in the first two categories.

For the macro elasticity, I first derive the response of endogenous variables to a change in aggregate productivity between two dates s and t, holding the sources of heterogeneity across households, (θ, χ, ν, ξ), constant. Suppose the allocation rule in equation (6) satisfies capital market clearing at some date s. Then, Appendix A.1 shows that for any other date t, household labor supply scales by a common factor:

$$\mu_t = \frac{n_t(\iota)}{n_s(\iota)} = \left(\frac{A_t}{A_s}\right)^{\frac{1-\gamma}{1/\varepsilon+\alpha+\gamma(1-\alpha)}}, \qquad \text{for all } \iota. \tag{11}$$

Aggregates scale as $L_t/L_s = \mu_t$, $w_t/w_s = (A_t/A_s)\mu_t^{-\alpha}$, $\Pi_t/\Pi_s = (A_t/A_s)\mu_t^{1-\alpha}$, and households' labor shares are constant, $\zeta_t(\iota) = \zeta_s(\iota)$ for all ι.

This key result underlies the expression for the macro elasticity in equation (10). The macro elasticity coincides with the Marshallian elasticity evaluated at $\zeta_t(\iota) = 1$ for all households. Intuitively, when each household's labor share is constant over time, non-labor income and labor income scale proportionally with the aggregate wage. Thus, consumption in the first-order condition (5) depends only on the aggregate wage and the labor share disappears from the formula for the macro elasticity. From the household's perspective, the income effect becomes stronger because changes in the wage scale both labor and non-labor income.

Equation (10) shows that the allocation of capital income is irrelevant for the cross-country or time-series response of labor supply to a change in the wage, provided the allocation rule for capital income belongs to the affine class described by equation (6). Provided they are calibrated similarly, the fixed-share rule ($\xi(\iota) = 0$) and the constant-labor-share rule ($\nu(\iota) = 0$) generate identical equilibria at every t. This reflects two properties of the model. First, the Cobb-Douglas technology generates a constant ratio of aggregate profits to aggregate labor income, $\Pi_t/(w_t L_t) = \alpha/(1-\alpha)$. Second, MaCurdy preferences generate a uniform scaling of labor supply and consumption across all households. As a corollary, the constancy of the household labor share is preserved across time even under the fixed-share rule. Further, any rule that scales household profits $\pi_t(\iota)$ with other aggregates, such as output or labor income, generates a constant labor share at the household level and a macro elasticity that is given by equation (10).

Empirical micro studies aim to identify the micro elasticity by exploiting cross-sectional wage variation while attempting to hold non-labor income and preferences fixed. I show that conditioning these regressions on the labor share, $\zeta_t(\iota)$, provides an estimate of the macro elasticity. Substituting $c_t(\iota) = w_t\theta(\iota)n_t(\iota)/\zeta_t(\iota)$ into equation (5) and taking logs yields the expression:

$$\ln n_t(\iota) = \frac{1-\gamma}{1/\varepsilon+\gamma}\ln(w_t\theta(\iota)) - \frac{1}{1/\varepsilon+\gamma}\ln\chi(\iota) + \frac{\gamma}{1/\varepsilon+\gamma}\ln\zeta_t(\iota), \tag{12}$$

Equation (12) shows that an OLS regression of $\ln n_t(\iota)$ on $\big(\ln(w_t\theta(\iota)), \ln\chi(\iota), \ln\zeta_t(\iota)\big)$ recovers the macro elasticity, η_w in equation (10), on the wage coefficient exactly. The reconciliation between micro and macro estimates therefore can be entirely resolved by conditioning the regression on the labor share. Replacing the labor share with proxies for non-labor income yields a different estimand, which sometimes is closer to the average Marshallian, as discussed in numerical examples below.[6]

2.3 General Utility Function

This section summarizes the results that generalize to utility functions outside the separable isoelastic case of MaCurdy (1981). The formal derivations are presented in Appendix A.2. Consider a general, well-behaved utility function $U(c,n)$ that satisfies usual properties. Define the marginal rate of substitution between consumption and hours by $\mathrm{MRS}(c,n) \equiv -U_n(c,n)/U_c(c,n)$, so the household's first-order condition is $\mathrm{MRS}(c,n) = W$, where $W = w\theta$ is the household's wage. Define the log-elasticities of the marginal rate of substitution with respect to consumption and hours by:

$$\gamma(c,n) \;\equiv\; \frac{\partial \log \mathrm{MRS}(c,n)}{\partial \log c}, \qquad \frac{1}{\varepsilon(c,n)} \;\equiv\; \frac{\partial \log \mathrm{MRS}(c,n)}{\partial \log n}. \tag{13}$$

For separable isoelastic preferences, $\gamma(c,n) = \gamma$ and $\varepsilon(c,n) = \varepsilon$ are constants. Under a general utility function, both objects depend on the household's allocation of consumption and hours. I abbreviate $\gamma(\iota) \equiv \gamma(c(\iota), n(\iota))$ and $\varepsilon(\iota) \equiv \varepsilon(c(\iota), n(\iota))$ for household ι.

The first result that generalizes to any utility function is the formula for the micro elasticity in equation (9). The only difference is that the constants γ and ε are replaced by $\gamma(\iota)$ and

[6] Cross-sectional regressions of hours on wages produce biased coefficients on wages, when measurement error in hours generates measurement error in wages, which are derived from the ratio of labor income to hours. The labor share has labor income in its numerator, which is directly observed, so measurement error in hours does not affect the labor share variable even if it affects the wage variable. However, measurement error in either consumption or labor income would affect the labor share variable.

$\varepsilon(\iota)$, where both elasticities are evaluated at the household's optimal consumption and hours as shown in equation (13). Thus, the micro elasticity varies not only because of heterogeneity in the labor share, but also because households may have different curvatures in their utility functions at their optimal solutions. This result generalizes the derivation of Keane (2011) for the Marshallian elasticity of labor supply in the nested case of separable isoelastic preferences.

The second result that generalizes to any utility function is the formula for the macro elasticity in equation (10), provided that the household labor share $\zeta_t(\iota)$ is held constant over time. Similar to the micro elasticity, the constants γ and ε are replaced by $\gamma(\iota)$ and $\varepsilon(\iota)$. Thus, the macro elasticity now potentially differs across households, $\eta_w(\iota)$. The response of aggregate hours to the aggregate wage is a weighted average of the household elasticities, $\bar{\eta}_w = \int \omega(\iota)\eta_w(\iota)\mathrm{d}F(\iota)$, where $\omega(\iota) = \theta(\iota)n(\iota)/L$ is the share of the wage bill that accrues to household ι.

The household labor share, $\zeta_t(\iota)$, is constant under the $\nu(\iota) = 0$ rule in equation (6), because this rule allocates profits proportionally to labor income. In this case, the constancy of the labor share does not depend on the utility function or whether labor and consumption scale uniformly across households.

To sum up, *for any utility function*, the micro elasticity generalizes with ι-specific curvature parameters. Provided that profits are distributed proportionally to labor income at every date, the macro elasticity also generalizes with ι-specific curvature parameters. The reconciliation of the gap between the two elasticities is qualitatively similar to the one under separable isoelastic preferences.

For a general utility function, the household labor share, $\zeta_t(\iota)$, is not necessarily constant under the $\xi(\iota) = 0$ rule in equation (6). Appendix A.2 shows that $\zeta_t(\iota)$ is constant if the marginal rate of substitution is homogeneous of degree one in consumption or it has a multiplicative-power form in consumption and hours. In that case, consumption and labor scale uniformly across households and so the rule that allocates profits according to a fixed share endogenously generates a constant labor share. Under separable isoelastic preferences, the constant-labor-share rule and the fixed-share rule yield identical allocations and macro elasticities. Under a general utility function, the two rules may differ. Apart from separable isoelastic preferences, other examples of utility functions that satisfy a constant labor share even with a fixed-share rule are Cobb-Douglas and GHH preferences. A counterexample is CES preferences over consumption and leisure with an elasticity of substitution different from one.

Boppart and Krusell (2020) show that MaCurdy preferences, which are outside the King, Plosser, and Rebelo (1988) class, belong to their broader class of preferences that admit a balanced growth path with falling hours. Appendix A.2 shows that, within the Boppart-Krusell class, MaCurdy preferences are the only preferences that admit a constant labor share for each household under the fixed-share rule ($\xi(\iota) = 0$) and generate a non-zero macro elasticity. Thus, for their broader class of preferences, my generalization of the macro elasticity formula requires that the distribution of capital income is such that each household has a constant labor share over time ($\nu(\iota) = 0$).[7]

Finally, I show that equation (12) continues to hold as a local approximation to the household's first-order condition. The difference is that, with a general utility function, the coefficients that multiply the wage, the disutility of work, and the labor share are ι-specific. Thus, the wage coefficient in an OLS regression recovers a covariance-weighted average of the local macro elasticities, $\eta_w(\iota)$, in the cross section.

2.4 Quantification

The analysis above explains how the gap between the micro and macro elasticities arises. This section takes the model with separable isoelastic preferences to the data in order to evaluate quantitatively the gap between micro and macro elasticities under a realistic heterogeneity across households. Beyond the quantitative reconciliation of micro with macro elasticities, this exercise generates distributions of labor shares and Marshallian elasticities, which can be compared to ones generated by the model with endogenous capital income that is disciplined by moments of the labor and capital income distribution.

I make the following parametric assumptions on the cross-sectional distribution of household heterogeneity, $(\theta(\iota), \chi(\iota), \zeta(\iota))$. The marginal distributions of productivity, θ, and the disutility of work, χ, are log-normal. The mean of each variable is normalized to one. The variances of the log of the variables are given by σ_θ^2 and σ_χ^2. The marginal distribution for the labor share, ζ, is shifted Generalized Pareto. I use two summary statistics of this distribution as parameters to be reported, which are the median labor share $\bar{\zeta}$ and the variance of the log labor share, σ_ζ^2.[8]

[7] A subtlety arises from the observation that Boppart and Krusell have a constant aggregate labor share. The difference with my framework is that they have a representative household. With heterogeneity, the distribution of capital income determines the evolution of each household's labor share. Keeping each household's labor share constant over time mimics the constant aggregate labor share in the representative household model of Boppart and Krusell. This is true even when households scale their hours and consumption at different rates because of heterogeneous local elasticities under a general utility function.

[8] Appendix A.3 shows that there is an explicit mapping between the three parameters of the shifted General-

The covariances between the sources of heterogeneity are given by $\sigma_{\theta\chi}$, $\sigma_{\theta\zeta}$, and $\sigma_{\chi\zeta}$.[9]

To summarize, the model has ten parameters to identify. These are the preference parameters ε and γ, the production function parameter α, the variances and covariances of the joint cross-sectional distribution of the sources of heterogeneity, σ^2_θ, σ^2_χ, σ^2_ζ, $\sigma_{\theta\chi}$, $\sigma_{\theta\zeta}$, and $\sigma_{\chi\zeta}$, and the median labor share $\bar{\zeta}$. To begin, suppose we observe the following nine moments:

(M1) the Frisch elasticity of labor supply, ε;

(M2) the macro elasticity of labor supply, η_w;

(M3) the aggregate labor share, s_ℓ;

(M4) the variance of log wages, σ^2_w;

(M5) the variance of log hours, σ^2_n;

(M6) the variance of log consumption, σ^2_c;

(M7) the covariance of log wages and log hours, σ_{wn};

(M8) the covariance of log wages and log consumption, σ_{wc};

(M9) the covariance of log hours and log consumption, σ_{nc}.

Then, parameters

$$\left(\varepsilon,\ \gamma,\ \alpha,\ \ \sigma^2_\theta,\ \sigma^2_\chi,\ \sigma^2_\zeta,\ \sigma_{\theta\chi},\ \sigma_{\theta\zeta},\ \sigma_{\chi\zeta}\right)$$

are identified from the nine moments by closed-form recursive expressions.

Appendix A.3 gives the formal proof. Briefly, the aggregate labor share identifies parameter α directly and inverting equation (10) identifies parameter γ, given ε and the targeted macro elasticity. Using the definition of the labor share at the household level and the log-linear hours equation (12) yields explicit expressions for parameters $\sigma^2_\theta, \sigma_{\theta\chi}, \sigma_{\theta\zeta}$ as a function of the

ized Pareto and the median labor share. Thus, I use the convention that $\bar{\zeta}$ is a parameter, which is more useful to report than the underlying parameters of the distribution. As discussed in more detail in the appendix, targeting the mean micro Marshallian elasticity while respecting aggregate market clearing implies that the marginal distribution of the labor share has to concentrate mass near a lower floor and admit a heavy tail extending to labor shares above one. A labor share above one allows for comparisons with the endogenous capital income model in which some households are borrowers. The Pareto distribution is not the only distribution that satisfies these requirements, and the results are very similar under distributions in the log-normal, Gamma, or Weibull families.

[9] The three marginal distributions are combined into a joint distribution through a Gaussian copula whose underlying normal correlation matrix is set equal to the calibrated correlation matrix of the logs of (θ, χ, ζ). The reported covariances of $\log\zeta$ with $\log\theta$ and $\log\chi$ in Table 2 are the closed-form values implied by the targeted moments. The realized covariances of these objects in a simulated sample are slightly attenuated when using the parametric marginal distribution of ζ, as discussed in Appendix A.3.

three moments that are related to wages, (M4), (M7), and (M8). Finally, equation (12) and the budget constraint (4) yield a system of equations for log hours and log consumption that express $\sigma_\chi^2, \sigma_\zeta^2, \sigma_{\chi\zeta}$ as a function of the three remaining moments, (M5), (M6), and (M9).

A tenth moment, (M10) the mean micro elasticity of labor supply, $\bar{\eta}_\theta \equiv \int \eta_\theta(\iota)\, \mathrm{d}F(\iota)$, and the parametric assumption on the distribution of the labor share allow me to back out the median labor share, $\bar{\zeta}$. Appendix A.3 shows that the three parameters of the labor share distribution can be calculated from the mean micro elasticity (M10), the variance of the log of labor share that is already known from the 9 moments above, and the capital market clearing condition (8). It also shows how the median labor share explicitly relates to these parameters.

Table 1 reports the values and sources for the targeted moments. The Frisch elasticity, ε, is taken from the meta-analysis of Chetty, Guren, Manoli, and Weber (2012). The macro elasticity is in the midpoint of estimates reported by Bick, Fuchs-Schündeln, and Lagakos (2018) and Boppart and Krusell (2020). The aggregate labor share is from Karabarbounis (2024). The six cross-sectional moments are taken from the analysis of the Panel Study for Income Dynamics and the Consumer Expenditure Survey in Heathcote, Storesletten, and Violante (2014). Finally, the micro elasticity is the central value reported in the meta-analysis of Keane (2011).[10]

Table 2 reports the ten parameters that allow the model to match perfectly the targeted moments. The curvature of the utility function with respect to consumption is $\gamma = 1.58$. Recall that this parameter affects both the micro and the macro elasticities of labor supply. For the latter to be negative, γ has to exceed one. For the former to be positive but small, the median household labor share is $\bar{\zeta} = 0.43$. There are two reasons why the median labor share is quite low. Inspection of equation (9) shows that if all households had the same labor share, then a value of 0.53 would have been required in order to generate a micro elasticity of 0.06. The Pareto tail of the labor share distribution, however, tends to increase the mean labor share.[11] This implies that the model requires a significant mass of households with a relatively low

[10]For the aggregate labor share, I use the mean value starting in 1989. I do so to concord this measurement with the measurements of moments from the Survey of Consumer Finances in the model with endogenous capital income that also begin in 1989. The same logic applies whenever I use long-term averages of other moments below, such as the capital-to-output ratio and the depreciation rate. I use the midpoint of the macro elasticity -0.2 from Boppart and Krusell (2020) (annual percent decline in hours divided by two percent wage growth) and the macro elasticity of -0.15 from Bick, Fuchs-Schündeln, and Lagakos (2018) (slope of log hours on log GDP across countries). The moments in Heathcote, Storesletten, and Violante (2014) are averages of within-age moments and thus should be interpreted as cross-sectional moments conditional on age.

[11]Using a simulated cross section of households, I find that roughly 8 percent of households have a labor share above one. In the model with exogenous capital income, this corresponds to negative profits allocated to these households. More realistically, in the model with endogenous capital income, a labor share above one occurs for some households that borrow to finance consumption that exceeds their labor earnings.

Table 1: Calibration Targets, Exogenous Capital Income

Moment	Symbol	Value	Source
Frisch elasticity	ε	0.54	Chetty, Guren, Manoli, and Weber (2012)
macro elasticity	η_w	-0.17	Bick, Fuchs-Schündeln, and Lagakos (2018) and Boppart and Krusell (2020)
aggregate labor share	s_ℓ	0.63	Karabarbounis (2024)
variance of log wages	σ_w^2	0.36	Heathcote, Storesletten, and Violante (2014)
variance of log hours	σ_n^2	0.12	Heathcote, Storesletten, and Violante (2014)
variance of log consumption	σ_c^2	0.16	Heathcote, Storesletten, and Violante (2014)
correlation of log wages, hours	ρ_{wn}	-0.10	Heathcote, Storesletten, and Violante (2014)
correlation of log wages, cons.	ρ_{wc}	0.50	Heathcote, Storesletten, and Violante (2014)
correlation of log hours, cons.	ρ_{nc}	0.22	Heathcote, Storesletten, and Violante (2014)
mean micro elasticity	$\bar{\eta}_\theta$	0.06	Keane (2011)

Notes. The table reports the values of the ten moments targeted in the calibration of the model with exogenous capital income and their sources from the literature.

Table 2: Identified Parameters, Exogenous Capital Income

Parameter	Description	Value
ε	Frisch elasticity	0.540
γ	curvature with respect to consumption	1.584
α	capital share in production	0.370
σ_θ^2	variance of $\ln\theta$	0.360
σ_χ^2	variance of $\ln\chi$	1.049
σ_ζ^2	variance of $\ln\zeta$	0.297
$\sigma_{\theta\chi}$	$\text{cov}(\ln\theta, \ln\chi)$, corr 0.339	0.208
$\sigma_{\theta\zeta}$	$\text{cov}(\ln\theta, \ln\zeta)$, corr 0.670	0.219
$\sigma_{\chi\zeta}$	$\text{cov}(\ln\chi, \ln\zeta)$, corr 0.192	0.107
$\bar{\zeta}$	median ζ	0.431

Notes. The table reports the ten parameters of the model with exogenous capital income, identified in closed form from the targeted moments in Table 1 using the recursive expressions in Appendix A.3.

labor share in order to generate a mean value of 0.06. The dispersion in the labor share is also significant, $\sigma_\zeta^2 = 0.30$. The labor share is quite dispersed because neither component of labor income—wages or hours—is strongly correlated with consumption. As equation (7) reveals, this observation implies that households must have quite heterogeneous labor shares.

I do not attempt to compare the labor share at the household level to empirical analogs, reflecting that ζ in this model is an input that, given other parameters, reconciles the gap between the micro and macro elasticities. I discuss below, in relation to the model with endogenous capital income, how second moments of the joint distribution of wages, hours, and consumption pin down second moments of the distribution of the labor share. The difference between the two models is that, with endogenous capital income, the labor share is an outcome. Thus, its distribution arises endogenously from the joint distribution of labor and capital income, as opposed to being specified parametrically as an input to the model.

The table also shows that the model requires a significant dispersion across households in their disutility of work, $\sigma_\chi^2 = 1.05$. The moments most informative about this parameter are the moderate correlation of wages with consumption and the low correlation of wages with hours. This is because variations in the disutility of work make the wage less important for a household's choice of consumption and labor. The same moments also lead the disutility of work to be positively correlated with both the wage and the labor share.

The mean micro elasticity targeted in the model is 0.06. If one used the aggregate labor share, $1 - \alpha$, to evaluate equation (9), one would have concluded that the mean elasticity is 0. There are two reasons why this gap arises. First, the aggregate labor share differs from the mean labor share, $\int \zeta_t(\iota)\mathrm{d}F(\iota)$, because the former equals an average of the household labor share weighted by the consumption share, $1 - \alpha = \int \zeta_t(\iota)(c_t(\iota)/C_t)\mathrm{d}F(\iota)$. The covariance between labor share and consumption is negative and thus the mean labor share exceeds the aggregate. Evaluating (9) at the mean labor share produces a negative value for the elasticity. Second, as this equation shows, the mean elasticity is a convex function of $\zeta_t(\iota)$. This implies that the mean of the elasticity across households is larger than the elasticity evaluated at the mean household labor share. As discussed above, there is a large dispersion in the labor shares across households, which tends to increase the mean value of the micro elasticity.

To conclude this section, Table 3 presents OLS estimates of the Marshallian elasticity of labor supply from a simulated cross section of 10,000 households in the model. The true micro elasticity in the model is 0.06, averaged across households using equation (9). The first and

Table 3: Estimates of Marshallian Elasticity of Labor Supply

Specification	Slope on $\ln(w\theta)$
wage only	-0.132
wage, π	-0.147
wage, π, $\ln\chi$	0.028
wage, $\ln\zeta$, $\ln\chi$	-0.170

Notes. The table reports OLS coefficients from regressions of log hours on log wages in a simulated cross section of 10,000 households drawn from the model with exogenous capital income. Each row varies the control variables included alongside log wages.

second rows show the standard bias that arises when regressing log hours on log wages with or without non-labor income, when not controlling for unobserved variables that are correlated with the wage. Because a higher disutility of work, χ, affects hours negatively and is positively correlated with the wage, the estimated micro elasticity from these regressions is downward biased at around -0.14. Controlling for χ reduces substantially the bias of the OLS coefficient.[12] The last row of the table demonstrates that replacing non-labor income with the labor share yields the macro elasticity as the estimated OLS coefficient. This follows from equation (12), which shows that equilibrium labor is exactly a log linear function of the wage, disutility of work, and the labor share.[13]

3 Model with Endogenous Capital Income

In this section I consider an Aiyagari (1994) model in which households choose their savings. The decreasing returns to scale technology in the model with exogenous capital income is replaced by a constant returns to scale technology with respect to both capital and labor. The

[12] The regression does not perfectly recover the true elasticity for two reasons. First, the functional form in the log-linear regression is not the same as the one that determines hours in the structural model. Second, with heterogeneity in household labor supply elasticities, the OLS regression does not recover the unweighted mean of the elasticity in the population. As the table shows, however, these two issues create only a 0.03 deviation between the unweighted mean and the estimated OLS coefficient.

[13] In implementing this regression, one has to be aware that survey measures of capital income do not have a direct counterpart to the notion of capital income in the model. This notion is simply all factor income in the economy that does not accrue to labor. Survey measures, by contrast, do not include unrealized capital gains and may suffer from measurement error due to unreported assets or uncertainty about the valuation of private businesses. Therefore, one would need to impute capital income in survey data in order to derive an empirical analog to the model variable ζ. See Birinci, Karabarbounis, and See (2026) for such an attempt in a static model. I also note that the notion of the labor share relevant to labor supply has consumption in the denominator and not income. In the static model, consumption and income are equal to each other, but in a dynamic model they do not because households can save and borrow. Thus, one would need to use data on consumption, as opposed to capital income, if the underlying model is dynamic.

ad-hoc rule for allocating capital income is replaced by an endogenously determined capital income at a rate of return that clears the capital market. Early work on incomplete asset market models with endogenous labor supply includes Pijoan-Mas (2006), Chang and Kim (2006), and Marcet, Obiols-Homs, and Weil (2007). Relative to these papers, I add heterogeneity in the disutility of work, in order to mimic the model with exogenous capital income; heterogeneity in the discount factor, in order to generate a realistic wealth distribution; and taxes and transfers, which generate a wedge between consumption and earned income.

3.1 The Model

I focus on a stationary equilibrium of the model in which aggregates and the distribution of households are constant and omit time subscripts for these objects. When calculating the macro elasticity of labor supply, I will perform comparative statics across stationary equilibria indexed by different levels of aggregate productivity, which approximates the experiment of comparing hours over distant time periods or across countries. Households face shocks and change their decisions over time and thus I retain time subscripts when describing household-level variables.

Firm. Production is of the Cobb-Douglas form with respect to capital and aggregate labor services:

$$Y = AK^{\alpha}L^{1-\alpha}. \tag{14}$$

The firm takes the aggregate return on capital, r, and the aggregate wage, w, as given and chooses K and L to maximize profits, $\Pi = Y - rK - wL$. The first-order condition for labor is the same as in equation (2). In combination with the first-order condition for capital, we obtain the optimal capital-to-labor ratio:

$$\frac{K}{L} = \frac{\alpha}{1-\alpha}\frac{w}{r}. \tag{15}$$

With a Cobb-Douglas production function, aggregate income is distributed with constant shares to capital income, $rK = \alpha Y$, and to labor income, $wL = (1-\alpha)Y$. What the exogenous capital income model labeled as profits, $\Pi = \alpha Y$, is now accruing to capital owners in the form of income from renting their capital to the firm.

Households. There is a continuum of households indexed by $\iota \in [0, 1]$ and distributed according to the function $F(\iota)$. Households are exogenously heterogeneous in three dimensions: their efficiency units of labor, $\theta_t(\iota)$, their disutility of work, $\chi_t(\iota)$, and their discount factor $\beta(\iota)$.

Relative to the static model, here I allow θ and χ to vary over time, because this gives households a motive to save. The heterogeneity in β is permanent.

Households derive utility from the same separable, isoelastic preferences in equation (3). They choose consumption $c_t(\iota)$, hours $n_t(\iota)$, and capital $k_{t+1}(\iota)$ to maximize their expected discounted utility. The Bellman equation is

$$V_t(k_t, \theta_t, \chi_t, \beta) = \max_{c_t, n_t, k_{t+1}} \left\{ \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \frac{\chi_t n_t^{1+1/\varepsilon}}{1+1/\varepsilon} + \beta \mathbb{E}_t V_{t+1}(k_{t+1}, \theta_{t+1}, \chi_{t+1}, \beta) \right\}, \tag{16}$$

where $\beta \in (0,1)$ is the discount factor. Households take the aggregate return on capital, r, the aggregate wage, w, and transfers, T, as given and maximize their value in equation (16) subject to the budget constraint:

$$c_t + k_{t+1} = (1-\tau)(w\theta_t n_t + r k_t) + (1-\delta)k_t + T, \tag{17}$$

where $\delta \in [0,1]$ is the depreciation rate of capital. Relative to the model in the previous section, non-labor income has been replaced by capital income from renting assets to the firm. The budget constraint also adds a proportional tax on all income, τ, and a lump-sum transfer to all households, T. Households are subject to a borrowing constraint:

$$k_{t+1} \geq -b, \tag{18}$$

where $b \geq 0$ is the exogenous borrowing limit. Optimal labor supply satisfies a modified version of the first-order condition in equation (5) that includes taxes, $\chi_t(\iota) n_t(\iota)^{1/\varepsilon} c_t(\iota)^{\gamma} = (1-\tau) w \theta_t(\iota)$. Optimal savings are determined by a standard Euler equation, $c_t(\iota)^{-\gamma} = \beta(\iota)\mathbb{E}_t(1 + (1-\tau)r - \delta)c_{t+1}(\iota)^{-\gamma}$ if a household is unconstrained, $k_{t+1} > -b$, and residually by the budget constraint in equation (17) if a household is constrained, $k_{t+1} = -b$.

Stationary Equilibrium. A stationary equilibrium consists of an aggregate return to capital, r, aggregate wage w, aggregate quantities (Y, K, L, T), and household decision rules $c_t(k_t, \theta_t, \chi_t, \beta)$, $n_t(k_t, \theta_t, \chi_t, \beta)$, and $k_{t+1}(k_t, \theta_t, \chi_t, \beta)$ such that:

1. taking as given factor prices, r and w, the firm maximizes profits by setting labor demand according to equation (2) and the capital-to-labor ratio according to equation (15);

2. taking as given factor prices, r and w, and transfers, T, each household $\iota \equiv (k, \theta, \chi, \beta)$ maximizes its value function in equation (16) subject to the budget constraint (17) and the borrowing constraint (18);

3. the government balances its budget, $T = \tau \int (w\theta_t(\iota)n_t(\iota) + rk_t(\iota))\mathrm{d}F(\iota) = \tau(wL + rK) = \tau Y$;

4. the goods market clears, $Y = \int (c_t(\iota) + k_{t+1}(\iota) - (1-\delta)k_t(\iota))\mathrm{d}F(\iota)$, the labor market clears, $L = \int \theta_t(\iota)n_t(\iota)\mathrm{d}F(\iota)$, and the capital market clears, $K = \int k_t(\iota)\mathrm{d}F(\iota)$;

5. the distribution of households, $F(\iota)$, is stationary.

Appendix B outlines the detailed steps to solve and quantify the model.[14]

3.2 Quantification

The strategy for quantifying the model with endogenous capital income is different from the one adopted for the model with exogenous capital income. In the latter, I identified parameters in closed form such that the model matches targets from the data, including the micro and the macro elasticities. The reason for doing so was to illustrate the key mechanisms by which a simple model of labor supply can match both statistics. I am now interested in a different question: how close does the model with endogenous savings and labor supply come to generating as outcomes the estimated micro and macro elasticities from the data?

In quantifying the model with endogenous capital income, I target the same six cross-sectional variances and covariances of wages, hours, and consumption as in the model with exogenous capital income. The next proposition formalizes how second moments of the joint distribution of $(\log\theta, \log n, \log c)$ discipline the second moments of the labor share distribution.

Proposition 2 (Labor Share Moments)**.** The cross-sectional moments of the labor share are given by

$$\mathrm{var}(\log\zeta) = \sigma_\theta^2 + \sigma_n^2 + \sigma_c^2 + 2\,\sigma_{\theta n} - 2\,\sigma_{\theta c} - 2\,\sigma_{nc}, \tag{19}$$

$$\mathrm{cov}(\log\zeta,\, \log\theta) = \sigma_\theta^2 + \sigma_{\theta n} - \sigma_{\theta c}, \tag{20}$$

$$\mathrm{cov}(\log\zeta,\, \log n) = \sigma_{\theta n} + \sigma_n^2 - \sigma_{nc}, \tag{21}$$

$$\mathrm{cov}(\log\zeta,\, \log c) = \sigma_{\theta c} + \sigma_{nc} - \sigma_c^2. \tag{22}$$

[14] To summarize, idiosyncratic productivity and disutility of work shocks are discretized using a variation of the method of Tauchen (1986) and asset holdings are discretized on an exponentially-spaced grid. I solve for the stationary equilibrium with a nested fixed point algorithm. The outer loop updates transfers using the tax rate times aggregate output. The second loop updates the capital-labor ratio with bisection on this ratio until firm and household optimal policies are consistent. Given aggregates, the inner loop solves for household policies using the endogenous grid method of Carroll (2006) and for the stationary distribution using the non-stochastic histogram of Young (2010).

The labor share is defined as $\zeta(\iota) = (1-\tau)w\theta(\iota)n(\iota)/c(\iota)$. Taking logs, $\log\zeta(\iota) = \log((1-\tau)w) + \log\theta(\iota) + \log n(\iota) - \log c(\iota)$, where $\log((1-\tau)w)$ does not vary across households. Given the cross-sectional variances and covariances of $(\log\theta,\ \log n,\ \log c)$, the variance of $\log\zeta$ and the covariances of $\log\zeta$ with each of $\log\theta$, $\log n$, and $\log c$ are uniquely determined by equations (19) to (22). This proposition follows from the definition of the labor share as a function of wages, hours, and consumption and the linearity of the covariance operator.

An important implication of this proposition is that second moments of the labor share distribution are not free parameters. These moments are already pinned down by the cross-sectional variances and covariances of $(\log\theta,\ \log n,\ \log c)$ that the model will target. Thus, even if one had direct measurements of the labor share in the data, these measurements would have no additional information about the second moments of the labor share distribution.[15]

The difference between the model with exogenous and the model with endogenous capital income is that the latter restricts cross-sectional moments of $(\log\theta,\ \log n,\ \log c)$ to be consistent with the savings decisions of households in general equilibrium. Thus, targeting moments of the wealth distribution provides an external validation of the model, because savings decisions affect the joint moments of $(\log\theta,\ \log n,\ \log c)$. In essence, the validation exercise is whether the model can generate the micro and macro elasticities, while being disciplined by moments of the wealth distribution. These moments did not exist in the model with exogenous capital income and so the distribution of the labor share was free to match the elasticities by design.

Table 4 presents the parameters of the model with endogenous capital income. The top panel of the table shows parameters that are set externally, before solving the model. Consistent with the strategy of validating the model against the micro and macro elasticities, I fix preference and technology parameters to the same values used in the model with exogenous capital income, $\varepsilon = 0.54$, $\gamma = 1.58$, and $\alpha = 0.37$. The depreciation rate, $\delta = 0.066$, is calculated using depreciation expenditures divided by fixed assets from the Bureau of Economic Analysis Fixed Asset Tables (Tables 1.1 and 1.3). The tax rate, $\tau = 0.212$, is calculated using current tax receipts less taxes on products and imports divided by GDP from the Bureau of Economic Analysis National Income and Product Accounts (Tables 1.1.5 and 3.1). Idiosyncratic productivity, $\theta_t(\iota)$, follows

[15] Further, the mean labor share is also disciplined by the covariance of the labor share with consumption, the aggregate labor share $1-\alpha$, the data I use on depreciation expenses, $\delta K/Y$, and taxes τ. Starting from the definition of the aggregate labor share, $\int w\theta(\iota)n(\iota)\mathrm{d}F(\iota) = (1-\alpha)Y$, and using the definition of the labor share we obtain $\int \zeta(\iota)c(\iota)\mathrm{d}F(\iota) = (1-\tau)(1-\alpha)Y$. Dividing by consumption and using the steady-state condition $Y/C = 1/(1-\delta K/Y)$, we obtain $\int \zeta(\iota)(c(\iota)/C)\mathrm{d}F(\iota) = (1-\tau)(1-\alpha)/(1-\delta K/Y)$. Thus, the mean labor share is $\int \zeta(\iota)\mathrm{d}F(\iota) = (1-\tau)(1-\alpha)/(1-\delta K/Y) - \operatorname{cov}(\zeta(\iota), c(\iota)/C)$.

Table 4: Parameters, Endogenous Capital Income

Fixed Parameter	Description	Value
ε	Frisch elasticity	0.540
γ	curvature with respect to consumption	1.584
α	capital share in production	0.370
δ	depreciation rate	0.066
τ	tax rate	0.212
ρ_θ	persistence of productivity	0.970
ρ_χ	persistence of disutility of work	0.730
Calibrated Parameter	**Description**	**Value**
σ_θ^2	variance of $\ln\theta$	0.217
σ_χ^2	variance of $\ln\chi$	1.167
$\sigma_{\theta\chi}$	$\text{cov}(\ln\theta, \ln\chi)$, corr 0.412	0.207
β_m	central discount factor	0.960
u_β	spread in discount factors	0.012
φ	share with low discount factor	0.684
b	borrowing limit	0.085

Notes. The table reports the parameters of the model with endogenous capital income. The top panel lists parameters set externally, before solving the model. The bottom panel lists parameters calibrated internally by minimizing the distance between the moments generated by the model and their data analogs in Table 5. The reported σ_θ^2 and σ_χ^2 are the unconditional variances of the continuous AR(1) processes used as inputs to the Tauchen discretization. The realized variances in the finite-state stationary distribution will in general differ. The model variance of log wages is reported in Table 5.

an AR(1) process with persistence $\rho_\theta = 0.97$ (Krueger, Mitman, and Perri, 2016). The disutility of work, $\chi_t(\iota)$, also follows an AR(1) process, with persistence $\rho_\chi = 0.73$ (Erosa, Fuster, and Kambourov, 2016).[16]

The lower panel of the table lists the seven parameters that are calibrated internally by minimizing an objective function that depends on the distance between moments generated by the model and their analogs in the data. The joint stochastic process for the innovations to the exogenous sources of heterogeneity is parameterized by the unconditional variance σ_θ^2

[16]The value of γ from the exogenous capital income model is consistent with several independent estimates in the literature, such as Pijoan-Mas (2006) at 1.46, Domeij and Flóden (2006) and Kaplan (2012) at 1.5, Aguiar and Hurst (2013) at 1.49, and Heathcote, Storesletten, and Violante (2014) at 1.71. Section 5 shows that the ability of the model to generate the gap between the micro and the macro elasticity is robust to a lower value of γ. Additionally, the section presents sensitivity analyses with respect to the Frisch elasticity, the depreciation rate, the tax rate, and the persistence of the stochastic processes. The conclusions are robust to reasonable perturbations of these parameters.

Table 5: Moments, Endogenous Capital Income

Targeted Moment	Data Value	Model Value
variance of log wages	0.36	0.35
variance of log hours	0.12	0.28
variance of log consumption	0.16	0.18
correlation of log wages, hours	−0.10	−0.10
correlation of log wages, consumption	0.50	0.82
correlation of log hours, consumption	0.22	0.10
capital-output ratio	3.25	3.22
top 10% wealth share	0.65	0.65
fraction with negative assets	0.20	0.20
fraction hand-to-mouth	0.11	0.13
Untargeted Moment	**Data Value**	**Model Value**
macro elasticity	−0.17	−0.17
micro elasticity	0.06	0.05
median ζ	0.43	0.56
variance of $\ln\zeta$	0.30	0.29
$\text{corr}(\ln\zeta, \ln\theta)$	0.67	0.36
$\text{corr}(\ln\zeta, \ln\chi)$	0.19	−0.61
$\text{corr}(\ln\zeta, \beta)$	–	−0.29

Notes. The upper panel reports the ten data moments targeted in the calibration of the model with endogenous capital income and the corresponding moments generated by the model. The bottom panel reports model-generated statistics of the household-level labor share distribution and the average micro and macro elasticities of labor supply. For comparability, the data values for the statistics of the household-level labor share are populated with the statistics of the exogenous capital income model.

for $\log\theta$, the unconditional variance σ_χ^2 for $\log\chi$, and the covariance $\sigma_{\theta\chi}$ between $\log\theta$ and $\log\chi$. Following Krueger, Mitman, and Perri (2016), there are two types of households, with discount factors given by $\beta_m - u_\beta$ and $\beta_m + u_\beta$, where β_m is a central value and u_β governs the dispersion in discounting between types. The population share of the impatient types is φ. The final parameter to be calibrated is the borrowing limit b.

I use ten moments to estimate these parameters, listed in the upper panel of Table 5.[17] Six of them are the same cross-sectional moments used in the model with exogenous capital income

[17] Section 5 presents sensitivity analyses with respect to the choice of cross-sectional moments and the target capital-to-output ratio. The conclusions are robust to using cross-sectional moments from Boerma and Karabarbounis (2021) instead of Heathcote, Storesletten, and Violante (2014) and to targeting a capital-output ratio that excludes consumer durables from fixed assets.

and, as discussed above, pin down the second moments of the labor share distribution. I add four additional targets in order to match the four new parameters of the model with endogenous capital income. The aggregate capital-output ratio, K/Y, calculated as the ratio of fixed assets to GDP, is closely related to the central value for the discount factor, β_m. The top 10 percent wealth share, calculated directly from the Distributional National Accounts, is related to the dispersion in discount factors, u_β. The fraction of households with negative net worth, taken from the calculations of Wolff (2021) from the Survey of Consumer Finances, and the share of households at the borrowing constraint, taken from Kaplan, Violante, and Weidner (2014), are informative about the share of impatient households, φ, and the borrowing limit, b.[18]

The estimated parameters that minimize the distance between model outcomes and data targets are presented in the lower panel of Table 4. The central value of the discount factor is 0.96, with a spread of 2.4 percentage points between the two types.[19] Around two-thirds of households have the low discount factor. The value of the borrowing limit implies that households can borrow up to roughly 4 percent of the average income in the economy. As in the model with exogenous capital income, productivity is positively correlated with the disutility of work, with the model with endogenous capital income generating a correlation of 0.41 as opposed to a correlation of 0.34 in the model with exogenous capital income.

With ten moments and seven parameters, the model does not necessarily match every targeted moment.[20] However, as the upper panel of Table 5 shows, the model does a good job matching most of the targets. It matches almost perfectly the variances of wages and consumption and the correlation of hours with wages. The model is consistent with the low correlation between consumption and hours that is observed in the data. Crucially for understanding the role of income effects for the divergence of the micro and macro elasticities, the model matches almost perfectly the four targets related to the distribution of capital income. The

[18] Kaplan, Violante, and Weidner (2014) calculate that about one-third of households are hand-to-mouth, with two-thirds of these households being wealthy in illiquid assets. The model cannot account for this type of household and, thus, I target the share of poor households that are hand-to-mouth. Given that b is expected to be positive, the target of 11 percent is also consistent with the 20 percent target for the share of households with negative net worth.

[19] The discount factor may appear high, but the model also includes taxes on capital income. For the patient household, $(\beta_m + u_\beta)(1 + (1 - \tau)r - \delta) < 1$, because there is positive mass of households at the borrowing limit and in the stationary distribution even patient households may face a sequence of negative shocks that drives them to the borrowing limit. At the equilibrium gross rate of return, $r = 0.115$, the after-tax net return on capital is about 2.5 percent.

[20] I do not perform a formal overidentification test, because the moments I target are derived from different studies or sources. For example, it is not feasible to calculate the covariance between the capital-output ratio from the national income and product accounts and the correlation of wages with hours from survey data.

capital-output ratio is related to the average capital share of households, whereas the top 10 percent wealth share, share of households with negative assets, and share of households at the borrowing constraint discipline the capital income distribution.[21]

The two most notable deviations of the model from the data concern the correlation of wages with consumption, which is around 0.8 in the model as opposed to 0.5 in the data, and the variance of log hours, with the model generating more than twice the dispersion in hours observed in the data. Section 5 recalibrates the model to match each of these moments exactly. In both exercises, the gap between micro and macro elasticities, the distribution of elasticities in the population, and the strength of the income effects are essentially unchanged. Therefore, the inability of the model to match these two statistics reflects that seven parameters cannot hit exactly ten moments, rather than a fundamental tension in the mechanism that reconciles the gap between micro and macro elasticities.

The headline result of this analysis is presented at the bottom panel of Table 5. The model with endogenous capital income comes very close to matching the estimated micro and macro elasticities from the data. This is an external validation of the core mechanism in the paper, because the model generates the gap between the micro and the macro elasticities observed in the data, while being disciplined by both moments of wages, hours, consumption (which pin down moments of the labor share) and by moments of wealth.

Beginning with the macro elasticity, the model with endogenous capital income matches perfectly its value in the data.[22] This elasticity was targeted in the model with exogenous capital income by choosing the value of the curvature parameter in the utility function, $\gamma = 1.58$. There is no theorem that the macro elasticities should match across models and so the result here is numerical. To gain intuition why the macro elasticity is the same across models, I note that the macro elasticity in the model with endogenous capital income would still be given by the formula in equation (10), if the equilibrium rate of return is invariant and if households increase their consumption proportionally in response to a change in aggregate productivity (see Appendix A.4 for a proof and a weaker sufficient condition). The similarity of the macro elasticity between the two models arises because the wedge between a deterministic Euler equation that pins down r as a function of only the discount factor and the Euler equation in

[21] The model also generates a wealth Gini coefficient of 0.81. Using various waves of the Survey of Consumer Finances, Kuhn and Ríos-Rull (2025) report an average Gini coefficient of 0.83.

[22] This elasticity is calculated by varying aggregate productivity A across stationary equilibria. The responses are almost linear in the size of the change in aggregate productivity. The macro elasticity coincides to the third digit when aggregate productivity changes by 1 percent or when it changes by 100 percent.

the model with incomplete markets is quantitatively unresponsive to aggregate productivity.

Turning to the micro elasticity, the model with endogenous capital income comes very close to matching the micro elasticity in the data. Recall that this elasticity was previously targeted in the model with exogenous capital income by choosing parameters of the distribution of the labor share ζ. The difference between the two models is that the model with endogenous capital income generates a labor share distribution as the outcome of optimal labor supply and savings decisions. The bottom panel of Table 5 presents some statistics of the labor share distribution. To enable comparisons across models, the data values for the statistics of the labor share are populated with the statistics of the exogenous capital income model. The model with endogenous capital income produces a higher median labor share than the model with exogenous capital income. The borrowing constraint precludes households from increasing their labor share too much, so the model with endogenous capital income does not feature a heavy tail similar to the one under the Pareto distribution. On the other hand, the two models feature almost the same dispersion in the labor share.[23]

The correlation between the labor share and the wage is positive in both models, although the model with endogenous capital income produces a weaker correlation of 0.36 relative to the model with exogenous capital income, which produces a correlation of 0.67. The most notable difference between the two models is that households with a higher disutility of work tend to have a lower labor share in the model with endogenous capital income, whereas this correlation was weak but positive in the model with exogenous capital income. The difference occurs because in the model with endogenous savings, capital income is predetermined and thus a higher disutility of work lowers labor relative to capital income. Finally, the model with endogenous capital income produces a negative correlation between the discount factor and the labor share, because more patient households tend to save more and have higher capital income.

The reconciliation between the micro and macro elasticities rests on the latter being calculated across stationary equilibria. Over the long run, capital income adjusts alongside labor

[23] The model with exogenous capital income does not feature taxes, because ζ was directly inferred as opposed to generated endogenously. No moment is affected by this choice, as long as the tax rate is common across households, as in the model with endogenous capital income. As discussed in more detail in Appendix A.3, under the exogenous capital income, targeting the mean micro Marshallian elasticity while respecting aggregate market clearing implies that the marginal distribution of the labor share has to concentrate mass near a lower floor and admit a heavy tail extending to labor shares above one. In the endogenous capital income model, the same restrictions are satisfied through different mechanisms. The labor share distribution is bounded above by the borrowing limit and is shaped by households' optimal savings choices and aggregate market clearing. The median labor share lands near the mean and the distribution does not feature a heavy tail.

income and the income effect becomes stronger. Comparisons across stationary equilibria are appropriate for the macro elasticity, because data for this elasticity come from secular declines in hours, as in Boppart and Krusell (2020), or cross-country comparisons, as in Bick, Fuchs-Schündeln, and Lagakos (2018), which implicitly allow for capital income to adjust with labor income. I do not solve for transitional dynamics because, in the short run, capital is essentially fixed and the macro elasticity converges toward the micro elasticity. This is the familiar pattern that occurs in the stochastic neoclassical growth model, populated by a representative household with log preferences over consumption and a representative firm that uses a Cobb-Douglas production function. With log preferences, hours do not change in the balanced growth path, as capital income scales one-to-one with labor income. In the short run, however, capital is fixed and thus hours respond positively to productivity shocks, even with log preferences. The results here generalize this benchmark to allow for different curvature in preferences with respect to consumption and for household heterogeneity.

4 Distribution of Elasticities and Income Effects

This section discusses the distribution of the micro elasticity of labor supply and the income effects that shape this distribution. Throughout my discussion, I focus on comparing the outcomes generated by my models to analogous statistics either generated in other structural models or estimated in studies with quasi-experimental variation. The main finding is that the model-generated statistics lie comfortably in the range of estimates reported in the literature. These comparisons show that the income effect on labor supply is a quantitatively plausible mechanism to reconcile the gap between the micro and macro elasticities.

Table 6 presents the distribution of the Marshallian elasticity of labor supply generated by the model with exogenous capital income and the model with endogenous capital income. While the models generate almost the same elasticity mean, they differ somewhat in their percentiles of elasticities, especially at the top of the distribution. Recall that the labor share is inversely related to the Marshallian elasticity of labor supply. The difference between models reflects differences at the bottom of the distribution of the labor share. In the model with endogenous capital income, there is a large mass of households with a low labor share. These are the households who have accumulated significant wealth and finance their consumption mostly with capital income. As a result, there is quite a bit of dispersion in the elasticities across households, with the interquartile range being 0.21 and the 90th percentile of the distribution

Table 6: Distribution of Marshallian Elasticity of Labor Supply

Moment	Exo capital income	Endo capital income	Attanasio et al. (2018)
10th	−0.11	−0.15	−0.14
25th	0.05	−0.06	0.01
50th	0.13	0.04	0.18
75th	0.15	0.15	0.39
90th	0.16	0.26	0.79
Average	0.06	0.05	N/A

Notes. The table reports percentiles of the Marshallian elasticity of labor supply across households in the model with exogenous capital income and the model with endogenous capital income. The third column reports the corresponding distribution from the structural model of Attanasio, Levell, Low, and Sánchez-Marcos (2018).

having an elasticity of 0.26. In the model with exogenous capital income, there is a large mass of households close to the median. This mass occurs because the model assumes a distribution of the labor share that belongs to the Pareto family, and the large mass close to the median compensates for the heavy tail in order for the model to match the target value for the mean micro elasticity. Thus, the interquartile range and the 90th percentile of the distribution are noticeably smaller.

The third column of the table adds the distribution of elasticities generated by the structural model of Attanasio, Levell, Low, and Sánchez-Marcos (2018). The authors develop a life-cycle model for women, featuring flexible preferences and rich heterogeneity, and estimate it using the Consumer Expenditure Survey. While their model differs quite substantially from my models, the distribution of elasticities in their model is quite similar to that in the endogenous capital income model at the bottom half of the population. The elasticities generated by their model are generally higher than the elasticities in my models, with most of the divergence occurring at the top of the distribution. The differences between their and my results arise from differences in the population of interest and in model structure. The estimates in Attanasio, Levell, Low, and Sánchez-Marcos (2018) concern women's labor supply only, whereas I targeted a micro elasticity of 0.06 from the meta-analysis of Keane (2011) that concerns men. Estimated elasticities are typically found to be larger for women than for men (Blau and Kahn, 2007; Gelber, 2014). Additionally, as equation (9) shows, the Marshallian elasticity in my model with separable preferences is bounded above by the Frisch elasticity $\varepsilon = 0.54$, with the bound corresponding to households with zero labor share. Attanasio, Levell, Low, and Sánchez-Marcos (2018) have

Table 7: Marginal Propensity to Earn

Size of Transfer	Mean	10th	25th	50th	75th	90th
Panel A: Exogenous capital income						
0.01	−0.31	−0.42	−0.31	−0.27	−0.25	−0.24
0.05	−0.30	−0.42	−0.31	−0.26	−0.24	−0.24
0.10	−0.30	−0.41	−0.30	−0.26	−0.24	−0.23
0.20	−0.28	−0.40	−0.29	−0.24	−0.22	−0.21
Panel B: Endogenous capital income						
0.01	−0.49	−0.80	−0.60	−0.45	−0.34	−0.24
0.05	−0.48	−0.78	−0.58	−0.44	−0.32	−0.23
0.10	−0.46	−0.76	−0.56	−0.42	−0.31	−0.22
0.20	−0.43	−0.72	−0.52	−0.39	−0.28	−0.20

Notes. The table reports statistics of the marginal propensity to earn, which is defined as the ratio of the change in earnings to the change in non-labor income. The statistics are reported across households in the model with exogenous capital income and the model with endogenous capital income and for hypothetical lump-sum transfers of different sizes relative to average income.

a complementarity between consumption and work in preferences that allows them to generate larger elasticities at the top half of the distribution.

In recent work, Graber, Håvarstein, Mogstad, Torsvik, and Vestad (2026) use Norwegian administrative data to examine earned income responses following a tax reform that reduced marginal tax rates. Combining these responses with responses to lottery winnings that isolate income effects, they estimate uncompensated elasticities of around zero at the median and 0.3 at the 90th percentile. The model with endogenous capital income comes very close to these estimates, as it produces an elasticity of 0.04 at the median and 0.26 at the 90th percentile.

The labor supply response from changes in non-labor income is the key mechanism that allows my models to simultaneously generate negative and large in magnitude macro elasticities and smaller but positive micro elasticities. How does the strength of the income effects generated by my models compare to the strength of the income effects estimated in quasi-experimental studies? To answer this question, Table 7 presents estimates of the marginal propensity to earn, which is the ratio of changes in earnings to changes in non-labor income. To calculate the marginal propensity to earn in the model with exogenous capital income, I hold constant the primitives that differentiate households and give every household an additional lump-sum transfer. I mimic the setup of the empirical papers these estimates compare to by computing the new level of labor supply while holding constant the aggregate wage and aggregate profits. The experiment is similar in spirit for the model with endogenous capital income. Holding constant aggregate factor prices and aggregate transfers, I solve for the new level of labor supply for every household that receives a hypothetical additional lump-sum transfer. Households treat this transfer as a new parameter in their problem, so they behave as if the transfer is permanent.

Each entry in Table 7 presents a statistic of the marginal propensity to earn, differentiated by model and the size of the lump-sum transfer relative to average income. The model with exogenous capital income generates a mean marginal propensity to earn of around -0.3. In this model, there is not much dispersion in the marginal propensity to earn across households. The model with endogenous capital income generates a lower mean marginal propensity to earn, at around -0.5. Here, the dispersion is significantly larger, with the interquartile range of the marginal propensities to earn being around 0.25. The dispersion in income effects generated by the model is accounted for by the heterogeneity in labor shares, $\zeta(\iota)$. Households with higher labor relative to capital income face stronger income effects and their earnings respond more negatively to a given lump-sum transfer. These are the same households with a lower micro elasticity with respect to the wage, as equation (9) shows. Finally, in both models, the marginal propensity to earn decreases in absolute value when the size of the transfer increases, because earnings are bounded below by zero.

Golosov, Graber, Mogstad, and Novgorodsky (2024) conduct an event study of the earnings of U.S. lottery winners. The typical lottery winner has a permanent windfall between 0.05 and 0.10 of average income, which are values that Table 7 includes for comparability. Their headline number is that the marginal propensity to earn is -0.52. Similar to my models, they also document that income effects decrease in absolute value when the size of the prize increases. Cesarini, Lindqvist, Notowidigdo, and Östling (2017) study lottery winners in Sweden. Golosov, Graber, Mogstad, and Novgorodsky (2024) show that converting the estimates from the Swedish experiment to be comparable with the estimates from the U.S. experiment implies a marginal propensity to earn of -0.33 in the study of Cesarini, Lindqvist, Notowidigdo, and Östling (2017). My estimates are near the range of these two papers.

5 Sensitivity Analyses

This section presents sensitivity analyses with respect to the parameters and targeted moments used in the baseline analysis of the model with endogenous capital income. I find that the

Table 8: Sensitivity Analyses, Persistence of Productivity and Disutility of Work

Statistic	Data	Baseline	$\rho_\theta = 0.95$	$\rho_\theta = 0.99$	$\rho_\chi = 0.00$	$\rho_\chi = 0.97$
$\text{var}(\ln\theta)$	0.36	0.35	0.33	0.47	0.58	0.35
$\text{var}(\ln n)$	0.12	0.28	0.27	0.29	0.32	0.26
$\text{var}(\ln c)$	0.16	0.18	0.14	0.24	0.20	0.24
$\text{corr}(\ln n, \ln\theta)$	−0.10	−0.10	−0.15	−0.04	−0.08	−0.11
$\text{corr}(\ln\theta, \ln c)$	0.50	0.82	0.76	0.90	0.87	0.69
$\text{corr}(\ln n, \ln c)$	0.22	0.10	0.08	0.13	−0.08	0.38
K/Y	3.25	3.22	3.26	3.14	3.31	3.23
top 10% k share	0.65	0.65	0.60	0.62	0.62	0.83
fraction $k < 0$	0.20	0.20	0.20	0.20	0.14	0.73
fraction $k = -b$	0.11	0.13	0.14	0.12	0.08	0.70
macro elasticity	−0.17	−0.17	−0.17	−0.17	−0.17	−0.17
mean micro elasticity	0.06	0.05	0.04	0.06	0.07	0.03
90-50 diff Marshallian	0.30	0.22	0.22	0.22	0.24	0.20
median ζ	0.43	0.56	0.56	0.54	0.51	0.63
variance of $\ln\zeta$	0.30	0.29	0.30	0.29	0.48	0.19
$\text{corr}(\ln\zeta, \ln\theta)$	0.67	0.36	0.39	0.40	0.47	0.46
$\text{corr}(\ln\zeta, \ln\chi)$	0.19	−0.61	−0.52	−0.70	−0.57	−0.35
$\text{corr}(\ln\zeta, \beta)$	–	−0.29	−0.29	−0.28	−0.24	−0.39
MPE at lot $= 0.05 \cdot Y$	−0.52	−0.48	−0.50	−0.45	−0.49	−0.43

Notes. The table reports the moments from Table 5, the difference between the 90th and the 50th percentiles of the Marshallian elasticity of labor supply, and the marginal propensity to earn in response to a lump-sum transfer of 5 percent of average income. The first two columns report the values in the data and in the baseline model with endogenous capital income. The remaining columns report the values under alternative calibrations that vary the persistence of productivity, ρ_θ, or the persistence of the disutility of work, ρ_χ, relative to the baseline. The data value for the difference between the 90th and the 50th percentiles of the Marshallian elasticity is from Graber, Håvarstein, Mogstad, Torsvik, and Vestad (2026), and the data value for the marginal propensity to earn is from Golosov, Graber, Mogstad, and Novgorodsky (2024).

model robustly generates the gap between the micro and macro elasticities. Additionally, the model always generates distributions of elasticities and a strength of income effects that is very similar to the ones discussed in the previous section.

I begin my analysis by changing selected parameters one at a time, while keeping all other parameters at their calibrated baseline values. This exercise clarifies how individual parameters affect targeted and untargeted moments. Table 8 presents the moments previously shown in Table 5 alongside the difference between the 90th and 50th percentile of the Marshallian elasticities and the marginal propensity to earn in response to a lump-sum transfer of 5 percent of average income. I benchmark the latter two moments to the quasi-experimental studies discussed above by Graber, Håvarstein, Mogstad, Torsvik, and Vestad (2026) and Golosov, Graber, Mogstad, and Novgorodsky (2024). The first two columns present the values of the moments in the data and the values generated by the baseline model. The next two columns show the sensitivity of the results when varying the persistence of productivity from $\rho_\theta = 0.97$ in the baseline to either $\rho_\theta = 0.95$ or to $\rho_\theta = 0.99$ and the last two columns show the sensitivity when varying the persistence of the disutility of work from $\rho_\chi = 0.73$ in the baseline to either $\rho_\chi = 0$ or to $\rho_\chi = 0.97$. The mean micro elasticity generated by the model is not very sensitive to these perturbations. It tends to increase with the persistence of productivity and to decrease with the persistence of disutility, but all mean elasticities are between 0.03 and 0.07, which is significantly higher than the macro elasticity of -0.17. The marginal propensity to earn also does not change significantly. More persistent shocks to the disutility of work reduce somewhat the gap between the variance of hours and the covariance of wages with consumption in the data and their respective values in the baseline model.

Table 9 presents sensitivity analyses with respect to the variance-covariance matrix of productivity and disutility of work shocks. A concern about the baseline model is that it produces a significantly higher correlation between wages and consumption than observed in the data. In the third column of the table, I significantly lower the variance of wages in order to match this correlation exactly. The gap between the micro and the macro elasticities remains significant, but the distribution of the micro elasticity centers around a somewhat smaller value and features somewhat less dispersion. The marginal propensity to earn is not significantly affected. A second concern about the baseline model is that it features hours that are too dispersed relative to the data. In the fourth column, I significantly lower the variance of the disutility of work, in order to match the variance of hours in the data. The results are very similar to those under a lower dispersion in productivity. Finally, as the last column shows, the model-generated values of the micro elasticity and marginal propensities to earn are not sensitive to the positive correlation between productivity and disutility of work.

Table 10 presents sensitivity analyses with respect to the tax rate and the preference parameters. For the tax rate, I choose a value of $\tau = 0.264$, which arises if consumption taxes are

Table 9: Sensitivity Analyses, Variance-Covariance Matrix

Statistic	Data	Baseline	$\sigma_\theta^2 = 0.054$	$\sigma_\chi^2 = 0.47$	$\sigma_{\theta\chi} = 0.0$
$\text{var}(\ln\theta)$	0.36	0.35	0.13	0.41	0.35
$\text{var}(\ln n)$	0.12	0.28	0.27	0.12	0.29
$\text{var}(\ln c)$	0.16	0.18	0.06	0.18	0.20
$\text{corr}(\ln n, \ln\theta)$	−0.10	−0.10	−0.29	−0.17	−0.01
$\text{corr}(\ln\theta, \ln c)$	0.50	0.82	0.50	0.86	0.85
$\text{corr}(\ln n, \ln c)$	0.22	0.10	0.14	−0.13	0.16
K/Y	3.25	3.22	3.13	3.21	3.26
top 10% k share	0.65	0.65	0.56	0.73	0.64
fraction $k < 0$	0.20	0.20	0.18	0.36	0.20
fraction $k = -b$	0.11	0.13	0.11	0.24	0.12
macro elasticity	−0.17	−0.17	−0.17	−0.17	−0.17
mean micro elasticity	0.06	0.05	0.03	0.03	0.05
90-50 diff Marshallian	0.30	0.22	0.21	0.20	0.22
median ζ	0.43	0.56	0.61	0.62	0.55
variance of $\ln\zeta$	0.30	0.29	0.22	0.20	0.31
$\text{corr}(\ln\zeta, \ln\theta)$	0.67	0.36	0.18	0.49	0.37
$\text{corr}(\ln\zeta, \ln\chi)$	0.19	−0.61	−0.59	−0.28	−0.68
$\text{corr}(\ln\zeta, \beta)$	–	−0.29	−0.37	−0.39	−0.26
MPE at lot $= 0.05 \cdot Y$	−0.52	−0.48	−0.48	−0.46	−0.49

Notes. The table reports the moments from Table 5, the difference between the 90th and the 50th percentiles of the Marshallian elasticity of labor supply, and the marginal propensity to earn in response to a lump-sum transfer of 5 percent of average income. The first two columns report the values in the data and in the baseline model with endogenous capital income. The remaining columns report the values under alternative calibrations of the variance-covariance matrix of the shocks to productivity and to the disutility of work relative to the baseline. The data value for the difference between the 90th and the 50th percentiles of the Marshallian elasticity is from Graber, Håvarstein, Mogstad, Torsvik, and Vestad (2026), and the data value for the marginal propensity to earn is from Golosov, Graber, Mogstad, and Novgorodsky (2024).

added to the model.[24] A higher tax rate increases the micro elasticity, because the after-tax labor share, $\zeta = (1-\tau)w\theta n/c$, which is relevant for this elasticity, falls.

The next two columns of Table 10 present a first analysis of varying the Frisch elasticity, ε. In the baseline, I adopted the value of $\varepsilon = 0.54$ from Chetty, Guren, Manoli, and Weber

[24] Let τ_c be the consumption tax and τ_y be the income tax. Then, up to transfers, the wedge between income and consumption is given by $\tau = (\tau_c + \tau_y)/(1+\tau_c)$. I measure τ_c using the ratio of taxes on products and imports divided by GDP from the Bureau of Economic Analysis National Income and Product Accounts (Tables 1.1.5 and 3.1).

Table 10: Sensitivity Analyses, Tax and Preference Parameters

Statistic	Data	Baseline	$\tau = 0.264$	$\varepsilon = 0.38$	$\varepsilon = 0.70$	$\gamma = 1.2$
$\text{var}(\ln \theta)$	0.36	0.35	0.35	0.35	0.35	0.35
$\text{var}(\ln n)$	0.12	0.28	0.28	0.15	0.45	0.29
$\text{var}(\ln c)$	0.16	0.18	0.16	0.18	0.18	0.21
$\text{corr}(\ln n, \ln \theta)$	−0.10	−0.10	−0.07	−0.11	−0.09	−0.00
$\text{corr}(\ln \theta, \ln c)$	0.50	0.82	0.81	0.84	0.80	0.81
$\text{corr}(\ln n, \ln c)$	0.22	0.10	0.14	0.04	0.14	0.23
K/Y	3.25	3.22	3.00	3.20	3.26	3.19
top 10% k share	0.65	0.65	0.67	0.71	0.59	0.69
fraction $k < 0$	0.20	0.20	0.24	0.27	0.17	0.25
fraction $k = -b$	0.11	0.13	0.15	0.17	0.11	0.17
macro elasticity	−0.17	−0.17	−0.17	−0.14	−0.19	−0.07
mean micro elasticity	0.06	0.05	0.08	0.02	0.08	0.13
90-50 diff Marshallian	0.30	0.22	0.21	0.14	0.31	0.18
median ζ	0.43	0.56	0.50	0.59	0.54	0.56
variance of $\ln \zeta$	0.30	0.29	0.31	0.19	0.42	0.30
$\text{corr}(\ln \zeta, \ln \theta)$	0.67	0.36	0.43	0.44	0.30	0.40
$\text{corr}(\ln \zeta, \ln \chi)$	0.19	−0.61	−0.61	−0.49	−0.67	−0.59
$\text{corr}(\ln \zeta, \beta)$	–	−0.29	−0.27	−0.35	−0.24	−0.30
MPE at lot $= 0.05 \cdot Y$	−0.52	−0.48	−0.47	−0.36	−0.58	−0.39

Notes. The table reports the moments from Table 5, the difference between the 90th and the 50th percentiles of the Marshallian elasticity of labor supply, and the marginal propensity to earn in response to a lump-sum transfer of 5 percent of average income. The first two columns report the values in the data and in the baseline model with endogenous capital income. The remaining columns report the values under alternative calibrations of the proportional tax rate, τ, the Frisch elasticity of labor supply, ε, and the curvature of utility with respect to consumption, γ, relative to the baseline. The data value for the difference between the 90th and the 50th percentiles of the Marshallian elasticity is from Graber, Håvarstein, Mogstad, Torsvik, and Vestad (2026), and the data value for the marginal propensity to earn is from Golosov, Graber, Mogstad, and Novgorodsky (2024).

(2012). I now consider the values of $\varepsilon = 0.70$ from Pistaferri (2003) and $\varepsilon = 0.38$ for symmetry. Increasing ε to 0.70, while holding constant γ, results in a higher micro elasticity of 0.08, as shown in equation (9). It also makes the macro elasticity more negative, at −0.19. Increasing ε to 0.70 significantly increases the dispersion in the elasticities, with the difference between the 90th percentile and the median matching almost exactly the estimate of Graber, Håvarstein, Mogstad, Torsvik, and Vestad (2026). The only targeted moment that is significantly affected by increasing ε is the variance of hours, which moves further away from its value in the data.

The final column of Table 10 sets the curvature of the utility function with respect to consumption to the lower value of $\gamma = 1.2$ relative to the baseline value of $\gamma = 1.58$. I do not consider lower values, because the macro elasticity would drift too far away from the estimate of -0.15 in Bick, Fuchs-Schündeln, and Lagakos (2018) and the estimate of -0.2 in Boppart and Krusell (2020). While the macro elasticity is now higher at -0.07, the micro elasticity is also higher at an average value of 0.13. Thus, the gap between the two elasticities is quite robust to the value of γ.

These two exercises clarify the role of ε and γ, but their disadvantage is that they do not allow for other parameters to adjust and absorb any gap between the model and the data in other dimensions (for example, the variance of hours). I now vary ε from 0.54 to either 0.38 or 0.70, while fully recalibrating the model. As shown in equation (10), changing the Frisch elasticity also affects the value of γ that targets the macro elasticity in the model with exogenous capital income. I set $\gamma = 1.50$ and $\gamma = 1.74$ respectively to match the same macro elasticity as in the baseline. Appendix C presents the detailed results, which report all the previously presented results in both the model with exogenous and the model with endogenous capital income for different values of ε. To summarize, the results are quite similar to the case in which I varied ε without adjusting other parameters.

Appendix C also reports two more robustness exercises with respect to the moments targeted in the models. The first exercise is to change the six cross-sectional moments that concern the variances and covariances of wages, hours, and consumption from the ones in Heathcote, Storesletten, and Violante (2014) to the ones in Boerma and Karabarbounis (2021). The latter focuses on a sample of married households in a later period of time and features a higher variance of hours, a higher variance of consumption, and a lower correlation of consumption with wages. Despite these differences, most estimated parameters are very similar to the baseline. The micro and macro elasticities generated by the model with endogenous capital income are identical to the baseline. The distribution of elasticities and marginal propensities to earn are very similar to the baseline.

Appendix C also shows robustness to the choice of fixed assets used to measure capital and depreciation. Excluding durables lowers the target capital-to-output ratio from 3.25 in the baseline to 2.94 and the depreciation rate from 0.066 to 0.053. These changes result in a lower central value for the discount factor, a lower borrowing limit, and a more dispersed disutility of work relative to the baseline. Despite these changes, the model generates a micro elasticity of 0.06 as opposed to 0.05 in the baseline and an identical macro elasticity to the baseline. The distribution of elasticities and marginal propensities to earn are also very similar to the baseline.

6 Conclusion

This paper has reconciled micro and macro estimates of the Marshallian elasticity of labor supply using a single utility specification in models with rich household heterogeneity. The reconciliation rests on the simple observation that micro-level estimates condition on non-labor income, while macro variation in wages over long horizons or across countries allows capital income to adjust with labor income. Once non-labor income is free to adjust, the income effect on hours becomes stronger, and the same preferences that deliver small and typically positive micro elasticities can deliver negative macro elasticities. The gap between the two elasticities is shaped by the joint distribution of labor and capital income across households, summarized by the labor share at the household level.

I formalized this reconciliation in two complementary models. A static model with exogenous capital income yields closed-form expressions for both elasticities and an identification result that maps moments of the joint distribution of wages, hours, and consumption into the underlying preference and heterogeneity parameters. A dynamic economy with incomplete asset markets reproduces both elasticities as outcomes when calibrated to the same cross-sectional moments together with statistics of the wealth distribution. The marginal propensities to earn implied by both models lie near the range of estimates from quasi-experimental lottery studies. The distribution of micro elasticities generated by the model with endogenous capital income is comparable to those from other structural and quasi-experimental analyses.

Beyond the quantitative reconciliation of the micro and macro elasticities, the paper offers methodological tools for empirical work. It generalizes the micro and macro elasticity formulas to general classes of preferences that imply heterogeneity in the curvature of the utility function. Without heterogeneity in the curvature of the utility function, I show that a cross-sectional regression of hours on wages that conditions on the labor share recovers the macro elasticity. With such heterogeneity, this regression recovers a weighted average of households' macro elasticities.

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Reconciling Micro Elasticities with the Macro Decline in Labor Supply

Online Appendix

Loukas Karabarbounis

A Model Appendix

Appendix A contains the proof of Proposition 1, the case of a general utility function, the identification of parameters, and sufficient conditions under which the macro-elasticity formula extends to the model with endogenous capital income.

A.1 Proof of Proposition 1

Micro elasticity. Fix a date t and a household ι and drop the time subscript and the household index to ease the notation. Define a household's wage, $W \equiv w\theta$ as the product of the aggregate wage and household-level efficiency units. A household's first-order condition (5) can be written as

$$F(n, W) \equiv \chi n^{1/\varepsilon} (Wn + \pi)^{\gamma} - W = 0.$$

The Marshallian elasticity holds π and χ fixed. Differentiating F with respect to n

$$\begin{aligned} F_n &= \chi \tfrac{1}{\varepsilon} n^{1/\varepsilon - 1} (Wn + \pi)^{\gamma} + \gamma \chi n^{1/\varepsilon} (Wn + \pi)^{\gamma - 1} W \\ &= \chi n^{1/\varepsilon - 1} (Wn + \pi)^{\gamma - 1} \left[\tfrac{1}{\varepsilon} (Wn + \pi) + \gamma Wn \right] \\ &= \chi n^{1/\varepsilon - 1} (Wn + \pi)^{\gamma} \left[\tfrac{1}{\varepsilon} + \gamma \zeta \right] = \frac{W}{n} \left[\tfrac{1}{\varepsilon} + \gamma \zeta \right], \end{aligned}$$

where the last equality uses the first-order condition $\chi n^{1/\varepsilon}(Wn+\pi)^{\gamma} = W$ and $\zeta = Wn/(Wn+\pi)$. Similarly, differentiating with respect to W

$$F_W = \gamma \chi n^{1/\varepsilon + 1} (Wn + \pi)^{\gamma - 1} - 1 = \gamma \zeta - 1.$$

By the implicit function theorem, we obtain

$$\eta_\theta \equiv \frac{\mathrm{d}n}{\mathrm{d}W} \frac{W}{n} = -\frac{F_W}{F_n} \frac{W}{n} = \frac{1 - \gamma \zeta}{1/\varepsilon + \gamma \zeta},$$

where ζ is in general indexed by time period and household type.

Macro elasticity. Let s denote the baseline date and t any other date. Define $g \equiv A_t/A_s$ as the gross growth rate of aggregate productivity between the two dates. Conjecture that $n_t(\iota) = \mu\, n_s(\iota)$ for all ι, for some scalar μ that needs to be determined.

Under the conjecture, we obtain $L_t = \mu L_s$. The firm's first-order condition in equation (2) yields

$$\frac{w_t}{w_s} = g\,\mu^{-\alpha}, \qquad \frac{\Pi_t}{\Pi_s} = g\,\mu^{1-\alpha}.$$

The two pieces of $\pi_t(\iota)$ in equation (6) scale by the same factor:

$$\nu(\iota)\,\Pi_t = g\,\mu^{1-\alpha}\,\nu(\iota)\,\Pi_s, \qquad \xi(\iota)\,w_t\,\theta(\iota)\,n_t(\iota) = g\,\mu^{1-\alpha}\,\xi(\iota)\,w_s\,\theta(\iota)\,n_s(\iota),$$

using the result above that $w_t\mu = w_s g\mu^{1-\alpha}$. Thus, from the household's budget constraint (4), we obtain

$$c_t(\iota) = w_t\,\theta(\iota)\,n_t(\iota) + \pi_t(\iota) = g\,\mu^{1-\alpha}\,c_s(\iota).$$

Substituting the expression for consumption into the household's first-order condition for labor supply (5) at date t and dividing by the same first-order condition at date s gives

$$\mu^{1/\varepsilon}\left(g\,\mu^{1-\alpha}\right)^{\gamma} \;=\; g\,\mu^{-\alpha} \;\;\Longrightarrow\;\; \mu \;=\; g^{\frac{1-\gamma}{1/\varepsilon+\alpha+\gamma(1-\alpha)}}.$$

This pins down μ, which is independent of ι and confirms the conjecture.

Aggregating labor we obtain, $L_t = \mu L_s$, which is consistent with labor from the firm's problem. Aggregating capital income we obtain

$$\int \pi_t(\iota)\,\mathrm{d}F(\iota) = g\,\mu^{1-\alpha}\int \pi_s(\iota)\,\mathrm{d}F(\iota) = g\,\mu^{1-\alpha}\,\Pi_s = \Pi_t,$$

using capital market clearing at date s. Goods market clearing then follows from the household's budget constraint (4) and the firm's profits, $\Pi_t = Y_t - w_t L_t$.

Next, I show that the labor share is constant over time for every household. Since $w_t\theta(\iota)n_t(\iota) = g\mu^{1-\alpha}w_s\theta(\iota)n_s(\iota)$ and $c_t(\iota) = g\mu^{1-\alpha}c_s(\iota)$,

$$\zeta_t(\iota) \;=\; \frac{w_t\,\theta(\iota)\,n_t(\iota)}{c_t(\iota)} \;=\; \frac{w_s\,\theta(\iota)\,n_s(\iota)}{c_s(\iota)} \;=\; \zeta_s(\iota).$$

To calculate the macro elasticity, use $w_t/w_s = g\,\mu^{-\alpha}$ and the expression for μ to arrive at

$$\frac{w_t}{w_s} = g^{\frac{1/\varepsilon+\gamma}{1/\varepsilon+\alpha+\gamma(1-\alpha)}}.$$

Eliminating g from the ratios for $n_t(\iota)$ and w_t gives

$$\frac{n_t(\iota)}{n_s(\iota)} = \mu = \left(\frac{w_t}{w_s}\right)^{\frac{1-\gamma}{1/\varepsilon+\gamma}}.$$

Taking logs and differentiating yields $\eta_w(\iota) = (1-\gamma)/(1/\varepsilon+\gamma)$, which is identical across households ι and independent of the primitives, $(\nu(\iota), \xi(\iota))$, that determine the allocation of capital income.

A.2 General Utility Function

In the main text and in Appendix A.1, I derived the micro and macro elasticities and the log-linear equation for labor supply (12) under MaCurdy (1981) preferences. This appendix clarifies which parts of the analysis survive the generalization to a utility function $U(c, n)$ and which depend on the separable isoelastic or other specific structures.

Utility $U(c, n)$ is increasing in c, decreasing in n, and satisfies the usual concavity and Inada conditions. Define the marginal rate of substitution between consumption and hours by $\mathrm{MRS}(c, n) \equiv -U_n(c, n)/U_c(c, n)$, so the household's first-order condition (5) is $\mathrm{MRS}(c, n) = W$, where $W = w\theta$ is the household's wage. Define the log-elasticities of the marginal rate of substitution with respect to consumption and hours by:

$$\gamma(c, n) \;\equiv\; \frac{\partial \log \mathrm{MRS}(c, n)}{\partial \log c}, \qquad \frac{1}{\varepsilon(c, n)} \;\equiv\; \frac{\partial \log \mathrm{MRS}(c, n)}{\partial \log n}.$$

For separable isoelastic preferences, $\gamma(c, n) = \gamma$ and $\varepsilon(c, n) = \varepsilon$ are constants and coincide with the curvature of utility with respect to consumption and the Frisch elasticity of labor supply. Under a general utility function, both objects depend on the household's allocation (c, n). I abbreviate $\gamma(\iota) \equiv \gamma(c(\iota), n(\iota))$ and $\varepsilon(\iota) \equiv \varepsilon(c(\iota), n(\iota))$ for household ι. Where it does not cause confusion, I drop the dependence on ι below.

Micro elasticity. Total differentiation of the first-order condition, $\log \mathrm{MRS}(c, n) = \log W$, for a given household and date gives

$$\gamma \,\mathrm{d}\log c \;+\; \frac{1}{\varepsilon}\,\mathrm{d}\log n \;=\; \mathrm{d}\log W.$$

The Marshallian elasticity holds non-labor income π and any preference shifters implicit in U fixed. The budget constraint, $c = Wn + \pi$, and the definition of the labor share, $\zeta = Wn/c$, imply $\mathrm{d}\log c = \zeta\,(\mathrm{d}\log W + \mathrm{d}\log n)$, given that $\mathrm{d}\pi = 0$. Substituting the change in log consumption into the previous equation and solving for the micro elasticity, $\mathrm{d}\log n/\mathrm{d}\log W$ holding π fixed, yields

$$\eta_\theta(\iota) \;=\; \frac{1-\gamma(\iota)\,\zeta(\iota)}{1/\varepsilon(\iota)+\gamma(\iota)\,\zeta(\iota)}.$$

This is identical to equation (9) in the main text, with the difference that γ and ε are now interpreted as the local elasticities $\gamma(c(\iota), n(\iota))$ and $\varepsilon(c(\iota), n(\iota))$ evaluated at the household's optimal solution for consumption and hours. Under separable isoelastic preferences, the micro elasticity varies across households only through the labor share, $\zeta(\iota)$. Under a general utility function, it additionally varies with consumption and hours through $\gamma(\iota)$ and $\varepsilon(\iota)$.

Macro elasticity. The macro elasticity can be defined locally for each household as the response of the household's hours to the aggregate wage. When the household's labor share is held constant as in the main text and in Appendix A.1, the macro elasticity has the same formula as in equation (10) for any general utility function, with the difference that it is potentially different for every household ι. Whether the model's general equilibrium actually implies constancy of $\zeta(\iota)$ for every household is a separate question that I address next.

To derive this elasticity, I use two equations. The first is the total log differential of the household's first-order condition, $\log \mathrm{MRS}(c, n) = \log W$, which by the definitions of γ and ε is

$$\gamma(\iota)\,\mathrm{d}\log c \;+\; \frac{1}{\varepsilon(\iota)}\,\mathrm{d}\log n \;=\; \mathrm{d}\log W.$$

The second is the constraint that the household's labor share is held constant. Totally differentiating $\log\zeta = \log W + \log n - \log c$ and using $\mathrm{d}\log\zeta = 0$ yields $\mathrm{d}\log c = \mathrm{d}\log W + \mathrm{d}\log n$. Substituting this relation into the first-order condition,

$$\big(\gamma(\iota) + 1/\varepsilon(\iota)\big)\,\mathrm{d}\log n \;=\; \big(1-\gamma(\iota)\big)\,\mathrm{d}\log W,$$

and solving for the elasticity,

$$\eta_w(\iota) \;=\; \frac{1-\gamma(\iota)}{1/\varepsilon(\iota)+\gamma(\iota)}.$$

The labor share $\zeta(\iota)$ does not appear in this expression because the imposed constraint $\mathrm{d}\log\zeta = 0$ implies that labor and capital income scale one-to-one when the aggregate wage increases, which strengthens the income effect. Equivalently, the macro elasticity for a household coincides with the micro-elasticity formula evaluated at $\zeta(\iota) = 1$. The expression is identical to equation (10) in the main text, with $\gamma(\iota)$ and $\varepsilon(\iota)$ now interpreted as local elasticities at the household's optimal allocation. Under separable isoelastic preferences, $\gamma(\iota)$ and $\varepsilon(\iota)$ are constants and $\eta_w(\iota)$ is the same across all households. Under a general utility function, $\eta_w(\iota)$ varies with the household's allocation. The aggregate response of hours to a change in the aggregate wage is the weighted average of the household responses,

$$\bar{\eta}_w = \int \omega(\iota)\eta_w(\iota)\mathrm{d}F(\iota),$$

where the weights are given by $\omega(\iota) = \theta(\iota)n(\iota)/L$.

The conceptual reconciliation of the gap between the micro and the macro elasticity, namely that the macro elasticity differs from the micro because the former features stronger income effects from changes in the aggregate wage, extends to any utility function as long as changes in the aggregate wage scale both labor and non-labor income proportionally.[1] What does not extend to a general utility function is the property that every household shares the same macro elasticity.

Constancy of the labor share under the $\nu(\iota) = 0$ rule. Under the $\nu(\iota) = 0$ rule, the affine allocation rule (6) and the budget constraint (4) imply $c_t(\iota) = (1 + \xi(\iota))\, w_t\, \theta(\iota)\, n_t(\iota)$ at every date. The household's labor share equals

$$\zeta_t(\iota) = \frac{w_t\, \theta(\iota)\, n_t(\iota)}{c_t(\iota)} = \frac{1}{1 + \xi(\iota)},$$

which depends only on the time-invariant primitive $\xi(\iota)$, which is constant over time. This conclusion holds for any utility function $U(c, n)$ and for any scaling of individual labor supply, uniform or otherwise. Then, the local macro elasticity, $\eta_w(\iota) = (1 - \gamma(\iota))/(1/\varepsilon(\iota) + \gamma(\iota))$, describes the actual response of the household's hours to a change in the aggregate wage.

Constancy of the labor share under the $\xi(\iota) = 0$ rule. Under the $\xi(\iota) = 0$ rule, the affine allocation rule (6) and the budget constraint (4) imply $c_t(\iota) = w_t\, \theta(\iota)\, n_t(\iota) + \nu(\iota)\, \Pi_t$. Unlike the constant-labor-share rule, the labor share $\zeta_t(\iota) = w_t\, \theta(\iota)\, n_t(\iota)/c_t(\iota)$ is not necessarily constant. I now examine conditions on the utility function that retain the constancy of the labor share.

Let s denote the baseline date and t any other date. Define $g \equiv A_t/A_s$ as the gross growth rate of aggregate productivity between the two dates and $\mu \equiv L_t/L_s$ as the growth of aggregate labor. From the firm's problem, we obtain $w_t/w_s = g\, \mu^{-\alpha}$ and $\Pi_t/\Pi_s = g\, \mu^{1-\alpha}$. Suppose for the moment that individual labor supply scales by the same factor across households, $n_t(\iota) =$

[1] The proportionality assumption makes the derivation transparent. Even if non-labor income scaled less than proportionally with labor income, the macro elasticity would feature stronger income effects than the micro elasticity, which always holds non-labor income constant.

$\mu\, n_s(\iota)$. Then the budget constraint and the fixed-share rule imply $c_t(\iota)/c_s(\iota) = g\,\mu^{1-\alpha}$ for every household, and the numerator and denominator of $\zeta_t(\iota)$ scale by the same factor:

$$\zeta_t(\iota) \;=\; \frac{w_t\,\theta(\iota)\,n_t(\iota)}{c_t(\iota)} \;=\; \frac{g\,\mu^{1-\alpha}\,w_s\,\theta(\iota)\,n_s(\iota)}{g\,\mu^{1-\alpha}\,c_s(\iota)} \;=\; \zeta_s(\iota).$$

Thus, if labor supply scales uniformly across households in equilibrium and the share of profits that households receive is fixed over time, then the labor share for every household is constant.

The functional form of the marginal rate of substitution determines whether uniform scaling is consistent with the household's first-order condition, $W_t(\iota) = \mathrm{MRS}(c_t(\iota), n_t(\iota))$. Substituting $w_t/w_s = g\,\mu^{-\alpha}$, $n_t(\iota) = \mu\, n_s(\iota)$, and $c_t(\iota)/c_s(\iota) = g\,\mu^{1-\alpha}$ into the first-order condition, uniform scaling is consistent with household optimality only if

$$\frac{\mathrm{MRS}\big(g\,\mu^{1-\alpha}\,c_s(\iota),\,\mu\,n_s(\iota)\big)}{\mathrm{MRS}\big(c_s(\iota),\,n_s(\iota)\big)} \;=\; g\,\mu^{-\alpha}$$

holds at every household allocation $(c_s(\iota), n_s(\iota))$. For a single μ to satisfy this equation across heterogeneous households, the marginal rate of substitution must either be homogeneous of degree one in consumption ($\gamma(c,n) = 1$) or take the multiplicative-power form

$$\mathrm{MRS}(c, n; \iota) \;=\; \overline{\mathrm{MRS}}(\iota)\, c^{\gamma}\, n^{1/\varepsilon},$$

where the exponents γ and $1/\varepsilon$ are common across households, while the preference shifter $\overline{\mathrm{MRS}}(\iota)$ may be heterogeneous but constant over time. If the exponents were household-specific, $\gamma(\iota)$ or $1/\varepsilon(\iota)$, the equation above would pin down a different scaling $\mu(\iota)$ for each household and the labor share would not in general be constant. Separable isoelastic preferences are in the class of preferences in which the exponents are common across households.[2]

Under the multiplicative-power form condition, the household's first-order condition implies that

$$\frac{w_t}{w_s} \;=\; \frac{\mathrm{MRS}\big(c_t(\iota),\,n_t(\iota)\big)}{\mathrm{MRS}\big(c_s(\iota),\,n_s(\iota)\big)} \;=\; \left(\frac{c_t(\iota)}{c_s(\iota)}\right)^{\gamma} \left(\frac{n_t(\iota)}{n_s(\iota)}\right)^{1/\varepsilon}.$$

Substituting $w_t/w_s = g\mu^{-\alpha}$, $c_t(\iota)/c_s(\iota) = g\mu^{1-\alpha}$, and $n_t(\iota)/n_s(\iota) = \mu$ and solving for μ gives

$$\mu \;=\; g^{\frac{1-\gamma}{1/\varepsilon+\alpha+\gamma(1-\alpha)}},$$

[2]Any monotone transformation of separable isoelastic preferences generates a MRS with the same multiplicative-power form. Two further cases admit a common scaling factor μ. The first is $\gamma(c,n) = 1$, as in the King, Plosser, and Rebelo (1988) class and applied variants such as Trabandt and Uhlig (2011), where the macro elasticity is zero and $\mu = 1$ holds trivially. The second is $\gamma(c,n) = 0$, which corresponds to GHH preferences and equates the micro and macro elasticities to the Frisch elasticity. Neither case can rationalize the simultaneous existence of small positive micro elasticities and a negative macro elasticity.

which is the same scaling factor under the MaCurdy preferences derived in Appendix A.1. Because γ and ε are constants under the power-function condition, the macro elasticity is the same across all households and equals $(1-\gamma)/(1/\varepsilon+\gamma)$, as in equation (10) in the main text.

For utility functions outside both the power-function class and the class of utilities homogeneous of degree one in consumption, no common μ solves the household's first-order condition under the fixed-share rule. The labor share is then not necessarily constant over time and the formula derived for $\eta_w(\iota)$ above is not the actual response of aggregate or household labor to a change in the aggregate wage. Under separable isoelastic preferences, the constant-labor-share rule and the fixed-share rule yield identical allocations and macro elasticities. Under a general utility function, the two rules differ.

Examples. Four examples illustrate when the condition on the marginal rate of substitution admits a single μ across households and when it does not.

1. Cobb-Douglas preferences over consumption and leisure:

$$U(c,n) = c^{\kappa}(1-n)^{1-\kappa}.$$

The marginal rate of substitution is $\mathrm{MRS}(c,n) = \frac{1-\kappa}{\kappa}\, c/(1-n)$. Then $\gamma(c,n) = 1$ is constant and $1/\varepsilon(c,n) = n/(1-n)$ depends on hours. The marginal rate of substitution admits a single $\mu = 1$ for every g. Plugging $\gamma = 1$ into the macro elasticity we find that it is zero for every household, so aggregate labor does not respond to permanent wage changes. This recovers the familiar balanced growth path implication under Cobb-Douglas preferences, but in a model with household heterogeneity.

2. GHH preferences over consumption and hours:

$$U(c,n) = \frac{\left(c - \chi\, \frac{n^{1+1/\varepsilon}}{1+1/\varepsilon}\right)^{1-\gamma} - 1}{1-\gamma}.$$

The marginal rate of substitution is $\mathrm{MRS}(c,n) = \chi\, n^{1/\varepsilon}$. The marginal rate of substitution depends only on hours: $\gamma(c,n) = 0$ and $1/\varepsilon(c,n) = 1/\varepsilon$ are constants. The marginal rate of substitution admits a single $\mu = g^{\varepsilon/(1+\varepsilon\alpha)}$, and the local macro elasticity is $\eta_w(\iota) = \varepsilon$, equal to the Frisch elasticity and independent of the household's labor share. Under GHH preferences, the micro and macro elasticities coincide for every household, reflecting the absence of an income effect on labor supply.

3. CES preferences over consumption and leisure:

$$U(c,n) = (\kappa\, c^{(\rho-1)/\rho} + (1-\kappa)(1-n)^{(\rho-1)/\rho})^{\rho/(\rho-1)}.$$

The marginal rate of substitution is $\mathrm{MRS}(c,n) = \frac{1-\kappa}{\kappa}\,(c/(1-n))^{1/\rho}$. Then $\gamma(c,n) = 1/\rho$ is constant but $1/\varepsilon(c,n) = n/(\rho(1-n))$ depends on hours. The equation determining μ reduces to $\mu^{1+\alpha(\rho-1)}(1-n_s) = g^{\rho-1}(1-\mu\, n_s)$, which depends on n_s and does not admit a single μ across heterogeneous households (except in the Cobb-Douglas limit $\rho = 1$). Thus, CES preferences do not in general feature a constant labor share under the fixed-share rule.

4. Boppart and Krusell (2020) preferences over consumption and hours:

$$U(c,n) \;=\; \frac{\big(c\, f\big(n\, c^{v/(1-v)}\big)\big)^{1-\gamma} - 1}{1-\gamma},$$

for some arbitrary function $f(\cdot)$, the marginal rate of substitution is given by

$$\mathrm{MRS}(c,n) \;=\; c^{1/(1-v)}\, q\big(n\, c^{v/(1-v)}\big),$$

for some function q and a scalar $v < 1$. To see this, let $x \equiv n\, c^{v/(1-v)}$ and $Z \equiv c\, f(x)$, so that $U = (Z^{1-\gamma} - 1)/(1-\gamma)$. Differentiating Z,

$$Z_n \;=\; c^{1/(1-v)}\, f'(x), \qquad Z_c \;=\; f(x) + \frac{v}{1-v}\, x\, f'(x),$$

and therefore

$$\mathrm{MRS}(c,n) \;\equiv\; -\frac{U_n}{U_c} \;=\; -\frac{Z_n}{Z_c} \;=\; c^{1/(1-v)}\; \underbrace{\frac{-f'(x)}{f(x) + \frac{v}{1-v}\, x\, f'(x)}}_{\equiv\; q(x)}.$$

The MRS thus splits into a power of c and a function q evaluated at the composite $x = n\, c^{v/(1-v)}$. The corresponding local elasticities of the MRS are

$$\gamma(c,n) \;=\; \frac{1}{1-v} + \frac{v}{1-v}\,\frac{q'(x)\, x}{q(x)}, \qquad \frac{1}{\varepsilon(c,n)} \;=\; \frac{q'(x)\, x}{q(x)}.$$

Both elasticities generally vary across households, because the composite x takes different values across the cross-section. Hence the marginal rate of substitution does not, in general, belong to the multiplicative-power class, which is necessary under the fixed-share rule ($\xi(\iota) = 0$) to deliver a constant household labor share. Consequently, in a model with heterogeneous households endowed with Boppart and Krusell (2020) preferences, the fixed-share rule generically fails to admit a single scaling factor μ that aligns labor and consumption growth across the cross-section, and the household labor share is not necessarily constant over time.

The class collapses to the multiplicative-power form only when q itself is a power function. Setting $q(x) = \kappa\, x^{1/\varepsilon}$ yields

$$\mathrm{MRS}(c,n) \;=\; \kappa\, c^{1/(1-v)} \left(n\, c^{v/(1-v)}\right)^{1/\varepsilon} \;=\; \kappa\, c^{(\varepsilon+v)/((1-v)\varepsilon)}\, n^{1/\varepsilon},$$

which is the MaCurdy (1981) form with $\gamma = (\varepsilon+v)/((1-v)\varepsilon)$, equivalently $v = (\gamma - 1)/(\gamma + 1/\varepsilon)$. In this case the ι-specific local elasticities collapse to the constants γ and ε, the macro elasticity reduces to $-v = (1-\gamma)/(\gamma + 1/\varepsilon)$ as in equation (10), and the household labor share is constant under either the fixed-share or the constant-labor-share rule, as derived in Appendix A.1. Thus separable isoelastic preferences are the unique intersection of the Boppart and Krusell (2020) class and the class for which the elasticity gap admits a closed-form representation invariant to the distribution of capital income.

This observation also clarifies why the cross-sectional "ideal experiment" in Boppart and Krusell (2020) scales nonwage income by g_w^{1-v} rather than by g_w, where g_w is the growth factor of wages. Along their balanced-growth path the wage scales by g_w while hours scale by g_w^{-v}, so labor income and consumption both scale by g_w^{1-v}, and the aggregate labor share is constant. With a representative household, this scaling is internally consistent. With cross-sectional heterogeneity in hours, however, the elasticities $\gamma(c,n)$ and $1/\varepsilon(c,n)$ vary with the household-specific composite x, and no single scaling factor for hours exists outside the MaCurdy utility function. The g_w^{1-v} scaling generates a constant aggregate labor share but does not imply a constant labor share for each household, because households with Boppart and Krusell (2020) preferences scale their consumption and hours at different rates under the fixed-share rule ($\xi(\iota) = 0$). In my framework, by contrast, the constant-labor-share rule ($\nu(\iota) = 0$) maintains a constant labor share for each household by construction for any utility function, and the fixed-share rule does the same whenever the MRS is in the multiplicative-power class.

Log-linear Equation (12). Equation (12) is derived in the main text by substituting $c = Wn/\zeta$ into the first-order condition under separable isoelastic preferences and then taking logs, where $W \equiv w\theta$. Under a general utility function, this substitution yields the equation $\log \mathrm{MRS}(Wn/\zeta, n; \chi) = \log W$. A local log-linearization, around $\gamma(\iota)$ and $\varepsilon(\iota)$, preserves the structure of equation (12). Assuming that the preference shifter χ enters the marginal rate of substitution multiplicatively with the bundle of consumption and labor, $\mathrm{MRS}(c, n, \chi) = \chi\widetilde{\mathrm{MRS}}(c, n)$, total differentiation of the first-order condition combined with $\mathrm{d}\log c = \mathrm{d}\log W + \mathrm{d}\log n - \mathrm{d}\log\zeta$ delivers

$$\mathrm{d}\log n(\iota) = \frac{1-\gamma(\iota)}{1/\varepsilon(\iota)+\gamma(\iota)}\mathrm{d}\log W(\iota) - \frac{1}{1/\varepsilon(\iota)+\gamma(\iota)}\mathrm{d}\log\chi(\iota) + \frac{\gamma(\iota)}{1/\varepsilon(\iota)+\gamma(\iota)}\mathrm{d}\log\zeta(\iota),$$

which is of the same form as equation (12), with the difference that γ and ε are replaced by their local values at the household's allocation. Under separable isoelastic preferences, this relationship holds globally because γ and ε are constants. In that case, an OLS regression of log hours on the wage, the disutility of work, and the labor share recovers the macro elasticity exactly on the wage coefficient. Under a general utility function, the relationship holds locally. An OLS regression recovers a covariance-weighted average of households' macro elasticities, $\eta_w(\iota)$. This applies to the cases enumerated in the previous paragraphs in which the macro elasticity is given by the formula $(1-\gamma(\iota))/(1/\varepsilon(\iota)+\gamma(\iota))$ for each household ι.

A.3 Identification

This appendix discusses the identification of the parameters from the moments listed in the main text. The discussion proceeds in two steps. The first step provides closed-form expressions that prove that 9 of the parameters are identified from 9 of the moments. The second step discusses how to identify the median labor share, $\bar{\zeta}$, from the last moment (M10) and an assumption on the marginal distribution of the labor share. The appendix concludes by discussing the necessity of a heavy-tailed distribution for the labor share.

Throughout this appendix, when I write the households' wage I mean $w(\iota) = \theta(\iota)w$, where $\theta(\iota)$ is the heterogeneous component of the wage and w is the aggregate wage that is common across households. I define σ_w^2 as the variance of log wages in the cross section of households and σ_{wx} as the covariance of log wages and log x, for $x \in \{c, n\}$.

A.3.1 Step 1: Recovering 9 parameters from 9 moments

Given the Frisch elasticity, ε, inverting equation (10) and using the macro elasticity moment, we obtain

$$\gamma = \frac{1-\eta_w/\varepsilon}{1+\eta_w}.$$

The production function parameter equals one minus the aggregate labor share, $\alpha = 1 - s_\ell$.

The variance of log wages is directly observed, thus

$$\sigma_\theta^2 = \sigma_w^2.$$

Next, I use the definition of the labor share, $\log \zeta = \log(w\theta) + \log n - \log c$. Taking the covariance of each variable with respect to the wage we obtain

$$\sigma_{\theta\zeta} = \sigma_\theta^2 + \sigma_{wn} - \sigma_{wc}.$$

Given σ_θ^2, the two moments on the right-hand side of this equation pin down $\sigma_{\theta\zeta}$.

Define the three coefficients that appear in the log-linear hours equation (12):

$$\beta_w \equiv \frac{1-\gamma}{1/\varepsilon + \gamma}, \quad \beta_\chi \equiv \frac{1}{1/\varepsilon + \gamma}, \quad \beta_\zeta \equiv \frac{\gamma}{1/\varepsilon + \gamma}.$$

Taking the covariance of each variable in the log-linear hours equation (12) with respect to the wage yields

$$\sigma_{\theta\chi} = \frac{1}{\beta_\chi}\left(\beta_w\,\sigma_\theta^2 + \beta_\zeta\,\sigma_{\theta\zeta} - \sigma_{wn}\right).$$

All objects on the right-hand side are known and so this equation pins down $\sigma_{\theta\chi}$.

Next, I use moments (M5), (M6), (M9) to identify the remaining three parameters, $\sigma_\chi^2, \sigma_\zeta^2, \sigma_{\chi\zeta}$. Taking the variance of $\log n$ in the log-linear hours equation (12) yields

$$\sigma_n^2 = \beta_w^2\,\sigma_\theta^2 + \beta_\zeta^2\,\sigma_\zeta^2 + \beta_\chi^2\,\sigma_\chi^2 + \ 2\beta_w\beta_\zeta\,\sigma_{\theta\zeta} - 2\beta_w\beta_\chi\,\sigma_{\theta\chi} - 2\beta_\zeta\beta_\chi\,\sigma_{\chi\zeta}.$$

Subtracting the three terms involving wages, σ_θ^2, $\sigma_{\theta\zeta}$, and $\sigma_{\theta\chi}$, which are already pinned down, yields the residual:

$$r_n \ \equiv \ \sigma_n^2 \ - \ \beta_w^2\,\sigma_\theta^2 \ + \ 2\beta_w\beta_\chi\,\sigma_{\theta\chi} \ - \ 2\beta_w\beta_\zeta\,\sigma_{\theta\zeta} = \beta_\zeta^2\,\sigma_\zeta^2 + \beta_\chi^2\,\sigma_\chi^2 - 2\beta_\zeta\beta_\chi\,\sigma_{\chi\zeta},$$

which involves only unknowns on the right-hand side.

Substituting the log-linear hours equation (12) into the budget constraint (4) yields an expression for log consumption:

$$\log c \ = \ (1+\beta_w)\log(w\theta) \ + \ (\beta_\zeta - 1)\log\zeta \ - \ \beta_\chi \log\chi.$$

Taking the variance of log consumption yields

$$\begin{aligned}\sigma_c^2 \ &= (1+\beta_w)^2\,\sigma_\theta^2 + (\beta_\zeta-1)^2\,\sigma_\zeta^2 + \beta_\chi^2\,\sigma_\chi^2 \\ &\quad + \ 2(1+\beta_w)(\beta_\zeta-1)\,\sigma_{\theta\zeta} - 2(1+\beta_w)\beta_\chi\,\sigma_{\theta\chi} - 2(\beta_\zeta-1)\beta_\chi\,\sigma_{\chi\zeta}.\end{aligned}$$

Subtracting the three terms involving wages, σ_θ^2, $\sigma_{\theta\zeta}$, and $\sigma_{\theta\chi}$, which are already pinned down, yields the residual:

$$r_c \equiv \sigma_c^2 - (1+\beta_w)^2\sigma_\theta^2 + 2(1+\beta_w)\beta_\chi\sigma_{\theta\chi} - 2(1+\beta_w)(\beta_\zeta - 1)\sigma_{\theta\zeta}$$
$$= (\beta_\zeta - 1)^2\sigma_\zeta^2 + \beta_\chi^2\sigma_\chi^2 - 2(\beta_\zeta - 1)\beta_\chi\sigma_{\chi\zeta},$$

which involves only unknowns on the right-hand side.

The covariance between log hours and log consumption is calculated using the log-linear hours equation (12) and the log-linear consumption equation above:

$$\sigma_{nc} = \beta_w(1+\beta_w)\sigma_\theta^2 + \beta_\zeta(\beta_\zeta - 1)\sigma_\zeta^2 + \beta_\chi^2\sigma_\chi^2 + (2\beta_w\beta_\zeta + \beta_\zeta - \beta_w)\sigma_{\theta\zeta}$$
$$- \beta_\chi(1+2\beta_w)\sigma_{\theta\chi} - \beta_\chi(2\beta_\zeta - 1)\sigma_{\chi\zeta}.$$

Subtracting the three terms involving wages, σ_θ^2, $\sigma_{\theta\zeta}$, and $\sigma_{\theta\chi}$, which are already pinned down, yields the residual:

$$r_{nc} \equiv \sigma_{nc} - \beta_w(1+\beta_w)\sigma_\theta^2 + \beta_\chi(1+2\beta_w)\sigma_{\theta\chi} - (2\beta_w\beta_\zeta + \beta_\zeta - \beta_w)\sigma_{\theta\zeta}$$
$$= \beta_\zeta(\beta_\zeta - 1)\sigma_\zeta^2 + \beta_\chi^2\sigma_\chi^2 - \beta_\chi(2\beta_\zeta - 1)\sigma_{\chi\zeta},$$

which involves only unknowns on the right-hand side.

Stacking the three residuals in a system of three equations in three unknowns yields

$$\begin{pmatrix} \beta_\chi^2 & \beta_\zeta^2 & -2\beta_\chi\beta_\zeta \\ \beta_\chi^2 & (\beta_\zeta - 1)^2 & -2\beta_\chi(\beta_\zeta - 1) \\ \beta_\chi^2 & \beta_\zeta(\beta_\zeta - 1) & -\beta_\chi(2\beta_\zeta - 1) \end{pmatrix} \begin{pmatrix} \sigma_\chi^2 \\ \sigma_\zeta^2 \\ \sigma_{\chi\zeta} \end{pmatrix} = \begin{pmatrix} r_n \\ r_c \\ r_{nc} \end{pmatrix}.$$

The determinant of the coefficient matrix is β_χ^3, which is strictly positive since $\beta_\chi = 1/(1/\varepsilon + \gamma) > 0$. This system has a unique solution and identifies $\sigma_\chi^2, \sigma_\zeta^2, \sigma_{\chi\zeta}$ given that all moments and other parameters are known.

A.3.2 Step 2: Recovering parameters of the distribution

The marginal distribution of the labor share ζ is a shifted Generalized Pareto distribution with three parameters, $\zeta_0, \zeta_1, \zeta_2$. The density is given by

$$f(\zeta;\, \zeta_0, \zeta_1, \zeta_2) = \frac{1}{\zeta_1}\left(1 + \zeta_2\,\frac{\zeta - \zeta_0}{\zeta_1}\right)^{-1/\zeta_2 - 1}, \qquad \zeta > \zeta_0,$$

where $\zeta_0 \geq 0$ is a location shift that bounds the support of ζ from below, $\zeta_1 > 0$ is a scale parameter, and $\zeta_2 > 0$ is a shape parameter that controls the heaviness of the right tail. The support of ζ is $[\zeta_0, \infty)$, which admits realizations with $\zeta > 1$.

The three parameters, $\zeta_0, \zeta_1, \zeta_2$, are identified by three conditions.

(C1) The mean micro elasticity equals the value of $\bar{\eta}_\theta$ in (M10):

$$\bar{\eta}_\theta \;=\; \int \frac{1-\gamma\,\zeta}{1/\varepsilon + \gamma\,\zeta}\, f(\zeta;\, \zeta_0, \zeta_1, \zeta_2)\, \mathrm{d}\zeta.$$

(C2) The variance of the log labor share equals the value σ_ζ^2 that I identified in the previous step:

$$\sigma_\zeta^2 \;=\; \int \left(\log\zeta - \overline{\log\zeta}\right)^2 f(\zeta;\, \zeta_0, \zeta_1, \zeta_2)\, \mathrm{d}\zeta.$$

(C3) Aggregate income consistency. Capital income across households aggregates to firm profits:

$$\int \pi(\iota)\, \mathrm{d}F(\iota) \;=\; \Pi.$$

Unlike (C1) and (C2), which match moments in the data, (C3) is a market-clearing restriction that any joint distribution implied by the model must satisfy in equilibrium. Equivalently, this restriction guarantees that the parameters of the underlying distribution of the labor share satisfy equation (8) in the main text.

The median of the labor share admits an explicit expression in terms of the three parameters:

$$\bar{\zeta} \;=\; \zeta_0 \;+\; \zeta_1\, \frac{2^{\zeta_2} - 1}{\zeta_2}.$$

Because $(\zeta_0, \zeta_1, \zeta_2)$ are pinned down as a function of the underlying moments, the median labor share $\bar{\zeta}$ is itself a function of the moments. In this sense, $\bar{\zeta}$ is almost a parameter and I report it as a useful moment that summarizes the labor share distribution.

These three conditions form a system of three equations in the three unknowns, $\zeta_0, \zeta_1, \zeta_2$. The integrals in conditions (C1) and (C2) and the aggregate condition (C3) do not admit closed-form expressions, so I evaluate them by simulation. I draw $N = 10{,}000$ households from a Gaussian copula whose underlying log-correlation matrix is implied by the covariance parameters identified in the closed-form steps above. For each household i in the simulation, I transform the third coordinate of the draw, $Z_i \sim \mathcal{N}(0,1)$, into a uniform variable $U_i = \Phi(Z_i)$ between 0 and 1, and then into a labor share by inverting the cumulative distribution function of the shifted Pareto:

$$\zeta_i \;=\; \zeta_0 \;+\; \zeta_1\, \frac{(1-U_i)^{-\zeta_2} - 1}{\zeta_2}.$$

I solve numerically for the values of $(\zeta_0, \zeta_1, \zeta_2)$ such that conditions (C1)–(C3) are satisfied in the simulated sample.[3]

A.3.3 The necessity of a heavy-tailed labor share distribution

To understand why the shifted Generalized Pareto distribution is helpful in matching these three conditions, note that condition (C3) can be written equivalently as

$$\begin{aligned}
\int w\theta(\iota)n(\iota)\mathrm{d}F(\iota) &= wL = (1-\alpha)Y \Longrightarrow \\
\int \zeta(\iota)c(\iota)\mathrm{d}F(\iota) &= (1-\alpha)C \Longrightarrow \\
\int \zeta(\iota)(c(\iota)/C)\mathrm{d}F(\iota) &= (1-\alpha) \Longrightarrow \\
\int \zeta(\iota)\mathrm{d}F(\iota) + \mathrm{cov}(\zeta(\iota), c(\iota)/C) &= (1-\alpha),
\end{aligned}$$

which says that the aggregate labor share on the right-hand side equals the unweighted mean labor share plus the covariance of the labor share with consumption on the left-hand side. Given that consumption is not very highly correlated with either hours or the wage, the definition of the labor share in equation (7) implies that consumption and the labor share are negatively correlated. Thus, the above equation implies that the mean of the labor share distribution has to exceed the aggregate labor share. Heavy-tailed distributions such as the shifted Generalized Pareto distribution are compatible with this requirement.

This requirement together with condition (C1) requires a large mass of households with low labor shares. For the model to produce a small but positive mean elasticity, the negative elasticity for the households with very high labor shares has to be offset by a positive elasticity for a large mass of households. This explains why half or more of households have an elasticity above 0.13 in the model with exogenous capital income as shown in Table 6. In addition, as equation (9) shows, the micro elasticity is convex in the labor share. Thus, the spread in the labor share distribution also helps increase the mean value of the micro elasticity.

[3]Because the marginal distribution of $\log\zeta$ is a nonlinear transformation from a Gaussian draw, the correlations of $\log\zeta$ with $\log\theta$ and $\log\chi$ in a simulated sample differ from the closed-form values that I identified from the covariance moments. In the baseline calibration the simulated correlations are 0.50 and 0.16 respectively, as opposed to 0.67 and 0.19 in the closed-form solutions. The gap reflects the difference between the shifted Generalized Pareto distribution and the log-normal distribution. The reconciliation between the micro and macro elasticities does not depend on this distinction, because the covariance moments themselves are matched by construction.

Other distributions that satisfy these guidelines are generalizations of the log-normal, the Gamma, and Weibull. For all three, results are very similar, so I report only results under the Pareto assumption. When I used distributions with thinner tails, such as Beta or Generalized Gamma, conditions (C1)–(C3) were impossible to satisfy simultaneously. However, the mean micro elasticity coming out of these simulations was very close to the targeted 0.06 value.

A.4 Macro Elasticity in the Aiyagari Economy

In this appendix I discuss some sufficient conditions such that the macro elasticity in the model with endogenous capital income coincides with that of the model with exogenous capital income given by equation (10). This is true if the following two conditions hold:

1. The equilibrium rate of return r does not change with A.

2. $\mathrm{d}\log C = \int \omega(\iota)\mathrm{d}\log c(\iota)\mathrm{d}F(\iota)$, where $\omega(\iota) = \theta(\iota)n(\iota)/L$ is household ι's share of the total wage bill.

To be clear, these conditions are not expressed in terms of primitives and so there is no analytical proof that the macro elasticity is given by (10). In numerical solutions, I find that it is up to the third or fourth digit. The point of this section is to guide intuition why this might be the case.

When r is constant, equation (15) and the first-order condition with respect to capital, $r = \alpha A(K/L)^{\alpha-1}$, imply that K/L and w grow $1/(1-\alpha)$ times the growth of A. Thus, K/Y and C/Y are constant. Given the constancy of the labor share, aggregate consumption changes by the sum of changes in the wage and labor:

$$\mathrm{d}\log C = \mathrm{d}\log w + \mathrm{d}\log L.$$

The definition of labor services in equation (14) implies that $\mathrm{d}\log L = \int \omega(\iota)\mathrm{d}\log n(\iota)\mathrm{d}F(\iota)$. Substituting in the first-order condition for household labor supply we obtain

$$\begin{aligned}
\mathrm{d}\log L &= \varepsilon\,\mathrm{d}\log w - \varepsilon\gamma \int \omega(\iota)\mathrm{d}\log c(\iota)\mathrm{d}F(\iota) \\
&= \varepsilon\,\mathrm{d}\log w - \varepsilon\gamma\,\mathrm{d}\log C \\
&= \varepsilon\,\mathrm{d}\log w - \varepsilon\gamma\,(\mathrm{d}\log w + \mathrm{d}\log L) \implies \\
(1+\varepsilon\gamma)\,\mathrm{d}\log L &= \varepsilon(1-\gamma)\,\mathrm{d}\log w,
\end{aligned}$$

where the second line uses the second sufficient condition, the third line uses the change in log consumption calculated above, and the fourth line rearranges the equation. Differentiating $\log L$ with respect to $\log w$ yields the formula in equation (10).

I conclude by examining the second sufficient condition, $\mathrm{d}\log C = \int \omega(\iota)\mathrm{d}\log c(\iota)\mathrm{d}F(\iota)$. Define the consumption weight $\tilde{\omega}(\iota) = c(\iota)/C$, so that $\mathrm{d}\log C = \int \tilde{\omega}(\iota)\mathrm{d}\log c(\iota)\mathrm{d}F(\iota)$. Household ι's share of the wage bill equals:

$$\omega(\iota) = \frac{\theta(\iota)n(\iota)}{L} = \frac{\zeta(\iota)c(\iota)}{(1-\tau)wL} = \frac{\zeta(\iota)}{(1-\tau)wL/C}\tilde{\omega}(\iota) = \mathcal{C}\,\zeta(\iota)\tilde{\omega}(\iota),$$

where $\mathcal{C} = ((1-\tau)wL/C)^{-1}$ is a constant in the stationary equilibrium. Thus, the condition becomes

$$\int \omega(\iota)\mathrm{d}\log c(\iota)\mathrm{d}F(\iota) = \mathcal{C} \int \zeta(\iota)\tilde{\omega}(\iota)\mathrm{d}\log c(\iota)\mathrm{d}F(\iota).$$

The integral on the left-hand side equals $\mathrm{d}\log C$ if the consumption-weighted covariance of $\zeta(\iota)$ with $\mathrm{d}\log c(\iota)$ is zero. This is implied by, but weaker than, the condition that all households scale their consumption proportionally.

B Quantitative Appendix

In this appendix I outline the steps to quantify and solve the model with endogenous capital income in Section 3. The first step is the calibration of parameters that allow the model to match data moments. The second step, which is invoked at every parameter evaluation, is the computation of the stationary equilibrium of the model. Inside the second step, there are four nested loops that solve the government budget constraint, the capital-to-labor ratio, the household problem, and the stationary distribution.

B.1 Calibration

Parameters ε, γ, α, δ, τ, ρ_θ, and ρ_χ are fixed externally as shown in the upper panel of Table 4. The remaining seven parameters,

$$x = (\sigma_\theta^2, \sigma_\chi^2, \sigma_{\theta\chi}, \beta_m, u_\beta, \varphi, b),$$

are calibrated jointly to the ten targeted moments listed in Table 5. The system is over-identified by three moments. Formally, I solve

$$\min_{x\in[\mathrm{lb},\mathrm{ub}]} \frac{1}{10}\sum_{j=1}^{10}\left|m_j^{\mathrm{model}}(x) - m_j^{\mathrm{data}}\right|,$$

where each evaluation of $m^{\text{model}}(x)$ requires the solution of the stationary equilibrium. I use the mean absolute deviation rather than the sum of squared residuals because it is more robust to moments that the model cannot match exactly.

The algorithm evaluates the criterion at 400 random perturbations around an initial guess. Then, it feeds the best candidate as the starting point to a nonlinear optimizer. It also uses a parallelized finite-difference Jacobian to accelerate the local search. I implement this minimization with MATLAB's `lsqnonlin` solver after a sign-preserving square-root transformation of the residuals, so that the sum-of-squared transformed residuals equals the mean absolute deviation.

B.2 Stationary Equilibrium

Given parameters, I solve for a stationary equilibrium by a nested fixed point with four levels. The outer loop is a damped Picard iteration on the lump-sum transfer T. The middle loop is a bisection (Brent's method) on the capital-labor ratio K/L with T held at its current value. Each candidate K/L determines the wage w and the rental rate r from the firm's first-order conditions, $r = \alpha A(K/L)^{\alpha-1}$ and $w = (1-\alpha)A(K/L)^{\alpha}$. The inner step solves the household problem and the stationary distribution at the candidate prices and transfer, (w, r, T). I describe each level in turn, starting from the outer loop.

Outer loop: government budget and the transfer T. The equilibrium lump-sum transfer equals $T = \tau Y = \tau w L/(1-\alpha)$, but L depends on T through household decisions. I solve this fixed point with a damped Picard iteration. At each pass I fix T, run the middle-loop K/L bisection to find the matching equilibrium aggregates, and then update T as

$$T_{\text{new}} \;=\; \omega_T \cdot \tau \frac{wL}{1-\alpha} \;+\; (1-\omega_T) \cdot T_{\text{old}},$$

with $\omega_T = 0.7$. I iterate until $|T_{\text{new}} - T_{\text{old}}| < 10^{-2}$, and verify $|T - \tau Y| < 10^{-2}$ as a post-equilibrium budget check.

Middle loop: capital-labor ratio K/L. With T fixed at its current outer value, the middle loop involves a comparison of the K/L implied by the firm's decisions and the K/L implied by the household decisions described in the inner loop below. For the first outer iteration, I bracket the bisection using the complete-markets ratio $(K/L)_{cm} = (\alpha A/r_{cm})^{1/(1-\alpha)}$ with $r_{cm} = (1/\beta_{\max} - 1 + \delta)/(1-\tau)$, where $\beta_{\max}$ is the largest discount factor across types. I use the bracket $[0.3\,(K/L)_{cm},\, 5\,(K/L)_{cm}]$. For subsequent outer iterations, I restart the bisection from the previous K/L equilibrium as a scalar initial guess rather than from the full bracket. With T moving only modestly between outer iterations, the new equilibrium K/L is close to the previous one, and starting with the previous household policies makes each evaluation cheap. I also check that at each candidate K/L, the implied gross return $1 + (1-\tau)r - \delta$ does not exceed $1/\beta_{\max}$, because that would lead the patient type to accumulate infinite assets. If the implied gross return exceeds the inverse of the discount factor of the patient type, I add a large positive gap to the K/L that pushes the bisection upward. This bypasses both the household solution and the stationary distribution iteration, which would otherwise not converge.

Household problem: endogenous grid method. For each candidate (w, r, T) and each β-type, I solve the household problem using the endogenous grid method of Carroll (2006). Because the intratemporal first-order condition gives a closed form for labor as a function of consumption and the exogenous (θ, χ) of the household,

$$n \;=\; \left(\frac{(1-\tau)w\theta}{\chi\, c^{\gamma}}\right)^{\varepsilon},$$

the only functional unknown is the consumption policy function, $c(k, \theta, \chi, \beta)$. The algorithm iterates the following steps to convergence:

1. Guess $c^{(0)}(k, \theta, \chi, \beta)$ on the asset grid.

2. For each next-period asset value and each current state for the shocks, evaluate the right-hand side of the Euler equation:

   $$\text{RHS}(k', \theta, \chi, \beta) \;=\; \beta\,(1 + (1-\tau)r - \delta)\,\mathbb{E}\big[c^{(j)}(k', \theta', \chi', \beta)^{-\gamma}\,\big|\,\theta, \chi\big]$$

   using the discretized transition matrix for the shocks which is described below.

3. Invert marginal utility to obtain $\hat{c} = \text{RHS}^{-1/\gamma}$, recover $\hat{n}$ from the intratemporal first-order condition, and back out the implied current asset $\hat{k}$ from the budget constraint.

4. Interpolate $(\hat{k}, \hat{c})$ onto the fixed asset grid linearly to obtain $c^{(j+1)}(k, \theta, \chi, \beta)$.

5. For grid points for which the implied $\hat{k}$ falls below $-b$, the borrowing constraint binds. Then, set $k' = -b$ and solve a one-dimensional Newton iteration for c from the budget constraint evaluated at $k' = -b$.

Convergence is declared when $\|c^{(j+1)} - c^{(j)}\|_\infty < 10^{-8}$. Each iteration starts from the previous parameter evaluation, which substantially reduces the number of iterations across bisection steps.

Stationary distribution: histogram method. Given the policy function, $\hat{k} = k'(k, \theta, \chi, \beta)$, I compute the stationary distribution over (k, θ, χ, β) using the histogram method of Young (2010). In general $\hat{k}$ will land between two adjacent asset-grid points, $k_j \leq \hat{k} \leq k_{j+1}$. To place $\hat{k}$ into the asset grid, I split the mass in proportion to how close $\hat{k}$ is to the two adjacent points of the grid, so that the linear average exactly reproduces the off-grid value. Thus, the distribution becomes

$$\lambda_{t+1}(k', \theta', \chi', \beta) \;=\; \sum_{(k,\theta,\chi)} \lambda_t(k, \theta, \chi, \beta)\, \mathbb{P}(\theta', \chi' \mid \theta, \chi)\, \psi(k', k, \theta, \chi, \beta),$$

where for two adjacent points k_j and k_{j+1}

$$\psi(k_j,\, k, \theta, \chi, \beta) \;=\; \frac{k_{j+1} - \hat{k}}{k_{j+1} - k_j} \quad \text{and} \quad \psi(k_{j+1},\, k, \theta, \chi, \beta) \;=\; \frac{\hat{k} - k_j}{k_{j+1} - k_j}.$$

I iterate on the distribution until $\|\lambda_{t+1} - \lambda_t\|_\infty < 10^{-10}$. Because the heterogeneity in β is permanent, I compute a separate stationary distribution for each type and then aggregate using the population shares φ and $1 - \varphi$.

Discretization. The exogenous shocks $(\log\theta, \log\chi)$ follow a bivariate AR(1) with diagonal persistence and (possibly correlated) Gaussian innovations. I discretize the joint process using multivariate Tauchen on a tensor grid of $n_\theta \times n_\chi$ points: each marginal log-grid is equispaced over $[-m\sigma_{\text{unc}}, +m\sigma_{\text{unc}}]$ in unconditional standard deviations (with $m = 3$), and the joint transition matrix is constructed by integrating the bivariate normal conditional density over rectangular bins centered at each grid point. In my baseline I set $n_\theta = n_\chi = 7$. Appendix Table A.1 shows that increasing the grid points for θ, χ affects the calibrated values of the variance parameters, but only slightly the elasticities or the income effects.

The asset grid is nonlinear with $n_k = 200$ points, denser near the borrowing constraint:

$$k_j \;=\; -b \;+\; (k_{\max} + b)\, \frac{(1+\kappa)^{j-1} - 1}{(1+\kappa)^{n_k - 1} - 1}, \qquad j = 1, \ldots, n_k,$$

with curvature parameter $\kappa = 0.04$ and $k_{\max} = 200$. The upper bound of the grid is chosen such that the mass of the stationary distribution at the upper bound is negligible. I have also confirmed that increasing n_k does not significantly affect the moments generated by the model.

Other details. I use loose inner-solver tolerances during the many evaluations during the calibration and tighten them for the final reporting of the results. Specifically, during calibration evaluations the K/L bisection, the household iteration, and the stationary distribution iteration use tolerances of 10^{-3}, 10^{-6}, and 10^{-7}, respectively. These are tightened to 10^{-4}, 10^{-8}, and 10^{-10} for the final reporting. The household consumption policy and the stationary distribution are initialized from the most recent solution at adjacent parameter or price values, which yields substantial speedups across both the inner bisection steps and the outer calibration evaluations.

C Additional Results

This Appendix presents results from sensitivity analyses. All sensitivity analyses fully recalibrate the models.

- Tables A.2, A.3, A.4, A.5, A.6, A.7, and A.8 repeat the analyses of Tables 1, 2, 3, 4, 5, 6, and 7 in the main text, when I fix the Frisch elasticity of labor supply to $\varepsilon = 0.70$, instead of $\varepsilon = 0.54$ in the main text.

- Tables A.9, A.10, A.11, A.12, A.13, A.14, and A.15 repeat the analyses of Tables 1, 2, 3, 4, 5, 6, and 7 in the main text, when I fix the Frisch elasticity of labor supply to $\varepsilon = 0.38$, instead of $\varepsilon = 0.54$ in the main text.

- Tables A.16, A.17, A.18, A.19, A.20, A.21, and A.22 repeat the analyses of Tables 1, 2, 3, 4, 5, 6, and 7 in the main text, when I use the cross-sectional moments estimated in the Consumer Expenditure Survey for married households by Boerma and Karabarbounis (2021) instead of the moments used in the main text from Heathcote, Storesletten, and Violante (2014).

- Tables A.23, A.24, A.25, and A.26 repeat the analyses of Tables 4, 5, 6, and 7 in the main text, when I target the ratio of fixed assets excluding durables to GDP ($K/Y = 2.94$ and $\delta = 0.053$) instead of fixed assets including durables to GDP used in the main text. The exogenous capital income model does not use these values, so Tables 1, 2, 3 are not affected.

Table A.1: Sensitivity Analyses, Size of Grids

Statistic	Data	Baseline	$n_\theta = 11$	$n_\chi = 11$	$n_\theta = 15$	$n_\chi = 15$
$\text{var}(\ln \theta)$	0.36	0.35	0.30	0.35	0.26	0.35
$\text{var}(\ln n)$	0.12	0.28	0.27	0.25	0.26	0.25
$\text{var}(\ln c)$	0.16	0.18	0.12	0.17	0.11	0.17
$\text{corr}(\ln n, \ln \theta)$	−0.10	−0.10	−0.22	−0.11	−0.26	−0.11
$\text{corr}(\ln \theta, \ln c)$	0.50	0.82	0.73	0.82	0.69	0.83
$\text{corr}(\ln n, \ln c)$	0.22	0.10	0.05	0.08	0.04	0.08
K/Y	3.25	3.22	3.21	3.22	3.19	3.21
top 10% k share	0.65	0.65	0.62	0.66	0.61	0.66
fraction $k < 0$	0.20	0.20	0.21	0.22	0.20	0.22
fraction $k = -b$	0.11	0.13	0.13	0.14	0.13	0.14
macro elasticity	−0.17	−0.17	−0.17	−0.17	−0.17	−0.17
mean micro elasticity	0.06	0.05	0.04	0.04	0.03	0.04
90-50 diff Marshallian	0.30	0.22	0.21	0.21	0.21	0.21
median ζ	0.43	0.56	0.59	0.56	0.59	0.57
variance of $\ln \zeta$	0.30	0.29	0.27	0.27	0.25	0.27
$\text{corr}(\ln \zeta, \ln \theta)$	0.67	0.36	0.34	0.37	0.30	0.37
$\text{corr}(\ln \zeta, \ln \chi)$	0.19	−0.61	−0.51	−0.58	−0.51	−0.57
$\text{corr}(\ln \zeta, \beta)$	–	−0.29	−0.32	−0.31	−0.34	−0.31
MPE at lot $= 0.05 \cdot Y$	−0.52	−0.48	−0.48	−0.48	−0.48	−0.47

Notes. The table reports the moments from Table 5, the difference between the 90th and the 50th percentiles of the Marshallian elasticity of labor supply, and the marginal propensity to earn in response to a lump-sum transfer of 5 percent of average income. The first two columns report the values in the data and in the baseline model with endogenous capital income. The remaining columns report the values under alternative discretizations of the joint productivity-disutility shock process relative to the baseline. The data value for the difference between the 90th and the 50th percentiles of the Marshallian elasticity is from Graber, Håvarstein, Mogstad, Torsvik, and Vestad (2026), and the data value for the marginal propensity to earn is from Golosov, Graber, Mogstad, and Novgorodsky (2024).

Table A.2: Calibration Targets, Exogenous Capital Income

Moment	Symbol	Value	Source
Frisch elasticity	ε	0.70	Pistaferri (2003)
macro elasticity	η_w	-0.17	Bick, Fuchs-Schündeln, and Lagakos (2018) and Boppart and Krusell (2020)
aggregate labor share	s_ℓ	0.63	Karabarbounis (2024)
variance of log wages	σ_w^2	0.36	Heathcote, Storesletten, and Violante (2014)
variance of log hours	σ_n^2	0.12	Heathcote, Storesletten, and Violante (2014)
variance of log consumption	σ_c^2	0.16	Heathcote, Storesletten, and Violante (2014)
correlation of log wages, hours	ρ_{wn}	-0.10	Heathcote, Storesletten, and Violante (2014)
correlation of log wages, cons.	ρ_{wc}	0.50	Heathcote, Storesletten, and Violante (2014)
correlation of log hours, cons.	ρ_{nc}	0.22	Heathcote, Storesletten, and Violante (2014)
mean micro elasticity	$\bar{\eta}_\theta$	0.06	Keane (2011)

Notes. The table reports the values of the ten moments targeted in the calibration of the model with exogenous capital income and their sources from the literature.

Table A.3: Identified Parameters, Exogenous Capital Income

Parameter	Description	Value
ε	Frisch elasticity	0.700
γ	curvature with respect to consumption	1.497
α	capital share in production	0.370
σ_θ^2	variance of $\ln\theta$	0.360
σ_χ^2	variance of $\ln\chi$	0.794
σ_ζ^2	variance of $\ln\zeta$	0.297
$\sigma_{\theta\chi}$	$\text{cov}(\ln\theta, \ln\chi)$, corr 0.393	0.210
$\sigma_{\theta\zeta}$	$\text{cov}(\ln\theta, \ln\zeta)$, corr 0.670	0.219
$\sigma_{\chi\zeta}$	$\text{cov}(\ln\chi, \ln\zeta)$, corr 0.278	0.135
$\bar{\zeta}$	median ζ	0.601

Notes. The table reports the ten parameters of the model with exogenous capital income, identified in closed form from the targeted moments in Table A.2 using the recursive expressions in Appendix A.3.

Table A.4: Estimates of Marshallian Elasticity of Labor Supply

Specification	Slope on $\ln(w\theta)$
wage only	-0.071
wage, π	-0.124
wage, π, $\ln\chi$	0.053
wage, $\ln\zeta$, $\ln\chi$	-0.170

Notes. The table reports OLS coefficients from regressions of log hours on log wages in a simulated cross section of 10,000 households drawn from the model with exogenous capital income. Each row varies the control variables included alongside log wages.

Table A.5: Parameters, Endogenous Capital Income

Fixed Parameter	Description	Value
ε	Frisch elasticity	0.700
γ	curvature with respect to consumption	1.497
α	capital share in production	0.370
δ	depreciation rate	0.066
τ	tax rate	0.212
ρ_θ	persistence of productivity	0.970
ρ_χ	persistence of disutility of work	0.730
Calibrated Parameter	**Description**	**Value**
σ^2_θ	variance of $\ln\theta$	0.207
σ^2_χ	variance of $\ln\chi$	1.034
$\sigma_{\theta\chi}$	$\mathrm{cov}(\ln\theta, \ln\chi)$, corr 0.550	0.255
β_m	central discount factor	0.959
u_β	spread in discount factors	0.013
φ	share with low discount factor	0.701
b	borrowing limit	0.119

Notes. The table reports the parameters of the model with endogenous capital income. The top panel lists parameters set externally, before solving the model. The bottom panel lists parameters calibrated internally by minimizing the distance between the moments generated by the model and their data analogs in Table A.6.

Table A.6: Moments, Endogenous Capital Income

Targeted Moment	Data Value	Model Value
variance of log wages	0.36	0.34
variance of log hours	0.12	0.39
variance of log consumption	0.16	0.17
correlation of log wages, hours	-0.10	-0.10
correlation of log wages, consumption	0.50	0.78
correlation of log hours, consumption	0.22	0.14
capital-output ratio	3.25	3.21
top 10% wealth share	0.65	0.64
fraction with negative assets	0.20	0.22
fraction hand-to-mouth	0.11	0.13
Untargeted Moment	**Data Value**	**Model Value**
macro elasticity	-0.17	-0.17
micro elasticity	0.06	0.09
median ζ	0.60	0.56
variance of $\ln \zeta$	0.30	0.37
$\text{corr}(\ln \zeta, \ln \theta)$	0.67	0.31
$\text{corr}(\ln \zeta, \ln \chi)$	0.28	-0.61
$\text{corr}(\ln \zeta, \beta)$	–	-0.29

Notes. The upper panel reports the ten data moments targeted in the calibration of the model with endogenous capital income and the corresponding moments generated by the model. The bottom panel reports model-generated statistics of the household-level labor share distribution and the average micro and macro elasticities of labor supply. For comparability, the data values for the statistics of the household-level labor share are populated with the statistics of the exogenous capital income model.

Table A.7: Distribution of Marshallian Elasticity of Labor Supply

Moment	Exo capital income	Endo capital income	Attanasio et al. (2018)
10th	−0.20	−0.16	−0.14
25th	−0.11	−0.05	0.01
50th	0.04	0.07	0.18
75th	0.22	0.22	0.39
90th	0.36	0.37	0.79
Average	0.06	0.09	N/A

Notes. The table reports percentiles of the Marshallian elasticity of labor supply across households in the model with exogenous capital income and the model with endogenous capital income. The third column reports the corresponding distribution from the structural model of Attanasio, Levell, Low, and Sánchez-Marcos (2018).

Table A.8: Marginal Propensity to Earn

Size of Transfer	Mean	10th	25th	50th	75th	90th
Panel A: Exogenous capital income						
0.01	−0.37	−0.53	−0.48	−0.38	−0.28	−0.20
0.05	−0.37	−0.52	−0.47	−0.38	−0.27	−0.19
0.10	−0.36	−0.52	−0.46	−0.37	−0.26	−0.19
0.20	−0.35	−0.50	−0.45	−0.35	−0.25	−0.18
Panel B: Endogenous capital income						
0.01	−0.57	−0.96	−0.70	−0.51	−0.38	−0.24
0.05	−0.55	−0.93	−0.68	−0.49	−0.36	−0.23
0.10	−0.53	−0.91	−0.65	−0.47	−0.34	−0.22
0.20	−0.49	−0.86	−0.60	−0.44	−0.31	−0.20

Notes. The table reports statistics of the marginal propensity to earn, which is defined as the ratio of the change in earnings to the change in non-labor income. The statistics are reported across households in the model with exogenous capital income and the model with endogenous capital income and for hypothetical lump-sum transfers of different sizes relative to average income.

Table A.9: Calibration Targets, Exogenous Capital Income

Moment	Symbol	Value	Source
Frisch elasticity	ε	0.38	
macro elasticity	η_w	-0.17	Bick, Fuchs-Schündeln, and Lagakos (2018) and Boppart and Krusell (2020)
aggregate labor share	s_ℓ	0.63	Karabarbounis (2024)
variance of log wages	σ_w^2	0.36	Heathcote, Storesletten, and Violante (2014)
variance of log hours	σ_n^2	0.12	Heathcote, Storesletten, and Violante (2014)
variance of log consumption	σ_c^2	0.16	Heathcote, Storesletten, and Violante (2014)
correlation of log wages, hours	ρ_{wn}	-0.10	Heathcote, Storesletten, and Violante (2014)
correlation of log wages, cons.	ρ_{wc}	0.50	Heathcote, Storesletten, and Violante (2014)
correlation of log hours, cons.	ρ_{nc}	0.22	Heathcote, Storesletten, and Violante (2014)
mean micro elasticity	$\bar{\eta}_\theta$	0.06	Keane (2011)

Notes. The table reports the values of the ten moments targeted in the calibration of the model with exogenous capital income and their sources from the literature.

Table A.10: Identified Parameters, Exogenous Capital Income

Parameter	Description	Value
ε	Frisch elasticity	0.380
γ	curvature with respect to consumption	1.744
α	capital share in production	0.370
σ_θ^2	variance of $\ln\theta$	0.360
σ_χ^2	variance of $\ln\chi$	1.648
σ_ζ^2	variance of $\ln\zeta$	0.297
$\sigma_{\theta\chi}$	$\text{cov}(\ln\theta, \ln\chi)$, corr 0.267	0.205
$\sigma_{\theta\zeta}$	$\text{cov}(\ln\theta, \ln\zeta)$, corr 0.670	0.219
$\sigma_{\chi\zeta}$	$\text{cov}(\ln\chi, \ln\zeta)$, corr 0.078	0.055
$\bar{\zeta}$	median ζ	0.399

Notes. The table reports the ten parameters of the model with exogenous capital income, identified in closed form from the targeted moments in Table A.9 using the recursive expressions in Appendix A.3.

Table A.11: Estimates of Marshallian Elasticity of Labor Supply

Specification	Slope on $\ln(w\theta)$
wage only	-0.187
wage, π	-0.192
wage, π, $\ln\chi$	-0.055
wage, $\ln\zeta$, $\ln\chi$	-0.170

Notes. The table reports OLS coefficients from regressions of log hours on log wages in a simulated cross section of 10,000 households drawn from the model with exogenous capital income. Each row varies the control variables included alongside log wages.

Table A.12: Parameters, Endogenous Capital Income

Fixed Parameter	Description	Value
ε	Frisch elasticity	0.380
γ	curvature with respect to consumption	1.744
α	capital share in production	0.370
δ	depreciation rate	0.066
τ	tax rate	0.212
ρ_θ	persistence of productivity	0.970
ρ_χ	persistence of disutility of work	0.730
Calibrated Parameter	**Description**	**Value**
σ^2_θ	variance of $\ln\theta$	0.202
σ^2_χ	variance of $\ln\chi$	1.479
$\sigma_{\theta\chi}$	$\mathrm{cov}(\ln\theta, \ln\chi)$, corr 0.218	0.119
β_m	central discount factor	0.960
u_β	spread in discount factors	0.012
φ	share with low discount factor	0.674
b	borrowing limit	0.041

Notes. The table reports the parameters of the model with endogenous capital income. The top panel lists parameters set externally, before solving the model. The bottom panel lists parameters calibrated internally by minimizing the distance between the moments generated by the model and their data analogs in Table A.13.

Table A.13: Moments, Endogenous Capital Income

Targeted Moment	Data Value	Model Value
variance of log wages	0.36	0.33
variance of log hours	0.12	0.19
variance of log consumption	0.16	0.17
correlation of log wages, hours	−0.10	−0.10
correlation of log wages, consumption	0.50	0.84
correlation of log hours, consumption	0.22	0.07
capital-output ratio	3.25	3.24
top 10% wealth share	0.65	0.66
fraction with negative assets	0.20	0.17
fraction hand-to-mouth	0.11	0.13
Untargeted Moment	**Data Value**	**Model Value**
macro elasticity	−0.17	−0.17
micro elasticity	0.06	−0.00
median ζ	0.40	0.57
variance of $\ln \zeta$	0.30	0.21
$\text{corr}(\ln \zeta, \ln \theta)$	0.67	0.40
$\text{corr}(\ln \zeta, \ln \chi)$	0.08	−0.61
$\text{corr}(\ln \zeta, \beta)$	–	−0.30

Notes. The upper panel reports the ten data moments targeted in the calibration of the model with endogenous capital income and the corresponding moments generated by the model. The bottom panel reports model-generated statistics of the household-level labor share distribution and the average micro and macro elasticities of labor supply. For comparability, the data values for the statistics of the household-level labor share are populated with the statistics of the exogenous capital income model.

Table A.14: Distribution of Marshallian Elasticity of Labor Supply

Moment	Exo capital income	Endo capital income	Attanasio et al. (2018)
10th	0.05	−0.14	−0.14
25th	0.08	−0.07	0.01
50th	0.09	−0.00	0.18
75th	0.09	0.07	0.39
90th	0.09	0.15	0.79
Average	0.06	−0.00	N/A

Notes. The table reports percentiles of the Marshallian elasticity of labor supply across households in the model with exogenous capital income and the model with endogenous capital income. The third column reports the corresponding distribution from the structural model of Attanasio, Levell, Low, and Sánchez-Marcos (2018).

Table A.15: Marginal Propensity to Earn

Size of Transfer	Mean	10th	25th	50th	75th	90th
Panel A: Exogenous capital income						
0.01	−0.23	−0.24	−0.21	−0.21	−0.21	−0.21
0.05	−0.23	−0.24	−0.21	−0.20	−0.20	−0.20
0.10	−0.22	−0.23	−0.21	−0.20	−0.19	−0.19
0.20	−0.21	−0.23	−0.20	−0.19	−0.18	−0.17
Panel B: Endogenous capital income						
0.01	−0.41	−0.64	−0.49	−0.38	−0.30	−0.22
0.05	−0.40	−0.62	−0.48	−0.37	−0.29	−0.21
0.10	−0.39	−0.61	−0.46	−0.36	−0.27	−0.20
0.20	−0.36	−0.57	−0.44	−0.33	−0.25	−0.18

Notes. The table reports statistics of the marginal propensity to earn, which is defined as the ratio of the change in earnings to the change in non-labor income. The statistics are reported across households in the model with exogenous capital income and the model with endogenous capital income and for hypothetical lump-sum transfers of different sizes relative to average income.

Table A.16: Calibration Targets, Exogenous Capital Income

Moment	Symbol	Value	Source
Frisch elasticity	ε	0.54	Chetty, Guren, Manoli, and Weber (2012)
macro elasticity	η_w	-0.17	Bick, Fuchs-Schündeln, and Lagakos (2018) and Boppart and Krusell (2020)
aggregate labor share	s_ℓ	0.63	Karabarbounis (2024)
variance of log wages	σ_w^2	0.33	Boerma and Karabarbounis (2021)
variance of log hours	σ_n^2	0.23	Boerma and Karabarbounis (2021)
variance of log consumption	σ_c^2	0.33	Boerma and Karabarbounis (2021)
correlation of log wages, hours	ρ_{wn}	-0.07	Boerma and Karabarbounis (2021)
correlation of log wages, cons.	ρ_{wc}	0.29	Boerma and Karabarbounis (2021)
correlation of log hours, cons.	ρ_{nc}	0.13	Boerma and Karabarbounis (2021)
mean micro elasticity	$\bar{\eta}_\theta$	0.06	Keane (2011)

Notes. The table reports the values of the ten moments targeted in the calibration of the model with exogenous capital income and their sources from the literature.

Table A.17: Identified Parameters, Exogenous Capital Income

Parameter	Description	Value
ε	Frisch elasticity	0.540
γ	curvature with respect to consumption	1.584
α	capital share in production	0.370
σ_θ^2	variance of $\ln\theta$	0.330
σ_χ^2	variance of $\ln\chi$	1.925
σ_ζ^2	variance of $\ln\zeta$	0.588
$\sigma_{\theta\chi}$	$\text{cov}(\ln\theta, \ln\chi)$, corr 0.269	0.214
$\sigma_{\theta\zeta}$	$\text{cov}(\ln\theta, \ln\zeta)$, corr 0.488	0.215
$\sigma_{\chi\zeta}$	$\text{cov}(\ln\chi, \ln\zeta)$, corr 0.193	0.206
$\bar{\zeta}$	median ζ	0.394

Notes. The table reports the ten parameters of the model with exogenous capital income, identified in closed form from the targeted moments in Table A.16 using the recursive expressions in Appendix A.3.

Table A.18: Estimates of Marshallian Elasticity of Labor Supply

Specification	Slope on $\ln(w\theta)$
wage only	-0.143
wage, π	-0.168
wage, π, $\ln\chi$	0.017
wage, $\ln\zeta$, $\ln\chi$	-0.170

Notes. The table reports OLS coefficients from regressions of log hours on log wages in a simulated cross section of 10,000 households drawn from the model with exogenous capital income. Each row varies the control variables included alongside log wages.

Table A.19: Parameters, Endogenous Capital Income

Fixed Parameter	Description	Value
ε	Frisch elasticity	0.540
γ	curvature with respect to consumption	1.584
α	capital share in production	0.370
δ	depreciation rate	0.066
τ	tax rate	0.212
ρ_θ	persistence of productivity	0.970
ρ_χ	persistence of disutility of work	0.730
Calibrated Parameter	**Description**	**Value**
σ_θ^2	variance of $\ln\theta$	0.204
σ_χ^2	variance of $\ln\chi$	1.150
$\sigma_{\theta\chi}$	$\text{cov}(\ln\theta, \ln\chi)$, corr 0.243	0.118
β_m	central discount factor	0.963
u_β	spread in discount factors	0.010
φ	share with low discount factor	0.780
b	borrowing limit	0.110

Notes. The table reports the parameters of the model with endogenous capital income. The top panel lists parameters set externally, before solving the model. The bottom panel lists parameters calibrated internally by minimizing the distance between the moments generated by the model and their data analogs in Table A.20.

Table A.20: Moments, Endogenous Capital Income

Targeted Moment	Data Value	Model Value
variance of log wages	0.33	0.33
variance of log hours	0.23	0.28
variance of log consumption	0.33	0.18
correlation of log wages, hours	−0.07	−0.06
correlation of log wages, consumption	0.29	0.82
correlation of log hours, consumption	0.13	0.12
capital-output ratio	3.25	3.23
top 10% wealth share	0.65	0.67
fraction with negative assets	0.20	0.20
fraction hand-to-mouth	0.11	0.12
Untargeted Moment	**Data Value**	**Model Value**
macro elasticity	−0.17	−0.17
micro elasticity	0.06	0.05
median ζ	0.39	0.56
variance of $\ln\zeta$	0.59	0.30
$\text{corr}(\ln\zeta, \ln\theta)$	0.49	0.35
$\text{corr}(\ln\zeta, \ln\chi)$	0.19	−0.64
$\text{corr}(\ln\zeta, \beta)$	–	−0.31

Notes. The upper panel reports the ten data moments targeted in the calibration of the model with endogenous capital income and the corresponding moments generated by the model. The bottom panel reports model-generated statistics of the household-level labor share distribution and the average micro and macro elasticities of labor supply. For comparability, the data values for the statistics of the household-level labor share are populated with the statistics of the exogenous capital income model.

Table A.21: Distribution of Marshallian Elasticity of Labor Supply

Moment	Exo capital income	Endo capital income	Attanasio et al. (2018)
10th	-0.20	-0.15	-0.14
25th	0.05	-0.06	0.01
50th	0.15	0.04	0.18
75th	0.18	0.15	0.39
90th	0.19	0.26	0.79
Average	0.06	0.05	N/A

Notes. The table reports percentiles of the Marshallian elasticity of labor supply across households in the model with exogenous capital income and the model with endogenous capital income. The third column reports the corresponding distribution from the structural model of Attanasio, Levell, Low, and Sánchez-Marcos (2018).

Table A.22: Marginal Propensity to Earn

Size of Transfer	Mean	10th	25th	50th	75th	90th
Panel A: Exogenous capital income						
0.01	-0.31	-0.48	-0.32	-0.25	-0.23	-0.22
0.05	-0.30	-0.47	-0.31	-0.25	-0.22	-0.22
0.10	-0.30	-0.46	-0.30	-0.24	-0.22	-0.21
0.20	-0.28	-0.44	-0.29	-0.23	-0.21	-0.19
Panel B: Endogenous capital income						
0.01	-0.50	-0.82	-0.61	-0.45	-0.34	-0.24
0.05	-0.48	-0.80	-0.59	-0.44	-0.32	-0.23
0.10	-0.47	-0.78	-0.57	-0.42	-0.31	-0.22
0.20	-0.43	-0.73	-0.53	-0.39	-0.28	-0.19

Notes. The table reports statistics of the marginal propensity to earn, which is defined as the ratio of the change in earnings to the change in non-labor income. The statistics are reported across households in the model with exogenous capital income and the model with endogenous capital income and for hypothetical lump-sum transfers of different sizes relative to average income.

Table A.23: Parameters, Endogenous Capital Income

Fixed Parameter	Description	Value
ε	Frisch elasticity	0.540
γ	curvature with respect to consumption	1.584
α	capital share in production	0.370
δ	depreciation rate	0.053
τ	tax rate	0.212
ρ_θ	persistence of productivity	0.970
ρ_χ	persistence of disutility of work	0.730
Calibrated Parameter	**Description**	**Value**
σ_θ^2	variance of $\ln \theta$	0.201
σ_χ^2	variance of $\ln \chi$	1.316
$\sigma_{\theta\chi}$	$\mathrm{cov}(\ln \theta, \ln \chi)$, corr 0.446	0.229
β_m	central discount factor	0.935
u_β	spread in discount factors	0.016
φ	share with low discount factor	0.747
b	borrowing limit	0.014

Notes. The table reports the parameters of the model with endogenous capital income. The top panel lists parameters set externally, before solving the model. The bottom panel lists parameters calibrated internally by minimizing the distance between the moments generated by the model and their data analogs in Table A.24.

Table A.24: Moments, Endogenous Capital Income

Targeted Moment	Data Value	Model Value
variance of log wages	0.36	0.33
variance of log hours	0.12	0.31
variance of log consumption	0.16	0.20
correlation of log wages, hours	−0.10	−0.11
correlation of log wages, consumption	0.50	0.73
correlation of log hours, consumption	0.22	0.06
capital-output ratio	2.94	2.90
top 10% wealth share	0.65	0.64
fraction with negative assets	0.20	0.18
fraction hand-to-mouth	0.11	0.15
Untargeted Moment	**Data Value**	**Model Value**
macro elasticity	−0.17	−0.17
micro elasticity	0.06	0.06
median ζ	0.43	0.57
variance of $\ln \zeta$	0.30	0.37
$\text{corr}(\ln \zeta, \ln \theta)$	0.67	0.31
$\text{corr}(\ln \zeta, \ln \chi)$	0.19	−0.51
$\text{corr}(\ln \zeta, \beta)$	–	−0.46

Notes. The upper panel reports the ten data moments targeted in the calibration of the model with endogenous capital income and the corresponding moments generated by the model. The bottom panel reports model-generated statistics of the household-level labor share distribution and the average micro and macro elasticities of labor supply. For comparability, the data values for the statistics of the household-level labor share are populated with the statistics of the exogenous capital income model.

Table A.25: Distribution of Marshallian Elasticity of Labor Supply

Moment	Exo capital income	Endo capital income	Attanasio et al. (2018)
10th	−0.11	−0.14	−0.14
25th	0.05	−0.05	0.01
50th	0.13	0.04	0.18
75th	0.15	0.16	0.39
90th	0.16	0.29	0.79
Average	0.06	0.06	N/A

Notes. The table reports percentiles of the Marshallian elasticity of labor supply across households in the model with exogenous capital income and the model with endogenous capital income. The third column reports the corresponding distribution from the structural model of Attanasio, Levell, Low, and Sánchez-Marcos (2018).

Table A.26: Marginal Propensity to Earn

Size of Transfer	Mean	10th	25th	50th	75th	90th
Panel A: Exogenous capital income						
0.01	−0.31	−0.42	−0.31	−0.27	−0.25	−0.24
0.05	−0.30	−0.42	−0.31	−0.26	−0.24	−0.24
0.10	−0.30	−0.41	−0.30	−0.26	−0.24	−0.23
0.20	−0.28	−0.40	−0.29	−0.24	−0.22	−0.21
Panel B: Endogenous capital income						
0.01	−0.47	−0.75	−0.57	−0.44	−0.33	−0.21
0.05	−0.46	−0.73	−0.56	−0.42	−0.32	−0.20
0.10	−0.44	−0.71	−0.54	−0.41	−0.31	−0.19
0.20	−0.41	−0.68	−0.51	−0.38	−0.28	−0.18

Notes. The table reports statistics of the marginal propensity to earn, which is defined as the ratio of the change in earnings to the change in non-labor income. The statistics are reported across households in the model with exogenous capital income and the model with endogenous capital income and for hypothetical lump-sum transfers of different sizes relative to average income.

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