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ABSTRACT

Standard New Keynesian models with time-consistent policy predict minimal debt responses to conventional shocks, as a debt stabilization bias dominates tax-smoothing motives. We show that two mechanisms can generate debt movements of the magnitude observed in the data: increases in policymaker myopia and declines in real interest rates, such as during flight-to-safety episodes. Other potential drivers—changes in markups, debt maturity, government transfers, or large recessions—cannot account for such fluctuations.

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1 Introduction

After falling steadily in many advanced economies following World War II, government debt has risen sharply since the early 1980s. In the United States, the ratio of debt held by the public to GDP fell from around 100% of GDP to about 20% in 1980, before increasing to 75% on the eve of the global financial crisis. It has climbed much further since – see Figure 4 below. Similar trends are observed in many other advanced economies, across very different macroeconomic environments – from the Great Moderation to the global financial crisis and the prolonged low interest rate period of the 2010s.

Historical decompositions of the data can help identify the drivers of these trends. Hall and Sargent (2011) show that the large postwar reduction in U.S. debt – from about 110% of GDP in 1945 to roughly 25% by the early 1970s – was driven primarily by sustained primary surpluses (accounting for around 40–50% of the decline) and robust economic growth (around 30–40%), while inflation contributed only modestly, on the order of 10–15 percentage points.

More recent work refines this accounting by incorporating the maturity structure of government debt and distinguishing between expected and unexpected returns. Acalin and Ball (2025) show that, even in the presence of substantial inflation, debt reduction in the postwar period relied heavily on primary surpluses and periods of low or negative real interest rates associated with financial repression, with growth alone explaining less than half of the observed decline in debt. These findings are consistent with broader historical evidence in Hall and Sargent (2021), who document that large increases in government debt following major shocks are typically financed through borrowing and are only gradually unwound through a combination of fiscal adjustment, growth, and favorable interest rate conditions, rather than rapid mean reversion.

Taken together, these facts suggest that the persistence and scale of debt movements pose an important challenge for standard theories of optimal fiscal policy, which typically imply either that debt should follow a random walk after shocks or revert toward a stable level.

Consider the random walk result. In a standard New Keynesian model, augmented with government debt, an optimizing Ramsey policy maker will engage in a policy of tax smoothing.[1] This implies that following shocks, tax rates will jump to ensure the solvency of the government's budget constraint. However, this new tax rate will not seek to return debt to its original level but to ensure that, once the effects of the shock have dissipated, tax revenues will be sufficient to sustain the new steady-state level of debt. That is, the steady-state debt-to-GDP ratio follows a random walk in the face of economic shocks with fiscal consequences. In the new steady-state the marginal costs of sustaining a higher debt level are balanced against the costs of the fiscal austerity needed to reduce debt. Implicitly, Barro's tax smoothing result relies on the government's ability to commit to maintain the new tax rate indefinitely.

In a similar framework, but removing the policy maker's ability to commit, the random walk result no longer applies. The existence of a stock of nominal government debt, which

[1] See, for example, the discussion in Leeper et al. (2021) which shows the relevance of Barro (1979)'s tax smoothing result across a variety of models.

requires costly tax increases or spending cuts to service, gives rise to an inflation bias problem. The policymaker faces the temptation to induce inflation surprises to reduce the real value of government debt. Economic agents anticipate this, and we end up worsening the inflation bias problem relative to the case of inefficiencies due to monopolistic competition and distortionary taxation alone. However, by reducing the debt level, the policymaker can reduce the inflationary bias associated with government debt. This means that, following shocks, the policy maker will return to a unique steady-state which balances the costs of the inflation bias against the costs of fiscal consolidation. This sub-optimal return of debt to a unique steady-state has been labeled the 'debt stabilization bias' (see Leeper and Leith (2016), Leeper et al. (2021) and Kirsanova et al. (2025).).

In this paper we explore some aspects of the debt stabilization bias which have not been reported in the literature. Specifically, we focus on the fact that the debt stabilization bias is so strong that the standard model cannot explain the observed fluctuations in debt – policy makers would not allow conventional economic shocks to generate more than modest movements in government debt. In fact, even if we consider a standard tax smoothing example – the economy experiences a temporary 50% increase in fiscal transfers – the time-consistent policy maker would actually *reduce* debt during the period of high fiscal transfers, where a tax smoothing policy would issue debt as a buffer – see Section 5 below.

These features also imply that the solution method matters. The policy problem is inherently nonlinear because the long-run allocation is itself endogenous and depends on the inherited level of debt, while policy incentives vary with debt through their effects on inflation expectations and bond pricing. As a result, the elements we study—such as large debt movements, regime switches in policymaker myopia, and crisis episodes involving sharp changes in real interest rates—cannot be fully captured by methods that approximate the economy only locally around a fixed steady state. A global solution is therefore important for our analysis, because it allows us to characterize equilibrium policy functions over the range of debt levels and regimes relevant for the mechanisms emphasized in the paper.

Leeper et al. (2021) show that the steady-state level of debt is influenced most by three factors – the size of firm markups, the maturity structure of the debt and the myopia of the policy maker. We explore the ability of *changes* in these three factors to explain movements in the debt-to-GDP ratio. We find that recent rises in markups should have led to debt falling, not rising, and that changes in debt maturity are not of the required magnitude to explain trends in government debt. However, movements in policy maker myopia can offer a partial explanation for fluctuations in debt, at least until the financial crisis.

We then, in a stylized way, consider the fiscal implications of the financial crisis and subsequent pandemic. We model these as a previously unexpected event which results in a combination of higher fiscal transfers, a significant recession and a flight-to-safety on the part of households, which reduces real interest rates. Can this combination of shocks, loosely chosen to mimic the financial crisis/pandemic, give rise to plausible movements in the debt-to-GDP ratio? We find that the reduction in interest rates resulting from the flight-to-safety gives the government an opportunity to issue significant amounts of debt. However, were interest rates ever to be re-normalized, the higher stock of debt will worsen the inflationary bias problem resulting in a period of high inflation and fiscal austerity until debt levels have returned to normal. In contrast, the recession and higher transfers, despite being examples of situations where one would expect tax smoothing motives to come to the fore, do not deliver significant increases in government debt. The debt stabilization bias dominates tax smoothing motives in these cases. Nevertheless, the combination of all three shocks does imply a combination of reduced inflation, initially, a significant recession and a large increase in government debt, helping us reconcile the model with the fiscal consequences of the financial crisis/pandemic. The crisis also implies a significant increase in inflation as debt accumulates and the policymakers' aversion to the increase in debt reasserts itself.[2]

We also consider the impact of conventional cost-push shocks in a variety of settings – high vs low debt and during periods of 'crisis' or normality. The cost-push shock is considerably more inflationary when we have just exited a period of crisis and debt levels are high. Moreover, estimating a Taylor rule using data generated by the simulated policy response would imply that monetary policy remains active, but significantly less so, when debt levels are high. In this sense, it is not surprising that the end of the pandemic brought about an increase in inflation.

Literature Review

Various papers have discussed the rise in government debt using mechanisms which complement the current paper, often focusing on political economy explanations of why governments may suffer from a deficit bias – for a comprehensive survey see Alesina and Passalacqua (2016). Yared (2019) argues that the rise in government debt in advanced economies since the 1970s is not supported by normative explanations, such as tax smoothing, dynamic inefficiency, and the need to supply safe assets. Instead, Yared (2019) argues that aging populations, increased political polarization and electoral uncertainty are more likely candidates to explain the rise in debt. Our paper makes a similar argument, but where the source of the time-consistency problem is not due to time-inconsistent policy maker preferences or other political economy frictions, rather the fact that government debt is nominal. Within our New Keynesian framework, we also find that standard economic shocks are not responsible for the rise in debt, but that increases in policy maker myopia (a short-hand for any mechanism which can drive a deficit bias) in combination with the debt stabilization bias, can do so, to some extent. Moreover, our analysis suggests that of the major shocks that have hit the world economy since 2008, it is the flight to safety that would have given governments the incentives to increase government debt. Increases in transfers or major recessions, would, in fact, have done the opposite.

[2] We confirm the robustness of these results to introducing additional costs to adjusting fiscal instruments and allowing for changes in debt maturity, in Section 7 below.

Halec and Yared (2025) also explore the impact of 'crises' on debt accumulation. This is the subject of the analysis in Section 7. Reis (2022) focuses on the convenience yield and debt dynamics – this is similar, in terms of its consequences, to the reduced interest rates experienced during a flight-to-safety.

Rather than coming from the literature which offers political economy explanations of excessive debt accumulation in real economies, our paper is closer to those papers which examine time-consistent policy problems in economies featuring both monetary and fiscal policy. In addition to the key papers cited in the Introduction which analyze the policy problem in the context of New Keynesian models, monetary frictions have also been used to generate a cost for inflation and generate trade-offs between the use of monetary and fiscal policy. For example, Schmitt-Grohé and Uribe (2004) study Ramsey policy in a flexible-price model with a cash-in-advance constraint, while Martin (2009) studies the time-consistency problems that arise from the interaction between debt and monetary policy, since inflation surprises reduce the real value of nominal liabilities. Martin (2011, 2014, 2015) examine time-consistent policies in variants of the monetary search model of Lagos and Wright (2005). Niemann et al. (2013b) combines both a cash-in advance constraint and sticky prices in the context of time-consistent policy with single-period debt – the monetary friction helps ensure the model can sustain a positive debt-to-GDP ratio in steady-state.

There is also a literature which looks at the strategic interactions between monetary and fiscal authorities. In the context of sticky-price New Keynesian economies like ours, notable examples include, Dixit and Lambertini (2003), Adam and Billi (2008, 2014) and Schreger et al. (2024).[3] [4] We abstract from this issue in the current paper, by assuming that the monetary and fiscal authorities share common objectives. Recent work by Kirsanova et al. (2025), using a framework similar to the current one, distinguishes between monetary and fiscal policy makers. In doing so, they find that the granting of central bank independence can offer an explanation for the observed increase in government debt – an inflation averse central bank reduces the inflationary costs of issuing debt. The government therefore issues more debt. However, eventually the higher debt leads to a worsening of the inflationary bias problem and inflation rises above its pre-independence level. Moreover, they find that it is the myopia of the central bank and not the government that is key to generating additional significant movements in the debt-to-GDP ratio. This work offers a complementary explanation to the current paper in explaining recent movements in government debt.

Roadmap. The paper proceeds as follows. We describe the benchmark model in Section 2. The non-linear time-consistent optimal policy problem is described in section 3. We undertake a numerical analysis of the equilibrium implied by our model in Section 4. We explore the extent

[3] Of these papers, only Schreger et al. (2024) include a meaningful role for government debt, although they consider a two-period economy that cannot assess the longer-term evolution of debt dynamics and inflation. Our stochastic model includes sticky prices, long-term debt, inflation conservatism and policy-maker myopia.

[4] Outside of the New Keynesian literature, flexible price economies subject to monetary frictions, are considered in Alvarez et al. (2004), Chari and Kehoe (2007), Niemann (2011), Niemann et al. (2013a) and Aguiar et al. (2015) and, using a Lagos-Wright monetary search model, Martin (2015).

to which the standard model can generate movements in government debt of the magnitude of those seen in the data in Section 5. In Section 6 we allow for switches in the degree of policy maker myopia and show that this can explain a large part of the increase in debt since the 1970s, at least until the financial crisis. We consider the impact of a crisis, consisting of a large recession, a 50% increase in fiscal transfers and a flight-to-safety which reduces interest rates – capturing, loosely, the combined impact of the financial crisis and pandemic – in Section 7. Section 7 also undertakes various extensions to demonstrate the robustness of the key results. We conclude in Section 8.

2 The Model

Our model follows that in Leeper et al. (2021). Specifically, it is a standard New Keynesian model but augmented to include the government's budget constraint where government spending is financed by distortionary taxation and/or borrowing. The policy makers are assumed to share the same objective, such that they operate as if they are a single policy maker.[5]

Other elements of the model include the fact that we allow the government to optimally vary government spending in the face of shocks, rather than simply treating government spending as an exogenous flow which must be financed. Additionally, our nominal debt is not of single-period maturity but consists of a portfolio of bonds of mixed maturities. This matters in highly indebted economies since even modest surprise changes in inflation and interest rates can have substantial effects on the market value of debt.[6]

We also allow for variation in the households' discount factor, which allows us to capture flight-to-safety dynamics. Finally, our solution algorithm permits us to consider distinct policy regimes. We use this to consider a classic tax-smoothing experiment where there is a temporary, but sustained, increase in government transfers. We also, subsequently, consider a crisis regime, where a combination of large recession, increase in transfers and a flight-to-safety capture some of the key elements of the recent financial crisis/pandemic.

We begin by deriving the model before turning to the detailed specification of the policy problem in the following section.

2.1 Households

There are a continuum of households of size one. Households appreciate private consumption as well as the provision of public goods and dislike supplying labor. We shall assume complete asset markets such that, through risk sharing, they will face the same budget constraint. As a result, the typical household will seek to maximize the following objective function,

[5] Using the same framework Kirsanova et al. (2025) consider the case of strategically interacting policy makers with different objectives.

[6] See Hall and Sargent (2011) and Sims (2013) for the empirical findings on the contribution of this kind of fiscal financing to the decline in the U.S. debt-GDP ratio from 1945 to 1974.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\prod_{i=0}^{t-1} \beta_i) U(C_t, N_t, G_t) \tag{1}$$

where C, G, and N are a consumption aggregate, a public goods aggregate, and labor supply, respectively. We have allowed the households' discount factor to vary over time.[7] Initially, we shall consider the discount factor to be fixed, but in Section 6 we shall introduce variation in the natural rate of interest through the device of exogenous Markov switching in households' regime-dependent steady-state discount factors. For the moment, it is important to note that the timing of the discount factor implies that β_t is the discount factor used to discount next period $t+1$ utility at time t. Therefore, households know their discount factor as it applies to next period's utility, but it may change beyond that, and this needs to be factored into their expectations.

The private consumption aggregate is defined as

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{2}$$

where j denotes the good's type or variety and $\epsilon_t > 1$ is the elasticity of substitution between varieties. $\ln(\epsilon_t)$ is AR(1) such that $\ln(\epsilon_t) = (1 - \rho_\epsilon)\ln(\epsilon) + \rho_\epsilon \ln(\epsilon_{t-1}) + e_{\epsilon t}$, with $0 \leq \rho_\epsilon < 1$, $e_{\epsilon t} \overset{i.i.d}{\sim} N(0, \sigma_\epsilon^2)$ and ϵ is the steady-state of ϵ_t. This serves as a device to introduce mark-up shocks to the model. The public goods aggregate takes the same form

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{3}$$

Minimizing the cost of achieving a given level of real consumption yields the private and public demand for individual goods,

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_t} C_t$$

and

$$G_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_t} G_t$$

respectively, where where $P_t(j)$ is the price of variety j and we have price indices given by,

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_t} dj \right)^{\frac{1}{1-\epsilon_t}}$$

The budget constraint at time t is given by

$$P_t^S B_t^S + P_t^M B_t^M \leq \Xi_t + (1 + \rho P_t^M) B_{t-1}^M + B_{t-1}^S + W_t N_t (1 - \tau_t) - P_t C_t + P_t Tr_t \tag{4}$$

[7] Therefore, utility in the initial period, $U(C_0, N_0, G_0)$, is undiscounted, in the following period, $U(C_1, N_1, G_1)$ is discounted by β_0, then in period t=2, $U(C_2, N_2, G_2)$ by $\beta_0 \beta_1$ and so on.

where $\int_0^1 P_t(j)C_t(j)dj = P_tC_t$, Ξ is the representative household's share of profits in the imperfectly competitive firms, P_tTr_t are the nominal value of lump-sum transfers from the government, W_t are wages, and τ_t is an wage income tax rate.[8] Households hold two basic forms of government bonds. The first is the familiar one period debt, B_t^S which has the price equal to the inverse of the gross nominal interest rate, $P_t^S = R_t^{-1}$. The second type of bond is actually a portfolio of many bonds which, following Woodford (2001) pay a declining coupon of ρ^j dollars $j+1$ periods after they were issued where $0 < \rho \leq \beta_j^{-1}$,$j = H, L$. The steady-state duration of the bond $(1-\beta_j\rho)^{-1}$, which allows us to vary ρ as a means of changing the implicit maturity structure of government debt. By using such a simple structure, we need only price a single bond, since any existing bond issued j periods ago is worth ρ^j new bonds. In the special case where $\rho = 1$ these bonds become infinitely lived consols.[9]

Throughout the analysis, one-period government bonds B_t^S will be in zero net supply with beginning-of-period price P_t^S, while the general portfolio of government bond B_t^M is in non-zero net supply with beginning-of-period price P_t^M. Higher ρ raises the maturity of the bond portfolio. In the special case, where $\rho = 0$, the debt portfolio collapses to one-period debt.[10]

To facilitate numerical implementation, we assume that (1) takes the specific form,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\prod_{i=-1}^{t-1} \beta_i) \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{5}$$

We can then maximize utility subject to the budget constraint (4) to obtain the optimal allocation of consumption across time based on the pricing of one period bonds,

$$\beta_t R_t \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \tag{6}$$

and the declining payoff consols,

$$\beta_t \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \left(1 + \rho P_{t+1}^M\right) \right\} = P_t^M \tag{7}$$

It is convenient to define the stochastic discount factor (for nominal payoffs) for later use,

$$\beta_t \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1}$$

Households also decide on labor supply which is given, optimally, by,

$$(1-\tau_t) \left(\frac{W_t}{P_t} \right) = N_t^{\varphi} C_t^{\sigma}$$

[8] Since fiscal policy is an important element of this paper, we do not assume any kind of lump-sum-tax-financed subsidy to offset the distortion arising from monopolistic competition, which is a not uncommon assumption in the optimal fiscal and monetary policy literature using New Keynesian models. Thus, the steady state of the model economy is not efficient. Moreover, it cannot be determined outside of the policy problem itself, requiring the use of global solution methods.

[9] This allows us to study long-duration bonds without increasing the dimensionality of the state space, and it is commonly adopted in the literature (e.g., Eusepi and Preston, 2012; Chen et al., 2012).

[10] Leeper et al. (2021) allow for optimal variation in the composition of debt between short and long-term bonds.

That is, the marginal rate of substitution between consumption and leisure equals the after-tax wage rate. Besides these FOCs, necessary and sufficient conditions for household optimization also require the household's budget constraints to bind with equality. The no-Ponzi-game condition can be written as,

$$\lim_{T\to\infty} \mathbb{E}_t \left[\frac{1}{R_{t,T}} \frac{D_T}{P_T}\right] \geq 0 \tag{8}$$

where household wealth brought into period t is given by,

$$D_t = (1 + \rho P_t^M) B_{t-1}^M + B_{t-1}^S$$

and

$$R_{t,T} = \prod_{s=t}^{T-1} \left(\frac{1 + \rho P_{s+1}^M}{P_s^M} \frac{P_s}{P_{s+1}} \right)$$

for $T \geq 1$ and $R_{t,t} = 1$. The no-Ponzi-game condition says that the present discounted value of household's real wealth at infinity is non-negative, that is, there is no overaccumulation of debt. In equilibrium, the condition holds with equality.

2.2 Firms

The production function is linear, so for firm j

$$Y_t(j) = N_t(j) \tag{9}$$

The real marginal costs of production is defined as $mc_t = W_t/(P_t)$. The demand curve they face is given by,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} Y_t$$

where $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj\right]^{\frac{\epsilon_t}{\epsilon_t - 1}}$. Firms are also subject to quadratic adjustment costs in changing prices, as in Rotemberg (1982).

We define the Rotemberg price adjustment costs for a monopolistic firm j as,

$$v_t(j) = \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t \tag{10}$$

where $\phi \geq 0$ measures the degree of nominal price rigidity. The adjustment cost, which accounts for the negative effects of price changes on the customer–firm relationship, increases in magnitude with the size of the price change and with the overall scale of economic activity Y_t.

The problem facing firm j is to maximize the discounted value of profits,

$$\max_{P_t(j)} \mathbb{E}_t \sum_{z=0}^{\infty} Q_{t,t+z} \Xi_{t+z}(j)$$

where profits are defined as,

$$\Xi_t(j) = P_t(j) Y_t(j) - mc_t Y_t(j) P_t - \frac{\phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t \tag{11}$$

So that, in a symmetric equilibrium where $P_t(j) = P_t$ the first order conditions are given by,

$$\begin{aligned} 0 &= (1-\epsilon_t) + \epsilon_t mc_t - \phi \Pi_t (\Pi_t - 1) \\ &+ \phi \beta_t \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} (\Pi_{t+1} - 1) \right] \end{aligned} \tag{12}$$

which is Rotemberg's version of the New Keynesian Phillips curve relationship.

Goods market clearing requires, for each good j,

$$Y_t(j) = C_t(j) + G_t(j) + v_t(j)$$

which allows us to write, in a symmetrical equilibrium,

$$Y_t \left[1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right] = C_t + G_t$$

There is also market clearing in the bond market, where we assume that one period bonds are in zero net supply, $B_t^S = 0$ and the remaining portfolio of longer-term bonds evolves according to the government's budget constraint which we will now describe.

2.3 The Government Budget Constraint

There are two policy makers – monetary and fiscal. The monetary authority controls the nominal interest rate on short-term nominal bonds. The fiscal authority chooses the level of government consumption, labor income taxes, and issues debt. In addition to government consumption, government expenditures also consist of exogenously determined transfers, $P_t Tr_t$, and the interest payments on outstanding debt.

Government expenditures are financed by levying labor income taxes at the rate τ_t, and by issuing one-period, risk-free (non-contingent), nominal obligations B_t^S, and long term bonds B_t^M. The government's sequential budget constraint is then, assuming the one-period bond is in zero net supply, $B_t^S = 0$, given by,

$$P_t^M B_t^M = (1 + \rho P_t^M) B_{t-1}^M - W_t N_t \tau_t + P_t G_t + P_t Tr_t \tag{13}$$

Note that $(1 + \rho P_t^M) B_{t-1}^M$ is outstanding government liabilities in period t. Distortionary taxation and spending adjustments are required to service government debt as well as stabilize the economy. Rewriting in real terms

$$P_t^M b_t = (1 + \rho P_t^M) \frac{b_{t-1}}{\Pi_t} - \frac{W_t}{P_t} N_t \tau_t + G_t + Tr_t \tag{14}$$

where $b_t \equiv B_t^M / P_t$ represents the ratio of the number of nominal bonds to the price level, a ratio that will be stationary despite the economy being subject to an endogenously determined rate of inflation.

Given that government debt is nominal and prices are sticky, monetary policy decisions affect the government budget through various channels: first, monetary policy affects debt service costs; second, in a sticky price economy, monetary policy has real effects which impact the tax base and, for a given tax rate, the government's primary surplus.

Monetary policy's impact on debt service costs depends on the maturity structure of the debt. In (14), the amount of outstanding real government debt is $P_t^M b_t$, and the period real return on holding government debt is $(1+\rho P_t^M)/\left(\Pi_t P_{t-1}^M\right)$. If $\rho = 0$, government debt b_t is reduced to one-period debt, and then the only way to adjust the real return on bonds *ex post* is through inflation in the current period Π_t. Large price fluctuations can be very costly in the presence of nominal rigidities. However, if government debt has a longer maturity, $0 < \rho < 1$, adjustments in the *ex post* real return can be engineered via changes in the bond price P_t^M, which depends on inflation in future periods. This means that changes in the real debt return can be produced by a small but sustained inflation, which is less costly than equivalent large fluctuations in inflation. In the context of the time-consistent policy making considered in this paper, economic agents understand that debt maturity affects the government's incentives to induce surprise inflation such that it impacts the inflationary bias problem, and the subsequent debt stabilization bias, even if policy makers have a limited ability to inflate away the debt in equilibrium.

That completes the description of our model, which contains the usual resource constraint, consumption Euler equation, and New Keynesian Phillips curve as well as the government's budget constraint and the bond pricing equation for longer-term bonds. These equations and the debt-dependent steady state are described in Appendix B1.

3 Time-Consistent Optimal Policy

In this section, we assume the two authorities possess the same objective functions and specify the time-consistent policy problem, which is what would emerge if policy was set by a single policy maker.

The policy maker's flow utility function is consistent with that of households and is given by,

$$U_t = \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi}\right) \tag{15}$$

will be reflected in policymaker behavior.

The solution concept we consider is that of a time-consistent optimum, where neither policymaker has the ability to commit to future policies. However, they can influence the future through the impact of their current actions on endogenous state variables, specifically government debt. We capture this by defining the auxiliary functions,

$$M(b_t, \beta_{t+1}, \epsilon_{t+1}) = (C_{t+1})^{-\sigma} Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1} - 1\right)$$

$$L(b_t, \beta_{t+1}, \epsilon_{t+1}) = (C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1}(1+\rho P_{t+1}^M)$$

which can then replace the relevant expectations terms in the constraints facing each policymaker, as in Anderson et al. (2010). This is the same time-consistent policy problem considered in Leeper et al. (2021), but augmented to allow for time-variation in the households' discount factor. It shall also be extended further to allow for different policy regimes, as described below.

3.1 The Policy Maker's Problem

The policy maker maximizes the following Lagrangian,

$$
\begin{aligned}
\mathcal{L}^P = & \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} + \beta^P \mathbb{E}_t[V_{t+1}^P(b_t, \beta_{t+1}, \epsilon_{t+1})] \right\} \\
& + \lambda_{1t} \left[Y_t \left(1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right) - C_t - G_t \right] \\
& + \lambda_{2t} \left[\begin{array}{c} (1-\epsilon_t) + \epsilon_t (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} - \phi \Pi_t (\Pi_t - 1) \\ + \phi \beta_t C_t^{\sigma} Y_t^{-1} \mathbb{E}_t \left[M(b_t, \beta_{t+1}, \epsilon_{t+1}) \right] \end{array} \right] \\
& + \lambda_{3t} \left[\begin{array}{c} \beta_t b_t C_t^{\sigma} \mathbb{E}_t \left[L(b_t, \beta_{t+1}, \epsilon_{t+1}) \right] - \frac{b_{t-1}}{\Pi_t} \left(1 + \rho \beta_t C_t^{\sigma} \mathbb{E}_t \left[L(b_t, \beta_{t+1}, \epsilon_{t+1}) \right] \right) \\ + \left(\frac{\tau_t}{1-\tau_t} \right) Y_t^{1+\varphi} C_t^{\sigma} - G_t - Tr_t \end{array} \right]
\end{aligned}
$$

where β^P is the policy maker's discount factor which may differ from that of the households[11], and $\mathbb{E}_t[V_{t+1}^P(b_t, \beta_{t+1}, \epsilon_{t+1})]$ are the expected future payoffs to the policy maker conditional on the state-variables bequeathed to the future.

The complete set of FOCs from this problem are described in the Appendix B2. Here we highlight a selection of those, most closely related to policy choices, to facilitate economic intuition.

Firstly, the FOC for inflation,

$$
\begin{aligned}
& -\lambda_{1t} \left[Y_t \phi (\Pi_t - 1) \right] - \lambda_{2t} \left[\phi (2\Pi_t - 1) \right] \\
& + \lambda_{3t} \left[\frac{b_{t-1}}{\Pi_t^2} \left(1 + \rho \beta_t C_t^{\sigma} \mathbb{E}_t \left[L(b_t, \beta_{t+1}, \epsilon_{t+1}) \right] \right) \right] = 0
\end{aligned}
\tag{16}
$$

which says that a higher inflation rate tightens the resource constraint ($\lambda_{1t} \geq 0$), has positive effects on the inflation-output trade-off at time t ($\lambda_{2t} \leq 0$), and relaxes the government budget constraint ($\lambda_{3t} \geq 0$). Therefore, the inflation bias balances the costs of inflation against the potential gains from surprise inflation in boosting output and reducing the real value of government debt.

Government consumption,

$$\chi G_t^{-\sigma_g} - \lambda_{1t} - \lambda_{3t} = 0 \tag{17}$$

implying that the government matches the marginal utility gain from higher government spending against the tightening of the resource constraint ($\lambda_{1t} \geq 0$), and government budget constraint ($\lambda_{3t} \geq 0$). Similarly for taxation,

$$\epsilon_t \lambda_{2t} + \lambda_{3t} Y_t = 0 \tag{18}$$

[11] A convenient interpretation of policy-maker myopia is political survival. Suppose that in each quarter the incumbent policymaker remains in office with probability γ, and conditional on remaining in office discounts future utility at the same rate as households, β. Then the policymaker's effective discount factor is $\beta^P = \gamma\beta$. Hence $\gamma = \beta^P/\beta$, and the implied expected remaining duration in office is $E[T] = \frac{1}{1-\gamma} = \frac{1}{1-\beta^P/\beta}$ quarters. In this sense, a lower β^P can be interpreted as a shorter effective planning horizon.

which is the marginal condition capturing the fact that a higher (distortionary) tax rate increases marginal costs and fuels inflation t ($\lambda_{2t} \leq 0$), while at the same time generating tax revenues which relaxes the government's budget constraint ($\lambda_{3t} \geq 0$).

Finally, we consider the FOC for government debt derived in Appendix B2,

$$P_t^M \lambda_{3t} - \beta^P \mathbb{E}_t \left[\frac{\lambda_{3t+1}}{\Pi_{t+1}} \left(1 + \rho P_{t+1}^M\right) \right]$$

$$-\beta_t \lambda_{3t} C_t^\sigma \left[\phi \epsilon_t^{-1} \mathbb{E}_t \left[M_1(b_t, \beta_{t+1}, \epsilon_{t+1}) \right] - (b_t - \rho \frac{b_{t-1}}{\Pi_t}) \mathbb{E}_t \left[L_1(b_t, \beta_{t+1}, \epsilon_{t+1}) \right] \right] = 0 \qquad (19)$$

The first line of this expression describes how the policy maker trades off the current and future distortions associated with the government budget constraint. This element of the government's FOC would also exist under commitment. The second line captures the debt stabilization bias, which does not exist under commitment.

In the absence of any policy maker myopia $\beta^P = \beta$, the first line would describe a policy of tax smoothing as, in Barro (1979). If the (risk-adjusted) real return to bonds equaled the rate of time preference of the household/government, then policy would ensure the distortions associated with the budget constraint are constant, and steady-state government debt would follow a random walk. In other words, following shocks, government debt would settle at a new steady state where, for example, in the case of a negative shock, the costs of undertaking a fiscal consolidation to reduce debt exactly balance the costs of servicing that debt. When the government is myopic, $\beta^P < \beta$ they would deviate from tax smoothing by allowing debt (and the distortions associated with debt, captured by λ_{3t}), to rise even when the real return to debt equaled the households' rate of time preference.[12]

Under the time-consistent policy, however, the policy maker's desired path of fiscal distortions also depends upon the influence debt has on expectations via the derivatives of the auxiliary functions on the second line of (19). The first term within the second set of square brackets, $M_1(b_t, \beta_{t+1}, \epsilon_{t+1}) > 0$, captures the increase in inflation expectations associated with an increase in debt. Since inflation is costly in our economy, this encourages the fiscal authority to further deviate from tax smoothing and reduce debt. The second term, $L_1(b_t, \beta_{t+1}, \epsilon_{t+1}) < 0$, instead measures the reduction in expected bond yields associated with increases in government debt. This also serves to discourage the accumulation of debt. Falling debt levels will imply rising bond prices, which make it cheaper to issue debt but more costly to repay that debt (when it is of longer maturity). Therefore, the shorter the maturity of debt, the greater incentive the government has to repay debt due to this mechanism.

In summary, a non-myopic government would like to smooth taxes and hold debt at a new sustainable level following shocks. However, when they cannot commit, they wish to deviate from tax smoothing and return debt to a pre-shock steady state due to the negative impact

[12] Note that the *ex ante* real return on government bonds, $\mathbb{E}_t \left[\frac{(1+\rho P_{t+1}^M)}{\Pi_{t+1} P_t^M} \right]$, will vary depending on the monetary policy set by the policy maker thereby tilting the intertemporal tax smoothing trade-offs they face. This will be an additional channel through which monetary policy interacts with fiscal.

debt has on both inflation expectations and the costs of bond issuance (particularly for shorter maturity debt). This tendency may be offset to the extent that the policy maker is myopic.

3.2 The Equilibrium

The solution to this policy problem is then obtained by simultaneously solving the FOCs for the policy maker alongside the three constraints. Therefore, we solve for the following 10 variables, $\{C_t, Y_t, \Pi_t, b_t, \tau_t, G_t, P_t^M, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}\}$ using the three constraints, the bond pricing equation (7), and the six first-order conditions: (19) and (B2.1)-(B2.5). Specifically, we need to find these ten time-invariant Markov-perfect equilibrium policy functions which depend on the three state variables $\{b_{t-1}, \beta_t, \epsilon_t\}$. That is, we need to find policy functions such as $b_t = b\left(b_{t-1}, \beta_t, \epsilon_t\right)$, $\tau_t = \tau\left(b_{t-1}, \beta_t, \epsilon_t\right)$, and $\Pi_t = \Pi\left(b_{t-1}, \beta_t, \epsilon_t\right)$ for each endogenous variable.

4 Numerical Analysis

4.1 Solution Method and Calibration

For the full policy problem described in Section 3, the equilibrium policy functions cannot be computed in closed form. We thus resort to computational methods. A local approximation around a fixed steady state is not well suited to our environment for two related reasons. First, the model's steady state is endogenously determined as part of the policy problem and depends on inherited debt, so the relevant long-run position is not known independently of the solution. Second, the mechanisms central to the paper are inherently nonlinear: the incentives to inflate away debt, the strength of the debt stabilization bias, and the pricing of long-term government debt all vary with the level of debt and with the regime in place. These features matter especially for the large movements in debt associated with switches in policymaker myopia and crisis episodes with sharp changes in real interest rates. A global solution method therefore allows us to characterize the Markov-perfect policy functions over the relevant range of states, rather than only in a neighborhood of a particular reference point. Specifically, we use Chebyshev collocation with time iteration to solve the model.[13] The detailed algorithm is presented in Appendix B3.

Before solving the model numerically, the benchmark values of structural parameters must be specified. The calibration of parameters is summarized in Table A.1. We set $\beta = (1/1.02)^{1/4} = 0.995$, which is a standard value for models with quarterly data and implies 2% annual real interest rate. Our policymakers are relatively myopic, and in the benchmark calibration we set $\beta^P = 0.982$. Interpreting $\beta^P = \gamma\beta$, where γ is the quarterly probability that the incumbent remains in office, this implies $\gamma = 0.982/0.995 = 0.987$. The associated expected planning horizon is therefore $1/(1-\gamma) = 76.5$ quarters, or about 20 years. We shall consider the implications of varying the myopia of the policymakers in Section 6, below. The intertemporal elasticity of substitution is set to one-half ($\sigma = \sigma^g = 2$), which is in the middle of the parameter

[13]See Judd (1998) for a textbook treatment.

range typically considered in the literature. The labor supply elasticity is set to $\varphi^{-1} = 1/3$. The elasticity of substitution between intermediate goods is chosen as $\epsilon_t = 14.33$, which implies a monopolistic markup of approximately 7.5%, similar to Siu (2004). The decay parameter defining the geometric maturity structure of government debt, $\rho = 0.95$, corresponds to $4 \sim 5$ years of debt maturity, consistent with US data.[14] The scaling parameter $\chi = 0.007$ ensures that the share of government spending in output is about 7.8%. The cost push shock parameters are set to $\rho_a = 0.95$ and $\sigma_a = 0.01$. The price adjustment cost parameter $\phi = 50$ – implying that on average, firms re-optimize prices every 6 months – is in line with empirical evidence. Transfers are fixed at a level that amounts to 9% of steady-state GDP in the benchmark calibration.

With this benchmark parameterization, we solve the fully nonlinear models using the Chebyshev collocation method.

4.2 Numerical Results

In this sub-section, we explore the properties of the equilibrium under the optimal time-consistent policy, which corresponds to the case considered in Leeper et al. (2021) and is reported in Table A.2. Leeper et al. (2021) also discuss how debt maturity, price stickiness and markups impact that steady-state. In Figure 1 we plot the transition from a high level of debt to the steady-state. The high initial level of debt worsens the inflationary bias problem, leading to a significant rise in inflation above 5%. In order to reduce this inflation, the policy maker raises interest rates to fight inflation, while simultaneously undertaking a fiscal consolidation to reduce the debt which is driving the inflation bias problem. The fiscal consolidation involves significant increases in taxation, but fairly limited reductions in government consumption. Gradually, as debt returns to its steady-state, inflation falls, the fiscal consolidation ends and monetary policy is relaxed. The transition from the higher level of debt to a unique steady-state is the debt-stabilization bias. Such a transition would not occur under commitment. Under commitment, with a myopic policy maker, the welfare costs of tax distortions would gradually increase over time.

5 Debt Fluctuations and Economic Shocks

Figure 1 shows how the economy would evolve starting from a level of debt significantly higher than its steady-state value. However, the question we are interested in is, how might the economy have come to have such a high level of debt in the first place?

We begin to answer that question, by asking to what extent can standard economic shocks generate sizable movements in government debt? To do so we undertake a stochastic simulation of the model in the face of cost-push shocks created by variation in the elasticity of substitution between goods in our model. We double the standard deviation of the shocks to the elasticity of substitution relative to their calibrated value. Figure 2 simulates the model 100 times for

[14] We shall consider the impactions of changes in debt maturity in Figure 7, in Section 7.

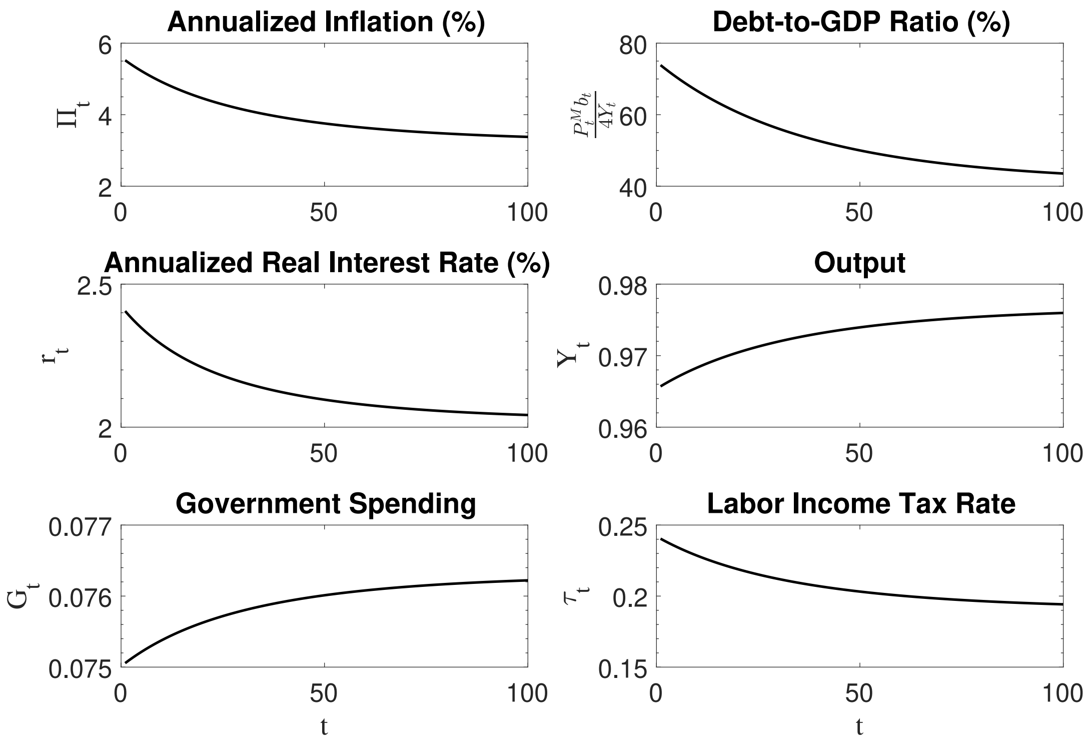

Figure 1: The Debt Stabilization Bias
Note: The figure plots the transition path from a high level of debt towards the steady-state described in Table A.2.

1000 periods, after dropping the first 100 periods of each simulation to avoid the simulations being influenced by starting conditions.

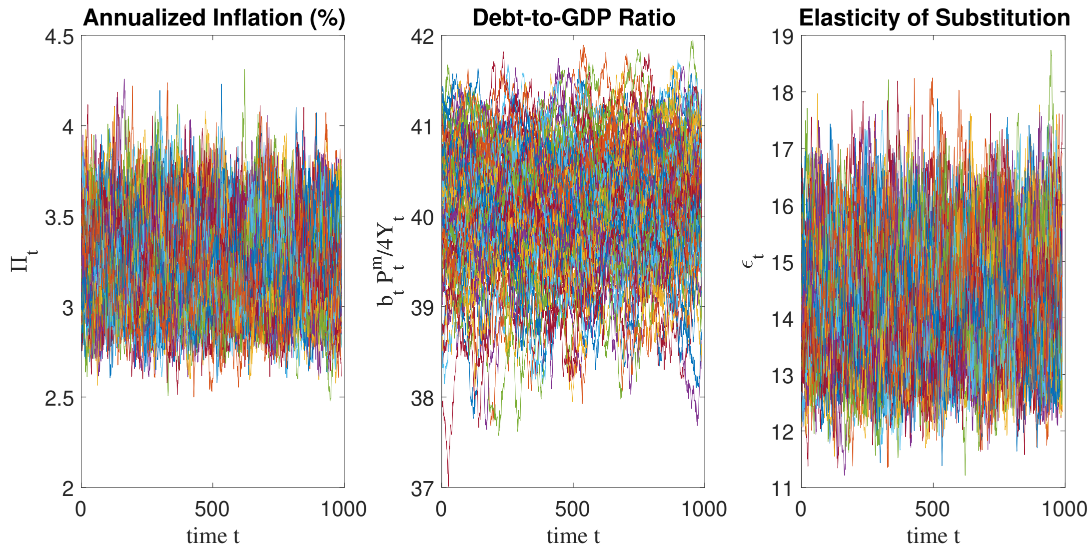

Figure 2: The Volatility of Government Debt in the Face of Standard Shocks
Note: We simulate the model 100 times for 1000 periods, after dropping the first 100 periods using random draws from distribution of the elasticity of substitution i.e. mark-up shocks.

In our benchmark calibration, the fluctuations in the debt-to-GDP ratio are only 0.3%, rising to 0.59% when we double the shock standard deviation. This gives rise to data consistent movements in inflation during the Great Moderation period, but does not allow debt to rise above 42% of GDP. The minuscule degree of movement in government debt remains if we consider productivity shocks rather than cost-push shocks.

Given that standard economic shocks cannot generate sizable movements in government debt, we turn to consider an experiment designed to highlight the features of tax smoothing. Specifically, we create a temporary regime where fiscal transfers are, unexpectedly, 50% above their usual level for a sustained but temporary period. This would be a classic motive for tax smoothing – taxes should rise to pay the interest on the debt issued to finance the temporary increase in fiscal transfers. We shall see that the debt stabilization bias completely overturns this result.

In order to model this we assume that the following transition probability matrix governs the evolution of a two-state Markov process, between normal times and episodes of high fiscal transfers, $\begin{bmatrix} 1 & 0 \\ 1-0.98 & 0.98 \end{bmatrix}$ where the first row indicates that we do not expect to leave, the 'normal' regime, while the second row indicates that were we to unexpectedly enter a regime of high fiscal transfers, this would be expected to last for 12.5 years. Upon exit from the high transfers regime, we would not expect to return to that regime.[15] The impact of such a fiscal shock is considered in Figure 3.

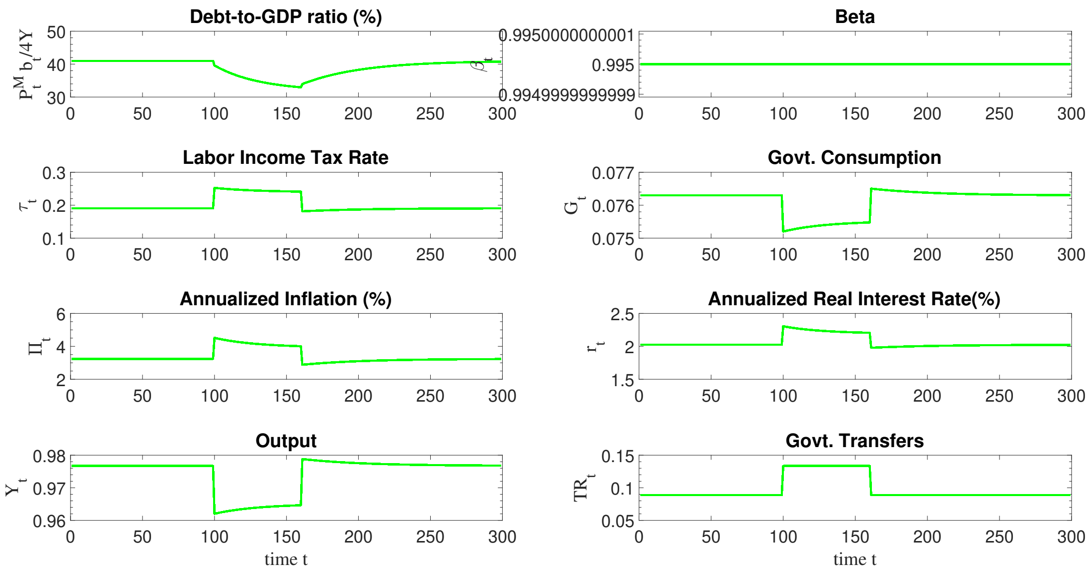

Figure 3: Large Temporary Fiscal Transfers Shock
Note: An unexpected shock which raises government transfers by 50%. The shock is expected to last for around 12.5 years, and is not expected to re-occur.

As noted above, since the regime of high transfers is entirely unexpected and no-one expects to return to that regime, tax smoothing would suggest that, when the increase in transfers

[15]Section 7 below considers what happens if we enter a crisis regime, which we then realise we are likely to return to at some point in the future. This can give rise to quite significant precautionary behavior on the part of the policy maker.

occurs, we raise taxes by just enough to service the debt that would need to be issued to pay for the temporary increase in transfers. Instead, what we find is that even during the period of raised transfers the government increases taxes to more than balance its budget and prevent any increase in government debt. The debt stabilization bias is so strong, that it more than dominates the conventional desire to smooth taxes.

Therefore, we have shown that standard economic shocks, as well as really quite extreme fiscal shocks, cannot generate the increases in government debt needed to match the data.[16]

6 Switches in Policy Maker Myopia

Leeper et al. (2021) show that policy maker myopia, monopolistic competition distortions and debt maturity are the key drivers of the steady-state equilibrium rate of inflation and debt-to-GDP ratio, while other endogenously determined steady-state fiscal ratios are largely unaffected by these and other parameters. This highlights the importance of the state-dependent inflationary bias and the associated debt stabilization bias in jointly determining the equilibrium outcomes for inflation and debt.

We begin this section by assessing the extent to which changes in these parameters can potentially explain movements in the debt-to-GDP ratio. In doing so, our immediate focus is on the period starting from the low point of the US debt-to-GDP ratio in 1980 to the pre-pandemic period 2018.

US government debt in 1980 was 20% of GDP. At this point in time the average maturity of US debt was 45 months (US Treasury, 2019) and Traina (2018) estimates the typical markup was 10%.[17] Adopting a degree of myopia of 0.9815, recreates the observed debt-to-GDP ratio in 1980. Since then, the quantity of debt in the hands of the public rose to 75% of GDP by 2018 – see Dallas Fed (2025). Over the same period Traina (2018) estimates the markup had risen to 15%, which would have encouraged the policy maker to accumulate assets such that the debt-to-GDP ratio would be -34.6%, clearly at odds with the data. However, at the same time the average maturity of debt rose to 69.2 months, which, alone, would have raised the debt-to-GDP ratio to 28.8%, qualitatively in the right direction but significantly short of the observed increase. Instead, to explain the rise in the debt-to-GDP ratio, alongside the increases in markup and debt maturity, a rise in policy maker myopia from $\beta^P = 0.982$ to $\beta^P = 0.929$ is required – a fall in the policymaker's quarterly survival probability from 0.987 to 0.934, and hence in the expected effective planning horizon from about 20 years to about 4 years. These implied horizons are of the same order of magnitude as political tenures and electoral cycles, especially at the more myopic end, where the effective horizon is close to a single electoral term.

Is this implied increase in policy maker myopia credible? There are measures of the typical

[16] We extend these results in Section 7 to allow for the possibility that (i) it is difficult to reduce transfers after they have been increased and (ii) it is costly to increase taxes and find that this reinforces these results.

[17] Traina (2018)'s estimates form part of a debate instigated by De Loecker and Eeckhout (2017) over the extent to which markups have been trending upwards since the 1980s.

duration of legislators' terms of office which currently stand at 10 years for US Senators and 9 years for Representatives (Congressional Research Service (2019)), lying between the implied effective time horizon of our myopic policy maker of 4-20 years, suggesting that it is not implausibly large. However, such a literal interpretation of myopia is at odds with the fact that the typical duration of membership of Congress has been increasing not decreasing since 1980. Therefore, it must be the case that some other motivation for the deficit bias is driving the increase in myopia (see Alesina and Passalacqua (2016) for a survey.) An interesting area for future research would be to assess whether or not such mechanisms, when embedded in the current model, can explain the observed movements in the debt-to-GDP ratio.

The benchmark model, despite matching key fiscal data averages, cannot capture the key trends in the debt-to-GDP ratio seen in the data. Therefore, in order to generate plausible movements in the debt-to-GDP ratio, there is a need to go beyond standard economic shocks and consider variations in political frictions. Specifically, in light of the comparative static results just presented, the degree of policy maker myopia is assumed to switch between two regimes, β_L, β_H where $\beta_L > \beta_H$ where the former 'L' regime has a low degree of myopia and correspondingly supports a lower level of debt. Conversely, the high myopia regime is consistent with a higher debt level. There is an associated transition probability matrix governing the evolution of this two-state Markov process, $\begin{bmatrix} p_L & 1-p_L \\ 1-p_H & p_H \end{bmatrix}$ where p_i is the probability of remaining in regime i $(i = H, L)$ given we are currently regime i and $1 - p_i$ is the probability of exit to the other regime j, ,$j = (H, L)$,$j \neq i$. The policy maker is assumed to not be in conflict with their future selves but to discount the future in line with whatever degree of myopia is in place at the time. As a result, the degree of myopia becomes an additional state variable so that the value function is defined as,

$$
\begin{aligned}
\mathcal{L}^f = & \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} + \beta_{i,t}\mathbb{E}_t[V_{t+1}(b_t, \beta_{t+1}, \epsilon_{t+1}, \beta_{i,t+1})] \right\} \\
& + \lambda_{1t} \left[Y_t \left(1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right) - C_t - G_t \right] \\
& + \lambda_{2t} \left[\begin{array}{c} (1-\epsilon_t) + \epsilon_t (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} - \phi \Pi_t (\Pi_t - 1) \\ + \phi \beta_t C_t^{\sigma} Y_t^{-1} \mathbb{E}_t [M(b_t, \beta_{t+1}, \epsilon_{t+1}, \beta_{i,t+1})] \end{array} \right] \\
& + \lambda_{3t} \left[\begin{array}{r} \beta_t b_t C_t^{\sigma} \mathbb{E}_t [L(b_t, \beta_{t+1}, \epsilon_{t+1}.\beta_{i,t+1})] - \frac{b_{t-1}}{\Pi_t} (1 + \rho \beta_t C_t^{\sigma} \mathbb{E}_t [L(b_t, \beta_{t+1}, \epsilon_{t+1}, \beta_{i,t+1})]) \\ + \left(\frac{\tau_t}{1-\tau_t} \right) Y_t^{1+\varphi} C_t^{\sigma} - G_t - Tr_t \end{array} \right]
\end{aligned}
$$

subject to the same constraints as before but where all auxiliary functions are based on this expanded state-space where $\beta_{i,t} = \beta_L$ or β_H.

The policy maker optimizes this Lagrangian by choosing C_t, G_t, Y_t, Π_t, τ_t, b_t and the multipliers, λ_{1t}, λ_{2t}, λ_{3t}. The only difference between these FOCs and those in the benchmark model are that the FOC for debt now depends upon the myopia of the current policy maker, $\beta_{i,t}$, such that

$$P_t^M \lambda_{3t} - \beta_{i,t} \mathbb{E}_t \left[\frac{\lambda_{3t+1}}{\Pi_{t+1}} \left(1 + \rho P_{t+1}^M\right) \right] + \lambda_{2t} \phi \beta_t C_t^{\sigma} Y_t^{-1} \mathbb{E}_t \left[M_1(b_t, \beta_{t+1}, \epsilon_{t+1}, \beta_{i,t+1}) \right]$$

$$+ \beta_t \lambda_{3t} C_t^{\sigma} (b_t - \rho \frac{b_{t-1}}{\Pi_t}) \mathbb{E}_t \left[L_1(b_t, \beta_{t+1}, \epsilon_{t+1}, \beta_{i,t+1}) \right] = 0 \tag{20}$$

The solution to the resultant system of FOCs is a set of time-invariant Markov-perfect equilibrium policy rules $y_t = H(s_{t-1})$ mapping the vector of states $s_{t-1} = \{b_{t-1}, \beta_t, \epsilon_t, \beta_{i,t}\}$ to the optimal decisions for $y_t = \{C_t, G_t, Y_t, \Pi_t, \tau_t, b_t, P_t^M, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}\}$ for all $t \geq 0$.

The calibration of this Markov switching process follows Chen et al. (2022) in identifying key shifts in the trend of the U.S. debt-to-GDP ratio – see Figure 4. Between 1954 and the first budget of the Reagan presidency in 1981 debt is on a downward trend. Similarly following Clinton's first budget until the first budget of the George W. Bush there is a sustained reduction in the debt-to-GDP ratio. We label these episodes as being periods of low myopia. In contrast the periods of rising debt-to-GDP ratios covering all other periods are labeled as high myopia. Given this labeling the implied transition matrices between the two regimes can be estimated as, $\begin{bmatrix} 0.9859 & 1-0.9859 \\ 1-0.9868 & 0.9868 \end{bmatrix}$ $\beta_L = 0.9866$ and $\beta_H = 0.9759$ are chosen to replicate the peaks and troughs of the debt-to-GDP ratio found in the data, while the remainder of the benchmark calibration is retained. The success of this exercise, in terms of tracking the debt-to-GDP ratio, can be seen in Figure 4 where the model implied dynamics of the ratio track the pre-financial crisis data both in the sense of ensuring the model can achieve the highs and lows seen in the data, but also the pace at which debt increases or decreases over time.

The movement in inflation implied by the model is, however, much more modest than found in the data where significant cost-push pressures, such as the oil price shocks of the 1970s, appear to dominate the data. Empirical estimates in Chen et al. (2022) and Kirsanova et al. (2025) also suggest that, alongside cost-push factors, much of the variation in inflation can be tied to varying degrees of inflation conservatism on the part of the US Fed.[18] Historical decompositions also suggest that this episode reflected not only fiscal and political forces, but also changes in the return on government debt. Hall and Sargent (2011) show that between 1981 and 1993 the U.S. debt-to-GDP ratio rose by 28.3 percentage points, with 17.8 percentage points attributable to primary deficits, while unusually high real returns on long-term government debt following the disinflation also made a substantial contribution.

The financial crisis and subsequent pandemic led to further increases in government debt. More broadly, the historical evidence suggests that important debt episodes are not driven by political frictions alone, but also by movements in real interest rates and realized returns on government debt. It could be that policymakers became even more myopic than in our calibrated high-myopia regime. However, the fact that the post-2008 period involved significant

[18]Kirsanova et al. (2025) explore this in an extension of the current model which allows for interactions between the government and an independent central bank. They find that an increase in central bank conservatism moderates inflation and reduces the debt stabiliZation bias. However, eventually the build up of debt this implies leads to a worsening of inflation relative to the situation prior to the increase in conservatism.

falls in interest rates, increases in fiscal transfers and large recessions suggests that these macro-financial forces may also have played an important role.

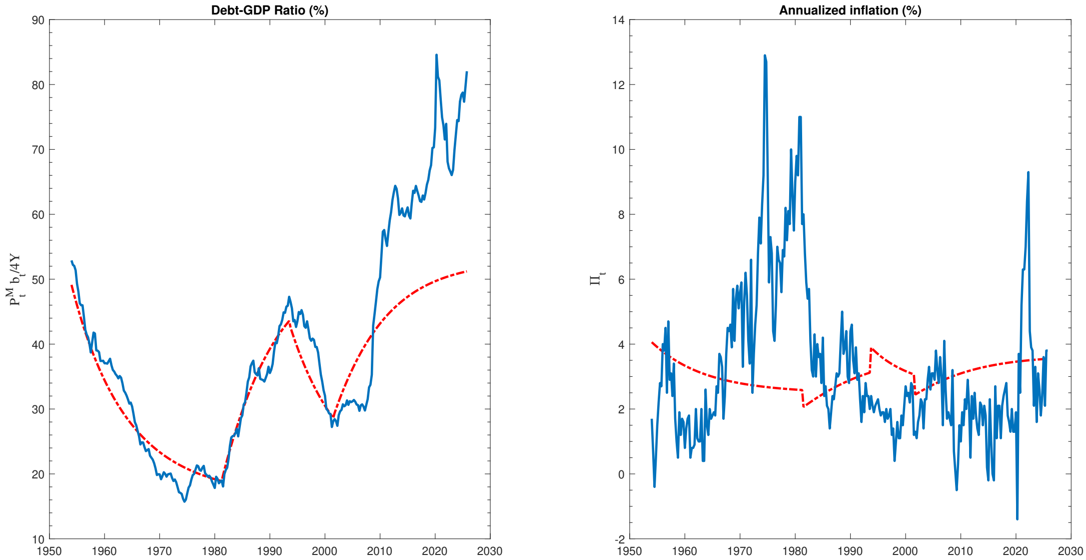

Figure 4: Simulated Debt-to-GDP Ratio with Switches in Policymaker Myopia vs Data
Note: In the first sub-plot, The red line plots the simulated path for the debt-to-GDP ratio with switches in policymaker myopia in line with the regimes described in Chen et al. (2022) prior to the financial crisis. The red line in the second sub-plot does the same for inflation. The blue line captures the data – the market value of the government debt in the hands of the public relative to GDP, taken from the Dallas Fed database: Dallas Fed (2025) and GDP and the GDP deflator from NIPA tables 1.1.5 and 1.1.9 from the Bureau of Economic Analysis, respectively.

7 An Unexpected Crisis

Our benchmark calibration (and any of the variants considered above) imply very limited movements in government debt in the face of our mark-up shock - the standard deviation of the debt-to-GDP ratio in our benchmark calibration is only 0.3%. The same is true for technology shocks. Shifts in myopia could explain larger movements in debt. However, there have also been some unusually large shocks, which may have contributed the observed increase in government debt: significant falls in GDP during the financial crisis and pandemic; large increases in government transfers at the same time; and large falls in government debt yields in line with a flight to safety during periods of crisis. To explore the impact of such effects, we split our model economy into two regimes. The first is as described in our benchmark calibration above. The second includes a variety of additional shocks that can occur either individually or together whenever the economy enters a 'crisis' episode: (1) A gradual 6% fall in productivity; (2) a 50% increase in government transfers; and (3) a flight to safety consistent with a fall in real interest rates from 2% to -2%. These are broadly in line with our experience during the financial crisis/pandemic.

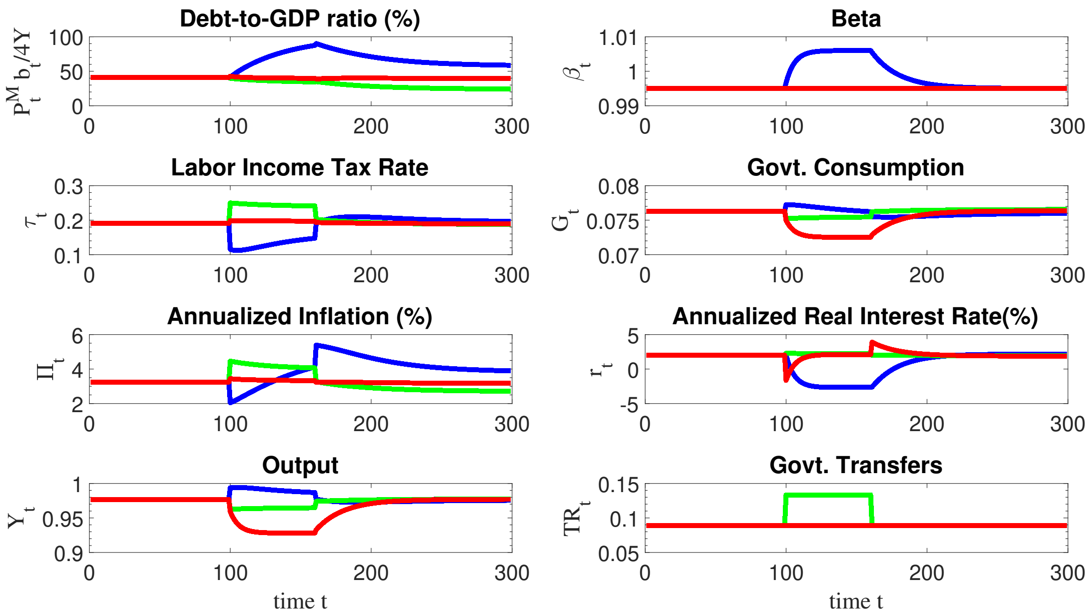

Figure 5: Simulated Crisis – Three Individual Shocks
Note: The Figure simulates the model three times in the face of three different shocks, respectively. The green line captures a 50% increase in transfers, the blue line a flight to safety and the red line a sustained fall in productivity resulting in a large recession. Once it happens the crisis is expected to re-occur.

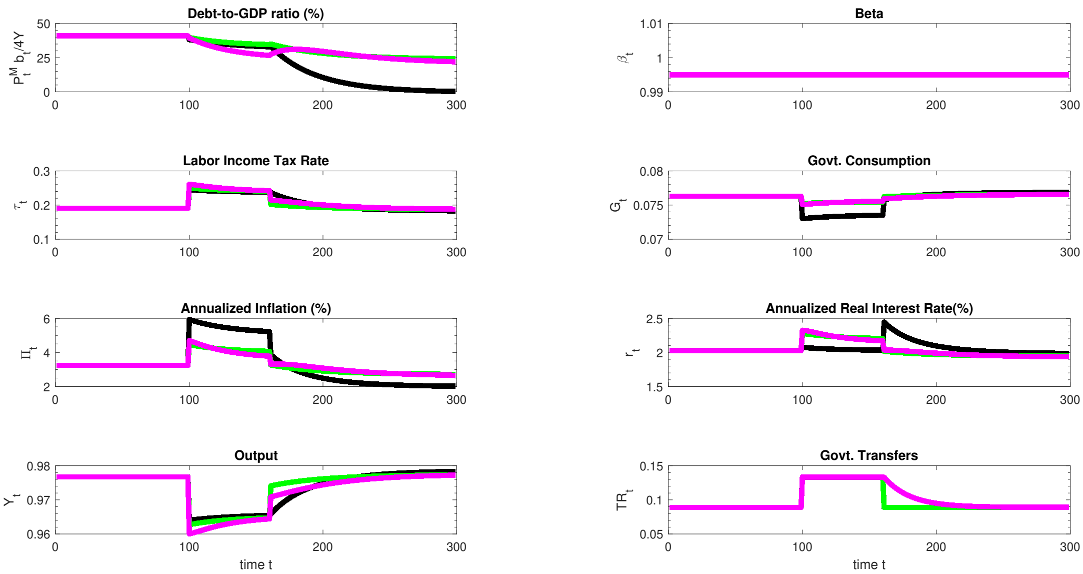

Figure 6: Fiscal Frictions and an Increase in Transfers.
Note: The Figure simulates the model three times in the face of a 50% increase in transfers. The green line recreates the same line from Figure 5. The magenta line considers the case where the rise in transfers takes time to return to normal. The black line imposes a quadratic cost on raising taxes during the time of crisis. not that, respectively. The rise in transfers is unexpected, but once it happens it is expected to re-occur.

Figure 5 plots the impact of these shocks when they occur individually. We assume that we are initially at the steady-state associated with our benchmark calibration, before a crisis unexpectedly occurs. Prior to the first occurrence of the crisis, no one expected the crisis to occur, but afterwards, it is expected to follow a two-state Markov process, $\begin{bmatrix} p_N & 1-p_N \\ 1-p_C & p_C \end{bmatrix}$ where p_j is the probability of remaining in regime j $(j = C, N)$ given we are currently regime j and $1 - p_j$ is the probability of exit to the other regime $k, j = (C, N), j \neq k$. To capture a large fall in output, we introduce time-varying productivity, A_t, such that the firm's production function becomes,

$$Y_t(j) = A_t N_t(j) \tag{21}$$

In a crisis (C), we assume that productivity falls by 2% immediately and then gradually by a further 4% at rate $\rho^C = 0.85$. After returning to the normal (N) regime, productivity rises to its original value of 1 at a rate, $\rho^N = 0.95$. In the case of transfers, they simply jump by 50% in the crisis and return to normal immediately upon exiting the crisis. Finally, $\beta > \beta^C$ is the 'flight-to-safety' regime where households have a greater desire to save in the form of government bonds, ceteris paribus. During the flight to safety, the discount factor rises to $\beta^C = 1.005$, and in normal times, it stays at the benchmark calibration of $\beta^N = 0.995$. This is captured by adopting the following process for the private sector discount factor,

$$\beta_t = \rho^i \beta_{t-1} + \left(1 - \rho^i\right) \beta^i$$

with $i = C, N$ respectively denoting the crisis and normal regimes, $\beta^C > \beta^N$, respectively. The persistence of beta within each regime is $\rho^N = 0.95$ and $\rho^C = 0.85$. The fact that we have adopted the same values of ρ^i for both productivity and time preference shocks means that we need only track one additional state variable which captures where in the evolution of the crisis we are. The probabilities of each regime are given by, $p_C = 0.98$ and $p_N = 0.99$. This implies the crisis is expected to last for 12.5 years and a crisis is expected to happen, on average, every 25 years.

Consider the green line in Figure 5 – this is the case where the crisis implies a sharp but temporary increase in government transfers. Under a conventional tax smoothing policy, this is the classic experiment where the policy would call for a permanent rise in government debt to facilitate a smaller but sustained rise in taxation to pay for the temporary increase in transfers. Instead, what happens here is that the government raises taxes by even more than the rise in transfers so that the debt-to-GDP ratio actually falls throughout the crisis. This is despite the fact that higher taxation reduces output, the denominator of the debt-to-GDP measure. The higher taxation makes the economy less efficient, which worsens the inflation bias and fuels inflation, driving the desire to decrease government debt. Similarly, the red line considers the case of a very large fall in productivity/output. Despite a significant contraction of the tax base, a decline in government consumption and higher tax rate result in a modest decrease in government debt. The output contraction is consistent with a fall in the real interest rate as households attempt to save to smooth the ongoing fall in output/consumption, but it re-normalizes relatively quickly, well before the crisis is over. Finally, the blue line considers a flight to safety, which drives real interest rates down to around -2% for the duration of the crisis, and they then recover, gradually, upon exiting the crisis. This is now consistent with a large increase in government debt. There is an immediate fall in inflation as the debt stabilization bias is weaker when the policymakers are relatively myopic. The fiscal authority does not feel as constrained by the consequences for inflation of its actions, such as cutting taxes and boosting public consumption. As debt accumulates towards 100% of GDP inflation rises, reflecting the worsening of the inflation bias problem as debt increases, surpassing the pre-crisis rate of inflation after around 10 years. However, it is only when the re-normalization of interest rates begins that the consequences of this higher debt are felt fully. Inflation immediately jumps up as we return to the normal regime of positive real interest rates. Therefore, upon exiting the flight to safety, there is a significant tightening of monetary policy and a severe fiscal consolidation as both policymakers seek to reduce debt levels and offset the inflation generated by high levels of debt.

It is interesting to note that the risk of further crises affects the new steady state that the economy enters outside of the crisis regime. In the case of a risk of a fall in output and an increase in transfers should a crisis episode re-emerge, the new steady-state in normal times implies a debt-to-GDP ratio that is lower than previously. This reflects a precautionary desire on the part of the policymaker to reduce debt levels prior to a new crisis, thereby facilitating better outcomes should the crisis emerge. In contrast, in the case of a flight to safety on its own, this is actually good news as it reduces debt service costs, implying that it is less costly to sustain an inherited level of debt. This increases the debt-to-GDP ratio in normal times. However, inflation remains elevated as a result.

The analysis above assumes there are no costs in changing fiscal instruments, beyond those implied by the economic mechanisms contained within the model. However, politicians face political constraints in either raising taxes or cutting government expenditure, particularly transfers. We briefly discuss the implications of such frictions by allowing for (i) a delay in reversing the increase in transfers when the economy exits the crisis period (the magenta line in Figure 6), and (ii) A quadratic cost to raising taxes during the crisis period (the black line). Otherwise the shock is a 50% increase in transfers captured by the identical green line in Figures 5 and 6 which was described above. Delaying the return on transfers to normal adds to the expected severity of the fiscal shock, leading the policy maker to respond even more aggressively by raising taxes during the period of crisis. As a result debt falls by more during the period of raised transfers, and continues to fall when the economy returns to normal to ensure the economy enjoys a lower debt stock when the next period of increased transfers hits. The black line then considers a situation where it is costly to raise taxes during the period of enhanced transfers. Despite this debt still falls during the 'crisis' period. The costs of raising taxes raises the inflation bias associated with a given stock of government debt implying a significant increase in inflation. As a result, despite the cost, the policy maker still raises taxes to reduce debt, while also failing to significantly increase interest rates to fight inflation. When the shock is over and the government's ability to increase taxes has returned, they keep taxes high for longer to reduce the debt-to-GDP ratio in preparation for another spell of temporarily high transfers where their ability to raise taxes is also constrained. Therefore, in both cases, adding fiscal frictions which might have been thought likely to lead to higher debt, actually leads the policy maker to reduce debt due to the debt stabilization bias being enhanced as a result of the friction.

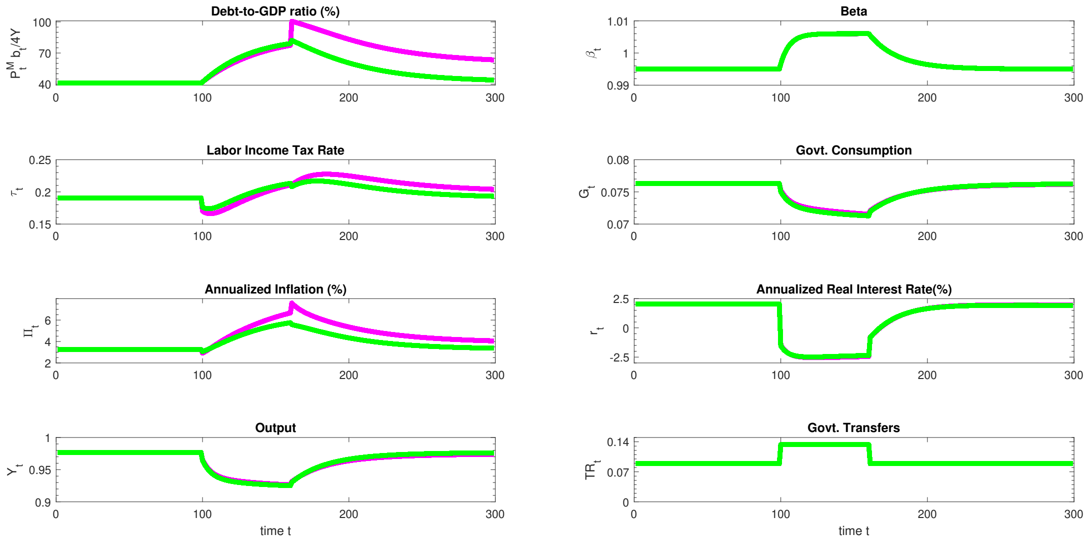

Figure 7: Crisis: All Shocks simultaneously
Note: The figure considers the response of the economy to a crisis regime where all shocks (the 50% increase in transfers, flight to safety and large recessions) occur simultaneously. The green line considers the benchmark calibration of debt maturity. The magenta line allows maturity to increase from 4.5 years to 10 years once the crisis hits. Once the crisis occurs it is expected to last for around 12.5 years and to re-occur, on average, every 20 years.

The impact of combining all the various shocks shown in Figure 5 is then shown as the green line in Figure 7. Here, we see a fall in inflation (which is modest), output, and interest rates at the outset of the crisis, which prompts a fiscal stimulus that raises the debt-to-GDP ratio rapidly. Relatively quickly, the rising debt levels increase inflation, and fiscal policy is tightened to slow the rise in debt. There is a negligible tightening of monetary policy, although interest rates remain not far from the ZLB as a result of the ongoing flight to safety. Inflation continues to rise as debt increases, and when the crisis period ends, interest rates and output re-normalize. However, now that crisis episodes are known to be possible, and there is a perceived risk that they will re-occur, debt levels will not return to their original pre-crisis level. Instead, both debt and inflation remain high as policymakers anticipate future crises. The reason for this is that there is little desire to incur the short-run costs of debt reduction when policymakers know that there are likely to be sustained periods where a flight to safety reduces debt service costs. This is despite the fact that lowering debt would facilitate the use of fiscal policy for stabilization purposes when a future crisis hits.

Although the response to all shocks does lead to a slight fall in inflation initially, it quickly starts to rise as rising debt worsens the inflationary bias problem. This does not correspond to the sustained reduction in inflation following the financial crisis. One factor which may play a role is that government debt management tended to use the low interest rate environment as an opportunity to lengthen debt maturity. We now turn to assess the implications of such policies.

The purple line assumes that the policy maker extends the duration of its debt when the crisis hits from 4.5 years to 10 years (i.e. the bond maturity parameter ρ increases from 0.95 to 0.98.). Since this occurs after the crisis hits there is no additional transfer of wealth from bond holders to the government due to bond price movements since the policy maker retires the debt of one maturity and re-issues at a longer maturity at the prevailing market prices. However, longer maturity reduces the inflation bias associated with a given level of government debt, thereby initially weakening the debt stabilization bias. As a result, taxes are lower initially during the crisis period. However, as debt levels rise, inflation rises too and when the crisis ends, the path for inflation is now expected to be lower (relative to the situation where the crisis continues), increasing bond prices leading to a sharp increase in the debt-to-GDP ratio. The weaker debt stabilization bias also implies that debt stabilizes at a higher level and this drives a high steady-state rate of inflation, which in turn feeds into earlier inflation thanks to the forward looking nature of the New Keynesian Phillips curve.

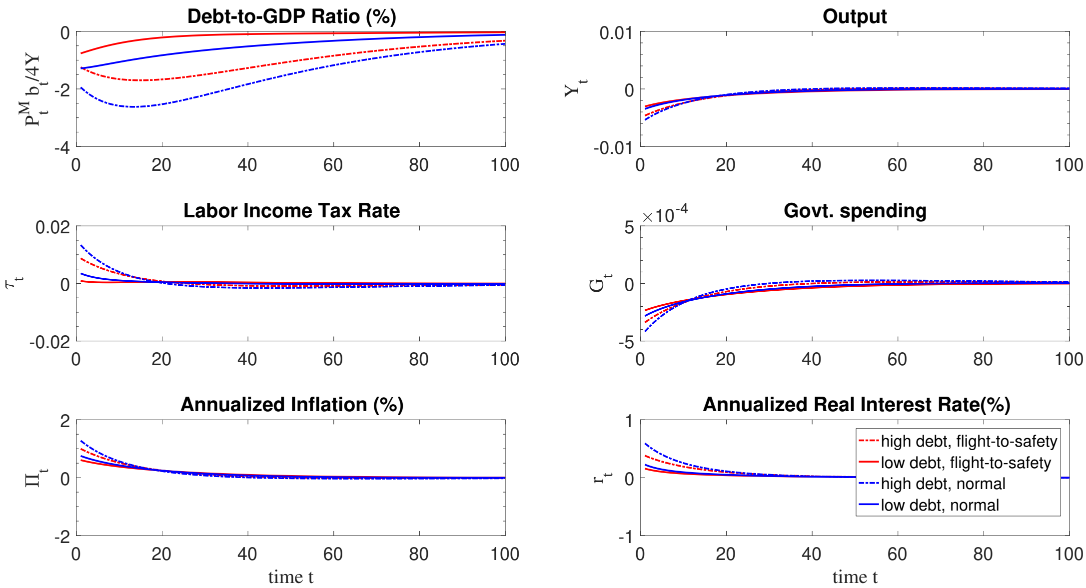

Figure 8: The impact of shock inside/outside of a crisis with high/low levels of debt.
Note: The red (blue) lines imply that the shock occurs when debt levels are low (high), and the solid (dotted) line denotes a flight-to-safety (normal) regime, respectively. All figures plot the deviation from a non-shock baseline.

The process also affects how the central bank responds to shocks. Figure 8 considers the marginal impact of a cost-push shock in normal vs flight-to-safety times, as well as with high or low levels of debt. The red (blue) lines imply that the shock occurs when debt levels are low (high), and the solid (dotted) line denotes a flight-to-safety (normal) regime, respectively. This indicates that the low debt stabilization bias during the flight-to-safety regime results in relatively low additional inflation from the cost-push shock, which further reduces output below its efficient level. In contrast, when we are in the normal regime, the cost-push shock is significantly more inflationary. When we turn to high levels of debt – the blue lines – the impacts are more pronounced. A cost-push shock occurring shortly after the re-normalization of interest rates (when debt levels are still high following the debt accumulation during the flight-to-safety episode) worsens an already large inflation bias problem fueling inflation by more than three times that observed under low debt levels. While the monetary policy response, in terms of real interest rates, rises with inflation, the ratio of the increase in inflation to the increase in real interest rates (consistent with the coefficient of a simple Taylor rule) falls significantly - in this sense, the high debt levels cause the central bank to moderate its anti-inflation response to cost-push shocks, although the Taylor principle would still be adhered to in observed data.

8 Conclusions

We utilized a new Keynesian model to explore the implications of the debt stabilization bias for movements in government debt. The standard model, hit by conventional economic shocks, does not generate data-consistent movements in government debt. Time-consistent policy makers' desire to avoid the inflation bias associated with high levels of government debt prevents them from using debt as a shock absorber in they way they would under a conventional tax smoothing policy. Although the tax smoothing motive is present, the incentive to return debt to its pre-shock level (the debt stabilization bias) dominates to such an extent that government debt barely rises in the face of standard economic shocks. Even considering a large, but temporary, increase in government transfers – a classic tax smoothing experiment – does not overturn this result, as the policy maker more than balances the budget during the period of high fiscal transfers. This is true even if the policy maker faces additional costs which limit its ability to raise taxes or undo increases in transfers.

Given that the obvious economic shocks which may have been thought able to explain movements in government debt, don't have traction when policy is time consistent, we turn to consider alternative explanations. Leeper et al. (2021) find that the steady-state debt-to-GDP ratio is largely determined by the maturity of the debt, the size of markups in the economy and the myopia of the policy maker. We explored how *changes* in these factors may have influenced the evolution of government debt. We found that lengthening debt maturity is insufficient to generate the observed increases in government debt and that the rise in markups would have worsened the inflation bias, giving incentives to reduce rather than increase that debt. However, movements in policy maker myopia can mimic the trends in government debt, at least until the financial crisis. Obviously, beyond the financial crisis a further increase in policy maker myopia could help explain further increases in debt. However, since the financial crisis/pandemic were instances of quite profound economic upheaval, we consider if there was something about these episodes which could also have contributed to the rise in debt.

In a stylized manner, we focused on three elements of these crises which may have had an impact on the debt-to-GDP ratio. Specifically, we examined the impact of a rise in fiscal transfers, significant recession and a flight to safety, which reduced real interest rates, on debt dynamics during the crisis period. We found that of these three elements, the flight to safety created conditions which would have encouraged the policy maker to issue significant quantities of debt. However, as the crisis ended and interest rates rose, the higher debt stock and the associated debt service costs, would be expected to raise inflation as policy makers regained their aversion to debt.

We also explored how the response to shocks differs with high vs low debt, whether in normal times or during a flight to safety. During a flight to safety, cost-push shocks are less inflationary. However, the same shock occurring in normal times increases inflation by much more. Observed outcomes when debt levels are high are also consistent with the central bank moderating its response to shocks. This would appear as a decrease in the estimated coefficient of a Taylor rule, although the Taylor principle would still be satisfied.

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A Tables

Table A.1: Parameterization

Parameter	Value	Definition
β	0.995	Quarterly discount factor (households)
$\beta^F = \beta^M$	0.982	Quarterly discount factor (policy maker)
σ	2	Relative risk aversion coefficient
σ^g	2	Relative risk aversion coefficient for government spending
φ	3	Inverse Frish elasticity of labor supply
ϵ	$14 + 1/3$	Elasticity of substitution between varieties
ρ	0.95	Debt maturity structure
χ	0.007	Scaling parameter associated with government spending
ρ_ϵ	0.95	AR-coefficient of cost push shock
σ_ϵ	0.01	Standard deviation of cost push shock
ϕ	50	Rotemberg adjustment cost coefficient

Table A.2: Steady state values under the benchmark parameterization

Variable	Steady State	Definition
b	0.10	real long term debt
Y	0.97	output
$P^M b/(4Y) \times 100$	41%	debt-GDP ratio in terms of annual output
G/Y	7.8%	government spending
$\left(\Pi^4 - 1\right) \times 100$	3.2%	annualized inflation rate
τ	19%	income tax rate
i	5.2%	annualized nominal interest rate
r	2%	annualized real interest rate

Online Appendix

to

Explaining Movements in Government Debt

by

Tatiana Kirsanova, Eric Leeper, Campbell Leith, and Ding Liu

B1 Summary of Model

We now summarize the model and its steady state before turning to the time-consistent policy problem.

Consumption Euler equation,

$$\beta_t R_t \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \tag{B1.1}$$

Pricing of longer-term bonds,

$$\beta_t \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) (1 + \rho P^M_{t+1}) \right\} = P^M_t \tag{B1.2}$$

Labor supply,

$$N_t^{\varphi} C_t^{\sigma} = (1 - \tau_t) \left(\frac{W_t}{P_t} \right) \equiv (1 - \tau_t) w_t$$

Resource constraint,

$$Y_t \left[1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right] = C_t + G_t \tag{B1.3}$$

Phillips curve,

$$\begin{aligned} 0 = {} & (1 - \epsilon_t) + \epsilon_t mc_t - \phi \Pi_t (\Pi_t - 1) \\ & + \phi \beta_t \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} (\Pi_{t+1} - 1) \right] \end{aligned} \tag{B1.4}$$

Government budget constraint,

$$\begin{aligned} P_t^M b_t &= (1 + \rho P_t^M) \frac{b_{t-1}}{\Pi_t} - \frac{W_t}{P_t} N_t \tau_t + G_t + Tr_t \\ &= (1 + \rho P_t^M) \frac{b_{t-1}}{\Pi_t} - \left(\frac{\tau_t}{1 - \tau_t} \right) N_t^{1+\varphi} C_t^{\sigma} + G_t + Tr_t \\ &= (1 + \rho P_t^M) \frac{b_{t-1}}{\Pi_t} - \left(\frac{\tau_t}{1 - \tau_t} \right) \left(\frac{Y_t}{A_t} \right)^{1+\varphi} C_t^{\sigma} + G_t + Tr_t \end{aligned} \tag{B1.5}$$

Technology,

$$Y_t = A_t N_t \tag{B1.6}$$

Marginal costs,

$$mc_t = W_t/\left(P_t A_t\right) = (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} A_t^{-1-\varphi}$$

The objective function for social welfare is given by,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} (\prod_{i=-1}^{t-1} \beta_i) \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(N_t)^{1+\varphi}}{1+\varphi} \right) \tag{B1.7}$$

There are two state variables in the baseline line model: debt b_t and the elasticity of substitution ϵ_t. When considering Markov switching between crisis and normal periods, we also allow for time variation in productivity, A_t and the households' time discount factor, β_t such that, alongside the state of being in a crisis or normal regime, these become state variables, too.

The Deterministic Steady State

Given the system of non-linear equations, the corresponding steady state system can be written as follows:

$$A = 1$$
$$\frac{\beta R}{\Pi} = 1$$
$$\frac{\beta}{\Pi}\left(1 + \rho P^M\right) = P^M$$
$$(1-\tau)w = N^{\varphi} C^{\sigma}$$
$$Y \left[1 - \frac{\phi}{2}\left(\Pi - 1\right)^2\right] = C + G$$
$$(1-\epsilon_t) + \epsilon_t mc + \phi\left(\beta - 1\right)\left[\Pi\left(\Pi - 1\right)\right] = 0$$
$$P^M b = (1 + \rho P^M)\frac{b}{\Pi} - \left(\frac{\tau}{1-\tau}\right) Y^{1+\varphi} C^{\sigma} + G + Tr$$
$$Y = N$$
$$mc = w = (1-\tau)^{-1} Y^{\varphi} C^{\sigma}$$

which implies,

$$P^M = \frac{\beta}{\Pi - \beta\rho}$$
$$mc = w = \frac{\epsilon - 1}{\epsilon}$$

$$\frac{C}{Y} = \left[(1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right)\right]^{1/\sigma} Y^{-\frac{\varphi+\sigma}{\sigma}}$$
$$\frac{G}{Y} = 1 - \frac{C}{Y} = 1 - \left[(1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right)\right]^{1/\sigma} Y^{-\frac{\varphi+\sigma}{\sigma}}$$
$$\frac{P^M b}{Y} = \frac{\beta}{1-\beta}\left[\tau\left(\frac{\epsilon-1}{\epsilon}\right) - \frac{G}{Y} - \frac{Tr}{Y}\right]$$

Note that,

$$Y^{\varphi+\sigma}\left(1 - \frac{G}{Y}\right)^{\sigma} = (1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right) \tag{B1.8}$$

which will imply a level of output below that which would be chosen by a social planner, assuming taxes are non-negative.

B2 First Order Conditions

We can write the first-order conditions (FOCs) for the policy problem as follows.[19]

Government spending,

$$\chi G_t^{-\sigma_g} - \lambda_{1t} - \lambda_{3t} = 0 \tag{B2.1}$$

The FOC for consumption is given by,

$$C_t^{-\sigma} - \lambda_{1t}^m + \lambda_{2t}\left[\sigma\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi}C_t^{\sigma-1} + \sigma\phi\beta_t C_t^{\sigma-1}Y_t^{-1}\mathbb{E}_t\left[M(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right]$$

$$+\lambda_{3t}\left[\begin{array}{c} \sigma\beta_t b_t C_t^{\sigma-1}\mathbb{E}_t\left[L(b_t,\beta_{t+1},\epsilon_{t+1})\right] - \rho\sigma\beta_t\frac{b_{t-1}}{\Pi_t}C_t^{\sigma-1}\mathbb{E}_t\left[L(b_t,\beta_{t+1},\epsilon_{t+1})\right] \\ +\sigma\left(\frac{\tau_t}{1-\tau_t}\right)(Y_t)^{1+\varphi}C_t^{\sigma-1} \end{array}\right] = 0 \tag{B2.2}$$

Here, the marginal conditions equate the marginal utility gain from consumption plus the relaxation of the government's budget constraint by reducing real interest rates, cet. par. ($\lambda_{3t} \geq 0$) against the tightening of the resource constraint ($\lambda_{1t} \geq 0$) and the worsening of the output-inflation trade-off at time t ($\lambda_{2t} \leq 0$).

Output,

$$-Y_t^{\varphi} + \lambda_{1t}\left[1 - \frac{\phi}{2}\left(\Pi_t - 1\right)^2\right]$$

$$+\lambda_{2t}\left[\epsilon_t\varphi(1-\tau_t)^{-1}Y_t^{\varphi-1}C_t^{\sigma} - \phi\beta_t C_t^{\sigma}Y_t^{-2}\mathbb{E}_t\left[M(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right]$$

$$+\lambda_{3t}\left[(1+\varphi)Y_t^{\varphi}C_t^{\sigma}\left(\frac{\tau_t}{1-\tau_t}\right)\right] = 0 \tag{B2.3}$$

such that the marginal costs of higher output – the reduction in utility due to the need to increase labor supply and the fuelling of inflation this implies,($\lambda_{2t} \leq 0$) – are equated to the marginal gains from relaxing the resource constraint ($\lambda_{1t} \geq 0$) (facilitating enhanced private and/or public consumption) and the fiscal benefits of increasing the tax base, ($\lambda_{3t} \geq 0$).

Taxation,

$$\lambda_{2t}\left[\epsilon_t(1-\tau_t)^{-2}Y_t^{\varphi}C_t^{\sigma}\right] + \lambda_{3t}\left[Y_t^{1+\varphi}C_t^{\sigma}(1-\tau_t)^{-2}\right] = 0$$

simplifying,

$$\epsilon_t\lambda_{2t} + \lambda_{3t}Y_t = 0 \tag{B2.4}$$

Inflation,

$$-\lambda_{1t}\left[Y_t\phi\left(\Pi_t - 1\right)\right] - \lambda_{2t}\left[\phi\left(2\Pi_t - 1\right)\right]$$

$$+\lambda_{3t}\left[\frac{b_{t-1}}{\Pi_t^2}\left(1 + \rho\beta_t C_t^{\sigma}\mathbb{E}_t\left[L(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right)\right] = 0 \tag{B2.5}$$

Government debt,

$$\beta^P\mathbb{E}_t\left[\frac{\partial V_{t+1}^P(b_t,\beta_{t+1},\epsilon_{t+1})}{\partial b_t}\right] + \lambda_{2t}\left[\phi\beta_t C_t^{\sigma}Y_t^{-1}\mathbb{E}_t\left[M_1(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right]$$

$$+\beta_t\lambda_{3t}\left[C_t^{\sigma}\mathbb{E}_t\left[L(b_t,\beta_{t+1},\epsilon_{t+1})\right] + b_t C_t^{\sigma}\mathbb{E}_t\left[L_1(b_t,\beta_{t+1},\epsilon_{t+1})\right] - \rho\frac{b_{t-1}}{\Pi_t}C_t^{\sigma}\mathbb{E}_t\left[L_1(b_t,\beta_{t+1},\epsilon_{t+1})\right]\right] = 0$$

[19] Note we are implicitly treating consumption as the monetary policy instrument since they can costlessly adjust interest rates to achieve any value for consumption they wish, given the values of all other variables.

where

$$L_1(b_t, \beta_{t+1}, \epsilon_{t+1}) \equiv \partial L(b_t, \beta_{t+1}, \epsilon_{t+1})/\partial b_t$$

$$M_1(b_t, \beta_{t+1}, \epsilon_{t+1}) \equiv \partial M(b_t, \beta_{t+1}, \epsilon_{t+1})/\partial b_t$$

Note that by the envelope theorem,

$$\frac{\partial V_t^P(b_{t-1}, \beta_t, \epsilon_t)}{\partial b_{t-1}} = -\frac{\lambda_{3t}}{\Pi_t}\left(1 + \rho\beta_t C_t^{\sigma}\mathbb{E}_t\left[L(b_t, \beta_{t+1}, \epsilon_{t+1})\right]\right)$$

the FOC for taxation (B2.4), and the definition of bond prices (7), we can write the FOC for government debt as shown in the main text.

B3 Numerical Algorithm

This subsection describes the Chebyshev collocation method with time iteration used in the paper. See Judd (1998) for a textbook treatment of the numerical techniques involved.

Let $s_t = (b_{t-1}, \beta_t, \epsilon_t)$ denote the state vector at time t, where real stock of debt b_{t-1} is endogenous, discount factor β_t and elasticity of substitution between goods ϵ_t are exogenous and respectively, with the following laws of motion:

$$P_t^M b_t = (1 + \rho P_t^M)\frac{b_{t-1}}{\Pi_t} - w_t N_t \tau_t + G_t + tr_t;$$

$$\beta_t = \rho^{\gamma}\beta_{t-1} + (1 - \rho^{\gamma})\,\beta^{\gamma}$$

with $\gamma = C, N$ respectively denoting the crisis and normal regime, $\beta^C > \beta^N$, the associated transition probability matrix governing the evolution of this two-state Markov process

$$\begin{bmatrix} p_N & 1 - p_N \\ 1 - p_C & p_C \end{bmatrix} \tag{B3.1}$$

and p_γ denoting the probability of remaining in regime γ given currently in regime γ;

$$\ln(\epsilon_t) = (1 - \rho_\epsilon)\ln(\epsilon) + \rho_\epsilon \ln(\epsilon_{t-1}) + e_{\epsilon t},$$

with $0 \leq \rho_\epsilon < 1$ and $e_{\epsilon_t} \overset{i.i.d}{\sim} N(0, \sigma_\epsilon^2)$.

There are 7 endogenous variables and 3 Lagrangian multipliers in the optimal policy problem. It is convenient to consider β_t as an endogenous variable, real interest rate r_t and the value function V_t for social welfare in the numerical algorithm. Correspondingly, there are 13 functional equations associated with the 13 variables

$$\{C_t, Y_t, \Pi_t, b_t, \tau_t, G_t, P_t^M, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}, \beta_t, r_t, V_t\}.$$

Let's define a new function $X : \mathbb{R}^3 \to \mathbb{R}^{16}$, in order to collect the policy functions of endogenous variables as follows:

$$X(s_t) = \big(C_t(s_t), Y_t(s_t), \Pi_t(s_t), b_t(s_t), \tau_t(s_t), P_t^M(s_t), G_t(s_t), \lambda_{1t}(s_t), ..., \lambda_{3t}(s_t), \beta_t(s_t), r_t(s_t), V_t(s_t))\big)$$

Given the specification of the function X, the equilibrium conditions can be written more compactly as,

$$\Gamma\left(s_t, X(s_t), \mathbb{E}_t\left[Z\left(X(s_{t+1})\right)\right], \mathbb{E}_t\left[Z_b\left(X(s_{t+1})\right)\right]\right) = 0$$

where $\Gamma : \mathbb{R}^{3+16+4+2} \to \mathbb{R}^{16}$ summarizes the full set of dynamic equilibrium relationships, and

$$Z\left(X(s_{t+1})\right) = \begin{bmatrix} Z_1\left(X(s_{t+1})\right) \\ Z_2\left(X(s_{t+1})\right) \\ Z_3\left(X(s_{t+1})\right) \\ Z_4\left(X(s_{t+1})\right) \end{bmatrix} \equiv \begin{bmatrix} M(b_t, \beta_{t+1}, \epsilon_{t+1}) \\ L(b_t, \beta_{t+1}, \epsilon_{t+1}) \\ (\Pi_{t+1})^{-1}\left(1+\rho P^M_{t+1}\right)\lambda^m_{3t+1} \\ (\Pi_{t+1})^{-1}\left(1+\rho P^M_{t+1}\right)\lambda^f_{3t+1} \end{bmatrix}$$

with

$$M\left(b_t, \beta_{t+1}, \epsilon_{t+1}\right) = \left(C_{t+1}\right)^{-\sigma} Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)$$

$$L\left(b_t, \beta_{t+1}, \epsilon_{t+1}\right) = \left(C_{t+1}\right)^{-\sigma}\left(\Pi_{t+1}\right)^{-1}\left(1+\rho P^M_{t+1}\right)$$

and

$$Z_b\left(X(s_{t+1})\right) = \begin{bmatrix} \frac{\partial Z_1(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_2(X(s_{t+1}))}{\partial b_t} \end{bmatrix} \equiv \begin{bmatrix} \frac{\partial M(b_t,\beta_{t+1},\epsilon_{t+1})}{\partial b_t} \\ \frac{\partial L(b_t,\beta_{t+1},\epsilon_{t+1})}{\partial b_t} \end{bmatrix}$$

More specifically,

$$L_1(b_t, \beta_{t+1}, \epsilon_{t+1}) = \frac{\partial\left[(C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1}(1+\rho P^M_{t+1})\right]}{\partial b_t}$$

$$= -\sigma(C_{t+1})^{-\sigma-1}(\Pi_{t+1})^{-1}(1+\rho P^M_{t+1})\frac{\partial C_{t+1}}{\partial b_t}$$
$$-(C_{t+1})^{-\sigma}(\Pi_{t+1})^{-2}(1+\rho P^M_{t+1})\frac{\partial \Pi_{t+1}}{\partial b_t} + \rho(C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1}\frac{\partial P^M_{t+1}}{\partial b_t}$$

and

$$M_1(b_t, \beta_{t+1}, \epsilon_{t+1}) = \frac{\partial\left[\left(C_{t+1}\right)^{-\sigma} Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\right]}{\partial b_t}$$

$$= -\sigma(C_{t+1})^{-\sigma-1}Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial C_{t+1}}{\partial b_t} + \left(C_{t+1}\right)^{-\sigma}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial Y_{t+1}}{\partial b_t}$$
$$+\left(C_{t+1}\right)^{-\sigma}Y_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial \Pi_{t+1}}{\partial b_t} + \left(C_{t+1}\right)^{-\sigma}Y_{t+1}\Pi_{t+1}\frac{\partial \Pi_{t+1}}{\partial b_t}$$

$$= -\sigma(C_{t+1})^{-\sigma-1}Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial C_{t+1}}{\partial b_t} + \left(C_{t+1}\right)^{-\sigma}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial Y_{t+1}}{\partial b_t}$$
$$+\left(C_{t+1}\right)^{-\sigma}Y_{t+1}\left(2\Pi_{t+1}-1\right)\frac{\partial \Pi_{t+1}}{\partial b_t}$$

Note we are assuming $\mathbb{E}_t\left[Z_b\left(X(s_{t+1})\right)\right] = \partial\mathbb{E}_t\left[Z\left(X(s_{t+1})\right)\right]/\partial b_t$, which is normally valid using the Interchange of Integration and Differentiation Theorem. Then, the problem is to find a vector-valued function X that Γ maps to the zero function. Projection methods can be used.

Following the notation convention in the literature, we simply use $s = (b, \beta, \epsilon)$ to denote the current state of the economy $s_t = (b_{t-1}, \beta_t, \epsilon_t)$, and s' to represent next period state that evolves according to the law of motion specified above. The Chebyshev collocation method with time iteration which we use to solve this nonlinear system can be described as follows:

1. Define the collocation nodes and the space of the approximating functions:

    - Choose an order of approximation (i.e., the polynomial degrees) n_b, n_β and n_ϵ for each dimension of the state space $s = (b, \beta, \epsilon)$, then there are $N_s = (n_b + 1) \times (n_\beta + 1) \times (n_\epsilon + 1)$ nodes in the state space. Let $S = (S_1, S_2, ..., S_{N_s})$ denote the set of collocation nodes.
    - Compute the $n_b + 1$, $n_\beta + 1$ and $n_\epsilon + 1$ roots of the Chebychev polynomial of order $n_b + 1$, $n_\beta + 1$ and $n_\epsilon + 1$ as

$$
\begin{aligned}
z_b^i &= \cos\left(\frac{(2i-1)\pi}{2(n_b+1)}\right), \text{ for } i = 1, 2, ..., n_b + 1. \\
z_\beta^i &= \cos\left(\frac{(2i-1)\pi}{2(n_\beta+1)}\right), \text{ for } i = 1, 2, ..., n_\beta + 1. \\
z_\epsilon^i &= \cos\left(\frac{(2i-1)\pi}{2(n_\epsilon+1)}\right), \text{ for } i = 1, 2, ..., n_\epsilon + 1.
\end{aligned}
$$

    - Compute collocation points β_i as

$$
\beta_i = \frac{\overline{\beta} + \underline{\beta}}{2} + \frac{\overline{\beta} - \underline{\beta}}{2} z_\beta^i = \frac{\overline{\beta} - \underline{\beta}}{2}\left(z_\beta^i + 1\right) + \underline{\beta}
$$

      for $i = 1, 2, ..., n_\beta + 1$, which map $[-1, 1]$ into $[\underline{\beta}, \overline{\beta}]$. Note that the number of collocation nodes is $n_\beta + 1$. Similarly, compute collocation points b_i as

$$
b_i = \frac{\overline{b} + \underline{b}}{2} + \frac{\overline{b} - \underline{b}}{2} z_b^i = \frac{\overline{b} - \underline{b}}{2}\left(z_b^i + 1\right) + \underline{b}
$$

      for $i = 1, 2, ..., n_b + 1$, which map $[-1, 1]$ into $[\underline{b}, \overline{b}]$. Compute collocation points ϵ_i as

$$
\epsilon_i = \frac{\overline{\epsilon} + \underline{\epsilon}}{2} + \frac{\overline{\epsilon} - \underline{\epsilon}}{2} z_\epsilon^i = \frac{\overline{\epsilon} - \underline{\epsilon}}{2}\left(z_\epsilon^i + 1\right) + \underline{\epsilon}
$$

      for $i = 1, 2, ..., n_\epsilon + 1$, which map $[-1, 1]$ into $[\underline{\epsilon}, \overline{\epsilon}]$. Note that

$$
S = \{(b_i, \beta_j, \epsilon_k) \mid i = 1, 2, ..., n_b + 1,\ j = 1, 2, ..., n_\beta + 1, k = 1, 2, ..., n_\epsilon + 1\}
$$

      that is, the tensor grids, with $S_1 = (b_1, \beta_1, \epsilon_1)$, $S_2 = (b_1, \beta_1, \epsilon_2)$, ..., $S_{N_s} = \left(b_{n_b+1}, \beta_{n_\beta+1}, \epsilon_{n_\epsilon+1}\right)$.
    - The space of the approximating functions, denoted as Ω, is a matrix of three-dimensional Chebyshev polynomials. More specifically,

$$
\Omega(S) = \begin{bmatrix} \Omega(S_1) \\ \Omega(S_2) \\ \vdots \\ \Omega(S_{n_\epsilon+1}) \\ \vdots \\ \Omega(S_{N_s}) \end{bmatrix} =
$$

$$
\begin{bmatrix}
1 & T_0(\xi(b_1))T_0(\xi(\beta_1))T_1(\xi(\epsilon_1)) & T_0(\xi(b_1))T_0(\xi(\beta_1))T_2(\xi(\epsilon_1)) & \cdots & T_{n_b}(\xi(b_1))T_{n_\beta}(\xi(\beta_1))T_{n_\epsilon}(\xi(\epsilon_1)) \\
1 & T_0(\xi(b_1))T_0(\xi(\beta_1))T_1(\xi(\epsilon_2)) & T_0(\xi(b_1))T_0(\xi(\beta_1))T_2(\xi(\epsilon_2)) & \cdots & T_{n_b}(\xi(b_1))T_{n_\beta}(\xi(\beta_1))T_{n_\epsilon}(\xi(\epsilon_2)) \\
\vdots & \vdots & \vdots & \cdots & \vdots \\
1 & T_0(\xi(b_1))T_0(\xi(\beta_1))T_1(\xi(\epsilon_{n_\epsilon+1})) & T_0(\xi(b_1))T_0(\xi(\beta_1))T_2(\xi(\epsilon_{n_\epsilon+1})) & \cdots & T_{n_b}(\xi(b_1))T_{n_\beta}(\xi(\beta_1))T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1})) \\
\vdots & \vdots & \vdots & \cdots & \vdots \\
1 & T_0(\xi(b_{n_b+1}))T_0(\xi(\beta_{n_\beta+1}))T_1(\xi(\epsilon_{n_\epsilon+1})) & \vdots & \cdots & T_{n_b}(\xi(b_{n_b+1}))T_{n_\beta}(\xi(\beta_{n_\beta+1}))T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1}))
\end{bmatrix}_{N_s \times N_s}
$$

      where $\xi(x) = 2\left(x - \underline{x}\right) / \left(\overline{x} - \underline{x}\right) - 1$ maps the domain of $x \in [\underline{x}, \overline{x}]$ into $[-1, 1]$.

- Then for a given regime $\gamma = C, N$, at each node $s \in S$, policy functions $X_\gamma(s)$ are approximated by $X_\gamma(s) = \Omega(s)\Theta_X^\gamma$,

  where

$$\Theta_X^\gamma = \left[\theta_\gamma^c, \theta_\gamma^y, \theta_\gamma^\pi, \theta_\gamma^b, \theta_\gamma^\tau, \theta_\gamma^{\widetilde{p}}, \theta_\gamma^g, \theta_\gamma^{\lambda_1}, \theta_\gamma^{\lambda_2}, \theta_\gamma^{\lambda_3}, \theta_\gamma^\beta, \theta_\gamma^r, \theta_\gamma^V\right]$$

  is a $N_s \times 16$ matrix of the approximating coefficients.

2. Formulate an initial guess for the approximating coefficients, $\Theta_X^{\gamma,0}$, and specify the stopping rule ϵ_{tol}, say, 10^{-6}.

3. For a given regime $\gamma = C, N$, at each iteration j, we can get an updated $\Theta_X^{\gamma,j}$ by implement the following time iteration step:

   - At each collocation node $s \in S$, compute the possible values of future policy functions $X_\gamma(s')$ for $k = 1, ..., q$. That is,

$$X_\gamma(s') = \Omega(s')\Theta_X^{\gamma,j-1}$$

     where q is the number of Gauss-Hermite quadrature nodes. Note that

$$\Omega(s') = T_{j_b}(\xi(b'))T_{j_\beta}(\xi(\beta'))T_{j_\epsilon}\left(\xi(\epsilon')\right)$$

     is a $q \times N_s$ matrix, with $b' = \widehat{b}(s;\theta_\gamma^b)$, $\beta' = \widehat{\beta}(s;\theta_\gamma^\beta)$, $\epsilon' = \rho_\epsilon \epsilon + z_k\sqrt{2\sigma_\epsilon^2}$, $j_b = 0, ..., n_b$, $j_\beta = 0, ..., n_\beta$, and $j_\epsilon = 0, ..., n_\epsilon$. The hat symbol indicates the corresponding approximate policy functions, so $\widehat{b}$ is the approximate policy for real debt, for example. Similarly, the two auxiliary functions can be calculated as follows:

$$M_\gamma(s') \approx \left(\widehat{C}(s';\theta_\gamma^c)\right)^{-\sigma}\widehat{Y}(s';\theta_\gamma^y)\widehat{\Pi}(s';\theta_\gamma^\pi)\left(\widehat{\Pi}(s';\theta_\gamma^\pi) - 1\right)$$

     and,

$$L_\gamma(s') \approx \left(\widehat{C}(s';\theta_\gamma^c)\right)^{-\sigma}\left(\widehat{\Pi}(s';\theta_\gamma^\pi)\right)^{-1}\left(1 + \frac{\rho\widehat{P^M}\left(s';\theta_\gamma^{\widetilde{p}}\right)}{1-\rho\beta^\gamma}\right)$$

     Note that we use $\widetilde{P}_t^M = (1-\rho\beta^\gamma)P_t^M$ rather than P_t^M in numerical analysis, since the former is far less sensitive to maturity structure variations.

   - Now calculate the expectation terms $E\left[Z\left(X(s')\right)\right]$ at each node s. Let ω_k denote the weights for the quadrature, then

$$E\left[M_\gamma(s')\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^q \omega_k\left(\widehat{C}(s';\theta_\gamma^c)\right)^{-\sigma}\widehat{Y}(s';\theta_\gamma^y)\widehat{\Pi}(s';\theta_\gamma^\pi)\left(\widehat{\Pi}(s';\theta_\gamma^\pi) - 1\right) \equiv \overline{M}_\gamma\left(s', q\right)$$

$$E\left[L_\gamma(s')\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^q \omega_k\left(\widehat{C}(s';\theta_\gamma^c)\right)^{-\sigma}\left(\widehat{\Pi}(s';\theta_\gamma^\pi)\right)^{-1}\left(1 + \frac{\rho\widehat{P^M}\left(s';\theta_\gamma^{\widetilde{p}}\right)}{1-\rho\beta^\gamma}\right) \equiv \overline{L}_\gamma\left(s', q\right)$$

and for $\varsigma = m, f$

$$\mathbb{E}_t\left[\left(\frac{1+\rho P^M_{t+1}}{\Pi_{t+1}}\right)\lambda^{\varsigma}_{3t+1}\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\frac{1+\frac{\rho\widehat{P^M}\left(s';\theta^{\widetilde{p}}_{\gamma}\right)}{1-\rho\beta^{\gamma}}}{\widehat{\Pi}(s';\theta^{\pi}_{\gamma})}\right)\widehat{\lambda^{\varsigma}_3}(s';\theta^{\lambda^{\varsigma}_3}_{\gamma}) \equiv \Lambda^{\varsigma}_{\gamma}\left(s',q\right).$$

Hence,

$$E\left[Z\left(X(s')\right)\right] \approx E\left[\widehat{Z}\left(X(s')\right)\right] = \begin{bmatrix} \overline{M}_{\gamma}\left(s',q\right) \\ \overline{L}_{\gamma}\left(s',q\right) \\ \Lambda^{\varsigma}_{\gamma}\left(s',q\right) \end{bmatrix}$$

- Next calculate the partial derivatives under expectation $E\left[Z_b\left(X(s')\right)\right]$.

- Note that we only need to compute $\partial C_{t+1}/\partial b_t$, $\partial Y_{t+1}/\partial b_t$, $\partial \Pi_{t+1}/\partial b_t$ and $\partial P^M_{t+1}/\partial b_t$, which are given as follows:

$$\frac{\partial C_{t+1}}{\partial b_t} \approx \sum_{j_b=0}^{n_b}\sum_{j_\beta=0}^{n_\beta}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^{c}_{j_b j_\beta j_\epsilon}}{\overline{b}-\underline{b}}T'_{j_b}(\xi(b'))T_{j_\beta}(\xi(\beta'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \widehat{C}_b\left(s'\right)$$

$$\frac{\partial Y_{t+1}}{\partial b_t} \approx \sum_{j_b=0}^{n_b}\sum_{j_\beta=0}^{n_\beta}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^{y}_{j_b j_\beta j_\epsilon}}{\overline{b}-\underline{b}}T'_{j_b}(\xi(b'))T_{j_\beta}(\xi(\beta'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \widehat{Y}_b\left(s'\right)$$

$$\frac{\partial \Pi_{t+1}}{\partial b_t} \approx \sum_{j_b=0}^{n_b}\sum_{j_\beta=0}^{n_\beta}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^{\pi}_{j_b j_\beta j_\epsilon}}{\overline{b}-\underline{b}}T'_{j_b}(\xi(b'))T_{j_\beta}(\xi(\beta'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \widehat{\Pi}_b\left(s'\right)$$

$$\frac{\partial P^M_{t+1}}{\partial b_t} \approx \sum_{j_b=0}^{n_b}\sum_{j_\beta=0}^{n_\beta}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^{\widetilde{p}}_{j_b j_\beta j_\epsilon}}{\overline{b}-\underline{b}}T'_{j_b}(\xi(b'))T_{j_\beta}(\xi(\beta'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \widehat{P}^M_b\left(s'\right)$$

Hence, we can approximate the two partial derivatives under expectation

$$\begin{aligned}
&\frac{\partial E\left[M_{\gamma}(s')\right]}{\partial b} \\
&\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\begin{bmatrix} -\sigma\left(\widehat{C}(s';\theta^{c}_{\gamma})\right)^{-\sigma-1}\widehat{Y}(s';\theta^{y}_{\gamma})\widehat{\Pi}(s';\theta^{\pi}_{\gamma})\left(\widehat{\Pi}(s';\theta^{\pi}_{\gamma})-1\right)\widehat{C}_b\left(s'\right) \\ +\left(\widehat{C}(s';\theta^{c}_{\gamma})\right)^{-\sigma}\widehat{\Pi}(s';\theta^{\pi}_{\gamma})\left(\widehat{\Pi}(s';\theta^{\pi}_{\gamma})-1\right)\widehat{Y}_b\left(s'\right) \\ +\left(\widehat{C}(s';\theta^{c}_{\gamma})\right)^{-\sigma}\widehat{\Pi}(s';\theta^{\pi}_{\gamma})\left(2\widehat{\Pi}(s';\theta^{\pi}_{\gamma})-1\right)\widehat{\Pi}_b\left(s'\right)\end{bmatrix} \\
&\equiv \widehat{M}_{\gamma,b}\left(s',q\right), \\
&\frac{\partial E\left[L_{\gamma}(s')\right]}{\partial b} \\
&\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\begin{bmatrix} -\sigma\left(\widehat{C}(s';\theta^{c}_{\gamma})\right)^{-\sigma-1}\left(\widehat{\Pi}(s';\theta^{\pi}_{\gamma})\right)^{-1}(1+\frac{\rho\widehat{P^M}\left(s';\theta^{\widetilde{p}}_{\gamma}\right)}{1-\rho\beta^{\gamma}})\widehat{C}_b\left(s'\right) \\ -\left(\widehat{C}(s';\theta^{c}_{\gamma})\right)^{-\sigma}\left(\widehat{\Pi}(s';\theta^{\pi}_{\gamma})\right)^{-2}(1+\frac{\rho\widehat{P^M}\left(s';\theta^{\widetilde{p}}_{\gamma}\right)}{1-\rho\beta^{\gamma}})\widehat{\Pi}_b\left(s'\right) \\ +\rho\left(\widehat{C}(s';\theta^{c}_{\gamma})\right)^{-\sigma}\left(\widehat{\Pi}(s';\theta^{\pi}_{\gamma})\right)^{-1}\widehat{P}^M_b\left(s'\right)\end{bmatrix} \\
&\equiv \widehat{L}_{\gamma,b}\left(s',q\right).
\end{aligned}$$

That is,

$$E\left[Z_b\left(X_{\gamma}(s')\right)\right] \approx E\left[\widehat{Z}_b\left(X_{\gamma}(s')\right)\right] = \begin{bmatrix} \widehat{M}_{\gamma,b}\left(s',q\right) \\ \widehat{L}_{\gamma,b}\left(s',q\right)\end{bmatrix}$$

4. At each collocation node s, solve for $X_\gamma(s)$ such that
$$\Gamma\left(s, X_\gamma(s), E\left[\widehat{Z}\left(X_\gamma(s')\right)\right], E\left[\widehat{Z}_b\left(X_\gamma(s')\right)\right]\right) = 0$$
The equation solver *csolve* written by Christopher A. Sims is employed to solve the resulting system of nonlinear equations. With $X_\gamma(s)$ at hand, we can get the corresponding coefficient
$$\widehat{\Theta}_X^{\gamma,j} = \left(\Omega\left(S\right)^T \Omega\left(S\right)\right)^{-1} \Omega\left(S\right)^T X_\gamma(s)$$

5. Update the approximating coefficients for both regimes, $\Theta_X^{\gamma,j} = \eta\widehat{\Theta}_X^{\gamma,j} + (1-\eta)\,\Theta_X^{\gamma,j-1}$, where $0 \leq \eta \leq 1$ is some dampening parameter used for improving convergence.

6. Check the stopping rules. If $\left\|\Theta_X^{\gamma,j} - \Theta_X^{\gamma,j-1}\right\| < \epsilon_{tol}$ for both regimes, then stop, else update the approximation coefficients and go back to step 3.

When implementing the above algorithm, we start from lower-order Chebyshev polynomials and some reasonable initial guesses. Then, we increase the order of approximation and take the solution from the previous lower-order approximation as starting value. This informal homotopy continuation idea ensures us to find a solution.

Remark. Given the fact that the price P_t^M under each regime fluctuates significantly for larger ρ, in numerical analysis, we scale the rule for P_t^M by $(1-\rho\beta^\gamma)$, that is, $\widetilde{P}_t^M = (1-\rho\beta^\gamma)P_t^M$. In this way, the steady state of $\widetilde{P}_t^M$ is very close to β^γ, and $\widetilde{P}_t^M$ does not differ hugely as we change the maturity structure.