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ABSTRACT

We show that major investor disagreement proxies have near zero correlation – making disagreement pricing inferences challenging. We develop a frictionless model that generates overpricing from latent disagreement, jointly rationalizes major proxies, and generates new testable predictions. Equilibrium interrelationships motivate a new disagreement measure, DIS, which is more predictive of returns cross-sectionally than major proxies and more consistently predictive across macroeconomic regimes. We provide evidence that the friction-based channel of Miller (1977) and a frictionless mechanism coexist and are empirically separable. DIS also provides a micro-founded, disagreement-based account of the IVOL puzzle, absorbing IVOL's predictability while revealing its regime and friction dependence.

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1. Introduction

Disagreement among investors is ubiquitous in financial markets but fundamentally unobservable. The literature has responded by proposing a wide array of proxies, each with theoretical motivation and empirical support in some settings. Yet, the proxies disagree substantially with each other. Average pairwise correlations are close to zero among the three major categories of firm-level disagreement measures: analyst forecast dispersion, idiosyncratic volatility (IVOL), and short interest volume.[1] This divergence means that empirical findings about the relationship between disagreement and asset returns depend heavily on which proxy is chosen, contradictory results across samples are a natural consequence of proxy selection, and no existing measure can claim to capture the latent disagreement variable rather than one of its observable projections. For example, volume-based measures could also proxy for liquidity (Benston and Hagerman, 1974; Petersen and Fialkowski, 1994) and volatility-based measures such as IVOL could also proxy for risk (Goyal and Santa-Clara, 2003) or firm transparency (Ferreira and Laux, 2007). We call this the *disagreement of disagreement* problem, and we offer a resolution.

Our resolution proceeds in three steps: (i) a theoretical framework that links all major proxies to a single latent disagreement variable, explains why existing disagreement proxies disagree with one another, and provides testable structural predictions; (ii) a composite measure guided by the framework's equilibrium interrelationships to produce an empirical vehicle to test those predictions; and (iii) an empirical implementation that is consistent with the framework's predictions. Our analyses provide new evidence on the nature of investor disagreement as a mispricing phenomenon with its model-implied structure as essential to its empirical performance, and shed new light on related unresolved questions including a potential micro-founded resolution of the IVOL puzzle.

[1] These three categories are among the most common firm-specific disagreement measures in the literature and are based on the least stale information for a broad cross-section of stocks (Huang, Li, and Wang, 2021). Boehme, Danielsen, and Sorescu (2006) and Berkman et al. (2009) build on prior theory relating belief heterogeneity to volatility to propose IVOL as a firm-level measure of disagreement. See Appendix C for a more detailed discussion of measures within these categories as well as other categories of disagreement proxies employed in the literature.

The theoretical framework. We analyze a simple two-period market for one risky asset in which the equilibrium price is determined endogenously by a standard market-clearing approach across a continuum of expected-utility-maximizing investors with differences of opinion.[2] We generalize this approach to a multi-asset framework under a standard factor structure in Appendix A, grounding our single-asset focus for cross-sectional predictions.

The framework imposes no short-sale frictions and no parametric assumptions on the asset's payoff distribution. Standard utility assumptions apply: investors prefer more to less, are risk averse, and have demand for precautionary savings when facing uncertainty or positive intensity of prudence (Kimball, 1990). The key mechanism is that when asset payoffs are positively skewed, as they are for the vast majority of stocks (Boyer, Mitton, and Vorkink, 2010), optimistic investors buy more aggressively than pessimists sell short. This asymmetry arises because a short position in a positively skewed asset is equivalent to a long position in a negatively skewed one, which under standard skewness preferences commands lower-magnitude demand. When investors also disagree, this demand asymmetry means that the market-clearing price is pushed above what fundamentals warrant, and more disagreement implies greater overpricing (mispricing) and lower subsequent expected returns. The model identifies the ratio of absolute prudence to absolute risk aversion as the structural coefficient governing the magnitude of the disagreement effect on expected returns, a distinction that collapses under the CARA utility standard but is operative under more general preferences including CRRA (power) utility, where it enters as a common scalar across assets.

The mechanism does not require short-sale frictions to operate, unlike the leading explanation in the literature following Miller (1977). We demonstrate that the combination of heterogeneous beliefs and positive asset skewness is sufficient to produce overpricing in a frictionless market, though frictions, where present, may provide additional amplification. This theoretical result motivates our empirical investigation of whether the disagreement–return relationship persists in the cross section among stocks where short-selling constraints are

[2] Our model adopts the differences-of-opinion paradigm in which price influences investor trades but not beliefs; see Varian (1985), Harris and Raviv (1993), Shalen (1993), Kandel and Pearson (1995), and Cao and Ou-Yang (2008). Differences of opinion are consistent with investors not conditioning their decisions on prices (Banerjee, 2011; Banerjee, Davis, and Gondhi, 2024). Alternatively, insensitivity of beliefs to price can be approximately true under slow information diffusion (Hong and Stein, 1999), or with the addition of noise traders (e.g., Grossman and Stiglitz, 1980; Diamond and Verrecchia, 1981).

least binding (supporting evidence in Section 5.5).

We model investor beliefs via two components: the relative distribution of beliefs across investors and the scale of those beliefs. This separation accommodates many possible shapes of heterogeneity, such as polarized, uniform, concentrated-with-tails, and asymmetric, while treating the overall level of disagreement as a separate, cleanly defined parameter. The model formally distinguishes analyst earnings forecast dispersion from investor disagreement. Forecast dispersion amplifies the scale of existing belief heterogeneity rather than creating it, a distinction reflected in our approach and overlooked by prior work that treats analyst dispersion as a direct disagreement substitute (Rees and Thomas, 2010; Dittmar and Thakor, 2007; Park, 2005; cf. Goulding, 2017, 2018). In our framework, dispersion and idiosyncratic disagreement are correlated but separate constructs, and the composite measure reflects their interaction rather than treating analyst dispersion as a standalone proxy.

The framework generates additional predictions that do not arise from prior models of disagreement and skewness: a correctly specified idiosyncratic disagreement measure should exhibit cross-sectional return predictability that is stable across macroeconomic regimes (from the multi-asset separation); and the specific nonlinear functional form of the composite measure discussed below is essential and not recoverable by linear alternatives. These predictions are tested in Section 5 and the evidence is consistent with our framework.

The composite measure. The model's equilibrium conditions relate the unobservable scale of disagreement to observable asset-specific variables: idiosyncratic volatility, short interest, analyst forecast dispersion, idiosyncratic skewness, and uncertainty about the mean payoff. By solving for disagreement as a function of these variables, we derive a composite disagreement measure, DIS, which is a nonlinear function of observable components, several of which correspond to common proxies of disagreement in the literature. This nonlinearity reflects the equilibrium interplay and implicit cross-validation among proxies.

Empirical findings. We employ DIS as the vehicle through which we empirically examine our theoretical framework and its structural predictions. Our sample spans January 1994 to December 2022, restricted to stocks followed by at least two analysts, a restriction that excludes the smallest, most illiquid, and most retail-dominated stocks where anomaly profits are typically concentrated. Our empirical analyses are organized into eight sets of interlocking tests of a single framework, providing mutually reinforcing cumulative evidence.

First, a value-weighted decile spread portfolio sorted by DIS produces 17–21% annualized alpha (dependent on the factor model), substantially exceeding the alphas produced by any individual disagreement proxy on the same sample. Alphas are driven primarily by the short leg, which the model predicts directly. Demand convexity from positive skewness amplifies disagreement's overpricing effects at the firm level, producing short-leg dominance independently of aggregate sentiment or probability weighting. The framework also provides a structural explanation for why these alphas should survive standard risk adjustments (Appendix A.3) and why they persist. The same skewness preferences that generate overpricing discourage the corrective arbitrage trades, providing a preference-based complement to friction-based limits to arbitrage (Shleifer and Vishny, 1997; Pontiff, 2006). Alphas are robust to further exclusion of the most illiquid and retail-owned stocks from our already relatively liquid sample (Section 5.1).

Second, in Fama and MacBeth (1973) regressions, DIS generates substantial return predictability in the cross section – almost four standard errors from zero. This result is robust to controls for other disagreement measures and standard cross-sectional predictors.

Third, DIS provides a potential disagreement-based solution to the idiosyncratic volatility puzzle. Ang et al. (2006) document that stocks with high idiosyncratic volatility earn anomalously low subsequent returns, a finding that Hou and Loh (2016) characterize, after an exhaustive survey, as unresolved. IVOL is, in our model, a closely linked individual proxy to latent disagreement, inheriting negative return predictability that latent disagreement generates. Empirically, IVOL exhibits the highest correlation with DIS among proxies, but DIS absorbs IVOL's predictability in Fama and MacBeth (1973) regressions. IVOL's coefficient becomes statistically insignificant once DIS is included and this absorption holds across uncertainty regimes and in subsamples of smaller stocks (among those covered by two or more analysts), where IVOL's standalone predictability is strongest.

Fourth, DIS behaves as the model predicts for an idiosyncratic disagreement measure rather than one contaminated by aggregate uncertainty. Our framework implies a testable restriction: a well-specified idiosyncratic disagreement measure should predict returns independently of aggregate macro conditions, because aggregate variation is absorbed into factor premia. We test this restriction using eight diverse aggregate indicators spanning market stress, aggregate uncertainty, and investor sentiment. DIS predicts returns consistently in both high and low states across all eight indicators, while the individual proxies of IVOL and short interest predict returns significantly primarily in low-uncertainty environments. These patterns offer a structural account of the conflicting results across samples using different proxies, each capturing a distinct and partial slice of latent disagreement whose predictive content varies with the macro environment. More broadly, regime-dependence of predictive power can serve as a diagnostic for aggregate contamination of idiosyncratic proxies, a principle that extends beyond the disagreement setting.

Fifth, DIS provides evidence on a question debated largely on theoretical grounds since Miller (1977) of whether disagreement can impact pricing without short-sale frictions. Using institutional ownership as an inverse proxy for short-selling frictions following Nagel (2005), we find that both channels are operative and empirically separable. IVOL and short interest exhibit return predictability concentrated where frictions are more binding, as the Miller (1977) channel predicts, whereas DIS exhibits significant and stable return predictability even as institutional ownership approaches 100%, with an interaction with frictions indistinguishable from zero. These contrasting patterns, obtained within a common specification on the same sample, provide what is to our knowledge the first empirical evidence that a frictionless disagreement channel operates alongside the friction-based channel.

Sixth, the model-implied nonlinear structure of DIS is essential to its performance. In Fama and MacBeth (1973) regressions, the return predictability embedded in DIS is not recoverable by linear combinations of the same underlying information. Neither equally weighted composites nor principal components of the same or other common disagreement proxies carry marginal predictive power once DIS is included.

Seventh, we show that DIS detects economically meaningful disagreement that is undetected by individual proxies. We report the characteristics typical of disagreement that goes undetected, underscoring that DIS isolates the latent variable more completely than standalone measures.

Finally, we document the firm-level characteristics associated with elevated disagreement and find that our model-implied measure aligns intuitively with the expected relationship between firm opacity and investor disagreement. For example, higher-DIS firms tend to exhibit increased financial constraints, younger age, intensified R&D expenditures, and reduced volumes of Google searches, both overall and specifically investment-related.

Relationship to the literature. A large literature studies the role of differing opinions in asset pricing (see Chen, Hong, and Stein, 2002; Diether, Malloy, and Scherbina, 2002; Yu, 2011; Carlin, Longstaff, and Matoba, 2014; Fischer, Kim, and Zhou, 2022; and the surveys of Curcuru et al., 2010, and Scheinkman and Xiong, 2004). Key strands include investor heterogeneity as a risk factor Barry and Brown (1985); Merton (1987); Varian (1986); static models of short-sale constraints and disagreement (Miller, 1977; Jarrow, 1980; Diamond and Verrecchia, 1987; Chen, Hong, and Stein, 2002); dynamic models and speculative bubbles (Harrison and Kreps, 1978; Scheinkman and Xiong, 2003; Hong, Scheinkman, and Xiong, 2006); alternative channels via illiquidity (Sadka and Scherbina, 2007), information risk (Johnson, 2004), financial distress Avramov et al. (2009), and rational learning (Goulding, 2017, 2018; Albagli, Hellwig, and Tsyvinski, 2024); connections with trading volume, volatility, and informed trading (Harris and Raviv, 1993; Kandel and Pearson, 1995; Cao and Ou-Yang, 2008; Dumas, Kurshev, and Uppal, 2009; Banerjee and Kremer, 2010; Ottaviani and Sørensen, 2015; Atmaz and Basak, 2018; Banerjee, Davis, and Gondhi, 2018; Chabakauri and Han, 2020; see Hong and Stein, 2007, and Xiong, 2013, for surveys). Our paper differs in that our results do not rely on firm characteristics, market frictions, or parametric assumptions on the payoff distribution, and we establish linkages among different disagreement measures rather than treating any single proxy as the object of study.

Our pricing model is related to, but distinct from, that of Goulding, Santosh, and Zhang (2025), which builds on the frictionless skewness-demand-convexity mechanism of Goulding, Santosh, and Zhang (2023) to explore pricing effects of skewness-disagreement interactions, taking disagreement as given by analyst forecast dispersion. Our paper inverts the order of analysis to recover latent disagreement from the joint equilibrium behavior of model variables. This approach yields an interlocking set of distinct structural predictions about latent disagreement that are not available in single-proxy settings: its nonlinear structure of major proxies, why these proxies disagree, and its signature of the skewness-demand-convexity mechanism itself.

This inversion unlocks analyses in the empirical program of Section 5, including our regime-stability diagnostic, IVOL puzzle account, and two-channel separation. Prior empirical work has documented that the disagreement–return relationship for various proxies is stronger in the presence of short-selling constraints (e.g., Boehme, Danielsen, and Sorescu, 2006; Nagel, 2005), consistent with the friction-based channel, but has not isolated a friction-independent channel in the same data. Our analyses in Section 5.5 provide new evidence that both channels coexist under the same specification and same sample, and are complementary rather than competing. Their empirical separability is a contribution that is distinct from the theoretical contribution of either channel alone.

Our equilibrium framework also distinguishes the composite from atheoretic aggregation approaches. Huang, Li, and Wang (2021) construct an aggregate disagreement index via linear partial least squares to predict aggregate market returns. Our approach provides instead a model-based nonlinear composite applicable to individual stocks, with a specific nonlinear structure whose necessity is empirically supported. Bali et al. (2025) develop a machine-learning-based composite using over 150 stock characteristics on a broad sample, whereas our measure is derived from a parsimonious model with a small set of theory-motivated inputs, generating testable structural predictions about regime-stability, the role of frictions, and which proxies are expected to be closely linked to latent disagreement. Appendix C enumerates a wide variety of disagreement proxies employed in the literature across accounting, economics, finance, and management, and provides additional background on the three major proxy categories.

Organization. Section 2 presents the pricing model. Section 3 derives the composite disagreement measure. Section 4 describes the data and empirical construction. Section 5 presents empirical results: portfolio sorts, Fama–MacBeth regressions, the IVOL absorption analysis, the uncertainty-regime analysis, the role of short-selling frictions, the nonlinearity tests, and the characteristics of disagreement that DIS captures that is undetected by extant proxies. Section 6 offers some conclusions. Appendices provide the multi-asset framework, variable definitions, a survey of disagreement measures in the literature, proofs, equilibrium demand derivations, higher-order sensitivity results, and robustness analyses.

2. Model

We analyze a single-asset framework, which might seem counterintuitive given the framework makes cross-sectional predictions. However, given the nature of idiosyncratic disagreement, we can obtain identical inferences from a single asset framework that separates out of a multi-asset framework with a standard factor structure, in which pricing is governed by the asset's own idiosyncratic statistics (Appendix A). The model of this section is that separated problem and has the benefit of expositional clarity. The parameters that appear below (payout mean, variance, and skewness, and investor beliefs) are all to be understood as idiosyncratic quantities relative to the factor structure, as Appendix A makes explicit. A reader wishing to start from the multi-asset foundation before proceeding may read Appendix A first; the main text takes the separation result as given and focuses on characterizing the equilibrium relationships among idiosyncratic variables that form the basis of our empirical analysis.

Consider a two-period market having dates $t \in \{0, 1\}$ with two assets: a risk-free asset with its price and payoff normalized to one; and one risky asset with price p at date $t = 0$ and payoff $\widetilde{y}$ at date $t = 1$. We follow a standard market-clearing approach to characterize the equilibrium price p. Specifically, payoff $\widetilde{y}$ is exogenously given and the equilibrium price p will be endogenously determined by market clearing of the demands of a continuum of expected-utility maximizing investors, whose utility functions and differences of opinion about $\widetilde{y}$ are also exogenously given.

2.1. Payoff Distribution

We model the payoff $\widetilde{y}$ as a random variable with idiosyncratic statistics of mean, variance, and skewness denoted by $\mu = \mathbf{E}[\widetilde{y}]$, $\sigma^2 = \mathbf{Var}[\widetilde{y}]$, and $s = \mathbf{Skew}[\widetilde{y}] = \frac{\mathbf{E}[(\widetilde{y}-\mu)^3]}{\sigma^3}$, respectively, and with marginal cumulative distribution function (cdf) denoted by $F(y) = \mathbf{P}[\widetilde{y} \leq y]$. We denote the corresponding net return random variable by $\widetilde{r}$, where $\widetilde{r} = \frac{\widetilde{y}-p}{p}$, with its variance denoted by $\sigma_r^2 = \mathbf{Var}[\widetilde{r}]$.[3] Note that the return mean satisfies $\mathbf{E}[\widetilde{r}] = \frac{\mu-p}{p}$, the return variance satisfies $\sigma_r^2 = \frac{\sigma^2}{p^2}$, and the return skewness is the same as the payoff skewness. We assume μ

[3] Division by p is well-defined, for example, under the natural condition of limited liability that entails the payoff $\widetilde{y}$ has only nonnegative support such that $p > 0$ in equilibrium.

is finite and σ is strictly positive to avoid a trivial solution.

2.2. Investor Utility

There is a continuum of investors, indexed by the unit interval $[0, 1]$, each with common utility function of wealth $u(w)$ and initial wealth $w_0 > 0$. Investors prefer more to less, $u'(w) > 0$; are strictly risk averse, $u''(w) < 0$; and have demand for precautionary savings or positive intensity of prudence, $u'''(w) > 0$. Let $\mathcal{A} = -\frac{u''(w_0)}{u'(w_0)}$ denote the coefficient of absolute risk aversion at initial wealth (Pratt, 1964, and Arrow, 1965). Let $\mathcal{P} = -\frac{u'''(w_0)}{u''(w_0)}$ denote the coefficient of absolute prudence at initial wealth (Kimball, 1990).[4] Our assumptions on utility embed the natural conditions of $\mathcal{A} > 0$ and $\mathcal{P} > 0$. This broad class of utility functions includes two of the most widely used classes of utility functions in the literature, namely, constant absolute risk aversion (CARA) utility,

$$u(w) = -e^{-aw}, \qquad a > 0, \tag{1}$$

where a is a scalar that governs absolute risk aversion, and constant relative risk aversion (CRRA) utility,

$$u(w) = \begin{cases} \frac{w^{1-\gamma}}{1-\gamma}, & \gamma \neq 1, \\ \ln(w), & \gamma = 1, \end{cases} \tag{2}$$

for $w > 0$, where $\gamma > 0$ is a scalar that governs the degree of relative risk aversion. For CARA, $\mathcal{A} = \mathcal{P} = a > 0$, and for CRRA, $\mathcal{A} = \gamma/w_0 > 0$ and $\mathcal{P} = (1 + \gamma)/w_0 > 0$.

2.3. Investor Disagreement

Investors have differences of opinion about the expected payoff of the risky asset and, accordingly, also disagree about the expected return. To model their disagreement, we specify separately the relative distribution of beliefs and the scale of those beliefs. This framework encompasses many shapes of belief heterogeneity (polarized, uniform, concentrated-with-tails, or asymmetric) while treating the overall level of disagreement as a separate parameter governed by the scale D.

[4] Leland (1968) and Sandmo (1970) connect a positive third derivative of a utility function to a preference for precautionary savings and Kimball (1990) indicates the coefficient of absolute prudence or intensity of prudence measures the strength of the precautionary saving motive.

Specifically, regarding the relative distribution, each investor's belief corresponds to an independent realization of the random variable $\widetilde{\Delta}$ at date $t = 0$, which is a unitless standardized variable governing belief heterogeneity. Specifically, $\widetilde{\Delta}$ is a discrete mean-zero random variable with variance normalized to one, which takes $i = 1, 2 \ldots, n$ possible values: $\widetilde{\Delta} = \Delta_i$ with probability π_i. Belief heterogeneity characterizes n types of investors, with fraction π_i of investors being of type i, and models the concentration or dispersion of beliefs relative to each other holding the overall variance fixed to one unit.

Regarding the scale of these beliefs, each type-i investor believes the expected payoff is $\mu_i = \mu + D\Delta_i$, where D is a positive scalar, in units of currency, that expands or contracts the relative distribution of beliefs to their absolute levels. Equivalently, each type-i investor believes that $\widetilde{y}$ is drawn from cdf $F(y - D\Delta_i)$.[5] We refer to D when discussing the role of disagreement in prices or payoffs, which are denominated in units of currency.

Likewise, each type-i investor believes the expected return is $\mathbf{E}[\widetilde{r}] + d\Delta_i$, where $d = \frac{D}{p}$ is a unitless scalar. We refer to d, when discussing the role of disagreement in the expected return, which is also unitless. The higher that D (or d) is above zero, the greater the scale of disagreement is within the investor population, all else equal.

2.4. Investor Demand and the Equilibrium Price

Each investor of type $i = 1, 2, \ldots, n$ demands $x_i(p)$ units of the risky asset given price p, where $x_i(p)$ maximizes investor i's expected utility of date–1 wealth,

$$x_i(p) = \arg\max_x \mathbf{E}_i[u(w_0 + x(\widetilde{y} - p))], \tag{3}$$

where $\mathbf{E}_i$ denotes the expectation is taken with respect to investor i's beliefs: $F(y - D\Delta_i)$. Under type-i investor's belief, the expected payoff is $\mu_i = \mathbf{E}_i[\widetilde{y}] = \mu + D\Delta_i$. For notational simplicity, we denote the variance under this investor's belief by $\sigma_i^2 = \mathbf{E}_i[(\widetilde{y} - \mu_i)^2]$ and the

[5] For example, suppose the expected payoff is $\mu = 10$. Consider the case of $n = 2$ with $\Delta_1 = -1$ and $\Delta_2 = 1$ equally likely. These Δ_i's are zero on average with unit variance, representing that type-1 investors are relatively "pessimistic" and type-2 investors are relatively "optimistic." If the scale of disagreement is $D = 0.5$, then each pessimistic investor believes the payoff distribution of the risky asset is shifted by -0.5 such that $\mu_1 = 9.5$ and therefore that $\widetilde{y}$ is drawn from cdf $\mathbf{P}[\widetilde{y} - 0.5 \leq y] = \mathbf{P}[\widetilde{y} \leq y + 0.5] = F(y + 0.5)$. Similarly, each optimistic investor believes the payoff distribution of the risky asset is shifted upward by 0.5 such that $\mu_2 = 10.5$ and therefore that $\widetilde{y}$ is drawn from cdf $\mathbf{P}[\widetilde{y} + 0.5 \leq y] = \mathbf{P}[\widetilde{y} \leq y - 0.5] = F(y - 0.5)$; i.e., so more probability mass is put on above μ.

skewness by $s_i = \frac{\mathbf{E}_i[(\widetilde{y}-\mu_i)^3]}{\sigma_i^3}$. It can be shown that variance and skewness do not depend on i because investors only disagree about the mean payoff via a shift of the cdf, $F(y - D\Delta_i)$. As a result, we drop the subscripts and write $\sigma = \sigma_i = \mathbf{SD}[\widetilde{y}]$ and $s = s_i = \mathbf{Skew}[\widetilde{y}]$.

Proposition 1 (Demand)**.** *The Taylor's expansion of type-i's demand for the risky asset around $p = \mu_i$ is*

$$x_i(p) = -\frac{1}{\mathcal{A}}\frac{1}{\sigma^2}(p - \mu_i) + \frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}^2}\frac{s}{\sigma^3}(p - \mu_i)^2 + h_i(p)(p - \mu_i)^2, \tag{4}$$

where $h_i(p)$ is an unknown function that converges to 0 as $p \to \mu_i$ by Taylor's theorem.[6]

Proposition 1 characterizes type-i demand via a Taylor expansion around $p = \mu_i$. Demand is zero at $p = \mu_i$, decreasing in price with slope $-\frac{1}{\mathcal{A}\sigma^2}$, and shaped by skewness through the quadratic term, whose influence is stronger the higher $\mathcal{P}$ and weaker the higher $\mathcal{A}$.

2.4.1. Skewness and Local Demand Convexity

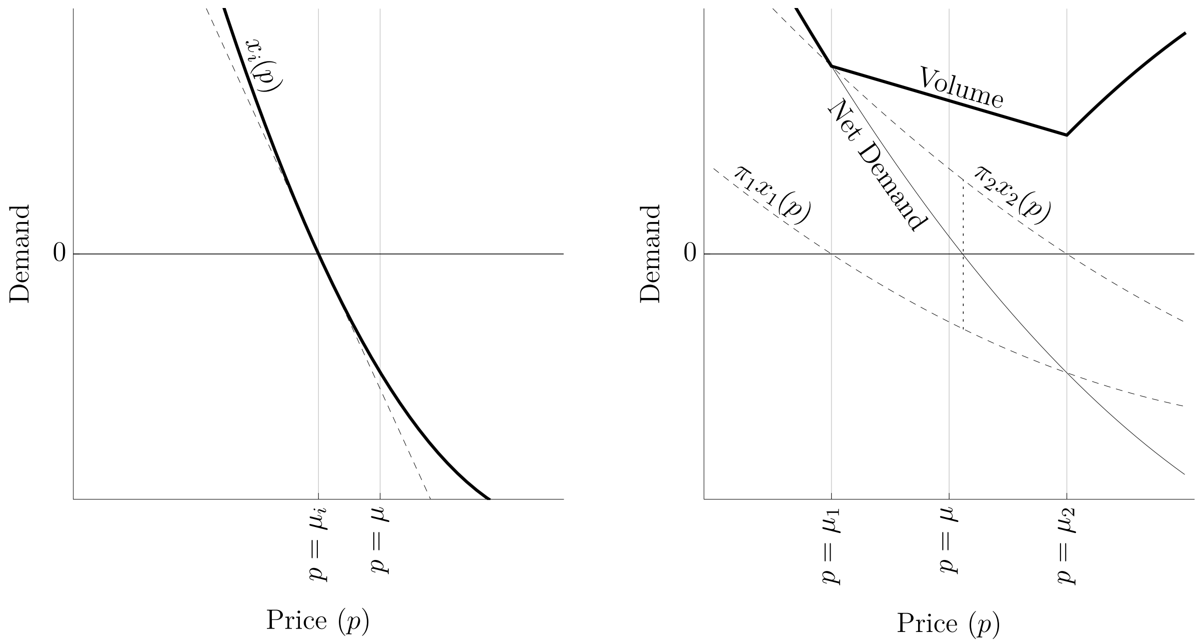
Figure 1a illustrates under CRRA utility the main properties of the demand function in the case of positive skewness: $s > 0$. Positive skewness produces nonlinear demand and, in particular, locally strictly convex demand around the investor's belief (solid curve in Figure 1a). This convexity generates asymmetric demand responses (to symmetric deviations in price) for all investor types. For example, for prices below the investor's belief, $p < \mu_i$, the investor has increasingly positive demand that accelerates as price decreases; whereas, for prices above, $p > \mu_i$, the investor has negative demand or short interest in the risky asset, which increases in magnitude at a diminishing rate as price increases because of the convexity.[7] Without skewness, demand would be a linear function (dashed tangent line in Figure 1a), which is the norm in the classic set-up of a normally-distributed payoff and

[6] A second-order Taylor expansion is sufficient to capture first- and second-order price sensitivity as well as generate the property of local convexity of demand generated by payout skewness. Higher-order expansions that reflect additional moments, such as kurtosis, could reduce approximation error for prices outside the range of investor beliefs, but at the expense of increased complexity. To preview, the equilibrium price of Proposition 2 falls within the span of beliefs for disagreement not too large, and expected returns have limited sensitivity to fourth or higher moments empirically. The composite measure DIS inherits this approximation; its empirical performance across subsamples (Section 5 and Appendix H) provides indirect evidence that the second-order expansion captures the economically relevant variation.

[7] This demand behavior reflects the preference for precautionary savings, for example, represented by CRRA utility of a positively skewed payoff.

Figure 1: Investor demand and trading volume as a function of price.

(a) Investor type i demand, pos. skew (b) Net demand and trading volume, pos. skew



Notes: Figure (a) plots the second-order Taylor polynomial of the demand $x_i(p)$ of a type-i investor with belief $\mu_i = \mu + D\Delta_i$ as a function of price (solid curve) and its tangent line at $p = \mu_i$ (dashed line) over the domain $p \in [\mu_i - \sigma, \mu_i + \sigma]$, where $\Delta_i = -1$, under CRRA utility with relative risk aversion parameter γ. Figure (b) plots the trading volume (bold solid curve), the second-order Taylor polynomials of the total demand functions of type-1 and type-2 investors with beliefs $\mu_1 = \mu + D\Delta_1$ and $\mu_2 = \mu + D\Delta_2$ (dashed curves), respectively, as well as the net demand (solid curve) over the domain $p \in [\mu - 0.5\sigma, \mu + 0.5\sigma]$, where $\Delta_1 = -1$, $\Delta_2 = 1$, $\pi_1 = \pi_2 = 0.5$. The vertical dotted line to the right of μ marks the equilibrium price at which type-1 and type-2 demands are equidistant from the price axis, so that positive and negative demands offset to clear the market. The figures have the following parameter values in common: $D = 0.05$, $\mu = 1$, $\sigma = 0.2$, $s = 0.6$, $\gamma = 2$, $w_0 = 1$.

CARA utility, and both positive demand and short interest would increase in magnitude symmetrically away from $p = \mu_i$ at a uniform rate.

2.4.2. Market Clearing Equilibrium Price and Expected Return

Combining the demand functions of all investor types in a market-clearing analysis determines the equilibrium price. Because investors of type i each trade $x_i(p)$ and are of size π_i in the population, we can express the net demand of all investors as

$$\pi_1 x_1(p) + \pi_2 x_2(p) + \cdots + \pi_n x_n(p), \tag{5}$$

where $\pi_i > 0$ and $\sum_{i=1}^{n} \pi_i = 1$. We assume that the net supply of the risky asset is $X = 0$ and determine the equilibrium price from market clearing: $\sum_{i=1}^{n} \pi_i x_i(p) = X$. Note that we do not require noisy supply to motivate trade as in market models with investors having common expectations. Differences in beliefs are sufficient to generate trade.

Figure 1b illustrates under CRRA utility the net demand (5) in a market with two types of investors who disagree: $\pi_1 = \frac{1}{2}$ of investors are type-1 investors, who are relatively pessimistic about the expected payoff, $\mathbf{E}_1[\widetilde{y}] = \mu - D$, and $\pi_2 = \frac{1}{2}$ of investors are type-2 investors, who are relatively optimistic about the expected payoff, $\mathbf{E}_2[\widetilde{y}] = \mu + D$. Figure 1b plots the case of a positively skewed payoff ($s = 0.6$). The total demands of each investor type, $\pi_1 x_1(p)$ and $\pi_2 x_2(p)$, respectively, are decreasing and convex in price p in a neighborhood overlapping their beliefs, by Proposition 1. In Figure 1b, this overlapping region of prices includes $p = \mu$, which is between $p = \mu_1$ and $p = \mu_2$, and a price above $p = \mu$ that perfectly offsets demands to clear the market. Again, because investors are risk averse, they are unwilling to trade a deterministic amount p for an uncertain payoff that equals p in expectation. Hence, demand is zero for all type-1 investors at price $p = \mu_1$. At $p = \mu_1$, type-2 investors, who believe the asset is worth more than p in expectation, demand a positive quantity, and therefore, net demand is positive. As price increases above $p = \mu_1$, type-1 demand becomes negative to offset the positive demand of type-2, increasing short interest and reducing net demand.

More generally, setting net demand (5) equal to zero determines the equilibrium price that offsets demands to clear the market. Because each $x_i(p)$ depends on the scale of disagreement, D, through belief $\mu_i = \mu + D\Delta_i$, the equilibrium price also depends on D. In Proposition 2, we use implicit differentiation of price in the market-clearing condition with respect to the D to obtain a Taylor expansion in terms of the scale of disagreement, for D not too large.[8]

Proposition 2 (Equilibrium price)**.** *The Taylor expansion of the equilibrium price p as a function of the scale of disagreement, around $D = 0$, is given by:*

$$p(D) = \mu + \frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D^2 + h(D)D^2, \tag{6}$$

where $h(D)$ is an unknown function that converges to 0 as $D \to 0$ by Taylor's theorem.[9]

[8] The phrase "for D not too large" means that there exists a $\overline{D} > 0$ such that the approximation is reasonable for all $D < \overline{D}$. See Appendix F for additional discussion in the context of equilibrium volume and price relationships.

[9] The second-order Taylor polynomial approximation of equilibrium price captures all disagreement sensitivity inherited from the second-order demand polynomials of Proposition 1 because of its determination via

Proposition 2 shows how the equilibrium price depends on the scale of disagreement.[10] If $s > 0$, then equilibrium price increases in D, is larger than expected payoff ($p > \mu$), and by the 2nd-order Taylor polynomial in (6), the expected return satisfies

$$\mathbf{E}[\widetilde{r}] = -\frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma_r}d^2, \tag{7}$$

which is decreasing in d. (Recall $d = D/p$ and $\sigma = \sigma_r p$.) By inspection of (6) and (7), we find that more disagreement means higher prices, and, therefore, lower expected returns for positive skewness, all else equal. The influence of disagreement on expected returns is stronger the higher the demand for precautionary savings $\mathcal{P}$ and weaker the higher the risk aversion $\mathcal{A}$. Intuitively, when $\mathcal{A}$ is higher, investors are more reluctant to take larger risky positions, reducing how aggressively buyers and sellers act on their views: disagreement translates into smaller price impact. When $\mathcal{P}$ is higher, investors react more sensitively to uncertainty by shifting toward safer positions, which moves prices more, and expected returns adjust more in equilibrium. The model identifies the $\mathcal{P}/\mathcal{A}$ ratio, the ratio of absolute prudence to absolute risk aversion, as the structural coefficient governing how strongly disagreement depresses expected returns. Standard models either use CARA utility, where $\mathcal{P} = \mathcal{A}$ and the ratio is identically one, or ignore higher-order preferences entirely. Our framework makes the two separable. This generalization matters theoretically because it identifies a structural parameter: the disagreement–return relationship is governed by the interaction of prudence and risk aversion, not by risk aversion alone. In our empirical application, however, the $\mathcal{P}/\mathcal{A}$ ratio does not drive the cross-sectional predictions we test, since $\mathcal{P}/\mathcal{A}$ enters as a scalar multiplier common to all assets.

Note that because of the standard simplifying assumption of zero supply, the expected return (7) is zero in the case of zero skewness. This result reflects the fact that positive expected returns typically are the result of a risk premium necessary to induce investors to have net positive demand for the asset in equilibrium. Therefore, with zero supply, we have isolated the impact of disagreement on expected returns net of the risk premium rather than

market clearing of such demands. That is, higher-order expansions for equilibrium price are relevant only for higher-order expansions of demand.

[10] Note that the asset's idiosyncratic statistics are relevant for pricing rather than, say, covariances with the market or aggregate-level disagreement, consistent with the interpretation given in Appendix A.

gross of the risk premium. Our empirical tests, which examine cross-sectional variation in returns conditional on standard factor controls, are designed to isolate precisely this deviation, making the zero-supply normalization appropriate for the cross-sectional predictions we test.

2.4.3. Demand Convexity and the Disagreement–Price Relationship

The local demand convexity established in Section 2.4.1 has a direct implication for equilibrium pricing. Because the distribution of investor beliefs is centered around fundamental value, a price at fundamental value cannot offset the asymmetric demand responses documented above: positive demand exceeds short interest in magnitude at any given deviation from the investor's belief. The market-clearing price must therefore be pushed higher. For example, in Figure 1b, net demand remains positive at $p = \mu$; price must increase above μ to clear the market. Higher scales of disagreement widen investor beliefs, amplify the asymmetry, and push the market-clearing price further above fundamental value.

Without demand convexity, as would obtain with zero skewness, long and short positions would perfectly offset at fundamental value regardless of the scale of disagreement. The departures from this benchmark documented in Proposition 2 are therefore entirely attributable to the interaction of skewness and disagreement.

This self-insulating property of disagreement-driven overpricing, in which the same skewness preferences that generate mispricing also discourage the arbitrage trades that would correct it, provides a preference-based complement to friction-based explanations for the persistence of anomaly returns (Shleifer and Vishny, 1997; Pontiff, 2006). The distinction is structural. In friction-based limits to arbitrage, rational agents are prevented from correcting mispricing by exogenous costs. In our framework, rational unconstrained agents endogenously choose smaller corrective positions because the short side of a positively skewed asset is itself negatively skewed.

2.5. Trading Volume

Denote trading volume as a function of price p by $V(p) = \int_0^1 |x_i(p)|di$; i.e., the sum of the absolute value of investors' trades. Because investors of type i each trade $x_i(p)$ and are of size π_i in the population, we have trading volume

$$V(p) = \pi_1|x_1(p)| + \pi_2|x_2(p)| + \cdots + \pi_n|x_n(p)|, \tag{8}$$

where $\pi_i > 0$ and $\sum_{i=1}^{n} \pi_i = 1$. Figure 1b shows trading volume as a function of price for a market with two investor types. There are two kinks in the trading volume curve: one occurs where type-1 investors' demand crosses zero (at μ_1) and the other occurs where type-2 investors' demand crosses zero (at μ_2).

We now write x_i and V to denote the equilibrium demand of investor type i and equilibrium volume, respectively, in which each of $x_i(p)$ and $V(p)$ are evaluated at equilibrium price $p(D)$, using the 2nd-order Taylor polynomials for demand, volume, and price:

$$x_i = \frac{1}{\mathcal{A}}\frac{1}{\sigma^2}D\left(\frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D - \Delta_i\right)\left[-1 + \frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D\left(\frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D - \Delta_i\right)\right], \tag{9}$$

$$V = \pi_1|x_1| + \pi_2|x_2| + \cdots + \pi_n|x_n|, \tag{10}$$

where $\pi_i > 0$ and $\sum_{i=1}^{n} \pi_i = 1$.

Relatedly, short interest is the absolute value of all negative demands. In the model, short interest is determined in equilibrium as a function of all other parameters and variables. It ends up being in constant proportion ($\frac{1}{2}$) to equilibrium volume because of our simplifying assumption of zero net demand, but it varies according to model parameters. Specifically, $\sum_{x_i<0} \pi_i|x_i| = -\sum_{x_i<0} \pi_i x_i = \sum_{x_i>0} \pi_i|x_i|$, so short interest is volume divided by two, and therefore, volume and short interest will have common sensitivities to other variables in the model. In our model, it is negative demand, or short interest, that translates the convexity of demand for a positively-skewed asset into an equilibrium price above fundamental value. So, although our analysis focuses on overall volume for simplicity of exposition, the sensitivity of volume in the model is interchangeable with that of short interest. Accordingly, we use a measure of short interest as our volume proxy in our empirical implementation.

Note that x_i in (9) depends on disagreement both via the underlying source of heterogeneity, Δ_i, and the overall scale of disagreement, D. Likewise, V depends on the scale of disagreement. To understand the equilibrium relationship between volume and the scale of disagreement, consider Figure 1b. At the market-clearing price, denoted by the vertical dotted line, trading volume is decreasing in price; i.e., an increase in price results in lower overall trading activity. If an increase in equilibrium price is the result of a higher scale of disagreement, however, demands expand away from the price axis at a rate faster than the trading volume decreases in price. The result is that equilibrium trading volume (as well as short interest) increases in the scale of disagreement. This result is consistent with other models in the literature, but we arrive at this result based on investors with different beliefs trading a skewed asset in a frictionless model, which to our knowledge is a novel mechanism. See Proposition 3 and Appendix F for a rigorous analysis.

Proposition 3. *Equilibrium trading volume, V, is increasing in the scale of disagreement, D: $\frac{\partial V}{\partial D} > 0$.*

Proposition 3 indicates that trading volume (and short interest) is a proxy for disagreement, consistent with the implications of earlier literature and the use of volume-based measures as proxies for disagreement, but arrived at under different assumptions and, hence, providing new support for the volume-based category of proxies. Higher levels of disagreement are associated with higher levels of trading volume and short interest, all else equal. See Appendix F for more discussion of the intuition for this result.

3. A Composite Proxy of Disagreement

In this section, we apply the model of Section 2 to develop a novel composite proxy of investor disagreement as a function of all model variables plus variables related to analyst forecast dispersion.

3.1. Two Symmetric Investor Types

First, we analyze the special case of disagreement between $n = 2$ symmetric investor types. Let $\pi_1 = \pi_2 = \frac{1}{2}$, $\Delta_1 = -1$, and $\Delta_2 = 1$, so that $\mathbf{E}[\widetilde{\Delta}] = 0$ and $\mathbf{Var}[\widetilde{\Delta}] = 1$. Because of market clearing, we have $x_1 < 0$ and $x_2 > 0$, and so $V = -\frac{1}{2}x_1 + \frac{1}{2}x_2$. Substituting (9) for x_1 and x_2 and simplifying, we find that trading volume equals

$$V = \frac{1}{\mathcal{A}} \frac{1}{\sigma^2} D \left[1 - \frac{1}{2} \frac{\mathcal{P}^2}{\mathcal{A}^2} \frac{s^2}{\sigma^2} D^2 \right]. \tag{11}$$

Equation (11) relates all key model variables in a simple expression. This expression relates volume to disagreement, volatility, and skewness, but does not yet incorporate the analyst forecast channel that our model identifies as a distinct amplifier of disagreement. We turn to this next.

3.2. Analyst Dispersion

We now augment our main model with a simple auxiliary model that incorporates the impact of analyst forecasts on the scale of disagreement. Analyst forecast dispersion and investor disagreement are correlated but distinct. Recall that investors agree about the payoff variance (σ^2) and, therefore, any impact of analyst forecast information on investor beliefs manifests in the variation of their means. Also, recall that the distribution of expected payoffs corresponds with $\mu + D\widetilde{\Delta}$, where $\widetilde{\Delta}$ is a standardized random variable that reflects relative differences and D governs scale. Analyst forecast dispersion thus impacts the scale of disagreement via a public information effect that does not alter the common payoff volatility belief, σ, and does not necessarily alter the underlying relative investor heterogeneity captured by $\widetilde{\Delta}$. Accordingly, we model the variance of means indexed to the common volatility level, ($\mathbf{Var}[(\mu + D\widetilde{\Delta})/\sigma]$), as proportional to the information conveyed by analyst forecasts via the posterior variance of the mean payoff given analyst forecasts. With this approach, analyst forecast dispersion will merge conveniently with our main model.

To fix ideas, let $j = 1, 2, \ldots, m$ denote financial analysts who each obtain a noisy signal about the true expected payoff and publish a corresponding forecast. These forecasts are observed by investors at date $t = 0$. Let $\widetilde{\mu} \sim \mathcal{N}(\mu, \sigma_\mu^2)$ be a random variable denoting the unknown expected payoff of the risky asset; i.e., $\widetilde{\mu}$ is normally distributed around the payoff's true mean μ with variance σ_μ^2. Via their research process, analysts obtain noisy observations of $\widetilde{\mu}$. Suppose analyst j publishes a forecast A_j, which is a realization of the random variable $\widetilde{A}_j = \widetilde{\mu} + \widetilde{a}_j$, where $\widetilde{a}_j \sim \mathcal{N}(0, \sigma_a^2)$ is independent, normally distributed noise with variance σ_a^2. Hence, analyst forecasts are correlated with each other because their signals partially reflect the true payoff $\widetilde{\mu}$, but each forecast differs because of analyst-specific noise; i.e., $\widetilde{a}_j$. Analysts publish forecasts conveying their noisy inferences about μ and the precision of their information is governed by $\frac{1}{\sigma_a^2}$.

Lemma 4. *Suppose the variance of investor beliefs about the expected payoff per unit of volatility is proportional to the variance of the expected payoff conditional on analysts fore-*

casts: $\mathbf{Var}[(\mu + D\widetilde{\Delta})/\sigma] \propto \mathbf{Var}\left[\widetilde{\mu}\middle|\widetilde{A}_1, \widetilde{A}_2, \ldots, \widetilde{A}_m\right]$. *Then,*

$$\frac{D^2}{\sigma^2} = \left(\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}\right)^{-1}, \tag{12}$$

when the proportionality constant is one.

Under the simplifying assumptions of analysts' signals, the scale of disagreement depends on the number of analysts, m, in Lemma 4. Because we model each analyst's signal as having independent noise for simplicity, analysts' collective forecasts reduce the posterior variance of the true payoff. If instead the noise in analyst signals were positively correlated across analysts or if the number of analysts increased with the uncertainty around the mean payoff, which are both plausible alternative assumptions, then the role of the number of analysts would drop out or be diminished. In our empirical implementation, we scale the number of analysts each month by the cross-sectional average number of analysts to allow for some variation across firms but to moderate the impact coming from our simplifying assumptions as well as to keep our disagreement measure unitless (see Section 4).

By Lemma 4, in absence of the influence of analyst forecasts ($m = 0$), risk-adjusted disagreement is captured by the fundamental uncertainty about μ: $D/\sigma = \sigma_\mu$. Thus, total heterogeneity among investors has a source that does not depend solely on heterogeneity of analysts (via σ_a^2) and instead reflects both idiosyncratic volatility (i.e., σ) and fundamental uncertainty about the mean (i.e., σ_μ). When there are two or more analysts, however, dispersion in analysts' forecasts can serve to amplify the magnitude of already-existing differences in investors' beliefs.[11] Specifically, based on the method-of-moments estimate of the variance of analyst forecasts, $\widehat{\sigma}_a^2$, the expression (12) characterizes explicitly the relationship between the scale of disagreement among investors, D, and analyst forecast dispersion, $\widehat{\sigma}_a$, as measured by the standard deviation of analyst forecasts. Although in this simple model, D and $\widehat{\sigma}_a^2$ are functionally related, analyst dispersion in the data is an imperfect proxy for disagreement. We can combine this result with our main model to capture interactions with trading-related components.

[11] Theoretically, this amplification is true for $m = 1$, but empirically we need $m \geq 2$ to estimate σ_a^2.

3.3. Composite Disagreement Measure

We now merge this simple auxiliary analyst dispersion model with our main trading model to develop a composite proxy of disagreement. Substituting equation (12) for D^2/σ^2 into the second term of (11) yields

$$V = \frac{1}{\mathcal{A}}\frac{1}{\sigma^2}D\left[1 - \frac{1}{2}\frac{\mathcal{P}^2}{\mathcal{A}^2}s^2\left(\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}\right)^{-1}\right]. \tag{13}$$

In our stylized model, supply and, therefore, shares outstanding are zero. In empirical applications of our model, however, shares outstanding are positive for each firm. To aid empirical implementation, let v denote volume traded as a fraction of shares outstanding (which is unitless) and suppose that aggregate initial wealth of all investors, w_0, is proportional to the market value of the risky asset, $w_0 = pX$, with unit proportionality constant. Applying identities $\sigma_r p = \sigma$ and $d = D/p$, normalizing initial wealth to one, and rearranging (13) to isolate d yields

$$d = \mathcal{A}\sigma_r^2 v\left[1 - \frac{1}{2}\frac{\mathcal{P}^2}{\mathcal{A}^2}s^2\left(\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}\right)^{-1}\right]^{-1}, \tag{14}$$

which is a single equation relating our unitless disagreement variable to all quantities of interest. Define $g \equiv \frac{1}{2}\frac{\mathcal{P}^2}{\mathcal{A}^2}s^2\left(\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}\right)^{-1}$. Note that the term $[1-g]^{-1}$ in (14) should be positive in order for d to be well-defined. This condition will be met as long as the term g is between 0 and 1. Note that $g \geq 0$ because the number of analysts is nonnegative ($m \geq 0$), the variance of analyst forecasts is positive ($\sigma_a^2 \geq 0$) and the variance of the true payoff is positive ($\sigma_\mu > 0$). However, depending on the scale of the quantities $\mathcal{A}$, $\mathcal{P}$, s, m, σ_a, and σ_μ, then g can potentially exceed 1. To ensure the condition $g \in [0,1]$, we apply a standard monotonic transformation $g \to 1 - e^{-g}$, which is increasing in g, equal to 0 for $g = 0$, and equal to 1 in the limit as $g \to \infty$. This preserves the essential multiplicative structure while keeping $g \in [0,1]$ and is robust for empirical implementation. Applying this to (14), after simplification we obtain the modified composite disagreement score:

$$d^* = \mathcal{A}\sigma_r^2 v \cdot \exp\left[\frac{\frac{1}{2}\frac{\mathcal{P}^2}{\mathcal{A}^2}s^2}{\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}}\right]. \tag{15}$$

We will use (15) to construct a unitless disagreement proxy for our main empirical analyses. Specifically, we use $\text{DIS} = \ln(1 + d^*)$, where d^* is as in (15), which helps to attenuate the skewness of d^* exhibited in our empirical implementation.

Economic interpretation. The composite measure of (15) is the product of a volatility-volume term, $\mathcal{A}\sigma_r^2 v$, and an exponential amplification term governed by skewness and analyst forecast dispersion. This multiplicative structure means that d^* registers high disagreement only when multiple components are simultaneously elevated: high idiosyncratic volatility alone, or high volume (short interest) alone, does not produce a high score unless the other components corroborate. For example, if σ_r rises broadly across stocks, as it does in periods of aggregate stress, but v and the dispersion-related amplification term do not rise commensurately, then d^* remains moderate. An additive composite, by contrast, could register elevated disagreement whenever any single component is high, regardless of the others. The co-occurrence requirement embedded in the multiplicative form thus provides a built-in filter against variation in any one component that is unrelated to firm-specific disagreement. Whether this structural property translates into empirical advantages over linear alternatives is a testable question, which we examine in Section 5.

3.4. Sensitivities

The composite proxy (15) is increasing in relative risk aversion, idiosyncratic volatility, idiosyncratic skewness, trading volume, analyst forecast dispersion, and in the scale of uncertainty about the mean payoff, all else equal; and is decreasing in the precision of analysts' information and the number of analysts, all else equal.[12] Moreover, this composite proxy reflects model interactions among these variables, which is missing from individual measures and otherwise difficult to account for in empirical analyses.

Disagreement d^* in (15) is a function of $\mathcal{A}$ (coefficient of absolute risk aversion), $\mathcal{P}$ (coefficient of absolute prudence), σ_r (idiosyncratic return volatility), v (short interest), s (idiosyncratic return skewness), m (number of analyst forecasts), σ_a (dispersion of analyst forecasts), and σ_μ (fundamental uncertainty) with the following sensitivities to key variables

[12] Uncertainty about mean returns is not observable in the data. In our empirical analyses, we use trailing volatility as a proxy, which overstates the uncertainty in the mean but can represent additional uncertainties absent in our simplified auxiliary model of analyst forecasts.

with observable proxies:

$$\frac{\partial d^*}{\partial \sigma_r} = 2\frac{d^*}{\sigma_r} > 0, \tag{16}$$

$$\frac{\partial d^*}{\partial v} = \frac{d^*}{v} > 0, \tag{17}$$

$$\frac{\partial d^*}{\partial \sigma_a} = d^* \frac{g}{\left(\frac{m}{\sigma_a^2} + \frac{1}{\sigma_\mu^2}\right)} \frac{2m}{\sigma_a^3} > 0. \tag{18}$$

Hence, each of idiosyncratic volatility, trading volume, and analyst forecast dispersion are proxies for disagreement, all else held equal. Moreover, each of the sensitivities (16) and (17) are consistent with those implied by (11) of the main model. In Appendix G, we explore higher-order sensitivities.

An important feature of (15) is that the identity and role of each component, the specific nonlinear interactions among them, and the absence of any additional variables are all determined by the conditions of the model. This distinguishes its empirical counterpart, DIS, from empirical composites whose components and weights are chosen to optimize in-sample performance.

4. Data and Construction of Composite Disagreement DIS

Our primary dataset is the intersection of CRSP and COMPUSTAT for all common stocks traded on the NYSE, AMEX, or NASDAQ. Following standard practices, we exclude stocks with month-end prices below \$5, require at least 15 valid daily observations per stock-month, and exclude stock-months with 10 or more zero daily returns. Because our composite measure incorporates analyst forecast dispersion, we restrict the sample to stocks covered by at least two analysts, using data from the I/B/E/S unadjusted details file. Analyst forecast dates are less reliable before 1994 (Clement and Tse, 2005; Kirk, Reppenhagen, and Tucker, 2014), and 2022 is the last year of available data for analyst forecasts and for our expected idiosyncratic skewness measure.[13] The resulting sample contains 719,940 monthly observations spanning January 1994 to December 2022.

[13] The ISKEW measure is obtained from the authors of Boyer, Mitton, and Vorkink (2010), who extended it through 2022.

4.1. Composite Disagreement DIS

We construct DIS following the form of equation (15) under CRRA utility with relative risk aversion parameter γ.[14] Each component is described below and all variables are constructed using only information available at the time of portfolio formation; see Appendix B for complete variable definitions.

Risk aversion. We set $\gamma = 2$, consistent with common values in the literature. Because $\mathcal{P}/\mathcal{A}$ enters (15) under CRRA utility only through the scalar multiplier $(\mathcal{P}/\mathcal{A})^2 = (1+1/\gamma)^2$, it scales the exponential amplification term uniformly across all stocks. Tables H.5 and H.6 present evidence consistent with the interaction structure of the components, not the preference calibration, driving cross-sectional predictions: DIS-decile spread CAPM alphas are stable and Fama–MacBeth coefficients are uniformly significant at the 1% level across $\gamma \in \{0.5, 1, 2, 4, 8\}$.

Idiosyncratic volatility. We compute IVOL as the standard deviation of daily residuals from a Fama and French (1993) three-factor model, following Ang et al. (2006), expressed in percentage: $\sigma_r = \mathrm{IVOL}_{i,t}$.

Trading volume. Volume is measured as monthly short interest, SIR, from the COMPUSTAT short interest file, the ratio of shares shorted to shares outstanding (Chen, Da, and Huang, 2019; Karpoff and Lou, 2010), plus one to ensure strict positivity. We supplement with NASDAQ short interest data prior to 2003 (Chen, Da, and Huang, 2019, 2022). Hence $v = 1 + \mathrm{SIR}_{i,t}$. Section 2.5 motivates SIR as the volume proxy rather than turnover because the model predicts that short interest and volume move proportionally in equilibrium (Section 2.5), and SIR more directly reflects the pessimistic demand that generates the overpricing mechanism. Section 5.8 (Table H.9) shows that turnover-based measures (TO, DTO, SUV) are also positively associated with DIS.

Expected idiosyncratic skewness. Because realized skewness lacks persistence (Hong and Stein, 1999; Harvey and Siddique, 1999, 2000), we use the expected idiosyncratic skewness metric ISKEW of Boyer, Mitton, and Vorkink (2010), derived from 60-month firm characteristics. Hence $s = \mathrm{ISKEW}_{i,t}$.

[14] Under CRRA utility at unit initial wealth, $\mathcal{A} = \gamma$ and $\mathcal{P} = 1 + \gamma$ so that $\frac{\mathcal{P}^2}{\mathcal{A}^2} = \left(1 + \frac{1}{\gamma}\right)^2$.

Analyst forecast dispersion. We use the Diether, Malloy, and Scherbina (2002) measure: the standard deviation of fiscal-year-end analyst EPS forecasts scaled by the mean absolute EPS forecast, expressed in percentage.[15] This measure is robust to the choice of scaling factor (Cen, Wei, and Yang, 2017). Hence $\sigma_a = \text{AFD}_{i,t}$.

Number of analysts. ANALYSTADJ is the number of analysts with valid forecasts for stock i in month t divided by the cross-sectional average number of analysts per firm in the same month, computed over firms with at least two analysts. This normalization also makes the measure unitless and comparable across time periods with different analyst coverage norms.

Return volatility. We proxy for the variance of the expected payout using one-month ex-post return volatility based on daily data, expressed in percentage: $\sigma_\mu = \text{RVOL}_{i,t}$.[16]

Composite disagreement DIS. Our composite measure is $\text{DIS} = \ln(1 + d^*)$, where d^* is as defined in equation (15), specialized to CRRA utility with the above empirical proxies. The logarithmic transformation retains nonnegativity and attenuates the right-skewness of d^*.

4.2. Other Variables

In addition to measuring disagreement and its impact on subsequent returns, we examine several firm- and stock-specific characteristics that explain the cross-sectional variation in returns which are also used as control variables in later analyses. We highlight the most important ones in this section. SIZE is computed as the natural logarithm of market capitalization and BTM as the book-to-market ratio as in Davis, Fama, and French (2000). Moreover, to account for the variations in returns as shown in the empirical asset pricing literature, we include MOM (Carhart, 1997) in our analysis defined as the cumulative returns from month $t - 12$ to $t - 1$. Notably, we omit the lagged return from MOM, in line with Jegadeesh and Titman (1993). BETA is the coefficient derived from regressing a stock's daily returns in month t on the market return. ILLIQ is based on Amihud (2002) illiquidity

[15] Following Hribar and McInnis (2012), we exclude observations with mean absolute forecast below \$0.10 EPS. We exclude stopped and excluded estimates and carry forward estimates released up to 105 days before the earnings announcement.

[16] Alternatives such as profitability-based measures would reduce sample size and introduce sensitivity to the choice of metric. Simple return volatility is widely available and offers a direct reflection of perceived variability.

Table 1: Descriptive Statistics

Variable	Code	Mean	SD	p10	p25	p50	p75	p90
Composite Disagreement	DIS	11.99	34.17	1.04	1.79	3.48	8.78	24.08
Short Interest	SIR	0.05	0.06	0.00	0.01	0.03	0.06	0.11
Idiosyncratic Volatility	IVOL	2.10	1.46	0.81	1.14	1.71	2.62	3.84
Exp. Idiosyncratic Skewness	ISKEW	0.80	0.58	0.15	0.45	0.76	1.07	1.46
Analyst Forecast Dispersion	AFD	13.18	64.07	0.74	1.51	3.58	9.61	25.24
Market Size ($\times 10^{-9}$)	SIZE	7.86	36.68	0.19	0.43	1.25	4.12	14.25
Book-to-Market Ratio	BTM	0.55	0.47	0.13	0.25	0.44	0.72	1.04
Momentum	MOM	19.02	66.62	-33.79	-12.32	9.75	34.83	70.80
Market Loading	BETA	1.08	0.95	0.11	0.55	1.00	1.52	2.15
Illiquidity	ILLIQ	0.10	1.90	0.00	0.00	0.00	0.01	0.06
Maximum Return	RMAX	3.27	2.22	1.32	1.82	2.68	4.04	5.92

Notes: This table presents the descriptive statistics for the relevant cross-sectional variables of interest. It includes the following statistical measures for each variable: mean, standard deviation, and percentiles at the 10th, 25th, 50th (median), 75th, and 90th levels. The sample ranges from January 1994 to December 2022 and comprises all common stocks (identified as shrcd = 10, 11) traded on the NYSE, AMEX, or NASDAQ. Stocks with month-end prices below $5 per share are excluded from the analysis. The data is collected on a monthly basis. For detailed definitions of the variables, please refer to Appendix B.

measure which averages the ratio of absolute daily returns to daily trading volume. Lastly, to account for the payoff of lottery stocks, we use RMAX, which is the average of the five highest daily returns in a given month t (Bali, Brown, and Tang, 2017). All variables, including our composite disagreement measure, are evaluated at a monthly level. A comprehensive overview of all variables utilized in this paper is provided in Appendix B.

4.3. Descriptive Statistics

First, Table 1 provides a descriptive overview of our composite disagreement measure, DIS, and selected characteristics of our sample. Note that stocks in our sample are relatively large because of our restriction to two or more analysts, resulting in average market size of $7.86B, which is more than 60% larger than the average market size for stocks without the analyst restrictions. Focusing on our key variable of interest, DIS, the sample exhibits an average disagreement of 11.99 with an average standard deviation of 34.17. Notably, the distribution of this measure is right-skewed, with the 10th percentile at 1.04 and 90th percentile at 24.08. Most stocks exhibit positive ISKEW and each of the variables in Table 1 is itself positively skewed (i.e., its mean above its median). In particular, analyst forecast dispersion, AFD, a widely-used measure of disagreement, is especially right-skewed. Our results are robust to winsorization of variables.

Table 2: Pooled Correlations

Panel A: Correlations with DIS	
SIR	0.12
IVOL	0.47
ISKEW	0.51
AFD	0.10
SIZE	−0.05
BTM	0.08
MOM	−0.05
BETA	0.12
ILLIQ	0.04
RMAX	0.46

Panel B: Correlations Among Disagreement Proxies			
	AFD	IVOL	SIR
AFD	1.00		
IVOL	0.08	1.00	
SIR	0.07	0.14	1.00

Notes: Panel A presents pooled correlations between our disagreement measure (DIS) and stock characteristics. Panel B presents pooled correlations among major disagreement proxies, AFD, IVOL, and SIR. The sample ranges from January 1994 to December 2022 and comprises all common stocks (identified as shrcd = 10, 11) traded on the NYSE, AMEX, or NASDAQ. For detailed definitions of the variables, please refer to Table 1 and Appendix B.

To further investigate the interplay between our empirical proxies for the components of our disagreement measure, Panel A of Table 2 presents pooled correlations of DIS with its individual components and various stock and firm characteristics, pooling all firm-months. In line with our model's predictions, there is a positive correlation between the measure and each of its individual components. DIS comoves relatively strongly with IVOL (0.47) and relatively weakly with AFD (0.10) and SIR (0.12). To the extent that DIS is an accurate proxy of disagreement, of the major disagreement proxies, IVOL is then the most aligned with disagreement and, therefore, we should expect DIS to absorb some of the return predictability of IVOL in empirical tests. We analyze this connection in Section 5.3. DIS also comoves relatively strongly with ISKEW (0.51), reflecting the important role of skewness in our model.

Panel B of Table 2 presents correlations among the major disagreement proxies, AFD, IVOL, and SIR. These cross-correlations are relatively low, highlighting the disagreement among different disagreement proxies.[17] These near-zero cross-correlations are the empirical face of the disagreement of disagreement: the proxies that purport to measure the same latent variable share almost none of their variation.

[17]In Appendix H, we report time-series average correlations corresponding to those of Table 2, from which we draw similar conclusions.

5. Empirical Results

In this section, we develop eight sets of analyses that are not independent findings but interlocking tests of a single theoretical framework. The logical progression is deliberate: Sections 5.1-5.2 show that DIS significantly predicts returns; Section 5.3 shows that DIS largely subsumes the predictability of IVOL, the individual proxy most closely linked to latent disagreement in the model; Section 5.4 tests whether DIS's signal is genuinely idiosyncratic, as the multi-asset separation of Appendix A implies; Section 5.5 provides support for the economic mechanism through which DIS operates, distinguishing it from the friction-based channel; Section 5.6 presents results consistent with the implication that the model's nonlinear structure is essential to these results; Sections 5.7 and 5.8 then characterize the conditions when DIS detects disagreement that is undetected by extant disagreement measures and the economic content of what DIS captures.

5.1. Portfolio Sorts: Disagreement and Return Predictability

To examine the link between disagreement and future returns, we perform portfolio sorts, reported in Table 3. Stocks are categorized monthly into deciles based on their level of disagreement, DIS, excluding firm-months with negative ISKEW to be consistent with Proposition 2, which predicts a negative disagreement–expected return relationship for positively skewed stocks. Because the vast majority of stocks are positively skewed, as reported in Table 1, our inferences are robust to the inclusion of negative ISKEW stocks and we include all stocks in subsequent analyses unless stated otherwise. See Table H.2 in the appendix for the version of Table 3 that includes all stocks. For each decile, value-weighted one-month-ahead returns are computed via five measures: raw returns, excess returns (stock i return minus risk-free rate), CAPM-α (market factor), FF3-α (adding size/value factors), and FFC-α (adding momentum per Carhart, 1997). Value-weighted decile portfolio returns per metric are calculated, with significance reported. Returns and alphas generally decrease across rising DIS deciles, though not monotonically, with the decline steepening in the upper deciles. All L–H long-short portfolios show positive, significant annualized alphas at the 1% level or better: 21.3% CAPM-α ($t = 4.95$), 20.3% FF3-α ($t = 5.53$), and 17.0% FFC-α ($t = 4.41$). We note that p-values throughout are unadjusted for multiple tests and should be inter-

Table 3: Portfolio Sorts on Disagreement, DIS

DIS	Raw Return	Excess Return	CAPM-α	FF3-α	FFC-α
Low	0.97	0.79	0.28	0.26	0.19
2	0.97	0.79	0.19	0.16	0.15
3	0.93	0.75	0.06	0.05	0.05
4	0.84	0.66	−0.10	−0.11	−0.07
5	0.79	0.61	−0.20	−0.20	−0.16
6	0.93	0.75	−0.11	−0.13	−0.02
7	0.66	0.48	−0.45	−0.46	−0.31
8	0.65	0.47	−0.55	−0.55	−0.36
9	0.30	0.12	−1.03	−0.99	−0.78
High	−0.04	−0.22	−1.49	−1.43	−1.23
Low-High	1.01	1.01	1.78	1.69	1.42
t-stat.	(2.09)	(2.09)	(4.95)	(5.53)	(4.41)
p-value	0.038	0.038	0.000	0.000	0.000

Notes: This table reports the mean values of value-weighted monthly returns at month $t + 1$, based on univariate portfolios sorted according to disagreement (DIS) at date t, excluding firm-months with negative ISKEW to match the theoretical prediction of Proposition 2 (which applies to positively skewed stocks). Table H.2 shows that results are robust to including all stocks. The sample ranges from January 1994 to December 2022. The row labeled "Low-High" presents the differences in monthly returns between the 10th decile (representing high disagreement) and the 1st decile (representing low disagreement). "Raw returns" refer to one-month-ahead monthly returns, whereas "Excess returns" are defined as one-month-ahead monthly returns minus the risk-free rate. The alphas (CAPM-α, FF3-α, FFC-α) represent intercepts from a time-series regression of monthly excess returns at month $t+1$ against the market return (CAPM), further adjusted for size and value factors (FF3), and additionally a momentum factor (FFC) at month t. t-statistics, adjusted according to Newey and West (1987), are presented in parentheses along with their corresponding p-values.

preted with appropriate caution, though the empirical work is based on implications from a theoretical model, which should be treated differently than an unstructured data mining approach (Harvey, 2017). That said, all three alpha t-statistics exceed the multiple-testing-adjusted threshold of 3.0 proposed by Harvey, Liu, and Zhu (2015). Although it is standard empirical practice to control for factor premia, note that our theoretical framework gives a structural explanation for why these alphas should survive standard factor risk adjustments (Appendix A.3).

As is typical in disagreement sorts, alphas are primarily driven by the short leg: the high-DIS decile generates −1.49% monthly CAPM alpha, while the low-DIS decile generates +0.28% (though statistically positive, t-stat. 2.88). The ratio of short-leg to long-leg contribution is approximately 5:1. This behavior is a nearly universal feature of anomaly portfolios that has been attributed to sentiment-driven overpricing (Stambaugh, Yu, and Yuan, 2012) and, relatedly, to probability-weighted demand for lottery-like payoffs (Barberis and Huang, 2008). Our model provides a complementary micro-foundation under standard expected utility preferences: for assets with positive idiosyncratic skewness (almost all stocks), disagreement generates overpricing, producing short-leg dominance through a mechanism that operates independently of both aggregate sentiment and probability weighting.

Notably, shorting is feasible and cost-effective for our sample's stocks. D'Avolio (2002) documents low shorting costs for most U.S. stocks (91% with annual fees $<$1%, value-weighted mean of 17 bps), with higher costs concentrated in small, illiquid names. Nagel (2005) uses institutional ownership (IO) as a short-sale constraint proxy, finding stronger return predictability in low-IO stocks; excluding the bottom 20% of IO yields weaker anomalies but easier shorting. Engelberg, Reed, and Ringgenberg (2018) note elevated costs and risks of maintaining short positions in the smallest size quintile. Our sample's restrictions (2+ analysts, price $>$ \$5) already omit the smallest, most illiquid, retail-heavy stocks, yielding relatively large stocks (average market cap 63% larger than unrestricted ones; the average size in our 10th DIS decile is larger than about 60% of unrestricted stocks). Moreover, over 99% of stocks in our sample have positive short interest, indicating that they are indeed shortable.

Further excluding the union of the 20% most illiquid and 20% most retail-owned stocks on breakpoints from the CRSP-COMPUSTAT universe (price $>$ \$5) removes only 9.4% of our observations (vs. 27.9% in the broader universe). DIS-decile spread alphas are robust to this exclusion: e.g., annualized CAPM-α stays at 21.5% (t-stat. 4.95), average ILLIQ in the 10th DIS decile drops from 0.46 (median 0.02) to $<$0.07 (median $<$0.01), and average IO rises from 0.55 to 0.65. See Table H.3 for other alphas under this exclusion, comparable to Table 3, and Tables H.9 and H.10 for average DIS-decile characteristics, with and without the exclusion. Because the spread portfolio is value-weighted, but the reported ILLIQ is an equal-weighted average that skews higher, effective illiquidity is lower than these already low reported averages (median ILLIQ 0.02 in the 10th DIS decile). DIS portfolios also exhibit relatively low turnover due to their value-weighted construction and the persistence of DIS decile assignments, especially in the extreme deciles (Table H.11).

Given our sample's composition, the alphas are economically large in magnitude, among the largest documented for disagreement proxies. For instance, Bali et al. (2025) employ a machine-learning-based disagreement measure on a broader sample that includes smaller and more illiquid stocks. They report the highest annualized CAPM-α among extant disagreement measures: about 17%. Direct comparison requires caution given sample differences. The median illiquidity in our 10th DIS decile (0.02; our short leg) is below that of 75% of their sample and less than one-eighth of the median in their 10th disagreement decile (0.17; their short leg), and our disagreement measure is also more persistent, implying lower turnover. That DIS generates alphas of comparable or greater magnitude on a sample that excludes the stocks where anomaly profits are typically most concentrated is consistent with the model's structural foundation isolating disagreement-related return variation rather than liquidity-related return patterns. Compared to other proxies evaluated on our sample, our CAPM-α exceeds that of an equivalent analyst forecast dispersion-sorted portfolio by 8.9%, an IVOL decile spread by 6.1%, and a short-interest decile spread by 13.1% (Table H.4).

The magnitude and persistence of these alphas raise a natural question: why does arbitrage not eliminate the overpricing that our model predicts? Our framework provides a structural answer that does not appeal to exogenous frictions (formally tested in Section 5.5). In the standard limits-of-arbitrage framework (Shleifer and Vishny, 1997; Pontiff, 2006), mispricing persists because arbitrageurs face costs or risks that prevent them from fully correcting it. Our model identifies a distinct mechanism. Even unconstrained arbitrageurs who short overpriced, high-disagreement stocks hold a position in a negatively skewed asset, because the short side of a positively skewed stock is itself negatively skewed. The demand convexity documented in Section 2.4.1 implies that such positions are unattractive relative to their expected payoff because the same prudence motive that generates overpricing in equilibrium also discourages the arbitrage trades that would correct it. This is not a friction-based limit but a preference-based one. Rational, unconstrained agents with standard preferences optimally choose smaller short positions than the magnitude of mispricing would warrant under symmetric payoffs. The result is that disagreement-driven overpricing in positively skewed stocks is partially self-insulating, and the alphas documented above are a prediction of the model.

Table 4: Fama and MacBeth (1973) Cross-Sectional Regressions

	(1) $EXRET_{t+1}$	(2) $EXRET_{t+1}$	(3) $EXRET_{t+1}$	(4) $EXRET_{t+1}$
DIS	-1.55^{***}	-0.97^{***}	-1.36^{***}	-0.96^{***}
	(-3.98)	(-3.23)	(-4.59)	(-3.27)
SIR		-3.75^{***}		-3.45^{**}
		(-2.64)		(-2.46)
IVOL		-0.04		-0.05
		(-0.56)		(-0.94)
ISKEW		-0.38^{**}		-0.23
		(-2.16)		(-1.39)
AFD		-0.16^{**}		-0.14^{**}
		(-2.22)		(-2.16)
LnSIZE			-0.04	-0.09^{**}
			(-1.00)	(-2.08)
BTM			0.09	0.06
			(0.58)	(0.45)
MOM			0.34	0.30
			(1.62)	(1.41)
BETA			-0.07	-0.03
			(-0.55)	(-0.26)
ILLIQ			-0.47	-0.02
			(-1.33)	(-0.07)
Constant	0.82^{***}	1.29^{***}	0.98^{**}	1.71^{***}
	(2.94)	(4.91)	(2.07)	(3.26)
Observations	719,940	719,940	701,159	701,159

Notes: This table displays the regression coefficients and t-statistics from a monthly Fama and MacBeth (1973) regression analysis conducted on excess returns, adhering to the methodologies in Fama and French (1993) and Carhart (1997). These regressions are performed against a lagged measure of disagreement and various lagged stock and firm characteristics. The sample ranges from January 1994 to December 2022. The coefficients on DIS, AFD, and MOM are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively, under a single test. For detailed definitions of the variables, please refer to Appendix B.

5.2. Fama and MacBeth (1973) Cross-Sectional Regressions

Next, we perform cross-sectional predictive regressions of excess returns (EXRET) at month $t+1$ in the spirit of Fama and MacBeth (1973). Table 4 reports regression coefficients with Newey and West (1987) adjusted t-stats in parentheses. Column (1) reveals a negative DIS impact on future EXRET (coeff. -1.55, t-stat. -3.98). Because DIS coefficients are reported multiplied by 100, the predicted monthly excess return spread equals the DIS spread times the reported coefficient divided by 100. For instance, from Table 10, the DIS spread between high and low deciles ($66.67 = 67.67 - 1.00$) multiplied by the DIS coefficient ($-1.55/100$) yields a predicted monthly return spread of approximately 1.03% (or 12.4% annualized), aligning with the decile-portfolio excess return spread in Table 3.

Column (2) shows that this negative relation holds after controlling for DIS's major components, with DIS still predicting lower returns with the coefficient somewhat reduced but still over three standard errors from zero, despite relatively high correlations with IVOL and ISKEW. DIS absorbs most of IVOL's negative variation, while ISKEW, AFD, and SIR remain negative predictors. We explore the relationship between DIS and IVOL in greater detail in Section 5.3.

Columns (3) and (4) add established return predictors (ILLIQ, LnSIZE, BTM, MOM, BETA) and then combine these with DIS's components; DIS's predictive power persists across both specifications with t-statistics exceeding three in magnitude. The results imply DIS's marginal contribution to a low-minus-high decile spread is 7.7% annualized, net of other cross-sectional determinants including extant disagreement proxies.

5.3. DIS and the Idiosyncratic Volatility Puzzle

Given the evidence that DIS predicts returns in the cross section (Sections 5.1-5.2), we now examine its relationship to the individual proxy most closely linked to latent disagreement in our model, idiosyncratic volatility. Ang et al. (2006) document a strong negative relation between IVOL and subsequent stock returns, which has proven difficult to rationalize. A substantial literature has sought explanations for the IVOL effect; Hou and Loh (2016) offer a comprehensive review, concluding that no approach fully resolves the puzzle. In our full sample, which comprises larger and more liquid stocks than those typically studied in the IVOL literature, we find a significant (at the 5% level) negative bivariate relation between IVOL and subsequent returns. As noted above, when DIS is included in the regression, it absorbs much of the variation.

To further investigate DIS's absorption of IVOL predictability, we examine smaller stocks in our sample using monthly SIZE breakpoints from the broader CRSP/COMPUSTAT universe (price $>$ \$5). Lower breakpoints yield more restrictive subsamples: e.g., below the midpoint corresponds to our sample's smallest 30.3%, and below the 25th percentile to the smallest 7.9%. Table 5 reports regressions for DIS and IVOL, individually and jointly, across four subsamples: full, smallest half, third, and quarter. As size decreases, IVOL's individual predictive strength grows (t-stat. from -2.14 full-sample to -4.70 in the smallest quarter), consistent with prior work. DIS shows a similar pattern. Yet, even as IVOL's effects

Table 5: Fama and MacBeth (1973) Cross-Sectional Regressions with DIS and IVOL for Subsamples Based on CRSP/COMPUSTAT Market Size Thresholds

Sample	DIS		IVOL		N	$N\%$
Full Sample	-1.55^{***}	(-3.98)			719,940	100.0%
			-0.16^{**}	(-2.14)		
	-1.26^{***}	(-3.64)	-0.08	(-1.00)		
Smallest Half	-1.51^{***}	(-4.65)			218,352	30.3%
			-0.16^{**}	(-2.45)		
	-1.56^{***}	(-4.35)	0.01	(0.18)		
Smallest Third	-1.46^{***}	(-5.03)			101,765	14.1%
			-0.23^{***}	(-3.62)		
	-1.18^{***}	(-2.96)	-0.06	(-0.72)		
Smallest Quarter	-1.74^{***}	(-5.58)			56,912	7.9%
			-0.32^{***}	(-4.70)		
	-1.47^{***}	(-3.01)	-0.08	(-0.85)		

Notes: This table displays the regression coefficients and t-statistics from a monthly Fama and MacBeth (1973) regression analysis conducted on excess returns, adhering to the methodologies in Fama and French (1993) and Carhart (1997). These regressions are performed against a lagged composite disagreement DIS, lagged IVOL, and a constant (not reported) for various subsamples based on monthly SIZE breakpoints from common stocks in CRSP/COMPUSTAT having price above \$5. N denotes the number of observations and $N\%$ indicates the observations count relative to our original sample that includes all stocks with 2+ analysts for which DIS is measurable. The sample ranges from January 1994 to December 2022. The coefficients on DIS are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively, under a single test. For detailed definitions of the variables, please refer to Appendix B.

strengthen in smaller stocks, DIS reliably absorbs its effects in that IVOL is insignificant in all size samples when DIS is included in the regression.

In Section 5.4, we further discuss how DIS reliably absorbs IVOL's predictability across different macro regimes. In Section 5.5, we document that IVOL's return predictability is also friction-dependent, whereas DIS's predictability appears to be robust to frictions, identifying another source of contaminating variation in IVOL that DIS's equilibrium structure appears to filter out. Tests in Section 5.7 show how DIS captures additional disagreement effects beyond what is detected by IVOL.

These results suggest a disagreement-based explanation for IVOL's return predictability. DIS's nonlinear equilibrium structure appears to be able to extract the disagreement-related component of IVOL's signal, while filtering the regime-dependent and friction-dependent components that contaminate IVOL as a standalone measure (Section 5.4-5.5). Our theoretical model, underpinning DIS, links IVOL closely to disagreement, incorporating it prominently in our composite proxy. Together, these findings provide a micro-founded view of IVOL's standalone effects arising in large part as an investor disagreement correlate. Relatedly, Bali et al. (2025) report that their machine-learning-based disagreement measure is also most highly correlated with IVOL among all inputs, consistent with our model's prediction that any well-specified measure of asset-specific disagreement should load heavily on idiosyncratic volatility.

5.4. Investor Disagreement and Macroeconomic Regimes

The absorption result of Section 5.3 raises a natural question: is DIS's predictive power genuinely idiosyncratic, or does it partly reflect aggregate variation that may contaminate the information in both DIS and IVOL? A key question for interpreting disagreement measures is whether they capture genuinely idiosyncratic, firm-level variation or instead reflect aggregate phenomena such as macroeconomic uncertainty or market-wide volatility, in which case their return predictability could partly reflect compensation for common risk rather than the firm-specific overpricing mechanism our model describes. The multi-asset separation result of Appendix A sharpens this into a testable restriction: a well-specified idiosyncratic disagreement measure should predict returns independently of the macroeconomic regime, since aggregate variation is absorbed into factor premia and does not enter the idiosyncratic pricing channel. We show that DIS is consistent with this restriction, but the individual proxies are not.

We split the sample into high and low states based on the median of eight aggregate indicators: the CBOE VIX Index; Volatility-of-Volatility, the realized volatility of daily VIX changes over the prior month (Bekaert and Hoerova, 2014); the CATFIN systemic risk measure of Allen, Bali, and Tang (2012); the Financial, Real, and Macroeconomic Uncertainty indices of Jurado, Ludvigson, and Ng (2015); the partial-least-squares Sentiment Index of Kelly and Pruitt (2013); and Aggregate IVOL, the value-weighted average of stock-level idiosyncratic volatilities. Within each regime, we estimate Fama and MacBeth (1973) regressions of one-month-ahead excess returns on each disagreement proxy with controls (BETA, LnSIZE, BTM, MOM). Table 6 reports the coefficient and t-statistic for each proxy (DIS, IVOL, AFD, SIR).

DIS is a significant negative predictor at the 5% level or better in both high and low

Table 6: **Return Predictability in High or Low Macroeconomic Regimes**

State	DIS		IVOL		AFD		SIR	
Macro Indicator	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
High								
VIX Index	-1.16^{**}	(-2.54)	-0.07	(-0.80)	-0.26^{*}	(-1.90)	-1.33	(-0.46)
Volatility-of-Volatility	-0.67^{**}	(-2.38)	-0.10	(-1.33)	-0.27^{**}	(-2.27)	-3.37^{*}	(-1.80)
CATFIN	-1.40^{***}	(-3.66)	-0.19^{**}	(-2.31)	-0.39^{***}	(-3.04)	-5.34^{***}	(-2.80)
Financial Uncertainty	-0.90^{**}	(-2.37)	-0.13	(-1.35)	-0.28^{**}	(-2.22)	-2.60	(-0.86)
Real Uncertainty	-0.99^{***}	(-3.30)	-0.09	(-1.13)	-0.23^{**}	(-2.48)	-0.78	(-0.27)
Macro Uncertainty	-1.34^{***}	(-3.54)	-0.15	(-1.65)	-0.28^{**}	(-2.45)	-2.01	(-0.70)
PLS Sentiment Index	-1.35^{***}	(-2.95)	-0.12	(-1.26)	-0.28^{*}	(-1.92)	-2.66	(-0.92)
Aggregate IVOL	-1.62^{***}	(-3.37)	-0.15	(-1.54)	-0.32^{**}	(-2.15)	-3.82	(-1.25)
Low								
VIX Index	-1.60^{***}	(-4.29)	-0.22^{***}	(-3.99)	-0.22^{**}	(-2.49)	-6.45^{***}	(-5.42)
Volatility-of-Volatility	-2.09^{***}	(-4.70)	-0.20^{***}	(-2.62)	-0.21^{*}	(-1.95)	-4.41^{*}	(-1.88)
CATFIN	-1.36^{***}	(-3.25)	-0.10	(-1.39)	-0.09	(-0.96)	-2.43	(-0.96)
Financial Uncertainty	-1.86^{***}	(-4.49)	-0.16^{***}	(-3.27)	-0.20^{**}	(-2.03)	-5.18^{***}	(-4.60)
Real Uncertainty	-1.77^{***}	(-3.48)	-0.20^{**}	(-2.47)	-0.25^{*}	(-1.79)	-6.99^{***}	(-5.00)
Macro Uncertainty	-1.42^{***}	(-3.10)	-0.15^{**}	(-2.30)	-0.20^{*}	(-1.69)	-5.77^{***}	(-4.17)
PLS Sentiment Index	-1.54^{***}	(-3.60)	-0.15^{**}	(-2.10)	-0.20^{**}	(-2.05)	-3.81^{**}	(-2.34)
Aggregate IVOL	-1.14^{***}	(-3.67)	-0.14^{***}	(-3.10)	-0.16^{**}	(-2.46)	-3.95^{***}	(-4.08)

Notes: This table displays the regression coefficient and t-statistic of each disagreement proxy (DIS, IVOL, AFD, or SIR; in columns) from a monthly Fama and MacBeth (1973) regression analysis performed on excess returns against the disagreement proxy, standard cross-sectional controls (BETA, LnSIZE, BTM, and MOM), and a constant (not reported). Regressions are performed for various subsamples based on median breakpoints of eight macro indicators: VIX; Volatility-of-Volatility following Bekaert and Hoerova (2014); CATFIN, the systemic risk measure of Allen, Bali, and Tang (2012); the Financial Uncertainty, Real Uncertainty, and Macroeconomic Uncertainty indexes of Jurado, Ludvigson, and Ng (2015); the PLS Sentiment Index of Kelly and Pruitt (2013); and Aggregate IVOL. For each indicator, high (low) states are defined as months in which the indicator is above (below) its full-sample median. The coefficients on DIS and AFD are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively, under a single test. For detailed definitions of the variables, please refer to Appendix B.

states across all eight indicators (16 of 16 total states), with coefficients broadly stable across regimes. The individual proxies are less consistent. IVOL is significant at the 5% level or better in low states for 7/8 indicators, but only 1/8 high-state subsamples (8 of 16 total states). SIR is significant at the 5% level or better in 6/8 low-state subsamples, but only 1/8 high-state subsamples (7 of 16 total states). AFD is more consistent, with significance at the 5% level or better across 6/8 high states and 4/8 low states (10 of 16 total states), though at uniformly less significance than DIS.

In further tests, regressing excess returns jointly on DIS, IVOL, and controls within each macro regime reveals that DIS absorbs or weakens IVOL's predictability in all high and low regimes except the VIX low state (see Table H.7 in Appendix H), extending the absorption result of Section 5.3 to the time-series dimension, but through two distinct mechanisms. In high-uncertainty states, IVOL is already non-predictive on its own, so its insignificance in the joint regression reflects a lack of independent content. In low-uncertainty states, IVOL carries a meaningful standalone signal that is nonetheless largely subsumed by DIS when the two enter jointly. This decomposition refines the potential micro-founded resolution of the IVOL puzzle offered in Section 5.3: IVOL's predictability is the regime-dependent portion of a broader disagreement effect, present when aggregate noise is low enough for IVOL to proxy reasonably for latent disagreement, and absent when aggregate volatility contamination severs that link.

The pattern among individual proxies has a structural explanation. In periods of aggregate stress, idiosyncratic volatility and short interest rise broadly across the cross section, compressing the informative variation among stocks and making it difficult to distinguish genuinely high-disagreement firms from those whose values are elevated for macro-related reasons. The cross-sectional predictive slope degrades because the *ranking* becomes contaminated. DIS's multiplicative structure provides insulation by requiring corroboration across components: a stock's position in the cross-sectional ranking is driven by the joint configuration of IVOL, SIR, and AFD rather than by any single component. Because analyst forecast dispersion is driven primarily by firm-specific information uncertainty and is less sensitive to market-wide volatility, this co-occurrence requirement preserves the cross-sectional variation that distinguishes genuinely high-disagreement firms even when aggregate conditions inflate the individual components broadly.

These results help provide a structural account of the conflicting empirical findings across samples using different proxies. Each proxy captures a partial, macro-uncertainty-contaminated slice of latent disagreement whose predictive content varies with the macro environment of the sample period. A researcher using IVOL in a low-volatility period will reach systematically different conclusions than one using SIR in a high-uncertainty period, not because the underlying phenomenon differs, but because the proxies are contaminated in ways that degrade their cross-sectional predictive content, whereas DIS's multi-component structure appears to mitigate this degradation.

Our use of regime-conditioning is the contrapositive logic of prior work such as that of Stambaugh, Yu, and Yuan (2012), who test whether a proxy's predictive power varies with regimes in a theoretically predicted direction. We instead assess whether it does not vary. This diagnostic principle may prove useful in other settings where researchers must distinguish idiosyncratic from aggregate signals using imperfect proxies. An empirical proxy that purports to capture a firm-specific, idiosyncratic phenomenon should, under the logic of factor-structure separation, exhibit return predictability that is consistent across aggregate macroeconomic states.

5.5. The Role of Short-Selling Frictions

Sections 5.3 and 5.4 have provided evidence suggesting that DIS largely subsumes IVOL's predictability and behaves consistently with a measure of idiosyncratic disagreement. We now ask through what economic mechanism DIS's predictive power operates, and in particular whether it reflects the frictionless skewness channel identified by our model or the friction-based channel of Miller (1977). Local demand convexity can arise from two sources: positive skewness, as in our model (Section 2.4.1), or short-sales frictions. For example, a prohibition on short sales would produce a kink in demand at the investor's belief, generating local convexity even without skewness. Since Miller (1977), the dominant explanation for disagreement-driven overpricing has relied on the latter source: short-sale constraints prevent pessimists from fully expressing their views, biasing prices toward optimists' valuations. Hong and Sraer (2016) show that overpricing can arise even without binding constraints if bearish investors simply hold zero rather than negative positions, though the mechanism still relies on the absence of short positions. Our model operates via a distinct channel in which the combination of heterogeneous beliefs and positive payoff skewness is sufficient to generate overpricing even when investors do short. They simply short less aggressively than optimists buy, because the short side of a positively skewed asset is itself negatively skewed. Both channels may contribute in practice. The question we address in this section is whether the frictionless channel is empirically operative, and whether the two channels can be distinguished in the data.

We follow Nagel (2005) in using institutional ownership proportion (IO) as an inverse proxy for short-selling frictions and estimate the Fama and MacBeth (1973) interaction regression

$$\text{EXRET}_{t+1} = \alpha + \beta_D \cdot D + \beta_{1-\text{IO}} \cdot (1 - \text{IO}) + \beta_{D\times(1-\text{IO})} \cdot (D \times (1 - \text{IO})) + \beta_Z' Z + \varepsilon, \quad (19)$$

for each disagreement measure D among DIS, IVOL, AFD, and SIR, where Z includes BETA, LnSIZE, BTM, and MOM. Because the data are not demeaned and the interaction term is included, β_D is correctly interpreted as the marginal effect of D as institutional ownership approaches 100% (Balli and Sørensen, 2013). Stocks with full or near-full institutional ownership are those for which locating shares to borrow is least costly and short-selling frictions are minimally binding. If the disagreement–return relationship operated primarily through constraints on short-selling, the marginal effect of DIS should be economically and statistically attenuated among these stocks.

This specification nests both channels and generates distinct empirical predictions for each. Under the friction-based channel alone, the disagreement–return relationship should intensify as frictions increase: the interaction coefficient $\beta_{D\times(1-\text{IO})}$ should be significantly negative, and the level effect as institutional ownership approaches 100%, β_D, should be zero, since the mechanism has no force when constraints are absent. Under the frictionless skewness channel, the relationship should be present even approaching full institutional ownership: β_D should be significantly negative, and the interaction should be zero or at least not necessary to generate the effect. The two channels are not mutually exclusive; both could appear simultaneously in the same measure. The empirical question is which pattern each measure exhibits.

Table 7 reports the results for all four measures. DIS is the only measure with a statistically significant marginal effect as institutional ownership approaches 100%: -2.46 ($t = -3.12$) unconditionally and -1.98 ($t = -3.09$) with standard cross-sectional controls, both significant at the 1% level. These magnitudes are comparable to the unconditional Fama–MacBeth results of Table 4, suggesting that the predictive content of DIS is not attenuated as short-selling frictions diminish. The interaction of DIS with $(1 - \text{IO})$ is statistically indistinguishable from zero in both specifications. This pattern is consistent with the frictionless skewness channel, for which the disagreement–return relationship operates through demand asymmetry that does not depend on constrained short-selling.

Table 7: Marginal Effect of Measure D as Institutional Ownership Approaches 100%: Fama and MacBeth (1973) Cross-Sectional Interaction Regressions

	D = DIS		D = IVOL		D = AFD		D = SIR	
β_D	-2.46^{***}	-1.98^{***}	-0.00	0.00	0.01	0.03	-0.53	-0.51
	(-3.12)	(-3.09)	(-0.00)	(0.02)	(0.07)	(0.24)	(-0.31)	(-0.32)
$\beta_{1-\text{IO}}$	-0.12	-0.18	0.70^{***}	0.65^{***}	-0.44^{**}	-0.39^{*}	-0.17	-0.12
	(-0.64)	(-0.98)	(2.72)	(2.68)	(-2.12)	(-2.03)	(-0.67)	(-0.52)
$\beta_{D\times(1-\text{IO})}$	0.81	0.31	-0.41^{***}	-0.40^{***}	-0.52	-0.44	-15.84^{***}	-13.99^{***}
	(0.92)	(0.39)	(-4.23)	(-4.16)	(-1.61)	(-1.61)	(-4.05)	(-3.99)
Controls	No	Yes	No	Yes	No	Yes	No	Yes

Notes: This table reports estimated coefficients for main effects and interaction terms with t-statistics in parentheses from Fama and MacBeth (1973) cross-sectional interaction regressions of subsequent excess returns on disagreement measure D (among DIS, IVOL, AFD, SIR), non-institutional ownership $1-\text{IO}$, and their interaction $D \times (1-\text{IO})$, first unconditionally (no controls) in the left-hand column for each measure and second conditionally on a vector of standard cross-sectional control variables Z in the right-hand column,

$$\text{EXRET}_{t+1} = \alpha + \beta_D \cdot D + \beta_{1-\text{IO}} \cdot (1-\text{IO}) + \beta_{D\times(1-\text{IO})} \cdot (D \times (1-\text{IO})) + \beta_Z' Z + \varepsilon,$$

where Z includes BETA, LnSIZE, BTM, and MOM, and β_Z is their corresponding coefficient vector; and where ε is an error term. The coefficient β_D represents the marginal effect of measure D when institutional ownership is 100% ($\beta_D = \frac{\partial \text{EXRET}_{t+1}}{\partial D}$ as IO $\nearrow 1$). The sample ranges from January 1994 to December 2022. Coefficients on DIS, AFD, and their interactions are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively, under a single test. For detailed definitions of the variables, please refer to Appendix B.

The individual proxies exhibit a different pattern. Neither IVOL, AFD, nor SIR generates a significant marginal effect as institutional ownership approaches 100%. The level coefficients are economically negligible and statistically insignificant across all specifications. In contrast, IVOL and SIR exhibit highly significant interactions with $(1-\text{IO})$. These patterns are consistent with the friction-based channel. The return predictability of these proxies is concentrated in stocks where short-selling constraints are more binding and vanishes where frictions are absent. AFD exhibits a marginally insignificant interaction, consistent with its distinct role in our model as an amplifier of existing disagreement, a public information effect, rather than a direct measure of the trading-based mechanism through which frictions operate.

Two channels of disagreement pricing. Taken together, the results in Table 7 provide evidence that disagreement generates overpricing through at least two distinct channels that coexist in the data and are empirically separable. These contrasting patterns emerge from a common specification applied to the same sample, providing identification that would not be available from studying any single measure in isolation. This identification is as clean as the institutional-ownership proxy permits: IO approaching 100% does not literally eliminate all shorting frictions, but represents the limiting case of the standard proxy in this literature (Nagel, 2005), and the contrasting patterns across measures within this common framework provide the empirical separation. The two channels are complementary rather than competing. The results suggest that IVOL and SIR each capture a partial slice of latent disagreement that is entangled with the friction mechanism, while DIS, whose non-linear multi-component structure filters these observables, exhibits the predictive behavior the model associates with the frictionless disagreement channel. DIS generating alphas exceeding those of any individual proxy on the same sample (Section 5.1) is consistent with this interpretation as it captures a channel that is operative broadly, rather than one that is active only conditionally.

The cross-sectional evidence of this section complements the macro-regime evidence of Section 5.4 in that periods of elevated aggregate uncertainty are also periods when shorting costs tend to rise (cf. Engelberg, Reed, and Ringgenberg, 2018), yet DIS predicts returns consistently across these states while the three standalone proxies do not. That DIS is supported by both tests, friction independence and regime stability, while IVOL and SIR are not, provides dual identification support that DIS captures a fundamentally different channel from its individual components. The union of both dimensions points to the skewness-based demand asymmetry identified by the model as the operative mechanism. This refines the limits-of-arbitrage interpretation of disagreement anomalies. The binding constraint is not that arbitrageurs cannot short overpriced stocks, but that shorting a positively skewed stock exposes the arbitrageur to negatively skewed payoffs, reducing optimal position sizes below what would be needed to eliminate the mispricing. Frictions, where present, provide an additional source of overpricing, and the individual proxies' sensitivity to IO variation confirms that this additional source is real. In sum, our analysis provides new evidence that the frictionless channel operates alongside the friction-based channel.

Joint implications for the IVOL puzzle. Sections 5.3 through 5.5 jointly characterize the relationship between IVOL and latent disagreement. Our model links IVOL closely to the latent variable, and DIS largely absorbs IVOL's return predictability even in the smaller-stock subsamples where that predictability is strongest (Section 5.3). Yet IVOL's standalone predictability is both regime-dependent, present in low-uncertainty states but absent in high-uncertainty states where aggregate volatility contaminates its signal (Section 5.4), and friction-dependent, concentrated where short-selling constraints bind (Section 5.5). DIS exhibits neither dependence. The complete picture is that IVOL inherits its negative relation with future returns from its proximity to latent disagreement, but that predictability is unreliable because IVOL captures one margin of a nonlinear multi-component latent variable while its predictive content remains exposed to aggregate and friction-related variation that DIS's equilibrium structure appears to filter out. We note that these results constitute a theory-grounded account rather than a formal causal decomposition; however, the conjunction of absorption, regime diagnosis, and friction diagnosis, each predicted by the model's characterization of IVOL's relationship to latent disagreement, provides evidence that is cumulative in a way not available to prior proposed explanations. In particular, Stambaugh, Yu, and Yuan (2015) attribute the IVOL puzzle to arbitrage asymmetry in that overpricing is harder to correct than underpricing because shorting is costlier than buying, so high-IVOL stocks are where mispricing persists. Our account is complementary but structurally distinct because the asymmetry in our model arises from preferences rather than costs, and the regime and friction diagnoses identify aggregate and friction-related variation in IVOL that an arbitrage-asymmetry framework, which relies on the same shorting frictions, does not address.

5.6. Role of Nonlinearity

The preceding sections have provided evidence that DIS's predictive power is both regime-stable and friction-independent, properties not shared by individual proxies and, as we now show, not absorbed by linear combinations of the same inputs. The composite disagreement measure DIS is a nonlinear function of its constituent proxies, with the functional form guided by the model's equilibrium conditions rather than selected from a flexible family of candidate specifications. As equation (15) makes explicit, the components interact multiplicatively. Idiosyncratic skewness scales the impact of idiosyncratic volatility on the equilibrium price bias, while analyst forecast dispersion amplifies the scale of existing heterogeneity rather than contributing additively. DIS therefore embeds a co-occurrence requirement: a stock registers

Table 8: Fama and MacBeth (1973) Cross-Sectional Regressions by DIS and Linear Composite Disagreement Alternatives

	(1) EXRET$_{t+1}$	(2) EXRET$_{t+1}$	(3) EXRET$_{t+1}$	(4) EXRET$_{t+1}$	(5) EXRET$_{t+1}$
DIS	-1.55^{***}	-1.20^{***}	-1.54^{***}	-1.01^{***}	-1.70^{***}
	(-3.98)	(-4.31)	(-4.59)	(-3.68)	(-4.83)
DEW		-0.08			
		(-1.39)			
DEW+			-0.04		
			(-0.79)		
DPCA				-0.20^{*}	
				(-1.80)	
DPCA+					-0.02
					(-0.30)
Constant	0.82^{***}	1.25^{***}	1.05^{***}	0.74^{***}	0.80^{***}
	(2.94)	(4.55)	(3.64)	(2.59)	(2.84)
Observations	719,940	719,940	717,344	719,940	717,344

Notes: This table displays the regression coefficients and t-statistics from a monthly Fama and MacBeth (1973) regression analysis conducted on excess returns, adhering to the methodologies in Fama and French (1993) and Carhart (1997). These regressions are performed against a lagged measure of disagreement and lagged alternative linear disagreement measures. The sample ranges from January 1994 to December 2022. DEW is a linear disagreement score constructed as follows: in each month, assign each stock to a decile based on each of AFD, IVOL, and SIR, and then take a simple average of these decile ranks. DEW+ is constructed similarly, except deciles are formed on AFD, IVOL, and ASIR, as well as four disagreement variables BIDASK, SUV, DTO, and DBREADTH. DPCA is the first component of a principal component analysis based on AFD, IVOL, and SIR. DPCA+ is the first component of a principal component analysis based on AFD, IVOL, ASIR, BIDASK, SUV, DTO, and DBREADTH. The specifications including DEW+ and DPCA+ have fewer observations because the additional variables (BIDASK, SUV, DTO, and DBREADTH) have more limited data availability than the three core proxies. The coefficients on DIS are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively, under a single test. For detailed definitions of the variables, please refer to Appendix B.

high disagreement only when multiple inputs are simultaneously elevated in proportions governed by their model-determined interactions.

To test whether this nonlinear structure matters empirically, we construct two families of linear alternatives. The first are equally weighted composites: DEW, the average of monthly decile ranks of AFD, IVOL, and SIR; and DEW+, which extends this to seven proxies by using abnormal short interest (ASIR) in place of SIR, and by additionally incorporating the high-low bid-ask spread (BIDASK), changes in mutual fund ownership (DBREADTH), and market-adjusted turnover changes (DTO), and standardized unexplained volume (SUV). The second are principal-component-based measures: DPCA, the first principal component of AFD, IVOL, and SIR (capturing 40% of joint variation); and DPCA+, its seven-proxy analogue (capturing 26%). Together these span the range from the most atheoretic equal-weighting scheme to the variance-maximizing principal component approach, and the expanded versions address the concern that the result is sensitive to the choice of three input proxies.

Table 8 reports Fama and MacBeth (1973) regressions of excess returns on DIS alongside each linear alternative in turn. In every specification, DIS remains a highly significant negative predictor, with coefficients ranging from -1.01 to -1.70 and t-statistics between -3.68 and -4.83. By contrast, none of the linear alternatives achieves conventional significance: DEW and DEW+ are insignificant at any standard level, and DPCA reaches only marginal significance ($t = -1.80$) while DIS alongside it remains strongly significant ($t = -4.31$). The broader seven-proxy versions perform similarly to their three-proxy counterparts. The return predictability embedded in DIS is not recoverable by these linear combinations of the same underlying information.

The model provides a structural account of this result. A linear composite weights its inputs additively, assigning high disagreement scores whenever any subset of proxies is elevated, including when IVOL and SIR are broadly inflated by aggregate stress without corresponding elevation in AFD. DIS's multiplicative structure requires corroboration across components, filtering the aggregate-driven signals that degrade linear alternatives in high-uncertainty states. The regime-stability of DIS documented in Section 5.4 and its dominance over linear alternatives here are two manifestations of the same structural feature of (15): the nonlinear structure is essential to DIS's performance, not incidental to it.

5.7. Detecting Disagreement Undetected by Extant Measures

In this section, we assess the degree to which and the nature of how DIS detects disagreement not detected by extant disagreement measures (IVOL, AFD, and SIR), i.e., when the extant measure approaches zero. For instance, what disagreement-associated characteristics might DIS be picking up that IVOL misses?

In Table 9, we report the estimated marginal effect β_{DIS} of composite disagreement DIS on characteristic Y when extant disagreement measure D approaches zero detection of disagreement using the interaction regression

$$Y = \alpha + \beta_{\text{DIS}} \cdot \text{DIS} + \beta_D \cdot D + \beta_{\text{DIS}\times D} \cdot (\text{DIS} \times D) + \beta_Z' Z + \varepsilon, \tag{20}$$

Table 9: Marginal Effect of DIS as Extant Disagreement Measure D Approaches Zero

	D = IVOL		D = AFD		D = SIR	
Y	DIS coeff.	t-stat.	DIS coeff.	t-stat.	DIS coeff.	t-stat.
EXRET_{t+1}	-2.384^{***}	(-3.46)	-1.531^{***}	(-4.26)	-1.074^{***}	(-3.00)
IVOL			0.044^{***}	(10.94)	0.042^{***}	(12.10)
AFD	0.784^{***}	(10.22)			0.330^{***}	(10.14)
SIR	-0.008	(-0.88)	0.017^{***}	(5.53)		
ISKEW	0.027^{***}	(13.75)	0.010^{***}	(19.01)	0.009^{***}	(19.54)
SIZE	-0.079^{***}	(-9.31)	-0.041^{***}	(-9.57)	-0.044^{***}	(-11.16)
BTM	0.111^{**}	(2.44)	-0.049^{*}	(-1.90)	-0.106^{***}	(-4.89)
MOM	-0.799^{***}	(-5.41)	-0.141^{**}	(-2.10)	-0.162^{***}	(-2.95)
BETA	0.014^{***}	(9.53)	0.007^{***}	(8.73)	0.006^{***}	(8.22)
ILLIQ	0.017^{***}	(4.10)	0.010^{***}	(3.28)	0.013^{***}	(3.89)
RMAX	0.019^{***}	(8.73)	0.052^{***}	(10.52)	0.050^{***}	(11.64)

Notes: This table reports the estimated marginal effect β_{DIS} of composite disagreement DIS on characteristic Y when extant disagreement measure D approaches zero ($\beta_{\text{DIS}} = \frac{\partial Y}{\partial \text{DIS}}$ as $D \searrow 0$) and its Newey and West (1987) adjusted t-statistic from the Fama and MacBeth (1973) cross-sectional interaction regression of characteristic Y against DIS, an extant disagreement measure D, and their interaction (DIS $\times$ D), as well as a vector of standard cross-sectional control variables Z,

$$Y = \alpha + \beta_{\text{DIS}} \cdot \text{DIS} + \beta_D \cdot D + \beta_{\text{DIS}\times D} \cdot (\text{DIS} \times D) + \beta_Z' Z + \varepsilon,$$

which are reported (by column) for each extant disagreement measure D among IVOL, AFD, and SIR; and (by row) for each characteristic Y among EXRET_{t+1}, IVOL, AFD, SIR, ISKEW, SIZE, BTM, MOM, BETA, ILLIQ, and RMAX; where Z includes BETA, LnSIZE, BTM, and MOM (control not included if same variable as Y), and β_Z is their corresponding coefficient vector; and where ε is an error term. The sample ranges from January 1994 to December 2022. Coefficients are multiplied by 100 for readability when Y = EXRET_{t+1}, SIR, and BTM. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively, under a single test. For detailed definitions of the variables, please refer to Table 1 and Appendix B.

for each extant disagreement measure D among IVOL, AFD, and SIR, and for each characteristic Y among EXRET_{t+1}, IVOL, AFD, SIR, ISKEW, SIZE, BTM, MOM, BETA, ILLIQ, and RMAX; where Z is a vector of standard cross-sectional control variables including BETA, LnSIZE, BTM, and MOM (control not included if same variable as Y).

The marginal effect of DIS on Y is $\beta_{\text{DIS}} + \beta_{\text{DIS}\times D} \cdot D$, a linear function of the extant disagreement proxy D. Because D is not de-meaned, as D approaches zero, the marginal effect is simply β_{DIS}, which is reported in Table 9 (Balli and Sørensen, 2013).

Table 9 shows that DIS is picking up disagreement associated with specific firm characteristics that extant disagreement measures fail to detect in the limit as it goes to zero. That is, in the limiting absence of disagreement detected by an extant disagreement measure (IVOL, AFD, and SIR), DIS exhibits a consistent negative relationship with EXRET_{t+1}, SIZE, and

MOM, and a consistent positive relationship with ISKEW, BETA, ILLIQ, and RMAX.[18] These relationships suggest that DIS detects a specific dimension of return-predictive variation consistent with disagreement, which would be undetected by each of the extant measures in the limit, that is fundamentally linked to lower future returns for relatively smaller, less trending, more positively skewed, riskier, and relatively more illiquid firms (among the relatively large and liquid firms in our sample restricted to 2+ analysts).

5.8. Characteristics of Disagreement DIS

Lastly, we look at characteristics associated with DIS. Table 10 reports monthly equally weighted average characteristics within DIS deciles. Consistent with our model's predictions, we observe an increase in individual components of DIS (SIR, IVOL, ISKEW, and AFD) as DIS rises. Moreover, stocks associated with higher DIS are characterized by relatively lower market capitalization (SIZE), among the larger stocks of our dataset restricted to 2+ analysts, and are consistent with potential market undervaluation (indicated by higher BTM ratios). Stocks with higher DIS also exhibit less momentum, although the relationship is statistically weaker than those of other characteristics. The relationship between DIS and future returns implies that high levels of disagreement may signal potential turning points in momentum (Goulding, Harvey, and Mazzoleni, 2023).

Moreover, we find that DIS aligns intuitively with the expected relationship between firm opacity and investor disagreement. Stocks with higher DIS have more systematic risk (as evidenced by higher beta values), helping to explain why DIS-spread portfolios diverge from CAPM predictions and yield large alphas in upper DIS deciles. High DIS stocks also exhibit relatively lower levels of liquidity, among the liquid stocks of our dataset. Stocks with more lottery-like behavior (RMAX; Zhang, 2006b), also associate with higher DIS. Additionally, higher-DIS firms tend to exhibit increased financial constraints, younger age, intensified R&D expenditures, greater opinion divergence among mutual funds, and reduced volumes of Google searches, both overall and specifically investment-related (Table H.9).

[18]Table H.8 in Appendix H extends the analysis of Table 9 to additional characteristics showing further that in the absence of disagreement detected by an extant disagreement measure (IVOL, AFD, and SIR), DIS exhibits a consistent negative relationship with EXRET_{t+1}, IO, and SV, and a consistent positive relationship with KZ, OHLSON, NI-IVOL, HHI, R&D, ISS, ASC, BIDASK, and SV-INV.

Table 10: **Average Stock Characteristics in DIS Deciles**

	Low	2	3	4	5	6	7	8	9	High	H−L	t-stat.
DIS	1.00	1.59	2.12	2.76	3.63	4.93	7.12	11.19	20.25	67.67	66.67***	(8.64)
SIR	0.02	0.03	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.07	0.04***	(13.79)
IVOL	0.83	1.16	1.41	1.64	1.87	2.10	2.33	2.58	2.96	4.13	3.30***	(34.86)
ISKEW	0.45	0.51	0.55	0.60	0.65	0.73	0.85	1.00	1.19	1.54	1.09***	(13.08)
AFD	3.03	4.15	5.47	7.19	9.24	11.61	14.88	19.51	24.73	33.15	30.12***	(22.01)
SIZE	24.80	15.69	11.33	8.36	6.12	4.81	3.48	2.28	1.50	0.79	−24.01***	(−10.91)
BTM	0.52	0.50	0.50	0.50	0.51	0.53	0.55	0.59	0.62	0.66	0.14***	(4.97)
MOM	15.07	17.39	20.03	21.79	23.52	24.20	22.17	19.41	14.50	8.08	−6.99*	(−1.77)
BETA	0.74	0.86	0.93	0.99	1.05	1.12	1.18	1.23	1.30	1.42	0.69***	(15.25)
ILLIQ	0.01	0.01	0.02	0.03	0.04	0.05	0.07	0.11	0.21	0.46	0.45***	(5.18)
RMAX	1.60	2.06	2.39	2.68	2.97	3.28	3.58	3.91	4.43	5.94	4.34***	(26.94)

Notes: This table reports the average values of various stock characteristics across univariate, equally weighted portfolios sorted based on disagreement (DIS). The sample ranges from January 1994 to December 2022. The categories "Low" and "High" refer to the portfolios within the lowest and highest disagreement deciles, respectively. Newey and West (1987) adjusted t-statistics for assessing the statistical difference between the highest and lowest disagreement decile are shown in the last column. For detailed definitions of the variables, please refer to Table 1 and Appendix B.

6. Conclusion

We address the disagreement of disagreement problem: the near-zero correlations among major investor disagreement proxies that make it challenging to infer how disagreement impacts prices. Our resolution proceeds through a parsimonious frictionless equilibrium model that links all major proxies to a single latent disagreement variable, a nonlinear composite disagreement measure DIS guided by the model's equilibrium interrelationships, and a comprehensive empirical program that employs DIS as the vehicle through which the model's structural predictions are tested. The empirical evidence is cumulative and mutually reinforcing, consistent with the model's predictions about cross-sectional predictability, proxy absorption, regime invariance, friction independence, and nonlinear necessity, with each test providing identification that is distinct from the others.

Empirically, our tests are broadly consistent with the model's predictions. DIS generates 17-21% annualized alpha on a value-weighted decile spread portfolio of relatively large stocks, well above stand-alone proxies on the same sample, with alphas robust to the exclusion of the most illiquid and retail-owned stocks. Our framework provides a structural explanation for why these alphas should survive standard risk adjustments. As with other disagreement proxies, alphas are concentrated in the short leg, but our model predicts this directly through a mechanism that operates independently of other channels such as aggregate sentiment and probability weighting.

Predictions extend to several additional dimensions. The model links IVOL closely to latent disagreement, and DIS absorbs IVOL's return predictability even in subsamples of smaller stocks where that predictability is strongest. The evidence extends beyond absorption. IVOL's standalone predictability is regime-dependent and friction-dependent, while DIS's is neither, identifying two specific sources of contamination in IVOL that the model's equilibrium structure appears to filter out. Together, these results provide a micro-founded, disagreement-based account of the IVOL puzzle that not only absorbs IVOL's signal but diagnoses why that signal can be unreliable.

Our framework implies that a well-specified idiosyncratic disagreement measure should predict returns independently of macro conditions. DIS is consistent with this restriction, predicting returns with consistent predictability across high and low states of eight diverse aggregate indicators, while individual proxies exhibit regime-dependence consistent with contamination by aggregate variation. This contrast offers a structural account of why studies using different proxies across different samples can reach conflicting conclusions.

The interaction regressions with institutional ownership provide evidence of a further distinction that disagreement depresses expected returns through two complementary channels. The individual proxies, IVOL and short interest in particular, exhibit return predictability concentrated where short-selling frictions bind and absent as institutional ownership approaches 100%, consistent with the friction-based channel of Miller (1977). In contrast, DIS exhibits significant predictability as institutional ownership approaches 100% that is insensitive to the level of frictions, consistent with the frictionless skewness channel identified by our model. Both patterns emerging from a common specification on the same sample establishes their empirical separability. The evidence that the frictionless channel operates alongside the friction-based channel provides an empirical answer to a question debated largely on theoretical grounds since Miller (1977), with implications for how the field interprets the large prior literature that conditions disagreement effects on short-sale constraints.

Our model is certainly not the last word on disagreement and there are several directions that merit future investigation. Whether the magnitude of the disagreement premium varies cross-sectionally with the ratio of prudence to risk aversion across investors, remains an open empirical question. Because the ratio governs the magnitude rather than the existence of the disagreement effect, such variation would not qualitatively alter our conclusions but could provide independent evidence for the prudence mechanism. The regime-dependence diagnostic developed here has broader applicability. In any setting where a firm-level proxy is intended to capture an idiosyncratic phenomenon (e.g., information asymmetry, financial constraints, investor attention, managerial quality) the same logic applies. If the underlying phenomenon is genuinely idiosyncratic, the proxy's predictive power should be invariant to aggregate conditions under a standard factor structure. Regime-dependence of predictive power is then diagnostic of contamination by aggregate variation, not a feature of the phenomenon itself. This principle may prove useful in other contexts where researchers must distinguish idiosyncratic from aggregate signals using imperfect proxies.

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Appendix A Multiple Risky Assets

This appendix grounds the single-asset model of Section 2 in a multi-asset foundation in which asset payoffs follow a standard factor structure. We show how the single-asset model, which characterizes how idiosyncratic investor disagreement affects the equilibrium pricing of the asset's idiosyncratic component, is an exact implication of a fully specified multi-asset equilibrium under CARA utility, and a close approximation under CRRA utility to the standard cubic Taylor expansion of Kraus and Litzenberger (1976). This foundation establishes that the single-asset model's parameters are idiosyncratic statistics (residual volatility, residual skewness, and disagreement about the residual mean) rather than total-return statistics; and delivers a key theoretical implication that an empirical measure correctly capturing idiosyncratic disagreement should be orthogonal to aggregate conditions, a prediction tested and supported in Section 5. Lastly, we discuss how our approach parallels related literature that uses independent assets or independent components of asset returns to derive separable sub-optimizations (e.g., Barberis and Huang, 2008; Goulding, Santosh, and Zhang, 2023, 2025) and why our single-asset approach is particularly appropriate for disagreement research.

A.1 The Multi-Asset Framework

Consider a market with assets that follow a factor structure, in which individual asset payoffs are the sum of a factor component (loadings on one or more factors) and an idiosyncratic component (intercept plus residual). Specifically, consider a two-period market with dates $t \in \{0, 1\}$ containing $1 + K + N$ assets: a risk-free asset with price and payoff normalized to one; K factors with price p_{f_k} and payoff $\widetilde{f}_k$ at dates $t = 0$ and $t = 1$, respectively, for $k = 1, 2, \ldots, K$; and N risky assets with price p_{θ_n} and payoff

$$\widetilde{\theta}_n = \sum_{k=1}^{K} b_{n,k}\widetilde{f}_k + \widetilde{\epsilon}_n \tag{A1}$$

at dates $t = 0$ and $t = 1$, respectively, for $n = 1, 2, \ldots, N$, where $b_{n,k}$ is asset n's loading on factor k, and $\widetilde{\epsilon}_n$ is an idiosyncratic component that is mutually independent across assets and independent of all factors, with mean μ_n, variance σ_n^2, skewness s_n, and cumulative distribution function $F_{\epsilon_n}(\epsilon) = \mathbf{P}[\widetilde{\epsilon}_n \leq \epsilon]$.

The factor structure in (A1) is intentionally general. It accommodates the CAPM, the Fama-French factor models, and any other factor structure the researcher wishes to impose.

The choice of factor model governs what counts as idiosyncratic, but does not affect the form of the separation result that follows, only the numerical values of the idiosyncratic statistics.

There is a continuum of investors, indexed by $[0, 1]$, each with initial wealth w_0 and common utility function $u(w)$ satisfying $u'(w) > 0$, $u''(w) < 0$, and $u'''(w) > 0$, the same assumptions as Section 2.2. Investors trade claims on the individual assets and on the factors that underlie them. Observe that in this setup there exists a combination of trades (replicating portfolio) that replicates the idiosyncratic component of each asset, and therefore each idiosyncratic component effectively can be traded and priced. For example, a portfolio that is long an individual asset and short its factors (in quantities proportional to the loadings for that asset) generates the same payoff as the idiosyncratic component of that asset. By the law of one price, the price of this portfolio is the price of the idiosyncratic component.

The mutual independence of idiosyncratic components $\{\widetilde{\epsilon}_n\}$ is the key structural assumption. It has strong empirical support, as average cross-sectional correlations among factor-model residuals are approximately zero. It implies that the variance of a portfolio of idiosyncratic components equals the sum of their individual variances, and analogously for third central moments, both of which are used in the separation derivation below.

Moreover, it is the idiosyncratic component of each asset that investors potentially disagree about: they agree on the factor structure but disagree about each asset's idiosyncratic expected payoff. For each asset, there is a different distribution of investor beliefs regarding the asset's idiosyncratic mean (i.e., alpha). These beliefs equivalently characterize differences of opinion about individual asset expected payoffs given the factor structure. For asset n, investor disagreement is modeled exactly as in Section 2.3: investors are of I_n types with type-i_n investors believing the idiosyncratic mean is $\mu_{i_n,n} = \mu_n + D_n\Delta_{i_n,n}$, where $\widetilde{\Delta}_n$ is a zero-mean unit-variance standardized belief variable and $D_n > 0$ is the scale of disagreement for asset n. The vector $i = (i_1, i_2, \ldots, i_N)$ denotes an investor's overall type across all assets, with the relative belief variables $\{\widetilde{\Delta}_n\}$ mutually independent across assets and independent of factors.

A.2 The Separation Result

Under commonly used utility functions, expected utility optimization across multiple asset demands splits into separable problems of choosing the exposure to the factors and choosing the exposures to each asset's residuals. Our single-asset model of the main text is the reduction of the multi-asset problem to one of its separable asset residual models, each of which has the same form but varies in its parameters.

An investor of overall type i chooses demands $\{x_{f_k}\}$ for factors and $\{x_{\theta_n}\}$ for risky assets to maximize expected utility of terminal wealth. Terminal wealth is

$$\widetilde{w}_i = w_0 + \sum_{k=1}^{K} x_{f_k}(\widetilde{f}_k - p_{f_k}) + \sum_{n=1}^{N} x_{\theta_n}(\widetilde{\theta}_n - p_{\theta_n}). \tag{A2}$$

By the law of one price, which holds under no-arbitrage, the price of the claim to asset n's idiosyncratic payoff $\widetilde{\epsilon}_n$ is $p_{\epsilon_n} = p_{\theta_n} - \sum_k b_{n,k} p_{f_k}$.

Substituting (A1) into (A2) and collecting terms, terminal wealth can be rewritten as

$$\widetilde{w}_i = w_0 + \sum_{k=1}^{K} \widehat{x}_{f_k}(\widetilde{f}_k - p_{f_k}) + \sum_{n=1}^{N} \widehat{x}_{\epsilon_n}(\widetilde{\epsilon}_n - p_{\epsilon_n}), \tag{A3}$$

where $\widehat{x}_{\epsilon_n} = x_{\theta_n}$ is the demand for asset n's idiosyncratic component and $\widehat{x}_{f_k} = x_{f_k} + \sum_n x_{\theta_n} b_{n,k}$ is the effective demand for factor k. Equation (A3) rewrites the investor's wealth as the sum of payoffs from a factor portfolio and N idiosyncratic components, with the cross terms eliminated by the law of one price. See Appendix E for detailed derivations.

Proposition A.1 (Separation under CARA)**.** *Under CARA utility $u(w) = -e^{-aw}$, the investor's expected utility maximization over $\{\widehat{x}_{f_k}, \widehat{x}_{\epsilon_n}\}$ separates exactly into $1+N$ independent problems: one for the factor portfolio and one for each idiosyncratic component n. The optimization for idiosyncratic component n is*

$$\widehat{x}_{\epsilon_n} = \arg\max_{x} \mathbf{E}_i[u(w_0 + x(\widetilde{\epsilon}_n - p_{\epsilon_n}))], \tag{A4}$$

which has precisely the form of the single-asset optimization in (3) of Section 2.4, where $\widetilde{y} = \widetilde{\epsilon}_n$, $x = \widehat{x}_{\epsilon_n}$, and $p = p_{\epsilon_n}$.

Proof. Under CARA utility, terminal wealth in (A3) enters the utility function through the exponential. Because the idiosyncratic components $\{\widetilde{\epsilon}_n\}$ are mutually independent and independent of the factors, the expectation of the product of exponential terms equals the product of the expectations. The joint maximization therefore factors into independent maximizations for the factor portfolio and each idiosyncratic component. See Appendix E for detailed derivations. □

The separation is exact under CARA because the exponential utility function transforms the product of independent payoff distributions into independent optimization problems without approximation error. Equation (A4) is not an approximation of the single-asset problem in Section 2.4, it is identical to it.

Proposition A.2 (Separation under CRRA)**.** *Under CRRA utility, the separation holds approximately to the cubic Taylor expansion of Kraus and Litzenberger (1976). Specifically, if investors care about the first three moments of portfolio returns, the expected utility of a type-i investor decomposes as*

$$\mathbf{E}_i[u(\widetilde{R})] \approx V_K + \sum_{n=1}^{N} \left[\widehat{z}_{\epsilon_n} E_i[\widetilde{r}_{\epsilon_n}] - \frac{\gamma}{2} \widehat{z}_{\epsilon_n}^2 \, Var[\widetilde{r}_{\epsilon_n}] + \frac{\gamma(1+\gamma)}{6} \widehat{z}_{\epsilon_n}^3 E_i[(\widetilde{r}_{\epsilon_n} - E_{\mathbf{i}}[\widetilde{r}_{\epsilon_n}])^3] \right], \quad \text{(A5)}$$

where V_k collects terms involving only the factor portfolio, $\widehat{z}_{\epsilon_n} = \hat{x}_{\epsilon_n} p_{\epsilon_n} / w_0$ is the portfolio weight on idiosyncratic component n, $\widetilde{r}_{\epsilon_n}$ is the net return on that component, and $\widetilde{R}$ is the gross total return on the investor's portfolio. Because the choice variable $\widehat{z}_{\epsilon_n}$ appears in a separable additive term for each n, the optimization over $\widehat{z}_{\epsilon_n}$ reduces to the independent maximization of the bracketed expression for asset n, which maps to cubic order to the single-asset CRRA demand of Section 2.4.

Proof. See the detailed derivation in Appendix E, which applies the independence of idiosyncratic components to decompose the variance and third central moment of total portfolio return into sums of asset-level components. □

Note that the cubic approximation is standard in the asset pricing literature and supported empirically: expected returns have limited sensitivity to fourth and higher moments.

A.3 Implications of the Separation

The separation results of Propositions A.1 and A.2 have three implications that carry through to the main text and the empirical analysis. These implications jointly support the theoretical framework, the measurement strategy, and the interpretation of the disagreement premium as a firm-level phenomenon distinct from aggregate phenomena.

A.3.1 From Single-Asset Equilibrium to Cross-Sectional Predictions

The separation result implies that each asset n in the multi-asset economy has its own idiosyncratic equilibrium, governed by its own idiosyncratic parameters, which vary across assets in the cross section. The comparative statics of the single-asset model translate directly into testable cross-sectional predictions. For example, an asset with higher idiosyncratic disagreement, all else equal, has a higher equilibrium price and lower subsequent expected return (Proposition 2). These predictions are cross-sectional in nature as they describe differences among assets at a point in time. The single-asset comparative statics are therefore not a simplification of a cross-sectional analysis but are theoretically founded.

A.3.2 Empirical Construction Should Use Idiosyncratic Statistics

The separation constrains how the model's parameters must be operationalized empirically. σ must be computed from factor-model residuals. s must be based on idiosyncratic skewness (ISKEW; Boyer, Mitton, and Vorkink, 2010) for the same reason. Analyst forecast dispersion, which pertains to beliefs about the firm's own earnings rather than market-wide outcomes, measures the component of D related to firm-specific public information. The composite measure DIS, constructed in Section 4 from idiosyncratic components, is the empirical realization of the model's composite disagreement variable d^*. A measure constructed from systematic rather than idiosyncratic components can conflate the idiosyncratic pricing channel with factor-level variation, a distinction that guides the specific empirical choices of Section 4 as theoretically motivated rather than arbitrary.

A.3.3 Orthogonality to Aggregate Conditions and Factor Premia

The separation result implies a testable prediction about how a correctly constructed idiosyncratic disagreement measure should behave in the data. Aggregate variation such as macroeconomic uncertainty, market-wide volatility, and investor sentiment, affects all assets through the common factor structure and is absorbed into factor premia. It therefore does not enter the idiosyncratic pricing channel. This implication is exact under CARA utility (Proposition A.1) and holds to the cubic approximation under CRRA utility (Proposition A.2); since the cubic expansion is standard in the asset pricing literature and expected returns have limited sensitivity to fourth and higher moments, the orthogonality prediction applies with high accuracy under both utility specifications. If an empirical disagreement measure is correctly capturing idiosyncratic disagreement, its return predictability should be stable across aggregate macro states: the disagreement effect should be present in both high- and low-uncertainty environments, both high- and low-sentiment environments, and both high- and low-volatility environments, because none of these aggregate conditions affect the idiosyncratic channel through which the mechanism operates.

Conversely, if an empirical measure exhibits strong regime-dependence by predicting returns in some aggregate states but not others, then this behavior is diagnostic of contamination by aggregate variation that the model's idiosyncratic channel excludes. Such a measure is, at least in part, capturing a macro-regime effect rather than genuine firm-level disagreement. This prediction is tested in Section 5.4, where we show that DIS predicts returns stably across eight distinct macro state variables while IVOL and SIR predict returns only in specific regimes and AFD's predictability is less stable.

Furthermore, there is a clean distinction between idiosyncratic equilibrium pricing and variation in factor loadings, which determines exposure to factor premia. The comparative statics of Section 2 characterize how idiosyncratic characteristics generate cross-sectional variation in expected returns beyond what factor loadings predict. This is precisely the variation that Fama-MacBeth regressions with factor-based controls are designed to isolate, and it is why the empirical tests of Section 5 consistently control for beta, size, book-to-market, and momentum: these controls absorb the factor-loading channel, leaving the idiosyncratic disagreement channel identified.

The large and robust alphas documented in Section 5, which survive CAPM, Fama-French three-factor, and Carhart four-factor adjustments, align with model predictions: factor models absorb the first source of cross-sectional return variation but leave the idiosyncratic disagreement channel unaddressed, because that channel operates through asset-specific equilibrium pricing rather than through covariance with priced factors. The multi-asset framework thus not only provides a precise theoretical foundation for the single-asset approach, but also provides the structural explanation for why disagreement alphas survive standard risk adjustments.

A.4 Single Risky Asset Analysis and Cross-Sectional Implications

This section situates our approach within the broader literature on single-asset frameworks as tools for generating cross-sectional predictions and highlights why the single-asset approach is particularly appropriate for disagreement research.

Relationship to the literature on single-asset frameworks and cross-sectional implications.

Our approach is consistent with a well-established tradition in behavioral asset pricing and market microstructure of deriving cross-sectional predictions from single-asset models via comparative statics. Kraus and Litzenberger (1976) introduce skewness into a single-asset valuation model and derive cross-sectional predictions about how distributional characteristics of assets affect their equilibrium prices. Glosten and Milgrom (1985) and Amihud and Mendelson (1986) model single assets with information asymmetry and transaction costs respectively, using comparative statics on bid-ask spreads and liquidity to derive cross-sectional liquidity premia. Mitton and Vorkink (2007) use a single representative skewed asset in a mean-variance-skewness framework to derive cross-sectional implications about skewness preference and underdiversification. Barberis and Huang (2008) study cross-sectional relationships between positive return skewness and expected returns by considering an independent skewed security added to a baseline multi-asset market. Goulding, Santosh, and Zhang (2023) study cross-sectional relationships among skewness, noise, and expected returns in a single-asset framework. Goulding, Santosh, and Zhang (2025) study cross-sectional relationships among expected returns, skewness, and disagreement as measured by analyst forecast dispersion, using a single-asset framework.

These papers share the use of independence assumptions, either explicit or implicit, to isolate idiosyncratic characteristics so that each asset's pricing can be treated separately, mirroring a single-asset setup. We make this structure fully explicit through the formal separation results of Propositions A.1 and A.2, which derive the independence of idiosyncratic pricing from the factor structure of the multi-asset economy.

Appropriateness for disagreement research.

A single-asset framework is especially well-suited to the study of idiosyncratic investor disagreement for reasons specific to the nature of the phenomenon. Disagreement, as modeled here and in the broader differences-of-opinion literature (Varian, 1985; Harris and Raviv, 1993; Kandel and Pearson, 1995; Cao and Ou-Yang, 2008) is inherently asset-specific: investors disagree about the expected payoff of a particular firm, not about aggregate market outcomes. The empirical proxies for disagreement studied in this paper such as analyst forecast dispersion for individual firms, idiosyncratic volatility, and firm-level short interest, are all firm-specific variables. The single-asset model, in which the scale of disagreement D is a parameter of the individual asset's equilibrium, captures this asset-specificity directly. A model in which disagreement were a market-wide variable would fundamentally mischaracterize the phenomenon and predict that all assets are simultaneously more or less disagreed upon as aggregate conditions change, contradicting both the near-zero cross-correlations among firm-level disagreement proxies documented in Table 2 and the empirical finding in Section 5.4 that DIS predicts returns stably across aggregate macro conditions.

This last point connects the methodological justification for the single-asset approach to our empirical findings in a way that goes beyond precedent in the literature. The framework, the measurement, and the empirical behavior of the measure are mutually consistent in a way that provides strong joint support for the paper's central claims.

Disagreement of Disagreement

Internet Appendix (IA)

IA B Empirical Variable Descriptions

All variables are properly lagged to be available in real time.

AFD is the standard deviation of fiscal year end analyst EPS forecasts valid in month t across analysts for stock i scaled by the average absolute EPS forecast, multiplied by 100.

AGE is the age of the firm in months following the methodology of Jiang, Lee, and Zhang (2005).

Aggregate IVOL is the value-weighted average of stock-level idiosyncratic volatilities (IVOL) across all stocks in the sample in month t.

ANALYSTADJ is the number of analysts that issued a valid fiscal year forecast in month t divided by the average number of analysts per firm with valid forecasts in month t.

ASC is the adverse selection component of the bid-ask spread in month t derived from Armstrong et al. (2011) using intraday data from TAQ. Data is available until 2020.

ASIR is the difference between a firm's actual and predicted SIR in month t. Predicted SIR is computed as the moving average SIR over the past twelve months.

BETA is the coefficient on daily market returns of the regression of daily returns on daily market returns for month t.

BIDASK is the high-low bid ask estimator for month t following Corwin and Schultz (2012).

BTM is the ratio of book value of equity (following Davis, Fama, and French, 2000) from the most recent quarter to the market value of equity which is defined as shares outstanding times the share price in month t.

CAPM-α is the intercept from the regression of monthly excess portfolio returns on a constant and excess market return for month $t+1$.

CATFIN is the systemic risk measure of Allen, Bali, and Tang (2012), which captures distress in the financial sector using the cross-sectional distribution of financial institution returns.

DBREADTH is the change in the ratio of mutual funds with long positions in stock i in quarter q to all mutual funds, multiplied by minus one.

DEW is the equally weighted average disagreement decile in month t based on same-month deciles of AFD, IVOL, and SIR.

DEW+ is the equally weighted average disagreement decile in month t based on same-month deciles of ASIR, BIDASK, DBREADTH, AFD, DTO, IVOL, and SUV.

DIS is the natural log of one plus our composite measure of disagreement in equation (15).

DPCA is the first component of a principal component analysis on AFD, IVOL, and SIR in month t.

DPCA+ is the first component of a principal component analysis on ASIR, BIDASK, DBREADTH, AFD, DTO, IVOL, and SUV in month t.

DTO is the average daily market-adjusted turnover in month t less the average daily market-adjusted turnover over the past three months.

EXRET_{t+1} is the difference between monthly raw returns and the risk-free rate for stock i in month $t+1$.

FF3-α is the intercept from the regression of monthly excess portfolio returns on a constant, the excess market return, a size factor, and a book-to-market factor in month $t+1$ following Fama and French (1993).

Financial Uncertainty is the financial uncertainty index of Jurado, Ludvigson, and Ng (2015), based on the common variation in the unforecastable component of a large panel of financial time series at a one-month horizon.

FFC-α is the intercept from the regression of monthly excess portfolio returns on a constant, the excess market return, a size factor, a book-to-market factor, and a momentum factor in month $t+1$ following Fama and French (1993) and Carhart (1997).

HHI is the Herfindahl-Hirschman Index defined as the summed, squared market shares of individual stock i in quarter q for the jth mutual fund.

ILLIQ averages the ratio of absolute daily returns to daily dollar trading volume, multiplied by 10^6, as in Amihud (2002).

IO is the ratio of shares held by institutional investors to total shares outstanding as in Chen, Hong, and Stein (2002).

IVOL is the standard deviation of the daily residuals estimated from the regression of excess portfolio returns on a constant, the excess market return, a size factor, and a book-to-market factor at a monthly level following Ang et al. (2006), multiplied by 100.

ISKEW is the expected idiosyncratic skewness based as defined in Boyer, Mitton, and Vorkink (2010).

ISS is the difference between a firm's equity issuance and its buybacks, scaled by total assets following Bradshaw, Richardson, and Sloan (2006).

KZ-INDEX is the Kaplan and Zingales (1997) Index as a measure of reliance on external financing which is based on a linear combination of cash flows, PP&E, Tobin's Q, Total Debt, Dividends, and Cash.

Macroeconomic Uncertainty is the macroeconomic uncertainty index of Jurado, Ludvigson, and Ng (2015), based on the common variation in the unforecastable component of a large panel of macroeconomic indicators at a one-month horizon.

NI-IVOL is the standard deviation of the residual of a regression of net income scaled by assets on the same measure lagged by a year.

MOM is the product of monthly gross returns for month $t-12$ to $t-1$, minus 1, multiplied by 100.

O-SCORE is the Ohlson O-Score, which is a measure of the probability of bankruptcy as defined in Ohlson (1980).

PLS Sentiment Index is the partial-least-squares investor sentiment index of Kelly and Pruitt (2013), which extracts the common sentiment component from a panel of proxies using a partial least squares methodology.

R&D is research and development expenses scaled by assets.

Real Uncertainty is the real activity uncertainty index of Jurado, Ludvigson, and Ng (2015), constructed analogously from a panel of real economic activity variables.

RET_{t+1} is the raw monthly return for month $t+1$, multiplied by 100.

RMAX is the average of the five highest daily returns for stock i in month t, multiplied by 100.

RVOL is the one-month ex-post return volatility based on daily data for stock i in month t, multiplied by 100.

SIR is the ratio of shares sold short to shares outstanding for stock i in month t as by Chen, Da, and Huang (2019).

SIZE is the product of shares outstanding and the share price of stock i in month t.

SUV is the standardized residual of a regression of trading volume on positive and negative returns over the past three months following Garfinkel (2009).

SV is the value of the Google search volume index for a specific ticker as provided by trends.google.com. Data is available until 2018.

SV-INV is the value of the modified Google search volume index for a specific ticker and investment-related searches as provided by trends.google.com and following the methodology of deHaan, Lawrence, and Litjens (2023). Data is available until 2018.

TO is the average ratio of daily trading volume to shares outstanding in month t.

VIX is the CBOE Volatility Index, which measures the implied volatility of S&P 500 index options over the next 30 days. Monthly observations correspond to the end-of-month VIX level.

Volatility-of-Volatility is the realized volatility of daily changes in the VIX over the prior calendar month, following Bekaert and Hoerova (2014).

IA C Measures of Disagreement

This section summarizes the most commonly used proxies for investor disagreement and gives examples of papers using the respective measure published in top tier journals across accounting, economics, finance, and management studies. Table C.1 tabulates this summary for key proxies.

In models of higher order beliefs, a key assumption for prices to move is that investors are uncertain about the opinions of other investors (Allen, Morris, and Shin, 2006; Banerjee, Kaniel, and Kremer, 2009). Houge et al. (2001) demonstrate how investors' opinions on an IPO vary depending on how they perceive the distribution of returns. Both Kandel and Pearson (1995) and Hsiao (2022) illustrate how disagreement can arise when traders have different interpretations of a public signal. Hsiao (2022) further argues that, due to traders' uncertainty regarding the information quality of the interpretation, they perceive other investors' interpretations of a signal as ambiguous. However, disagreement caused by ambiguity can also simply refer to the signal's ambiguous information quality (Illeditsch, 2011). Additionally, differences of opinion can occur when investors do not condition their decisions on prices (Banerjee, 2011). In other words, unlike a rational expectation model where investors agree on the interpretation of signals and condition on prices effectively to infer the private information of others, investors may not agree on the return distribution and, as a result, may not use prices to update their beliefs. Hong and Stein (1999) also make a claim about information diffusion, suggesting that information diffuses only gradually into prices among diversely informed investors. Consequently, investors may disagree because they perceive the same information differently, receive different information, or view the information received from other investors as ambiguous. Finding a comprehensive measure of disagreement is challenging due to these numerous sources. Nevertheless, the literature has created a number of measures to capture disagreement, each with its own set of advantages and disadvantages, which are discussed in this section.

C.A Measures Based on Analyst Forecasts

Banerjee, Kaniel, and Kremer (2009) as well as Verardo (2009) make the case that pro-

Table C.1: Table C.1: Summary of Disagreement Proxies in the Literature

Category	Proxy	Source / Frequency	Role in Eq. (15)	Key Limitations as Standalone Proxy	Representative Citations
Panel A: Direct Inputs to Composite Disagreement DIS (Equation 15)					
Analyst Forecast Dispersion	**AFD**: Std. dev. of EPS forecasts scaled by |mean forecast|. Variants use alternative scalings (price, book equity, assets, actual earnings) or BKLS diversity decomposition.	I/B/E/S; monthly; firm-level	*Exponential amplifier of existing belief heterogeneity via analyst signal precision (Lemma 4). Enters exponent of eq. (15), not additively.*	Requires 2+ analysts, excluding ∼1/3 of stocks. Subject to herding, stale forecasts, IBES biases, overoptimism. May capture uncertainty rather than disagreement (Barron, Stanford, and Yu, 2009). Robust to scaling choice (Cen, Wei, and Yang, 2017).	Diether, Malloy, and Scherbina (2002); Johnson (2004); Cen, Wei, and Yang (2017); Barron et al. (1998); Park (2005)
Idiosyncratic Volatility	**IVOL**: Std. dev. of daily residuals from a factor model (e.g., FF3). Variants use intraday TAQ data or market model residuals.	CRSP; monthly; firm-level	*Enters directly as σ_r with quadratic sensitivity. Most closely linked individual proxy to latent disagreement in the model.*	Also proxies for risk, firm transparency, or aggregate volatility. Return predictability is regime-dependent (Sec. 5.4) and friction-dependent (Sec. 5.5), both filtered by DIS.	Ang et al. (2006); Boehme, Danielsen, and Sorescu (2006); Berkman et al. (2009)
Short Interest	**SIR**: Shares shorted as fraction of shares outstanding. Variants include abnormal short interest (ASIR: actual minus predicted SIR).	Compustat / NASDAQ; biweekly–monthly; firm-level	*Enters as $v = 1 + \text{SIR}$. Proportional to model trading volume under zero supply (Sec. 2.5, Prop. 3).*	Proportionality to volume is approximate with positive net supply. Strategic shorting may be unrelated to disagreement. Return predictability is friction-dependent (Sec. 5.5).	(Chen, Da, and Huang, 2019; Karpoff and Lou, 2010; Chang et al., 2022)
Panel B: Other Disagreement Proxies in the Literature					
Other Volume Measures	Turnover (TO), change in market-adjusted turnover (ΔTO), standardized unexplained volume (SUV). Each refines raw volume by removing liquidity or market-wide components.	CRSP; monthly; firm-level	*Alternative proxies for the model's volume variable v. Model predicts common sensitivities with SIR (Sec. 2.5). Positively associated with DIS (Table H.9).*	TO confounds disagreement with liquidity and market-wide factors. ΔTO assumes stable price dynamics across windows. SUV requires regression estimation.	(Garfinkel and Sokobin, 2006; Lee and Swaminathan, 2000; Garfinkel, 2009)
Mutual Fund Holdings	Change in breadth of ownership (DBREADTH); std. dev. of funds' active holdings relative to benchmark.	Thomson Reuters / CRSP MF; quarterly; firm-level	*Not directly in eq. (15). Captures institutional belief heterogeneity through a different investor population.*	Quarterly frequency creates staleness. Covers institutional investors only. Breadth may also reflect information or liquidity motives.	(Chen, Hong, and Stein, 2002; Jiang and Sun, 2014; Golez and Goyenko, 2022)
Bid-Ask Spreads	High-low spread estimator (BIDASK); relative spreads; intraday order-level dispersion.	CRSP / TAQ; daily–monthly; firm-level	*Not directly in eq. (15). Related to the information asymmetry channel, distinct from belief heterogeneity.*	Confounds disagreement with liquidity and adverse selection. Does not capture non-executed limit orders (Garfinkel, 2009).	(Corwin and Schultz, 2012; Garfinkel, 2009; Handa, Schwartz, and Tiwari, 2003)
Options-Based	Disagreement implied by options trades (e.g., put–call implied volatility spread, option volume ratios).	OptionMetrics; monthly; firm-level	*Not in eq. (15). Captures belief heterogeneity through a distinct market, complementary to equity-based proxies.*	Limited to optionable (larger) stocks. Confounds disagreement with volatility risk and hedging demand.	(Golez and Goyenko, 2022; Ge, Lin, and Pearson, 2016)
Social Media / Text-Based	Disagreement from social media posts (StockTwits, Twitter/X); NLP-based measures from earnings calls or filings.	StockTwits / Twitter; daily; firm-level	*Not in eq. (15). Captures retail investor disagreement directly at high frequency.*	Short history. Selection bias toward retail and attentive investors. Sentiment classification noise.	(Cookson and Niessner, 2020; Cookson et al., 2023)
Machine Learning	ML-based composite disagreement from 150+ stock characteristics.	Multiple; monthly; firm-level	*Atheoretic composite. No structural functional form or falsifiable predictions about regime stability or friction independence.*	Black-box construction. Broad sample includes illiquid, retail-dominated stocks. Most correlated with IVOL among inputs, consistent with our model.	(Bali et al., 2025)
Other	Geographic dispersion of EDGAR requests; household survey data; macro indicator dispersion; historical income volatility.	Various; various; firm or aggregate	*Not in eq. (15). Alternative identification strategies exploiting distinct data sources.*	Availability and coverage vary widely. Macro-level measures cannot isolate firm-level disagreement.	(Chen, 2023; Li and Li, 2021; Anderson, Ghysels, and Juergens, 2009)

Notes: This table summarizes the major categories of investor disagreement proxies employed in the literature and their relationship to the composite measure DIS derived in Section 3. Panel A lists the three proxy categories that enter equation (15) as direct inputs; Panel B lists other measures that capture aspects of disagreement through channels not directly embedded in the model's equilibrium conditions. The near-zero cross-correlations among the three Panel A proxy categories are documented in Table 2. See Appendix B for precise variable definitions used in the empirical implementation.

fessional analysts' forecasts are the result of their underlying assumptions and information-processing models. Additionally, forecast dispersion is also likely to be free of any liquidity, trading costs, and/or size-related bias (Alexandridis, Antoniou, and Petmezas, 2007). Analyst forecasts are also frequently released, creating a near-constant flow of information (Ajinkya, Atiase, and Gift, 1991). As such, dispersion can be a suitable proxy for disagreement, provided that analysts express their unbiased opinion and that investors base their opinions on those of analysts (Garfinkel, 2009). Diether, Malloy, and Scherbina (2002) propose a disagreement measure based on the dispersion of analysts' forecasts, computed as the standard deviation across analysts divided by the mean absolute value of EPS forecasts (e.g. Ali et al., 2019; Barinov, 2013; Daniel, Klos, and Rottke, 2023). This measure has been used with various scaling factors, including the stock price (e.g., Banerjee, 2011; Cheong and Thomas, 2011), the book value of equity (e.g., Cen, Wei, and Yang, 2017; Dittmar and Thakor, 2007) or assets (e.g., Golez and Goyenko, 2022; Johnson, 2004), and actual earnings (Park, 2005). According to Cen, Wei, and Yang (2017), this measure is relatively robust to the selection of the scaling factor, suggesting that the standard deviation in the numerator reflects the degree of disagreement.

However, the metric has a drawback in that it can be only computed for reasonably large companies with at least two analysts following them, leaving out roughly one-third of tradeable equity securities (Danielsen and Sorescu, 2001). Depending on the normalization term, dispersion may not only capture disagreement but also other information (Banerjee, 2011; Cen, Wei, and Yang, 2017). Additionally, it is influenced by an IBES data bias (Dechow, Hutton, and Sloan, 2000; Garfinkel, 2009), stale forecasts (Garfinkel, 2009), conflicts of interest and career concerns (Johnson, 2004), overoptimism (Hong and Stein, 2003), underreaction to public information (Erturk, 2006), overconfidence in private information (Abarbanell and Bernard, 1992), and herding (Bernhardt, Campello, and Kutsoati, 2006; Hong, Lim, and Stein, 2000). While Johnson (2004) argues that analyst forecast disagreement is a nonsystematic risk, the opposing literature contends that dispersion only serves as a proxy for disagreement of opinion caused by asymmetric information (e.g., Berkman et al., 2009; Diether, Malloy, and Scherbina, 2002; Park, 2005). However, later studies suggest that dispersion is more closely related to uncertainty than to information asymmetry. Barron, Stanford, and Yu (2009) show that fluctuations in analysts' forecast dispersion reflect changes in information asymmetry around earnings announcements, whereas levels of dispersion reflect levels of uncertainty before earnings announcements. Furthermore, information asymmetry issues seem to be infrequent and unrelated to divergent investor opinions when analysts monitor a company, whereas concerns about information asymmetry are much greater and are strongly correlated with diverging investor opinions when there is no analyst coverage (Garfinkel, 2009).

Measures Based on Analyst Forecast Dispersion for Individual Firms

a) Standard deviation of EPS estimates for the currently unreported fiscal year scaled by the absolute value of the mean EPS forecast for the same fiscal year. The same measure can be used on a monthly, quarterly, or semi-annually basis: e.g., Ajinkya and Gift (1985); Ajinkya, Atiase, and Gift (1991); Ali et al. (2019), Barinov (2013); Boehme, Danielsen, and Sorescu (2006); Cen, Wei, and Yang (2017); Daniel, Klos, and Rottke (2023); Diether, Malloy, and Scherbina (2002); Dittmar and Thakor (2007) (as measure of overvaluation); Doukas, Kim, and Pantzalis (2006); Erturk (2006); Garfinkel (2009); Goetzmann and Massa (2005); Hibbert et al. (2020); Hong and Sraer (2016); Jiang and Sun (2014); Johnson (2004); Park (2005); Sadka and Scherbina (2007); Verardo (2009).

b) Unadjusted analyst forecast dispersion measured by the standard deviation of EPS estimates. Similar to a) where forecast period can be chosen: e.g., Alexandridis, Antoniou, and Petmezas (2007); Banerjee (2011); Hong and Sraer (2016); Moeller, Schlingemann, and Stulz (2007); Sheng and Thevenot (2012); Yu (2011).

c) Standard deviation of EPS forecasts scaled by absolute median forecast: e.g., Banerjee (2011); Sheng and Thevenot (2012).

d) Standard deviation of EPS forecasts scaled by lagged price for annual estimates or by price at estimation window end or by the beginning of estimation window: e.g., Banerjee (2011); Chatterjee, John, and Yan (2012); Cheong and Thomas (2011); Danielsen and Sorescu (2001); Doukas and McKnight (2005); Garfinkel (2009); Ge, Lin, and Pearson (2016); Rees and Thomas (2010); Zhang (2006); Zhang (2006b).

e) Variance of EPS forecasts scaled by stock price: e.g., Barron, Stanford, and Yu (2009).

f) Difference between high and low forecasts standardized by average EPS forecast: e.g., Goetzmann and Massa (2005).

g) Standard deviation of EPS forecasts scaled by the book value of equity per share: e.g., Cen, Wei, and Yang (2017); Dittmar and Thakor (2007).

h) Standard deviation of EPS scaled by value of total assets per share: e.g., Cen, Wei, and Yang (2017); Daniel, Klos, and Rottke (2023); Golez and Goyenko (2022); Johnson (2004).

i) Difference between high and low forecasts standardized by the time-series standard-deviation of EPS: e.g., De Bondt and Forbes (1999).

j) Standard deviation of EPS forecasts scaled by time-series standard deviation of EPS: e.g., De Bondt and Forbes (1999).

k) Difference between a firm's actual EPS from the quarter prior to the issue and the mean analyst EPS forecast, divided by the actual EPS (Forecast Error); e.g., Dittmar and Thakor (2007).

l) Absolute value of the difference between the actual earnings and the mean of the forecasts for the current fiscal year end, and scaled by the absolute value of the actual earnings (Forecast error): e.g., Hibbert et al. (2020).

m) Standard deviation of EPS forecasts normalized by actual earnings: e.g., Park (2005).

n) Change in standard deviation (in log) of analysts' earnings forecasts: e.g., Cen, Wei, and Yang (2017).

o) Diversity (disagreement) of opinion measures: 1 minus the consensus measured by the correlation in forecast errors across analysts (known as the diversity part of the BKLS 1998 measure which defines dispersion as the sum of diversity and uncertainty): e.g., Barron et al. (1998); Barron, Stanford, and Yu (2009); Doukas, Kim, and Pantzalis (2006).

Measures Based on Analyst Forecast Dispersion for Portfolios/Market

a) Standard deviation of EPS forecasts value-weighted by market cap (focus on long-term forecasts: 3-5 years); this measure is on the portfolio/market level: e.g., Beddock (2021); Huang, O'Hara, and Zhong (2021); Yu (2011).

b) Standard deviation of long-term forecasts (3-5 years) weighting each stock by pre-ranking beta; this measure is at the portfolio/market level: e.g., Hong and Sraer (2016); Huang, O'Hara, and Zhong (2021).

Other Measures Based on Analyst Forecasts

a) The categorized coefficient from regressing each analyst's quarterly EPS forecast on the most recent forecast issued by a different analyst for the same firm-quarter. Under agreement, the slope coefficient equals 1. The measure takes the value 1 if the slope is significantly below 1, -1 if significantly above 1, and 0 if insignificant, at the 1% level in a two-tailed test, where significance is based on the deviation of the slope from 1 scaled by its standard error: Fischer, Kim, and Zhou (2022).

C.B Measures Based on Volatility

Other studies propose idiosyncratic volatility as a measure of differences in opinion which is frequently used at the same time to quantify information asymmetry (e.g., Berkman et al., 2009; Boehme, Danielsen, and Sorescu, 2006; Dierkens, 1991). Theoretical evidence suggests that return volatility and divergent investor opinions are positively correlated (Shalen, 1993; Wang, 1998). Hence, measures of volatility should act as a proxy for dispersion in opinions. One of the most frequently used methods to derive idiosyncratic volatility is calculating the standard deviation of the residuals from a market model (e.g., Fama and French, 1993; Brown and Warner, 1985; Ang et al., 2006; Danielsen and Sorescu, 2001; Jiang and Sun, 2014). The use of stock market data enables the inclusion of even small firms in the sample, providing an accurate representation of market volatility. While most studies use a firm-level setting to proxy for disagreement, some propose a value-weighted approach to proxy for market disagreement (e.g., Boehme, Danielsen, and Sorescu, 2006; Huang, Li, and Wang, 2021).

Measures Based on Volatility

a) Standard deviation of daily market adjusted residuals (e.g., from a regression w/ FF3 factors): e.g., Alexandridis, Antoniou, and Petmezas (2007); Boehme, Danielsen, and Sorescu (2006); Chatterjee, John, and Yan (2012); Danielsen and Sorescu (2001); Ge, Lin, and Pearson (2016); Huang, O'Hara, and Zhong (2021); Jiang and Sun (2014); Moeller, Schlingemann, and Stulz (2007); Zhang (2006b).

b) Stock return volatility using TAQ as sample standard deviation or stock return volatility over a period: e.g., Chang, Cheng, and Yu (2007); Garfinkel (2009).

C.C Measures Based on Trading Volume

Numerous studies demonstrate that trading volume is indicative of the degree of disagreement between investors regarding the value of a security, based on either different information or divergent opinions (Bamber, 1987; Bamber, Barron, and Stober, 1999; Lee and Swaminathan, 2000; Banerjee, 2011; Boehme, Danielsen, and Sorescu, 2006; Varian, 1986). Banerjee (2011) distinguishes between two forms of divergence of opinion: disagreement among investors, which correlates positively with expected trading volume, and analyst forecast dispersion, which correlates negatively with returns. Because the two measures relate differently to returns, they should capture distinct underlying sources of disagreement. A common implementation is monthly average turnover, TO, defined as the ratio of trading volume to average shares outstanding for stock i in month t (Boehme, Danielsen, and Sorescu, 2006; Garfinkel, 2009).

The volume measure may not be the only way to measure disagreement. Benston and Hagerman (1974) as well as Petersen and Fialkowski (1994) suggest that volume serves as a proxy for liquidity if a stock consistently trades at a high volume. Garfinkel (2009) argues that volume only includes executed market orders, making it a poor substitute for all investors' private valuations. Holthausen and Verrecchia (1994) and Karpoff (1987) posit that the news that investors trade on may have a higher information content, implying that volume reflects informedness. Finally, Tkac (1999) shows that stock volume is positively correlated with market volume, indicating that market-wide factors influence volume. To address this, Garfinkel and Sokobin (2006) develop the change in market-adjusted turnover measure, ΔTO, which considers both volume attributable to liquidity trading and market-wide factors. ΔTO in this paper is defined as the average daily market-adjusted turnover in month t less an earlier measure similarly calculated for stock i over the past three months (Chen et al., 2023; Garfinkel, 2009):

However, because ΔTO assumes similar price movements during the measurement and control window, it does not completely rule out a potential component in volume related to the market's reaction to the information content of news. On the other hand, standardized unexplained volume, SUV, controls for all components of volume that are not attributable to differences in opinion and are not completely captured by ΔTO (Chen et al., 2023; Garfinkel, 2009; Garfinkel and Sokobin, 2006).

However, an attenuation bias may impede the capacity of volume-based measures and bid-ask spreads to serve as a proxy for differences in opinions since both measures do not account for non-executed limit orders, which provide a more complete picture of the market participants' willingness to buy or sell a security. Hollifield et al. (2006) and Garfinkel (2009) argue that execution prices might not accurately reflect all investors' private valuations. To this end, Garfinkel (2009) proposes a new proxy to measure differences in opinion by utilizing investors' expressions of interest in stocks through their market and limit orders in order to capture additional information on investors' private information.

Measures Based on Volume

a) Average turnover: e.g., Banerjee (2011); Boehme, Danielsen, and Sorescu (2006) (scaled by shares outstanding); Danielsen and Sorescu (2001); Dittmar and Thakor (2007) (as measure of overvaluation); Lee and Swaminathan (2000); Ge, Lin, and Pearson (2016); Golez and Goyenko (2022).

b) Unexplained volume: average daily market-adjusted turnover over a control window minus a similarly calculated measure before the control window: e.g., Chen (2023); Garfinkel (2009); Garfinkel and Sokobin (2006).

c) Standard unexplained volume obtained from subtracting a predicted turnover measure derived from a regression (turnover = positive return + negative return; for each company) from the real turnover measure and dividing the result by the standard deviation of the regression residuals: e.g., Chen et al. (2023); Garfinkel (2009); Garfinkel and Sokobin (2006); Huang, O'Hara, and Zhong (2021).

d) Daily Short Interest: proposed by, e.g., Bessembinder, Chan, and Seguin (1996) but daily data was not available at that time.

e) Abnormal Short Interest: e.g. Chang et al. (2022); Chen, Da, and Huang (2019); Karpoff and Lou (2010).

f) Contemporaneous correlation coefficient (at the end of a given month) of daily trading volume and absolute price change over the past two months, multiplied by -1: Hsiao (2022).

C.D Other Disagreement Measures

C.D.1 Measures Based on Mutual Funds

a) Breadth: Number of mutual funds that hold a long position in the target's stock scaled by the total number of mutual funds (on a quarterly level): e.g., Jiang and Sun (2014); Moeller, Schlingemann, and Stulz (2007).

b) Change in breadth of ownership: the change in the number of funds divided by the total number of funds (on a quarterly level): e.g., Chatterjee, John, and Yan (2012); Chen, Hong, and Stein (2002); Dittmar and Thakor (2007) (as measure of overvaluation).

c) Standard deviation of the funds' active holdings of a particular stock: the difference between the weight of the stock in each fund manager's portfolio and that in the manager's benchmark index: e.g., Golez and Goyenko (2022); Jiang and Sun (2014).

d) Change in mutual fund dispersion: Standard deviation of the funds' active holdings of a particular stock: the difference between the weight of the stock in each fund manager's portfolio and that in the manager's benchmark index: e.g., Jiang and Sun (2014).

C.D.2 Measures Based on Bid-Ask Spreads

a) Spreads and relative spreads: e.g., Garfinkel (2009); Handa, Schwartz, and Tiwari (2003); Houge et al. (2001).

C.D.3 Measures Based on Intraday Distances

a) Standard deviation of the distance between each order's requested price and the most recent trade price preceding that order using TAQ: e.g., Garfinkel (2009).

C.D.4 Other Disagreement Measures

a) Dispersion measure using investor accounts data: e.g., Goetzmann and Massa (2005).

b) Disagreement measure from options trades: e.g., Ge, Lin, and Pearson (2016); Golez and Goyenko (2022); Huang, O'Hara, and Zhong (2021).

c) Dispersion measure from household survey data: e.g., Huang, O'Hara, and Zhong (2021); Li and Li (2021).

d) Disagreement as uncertainty which is the sum of the projected variance of mean forecast errors (obtained from a GARCH model) and the observed dispersion: e.g., Sheng and Thevenot (2012).

e) Disagreement based on macroeconomic indicators: e.g., Anderson, Ghysels, and Juergens (2009); Berkman et al. (2009); Huang, O'Hara, and Zhong (2021).

f) Standard deviation of stock returns using all transactions during normal trading hours (with TAQ data): e.g., Garfinkel (2009).

g) Historical income volatility: e.g., Berkman et al. (2009).

h) Disagreement from social media posts (such as Twitter, Stocktwits, etc.): e.g., Cookson et al. (2023); Cookson and Niessner (2020).

i) Standard deviation of weekly raw returns: e.g., Danielsen and Sorescu (2001).

j) Estimated analyst dispersion from an OLS model for firms with no analyst data by running a regression of dispersion (when it is available) on idiosyncratic volatility and turnover: e.g., Boehme, Danielsen, and Sorescu (2006).

k) Geographic dispersion of a firm's investors as the variation in the locations of requests for the firms' filings to EDGAR: e.g., Chen (2023).

l) Distribution of the trades of institutional investors: e.g., Hibbert et al. (2020).

m) Machine-learning based forecasts: Bali et al. (2025).

IA D Proofs

Proof of Proposition 1. Denote date–1 wealth for investor type i as a function of price p by

$$w_i(p) = w_0 + x_i(p)(\widetilde{y} - p) \tag{D1}$$

so that the first-order condition of type-i investor's expected utility maximization is

$$\mathbf{E}_i[u'(w(p))(\widetilde{y} - p)] = 0. \tag{D2}$$

We will use (D1) and (D2) and differentiation applied twice to (D1) and (D2) to derive exact expressions for the coefficients $x_i(\mu_i)$, $x_i'(\mu_i)$, and $x_i''(\mu_i)$ of the Taylor expansion of

$x_i(p)$ around $\mu_i = \mu + D\Delta_i$, namely,

$$x_i(p) = x_i(\mu_i) + x_i'(\mu_i)(p - \mu_i) + \frac{x_i''(\mu_i)}{2}(p - \mu_i)^2 + h_i(p)(p - \mu_i)^2, \tag{D3}$$

where $h_i(p)$ is an unknown function that converges to 0 as $p \to \mu_i$ by Taylor's theorem.

First, because $w_i(\mu_i) = w_0$ for $x_i(\mu_i) = 0$, it is clear that

$$x_i(\mu_i) = 0 \tag{D4}$$

satisfies (D2) and this solution is unique because u is strictly concave.

Second, we differentiate both sides of (D2) with respect to p to obtain:

$$\mathbf{E}_i[u''(w_i(p))w_i'(p)(\widetilde{y} - p) - u'(w_i(p))] = 0. \tag{D5}$$

Note that the first derivative of wealth is $w_i'(p) = x_i'(p)(\widetilde{y} - p) - x_i(p)$, which evaluates to $w_i'(\mu_i) = x_i'(\mu_i)(\widetilde{y} - \mu_i)$ because of (D4). From (D5) evaluated at $p = \mu_i$, we derive that $u''(w_0)x_i'(\mu_i)\mathbf{E}_i[(\widetilde{y} - \mu_i)^2] - u'(w_0) = 0$, and therefore that

$$x_i'(\mu_i) = -\frac{1}{\mathcal{A}}\frac{1}{\sigma^2}, \tag{D6}$$

where we have used the identity $\sigma^2 = \mathbf{E}_i[(\widetilde{y} - \mu_i)^2]$ and the definition of absolute risk aversion at initial wealth, $\mathcal{A} = -\frac{u''(w_0)}{u'(w_0)}$. Note that $x_i(p)$ is locally strictly decreasing because $x_i'(\mu_i) < 0$, which follows by (D6) and the facts that $\mathcal{A} > 0$ and $\sigma^2 > 0$.

Third, we differentiate both sides of (D5) with respect to p to obtain:

$$\mathbf{E}_i[u'''(w_i(p))(w_i'(p))^2(\widetilde{y} - p) + u''(w_i(p))(w_i''(p)(\widetilde{y} - p) - 2w_i'(p))] = 0. \tag{D7}$$

Note that the second derivative of wealth is $w_i''(p) = x_i''(p)(\widetilde{y} - p) - 2x_i'(p)$, which evaluates to $w_i''(\mu_i) = x_i''(\mu_i)(\widetilde{y} - \mu_i) - 2x_i'(\mu_i)$. From (D7) evaluated at $p = \mu_i$, we derive that $u'''(w_0)(x_i'(\mu_i))^2\mathbf{E}_i[(\widetilde{y} - \mu_i)^3] + u''(w_0)x_i''(\mu_i)\mathbf{E}_i[(\widetilde{y} - \mu_i)^2] - 4u''(w_0)u'(w_0)\mathbf{E}_i[\widetilde{y} - \mu_i] = 0$, and therefore that

$$x_i''(\mu_i) = \frac{\mathcal{P}}{\mathcal{A}^2}\frac{s}{\sigma^3}, \tag{D8}$$

where we have used the identities (D6), $s\sigma^3 = \mathbf{E}_i[(\widetilde{y} - \mu_i)^3]$, $\sigma^2 = \mathbf{E}_i[(\widetilde{y} - \mu_i)^2]$, and $0 = \mathbf{E}_i[\widetilde{y} - \mu_i]$; the definition of absolute risk aversion at initial wealth, $\mathcal{A} = -\frac{u''(w_0)}{u'(w_0)}$; and the definition of absolute risk prudence at initial wealth, $\mathcal{P} = -\frac{u'''(w_0)}{u''(w_0)}$. For $s > 0$,we have

$x_i''(\mu_i) > 0$, which follows by (D8) and the facts that $\mathcal{P} > 0$ and $\sigma^3 > 0$.

Finally, we plug in (D4), (D6), and (D8) into (D3) to obtain:

$$x_i(p) = -\frac{1}{\mathcal{A}}\frac{1}{\sigma^2}(p-\mu_i) + \frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}^2}\frac{s}{\sigma^3}(p-\mu_i)^2 + h_i(p)(p-\mu_i)^2, \tag{D9}$$

where $h_i(p)$ is an unknown function that converges to 0 as $p \to \mu_i$ by Taylor's theorem. $\square$

Proof of Proposition 2. In the Taylor expansion for $x_i(p)$,

$$x_i(p) = -\frac{1}{\mathcal{A}}\frac{1}{\sigma^2}(p-\mu_i) + \frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}^2}\frac{s}{\sigma^3}(p-\mu_i)^2 + h_i(p)(p-\mu_i)^2, \tag{D10}$$

note that each of the expressions $x_i'(\mu_i) = -\frac{1}{\mathcal{A}}\frac{1}{\sigma^2}$ and $x_i''(\mu_i) = \frac{\mathcal{P}}{\mathcal{A}^2}\frac{s}{\sigma^3}$ do not depend on i. Thus, for simplicity we drop the explicit reference to argument μ_i and write X_1 and X_2 to denote $x_i'(\mu_i)$ and $x_i''(\mu_i)$, respectively, to express (D10) equivalently as

$$x_i(p) = X_1(p-\mu_i) + \frac{1}{2}X_2(p-\mu_i)^2 + h_i(p)(p-\mu_i)^2. \tag{D11}$$

The market-clearing condition states

$$\sum_{i=1}^{n} \pi_i x_i(p) = 0. \tag{D12}$$

Substituting $x_i(p)$ from (D11) into (D12) yields

$$X_1 \sum_{i=1}^{n} \pi_i(p-\mu_i) + \frac{1}{2}X_2 \sum_{i=1}^{n} \pi_i(p-\mu_i)^2 + H(p) = 0, \tag{D13}$$

where we define

$$H(p) = \sum_{i=1}^{n} \pi_i h_i(p)(p-\mu_i)^2. \tag{D14}$$

First, note that $\sum_{i=1}^n \pi_i(p-\mu_i) = p-\mu$ because $\widetilde{\Delta}$ has zero mean: $\sum_{i=1}^n \pi_i\Delta_i = 0$. Second, note that $\sum_{i=1}^n \pi_i(p-\mu_i)^2 = \sum_{i=1}^n \pi_i(p-\mu-D\Delta_i)^2 = \sum_{i=1}^n \pi_i[(p-\mu)^2 - 2D\Delta_i(p-\mu) + D^2\Delta_i^2] = (p-\mu)^2 + D^2$ because $\widetilde{\Delta}$ has zero mean and unit variance: $\sum_{i=1}^n \pi_i\Delta_i^2 = 1$. So, (D13) becomes

$$X_1(p-\mu) + \frac{1}{2}X_2[(p-\mu)^2 + D^2] + H(p) = 0. \tag{D15}$$

We now treat price as a function of D and use (D15) to compute the Taylor expansion of price around $D = 0$:

$$p(D) = p(0) + p'(0)D + \frac{1}{2}p''(0)D^2 + h(D)D^2, \tag{D16}$$

where $h(D)$ is an unknown function that converges to 0 as $D \to 0$ by Taylor's theorem.

First, at $D = 0$ it is clear that

$$p(0) = \mu \tag{D17}$$

satisfies (D13) because $\mu_i = \mu$ at $D = 0$.

Second, we differentiate both sides of (D15) with respect to D to obtain:

$$X_1 p' + \frac{1}{2}X_2\left[2(p-\mu)p' + 2D\right] + \frac{\partial H(p)}{\partial D} = 0. \tag{D18}$$

Note that $\frac{\partial H(p)}{\partial D}$ goes to zero as $D \to 0$ because it is the weighted sum of terms of the form $\frac{\partial h_i(p)(p-\mu_i)^2}{\partial D}$, which each go to zero as $D \to 0$. Therefore, evaluating (D18) at $D = 0$ yields

$$p'(D) = 0. \tag{D19}$$

Third, we differentiate both sides of (D18) with respect to D to obtain:

$$X_1 p'' + \frac{1}{2}X_2\left[2(p')^2 + 2(p-\mu)p'' + 2\right] + \frac{\partial^2 H(p)}{\partial D^2} = 0. \tag{D20}$$

Note that $\frac{\partial^2 H(p)}{\partial D^2}$ goes to zero as $D \to 0$ because it is the weighted sum of terms of the form $\frac{\partial^2 h_i(p)(p-\mu_i)^2}{\partial D^2}$, which each go to zero as $D \to 0$. Therefore, evaluating (D20) at $D = 0$ yields

$$p''(0) = \frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}. \tag{D21}$$

Therefore, by (D16), (D17), (D19), and (D21), the Taylor expansion of price around $D = 0$ is given by:

$$p(D) = \mu + \frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D^2 + h(D)D^2, \tag{D22}$$

where $h(D)$ is an unknown function that converges to 0 as $D \to 0$ by Taylor's theorem. □

Proof of Lemma F.1. Note that $\frac{\partial x_i(D)}{\partial \Delta_i} > 0 \Leftrightarrow 1 - \frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}\left(\frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D^2 - D\Delta_i\right) > 0$, where $\frac{\mathcal{P}}{\mathcal{A}} > 0$. For $s = 0$, this condition is satisfied. For $s > 0$, we have $\frac{\partial x_i(D)}{\partial \Delta_i} > 0 \Leftrightarrow \Delta_i > \frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D - \frac{1}{\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D}$. For $s < 0$, we have $\frac{\partial x_i(D)}{\partial \Delta_i} > 0 \Leftrightarrow \Delta_i < \frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D - \frac{1}{\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D}$. □

Proof of Lemma F.2. The equilibrium demand of a type-i investor satisfies: $\Delta_i \frac{\partial x_i}{\partial D} > 0$. To see this, note the following. If $\Delta_i < 0$, then

$$x_i' < 0 \Leftrightarrow \begin{cases} D < \frac{\mathcal{A}}{\mathcal{P}} \frac{\sigma}{|s|}(\Delta_i + \sqrt{2 + \Delta_i^2}), & s > 0, \\ D < \frac{\mathcal{A}}{\mathcal{P}} \frac{\sigma}{|s|}|\Delta_i|, & s < 0, \\ s = 0, \end{cases} \tag{D23}$$

If $\Delta_i > 0$, then

$$x_i' > 0 \Leftrightarrow \begin{cases} D < \frac{\mathcal{A}}{\mathcal{P}} \frac{\sigma}{|s|}(\Delta_i + \sqrt{2 + \Delta_i^2}), & s < 0, \\ D < \frac{\mathcal{A}}{\mathcal{P}} \frac{\sigma}{|s|}|\Delta_i|, & s > 0, \\ s = 0, \end{cases} \tag{D24}$$

Thus, if $s = 0$ or $D < \frac{\mathcal{A}}{\mathcal{P}} \frac{\sigma}{|s|} \min\{||\Delta_i|, \sqrt{2 + \Delta_i^2} - |\Delta_i|\}$, then $\Delta_i x_i'(D) > 0$. □

Proof of Proposition 3. Denote $x_i(D)$ as x_i for simplicity of notation. Without loss of generality, we assume that Δ_i is distinct for each i, and relabel investor types $i = 1, \ldots, n$ so that $\Delta_1 < \Delta_2 < \cdots < \Delta_n$. We assume that all Δ_i are nonzero. By Lemma F.1, $x_1 < x_2 < \cdots < x_n$, which means that the equilibrium demands of different type investors line up in the same order as their Δ's.

Because $\sum_{i=1}^n \pi_i x_i = 0$ by market clearing, there exists some investor type $i_x < n$ such that $x_i < 0$ for all $i \leq i_x$ and $x_i > 0$ for all $i > i_x$. Because $\mathbf{E}[\widetilde{\Delta}] = 0$, there exists some investor type $i_\Delta < n$ such that $\Delta_i < 0$ for all $i \leq i_\Delta$ and $\Delta_i > 0$ for all $i > i_\Delta$. If $i_x \leq i_\Delta$, then $x_i < 0$ and $\Delta_i < 0$ for all $i \leq i_x$. If $i_x > i_\Delta$, then $x_i > 0$ and $\Delta_i > 0$ for all $i > i_x$. In either case, $-\sum_{i \leq i_x} \pi_i x_i = \sum_{i > i_x} \pi_i x_i$. In the first case, $\sum_{i \leq i_x} \pi_i |x_i| = -\sum_{i \leq i_x} \pi_i x_i$, which is increasing in D because $\Delta_i < 0$ for all $i \leq i_x$ by Lemma F.2. In the second case, $\sum_{i > i_x} \pi_i |x_i| = \sum_{i > i_x} \pi_i x_i$, which is increasing in D because $\Delta_i > 0$ for all $i > i_x$ by Lemma F.2. Either case equals exactly half of the trading volume because of the market clearing condition. Therefore, trading volume is increasing in D, □

IA E Formal Derivations of the Multi-Asset Separation Result

Consider a two-period market having dates $t \in \{0, 1\}$ with $1 + K + N$ assets: a risk-free asset with its price and payoff normalized to one; K "factors" with price p_{f_k} and payoff $\widetilde{f}_k$ at dates $t = 0$ and $t = 1$, respectively, for $k = 1, 2, \ldots, K$; and N risky assets with price p_{θ_n}

and payoff

$$\widetilde{\theta}_n = \sum_{k=1}^{K} b_{n,k} \widetilde{f}_k + \widetilde{\epsilon}_n \tag{E1}$$

at dates $t = 0$ and $t = 1$, respectively, for $n = 1, 2, \ldots, N$, where $b_{n,k}$ is the asset's exposure to factor k, and where $\widetilde{\epsilon}_n$ is an idiosyncratic component that is independent of all factor payoffs and mutually independent across assets. Idiosyncratic payoff $\widetilde{\epsilon}_n$ is a random variable having objective cumulative distribution function (cdf) denoted by $F_{\epsilon_n}(\epsilon) = \mathbf{P}[\widetilde{\epsilon}_n \leq \epsilon]$, with mean, variance, and skewness denoted by $\mu_n = \mathbf{E}[\widetilde{\epsilon}_n]$, $\sigma_n^2 = \mathbf{Var}[\widetilde{\epsilon}_n]$, and $s_n = \mathbf{Skew}[\widetilde{\epsilon}_n]$, respectively, which are the idiosyncratic statistics of asset payoff $\widetilde{\theta}_n$ with respect to the factors term. Factor payoffs $\widetilde{f}_1, \ldots, \widetilde{f}_K$ are random variables with an arbitrary joint distribution and, therefore, $\widetilde{\theta}_n$ is a random variable with distribution corresponding to the convolution of the distributions of the factors term and the idiosyncratic term. Note that the marginal distribution of the idiosyncratic component for each asset n corresponds to the payoff distribution of Section 2.1 in the main text, for $\widetilde{y} = \widetilde{\epsilon}_n$, with subscripts n dropped (e.g., μ, σ, s, etc.).

There is a continuum of investors, indexed by the unit interval $[0, 1]$, each with common utility function of wealth $u(w)$ and initial wealth $w_0 > 0$. Investors prefer more to less, $u'(w) > 0$; are risk averse, $u''(w) < 0$; and have a demand for precautionary savings or positive intensity of prudence, $u'''(w) > 0$. These natural properties hold for the most commonly studied utility functions in the literature such as CARA utility and CRRA utility. Note that these assumptions on utility coincide with those of Section 2.2 in the main text.

Investors agree on the factor structure of returns but have differences of opinion about the expected payoff of individual assets and, equivalently, disagree about each asset's expected idiosyncratic payoff. We model this disagreement using the same construction as in Section 2.3 (separate specifications for the relative distribution of beliefs and scale of beliefs) but generalized to multiple assets. Note that disagreement here is about the mean of the residual $\widetilde{\epsilon}_n$, which for a given n is the counterpart to $\widetilde{y}$ in the main text.

Specifically, each investor is one of I_n types for each asset n, reflecting the investor's subjective belief for that asset (e.g., optimistic, pessimistic, etc.), and the vector $i = (i_1, i_2, \ldots, i_N)$ represents the investor's overall type i, where $i_n = 1, 2, \ldots, I_n$ and $n = 1, 2, \ldots, N$. The relative distribution of these I_n types is governed by $\widetilde{\Delta}_n$, a zero-mean unit-variance random variable that is mutually independent across assets, and takes I_n possible values: $\widetilde{\Delta}_n = \Delta_{i_n,n}$ with probability $\pi_{i_n,n}$. Each investor's relative belief is an independent realization of this relative belief random variable so that fraction $\pi_{i_n,n}$ of investors are of type i_n for asset n and the probability of being type i is $\pi_i = \prod_{n=1,2,\ldots,N} \pi_{i_n,n}$. Let D_n denote the scale of beliefs for asset n such that a type-i_n investor believes the idiosyncratic mean for asset n is $\mu_{i_n,n} = \mu_n + D_n \Delta_{i_n,n}$.

Observe that final wealth $\widetilde{w}$ at date $t = 1$ for an investor who demands x_{f_k} units of factor k at date $t = 0$ for price p_{f_k} for $k = 1, 2, \ldots, K$ and x_{θ_n} units of risky asset n at date $t = 0$ for price p_{θ_n} for $n = 1, 2, \ldots, N$ satisfies

$$\widetilde{w} = w_0 + \sum_{k=1}^{K} x_{f_k} \left(\widetilde{f}_k - p_{f_k} \right) + \sum_{n=1}^{N} x_{\theta_n} \left(\widetilde{\theta}_n - p_{\theta_n} \right). \tag{E2}$$

Each investor chooses these demands to maximize subjective expected utility of final wealth. For an investor of type i, optimal demands follow

$$\left(x_{i,f_1}, \ldots, x_{i,f_K}, x_{i,\theta_1}, \ldots, x_{i,\theta_N} \right) = \arg \max_{x_{f_1},\ldots,x_{f_K},x_{\theta_1},\ldots,x_{\theta_N}} \mathbf{E}_i \left[u \left(\widetilde{w} \right) \right], \tag{E3}$$

where x_{i,f_k} is the investor's optimal demand for factor k, x_{i,θ_n} is the investor's optimal demand for asset n, and $\mathbf{E}_i$ denotes the expectation under the subjective beliefs of type $i = (i_1, i_2, \ldots, i_N)$.

Terminal wealth (E2) for each investor of type-i can be expressed equivalently in terms of demands for factors and idiosyncratic asset components as follows:

$$\begin{aligned}
\widetilde{w}_i &= w_0 + \sum_{k=1}^{K} x_{i,f_k} \left(\widetilde{f}_k - p_{f_k} \right) + \sum_{n=1}^{N} x_{i,\theta_n} \left(\sum_{k=1}^{K} b_{n,k} \widetilde{f}_k + \widetilde{\epsilon}_n - \left(\sum_{k=1}^{K} b_{n,k} p_{f_k} + p_{\epsilon_n} \right) \right) \\
&= w_0 + \sum_{k=1}^{K} \left(x_{i,f_k} + \sum_{n=1}^{N} x_{i,\theta_n} b_{n,k} \right) \left(\widetilde{f}_k - p_{f_k} \right) + \sum_{n=1}^{N} x_{i,\theta_n} \left(\widetilde{\epsilon}_n - p_{\epsilon_n} \right) \\
&= w_0 + \sum_{k=1}^{K} \widehat{x}_{i,f_k} \left(\widetilde{f}_k - p_{f_k} \right) + \sum_{n=1}^{N} \widehat{x}_{i,\epsilon_n} \left(\widetilde{\epsilon}_n - p_{\epsilon_n} \right),
\end{aligned} \tag{E4}$$

where the first equality follows by substitution for $\widetilde{\theta}_n$ using (E1) and the law of one price, namely, $p_{\theta_n} = \sum_{k=1}^{K} b_{n,k} p_{f_k} + p_{\epsilon_n}$, where p_{ϵ_n} denotes the implied price of the claim to residual payoff $\widetilde{\epsilon}_n$;[19] the second equality follows by collecting terms; and the third equality follows by substitution of the following definitions of the demand for residual n, $\widehat{x}_{i,\epsilon_n} = x_{i,\theta_n}$, and the effective demand for factor k, $\widehat{x}_{i,f_k} = x_{i,f_k} + \sum_{n=1}^{N} x_{i,\theta_n} b_{n,k}$.

[19]By the no-arbitrage principle, the law of one price holds, and there exists a linear pricing functional for all assets, factors, and asset components.

Applying equivalence (E4), the investor's maximization in (E3) is equivalent to the following maximization:

$$
\begin{aligned}
&(\widehat{x}_{i,f_1}, \ldots, \widehat{x}_{i,f_K}, \widehat{x}_{i,\epsilon_1}, \ldots, \widehat{x}_{i,\epsilon_N}) \\
&\quad = \arg \max_{\widehat{x}_{f_1},\ldots,\widehat{x}_{f_K},\widehat{x}_{\epsilon_1},\ldots,\widehat{x}_{\epsilon_N}} \mathbf{E}_i \left[u \left(w_0 + \sum_{k=1}^{K} \widehat{x}_{f_k} \left(\widetilde{f}_k - p_{f_k} \right) + \sum_{n=1}^{N} \widehat{x}_{\epsilon_n} \left(\widetilde{\epsilon}_n - p_{\epsilon_n} \right) \right) \right],
\end{aligned} \tag{E5}
$$

where $\widehat{x}_{i,f_k}$ is the investor's optimal demand for factor k, $\widehat{x}_{i,\epsilon_n}$ is the investor's optimal demand for idiosyncratic component n.

E.A CARA Utility

Under CARA utility, $u(w) = -e^{-aw}$ with absolute risk aversion parameter a, a type-i investor's expected utility maximization (E5) is separable into $1+N$ optimization programs as we show in the following equivalence of the investor's maximum expected utility:

$$
\begin{aligned}
&\max_{\widehat{x}_{f_1},\ldots,\widehat{x}_{f_K},\widehat{x}_{\epsilon_1},\ldots,\widehat{x}_{\epsilon_N}} \mathbf{E}_i \left[-e^{-a\left(w_0 + \sum_{k=1}^{K} \widehat{x}_{f_k}\left(\widetilde{f}_k - p_{f_k}\right) + \sum_{n=1}^{N} \widehat{x}_{\epsilon_n}\left(\widetilde{\epsilon}_n - p_{\epsilon_n}\right)\right)} \right] \\
&= -e^{-aw_0} \min_{\widehat{x}_{f_1},\ldots,\widehat{x}_{f_K},\widehat{x}_{\epsilon_1},\ldots,\widehat{x}_{\epsilon_N}} \mathbf{E}_i \left[e^{-a\left(\sum_{k=1}^{K} \widehat{x}_{f_k}\left(\widetilde{f}_k - p_{f_k}\right)\right)} \cdot \prod_{n=1}^{N} e^{-a\widehat{x}_{\epsilon_n}\left(\widetilde{\epsilon}_n - p_{\epsilon_n}\right)} \right] \\
&= -e^{-aw_0} \min_{\widehat{x}_{f_1},\ldots,\widehat{x}_{f_K}} \mathbf{E}_i \left[e^{-a\left(\sum_{k=1}^{K} \widehat{x}_{f_k}\left(\widetilde{f}_k - p_{f_k}\right)\right)} \right] \cdot \prod_{n=1}^{N} \min_{\widehat{x}_{\epsilon_n}} \mathbf{E}_i \left[e^{-a\widehat{x}_{\epsilon_n}\left(\widetilde{\epsilon}_n - p_{\epsilon_n}\right)} \right] \\
&= -\min_{\widehat{x}_{f_1},\ldots,\widehat{x}_{f_K}} \mathbf{E}_i \left[e^{-a\left(\sum_{k=1}^{K} \widehat{x}_{f_k}\left(\widetilde{f}_k - p_{f_k}\right)\right)} \right] \cdot \prod_{\substack{m=1 \\ m \neq n}}^{N} \min_{\widehat{x}_{\epsilon_m}} \mathbf{E}_i \left[e^{-a\widehat{x}_{\epsilon_m}\left(\widetilde{\epsilon}_m - p_{\epsilon_m}\right)} \right] \\
&\qquad \cdot \max_{\widehat{x}_{\epsilon_n}} \mathbf{E}_i \left[u \left(w_0 + \widehat{x}_{\epsilon_n} \left(\widetilde{\epsilon}_n - p_{\epsilon_n} \right) \right) \right],
\end{aligned} \tag{E6}
$$

where the first equality follows from standard properties of the exponential function and the fact that the maximum of a negative function is the negative of its minimum; the second equality follows from mutual independence of the idiosyncratic components and their independence from the factors, which separates the optimization of the factors component of final wealth and each of the optimizations of idiosyncratic components; and the last equality follows by isolating the optimization for idiosyncratic component n and expressing it with respect to expected utility over final wealth of an individual just considering investment in that component. The optimization in Section 2.4 corresponds to this reduced problem above for asset n, which can be seen by setting $\widetilde{y} = \widetilde{\epsilon}_n$, $x = \widehat{x}_{\epsilon_n}$, and $p = p_{\epsilon_n}$. Note that we intentionally set aside the factor optimization part of the problem as our focus is on the idiosyncratic component. All the results of Section 2 carry through for each idiosyncratic component taken one at a time.

E.B CRRA Utility

Recall terminal wealth equivalence:

$$\widetilde{w}_i = w_0 + \sum_{k=1}^{K} \widehat{x}_{i,f_k} \left(\widetilde{f}_k - p_{f_k} \right) + \sum_{n=1}^{N} \widehat{x}_{i,\epsilon_n} \left(\widetilde{\epsilon}_n - p_{\epsilon_n} \right). \quad \text{(E4)}$$

Return is a sufficient statistic for wealth under CRRA utility, so we work with return without loss of generality. Final wealth can be recast equivalently in terms of asset returns and fractions of wealth invested in assets as follows:

$$\widetilde{w} = w_0 \left(1 + \sum_{k=1}^{K} z_{f_k} \widetilde{r}_{f_k} + \sum_{n=1}^{N} z_{\epsilon_n} \widetilde{r}_{\epsilon_n} \right), \quad \text{(E7)}$$

where $z_{f_k} = \frac{\widehat{x}_{f_k} p_{f_k}}{w_0}$ and $z_{\epsilon_n} = \frac{\widehat{x}_{\epsilon_n} p_{\epsilon_n}}{w_0}$ are the fractions of initial wealth invested in the correspondings factors and assets, respectively; and where we define the corresponding net return random variables by $\widetilde{r}_{\theta_n}$, $\widetilde{r}_{f_k}$, and $\widetilde{r}_{\epsilon_n}$, where $\widetilde{r}_\eta = (\widetilde{\eta} - p_\eta)/p_\eta$ for $\eta \in \{\theta_n, f_k, \epsilon_n\}$. Relatedly, gross total portfolio return, $\widetilde{R}$, satisfies

$$\widetilde{R} = \frac{\widetilde{w}}{w_0} = \left(1 + \sum_{k=1}^{K} z_{f_k} \widetilde{r}_{f_k} + \sum_{n=1}^{N} z_{\epsilon_n} \widetilde{r}_{\epsilon_n} \right). \quad \text{(E8)}$$

The optimal portfolio weights for each investor, $\{z_{f_k}\}$ and $\{z_{\epsilon_n}\}$, are independent of the investor's initial wealth w_0 due to the homothetic nature of CRRA preferences.

Following Kraus and Litzenberger (1976) and Goulding, Santosh, and Zhang (2023), suppose that investors care only about the first three moments of returns as a cubic Taylor expansion of CRRA utility around their expected return. For a type-i investor, corresponding expected utility expansion is of the form

$$\mathbf{E}_i[\widetilde{R}] - \frac{1}{2}\gamma \mathbf{Var}_i[\widetilde{R}] + \frac{1}{6}\gamma(1+\gamma)\mathbf{E}_i \left[(\widetilde{R} - \mathbf{E}_i[\widetilde{R}])^3 \right]. \quad \text{(E9)}$$

Now, express the gross total portfolio return equivalently as

$$\widetilde{R} = \widetilde{R}_K + \sum_{n=1}^{N} z_{\epsilon_n} \widetilde{r}_{\epsilon_n}, \quad \text{(E10)}$$

where $\widetilde{R}_K = 1 + \sum_{k=1}^{K} z_{f_k}\widetilde{r}_{f_k}$ denotes the gross return of investments in the K factors. The first moment and the second and third central moments, respectively, can be decomposed as follows:

$$\mathbf{E}_i[\widetilde{R}] = \mathbf{E}_i[\widetilde{R}_K] + \sum_{n=1}^{N} z_{\epsilon_n}\mathbf{E}_i[\widetilde{r}_{\epsilon_n}], \tag{E11}$$

$$\mathbf{Var}_i[\widetilde{R}] = \mathbf{Var}_i[\widetilde{R}_K] + \sum_{n=1}^{N} z_{\epsilon_n}^2\mathbf{Var}_i[\widetilde{r}_{\epsilon_n}], \tag{E12}$$

$$\mathbf{E}_i[(\widetilde{R} - \mathbf{E}_i[\widetilde{R}])^3] = \mathbf{E}_i[(\widetilde{R}_K - \mathbf{E}_i[\widetilde{R}_K])^3] + \sum_{n=1}^{N} z_{\epsilon_n}^3\mathbf{E}_i\left[(\widetilde{r}_{\epsilon_n} - \mathbf{E}_i[\widetilde{r}_{\epsilon_n}])^3\right], \tag{E13}$$

where we have applied the mutual independence of $\{\widetilde{r}_{\epsilon_n}\}$ and their independence from $\widetilde{R}_K$ together with the facts that the variance of a sum of independent random variables is the sum of their variances, and the third moment of a sum of independent mean-zero random variables ($\widetilde{R}_K - \mathbf{E}_i[\widetilde{R}_K]$ and $\widetilde{r}_{\epsilon_n} - \mathbf{E}_i[\widetilde{r}_{\epsilon_n}]$ are mean-zero) is the sum of their third moments.

Substitution of (E11), (E12), and (E13) into (E9) with rearrangement of terms yields the following expression for the expected utility of a type-i investor:

$$\begin{aligned}\mathbf{E}_i[\widetilde{R}_K] - \frac{\gamma}{2}\mathbf{Var}_i[\widetilde{R}_K] + \frac{\gamma(1+\gamma)}{6}\mathbf{E}_i\left[(\widetilde{R}_K - \mathbf{E}_i[\widetilde{R}_K])^3\right]& \\ + \sum_{n=1}^{N}\left(z_{\epsilon_n}\mathbf{E}_i[\widetilde{r}_{\epsilon_n}] - \frac{\gamma}{2}z_{\epsilon_n}^2\mathbf{Var}_i[\widetilde{r}_{\epsilon_n}] + \frac{\gamma(1+\gamma)}{6}z_{\epsilon_n}^3\mathbf{E}_i\left[(\widetilde{r}_{\epsilon_n} - \mathbf{E}_i[\widetilde{r}_{\epsilon_n}])^3\right]\right)&.\end{aligned} \tag{E14}$$

Because the choice variable z_{ϵ_n} regarding allocation to the idiosyncratic component n appears separately of all other choice variables in an additive expression, the investor's optimization problem in z_{ϵ_n} reduces to maximization over z_{ϵ_n} of the term

$$\arg\max_{z_{\epsilon_n}}\left\{z_{\epsilon_n}\mathbf{E}_i[\widetilde{r}_{\epsilon_n}] - \frac{\gamma}{2}z_{\epsilon_n}^2\mathbf{Var}_i[\widetilde{r}_{\epsilon_n}] + \frac{\gamma(1+\gamma)}{6}z_{\epsilon_n}^3\mathbf{E}_i\left[(\widetilde{r}_{\epsilon_n} - \mathbf{E}_i[\widetilde{r}_{\epsilon_n}])^3\right]\right\},$$

which maps to optimal demand via $\widehat{x}_{\epsilon_n} = z_{i,\epsilon_n}w_0/p_{\epsilon_n}$. Each of these $n = 1, 2, \ldots, N$ separable optimization problems is equivalent, up to cubic Taylor expansion, to maximization of CRRA demand of single idiosyncratic component n. Again, the optimization in Section 2.4 corresponds to this reduced problem above for asset n, which can be seen by setting $\widetilde{y} = \widetilde{\epsilon}_n$, $x = \widehat{x}_{\epsilon_n}$, and $p = p_{\epsilon_n}$. Again, all the results of Section 2 carry through for each idiosyncratic component taken one at a time.

IA F Equilibrium Demands

In this section, we develop the argument and intuition behind the relationship between equilibrium volume and the scale of disagreement. For simplicity, we consider the case of CRRA utility but the same approach extends to more general utility. First, we analyze the sensitivity of each investor's equilibrium demand to the investor's underlying source of heterogeneity in belief, i.e., how x_i varies with Δ_i:

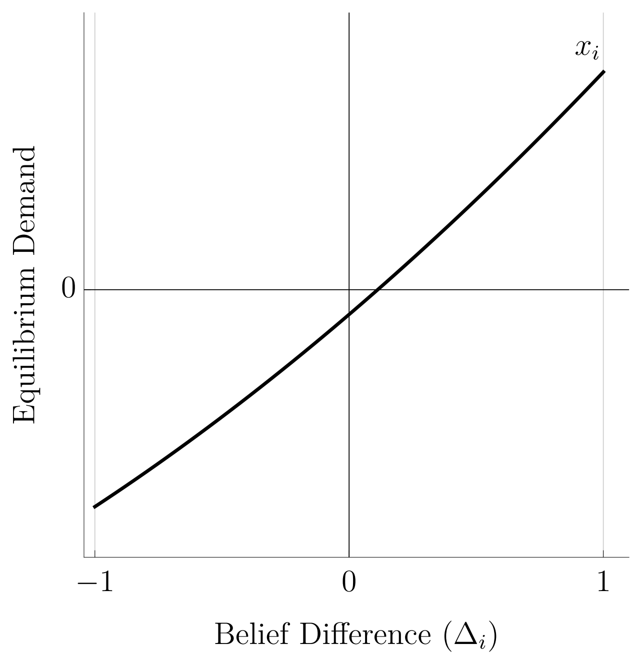
$$\frac{\partial x_i}{\partial \Delta_i} = \frac{1}{\gamma}\frac{1}{\sigma^2}D - \frac{1}{\gamma}\left(1+\frac{1}{\gamma}\right)\frac{s}{\sigma^3}D^2\left(\frac{1}{2}\left(1+\frac{1}{\gamma}\right)\frac{s}{\sigma}D - \Delta_i\right). \tag{F1}$$

Lemma F.1 indicates that (F1) is positive, and hence equilibrium demand x_i is increasing in Δ_i.

Lemma F.1. *Equilibrium demand x_i is increasing in Δ_i for D not too large.*

Proof of Lemma F.1. See Appendix D. □

Figure F.1: Equilibrium demand x_i as a function of underlying belief heterogeneity Δ_i.



Notes: This figure plots the 2nd-order Taylor polynomial of the equilibrium demand x_i as a function of Δ_i over the domain $\Delta_i \in [-1, 1]$ for the following parameter values: $D = 0.05$, $\mu = 1$, $\sigma = 0.2$, $s = 0.6$, $\gamma = 2$, $w_0 = 1$.

Figure F.1 shows how x_i increases with Δ_i. Because equilibrium price p is higher than μ for $s > 0$ and small $D > 0$, equilibrium demand is negative at $\Delta_i = 0$, which corresponds to belief $\mu_i = \mu < p$. At $\Delta_i = \frac{p-\mu}{D} > 0$, we have $\mu_i = p$ so that $x_i = 0$.

The intuition that Lemma F.1 supports is that equilibrium demands line up in the same order as underlying beliefs, $\{\Delta_i\}$. Without loss of generality, we relabel the beliefs in order from lowest to highest belief difference:

$$\Delta_1 < \Delta_2 < \cdots < \Delta_n. \tag{F2}$$

Without loss, we also assume each of the Δ_i's are unique. Lemma F.1 tells us that the same ordering will apply to equilibrium demands:

$$x_1 < x_2 < \cdots < x_n. \tag{F3}$$

This intuition is also evident in Figure 1b, which has $\Delta_1 < \Delta_2$ and shows that demands at the vertical dotted line, which corresponds to the market-clearing price, satisfy $x_1 < x_2$.

Second, we analyze the sensitivity of equilibrium demand to the scale of disagreement, i.e., how x_i varies with D:

$$\frac{\partial x_i}{\partial D} = \left(\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D - \Delta_i\right)\left[-\frac{1}{\mathcal{A}}\frac{1}{\sigma^2} + \frac{\mathcal{P}}{\mathcal{A}^2}\frac{s}{\sigma^3}D\left(\frac{1}{2}\frac{\mathcal{P}}{\mathcal{A}}\frac{s}{\sigma}D - \Delta_i\right)\right]. \tag{F4}$$

Lemma F.2 indicates that (F4) has the same sign as Δ_i, and hence the product of Δ_i with equilibrium demand x_i is increasing in D.

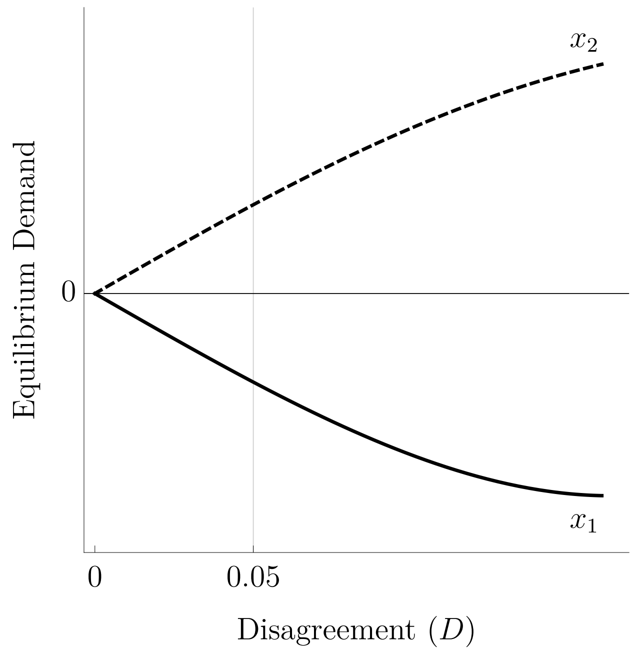
Lemma F.2. *The product of Δ_i with equilibrium demand x_i, $\Delta_i x_i$, is increasing in D for D not too large.*

$$\Delta_i \frac{\partial x_i(D)}{\partial D} > 0. \tag{F5}$$

Proof of Lemma F.2. See Appendix D. □

Figure F.2 shows how x_i increases with D for $\Delta_i > 0$ and decreases with D for $\Delta_i < 0$, for D not too large. The intuition is as follows. Because demand is strictly decreasing in price in a neighborhood of μ_i and crosses the price axis at $p = \mu_i = \mu + D\Delta_i$ (see Figure 1b), then as D increases, demand will cross the price axis at a higher price if $\Delta_i > 0$, and hence be higher at all prices less than this new price. Likewise if $\Delta_i < 0$, then demand will cross the price axis at a lower price and demand will be lower at all prices higher than this new price. Thus, demands expand away from the price axis as D increases. For D not too large,

Figure F.2: Equilibrium demand x_i as a function of disagreement D.



Notes: This figure plots under CRRA utility with risk aversion parameter γ the 2nd-order Taylor polynomial of the equilibrium demands x_1 (solid curve) and x_2 (dashed curve) as functions of D over the domain $D \in [0, 0.16]$ for the following parameter values: $\Delta_1 = -1$, $\Delta_2 = 1$, $\mu = 1$, $\sigma = 0.2$, $s = 0.6$, $\gamma = 2$, $w_0 = 1$.

the corresponding increase in the equilibrium price does not overcome this expansion effect of demands away from the price axis around the equilibrium price.

The intuition for the relationship of equilibrium volume to the scale of disagreement is related to the intuition for Lemma F.2. Because demands expand away from the price axis as D increases, negative demands become more negative and positive demands become more positive, and their absolute values increase, which increases equilibrium volume V.

IA G Higher-Order Sensitivities

G.A Sensitivity of Trading-Volume–Volatility Relationship to Disagreement in Equilibrium

For simplicity, we consider the case of CRRA utility but the same approach extends to more general utility. Differentiating V with respect to σ in (11), we obtain the following relationship between trading volume and payoff volatility:

$$\frac{\partial V}{\partial \sigma} = \frac{2}{\gamma} \frac{D}{\sigma^3} \left[-1 + \left(1 + \frac{1}{\gamma}\right)^2 \frac{s^2}{\sigma^2} D^2 \right]. \tag{G1}$$

Equation (G1) characterizes the nature of comovement between trading volume and volatility, holding all else equal. This relationship, multiplied by -1, increases in the scale of disagreement:

$$\frac{\partial}{\partial D}\left(-\frac{\partial V}{\partial \sigma}\right) = \frac{2}{\gamma}\frac{1}{\sigma^3}\left[1 - 3\left(1+\frac{1}{\gamma}\right)^2 \frac{s^2}{\sigma^2}D^2\right] > 0. \tag{G2}$$

This result indicates that the correlation between trading volume and volatility, multiplied by -1, is a proxy for disagreement.

Compare this result with Hsiao (2022), who proposes correlation between absolute returns and dollar volume, multiplied by -1, as an investor disagreement proxy. The behavior of absolute returns is tied to volatility, establishing a link to our results, but arrived at under distinct assumptions.

G.B Sensitivity of Trading-Volume–Skewness Relationship to Disagreement in Equilibrium

Differentiating V with respect to s in (11), we obtain the following relationship between trading volume and skewness:

$$\frac{\partial V}{\partial s} = -\frac{1}{\gamma}\frac{D^3}{\sigma^4}\left(1+\frac{1}{\gamma}\right)^2 s. \tag{G3}$$

Equation (G3) characterizes the nature of comovement between trading volume and skewness, holding all else equal. This relationship, multiplied by -1, increases in the scale of disagreement:

$$\frac{\partial}{\partial D}\left(-\frac{\partial V}{\partial s}\right) = \frac{3}{\gamma}\frac{D^2}{\sigma^4}\left(1+\frac{1}{\gamma}\right)^2 s > 0, \tag{G4}$$

for $s > 0$. This result indicates a novel proxy for investor disagreement, namely, the correlation between trading volume and skewness, multiplied by -1. Likewise, the correlation between trading volume and cubed returns, multiplied by -1, is suggested as an investor disagreement proxy.

IA H Robustness

H.A Additional Correlation Analyses

Table H.1: Time-Series Distributions of Cross-Sectional Correlations

Panel A: Correlations with DIS

Variable	Mean	SD	Min	p10	p25	p50	p75	p90	Max
SIR	0.13	0.08	−0.07	0.02	0.07	0.13	0.18	0.23	0.42
IVOL	0.62	0.07	0.29	0.53	0.58	0.63	0.66	0.70	0.77
ISKEW	0.48	0.13	−0.11	0.32	0.41	0.50	0.56	0.62	0.76
AFD	0.15	0.07	0.02	0.06	0.09	0.14	0.20	0.25	0.37
SIZE	−0.09	0.02	−0.15	−0.13	−0.11	−0.09	−0.08	−0.07	−0.03
BTM	0.07	0.09	−0.12	−0.02	0.01	0.05	0.11	0.21	0.38
MOM	−0.08	0.13	−0.39	−0.24	−0.16	−0.09	0.00	0.09	0.46
BETA	0.14	0.14	−0.25	−0.03	0.05	0.14	0.22	0.33	0.54
ILLIQ	0.12	0.11	−0.01	0.02	0.04	0.09	0.16	0.27	0.59
RMAX	0.55	0.07	0.35	0.45	0.50	0.55	0.60	0.65	0.72

Panel B: Correlations Among Major Disagreement Proxies

Variables	Mean	SD	Min	p10	p25	p50	p75	p90	Max
AFD, IVOL	0.13	0.05	0.02	0.06	0.09	0.13	0.16	0.19	0.25
AFD, SIR	0.07	0.04	−0.01	0.03	0.04	0.07	0.10	0.12	0.22
IVOL, SIR	0.24	0.08	0.01	0.13	0.18	0.24	0.31	0.35	0.44

Notes: Panel A presents the time-series distribution of the cross-sectional monthly correlation between DIS and various variables of interest. Panel B presents the time-series distribution of cross-sectional monthly correlations between each pair of major disagreement proxies: AFD, IVOL, and SIR. Each panel includes the following statistical measures for each variable: mean, standard deviation, minimum, maximum, and percentiles at the 10th, 25th, 50th (median), 75th, and 90th levels. The sample ranges from January 1994 to December 2022 and comprises all common stocks (identified as shrcd = 10, 11) traded on the NYSE, AMEX, or NASDAQ. For detailed definitions of the variables, please refer to Table 1 and Appendix B.

H.B Additional Portfolio Sorts

Table H.2: Portfolio Sorts on Disagreement, DIS, All Stocks

DIS	Raw Return	Excess Return	CAPM-α	FF3-α	FFC-α
Low	0.96	0.78	0.27	0.24	0.18
2	1.00	0.82	0.21	0.19	0.17
3	0.93	0.75	0.07	0.06	0.06
4	0.90	0.72	−0.04	−0.05	−0.01
5	0.85	0.67	−0.15	−0.15	−0.13
6	0.87	0.69	−0.17	−0.17	−0.07
7	0.75	0.56	−0.35	−0.36	−0.22
8	0.71	0.53	−0.50	−0.48	−0.31
9	0.46	0.28	−0.87	−0.82	−0.65
High	−0.06	−0.24	−1.55	−1.49	−1.31
Low-High	1.01	1.01	1.82	1.73	1.49
t-stat.	(2.02)	(2.02)	(4.90)	(5.57)	(4.65)
p-value	0.04	0.04	0.00	0.00	0.00

Notes: This table reports the mean values of value-weighted monthly returns at month $t+1$, based on univariate portfolios sorted according to disagreement (DIS) at date t. The sample ranges from January 1994 to December 2022, comparable to Table 3, but for all stocks including negative ISKEW stocks. The row labeled "Low-High" presents the differences in monthly returns between the 10th decile (representing high disagreement) and the 1st decile. "Raw returns" refer to one-month-ahead monthly returns, whereas "Excess returns" are raw returns minus the risk-free rate. The alphas (CAPM-α, FF3-α, FFC-α) represent intercepts from a time-series regression of monthly excess returns at month $t+1$ against the market return (CAPM), further adjusted for size and value factors (FF3), and additionally a momentum factor (FFC) at month t. t-statistics, adjusted according to Newey and West (1987), are presented in parentheses along with their corresponding p-values.

Table H.3: DIS-Decile-Spread Portfolio α's, Excluding the Union of 20% Most Illiquid and 20% Most Retail-Owned Stocks

DIS	CAPM-α	FF3-α	FFC-α
Low-High	1.79	1.71	1.43
t-stat.	(4.95)	(5.39)	(4.31)
p-value	0.000	0.000	0.000

Notes: This table reports the mean values of value-weighted monthly alphas at month $t+1$, based on univariate portfolios sorted according to disagreement (DIS) at date t, excluding firm-months with negative ISKEW (cf. Table 3) and excluding trailing 6-month illiquidity (Amihud, 2002) in the top 20% or institutional ownership (IO) in the bottom 20% (equivalently, the top 20% retail-owned stocks) each month using breakpoints from CRSP/COMPUSTAT (price >\$5). The sample ranges from January 1994 to December 2022. The row labeled "Low-High" presents the differences between the 1st and 10th DIS deciles. The alphas (CAPM-α, FF3-α, FFC-α) represent intercepts from a time-series regression of monthly excess returns at month $t+1$ against the market return (CAPM), further adjusted for size and value factors (FF3), and additionally a momentum factor (FFC) at month t. t-statistics, adjusted according to Newey and West (1987), are presented in parentheses along with their corresponding p-values.

Table H.4: CAPM-α of Portfolio Sorts on Various Disagreement Proxies

	DIS	AFD	IVOL	SIR
Low	0.27	0.32	0.24	0.15
2	0.21	0.11	0.29	0.00
3	0.07	0.13	0.09	0.02
4	−0.04	−0.08	−0.08	−0.01
5	−0.15	0.00	−0.07	−0.06
6	−0.17	0.03	−0.31	−0.04
7	−0.35	−0.13	−0.29	−0.04
8	−0.50	−0.15	−0.33	−0.26
9	−0.87	−0.21	−0.56	−0.46
High	−1.55	−0.77	−1.07	−0.58
Low-High	1.82	1.08	1.31	0.73
t-stat.	(4.90)	(4.23)	(3.94)	(2.66)
p-value	0.00	0.00	0.00	0.01

Notes: This table reports the CAPM-α estimated as the intercept from a time-series regression of value-weighted monthly excess returns at month $t+1$ against the market excess return at month t for univariate portfolios sorted according to various disagreement proxies at date t. Each column represents a different sorting variable listed at the top of the column: disagreement (DIS), analyst forecast dispersion (AFD), IVOL, or short interest (SIR). The sample ranges from January 1994 to December 2022. The row labeled "Low-High" presents the differences in monthly returns between the 10th decile (representing high values of the sorting variable) and the 1st decile (representing low values of the sorting variable). t-statistics, adjusted according to Newey and West (1987), are presented in parentheses along with their corresponding p-values.

H.C Robustness to Relative Risk Aversion

Table H.5: CAPM-α of Portfolio Sorts on Disagreement, DIS, for Various Levels of Relative Risk Aversion

DIS	$\gamma = \frac{1}{2}$	$\gamma = 1$	$\gamma = 2$	$\gamma = 4$	$\gamma = 8$
Low	0.28	0.27	0.27	0.29	0.28
2	0.18	0.17	0.21	0.23	0.23
3	0.08	0.13	0.07	0.08	0.10
4	−0.08	−0.04	−0.04	−0.10	−0.11
5	−0.15	−0.22	−0.15	−0.12	−0.13
6	−0.09	−0.09	−0.17	−0.20	−0.18
7	−0.29	−0.27	−0.35	−0.34	−0.33
8	−0.54	−0.56	−0.50	−0.54	−0.54
9	−0.72	−0.80	−0.87	−0.82	−0.86
High	−1.46	−1.57	−1.55	−1.56	−1.56
Low-High	1.74	1.85	1.82	1.84	1.84
t-stat.	(4.98)	(5.14)	(4.90)	(4.94)	(5.00)

Notes: This table reports the CAPM-α estimated as the intercept from a time-series regression of value-weighted monthly excess returns at month $t+1$ against the market excess return at month t, for univariate portfolios sorted according to disagreement (DIS) at date t, computed using various values of the relative risk aversion coefficient, γ, which varies by column. The sample ranges from January 1994 to December 2022. The row labeled "Low-High" presents the differences in monthly returns between the 10th decile (representing high disagreement) and the 1st decile (representing low disagreement). t-statistics, adjusted according to Newey and West (1987), are presented in parentheses along with their corresponding p-values.

Table H.6: Fama-MacBeth Cross-Sectional Regressions for Various Levels of Relative Risk Aversion

γ	($\gamma = \frac{1}{2}$) EXRET$_{t+1}$	($\gamma = 1$) EXRET$_{t+1}$	($\gamma = 2$) EXRET$_{t+1}$	($\gamma = 4$) EXRET$_{t+1}$	($\gamma = 8$) EXRET$_{t+1}$
DIS	−0.58***	−1.05***	−1.55***	−2.18***	−2.62***
	(−4.23)	(−4.35)	(−3.98)	(−4.04)	(−3.97)
Constant	0.84***	0.82***	0.82***	0.85***	0.88***
	(3.09)	(2.96)	(2.94)	(3.05)	(3.16)

Notes: This table displays the regression coefficients and t-statistics from a monthly Fama and MacBeth (1973) regression analysis conducted on excess returns, adhering to the methodologies in Fama and French (1993) and Carhart (1997). These regressions are performed against a lagged measure of disagreement, computed based on one of five levels of relative risk aversion, γ. The sample ranges from January 1994 to December 2022. The coefficients on DIS are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively, under a single test.

H.D DIS, IVOL, and Macro Regimes

Table H.7: Return Predictability in High or Low Macroeconomic Regimes

	High State				*Low State*			
	DIS		IVOL		DIS		IVOL	
Macro Indicator	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
VIX Index	−0.97***	(−2.85)	0.01	(−0.07)	−1.04**	(−2.33)	−0.19***	(−2.88)
Volatility-of-Volatility	−0.44	(−1.52)	−0.06	(−0.83)	−1.57***	(−3.56)	−0.12	(−1.46)
CATFIN	−0.76**	(−2.25)	−0.13	(−1.42)	−1.25***	(−2.71)	−0.06	(−0.70)
Financial Uncertainty	−0.72***	(−2.65)	−0.07	(−0.71)	−1.29***	(−2.66)	−0.11*	(−1.77)
Real Uncertainty	−0.92**	(−2.17)	−0.04	(−0.42)	−1.09***	(−2.91)	−0.14*	(−1.75)
Macro Uncertainty	−1.02**	(−2.41)	−0.09	(−0.96)	−0.99**	(−2.46)	−0.10	(−1.29)
PLS Sentiment Index	−1.00**	(−2.56)	−0.07	(−0.66)	−1.13**	(−2.61)	−0.09	(−1.19)
Aggregate IVOL	−1.03**	(−2.56)	−0.09	(−0.88)	−0.98**	(−2.51)	−0.09	(−1.60)

Notes: This table displays the regression coefficient and t-statistic of each disagreement measure DIS and IVOL from a monthly Fama and MacBeth (1973) regression analysis performed on excess returns against DIS and IVOL, standard cross-sectional controls (BETA, LnSIZE, BTM, and MOM), and a constant (not reported). Regressions are performed for various subsamples based on median breakpoints of eight macro indicators: VIX; Volatility-of-Volatility following Bekaert and Hoerova (2014); CATFIN, the systemic risk measure of Allen, Bali, and Tang (2012); the Financial Uncertainty, Real Uncertainty, and Macroeconomic Uncertainty indexes of Jurado, Ludvigson, and Ng (2015); the PLS Sentiment Index of Kelly and Pruitt (2013); and Aggregate IVOL. For each indicator, high (low) states are defined as months in which the indicator is above (below) its full-sample median. The coefficients on DIS are multiplied by 100 for readability. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively, under a single test. For detailed definitions of the variables, please refer to Appendix B.

H.E Detecting Undetected Disagreement

Table H.8: Marginal Effect of DIS when Extant Disagreement Measure D Indicates No Disagreement: Fama and MacBeth (1973) Cross-Sectional Interaction Regressions, Additional Characteristics

	D = IVOL		D = AFD		D = SIR	
Y	DIS coeff.	t-stat.	DIS coeff.	t-stat.	DIS coeff.	t-stat.
EXRET$_{t+1}$	−2.384***	(−3.46)	−1.531***	(−4.26)	−1.074***	(−3.00)
IVOL			0.044***	(10.94)	0.042***	(12.10)
AFD	0.784***	(10.22)			0.330***	(10.14)
SIR	−0.008	(−0.88)	0.017***	(5.53)		
ISKEW	0.027***	(13.75)	0.010***	(19.01)	0.009***	(19.54)
SIZE	−0.079***	(−9.31)	−0.041***	(−9.57)	−0.044***	(−11.16)
BTM	0.111**	(2.44)	−0.049*	(−1.90)	−0.106***	(−4.89)
MOM	−0.799***	(−5.41)	−0.141**	(−2.10)	−0.162***	(−2.95)
BETA	0.014***	(9.53)	0.007***	(8.73)	0.006***	(8.22)
ILLIQ	0.017***	(4.10)	0.010***	(3.28)	0.013***	(3.89)
RMAX	0.019***	(8.73)	0.052***	(10.52)	0.050***	(11.64)
KZ	0.051***	(7.19)	0.039***	(9.39)	0.032***	(9.36)
OHLSON	0.525***	(13.83)	0.296***	(14.58)	0.244***	(14.49)
AGE	0.441**	(2.41)	−0.269***	(−5.58)	−0.401***	(−9.99)
NI-IVOL	0.308***	(7.54)	0.224***	(5.74)	0.222***	(3.47)
HHI	0.145***	(7.20)	0.065***	(7.69)	0.074***	(9.19)
IO	−0.451***	(−10.38)	−0.175***	(−12.58)	−0.123***	(−13.18)
R&D	0.256***	(13.33)	0.143***	(14.76)	0.124***	(14.95)
ISS	0.253***	(10.24)	0.170***	(12.33)	0.140***	(10.73)
ASC	0.014***	(6.83)	0.006***	(6.80)	0.007***	(7.82)
SUV	−0.016***	(−6.05)	0.010***	(8.62)	0.009***	(9.69)
TO	−0.088***	(−4.31)	0.114***	(9.66)	0.058***	(8.12)
DTO	−0.115***	(−8.92)	0.058***	(7.03)	0.035***	(7.45)
DBREADTH	−0.014***	(−3.44)	−0.001	(−0.85)	−0.006***	(−3.31)
BIDASK	0.014***	(7.20)	0.012***	(6.34)	0.012***	(6.80)
SV	−0.081***	(−3.96)	−0.030***	(−5.30)	−0.034***	(−7.17)
SV-INV	0.062***	(2.94)	0.028***	(5.48)	0.038***	(2.95)

Notes: This table reports the estimated marginal effect β_{DIS} of composite disagreement DIS on characteristic Y when extant disagreement measure D detects no disagreement ($\beta_{\text{DIS}} = \frac{\partial Y}{\partial \text{DIS}}$ as $D \searrow 0$) and its Newey and West (1987) adjusted t-statistic from the Fama and MacBeth (1973) cross-sectional interaction regression of characteristic Y against DIS, an extant disagreement measure D, and their interaction (DIS $\times$ D), as well as a vector of standard cross-sectional control variables Z,

$$Y = \alpha + \beta_{\text{DIS}} \cdot \text{DIS} + \beta_D \cdot D + \beta_{\text{DIS}\times D} \cdot (\text{DIS} \times D) + \beta_Z' Z + \varepsilon,$$

which are reported (by column) for each extant disagreement measure D among IVOL, AFD, and SIR; and (by row) for each variable Y among EXRET$_{t+1}$, IVOL, AFD, SIR, ISKEW, SIZE, BTM, MOM, BETA, ILLIQ, RMAX, KZ, OHLSON, AGE,NI-IVOL, HHI, IO, R&D, ISS, ASC, SUV, TO, DTO, DBREADTH, BIDASK, SV, and SV-INV; where Z includes BETA, LnSIZE, BTM, and MOM, and β_Z is their corresponding coefficient vector; and where ε is an error term. The sample ranges from January 1994 to December 2022. Coefficients are multiplied by 100 for readability when Y = EXRET$_{t+1}$, SIR, BTM, OHLSON, HHI, IO, R&D, ISS, and BIDASK. The significance levels are denoted as ***, **, and *, indicating significance at the 1%, 5%, and 10% levels, respectively, under a single test. For detailed definitions of the variables, refer to Table 1 and Appendix B.

Table H.8 extends the analysis of Table 9 to additional characteristics showing further that in the absence of disagreement detected by an extant disagreement measure (IVOL, AFD, and SIR), DIS exhibits a consistent negative relationship with EXRET_{t+1}, IO, and SV, and a consistent positive relationship with KZ, OHLSON, NI-IVOL, HHI, R&D, ISS, ASC, BIDASK, and SV-INV. These relationships further suggest that DIS detects a specific dimension of return-predictive variation consistent with disagreement, undetected by extant measures, that is fundamentally linked to firms having less institutional ownership, lower Google search volume, lower financial health, higher net income idiosyncratic volatility, less competition, more research and development expenditures, augmented net equity issuance, greater information asymmetry, wider bid-ask spreads, and higher investment-specific Google search volume.

H.F Additional Stock Characteristics

In Table H.9, we analyze the relationship between DIS and various additional firm characteristics, similar to Table 10. Our findings indicate that elevated levels of DIS are associated with several attributes: increased financial constraints (OHLSON, Ohlson, 1980; and KZ, Kaplan and Zingales, 1997); younger firms (AGE); increased net income idiosyncratic volatility (NI-IVOL); heightened concentration of institutional investors (as measured by the Herfindahl and Hirschman Index, HHI); intensified research and development expenditures (R&D); augmented net equity issuance (ISS); greater information asymmetry (ASC); elevated standardized unexplained volume (SUV); increased turnover (TO); higher change in market-adjusted turnover (DTO); greater opinion divergence among mutual funds (DBREADTH; see Chen, Hong, and Stein, 2002); wider bid-ask spreads (BIDASK); and reduced volumes of Google searches, both overall (SV) and specifically investment-related (SV-INV). For details of these variables, refer to Appendices B and C.

Table H.10 repeats the analyses of Table H.9, excluding firm-months with trailing 6-month illiquidity (Amihud, 2002) in the top 20% or institutional ownership (IO) in the bottom 20% (equivalently, the top 20% retail-owned stocks) each month using breakpoints from CRSP-COMPUSTAT databases having price above $5.

Table H.9: Average Stock Characteristics of DIS-Sorted Portfolios: Additional Characteristics

	Low	2	3	4	5	6	7	8	9	High	H−L	t-stat.
DIS	1.00	1.59	2.12	2.76	3.63	4.93	7.12	11.19	20.25	67.67	66.67***	(8.64)
SIR	0.02	0.03	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.07	0.04***	(13.79)
IVOL	0.83	1.16	1.41	1.64	1.87	2.10	2.33	2.58	2.96	4.13	3.30***	(34.86)
ISKEW	0.45	0.51	0.55	0.60	0.65	0.73	0.85	1.00	1.19	1.54	1.09***	(13.08)
AFD	3.03	4.15	5.47	7.19	9.24	11.61	14.88	19.51	24.73	33.15	30.12***	(22.01)
SIZE	24.80	15.69	11.33	8.36	6.12	4.81	3.48	2.28	1.50	0.79	−24.01***	(−10.91)
BTM	0.52	0.50	0.50	0.50	0.51	0.53	0.55	0.59	0.62	0.66	0.14***	(4.97)
MOM	15.07	17.39	20.03	21.79	23.52	24.20	22.17	19.41	14.50	8.08	−6.99*	(−1.77)
BETA	0.74	0.86	0.93	0.99	1.05	1.12	1.18	1.23	1.30	1.42	0.69***	(15.25)
ILLIQ	0.01	0.01	0.02	0.03	0.04	0.05	0.07	0.11	0.21	0.46	0.45***	(5.18)
RMAX	1.60	2.06	2.39	2.68	2.97	3.28	3.58	3.91	4.43	5.94	4.34***	(26.94)
KZ	3.37	3.78	4.09	4.29	4.52	4.81	4.77	4.94	5.40	6.21	2.84***	(7.76)
OHLSON	0.05	0.05	0.06	0.08	0.09	0.11	0.14	0.18	0.25	0.35	0.31***	(18.24)
AGE	468.8	399.6	355.0	322.5	292.3	268.9	247.7	226.7	199.2	167.4	−301.4***	(−49.52)
NI-IVOL	1.44	1.88	2.29	2.78	3.42	3.97	5.23	6.64	9.67	15.22	13.79***	(8.62)
HHI	0.05	0.06	0.06	0.06	0.07	0.07	0.08	0.08	0.10	0.12	0.07***	(18.95)
IO	0.65	0.70	0.71	0.72	0.71	0.71	0.69	0.66	0.62	0.55	−0.10***	(−15.88)
R&D	0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.10	0.13	0.18	0.14***	(19.03)
ISS	−0.02	−0.02	−0.01	−0.01	0.00	0.01	0.03	0.04	0.08	0.15	0.17***	(14.74)
ASC	0.24	0.30	0.35	0.39	0.44	0.48	0.53	0.60	0.72	0.95	0.70***	(12.33)
SUV	−0.04	0.01	0.05	0.09	0.12	0.15	0.17	0.18	0.18	0.37	0.40***	(16.28)
TO	4.74	5.80	6.75	7.68	8.50	9.23	9.56	9.79	10.20	12.68	7.94***	(15.53)
DTO	−0.51	−0.43	−0.35	−0.25	−0.13	0.04	0.15	0.16	0.14	1.09	1.59***	(9.06)
DBREADTH	−1.14	−1.30	−1.27	−1.17	−1.17	−1.08	−0.84	−0.62	−0.33	−0.01	1.13***	(5.27)
BIDASK	0.005	0.006	0.007	0.008	0.009	0.009	0.010	0.011	0.013	0.017	0.011***	(19.19)
SV	43.62	41.12	39.51	38.29	36.89	35.64	33.92	32.49	30.52	28.23	−15.33***	(−30.32)
SV-INV	7.73	7.65	7.54	7.33	7.16	7.15	6.67	6.74	6.42	6.03	−1.77***	(−7.17)

Notes: This table reports the average values of various stock characteristics across univariate, equally weighted portfolios sorted based on disagreement (DIS). The sample ranges from January 1994 to December 2022. The categories “Low” and “High” refer to the portfolios within the lowest and highest disagreement deciles, respectively. NI-IVOL is multiplied by 100 and ASC, DTO, DBREADTH, and TO are multiplied by 1,000. Newey and West (1987) adjusted t-statistics for assessing the statistical difference between the highest and lowest disagreement decile are shown in the last column. For detailed definitions of the variables, please refer to Appendix B.

Table H.10: Average Stock Characteristics of DIS-Sorted Portfolios, Excluding the Union of 20% Most Illiquid and 20% Most Retail-Owned Stocks

	Low	2	3	4	5	6	7	8	9	High	H−L	t-stat.
DIS	0.98	1.55	2.05	2.63	3.40	4.54	6.40	9.81	17.23	55.20	54.22***	(8.72)
SIR	0.02	0.03	0.04	0.04	0.05	0.05	0.06	0.06	0.07	0.08	0.05***	(13.60)
IVOL	0.82	1.14	1.38	1.61	1.82	2.05	2.29	2.53	2.88	3.97	3.15***	(33.23)
ISKEW	0.44	0.50	0.54	0.58	0.63	0.70	0.79	0.93	1.10	1.40	0.95***	(13.58)
AFD	2.98	3.95	5.21	6.78	8.46	10.94	13.93	18.80	23.81	33.74	30.76***	(19.41)
SIZE	23.18	15.11	11.24	8.50	6.61	5.08	3.93	2.63	1.83	1.03	−22.15***	(−12.90)
BTM	0.52	0.50	0.49	0.50	0.50	0.51	0.53	0.57	0.60	0.65	0.13***	(4.66)
MOM	15.05	17.27	19.70	21.68	22.97	24.33	23.09	20.00	15.79	8.35	−6.70*	(−1.67)
BETA	0.74	0.86	0.93	0.99	1.06	1.13	1.19	1.25	1.34	1.53	0.79***	(17.13)
ILLIQ	0.01	0.01	0.01	0.01	0.02	0.02	0.03	0.03	0.05	0.07	0.06***	(4.10)
RMAX	1.60	2.04	2.36	2.65	2.93	3.22	3.54	3.85	4.34	5.74	4.14***	(25.65)
KZ	3.42	3.82	4.13	4.33	4.58	4.72	4.92	5.06	5.50	6.29	2.87***	(7.91)
OHLSON	0.05	0.05	0.06	0.07	0.09	0.11	0.13	0.16	0.22	0.32	0.28***	(18.49)
AGE	476.1	407.8	363.7	333.3	302.9	280.1	258.5	237.8	211.5	179.2	−296.9***	(−50.57)
NI-IVOL	1.40	1.84	2.24	2.73	3.29	3.93	4.60	5.53	7.14	12.38	10.97***	(14.50)
HHI	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.07	0.08	0.09	0.04***	(18.09)
IO	0.67	0.72	0.74	0.75	0.75	0.75	0.74	0.72	0.69	0.65	−0.02***	(−3.40)
R&D	0.03	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.12	0.16	0.13***	(22.30)
ISS	−0.02	−0.02	−0.02	−0.01	0.00	0.01	0.02	0.04	0.07	0.13	0.15***	(15.44)
ASC	0.23	0.29	0.33	0.37	0.41	0.44	0.49	0.55	0.63	0.79	0.56***	(9.51)
SUV	−0.04	0.01	0.05	0.08	0.11	0.14	0.17	0.17	0.17	0.33	0.37***	(16.72)
TO	4.80	5.83	6.78	7.73	8.59	9.41	10.00	10.33	10.89	13.99	9.19***	(17.35)
DTO	−0.52	−0.45	−0.36	−0.28	−0.16	0.00	0.19	0.17	0.17	1.26	1.78***	(12.93)
DBREADTH	−1.29	−1.46	−1.43	−1.33	−1.26	−1.25	−1.03	−0.73	−0.45	−0.03	1.27***	(6.36)
BIDASK	0.005	0.006	0.007	0.008	0.008	0.009	0.010	0.011	0.012	0.015	0.010***	(22.59)
SV	43.98	41.51	39.88	38.94	37.64	36.44	34.83	33.52	31.84	29.92	−14.01***	(−24.46)
SV-INV	7.69	7.66	7.56	7.41	7.23	7.14	6.87	6.74	6.57	6.22	−1.54***	(−6.73)

Notes: This table reports the average values of various stock characteristics across univariate, equally weighted portfolios sorted based on disagreement (DIS), excluding firm-months with trailing 6-month illiquidity (Amihud, 2002) in the top 20% or institutional ownership (IO) in the bottom 20% (equivalently, the top 20% retail-owned stocks) each month among all stocks in the intersection of CRSP and COMPUSTAT databases having price above $5. The sample ranges from January 1994 to December 2022. The categories "Low" and "High" refer to the portfolios within the lowest and highest disagreement deciles, respectively. NI-IVOL is multiplied by 100 and ASC, DTO, DBREADTH, and TO are multiplied by 1,000. Newey and West (1987) adjusted t-statistics for assessing the statistical difference between the highest and lowest disagreement decile are shown in the last column. For detailed definitions of the variables, please refer to Appendix B.

H.G DIS-Decile Transition Relative Frequency Matrix

Table H.11: Relative Frequency Monthly Transition Matrix for Deciles of DIS

DIS Decile	DIS Deciles at t									
at $t-1$	Low	2	3	4	5	6	7	8	9	High
Low	50.1	23.8	12.0	6.7	3.6	2.1	1.1	0.5	0.1	0.0
2	25.6	27.4	19.6	12.2	7.1	4.3	2.3	1.1	0.4	0.1
3	13.3	21.5	22.8	17.3	11.3	6.9	4.0	1.9	0.8	0.3
4	7.0	13.9	19.4	20.7	16.7	11.0	6.3	3.1	1.5	0.5
5	3.8	8.1	13.3	19.0	21.0	16.3	10.0	5.2	2.3	0.9
6	2.1	4.7	8.2	13.2	19.1	21.4	16.4	9.1	4.2	1.6
7	1.1	2.5	4.6	7.8	12.7	19.8	23.3	17.0	8.2	3.0
8	0.5	1.1	2.1	3.9	6.8	12.2	21.2	26.9	18.4	7.1
9	0.2	0.4	0.8	1.5	3.0	5.6	11.7	23.9	33.7	19.3
High	0.1	0.1	0.2	0.4	0.8	1.7	3.8	9.8	26.6	56.5

Notes: This table reports for stocks in each DIS decile i in month $t-1$ (rows) their relative frequency of being in DIS decile j next month t (columns), averaged across time. Cells with higher relative frequency are shaded darker. Rows sum to 100%.

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