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ABSTRACT

Recessions that follow asset price booms accompanied by high credit growth are deeper and longer-lasting than those following asset price booms without strong debt accumulation. We develop a dynamic general equilibrium model with a rational asset bubble and an occasionally binding borrowing constraint that reproduces these empirical regularities. The bubble raises collateral values and relaxes borrowing limits during upswings, but tightens them when it bursts. We derive the time-consistent optimal policy and characterize simple borrowing-tax rules approximating it, showing such policies substantially reduce frequency and severity of recessions triggered by bursting of leveraged bubbles.

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1 Introduction

A large empirical literature documents that when asset price booms are accompanied by credit expansions, subsequent downturns tend to be deeper and longer-lasting than downturns preceded by just asset price booms. Using a cross-country dataset spanning 140 years, Jordà, Schularick, and Taylor (2015) document that *among recessions following an asset price boom*, output falls by more than twice as much when the preceding boom was accompanied by high credit growth–what they term *leveraged bubbles*. Similarly, Dell'Ariccia, Igan, Laeven, and Tong (2016) and others point to the key role of leverage-driven asset price booms in amplifying macroeconomic fluctuations. The mechanism is intuitive: when borrowing rises alongside asset valuations, it becomes more exposed to future price corrections. When asset valuations deflate, collateral values shrink, borrowing constraints tighten, and deleveraging amplifies the downturn (Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999).

These stylized facts illustrate the relevance of the macro-financial feedback between asset valuations, credit, and economic activity. Although many existing theoretical models feature a feedback loop between credit and asset prices, they rely on exogenous changes in credit conditions or in the tightness of borrowing constraints to replicate big downturns (Jermann and Quadrini, 2012; Bianchi and Mendoza, 2018; Jeanne and Korinek, 2020). A key question is whether these amplification patterns can instead emerge through movements in asset valuations, without assuming exogenous shifts in financial conditions, such as a tightening of lending standards, which reduce the pledgeability of collateral. Naturally, a sudden drop in asset valuations may coincide with a tightening in lending standards, but while the latter has been extensively studied in the literature, the former has not been broadly explored due to difficulties in incorporating asset price bubbles into dynamic general equilibrium models.[1]

This paper develops a dynamic general equilibrium model with an occasionally binding borrowing constraint and an endogenously growing asset price bubble that generates dynamics of leveraged asset price bubbles consistent with the empirical evidence. Our focus is on the non-linear dynamics of bubbles and credit, which can result in infrequent but severe drops in output. We solve globally for the economy's recursive equilibrium to examine how output losses are amplified through the state-dependent interaction of high asset valuations and credit, rather than focusing on long-term dynamics or perturbations around the steady-state. To that extent, we complement the existing literature on the macroeconomics of bubbles that study their implications for growth and resource misallocation (Martin and Ventura, 2012, 2016), as well as for demand stabilization in response to deviations from the balanced growth path of the economy (Galí, 2014).

Our framework extends existing models of a small open economy with occasionally binding borrowing constraints (Mendoza, 2010; Bianchi, 2011; Bianchi and Mendoza, 2018) by introducing a rational bubble that can serve as collateral. Agents are infinitely lived, produce output using labor, capital, and intermediate inputs, and borrow externally by pledging both capital and bubble holdings as collateral. The bubble itself does not yield dividends but its potential use as collateral allows its existence despite the assumption of an infinitely-lived representative

[1] We refer to an asset price bubble as an asset whose price exceeds its fundamental value.

consumer. The bubble emerges and bursts exogenously: each period, it can pop with a fixed probability. If it survives, however, its price grows endogenously with economic conditions, increasing collateral values and relaxing the borrowing constraint. When the bubble bursts, the opposite occurs: the borrowing constraint tightens sharply, leading to deleveraging and a recession. As a result, fluctuations in bubble valuations translate directly into borrowing capacity, introducing a powerful amplification mechanism. Contrary to models with exogenous tightening of the borrowing constraint, the endogeneity of the bubble dynamics in our framework implies that financial conditions and, hence, the incidence and severity of financial crises, can be influenced by policy.

We solve the model quantitatively using a fixed-point iteration algorithm with two endogenous state variables—debt and bubble value—following Mendoza and Villalvazo (2020). The model parameters are calibrated as in Bianchi and Mendoza (2018) to match the magnitude and frequency of *Sudden Stop* episodes observed in the data,[2] while the parameters for the bubble process are set to match the frequency and magnitude of the leveraged bubble episodes documented in Jordà et al. (2015).

Our model generates key features of leveraged bubble episodes documented in the literature. First, recessions that follow periods of elevated collateral values and rapid debt accumulation exhibit output declines around 1.5 times larger, on average, than downturns that occur after high asset price valuations with limited leverage. Output also takes longer to recover. Second, these recessions become markedly more severe and persistent when the bubble burst coincides with a negative productivity shock. The two shocks interact through the occasionally binding borrowing constraint, generating losses that exceed the sum of their individual effects. From peak, output falls, on average, by about 4% when the downturn is driven solely by a bursting of a bubble, 6.1% when triggered solely by negative productivity shocks, and 11.7% when the two occur jointly. Third, in our simulations leveraged bubble episodes typically emerge during extended periods of output expansion driven by positive productivity shocks. This pattern is consistent with leveraged bubbles forming during "good times"—periods of sustained positive fundamental shocks to the economy (Kindleberger and Aliber, 2005)—which facilitates greater debt accumulation and subsequent downturn amplification. On the contrary, unleveraged bubbles can form during periods of low productivity growth.

Finally, we examine the optimal policy responses. Because borrowing decisions and asset valuations are interlinked through the collateral constraint, private agents do not internalize how their choices today affect future borrowing capacity, giving rise to externalities that policy can partially correct. Specifically, agents do not internalize how their borrowing today (i) affects the fundamental price of capital (present in Bianchi and Mendoza, 2018), and (ii) the growth rate of the asset price bubble—both of which feed into the collateral value and hence the borrowing capacity. Importantly, we show that the bubble-related externality differs from the externality operating through the fundamental price of capital: whereas the latter influences only the tightness of the current-periods borrowing constraint, the former affects the entire future path

[2] A *Sudden Stop* episode is typically defined as a reversal of net capital inflows that exceeds two standard deviations of its historical mean (or a current account reversal of at least 5% of GDP within one year), accompanied by a fall in asset prices and binding external borrowing constraints (see Bianchi and Mendoza, 2020 for a recent review of the literature.

of borrowing constraints because the bubble growth influences the future bubble state. We characterize the optimal policy, derive a Pigouvian tax on borrowing that decentralizes the efficient allocation, and study simple policy rules that approximate the optimal policy response to leveraged bubbles. In the simulated economy, the optimal tax policy reduces the frequency of leveraged bubbles to nearly zero and lowers the volatility of both output and consumption. As a result, the planner can achieve non-trivial welfare gains of about 0.3 percent in terms of compensating consumption variation.

Related literature—This paper relates to several strands of the literature. First, it builds on models of borrowing constraints and endogenous amplification mechanisms (Kiyotaki and Moore, 1997; Bernanke et al., 1999; Mendoza, 2010; Jermann and Quadrini, 2012; Bianchi and Mendoza, 2018), and connects to the literature on endogenous financial crises (Lorenzoni, 2008; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Boissay and Collard, 2016; Boissay, Collard, Galí, and Manea, 2026), by showing how a rational bubble interacts with borrowing constraints to generate boom-bust episodes.

Second, the paper contributes to the literature on rational bubbles with a macro-finance focus (Kocherlakota, 2009; Farhi and Tirole, 2012; Martin and Ventura, 2012; Hirano and Yanagawa, 2017; Miao and Wang, 2018;Biswas, Hanson, and Phan, 2020; Allen, Barlevy, and Gale, 2022). In particular, we embed a bubble into a representative-agent dynamic general equilibrium model with occasionally binding borrowing constraints suitable for explaining empirical facts and conducting optimal policy. We show that the bubble can be sustained in equilibrium—even without intrinsic value—as long as the borrowing constraint binds with positive probability. In our setting, the bubble endogenously relaxes financial constraints, raising leverage, but also magnifying downturns once it bursts.

Finally, the paper contributes to the macroprudential policy literature (Bianchi, 2011; Benigno, Chen, Otrok, Rebucci, and Young, 2013; Bianchi and Mendoza, 2018; Dávila and Korinek, 2018; Jeanne and Korinek (2020)), and to recent work on policy and bubbles (Farhi and Tirole, 2012; Martin and Ventura, 2016; Galí, 2014; Galí, 2021; Allen, Barlevy, and Gale, 2025), by deriving a time-consistent optimal macroprudential policy in the presence of leveraged bubbles. We solve a fully-fledged Ramsey optimal policy problem in a stochastic environment with occasionally binding constraints and endogenously evolving bubbles, identify the externalities that the bubbles introduce, and show that a countercyclical borrowing tax mitigates the buildup of leverage in good times, reduces the frequency and severity of crises, and delivers substantial welfare gains.

Section 2 presents the model economy. Section 3 describes the quantitative calibration and analyzes the competitive equilibrium. Section 4 introduces the planner's problem, characterizes the optimal policy, and discusses its implementation through a Pigouvian borrowing tax. Section 5 concludes.

2 A Model of Leveraged Bubbles

Consider a small open economy with a continuum of infinitely-lived identical agents of unit mass. The representative agent owns a firm that utilizes labor, intermediate goods, and capital to produce a (perishable) consumption good. Apart from capital, there is an additional durable asset in the economy, which does not yield any dividends and is not used as a factor of production. We call this asset a bubble. Moreover, agents have access to an international credit market where they can borrow by pledging their holdings of capital and of the bubble as collateral. Within each periods, the representative agent supplies labor, purchases intermediate goods from abroad, purchases and sells capital and the bubble in domestic capital markets, rolls over debt in the international credit market, and consumes. We proceed by outlining the model, the agent's optimization decision, and the definition of the competitive equilibrium.

2.1 Representative Agent's Optimization Problem.

The preferences of the representative agent are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t - G(l_t)\right), \tag{1}$$

where $E(\cdot)$ denotes the expectations operator and β is the subjective discount factor. The utility function $U(\cdot)$ is concave, twice continuously differentiable, and satisfies the Inada conditions. It depends on consumption, c_t, and labor supply, l_t, combined in a composite commodity $c_t - G(l_t)$, defined by Greenwood, Hercowitz, and Huffman (1988). $G(\cdot)$ is a convex, strictly increasing, and continuously differentiable function, measuring the disutility of labor. This form of utility function removes the wealth effect on labor supply, which prevents a counterfactual increase in labor supply in bad times. We consider a standard CRRA utility function $U(c_t - G(l_t)) = \left((c_t - G(l_t))^{1-\sigma} - 1\right)/(1-\sigma)$ with risk aversion coefficient σ, and $G(l_t) = \psi l_t^{1+\varphi}/(1+\varphi)$ with Frisch elasticity of labor supply equal to $1/\varphi$.

The agent produces $y_t = F(z_t, k_t, l_t, v_t)$ each periods. $F(\cdot)$ is a Cobb-Douglas production function, which combines labor, l_t, with capital purchased in the previous periods, k_t, and the intermediate good, v_t; z_t is an aggregate productivity shock. In particular, $F(z_t, k_t, l_t, v_t) = e^{z_t} k_t^{\alpha_k} v_t^{\alpha_v} l_t^{\alpha_l}, \quad \alpha_k, \alpha_v, \alpha_l > 0, \quad \alpha_k + \alpha_v + \alpha_l \leq 1$ and the TFP shock follows an independent AR(1) process, given by $z_t = \bar{z} + \rho_z z_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon)$.

Capital does not depreciate and aggregate capital is in unit fixed supply: $K = 1$. The intermediate good is traded in competitive international markets at a fixed exogenous price, p^v. The budget constraint of the representative agent is given by

$$c_t \leq y_t - p^v v_t + L_{t+1}/R - L_t - q_t^k(k_{t+1} - k_t) - q_t^b(x_{t+1} - 1 - (x_t - 1)\mathcal{I}_t), \tag{2}$$

where L_t denotes the beginning-of-periods stock of one-periods, non-state contingent bonds issued last periods, q_t^k is the price of capital, and q_t^b is the price of the bubble. R is the (exogenously given) world-determined gross real interest rate that satisfies $\beta R < 1$. The agent's consumption, c_t, is less than or equal to the output, y_t, net of outlays for the purchase of the intermediate good, $p^v v_t$, the net debt issuance, $L_{t+1}/R - L_t$, the net capital expenditure, $q_t^k(k_{t+1} - k_t)$, and the net bubble expenditure, $q_t^b(x_{t+1} - 1 - (x_t - 1)\mathcal{I}_t)$. x_{t+1} are agent's bubble-holdings at the end of periods t. We explain in detail below the bubble process and how it affects agents' bubble-holdings.

Bubble process. At $t = 0$, a bubble emerges with exogenously given value (price) equal to u and aggregate supply $X = 1$. We assume that the bubble is distributed equally among agents, so each agent receives a unit of the bubble. The survival of a bubble that was present at $t - 1$ into periods t is governed by the realization on a random variable $\mathcal{I}_t$ which follows a two-state Markov process. With probability π, and independent of $\mathcal{I}_{t-1}$ for simplicity, $\mathcal{I}_t = 1$ and the $t - 1$-bubble survives, while with probability $1 - \pi$, and again independent of $\mathcal{I}_{t-1}$, the $t - 1$ bubble bursts and a new bubble emerges. The new bubble is also in unitary aggregate supply, it is distributed equally among agents at the beginning of the periods that it emerges, and it is exogenously valued at u.[3]

Denote by b_t the periods t price of a bubble that existed in the previous periods. If the bubble bursts at t, i.e., $\mathcal{I}_t = 0$, then we set $b_t = 0$. Otherwise, b_t is determined endogenously to satisfy the representative agent's optimality conditions, specified below. At the end of the periods, the agent can re-optimize the holdings of surviving bubbles or their holdings of newly distributed bubbles. Denote by x_{t+1} the agent's re-optimized holdings of the bubble at t. If the bubble from the previous periods survives, then the net expenditure after re-optimizing the bubble holdings is equal to $b_t(x_{t+1} - x_t)$, where x_t is the agent's bubble holdings from the previous periods. If the bubble from the previous periods bursts, then the net bubble expenditure is equal to $u(x_{t+1} - 1)$, given that the new bubble is distributed equally among agents and each agent receives one unit of it.

Then, the price of the bubble, q_t^b, that appears in (2) is defined as

$$q_t^b = \begin{cases} b_t & \text{if } \mathcal{I}_t = 1 \text{ with prob. } \pi \\ u & \text{if } \mathcal{I}_t = 0 \text{ with prob. } 1 - \pi \end{cases} \tag{3}$$

Borrowing constraint. The amount of borrowing from international markets is limited by a borrowing constraint that is endogenously derived from a standard renegotiation problem (see Section A.1 in the Appendix). An agent can obtain two types of loans: An intertemporal loan, L_{t+1}/R, and an intra-temporal loan. The latter is used to finance, ahead of production, a portion θ of the intermediate good purchases. The total liabilities at the beginning of the periods are $L_{t+1}/R + \theta p^v v_t$. While the inter-temporal loan bears an interest payment, the intra-temporal one does not as it is repaid within the same periods. All borrowed funds can be diverted, which is precluded by requiring agents to post collateral. Importantly, the collateral agents can post includes not only their capital holdings but also their holdings of the bubble. Hence, the bubble—while not useful as an input in the production function—it is useful as collateral for borrowing. Because borrowing takes place at the beginning of periods t, i.e. before agents re-optimize their capital and bubble holdings, the borrowing constraint is given by

$$L_{t+1}/R + \theta p^v v_t \leq m(q_t^k k_t + q_t^b(1 + (x_t - 1)\mathcal{I}_t)) \tag{4}$$

where m is the pledgeable portion of capital and the bubble.

[3]The assumption that a new bubble always has the same price u is made for simplicity; alternatively it could be derived from a random distribution.

Optimality conditions. The representative agent chooses $\{c_t, L_{t+1}, k_{t+1}, x_{t+1}, v_t, l_t\}$ to maximize their expected utility (1) subject to the budget constraint (2), the borrowing constraint (4), and the process (3) for q_t^b. Denote by $\beta^t \lambda_t$ and $\beta^t \mu_t \lambda_t$ the Lagrange multipliers on(2) and (4), respectively. The first-order optimality conditions with respect to (*wrt*) each choice variable are given by

$$wrt \text{ c}_\text{t} \quad : \quad \lambda_t = U_{c,t}, \tag{5}$$

$$wrt \text{ L}_{\text{t+1}} : \quad U_{c,t}(1-\mu_t) = \beta R E_t U_{c,t+1}, \tag{6}$$

$$wrt \text{ k}_{\text{t+1}} : \quad q_t^k U_{c,t} = \beta E_t U_{c,t+1} \left(F_{k,t+1} + q_{t+1}^k (1 + m\mu_{t+1}) \right), \tag{7}$$

$$wrt \text{ v}_\text{t} : \quad p^v (1 + \theta \mu_t) = F_{v,t}, \tag{8}$$

$$wrt \text{ l}_\text{t} : \quad G_{l,t} = F_{l,t}, \tag{9}$$

$$\begin{aligned} wrt \text{ x}_{\text{t+1}} : \quad q_t^b U_{c,t} &= \beta E_t \left(b_{t+1} U_{c,t+1} (1 + m\mu_{t+1}) \right) \\ &= \beta \pi \tilde{E}_t \left(b_{t+1} U_{c,t+1} (1 + m\mu_{t+1}) \right), \end{aligned} \tag{10}$$

where $\tilde{E}_t(\cdot) \equiv E_t(\cdot|\mathcal{I}_{t+1} = 1)$. In other words, the expectation on the right-hand side of (10) is over states at $t+1$ in which the t-periods bubble has not burst, i.e., $\mathcal{I}_{t+1} = 1$ with probability π. As explained earlier, if the bubble bursts at $t+1$, i.e., $\mathcal{I}_{t+1} = 0$, then the new bubble emerges at $t+1$, distributed equally among agents, irrespective of the holdings (x_{t+1}) at the end of periods t. Thus, q_t^b appears on the left-hand side and b_{t+1} on the right-hand side of (10).

The complementarity slackness condition is given by

$$\mu_t \left(m \left(q_t^k k_t + q_t^b (1 + (x_t - 1)\mathcal{I}_t) \right) - (L_{t+1}/R + \theta p^v v_t) \right) = 0, \tag{11}$$

The borrowing constraint distorts both the optimal intra- and inter-temporal margins when binding. Condition (8), defining the choice of the intermediate good, embeds an additional cost, i.e., the cost of collateral financing equal to $\mu_t \theta p^v$. In addition, all Euler equations are distorted. The Euler equation for borrowing, (6), implies that the marginal benefit from increasing borrowing today outweighs the expected future marginal cost by an amount equal to the shadow price of relaxing the current borrowing constraint, valued at $U_{c,t}\mu_t$. The Euler equations with respect to capital and the bubble, (7) and (10), equating the marginal cost of an extra unit of capital and the bubble with their marginal benefit, embed the additional benefits derived from relaxing future borrowing constraints, valued at $\beta E_t U_{c,t+1} m \mu_{t+1} q_{t+1}^k$ and $\beta \pi \tilde{E}_t U_{c,t+1} m \mu_{t+1} b_{t+1}$. The latter is especially important for the bubble existence as explained below.

Optimality also requires that the transversality conditions for the paths of capital and the bubble are satisfied, i.e.,

$$\lim_{T\to\infty} E_t \beta^T U_{c,t+T} q_{t+T}^k k_{t+T} = \lim_{T\to\infty} E_t \beta^T U_{c,t+T} q_{t+T}^k = 0, \tag{12}$$

$$\lim_{T\to\infty} E_t \beta^T U_{c,t+T} b_{t+T} x_{t+T} = \lim_{T\to\infty} E_t \beta^T U_{c,t+T} b_{t+T} = 0. \tag{13}$$

Condition (13) implies that a positive bubble valuation cannot be sustained in equilibrium if the borrowing constraint never binds, i.e., $\mu_t = 0$ for all t (Tirole, 1982; see, also, Santos and Woodford, 1997). This is easy to see by iterating (10) forward, setting all Lagrange multipliers on the borrowing constraint to zero, and applying (13). By contrast, a positive price of capital can be sustained in equilibrium, without violating (12), even if the collateral borrowing never binds because capital is used directly in production and thus delivers a dividend equal to its marginal product.

Bubble existence. We show that a positive bubble price can be supported in equilibrium without violating (13) as long as there is positive probability that the borrowing constraint continues to bind as $t \to \infty$, even if it does so occasionally. In other words, a bubble continues to be valued as long as it remains useful in mitigating financial frictions. The following proposition establishes this result.

Proposition 1. *There exists $0 < \overline{m} < 1$ such that for $m < \overline{m}$ the borrowing constraint binds infinitely often along equilibrium paths.*

Proof: See Appendix A.2.

Note that if m is sufficiently close to zero then the borrowing constraint would always be binding. We are interested in cases where m is such that the constraint only binds occasionally.

2.2 Recursive competitive equilibrium

We now define the competitive equilibrium in recursive form. This representation is needed for our quantitative analysis in Section 3—where we solve for global, non-linear, dynamics of leverage and bubbles,—and for studying optimal time-consistent policy in Section 4. We discuss the state variables and define the recursive competitive equilibrium.

State variables. Absent a bubble, the endogenous state variable is L_t and the exogenous one is z_t. Aggregate capital is not a state variable because it is in fixed supply. With a bubble, additional state variables arise.

First, the previous periods bubble may burst and a new one emerges, so $\mathcal{I}_t$ is an (additional) exogenous state variable.[4] Let $\Xi_t = \{z_t, \mathcal{I}_t\}$ denote the set of exogenous state variables at t, which evolves following a Markov transition matrix into next periods exogenous states Ξ_{t+1}. Second, when a bubble survives from $t-1$ ($\mathcal{I}_t = 1$), its value b_t grows endogenously, satisfying (10), iterated one periods back:

$$q^b_{t-1} U_{c,t-1} = \beta \pi \tilde{E}_{t-1} \left(b_t U_{c,t} (1 + m\mu_t) \right). \tag{14}$$

Equation (14) is the equilibrium condition that any rational bubble should satisfy and is consistent with multiple b_t's. To pin down b_t, assumptions are needed about bubble price innovations and their relation to other variables. We shut down that additional dimension of indeterminacy

[4] The value of a newly emerged bubble is always u, so it need not be defined as a separate state variable.

by assuming no surprises in the bubble, i.e., $b_t = \tilde{E}_{t-1}(b_t)$. Under this assumption, Lemma 1 derives the bubble price at t, which becomes an additional state variable, endogenously determined at $t-1$.

Lemma 1. *Assuming zero innovation in the price of the bubble, i.e., $b_t = \tilde{E}_{t-1}(b_t)$,* (14) *yields the following bubble growth process:*

$$b_t = \frac{q^b_{t-1} U_{c,t-1}}{\beta \pi \tilde{E}_{t-1} \left(U_{c,t}(1 + m\mu_t) \right)} \tag{15}$$

The Lemma follows directly from the assumption $b_t = \tilde{E}_{t-1}(b_t)$. The online appendix considers more general cases with non-zero bubble innovations. We show that a similar growth process for b_t arises if bubble innovations are zero-in-expectations and are orthogonal to fundamentals; yet the right-hand side of (15) would include an additional term for the exogenous bubble innovation, drawn by a distribution that would need to be specified. For tractability and without loss of generality, we restrict attention here to the benchmark case of zero innovations.

The state space at t is given by (Ξ_t, L_t, b_t), or in extended form, $(z_t, \mathcal{I}_t, L_t, b_t)$.[5] In the recursive formulation, we denote the state space in the current periods by (Ξ, L, b), and the next-periods state space by (Ξ', L', b').

To link the bubble growth process to other time-varying variables, rewrite (15) by iterating it one periods forward and rearranging terms:

$$\frac{b_{t+1}}{q^b_t} = \frac{1}{\pi \tilde{E}_t M_{t+1} \left(1 + m\mu_{t+1}\right)} \tag{16}$$

where $M_{t+1} = \beta \left(U_{c,t+1} \right) / U_{c,t}$. The left-hand side denotes the growth rate of the bubble from periods t to $t+1$. This expression shows that the bubble grows faster when:

(i) Current consumption is lower relative to expected future consumption. Higher marginal utility today ($U_{c,t}$) lowers the stochastic discount factor (M_{t+1}); agents require higher growth to be willing to hold the bubble.

(ii) Future borrowing constraints are expected to bind less. If borrowing constraints are expected to be slack at $t+1$ (lower μ_{t+1}), the bubble provides less value as collateral; agents require higher growth to be willing to hold the bubble. Conversely, if constraints are expected to bind more tightly, the bubble's collateral value is higher; hence, agents require lower growth to be willing to hold the bubble.

(iii) The probability that the bubble survives into the next periods is lower. A lower survival probability (π) increases the risk of holding the bubble; agents therefore require higher growth to be willing to hold the bubble.

Following (i) and (ii), borrowing affects bubble growth endogenously through its effects on consumption and the stochastic discount factor, and through the tightness of future borrowing constraints, which together determine the growth required for agents to hold the bubble. The probability that the bubble survives into the next periods (π) instead affects the bubble growth

[5]As an aside, in a bubbleless economy, the state space is simply (z_t, L_t). See also Farhi and Tirole (2012) on bubbles as state variables.

exogenously; a typical assumption in the literature (Blanchard and Watson, 1982; Galí, 2021). If, counterfactually, the survival probability were declining in borrowing, borrowing would affect bubble growth through an additional margin. Lower borrowing would raise required growth via discounting and the collateral value, while simultaneously lowering required growth by increasing the likelihood of survival through a higher π. In that case, equilibrium borrowing reflects a trade-off between faster growth conditional on survival and a higher probability of bubble survival. By treating π as exogenous, the model shuts down this trade-off and focuses on the channels operating through intertemporal consumption and the collateral value.

Definition of recursive equilibrium. We define the competitive equilibrium in recursive form. Herein we will express the state space in aggregate terms given that there is a representative agent, but a more detailed representation of the recursive competitive equilibrium could be defined, though that requires distinguishing between individual and aggregate debt and capital (as in Bianchi-Mendoza).

Let $q^k(\Xi, L, b)$ denote the conjectured pricing function for capital and $L' = \Gamma_L(\Xi, L, b)$ the perceived law of motion for borrowing.

To derive the law of motion for the endogenous bubble state b, we iterate (15) one periods forward to determine the bubble growth to get:

$$b' = \Psi(\Xi, L, b, b') \tag{17}$$

$$\text{where } \Psi(\Xi, L, b, b') = \frac{q^b U_c(\hat{c}(\Xi, L, b) - G(\hat{l}(\Xi, L, b)))}{\beta\pi\tilde{E}_{\Xi'|\Xi} U_c(\hat{c}(\Xi', L', b') - G(\hat{l}(\Xi', L', b')))(1 + m\hat{\mu}(\Xi', L', b'))}, \tag{18}$$

and $\hat{c}$, $\hat{l}$, and $\hat{\mu}$ are the competitive equilibrium policy functions for consumption, labor, and the Lagrange multiplier on the borrowing constraint.

Define by $\Gamma_b(\Xi, L, b)$ the value of b' that solves $b' = \Psi(\Xi, L, b, b')$ given (Ξ, L, b). The following Proposition establishes that $\Gamma_b(\Xi, L, b)$ is a well-defined (unique) function that can be used to characterize the recursive equilibrium.

Proposition 2. *There exists $\widehat{m} > 0$ such that, if $m \leq \widehat{m}$, there exists a function $\Gamma_b(\Xi, L, b)$ governing the growth of the bubble.*

The recursive problem of the representative agent is $\overline{\mathbb{V}}(\Xi, L, b, k) = \max_{c,L',k',x',v,l}\{U(c - G(l)) + \beta E_{\Xi'|\Xi}\overline{\mathbb{V}}(\Xi', L', b', k')\}$, subject to $L' = \Gamma_L(\Xi, L, b)$, $b' = \Gamma_b(\Xi, L, b)$, the recursive representations of the budget and borrowing constraints, (2) and(4), given by $c \leq F(z, k, l, v) - p^v v + L'/R - L - q^k(k' - k) - q^b(x' - 1 - (x - 1)\mathcal{I})$ and $L'/R + \theta p^v v \leq m(q^k k + q^b(1 + (x - 1)\mathcal{I})$, and the recursive representation of (3), $q^b = \mathcal{I} \cdot b + (1 - \mathcal{I})u$.

Definition 1. *A recursive competitive equilibrium is defined by asset pricing function $q^k(\Xi, L, b)$, perceived law of motions $L' = \Gamma_L(\Xi, L, b)$ and $b' = \Gamma_b(\Xi, L, b)$ for borrowing and bubble growth, policy rules $\hat{c}(\Xi, L, b)$, $\hat{v}(\Xi, L, b)$, $\hat{l}(\Xi, L, b)$, $\hat{\lambda}(\Xi, L, b)$, $\hat{\mu}(\Xi, L, b)$ for consumption, intermediate good demand, labor, and the Lagrange multiplier on budget and borrowing constraints, as well as value function $\overline{\mathbb{V}}(\Xi, L, b)$ such that:*

1. *The policy rules and value function solve the recursive problem of the representative agent, taking as given $q^k(\Xi, L, b)$, $q^b(\Xi, L, b)$, $\Gamma_L(\Xi, L, b)$, and $\Gamma_b(\Xi, L, b)$.*

2. *Markets clear:* $k = k' = 1$ *and* $x = x' = 1$.

3. *The resource constraint is satisfied:* $\hat{c}(\Xi, L, b) \leq F(z, 1, \hat{l}(\Xi, L, b), \hat{v}(\Xi, L, b)) - p^v \hat{v}(\Xi, L, b) + \Gamma_L(\Xi, L, b)/R - L$.

4. *The complementarity slackness condition is satisfied:* $\hat{\mu}(\Xi, L, b)(m(q^k(\Xi, L, b) + \mathcal{I} \cdot b + (1 - \mathcal{I})u) - \Gamma_L(\Xi, L, b)/R - \theta p^v \hat{v}(\Xi, L, b)) = 0$

5. *The asset pricing* $q^k(\Xi, L, b)$ *solve the recursive forms of* (7).

3 Quantitative Analysis of Competitive Equilibrium

We now turn to the quantitative analysis of the equilibrium in the absence of policy intervention, which we refer to as the *baseline model*. Section 3.1 describes the calibration, with emphasis on the bubble dynamics, Section 3.2 reports key summary statistics, and Sections 3.3-3.5 detail the policy functions and responses to shocks.

We solve the model using a fixed-point iteration algorithm over a fixed grid, following Mendoza and Villalvazo (2020), who propose an algorithm for solving macro models with two endogenous state variables and occasionally binding constraints. Our two endogenous state variables are outstanding debt, L_t, and the bubble, b_t. The competitive equilibrium solution is obtained by iterating over the first-order conditions.

3.1 Calibration

Table 1 lists the parameter values for the baseline calibration. Each time period in the model corresponds to a year. Most of the parameters follow Bianchi and Mendoza (2018) to match similar moments in the data, except for the bubble process.[6]

Relative to Bianchi-Mendoza, we introduce the bubble and depart in two ways to simplify the analysis and highlight the bubble as a source of amplification. First, we abstract from a stochastic global interest rate process to limit the number of exogenous states and fix $R = 1.0128$, equal to the average interest rate in their calibration. Second, while they allow the pledgeability parameter m to vary exogenously, we keep it constant. In their framework, the variation in m is an important driver of output and credit fluctuations, and helps match Sudden Stop dynamics. Since our focus is on the real effects of bubbles, we purposely switch off this exogenous source of variation. We set $m = 0.8$, consistent with US private-sector leverage post-GFC.[7]

Bubble process calibration. We calibrate the bubble process to capture the type of bubbles identified in Jordà et al. (2015). We refer to these as *JST bubbles*. Jordà et al. (2015) use two criteria to identify a bubble. The first criterion sets the bubble's start, t_0, as the point when the real value of all assets q_t is at least one standard deviation above its Hodrick-Prescott (HP) trend. Given the stationarity in our model, we define t_0 as the first period when the collateral

[6] See section III.A of their paper for details.

[7] Median loan-to-value (LTV) for mortgage originations in the US has been 75%-80% after the GFC, below the 90% LTV used by Bianchi-Mendoza, corresponding to the pre-crisis period. See St. Louis Fed here.

Table (1) Calibration

Parameter	Value
Risk aversion	$\sigma = 1$
Labor disutility coefficient	$\psi = 0.352$
Frisch elasticity of labor supply	$1/\varphi = 2$
Share of intermediate good in output	$\alpha_v = 0.45$
Share of labor in output	$\alpha_l = 0.352$
Share of assets in output	$\alpha_k = 0.008$
Interest rate	$R = 1.0128$
TFP process	$\rho_z = 0.78$, $\sigma_\epsilon = 0.01$
Discount factor	$\beta = 0.95$
Working capital coefficient	$\theta = 0.14$
Collateral pledgeability	$m = 0.80$
Probability of a bubble surviving	$\pi = 0.67$
New bubble value	$u = 0.002$

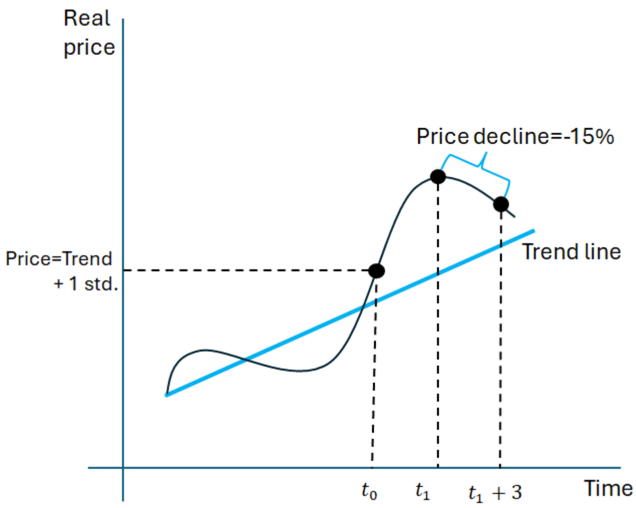


Figure (1) Graphical representation of a JST bubble

price is one standard deviation above its mean, $q_t > \bar{q}_t + \sigma(q_t)$, where $q_t = q_t^k + q_t^b$. The second criterion defines the end of the bubble episode, t_1, as the point after which the real asset value of all assets q_t falls by at least 15 percent over the subsequent 3 years. Figure 1 presents a graphical example of a JST bubble.

Based on these criteria, Jordà et al. (2015) define two characteristics of bubble episodes that guide our calibration of the bubble process: (i) *duration*-the average time from t_0 to t_1, (ii) *amplitude*-the percentage increase in the real asset prices from t_0 to t_1, captured in our model by the collateral price growth $(q_{t_1}/q_{t_0} - 1)$. They document an average duration of 3.2 years and an average amplitude of 17.3 percent for house-price bubbles in the post World War II period.[8]

To match these JST bubble moments, we jointly calibrate the bubble survival probability and the value of a new bubble to $\pi = 0.67$ and $u = 0.002$, respectively. The new bubble value is small, equal to around 1.4% of the average price of capital in our simulations. We gauge this as a sensible choice because it implies that larger bubbles take time to build up, consistent with

[8]Our bubble is not tied to a dividend-paying firm, so we focus on real estate rather than stock price bubbles. See Miao and Wang (2018) and Hirano and Toda (2025) for bubbles on dividend-paying assets.

episodes of leveraged bubbles, rather than being introduced exogenously.[9]

3.2 Summary statistics

Table 2 reports key moments of the baseline economy, based on simulations of the equilibrium policy functions for 500,000 periods.

Table (2) Summary statistics for the baseline model

PANEL A: Credit and asset value statistics	mean	st. dev
Debt to GDP	16.3 %	1.5%
Capital value to GDP	29.3%	1.7%
Collateral value to GDP	30.9%	3.3%
PANEL B: Recession statistics		
Incidence of a recession	20.3%	
Incidence of a severe recession	4.4%	
PANEL C: Bubble statistics		
Incidence of JST bubbles	7.2%	
Average duration of JST bubbles	3.3 years	
Average amplitude of JST bubbles	17.0%	

Notes: A recession is defined as a period with an output decline compared to the period before. A severe recession is a recession with an output decline of -10% or more.

Debt accumulation and growth. The equilibrium is characterized by a modest average debt-to-GDP ratio (Panel A). This reflects the fact that the model captures only external borrowing, which represents just a share of total private sector debt, as domestic lending is excluded. Even so, the resulting average debt-to-GDP ratio is comparable to standard models of Sudden Stops.

Turning to asset values, the presence of the bubble raises modestly the average value of collateral as seen in Panel A by comparing the value of capital alone and the total collateral value (in percent of GDP). Importantly, however, the bubble also increases significantly the volatility of collateral, which will be a source of amplification that we analyze below.

Recession frequencies. The model matches the post-war OECD share of years classified as recessionary, which is about 20 percent (Panel B) (Jordà, Schularick, and Taylor, 2017). In the model, a recession is defined as a decline in output relative to the previous period. Severe recessions (output falls by at least 10%) occur about 4.4% of the time, close to the incidence of Sudden Stop events as in Mendoza and Terrones (2012) and in Bianchi and Mendoza (2020), associated with a reversal in the current account and a severe drop in output. Our model generates these events without exogenous shocks in the collateral pledgeability m allowing us to study collapses in asset prices and severe recessions without a contemporaneous tightening in lending standards. Note, however, that our model can easily be extended to account for

[9] When simulating the model, we also verify that the maximum ratio of the bubble price to collateral price is 33%, which is close to the largest asset price deviations from their fundamentals observed in the past. See, for example, Figure 1.18 in the Federal Reserve Board's Financial Stability Report.

stochastic m in order to study the joint effect of bursting bubbles and tightening of lending standards on recessions and crises.

Bubble incidence. Panel C confirms that JST bubbles are infrequent: the economy is in a JST bubble episode 7.2% of the time. Recall that a bubble is always present in the model (a new one appears when the old one bursts), but the JST criterion flags only the large, empirically salient episodes that are the focus of the analysis below.

3.3 Policy functions

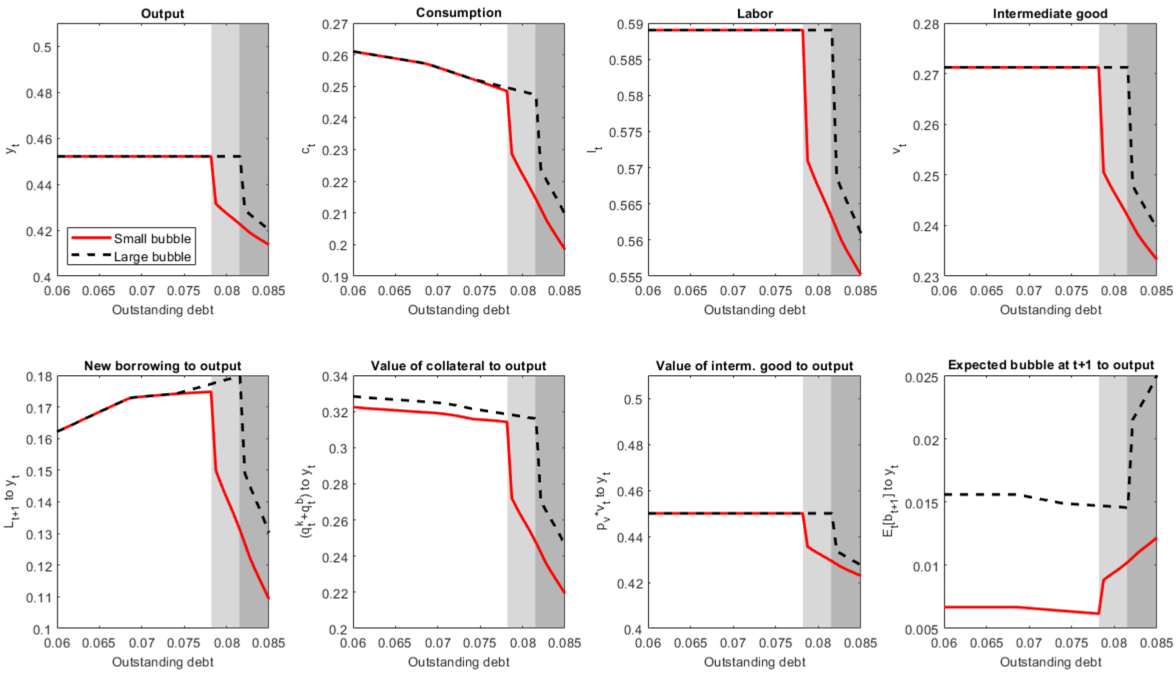
Figure 2 shows policy functions for selected macro variables for different levels of outstanding debt, L_t, and two bubble states: a "large" bubble (black dashed lines), and a "small" bubble of price u (red solid lines). Productivity is fixed at its lowest level, $z_t = z_l$. The light shaded region marks outstanding debt values where the borrowing constraint binds with a new, small bubble. This area is always larger than when the bubble is large (dark shaded region). A larger bubble supports higher debt without triggering a binding borrowing constraint.

Next, consider the region where the borrowing constraint is not binding. The policy functions coincide across bubble states because both the intra- and inter-temporal margins are undistorted (Section 2.1). The only two differences are the collateral value and the expected bubble price in the next period conditional on not bursting. The collateral value is equal to the sum of the value of capital and the bubble ($q_t^k + q_t^b$), which is always higher with a larger bubble for a given L_t. The expected bubble price is always higher for a higher price of today's bubble, given the bubble growth process (16). Factors of production and output are pinned down by the technology parameters when $\mu_t = 0$ (solve (8) and (9) with $k_t = 1$). Consumption, however, declines with L_t due to higher rollover costs and consumption smoothing.

Finally, when the borrowing constraint binds, distortions appear in both the intra- and intertemporal margins. New borrowing falls and consumption declines more steeply with L_t when the bubble is small. Lower consumption depresses q_t^k and the value of collateral, further tightening borrowing capacity. Turning to the factors of production, the constraint reduces intermediate input demand v_t because its purchase partly requires working capital financing. Output falls accordingly. GHH preferences—suppressing wealth effects—imply labor supply also contracts, reinforcing the decline in y_t. In sum, when the constraint binds, v_t, l_t, and y_t all drop. Additionally, the expected bubble price in the next period conditional on its survival increases nonlinearly once the borrowing constraint binds today. This is due to the rise in the marginal utility of consumption $U_{c,t}$ in equation (16), driven by the decline in current consumption.

Finally, Figure 3 plots the new debt, L_{t+1} for selected values of both the outstanding debt L_t and the price of the bubble q_t^b, when the productivity shock is set to low, $z_t = z_l$. The Figure clearly shows that the new debt increases in the value of outstanding debt (yellow area) until certain point, beyond which the new debt collapses (green-blue area). This threshold value of outstanding debt is the value above which the borrowing constraint becomes binding. As Figure 3 shows, it is increasing in the price of the bubble.

Figure (2) Policy functions for selected macro variables



Notes: The figure shows output y_t, consumption c_t, labor l_t, intermediate good demand v_t, the new debt to output ratio L_{t+1}/y_t, the ratio of the collateral value to output $(q_t^k + q_t^b)/y_t$, the ratio of the intermediate good expenditure to output $p^v v_t/y_t$, and the ratio of the expected future bubble price (conditional on surviving) to output $\tilde{E}_t(b_{t+1})/y_t$, as functions of outstanding debt L_t, in the competitive equilibrium. The two lines correspond to policy functions for a large bubble with the price b_t (black dashed line) and for a small bubble, with the price $u < b_t$ (red solid lines). Shaded areas correspond to values of outstanding debt for which the borrowing constraint binds when the bubble is small (light gray areas) and when the bubble is large (dark gray areas). Productivity shock is set to low, $z_t = z_l$.

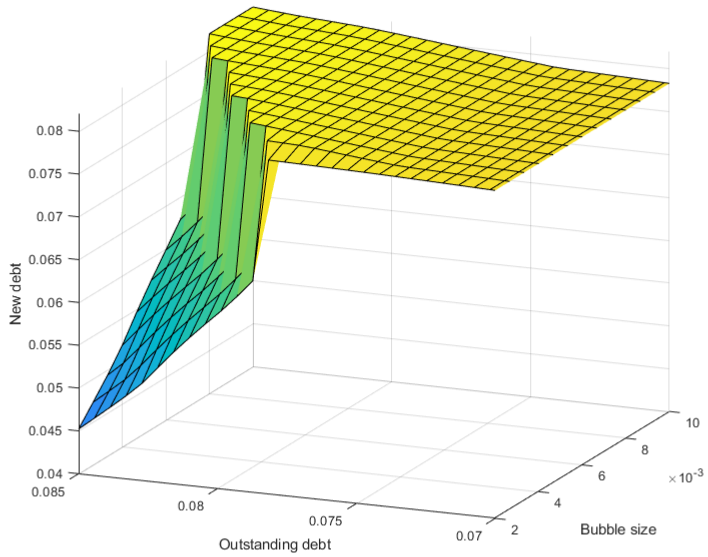
3.4 Impulse response functions

We study impulse response functions (IRFs) to a bursting-bubble shock under different outstanding debt levels. After the burst of the bubble, the economy is assumed to return to its initial state and remain there. This means that the price of the new bubble that emerges continues to grow. The purpose of the exercise is to isolate the impact of a bursting bubble in a controlled manner, and complement the event analysis in the next subsection, that averages over the full ergodic distribution of the simulated economy.

We consider a bubble burst that, absent other shocks, generates a recession. In all periods, productivity is fixed at a low level $z = z_l$ throughout. The bubble at $T = 0$, which is the period before bursting, equals a median JST bubble in the simulated economy; at $T = 1$ it bursts. We compare two initial outstanding debt levels at $T = 1$; 80th and 20th percentiles of the ergodic distribution. We assess whether the effects of a bursting bubble are amplified by high outstanding debt levels. Figure 4 shows the IRFs.

As Figure 4 shows, with $z_t = z_l$, a bubble burst leads to a larger contraction in output and consumption when the outstanding debt level is high (solid red) than when debt is low (dashed black). This occurs because a bursting of the bubble tightens the borrowing constraint and forces stronger deleveraging when outstanding debt is high. The need to reduce debt amplifies the downturn through its effect on the collateral value. While Figure 4 focuses on the case where $z_t = z_l$ over the entire horizon, the qualitative results hold across the other two productivity

Figure (3) New debt for selected values of outstanding debt and the bubble price



Notes: The figure shows new debt as a function of outstanding debt L_t and the bubble price q_t^b. Productivity shock is set to low, $z_t = z_l$.

states. One notable difference is that when the bubble bursts when productivity is fixed at a high level, the decline in output is only marginal when debt level is low as the borrowing constraint does not bind.

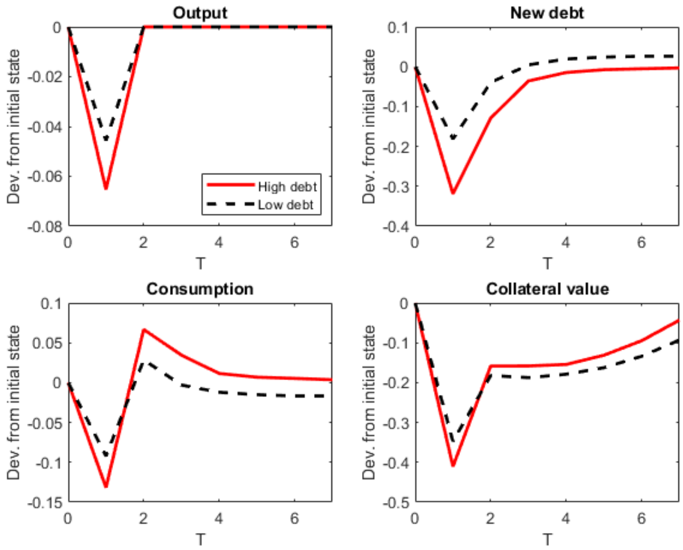
3.5 Event analysis

We perform an event analysis to examine recessions in the simulated baseline economy. As before, a recession is a period when output declines relative to the previous period, $y_t < y_{t-1}$. We focus on recessions that coincide with a bursting JST bubble, and compare outcomes across different credit conditions, measured by the outstanding debt, L_t. Our purpose is to characterize the effect from the interaction of asset valuations and indebtedness on recessions, as documented in the empirical literature. In particular, we relate our findings to Jordà et al. (2015) who show that recessions preceded by leveraged bubbles are deeper and longer than those preceded by bubbles without strong debt accumulation. We verify whether our model is able to match this empirical fact.

Recessions that follow leveraged bubble episodes. Figure 5 shows average dynamics of output, consumption, borrowing, and the collateral value during recessions ($T = 0$) that coincide with a bursting of a JST bubble. The underlying cause of the recession could be either a bursting of an old bubble, a decline in productivity, or both.[10] Figure 5 differentiates between recessions preceded by high credit growth, as captured by high outstanding debt level in the model (L_0 above the median across JST-bubble recessions; solid red lines) and recessions

[10]Our event analysis is motivated by the empirical properties of bubbly episodes documented in Jordà et al. (2015). We want to remain as close as possible to their approach of identifying bubble episodes, so implicitly we assume that the true bubble process is not observable to the econometrician.

Figure (4) IRFs to a bursting-bubble shock for low z



Notes: The figure shows output y_t, new borrowing L_{t+1}, consumption c_t and the collateral value $q_t^k + q_t^b$ following a bursting of a bubble in period $T = 1$. The price of the bubble at $T = 0$ is set to the median value during JST-bubble episodes in the simulated economy. The level of outstanding debt in period $T = 1$ is set to the 20th (dashed black line) and 80th percentiles of its ergodic distribution (red solid line). Throughout the periods $T = 0, 1, 2..7$ productivity shock is set to the low value, $z_t = z_l$.

preceded by low credit growth (L_0 below the median across JST-bubble recessions; black dashed lines).[11]

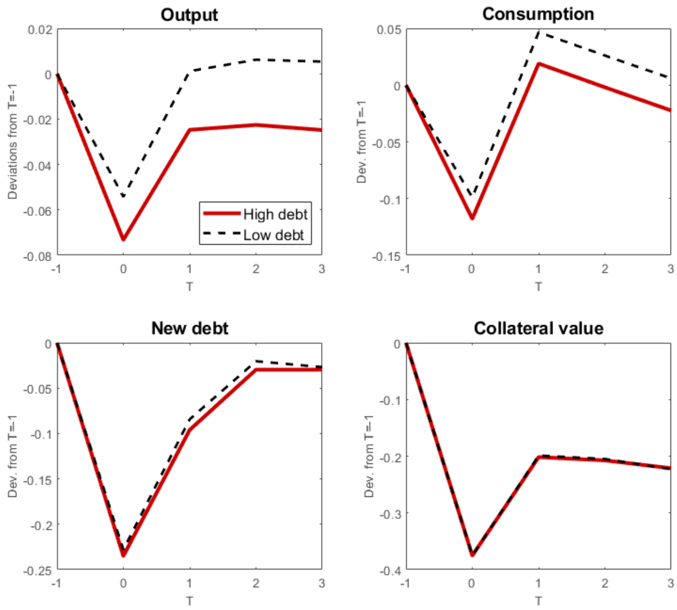
Leveraged JST-bubble recessions feature output declines roughly 1.5 times larger than in a low-debt scenario as well as slower recoveries. This is in line with the main finding in Jordà et al. (2015), i.e. that episodes of leveraged bubbles amplify the macroeconomic costs of recessions, both in terms of depth and duration. Consumption and borrowing show a similar pattern, albeit less pronounced. Interestingly, as shown in the top panel of Figure 6, average drop in productivity is higher for leveraged bubbles. This is consistent with the evidence of leveraged bubbles forming during "good times", i.e. periods of continued positive fundamental shocks to the economy—Kindleberger and Aliber (2005),—which in turn enable more pronounced debt accumulation (Figure 6, bottom panel). On the contrary, unleveraged bubbles can form during periods of low productivity growth.

Appendix Figure A.2 shows dynamics of output and other variables before, during, and after recessions, this time relative to their ergodic means. As expected, pre-recession output and consumption are higher when accompanied by leveraged JST bubbles and fall more during recessions. Both output and consumption recover subsequently, although to marginally higher levels when the recession was preceded by a leveraged bubble.

Types of recessions. Figure 7 shows the dynamics of output in recessions conditional on the underlying cause. We report the means of output, consumption, borrowing, and value of collateral in the simulated economy for three types of recessions: first, recessions caused by both the bursting of a JST bubble and productivity shocks (red solid lines); second, recessions

[11]As we are interested in matching the Jordà et al. (2015) evidence, in Figure 5 we present the dynamics during recessions relative to the pre-recession levels.

Figure (5) JST bubbles, outstanding debt and recessions



Notes: The figure shows average dynamics for output, consumption, borrowing, and collateral value during JST-bubble recession events. A recession ($T = 0$) is a period with an output decline compared to the period before, $T = -1$. High (low) credit is defined as L_0 above (below) the 50th percentile of borrowing (L_t) across all JST-defined bubbly recessions.

whereby a JST bubble bursts but productivity is fixed at the medium level in all periods (dashed lines); and, third, recessions caused by an adverse productivity shock whereby the bubble price is always fixed at its lowest value, u (dotted lines).

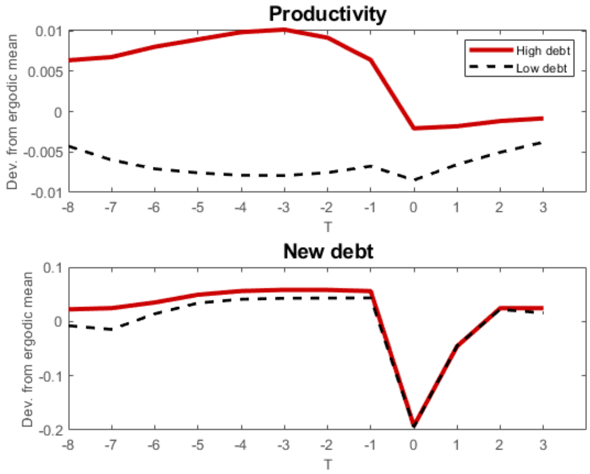
Clearly, recessions are most severe and persistent when a large bubble burst coincides with a negative productivity shock: the two forces (fundamental and non-fundamental) interact through the occasionally binding borrowing constraint, producing losses *larger than the sum of their separate effects*. Peak output declines are about 4% for recessions induced solely by a bursting of a bubble, 6.1% for recessions induced by negative productivity shocks, and 11.7% when the two shocks hit jointly. Consumption and borrowing exhibit similar super-additive drops when the recession is induced by both shocks.

Recessions induced solely by a bursting of a bubble are milder and shorter—in terms of output and consumption drops—than productivity-driven ones, as fundamentals remain intact and output rebounds once financial conditions stabilize. During these episodes, collateral values fall more than under recessions caused by adverse productivity shocks because the bubble component collapses to the level of a new bubble, u.

4 Time-consistent Optimal Policy

When making consumption and saving decisions, private agents do not internalize their impact on the capital and bubble valuations and, hence, on the borrowing constraint, thus generating an externality. In this section we identify the externalities associated with the price of capital and the bubble growth, discuss how they differ, and examine policies that can be employed to address them. To do so, we examine the optimal choices of a social planner

Figure (6) Average productivity during a bubble build-up



Notes: The figure shows average productivity and new debt patterns, as deviations from the ergodic mean, before and after JST-bubble recession events. A recession ($T = 0$) is a period with an output decline compared to the period before, $T = -1$. High (low) credit is defined as L_0 above (below) the 50th percentile of borrowing (L_t) across all JST-bubble recessions.

who is constrained by the same frictions as the competitive economy, namely the borrowing constraint, but who internalizes how consumption, borrowing, and other choices affect the value of collateral. Section 4.1 presents the planner's problem and characterizes the externalities in the competitive economy. Section 4.2 shows how the planner's solution can be implemented with a Pigouvian tax on borrowing. Section 4.3 presents the quantitative solution to the planner's problem and compares it to the competitive equilibrium.

4.1 Planner's problem

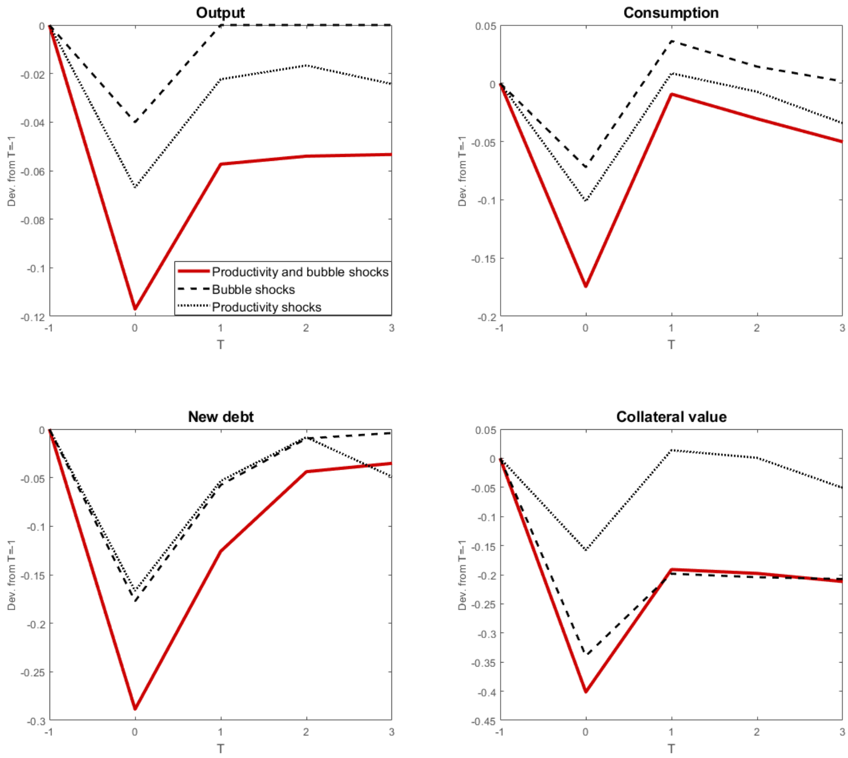
We formulate the social planner's problem similarly to Bianchi and Mendoza (2018). The planner chooses allocations to maximize the welfare of the representative agent subject to budget, borrowing, and implementability constraints as well as the process (3) for q_t^b.

Some remarks are in order. First, the price of capital and the growth rate of the bubble are market-determined. Hence, the Euler condition with respect to capital, (7), and the bubble growth rate, (16), enter the social planner's problem as implementability constraints. Second, the planner internalizes market clearing and does not optimize over capital (k_{t+1}) or bubble holdings (x_{t+1}) as they are held by the representative agent and are in fixed supply. Third, agents continue to choose their labor supply optimally and face the working capital constraint when choosing the intermediate good. Hence, (8) and (9) are constraints for the planner, but never bind and, thus, can be omitted (see Appendix A.5 for the proof).[12] Fourth, the planner chooses L_{t+1} on behalf of agents, and the Euler equation with respect to borrowing, (6), does not enter the planner's problem as an additional constraint.[13] Lastly, the planner cannot commit to future policies. We focus on Markov-stationary (time-consistent) policy rules, as in Klein, Krusell, and Ríos-Rull (2008) and Bianchi and Mendoza (2018), that are expressed as functions

[12](8) will be used to infer the Lagrange multiplier μ in the competitive economy evaluated at the planner's allocation, which is different than the Lagrange multiplier that the planner assigns to the borrowing constraint.

[13]See Section 4.2 for a description of a policy tool to implement the planner's choice of L_{t+1} in the competitive equilibrium.

Figure (7) JST bubbles, productivity shocks and recessions



Notes: The figure shows average dynamics for output, consumption, new debt, and collateral value during recession events along three simulated paths of the baseline economy. A recession ($T = 0$) is defined as a period with an output decline compared to the period before, $T = -1$. Red solid line shows the average path of the respective variable relative to the $T = -1$ period for recessions which were caused by a decline in the productivity shock *and* a bursting of a JST bubble between $T = -1$ and $T = 0$ in our baseline economy where both productivity and bubble shocks can materialize. Dashed line depicts dynamics during recessions caused solely by a JST bubble bursting at $T = 0$ in a simulated economy where productivity is always kept fixed at the normal level (i.e. no productivity shocks happen). Dotted line corresponds to recessions in a simulated economy with productivity shocks only, in which the bubble size is fixed at the minimum size, u.

of the payoff-relevant state variables (Ξ, L, b). The planner chooses policy rules at any period taking as given the policy rules of future planners.

Denote by c, L', v, l, q^k the choices of the current planner and by $\mathbb{C}(\Xi, L, b)$, $\mathbb{L}(\Xi, L, b)$, $\mathbb{V}(\Xi, L, b)$, $\mathbb{Q}^k(\Xi, L, b)$, $\mathbb{M}(\Xi, L, b)$ the policy rules of future planners for consumption, labor, intermediate good demand, capital price, and borrowing constraint multiplier, which the current planner takes as given.

The planner's recursive optimization problem can be written as

$$V(\Xi, L, b) = max_{c,L',v,l,q^k,b'} U(c - G(l)) + \beta E_{\Xi'|\Xi} V(\Xi', L', b') \text{ subject to} \tag{19}$$

$$c \leq y - p^v v + L'/R - L \quad (\lambda^P) \tag{20}$$

$$L'/R + \theta p^v v \leq m \left(q^k + \mathcal{I} \cdot b + (1 - \mathcal{I}) u \right) \quad (\mu^P) \tag{21}$$

$$q^k U_c(c - G(l)) = \beta E_{\Xi'|\Xi} \mathcal{H}^k(\Xi', L', b') \quad (\xi) \tag{22}$$

$$\text{with} \quad \mathcal{H}^k(\Xi', L', b') = U_c\left(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b'))\right)\Big(F_k\left(z', 1, \mathbb{L}(\Xi', L', b'), \mathbb{V}(\Xi', L', b')\right) + \mathbb{Q}^k(\Xi', L', b')(1 + m\mathbb{M}(\Xi', L', b'))\Big)$$

$$(\mathcal{I} \cdot b + (1-\mathcal{I})u)U_c(c - G(\ell)) = \beta\pi\tilde{E}_{\Xi'|\Xi}\mathcal{H}^b(\Xi', L', b') \quad (\phi) \tag{23}$$

$$\text{with} \quad \mathcal{H}^b(\Xi', L', b') = b'U_c\left(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b'))\right)(1 + m\mathbb{M}(\Xi', L', b'))$$

The Lagrange multipliers are in parentheses. We have substituted $q^b(\Xi, L, b) = \mathcal{I}\cdot b + (1-\mathcal{I})u$, using (3), in the borrowing constraint and the bubble growth implementability constraint, to facilitate the derivations of the first-order conditions.

Constraints (22) and (23) are the recursive representations of the implementability constraints that govern the price of capital and the growth of the bubble. The two take similar form but also differ in important ways. First, while both have the current price of capital, q^k, and the bubble, q^b, on the left-hand side, the planner optimizes over the former in (22), while taking as given the latter in (23), since it is a state variable. Second, the expected payoff from holding capital on the right-hand side of (22) is the sum of the marginal utility of dividends and the future price of capital, while the expected payoff in (23) from holding the bubble, conditional on its survival, just accrues from the marginal utility of the future bubble. Third, the planner takes as given the future price of capital, $\mathbb{Q}^k$, as it is chosen by future planners but decides how the bubble grows by choosing b'.

Finally, even though the planner takes as given the decision of future planners, the planner internalizes how current decisions affect the actions of future planners via the endogenous states L' and b' appearing in the right-hand side of both implementability constraints, reflecting the "time-consistent" policy rules.

Definition 2. *The recursive equilibrium of the planner's optimization problem is defined by the policy function for borrowing $L'(\Xi, L, b)$ with associated decision rules $c(\Xi, L, b)$, $v(\Xi, L, b)$, $l(\Xi, L, b)$, $\mu(\Xi, L, b)$, asset pricing function $q^k(\Xi, L, b)$, bubble growth function $b'(\Xi, L, b)$, value function $V(\Xi, L, b)$, the conjectured function characterizing the borrowing decision rule of future planners $\mathcal{L}(\Xi, L, b)$ and the associated decision rules $\mathbb{C}(\Xi, L, b)$, $\mathbb{V}(\Xi, L, b)$, $\mathbb{L}(\Xi, L, b)$, $\mathbb{M}(\Xi, L, b)$, asset prices $\mathbb{Q}^k(\Xi, L, b)$, and bubble growth $\mathbb{B}(\Xi, L, b)$, such that these conditions hold:*

1. *$V(\Xi, L, b)$ and functions $\{c(\Xi, L, b), L'(\Xi, L, b), v(\Xi, L, b), l(\Xi, L, b), q^k(\Xi, L, b), b'(\Xi, L, b)\}$ solve the Bellman equation in (19) subject to (20)-(23), given $\mathcal{L}(\Xi, L, b)$,$\mathbb{C}(\Xi, L, b)$, $\mathbb{V}(\Xi, L, b)$, $\mathbb{L}(\Xi, L, b)$, $\mathbb{M}(\Xi, L, b)$, $\mathbb{Q}^k(\Xi, L, b)$, $\mathbb{B}(\Xi, L, b)$, and $\mu(\Xi, b, l)$ satisfies (8).*

2. *Policy rules are time-consistent, that is the conjectured functions policy rules that characterize the optimal choices of future planners match the recursive functions that characterize the optimal choices of the current planner: $L'(\Xi, L, b) = \mathcal{L}(\Xi, L, b)$, $c(\Xi, L, b) = \mathbb{C}(\Xi, L, b)$, $v(\Xi, L, b) = \mathbb{V}(\Xi, L, b)$, $l(\Xi, L, b) = \mathbb{L}(\Xi, L, b)$, $\mu(\Xi, L, b) = \mathbb{M}(\Xi, L, b)$, $q^k(\Xi, L, b) = \mathbb{Q}^k(\Xi, L, b)$, $b'(\Xi, L, b) = \mathbb{B}(\Xi, L, b)$*

We next take first-order conditions. As it is standard we express them in a *sequential form*, i.e., with a time-subscript. After taking the first-order conditions we substitute back the definition of q_t^b for brevity. The first-order condition with respect to consumption is

$$\lambda_t^P = U_{c,t} - \xi_t q_t^k U_{cc,t} - \phi_t q_t^b U_{cc,t}. \tag{24}$$

Relative to the competitive equilibrium, the social planner's marginal value of resources, λ_t^p, incorporates not only the marginal utility of consumption, $U_{c,t}$ but also two extra terms reflecting the effects of consumption on the economy's constraints. The first extra term, $-\xi_t q_t^k U_{cc,t}$, captures how an increase in c_t affects the price of capital, q_t^k, and in turn the planner's borrowing capacity. As shown below in the first-order condition for q_t^k, (27), when the borrowing constraint binds, a marginal increase in c_t can raise q_t^k, relaxing the constraint. This indirect effect makes an additional unit of consumption more socially valuable. This mechanism parallels that in Bianchi and Mendoza (2018).

The second extra term, $-\phi_t q_t^b U_{cc,t}$, captures how an increase in c_t affects the bubble. Recall that q_t^b is a state that the planner takes as given, hence the effect of consumption operates through the future growth of the bubble embedded in ϕ_t as we show below in the first-order condition for b_{t+1}, (28).

The first-order condition with respect to borrowing L_{t+1} is:

$$\lambda_t^p/R - \mu_t^P/R - \beta E_t \lambda_{t+1}^P + \xi_t \beta E_t \mathcal{H}_{L,t+1}^k + \phi_t \beta \pi \tilde{E}_t \mathcal{H}_{L,t+1}^b = 0. \tag{25}$$

To better understand this optimality condition, it is useful to substitute in the optimal consumption condition (24), yielding:

$$\begin{aligned} U_{ct} - \xi_t q_t^k U_{cc,t} - \phi_t q_t^b U_{cc,t} = \beta R E_t \left(U_{c,t+1} - \xi_{t+1} q_{t+1}^k U_{cc,t+1} - \phi_{t+1} q_{t+1}^b U_{cc,t+1} \right) + \mu_t^p \\ - \xi_t \beta R E_t \mathcal{H}_{L,t+1}^k - \phi_t \pi \beta R \tilde{E}_t \mathcal{H}_{L,t+1}^b. \end{aligned} \tag{26}$$

Unlike the baseline economy, where agents consider only the current and future marginal utility of consumption and the tightness of the borrowing constraint, the planner also internalizes how borrowing affects the current and future *price* of capital (via ξ_t and ξ_{t+1}) and the current and future *growth* of the bubble (via ϕ_t and ϕ_{t+1}). Higher borrowing L_{t+1} increases current consumption and the current price of capital, q_t^k. A higher q_t^k, in turn, relaxes a binding borrowing constraint at t, enhancing the ability of the planner to borrow. At the same time, higher L_{t+1} tightens future borrowing capacity as agents need to rollover a higher debt, which decreases future consumption and the future price of capital, q_{t+1}^k. A lower q_{t+1}^k further tightens borrowing constraints at $t+1$, curtailing borrowing ability. The planner balances these two effects when choosing L_{t+1}, which are captured by the terms $\xi_t q_t^k U_{cc,t}$ and $\xi_{t+1} q_{t+1}^k U_{cc,t+1}$ and correspond to the pecuniary externality in Bianchi and Mendoza (2018). We show that these terms are positive insofar as the concurrent borrowing constraint is binding. Hence, if the constraint at t is not binding, the planner only cares about the effect of L_{t+1} on the future price of capital when the constraint may be binding.

With respect to the bubble, the planner internalizes how L_{t+1} affects the bubble growth in the current and next period. This, in turn, matters for the future tightness of the borrowing constraint, distinguishing the bubble externalities from the pecuniary externalities operating via the price of capital. This point can be made clearer when we solve of the Lagrange multiplier ϕ_t below. Intuitively, a higher L_{t+1} increases consumption today, which decreases the current growth of the bubble (see discussion on (16) in section 2). A lower bubble value tomorrow tightens future borrowing constraint. At the same time, higher L_{t+1} decreases future consumption, which boosts the bubble growth from $t+1$ to $t+2$, helping relax borrowing constraints at $t+2$. The planner weighs these two effect when choosing L_{t+1}, captured by the terms $\phi_t q_t^b U_{cc,t}$ and $\phi_{t+1} q_{t+1}^b U_{cc,t+1}$. As we establish below, ϕ_t (and ϕ_{t+1}) depends on how binding all future constraints are; not just the one of next period. Hence, the planner will consider the effect on all future bubble growth paths until the bubble bursts when choosing L_{t+1}.

Finally, the terms $\xi_t \beta R E_t \mathcal{H}_{L,t+1}^k$ and $\phi_t \pi \beta R \tilde{E}_t \mathcal{H}_{L,t+1}^b$ capture the time-consistency of the planner's policy.[14] The current planner internalizes that L_{t+1} is a state variable for future planner's and how it will affect their choices, taking their policy functions as given.

The first-order condition with respect to the capital price q_t^k is:

$$-\xi_t U_{c,t} + m\mu_t^P = 0, \tag{27}$$

This condition shows that higher q_t^k tightens the implementability constraint (first term) and relaxes the borrowing constraint (second term). The planner weighs these opposing forces using the shadow values, ξ_t and μ_t^p. If the borrowing constraint is not binding in the current period, then, $\xi_t = 0$.

The first-order condition with respect to the bubble component b_{t+1} is:

$$\beta\pi m \tilde{E}_t \mu_{t+1}^P + \xi_t \beta\pi \tilde{E}_t \mathcal{H}_{b,t+1}^k + \phi_t \pi\beta \tilde{E}_t \mathcal{H}_{b,t+1}^b - \beta\pi \tilde{E}_t \phi_{t+1} U_{c,t+1} = 0 \tag{28}$$

The economic interpretation of this condition is less direct. The first term reflects how higher bubble growth from t to $t+1$ relaxes future borrowing constraints. The second term captures the effect of the bubble growth on the current capital price (through expectations in the current implementability constraint (22)). This arises because b_{t+1} is a state variable for the future planners, and the current planner understands that choices of b_{t+1} shape future decisions. The third term captures the impact of the bubble growth from t to $t+1$ on the current implementability constraint (23) (weighted by ϕ_t), which governs the bubble growth rate. This reflects that the planner's choice of the bubble growth must remain consistent with the agents' optimal behavior in the competitive equilibrium.[15] Finally, the fourth term captures how the bubble growth from t to $t+1$ affects the future implementability constraint (23) (weighted by ϕ_{t+1}), since b_{t+1} influences the bubble growth from $t+1$ to $t+2$.

In sum, the planner's choice of the current price of capital (27) relative to the choice of the bubble growth (28) differs in three important ways: (i) the price of capital matters for the current borrowing constraint at t, while the bubble growth matters for the future borrowing constraints at $t+1$, (ii) the bubble growth governs the future bubble state at $t+1$ and, thus, matters for the choices of future planners that the current planner takes into account when

[14] $\mathcal{H}_{L,t+1}^k$ and $\mathcal{H}_{L,t+1}^b$ collect all partial derivatives with respect to L_{t+1} on the right-hand side of (22) and (23). See Appendix A.6 for a recursive representation.

[15] $\mathcal{H}_{b,t+1}^k$ and $\mathcal{H}_{b,t+1}^b$ collect all partial derivatives with respect to b_{t+1} on the right-hand side of (22) and (23). See Appendix A.6 for a recursive representation.

choosing time-consistent policies, and (iii) the current bubble growth from t to $t+1$ matters for the future growth of the bubble from $t+1$ to $t+2$ since it determines the $t+1$ value of the bubble, which the future planners cannot manipulate. The latter effect implies that the choice of the current bubble growth from t to $t+1$ will matter for all future borrowing constraints at $t+1$, $t+2$, and so on until the bubble bursts, which is in contrast to the effect of the price of capital that only matters for the current borrowing constraint at t.

To illustrate this latter effect, observe that equation (28) is a forward-looking difference equation with expectations. Using standard rational expectations methods, we get that

$$\phi_t = \tilde{E}_t \sum_{s=1}^{\infty} \left(\prod_{j=1}^{s-1} \frac{U_{c,t+j}}{\tilde{E}_{t+j-1} H^b_{b,t+j}} \right) \left(-\frac{m\,\mu^P_{t+s}}{\tilde{E}_{t+s-1} H^b_{b,t+s}} - \frac{m\,\mu^P_{t+s-1}}{U_{c,t+s-1}} \frac{H^k_{b,t+s}}{\tilde{E}_{t+s-1} H^b_{b,t+s}} \right), \tag{29}$$

where we have substituted (27) to eliminate ξ_t. This expression shows that ϕ_t depends on the anticipated path of borrowing constraints' tightness.

The first-order condition with respect to intermediate good demand v_t is:

$$-\lambda^p_t p^v - \mu^p_t \theta p^v + \lambda^p_t F_{v,t} = 0. \tag{30}$$

This condition, in its functional form, coincides with the one in the competitive equilibrium.

The first-order condition with respect to labor l_t is:

$$-U_{c,t} G_{l,t} + \lambda^p_t F_{l,t} + \xi_t q^k_t U_{cc,t} G_{l,t} + \phi_t q^b_t U_{cc,t} G_{l,t} = 0. \tag{31}$$

Substituting in equation (24) yields $F_{l,t} = G_{l,t}$, which is the same optimality condition for labor as in the competitive economy.

4.2 Implementation via a Pigouvian tax on borrowing

To implement the planner's solution as a competitive equilibrium, we introduce a state contingent tax on debt, τ_t, which is applied when debt is repaid at $t+1$, i.e., the agent need to repay $(1+\tau_t)L_{t+1}$ in total at $t+1$. The budget constraint (2), then, becomes

$$c_t \leq y_t - p^v v_t + L_{t+1}/R - (1+\tau_{t-1})L_t - q^k_t (k_{t+1} - k_t) - q^b_t (x_{t+1} - 1 - (x_t - 1)\mathcal{I}_t) + T_t, \tag{32}$$

where T_t is the lump-sum transfer rebating the tax revenue equal to $\tau_{t-1} L_t$. The agents' Euler equation for bonds (6) becomes:

$$U_{c,t}(1-\mu_t) = \beta R E_t U_{c,t+1}(1+\tau_t) \tag{33}$$

Combining equation (33) with (24) and (25) the tax of borrowing is

$$\tau_t = \frac{\mu_t^p - U_{c,t}\mu_t + \xi_t q_t^k U_{cc,t} + \phi_t q_t^b U_{cc,t}}{\beta R E_t U_{c,t+1}} - \frac{E_t\left(\xi_{t+1} q_{t+1}^k U_{cc,t+1} + \phi_{t+1} q_{t+1}^b U_{cc,t+1}\right)}{E_t U_{c,t+1}}$$
$$- \frac{\xi_t E_t \mathcal{H}_{L,t+1}^k + \phi_t \pi \tilde{E}_t \mathcal{H}_{L,t+1}^b}{E_t U_{c,t+1}}.$$

The optimal tax rate on borrowing implements the planner solution as a competitive equilibrium by accounting for all the aforementioned effects in section 4.1 that operate through the price of capital and the growth of the bubble, which the planner internalizes.

If the borrowing constraint does not bind at time t, i.e., $\mu_t = \mu_t^p = \xi_t = 0$, the borrowing tax takes on a macroprudential interpretation: it is levied during good times—when the borrowing constraint does not bind—to prevent excessive borrowing and mitigate the consequences of high indebtedness during bad times—when the borrowing constraint binds in the future. In other words, the tax tames credit growth in good times to reduce the risk of a future credit crunch resulting in output losses. Moreover, by doing so the tax curtails current consumption, which increases the bubble growth through (16) and, hence, further relaxes future borrowing constraints (see discussion on (16) in section 2); a mechanism absent in Bianchi and Mendoza (2018). After re-arranging terms, the macroprudential debt tax takes the following form:

$$\tau_t^{MP} = \overbrace{\frac{\phi_t}{\beta R E_t U_{c,t+1}}\left(q_t^b U_{cc,t} - \pi\beta R \tilde{E}_t \mathcal{H}_{L,t+1}^b\right)}^{\substack{\text{Bubble growth externality} \\ \text{from } t \text{ to } t+1}} \overbrace{- \frac{E_t\left(\phi_{t+1} q_{t+1}^b U_{cc,t+1}\right)}{E_t U_{c,t+1}}}^{\substack{\text{Bubble growth externality} \\ \text{from } t+1 \text{ to } t+2}} \overbrace{- \frac{E_t\left(\xi_{t+1} q_{t+1}^k U_{cc,t+1}\right)}{E_t U_{c,t+1}}}^{\substack{\text{Fundamental price} \\ \text{externality at } t}}. \tag{34}$$

The optimal macroprudential tax τ_t^{MP} addresses the typical pecuniary externality from the fundamental price of capital described in Bianchi and Mendoza (2018) (last term in (34)), which calls for a positive tax. But the tax also incorporates two additional components associated with the bubble growth. The first component in (34) captures how higher borrowing at t affects the growth of the bubble from t to $t+1$, which matters for the tightness of all future borrowing constraints from $t+1$ onward (through ϕ_t, as defined in equation (29)). The second component in (34) relates to how higher borrowing at t affects the bubble growth from $t+1$ to $t+2$, which may mitigate the effect from the bubble growth from t to $t+1$. As mentioned, less borrowing at t allows the current bubble to grow faster, relaxing future borrowing constraint and increasing future consumption. All else equal, a higher consumption tomorrow puts downwards pressure on the bubble growth from $t+1$ to $t+2$, leaning against the positive effect of the current bubble growth on all future borrowing constraints. Taken together, the optimal macroprudential tax internalizes not only the standard pecuniary externality that operates through the price of capital, but also the dynamic effects—reflecting an inter-temporal trade-off—of borrowing on the bubble growth.

4.3 Quantitative analysis of planner's equilibrium

We solve the planner's problem by applying a value function iteration algorithm. To obtain the ergodic distribution, we simulate the planner's economy for 500,000 periods and for the same path of exogenous shocks as in the simulated baseline economy in Section 3. We compare the economy under optimal policy with the baseline economy along several dimensions.

Summary Statistics. Compared to the baseline economy, the planner reduces the volatility of output and consumption by curtailing the average level of debt to GDP and by reducing their volatility (Panel A in Table 3). Because the average debt level is lower, the average value of collateral declines too, mainly driven by a decrease in the value of capital as we elaborate below. All else equal, lower collateral values should result in a more binding borrowing constraint and in more frequent and deeper recessions. However, this is not necessarily the case because by curtailing credit the planner also moves the economy away from the region where the borrowing constraint binds.

The incidence and severity of recessions relative to the baseline economy are shown in Panel B. Mild recessions are more frequent under optimal policy because, to maintain borrowing capacity in case bigger shocks materialize, the planner restricts the ability to borrow when small shocks hit the economy. At the same time, mild recessions are considerably less severe for the planner given that the average debt level is lower. Moreover, the planner eliminates the more severe recessions almost entirely. As a result, the planner can achieve non-trivial welfare gains of about 0.3 percent in terms of consumption compensating variation (Panel D). The tax on borrowing that implements the planner's solution has an average value of 2.2 percent and a standard deviation of 1.3 percent.

Restricting borrowing has a direct effect on the build-up of JST bubbles: As reported in Panel C, the incidence of JST bubbles falls dramatically. In sum, by levying borrowing taxes, the planner does not allow collateral valuations to grow up to a size that can have detrimental consequences for output and consumption when the underlying bubble burst. Indeed, the average collateral value relative to GDP during JST bubbles is lower under optimal policy (33.1%) compared to the baseline model (38.9%). However, the contribution of the bubble in the collateral value is higher in the model with optimal policy. To understand why that is the case, it is useful to consider the policy functions for the social planner.

Policy functions. Figure 8 plots key policy functions for the social planner for different L_t and for a fixed price of a bubble b_t. The productivity level is again set to low, $z_t = z_l$.

There are notable differences between the social planner's policy functions and those in the baseline economy. The planner internalizes the impact of her consumption and borrowing decisions on the borrowing constraint, through their effect on capital and bubble values, and chooses new debt L_{t+1} that is always lower than in the baseline economy for any L_t. Note that the borrowing constraint becomes binding at lower levels of L_t in the planner's equilibrium (light gray areas) than in the baseline (dark gray areas). This is because lower new debt under optimal policy leads to lower consumption, which depresses the price of capital and, thus, the total value of collateral, tightening the borrowing constraint. As long as the borrowing

Table (3) Model simulations: summary statistics

PANEL A	Baseline economy mean	Baseline economy st. dev	Optimal policy mean	Optimal policy st. dev
Output (GDP)		2.5%		2.3%
Consumption		1.7%		1.3%
Debt to GDP	16.3 %	1.5%	9.5%	0.5%
Collateral asset value to GDP	30.9%	3.3%	22.1%	3.0%
PANEL B				
Incidence of a recession	20.3%		30.2%	
Output decline in a recession	-6.5%		-3.3%	
Incidence of a severe recession	4.4%		0.03%	
Output decline in a severe recession	-11.3%		-13.2%	
PANEL C				
Incidence of JST bubbles	7.2%		0.6%	
Collateral value to GDP in JST bubbles	38.9%		33.1%	
Bubble value to the collateral value in JST bubbles	24.1%		39.9%	
PANEL D				
Tax on borrowing			2.2%	1.3%
Welfare gains			0.3%	

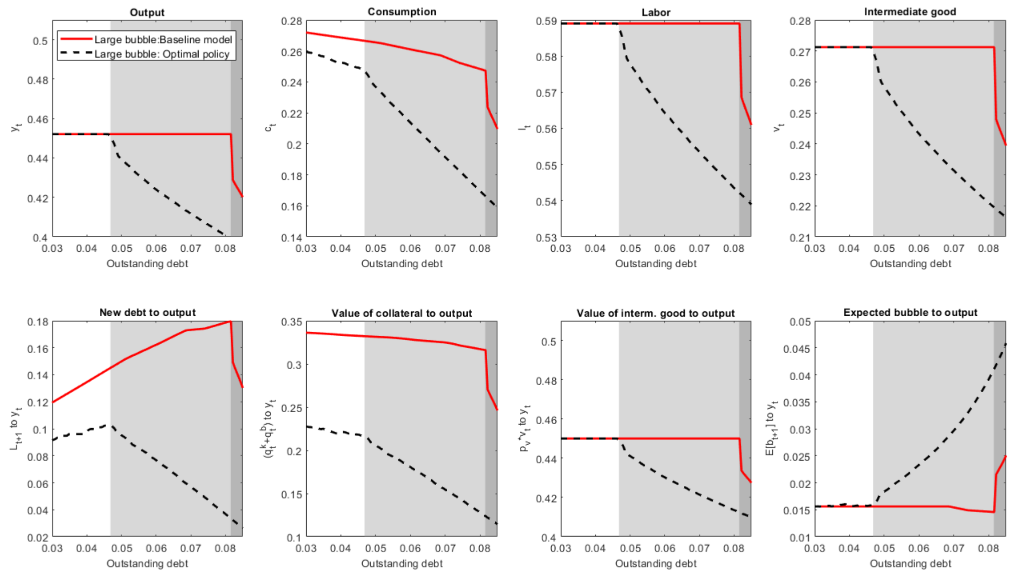
Notes: A recession is defined as a period with an output decline compared to the period before. A severe recession is a recession with an output decline of -10% or more.

constraint does not bind, output, labor supply, and intermediate good demand are the same for the planner and the baseline economy. Nevertheless, consumption is lower for the planner. Once the borrowing constraint binds, then output also declines. This may seem counterintuitive given the planner's higher welfare outcome. The key difference lies in the distribution of states between the planner's and the baseline equilibrium. Although for the same levels of state variables the planner implements lower consumption than in the baseline economy, the planner's equilibrium is more frequently in the states in which the borrowing constraint does not bind. Hence, the fluctuations of consumption are less pronounced for the planner, resulting in higher welfare.

Interestingly, for a given outstanding debt L_t and current bubble value b_t, the bubble value in the next period, or equivalently the bubble growth, can be higher in the planner's solution compared to the baseline economy. The reason is that the bubble growth is tied to the inverse of the expected stochastic discount factor from (16). When the borrowing constraint binds in the current period, consumption falls and the stochastic discount factor becomes smaller, leading to faster bubble growth. This result is intuitive: agents require higher expected return from buying a bubble of a given value when their consumption is lower.[16] Since we have restricted the planner to respect the pricing of capital and the bubble of the baseline economy, the planner can directly affect agents' borrowing choice only. The optimal policy is to borrow less, but leaning against credit growth cannot at the same time facilitate leaning against the bubble growth. As a result, in the absence of other tools that can directly impact bubble growth, the bubble grows faster under the optimal policy (for a given outstanding debt and current bubble value)

[16]Agents would similarly require a higher return on holding capital, which is achieved by a downward adjustment in its fundamental price in the current period. Such adjustment is not feasible for the bubble, which needs to grow faster to incentivize agents to hold it.

Figure (8) Policy functions under optimal policy



Notes: The figure shows output y_t, consumption c_t, labor l_t, intermediate good demand v_t, the ratio of new debt to output L_{t+1}/y_t, the ratio of the collateral value to output $(q_t^k + q_t^b)/y_t$, the ratio of intermediate good expenditure to output $p^v v_t/y_t$, and the ratio of expected future bubble (conditional on surviving) to output $E_t(b_{t+1})/y_t$, as functions of outstanding debt L_t, in the baseline model (red solid lines) and in the model with optimal policy (black dashed lines), for the same size of a bubble. Shaded areas correspond to values of L_t for which the borrowing constraint binds in the baseline model (dark gray areas) or under optimal policy (light gray areas). Productivity shock is set to low, $z_t = z_l$.

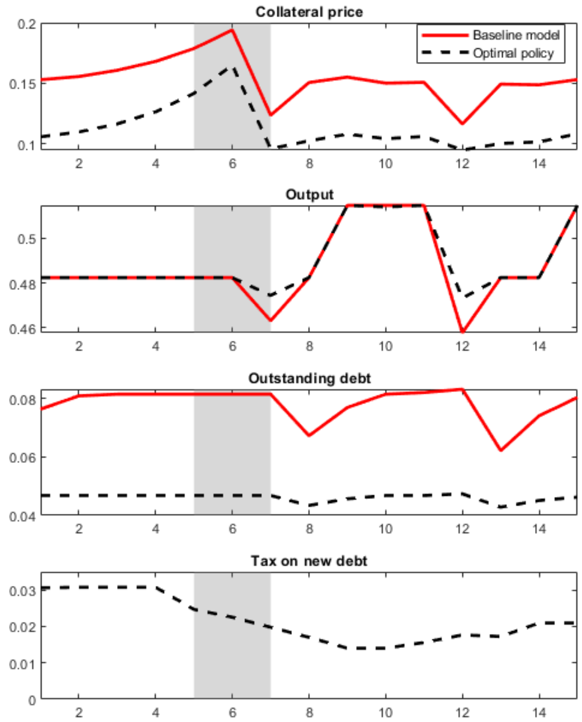
once the borrowing constraint binds.[17] This is the reason why in the simulated economy under optimal policy the contribution of the bubble component to the collateral value during JST bubbles episodes is higher than in the baseline model (Panel C of Table 3). Finally, the reason why the planner reduces the incidence of JST bubbles, i.e. episodes of strong bubble growth *and* excessive debt accumulation, is because the distribution of debt shifts to the left in the planner's solution compared to the baseline economy.

Simulated paths with bubble bursting. To illustrate the amplification when a bubble bursts, Figure 9 plots 15-period simulated paths of key variables in the baseline and planner's equilibria. The realizations of exogenous processes—productivity z_t and the bubble bursting process $\mathcal{I}_t$—are identical for both equilibria. Over the horizon shown in Figure 9, a JST bubble builds up and subsequently burst in the baseline economy between $t = 5$ to $t = 7$ (shaded area). In both equilibria, the collateral value initially increases due to a sequence of positive productivity shocks and collapses when the bubble bursts at $t = 7$.

Prior to the burst, outstanding borrowing remains elevated in the baseline equilibrium, as the presence of the bubble allows debt to be continuously rolled over. Output, however, does not increase during the bubble build-up, since it is driven solely by productivity and is independent

[17]This result relates to the one in Galí (2014) that the bubble grows faster when the policymaker raises the interest rate. In our case the interest rate is constant but policy affects the bubble part by altering the stochastic discount factor through changes in the path of consumption, which matters in the global, non-linearized, dynamics we examine. Allen et al. (2025) show that setting a sufficiently high interest rate can restrict the set of bubbles values that can be supported in equilibrium. Unlike these papers, our analysis does not attempt to answer whether policymakers can dampen bubbles by setting a higher interest rate but rather to derive the optimal policy to maximize welfare.

Figure (9) Simulations of the economy with and without policy.



Notes: The figure shows simulated 15-period paths of the collateral value q_t, output y_t, outstanding debt L_t for the baseline economy (red solid lines) and the economy with optimal policy (dashed black lines). The bottom panel shows the simulated path for and the optimal tax on new debt.

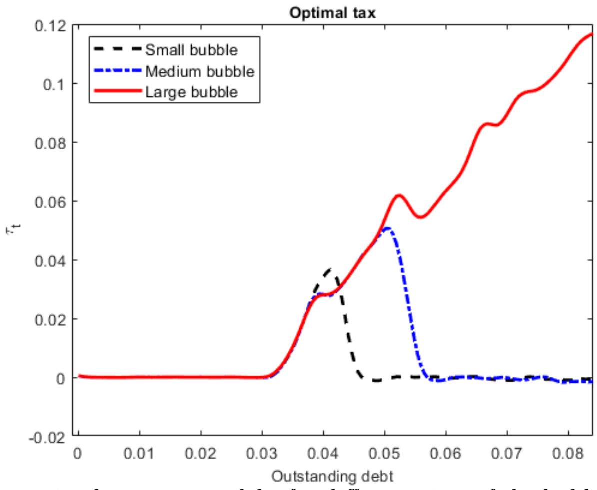
of borrowing as long as the borrowing constraint does not bind. When the bubble bursts, output falls sharply in the baseline equilibrium, reflecting the large and abrupt deleveraging that follows the collapse in collateral values. By contrast, the equilibrium with optimal policy is characterized by a lower outstanding debt in the run-up to the bubble burst. As a result, the decline in collateral value leads to a more moderate tightening of the borrowing constraint, dampening the contraction in output and consumption.[18]

Optimal tax on borrowing. Figure 10 plots the optimal tax on borrowing in(33) as a function of outstanding debt (x-axis) for three bubble sizes: a small bubble equal to the lowest grid value, u (dashed black line), an average bubble in the simulated competitive economy (dash-dotted blue line) and a median JST bubble in the competitive economy (solid red line). A few observations are worth noting. First, for very low levels of debt the tax on borrowing is close to zero, independently of the bubble size. The reason is that for very low levels of outstanding debt, the borrowing constraint does not bind in the current period and the probability of it binding sometime in the future is very low. Thus, the optimal tax oscillates around zero.[19] Second, for intermediate levels of debt, the probability that the borrowing constraint starts to

[18]The decline in output does not coincide with the decline in borrowing because the figure plots outstanding debt rather than new borrowing. While new borrowing contracts at the time of the bubble burst, outstanding debt adjusts in the following period.

[19]The optimal tax can have negative values for low levels of debt, if the probability of the borrowing constraint binding in the future periods is sufficiently high

Figure (10) Total tax on new debt for selected values of state variables



Notes: The figure plots the optimal tax on new debt for different sizes of the bubble in the simulated competitive economy: the lowest value, u (dashed black line), the average bubble size (dash-dotted blue line), and a large bubble (solid red line). Productivity level is set to low, $z_t = z_l$.

bind increases and the optimal tax turns positive to mitigate the fundamental-price and bubble-growth externalities. Finally, for sufficiently high levels of outstanding debt, the optimal tax starts to decline until it reaches zero again. For such high levels of L_t, the borrowing constraint becomes binding in the current period, and the optimal tax is lower, weighing the benefit of relaxing the constraint today against the cost of making it more binding in the future. Note that, for the JST bubble in Figure 10, the borrowing constraint in the current period remains slack even for very high levels of outstanding debt. Hence, the planner continues to levy a positive tax, which is higher the higher the level of outstanding debt is.

Simple rules. The optimal policy is implemented through a state-contingent tax, which may raise concerns about how feasible its implementation is in practice. We examine whether policies based on simple rules can generate welfare gains too. Simple rules could be beneficial if they share some characteristics of the optimal policy, such as incorporating credit imbalances and asset overvaluations. We consider a simple rule that take the form $\tau_t = (1+\bar{\tau})(\bar{L}/L_t)^{\eta_1}(q_t/\bar{q})^{\eta_2} - 1$; $\bar{\tau}$ is average optimal tax in the simulated economy under optimal policy, $\bar{L}$ and $\bar{q}$ are the ergodic means of debt and the value of collateral, and $\eta_1, \eta_2 \geq 0$ are parameters. A time-invariant tax corresponds to $\eta_1 = \eta_2 = 0$, while varying η_1 and η_2 we can obtain time-varying taxes that depend on the deviations of debt and collateral values from their ergodic means. Note that this simple tax specification results in a higher tax when current debt level are below and collateral valuation above their respective ergodic means and, thus, is leaning against credit growth at times that the borrowing constraint is less likely to bind.

Table 4 reports the welfare gains under these three simple rules for combinations of $\eta_1 \in \{0, 1, 2\}$ and $\eta_2 \in \{0, 1, 2\}$. There are four key takeaways. First, a small invariant tax, corresponding to $\eta_1 = \eta_2 = 0$, does not increase welfare in our simulated economy.[20] Second, incorporating credit imbalances can improve upon the invariant tax for high enough η_1. Third, incorporating account for valuation imbalances on top of credit imbalances increases the welfare

[20]This is not a general result and may depend on the specifics of our simulation.

gains. Fourth, the gains from the simple rules under consideration are lower than the gain under the optimal policy, about 0.3%.

Table (4) Simple rules' welfare gains.

		η_2		
		0	*1*	*2*
	0	-0.0014%	-0.0396%	0.0297%
η_1	*1*	-0.0772%	0.0945%	0.0892%
	2	0.0860%	0.0876%	0.0884%

Note: The table shows welfare gains in terms of compensating consumption variations.

5 Conclusion

This paper developed a dynamic general equilibrium model with an occasionally binding borrowing constraint and an endogenously growing asset price bubble that can account for the main regularities of leveraged bubble cycles. When asset valuations rise, collateral values increase and borrowing constraints relax, amplifying credit and output. When the bubble bursts, collateral values fall and the constraint tightens, leading to deleveraging and a sharper downturn. The model reproduces the empirical finding that recessions following leveraged asset price booms are deeper and more persistent than those following pure asset price corrections. Amplification arises not from exogenous changes in financial tightness, but from movements in asset valuations that affect borrowing capacity.

The policy analysis shows that these dynamics call for a preventive approach to macroprudential policy. Because agents do not internalize how their borrowing decisions affect future collateral values and systemic risk, credit expands excessively in good times. The optimal time-consistent policy takes the form of a countercyclical borrowing tax that restrains leverage during booms and mitigates downturns when bubbles burst. Implementing such a policy substantially reduces the frequency of severe recessions and raises welfare, implying that addressing asset-price-driven amplification in the design of macroprudential frameworks is an important consideration.

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A Appendix

A.1 Limited Commitment Problem and Borrowing Constraint

The borrowing constraint can be derived from a debt renegotiation problem between borrowers and lenders as in Bianchi and Mendoza (2018) and Jermann and Quadrini (2012). At the beginning of period t, after previous period borrowing, L_t, has been repaid, the total liabilities of the borrower are $L_{t+1}/R + \theta p^v v_t$, which comprise of the inter-period debt plus the intra-period debt to fund working capital. Within period, the borrower can decide to divert the borrowed funds and avoid any costs from defaulting next period when debt becomes due. At the end of the period, there are no more opportunities to divert revenues and repayment of previous bonds is enforced.

Using the threat to divert, the borrower can try to renegotiate the debt. If the lender does not agree to renegotiate the debt, it can decide to monitor the borrower, and seize and sell the borrower's collateral at market prices. The market value of the collateral consists of the value of the capital holdings, $q_t^k k_t$, and of the bubble, $q_t^b(1+(x_t-1)\mathcal{I}_t)$. Lenders can only successfully seize the collateral with probability m, which can be interpreted as the probability that the lenders can indeed monitor and enforce the relevant covenants. Given that lenders are risk-neutral, the expected value from monitoring and seizing the collateral is $m\left(q_t^k k_t + q_t^b(1+(x_t-1)\mathcal{I}_t)\right)$, which is equal to their outside option.

The borrower has full negotiation power. Hence, the net benefit from diversion is $\tilde{d} = L_{t+1}/R + \theta p^v v_t - m\left(q_t^k k_t + q_t^b(1+(x_t-1)\mathcal{I}_t)\right)$, i.e., equal to the borrowed funds that can be diverted minus the outside option of lenders, since borrowers would need to compensate lenders for their outside option to avoid liquidation.

Given that debt contracts are not exclusive and there is no other penalty for default, the value from renegotiating, $\overline{\mathbb{V}}^R$, and the value from not renegotiating, , $\overline{\mathbb{V}}^{NR}$ are given by

$$\overline{\mathbb{V}}^R = \max U(c+\tilde{d}) + \beta E\overline{\mathbb{V}}', \tag{A.1}$$

$$\overline{\mathbb{V}}^{NR} = \max U(c) + \beta E\overline{\mathbb{V}}', \tag{A.2}$$

subject to the same constraints, where $\overline{\mathbb{V}}'$ is the continuation value for both cases.

The incentive compatibility constraint requires that $\overline{\mathbb{V}}^R \leq \overline{\mathbb{V}}^{NR}$, which can be simplified to $\tilde{d} \leq 0$, giving rise to the borrowing constraint $L_{t+1}/R + \theta p^v v_t \leq m\left(q_t^k k_t + q_t^b(1+(x_t-1)\mathcal{I}_t)\right)$.

A.2 Proof of Proposition 1

We outline the steps for bubble existence, which rely on showing that the borrowing constraint binds infinitely often. In other words, we need to show that there are no paths that the constraint stops binding and thus the bubble cannot be supported along those paths.

We check whether a positive bubble can be supported in equilibrium, i.e., that it satisfies (10), without violating the transversality condition (13). Iterate (10) forward to get

$$q_t^b = \lim_{T\to\infty} E_t \beta^T \Pi_{i=1}^T (1+m\mu_{t+i}) \frac{U_{c,t+T}}{U_{c,t}} b_{t+T}. \tag{A.3}$$

If $\mu_{t'} = 0$ for all $t' > t$, then the right-hand side in (A.3) is zero from the transversality condition (13) and a zero bubble value is the only consistent with equilibrium. Similarly, assume that there is a $T' > t$ such that $\mu_t' = 0$ for $t' \geq T'$, but may be positive or zero for $t' \in (t, T')$. Then, (A.3) is again zero—$E_t \beta^{T'} \Pi_{i=1}^{T'} (1+m\mu_{t+i}) \frac{U_{c,t+T'}}{U_{c,t}} b_{t+T'} = 0$—because from (13) applied to time $t+T'$ when $\mathcal{I}_{t+T'} = 1$, we have that $b_{t+T'} = \lim_{T\to\infty} E_{t+T'} \left(\beta^T \frac{U_{c,t+T'+T}}{U_{c,t+T'}} b_{t+T'+T}\right) = 0$. The first case reflects the situation where the borrowing constraint never binds after time t, while the second case reflects the situation where it occasionally binds for a finite period of time. In both cases, $lim_{n\to\infty} \sup S_n = 0$, where $S_n = \{\mu_n, \mu_{n+1}, \mu_{n+2}, ...\} = 0$. If, instead, the borrowing constraint binds infinitely often, or $lim_{n\to\infty} \sup S_n > 0$ for all t', then (13) no longer excludes rational bubbles in equilibrium. Next, we show that this is the case.

Observe that $M_t \equiv (\beta R)^t U_{c,t}$ is a super-martingale, because $U_{c,t}(1-\mu_t) = \beta R E_t U_{c,t+1}$, and, thus it converges to a nonnegative random variable. Since $\beta R < 1$, $U_{c,t}$ does not need to converge asymptotically but can remain finite and continue to vary randomly (see Ljungqvist and Sargent, 2004, p.574). So, the Euler equations together with the transversality conditions (7), (10),(12), and (13) rule out explosive price paths; with bounded equilibrium allocations, this implies q_t^k and q_t^b are finite, and thus the pledgeable collateral value in (4) is finite as well.

Finally, we derive conditions under which the borrowing constraint binds infinitely often for any path that the bubble survives. Suppose the borrowing constraint is slack, i.e., $\mu_{t+i} = 0$ for all i in (A.3). The Euler equation for borrowing implies $U_{c,t+i} = \beta R E_{t+i} U_{c,t+i+1}$. Since $\beta R < 1$, agents are impatient and optimally tilt consumption toward the present by borrowing whenever feasible. Consequently, during periods in which the bubble survives, productivity is high, and collateral values are elevated, equilibrium debt increases. Consider now an adverse productivity realization (or a series of adverse productivity realizations), which suppresses collateral values through its direct effect on the price of capital. Because the economy enters the period with debt accumulated from the previous period, the newly lower collateral value may constrain borrowing. Note that for $m \to 0$, the pledgeable value of collateral goes to zero, while agents still want to borrow, and the borrowing constraint must bind. Hence, by continuity, we can always find a $\bar{m} > 0$ such that for all $m < \bar{m}$ the constraint binds following the bad productivity shock. Given the Markov structure of the productivity shocks and their independence from the bubble survival process, bad productivity realizations occur infinitely often along paths that the bubble survives. Hence, such events that the collateral value drops sharply and the borrowing constraint binds under sufficiently low m occur infinitely often.

A.3 Generalization of Lemma 1

The realized bubble price b_t for $\mathcal{I}_t = 1$ can be disentangled into a pre-determined at time $t-1$ part (measurable with respect to $t-1$ information), denoted by $\bar{b}_t$, and an innovation ι_t (measurable with respect to time t information). Hence, $b_t = \bar{b}_t + \iota_t$. Substituting this relation in (14) we get

$$
\begin{aligned}
q_{t-1}^b U_{c,t-1} &= \beta\pi \tilde{E}_{t-1}\left((\bar{b}_t + \iota_t) U_{c,t}(1+m\mu_t)\right) \Rightarrow \\
q_{t-1}^b U_{c,t-1} &= \beta\pi \tilde{E}_{t-1}\left(\bar{b}_t U_{c,t}(1+m\mu_t)\right) + \beta\pi \tilde{E}_{t-1}\left(\iota_t U_{c,t}(1+m\mu_t)\right) \Rightarrow \\
q_{t-1}^b U_{c,t-1} &= \beta\pi \bar{b}_t \tilde{E}_{t-1}\left(U_{c,t}(1+m\mu_t)\right) \\
&\quad + \beta\pi \tilde{E}_{t-1}\left(\iota_t\right) \tilde{E}_{t-1}\left(U_{c,t}(1+m\mu_t)\right) + \beta\pi \widetilde{Cov}_{t-1}\left(\iota_t, U_{c,t}(1+m\mu_t)\right), \qquad \text{(A.4)}
\end{aligned}
$$

where we moved $\bar{b}_t$ outside the expectation operator since it is measurable with $t-1$ information. We have used the notation $\widetilde{Cov}_{t-1}(\cdot,\cdot) \equiv Cov_{t-1}(\cdot,\cdot|\mathcal{I}_t = 1)$.

We make the following two assumptions about the innovations:

1. The innovations are zero in expectation, i.e., $\tilde{E}_{t-1}(\iota_t) = 0$
2. The innovations are only measurable/depend on the realization of $\mathcal{I}_t = 1$ and are independent of fundamentals, i.e., $\widetilde{Cov}_{t-1}\left(\iota_t, U_{c,t}(1+m\mu_t)\right) = 0$

Under these assumptions we can use (A.4) to determine the value of $\bar{b}_t$ using $t-1$ information:

$$
\bar{b}_t = \frac{q_{t-1}^b U_{c,t-1}}{\beta\pi \tilde{E}_{t-1}\left(U_{c,t}(1+m\mu_t)\right)} \qquad \text{(A.5)}
$$

Finally, there is still some indeterminacy with respect to the realization of the innovation ι_t. To resolve this indeterminacy we can specify an exogenous distribution F_ι that ι_t follows satisfying the aforementioned assumptions. We opt for the limiting case that $\iota_t \to 0$. Under this additional assumption, $b_t \to \bar{b}_t$—or $b_t = \tilde{E}_{t-1}(b_t)$—and hence b_t is given by (15).

A.4 Proof of Proposition 2

Given current states (Ξ, L, b), $\Gamma_b(\Xi, L, b)$ is a fixed point of $\Psi(\Xi, L, b, b')$. Figure A.1 plot (a hypothetical) function Ψ for different values of b'.

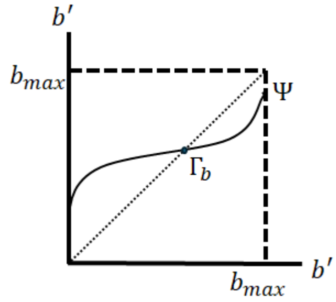


Figure (A.1) Function $\Psi(\Xi, L, b, b')$

We first establish the existence of a fixed point. Consider a range $[0, b_{max}]$ of possible values for b'; b_{max} is the maximum value of the bubble, which is finite in equilibrium, as established earlier. Ψ is continuous and is larger than zero as $b' \to 0$, because consumption is finite (i.e. $U_{c,t}$ is not zero) and bounded away from zero (i.e. $U_{c,t+1}$ is not infinite). Moreover, Ψ is smaller than b' as $b' \to b_{max}$, because it is bounded by b_{max}. So a solution $\Gamma_b \in (0, b_{max}]$ exists.

To establish uniqueness we first compute the first derivative of Ψ, which is given by (note the simplified notation for brevity)

$$\frac{d\Psi}{db'} = -\Psi \cdot \frac{1}{E_{\Xi'|\Xi} U_c} \cdot E_{\Xi'|\Xi} \left(U_{cc} \left(\frac{d\hat{c}}{db'} - G_l \frac{d\hat{l}}{db'} \right) (1 + m\hat{\mu}) + U_c \left(\hat{c} - G(\hat{l}) \right) m \frac{d\hat{\mu}}{db'} \right). \tag{A.6}$$

If the bracketed term in (A.6) including the derivatives of $\hat{c}$, $\hat{l}$, and $\hat{\mu}$ is negative, then $d\Psi/db' < 0$ and the fixed point is unique. The reason is that Ψ is larger than zero (and lower than b_{max}) for $b' \to 0$ and, thus, it would have unambiguously crossed the 45-degree line from above at exactly one point. However, the case that the bracketed term is positive, requires deeper examination because Ψ could be S-shaped and cross the 45-degree line in multiple points from above and below. Usually one would compute the second derivative of Ψ to characterize its curvature but this derivative cannot be unambiguously signed. Instead, we follow a different approach to prove uniqueness when $d\Psi/db' > 0$, which does not rely on the curvature of Ψ and only relies on the fact that Ψ is larger than zero for $b' \to 0$. But, as it will be clear, this approach also requires to consider a m less than a certain threshold.

The intuition is easy to follow from Figure A.1: Ψ will necessarily cross the 45-degree line at least once from above because a fixed point exists and Ψ is strictly positive for $b' \to 0$.

Given that Ψ is continuous, a second fixed point would require that it crosses (or touches) the 45-degree line from below, which is turn requires that $d\Psi/db'$ evaluated at that fixed point is higher than or equal to one. Hence, to ensure fixed-point uniqueness, it suffices that $d\Psi/db'$ evaluated at any candidate fixed point is strictly smaller than one. Given concave utility, U_{cc}/U_c is negative and bounded. Moreover, $\hat{c}$ and $\hat{l}$ depend on the bubble insofar $m > 0$. Take the limit $m \to 0$. Then, the bracketed term in (A.6), and hence $d\Psi/db'$, converge to zero. By continuity, there exists a threshold $\hat{m}$ such that $d\Psi/db' < 1$ at any fixed point even when the bracketed term is positive. Note that this threshold may be larger than one and, thus, may not impose any restrictions.

A.5 Complete Planner's Problem

We derive the complete recursive problem of the time-consistent planner and shows that it is equivalent to the "relaxed" planner's problem derived in Section 4.1 in the paper. Recall that for the latter we omitted some optimality conditions from the competitive equilibrium: (a) the first-order condition with respect to the intermediate good, the first-order condition with respect to labor, and the complementarity slackness constraints, because as we will show below they are not binding; and (b) the first-order condition with respect to borrowing because the planner controls directly this decision and, as we show below, can implement it with a state-contingent tax on borrowing, τ. Denote by $T(\Xi, L, b)$ the future tax chosen by future planners (regulators).

The complete recursive problem of the time-consistent planner (regulator), who is subject to all the omitted constraints and can also choose taxes, is

$$V(\Xi, L, b) = \max_{c,l,v,L',q^k,b',\mu,\tau} \quad U(c - G(l)) + \beta \mathbb{E}_{\Xi'|\Xi} V(\Xi', L', b') \quad \text{s.t.} \tag{A.7}$$

$$c \leq y - p^v v + \frac{L'}{R} - L, \tag{A.8}$$

$$\frac{L'}{R} + \theta p^v v \leq m\Big(q^k + \mathcal{I} \cdot b + (1 - \mathcal{I})u\Big), \tag{A.9}$$

$$q^k U_c(c - G(l)) = \beta E_{\Xi'|\Xi} \mathcal{H}^k(\Xi', L', b'), \tag{A.10}$$

$$\Big(\mathcal{I} \cdot b + (1 - \mathcal{I})u\Big) U_c(c - G(l)) = \beta\pi\, \tilde{E}_{\Xi'|\Xi} \mathcal{H}^b(\Xi', L', b'), \tag{A.11}$$

$$F_v = p^v(1 + \theta\mu), \tag{A.12}$$

$$\mu \left(m\big(q^k + \mathcal{I} \cdot b + (1 - \mathcal{I})u\big) - \big(\frac{L'}{R} + \theta p^v v\big) \right) = 0, \tag{A.13}$$

$$\mu \geq 0, \tag{A.14}$$

$$F_l = G_l. \tag{A.15}$$

$$U_c(c - G(l))(1 - \mu) = \beta R(1 + \tau)\, E_{\Xi'|\Xi} U_c\Big(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b'))\Big). \tag{A.16}$$

with (A.12)-(A.16) being the omitted constraints in the "relaxed" problem.

First, we examine whether constraints (A.12)-(A.14) are satisfied in the "relaxed" planner's solution. Denoting by λ^P and μ^P the Lagrange multipliers the planner assigns on (A.8) and (A.9) we have that $F_v = p_v \left(1 + \theta\mu^P/\lambda^P\right)$ from the optimality condition with respect to v and $\mu^P \left(L'/R + \theta p_v v - m\big(q^k + I \cdot b + (1-I)u\big)\right) = 0$ from the complementarity slackness condition in the planner's solution. Set $\mu = (F_v/p_v - 1)/\theta$ implying that $\mu = \mu^P/\lambda^P$. Along with $\mu^P/\lambda^P \geq 0$ and the slackness condition above, $\mu = \mu^P/\lambda^P$ implies that (A.13) and (A.14) are also satisfied. By definition, (A.12) is also satisfied. Hence, we do not include (A.12)-(A.14) as constraints because they do not bind, while the value of μ can be retrieved using the value of μ^P/λ^P from the "relaxed" planner's solution.

Second, we examine whether constraint (A.15) is satisfied in the "relaxed" planner's solution. The planner's optimality condition with respect to labor implies $U_c G_l - \xi q^k U_{cc} G_l - \phi\Big(\mathcal{I} \cdot b + (1-\mathcal{I})u\Big)U_{cc} G_l = \lambda^P F_l$. From the optimality condition with respect to consumption we get that $\lambda^P = U_c - \xi q^k U_{cc} - \phi\Big(\mathcal{I} \cdot b + (1-\mathcal{I})u\Big)U_{cc}$, where ξ and ϕ are the multipliers on (A.10) and (A.11), respectively Combining the two yields $F_l = G_l$, which corresponds exactly to (A.15). Hence, the labor optimality condition is also satisfied at the "relaxed" planner's allocation and does not bind as an additional constraint.

Third, we examine whether constraint (A.16) is binding in the "relaxed" planner's solution. Because τ is state-contingent and Pigouvian (that is, the tax proceeds are returned back to the same agent lump-sum in order to neutralize wealth effects), it is easy to see that it can be chosen by the current planner/regulator to satisfy (A.16) at the "relaxed" planner's allocations. Of course, the optimal tax choice should match the optimal tax choice of future planners/regulators, i.e., $\tau(\Xi, L, b) = T(\Xi, L, b)$, which is easily satisfied given the definition of the recursive equilibrium in the "relaxed" planner's problem.

A.6 Recursive representation of $\mathcal{H}^k_{L,t+1}$, $\mathcal{H}^b_{L,t+1}$, $\mathcal{H}^k_{b,t+1}$, and $\mathcal{H}^b_{b,t+1}$

In recursive form $\mathcal{H}^k_{L,t+1}$ and $\mathcal{H}^b_{L,t+1}$ are given by:

$$\begin{aligned}
\mathcal{H}'^k_L = {} & U_{cc}(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b'))) \\
& \times \left(F_k(z', 1, \mathbb{L}(\Xi', L', b'), \mathbb{V}(\Xi', L', b')) + \mathcal{Q}(\Xi', L', b')(1 + m\mathcal{M}(\Xi', L', b'))\right) \\
& \times \left(\mathbb{C}_L(\Xi', L', b') - G'(\mathbb{L}(\Xi', L', b'))\mathbb{L}_L(\Xi', L', b')\right) \\
& + U_c(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b'))) \\
& \times \Big(F_{kv}(z', 1, \mathcal{L}(\Xi', L', b'), \mathcal{V}(\Xi', L', b'))\mathcal{V}_L(\Xi', L', b') \\
& \quad + F_{kl}(z', 1, \mathcal{L}(\Xi', L', b'), \mathcal{V}(\Xi', L', b'))\mathcal{L}_L(\Xi', L', b') \\
& \quad + \mathcal{Q}^k_L(\Xi', L', b')(1 + m\mathcal{M}(\Xi', L', b')) \\
& \quad + \mathcal{Q}^k(\Xi', L', b')m\mathcal{M}_L(\Xi', L', b')\Big)
\end{aligned}$$

and

$$
\begin{aligned}
{\mathcal{H}'}^b_L &= U_{cc}(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b')))b'(1 + m\mathcal{M}(\Xi', L', b')) \\
&\quad \times \big(\mathbb{C}_L(\Xi', L', b') - G'(\mathbb{L}(\Xi', L', b'))\mathbb{L}_L(\Xi', L', b')\big) \\
&\quad + U_c(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b')))b' m\mathcal{M}_L(\Xi', L', b').
\end{aligned}
$$

In recursive form $\mathcal{H}^k_{b,t+1}$ and $\mathcal{H}^b_{b,t+1}$ are given by:

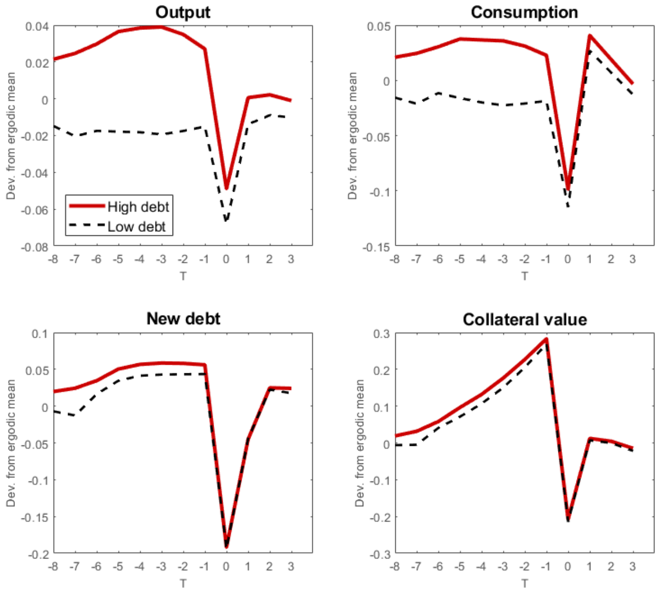
$$
\begin{aligned}
{\mathcal{H}'}^k_b &= U_{cc}(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b'))) \\
&\quad \times \big(F_k(z', 1, \mathbb{L}(\Xi', L', b'), \mathbb{V}(\Xi', L', b')) + \mathcal{Q}(\Xi', L', b')(1 + m\mathcal{M}(\Xi', L', b'))\big) \\
&\quad \times \big(\mathbb{C}_b(\Xi', L', b') - G'(\mathbb{L}(\Xi', L', b'))\mathbb{L}_b(\Xi', L', b')\big) \\
&\quad + U_c(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b'))) \\
&\quad \times \Big(F_{kv}(z', 1, \mathcal{L}(\Xi', L', b'), \mathcal{V}(\Xi', L', b'))\mathcal{V}_b(\Xi', L', b') \\
&\qquad + F_{kl}(z', 1, \mathcal{L}(\Xi', L', b'), \mathcal{V}(\Xi', L', b'))\mathcal{L}_b(\Xi', L', b') \\
&\qquad + \mathcal{Q}^k_b(\Xi', L', b')(1 + m\mathcal{M}(\Xi', L', b')) \\
&\qquad + \mathcal{Q}^k(\Xi', L', b') m\mathcal{M}_b(\Xi', L', b')\Big)
\end{aligned}
$$

and

$$
\begin{aligned}
{\mathcal{H}'}^b_L &= U_{cc}(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b')))b'(1 + m\mathcal{M}(\Xi', L', b')) \\
&\quad \times \big(\mathbb{C}_L(\Xi', L', b') - G'(\mathbb{L}(\Xi', L', b'))\mathbb{L}_L(\Xi', L', b')\big) \\
&\quad + U_c(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b')))b' m\mathcal{M}_b(\Xi', L', b') \\
&\quad + U_c(\mathbb{C}(\Xi', L', b') - G(\mathbb{L}(\Xi', L', b')))(1 + m\mathcal{M}(\Xi', L', b')).
\end{aligned}
$$

A.7 Figures

Figure (A.2) JST bubbles, outstanding debt and recessions: Dynamics relative to the ergodic means



Notes: The figure shows average dynamics for output, consumption, borrowing, and collateral values relative to their ergodic means in the simulated economy, during JST-bubble recessions. A recession ($T = 0$) is a period with an output decline compared to the period before, $T = -1$. High (low) credit is defined as L_1 above (below) the 50th percentile of borrowing (L_t) across all JST-bubble recessions.