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ABSTRACT

While standard sudden-stop models explain well the sharpness of financial crises, it remains challenging to account for the persistent growth stagnation that typically follows credit-driven capital account liberalizations in emerging markets. This paper presents a small open economy model with endogenous growth and borrowing constraints to ex- plain this phenomenon. The model highlights a valuation optimistic growth expectations, fueled by foreign credit with perfectly elastic supply, raise asset prices and relax liquidity constraints, inducing investment that validates the initial optimism. This positive feed- back loop generates multiple balanced growth paths and self-fulfilling cycles. However, in closed economies or with only FDI inflows, funding supply for investment is limited by consumption smoothing, dampening this feedback and leading to a unique growth path. Belief-driven regime switches can replicate crisis dynamics, including sharp drops in asset prices and growth. The framework also exhibits periodic orbits, producing endogenous deterministic fluctuations under perfect foresight. Countercyclical macroprudential measures can disrupt the detrimental feedback loop, while policies prioritizing FDI over credit prevent bad equilibria altogether, guiding the economy toward a stable, high-growth trajectory.

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1 Introduction

Small open economies (SOEs) routinely experience powerful boom cycles in the wake of capital account liberalization (Aoki et al., 2007; Martin and Rey, 2006; Broner and Ventura, 2016). Capital inflows initially fuel credit expansion and elevate asset valuations, only to be followed by abrupt reversals that trigger financial crashes and deep recessions. This pattern is particularly pronounced in emerging markets, where capital inflow bonanzas are more likely to end in sudden stops or crises, as documented by Reinhart and Reinhart (2009). Crucially, this destructive pattern is primarily associated with volatile debt inflows, which amplify financial fragility, rather than with foreign direct investment (FDI) inflows (Rogoff, 1999; Aghion et al., 2004; Ghosh et al., 2016; Caballero, 2016). For instance, IMF (1998) argues that China's economy exhibited stability and resilience during the Asian Financial Crisis as its capital inflows are primarily direct investments. [1] Consistent with IMF (1998), Panels (a) and (b) of Figure 1 compare Thailand and China during the Asian Financial Crisis. Prior to the crisis, most capital inflows to Thailand were in the form of debt, whereas China attracted predominantly foreign direct investment (FDI). After the crisis, capital outflows were more drastic and GDP growth fell more sharply in Thailand.

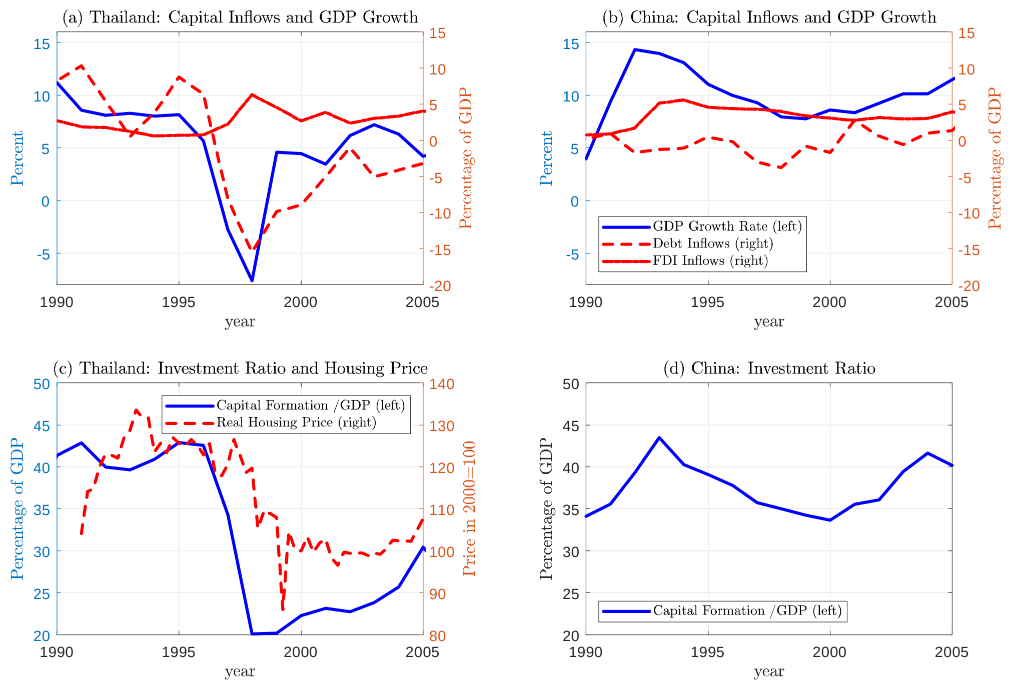

Figure1: Thailand and China

Note: This figure shows the capital flows and other macroeconomic variables for Thailand and China after the Asian Financial Crisis. We use the same scale of y-axis to compare. China did not begin its housing marketization reform until 1998, when the welfare-based housing allocation system was abolished. The housing price data in China is not available during the episode.

[1]Other international organizations, like UNCTAD also held similar opinion. See "https://press.un.org/en/1999/19990428.tad1873.html".

A defining characteristic of financial crises in emerging markets is not merely the severity of the crash, but the persistence of the stagnation that follows. Following the Asian Financial Crisis and the Latin American Debt Crisis, economies such as Thailand, Malaysia, and Mexico did not simply experience a temporary deviation from a stable trend. They suffered a permanent downward shift in their long-run growth trajectories. As shown in Table 1, comparing average economic growth rates over the decade before and after a crisis, while excluding the crisis years themselves, these countries exhibit a noticeable downward shift in post-crisis trend growth. As documented by Aguiar and Gopinath (2007), emerging market business cycles are dominated by shocks to trend growth rather than transitory fluctuations. Yet, the standard canonical models of small open economies, which emphasize pecuniary externalities and balance-sheet constraints, typically treat long-run growth as exogenous. Consequently, while these frameworks excel at explaining the bust, it remains challenging to account for the long-run stagnation. Why do temporary financial dislocations cast such long shadows over future economic growth?

Average Growth	Mexico		Thailand		Malaysia	
Before	1972-1981	7.24%	1987-1996	9.28%	1988-1997	9.27%
After	1984-1993	2.41%	1999-2008	4.84%	1999-2008	5.55%

Table 1: Long-run Growth after Crises

Note: The table reports the average growth rates over the decade before and after a crisis, while excluding the crisis years themselves.

This paper aims to fill this gap by developing an endogenous growth framework to analyze the impacts of different types of inflows on crises and long-term growth. To articulate the disparate impacts, our model is centered on a valuation-liquidity feedback mechanism. A key distinction from canonical sudden-stop models is our introduction of capital accumulation and an endogenous growth mechanism. This setup explicitly links changes in the expected growth rate to shifts in asset valuation, as growth prospects alter the present value of future asset returns, which is central to the feedback we analyze. The behavior of this mechanism, whether it acts as an engine of instability or a stabilizer, hinges fundamentally on the elasticity of the funding supply in different open regimes. In a closed economy or FDI-open economy, higher growth expectations increase current consumption driven by the permanent income effect and consumption smoothing, and raise the domestic real interest rate due to competition for scarce savings. This leads to a higher cost of borrowing, which in turn dampens asset valuations, tightens credit constraints and discourages investment, creating a negative feedback loop that puts the economy on a unique balanced growth path. By contrast, in a credit-open economy, capital flows are perfectly elastic at the world interest rate. Consequently, improved growth expectations boost asset prices without raising borrowing costs, relax collateral constraints and stimulate investment. This validates the initial optimism in a self-fulfilling way. The same mechanism works in reverse under pessimistic expectations. This positive feedback loop, which is specific to credit open regimes, gives rise to multiple balanced growth paths. [2] It thereby complements the important analysis of Schmitt-Grohé and Uribe (2021a,b) by shifting the focus from multiplicity in transition paths to multiplicity in long-run balanced growth paths, which is key to explaining the permanent growth shifts behind the post-crisis markets.

Multiple balanced growth paths provide a natural theory of the sudden stop as a regime switch. A shift in market sentiment can trigger an abrupt transition from the high-growth to the low-growth path. Unlike standard sudden-stop models, where crises are temporary deviations along a unique transition path, our framework demonstrates that a belief-driven crisis triggers a permanent regime shift across multiple balanced growth paths. Because perfectly elastic foreign credit neuters the domestic interest rate's natural stabilizing role, a financial crash traps the economy in a permanently lower steady-state growth trajectory, mirroring the persistent stagnation observed in the data. Importantly, to generate multiplicity, the productivity cannot be very high. Otherwise, the economy possesses a unique balanced growth path without self-fulfilling crises. This is consistent with the evidence in Reinhart and Reinhart (2009) that bonanzas are more destabilizing in emerging economies. Our theoretical contrast between FDI and debt inflows further clarifies that governments should encourage FDI, whereas debt inflows require countercyclical policies to avoid the pitfalls of multiplicity.

Specifically, we first build a closed economy model, which comprises two types of agents: households and entrepreneurs. Households consume and rent houses. They cannot borrow from the credit market. Entrepreneurs play a core role in our model. They invest in productive capital or purchase housing assets to collect rental income, and they can borrow or save from the credit market. Borrowing requires capital or housing assets as collateral. A key feature in this model is the presence of idiosyncratic, stochastic investment opportunity shocks. Only a fraction of entrepreneurs has investment opportunities in each period. These idiosyncratic shocks, combined with collateral constraints, lead to an incomplete market framework. In this environment, housing assets provide liquidity and collateral for entrepreneurs, enabling them to collect funds by selling houses to seize investment opportunities. Therefore, the balanced growth rate itself increases monotonically with housing prices. An increase in housing values enhances entrepreneur liquidity reserves, enabling greater investment upon investment opportunities, which fuels economic growth. However, due to permanent income effect and consumption smoothing, expectations of higher future growth lead to higher current consumption, which creates a shortage of savings as resources are limited in the closed economy. Therefore, supply of funds is not perfectly elastic. To clear the market, the equilibrium cost of funds (the real interest rate) must rise sharply, which acts as a powerful dampening force. It counteracts the rise in expected rents: although the houses promise higher future cash flows, those flows are discounted at a much steeper rate representing the higher fund cost. Consequently, the valuation of the houses is dragged down. Therefore, the interest rate plays a role of stabilizer

[2] This mechanism differs from rational bubbles. In models of rational bubbles, an asset that yields dividends leads to a unique equilibrium. In our model, although housing generates rents, its interaction with endogenous growth can produce multiple equilibrium growth paths.

and the economy is a negative feedback system. The balanced growth path is unique.

Furthermore, we consider the openness to foreign direct investment (FDI) in our model. We assume that FDI inflows face entry costs that increase with the share of foreign capital in the economy, reflecting potential barriers such as regulatory hurdles or market saturation. Additionally, we introduce complementarity between domestic and foreign capital. A key feature of our model is that the balanced growth path remains unique, irrespective of whether technological complementarity between foreign and domestic capital exists. The intuition behind this is that FDI funds do not flow into the housing market and they cannot fuel the housing price when the growth expectation increases. The interest rate still plays a role of stabilizer. However, by incorporating complementarity, we can better capture the growth-enhancing benefits of FDI, such as technology diffusion and improvements in the overall business environment, as documented in empirical studies (Javorcik, 2004; Haskel et al., 2007). We show that FDI inflows can raise the economic growth rate by spillover effects.

We further consider the openness of credit. In this setting, the system has multiple balanced growth paths due to the valuation-liquidity feedback loop. The distinctiveness of this regime lies in the elasticity of the funding supply. In a financially integrated small open economy, agents can borrow at the world interest rate, which is fixed and largely insensitive to domestic conditions. Therefore, supply of funds is perfectly elastic, which creates a decoupling between domestic growth dynamics and the cost of capital. Consider a scenario where agents become optimistic about future growth rates. In a credit-open regime, this optimism does not trigger a rise in the interest rate that would otherwise cool the economy. Because the cost of borrowing remains anchored at the low world rate, the increase in expected future rents translates into soaring housing valuations, which relax the liquidity constraints for entrepreneurs. This influx of imported liquidity fuels immediate investment, generating the very growth that agents anticipated. Thus, the combination of fixed borrowing costs and endogenous collateral values generates a self-fulfilling positive feedback loop, giving rise to multiple balanced growth paths in our model. The main mechanism of the model is consistent with the macro variables we observe. As shown in Figure 1, in the aftermath of the Asian Financial Crisis, Thailand experienced capital outflows and economic downturns, along with a sharp decline in housing prices and a significant drop in the investment rate in the economy.

We compare the growth rates in the credit-open economy with the closed economy. While in most cases capital account opening leads to higher growth rates on a balanced growth path, our model suggests that if foreign interest rates are too low and lead to a decline in the return on liquid assets, the entrepreneurs become less willing to hold these liquidity assets. This undermines their role as liquidity buffers, reduces investment, and can push the economy onto a growth trajectory that is even lower than that under financial autarky.

Furthermore, we employ a regime-switching approach to capture the stochastic cyclical patterns observed in the economy. Our model features two distinct balanced growth paths, each consistent with equilibrium conditions. A shift in market sentiment, such as the pessimism regarding future growth, can trigger a transition from the high-growth to the low-growth equilibrium. During this transition, the dynamic system leaves the high-growth path and converges to the low-growth path. This adjustment is marked by a sharp decline in both housing price and the economic growth rate. The economy may experience persistent stagnation as the system can stay on the low balanced growth path for arbitrarily long time. Additionally, small changes in structural parameters, such as foreign interest rate or collateral ratio, can alter the number of balanced growth paths. Specifically, such changes may cause the high-growth equilibrium to vanish, leaving the low-growth path as the only stable outcome. In this scenario, the economy transitions to a permanently lower growth trajectory, even without a shift in beliefs.

Our model can even present deterministic cycles or periodic orbits in a perfect foresight equilibrium. The high-growth path typically acts as a saddle point, while the low-growth equilibrium may function as either a sink or a source. Using bifurcation theory and global dynamic analysis, we find that under certain parameterizations, specifically near critical points separating sink and source regimes, a three-dimensional Hopf bifurcation arises. This bifurcation gives rise to stable periodic orbits, implying that the economy can exhibit endogenous, self-sustaining cycles in a perfect-foresight equilibrium. These cyclical dynamics, generated entirely within the model without relying on exogenous shocks, align with the view of Beaudry et al. (2020) that economic fluctuations can be driven by internal cyclical mechanisms and coordination forces. Our results thus provide a novel theoretical foundation for understanding endogenous cyclical fluctuations in emerging markets and underscoring the critical role of global dynamics.

Our model suggests potential policy implications that warrant consideration. First, it highlights a possible framework for managing capital account liberalization. Rather than treating all capital inflows as equivalent, policymakers may benefit from prioritizing foreign direct investment (FDI) over riskier debt flows. Measures such as improving the business environment, strengthening investor protections, and selectively applying tools like leverage limits or capital flow management (CFMs) could help balanced growth objectives with financial stability. This dual approach may harness the growth-enhancing aspects of FDI while mitigating risks associated with debt-driven cycles. Second, in economies where credit openness is already advanced, the analysis points toward the advisability of countercyclical policies. This contrasts with the procyclical fiscal responses often observed during capital inflow surges, as highlighted by Reinhart and Reinhart (2009), which can exacerbate boom-bust dynamics. Our framework suggests that timely macroprudential tools or state-contingent subsidies might disrupt valuation-liquidity feedback loops, reduce the risk of low-growth equilibria, and potentially guide economies toward more stable growth trajectories.

This paper makes several contributions to small open economies. First, we identify a novel mechanism through which liquidity effects and valuation effects interact to generate a positive feedback loop, leading to multiple balanced growth paths and self-fulfilling cycles. Second, our model provides a theoretical explanation for the patterns observed in many emerging markets after financial crises: both the differing impacts of FDI versus debt inflows on financial stability, and the persistent growth slowdowns that follow. Third, we are among the first to systematically apply bifurcation analysis in a small open economy, uncovering rich global dynamics such as global indeterminacy and stable periodic orbits, thereby offering a novel theoretical benchmark for understanding the origins of economic fluctuations and endogenous cycles in emerging markets. Finally, our policy analysis shows that countercyclical macroprudential measures and policies promoting FDI inflows can help anchor expectations, mitigate coordination failures, and avoid self-fulfilling crises.

Related Literature: Capital account liberalization in emerging markets may induce sudden stops characterized by large, abrupt current-account reversals, deep recessions, and real exchange-rate depreciation (Calvo et al., 2004; Edwards, 2004). Early studies identified borrowing constraints and currency mismatches as key drivers of sudden stops (Krugman, 1999; Mendoza, 2002; Schneider and Tornell, 2004). Subsequent literature has largely adopted a Fisherian approach to model and analyze sudden-stop dynamics (Bianchi and Mendoza, 2020). In this framework, a decline in asset prices or income tightens borrowing constraints, forcing the economy to deleverage, which can further depress prices. Two main types of borrowing constraints have been examined. The first is the loan-to-value (collateral) constraint. Mendoza and Smith (2006) develops an open-economy model with fixed capital as collateral, showing that when the constraint binds, a debt-deflation mechanism triggers sharp deleveraging and a drop in consumption. Mendoza (2010) extends this by incorporating endogenous investment and time-varying capital, demonstrating that when leverage exceeds a threshold and borrowing constraints bind, Fisherian deflation and output recessions can occur even without exogenous fundamental shocks. This framework has been applied in studies such as Bianchi and Mendoza (2018) and Jeanne and Korinek (2019) to analyze policies for preventing sudden-stop crises. The second type is the debt-to-income (flow) constraint. Bianchi (2011) introduces financial constraints tied to market-determined prices of tradables and nontradables, highlighting how feedback between debt levels and endogenous prices can create systemic externalities. When investors neglect these externalities, overborrowing may arise, escalating into self-fulfilling financial crises through a debt-deflation spiral. Within this setting, Benigno et al. (2013) and Benigno et al. (2016) examine macroprudential and exchange-rate policies in a two-sector model, showing that such policies can eliminate crisis equilibria.

Schmitt-Grohé and Uribe (2021a,b) (hereinafter SGU) make a fundamental contribution by showing that such economies can exhibit belief-driven multiple equilibria and deterministic cycles. Building on these insights, our paper shares the same intellectual motivation as the canonical SGU framework while developing a distinct mechanism. First, the core setups differ in complementary ways. SGU elegantly demonstrates multiplicity in an endowment economy through tradable-nontradable complementarity. In a complementary spirit, we develop a production economy with housing and capital as collateral, incorporating endogenous growth and asset price effects on investment, a different channel than SGU's real exchange rate mechanism. Multiplicity here arises from interaction between endogenous growth and asset pricing, generating multiple long-run growth paths. Second, the implications differ. While multiplicity in SGU manifests along transition dynamics with a unique long-run steady state in general case, our model generates two distinct balance growth paths, implying self-fulfilling crises can permanently trap economies in stagnation, consistent with post-crisis experiences in Mexico, Thailand, and Malaysia. Third, we adopt a continuous-time framework revealing Hopf bifurcations and limit cycles through a different mechanism than SGU's deterministic cycles. Our model also matches an underemphasized empirical pattern: firms, not households, are the primary foreign borrowers (see Table A.2).

Our paper also contributes to the literature on global dynamics and endogenous cycles. Benhabib and Farmer (1994) generates indeterminacy in a model with aggregate externality and the economy can present endogenous cycles locally around the steady state. Recent literature has further delved into the discussions of global dynamics and global endogenous cycles. One type of global endogenous cycle involves regime switches between multiple equilibria driven by sentiment. For instance, Kaplan and Menzio (2016) presents a model with multiple equilibria due to search externalities, the shifts in sentiment can drag the economy into a crisis state with substantially increased unemployment rates. Cui and Kaas (2021) constructs a default model encompassing multiple equilibria, where changes in sentiment significantly alter the default rates within the economy. Kalantzis (2015) and Schmitt-Grohé and Uribe (2021b) introduce borrowing constraints in a small open economy, where financial externalities result in multiple equilibria. Changes in sentiment can trigger endogenous financial crises. Another form of global endogenous cycles considers the periodic movements in the case of a perfect foresight equilibrium. Benhabib et al. (2001) studies the Taylor rule and proves that the system can exhibit periodic orbits through analysis of Bogdanov-Takens bifurcation. Similarly, Antoci et al. (2011), Sniekers (2018), and Gu et al. (2024) use Hopf bifurcation or Bogdanov-Takens bifurcation to analyze the periodic motions in the economy. Our paper is also concerned with the two types of endogenous cycles mentioned above. In our model, a sentiment-driven regime switch allows the system to switch between two balanced growth paths, generating endogenous economic fluctuations and self-fulfilling crises. At the same time, the system can generate deterministic periodic trajectories, based on Hopf bifurcation analysis. This finding is consistent with the view in Beaudry et al. (2020) that business cycles are largely driven by endogenous cyclical mechanisms.

The remainder of the paper is structured as follows. Section 2 provides motivating facts. Section 3 presents the baseline model of the closed economy. Section 4 presents the model with FDI openness. Section 5 presents the model with credit openness. Section 6 discusses regime-switching behavior and self-fulfilling crises. Section 7 discusses sudden stops. Section 8 explores global dynamics, including bifurcations and endogenous cycles. Section 9 analyzes policy implications. Section 10 concludes.

2 Motivating Evidence

This section presents a set of motivating facts that underpin the key mechanisms of our theoretical model. We first document the literature that shows the debt inflows are more likely to cause crises compared with FDI inflows. To rationalize this fact, our model highlights a valuation-liquidity channel in an economy with debt openness. To ground these mechanisms empirically, we first review established facts in the literature regarding the role of real estate collateral values in amplifying business investment. Then, we demonstrate the significant impact of economic growth on asset valuations in small open economies and the amplification effect of capital inflows using country-level panel data and local projection methods.

2.1 Inflow Types and Crises

Extensive research has documented that debt inflows are associated with a significantly higher likelihood of financial crises compared to foreign direct investment (FDI).

After the Asian Financial Crisis, Rogoff (1999) comments that FDI and equity inflows hold a significant advantage over debt inflows due to their inherent mechanism for automatic risk-sharing. Unlike debt, which requires fixed repayments and can lead to liquidity crises and destructive runs during economic downturns, equity investments automatically adjust their value, causing investors to share the burden of poor performance. This makes countries relying on equity and FDI far less vulnerable to the sudden capital flight that characterizes debt crises. However, due to institutional biases, such as deposit insurance subsidies and a legal system that more easily enforces debt contracts, the pre-crisis East Asian economies relied heavily on short-term bank debt.

This idea is supported by the empirical evidence. Ghosh et al. (2016) conducts an empirical analysis of capital inflow surges in 53 emerging market economies over the period 1980-2014, identifying 152 completed surge episodes. The study examines how various surge characteristics influence whether an episode ends up with a financial crisis or a soft landing. A critical aspect of the analysis is the composition of inflows (Columns 7 and 8 of Table 1 in Ghosh et al., 2016). The results demonstrate that surges dominated by FDI are significantly less likely to end in a crisis, whereas those dominated by other investment liabilities (mostly bank flows) substantially increase the probability of a crash. Caballero (2016) analyzes an extensive dataset covering 146 countries over the period 1973-2008, which includes 125 systemic banking crises. The author employs several complementary methods to identify capital inflow bonanzas. The paper finds that a bonanza raises the probability of crisis from an unconditional level of 4% to about 14%. Meanwhile, the composition of flows matters crucially (Table 6 of Caballero, 2016). Surges in debt and portfolio-equity inflows are associated with a higher likelihood of banking crises. However, in contrast, the study finds no statistically significant association between surges in FDI and the subsequent occurrence of banking crises.

In terms of the consequence of the crisis, Tong and Wei (2011) study the Global Financial Crisis and demonstrate that the composition of pre-crisis capital inflows was a critical determinant of crisis severity. Their analysis reveals that countries with larger pre-crisis stocks of FDI experienced significantly milder capital flow reversals, as the stable nature of FDI provided a buffer during the global downturn (Figure 2 of Tong and Wei, 2011). This stabilizing effect extended to the corporate sector. Firms intrinsically dependent on external finance suffered a less severe liquidity crunch if they were located in countries with a high pre-crisis FDI ratio. In stark contrast, a pre-crisis reliance on more volatile flows, such as foreign loans and portfolio equity, amplified the negative impact on these vulnerable firms, exacerbating the credit crunch they faced (Table 6 of Tong and Wei, 2011).

In summary, extensive research establishes that the composition of capital inflows is a critical determinant of financial stability, with debt inflows heightening crisis risk and FDI providing a stabilizing effect. In line with this literature, our theoretical model developed in the subsequent sections demonstrates that, while debt inflows may undermine economic stability, FDI inflows do not pose the same threat.

2.2 Liquidity Effect on Investment

A substantial body of literature demonstrates that real estate serves as a critical collateral asset, enhancing corporate lending capacity and thereby stimulating real investment through the alleviation of financing constraints.

In the United States, seminal work by Chaney et al. (2012) reveals that 1-dollar increase in real estate collateral value boosts corporate investment by approximately 0.06 dollar, primarily financed through long-term debt issuance rather than equity or internal funds. This effect is particularly pronounced in financially constrained firms, such as small or non-investment-grade entities, highlighting the role of collateral in easing credit frictions. Complementing this, Cvijanović (2014) s shows that rising real estate prices lead to higher leverage ratios. A one standard deviation increase in prices raises leverage by 3 percentage points and a shift toward secured debt, consistent with theories where collateral value lowers default risk and financing costs.

International studies reinforce these findings. Using Japanese land price collapse as a natural experiment, Gan (2007a,b) finds that banks with high real estate exposure reduce lending, causing investment declines and valuation drops among affected firms. This lending channel is especially severe for firms without bond market access, illustrating how asset bubbles transmit shocks via credit constraints. Cross-border evidence from Peek and Rosengren (2000) indicates that the Japanese banking crisis contracted commercial real estate lending in the U.S., depressing construction activity and affirming the global interconnectedness of collateral effects. Similarly, Schmalz et al. (2017) document that in France, housing price growth increases entrepreneurship rates for homeowners, with new ventures starting larger and more leveraged, underscoring collateral's role in easing startup financing.

Theoretical models, such as the DSGE framework by Liu et al. (2013), quantify these dynamics, showing that positive co-movements between land prices and business investment are a driving force behind the broad impact of land-price dynamics on the macroeconomy.

In summary, the literature consistently supports the function of real estate as a financial accelerator, where collateral value increase promotes lending and investment, particularly under financing frictions. This mechanism is consistent with the liquidity effect of housing price increasing on real investment in our model. We also present some evidence on the relationship between housing price and real investment using the data from Thailand listed company. (see Appendix A.3)

2.3 Growth Effect on Valuation

In this section, we present the relationship between asset valuation, economic growth, and capital inflows using the Jord Database (JST) (Jordá et al., 2017). Our sample spans from 1950 to 2020 and includes 14 small open economies with available rent-to-price ratio variables.[3] The primary empirical methodology we employ is local projection, which allows us to trace out the dynamic responses of asset valuation to economic growth shocks over various horizons.

Our theoretical framework in Section 5 predicts that higher economic growth rates should lead to increased housing asset valuations in small open economies with credit openness. The underlying intuition is that when economic growth accelerates, the growth rate of asset dividends (housing rents) also rises. Meanwhile, the interest rate will not rise too much due to the feature of small open economies. These lead investors to assign higher valuations to these assets due to improved future income prospects.

Let RP_{jt} denote the rent-to-price ratio of country j in year t. A lower RP_{jt} indicates a higher valuation of housing assets. We test whether an increase in the economic growth rate Growth_{jt} reduces the rent-to-price ratio RP_{jt} using the following specification:

$$\Delta RP_{jt+h} = \beta_1^h \text{Growth}_{jt} + \theta^h \text{Controls}_{jt} + v_t^h + \mu_j^h + \varepsilon_{jt+h}. \tag{1}$$

where $\Delta RP_{jt+h} = RP_{jt+h} - RP_{jt}$ represents the change in the rent-to-price ratio over horizon h. The control variables Controls_{jt} include capital inflows to GDP ratio ($\text{Capital Inflow}_{jt}$), logarithmic real GDP per capita ($\log(\text{GDP per capita}_{jt})$), deficit to GDP ratio (Deficit/GDP_{jt}), total loans to GDP ratio (Loans/GDP_{jt}), short-term interest rate ($\text{Short Term Rate}_{jt}$), yearly changes in the exchange rate ($\text{Exchange Rate Change}_{jt}$), and the rent-to-price ratio itself (RP_{jt}). We also include year fixed effects (v_t^h) and country fixed effects (μ_j^h) to account for unobserved time-invariant heterogeneity and common temporal shocks.

To address potential endogeneity concerns arising from reverse causality and omitted variables, we replace Growth_{jt} in Equation (1) with its forecast error $\hat{\epsilon}_{growth,jt}$, which is estimated from the following regression:

$$\text{Growth}_{jt} = \alpha_1 \text{Growth}_{jt-1} + \alpha_2 \text{Controls}_{jt-1} + u_j + \epsilon_{growth,jt}.$$

This strategy helps isolate exogenous variation in economic growth that is orthogonal to other determinants of asset valuations.

[3] We focus on the post-World War II period and include all countries in the dataset except the USA (not considered a small open economy) and Germany (due to data separation issues before reunification).

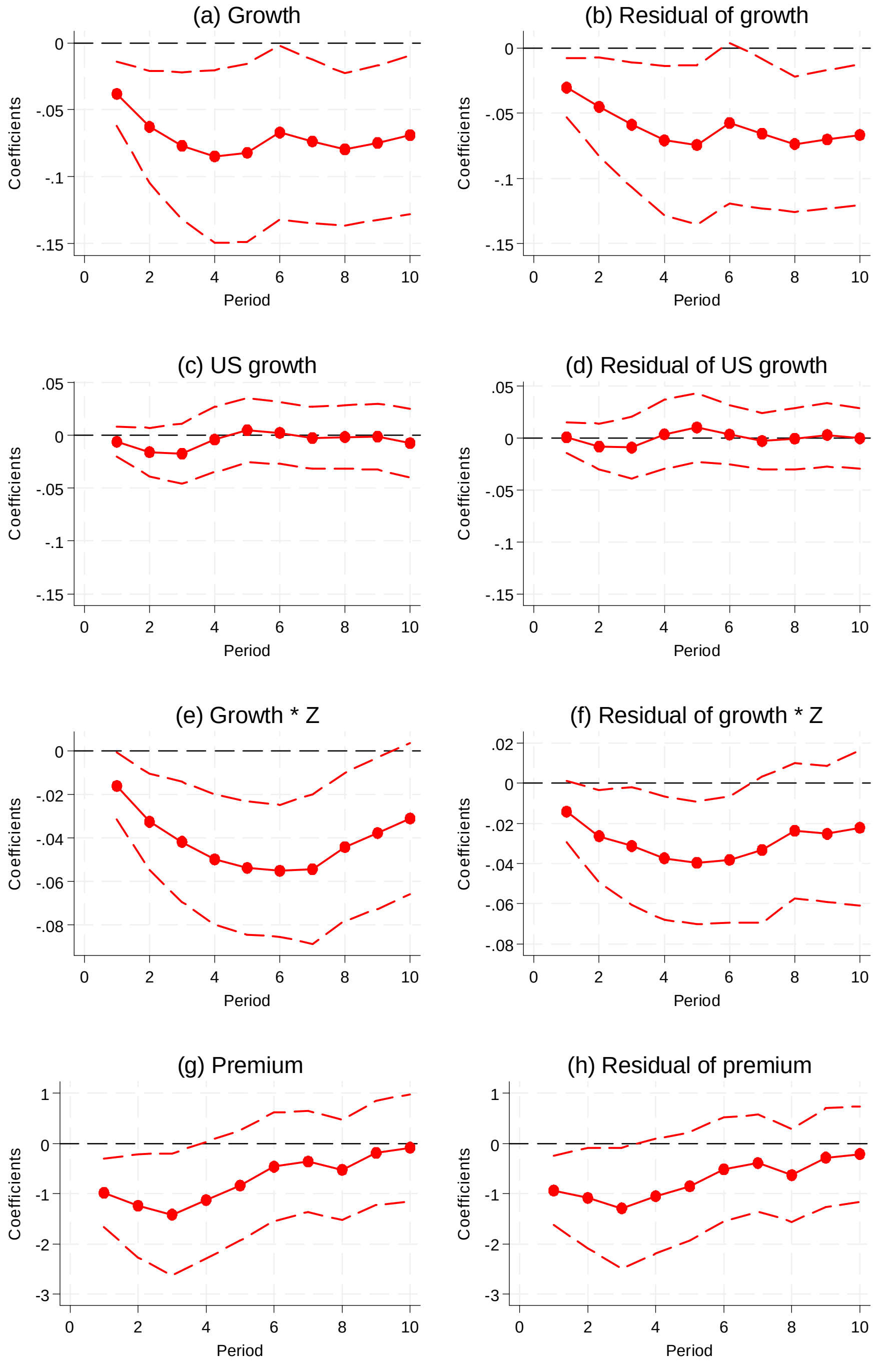

Figure 2: Results of Local Projections

Note: This table presents the estimation results from local projection models. The first row corresponds to Equation (1) with data of 14 open economies. The second row corresponds to Equation (1) with US data. The third row corresponds to Equation (2). The fourth row corresponds to Equation (3). All specifications use Driscoll-Kraay standard errors with a lag of 2 periods, and the confidence bands are at the 95% level.

The empirical results, presented in Figure 2, provide strong support for our theoretical predictions. Panel (a) shows that positive growth shocks lead to significant declines in the rent-to-price ratio across multiple horizons, indicating that higher economic growth indeed boosts housing valuations. This effect persists even when using growth forecast errors (Panel b), confirming the robustness of our findings to endogeneity concerns.

By contrast, if we estimate Equation (1) using US data, we find that the valuation effect is not significant. [4] In the theoretical context of a large open economy or a closed economy, a rise in the economic growth rate causes the equilibrium interest rate to rise. This linkage is governed by the consumption Euler equation, which necessitates a higher interest rate to equilibrate savings and investment demand when agents anticipate higher future income. Consequently, this endogenous rise in the interest rate serves as a stabilizing force, dampening the valuation effect of higher dividend growth. As demonstrated in Panels (c) and (d), US data are consistent with this general equilibrium effect: growth shocks do not generate statistically significant asset price booms. In sharp contrast, small open economies facing a fixed world interest rate are unconstrained by this feedback loop. This is consistent with the discussion in our model. Other empirical literature, like Cesa-Bianchi et al. (2015), also argues that house prices in emerging economies are more volatile compared with advanced economies and capital inflows play an important role in this comparison.

Second, our theoretical model suggests that foreign capital inflows amplify the impact of economic growth on asset valuation through leverage effects. To test this amplification channel, we interact economic growth with standardized capital inflows (Z_{jt}, obtained as the z-score of Capital Inflow$_{jt}$) using the following specification:

$$\Delta RP_{jt+h} = \beta_1^h \text{Growth}_{jt} + \beta_2^h \text{Growth}_{jt} \times Z_{jt} + \beta_3^h Z_{jt} + \theta^h \text{Controls}_{jt} + v_t^h + \mu_j^h + \varepsilon_{jt+h}. \quad (2)$$

For robustness, we also estimate this specification using the forecast error $\hat{\epsilon}_{growth,jt}$ and its interaction with Z_{jt}.

The interaction effects in Panels (e) and (f) reveal that capital inflows significantly amplify the valuation effects of economic growth. The negative coefficients on the interaction terms (Growth$_{jt}$ $\times$ Z_{jt}) indicate that the growth-valuation relationship is stronger in economies experiencing larger capital inflows, consistent with the leverage amplification mechanism in our theoretical model.

Third, our theoretical model suggests that liquidity premium can elevate the valuation. We examine the role of liquidity premium channels by estimating the following specification:

$$\Delta RP_{jt+h} = \beta_1^h \text{Premium}_{jt} + \theta^h \text{Controls}_{jt} + v_t^h + \mu_j^h + \varepsilon_{jt+h}. \quad (3)$$

where Premium$_{jt}$ measures the liquidity premium. It is defined as the spread between the return on total assets (which proxies for the stochastic discount factor) and the return on bonds and bills (which are highly liquid assets). This spread serves as a proxy for the liquidity

[4] This regression has only one country and therefore we do not add year fixed effects.

premium. We also calculate $\hat{\epsilon}_{premium,jt}$ using the following method for robustness checks.

$$\text{Premium}_{jt} = \alpha_1 \text{Premium}_{jt-1} + \alpha_2 \text{Controls}_{jt-1} + u_j + \epsilon_{premium,jt}.$$

Panels (g) and (h) demonstrate that liquidity premium shocks also reduce rent-to-price ratios, supporting that liquidity premium plays an important role in asset valuation dynamics.

Economically, these findings validate the core mechanism of our model that growth expectations can drive asset valuations in small open economies and highlight how capital inflows can amplify this effect. The persistent effects across horizons suggest that these valuation effects have long-lasting implications for economic dynamics, consistent with the persistent growth downgrades observed following financial crises in emerging markets.

3 Closed Economy

In this section, we construct a continuous-time general equilibrium model characterized by discrete time periods denoted as $t = 0, \Delta, 2\Delta, \ldots$, where each period spans a duration of Δ. We then investigate the limit as Δ tends to zero. This approach enables us to analyze both local and global dynamics within a continuous-time framework. The model comprises two sectors: households and entrepreneurs. Households obtain utility from consumption and housing services. Entrepreneurs play a pivotal role in our model. They are subject to idiosyncratic investment opportunity shocks and face financial frictions in the credit market. Under such circumstances, entrepreneurs will invest in housing assets to secure liquidity. The economy in this section focuses on the closed environment. Later we will consider the FDI openness and credit openness.

3.1 Households

A representative household derives utility from consumption C_t and housing services H_t. The utility function is

$$\sum_{s=0}^{\infty} \exp(-\rho s\Delta) \frac{[C_{t+s\Delta}^{1/(1+\psi)} H_{t+s\Delta}^{\psi/(1+\psi)}]^{(1-\sigma)(1+\psi)}}{(1-\sigma)(1+\psi)} \Delta$$

where $\exp(-\rho)$ is the subjective discount rate, σ is the inverse of the intertemporal elasticity of substitution, and ψ is the relative weight of housing services to consumption in the utility function. To ensure the existence of balanced growth paths, we choose the Cobb-Douglas form of combination function.

The budget constraint of the households is

$$C_t\Delta + M_t H_t \Delta = \int_j V_{jt}(s_{jt+\Delta} - s_{jt}) dj + \int_j D_{jt}\Delta s_{jt} dj + Tr_t \Delta$$

where M_t is the rent rate of housing services, V_{jt} is the stock price of entrepreneur j, D_{jt} is the dividend, s_{jt} is the share held by the households, and Tr_t is the lump sum transfer from governments. We assume that households cannot borrow from the credit market. Later we will show that as the stochastic discount rate of households is higher than the interest rate, households do not save on the credit market.

The first order condition of housing services H_t is

$$\frac{1}{C_t} = \frac{\psi}{M_t H_t}$$

Therefore, the rent is a fraction of consumption $M_t H_t = \psi C_t$. The first order condition of the pricing equation of V_{jt} is

$$\exp(-\rho\Delta) E_t \frac{[C_{t+\Delta}^{1/(1+\psi)} H_{t+\Delta}^{\psi/(1+\psi)}]^{(1-\sigma)(1+\psi)-1} C_{t+\Delta}^{-\psi/(1+\psi)}}{[C_t^{1/(1+\psi)} H_t^{\psi/(1+\psi)}]^{(1-\sigma)(1+\psi)-1} C_t^{-\psi/(1+\psi)}} \frac{V_{j,t+\Delta} + D_{j,t+\Delta}}{V_t} = 1$$

Later we will assume that the supply of housing is fixed, $H_t = H_0 = 1$. We can rewrite the equation to be

$$\exp(-\rho\Delta) E_t \frac{C_{t+\Delta}^{-\sigma}}{C_t^{-\sigma}} \frac{V_{j,t+\Delta} + D_{j,t+\Delta}}{V_t} = 1.$$

The term $\exp(-\rho\Delta) E_t \frac{C_{t+\Delta}^{-\sigma}}{C_t^{-\sigma}}$ is the stochastic discount factor. Micro evidence shows that σ should be larger than 1 (see Havránek, 2015). Throughout our paper, we focus on the case that $\sigma > 1$.

Assumption 1 *The inverse of the elasticity of intertemporal substitution σ is larger than* 1*.*

3.2 Entrepreneurs

The entrepreneurs are the central actors in our model. They invest I_{jt} in capital or buy houses H_{jt}. They get capital return R_t^k and housing rent M_t. They borrow $L_{jt+\Delta}/(1 + R_t\Delta)$ at the interest rate R_t in the credit market. In the closed economy, the interest rate R_t is endogenous. If $L_{jt+\Delta}/(1 + R_t\Delta) < 0$, it means that the entrepreneur saves in the credit market. Meanwhile, they also pay dividend D_{jt} to the households. The budget constraint of the entrepreneurs is

$$D_{jt}\Delta + I_{jt} + P_t(H_{jt+\Delta} - H_{jt}) - \frac{L_{jt+\Delta}}{1 + R_t\Delta} + L_{jt} = R_t^k K_{jt}\Delta + M_t H_{jt}\Delta. \tag{4}$$

where P_t is the price of houses, K_{jt} is the capital stock.

As this is an endogenous growth economy, for tractability, we assume AK-type production function form in the baseline model

$$Y_{jt} = Z K_{jt}.$$

Therefore the capital return is constant in the baseline model, $R_t^k = Z$.

Our model captures idiosyncratic investment opportunity shocks. In each period (during t to $t + \Delta$), $\mu\Delta$ fraction of entrepreneurs have opportunity to invest in new capital. For them, the capital investment can be larger than zero $I_{jt} \geq 0$ and law of motion for capital K_{jt} is

$$K_{jt+\Delta} = (1 - \delta\Delta)K_{jt} + I_{jt}. \tag{5}$$

For the other $1-\mu\Delta$ fraction of entrepreneurs, they do not have capital investment opportunity and $I_{jt} = 0$. The entrepreneurs have no restrictions on buying houses but they can not short houses $H_{jt} \geq 0$. The investment decision is made after the investment opportunity shock is realized.

There are two types of financial frictions. First, entrepreneurs have borrowing constraints. They can use capital and houses as collateral. The borrowing constraint is [5]

$$\frac{L_{jt+\Delta}}{1 + R_t\Delta} \leq \theta K_{jt} + \theta_h P_t H_{jt+\Delta}, \tag{6}$$

where θ and θ_h are collateral ratios of capital and houses in the credit market. In addition to the borrowing constraint, we assume that entrepreneurs cannot pay negative dividends to households, i.e. $D_{jt} \geq 0$, which captures the equity financing restriction.

Given the above settings, the entrepreneurs maximize the present value of the dividend flows. The Bellman equation of an entrepreneur is

$$V_{jt}(K_{jt}, H_{jt}, L_{jt}) = \max_{D_{jt}, I_{jt}, L_{jt+\Delta}, H_{jt+\Delta}} D_{jt}\Delta + \exp(-\rho\Delta)E_t\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\sigma} V_{jt+\Delta}(K_{jt+\Delta}, H_{jt+\Delta}, L_{jt+\Delta})$$

subject to constraints (4), (5), (6), and $I_{jt}, H_{jt}, D_{jt} \geq 0$.

3.3 Market Clearing

In this economy, the aggregate variables can be defined as $I_t\Delta = \int_j I_{jt}dj$, $K_t = \int_j K_{jt}dj$, $Y_t = \int_j Y_{jt}dj$, and $L_t = \int_j L_{jt}dj$. The law of motion for capital satisfies

$$K_{t+\Delta} = (1 - \delta\Delta)K_t + I_t\Delta.$$

The total output satisfies $Y_t = ZK_t$. The clearing condition of goods market is

$$C_t + I_t = Y_t.$$

As this is a closed economy, credit supply should equal credit demand. The aggregate credit is zero

$$L_t = \int_j L_{jt}dj = 0.$$

[5] Housing assets are crucial in our model. As will be shown in Section 5.4, the main results hinge on housing assets $H_{jt+\Delta}$, regardless of whether the capital collateral takes the form θK_{jt} or $\theta Q_t K_{jt}$.

3.4 Characterizations of the Equilibrium

To characterize the equilibrium, we first solve the problem of entrepreneurs. As all settings and constraints of entrepreneurs are linear, we conjecture and verify a linear function of the expected discounted value in the next period as [6]

$$\exp(-\rho\Delta)E_t\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\sigma}V_{jt+\Delta}(K_{jt+\Delta},H_{jt+\Delta},L_{jt+\Delta})=Q_tK_{jt+\Delta}+P_tH_{jt+\Delta}-\frac{L_{jt+\Delta}}{1+R_t\Delta}.$$

where Q_t is Tobin's Q. Generally, $Q_t>1$ in this economy due to underinvestment resulting from liquidity constraint. The parameter μ introduces a form of heterogeneity and search friction in accessing profitable projects. This implies that not all entrepreneurs can immediately deploy new funds, creating a premium on liquidity for those who currently have viable projects. This liquidity premium, captured by Q_t-1, becomes one of the important drivers of asset valuation in our model. The coefficients of $H_{jt+\Delta}$ and $L_{jt+\Delta}$ are P_t and $1/(1+R_t\Delta)$ respectively. The intuition behind this is that the entrepreneurs without investment opportunity are indifferent between paying dividend and holding housing assets or saving on the credit market.

Given the above value functions, we first figure out the decisions of entrepreneurs. If an entrepreneur j has opportunity to invest capital, she will use all sources of funds, including selling housing assets, and borrowing loans with capital as collateral in credit markets, to invest in capital. The reason behind this decision is that the value of capital (Q_t) is higher than the price of investment goods (1 unit), and the entrepreneur can transform one unit of final good into one unit of new capital with net gains Q_t-1. Therefore, she will not issue dividend $D_{jt}\Delta=0$ or hold houses $H_{jt+\Delta}=0$. The loan and the investment chosen by an entrepreneur with investment opportunity are

$$\frac{L_{jt+\Delta}}{1+R_t\Delta}=\theta K_{jt},$$

$$I_{jt}=R^kK_{jt}\Delta+M_tH_{jt}\Delta+P_tH_{jt}+\theta K_{jt}-L_{jt}.$$

However, for the entrepreneurs who cannot invest $I_{jt}=0$, there is no difference between paying dividend $D_{jt}\Delta$, buying housing assets $H_{jt+\Delta}$, or saving on the credit market $L_{jt+\Delta}/(1+R_t\Delta)$. Substituting the decisions of entrepreneurs into the value function allows us to verify the previously conjectured value function and get the motions of prices.

We now let $\Delta\to 0$. Proposition 1 summarizes the motions of prices in the economy.

Proposition 1 *When $\Delta\to 0$ and $Q_t>1$, the pricing equations of the housing price, interest rate, and Tobin's Q follow*

$$\rho+\sigma\frac{\dot{C}_t}{C_t}=\frac{\dot{P}_t+M_t}{P_t}+\underbrace{\mu(Q_t-1)}_{\textit{Liquidity premium}}, \tag{7}$$

[6] The expected value in the next period is not related to the investment opportunity in the current period as it is an i.i.d shock.

$$\rho + \sigma \frac{\dot{C}_t}{C_t} = R_t + \mu(Q_t - 1), \tag{8}$$

$$\left(\rho + \sigma \frac{\dot{C}_t}{C_t}\right) Q_t = \dot{Q}_t + R^k - \delta Q_t + \mu\theta(Q_t - 1). \tag{9}$$

Equation (7) is the pricing equation of housing assets. The stochastic discount factor is $\rho + \sigma \dot{C}_t / C_t$. It should equal two parts of return. The first is the housing appreciation and the rent. Meanwhile, the housing asset also has liquidity premium. When the investment opportunity comes at the probability μ, entrepreneurs can sell the houses and transfer one unit of investment into the capital with value Q_t. Therefore, we have the other liquidity premium term $\mu(Q_t - 1)$.

Similarly, Equation (8) is the pricing equation of the interest rate. Saving one unit of funds can yield the interest R_t. Meanwhile, the saved funds can also be transferred into capital at probability μ, we also have liquidity premium $\mu(Q_t - 1)$.

Equation (9) is the pricing equation of Tobin's Q. Holding one unit of capital can enjoy the capital return R^k but the capital depreciates at δ. Meanwhile, when the investment opportunity comes, the entrepreneur can borrow against collateral at the collateral rate θ to invest.

From the perspective of aggregation, the total investment in the economy is the sum of the investments of all entrepreneurs with investment opportunities. The total investment is

$$I_t \Delta = \mu \Delta \left(R^k K_{jt} \Delta + M_t H_{jt} \Delta + P_t H_{jt} + \theta K_{jt} - L_{jt} \right).$$

Given the credit market clearing condition $\int L_{jt} dj = 0$, and let $\Delta \to 0$, we have

$$I_t = \mu(P_t H_t + \theta K_t).$$

For consumption,

$$C_t = R^k - I_t.$$

We use lower case letters to denote variables that are detrended by K_t. Let $l_t = L_t / K_t$, $p_t = P_t H_t / K_t$, $c_t = C_t / K_t$, and we assume that $g_t = \dot{K}_t / K_t$. The pricing equation of houses and capital form a two-dimensional dynamic system.

Proposition 2 *In the closed economy, the dynamic system can be summarized by the following equations of* $\{Q_t, p_t\}$*:*

$$\rho + \sigma \frac{\dot{c}_t}{c_t} + \sigma g_t = \underbrace{\frac{\dot{p}_t + \psi c_t}{p_t} + g_t}_{Return} + \underbrace{\mu(Q_t - 1)}_{Liquidity\ premium}, \tag{10}$$

$$\left(\rho + \sigma \frac{\dot{c}_t}{c_t} + \sigma g_t\right) Q_t = \dot{Q}_t + R^k - \delta Q_t + \mu\theta(Q_t - 1), \tag{11}$$

where $g_t = \mu(p_t + \theta) - \delta$ *and* $c_t = R^k - (g_t + \delta)$.

3.5 Balanced Growth Paths

Now, we proceed to calculate the balanced growth paths of the system. We employ lowercase letters without the subscript t to represent variables along the balanced growth path. It can be proven that the system has at most one balanced growth path.

Theorem 1 *If* $\sigma > 1$*, the closed economy with endogenous growth has at most one balanced growth path.*

When considering partial equilibrium, with c and Q held constant, we obtain the following two equations. The first is the growth equation:

$$g = \mu(p + \theta) - \delta.$$

This equation illustrates that the economic growth rate g is an increasing function of the housing price p. A higher housing price can stimulate investment and, consequently, the economic growth rate.

The second equation is the pricing equation for housing assets:

$$\rho + \sigma g = \frac{\psi c}{p} + g + \mu(Q - 1),$$

When $\sigma > 1$, the housing price p is a decreasing function of the economic growth rate g. The underlying intuition is that although a higher economic growth rate leads to an increase in future rent income, it also raises the stochastic discount rate $\rho + \sigma g$. When σ is sufficiently large, an increase in the growth rate will drive down the asset price.

Thus, while an increase in asset prices contributes to an increase in economic growth through liquidity effects, an increase in economic growth undermines the rise in asset prices. Consequently, there exists a unique equilibrium growth path for the system.

4 FDI Openness

In the previous section, we discussed the baseline model of a closed economy. In this section, we discuss the impact of FDI openness on the economy. Our analysis shows that the equilibrium is unique in the economy after FDI inflows and the growth rate of the economy increases.

FDI liberalization operates on the production side. It benefits technology diffusion and overall business environment improvements (Javorcik, 2004; Haskel et al., 2007). We assume that the final production uses the domestic capital K_t and foreign capital K_t^f. The production function is of CES form

$$Y_t = Z\left[\omega K_t^{(\epsilon-1)/\epsilon} + (1-\omega) {K_t^f}^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)}.$$

where Y_t is the aggregate output and $\epsilon > 1$ is the elasticity of substitution. When $\epsilon < \infty$, the domestic and foreign capital are not perfectly substitutable. Agosin and Machado (2005) analyze panel data and find that in Asia and Africa, FDI exerts a strong crowding in effect on domestic investment. Our assumption captures the complementarity between domestic and foreign capital. [7] The demand function of each variety is

$$R_t^k = \omega Z \left[\omega + (1-\omega){\kappa_t}^{(\epsilon-1)/\epsilon}\right]^{1/(\epsilon-1)}, \tag{12}$$

$$R_t^{kf} = (1-\omega) Z \left[(1-\omega) + \omega(1/\kappa_t)^{(\epsilon-1)/\epsilon}\right]^{1/(\epsilon-1)}. \tag{13}$$

where $\kappa_t = K_t^f / K_t$. Equation (12) shows the positive relationship between R^k and κ, which implies a technological or demand-side complementarity between domestic capital and FDI. Foreign Direct Investment (K_t^f) interacts with domestic capital (K_t) in the production process. This interaction can take the form of technological spillovers, enhanced competition, or by augmenting the overall productive capacity, thereby influencing domestic capital returns (R_t^k) and growth dynamics. This result is consistent with the empirical studies. For instance, Haskel et al. (2007) find that FDI boosts the productivity of domestic firms in the UK through industry-level spillovers, while Javorcik (2004) provides evidence from Lithuania that FDI increases domestic firm productivity via backward linkages along supply chains. At the same time, Equation (13) shows that the return to foreign capital falls as its share rises, which is due to diminishing margins.

The foreign investors can invest at the opportunity cost of R^f. For foreign investors, FDI has entry costs. Head et al. (1995) provide empirical evidence that while agglomeration benefits exist, high densities of foreign activity lead to congestion costs that raise the effective cost of entry for latecomers. The profit from directly investing K_{jt}^f is

$$\max_{K_{jt}^f} (R_t^{kf} - \delta) K_{jt}^f - \Omega \left(\frac{K_t^f}{K_t + K_t^f}\right)^{\phi} K_{jt}^f - R^f K_{jt}^f$$

where $\Omega \left(\frac{K_t^f}{K_t + K_t^f}\right)^{\phi} K_{jt}^f$ is the investment cost and it is a convex function. The first order condition of the foreign investors is

$$(1-\omega) Z \left[(1-\omega) + \omega \left(\frac{1}{\kappa_t}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}} = \Omega \left(\frac{\kappa_t}{1+\kappa_t}\right)^{\phi} + R^f + \delta \tag{14}$$

Equation (14) shows that the share of FDI decreases with the entry cost. Therefore, if the government can relax the restrictions on FDI and optimize the business environment for foreign investment, then it can promote the inflow of FDI and increase the rate of return on domestic

[7] We restrict $\epsilon > 1$ so that the output $Y_t > 0$ even if $K_t^f = 0$.

capital. The above result can be summarized as the following Lemma

Lemma 1 *R^{kf} is an increasing function of Ω while κ and R^k are decreasing functions of Ω.*

From the above analysis, it can be seen that the inflow of FDI capital affects only the return on domestic capital R^k and does not influence the other equations of the economy. Therefore, according to Proposition 1, we can conclude that there exists at most one balanced growth path in the economy. Moreover, with the inflow of FDI, the rate of return on domestic capital R^k rises, which increases the amount of resources available for investment in the economy. We can further show that during the gradual liberalization process, that is, as Ω decreases from ∞ downward, the economy still exhibits at most one balanced growth path, and the growth rate along this path increases continuously.

Theorem 2 *If $\sigma > 1$, an economy with FDI openness has at most one balanced growth path. Meanwhile, if the balanced growth path exists, the growth rate increases when the inflow cost Ω decreases.*

The uniqueness of the balanced growth path in our model, even after opening to FDI, does not rely on the presence of complementarity between foreign and domestic capital. Instead, it is guaranteed by a fundamental counteracting mechanism inherent in the system. Specifically, while rising asset prices stimulate investment and accelerate economic growth, this very growth leads to an increase in the equilibrium interest rate and the households' stochastic discount factor. The higher interest rate, in turn, exerts downward pressure on asset valuations. These opposing forces, where asset prices fuel growth but are simultaneously dampened by the resulting increase in the cost of capital, create a negative feedback effect that ensures the economy converges to a single, unique balanced growth path, irrespective of whether technological complementarity exists.

Remark 1 *If $\sigma > 1$ and $\epsilon = \infty$, an economy with FDI openness has at most one balanced growth path. Meanwhile, the growth rate is independent of κ_t or Ω.*

However, when there is no complementarity between the two types of capital, a rise in FDI fails to raise the return on domestic capital R^k, and therefore fails to raise the rate of economic growth.

5 Credit Openness

Now, we consider financial openness. We assume that the domestic credit market becomes integrated with the international credit market. Following openness, the borrowing rate is no longer determined endogenously but becomes exogenous. Subsequently, we will demonstrate that in such an environment, rising housing prices contribute to economic growth, and economic growth in turn further fuels the increase in housing prices. This positive feedback mechanism may give rise to multiple balanced growth paths in the economy.

5.1 Setups

The impact of credit openness is mainly on the entrepreneur sector. As the domestic credit market has been integrated with the international credit market, the interest rate becomes an exogenous constant R^f.

In this way, the budget constraint of an entrepreneur becomes

$$D_{jt}\Delta + I_{jt} + P_t(H_{jt+\Delta} - H_{jt}) - \frac{L_{jt+\Delta}}{1+R^f\Delta} + L_{jt} = R_t^k K_{jt}\Delta + M_t H_{jt}\Delta. \tag{15}$$

Meanwhile, the borrowing constraint becomes

$$\frac{L_{jt+\Delta}}{1+R^f\Delta} \leq \theta K_{jt} + \theta_h P_t H_{jt+\Delta}. \tag{16}$$

Housing assets are crucial in our model. As will be shown in Section 5.4, the main results hinge on housing assets $H_{jt+\Delta}$, regardless of whether the capital collateral takes the form θK_{jt} or $\theta Q_t K_{jt}$. Meanwhile, entrepreneurs are assumed to be the ultimate borrowers of external debt in our model. They can issue bonds directly or secure loans via domestic financial intermediaries, which is an assumption supported by empirical evidence. Based on data from the Quarterly External Debt Statistics (QEDS) and the Bank for International Settlements (BIS), we calculate the share of non-financial corporations in total private external debt, which includes borrowing by both non-financial corporations and households. As presented in Appendix Table A.2, non-financial corporations constitute the vast majority of private non-financial external debt. Whether considering direct borrowing alone or total borrowing that incorporates indirect exposure through domestic financial intermediation, the corporate share generally exceeds 70%

It should be noted that given the credit market openness, the aggregate credit $L_t = \int_j L_{jt}dj$ is not necessarily equal to zero. The resource constraint of the economy becomes

$$C_t\Delta = Y_t\Delta - I_t\Delta + \frac{L_{t+\Delta}}{1+R^f\Delta} - L_t.$$

5.2 Characterizations of the Equilibrium

To analyze the system, we let $\Delta \to 0$. Before we solve the Bellman equation, we can first analyze the borrowing constraint. Denote the return on housing asset as R_t^H where

$$R_t^H\Delta = \frac{P_{t+\Delta} - P_t + M_t\Delta}{P_t}.$$

If $R_t^H > R^f$, borrowing constraint (16) binds for any entrepreneur and at any time as entrepreneurs can borrow credit and invest in the housing market otherwise. However, if $R_t^H = R^f$, the borrowing constraint (16) does not necessarily bind for all entrepreneurs. For the entrepreneurs who have investment opportunity, they will borrow as much as possible to invest. However, for the entrepreneurs who do not have investment opportunity, they have no difference between borrowing or investing in houses.

Case of $R_t^H > R^f$ We mainly focus on the case that R^f is very small and $R_t^H > R^f$ in the equilibrium, and this is also the typical case in the small open economy. As $R_t^H > R^f$, borrowing constraint (16) binds for any entrepreneur and at any time. For any entrepreneur, we have

$$\frac{L_{jt+\Delta}}{1+R^f\Delta} = \theta K_{jt} + \theta_h P_t H_{jt+\Delta}.$$

As all settings and constraints of the entrepreneurs are linear, we conjecture and verify a linear value function that

$$\exp(-\rho\Delta)E_t\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\sigma} V_{jt+\Delta}\left(K_{jt+\Delta}, H_{jt+\Delta}, L_{jt+\Delta}\right) = Q_t K_{jt+\Delta} + Q_{ht}H_{jt+\Delta} - Q_{lt}L_{jt+\Delta}.$$

where $Q_t > 1$ is Tobin's Q . Q_{ht} and Q_{lt} are the marginal value of houses and marginal cost of credit respectively. Due to foreign credit, these two marginal values are no longer P_t or $1/\left(1+R_t\Delta\right)$.

Similarly to the case of the closed economy, when $Q_t > 1$, the entrepreneur can transform one unit of final good into one unit of new capital with value Q_t. Therefore, she sells all housing asset $H_{jt+\Delta} = 0$ and pays no dividend $D_{jt}\Delta = 0$. She borrows as much as possible with capital as collateral to invest. She chooses

$$\frac{L_{jt+\Delta}}{1+R^f\Delta} = \theta K_{jt},$$

$$I_{jt} = R^k K_{jt}\Delta + M_t H_{jt}\Delta + P_t H_{jt} + \theta K_{jt} - L_{jt}.$$

However, for an entrepreneur without investment opportunity, she does not invest $I_{jt} = 0$ but has no difference between dividend $D_{jt}\Delta$ and houses $H_{jt+\Delta}$. The borrowing constraint binds as $R_t^H > R^f$,

$$\frac{L_{jt+\Delta}}{1+R^f\Delta} = \theta K_{jt} + \theta_h P_t H_{jt+\Delta}.$$

We let $\Delta \to 0$ and give Proposition 3 to summarize the motion of prices in the economy.

Proposition 3 *When $\Delta \to 0$, in an economy with credit openness and $R_t^H > R^f$, the pricing equations of the housing price and Tobin's Q follow*

$$\left(\rho + \sigma\frac{\dot{C}_t}{C_t}\right)(1-\theta_h)P_t = \dot{P}_t + M_t - \theta_h P_t R^f + \mu(Q_t - 1)(1-\theta_h)P_t, \tag{17}$$

$$\left(\rho + \sigma\frac{\dot{C}_t}{C_t}\right)(Q_t - \theta) = \dot{Q}_t + R^k - \delta Q_t - R^f\theta. \tag{18}$$

Equation (17) is the pricing equation of housing assets. The purchase of a unit of housing in the current period requires a payment of $(1-\theta_h)P_t$ while borrowing $\theta_h P_t$. The cash flow available in the next period will be the appreciated house $\dot{P}_t$, the rent M_t minus the interest to be paid $R^f\theta_h P_t$. Cash flows have not only their own value, but also a liquidity premium. Therefore, its future value needs to be added by $\mu(Q_t-1)$.

Similarly, Equation (18) is the pricing equation of Tobin's Q. As the borrowing constraint binds, holding one unit capital can borrow θ unit of credit. The net value is $Q_t-\theta$. Holding the capital can enjoy the capital value appreciation $\dot{Q}_t$, capital return R^k, but suffers from the depreciation δQ_t and has to pay interest R^f.

When $\Delta \to 0$, the aggregate international loan is

$$L_t = \theta K_t + \theta_h P_t H_t.$$

From the perspective of aggregation, the total investment in the economy is the sum of investments of all entrepreneurs with investment opportunities. The total investment is

$$I_t = \mu(P_t H_t + \theta K_t - L_t).$$

Let us use lowercase letters to denote variables that are de-trended by K_t. The pricing equation of houses and capital, and the consumption equation form a three-dimensional dynamic system.

Proposition 4 *If $R_t^H > R^f$, the dynamic system can be summarized by the following equations of $\{c_t, Q_t, p_t\}$:*

$$\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t = \underbrace{\frac{1}{1-\theta_h}\left(\frac{\dot{p}_t + \psi c_t}{p_t} + g_t\right) - \frac{\theta_h}{1-\theta_h}R^f}_{\text{Leveraged return}} + \underbrace{\mu(Q_t-1)}_{\text{Liquidity premium}}, \tag{19}$$

$$\left(\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t\right)(Q_t-\theta) = \dot{Q}_t + R^k - \delta Q_t - R^f\theta, \tag{20}$$

$$c_t = R^k - (g_t+\delta) + (\theta + \theta_h p_t)(g_t - R^f) + \theta_h \dot{p}_t, \tag{21}$$

where

$$g_t = \mu(1-\theta_h)p_t - \delta. \tag{22}$$

Equation (19) is one of the key equations. It presents the pricing function of housing assets and is central to our mechanism. The left-hand side is the required rate of return for a household, which is their rate of time preference adjusted for consumption growth (the standard stochastic discount factor in continuous time). The right-hand side is the total return from investing one unit of internal funds in housing. This return has three components. First, the term $1/(1-\theta_h)$ is a collateral multiplier. When an agent has one dollar of her own funds, she could borrow $\theta_h/(1-\theta_h)$ dollar and buy houses with value $1/(1-\theta_h)$. The return on the asset's capital gains $(\dot{p}_t/p_t + g_t)$ and rent yield $(\psi c_t/p_t)$ is magnified by $1/(1-\theta_h)$ times. This term embodies a financial accelerator. Second, they must pay the world interest rate R^f on the leveraged portion $\theta_h/(1-\theta_h)$. Third, holding the asset provides a liquidity service, valued at $\mu(Q_t - 1)$. This liquidity premium captures the investment option value entrepreneurs with profitable projects (arriving with intensity μ) are willing to pay a premium for scarce liquidity that allows them to invest. The equation thus states that the required return must equal the leverage-adjusted asset return plus its liquidity value.

Equation (20) demonstrates that the net value of a unit of capital equals Tobin's Q_t minus debt θ. Its total return should consist of capital appreciation, capital gains, minus interest on debt. Equation (21) shows how consumption is decided. It can be derived from the resource constraint.

Equation (22) is the other key equation. It shows how growth is decided. Total investment in the economy depends on the funds available to all investable entrepreneurs. The amount of funds available to entrepreneurs depends on the net value of the houses $(1-\theta_h)p_t$. If there is more investment, then the economy grows faster.

Case of $R_t^H = R^f$ Up to now, we have discussed the case that $R_t^H > R^f$ where the borrowing constraints of all entrepreneurs bind. If $R_t^H = R^f$, the borrowing constraint binds for the entrepreneurs who have investment opportunities but does not bind for those without investment opportunity. We can also provide a set of equations to discuss the case of $R_t^H = R^f$. Scaled by K_t, the pricing equation of houses and capital, and the consumption equation form a four-dimensional dynamic system.

Proposition 5 *If $R_t^H = R^f$, the dynamic system can be summarized by the following equations of $\{c_t, Q_t, p_t, l_t\}$:*

$$R^f = \frac{\dot{p}_t + \psi c_t}{p_t} + g_t, \tag{23}$$

$$\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t = \underbrace{R^f}_{return} + \underbrace{\mu(Q_t - 1)}_{liquidity\ premium}\ , \tag{24}$$

$$\left(\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t\right) Q_t = \dot{Q}_t + R^k - \delta Q_t + \mu\theta(Q_t - 1), \tag{25}$$

$$c_t = R^k - (g_t + \delta) + (g_t - R^f)l_t + \dot{l}_t, \tag{26}$$

where

$$g_t = \mu(p_t - l_t + \theta) - \delta.$$

Note that when the borrowing constraint does not bind for all entrepreneurs, there is no difference between investing housing assets and saving in the market (Equation 23). The return and liquidity premium equal the stochastic discount rate (Equations 24 and 25). Meanwhile, the foreign credit is no longer equal to $\theta + \theta_h p_t$. We use l_t to express the foreign debt.

5.3 Balanced Growth Paths

In this part, we examine the balanced growth paths of the system and analyze the local dynamics around each. We highlight a positive feedback mechanism that can lead to multiple balanced growth paths in the open economy: foreign credit amplifies economic growth, which in turn raises asset prices through a valuation effect; higher asset prices then further stimulate growth via liquidity effects. In contrast, in a closed economy lacking foreign credit, the valuation effect remains negative to generate such multiplicity. Additionally, local indeterminacy may emerge around certain balanced growth paths in open economies.

We first consider the case that the foreign borrowing constraint binds for all entrepreneurs when $R_t^H > R^f$ and then consider the case that the foreign borrowing constraint does not bind for all entrepreneurs when $R_t^H = R^f$.

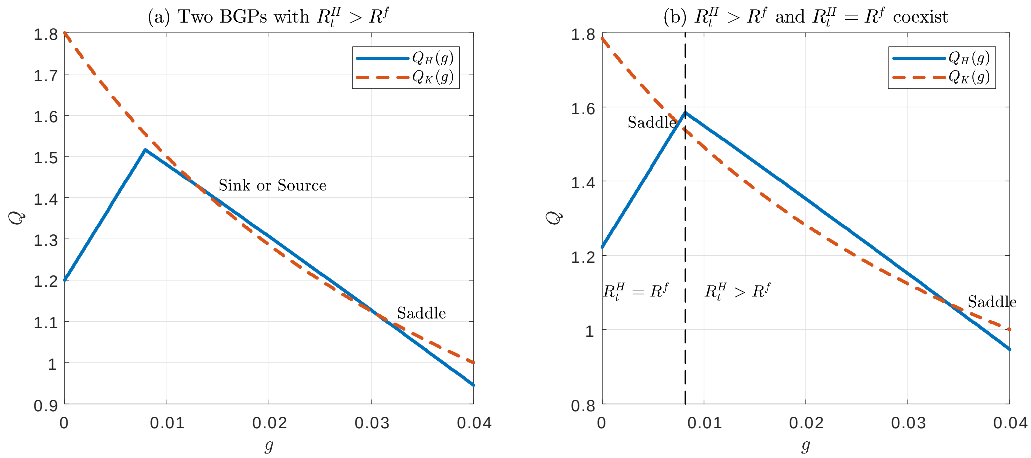

Figure 3: Two types of Multiple Equilibria

Note: In Panel (a), the borrowing constraints bind for all entrepreneurs in both balanced growth paths. In Panel (b), the borrowing constraints bind for all entrepreneurs only in the high growth path. The blue curve $Q_H(g)$ is the housing pricing curve while red curve $Q_K(g)$ is the capital pricing curve.

Case of $R_t^H > R^f$ Substituting Equations (21) and (22) into Equation (19), we have

$$\rho + \sigma g = \frac{\psi\mu[R^k - (g+\delta) + \theta(g - R^f)]}{g+\delta} + \frac{1+\psi\theta_h}{1-\theta_h} g - \frac{(1+\psi)\theta_h}{1-\theta_h} R^f + \mu(Q-1),$$

and

$$(\rho + \sigma g)(Q - \theta) = R^k - \delta Q - R^f \theta,$$

The balanced growth paths should be the intersections of the following two curves

$$Q_H(g) = \frac{1}{\mu}\left[\rho + \mu - \frac{\psi\mu[R^k - (g+\delta) + \theta(g - R^f)]}{g+\delta} - \left(\frac{1+\psi\theta_h}{1-\theta_h} - \sigma\right) g + \frac{(1+\psi)\theta_h}{1-\theta_h} R^f\right],$$

$$Q_K(g) = \frac{R^k + (\rho + \sigma g - R^f)\theta}{\rho + \delta + \sigma g}.$$

Note that the feasible balanced growth paths also require that the liquidity constraint binding condition $Q > 1$, the foreign borrowing constraint binding condition $g + \psi c/p > R^f$. It can be shown that these two conditions can also imply that $c > 0$.

Note that $Q_K(g)$ is derived from the condition $\dot{Q} = 0$ and $\dot{c}_t = 0$. It describes how the value of capital, Q_t, is priced in the steady state. We call $Q_K(g)$ the *capital pricing curve.* As Figure 3 shows, the capital pricing curve $Q_K(g)$ is a monotonically decreasing function. The economic intuition is that when the economy grows faster, the higher the stochastic discount factor $\rho + \sigma\left(\frac{\dot{C}_t}{C_t}\right)$ of the economic agent, then the lower the value of capital given its return R^k.

It is even more complicated for the other equation $Q_H(g)$, which is derived from the condition $\dot{p} = 0$ and $\dot{c}_t = 0$. It describes how the pricing of houses, p_t, is priced in the steady state. We call $Q_H(g)$ the *housing pricing curve.* The economic implication is that the investment return on the house plus the liquidity premium should equal the stochastic discount factor. When $1 + \psi\theta_h \geq \sigma(1 - \theta_h)$ is satisfied, $Q_H(g)$ is also a decreasing function when g is large enough. $(1 + \psi\theta_h)/(1 - \theta_h)$ measures the impact of g on return while σ measures the impact of g on discount rate. When economic growth rises, the net return on investment in housing rises, but so does the stochastic discount factor. $1 + \psi\theta_h \geq \sigma(1 - \theta_h)$ implies that the net return on houses rises faster than the stochastic discount factor when economic growth rises. Thus, when the economic growth rate g is particularly high, the required liquidity premium is smaller and Q is smaller.

On a diagram, two monotonically decreasing functions may have more than one intersection. Panel (a) of Figure 3 presents this case. We can see that the system has two balanced growth paths and $R_t^H > R^f$ on both paths.

Case of $R_t^H = R^f$ Now we consider the case of $R_t^H = R^f$. In this case, Equations (23) to (25) become

$$R^f = \frac{\psi c}{p} + g, \tag{27}$$

$$\rho + \sigma g = R^f + \mu(Q - 1), \tag{28}$$

$$(\rho + \sigma g)\, Q = R^k - \delta Q_t + \mu\theta(Q_t - 1), \tag{29}$$

Rearranging Equation (28), we have

$$Q_H(g) = \frac{1}{\mu}\left[\rho + \mu - R^f + \sigma g\right],$$

Combining Equations (28) and (29) to eliminate μ, we have

$$Q_K(g) = \frac{R^k + (\rho + \sigma g - R^f)\theta}{\rho + \delta + \sigma g}.$$

where $Q_H(g)$ is the *housing pricing curve* in $R^H = R^f$, which says that the returns on the housing investment, which equals the foreign interest rate R^f, plus the liquidity premium should equal the stochastic discount factor. As Figure 3 shows, when the interest rate R^f is given, the stochastic discount factor rises as economic growth rises, so do the required liquidity premium and Q. Function $Q_K(g)$ is the *capital pricing curve* in $R^H = R^f$, which is obtained by recombining the above two equations and is the same as in the binding case, which facilitates our comparison.

Panel (b) of Figure 3 presents a case that the *housing pricing curve* $Q_H(g)$ can also interact with *capital pricing curve* $Q_K(g)$ when $R^H = R^f$. The system may have a balanced growth path with $R^H = R^f$ and a balanced growth path with $R^H > R^f$.

We formalize the above results in the following proposition

Theorem 3 (Multiple Equilibria) *If $(1 + \psi\theta_h)/(1 - \theta_h) > \sigma$ and $R^k - \delta > R^f$, the small open economy with endogenous growth has*
(a) two balanced growth paths with $R_t^H > R^f$ if $\max\{\underline{R}^k, \tilde{R}^k\} < R^k < \overline{R}^k$.
(b) one balanced growth path with $R_t^H > R^f$ and one balanced growth path with $R_t^H = R^f$ if $\underline{R}^k < R^k < \tilde{R}^k$.
where $\underline{R}^k$, $\overline{R}^k$, and $\tilde{R}^k$ are given in the Appendix B.5.

The intuition for multiple balanced growth paths relies on the valuation effect of economic growth on asset prices and the liquidity effect of asset prices on economic growth.

Different from the closed economy, an economic growth rate increase may raise the valuation of housing price even when $\sigma > 1$. Recall the key equation (19),

$$\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t = \underbrace{\frac{1}{1-\theta_h}\left(\frac{\dot{p}_t + \psi c_t}{p_t} + g_t\right) - \frac{\theta_h}{1-\theta_h}R^f}_{\text{Leveraged return}} + \underbrace{\mu(Q_t - 1)}_{\text{Liquidity premium}},$$

where the left hand side is the stochastic discount rate, the first two terms on the right hand side are the leveraged return of assets, the last term is the liquidity premium. On the balanced growth path, we have

$$\rho = \left(\frac{1}{1-\theta_h} - \sigma\right)g + \frac{1}{1-\theta_h}\frac{\psi c}{p} - \frac{\theta_h}{1-\theta_h}R^f + \mu(Q - 1).$$

If θ_h is large enough, when the growth rate g increases, the leveraged capital return increases faster than the stochastic discount rate, and the investors are willing to accept a lower dividend ratio $\psi c/p$. As a result, the investors have a higher valuation of the asset and the asset price p increases. Therefore, economic growth, amplified by foreign credit, has a positive valuation effect on asset prices.

The term $1/(1-\theta_h)$ determines the valuation effect of growth on housing prices. Specifically, $1/(1-\theta_h)$ captures the amplification effect of foreign borrowing leverage on economic growth, while σ measures the extent to which the stochastic discount factor increases with growth. When the growth rate g rises, the return on housing assets also increases. The use of leverage can amplify the net return, thereby pushing up the valuation. However, this upward pressure can be offset by a rise in the stochastic discount factor, which dampens the valuation. For the net valuation effect to be positive, θ_h must be sufficiently high.

In contrast, in a closed economy, the interest rate R_t equals the return on housing assets, and the system becomes equivalent to the case where $\theta_h = 0$. This explains why the valuation effect does not operate in a closed economy.

On the other hand, Equation (22)

$$g_t = \mu(1-\theta_h)p_t - \delta,$$

says that an increase of asset price p_t can relax the borrowing constraint and increase the growth rate g_t further. This positive feedback loop can generate multiple balanced growth paths.

Mathematically, a rise in σ will allow the housing pricing curve $Q_H(g)$ to rise, and the system may have a unique balanced growth path. Conversely, a rise in θ_h or a fall in R^f will cause the housing pricing curve $Q_H(g)$ to fall, offsetting the effect of a rise in σ. Thus, given productivity Z, our model generates multiple balanced growth paths conditional on low interest rates and high collateral ratios of foreign credit.

It follows from Theorem 3 that the existence of multiple balanced growth paths requires the domestic capital return R^k, as determined by productivity Z, to fall within a specific interval. When productivity is too low, the economy may exhibit only one balanced growth path where $R^H = R^f$ along the balanced growth path. This implies that productivity and the return on capital are so low that the economy cannot attract too many capital inflows. This is consistent with the empirical result in Ghosh et al. (2014) that countries with bad GDP performance or low domestic real interest rate are less likely to experience capital inflow surges. Conversely, when R^k is too high, as demonstrated in Proposition B.1 of the Appendix, the balanced growth path is unique and $Q = 1$. When the capital return or the productivity is very high, household incomes and property prices are high, liquidity is no longer scarce, and the positive feedback loop described above naturally dissipates. When the equilibrium is unique, the economy will not experience self-fulfilling crises. This result implies that developed economies where productivity is relatively high are less likely to experience such crises. As

Reinhart and Reinhart (2009) document, inflow bonanzas in developing economies are very likely contributing to economic vulnerability. However, the influence on advanced economies is not very significant. Our theory supports this result.

5.4 Model Comparisons

In the preceding analysis, we have demonstrated that our model with credit openness gives rise to multiple balanced growth paths. The model incorporates several key features, including financial openness, an endogenous growth mechanism, idiosyncratic investment opportunity shocks, and the presence of housing assets. Our previous comparisons have indicated that, in contrast to both the closed economy and the economy with FDI openness, credit openness is a necessary condition for generating such multiplicity. In this section, we conduct counterfactual experiments to examine the specific role each of the other three features plays in enabling the emergence of multiple balanced growth paths.

No growth model If the economy does not have an endogenous growth mechanism, and we assume that the production function is Cobb-Douglas $y_t = ZK_t^\alpha$, the system becomes

$$\rho + \sigma\frac{\dot{c}_t}{c_t} = \frac{1}{1-\theta_h}\left(\frac{\dot{p}_t + \psi c_t}{p_t}\right) - \frac{\theta_h}{1-\theta_h}R^f + \mu(Q_t - 1),$$

$$\left(\rho + \sigma\frac{\dot{c}_t}{c_t}\right)(Q_t - \theta) = \dot{Q}_t + \alpha Z k_t^{\alpha-1} - \delta Q_t - R^f\theta,$$

$$c_t = Zk_t^\alpha - i_t - R^f(\theta k_t + \theta_h p_t) + \theta\dot{k}_t + \theta_h\dot{p}_t,$$

$$i_t = \mu(p_t + \theta k_t),$$

and

$$\dot{k}_t = i_t - \delta k_t.$$

We can show that the balanced growth path is unique.

Proposition 6 *The small open economy without endogenous growth has at most one balanced growth path.*

The mechanism of our model hinges critically on growth. If there is no change in the economic growth rate, then the valuation effects generated by the growth rate discussed earlier will not occur. Therefore, the model does not have multiple balanced growth paths when there is no endogenous growth in the economy.

No idiosyncratic shock model If the economy does not have idiosyncratic investment opportunity shocks, all entrepreneurs can invest capital at any time. It is not an underinvestment economy and $Q_t = 1$. The system becomes

$$\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t = \frac{1}{1-\theta_h}\left(\frac{\dot{p}_t + \psi c_t}{p_t} + g_t\right) - \frac{\theta_h}{1-\theta_h}R^f,$$

$$\left(\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t\right)(1-\theta) = R^k - \delta - R^f\theta,$$

$$c_t = R^k - (g_t + \delta) + (\theta + \theta_h p_t)(g_t - R^f) + \theta_h \dot{p}_t.$$

From the second equation, we can see that the balanced growth path is unique.

Proposition 7 *The small open economy without idiosyncratic investment opportunity shocks has a unique balanced growth path.*

In our model, the reason why asset prices can generate a liquidity effect on investment and growth is that idiosyncratic investment opportunity shocks lead to a situation where some entrepreneurs in the economy are willing to invest but unable to do so, resulting in insufficient investment. In such an environment, housing assets can provide liquidity to entrepreneurs with investment opportunities, thereby promoting investment. If idiosyncratic investment opportunity shocks disappear, the problem of insufficient investment also disappears, and the role of housing assets in easing liquidity constraints vanishes as well. In that case, the model would have at most one balanced growth path.

No housing model If the economy lacks housing assets, the condition $R_t^H > R^f$ cannot occur. Otherwise, as indicated by Equation (22), investment would be zero, leading to a growth rate of $-\delta$. In the absence of housing, the no-arbitrage condition requires $R_t^H = R^f$, and the borrowing constraints will not bind for all entrepreneurs. The economic intuition is that entrepreneurs without current investment opportunities will preserve some borrowing capacity or even hold savings to ensure liquidity for future investment prospects. Following Proposition 5, the dynamic system without housing simplifies to

$$\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t = R^f + \mu(Q_t - 1),$$

$$\left(\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t\right)Q_t = \dot{Q}_t + R^k - \delta Q_t + \mu\theta(Q_t - 1),$$

$$c_t = R^k - (g_t + \delta) + (g_t - R^f)l_t + \dot{l}_t,$$

where

$$g_t = \mu(\theta - l_t) - \delta.$$

The balanced growth path of the above system is unique.

Proposition 8 *The small open economy without housing assets has at most one balanced growth path.*

In our model, there is a valuation effect of economic growth on real estate assets, as the growth rate of house prices rises with economic growth. In contrast, if there is only capital in the economy, economic growth does not affect the return on capital since the return on capital is constant. Therefore, the model does not have multiple balanced growth paths when there is no housing in the economy. In Proposition B.2, we show that this result is robust if the borrowing constraint becomes $L_{jt+\Delta}/(1+R^f\Delta) \leq \theta Q_t K_{jt}$ and there is no housing asset.

Through the above comparisons, we can see that the ingredients of an endogenous growth mechanism, idiosyncratic investment opportunity shocks, and housing assets play important roles in generating multiple balanced growth paths.

5.5 Growth Rate Comparisons

Another question we have is given we have multiple balanced growth paths, how should the growth rate on the balanced growth path change when an economy changes from closed to open. We present the results first and then explain the intuition.

Proposition 9 *When $\theta = 0$, if the foreign interest rate is smaller than the growth rate of closed economy $R^f < g_c$ respectively:*
(a) The growth rate of any steady state with $R^H > R^f$ is higher than g_c if $\theta_h < \theta_h^*$ while lower than g_c if $\theta_h > \theta_h^*$ where θ_h^* is given in the Appendix B.8.
(b) The growth rate of a steady state with $R^H = R^f$ is lower than g_c.

To present the intuition behind the growth comparison, we can consider the equation of housing pricing

$$\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t = R_t^H + \mu(Q_t - 1),$$

where R_t^H is the net return of investing housing assets. If R_t^H is high, then the liquid asset (housing) itself has a high rate of return, and then entrepreneurs are willing to hold the asset. Since the liquid asset provides liquidity for investment, the rate of economic growth is higher. In the closed economy,

$$R^H = \frac{\dot{p}_t + \psi c_t}{p_t} + g_t.$$

In the open economy with $R^H > R^f$ and all borrowing constraints binding for all entrepreneurs, the net return of housing asset investment R_{cstr}^H satisfies

$$R_{cstr}^H = \frac{1}{1-\theta_h}\left(\frac{\dot{p}_t + \psi c_t}{p_t} + g_t\right) - \frac{\theta_h}{1-\theta_h}R^f,$$

In the open economy with $R^H = R^f$ and borrowing constraints binding for part of entrepreneurs, the net return of housing asset investment R^H_{uncs} is

$$R^H_{uncs} = R^f.$$

In the $R^H = R^f$ case, the return on housing investment is equal to foreign interest rate. Generally, foreign interest rates are low. Therefore, returns on liquid assets are low and entrepreneurs are less willing to hold them. Consequently, liquid assets will not promote investment too much and the economic growth rate is even lower.

For the $R^H > R^f$ case, the effect of openness is ambiguous. When θ_h is not too large, credit inflows may amplify the return on the housing asset. The economic growth rate will also increase. Meanwhile, if θ_h is very large, too much inflow may decrease the return on the housing asset, making it close to the foreign interest rate R^f. Therefore, the economic growth rate decreases.

Corollary 1 (Perils of Liberalization) *Capital account liberalization may create the possibility of an even lower growth rate.*

A corollary to Proposition 9 is that the liberalization of capital account is not necessarily a good thing for an economy. Even if the volatility and risks associated with capital flows are not taken into account, capital account liberalization may make the economy grow at a lower rate in terms of the balanced growth path.

5.6 Local Dynamics

To analyze the local dynamics of the system, we can make a linear approximation near the steady state,

$$\dot{\mathbf{x}}_t \approx \mathcal{J} \cdot \mathbf{x}_t$$

where $\mathbf{x}_t$ is $\{c_t, Q_t, p_t\}$ in the case of $R^H > R^f$ while $\mathbf{x}_t$ is $\{c_t, Q_t, p_t, l_t\}$ in the case of $R^H = R^f$. The sign of the eigenvalues of the Jacobian matrix determines the dynamical property of the steady states.

Proposition 10 *When there are two steady states, the steady state with high growth rate is a saddle point, the steady state with low growth rate is a sink or a source point if $R^H > R^f$ while it is a saddle point if $R^H = R^f$.*

In this system, the stock of external debt l_t is predetermined and cannot jump. Therefore, in the system with $R^H = R^f$, the state variable is l_t. Meanwhile, in the system with $R^H > R^f$, as $l_t = \theta + \theta_h p_t$, the state variable is p_t. When only one of the eigenvalues of the steady state has a negative real part, the steady state is a saddle point. The steady state is a sink point if two of the eigenvalues have negative real parts. When no eigenvalue has a negative real part, the steady state is a source point.

The local dynamic property is also marked on Figure 3. When the steady state is a source point, it is not locally stable. When the steady state is a sink point, the local dynamics of the steady state exhibits one degree of indeterminacy. Given the initial debt level close to the low growth steady state, the system has infinite paths converging to the steady state.

6 Regime Switching and Endogenous Crises

Our model with credit openness has two balanced growth paths that satisfy the equilibrium conditions. When the confidence of market agents changes and they anticipate that the rate of economic growth may decline in the future, the economy may shift from the high growth path to the low growth path in a self-fulfilling way. In this section, we discuss the impact of changes in the sentiment of market agents in the economic system.

When a crisis occurs, asset prices drop abruptly. This collapse pushes collateral values below the outstanding debt level, potentially triggering corporate defaults. In this section, we first incorporate this default mechanism at the crisis onset into our model. We then simulate a scenario representative of a developing economy: a high-growth phase is interrupted by a crisis, followed by a prolonged period of low growth. Finally, we introduce repeated regime switching into the model, allowing the economy to alternate between high-growth and low-growth equilibria.

6.1 Default Settings

We first reformulate the model and add the settings of default. Suppose the economy is in the good state at time t, where the price of housing is P_t. With probability $\pi_{GB}\Delta$ between time t and $t+\Delta$, the economy may transition to the bad state, in which the housing price drops to $\tilde{P}_t$. An entrepreneur holds collateral in the form of housing stock H_{jt}, and borrows an amount $L^h_{jt+\Delta}/(1+\tilde{R}^f_t\Delta)$ at the loan interest rate $1+\tilde{R}^f_t\Delta$, with the obligation to repay $L^h_{jt+\Delta}$ at time $t+\Delta$.

We introduce a limited commitment problem: the entrepreneur can divert a fraction $(1-\theta_h)$ of the asset value in case of default. Accordingly, the lender whereas if a price jump occurs, the recovery value is reduced to $\min\left\{L^h_{jt+\Delta}, \theta_h\tilde{P}H_{jt+\Delta}\right\}$.

The lender and competitive lending, the no-arbitrage condition for foreign lenders requires the expected repayment to equal the cost of funds:

$$\frac{1+R^f\Delta}{1+\tilde{R}^f_t\Delta}L^h_{jt+\Delta} = (1-\pi_{GB}\Delta)\min\left\{L^h_{jt+\Delta},\ \theta_h P_{t+\Delta}H_{jt+\Delta}\right\} + \pi_{GB}\Delta\ \min\left\{L^h_{jt+\Delta},\ \theta_h\tilde{P}_t H_{jt+\Delta}\right\}.$$

Entrepreneurs choose the pair $\left\{\tilde{R}^f_t, L^h_{jt+\Delta}\right\}$ to maximize profit. Under the assumption that both R^f and the jump probability $\pi_{GB}\Delta$ are sufficiently small, borrowers have an incentive to borrow up to their collateral constraint. The maximum loan amount satisfies:

$$\frac{L^h_{jt+\Delta}}{1+\tilde{R}^f_t\Delta} \leq \frac{(1-\pi_{GB}\Delta)\theta_h P_{t+\Delta}H_{jt+\Delta} + \pi_{GB}\Delta \cdot \theta_h \tilde{P}_t H_{jt+\Delta}}{1+R^f\Delta}. \tag{30}$$

This constraint holds with equality if and only if $L^h_{jt+\Delta} \geq \theta_h P_{t+\Delta}H_{jt+\Delta}$. Moreover, in this contract, the funds borrowers can get $L^h_{jt+\Delta}/(1+\tilde{R}^f_t\Delta)$ are restricted by Equation (30), and borrowers have to pay back $L^h_{jt+\Delta}$. Therefore, borrowers have no incentive to choose $L^h_{jt+\Delta} > \theta_h P_{t+\Delta}H_{jt+\Delta}$, in equilibrium:

$$L^h_{jt+\Delta} = \theta_h P_{t+\Delta}H_{jt+\Delta},$$

$$\frac{1+R^f\Delta}{1+\tilde{R}^f_t\Delta} = 1-\pi_{GB}\Delta + \pi_{GB}\Delta \cdot \frac{\tilde{P}_t}{P_{t+\Delta}}.$$

Now consider a second type of loan, L_{jt}, collateralized by physical capital K_{jt}. Since the lender can always sell the collateral capital at the investment price of 1 and this price does not drop in the bad state, the recovery amount remains θK_{jt} regardless of the regime. Therefore, the lender In this case, the lending rate equals the risk-free rate R^f, and the borrowing constraint is:

$$L_{jt+\Delta} = \theta K_{jt}.$$

The other settings of the entrepreneurs are the same as the baseline model. When $\Delta \to 0$, the pricing equation of the housing asset becomes

$$\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t = \frac{1}{1-\theta_h}\left(\frac{\dot{p}_t + \psi c_t}{p_t} + g_t\right) - \frac{\theta_h}{1-\theta_h}\tilde{R}^f_t + \mu(Q_t - 1) + \pi_{GB}\left[\left(\frac{\tilde{c}}{c_t}\right)^{-\sigma}\frac{\tilde{p}}{p_t} - 1\right],$$

where

$$\tilde{R}_t = R^f + \pi_{GB}\left(1 - \frac{\tilde{p}}{p_t}\right).$$

Compared with the baseline model, the interest rate of the loan with housing collateral is no longer the foreign riskless rate R^f, but becomes $\tilde{R}^f_t$ which adds the default risk premium. Meanwhile, the pricing equation should also consider the case where the housing price jumps to the bad state. Therefore, the system has one more term $\pi_{GB}\left[\left(\frac{\tilde{c}}{c_t}\right)^{-\sigma}\frac{\tilde{p}}{p_t} - 1\right]$.

The economy is required to pay higher interest rates on loans than in the baseline model because there is a risk premium on borrowing. Therefore, the consumption equation needs to be changed to

$$c_t = R^k - (g_t + \delta) + (\theta + \theta_h p_t)(g_t - R^f) - \pi_{GB}\theta_h\left(p_t - \tilde{p}\right) + \theta_h \dot{p}_t,$$

where $\pi_{GB}\theta_h\left(p_t - \tilde{p}\right)$ is the additional interest payment.

The pricing equation of Tobin's Q becomes

$$\rho + \sigma \frac{\dot{c}_t}{c_t} + \sigma g_t = \frac{\dot{Q}_t + R_t^k - \delta Q_t - R^f \theta}{Q_t - \theta} + \pi_{GB} \left[\left(\frac{\tilde{c}}{c_t} \right)^{-\sigma} \frac{\tilde{Q} - \theta}{Q_t - \theta} - 1 \right],$$

which considers the case that the housing price jumps to the bad state. The growth equation is the same as the baseline model.

$$g_t = \mu(1 - \theta_h) p_t - \delta.$$

When the system jumps to the bad state, we assume that the system follows the equations in the baseline model.

6.2 Permanent Transition

A canonical feature of emerging economies is their tendency to experience episodes of high growth that are abruptly interrupted by financial crises, leading to prolonged periods of low growth. To analyze this phenomenon, we simulate an economic environment where an economy in a high-growth state confronts a constant hazard rate of permanently shifting to a low-growth state. We begin by examining the case where the low-growth state is an equilibrium with $R_t^H = R^f$, a specification that demonstrates a good fit with the empirical data. We then consider an alternative where the low-growth state is an equilibrium with $R_t^H > R^f$.

We try to use the case similar to Figure 3 (b) to simulate the crisis pattern. Specifically, we assume that when the economy transitions into the bad state, the system lands on the saddle path of the low-growth equilibrium. For the transition at the moment of crisis to be well-defined, that is, to prevent an infinite spike in instantaneous consumption, the following condition must be satisfied:

$$\tilde{L}_{jt} = \theta_h \tilde{P} H_{jt} + \theta K_{jt}, \tilde{l} = \theta_h \tilde{p} + \theta.$$

On the low-growth equilibrium saddle path, there is a unique point that satisfies $\tilde{l} = \theta_h \tilde{p} + \theta$, and it is to this point that the system jumps at the time of the crisis. Let (p^b, l^b) denote the steady state of the bad state. Note that as $R_t^H = R^f$, l^b might be smaller than $\theta_h p^b + \theta$. Relative to this steady state, the jump point is characterized by a lower collateral price $\tilde{p} < p^b$ but a higher aggregate debt level $\tilde{l} > l^b$. After the discontinuous jump, the system converges gradually along the saddle path: the price p rises toward p^b, while the debt level l declines toward l^b.

We focus on the case that only housing assets can serve as collateral and assume that $\theta = 0$, as information asymmetry of capital collateral quality is severe in emerging economies. Following Miao et al. (2022), we take $\delta = 0.05$, $\rho = 0.02$. Following Smets and Wouters (2007), we take $\sigma = 1.5$. We take $R^f = 1.125\%$ so that the yearly interest rate is about

Symbol	Economic Meaning	Value	Target or Source
σ	Inverse of EIS	1.5	Smets and Wouters (2007)
ρ	Discount rate	0.02	Miao et al. (2022)
δ	Depreciation rate	0.05	Miao et al. (2022)
ψ	Rent-to-consumption ratio	0.046	WIOD and World Bank
θ	Collateral rate of capital	0	Severe frictions
R^f	Foreign interest rate	1.125%	Federal funds effective rate
θ_h	Collateral rate of houses	0.53	Growth target
μ	Investment opportunity rate	0.018	Growth target
Z	Productivity	0.2	Investment rate
π_{GB}	Jumping probability	0.01	25-year duration of crises

Table 2: Calibrations of Parameters

4.5%, consistent with the federal funds effective rate in the 1990s. Data from the World Input Output Database (WIOD) indicates that in Thailand, the ratio of value-added of real estate sector to GDP is approximately 2.97% in 1997, while the consumption-to-GDP ratio is about 65.1% (World Bank). Consequently, we set $\psi = 0.046$. [8] To capture that the investment ratio is about 30% after the crisis, we choose $Z = 0.2$. To match that the annualized growth rate is about 9%-10% and 4%-5% before and after the crisis, we choose $\mu = 0.018$ while $\theta_h = 0.53$. We take $\pi_{GB} = 0.01$ so that the average interval between crises is 25 years. The parameters are summarized in Table 2.

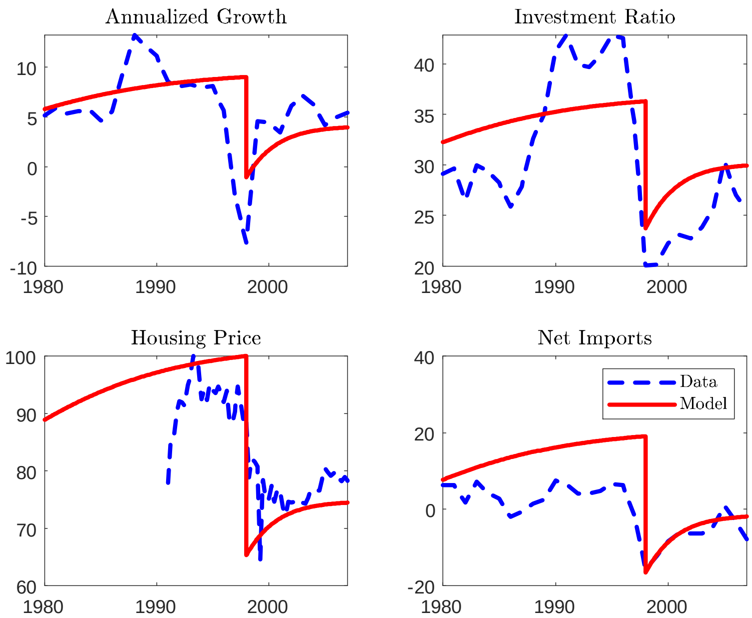

Figure 4: Crisis in 1998 of Thailand

Note: The parameters are summarized in Table 2. We assume that the regime switching happens in 1998.

[8] In our model, $M_t H_t = \psi C_t$ can be interpreted as the value-added of the real estate sector, where ψ signifies the ratio of real estate value to consumption.

We assume that the crisis happens in 1998. Figure 4 illustrates the simulation results for Thailand during the 1998 Asian Financial Crisis, comparing model outputs with empirical data. The simulation begins with the economy on a high-growth balanced growth path, characterized by a saddle path where economic growth, asset prices, and investment rates gradually increase as the system converges toward the high-growth steady state. However, upon the occurrence of a crisis, triggered by an exogenous shift in beliefs, the system abruptly transitions to the low-growth equilibrium saddle path. This transition involves an instantaneous spike in the liquidity premium, while asset prices, investment, and the growth rate sharply decline. Post-crisis, the economy exhibits short-term overshooting; for instance, asset prices may excessively decline before gradually converging to the low-growth steady state. This dynamic aligns with empirical observations, including the capital outflows, GDP contraction, housing price collapses, and reduced investment rates witnessed in Thailand after 1998.

The simulation results demonstrate a close fit with real-world data, validating the ability of the model to replicate crisis patterns. The convergence path post-crisis reflects persistent stagnation, consistent with the long-term growth downgrades observed in emerging markets following financial crises. By integrating default risks and regime switches, the model provides a nuanced understanding of how small open economies can experience endogenous fluctuations without fundamental shocks, underscoring the importance of such vulnerabilities.

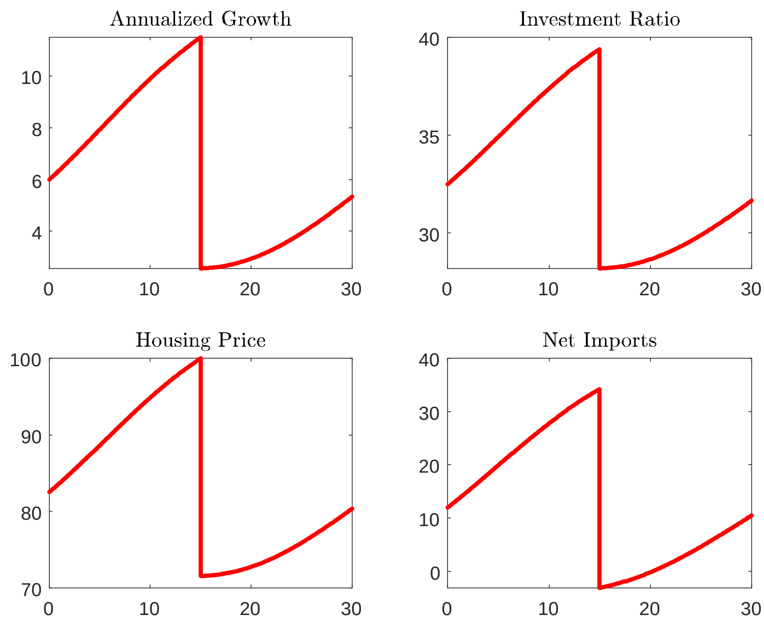

Figure 5: Alternative Case

Note: The parameters are summarized in Table 2 except that $R^f = 0.009$ and $\theta_h = 0.45$. We assume that the regime switching happens in year 15.

Then, we turn to the analysis of the case similar to Figure 3 (a) where the low state satisfies $R_t^H > R^f$. In the low balanced growth path, the corresponding steady state can be either a sink or a source. If it is a sink, then a crisis can trigger a shift in the system: it falls away from the high-growth saddle path and onto an arbitrary trajectory converging to the low-growth balanced growth path.

If we take $R^f = 0.009$ and $\theta_h = 0.45$, the low-growth path becomes a sink. As Figure 5 shows, initially, the system is positioned on a high-growth saddle path that converges to a high-growth balanced growth path, characterized by elevated asset prices, investment rates, and consumption levels. However, following a crisis, modeled as an exogenous belief shock, the system shifts to the low-growth equilibrium. During this transition, key variables such as asset prices, consumption, and the growth rate decline sharply with a relatively mild overshoot. The economy gradually converges to the low-growth balanced growth path, illustrating how financial frictions and foreign leverage interact to generate lasting adverse effects.

6.3 Repeated Regime Switching

We now turn to the case of repeated regime switching, where the economy can alternate between a good state and a bad state. Our analysis primarily considers situations in which the bad state is a sink. It is important to note that when the system jumps from the bad state back to the good state, the state variable, debt, cannot jump, otherwise the consumption would be infinite at the point of transition. This continuity implies that asset prices also do not jump at the point of transition. The dynamics in the bad state are described by the following equations:

$$\rho + \sigma \frac{\dot{c}_t}{c_t} + \sigma g_t = \frac{1}{1-\theta_h}\left(\frac{\dot{p}_t + \psi c_t}{p_t} + g_t\right) - \frac{\theta_h}{1-\theta_h} R^f + \mu(Q_t - 1) + \pi_{BG}\left[\left(\frac{\tilde{c}_t}{c_t}\right)^{-\sigma} - 1\right],$$

$$\rho + \sigma \frac{\dot{c}_t}{c_t} + \sigma g_t = \frac{\dot{Q}_t + R_t^k - \delta Q_t - R^f \theta}{Q_t - \theta} + \pi_{BG}\left[\left(\frac{\tilde{c}_t}{c_t}\right)^{-\sigma} \frac{\tilde{Q}_t - \theta}{Q_t - \theta} - 1\right],$$

$$c_t = R_t^k - (g_t + \delta) + (\theta + \theta_h p_t)(g_t - R^f) + \theta_h \dot{p}_t,$$

where

$$g_t = \mu(1-\theta_h)p_t - \delta.$$

Compared with the case of the good state, the pricing equation does not have the term $\tilde{p}/p$. The consumption function does not have $\pi\theta_h\left(p_t - \tilde{p}_t\right)$. The reason is that the price cannot jump.

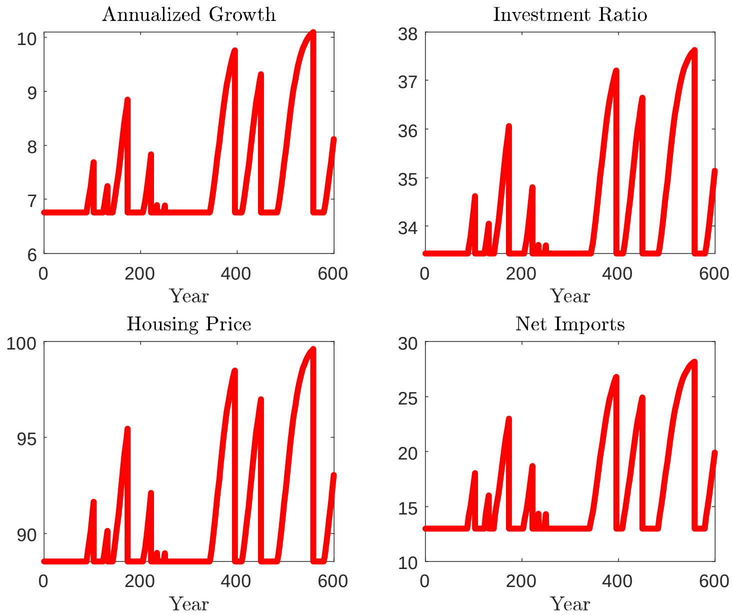

Figure 6: Repeated Regime Switching

Note: The parameters are summarized in Table 2 except that $R^f = 0.009$, $\theta_h = 0.47$, and $\pi_{BG} = 0.01$.

Figure 6 illustrates the dynamic pattern of repeated regime switching between high-growth and low-growth equilibria in the model economy. The graph captures the asymmetric nature of these transitions: when the economy shifts from a good state (high-growth path) to a bad state (low-growth path), the crisis occurs abruptly, with key variables like asset prices, consumption, and growth rates plummeting almost instantaneously. This rapid decline reflects the sudden collapse in market confidence and the immediate tightening of liquidity constraints after the default. In contrast, the recovery process from the bad state back to the good state is gradual and prolonged, characterized by a slow rebound in economic indicators along the saddle path of the system. This asymmetry is consistent with the real world.

6.4 Transitions without Default

We also simulate the regime-switching dynamics without default in Appendix C. In the no-default setting, where debt serves as a state variable and must remain continuous, asset prices cannot jump abruptly without violating borrowing constraints. As a result, transitions between equilibria occur gradually. Asset prices, debt levels, and growth rates adjust slowly over time, with no sudden discontinuities. For instance, during the shift from a high-growth path to a low-growth path with $R_t^H > R^f$, the economy exhibits a prolonged adjustment phase, where variables like growth rate, housing prices, and investment evolve smoothly. Interestingly, we even observe a temporary increase in growth rates at the onset of the transition to the bad equilibrium with $R_t^H = R^f$. In contrast, the model with default mechanism in this section allows for abrupt jumps in asset prices and economic growth rates during crises, which is more consistent with the patterns in the real world.

7 Small Shocks and Sudden Stops

The coexistence of multiple balanced growth paths in our credit openness case implies that the economy is vulnerable. In particular, even a minor shift in certain parameters, such as a sudden decline in leverage capacity or an increase in interest rates, can lead to the disappearance of the high-growth balanced growth path. When this occurs, the system is forced to converge to the remaining low-growth balanced growth path, resulting in a sharp and persistent decline in the growth rate. This phenomenon illustrates a key feature of systems with multiple equilibria: the relationship between fundamentals and outcomes is not always continuous or smooth. Rather, a small fundamental change can induce a discontinuous jump across balanced growth paths, reflecting the nonlinearities in the model.

We interpret the exogenous shifts in our model parameters through the lens of the global financial cycle (Rey, 2013). As documented by Miranda-Agrippino and Rey (2020), a tightening of US monetary policy transmits globally not just through interest rates, but through a retrenchment of global risk appetite. In our model, this acts as a simultaneous shock to the cost of credit (R^f) and the availability of leverage (θ_h), pushing the small open economy from a stable high-growth region into a region of low growth and collapse.

Such a transition embodies the classic characteristics of a *sudden stop* episode: an abrupt reversal of capital flows, a rapid deleveraging process, and a sharp contraction in economic activity. A rise in the foreign interest rate R^f can prompt a sudden withdrawal of international lending, exacerbating financial constraints and pushing the economy into a bad equilibrium. Similarly, the loss of confidence among foreign creditors, reflected in the drop of θ_h, can trigger large-scale capital outflows, leading to a collapse in asset prices and domestic investment. These mechanisms align closely with the empirical patterns of sudden stops observed in emerging markets.

Panel (a) of Figure 7 shows the outcome of a sudden increase in the foreign interest rate R^f. A modest rise can eliminate the high-growth equilibrium entirely. The adjustment is discontinuous: the economy does not experience a gradual decline but rather a regime shift, moving swiftly from the high to the low steady state. This reinforces the idea that in a multi-equilibrium setting, small parameter changes can provoke large and abrupt collapses in growth, with no intermediate outcomes. The rise in R^f leads to a reversal in capital inflows, accelerating the contraction and underscoring the role of financial fragility in sudden stop dynamics.

Similarly, Panel (b) of Figure 7 illustrates the effects of a sudden drop in θ_h, which may stem from a shift in foreign creditors change in the parameter is small, its consequences are not marginal: the high-growth steady state ceases to exist, and the economy undergoes an abrupt transition to the low-growth regime. This highlights the fact that the system does not adjust continuously in the neighborhood of the high equilibrium; instead, the equilibrium vanishes, compelling a discontinuous jump to the inferior equilibrium. The decline in θ_h effectively reduces the availability of foreign credit, inducing capital flight and a sudden stop in financing.

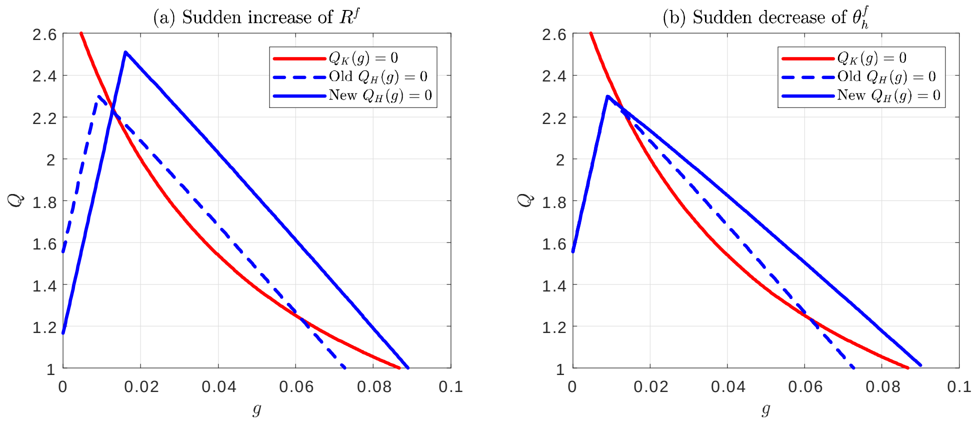

Figure 7: Sudden Stops

Note: Panel (a) presents that the good balanced growth path disappears after a rise in R^f. Panel (b) presents that the good balanced growth path disappears after a fall in θ_h.

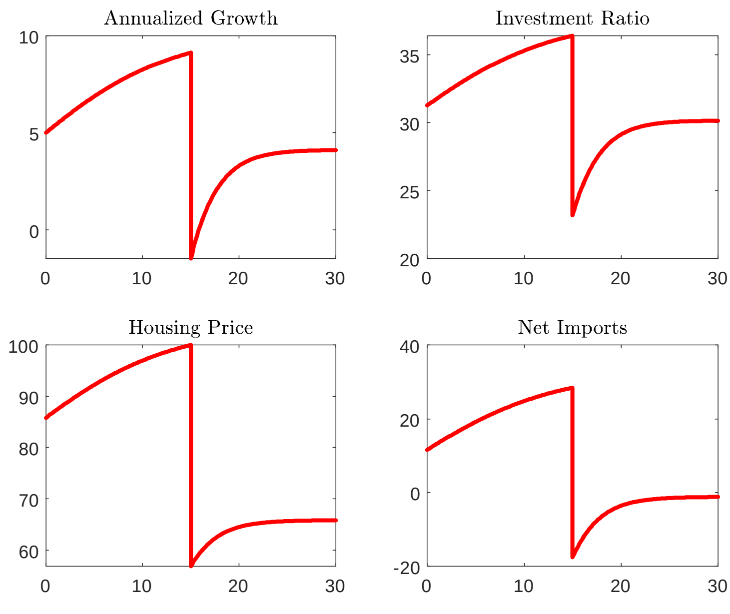

Figure 8: Transition of Sudden Stops

Note: The parameters are summarized in Table 2 except that $\pi_{GB} = 0$. Sudden stops happen in year 15 when R^f increases from 0.01 to 0.01125 while θ_h decreases from 0.50 to 0.46.

Figure 8 illustrates the dynamic adjustment of the economy following a simultaneous reduction in the foreign collateral parameter θ_h and an increase in the world interest rate R^f, which collectively simulate a sudden stop episode. Initially, the economy resides on a high-growth balanced growth path, characterized by saddle-path dynamics where key macroeconomic variables, including economic growth, asset prices, and investment rates, exhibit gradual increases as the system converges toward the high-growth steady state. However, the onset of a sudden stop, triggered by the adverse parameter shifts, radically alters the equilibrium structure: the high-growth path ceases to exist, leaving the low-growth balanced growth path as the only feasible equilibrium. This disruption forces an abrupt transition toward the inferior steady state. The immediate aftermath is marked by a sharp spike in the liquidity premium, reflecting intensified financial constraints, while asset valuations, investment activity, and the growth rate experience pronounced declines. During the subsequent adjustment phase, the economy exhibits short-term overshooting, a common feature in crisis transitions state. This trajectory underscores the nonlinear and disruptive nature of sudden stops, where modest parameter changes can induce large-scale reallocations and persistent stagnation.

8 Cycles in Perfect Foresight Equilibrium

The previous sections simulate the endogenous cycles via regime switching. We now demonstrate that the model can generate deterministic cycles even under perfect foresight equilibrium. To do so, we shift the analysis from the local dynamics around steady states (balanced growth paths) to the global dynamics of the system. This broader perspective allows us to uncover not only multiple equilibria but also periodic orbits, which would remain undetected through local analysis alone. As noted in Beaudry et al. (2020), the emergence of such periodic trajectories can offer important insights into the endogenous mechanisms driving business cycle fluctuations.

Within this global framework, we also document the existence of global indeterminacy, a phenomenon that may persist even when steady states exhibit local determinacy. While local determinacy implies a unique equilibrium path near a steady state, the system as a whole can still admit multiple feasible trajectories. For instance, in a source-saddle system, although dynamics are locally determinate around the source, global indeterminacy can arise from the coexistence of distinct convergence paths toward the saddle point.

Analyzing global dynamics requires advanced mathematical tools. Following the economic applications introduced in Benhabib et al. (2003), we employ bifurcation analysis as the principal method for this investigation.

8.1 Hopf Bifurcation and Periodic Orbits

In Proposition 10, we show that when foreign borrowing binds for all entrepreneurs ($R_t^H = R^f$), the equilibrium with a low growth rate may be a source or a sink. The product of the three eigenvalues is a positive real number, one of the eigenvalues is positive ($\lambda_3 > 0$), and the real parts of the other two eigenvalues may both be positive or both be negative. Then if the parameter is taken such that the real part of the other two eigenvalues is 0, there may be a pair of conjugate imaginary eigenvalues ($\lambda_{1,2} = \pm i\omega$). We assume that when $R^k = R^k_{HB}$, the steady state has a pair of conjugate imaginary eigenvalues. Then as long as some other regular conditions are satisfied, the system displays the Hopf bifurcation (see Theorem 3.4.2 of Guckenheimer and Holmes (1983)). We call R^k_{HB} the Hopf bifurcation point.

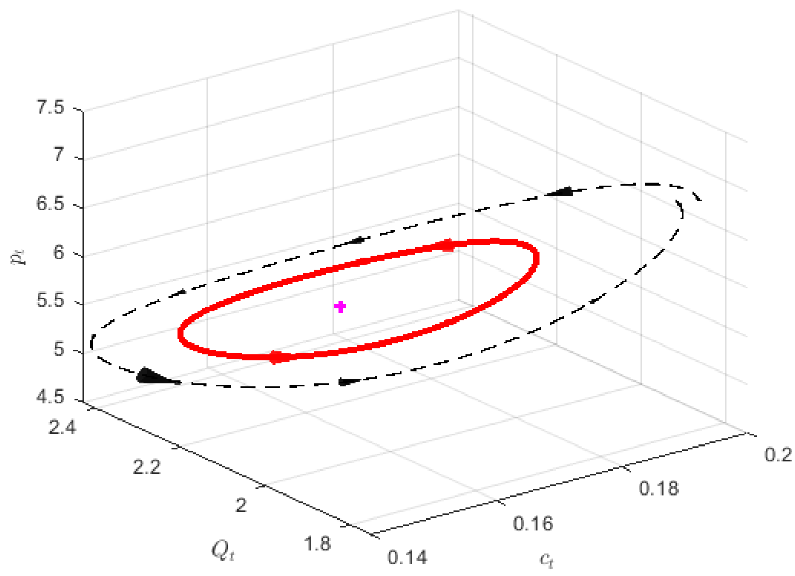

Figure 9: Periodic Orbit

Note: The parameters are summarized in Table 2 except $R^f = 0.005$ and $\theta_h = 0.363$.

Theorem 4 (Existence of Periodic Orbits) *When the steady state with low growth rate can be either a sink or a source as R^k continuously changes, the dynamic system admits a Hopf bifurcation at a critical value of the domestic capital return, R^k_{HB}. For a range of R^k in the neighborhood of R^k_{HB}, a periodic orbit exists, generating deterministic, endogenous fluctuations in all macroeconomic aggregates.*

When the borrowing constraints bind for all entrepreneurs in the low steady state, it is a sink point or a source point, depending on the parameters. When it changes from stable to unstable, the path around the steady state changes from oscillatory convergence to oscillatory divergence from the steady state, with possible periodic orbits in the parameter space on one side of the critical point. Figure 9 presents an example of the periodic orbit. The emergence of a periodic orbit implies that, even in a perfect foresight equilibrium, the economy does not necessarily converge to a steady state. Rather, it undergoes cyclical motion. This finding is consistent with empirical literature (Beaudry et al., 2020) that economic cycles are largely driven by endogenous cyclical forces.

Numerically, we can show that it is a supercritical Hopf bifurcation. Given the surface where the steady state point and the periodic orbit are located, with a starting point outside the periodic orbit on that surface, the system will converge to the periodic orbit from the outside in. Conversely, with a starting point inside the periodic orbit on that surface, the system will converge to the periodic orbit from the inside out.

We provide evidence of cyclical components in the data. Building on the methodological framework of Beaudry et al. (2020), which identifies cyclical components in economic activity using spectral analysis, we apply a similar approach to examine cyclical patterns in capital flows. The results, presented in Appendix Figure A.1, reveal significant cyclicality in capital flows across several small open economies, with distinct spectral peaks observed for Thailand, Malaysia, Spain, and Mexico. These findings are consistent with our theoretical predictions that small open economies are prone to deterministic cycles driven by endogenous mechanisms.

8.2 More Complex Bifurcations

In the previous part, we use the Hopf bifurcation to show that periodic orbits may occur in the system. Considering that there are two steady states in our system, the system may exhibit more complex dynamics. It involves the interaction between the two steady states.

Specifically, in this part, we numerically apply the Theorem in Kopell and Howard (1975). The theorem tells us that if a two-parameter dynamic system has a steady state with two zero eigenvalues for some parameter values, and some regular conditions are satisfied, there may exist homoclinic orbits, periodic orbits, or paths connecting the two steady states after changing parameter values.

Figure 10 numerically shows the bifurcations described in Kopell and Howard (1975). In Panel (a), the system has two steady states: a source point and a saddle point. The system can diverge from the low-growth steady state owing to the characteristics of the source point. Furthermore, a saddle-point path originating from the source point and converging towards the saddle point exists. To satisfy the transversality condition, all trajectories must align with this saddle-point path, and failure to do so may result in p_t or Q_t diverging to infinity. It is important to note that, while local indeterminacy near the low-growth steady state is absent, a global indeterminacy emerges. This occurs because there are multiple points near the steady state that lie on the saddle-point path and can serve as starting points, guiding the system towards convergence to the high- growth steady state along the saddle-point path.

When we decrease R^f, we can get Panel (b). The system reaches a critical threshold at which a periodic orbit appears, and the system undergoes a saddle-loop bifurcation. As illustrated in the diagram, there is a distinct homoclinic orbit that originates from the saddle point and eventually returns to itself. It is important to note that the occurrence of the saddle-loop bifurcation necessitates R^f being precisely equal to R^f_{SL}, a condition that holds a measure of zero in the parameter space.

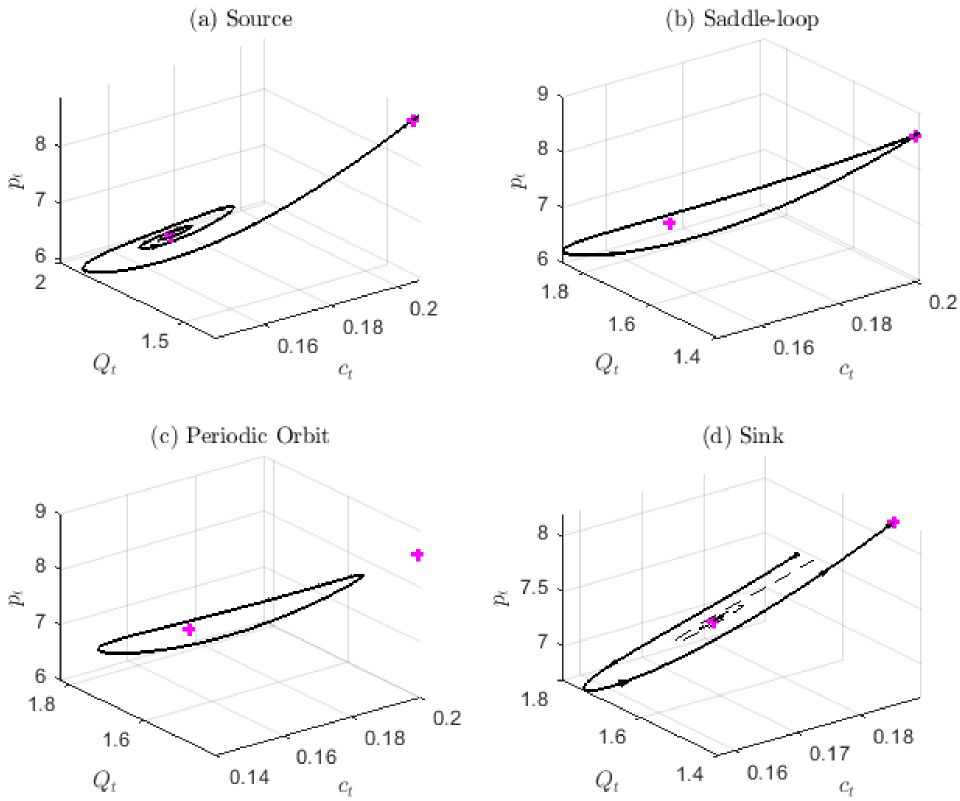

Figure 10: Bifurcations

Note: The parameters are summarized in Table 2 except $\sigma = 1.1$, $\theta_h = 0.25$. We take R^f as 0.0126, 0.0123, 0.0122 and 0.0117 respectively.

If we decrease R^f further, we illustrate the periodic orbit in case Panel (c). In this scenario, the low-growth steady state behaves as a source point, while the high-growth steady state is a saddle point. As depicted in the diagram, a periodic orbit surrounds the bad steady state, causing the economy to oscillate around it in a consistent, non-decaying way. This pattern suggests the presence of endogenous cycles within the economy, where economic variables fluctuate continuously without external disturbances. Given that it is supercritical, the cycle is a stable periodic orbit. Trajectories starting within the cycle can converge to this orbit, and those initiating near the exterior of the cycle also converge to it. Consequently, this periodic orbit is referred to as a limit cycle. The appearance of periodic orbits has profound economic implications. It demonstrates that the economy can produce endogenous fluctuations in a perfect foresight equilibrium, without the influence of external or internal sentiment shocks. This insight offers a fresh perspective on understanding business and financial cycles. Our findings resonate with the viewpoint presented in Beaudry et al. (2020), which posits that business cycles primarily arise due to internal forces that endogenously generate cyclical mechanisms. It is important to note that limit cycles can only be identified through a global analysis and cannot be constructed using a local analysis, highlighting one of the key motivations for adopting a global analysis approach in this paper.

If we continue to decrease R^f, we get Panel (d). As illustrated in the diagram, the low-growth steady state is a sink point, whereas the high-growth is a saddle point. The figure reveals that any initial condition within the surface bounded by the saddle path leads the system to converge towards the bad steady state. As the system nears this low-growth steady state, its variables oscillate and gradually converge, causing endogenous fluctuations. Meanwhile, multiple starting points along the saddle point path can lead to the bad steady state. This rich indeterminacy suggests that entrepreneurs can coordinate to form a variety of rational expectations. Such expectations influence the initial value of housing assets, and the subsequent economic dynamics. The expectations become self-fulfilling prophecies. Indeterminacy influences both the endpoints and the paths taken to reach them.

In this section, we study the global indeterminacy through both theoretical and numerical analysis. Our system encompasses two steady states: one is a saddle point, while the other can be classified as either a sink or a source point. Our findings indicate that, in both scenarios, the system exhibits indeterminacy. Specifically, there exists an infinite array of paths that fully satisfy all equilibrium conditions, including the transversality condition. What is particularly intriguing is that the system demonstrates periodic orbits when the parameters fall within specific ranges, hinting at the potential for endogenous economic cycles.

9 Policies

The analysis thus far has established that economies with multiple equilibria are susceptible to belief-driven fluctuations, self-fulfilling crises, and sudden stops, often with severe and persistent effects on growth. These findings naturally call for policy interventions that can mitigate such instability. In this section, we evaluate two types of policies aimed at stabilizing the economy and improving long-term growth outcomes. First, we examine countercyclical policies, which are designed to dampen endogenous cycles and eliminate the low-growth balanced growth path through state-contingent subsidies or macroprudential tools. Second, we consider policies that encourage foreign direct investment (FDI), which can raise the domestic return to capital and shift the economy toward a higher growth path. Each policy is evaluated in terms of its theoretical mechanism and equilibrium effects within our dynamic framework.

9.1 Countercyclical Policy

To address the instability caused by multiple equilibria and self-fulfilling cycles, we propose two forms of countercyclical policy: a subsidy to entrepreneurs and a macroprudential tool applied to foreign borrowing. Both policies are designed to eliminate the bad equilibrium by introducing a stabilizing negative feedback mechanism that counteracts the positive feedback loop described earlier. Specifically, each policy responds to the growth gap $(g_t - g_0)$, effectively against the wind these policies can remove the low-growth equilibrium altogether, ensuring that the economy converges uniquely to the high-growth steady state.

Countercyclical Subsidy Policy To mitigate self-fulfilling downturns and eliminate the bad equilibrium, we introduce a countercyclical subsidy policy financed by lump-sum taxes targeted at entrepreneurs. The subsidy is designed to offset pessimistic expectations and disrupt the destabilizing feedback loop between falling growth and declining asset values.

Formally, let the realized return on capital of entrepreneurs be $\tilde{R}_t^k$. We define a subsidy rule contingent on the growth gap:

$$\tilde{R}_t^k = R_t^k + \tau(g_0 - g_t)$$

where g_0 denotes the growth rate in the high steady state and $\tau > 0$ is the policy intensity parameter. The subsidy is thus higher when the economy is further below its potential growth.

Incorporating this subsidy, the modified equation governing the dynamics of Tobin

$$\left(\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t\right)(Q_t - \theta) = \dot{Q}_t + R^k + \tau(g_0 - g_t) - \delta Q_t - R^f\theta.$$

The policy alters the structure of the system: the capital pricing curve $Q_K(g) = 0$ rotates clockwise around the good equilibrium, as shown in Figure 11a. When the growth rate is lower than the target, the value of capital Q_t is higher than the case without policy as subsidy increases the capital value. When τ is sufficiently large, the bad equilibrium disappears, leaving only the high-growth balanced growth path.

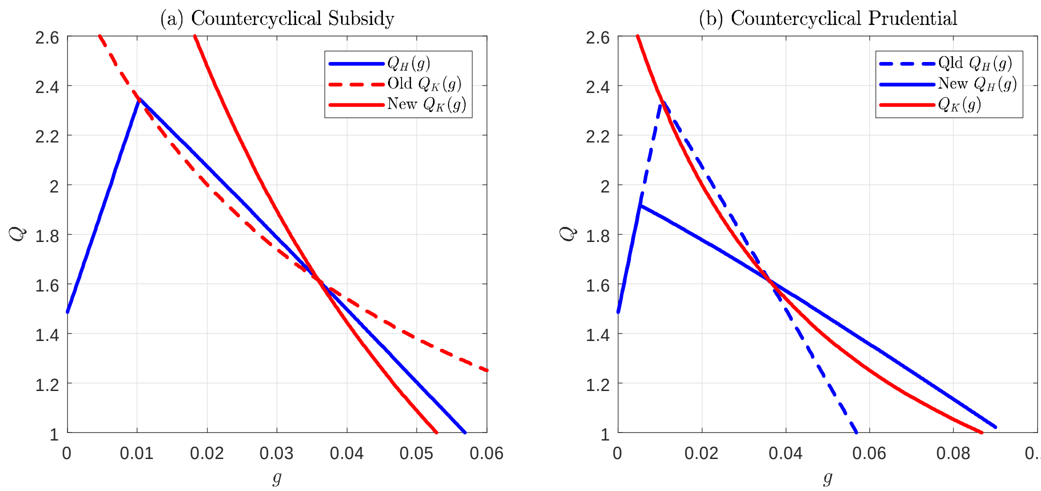

Figure 11: Policy

Note: Panel (a) presents that the bad balanced growth path disappears with countercyclical subsidy policy. Panel (b) presents that the bad balanced growth path disappears countercyclical prudential policy.

The key mechanism through which this policy works is by breaking the loop in which low growth reduces asset values, tightens financing constraints, and further dampens economic activity. By contrast, the countercyclical subsidy dampens fluctuations: when growth falls, the subsidy increases, raising the effective return on capital and Tobin's Q_t. This supports the rise in liquidity premium and the valuation of housing assets. Consequently, the economy avoids coordination failure and converges uniquely to the high-growth path.

Countercyclical Macroprudential Policy To further mitigate equilibrium multiplicity and stabilize the financial cycle, we propose a countercyclical macroprudential policy applied to foreign borrowing. This policy aims to moderate capital flow volatility and prevent deleveraging-driven downturns by making the cost of foreign borrowing countercyclical to the domestic business cycle.

The baseline foreign interest rate is R^f. We introduce a time-varying component that depends on the growth gap:

$$R^f(g) = R^f + \tau(g_t - g_0)$$

where g_0 is the growth rate in the high steady state and τ measures policy intensity. During periods of high growth ($g_t > g_0$), governments tax the foreign credit and increase the borrowing cost. During downturns ($g_t < g_0$), governments subsidize the foreign credit and reduce the borrowing cost.

This countercyclical adjustment modifies the dynamics of asset price p_t. Specifically, the housing pricing curve $Q_H(g) = 0$ rotates counterclockwise around the good equilibrium (see Figure 11b). When the growth rate is lower than the target, the subsidy decreases the foreign borrowing cost and increases the net return on housing investment. Therefore, entrepreneurs require a lower liquidity premium. For a sufficiently large τ, the low-growth equilibrium is eliminated, and the economy converges uniquely to the high-growth path.

The policy operates by breaking the harmful feedback loop in which low growth leads to lower asset valuation and further contraction. By lowering foreign borrowing costs during downturns, the policy supports asset valuations and prevents self-fulfilling credit crunches. Thereby, it ensures macroeconomic stability and rules out coordination failures.

In summary, the two countercyclical policies proposed are capable of breaking the positive feedback loop, thereby eliminating the low-growth equilibrium and steering the economy toward a unique high-growth path. This policy approach stands in sharp contrast to the historical pattern documented by Reinhart and Reinhart (2009), in which many developing economies adopted procyclical fiscal policies during capital inflow bonanzas, amplifying financial cycles and ultimately increasing the risk of sudden stops or crises. Our model provides a practical framework to avoid the self-fulfilling crises associated with procyclical policymaking.

9.2 FDI Policy

To complement the countercyclical measures, we now examine how policies encouraging foreign direct investment (FDI) can alter equilibrium outcomes by raising the domestic return to capital and shifting the economy toward a higher growth path. The core mechanism hinges on FDI it enhances the marginal product of domestic capital through technological spillovers and complementarity (Javorcik, 2004; Haskel et al., 2007), while avoiding the destabilizing valuation-liquidity feedback loop inherent in foreign credit inflows. By raising the domestic return to capital (R^k), FDI shifts the equilibrium dynamics, specifically modifying the capital pricing curve ($Q_K(g)$) and housing pricing curve ($Q_H(g)$), to eliminate low-growth equilibria and anchor the economy on a unique, high-growth trajectory.

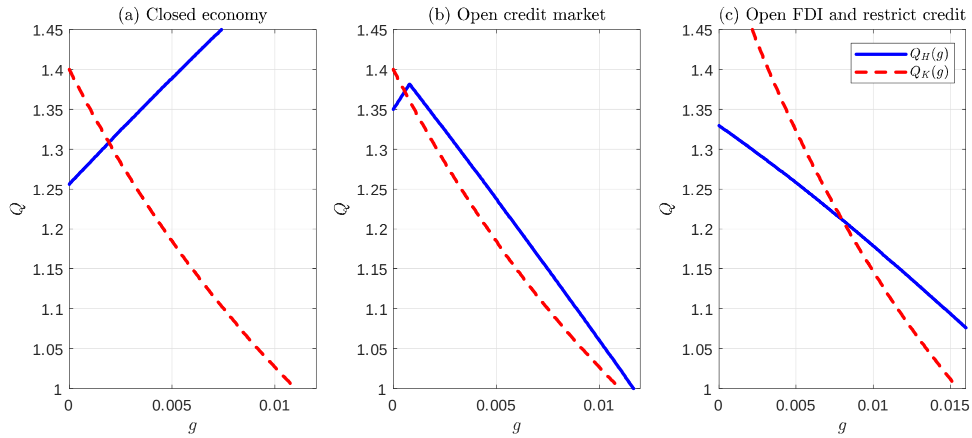

Figure 12: Credit inflows and FDI inflows

Note: Panel (a) is the diagram of closed economy. Panel (b) is the diagram that the economy only opens the credit market. Panel (c) is the diagram that the economy partially opens the credit market and attracts FDI inflows .

Figure 12 illustrates this mechanism through three comparative scenarios. Panel (a) depicts the closed economy benchmark: the system features a unique balanced growth path where the capital pricing curve $Q_K(g)$ (downward-sloping, as higher growth raises the stochastic discount factor and reduces capital valuation) intersects the housing pricing curve $Q_H(g)$ (upward-sloping, as higher growth increases the interest rate and dampens housing valuations). The balanced growth path is unique, determined by the negative feedback loop between growth, interest rates, and asset prices (Theorem 1).

Panel (b) shows the consequence of opening the economy only to foreign credit without FDI. As the productivity is not high enough, the openness of credit only yields a unique balanced growth path where $R_t^H = R^f$. Critically, $Q_H(g)$ intersects $Q_K(g)$ at a lower growth rate. As Corollary 1 highlights, this low-growth trap arises because the openness makes the return on housing assets equal to the foreign interest rate and entrepreneurs hold fewer housing assets, limiting investment and growth even below the closed-economy benchmark.

Panel (c) presents the welfare-improving outcome of combining FDI openness with restricted foreign credit (lower θ_h to curb debt inflows). This policy has two key effects on the equilibrium curves: First, FDI inflows raise R^k as stated in Section 4. The capital pricing curve $Q_K(g)$ shifts upward, reflecting higher valuations of domestic capital for any given growth rate. Second, the housing pricing curve $Q_H(g)$ becomes flattened, as credit inflows are restricted and θ_h is low. The result is a unique balanced growth path, where $Q_K(g)$ and $Q_H(g)$ intersect at a growth rate higher than the closed case.

In summary, inducing FDI inflows and restricting credit inflows alter the equilibrium structure by raising R^k to shift $Q_K(g)$ upward and preserving the negative feedback loop between growth and asset prices (via restricted credit inflows). The transition from Panel (b) to Panel (c) demonstrates that FDI not only raises the level of growth but also stabilizes the economy against self-fulfilling crises.

10 Conclusion

This paper formulates a theory of self small open economies, driven by distinct dynamic mechanisms depending on the composition of capital inflows. When openness is dominated by foreign credit, which supplies perfectly elastic liquidity at the world interest rate, optimistic growth expectations unleash a valuation feedback loop: asset prices rise without being checked by higher domestic borrowing costs, collateral constraints relax, and surging investment validates the initial optimism. This mechanism creates multiple balanced growth paths. In contrast, under FDI openness or in closed economies, growth optimism increases the current consumption due to the permanent income effect and creates a shortage of savings as resources are limited in the closed economy. It drives up the interest rate and dampens asset valuations, which ensures a unique, stable growth path. Belief explain sudden stop crises, where coordinated pessimism can trigger a persistent transition to a low growth equilibrium with collapsing asset prices and consumption. Furthermore, the system exhibits deterministic endogenous cycles via Hopf bifurcations, showing that fluctuations can arise even under perfect foresight without exogenous shocks.

The framework carries clear policy implications. State-contingent capital-inflow levies can dampen leveraged returns to collateral in bad times, collapsing the low-growth equilibrium and restoring a unique, high-growth path. Policies that raise the return to capital, such as tilting inflows toward FDI with technology spillovers, shift the system away from multiplicity and strengthen long-run growth. These recommendations target the loop, not just its symptoms, and can be implemented with observable triggers.

The theory yields testable predictions. Economies with higher foreign collateralizability of housing should exhibit larger cycle amplitudes and a greater likelihood of belief-driven regime switches. Modest increases in foreign rates can precipitate discontinuous transitions when the high-growth path disappears. We view these as opportunities for measurement using cross-country variation in collateral institutions and exposure to global rates.

References

1. Aghion, Philippe, Philippe Bacchetta, and Abhijit Banerjee (2004), "Financial development and the instability of open economies," *Journal of Monetary Economics*, 51(6), 1077–1106.

2. Agosin, Manuel R and Roberto Machado (2005), "Foreign investment in developing countries: does it crowd in domestic investment?," *Oxford Development Studies*, 33(2), 149–162.

3. Aguiar, Mark and Gita Gopinath (2007), "Emerging market business cycles: The cycle is the trend," *Journal of Political Economy*, 115(1), 69–102.

4. Antoci, Angelo, Marcello Galeotti, and Paolo Russu (2011), "Poverty trap and global indeterminacy in a growth model with open-access natural resources," *Journal of Economic Theory*, 146(2), 569–591.

5. Aoki, Kosuke, Gianluca Benigno, Nobuhiro Kiyotaki, Andrew G Haldane, and Şebnem Kalemli-Ozcan (2007), "Capital flows and asset prices," in *NBER International Seminar on Macroeconomics*, Vol. 2007, The University of Chicago Press, Chicago, IL, pp. 175–229.

6. Beaudry, Paul, Dana Galizia, and Franck Portier (2020), "Putting the cycle back into business cycle analysis," *American Economic Review*, 110(1), 1–47.

7. Benhabib, Jess and Roger EA Farmer (1994), "Indeterminacy and increasing returns," *Journal of Economic Theory*, 63(1), 19–41.

8. Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe (2001), "The perils of Taylor rules," *Journal of Economic Theory*, 96(1-2), 40–69.

9. Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe (2003), "Backward-looking interest-rate rules, interest-rate smoothing, and macroeconomic instability," *Journal of Money, Credit & Banking*, 35(6), S1379–S1379.

10. Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R Young (2013), "Financial crises and macro-prudential policies," *Journal of International Economics*, 89(2), 453–470.

11. Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R Young (2016), "Optimal capital controls and real exchange rate policies: A pecuniary externality perspective," *Journal of Monetary Economics*, 84, 147–165.

12. Bianchi, Javier (2011), "Overborrowing and systemic externalities in the business cycle," *American Economic Review*, 101(7), 3400–3426.

13. Bianchi, Javier and Enrique G Mendoza (2018), "Optimal time-consistent macroprudential policy," *Journal of Political Economy*, 126(2), 588–634.

14. Bianchi, Javier and Enrique G Mendoza (2020), "A fisherian approach to financial crises: Lessons from the sudden stops literature," *Review of Economic Dynamics*, 37, S254–S283.

15. Broner, Fernando and Jaume Ventura (2016), "Rethinking the effects of financial globalization," *The Quarterly Journal of Economics*, 131(3), 1497–1542.

16. Caballero, Julián A (2016), "Do surges in international capital inflows influence the likelihood of banking crises?," *The Economic Journal*, 126(591), 281–316.

17. Calvo, Guillermo A, Alejandro Izquierdo, and Luis-Fernando Mejia (2004), "On the empirics of sudden stops: the relevance of balance-sheet effects."

18. Cesa-Bianchi, Ambrogio, Luis Felipe Céspedes, and Alessandro Rebucci (2015), "Global liquidity, house prices, and the macroeconomy: Evidence from advanced and emerging economies," *Journal of Money, Credit and Banking*, 47(S1), 301–335.

19. Chaney, Thomas, David Sraer, and David Thesmar (2012), "The collateral channel: How real estate shocks affect corporate investment," *American Economic Review*, 102(6), 2381–2409.

20. Cui, Wei and Leo Kaas (2021), "Default cycles," *Journal of Monetary Economics*, 117, 377–394.

21. Cvijanović, Dragana (2014), "Real estate prices and firm capital structure," *The Review of Financial Studies*, 27(9), 2690–2735.

22. Edwards, Sebastian (2004), "Financial openness, sudden stops, and current-account reversals," *American Economic Review*, 94(2), 59–64.

23. Gan, Jie (2007a), "Collateral, debt capacity, and corporate investment: Evidence from a natural experiment," *Journal of Financial Economics*, 85(3), 709–734.

24. Gan, Jie (2007b), "The real effects of asset market bubbles: Loan-and firm-level evidence of a lending channel," *The Review of Financial Studies*, 20(6), 1941–1973.

25. Ghosh, Atish R, Jonathan D Ostry, and Mahvash S Qureshi (2016), "When do capital inflow surges end in tears?," *American Economic Review*, 106(5), 581–585.

26. Ghosh, Atish R, Mahvash S Qureshi, Jun Il Kim, and Juan Zalduendo (2014), "Surges," *Journal of International Economics*, 92(2), 266–285.

27. Gu, Chao, Lu Wang, and Randall Wright (2024), "Intermediaries, Inventories and Endogenous Dynamics in Frictional Markets," Available at SSRN 4716108.

28. Guckenheimer, John and Philip Holmes (1983), *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields*, Vol. 42, New York: Springer.

29. Haskel, Jonathan E, Sonia C Pereira, and Matthew J Slaughter (2007), "Does inward foreign direct investment boost the productivity of domestic firms?," *The Review of Economics and Statistics*, 89(3), 482–496.

30. Havránek, Tomáš (2015), "Measuring intertemporal substitution: The importance of method choices and selective reporting," *Journal of the European Economic Association*, 13(6), 1180–1204.

31. Head, Keith, John Ries, and Deborah Swenson (1995), "Agglomeration benefits and location choice: Evidence from Japanese manufacturing investments in the United States," *Journal of International Economics*, 38(3-4), 223–247.

32. IMF (1998), *World Economic Outlook, May 1998: Financial Crises: Causes and Indicators*, International Monetary Fund.

33. Javorcik, Beata Smarzynska (2004), "Does foreign direct investment increase the productivity of domestic firms? In search of spillovers through backward linkages," *American Economic Review*, 94(3), 605–627.

34. Jeanne, Olivier and Anton Korinek (2019), "Managing credit booms and busts: A Pigouvian taxation approach," *Journal of Monetary Economics*, 107, 2–17.

35. Jordà, Òscar, Moritz Schularick, and Alan M Taylor (2017), "Macrofinancial history and the new business cycle facts," *NBER Macroeconomics Annual*, 31(1), 213–263.

36. Kalantzis, Yannick (2015), "Financial fragility in small open economies: firm balance sheets and the sectoral structure," *The Review of Economic Studies*, 82(3), 1194–1222.

37. Kaplan, Greg and Guido Menzio (2016), "Shopping externalities and self-fulfilling unemployment fluctuations," *Journal of Political Economy*, 124(3), 771–825.

38. Kopell, Nancy and LN Howard (1975), "Bifurcations and trajectories joining critical points," *Advances in Mathematics*, 18(3), 306–358.

39. Krugman, Paul (1999), "Balance sheets, the transfer problem, and financial crises," *International Tax and Public Finance*, 6(4), 459–472.

40. Liu, Zheng, Pengfei Wang, and Tao Zha (2013), "Land-price dynamics and macroeconomic fluctuations," *Econometrica*, 81(3), 1147–1184.

41. Martin, Philippe and Helene Rey (2006), "Globalization and emerging markets: With or without crash?," *American Economic Review*, 96(5), 1631–1651.

42. Mendoza, Enrique G (2002), "Credit, prices, and crashes: Business cycles with a sudden stop," in *Preventing currency crises in emerging markets*, University of Chicago Press, pp. 335–392.

43. Mendoza, Enrique G (2010), "Sudden stops, financial crises, and leverage," *American Economic Review*, 100(5), 1941–1966.

44. Mendoza, Enrique G and Katherine A Smith (2006), "Quantitative implications of a debt-deflation theory of sudden stops and asset prices," *Journal of International Economics*, 70(1), 82–114.

45. Miao, Jianjun, Pengfei Wang, and Jing Zhou (2022), "Asset bubbles and foreign interest rate shocks," *Review of Economic Dynamics*, 44, 315–348.

46. Miranda-Agrippino, Silvia and Hélène Rey (2020), "US monetary policy and the global financial cycle," *The Review of Economic Studies*, 87(6), 2754–2776.

47. Peek, Joe and Eric S Rosengren (2000), "Collateral damage: Effects of the Japanese bank crisis on real activity in the United States," *American Economic Review*, 91(1), 30–45.

48. Reinhart, Carmen M and Vincent R Reinhart (2009), "Capital flow bonanzas: An encompassing view of the past and present," in *NBER international seminar on macroeconomics*, Vol. 5, The University of Chicago Press, Chicago, IL, pp. 9–62.

49. Rey, Hélène (2013), "Dilemma not trilemma: the global cycle and monetary policy independence," *Proceedings - Economic Policy Symposium - Jackson Hole*.

50. Rogoff, Kenneth (1999), "International institutions for reducing global financial instability," *Journal of Economic perspectives*, 13(4), 21–42.

51. Schmalz, Martin C, David A Sraer, and David Thesmar (2017), "Housing collateral and entrepreneurship," *The Journal of Finance*, 72(1), 99–132.

52. Schmitt-Grohé, Stephanie and Martín Uribe (2021a), "Deterministic debt cycles in open economies with flow collateral constraints," *Journal of Economic Theory*, 192, 105195.

53. Schmitt-Grohé, Stephanie and Martín Uribe (2021b), "Multiple equilibria in open economies with collateral constraints," *The Review of Economic Studies*, 88(2), 969–1001.

54. Schneider, Martin and Aaron Tornell (2004), "Balance sheet effects, bailout guarantees and financial crises," *The Review of Economic Studies*, 71(3), 883–913.

55. Smets, Frank and Rafael Wouters (2007), "Shocks and frictions in US business cycles: A Bayesian DSGE approach," *American Economic Review*, 97(3), 586–606.

56. Sniekers, Florian (2018), "Persistence and volatility of Beveridge cycles," *International Economic Review*, 59(2), 665–698.

57. Tong, Hui and Shang-Jin Wei (2011), "The composition matters: Capital inflows and liquidity crunch during a global economic crisis," *The Review of Financial Studies*, 24(6), 2023–2052.

Appendix

A Details

A.1 Key Notation

Symbol	Meaning
R^f	World (foreign) interest rate
R^k	Return on capital
θ_h	Pledgeability of housing to lenders
θ	Pledgeability of capital to lenders
Q	Tobin's Q (valuation)
p	Housing price index (scaled)
g	Endogenous growth rate
μ	Arrival rate of investment opportunities (liquidity)
σ	Intertemporal elasticity of substitution inverse
ρ	Subjective discount rate
ψ	Rent-to-consumption share parameter
δ	Depreciation rate

Table A.1: Key notation

A.2 External Debt Components

Country	Direct	Total
Austria	94.22%	73.44%
Brazil	96.90%	77.46%
Bulgaria	98.98%	
Canada	92.79%	60.79%
Chile	99.99%	92.07%
Colombia	99.78%	86.26%
Croatia	99.45%	
Cyprus	88.97%	
El Salvador	98.98%	
Estonia	99.83%	
Finland	90.09%	68.69%
Hungary	98.42%	89.41%
India	100.00%	77.53%
Kazakhstan	98.11%	
Lithuania	99.79%	
Moldova	97.79%	
Netherlands	95.30%	68.85%
North Macedonia	97.50%	
Poland	99.88%	76.59%
Romania	95.37%	
Slovenia	97.49%	
Thailand	99.38%	74.70%

Table A.2: The Ratio of Non-Financial Firms in Private Credit

Note: This table shows the external debt of the non-financial corporate sector as a proportion of all private external debt (non-financial corporations plus households). Data on direct external debt are derived from the QEDS items on external debt of households and non-financial corporations. Total debt is direct external debt plus indirect external debt, where indirect external debt is estimated using the allocation of external debt of the financial sector in proportion to total credit to households and corporations (BIS). This table lists countries meeting the criterion of having positive direct household foreign debt in the QEDS database. The vacancy in the second row reflects the unavailability of corresponding BIS credit data for that particular country.

A.3 Firm-Level Evidence

To complement the country-level analysis and provide microeconomic validation of the liquidity transmission mechanism, we examine firm-level investment behavior using a dataset of Thailand

	I_{jt}/K_{jt}					
	(1)	(2)	(3)	(4)	(5)	(6)
I_{jt-1}/K_{jt-1}	0.050	0.050	0.050	0.031	0.031	0.032
	(0.049)	(0.049)	(0.049)	(0.055)	(0.055)	(0.055)
IP_{jt-1}	-0.687***	-0.638***	-0.192***	-0.140	-0.145	-0.151
	(0.227)	(0.199)	(0.072)	(0.104)	(0.103)	(0.112)
$\text{IP}_{jt-1} \times P_t$	0.005***					
	(0.002)					
$\text{IP}_{jt-1} \times P_{t-1}$		0.005***				
		(0.002)				
$\text{IP}_{jt-1} \times$ (Year>2012)			0.172**			
			(0.084)			
$\text{IP}_{j2010} \times P_t$				0.008**		
				(0.004)		
$\text{IP}_{j2010} \times P_{t-1}$					0.008**	
					(0.004)	
$\text{IP}_{j2010} \times$ (Year>2012)						0.161
						(0.178)
Control Variables	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	11,027	11,027	11,027	7,389	7,389	7,389
R-squared	0.143	0.143	0.143	0.145	0.145	0.144

Table A.3: Firm-level Regression Results

Note: The estimated parameters are provided in the top row, and the standard error (clustered to firm level) is shown in parentheses. The sample of (1)-(3) is from 2000 to 2024 while the sample of (4)-(6) is from 2010 to 2024. *, ** and *** indicate significance at the 10%, 5% and 1% levels.

listed corporations from 2000 to 2024 from Orisis Database. Real house prices in Thailand fluctuated within a range of 5% between 2000 and 2012, and rose by more than 40% from 2012 to 2024. This natural experiment allows us to test the core tenet of our theory: that fluctuations in real estate prices directly impact corporate investment sensitivity by liquidity channel.

We construct an annual firm-level panel and specify the following regression model to test for the presence of liquidity channel:

$$\text{I/K}_{jt} = \beta_1 \text{IP}_{jt-1} + \beta_2 \text{IP}_{jt-1} \times \text{P}_t + \text{Controls}_{jt-1} + u_j + \delta_t + \varepsilon_{jt} \tag{A.1}$$

where the dependent variable $(\text{I/K})_{jt}$ denotes the investment rate of firm j in year t, calculated as the investment scaled by the lagged fixed asset. [9] The key explanatory variable, $(\text{IP ratio})_{jt-1}$, measures the intensity of a firm's collateralizable assets, defined as the ratio of

[9] Investment is measured by the term "additions to fixed assets" in the cash flow statement.

investment property (real estate assets held on the balance sheet) to total assets. The interaction term captures the effect of aggregate housing price on the link between real estate holding and investment. The coefficient of interest, β_2, is predicted to be positive and significant, indicating that rising asset prices strengthen the role of collateral in facilitating investment. The specification incorporates a comprehensive set of firm-level control variables, including leverage ratio, return of assets, cash-to-asset ratio, and price-to-sales ratio, all lagged by one period to mitigate concerns of reverse causality. We further include firm fixed effects and year fixed effects.

As shown in Column (1), the coefficient on the interaction term $\text{IP}_{jt-1} \times P_t$ is positive and statistically significant at the 1% level, supporting the view that firms with greater investment property holdings increase fixed investment more strongly during periods of high real estate prices. To mitigate potential simultaneity concerns, we lag the house price variable by one period in Column (2). The coefficient on $\text{IP}_{jt-1} \times P_{t-1}$ remains positive and significant at the 1% level, confirming the robustness of the liquidity channel. Given that real house prices in Thailand were stable before 2012 and rose sharply thereafter, we replace the continuous price measure with a post-2012 dummy in Column (3). The significantly positive coefficient on $\text{IP}_{jt-1} \times (\text{Year} > 2012)$ further reinforces the baseline result.

To address potential endogeneity in firms leverage the cross-sectional variation in pre-boom exposure. Since IP_{jt-1} is measured at historical cost, it may not accurately reflect contemporary market-value exposure. We therefore use the investment property ratio measured in 2010 (IP_{j2010}), a year preceding the housing boom and characterized by relatively flat prices, which helps ensure comparability in real estate valuation across firms. Restricting the sample to the period from 2010 onward, we find qualitatively similar results in Columns (4) to (6): the coefficients on $\text{IP}_{j2010} \times P_t$ and $\text{IP}_{j2010} \times P_{t-1}$ are positive and significant at the 5% level, while the coefficient on $\text{IP}_{j2010} \times (\text{Year} > 2012)$ remains positive though not significant, likely due to reduced sample size.

A.4 Cyclical Components

Beaudry et al. (2020) identify cyclical components in economic activity through spectral analysis of per capita labor hours data. Building on their methodological framework, we extend the spectral analysis approach to capital flows. Our spectral estimation procedure follows Beaudry et al. (2020) with two technical specifications: (1) raw spectral density estimation and (2) filtered series analysis. We use a fast Fourier transform algorithm with $N = 1024$ frequency points to compute the discrete Fourier transform (DFT) and spectral density estimates are obtained by applying a Hamming smoothing kernel with window length $W = 5$.

The black solid line in Figure A.1 displays the baseline spectral density estimate for periodic components between 2-14 years. To address low-frequency trends, we implement a high-pass filter that removes fluctuations with periods exceeding P years, where $P \in [25, 50]$. The filtered spectral densities (gray lines) demonstrate remarkable consistency with the baseline estimate. Estimations of Thailand and Spain reveal a spectral peak at 8-10 year frequencies while estimations of Malaysia and Mexico reveal a spectral peak at 10-12 year frequencies. This persistent cyclical pattern provides empirical evidence for the existence of endogenous cyclical mechanisms in capital inflows.

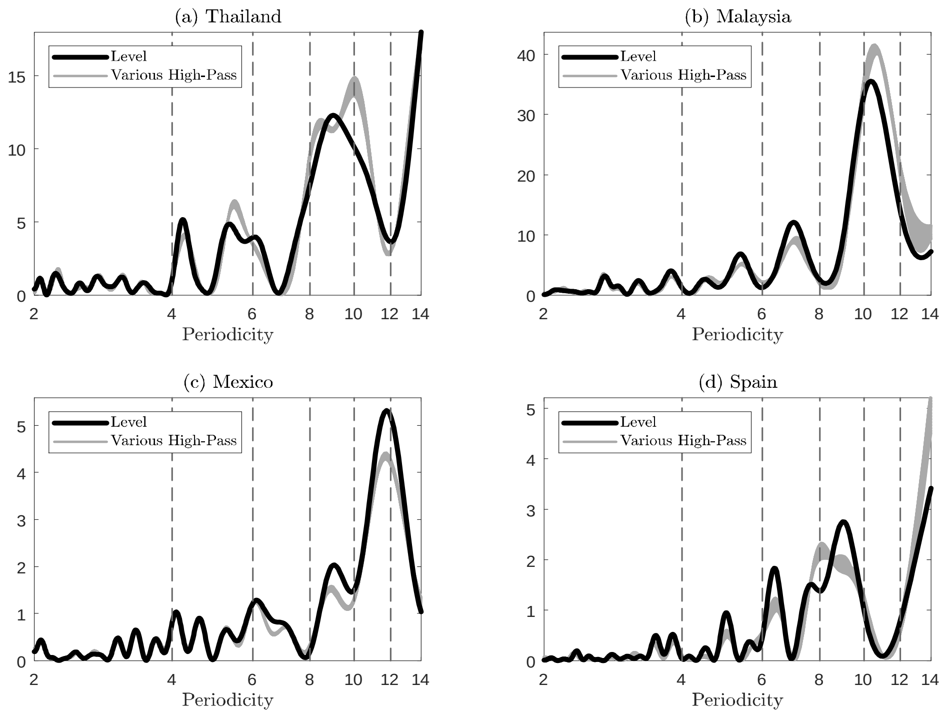

Figure A.1: Cyclical Components

Note:The black line: a fast Fourier transform algorithm with $N = 1024$ frequency points to compute the discrete Fourier transform (DFT) and applying a Hamming smoothing kernel with window length $W = 5$. The dashed shade uses data with a high-pass filter that removes fluctuations with periods exceeding [25,50] years.

B Proof

B.1 Proof of Proposition 1

We first give the decisions and pricing equations in the discrete-time form and then let $\Delta \to 0$. If an entrepreneur does not have investment opportunity, she does not invest $I_{jt} = 0$ and has no difference between $D_{jt}\Delta$, $H_{jt+\Delta}$, and $L_{jt+\Delta}$. Otherwise there will be no supplier of housing assets or the credit. For an entrepreneur who has investment opportunity, she borrows as much as she can and hold no housing asset, $H_{jt+\Delta} = 0$. The reason is that $Q_t > 1$ and she will try her best to invest. She chooses

$$\frac{L_{jt+\Delta}}{1+R_t\Delta} = \theta K_{jt},$$

$$I_{jt} = R^k K_{jt}\Delta + M_t H_{jt}\Delta + P_t H_{jt} + \theta K_{jt} - L_{jt}.$$

If we place these equations into value functions,

$$V_{jt}(K_{jt}, H_{jt}, L_{jt}) = D_{jt}\Delta + \exp(-\rho\Delta)E_t\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\sigma} V_{jt+\Delta}(K_{jt+\Delta}, H_{jt+\Delta}, L_{jt+\Delta})$$

$$= D_{jt}\Delta + Q_t K_{jt+\Delta} + P_t H_{jt+\Delta} - \frac{L_{jt+\Delta}}{1+R_t\Delta}.$$

we have pricing equations of the housing price, marginal cost of credit, and Tobin's Q follow

$$P_t = \exp(-\rho\Delta)E_t\left(\tfrac{C_{t+\Delta}}{C_t}\right)^{-\sigma}\left[1+\mu\Delta(Q_{t+\Delta}-1)\right]\left(P_{t+\Delta}+M_{t+\Delta}\Delta\right),$$

$$\frac{1}{1+R_t\Delta} = \exp(-\rho\Delta)E_t\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\sigma}\left[1+\mu\Delta(Q_{t+\Delta}-1)\right],$$

$$Q_t = \exp(-\rho\Delta)E_t\left(\tfrac{C_{t+\Delta}}{C_t}\right)^{-\sigma}\left[R^k\Delta + Q_{t+\Delta}(1-\delta\Delta) + (R^k\Delta+\theta)\mu\Delta(Q_{t+\Delta}-1)\right].$$

From the perspective of aggregation, the total investment in the economy is the sum of the investments of all entrepreneurs with investment opportunities. The total investment is

$$I_t\Delta = \mu\Delta(R^k K_t\Delta + M_t H_t\Delta + P_t H_t + \theta K_t).$$

When $\Delta \to 0$, we have continuous-time form. The above equations become

$$\rho + \sigma\frac{\dot{C}_t}{C_t} = \frac{\dot{P}_t + M_t}{P_t} + \mu(Q_t - 1).$$

$$\left(\rho + \sigma\frac{\dot{C}_t}{C_t}\right) Q_t = \dot{Q}_t + R^k - \delta Q_t + \mu\theta(Q_t - 1).$$

$$I_t = \mu(P_t H_t + \theta K_t).$$

and

$$C_t = R^k - I_t.$$

B.2 Proof of Theorem 1

In the steady state, q and g should satisfy the following functions

$$\rho + (\sigma - 1)g = \mu\psi\left(\frac{R^k - \mu\theta}{g + \delta - \mu\theta} - 1\right) + \mu(Q - 1),$$

$$Q = \frac{R^k - \mu\theta}{\rho + \delta - \mu\theta + \sigma g}.$$

or

$$q = Q_H(g) = \frac{1}{\mu}\left[\rho + \mu + (\sigma - 1)g - \psi\mu\left(\frac{R^k - \mu\theta}{g + \delta - \mu\theta} - 1\right)\right],$$

$$q = Q_K(g) = \frac{R^k - \mu\theta}{\rho + \delta - \mu\theta + \sigma g}.$$

Given $p = \frac{1}{\mu}(g + \delta) - \theta > 0$, we have $g + \delta - \mu\theta > 0$. Meanwhile, as $c = R^k - (g + \delta) > 0$, $R^k - \mu\theta > R^k - (g+\delta) > 0$. Therefore, $Q'_H(g) > 0$. Meanwhile, as $\rho + \sigma g - g = \psi c/p + \mu(Q-1) > 0$, we have $\rho + \delta - \mu\theta + \sigma g = g + \delta - \mu\theta + \rho + (\sigma - 1)g > 0$. Therefore, $Q'_K(g) < 0$.

If $\sigma \geq 1$, we have $Q'_H(g) > 0$ while $Q'_K(g) < 0$. The intersection of these two functions is unique.

B.3 Proof of Theorem 2

After FDI openness, the system is the same as the closed economy. The balanced growth path still follows

$$q = Q_H(g) = \frac{1}{\mu}\left[\rho + \mu + (\sigma - 1)g - \psi\mu\left(\frac{R^k - \mu\theta}{g + \delta - \mu\theta} - 1\right)\right],$$

$$q = Q_K(g) = \frac{R^k - \mu\theta}{\rho + \delta - \mu\theta + \sigma g}.$$

where R^k increases when Ω decreases. In the proof of Theorem 1, we have shown that balanced growth path is unique. Meanwhile, when $p > 0$ and $Q > 1$, we have $Q'_H(g) > 0$ and $Q'_K(g) < 0$.

When R^k increases, $Q_H(g)$ decreases while $Q_K(g)$ increases. The intersection g will also increase.

B.4 Proof of Proposition 3

Solving the Decisions If the borrowing constraint (16) binds for all entrepreneurs, then the problem becomes

$$
\begin{aligned}
&\max_{D_{jt},I_{jt},H_{jt+\Delta}} \\
&D_{jt}\Delta + Q_t(1-\delta\Delta)K_{jt} + Q_t\varepsilon_{jt}I_{jt} + Q_{ht}H_{jt+\Delta} - Q_{lt}(1+R^f\Delta)[\theta K_{jt} + \theta_h P_t H_{jt+\Delta}] \\
&+\lambda_{1jt}\left\{R_t^k K_{jt}\Delta + M_t H_{jt}\Delta + P_t H_{jt} - P_t H_{jt+\Delta} + \theta K_{jt} + \theta_h P_t H_{jt+\Delta} - L_{jt} - I_{jt} - D_{jt}\Delta\right\} \\
&+\lambda_{2jt}I_{jt} + \lambda_{3jt}H_{jt+\Delta} + \lambda_{4jt}D_{jt}\Delta
\end{aligned}
$$

where $\varepsilon_{jt} = 1$ denotes that the entrepreneur can invest while $\varepsilon_{jt} = 0$ means the entrepreneur cannot. The FOCs: $D_{jt}\Delta$ is

$$1 - \lambda_{1jt} + \lambda_{4jt} = 0,$$

$I_{jt}\Delta$ is

$$Q_t\varepsilon_{jt} - \lambda_{1jt} + \lambda_{2jt} = 0,$$

$H_{jt+\Delta}$ is

$$Q_{ht} - Q_{lt}(1+R^f\Delta)\theta_h P_t - \lambda_{1jt}(1-\theta_h)P_t + \lambda_{3jt} = 0.$$

1. $\varepsilon_{jt} = 0$: $Q_t\varepsilon_{jt} < 1$, $\lambda_{1jt} = 1$, $\lambda_{2jt} > 0$. Meanwhile, $\lambda_{3jt} = 0$, $\lambda_{4jt} = 0$. Otherwise, there will be no dividend, or asset holding for all time, the market cannot clear. There is no difference between $D_{jt}\Delta$ or $H_{jt+\Delta}$. We have the first order condition for $H_{jt+\Delta}$

$$Q_{ht} - Q_{lt}(1+R^f\Delta)\theta_h P_t - (1-\theta_h)P_t = 0, \tag{B.1}$$

Then the value function becomes

$$
\begin{aligned}
V_{jt}(K_{jt},H_{jt},L_{jt}) \ &= R^k K_{jt}\Delta + M_t H_{jt}\Delta + P_t H_{jt} - L_{jt} + \theta K_{jt} \\
&+Q_t(1-\delta\Delta)K_{jt} - Q_{lt}(1+R^f\Delta)\theta K_{jt}.
\end{aligned}
$$

2. $\varepsilon_{jt} = 1$: $Q_t\varepsilon_{jt} > 1$, $\lambda_{1jt} = Q_t\varepsilon_{jt}$, $\lambda_{2jt} = 0$, $\lambda_{4jt} = Q_t - 1 > 0$, $D_{jt}\Delta = 0$. Meanwhile,

$$Q_{ht} - Q_{lt}(1+R^f\Delta)\theta_h P_t - Q_t(1-\theta_h)P_t + \lambda_{3jt} = 0$$

or the equation is

$$Q_{ht} - Q_{lt}(1+R^f\Delta)\theta_h P_t - (1-\theta_h)P_t + \lambda_{3jt} = (Q_t - 1)(1-\theta_h)P_t$$

As $Q_{ht} - Q_{lt}(1+R^f\Delta)\theta_h P_t - (1-\theta_h)P_t = 0$ in (B.1) and $\theta_h < 1$, $\lambda_{3jt} = (Q_t-1)(1-\theta_h)P_t > 0$. Therefore, $H_{jt+\Delta} = 0$. We have that

$$I_{jt} = R^k K_{jt}\Delta + M_t H_{jt}\Delta + P_t H_{jt} + \theta K_{jt} - L_{jt}$$

The intuition is that they will not hold housing assets, as selling one unit housing asset can finance P_t investment while borrowing credit with one unit housing asset as collateral can only finance $\theta_h P_t$ liquidity. We have

$$\begin{aligned}&V_{jt}(K_{jt}, H_{jt}, L_{jt}) = Q_t\left[(1-\delta\Delta)K_{jt} + (R^k K_{jt}\Delta + M_t H_{jt}\Delta + P_t H_{jt} + \theta K_{jt} - L_{jt})\right]\\ &-Q_{lt}(1+R^f\Delta)\theta K_{jt}\end{aligned}$$

Aggregate the probability Combining the value equations of the two types of firms, we can calculate that

$$Q_{ht} = \exp(-\rho\Delta)E_t\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\sigma}\left[P_{t+\Delta} + M_{t+\Delta}\Delta\right]\left[1+\mu\Delta(Q_{t+\Delta}-1)\right],$$

$$Q_{lt} = \exp(-\rho\Delta)E_t\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\sigma}\left[1+\mu\Delta(Q_{t+\Delta}-1)\right].$$

Then we have the pricing equations of the housing price, and Tobin's Q.

$$(1-\theta_h)P_t = \exp(-\rho\Delta)E_t\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\sigma}\left[1+\mu\Delta(Q_{t+\Delta}-1)\right]\left[P_{t+\Delta} + M_{t+\Delta}\Delta - (1+R^f\Delta)\theta_h P_t\right],$$

$$\begin{aligned}Q_t = \exp(-\rho\Delta)E_t\left(\tfrac{C_{t+\Delta}}{C_t}\right)^{-\sigma}&\left[R^k\Delta + Q_{t+\Delta}(1-\delta\Delta) + (1-Q_{lt+\Delta}(1+R^f\Delta))\theta\right.\\ &\left.+(R^k\Delta+\theta)\mu\Delta(Q_{t+\Delta}-1)\right].\end{aligned}$$

When $\Delta \to 0$, the pricing equation of P_t becomes

$$\left(\rho + \sigma\frac{\dot{C}_t}{C_t}\right)(1-\theta_h)P_t = \dot{P}_t + M_t - \theta_h P_t R^f + \mu(Q_t-1)(1-\theta_h)P_t,$$

Note that $Q_{lt}-1$ is the First-order term of Δ, let

$$\frac{1}{1+\tilde{R}_t\Delta} = Q_{lt} = \exp(-\rho\Delta)E_t\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\sigma}\left[1+\mu\Delta(Q_{t+\Delta}-1)\right].$$

We have

$$\rho + \sigma\frac{\dot{C}_t}{C_t} = R_t + \mu(Q_t-1),$$

and

$$\left(\rho + \sigma\frac{\dot{C}_t}{C_t}\right)Q_t = \dot{Q}_t + R^k - \delta Q_t + (R_t - R^f)\theta + \theta\mu(Q_t-1),$$

Combining the above two equations, we have

$$\left(\rho + \sigma\frac{\dot{C}_t}{C_t}\right)(Q_t-\theta) = \dot{Q}_t + R^k - \delta Q_t - R^f\theta.$$

B.5 Proof of Theorem 3

To prove this theorem, we first consider the case that both balanced growth paths with $R_t^H > R^f$. In this case, the following two curves should have two intersections

$$Q_H(g) = \frac{1}{\mu}\left[\rho + \mu - \frac{\psi\mu[R^k - (g+\delta) + \theta(g - R^f)]}{g+\delta} - \left(\frac{1+\psi\theta_h}{1-\theta_h} - \sigma\right) g + \frac{(1+\psi)\theta_h}{1-\theta_h} R^f\right],$$

$$= \frac{1}{\mu}\left[\rho - \frac{\psi\mu[R^k - \theta(\delta + R^f)]}{g+\delta} - \left(\frac{1+\psi\theta_h}{1-\theta_h} - \sigma\right) g + \frac{(1+\psi)\theta_h}{1-\theta_h} R^f\right] + 1 + \psi(1-\theta),$$

$$Q_K(g) = \frac{R^k + (\rho + \sigma g - R^f)\theta}{\rho + \delta + \sigma g} = \frac{R^k - \theta(\delta + R^f)}{\rho + \delta + \sigma g} + \theta.$$

Note that the feasible balanced growth paths also require the liquidity constraint binding condition $Q > 1$, and the foreign borrowing constraint binding condition $g + \psi c/p > R^f$.

To make sure the question well defined, we restrict $g + \delta > 0$ so that the investment is larger than zero. As we assume that $R^k - \delta > R^f$, $\rho + \delta + \sigma g$ should be larger than 0 so that $Q_K(g) > 1$. Meanwhile, $\rho + \sigma g > R^f$ so that the stochastic discount factor should be larger than foreign interest rate. That is, $g > g_0 = \max\{(R^f - \rho)/\sigma, -\delta, -(\rho+\delta)/\sigma\}$.

As we assume that $R^k - \delta > R^f$ and $(1+\psi\theta_h)/(1-\theta_h) > \sigma$, $Q_H(g)$ is an inverted-U function with $Q_H''(g) < 0$. $Q_K(g)$ is a decreasing function with $Q_K'(g) < 0$ and $Q_K''(g) > 0$. The system has at most two intersections.

Combining $Q_H(g)$ and $Q_K(g)$, we have

$$Q - \theta = X_1(g) = \frac{\frac{1}{\mu}\left[\rho + \mu(1+\psi)(1-\theta) - \left(\frac{1+\psi\theta_h}{1-\theta_h} - \sigma\right) g + \frac{(1+\psi)\theta_h}{1-\theta_h} R^f\right]}{1 + \psi\left[\frac{\rho + (1-\sigma)\delta}{g+\delta} + \sigma\right]},$$

Meanwhile, $Q_K(g)$ can be written as

$$Q - \theta = X_2(g) = \frac{R^k - \theta(\delta + R^f)}{\rho + \delta + \sigma g}.$$

The intersections of $Q_H(g)$ and $Q_K(g)$ should also be the intersections of $X_1(g)$ and $X_2(g)$. The change of R^k only influence $X_2(g)$ but not $X_1(g)$.

Suppose $(g, Q - \theta)$ is the intersection of $X_1(g)$ and $X_2(g)$. We want $g + \psi c/p > R^f$, that is

$$g + \frac{\psi(R^k - (g+\delta) + (\theta + \theta_h p)(g - R^f))}{p} > R^f,$$

which can be transferred as

$$g + \frac{\mu\psi(1-\theta_h)(R^k - (g+\delta) + \theta(g - R^f))}{g+\delta} + \psi\theta_h(g - R^f) > R^f,$$

$$g+\frac{\mu\psi(1-\theta_h)(R^k-(g+\delta)+\theta(g-R^f))}{g+\delta}+\psi\theta_h(g-R^f)>R^f,$$

$$\frac{\mu\psi[R^k-\theta(R^f+\delta)]}{g+\delta}-\mu\psi(1-\theta)+\frac{(1+\psi\theta_h)(g-R^f)}{1-\theta_h}>0,$$

which can be written as

$$\mu[Q_H(g)-1]-\rho+R^f-\sigma g<0,$$

as $Q_H(g)-\theta=X_1(g)$ on the intersection, we only require

$$X_1(g)<\frac{\rho+\sigma g-R^f}{\mu}+1-\theta,$$

If we want $Q>1$, we require that

$$X_1(g)>1-\theta.$$

When $g\in[\psi[(\sigma-1)\delta-\rho]/(1+\psi\sigma)-\delta,+\infty]$, the function $X_1(g)$ is a decreasing function with range $(-\infty,+\infty)$ if $(\sigma-1)\delta-\rho>0$. Otherwise, $X_1(g)$ is an inverted-U function if $(\sigma-1)\delta-\rho<0$.

Case 1 : $(\sigma-1)\delta-\rho>0$. As $X_1(g)$ is a decreasing function with range $(-\infty,+\infty)$, we define g_1 as the unique root of

$$X_1(g)=\frac{\rho+\sigma g-R^f}{\mu}+1-\theta,$$

when $g\in[\psi[(\sigma-1)\delta-\rho]/(1+\psi\delta)-\delta,+\infty]$. We define g_2 as the unique root of

$$X_1(g)=1-\theta,$$

when $g\in[\psi[(\sigma-1)\delta-\rho]/(1+\psi\delta)-\delta,+\infty]$.

Therefore, if the system has two constrained balanced growth paths, it means $X_2(g)$ and $X_1(g)$ has two intersections between $(g_1,g_2)\cap(g_0,+\infty)$. The sufficient condition is $X_2(g)>X_1(g)$ when $g=g_0$, $g=g_1$ and $g=g_2$, but there exists a $g\in(\max\{g_0,g_1\},g_2)$ such that $X_2(g)<X_1(g)$.

Define function

$$\Gamma(g)=X_1(g)(\rho+\delta+\sigma g)+\theta(\delta+R^f).$$

when $R^k=\Gamma(g)$, g is a solution of $X_1(g)=X_2(g)$.

Define $\underline{R}^k=\max\{\Gamma(g_0),\Gamma(g_2)\}$, $\tilde{R}^k=\Gamma(g_1)$, and

$$\overline{R}^k=\max_{(g_1,g_2)\cap(g_0,+\infty)}\Gamma(g)$$

Therefore, if $\max\{\underline{R}^k,\tilde{R}^k\}<R^k<\overline{R}^k$, the system has two balanced growth paths with $R_t^H>$

R^f.

If the system has one BGP with $R_t^H > R^f$ and one BGP with with $R_t^H = R^f$, it means $X_2(g)$ and $X_1(g)$ has one intersection at (g_0, g_1) and (g_1, g_2), respectively. We require $\underline{R}^k < R^k < \tilde{R}^k$.

Case 2 : $(\sigma - 1)\delta - \rho < 0$. As $X_1(g)$ is an inverted-U function, we define g_{11} and g_{12} ($g_{11} < g_{12}$) as the roots of

$$X_1(g) = \frac{\rho + \sigma g - R^f}{\mu} + 1 - \theta,$$

when $g \in [\psi[(\sigma - 1)\delta - \rho]/(1 + \psi\sigma) - \delta, +\infty]$. We define g_{21} and g_{22} ($g_{21} < g_{22}$) as the roots of

$$X_1(g) = 1 - \theta,$$

when $g \in [\psi[(\sigma-1)\delta-\rho]/(1+\psi\delta)-\delta, +\infty]$. Given we restrict $g > g_0$, we have $(\rho+\sigma g-R^f)/\mu > 0$. Therefore, if $g_{12} > g_0$ and $g_{22} > g_0$, $X_1(g_{12}) > X_1(g_{21}) = X_1(g_{22}) = 1-\theta$ and $g_{21} < g_{12} < g_{22}$.

Therefore, if the system has two constrained balanced growth paths, it means $X_2(g)$ and $X_1(g)$ has two intersections between $(g_{12}, g_{22}) \cap (g_0, +\infty)$. The sufficient condition is $X_2(g) > X_1(g)$ when $g = g_0$, $g = g_{11}$,$g = g_{12}$, $g = g_{21}$, and $g = g_{22}$, but there exists a $g \in (\max\{g_{12}, g_0\}, g_{22})$ such that $X_2(g) < X_1(g)$.

Define function

$$\Gamma(g) = X_1(g)(\rho + \delta + \sigma g) + \theta(\delta + R^f).$$

when $R^k = \Gamma(g)$, g is a solution of $X_1(g) = X_2(g)$.

Define $\underline{R}^k = \max\{\Gamma(g_0), \Gamma(g_{21}), \Gamma(g_{22}), \Gamma(g_{11})\}$, $\tilde{R}^k = \Gamma(g_{12})$, and

$$\overline{R}^k = \max_{(g_{12},g_{22})\cap(g_0,+\infty)} \Gamma(g)$$

Therefore, if $\max\{\underline{R}^k, \tilde{R}^k\} < R^k < \overline{R}^k$, the system has two constrained balanced growth paths.

If the system has one BGP with $R_t^H > R^f$ and one BGP with $R_t^H = R^f$, it means $X_2(g)$ and $X_1(g)$ has one intersection at $(\max\{g_0, g_{11}, g_{21}\}, g_{12})$ and (g_{12}, g_{22}), respectively. We require $\underline{R}^k < R^k < \tilde{R}^k$.

Proposition B.1 *If $R^k > \overline{R}^k$, there exists a unique balanced growth path and $Q = 1$.*

If $Q = 1$, the dynamic system becomes:

$$\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t = \frac{1}{1-\theta_h}\left(\frac{\dot{p}_t + \psi c_t}{p_t} + g_t\right) - \frac{\theta_h}{1-\theta_h}R^f,$$

$$\left(\rho + \sigma\frac{\dot{c}_t}{c_t} + \sigma g_t\right)(1-\theta) = R^k - \delta - R^f\theta,$$

$$c_t = R^k - (g_t + \delta) + (\theta + \theta_h p_t)(g_t - R^f) + \theta_h \dot{p}_t,$$

where g_t should satisfy

$$g_t \leq \mu(1-\theta_h)p_t - \delta.$$

The steady state requires

$$\rho + \sigma g = \frac{1}{1-\theta_h}\left(\frac{\psi c}{p} + g\right) - \frac{\theta_h}{1-\theta_h}R^f,$$

$$(\rho + \sigma g)(1-\theta) = R^k - \delta - R^f\theta,$$

$$c_t = R^k - (g+\delta) + (\theta + \theta_h p)(g - R^f),$$

where g_t should satisfy

$$g \leq \mu(1-\theta_h)p - \delta.$$

Combining the above three equations, we have

$$\rho = \frac{\psi(1-\theta)[\rho + (\sigma-1)g]}{(1-\theta_h)p} + \left(\frac{1+\psi\theta_h}{1-\theta_h} - \sigma\right)g - \frac{(1+\psi)\theta_h}{1-\theta_h}R^f,$$

As $p \geq \frac{g+\delta}{\mu(1-\theta_h)}$, we have

$$\rho \leq \frac{\mu\psi(1-\theta)[\rho + (\sigma-1)g]}{g+\delta} + \left(\frac{1+\psi\theta_h}{1-\theta_h} - \sigma\right)g - \frac{(1+\psi)\theta_h}{1-\theta_h}R^f,$$

or

$$\rho + \mu(1+\psi)(1-\theta) - \left(\frac{1+\psi\theta_h}{1-\theta_h} - \sigma\right)g + \frac{(1+\psi)\theta_h}{1-\theta_h}R^f \leq \frac{\mu\psi(1-\theta)[\rho + (\sigma-1)g]}{g+\delta} + \mu(1+\psi)(1-\theta),$$

or

$$1-\theta \geq X_1(g) = \frac{\frac{1}{\mu}\left[\rho + \mu(1+\psi)(1-\theta) - \left(\frac{1+\psi\theta_h}{1-\theta_h} - \sigma\right)g + \frac{(1+\psi)\theta_h}{1-\theta_h}R^f\right]}{1 + \psi\left[\frac{\rho + (1-\sigma)\delta}{g+\delta} + \sigma\right]},$$

From the proof of Theorem 3, we know that the above inequality should satisfy $g > g_2$ in case 1 or $g > g_{22}$ in case 2. Given g can be determined by R^k, we have

$$R^k > (\rho + \sigma g_2)(1-\theta) + \delta + R^f\theta = X_1(g_2)(\rho + \delta + \sigma g_2) + \theta(\delta + R^f) = \Gamma(g_2),$$

in case 1 or $R^k > \Gamma(g_{22})$ in case 2.

When $R^k > \overline{R}^k$, we have no balanced growth paths for $Q < 1$. Meanwhile, we have $R^k > \overline{R}^k \geq \Gamma(g_{22})$ or $\Gamma(g_2)$. There exists a unique balanced growth path that $Q = 1$.

B.6 Proof of Proposition 6

In the steady state,

$$\rho = \frac{1}{1-\theta_h}\frac{\psi c}{p} - \frac{\theta_h}{1-\theta_h}R^f + \mu(Q-1),$$

$$\rho(Q-\theta) = \alpha Z k^{\alpha-1} - \delta Q - R^f\theta,$$

$$c = Zk^\alpha - i - R^f(\theta k + \theta_h p),$$

$$i = \mu(p+\theta k),$$

and

$$i = \delta k.$$

Combining the above equations, we have

$$\rho = \frac{\psi}{1-\theta_h}\left(\frac{Zk^{\alpha-1} - \delta - R^f\theta}{\delta/\mu - \theta} - R^f\theta_h\right) - \frac{\theta_h}{1-\theta_h}R^f + \mu(Q-1),$$

and

$$Q = (\alpha Z k^{\alpha-1} + \rho\theta - R^f\theta)/(\rho+\delta),$$

The first equation shows that Q is increasing with k while the second shows that Q is decreasing with k. Therefore, the system has at most one intersection.

B.7 Proof of Proposition 8

In the steady state, we have

$$\rho + \sigma g = R^f + \mu(Q-1),$$

$$(\rho+\sigma g)Q = R^k - \delta Q + \mu\theta(Q-1),$$

Combining the above two equations, we have

$$R^f + \mu(Q-1) + \delta - \mu\theta = \frac{R^k - \mu\theta}{Q}.$$

If $R^k - \mu\theta > 0$, the steady state is unique as the left hand side is an increasing function while the right hand side is a decreasing function for $Q > 1$.

If $R^k - \mu\theta < 0$, the right hand side is an increasing concave function of Q while the left hand side is an increasing linear function. A necessary condition for multiple roots $Q > 1$ is that the derivative of the left hand side is smaller than the derivative of the right hand side when $Q = 1$, which means that $\mu\theta - R^k > \mu$, or $R^k < \mu(\theta - 1)$. However, $R^k > 0$ while

$\mu(\theta - 1) < 0$. Therefore, this condition cannot hold and the system will not have two steady states for $Q > 1$.

We can also show that

Proposition B.2 *If there is no housing asset ($H_t = 0$) and the borrowing constraint becomes*

$$\frac{L_{jt+\Delta}}{1 + R^f \Delta} \leq \theta Q_t K_{jt},$$

the small open economy still has at most one balanced growth path.

When the borrowing constraint changes, the steady state conditions become

$$\rho + \sigma g = R^f + \mu(Q - 1),$$

$$(\rho + \sigma g)Q = R^k - \delta Q + \mu \theta Q(Q - 1),$$

Combining the above two equations, we have

$$R^f + \delta + \mu(1 - \theta)(Q - 1) = R^k / Q.$$

We can only have at most one steady state given $Q > 1$.

B.8 Proof of Proposition 9

We first consider the case that $\theta = 0$. In this case, $Q_K(g)$ can always be expressed as

$$Q_K(g) = \frac{R^k}{\rho + \delta + \sigma g}.$$

Meanwhile, in the closed economy,

$$Q_{1c}(g) = \frac{1}{\mu}\left[\rho + \mu - \frac{\psi\mu[R^k - (g + \delta)]}{g + \delta} + (\sigma - 1)g\right],$$

In the case of $R_t^H > R^f$,

$$Q_{1cstr}(g) = \frac{1}{\mu}\left[\rho + \mu - \frac{\psi\mu[R^k - (g + \delta)]}{g + \delta} + (\sigma - 1)g - \frac{(1 + \psi)\theta_h}{1 - \theta_h}(g - R^f)\right],$$

In the case of $R_t^H = R^f$,

$$Q_{1uncs}(g) = \frac{1}{\mu}(\rho + \mu + \sigma g - R^f),$$

When $\theta_h = 0$, $Q'_H(g) > 0$ while $Q'_K(g) < 0$. $Q_{1c}(g_c) = Q_K(g_c)$.

We first consider the case of $R_t^H = R^f$. As $g_c > R^f$, and $R^k > g_c + \delta$, we have $Q_{1uncs}(g_c) > Q_{1c}(g_c) = Q_K(g_c)$. As $Q_{1uncs}(g)$ is an increasing function, the intersection of $Q_{1uncs}(g) = Q_2(g)$ is smaller than g_c.

When θ_h rises from 0 to positive, $Q_H(g)$ decreases, the new intersections should be larger than g_c.

Now we discuss the case of $R_t^H > R^f$. Note that when $g = R^f$, $Q_{1uncs}(R^f) = Q_{1c}(R^f) < Q_K(g_c)$. Note that $Q_K''(g) > 0$ while $Q_{1uncs}''(g) < 0$. Therefore, if $Q'^{f}_{1uncs}) > Q'^{f}_{K})$, then the growth rate in the steady state must be larger than g_{close}. If $Q'^{f}_{1uncs}) < Q'^{f}_{K})$, then the growth rate in the steady state must be smaller than g_c. Note that

$$Q'^{f}_{K}) = -\frac{R^k \sigma}{(\rho + \delta + \sigma g)^2}$$

and

$$Q'_{1cstr}(g) = \frac{\psi R^k}{(g+\delta)^2} + \frac{\sigma - 1}{\mu} - \frac{(1+\psi)\theta_h}{\mu(1-\theta_h)}.$$

$Q'^{f}_{1cstr}) > Q'^{f}_{K})$ requires that $\theta_h < \theta_h^*$, where θ_h^* is the solution of

$$\frac{\psi R^k}{(R^f+\delta)^2} + \frac{\sigma - 1}{\mu} + \frac{R^k \sigma}{(\rho + \delta + \sigma R^f)^2} = \frac{(1+\psi)\theta_h}{\mu(1-\theta_h)}.$$

B.9 Proof of Proposition 10

Case of $R_t^H > R^f$ The system can be rearranged as

$$\frac{\sigma}{c_t}\dot{c}_t - \frac{1}{(1-\theta_h)p_t}\dot{p}_t = \frac{1}{1-\theta_h}(g_t + \frac{\psi c_t}{p_t}) - \frac{\theta_h}{1-\theta_h}R^f + \mu(Q_t - 1) - \rho - \sigma g_t$$

$$\frac{\sigma(Q_t - \theta)}{c_t}\dot{c}_t - \dot{Q}_t = Z - \delta Q_t - R_f\theta - (\rho + \sigma g_t)(Q_t - \theta)$$

$$\dot{p}_t = 1/\theta_h(c_t + g_t + \delta - Z + R_f\theta + R_f\theta_h p_t - \theta g_t) - g_t p_t$$

where

$$g_t = \mu(1-\theta_h)p_t - \delta$$

$$\mathcal{J} = \begin{bmatrix} \frac{\sigma}{c} & 0 & -\frac{1}{(1-\theta_h)p} \\ \frac{\sigma(Q-\theta)}{c} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{\psi}{(1-\theta_h)p} & \mu & \mu - \sigma\mu(1-\theta_h) - \frac{\psi c}{(1-\theta_h)p^2} \\ 0 & -\delta - (\rho + \sigma g) & -\sigma\mu(Q-\theta)(1-\theta_h) \\ \frac{1}{\theta_h} & 0 & \left(\frac{1-\theta}{\theta_h} - p\right)\mu(1-\theta_h) + R_f - g \end{bmatrix}$$

The eigenvalues should be the solutions to the

$$\begin{bmatrix} \frac{\psi}{(1-\theta_h)p} - \frac{\sigma}{c}\lambda & \mu & \mu - \sigma\mu(1-\theta_h) - \frac{\psi c}{(1-\theta_h)p^2} + \frac{1}{(1-\theta_h)p}\lambda \\ -\frac{\sigma(Q-\theta)}{c}\lambda & -\delta-(\rho+\sigma g)+\lambda & -\sigma\mu(Q-\theta)(1-\theta_h) \\ \frac{1}{\theta_h} & 0 & \left(\frac{1-\theta}{\theta_h}-p\right)\mu(1-\theta_h)+R_f-g-\lambda \\ & & \end{bmatrix}$$

or

$$\left(\frac{\psi}{(1-\theta_h)p} - \frac{\sigma}{c}\lambda\right)[\mu\theta-\delta-(\rho+\sigma g)+\lambda]\left[\left(\frac{1-\theta}{\theta_h}-p\right)\mu(1-\theta_h)+R_f-g-\lambda\right]$$

$$-\sigma\mu^2(Q-\theta)(1-\theta_h)/\theta_h = -\frac{\sigma(Q-\theta)}{c}\lambda\mu\left[\left(\frac{1-\theta}{\theta_h}-p\right)\mu(1-\theta_h)+R_f-g-\lambda\right]$$

$$+\left[\mu-\sigma\mu(1-\theta_h)-\frac{\psi c}{(1-\theta_h)p^2}+\frac{1}{(1-\theta_h)p}\lambda\right][\lambda-\delta-(\rho+\sigma g)]/\theta_h$$

or

$$\left[-\frac{\sigma}{c}\lambda^2+\left(\frac{\psi}{(1-\theta_h)p}+\frac{\sigma}{c}(\delta+\rho+\sigma g)+\frac{\sigma(Q-\theta)}{c}\mu\right)\lambda-\frac{\psi(\delta+\rho+\sigma g)}{(1-\theta_h)p}\right]$$

$$\left[\left(\frac{1-\theta}{\theta_h}-p\right)\mu(1-\theta_h)+R_f-g-\lambda\right]-\sigma\mu^2(Q-\theta)(1-\theta_h)/\theta_h$$

$$-\left[\mu-\sigma\mu(1-\theta_h)-\frac{\psi c}{(1-\theta_h)p^2}+\frac{1}{(1-\theta_h)p}\lambda\right][\lambda-\delta-(\rho+\sigma g)]/\theta_h = 0$$

If we write the above equation in polynomial form, then the coefficient of λ^3 is $\sigma/c > 0$. The coefficient of λ^2 is

$$-\frac{\sigma}{c}\left[\left(\frac{1-\theta}{\theta_h}-p\right)\mu(1-\theta_h)+R_f-g+\frac{\psi c+c/\theta_h}{\sigma(1-\theta_h)p}+(\delta+\rho+\sigma g)+\mu(Q-\theta)\right]$$

The constant term is

$$-\frac{\psi(\delta+\rho+\sigma g)}{(1-\theta_h)p}\left[\left(\frac{1-\theta}{\theta_h}-p\right)\mu(1-\theta_h)+R_f-g\right]-\sigma\mu^2(Q-\theta)(1-\theta_h)/\theta_h$$

$$+\left[\mu-\sigma\mu(1-\theta_h)-\frac{\psi c}{(1-\theta_h)p^2}\right][\delta+(\rho+\sigma g)]/\theta_h$$

We can consider the pattern of $Q_H(g)$ and $Q_K(g)$ to analyze the sign of the constant term.

$$Q_H(g) = \frac{1}{\mu}\left[\rho+\mu+\sigma g-\frac{1}{1-\theta_h}(g+\frac{\psi c(g)}{p(g)})+\frac{\theta_h}{1-\theta_h}R^f\right]$$

$$Q_K(g) = \frac{R^k + (\rho + \sigma g - R^f)\theta}{\rho + \delta + \sigma g}.$$

$$Q_1'(g) = \frac{1}{\mu}\left[\sigma - \frac{1}{1-\theta_h}(1 + \frac{\psi c'(g)}{p} - \frac{\psi c}{p^2}p'(g))\right] = \frac{1}{\mu}\left[\sigma - \frac{1}{1-\theta_h}(1 + \frac{\psi c'(g)}{p} - \frac{\psi c}{\mu(1-\theta_h)p^2})\right]$$

where

$$c'(g) = \theta_h p + \theta + \frac{\theta_h(g - R^f)}{\mu(1-\theta_h)} - 1$$

$$Q_K'(g) = -\frac{\sigma(Q-\theta)}{\rho + \delta + \sigma g}$$

The constant term equals

$$-[Q_1'(g) - Q_2'(g)](\delta + \rho + \sigma g)\mu^2(1-\theta_h)/\theta_h$$

If $\rho + \sigma g + \delta > 0$ For the good equilibrium, $Q_1'(g) < Q_2'(g)$, therefore, the constant term is larger than zero, $\lambda_1^g\lambda_2^g\lambda_3^g < 0$. Meanwhile, For the bad equilibrium, $Q_1'(g) > Q_2'(g)$, therefore, the constant term is smaller than zero, $\lambda_1^b\lambda_2^b\lambda_3^b > 0$.

We should also focus on the key term λ^2.

$$\begin{aligned}
& -\tfrac{\sigma}{c}\left[\left(\tfrac{1-\theta}{\theta_h}\right)\mu + R_f - 2g + \tfrac{\psi c + c/\theta_h}{\sigma(1-\theta_h)p} + (\rho + \sigma g) + \mu(Q-1)\right] \\
< \; & -\tfrac{\sigma}{c}\left[\left(\tfrac{1-\theta}{\theta_h}\right)\mu + R_f - 2g + \tfrac{R^k - g - \delta + (\theta + \theta_h p)(g - R^f)}{\theta_h p} + (\rho + \sigma g) + \mu(Q-1)\right] \\
= \; & -\tfrac{\sigma}{c}\left[\left(\tfrac{1-\theta}{\theta_h}\right)\mu - g + \tfrac{R^k - g - \delta + \theta(g - R^f)}{\theta_h p} + (\rho + \sigma g) + \mu(Q-1)\right] \\
< \; & -\tfrac{\sigma}{c}\left[\left(\tfrac{1-\theta}{\theta_h}\right)\mu + \tfrac{[\rho + (\sigma - 1)g](1-\theta)}{\theta_h p} + [\rho + (\sigma-1)g] + \mu(Q-1)\right] \\
< \; & 0
\end{aligned}$$

The first $<$ is due to $\frac{1+\psi\theta_h}{(1-\theta_h)\sigma} > 1$. The second $<$ is due to $Q_K(g) > 1$, which implies that

$$\frac{R^k + (\rho + \sigma g - R^f)\theta}{\rho + \delta + \sigma g} > 1 \Rightarrow R^k - g - \delta + \theta(g - R^f) > [\rho + (\sigma-1)g](1-\theta)$$

The last $<$ is due to

$$\rho + \sigma g = \frac{1}{1-\theta_h}(g + \frac{\psi c}{p}) - \frac{\theta_h}{1-\theta_h}R^f + \mu(Q-1) > g + \frac{\psi c}{p} + \mu(Q-1) > g$$

Therefore, we have $\lambda_1^g + \lambda_2^g + \lambda_3^g > 0$ and $\lambda_1^b + \lambda_2^b + \lambda_3^b > 0$.

From $\lambda_1^g + \lambda_2^g + \lambda_3^g > 0$ and $\lambda_1^g\lambda_2^g\lambda_3^g < 0$, we know that the only possible combination is $(+,+,-)$ for the good steady state. From $\lambda_1^b + \lambda_2^b + \lambda_3^b > 0$ and $\lambda_1^b\lambda_2^b\lambda_3^b > 0$, we know that the possible combinations are $(+,+,+)$ or $(-,-,+)$ for the bad steady state.

Case of $R_t^H = R^f$ For this case,

$$\dot{p}_t = [R^f + \delta - \mu(p_t - l_t + \theta)]p_t - \psi c_t$$

$$\dot{c}_t = [R^f + \mu(Q_t - 1) - \rho + \sigma\delta - \sigma\mu(p_t - l_t + \theta)]c_t/\sigma,$$

$$\dot{l}_t = c_t - R_t^k + \mu(p_t - l_t + \theta)(1 - l_t) + (\delta + R^f)l_t$$

$$\dot{Q}_t = [R^f + \mu(Q_t - 1) + \delta]Q_t - \mu\theta(Q_t - 1) - R_t^k,$$

The variable Q_t is independent to other three variables. The Jacobian matrix of other three equations are

$$\mathcal{J} = \begin{bmatrix} R^f - g - \mu p & -\psi & \mu p \\ -\mu c & 0 & \mu c \\ \mu(1-l) & 1 & -\mu(1-l) - g + R^f \end{bmatrix}$$

$$\lambda_1\lambda_2\lambda_3 = \mu c(1+\psi)(g - R^f)$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 2R^f - 2g - \mu(p + 1 - l)$$

$$\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_1\lambda_3 = (R^f - g)^2 - \mu(p + 1 - l)(R^f - g) - \mu c(1 + \psi)$$

As $g > R^f$, we have $\lambda_1\lambda_2\lambda_3 < 0$. If $\lambda_1 + \lambda_2 + \lambda_3 > 0$, then the sign of eigenvalues are $(+, +, -)$. If If $\lambda_1 + \lambda_2 + \lambda_3 < 0$, that is $(R^f - g) + [R^f - g - \mu(p + 1 - l)] < 0$, we have $R^f - g - \mu(p + 1 - l) < 0$. Then

$$\begin{aligned} \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_1\lambda_3 &= (R^f - g)^2 - \mu(p + 1 - l)(R^f - g) - \mu c(1 + \psi) \\ &= (R^f - g)[R^f - g - \mu(p + 1 - l)] - \mu c(1 + \psi) \\ &< 0 \end{aligned}$$

then the sign of eigenvalues are $(+, +, -)$.

B.10 Proof of Theorem 4

In the proof of Proposition 10, we show that the system always has a positive eigenvalue λ_3. The system has other two eigenvalues $\lambda_1(R^k)$ and $\lambda_2(R^k)$. If the steady state with low growth rate can be either a sink or a source as R^k continuously changes, then $\lambda_1(R^k)\lambda_2(R^k)$ is always larger than zero. Meanwhile, $\lambda_1(R^k) + \lambda_2(R^k)$ can be either larger than zero or not when R^k continuously changes. There exists a critical point R_{HB}^k such that $\lambda_1(R^k) + \lambda_2(R^k) = 0$. If $\lambda_1(R_{HB}^k)\lambda_2(R_{HB}^k) > 0$ while $\lambda_1(R_{HB}^k) + \lambda_2(R_{HB}^k) = 0$, then $\lambda_1(R^k) = \omega(R_{HB}^k)i$ while $\lambda_2(R^k) =$

$-\omega(R^k_{HB})i$. As $\lambda_1(R^k)+\lambda_2(R^k)$ can be either larger than zero or not, $\frac{d}{dR^k}Re(\lambda_1(R^k))|_{R^k=R^k_{HB}} \neq 0$. Therefore, the system exhibits Hopf bifurcation.

C No Default Transition

Two BGPs with $R_t^H > R^f$ Under the specified parameterization, the economy admits multiple balanced growth paths with $R_t^H > R^f$. As illustrated in Figure C.1, a self-fulfilling crisis can occur without any exogenous shock to economic fundamentals. Initially, the economy resides on a high-growth balanced growth path. However, purely due to a shift in market sentiment, a wave of pessimism that coordinates expectations among agents, the economy abruptly transitions toward an alternative equilibrium characterized by persistently lower growth.

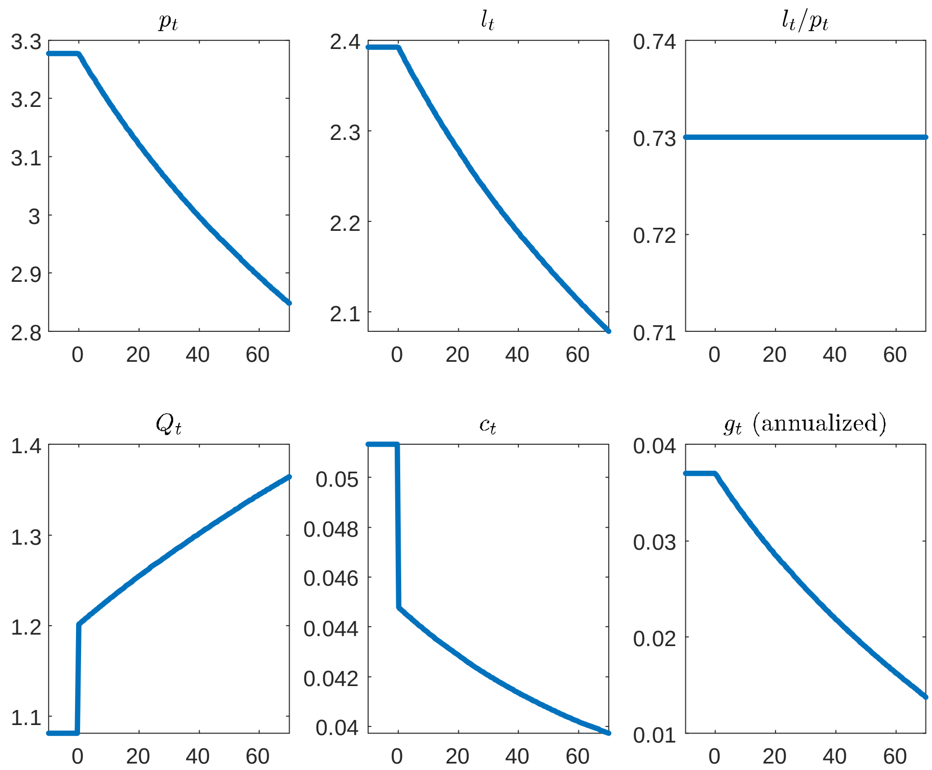

Figure C.1: Transitions in Panel (a) of Figure 3

Note: We take $\sigma = 2$, $\theta_h = 0.73$, $\theta = 0.3$, $\mu = 0.05$, $\rho = 0.02$, $\delta = 0.05$, $R^k = 0.0944$, $R^f = 0.0025$, $\psi = 0.12$. The horizontal axis represents quarters. The transition starts at period 0.

This transition is triggered by a sudden reassessment of liquidity risk: agents collectively demand higher liquidity premiums, causing Q_t to spike upward upon impact. Thereafter, Q_t gradually adjusts along the transitional path toward its new steady state. In contrast, asset prices p_t and the growth rate g_t decline steadily during the transition. Consumption c_t drops immediately at the onset of the crisis, then continues to adjust downward as the economy undergoes a prolonged phase of deleveraging.

Crucially, this shift between equilibria is driven solely by endogenous sentiment, a coordination failure without any change in underlying fundamentals. Once the economy settles into the new balanced growth path, it exhibits persistent stagnation. This is different from the crises in Schmitt-Grohé and Uribe (2021b) where the crises and consumption decrease is temporary. The multiplicity implies that the low-growth regime can endure for an extended period, potentially resulting in a sustained low growth from the earlier growth trajectory. This phenomenon highlights the role of belief-driven dynamics in generating long-run stagnation episodes without external triggers.

The above transition can be considered a financial crisis, with falling asset prices, lower economic growth, and an outflow of credit from the economy. It is worth noting that the above transition path is not the only path. As the low-growth steady state is a sink point with numerous paths converging to that steady state around it, the transition path is not unique.

From $R_t^H > R^f$ to $R_t^H = R^f$ Our framework is also capable of generating self-fulfilling transitions from a balanced growth path with $R_t^H > R^f$ to $R_t^H = R^f$. To simulate the equilibrium, we set $\theta_h = 0.74$ and $R^k = 0.0932$, resulting in a configuration similar to that in panel (b) of Figure 3. As depicted in Figure C.2, this setup produces an endogenous boom-bust cycle absent any exogenous shocks.

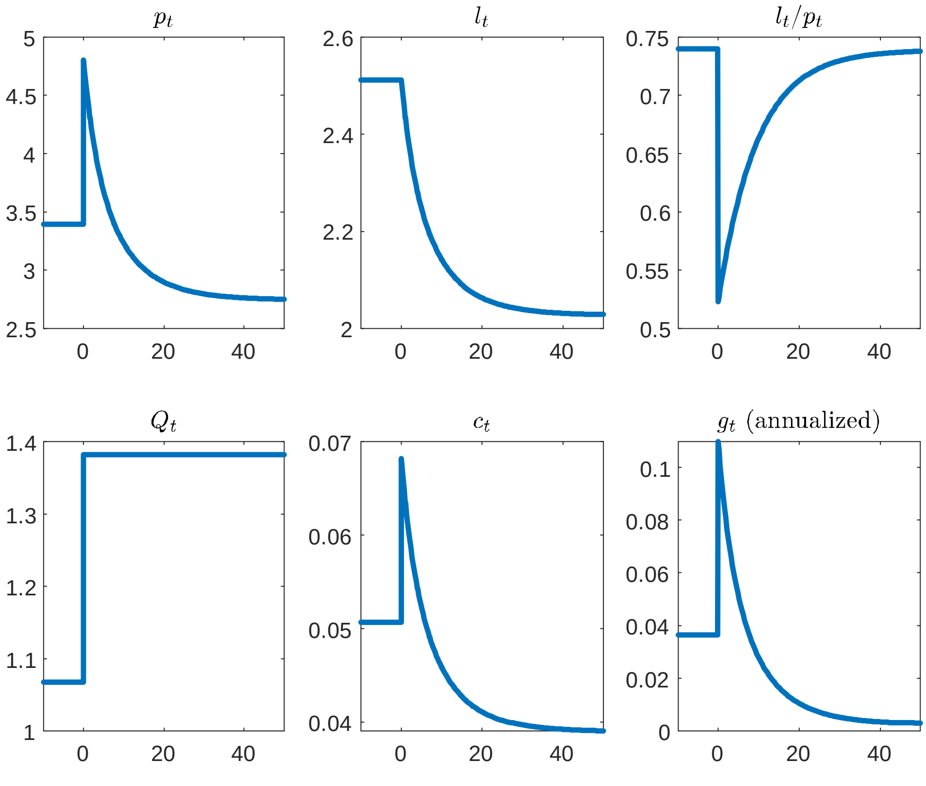

Figure C.2: Transitions in Panel (b) of Figure 3

Note: We take $\sigma = 2$, $\theta_h = 0.74$, $\theta = 0.3$, $\mu = 0.05$, $\rho = 0.02$, $\delta = 0.05$, $R^k = 0.0932$, $R^f = 0.0025$, $\psi = 0.12$. The horizontal axis represents quarters. The transition starts at period 0.

Initially, the economy is in a steady state with high growth. A shift in market sentiment then triggers a coordinated move among agents, leading the system toward a new equilibrium with lower long-run growth. In the short term, there is a temporary surge in asset prices, consumption, and economic growth. Credit constraints loosen during this phase, reflecting improved financial conditions.

However, this boom is ultimately self-limiting. All three variables eventually decline and converge to lower steady-state levels. It is important to emphasize that the entire transition is uniquely determined and saddle-path stable. Notably, the cycle is driven solely by changes in sentiment: the boom arises from coordinated optimism, and the subsequent bust from a self-fulfilling reversal in expectations. This exercise illustrates how belief fluctuations alone can generate endogenous economic cycles without underlying fundamental shocks.