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ABSTRACT

Sectoral shifts require costly labor reallocation for workers, fueling concerns about how quickly they occur. We study how the pace of such sectoral shifts affects workers at risk of displacement. We develop a life-cycle model with skill heterogeneity and job ladders where labor demand gradually rises in one sector and declines in another. The model reveals three novel insights. First, workers' lifetime earnings are strongly non-linear and even nonmonotonic in the horizon over which the transition unfolds. Second, more and more workers benefit on the extensive margin as the transition accelerates, but the tail of losses becomes thicker on the intensive margin. Third, labor market frictions are important to quantify these non-linearities. We apply our model to the climate transition and find substantial earnings losses from a transition ending in 2060. Completing the transition ten years earlier reduces average losses, but raises losses in the tail by a fifth.

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1 Introduction

Secular changes in economic activity require substantial labor reallocation from declining sectors to growing ones. These sectoral shifts happen constantly, and include the contraction of manufacturing and the rise of services, sectoral reallocation following a trade liberalization, or, looking ahead, the transition from polluting to clean industries. The labor reallocation that accompanies these sectoral shifts is often costly for workers attached to the declining sector, who forgo specific human capital and experience periods of unemployment over the transition, with lasting effects on their earnings (Jacobson et al., 1993; Davis and von Wachter, 2011). These reallocation costs fuel concern about how quickly sectoral shifts take place. For instance, survey evidence and voting behavior suggest that labor market concerns are a significant obstacle for the support to ambitious climate goals (Dechezleprêtre et al., 2025; Bombardini et al., 2025). Despite these crucial political economy implications, we know surprisingly little about how different transition timelines affect workers' outcomes.

This paper studies how the *pace* at which a sectoral shift occurs affects the earnings distribution among workers initially employed in the declining sector. For instance, in the climate context, we study how completing the energy transition by 2060, 2050, 2045, etc. affects the distribution of earnings losses among workers initially attached to dirty industries, who are the most at risk of displacement and the most likely to oppose this transition. It is essential to determine how these workers fare under different scenarios and how expensive it might be to compensate them to raise support for a faster transition. While much of the literature focuses on particular historical transitions,[1] we conduct a *counterfactual* analysis asking how workers' earnings would change if the government were to intervene to accelerate or slow down a transition. Our goal is to develop general conceptual insights that are portable to a variety of sectoral shifts, and apply these insights to a context which is relevant for today's policymakers: the climate transition.

We develop a model that captures the displacement and reemployment experience of workers initially attached to a declining industry. The model features finitely-lived workers with heterogeneous skills. Changes in preferences and technology induce a transition of economic activity away from a declining sector and

[1] See, e.g., Traiberman (2020), Caliendo et al. (2019), Dix-Carneiro (2014), and Adão et al. (2024).

towards a growing sector. As the first sector declines, workers may keep looking for employment in this sector where jobs are destroyed, transition to the growing sector where jobs are created, or claim an outside option which captures the rest of the economy and early retirement.

The government can accelerate or slow down this transition by implementing sector-specific subsidies financed by lump sum taxes, thereby shifting the path of marginal revenue products of labor (MRPL) and firms' incentives to create jobs across sectors. We study how government interventions that change these paths of MRPL — corresponding to more or less ambitious timelines — affect the distribution of workers' present discounted value of future earnings or "lifetime earnings."

Three novel conceptual insights emerge from our analysis. First, workers' lifetime earnings are highly *non-linear* or even *non-monotonic* in the horizon over which the transition unfolds. The non-linearity implies that shortening a transition by 1, 5 or 10 years has dramatically different implications for workers. Indeed, workers will experience a given shift in labor demands by the time they retire. A faster transition not only means that workers will be exposed to this shift for longer; it also exposes them to a larger shift which they otherwise would not have experienced before retirement. These two effects reinforce one another and imply that a faster transition has a non-linear effect on lifetime earnings. Extrapolating from small changes around a status quo transition can thus be very misleading.

Workers' earnings changes can even be *non-monotonic* in the pace of the transition: some workers suffer at first when the transition accelerates, but benefit from an even faster one. This non-monotonicity results from the tension between two opposing forces: a faster transition displaces workers earlier from the declining sector, but it also creates more high-paying jobs in the growing sector before they retire. We show analytically that the relative importance of the second effect grows as the transition accelerates. In particular, workers generally suffer the most for *medium-speed* transitions which are fast enough that workers experience all of the earnings drop in the declining sector, but slow enough that workers retire before benefiting from good job opportunities in the growing sector.

The second insight is that the government faces a trade-off as it accelerates the transition: on the one hand, more and more workers benefit on the *extensive margin* as the transition accelerates further; on the other hand, the tail of losses becomes thicker on the *intensive margin*. These results have important political economy implications: the median worker might prefer a faster transition, but a minority of workers might oppose it more and more vocally. We also find that older workers — who participate more in the political process — are *less* likely to benefit from a faster transition, as they have a shorter horizon over which to recoup the short-run losses experienced after being displaced from the declining sector.

After establishing these results analytically in a frictionless model with perfectly competitive labor markets, we develop a quantitative version of our model that nests this frictionless benchmark and incorporates features that are important empirically to account for earnings losses. First, unemployment spells and job ladders are crucial to explain workers' earnings losses after separation (Jacobson et al., 1993; Huckfeldt, 2022). We therefore allow for costly vacancy creation and directed search on-the-job (Menzio and Shi, 2011), which generate unemployment scarring as workers fall from the ladder. Second, human capital depreciation has long been recognized as another important source of unemployment scarring (Ljungqvist and Sargent, 1998). We therefore allow workers to accumulate multidimensional skills on-the-job and dis-accumulate them off-the-job (Lise and Postel-Vinay, 2020). Finally, there are large gross flows between industries (Pilossoph, 2014) and job-to-job switches often entail wage cuts (Sorkin, 2018). Accordingly, we assume that workers face a noisy discrete choice across jobs so that their search is partially directed.

Our third finding is that accounting for labor market frictions is important to quantify the non-linearities stemming from a faster transition. We repeat our counterfactual analysis in the quantitative model and find that workers' earnings remain highly non-linear in the transition horizon. We decompose the contribution of each quantitative feature by adding them one at a time, starting from the frictionless benchmark, until we reach the full model. The two key additions that most affect the non-linearity are the presence of hiring costs and the existence of a job ladder. Creating and filling a vacancy is costly in the model, as in the data (Hagedorn and Manovskii, 2008; Muehlemann and Strupler Leiser, 2018). Therefore, firms only create vacancies in the growing sector once the marginal revenue product of labor rises sufficiently rapidly to recoup this cost, which introduces additional non-linearities as the transition accelerates. Meanwhile, job ladders imply that high-paying jobs are no longer posted in the declining sector as the transition accelerates, resulting in large permanent losses for workers who do not have appropriate skills for the growing sector. Overall, we conclude that labor market frictions are key to quantify not just the *level* of earnings losses from displacement, but also how they *change* as a transition speeds up.

While these insights apply to any shifts in economic activity where one sector expands at the expense of another one, we apply the model empirically to one particular context: the climate transition. This application is especially relevant for today's policymakers: it concerns 10-15 million workers who are currently employed in polluting industries, roughly the size of manufacturing employment in the early 2000s; its pace is partly under the control of the government; and most of the transition has yet to take place. Whether the government decides to speed up or slow down the climate transition, it needs to know how workers' outcomes vary with the horizon over which it takes place. The prospective nature of our analysis means that it relies on long-term projections about the climate transition, and the literature offers a wide range of estimates. We therefore consider a menu of scenarios in line with existing predictions.

We start by estimating the distribution of workers' skill endowments in this particular context. To do so, we develop a novel structural estimation strategy leveraging the block recursive property of competitive search equilibria, meaning that workers' policy functions can be solved independently from the distribution of skill endowments. The extent and timing at which workers reallocate between polluting and clean industries in the phase of the transition observed so far allows us to trace the distribution of skill endowments non-parametrically, accounting for all the model's frictions and the presence of substantial gross flows.

In our main scenario, we find that 80% of workers initially attached to polluting industries lose from a status quo transition completed in 2060. These workers lose on average 5.5% in lifetime earnings, with some workers losing more than 10%. These numbers lie in the range of estimates available in the context of trade liberalization and import competition (Dix-Carneiro, 2014; Autor et al., 2016; Traiberman, 2020). We then use our model to perform counterfactuals and ask how changing the pace of the remaining transition, from 2025 onward, would affect workers' earnings. We find that a slight majority of workers prefers to accelerate the transition so it ends in 2050 or 2040 instead of 2060. Consistent with our conceptual insights, we find that policymakers face a tradeoff when accelerating the transition: doing so creates more winners on the extensive margin, but thickens the tail of lifetime earnings losses on the intensive margin with some workers losing an additional 3% relative to the status quo transition ending in 2060.

Our work connects to a literature studying historical transitions associated with particular labor demand shocks. Dix-Carneiro (2014) and Traiberman (2020) quantify the impact of trade liberalizations in Brazil and Denmark, respectively, and emphasize the role of specific human capital. Caliendo et al. (2019) study trade liberalizations in a setting in which workers face fixed costs of moving across sectors and regions, but without heterogeneous skill endowments or skill accumulation and without costly vacancy creation, job ladders and the associated unemployment scarring. Braxton and Taska (2023) find that workers incur substantial earnings losses when technological changes make their skills obsolete. Adão et al. (2024) show that the adoption of new technologies takes more time when workers do not possess the right skills to operate these technologies. In contrast to this literature, we do not study one particular historical episode. Instead, we ask a counterfactual question: how large would earnings losses be if the government intervened to speed up or slow down a given transition? We uncover strong non-linearities and non-monotonicities in earnings losses as transitions get faster, and show that accounting for skill heterogeneity, costly hiring, and unemployment scarring is key to quantify these non-linearities. Our focus on non-linearities also sets our paper apart from recent work considering first order deviations from status quo transitions (Lehr and Restrepo, 2023; Seo and Oh, 2024).

We contribute to a literature asking whether policymakers should slow the adoption of new technologies. Guerreiro et al. (2021) and Costinot and Werning (2023) argue that taxing automation can be optimal to mitigate their distributional consequences. Beraja and Zorzi (2024) show that the pace of technological transitions is inefficiently fast when workers cannot smooth consumption after being displaced. Braxton and Taska (2024) investigate the role of retraining subsidies to insure workers against technological change. Studying sectoral shifts where one sector grows at the expense of another, we find that some workers who suffer from a faster transition *locally* might, in fact, prefer an even faster one *globally* if it accelerates sufficiently to create enough high-paying jobs in the growing sector.

Our climate application adds to a growing body of work on the costs and benefits of the energy transition. A large literature documents substantial societal gains from a faster climate transition (Barrage and Nordhaus, 2023; Bilal and Känzig, 2025; Nath et al., 2024). At the same time, Bilal and Stock (2025) note that there has been comparatively little work on worker reallocation and earnings losses due to this transition, which motivates our application. Empirical work documents the ongoing reallocation of workers towards green industries (Curtis and Marinescu, 2023; Curtis et al., 2023). Consoli et al. (2016) and OECD (2024) argue that green jobs typically require more education and training, and are more intensive in cognitive skills. Walker (2011, 2013) and Känzig (2023) document substantial and long-lasting earnings losses due to environmental regulation, confirming that workers' skills are imperfectly transferable to clean industries.[2] Hafstead and Williams (2018) study the effect of carbon taxes on unemployment in models where workers have uniform skill across sectors.[3] Relative to the existing literature, our analysis estimates the distribution of workers' skills that matches the observed labor reallocation to clean industries, conducts counterfactuals to explore how the set of winners and losers changes as the transition accelerates from 2025 onward, and emphasizes the importance of non-linear effects.

2 Frictionless Benchmark

We introduce a frictionless model to illustrate the core economic mechanisms as transparently as possible. We will extend this model in Section 5 to account for realistic labor market frictions. Time is continuous and indexed by $t \geq 0$. There are two sectors: $k = C$ where labor demand grows over time, and $k = D$ where labor demand declines. With an eye toward our application, sector $k = C$ can be thought of as the "clean'" sector where jobs are "created," and sector $k = D$ can be thought of as the "dirty" sector where jobs are "destroyed." We also allow for an outside option $k = O$, which represents sectors unaffected by the transition.

2.1 Workers

Demographics and preferences. Workers live for J years and are replaced by new ones upon retirement. We let τ denote a worker's cohort or birth date. Workers supply

[2] A complementary set of papers studies how policy can influence the speed of this transition (Acemoglu et al., 2012, Acemoglu et al., 2023).

[3] Hafstead and Williams (2026) review this literature and emphasize labor market frictions and skill mismatch as avenues for future research, both of which play a central role in our paper.

labor inelastically and consume in ages $j \in [0, J)$. Their preferences are given by

$$\int_0^J \exp(-\rho j)\, u\left(c_j\right) dj \tag{2.1}$$

where c_j is consumption of the final good at age j, $u(c) \equiv c^{1-\sigma} / (1-\sigma)$ is the flow utility function, and $0 < \rho < \infty$ is the discount rate.

Skills. Workers have heterogeneous skills across sectors. Let ψ be their skill in sector D, and $\hat{\psi}$ be their relative skill in sector C. Workers transfer a share $b^O \geq 0$ of their skill ψ to the outside option. Therefore, a worker's skills are

$$h\left(\psi, \hat{\psi}, k\right) = \begin{cases} \psi & \text{if } k = D \\ \psi \times \hat{\psi} & \text{if } k = C \\ \psi \times b^O & \text{if } k = O \end{cases} \tag{2.2}$$

All workers are endowed with a reference skill $\psi = 1$ and draw an endowment of relative skill $\hat{\psi}$ from a distribution $G\left(\hat{\psi}\right)$. We suppose for now that these skills are fixed over a worker's life, and relax this assumption in our quantitative model.

2.2 Firms

Final good. A representative firm combines the output of the growing and declining sectors to produce a final good according to $Y_t = F_t\left(X_t^D, X_t^C\right)$, where X_t^k are intermediate goods produced in these sectors. The production function F_t has constant returns-to-scale and is concave in each argument. As usual, the aggregate production function can alternatively be interpreted as households' preferences over these two goods. The sectoral shift is driven by an exogenous change in F_t in favor of X_t^C over time. In our application to the climate transition, we will think of the two goods as being imperfect substitutes, e.g., electric vehicles (EVs) and internal combustion engines (ICEVs) to be concrete.[4]

Intermediates. The two intermediate goods X_t^k are produced by combining a continuum of mass 1 of symmetric varieties x_t^k using a CES aggregator with elasticity of substitution ε^k larger than one. Each variety is produced by a firm using la-

[4] Of course, the two intermediate goods cannot be perfect complements. Otherwise, they would perfectly co-move and there would be no sectoral shift in the first place.

bor services so $x_t^k = a_t^k y^k\left(n_t^k\right)$, where a_t^k is a sectoral TFP, $y^k(\cdot)$ is an increasing and concave function which captures decreasing returns to scale in production in that sector. In turn, $n_t^k = \int h\left(\psi,\hat{\psi},k\right) d\mu_t^k\left(\psi,\hat{\psi}\right)$ is the amount of labor used in that sector, where $\mu_t^k\left(\psi,\hat{\psi}\right)$ denotes the measure of workers employed in sector k and period t with skills $\left(\psi,\hat{\psi}\right)$.[5] These firms are price takers in the labor market but compete monopolistically in product markets, and sell their output to the final goods producer at price P_t^k in a symmetric equilibrium.

Productivities. Firms producing varieties of intermediate goods can invest resources to improve their productivity a_t^k, thereby increasing profits. We allow for knowledge spillovers across sectors. The shift in labor demands induced by changes in F_t is thus amplified endogenously, as it affects firms' incentives to innovate in each sector. None of our conceptual results rely on the specifics of the innovation process; we therefore relegate the exposition of these details to Appendix B.2. We show in Appendix B.5 that our economy admits a balanced growth path (BGP) where output grows at rate $g > 0$ in both sectors.

Ownership. All firms are owned by a continuum of infinitely-lived, risk-neutral and deep-pocket entrepreneurs who discount future flows at rate ρ.

2.3 Markets and Government

Labor markets. Labor markets are perfectly competitive in our frictionless benchmark. Workers may supply labor to growing sector $k = C$ or the declining one $k = D$ at a competitively determined wage per efficiency unit w_t^k. They can alternatively claim the outside option and receive a flow endowment of $h\left(\psi,\hat{\psi},O\right)$ of final goods which grows exogenously at the rate g that prevails along a BGP. Workers freely choose between $k = C, D, O$ at every instant.

Government. The government has two instruments to influence the transition once it is underway. First, it can introduce a subsidy or tax τ_t^k on the purchase of intermediate goods, e.g., a subsidy on EV sales. Second, it can use subsidies ς_t^k on innovation or investment in each sector, e.g., a subsidy on R&D to develop efficient battery cells used in EVs. We suppose that the government balances its budget in every period using lump sum taxes on workers or entrepreneurs.

[5] The demand for labor services is symmetric across firms producing varieties in each sector.

Financial Markets. Our analysis focuses on the case with risk-neutral workers, i.e., $\sigma = 0$, which is standard in the type of search models that we use in our quantitative analysis (Menzio and Shi, 2011; Bilal et al., 2022). It is equivalent to assume that $\sigma > 0$ but that workers can borrow or save freely. In both cases, workers' objective and what we characterize is the present discounted value of earnings.

2.4 Equilibrium

We define an equilibrium formally in Appendix B.3 in the context of our full model presented in Section 5, which nests the frictionless benchmark considered here. Appendix B.4 proves that worker values scale linearly with the reference skill ψ and grow at the same rate g along a BGP. This property also allows us *not* to track ψ as an explicit state variable. Next, we describe how the transition unfolds.

3 Status Quo and Counterfactual Transitions

We describe the status quo transition that would prevail without further intervention. We then characterize the set of allocations that the government can implement given its instruments, and we explain our counterfactuals.

3.1 Status Quo Transition

The *status quo* transition is the one that prevails when the government sticks to the path of policy interventions that is currently expected. The economy rests on its initial BGP before this transition begins in period $t = 0$. We suppose that sector C is not active along this initial BGP: there is no demand for the good it produces and no workers are employed there, e.g., EVs have not been invented or are not valued yet.[6] The transition starts in $t = 0$ due to an exogenous change in the function F_t in favor of the good produced in sector C, e.g., increased awareness of climate change or a change in government mandates for greener goods. The price of the good produced in that sector becomes positive as a result. It increases gradually over time as F_t changes further and firms adjust their innovation in each sector.

[6] We assume that the marginal product of labor remains bounded as $n_t^k \to 0$. Otherwise, many workers would "jump" to sector C when it appears in $t = 0$, whereas labor reallocation is gradual in the data. This assumption can be justified by interpreting $y^k(n_t^k)$ as output after accounting for some fixed sweat equity labor from the entrepreneurs producing the varieties in sector k.

To illustrate how the sectoral shift affects workers, it is useful to define the following de-trended marginal revenue products of labor in the two sectors

$$\mathrm{MRPL}_t^k \equiv A_t^k \equiv \frac{\varepsilon^k - 1}{\varepsilon^k}\left(1+\tau_t^k\right)\tilde{a}_t^k P_t^k y^{k,\prime}\left(n_t^k\right), \tag{3.1}$$

which depend on the price of intermediate goods P_t^k, the subsidies τ_t^k that firms receive for each sale, the sectoral TFPs $\tilde{a}_t^k \equiv \exp\left(-gt\right) a_t^k$ de-trended using the BGP growth rate $g > 0$, and sectoral returns to scale $y^{k,\prime}\left(n_t^k\right)$. Finally, we normalize these marginal revenue products of labor by the inverse product markups so A_t^k can be directly interpreted as workers's wages in our frictionless benchmark. These sectoral MRPLs play an important role because they are *sufficient statistics* to characterize workers' outcomes over the transition: other features of the economy matter only to the extent that they affect these MRPLs.

Lemma 1. *Workers' labor market outcomes depend exclusively on the sequences of de-trended MRPLs A_t^k given by expression (3.1).*

Proof. As we show in Appendix B.2, sectoral MRPLs A_t^k coincide with sectoral wages in this model with frictionless labor markets. Workers' labor supply decisions, and hence their outcomes, can thus be characterized exclusively as a function of their potential earnings A_t^D, $\hat{\psi} A_t^C$ and b^O. As we establish in Appendix B.3, the result also holds in our full model of Section 5 with costly vacancy posting and job ladders, which nests our frictionless benchmark. □

In the spirit of Kongsamut et al. (2001) and Ngai and Pissarides (2007), we model sectoral shifts as transitions that affect the composition of economic activity across sectors, but not the long-term growth rate g of the economy.[7] We can therefore represent the paths of sectoral MRPLs under the status quo as

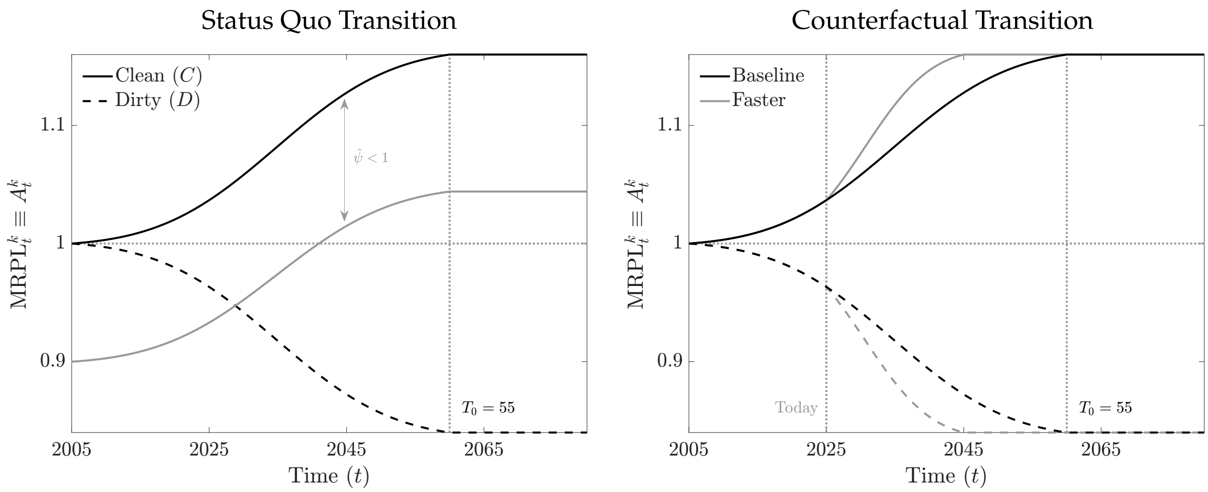
$$A_t^k = A_0^k + \Phi^k\left(t\right) \times \left(A_\infty^k - A_0^k\right), \tag{3.2}$$

where A_0^k and A_∞^k are the start and end values of these MRPLs. By assumption, $A_\infty^C > A_0^C$ in the growing sector and $A_\infty^D < A_0^D$ in the declining sector.[8] The increasing functions $\Phi^k : \mathbb{R}_+ \to [0,1]$ reflect the share of the status quo transition that is completed as a function of calendar time t.

[7] Appendix B.5 provides conditions ensuring that the initial and final BGPs grow at the same rate.

[8] Productivity in sector D declines relative to its original trend, but might keep growing in level.

Figure 3.1: Marginal revenues product of labor A_t^k



Notes: The left panel illustrates the paths of MRPLs A_t^k in the two sectors under the status quo transition. The solid and dashed curves indicate the growing sector C and the declining sector D, respectively. The gray curve indicates the earnings of a worker with $\hat{\psi} < 1$ who supplies labor to sector C. The right panel illustrates a faster transition parametrized by expression (3.3) with $\eta > 1$.

The left panel of Figure 3.1 illustrates how these MRPLs evolve over the status quo transition. The solid black line plots a path of A_t^C in the growing sector and the dashed line a path of A_t^D in the declining sector. By definition, a sectoral shift is a transition where labor demand (i.e., the MRPL) rises in one sector and falls in the other, leading to a labor reallocation between the two. For example, labor demand rises in EV production and falls in ICEV production; the service sector expands, while manufacturing declines relative to trend; a new and more productive technology takes over an aging one (Violante, 2002); or a trade shock induces a labor reallocation across industries (Dix-Carneiro, 2014; Traiberman, 2020). In our frictionless benchmark, all workers have $\psi = 1$ and are paid their marginal product, and thus earn A_t^D in sector D and $\hat{\psi} A_t^C$ in sector C. The gray curve in the left panel of Figure 3.1 depicts the earnings of a given worker in sector C.

We choose the scale of the economy by normalizing the initial MRPL in the declining sector to $A_0^D = 1$, without loss of generality. In turn, the initial MRPL in the growing sector A_0^C is not separately identified from the mean of the distribution of relative skills G: workers may decide to supply labor to the clean sector either because sectoral TFP A_0^C is high, or because they have high idiosyncratic skills there. We thus normalize $A_0^C = 1$.[9] With our application in mind, Figure 3.1

[9] Knowledge spillovers across sectors imply that $a_0^C > 0$ in sector C when the transition kicks off,

assumes that the transition is completed by a terminal period $T_0 \geq 0$.[10]

In our example, the de-trended MRPLs follow S-curves in the two sectors. These S-curves are characteristic of technology adoption or diffusion (Griliches, 1958), i.e., they can reflect changes in the function F_t itself. For instance, information diffuses slowly and car adjustments are infrequent, so EV adoption is slow. These S-curves also emerge endogenously due to the innovation process: productivity grows faster in sector C relative to trend before settling on a new balanced-growth path which, after de-trending, takes the form of an S-curve (Appendix B.7). The opposite happens in sector D. While the S-shapes are not crucial to our analytical results, we will maintain them throughout for illustration.

3.2 Counterfactuals

Our goal is to study how workers' outcomes change as the government intervenes to speed up a sectoral shift. The government may want to intervene for various reasons that depend on the context and that are not modelled explicitly, e.g., meeting climate commitments or improving terms of trade. We suppose that the government intervenes from today's date $t_0 \in [0, T_0)$ onward.

Implementability. What is the set of workers' outcomes that the government can implement using its instruments? In light of Lemma 1, this question is equivalent to asking: what is the set of MRPLs that the government can implement?

Proposition 1 (Implementability)**.** *The government can implement any arbitrary path of marginal revenue products of labor $MRPL_t^k$ in each sector.*

Proof. The proof is the same in our frictionless benchmark and in our full model (Section 5). We provide it in Appendix B.6. $\square$

This result states that the government can choose its instruments to support *any* paths of MRPLs (3.1), even if prices, innovations, spillovers and decreasing returns respond endogenously. Of course, implementing different paths of MRPLs means that the government must provide different incentives, and hence comes at different fiscal costs rebated lump sum to workers or entrepreneurs.

despite the lack of direct innovation in the past.

[10] When the convergence is asymptotic, we can define T_0 such that the transition is mostly completed, i.e., $\Phi^k(T_0) = 1 - \epsilon$ for a small ϵ of our choice.

Taken together, Lemma 1 and Proposition 1 fully characterize the set of implementable allocations: the government can choose arbitrary paths of MRPLs across sectors; given those, it knows workers' labor market outcomes. These two results also imply that policymakers can answer two distinct questions. The first question is the focus of our paper: what are the distributional consequences for workers associated to different paths of MRPLs? In other words, how expensive would it be to compensate workers who suffer from a more ambitious transition? The second question is: what is the fiscal cost of implementing particular paths of MRPLs?

We focus on the first question in this paper, for two reasons. First, assessing the welfare implications of this fiscal cost would require taking a stand on how it is financed (e.g. whether the government balances its budget or runs a deficit) and spread within and across generations, which is outside the scope of our paper. Second, the information requirement is much lower for the first question. Starting from a status quo transition, the government can perturb the paths of MRPLs without knowledge of the production side of the economy. It can adjust its subsidies by tâtonnement until it brings sectoral employments and wages in-line with the levels consistent with the targeted paths of MRPLs (Lemma 1).[11] All the government needs to know is the labor market block of our model and how paths of MRPLs translate into sectoral employment and wages. In contrast, the government needs to know everything about the production side of the economy to quantify the fiscal cost of its intervention. Many of these parameters are difficult to estimate in practice, including: the elasticity of sectoral innovation to R&D subsidies, the strength of innovation spillovers, how preferences F_t change over time, etc.

Effectively, our goal is to conduct a *positive* analysis: we perturb the paths of MRPLs across sectors, considering various transitions where labor demands shift more or less rapidly across sectors — with the understanding that the government can achieve these perturbations using short-run subsidies — and characterize how workers' outcomes change as a result.

Counterfactual transitions. For our counterfactual analysis, we suppose that, starting from today's date t_0, the sectoral MRPLs evolve according to

$$A_t^k(\eta) = A_0^k + \Phi_t^k\left(t_0 + \eta\,(t - t_0)\right) \times \left(A_\infty^k - A_0^k\right), \quad \text{for } t \geq t_0, \tag{3.3}$$

[11] For instance, suppose that labor supply n_t^k is lower than implied by the targeted paths of MRPLs in our frictionless benchmark. Then, the government should subsidize sector k more.

where the parameter $\eta > 0$ governs the speed of transition. The paths of A_t^k coincide with the status quo transition when $\eta = 1$. Increasing η compresses the timeline of the transition, ensuring that it completes sooner at some horizon $T < T_0$. The government can implement a more ambitious transition using short-run interventions so the economy converges earlier to its long-run BGP (Appendix B.7).[12] When the goods are substitutes, as in our application, subsidizing one good raises labor demand in that sector, at the expense of labor demand in the other, e.g., selling more EVs comes at the expense of ICEVs. The gray curves in the right panel of Figure 3.1 illustrate a counterfactual transition associated with some $\eta > 1$. While specification (3.3) provides a convenient way to parameterize the counterfactual pace of transition, we show in Appendix A that our conceptual results hold under more general perturbations of the paths of A_t^k.

For each counterfactual, we compute changes in workers' *lifetime earnings* from t_0 onward, relative to the status quo transition, as

$$\mathcal{W}\left(j, \hat{\psi}; \eta\right) \equiv \frac{W_{t_0}\left(j, \hat{\psi}; \eta\right)}{W_{t_0}\left(j, \hat{\psi}; \eta_0\right)\big|_{\eta_0 = 1}} - 1, \tag{3.4}$$

where $W_{t_0}\left(j, \hat{\psi}\right)$ denotes the value function of a worker with age j and relative skill $\hat{\psi}$ evaluated in t_0 (Appendix A) and η parametrizes the MRPLs (3.3). In Section 6.3, we also compare workers' values as of t_0 under the status quo transition with $\eta = 1$ relative to no transition at all. Our goal is to understand how workers initially attached to the declining sector D — those who are most likely to suffer from the sectoral shift and oppose it — fare along the status quo transition and as the government intervenes to accelerate it.

Climate application. While our conceptual insights apply to any sectoral shifts, it is useful to specialize equations (3.2)–(3.3) to our climate application to fix ideas. The Kyoto Protocol was the first major climate agreement. It established binding targets for greenhouse gas (GHG) emissions for the 192 countries that ratified it, including the US. The Protocol entered into force in 2005. We therefore interpret period $t = 0$ as January 2005. In turn, the Paris Climate Agreement of 2015 set a goal of net-zero emissions for developed economies by the year 2050. The US,

[12] We assume here that the government implements the same long-run allocation as in the status quo transition, by reverting to the same *long-run* subsidies/taxes. In Appendix B.5, we discuss exercises where the government intervenes to change the long-run level of the MRPLs.

however, pulled out of the Paris Agreement and is not currently on track to meet this target by 2050. The median response in a recent survey of US energy CEOs suggest that 2060 is a more realistic timeline (Bain & Company, 2025). We therefore assume that the status quo transition in our model runs from 2005 to 2060, corresponding to $T_0 = 55$ in Figure 3.1. For concreteness, we refer to these dates throughout our analysis. Our counterfactuals consist of speeding up the transition starting in $t_0 = 2025$ — the exercise that is most relevant to policymakers today.[13]

4 Economic Mechanisms

We use our frictionless benchmark to build new conceptual insights about how different transition timelines affect workers' outcomes during a sectoral shift.

4.1 The Non-Linear Effects of Rapid Transitions

Consider a worker initially attached to the declining sector. The left panel of Figure 4.1 plots their path of earnings over time under the status quo transition, in the case where the worker has a relative productivity $\hat{\psi} < 1$ in the growing sector. To keep the exposition simple, we abstract momentarily from the outside option, e.g., workers can only switch between manufacturing and services, or within car-manufacturing from ICEVs to EVs.[14] For illustration, we assume that workers are active between the ages of 25 to 65, i.e., $J = 40$.

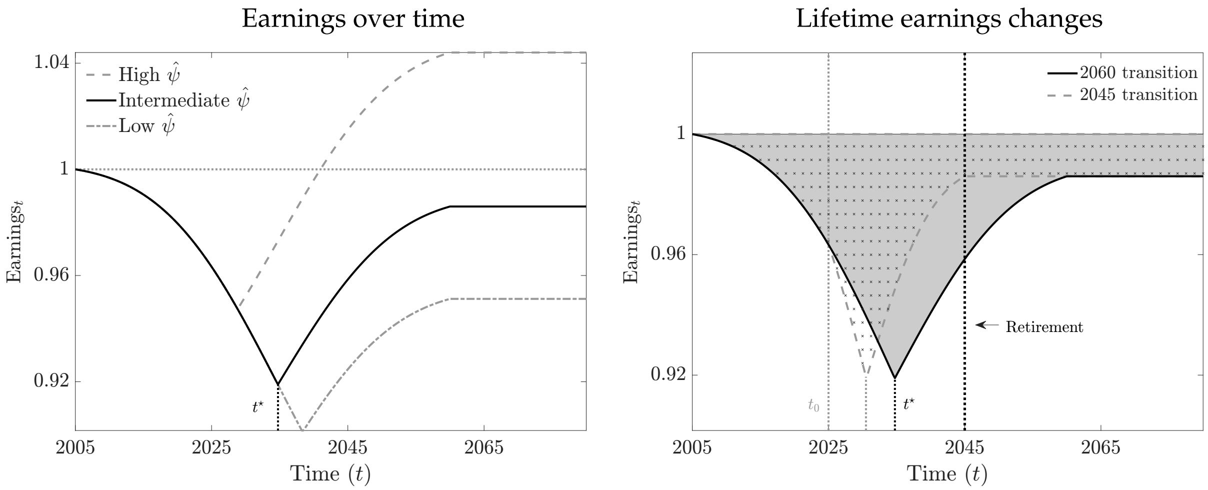
Workers supply labor to the growing sector if and only if $\hat{\psi} \geq A_t^D / A_t^C$. The right-hand side of this inequality shrinks over time. Therefore, a worker remains in the declining sector D until an idiosyncratic switching time $t^\star \geq 0$, which depends on their relative skill $\hat{\psi}$ and the particular transition that they face.[15] The worker supplies labor to the growing sector C after that. A worker actually reallocates across sectors if their switching time occurs during their life, i.e., $t^\star < J + \tau$.

[13] We treat the long-run levels as fixed, as discussed above. Of course, coordinated interventions across the globe can reduce aggregate emissions and climate damages, thereby raising aggregate output (in F_t) and the MRPL in both sectors in the long-run. Instead, we focus on unilateral interventions by a single country, effectively treating the path of climate damages as given. Despite being a large economy, the US accounts for only 13% of global CO2 emissions, for instance.

[14] That is, we set $b^O < A_\infty^D$ for now. We will discuss the role of this outside option shortly.

[15] Formally, the idiosyncratic t^* is the period when $\hat{\psi} = A_t^D / A_t^C$, with $t^\star \equiv 0$ or $t^\star \equiv \infty$ if no solution exists and the worker either switches immediately or never switches.

Figure 4.1: Lifetime earnings changes for various transition speeds



Notes: The left panel plots a worker's earnings over time in our frictionless benchmark, for various productivities $\hat{\psi}$ in the clean sector. The right panel indicates lifetime earnings changes under the status quo transition and when transition ends 15 years earlier.

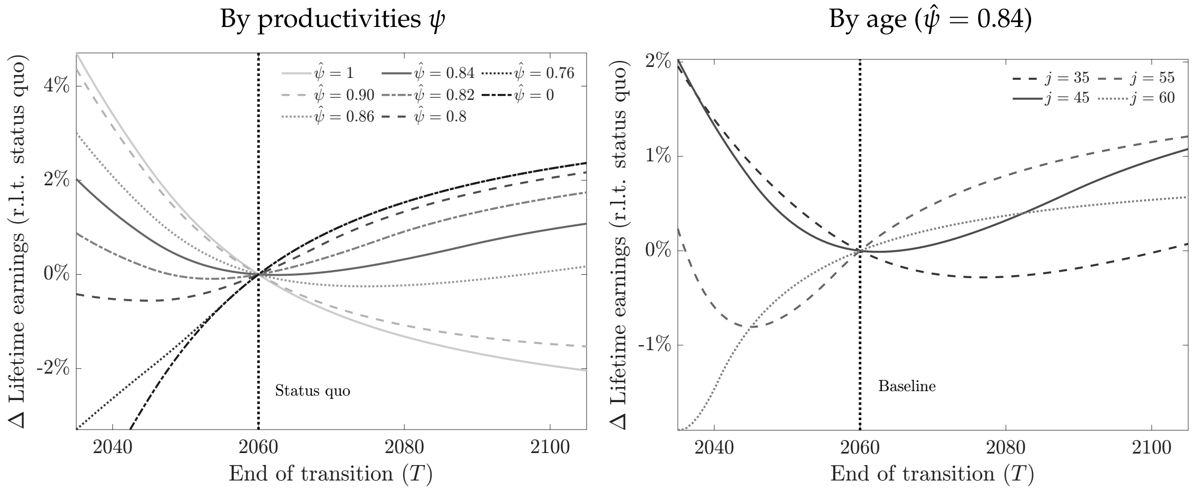
Workers' earnings fall while they remain in the declining sector, reflecting the path of A_t^D. Their earnings rise after they switch to the growing sector, reflecting the path of $\hat{\psi} A_t^C$, but may never fully recover. Workers with a higher relative skill $\hat{\psi}$ switch earlier and benefit more in the long-run, as depicted by the dashed line. The opposite happens when the worker has a lower relative skill.

The effect of the status quo transition on workers' lifetime earnings is given by the area between the realized earnings path and the one that they would have experienced without the transition, discounted at rate ρ until retirement. We shade this area in gray on the right panel of Figure 4.1. This area is negative when a worker's relative skill $\hat{\psi}$ is relatively low, since they lose from the early phase of the transition and do not make up for that before retirement.

Consider now a faster transition, as depicted by the checked region on the right panel of Figure 4.1. It has two opposing effects on a worker's lifetime earnings. On the one hand, it steepens the fall in their earnings while they remain in the declining sector. On the other hand, it means that the worker switches to the growing sector earlier, that their earnings grow faster once they supply labor there, and that a larger share of the transition is completed before they retire.

Our goal is to understand the interplay between these two forces as the transition accelerates. Figure 4.2 plots changes in lifetime earnings as a function of the year T when the transition is complete. Faster transitions are parametrized as in

Figure 4.2: Average lifetime earnings changes



Notes: The left panel plots workers' average lifetime earnings changes in our frictionless benchmark as a function of the transition horizon T, for various values of the relative skill $\hat{\psi}$. Earnings changes are relative to the status quo transition ending in 2060, and transitions are parameterized as in equation (3.3). All workers have 20 years remaining in their careers (median age). The right panel repeats the same exercise for various ages as of 2025, fixing relative productivity to $\hat{\psi} = 0.84$.

equation (3.3) and we set $\eta \equiv (T_0 - t_0) \,/\, (T - t_0)$ so the transition ends in period T. For each of these transitions, we compute the change in lifetime earnings, from 2025 onward, relative to the status quo transition ending in 2060. By construction, all lifetime earnings changes are zero for a transition ending in 2060. The left panel repeats this experiment for various workers with different relative skills $\hat{\psi}$ and who have 20 years of work remaining. In turn, the right panel does the same for workers of different cohorts with $\hat{\psi} = 0.84$.

This figure conveys our first main insight: changes in lifetime earnings are highly non-linear or even non-monotonic as a function of the horizon over which the sectoral shift unfolds. We now explain why this is the case.

Non-linearities. A faster transition creates winners and losers among workers, depending on their relative skill $\hat{\psi}$. Figure 4.2 reveals that, at the margin, a worker can gain or lose much more as the transition accelerates further. For instance, a worker with $\hat{\psi} = 0.86$ is mostly indifferent at first when the transition accelerates starting from 2060, but they benefit more and more as the pace of the transition picks up. In other words, accelerating the transition by 1, 5, or 10 years has a vastly different effect on workers' lifetime earnings. Policymakers should be cautious about extrapolating from small changes around the status quo ending in 2060.

To understand the intuition behind these non-linearities, consider a worker with $\hat{\psi} = 0$ who never switches to the growing sector C, regardless of the transition speed. This worker's change in lifetime earnings is given by the negative "triangle" between their earnings with no transition (1) and $A_t^D < 1$ which declines over time. In our example, $A_t^D \simeq 0.88$ under the status quo by the time the worker retires (Figure 3.1). As the transition accelerates, this negative area expands for two reasons: the economy reaches $A_t^D \simeq 0.88$ sooner, so the worker experiences a lower income for longer before retirement; and the economy experiences an *even larger* decline in A_t^D by the time the worker retires. These effects compound one another and translate into non-linear changes in lifetime earnings.[16] The opposite happens for a workers with $\hat{\psi} = 1$ who switches immediately to the growing sector. These strong non-linearities arise even when the transition evolves linearly over time (as opposed to being S-shaped), as we establish in Appendix A.[17]

Non-monotonicities. A faster transition can even have a *non-monotonic* effect on workers' lifetime earnings: some workers lose from a faster transition at first, but eventually benefit from an even faster one. In particular, consider the worker with $\hat{\psi} = 0.84$ on the left panel of Figure 4.2. Their earnings changes are U-shaped: they would benefit from either a slower or a faster transition relative to the status quo. For this worker, a *medium-speed* transition that ends in 2060 represents the worst of all worlds. It is sufficiently fast that the worker experiences maximal losses in the declining sector before they switch sectors, but sufficiently slow that they retire before benefiting meaningfully from earnings gains in the growing sector.

To understand the intuition behind these U-shapes, consider a worker who retires at the exact moment when they would have switched to the growing sector, i.e., $t^* = J + \tau$. This worker spends their whole career in the declining sector and thus clearly prefers a slower transition. Now consider a marginally faster transition that leads the worker to switch to sector C before retirement. As the right panel of Figure 4.1 illustrates, a faster transition has two effects: workers experience earnings losses earlier when they discount them less; but they also benefit from stronger gains in the growing sector C before they retire. One can show that that the first effect dominates if the transition accelerates sufficiently relative to the

[16] If the transition finishes before the worker's retirement, the earnings loss becomes the area of a trapezoid, generating an additional non-linearity.

[17] The result supposes *some* form of discounting, coming from the discount rate $\rho > 0$ or finite lives.

status quo. As a result, changes in lifetime earnings $\mathcal{W}\left(j, \hat{\psi}; \eta\right)$ defined in (3.4) are U-shaped as a function of the speed of transition.

All workers will experience these non-monotonicities for *some* transition horizon, except those with very large productivities $\hat{\psi} \geq \hat{A}_{t_0}^{-1}$ who switch immediately to sector C and those with very low productivities $\hat{\psi} \leq \hat{A}_{\infty}^{-1}$ who never switch for any horizon, where $\hat{A}_t \equiv A_t^C / A_t^D$ is the relative MRPL in sector C. In our quantitative application, roughly half of workers would experience non-monotonicities for some transition speed.

Proposition 2 (Non-monotonicity)**.** *Lifetime earnings $\mathcal{W}\left(j, \hat{\psi}; \eta\right)$ are non-monotonic, i.e. U-shaped, in the speed of sectoral shifts η for all workers with $\hat{\psi} \in (\hat{A}_{\infty}^{-1}, \hat{A}_{t_0}^{-1})$. In contrast, they increase monotonically in η for workers with $\hat{\psi} \geq \hat{A}_{t_0}^{-1}$ and decrease monotonically for those with $\hat{\psi} \leq \hat{A}_{\infty}^{-1}$.*

Proof. See Appendix A. □

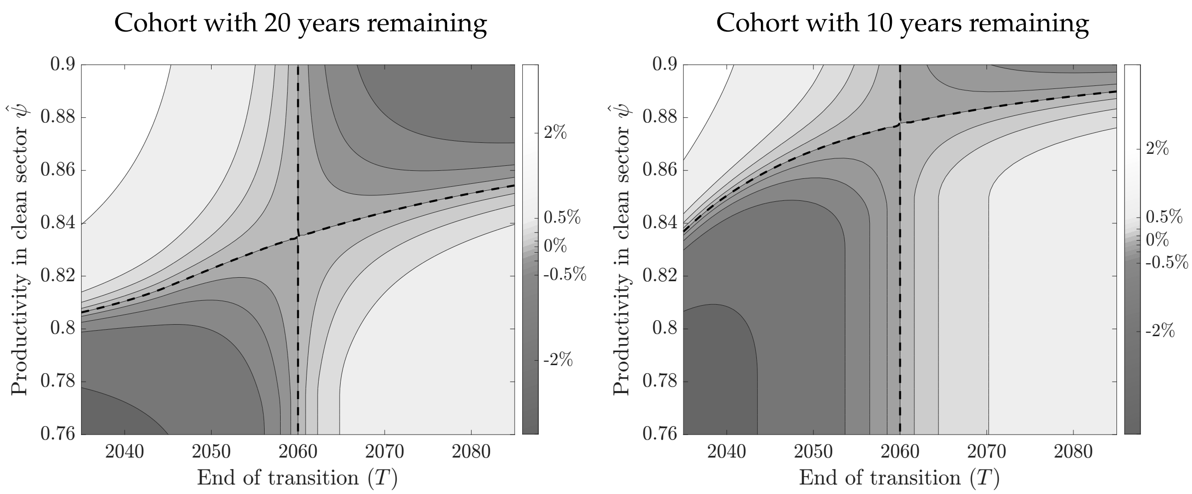
The right panel of Figure 4.2 shows that a worker's age also matters for these non-monotonicities. We repeat the same exercise as above, for various ages in 2025, fixing relative productivity $\hat{\psi} = 0.84$. Older workers have a shorter remaining horizon over which they can benefit from earnings gains in the growing sector. They are therefore more likely to experience earnings losses as the transition accelerates and it takes a faster transition for them to reach the trough of their U-shape.

The introduction of an outside option does not change these qualitative insights — only the *quantitative* predictions of the model, as we discuss in Section 5. This outside option puts a lower bound on workers' earnings; it is akin to truncating the MRPL in the declining sector D from below. Workers who never or always switch to the growing sector C still experience non-linear earnings changes as the transition accelerates further, and workers who switch to the growing sector at some point still experience non-monotonic earnings changes.

4.2 Political Economy Implications

An important corollary to the non-monotonicity result is that the set of workers who prefer a faster transition is endogenous and increases as the transition accelerates further. To understand this corollary, we plot in Figure 4.2 isoquants of lifetime earnings changes relative to the status quo transition in terms of the end

Figure 4.3: Lifetime earnings change isoquants



Notes: This figure plots isoquants of lifetime earnings changes in terms of the end date of the transition T and workers' relative productivities $\hat{\psi}$. All earnings changes are relative to a baseline transition ending in 2060, and faster transitions are parameterized as in equation (3.3). The left panel plots isoquants for workers with 20 years remaining in their careers, while the right panel considers workers with 10 years remaining.

date of the transition T (x-axis) and workers' relative productivities $\hat{\psi}$ (y-axis). The left panel corresponds to workers with 20 years of remaining work life, while the right panel corresponds to those with only 10 years left. The region on the left of 2060 corresponds to a faster transition, and the region on the right corresponds to a slower one.[18] Darker colors indicate larger losses, while lighter ones indicate smaller losses or even gains relative to the status quo. Intuitively, workers with a high relative skill $\hat{\psi}$ in sector C benefit more from, or are hurt less by, a faster transition and move to lighter and lighter regions. Those with a low $\hat{\psi}$ lose more and gradually move to darker and darker regions. For intermediate values of $\hat{\psi}$, workers experience non-monotonic earnings changes: they move to a darker region at first, cross the trough of their U-shape, and then move to ever lighter regions.

This figure reveals our second main insight: as the transition accelerates further, more workers prefer an even faster transition at the margin. Graphically, more workers cross from a darker region to a lighter one as we move to the left starting from 2060; either because they always benefit from a faster transition (high $\hat{\psi}$), or because more workers cross the trough of their U-shape (intermediate $\hat{\psi}$) and then

[18] The central symmetry reflects that workers who prefer a faster transition locally also dislike a slower one, for instance.

prefer a faster transition. In other words, if the government already puts measures in place to accelerate the transition, it might be easier to garner more votes on the *extensive margin* to accelerate it further.

Similarly, the share of workers who benefit *on net* relative to 2060, i.e., globally and not just locally, increases as the transition accelerates. Graphically, more and more workers cross the dashed curves that separate positive and negative isoquants as we move to the left starting in 2060. These dashed curves are upward-sloping in Figure 4.3, reflecting again the U-shapes discussed in Section 4.1.

At the same time, a faster transition worsens losses on the *intensive margin* among workers with low skill in sector C who do not reallocate. Therefore, the government faces a trade-off as it considers accelerating the transition: doing so increases the mass of workers who benefit on the extensive margin; but it also generates a thicker tail of losses on the intensive margin among workers who stay in place in the declining sector. This trade-off has important political economy considerations: for instance, the median worker might prefer a faster transition, but a smaller and vocal share of workers might oppose it increasingly.

Comparing the left and right panel reveals that accelerating the transition tends to benefit younger cohorts at the expense of older ones, since older workers are less likely to reach the trough of their U-shapes for a given transition horizon. In practice, older workers are more likely to participate in the political process (Census Bureau, 2025), potentially hindering public support for rapid transitions, which is particularly relevant for the climate transition (Dechezleprêtre et al., 2025).

4.3 Summary and Discussion

Our analysis reveals two novel conceptual insights. First, workers' lifetime earnings are highly non-linear or even U-shaped as a function of the horizon over which a sectoral shift unfolds. In particular, medium-speed transitions can maximize losses for workers — by displacing them early from the declining sector, without improving their prospects sufficiently in the growing sector before they retire. Second, a faster transition increases the mass of workers who benefit on the extensive margin, but generates a thicker tail of losses on the intensive margin.

A few additional remarks are in order. First, the qualitative insights developed here do *not* depend on workers' expectations about the future. Indeed, workers base their labor supply decisions on current MRPLs (or wages) in our frictionless benchmark. Second, our conceptual results also go through if the government changes the end point of the transition, as opposed to its speed. In both cases, incumbent workers are exposed to a larger shift in MRPLs over their lifetime. Third, this paper studies sectoral shifts, i.e., scenarios where one sector rises at the expense of another. This scenario is the natural case to think about job displacement and captures numerous transitions in practice. Still, one could imagine *another* form of transitions where one sector takes off without affecting the other, so that no worker is ever displaced. Our non-linearity result still carries through in this case (Appendix A). But, by construction, there cannot be any non-monotonicity, since no worker ever loses from that alternative transition.

Finally, note that the boundary of firms is not defined in this model. Therefore, $k = C, D$ really represent *technologies* — not necessarily sectors. In particular, the two technologies might co-exist in the same industry or firm, e.g., the same car manufacturer might produce ICEVs and EVs. The paths of A_t^k reflect firms' incentives to operate one technology or the other. In turn, the relative skill $\hat{\psi}$ reflects workers' comparative advantage at operating one or the other. Workers with a high $\hat{\psi}$ are more likely to operate the C technology, either by switching between employers or by being re-assigned to the new technology by their current employer.[19] In both cases, technology adoption is disruptive for workers. Empirically, Braxton and Taska (2023) find that technology adoption leads to displacement and substantial earnings losses for incumbent workers who do not possess the right skills. Kogan et al. (2020) document a higher separation risk in industries that innovate more, as workers' skills become obsolete more rapidly. This dynamic also plays out in the climate context: environmental regulations lead to layoffs (Greenstone, 2002; Walker, 2011) and green jobs tend to require more cognitive skills and experience (Consoli et al., 2016; Marin and Vona, 2019).[20]

[19] Our full model accounts for the fact that these switches are costly for the firm and worker. For instance, the firm must train the worker on a specific production process. Training costs are a large component of hiring costs in practice (Muehlemann and Strupler Leiser, 2018) and are incurred both when hiring outside the firm or re-assigning a worker within the firm. Similarly, workers must typically apply for jobs and wait to be matched, both for different firms and for openings in a different department or plant.

[20] For instance, ICEVs use many components that are not present in EVs and vice versa, such as catalytic converters, fuel tanks, engines, and batteries, which all rely on specific skills. The European Association of Automotive Suppliers projects half a million jobs will be lost from an EV transition, while the Economic Policy Institute similarly projects large layoffs in the US. Layoffs and retraining are already occurring at Ford and Volkswagen (accessed July 2, 2025).

5 The Role of Labor Market Frictions

To derive clear analytical insights, our frictionless benchmark abstracted from important empirical features. In particular, there was no proper job destruction or creation over the transition; and workers did not experience unemployment or the scarring that comes with it, both of which are crucial to explain earnings losses after displacement (Jacobson et al., 1993; Huckfeldt, 2022). Accordingly, we now augment the labor market block of our model by introducing costly vacancy creation, job ladders, and skill accumulation. We show that accounting for labor market frictions is crucial to quantify not only the level of earnings losses from displacement, but also how they *change* non-linearly with the transition speed.

5.1 A Realistic Model of the Labor Market

Skill accumulation. Workers have multi-dimensional skills: a general skill $\left(\psi^G\right)$ and skills that are specific to each sector $\left(\psi^D, \psi^C\right)$. Workers' idiosyncratic productivity in the declining and growing sectors are thus $\psi^G\psi^D$ and $\psi^G\psi^C$, respectively. At birth, workers are endowed with $\psi^G = \psi^D = 1$, and draw an initial $\psi^C \sim G$. The general skill accumulates at rate $\delta_j^+ \geq 0$ when the worker is employed in sectors D or C, at rate $\delta_j^O \geq 0$ when the worker claims the outside option, and $\delta^U \leq 0$ when unemployed. Specific skills grow at rate $\hat{\delta} \geq 0$ when employed in the corresponding sector. As before, we define a workers' reference skill as their skill in the declining sector $\psi \equiv \psi^G\psi^D$ and their relative skill as $\hat{\psi} \equiv \psi^C/\psi^D$. Workers' productivity $h\left(\psi, \hat{\psi}, k\right)$ is still defined as in equation (2.2).

Labor services. There is a continuum of atomistic firms in each sector, which can hire one worker each to produce labor services. Each atomistic firm produces an amount of labor services equal to the productivity $h\left(\psi, \hat{\psi}, k\right)$ of the worker they hire. They sell these labor services at price $\mathcal{P}_t^k$ to firms in their sector, which use them to produce varieties of the corresponding intermediate good. The total labor units available in sector k are $n_t^k = \int h\left(\psi, \hat{\psi}, k\right) d\mu_t^k\left(\psi, \hat{\psi}\right)$ where $\mu_t^k\left(\psi, \hat{\psi}\right)$ is the measure of workers employed in sector k and period t with skills $\left(\psi, \hat{\psi}\right)$.

Directed search and matching. Workers can become unemployed and experience earnings scarring, as they slide off the job ladder and dis-accumulate skills while unemployed. We formalize this idea by assuming that workers and firms producing labor services search for each other in frictional labor markets. Search is directed, as in Menzio and Shi (2011), both within and across sectors. Workers can search on-the-job, giving rise to a job ladder. Specifically, labor markets are indexed by a worker's age j, their skills $(\psi, \hat{\psi})$, the sector k of the job, and the wage rate w that workers and firms agree on throughout the duration of the match. We abstract from renegotiation in w since wages rarely fall on-the-job in practice (Grigsby et al., 2021) and discussions about pay cuts rarely take place even if they could save jobs (Davis and Krolikowski, 2025).[21] The wage rate w is paid for each unit of effective labor $h(\psi, \hat{\psi}, k)$ that workers supply, and effective wages grow on-the-job at the rate g that prevails along a balanced-growth path.[22] Employed workers therefore earn $wh(\psi, \hat{\psi}, k)\exp(gt)$ and firms claim the rest of revenues.

A constant returns to scale matching function yields a job-finding probability for workers $f(\theta)$ within each market, where θ is the market tightness defined as the ratio of vacancies to job-seekers, and f is increasing and concave. The corresponding job-filling probability for firms is $q(\theta) \equiv f(\theta)/\theta$ which decreases with θ. Workers search for jobs with intensity $\lambda^U > 0$ off-the-job and $\lambda^E > 0$ on-the-job, giving rise to a job-ladder in the declining and growing sectors.

We allow for imperfectly directed search to rationalize the fact that job switches often entail wage cuts (Sorkin, 2018) and that there are gross flows across occupations and sectors (Pilossoph, 2014). Specifically, we suppose that a worker of type $\mathbf{x} \equiv (j, \psi, \hat{\psi})$ in market $s \equiv (k, w)$ searches in a market s' with probability

$$p_t(s'; \mathbf{x}, s) \equiv \frac{\exp\left(f\left(\theta_t(\mathbf{x}, s')\right)\left[W_t(\mathbf{x}, s') - W_t(\mathbf{x}, s)\right]/\zeta\right)}{\sum_{\hat{s}} \exp\left(f\left(\theta_t(\mathbf{x}, \hat{s})\right)\left[W_t(\mathbf{x}, \hat{s}) - W_t(\mathbf{x}, s)\right]/\zeta\right)}, \tag{5.1}$$

where the sum over $\hat{s}$ includes all sectors $k = D, C$ and wages $w \in \{w_1, \ldots, w_N\}$ as well as the outside option O and unemployment U.[23] Workers are more likely to search in a given market or option s' if the expected improvement in their value is larger. This search behavior can be microfounded by introducing idiosyncratic amenity shocks that are distributed i.i.d. according to a Type-1 Extreme Value distribution with scale parameter ζ (Artuç et al., 2010) or by assuming that workers

[21] A typical job tenure is much shorter than the duration of a transition. Wage rigidities are important to explain earnings losses from sectoral reallocations (Rodriguez-Clare et al., 2025).

[22] This assumption ensures that w admits a stationary distribution.

[23] We suppose that workers only search in markets with positive vacancies so as not to "spend" their search effort on markets that they have no chance of finding a job in.

have to acquire costly information about the jobs that they apply to (Matêjka and McKay, 2015).[24] The parameter ζ captures the degree to which search is directed: search is purely directed as $\zeta \to 0$ as in canonical directed search, and purely random when $\zeta \to \infty$.[25] Matches are destroyed exogenously at rate $\chi > 0$.

Firm entry. We let $V_t\left(j, \psi, \hat{\psi}, s\right)$ denote the value of a firm employing a worker of age j with skills $\left(\psi, \hat{\psi}\right)$ in market $s = (k, w)$. Firms can post vacancies for a worker with reference skill ψ at a cost $\nu\psi \exp\left(gt\right) > 0$, where ν captures both the vacancy and hiring cost.[26] This cost grows at rate g to ensure that the economy admits a balanced-growth path. There is free entry in each market s so that

$$q\left(\theta_t\left(j, \psi, \hat{\psi}, s\right)\right) V_t\left(j, \psi, \hat{\psi}, s\right) \leq \nu\psi \exp\left(gt\right) \tag{5.2}$$

in every period t, with equality if firms post a positive measure of vacancies.

Outside sector and unemployment. The rest of the economy is unaffected by the transition. Workers receive an income of $\psi b^O \exp\left(gt\right)$ of final goods when claiming the outside option, and $\psi b^U \exp\left(gt\right)$ when they are unemployed, with $b^O, b^U \geq 0$. We think of the outside option as capturing self-employment and jobs with limited scope for professional progression that workers initially attached to declining sectors can easily transition to. Accordingly, we assume that workers can freely transition to the outside option and unemployment so $f^O = f^U = 1$ conditional on search (5.1) and we abstract from the job-ladder in the outside option. New cohorts ($\tau > 0$) are born unemployed.

Equilibrium. We assume that firms and workers have rational expectations. We state workers' and firms' values recursively and define an equilibrium in Appendix B. We also establish that firms' and workers' values are linear in $\psi \exp\left(gt\right)$, which allows us to de-trend the model using the growth rate g as in Section 3. This

[24] The first formulation would increase workers' values by allowing them to enjoy job-specific amenities. On the contrary, the second would decrease them by assuming that workers have to acquire costly information. We thus follow Traiberman (2020) and do not include the amenity value or information cost in workers' values. These values can therefore be directly interpreted as workers' lifetime earnings.

[25] Firms know the characteristics of the worker they are matching with, which preserves the block-recursive nature of the equilibrium (as in Menzio and Shi, 2011).

[26] Workers' accumulate the skill ψ on-the-job over their lifecycle. Assuming that the vacancy posting cost scales with ψ captures the fact that searching for a junior employee is cheaper compared to a more experienced, senior manager.

Table 5.1: External calibration

Parameter	Description	Value	Source
J	Lifespan	40	Career length
ρ	Discount rate	0.03	3% interest rate
α	Elast. of matching function	1.6	Schaal (2017)
δ_j^+	General skill accum. (D, C)	Age-specific	CPS ORG
δ^U	General skill accum. (U)	-0.096	Schmieder et al. (2016)
δ^O	General skill accum. (O)	0	Earnings grow at rate g

property also implies that it is without loss of generality to suppose that workers draw $\psi = 1$ at birth, and allows us *not* to track ψ as an explicit state variable in our numerical implementation, making the simulation and estimation of the model computationally manageable. Given paths of MRPLs across sectors, the labor market part of the model satisfies the usual block recursive property of directed search models (Menzio and Shi, 2011), which also allows us to solve it efficiency. Workers now face idiosyncratic risk and maximize expected utility. We continue to assume that workers are risk neutral. Workers' search decisions, and hence the distributional impact of the transition on earnings, would be the same if workers were risk-averse ($\sigma > 0$) but faced complete markets for idiosyncratic risk.

Finally, note that the frictionless benchmark of Section 2 obtains as a limit of our full model when vacancy posting is free ($\nu \to 0$), search is perfectly directed ($\zeta \to 0$), unemployment pays less than employment ($b^U \leq A_\infty^D$) and skills are fixed over workers' lives ($\delta_j^+ = \delta_j^O = \delta^U = \hat{\delta} = 0$). Our implementability result from Section 3.2 (Proposition 1) holds in our richer model, as we prove in Appendix B.6.

5.2 Calibration

External Calibration. Table 5.1 summarizes the external calibration. Workers live for $J = 40$ years and discount the future at rate $\rho = 3\%$.[27] Following Den Haan et al. (2000), the matching function is $f(\theta) = (1 + \theta^{-\alpha})^{-1/\alpha}$ with $\alpha = 1.6$ as in Schaal (2017). We directly measure the skill accumulation rate $\delta_j^+ + \hat{\delta}$ in the Current Pop-

[27] This is the effective discount rate net of the growth rate g along a BGP (see Appendix B.4).

Table 5.2: Internal calibration

Parameter	Description	Value	Moment	Source	Target	Model
Search						
λ^U	Search intensity (U)	5.973	UE rate (M)	Shimer (2005)	25%	25%
λ^E	Search intensity (E)	1.185	EE rate (M)	Fujita et al. (2024)	2.5%	2.5%
ζ	Search directedness	0.035	Share of wage cuts	Grigsby et al. (2021)	30%	30%
Job creation & destruction						
ν	Vacancy post. cost	0.437	Average markdown	Berger et al. (2022)	20%	20%
χ	Job destruction rate	7.04%	Unemployment rate	-	5%	5%
Benefits and Skill						
b^O	Benefit in O	0.642	Wage change (EUO)	CPS ORG	-16%	-16%
b^U	Benefit in U	0.343	Average UI replac.	US UI rules	45%	45%
$\hat{\delta}$	Specific skill accum.	0.12%	Net realloc. by age	CPS	–	–

ulation Survey (CPS) using on-the-job wage growth.[28] We let $\delta_j^U = -0.096$, corresponding to a 0.8% loss in human capital per month of unemployment (Schmieder et al., 2016). Finally, we set $\delta_j^O = 0$ so workers' income increases at the same rate at which the economy grows when they claim the outside option.

Internal Calibration. We calibrate the remaining parameters to match various features of US labor markets. Table 5.2 summarizes the internal calibration. We calibrate the search intensity for unemployed workers λ^U by targeting a monthly UE rate of 25% (Shimer, 2005), and the one for employed workers λ^E by matching a monthly EE rate of 2.5% (Fujita et al., 2024). We pick the degree of noise in search ζ so that 30% of job-switchers experience a wage cut (Grigsby et al., 2021; Sorkin, 2018). We obtain a scale parameter of 0.035, which is small relative to the average wage but still generates gross flows in line with the data (see Section 6).

Firms' markdowns determine firms' profits and hence their incentives to create vacancies during the transition. The vacancy-posting cost ν determines the barriers to entry in the labor markets and hence the average markdown: a higher ν will imply high markdowns in equilibrium by the free entry condition (5.2). We target an average markdown of 20% (Berger et al., 2022) and obtain $\nu = 43.7\%$ of annual average output, which lies between the values in Shimer (2005) and Hagedorn and Manovskii (2008). We choose the separation rate χ to match a steady state unemployment rate of 5%. We set the unemployment benefit b^U to obtain an average

[28] With an eye towards our application in Section 6, we measure this wage growth for workers attached to polluting sectors before the start of the climate transition.

replacement rate of 45% over workers' wage before separation, reflecting a mix of home production and unemployment insurance.

The two remaining parameters are the flow payoff of the outside option b^O and the rate at which workers accumulate specific skills $\hat{\delta}$. These two parameters are application-specific and depend on which occupations and sectors are affected by the transition. To illustrate the mechanics of the model, we set these parameters to the values that we calibrate internally in our climate application. We obtain $b^O = 0.642$ and $\hat{\delta} = 0.12\%$ in this case, as we explain in Section 6.

5.3 The Role of Labor Market Frictions

The left panel of Figure 5.1 reproduces the analysis of Figure 4.2, using our full model. We plot lifetime earnings changes in 2025 as a function of the transition horizon, relative to the status quo transition ending in 2060, for workers with various relative skills $\hat{\psi}$ in 2025 and 20 years of remaining work life.[29] We reach the same qualitative conclusion as in Section 4: workers' outcomes are strongly non-linear and even non-monotonic in the transition horizon.

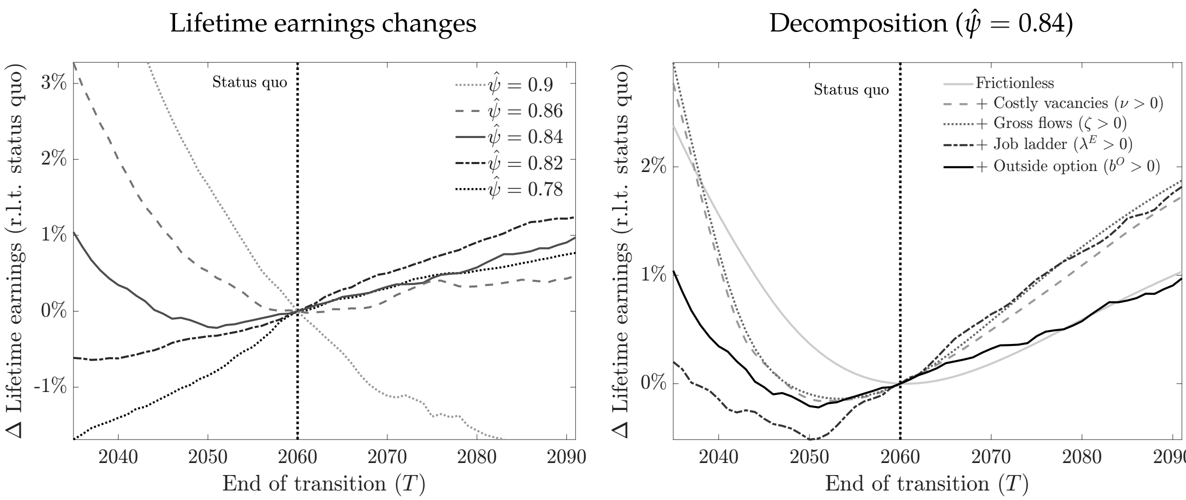
At the same time, there are obvious *quantitative* differences between our full model and its frictionless counterpart. The right panel of Figure 5.1 illustrates these differences by plotting lifetime earnings changes in the frictionless model with skill accumulation (solid gray line) and in our full model (solid black line), focusing on a worker with $\hat{\psi} = 0.84$ and 20 years of remaining work life. This worker experiences much lower earnings gains in our full model as the transition accelerates, compared to its frictionless counterpart. In other words, the additional labor market frictions are important to quantify the implications of a faster transition and the associated non-linearities.

To understand which features of the labor market explain these differences, we introduce one feature after another — namely, costly vacancy creation, gross flows, job-ladders, and the outside option — starting with the frictionless model, until we reach our full model. As we add each additional feature, we recalibrate all other parameters in order to match the moments described in Section 5.2.[30] We

[29] Since workers can be employed in different markets s, we must condition on some market to construct this plot. We look here at workers who are unemployed at the beginning of 2025. Appendix Figure E.2 averages instead over s using the 2025 equilibrium employment distribution in our quantitative application of Section 6.

[30] Table C.1 in Appendix C.4 describes these successive calibrations.

Figure 5.1: The role of labor market frictions



Notes: The left panel plots workers' average lifetime earnings changes in 2025 in our full model as a function of the transition horizon T, for various values of relative productivity $\hat{\psi}$. Earnings changes are relative to the status quo transition ending in 2060, and transitions are parameterized as in equation (3.3). All workers have 20 years remaining in their careers. We consider workers who are unemployed as of January 2025. The right panel decomposes these earnings changes for a worker with relative productivity $\hat{\psi} = 0.84$. The solid gray line corresponds to our frictionless benchmark from Section 2 augmented with skill accumulation. The other lines add one feature after another until we obtain our full model corresponding to the solid black line.

first describe how these features affect workers' lifetime earnings changes, before discussing the exact mechanisms.

First, we introduce costly vacancy creation ($\nu > 0$), which corresponds to the dashed gray line. Because of these costs, firms post a finite measure of vacancies and workers can experience unemployment. In this scenario, workers search only when they are unemployed ($\lambda^U > 0$); they do not search on-the-job, their search is perfectly directed, and we abstract from the outside option ($\lambda^E = 0, \zeta \to 0, b^O = 0$). Compared to the frictionless model, lifetime earnings gains are lower as the transition speeds up starting from 2060; however, these gains build up much faster as the transition accelerates further and the non-linearities are more pronounced. Second, we introduce partially directed search ($\zeta > 0$), which corresponds to the dotted gray curve. While gross flows are substantial in the model and in the data (see Section 6), their impact on earnings is relatively modest as the transition accelerates.[31] Third, we introduce on-the-job search ($\lambda^E > 0$) which gives rise to a

[31] Still, accounting for gross flows is important because they help the model generate large monthly EE flows and realistic dispersion in wages once we introduce on-the-job search. Generating a

job-ladder, corresponding to the dash-dotted gray curve. Earnings gains are much lower in this case as the transition accelerates. Finally, we introduce the outside option ($b^O > 0$) in the solid black curve, obtaining our full model. As expected, the presence of this outside option mitigates the losses associated with a rapid transition and therefore raises the expected earnings gains. Overall, costly vacancy creation and the existence of a job-ladder are particularly important to quantify workers' earnings changes as the transition accelerates — and the non-linearities and non-monotonicities associated with it.

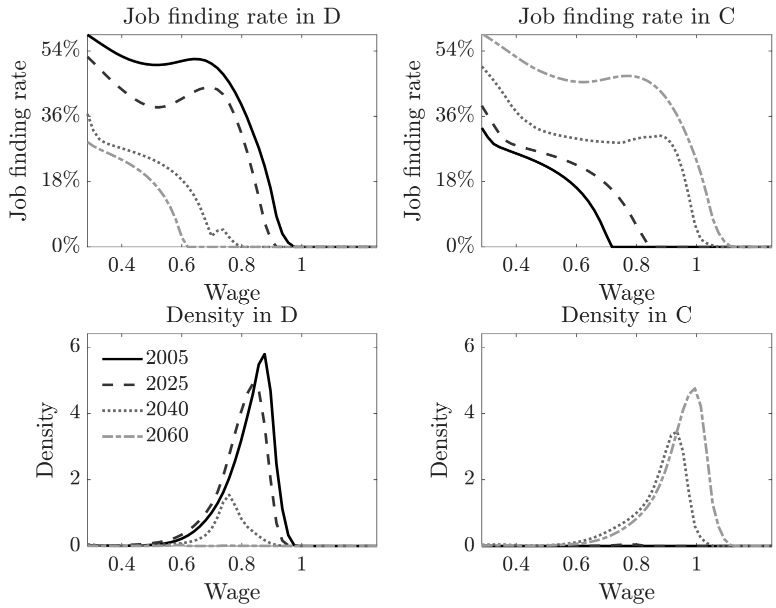
To understand why costly vacancies and the job-ladder generate such strong non-linearities, Figure 5.2 plots the evolution of the job finding rate (first row) and the wage distribution (second row) in our full model at various points of the status quo transition for a worker with $\hat{\psi} = 0.84$. The first column corresponds to the declining sector D and the second corresponds to the growing sector C. As the MRPL falls in the declining sector D, firms create fewer vacancies there and the job-finding rate plummets. The opposite happens in the growing sector C. Workers leave sector D gradually to join sector C. Over time, the wage distribution shifts to the left in the declining sector, and to the right in the growing sector. Accelerating the transition causes these shifts to occur sooner.[32]

Figure 5.2 illustrates the role that costly vacancy creation plays in shaping workers' earnings changes. Creating a vacancy is costly ($\nu > 0$), both in the model and in the data, and firms only post vacancies if they expect to recoup these costs. As the transition unfolds or accelerates, the job finding rate decreases immediately in sector D. However, it takes time to rise in sector C since the MRPL is too low initially to recoup the hiring cost: at first, firms do not post *any* vacancies in the growing sector at wages comparable to those that prevailed originally in the declining sector. Unemployed workers thus face an unpleasant choice: they can either be structurally unemployed while searching for jobs that are now difficult to find, or accept large wage cuts from the few jobs that are posted for them. Either way, they experience lower incomes — explaining why the earnings gains are lower at first on the right panel of Figure 5.1 as the transition accelerates, compared to the frictionless model. As the transition accelerates further, the MRPL finally rises to a point where firms in sector C find it profitable to post high-wage jobs before

realistic job-ladder is essential to quantify the scarring effect of unemployment spells.

[32] Of course, workers and firms anticipate the future evolution of the sectoral MRPLs in this frictional model when making their labor supply and job posting decisions.

Figure 5.2: Job finding rates and the wage distribution



Notes: The first row plots the job finding rate as a function of the wage posted by firms, in various periods under the status quo transition, for workers born with relative productivity $\hat{\psi} = 0.84$. The second row plots the equilibrium wage distribution, for the same periods and the same productivity. The first column corresponds to the declining sector, and the second to the growing sector.

workers retire — generating the strong non-linearities observed in Figure 5.1.

Figure 5.2 also illustrates the importance of on-the-job search ($\lambda^E > 0$) and the job-ladder that it induces. Before the transition begins, workers climb the ladder in sector D and slowly make their way up to the top of the wage distribution until they get separated again. As the transition unfolds or accelerates, firms stop posting these high-wage jobs in the declining sector. As a result, the transition not only reduces the median wage offered in sector D; it also trims the top of the job ladder, further depressing the average wage in that sector. The faster the transition, the sooner the job-ladder collapses in the declining sector — generating lower earnings gains on the right panel of Figure 5.1.

So far, we have focused on a particular worker with $\hat{\psi} = 0.84$. Appendix Figure E.1 considers workers with different relative skills. The effect of costly vacancy creation ($\nu > 0$) is larger for workers with lower productivity $\hat{\psi}$, since they are less likely to be hired in the growing sector C. The outside option ($b^O > 0$) mostly benefits workers with lower productivity $\hat{\psi}$, who are most likely to claim this outside option in order to avoid longer unemployment spells or large wage cuts. Finally, the collapse of the job ladder ($\lambda^E > 0$) matters most for workers with intermediate productivity, e.g., $\hat{\psi} = 0.84$, since they have a relatively hard time finding a job in the growing sector and are also less likely to claim the outside option.

In summary, our full model offers a third conceptual insight: accounting for labor market frictions is important to quantify how the earnings distribution changes with the horizon over which a transition unfolds, and the associated non-linearities and non-monotonicities. These frictions generate longer unemployment spells, large wage cuts, and a collapse of the job-ladder in the declining sector.

6 Application to the Climate Transition

Having developed general insights that apply to various sectoral shifts, we now present a quantitative application to the climate transition. This application is particularly relevant for today's policymakers: the climate transition carries direct environmental benefits, providing a motive to accelerate it; at the same time, there are 10-15 million workers currently employed in polluting industries who are at risk of displacement; furthermore, most of the transition has yet to happen, and the government can affect its pace using various taxes and subsidies. Appendix D provides more details on the empirical component of our analysis.

6.1 Sector Definitions and Paths of MRPLs

Sector definitions. We interpret the growing sector C as "clean" industries, and the declining sector D as polluting or "dirty" industries. Following OECD (2024), polluting industries are those in the top decile by emissions intensity. In turn, "clean" industries either produce green goods (Bontadini and Vona, 2023) or involve green tasks (Granholm, 2024).

Paths of A_t^k. Our climate application is a *prospective* analysis: it relies on assumptions about the trajectory of the US economy several decades in the future. We use existing projections available in the literature, recognizing that they do not always agree with one another. As before, we suppose that the transition begins in 2005, that the government can intervene starting in 2025, and that the status quo transition is complete by 2060. We must next choose a *shape* for the path of MRPLs, i.e., calibrate a function $\Phi(t)$ which describes the share of the transition that is completed as a function of calendar time.[33] As we argued in Section 3.1, the model naturally generates S-shapes which are characteristic of innovation and technology adoption. We show in Figure D.1 in Appendix that this shape describes the data well. Specifically, we plot realized data and forecasts from the U.S. Energy Information Administration (EIA) on the share of power generated by fossil fuels and renewable sources between 2005 and 2050.[34] We estimate a truncated normal CDF with mean μ and standard deviation σ, which fits the data well.

We next choose the final *levels* that the de-trended MRPLs converge to. There is substantial empirical uncertainty about long-run outcomes that have yet to occur. We therefore test the sensitivity of our results under a menu of different scenarios. Our baseline scenario assumes that, in the long-run steady state associated with zero net carbon, the MRPL in the polluting is driven down to its lowest value corresponding to the outside option ($A_\infty^D = 0.84$); and that changes in MRPL are symmetric across sectors in the long-run ($A_\infty^C = 1.16$), which is consistent with the view that the transition should not affect long-term unemployment meaningfully (Hafstead and Williams, 2018; Causa et al., 2024).[35] We present a battery of sensitivity analyses and find that the main takeaways are consistent across scenarios.

6.2 Estimating the Skill Distribution

Applying our framework to a particular episode requires one additional piece of information relative to the steady state calibration of Section 5.2: the distribution of relative skills $\hat{\psi}$. In our model, this distribution is determined by two objects: the distribution of skill endowments at birth $G\left(\hat{\psi}\right)$, and the subsequent rate at which workers accumulate specific skills $\hat{\delta}$. We now estimate these objects by matching workers' reallocation in the phase of the transition for which we have data (2005–2025), given the paths of MRPLs A_t^k that prevail under the status quo transition.

Identification. We identify the distribution of skill endowments non-parametrically by matching the observed labor reallocation between the polluting and clean sectors over the period 2005–2025. The idea is best understood by inspecting the

[33] Due to the lack of available data, we assume that this function is common across sectors, i.e., the declining and growing sectors converge to their new respective BGPs with the same half-life.

[34] We use the *production* share as a proxy for relative labor demands in the clean and polluting sectors. The underlying assumption is that the labor intensities are comparable in the two sectors, which appears consistent with the data (Weng et al., 2024).

[35] Decreasing returns ensure that the sectoral MRPLs would never go below b^O absent gross flows.

left panel of Figure 4.1. In our frictionless benchmark, workers switch sectors if and only if $\hat{\psi} \geq A_t^D / A_t^C$, so the share of workers who have switched by period t is $1 - G\left(A_t^D / A_t^C\right)$. We can therefore use changes in the share of workers who have reallocated over time to trace the distribution $G\left(\hat{\psi}\right)$ non-parametrically. Of course, the process of labor reallocation is richer in our full model: workers have different employment statuses, they accumulate skills, there are job ladders, large gross flows, etc. However, the same idea applies: workers endowed with a larger $\hat{\psi}$ are more likely to switch earlier to clean industries, allowing us to estimate the distribution G based on reallocation flows.

To disentangle the distribution of endowments G from the skill accumulation rate $\hat{\delta}$, we compare two cohorts born in different periods. Older workers have spent more time accumulating specific skills in polluting industries by 2005. As a result, they are less likely to reallocate to the clean sector, as in the data.[36]

Discussion. One possible concern is that, in our frictionless model, workers with skill $\hat{\psi}$ below a certain cutoff do not reallocate by 2025, so the distribution may not be identified below this cutoff. The presence of noisy search and substantial gross flows circumvents this issue, as workers have a positive probability of reallocating even below this cutoff. Of course, workers endowed with extremely low $\hat{\psi}$ — much lower than the frictionless cutoff — reallocate with vanishingly small probability even by 2060. We assume that these workers have $\hat{\psi} = 0$, since they remain in polluting industries regardless. Our approach therefore consists of estimating the *mass* of "never switchers" with $\hat{\psi} = 0$ and the distribution G *conditional on* $\hat{\psi} > 0$. The conditional distribution allows us to match workers' reallocation over the period 2005–2025, while the mass of never switchers helps us match long-run projections of relative employment in clean industries. As shown below, the estimation of G puts positive probability only on parts of the support that are well-identified — even values of $\hat{\psi}$ that are well below the frictionless cutoff.

Note that $\hat{\psi}$ captures any reason why a worker's productivity may differ in clean and polluting industries, such as the relative productivity of a worker's re-

[36] An alternative approach would be to target wage changes, as opposed to employment changes. The issue with this approach is twofold. First, wage changes *conditional on switching* are not a particularly informative moment — they are exactly zero in the frictionless model, for instance. Second, average sectoral wages are subject to strong composition effects in the data, e.g., workers' relative skill $\hat{\psi}$ correlates with their reference skill ψ. Addressing these compositional effects using our model would require tracking workers' reference skill ψ over time, making our analysis infeasible numerically. We do not currently have to track this variable as explained in Section 5.1.

gion for clean industries, e.g., a coal-rich region or a sunny desert. While the evidence suggests that migration of incumbents is mostly unresponsive to local labor demand shocks (Autor et al. 2025; Greenland et al. 2019), future work can extend our analysis by introducing a richer geography.[37] Finally, in Appendix D, we explain why our conclusions hold when workers reallocate not only based on earnings, but also on the amenity value of supplying labor to clean industries.

Estimation. The estimation problem consists of choosing a distribution G and a rate of specific skill accumulation $\hat{\delta}$ to solve

$$\min_{\{G,\hat{\delta}\}} \sum_{\tau} \int_{2005}^{2025} \left(z_{\tau,t}^{\text{data}} - \int_{0}^{\infty} z_{\tau,t}^{\text{model}} \left(\hat{\psi}^{\text{endow}}; \hat{\delta} \right) G \left(\mathrm{d}\hat{\psi}^{\text{endow}} \right) \right)^{2} dt, \tag{6.1}$$

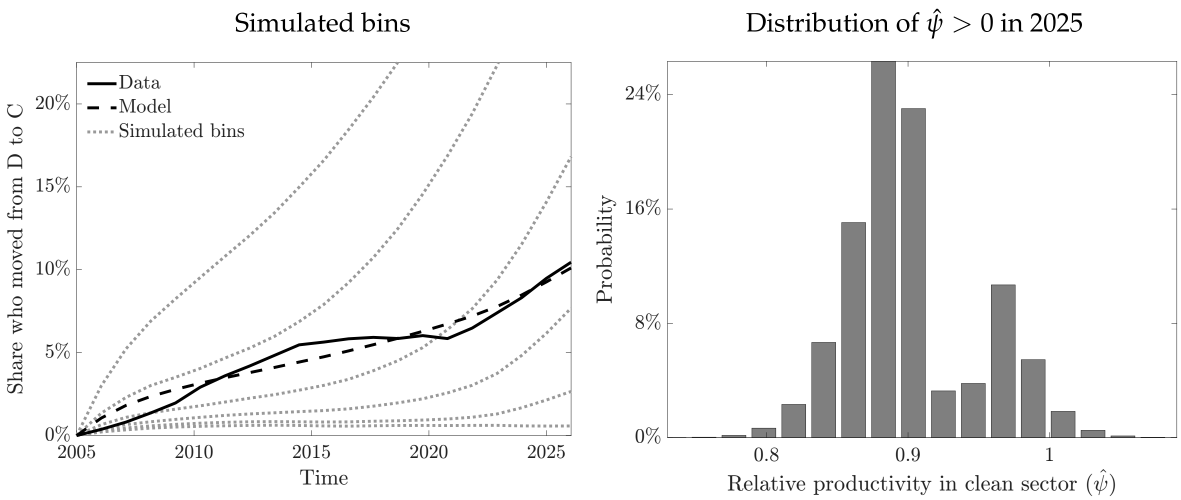
where $z_{\tau,t}^{\text{data}}$ is the empirical share of workers in cohort τ who have reallocated from polluting to clean industries between period 0 and t, while $z_{\tau,t}^{\text{model}} \left(\hat{\psi}^{\text{endow}}; \hat{\delta} \right)$ is the same share computed in the model among workers in that cohort who were endowed with relative skill $\hat{\psi}^{\text{endow}}$ at birth. The model-induced series depend on the skill accumulation rate $\hat{\delta}$. Empirically, we measure $z_{\tau,t}^{\text{data}}$ in the CPS as the share of workers initially attached to polluting industries who reallocated to a clean industry. By definition, a worker is initially "attached" to polluting industries if they are employed there for four consecutive months during their first CPS rotation.[38] As discussed throughout this paper, we focus on these workers because they are the most at risk of displacement by the climate transition and the most likely to oppose it, so predicting their outcomes and understanding how expensive it would be to compensate them is of the utmost importance.

We leverage the block recursivity of our directed search model (Menzio and Shi, 2011) to solve the problem (6.1) efficiently. This property allows us to solve for workers' optimal reallocation $z_{\tau,t}^{\text{model}} \left(\hat{\psi}^{\text{endow}}; \hat{\delta} \right)$ separately for each value of $\hat{\psi}^{\text{endow}}$ and *independently* from the distribution G. We then choose the weights G to assign to these optimal policies in order to match workers' reallocation over time.

[37] Monetary migration costs are absorbed in our estimates of the $\hat{\psi}$ distribution. As we will soon discuss, we estimate large costs of switching from D to C — on the order of 10% of lifetime earnings or \$114,000 — which is much larger than the typical monetary costs involved in moving, justifying the interpretation that $\hat{\psi}$ largely reflects productivity differences.

[38] The CPS does not allow us to track workers over long periods. We therefore weight individuals in later waves by the probability that they would have been attached to the polluting sector in the pre-2005 period according to their observables (see Appendix D for details).

Figure 6.1: Estimation of the skill distribution



Notes: The left panel plots the share of workers initially attached to polluting industries who moved to clean industries. The solid curve is the data from the CPS (see Appendix D). Workers are weighted by the probability that they are attached to the polluting sector based on observables in the data. We pool the two cohorts used for the estimation (aged 25-30 and 35-40 in 2005) for legibility. The dotted curves correspond to simulated bins associated to various values of $\hat{\psi}^{\text{endow}}$ under the estimated $\hat{\delta}$. The dashed curve corresponds to the model-generated reallocation when using the estimated distribution of $\hat{\psi}^{\text{endow}}$. The right panel plots the estimated cross-sectional distribution of relative productivity $\hat{\psi}$ in the population in 2025.

We focus on workers aged 25 to 40 when the transition began in 2005, since these cohorts have not retired yet by 2025. We repeat this experiment for various values of $\hat{\delta}$ and keep the one that best predicts the reallocation across different cohorts.

We illustrate our approach on the left panel of Figure 6.1. The solid black line corresponds to the empirical reallocation of the workers initially attached to polluting industries.[39] Their reallocation is relatively slow at first and picks up towards the end of the period. Roughly 10% of these workers had reallocated to clean industries by 2025. The overall timing of the reallocation and the relatively small annual reallocation rates in this early phase of the transition are consistent with existing findings in the literature (Curtis and Marinescu, 2023).

The dotted curves in this plot correspond to the model-implied net reallocations associated with different bins of relative skill endowment $\hat{\psi}$ at birth. The

[39] The CPS has less granular industry codes than 6-digit NAICS, which prevents us from distinguishing ICEV and EV production, for instance. We therefore attribute a share of dirty manufacturing employment to clean manufacturing over time in proportion to the share of cars sold which use alternative fuels, i.e., a proxy for clean technology uptake in the economy. This correction also allows us to capture technology adoption within industries over time.

estimation problem consists of choosing the weights on these dotted curves to best match the empirical series. This step can be achieved efficiently by projecting the empirical reallocation onto the reallocation paths predicted by each bin, using an OLS regression that constrains the coefficients to add up to 1. We plot reallocations associated with values of $\hat{\psi}$ that fall in the range that the estimation puts positive probability on. The levels and shapes of these curves are different from one another and do not overlap, allowing us to identify the mass assigned to each skill endowment $\hat{\psi}$. This procedure backs out the distribution G conditional on $\hat{\psi} > 0$. In turn, we estimate the mass of never switchers with $\hat{\psi} = 0$ by matching a ratio of employment in clean to polluting industries of 2:1 along the final BGP, which lies in the range spanned by McKinsey & Company (2022), IRENA (2023) and Mayfield et al. (2023). The reallocation implied by our model matches the data well, as depicted by the dashed line on the left panel of Figure 6.1.

We repeat this exercise for various values of the skill accumulation rate $\hat{\delta}$ to best match the reallocation for cohorts aged 25-30 years and 35-40 years in 2005. Figure E.3 in Appendix plots the model's fit of reallocation by cohort.

The right panel of Figure 6.1 plots the cross-sectional distribution of relative skills $\hat{\psi}$ induced by our estimates of G and $\hat{\delta}$ in 2025, conditional on $\hat{\psi} > 0$. Our estimates suggest that most workers initially attached to polluting industries exhibit substantial skill specificity (i.e., $\hat{\psi} < 1$), reflecting the relatively slow reallocation observed in the early phase of the transition. This finding echoes existing evidence showing that skill requirements are different in clean and polluting industries and that workers lose specific human capital when leaving polluting industries (Walker, 2013; Consoli et al., 2016; OECD, 2024). The distribution also reveals two types of workers. There is a mass with relatively high productivity in the clean sector ($\hat{\psi} \geq 0.95$), corresponding to workers who start reallocating early in the data. There is also a larger mode at an intermediate value ($\hat{\psi} \simeq 0.9$), corresponding to workers who begin reallocating in later years. The estimation requires a mix between these two types to explain the slow start in labor reallocation and its recent uptick. Roughly 45% of workers are never-switchers with $\hat{\psi} = 0$. Overall, more than half of workers have a $\hat{\psi}$ between $\hat{A}_\infty^{-1} = 0.84/1.16$ and $\hat{A}_0^{-1} = 1$; these workers are the ones who, if the transition accelerates sufficiently, will reallocate in some $t > 0$ and experience non-monotonic earnings changes.[40] We obtain a

[40] This range does not account for endogenous skill accumulation. Below, we use our model to

skill accumulation rate of $\hat{\delta} = 0.12\%$ per year, which lies in the range spanned by Kambourov and Manovskii (2009).

Aside from the steady state moments discussed in Section 5.2, our model replicates many labor market features specific to the climate application. Our estimation ensures that the model matches the reallocation to clean industries from workers initially attached to polluting ones, both overall and by cohort.[41] In addition, net flows to the C sector are much smaller (less than 1% per year) than gross flows between sectors (around 2.5% per *month*), as in the data. In the model, the probability of an E-E switch from polluting industries to the rest of the economy, as captured by the outside option, is 0.3% per month in steady state and 0.5% in 2025. In the data, those numbers are 0.4% before the transition (1996–2005) and 0.5% in 2025. In summary, our model matches well the flows between C, D and O.

6.3 Earnings Losses from the Climate Transition

Losses under the status quo transition. We start by comparing workers' lifetime earnings in 2025 under the status quo transition ending in 2060 to those that would have prevailed if there had been no transition at all (i.e., $A_t^k = 1$ forever in both sectors).[42] The left panel of Figure 6.2 shows that most of the workers we study lose from the status quo transition (the gray area in Figure 4.1 is negative for them), reflecting that $\hat{\psi} < 1$ for these workers.

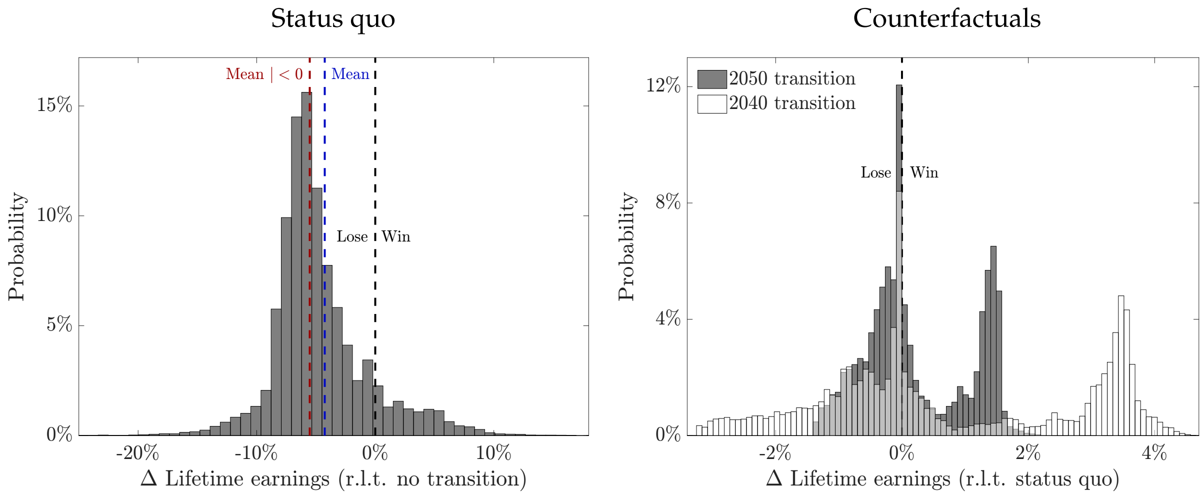
The average worker loses 4.3% of lifetime earnings as a result of the status quo transition. The average loss is 5.5% conditional on losing. Some workers experience substantially larger losses, on the order of 10–20% of remaining lifetime income. These figures reflect earnings changes *in expectation* (workers' values). Some workers experience even larger changes *ex post* if they are displaced and remain unemployed for a while. These large losses can explain why many workers oppose the climate transition (Dechezleprêtre et al., 2025).

quantify the share of workers who experience non-monotonicites for each transition horizon.

[41] Our model also does well at predicting the untargeted reallocation of cohorts born *after* the ones that we include in our estimation. In the model, the cohort born in 2015 is 5 p.p. more likely than the 2005 cohort to supply labor to clean industries 2 years after they enter the labor market (after they have had time to leave their initial unemployment state). In the data, this figure is 5 p.p. for the workers we focus on. In turn, these numbers are 11 p.p. in the model for the cohort born in 2020 and 9 p.p. in the data.

[42] We simulate the same workers from 2005–2025 under these two scenarios, subject to the same sequences of idiosyncratic shocks.

Figure 6.2: Distribution of expected lifetime earnings changes



Notes: The left panel plots the cross-sectional distribution of lifetime earnings changes in 2025 associated with the status quo transition ending in 2060, relative to no transition at all. The right panel plots lifetime earnings changes associated with faster transitions ending in 2050 or 2040, relative to the status quo transition.

For context, these losses are comparable to those estimated by Autor et al. (2016) due to exposure to Chinese imports, or Dix-Carneiro (2014) and Traiberman (2020) following trade liberalizations in Denmark and Brazil. Davis and von Wachter (2011) find that workers experience a reduction in lifetime earnings of roughly 18.6% if they lose their job during an economic downturn. In terms of earnings, the average loss of 4.3% that we obtain would be equivalent to a quarter of polluting-attached workers experiencing a job loss during a recession.

Compensating workers who lose from the transition would be relatively expensive. Given that average annual earnings in polluting industries was $80,000 in the Quarterly Census of Employment and Wages in 2025, the average worker initially attached to polluting-industries loses $51,860 in lifetime earnings due to the status quo transition. Multiplying this figure by the 9.7 million workers attached to these industries in 2025 implies a total cost of compensating these incumbent workers of $506 billion in present discounted terms,[43] which is equivalent to a $15.2 billion annual cost in perpetuity in our model.[44] For comparison, the U.S. Congress

[43] There are 11.1 million workers employed in polluting industries in 2025, but only 9.7 million are categorized as *attached* to these industries (Appendix D). The interest rate is 3%.

[44] These numbers account for workers' remaining work life and how it correlates with their lifetime earnings until retirement. Figure E.4 in Appendix reports the distribution of lifetime earnings changes for young and old workers separately.

budgets roughly half a billion per year for the Trade Adjustment Assistance for Workers (TAA), which assists workers who lost their job to foreign competition. While sizable, the numbers that we obtain are small relative to the social benefits of reducing carbon emissions estimated in the literature (Bilal and Känzig, 2025).

Counterfactuals. We now turn to our counterfactual analysis. The right panel of Figure 6.2 plots the distributions of earnings changes in 2025 from speeding up the transition such that it ends in 2050 (in gray) or 2040 (in white), relative to maintaining the status quo transition ending in 2060. Two main features stand out. First, earnings changes are indeed non-linear in the speed of the transition: the distribution associated with the 2040 transition is not simply a linearly "stretched" version of the one for the 2050 transition. In particular, the left tail gets thicker as the transition accelerates and a transition ending in 2040 leads some workers to lose an additional 2.5% of lifetime earnings relative to the status quo transition.

Second, a majority of workers would prefer the transition to end earlier than 2060. While a slight majority of workers (0.3% gap) would support a faster transition ending in 2050, this gap grows to 2.2% for the 2040 transition. Echoing the insight of Section 4.2, policymakers face a tradeoff between increasing the share of winners on the extensive margin, and generating a thicker tail of losers on the intensive margin. Overall, workers gain on average 0.3% in lifetime earnings from the 2050 transition relative to the status quo, and 0.8% from the 2040 transition.

Appendix E presents a number of additional results. First, Figure E.5 reports *average* changes in lifetime earnings by age, which increase non-linearly as the transition accelerates. Next, Figure E.6 plots the share of workers who experience U-shaped changes in lifetime earnings as the transition accelerates starting from 2060. In particular, 16% of workers who initially prefer a slower transition starting from 2060 would, in fact, prefer to speed up the transition further relative to 2050. Finally, Figure E.7 shows that the evolution of the job finding rates and wage distributions discussed in Figure 5.2 carry through in our application.

Sensitivity. We explore the sensitivity of our results in Figure E.8 in Appendix. Specifically, we consider alternative values for A^k_∞ and for long-run employment targets. The losses from the status quo and counterfactual transitions are largely unchanged once we re-estimate the distribution of relative skill endowments to match the realized reallocation over the period 2005–2025.

7 Conclusion

This paper studies how the pace of sectoral shifts affects workers at risk of displacement. We develop a life-cycle model with skill heterogeneity, costly hiring and job ladders in which labor demand gradually rises in one sector and declines in another. Our analysis reveals three novel insights. First, workers' lifetime earnings are strongly non-linear and even non-monotonic in the horizon over which the transition unfolds. Second, more and more workers benefit on the extensive margin as the transition accelerates, but the tail of losses becomes thicker on the intensive margin. Third, accounting for labor market frictions is important to quantify how earnings losses change non-linearly as the transition accelerates.

We apply our model in the context of the climate transition. We estimate the skill distribution using a novel estimation technique leveraging the block recursivity of directed search models. We find that, under a status quo climate transition ending in 2060, workers attached to polluting sectors lose about 4.5% of lifetime earnings on average, with substantial heterogeneity. Compensating these workers would cost in the hundreds of billions of dollars. Accelerating the transition so it ends in 2050 benefits the median worker, but exacerbates substantially the losses suffered by those with the least transferable skills.

The goal of our paper is to explain transparently the source and determinants of the non-linear and non-monotonic relationship between the pace of sectoral shifts and workers' outcomes. Future work can build on our analysis and enrich it quantitatively by introducing many sectors, aggregate uncertainty about the transition path, a rich geography with local demand spillovers, or by carrying out a broader optimal policy analysis accounting for the fiscal cost of policy interventions.

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Online Appendix

A Frictionless Benchmark

In this appendix, we derive formally the results that we presented in Sections 3–4 using our frictionless benchmark, and under weaker restrictions on the transitions than the ones we considered in the main text. Throughout, we index workers by their birth date τ and their relative skill $\hat{\psi}$ in the growing C sector. We define t_0 to be the date at which the government first intervenes to affect the speed of transition. We let $A^k(t, \eta)$ be the de-trended marginal product of labor in sector k, where we have made explicit the dependence on both calendar time t and η, which parametrizes how quickly sectoral MRPLs change after t_0. We define $\hat{A}(t, \eta) \equiv A^C(t, \eta) / A^D(t, \eta)$ as the relative MRPL between the growing and declining sectors. Finally, we maintain minimal regularity assumptions on $A^k(t, \eta)$, summarized below

Assumption 1. *De-trended marginal revenue products of labor $A^k(t, \eta)$ satisfy:*

1. *$A^C(t, \eta)$ increases with t and η and $A^D(t, \eta)$ decreases in t and η, for all $t \geq t_0$, and strictly so over some interval in the neighborhood of t_0;*
2. *$A^k(t, \eta)$ is continuous in t and η for all k; and*
3. *$\lim_{\eta \to \infty} A^k(t, \eta) = A^k_\infty$ and $A^k(t, 0) = A^k_{t_0}$ for all $t \geq t_0$ and k.*

The first assumption defines the kinds of transitions that we consider: productivity falls in the declining D sector (relative to trend) while it rises in the growing C sector. A larger η speeds up the transition in that it implies a faster MRPL growth in the growing sector and a faster decline in the declining sector starting from t_0. The second assumption is standard. The third assumption states that, in the limit as η goes to infinity, the MRPLs instantly jump to their long run level A^k_∞, and permanently stay at their initial level $A^k_{t_0}$ if $\eta = 0$. We define $\hat{A}_\infty$ and $\hat{A}_{t_0}$ similarly. Whenever is convenient, we also assume that $A^k(t, \eta)$ is differentiable almost everywhere in its arguments. This set of assumptions is strictly weaker than those imposed in Section 3.

A.1 Always, Never and Occasional Switchers

It is useful to first partition the workers into three groups based on their reallocation patterns. Let $t^{*}\left(\hat{\psi};\eta\right)$ denote the period in which a worker of skill $\hat{\psi}$ would switch from sector D to C under transition speed η if they lived forever, and let $\bar{t}\left(\hat{\psi};\eta\right)$ denote the final period after t_0 when the worker is employed in sector D.

Lemma 2. *The set of workers $\left(\tau,\hat{\psi}\right)$ can be partitioned into:*

1. ***Always switchers*** *with $\hat{\psi}\geq\hat{A}_{t_0}^{-1}$ who move immediately to the C sector so $\bar{t}\left(\tau,\hat{\psi};\eta\right)=t^{\star}\left(\hat{\psi};\eta\right)=t_0$ irrespective of τ and η.*
2. ***Never switchers*** *with $\hat{\psi}\leq\hat{A}_{\infty}^{-1}$ who never move to the C sector so $\bar{t}\left(\tau,\hat{\psi};\eta\right)=t^{\star}\left(\hat{\psi};\eta\right)=\infty$ irrespective of τ and η.*
3. ***Occasional switchers*** *with $\hat{\psi}\in\left(\hat{A}_{\infty}^{-1},\hat{A}_{t_0}^{-1}\right)$ who potentially move to sector C if they are sufficiently young and the transition is sufficiently short. For these workers,*

$$\bar{t}\left(\tau,\hat{\psi};\eta\right)=\min\left\{\max\left\{t^{\star}\left(\hat{\psi};\eta\right),t_0\right\},J+\tau\right\} \tag{A.1}$$

where the cutoff $t^{\star}\left(\hat{\psi};\eta\right)$ satisfies

$$\hat{\psi}=\hat{A}\left(t^{\star}\left(\hat{\psi};\eta\right),\eta\right)^{-1} \tag{A.2}$$

and is therefore continuous, strictly decreasing in $\hat{\psi}$, strictly decreasing in η, and satisfies $t^{\star}\left(\hat{\psi};\eta\right)\to t_0$ as $\eta\to\infty$.

Proof. A worker supplies labor to the growing sector if and only if

$$\hat{\psi}A^{C}\left(t,\eta\right)\geq A^{D}\left(t,\eta\right)\quad\text{or}\quad\hat{\psi}\geq\hat{A}\left(t,\eta\right)^{-1} \tag{A.3}$$

during their lifetime. Note that the relative MRPL $\hat{A}_t\left(\eta\right)^{-1}$ decreases over time. Condition (A.3) always holds for workers with $\hat{\psi}\geq\hat{A}_{t_0}^{-1}$ so they always supply labor to sector C for any $t\geq t_0$. On the contrary, this condition never holds for workers with $\hat{\psi}\leq\hat{A}_{\infty}^{-1}$ so they never supply labor to sector C. For any intermediate $\hat{\psi}\in\left(\hat{A}_{\infty}^{-1},\hat{A}_{t_0}^{-1}\right)$, there exists a cutoff $t^{\star}\left(\hat{\psi};\eta\right)$ that satisfies (A.2) since $\hat{A}\left(t,\eta\right)$ is a continuous function of t. Workers supply labor to the declining sector until $t^{\star}\left(\hat{\psi};\eta\right)$ and then supply labor to the growing sector. It is clear from condition

(A.3) that $t^{\star}\left(\hat{\psi},\eta\right)$is continuous because $\hat{A}\left(t,\eta\right)$ is itself continuous. Furthermore, it decreases with the relative productivity $\hat{\psi}$ since $\hat{A}\left(t,\eta\right)^{-1}$ decreases over time. In turn, a faster transition implies a smaller cutoff since $\hat{A}\left(t,\eta\right)^{-1}$ decreases with the pace of the transition η. Furthermore, the cutoff $t^{\star}\left(\hat{\psi};\eta\right)\rightarrow t_0$ as $\eta\rightarrow\infty$ by Assumption 1 since $\hat{\psi}>\hat{A}_{\infty}^{-1}$. Finally, a worker only switches in their lifetime if the cutoff $t^{\star}\left(\hat{\psi};\eta\right)$ occurs before they retire in period $J+\tau$. □

A.2 Non-Linearity of Lifetime Earnings

We now establish that earnings changes are non-linear for always and never switchers as a function of the horizon over which the transition unfolds. In other words, workers are affected non-linearly as the transition ends 5, 10 or 15 years earlier, and extrapolating from small changes around the status quo transition can be misleading. We will turn our attention to occasional switchers later in this appendix.

A worker's lifetime earnings starting from period t_0 are given by

$$W_{t_0}\left(\tau,\hat{\psi};\eta\right)=\int_{t^{+}(\tau)}^{\bar{t}\left(\tau,\hat{\psi};\eta\right)}e^{-\rho\left(t-t^{+}(\tau)\right)}A^{D}\left(t,\eta\right)dt+\hat{\psi}\int_{\bar{t}\left(\tau,\hat{\psi};\eta\right)}^{J+\tau}e^{-\rho\left(t-t^{+}(\tau)\right)}A^{C}\left(t,\eta\right)dt, \tag{A.4}$$

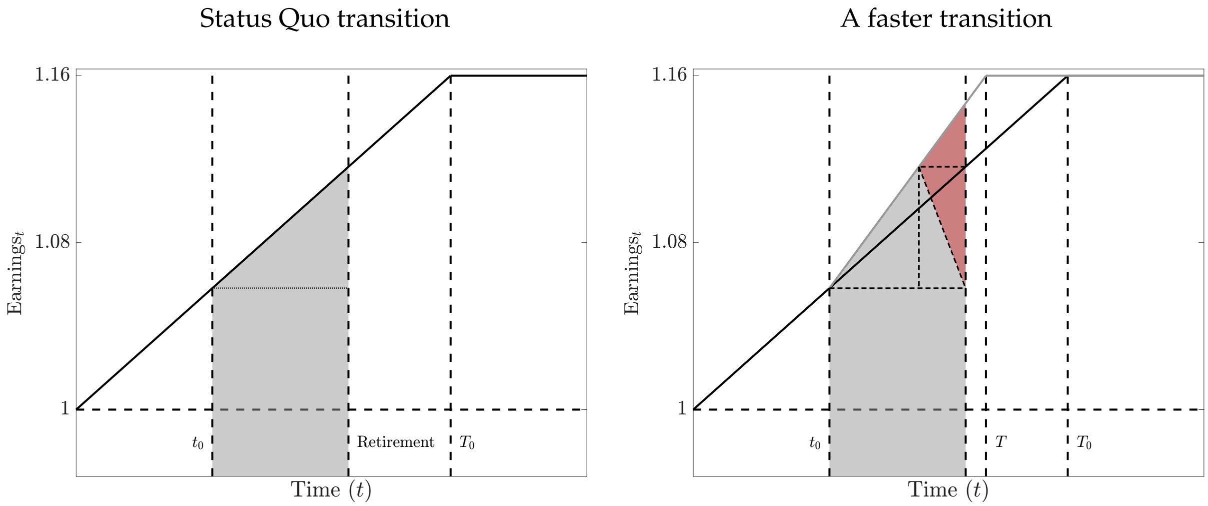
where $t^{+}\left(\tau\right)\equiv\max\left\{\tau,t_0\right\}$ is the first period where worker is present after t_0.[45] We focus on incumbent workers so $\tau\in\left(t_0-J,t_0\right]$ and $t^{+}\left(\tau\right)=t_0$.

We start by inspecting always switchers whose outcomes are unrelated to sector D. Their lifetime earnings become $W_{t_0}\left(\tau,\hat{\psi};\eta\right)=\hat{\psi}\int_{t_0}^{J+\tau}e^{-\rho(t-t_0)}A^{C}\left(t,\eta\right)dt$. Starting from a status quo horizon that ends in period T_0, we are interested in characterizing changes in lifetime earnings as a function of the horizon T over which the transition is completed. Figure A.1 illustrates the path of earnings for an always switcher with $\hat{\psi}=1$. The left panel corresponds to the status quo transition. As a convenient benchmark, suppose that the transition is linear over time. The shaded gray area corresponds to workers' lifetime earnings (A.4) in the case where there is no discounting, i.e., $\rho=0$.[46] The right panel of Figure A.1 considers a faster transition ending in period $T<T_0$. In the following, we let $\Delta T\equiv T_0-T$. The sum of the shaded gray and red areas correspond to lifetime earnings in this case.

[45] Note we index workers' values by cohort τ rather than age j for the purposes of Appendix A. One could equivalently index workers by their age as of time 0 given by $j_0\equiv\tau+t_0$.

[46] The gray area extends down all the way to zero. We truncated the plot to improve readability.

Figure A.1: Non-linear earnings changes



Notes: This figure plots the path of earnings over time for an always switcher worker with $\hat{\psi} = 1$. The left panel illustrates a status quo transition ending in periods T_0. The shaded area corresponds to lifetime earnings when there is no discounting, i.e., $\rho = 0$. The right panel illustrates a faster transition ending in $T < T_0$. The two shaded areas correspond to lifetime earnings in this case.

It is straightforward to show that lifetime earnings are convex in the horizon ΔT. This result can be illustrated geometrically. By construction, the shaded gray areas are the same in the two panels of Figure A.1 since they have the same base and height. Thus, lifetime earnings increase by the two shaded red triangles on the right panel of Figure A.1 as the transition accelerates. The area of the bottom triangle scales linearly with ΔT. On the contrary, the area of the top triangle grows non-linearly with ΔT since its base is linear in ΔT but its height increases too. As the transition ends earlier and earlier, every additional $\Delta T = 1$ year steepens the line in Figure A.1 more and more, leading to non-linear changes in lifetime earnings.[47] Note that this result holds regardless of how A_t^D changes over time. The case where the worker is a never switcher is symmetric.

Finally, it is worth noting that the transition above made two assumptions to convey the insights transparently. First, we abstracted from discounting. Obviously, exponential discounting can reinforce the non-linearities documented above. Second, we considered a linear transition. Naturally, non-linearities in the functions $\Phi^k(t)$ in expression (3.3) can strengthen the non-linearities in earnings.

[47] Of course, workers' earnings are bounded by the flat region in Figure A.1, which might itself introduce additional non-linearities.

A.3 Proof of Proposition 2

We now establish that earnings changes are non-monotonic for occasional switchers as a function of the transition horizon. In the following, we use the terminology, assumptions and notation previously introduced in this Appendix.

Before we establish this non-monotonicity result, let us briefly discuss the case of always switchers with $\hat{\psi} \geq \hat{A}_{t_0}^{-1}$, who switch immediately to the growing sector (Lemma 2). Their earnings increase over time, so they are trivially made better off when the transition is completed faster. On the contrary, never switchers with $\hat{\psi} \leq \hat{A}_{\infty}^{-1}$ stay in the declining sector. Their earnings decrease over time, so they are made worse off by a faster transition.

Consider now an occasional switcher with $\hat{\psi} \in \left(\hat{A}_{\infty}^{-1}, \hat{A}_{t_0}^{-1}\right)$. The simple idea behind the non-monotonicity is that occasional switchers spend their whole careers in sector D when η is small enough and thus behave like never switchers over that range, but essentially behave like always-switchers when η is high enough that they work most of their career in sector C.

Formally, consider an incumbent worker born in $\tau \in (t_0 - J, t_0]$. By Lemma 2 and the intermediate value theorem, there exists some $\eta_l \equiv \eta_l\left(\tau, \hat{\psi}\right)$ for which $t^{\star}\left(\hat{\psi}; \eta_l\right) = J + \tau$ so the worker switches to the growing sector exactly at retirement. The worker never switches in their lifetime if $\eta < \eta_l$. Over that region, lifetime earnings (A.4) are $W_{t_0}\left(\tau, \hat{\psi}; \eta\right) = \int_{t_0}^{J+\tau} e^{-\rho(t-t_0)} A^D\left(t, \eta\right) dt$ and decrease with η by Assumption 1, so the worker is worse off as the transition accelerates.

Now consider a pace η with $0 \leq t^{\star}\left(\hat{\psi}; \eta\right) < J + \tau$, i.e., the worker switches at some point before retirement. There exists such a pace by Lemma 2 and Assumption 1. In this case, the change in lifetime earnings (A.4) is

$$\frac{dW_{t_0}\left(\tau, \hat{\psi}; \eta\right)}{d\eta} = \int_{t_0}^{t^{\star}\left(\hat{\psi};\eta\right)} e^{-\rho(t-t_0)} \frac{\partial A^D\left(t, \eta\right)}{\partial \eta} dt + \hat{\psi} \int_{t^{\star}\left(\hat{\psi};\eta\right)}^{J+\tau} e^{-\rho(t-t_0)} \frac{\partial A^C\left(t, \eta\right)}{\partial \eta} dt, \tag{A.5}$$

where we have used the fact that the switching time $t^{\star}\left(\hat{\psi}; \eta\right)$ is chosen optimally. We will show that there exists some $\eta_h \equiv \eta_h\left(\tau, \hat{\psi}\right)$ with $\eta_h > \eta_l$ such that the expression above is positive for all $\eta > \eta_h$.

First, note that the second term of the right-hand side of (A.5) is positive since $A^C\left(t, \eta\right)$ increases with η. In particular, it converges to $\hat{\psi}\left(A_{\infty}^C - A_0^C\right)$ as $\eta \to \infty$ by

Assumption 1. Therefore, it suffices to show that

$$\lim_{\eta \to \infty} \int_{t_0}^{t^\star(\hat{\psi};\eta)} e^{-\rho(t-t_0)} \frac{\partial A^D(t,\eta)}{\partial \eta} dt = 0 \tag{A.6}$$

Note that, for any finite η, Assumption 1 implies

$$0 > \int_{t_0}^{t^\star(\hat{\psi};\eta)} e^{-\rho(t-t_0)} \frac{\partial A^D(t,\eta)}{\partial \eta} dt > \int_{t_0}^{t^\star(\hat{\psi};\eta)} \frac{\partial A^D(t,\eta)}{\partial \eta} dt \tag{A.7}$$

since $\partial A^D(t,\eta)/\partial \eta \leq 0$ and strictly so in a neighborhood of t_0. Moreover,

$$\int_{t_0}^{t^\star(\hat{\psi};\eta)} \frac{\partial A^D(t,\eta)}{\partial \eta} dt > \frac{\partial}{\partial \eta} \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt \tag{A.8}$$

since $t^\star(\hat{\psi};\eta)$ decreases with η. We will now establish that

$$\lim_{\eta \to \infty} \frac{\partial}{\partial \eta} \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt = 0 \tag{A.9}$$

Note that

$$\frac{\partial}{\partial \eta} \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt < 0 \tag{A.10}$$

both because $t^\star(\hat{\psi},\eta)$ and $A^D(t,\eta)$ decrease with η. Suppose by contradiction that expression (A.9) does not hold. Then, for each $\bar{\eta}$, there exists an $\epsilon < 0$ and an infinite number of subsets with positive measure in $(\bar{\eta},\infty)$ such that

$$\frac{\partial}{\partial \eta} \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt < \epsilon \tag{A.11}$$

for each η in these subsets. Then,

$$\begin{aligned} 0 &= \left. \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt \right|_{\eta=\infty} \\ &= \left. \int_{t_0}^{t^\star(\hat{\psi};\eta)} A^D(t,\eta)\, dt \right|_{\eta=\bar{\eta}} + \int_{\bar{\eta}}^{\infty} \left\{ \frac{\partial}{\partial \tilde{\eta}} \int_{t_0}^{t^\star(\hat{\psi};\tilde{\eta})} A^D(t,\tilde{\eta})\, dt \right\} d\tilde{\eta} \\ &= -\infty \end{aligned} \tag{A.12}$$

which yields the desired contradiction. Therefore, expression (A.9) holds. Combining (A.7)–(A.8) and using the squeeze theorem, it must be that equation (A.6) holds as well. Thus,

$$\lim_{\eta\to\infty} \frac{dW_{t_0}\left(\tau,\hat{\psi};\eta\right)}{d\eta} > 0, \tag{A.13}$$

which show that there exists indeed a pace $\eta_h > \eta_l$ above which workers benefit from a transition that ends sooner. Workers' welfare is therefore non-monotone in the transition horizon for incumbent workers.

Having established the desired result for incumbent cohorts, we now consider a future cohort born in $\tau \in (t_0, \infty)$. We derive η_l as above, such that the worker would switch only at retirement and thus prefers a slower transition for $\eta < \eta_l$. We may now define η_h as the speed such that the worker would switch to the growing sector at birth; at this point, the worker spends their whole life in the growing sector and prefers a faster transition for $\eta > \eta_h$. Thus, lifetime earnings are also non-monotonic in the transition horizon for these new cohorts. □

B Full Model

This appendix provides additional derivations and proofs for our full model introduced in Section 5. Section B.1 states workers' and firms' values recursively. Section B.2 characterizes firms' optimal choices of production and innovation. Section B.3 defines the equilibrium formally. Section B.4 shows that workers' and firms' values scale linearly with $\psi \exp{(gt)}$, where for g is the long-run growth rate of the economy and ψ is a worker's reference skill. Section B.5 characterizes the balanced-growth path of the economy before the transition begins. Section B.6 proves the implementability result (Proposition 1) in our full model. Section B.7 shows that the sectoral marginal revenues of product tend to be S-shaped along the status quo transition and describes how the government can speed up transitions.

B.1 Workers' and Firms' Values

Skill accumulation. Before stating workers' and firms' values, it is useful to perform a change of variable and define the "reference" skill $\psi \equiv \psi^G \psi^D$ and the "relative" skill $\hat{\psi} \equiv \psi^C/\psi^D$. Using the laws of motions that we described in Section 5, the reference skill ψ grows at rate

$$\delta_j^\star(k) \equiv \begin{cases} \delta_j^+ + \hat{\delta}_j & \text{if } k = D \\ \delta_j^+ & \text{if } k = C \\ \delta_j^k & \text{if } k = O, U \end{cases} \tag{B.1}$$

depending on the worker's employment status. The relative skill grows at rate

$$\hat{\delta}_j^\star(k) \equiv \begin{cases} -\hat{\delta}_j & \text{if } k = D \\ \hat{\delta}_j & \text{if } k = C \\ 0 & \text{if } k = O, U \end{cases} \tag{B.2}$$

In the following, we let

$$h\left(\psi, \hat{\psi}, k\right) = \begin{cases} \psi & \text{if } k = D \\ \psi \times \hat{\psi} & \text{if } k = C \\ \psi \times b^k & \text{if } k = O, U \end{cases} \tag{B.3}$$

denote a workers' productivity depending on their employment status. Workers are now indexed by five states: their age j; their reference skill ψ; their relative skill in the clean sector $\hat{\psi}$; their employment status $k = D, C, O, U$; and, if employed, their wage w. The last two states can be combined into a single state s which corresponds to a market (k, w) when the worker is employed, or to O or U when the worker claims the outside option or is unemployed. We omit the dependence of values on η for notational convenience.

Workers' values. The value of an employed worker of age j with skills ψ and $\hat{\psi}$ who is currently in submarket s satisfies

$$\begin{aligned} \rho W_t\left(j, \psi, \hat{\psi}, s\right) = {} & w(s)\, h\left(\psi, \hat{\psi}, k(s)\right) \exp(gt) \\ & + \lambda^E \sum_{s'} p_t\left(s'; j, \psi, \hat{\psi}, s\right) f\left(\theta_t\left(j, \psi, \hat{\psi}, s'\right)\right) \left[W_t\left(j, \psi, \hat{\psi}, s'\right) - W_t\left(j, \psi, \hat{\psi}, s\right)\right] \\ & + \chi\left[W_t\left(j, \psi, \hat{\psi}, U\right) - W_t\left(j, \psi, \hat{\psi}, s\right)\right] \\ & + \frac{d}{d\epsilon} W_{t+\epsilon}\left(j + \epsilon, \psi \exp\left(\delta_j^\star(k(s))\, \epsilon\right), \hat{\psi} \exp\left(\hat{\delta}_j^\star(k(s))\, \epsilon\right), s\right)\Bigg|_{\epsilon=0} \end{aligned} \tag{B.4}$$

for each age $j \in [0, J)$, where $W_t\left(j, \psi, \hat{\psi}, U\right)$ is the value of being unemployed, which is defined below. Workers discount flows at rate ρ. When employed, they earn $w(s)\exp(gt)$ for each productive unit of labor $h(\psi,\hat{\psi},k(s))$ that they provide, with $h(\cdot)$ given by expression (B.3).[48] Workers have the opportunity to search for on-the-job at rate λ^E. They search in submarket s' with probability $p_t(s';j,\psi,\hat{\psi},s)$ given by expression (5.1). Matches are dissolved exogenously at rate χ. Finally, there is drift in age, over time, and in the skills ψ and $\hat{\psi}$ at rates (B.1)–(B.2).[49] The boundary condition is $W_t(J,\psi,\hat{\psi},s)=0$ upon retirement at age $j=J$. The terminal conditions are similar for the values below so we omit them. Lump sum transfers from the government do not distort workers' labor supply decisions when workers are risk-neutral or have access to perfect borrowing and saving, as we assume in this paper (Sections 2.3 and 5.1). Therefore, we exclude these transfers from workers' values, allowing us to interpret them as the present discounted value of future earnings — the object of interest in this paper.

The value of claiming the outside option satisfies

$$\begin{aligned}\rho W_t(j,\psi,\hat{\psi},O) = {} & h(\psi,\hat{\psi},O)\exp(gt) \\ & + \lambda^E \sum_s p_t(s;j,\psi,\hat{\psi},O)\, f(\theta_t(j,\psi,\hat{\psi},s))\left[W_t(j,\psi,\hat{\psi},s)-W_t(j,\psi,\hat{\psi},O)\right] \\ & + \chi\left[W_t(j,\psi,\hat{\psi},U)-W_t(j,\psi,\hat{\psi},O)\right] \\ & + \frac{d}{d\epsilon}W_{t+\epsilon}\left(j+\epsilon,\psi\exp\left(\delta_j^\star(O)\,\epsilon\right),\hat{\psi},O\right)\bigg|_{\epsilon=0}\end{aligned} \tag{B.5}$$

for each age $j\in[0,J)$, since they receive benefits $h(\psi,\hat{\psi},O)\exp(gt)$, search for jobs with intensity λ^E, and separate from the outside option exogenously at rate χ. Similarly, the value of an unemployed worker satisfies

$$\begin{aligned}\rho W_t(j,\psi,\hat{\psi},U) = {} & h(\psi,\hat{\psi},U)\exp(gt) \\ & + \lambda^U \sum_s p_t(s;j,\psi,\hat{\psi},U)\, f(\theta_t(j,\psi,\hat{\psi},s))\left[W_t(j,\psi,\hat{\psi},s)-W_t(j,\psi,\hat{\psi},U)\right] \\ & + \frac{d}{d\epsilon}W_{t+\epsilon}\left(j+\epsilon,\psi\exp\left(\delta_j^\star(U)\,\epsilon\right),\hat{\psi},s\right)\bigg|_{\epsilon=0}\end{aligned} \tag{B.6}$$

for each age $j\in[0,J)$, since they receive benefits $h(\psi,\hat{\psi},U)\exp(gt)$ and search for

[48] As discussed in Section 5.1, workers' wage compensation grows at rate g on-the-job to ensure that wages w admit a stationary distribution along a balanced-growth path.

[49] These drifts are characterized by the single index ϵ.

jobs with intensity λ^U. Workers' search probabilities are then given by

$$p_t\left(s';\mathbf{x},s\right) \equiv \frac{\exp\left(f\left(\theta_t\left(\mathbf{x},s'\right)\right)\left[W_t\left(\mathbf{x},s'\right)-W_t\left(\mathbf{x},s\right)\right]/\zeta\right)}{\sum_{\hat{s}}\exp\left(f\left(\theta_t\left(\mathbf{x},\hat{s}\right)\right)\left[W_t\left(\mathbf{x},\hat{s}\right)-W_t\left(\mathbf{x},s\right)\right]/\zeta\right)}, \tag{B.7}$$

where $\mathbf{x} \equiv \left(j,\psi,\hat{\psi}\right)$ denotes a worker's state excluding its current market s.

Firms' values. The value of a filled job in submarket s is given by

$$\begin{aligned} rV_t\left(j,\psi,\hat{\psi},s\right) = & \left(\mathcal{P}_t^{k(s)}-w\left(s\right)\right)h\left(\psi,\hat{\psi},k\left(s\right)\right)\exp\left(gt\right)-\chi V_t\left(j,\psi,\hat{\psi},s\right) \\ & -\lambda^E\sum_{s'}p_t\left(s';j,\psi,\hat{\psi},s\right)f\left(\theta_t\left(j,\psi,\hat{\psi},s'\right)\right)V_t\left(j,\psi,\hat{\psi},s\right) \\ & +\frac{d}{d\epsilon}V_{t+\epsilon}\left(j+\epsilon,\psi\exp\left(\delta_j^\star\left(k\left(s\right)\right)\epsilon\right),\hat{\psi}\exp\left(\hat{\delta}_j^\star\left(k\left(s\right)\right)\epsilon\right),s\right)\Bigg|_{\epsilon=0} \end{aligned} \tag{B.8}$$

for each age $j \in [0,J)$, where $\mathcal{P}_t^k$ denotes the de-trended prices of labor services produced in the declining or growing sectors. These prices are given by the equilibrium conditions (B.14) below. Firms discount flows using the equilibrium interest rate $r \equiv \rho$.[50] The match is dissolved either exogenously at rate χ, or because workers voluntarily quit due to on-the-job search. The opportunity for on-the-job search arrives at rate λ^E and the probability of separating conditional on searching is given by $f\left(\theta_t(\cdot)\right)$ in each market s', weighted by the probability that the individual searches there $p_t\left(s';j,\psi,\hat{\psi},s\right)$. The firm's value also features a drift in workers' skills, in their age, and over time. The boundary condition is $V_t\left(J,\psi,\hat{\psi},s\right) = 0$ when the worker retires at age $j = J$.

Tightnesses. Free entry in each market implies that equilibrium tightnesses $\theta_t\left(j,\psi,\hat{\psi},s\right)$ satisfy

$$q\left(\theta_t\left(j,\psi,\hat{\psi},s\right)\right)V_t\left(j,\psi,\hat{\psi},s\right) \leq \nu\psi\exp\left(gt\right) \tag{B.9}$$

in every period t and market s, with equality if a positive measure of vacancies are posted. Posting a vacancy for a worker with reference skill ψ comes at a cost $\nu\psi\exp\left(gt\right) > 0$ which grows at rate g to ensure that the economy admits a balanced-growth path where vacancy posting costs do not vanish. Firms post vacancies until the expected value of a vacancy equals the vacancy-posting cost. This expected value is the product of the value of the match $V_t\left(j,\psi,\hat{\psi},s\right)$ and the prob-

[50] The equilibrium interest rate is $r = \rho$ since entrepreneurs have linear utility.

ability of finding match $q\left(\theta_t\left(j,\psi,\hat{\psi},s\right)\right)$. Firms do not post any vacancies if the expected value of the vacancy is lower than its cost.

B.2 Production, Innovation and Government

Production. The representative final good producer combines the intermediate goods produced by the declining and growing sectors using the technology

$$Y_t = F_t\left(X_t^D, X_t^C\right) \tag{B.10}$$

As usual, the technology (B.10) can alternatively be interpreted as workers' preferences over these two goods. Intermediate goods are produced by combining a continuum of mass 1 of symmetric varieties x_t^k using a CES aggregator with elasticity of substitution $\epsilon^k > 1$. These firms use labor services to produce $x_t^k = a_t^k y^k\left(n_t^k\right)$, where a_t^k is TFP in sector k and n^k is the amount of labor services used.

Equilibrium prices. The final good producer chooses its demand for intermediates so that

$$P_t^k = \frac{d}{dX_t^k} F_t\left(X_t^D, X_t^C\right), \tag{B.11}$$

for each sector $k = D, C$, where P_t^k is the purchase price of intermediates. In turn, the supply of intermediates is

$$X_t^k = a_t^k y^k\left(n_t^k\right), \tag{B.12}$$

where

$$n_t^{k'} = \int \mathbf{1}_{\{k(s)=k'\}} h\left(\psi,\hat{\psi},k\left(s\right)\right) d\mu_t\left(j,\psi,\hat{\psi},s\right) \tag{B.13}$$

is the labor supply in a sector $k' = D, C$, with $k\left(s\right)$ denoting the sector that a given worker in market s is employed in. The previous expression uses the fact that there is a continuum of symmetric monopolistic firms of mass 1 which produce each type of intermediate. In turn, the optimal price setting decision of these monopolistic firms implies that

$$\left(1+\tau_t^k\right) a_t^k P_t^k y^{k,\prime}\left(n_t^k\right) = \frac{\varepsilon^k}{\varepsilon^k - 1}\mathcal{P}_t^k \exp\left(gt\right). \tag{B.14}$$

As usual, the monopolistic firms' set a markup $\varepsilon^k / \left(\varepsilon^k - 1\right)$ over their marginal cost, which includes the price of labor services $\mathcal{P}_t^k$, the sectoral TFP a_t^k, and decreasing returns at the firm-level $y^{k,\prime}\left(n_t^k\right)$, and each firm takes into account that it receives a subsidy τ_t^k from the government for every unit sold. Finally, we de-trend the price of labor services $\mathcal{P}_t^k$ using the growth rate g, so $\mathcal{P}_t^k \exp\left(gt\right)$ is the price that firms actually pay. As we show in Appendix B.5, this normalization ensures that these prices are stationary along a balanced-growth path. Comparing equations (3.1) and (B.14) reveals that the price of labor services is given by $\mathcal{P}_t^k = A_t^k$ in equilibrium. In our frictionless benchmark of Section 2, the price of labor services coincides with workers' wage w_t^k per efficiency unit of labor.

Innovation. Entrepreneurs who own the firms producing varieties of intermediate goods can invest resources to improve their TFP a_t^k, thereby increasing profits. Each entrepreneur is infinitesimal, but symmetric so that they solve the same problem and make the same choices. Their TFP evolves according to $da_t^k = \left(z_t^k - \delta_a^k a_t^k\right) dt$, where z_t^k is gross investment and δ_a^k is the rate at which a firm's technology becomes obsolete relative to newer ideas (Caballero and Jaffe, 1993; Akcigit et al., 2022). We allow for technological spillovers across sectors

$$z_t^k = \sum_{k'} \beta_{k'}^k \times \iota\left(\frac{I_t^{k'}}{a_t^{k'}}\right) a_t^{k'}, \tag{B.15}$$

where I_t^k denotes investment on each technology.[51] The investment function $\iota\left(x\right) = \omega_0 x^{\omega_1}$ with $\omega_0 > 0$ and $\omega_1 \in \left(0,1\right)$ reflects decreasing returns to R&D. The coefficients $\beta_{k'}^k > 0$ capture the innovation spillovers across sectors.[52]

As we show in Appendix B.5, this specification ensures there exists a balanced-growth path where output in the two sectors grow at the same rate $g > 0$. We model sectoral shifts as transitions that affect the composition of economic activity across sectors, but not the long-term growth rate. We therefore place appropriate restrictions on the innovation technology ι and $\beta_{k'}^k$ to ensure that the economy con-

[51] The literature sometimes considers specifications where the productivities a_t^k enter multiplicatively across sectors, e.g., Acemoglu (2002). In our context, these formulations have the disadvantage that they allow laggard sectors to bring down the rest of the economy; this feature is problematic in our context since we consider an initial balanced-growth path where the C sector is not active yet (Section 3).

[52] The spillovers (B.15) can easily be augmented to allow for a slower diffusion (as in Eaton and Kortum, 1999), where lagged investments affect current TFP changes.

verges to a new balanced-growth path which grows at the same rate g as the one that prevailed before the transition.

Finally, note that, while we interpret the stock as a_t^k as intangible capital, it could alternatively be interpreted as tangible or physical capital. There are no technological spillovers in this case, and the initial and final growth rates are $g = 0$.

Entrepreneurs' optimal innovation solves

$$\max_{\left\{I_t^k, a_t^k\right\}} \int_0^\infty \exp\left(-rt\right)\left[\left(1+\tau_t^k\right) a_t^k P_t^k y^k \left(n_t^k\right) - \left(1-\varsigma_t^k\right) I_t^k\right] dt \tag{B.16}$$
$$\text{s.t. } da_t^k = \left(\sum_{k'} \beta_{k'}^k \times \iota\left(\frac{I_t^{k'}}{a_t^{k'}}\right) a_t^{k'} - \delta_a^k a_t^k\right) dt,$$

in each sector $k = D, C$, given some initial condition for a_0^k and taking innovations $I_t^{k'}$ and technologies $a_t^{k'}$ as given in the other sector $k' \neq k$. By the Principle of Optimality, entrepreneurs can choose their investment rate I_t^k and stock a_t^k, keeping fixed their optimal labor demands n_t^k.

At optimum, the marginal benefit of investing equals the marginal cost

$$\vartheta_t^k \beta_k^k \iota'\left(\frac{I_t^k}{a_t^k}\right) = 1 - \varsigma_t^k \tag{B.17}$$

where ϑ_t^k is the multiplier on the law of motion for a_t^k, which evolves as

$$\left(1+\tau_t^k\right) P_t^k y^k \left(n_t^k\right) + \vartheta_t^k \left[\beta_k^k \left(\frac{1}{\omega_1} - 1\right) \iota'\left(\frac{I_t^k}{a_t^k}\right) \frac{I_t^k}{a_t^k} - \delta_a^k\right] = -d\vartheta_t^k + r\vartheta_t^k, \tag{B.18}$$

where we have used the fact that $\iota'(x) = \omega_1 \iota(x)/x$ with $\omega_1 \in (0,1)$. Combining the two previous expressions yields

$$\left(1+\tau_t^k\right) P_t^k y^k \left(n_t^k\right) + \left(\frac{1}{\omega_1} - 1\right)\left(1-\varsigma_t^k\right) \frac{I_t^k}{a_t^k} = -d\vartheta_t^k + \left(r + \delta_a^k\right) \vartheta_t^k \tag{B.19}$$

The optimum to the innovation problem (B.16) is fully characterized by (B.17) and (B.19) together with the law of motion

$$da_t^k = \left(\sum_{k'} \beta_{k'}^k \times \iota\left(\frac{I_t^{k'}}{a_t^{k'}}\right) a_t^{k'} - \delta_a^k a_t^k\right) dt, \tag{B.20}$$

the usual transversality condition, and the initial condition a_0^k.

Government. The government chooses a path of subsidies $\{\tau_t^k, \varsigma_t^k\}$ and sets lump sum taxes to balance its flow budget

$$T_t = \sum_k \left(\tau_t^k P_t^k X_t^k + \varsigma_t^k I_t^k\right) \tag{B.21}$$

B.3 Equilibrium Definition

Definition 1. Given a path of government subsidies $\{\tau_t^k, \varsigma_t^k\}$, an equilibrium consists of sequences of workers' and firms' values $W_t\left(j, \psi, \hat{\psi}, s\right)$ and $V_t\left(j, \psi, \hat{\psi}, s\right)$, search probabilities $p_t\left(s'; j, \psi, \hat{\psi}, s\right)$, tightnesses $\theta_t\left(j, \psi, \hat{\psi}, s\right)$, a distribution of workers' states $\boldsymbol{\mu}_t\left(j, \psi, \hat{\psi}, s\right)$, sectoral outputs X_t^k, sectoral labor supplies n_t^k, sectoral TFPs a_t^k, prices of intermediates P_t^k , prices of labor services $\mathcal{P}_t^k$, and lump sum taxes T_t such that:

1. Workers' values satisfy (B.4)–(B.6)
2. Firms' values satisfy (B.8)
3. The search probabilities satisfy (B.7)
4. The tightnesses satisfy (B.9)
5. The distribution is consistent with the flows implied by workers' and firms' values (see Appendix C.3)
6. Sectoral outputs are given by (B.12)
7. Sectoral labor supplies are given by (B.13)
8. The prices of intermediates satisfy (B.11)
9. The prices of labor services satisfy (B.14)
10. Sectoral TFPs satisfy (B.17)–(B.20)
11. Lump sum taxes balance the government's budget (B.21)

Before characterizing the equilibrium further, it is worth noting that firms' vacancy posting and workers' labor supplies are pinned down by the coupled system of partial differential equations that define workers' and firms' values (B.4)–(B.6) and (B.8), together with the search probabilities (B.7) and the free entry condition (B.9). This is a closed system that is parametrized by the prices of labor services $\mathcal{P}_t^k$, i.e., the rest of the economy matters for workers' labor market outcomes only to the extent that it affects $\mathcal{P}_t^k$. As shown in Appendix B.2, $\mathcal{P}_t^k = A_t^k$ in equilibrium so MRPLs fully characterize workers' outcomes. Finally, note that lump sum taxes T_t are non-distortionary since workers are risk-neutral or have access to perfect borrowing and saving (Sections 2.3 and 5.1). Therefore, these taxes do not affect workers' labor supplies or any equilibrium variables characterized in this appendix.

B.4 Normalized and De-Trended Values

This appendix establishes that workers' and firms' are linearly homogenous in $\psi \exp(gt)$, where g still denotes the rate at which output grows along a balanced-growth path. This homogeneity property makes our analysis computationally feasible because it allows us to 1) characterize de-trended values that are constant over a balanced-growth path and 2) not keep track of the reference skill ψ in our state space. Formally, we will show that workers' and firms' values satisfy

$$W_t\left(j, \psi, \hat{\psi}, s\right) = \psi \exp(gt)\, W_t^{\star}\left(j, \hat{\psi}, s\right) \tag{B.22}$$

$$V_t\left(j, \psi, \hat{\psi}, s\right) = \psi \exp(gt)\, V_t^{\star}\left(j, \hat{\psi}, s\right), \tag{B.23}$$

for some $W_t^{\star}$ and $V_t^{\star}$ that do not depend on workers' reference skill ψ, and depend on time only through the de-trended prices of labor services $\mathcal{P}_t^k$, which are constant along a balanced-growth path (as we will establish in Section B.5). While establishing this property, we will also show that

$$\theta_t\left(j, \psi, \hat{\psi}, s\right) = \theta_t^{\star}\left(j, \hat{\psi}, s\right) \quad , \quad p_t\left(s'; j, \psi, \hat{\psi}, s\right) = p_t^{\star}\left(s'; j, \hat{\psi}, s\right) \tag{B.24}$$

for some $\theta_t^{\star}$ and $p_t^{\star}$ that do not depend on workers' reference skill ψ and depend on time only through $\mathcal{P}_t^k$. In the following, we use the fact that $h\left(\psi, \hat{\psi}, k\right) \equiv \psi h^{\star}\left(\hat{\psi}, k\right)$ where $h^{\star}\left(\hat{\psi}, k\right) \equiv h\left(1, \hat{\psi}, k\right)$ using the definition (B.3).

We proceed following a guess and verify approach. First, consider the value of a worker employed in market s. Using the definition (B.4), the properties above and expanding the time derivative with respect to ϵ, we obtain

$$\begin{aligned}
\rho \frac{W_t\left(j,\psi,\hat{\psi},s\right)}{\psi \exp\left(gt\right)} &= w\left(s\right) h^{\star}\left(\hat{\psi},k\left(s\right)\right) \\
&+ \lambda^E \sum_{s'} p_t^{\star}\left(s';j,\hat{\psi},s\right) f\left(\theta_t^{\star}\left(j,\hat{\psi},s'\right)\right)\left[W_t^{\star}\left(j,\hat{\psi},s'\right) - W_t^{\star}\left(j,\hat{\psi},s\right)\right] \\
&+ \chi\left[W_t^{\star}\left(j,\hat{\psi},U\right) - W_t^{\star}\left(j,\hat{\psi},s\right)\right] \\
&+ \left(g + \delta_j^{\star}\left(k\left(s\right)\right)\right) W_t^{\star}\left(j,\hat{\psi},s\right) + \hat{\psi}\hat{\delta}_j^{\star}\left(k\left(s\right)\right)\frac{\partial}{\partial\hat{\psi}} W_t^{\star}\left(j,\hat{\psi},s\right) \\
&+ \frac{\partial}{\partial j} W_t^{\star}\left(j,\hat{\psi},s\right) + \frac{\partial}{\partial t} W_t^{\star}\left(j,\hat{\psi},s\right),
\end{aligned} \tag{B.25}$$

which confirms that $W_t\left(j,\psi,\hat{\psi},s\right)$ can indeed be expressed as (B.22), where the normalized and de-trended value evolves as

$$\begin{aligned}
\rho^{\star} W_t^{\star}\left(j,\hat{\psi},s\right) &= w\left(s\right) h^{\star}\left(\hat{\psi},k\left(s\right)\right) \\
&+ \lambda^E \sum_{s'} p_t^{\star}\left(s';j,\hat{\psi},s\right) f\left(\theta_t^{\star}\left(j,\hat{\psi},s'\right)\right)\left[W_t^{\star}\left(j,\hat{\psi},s'\right) - W_t^{\star}\left(j,\hat{\psi},s\right)\right] \\
&+ \chi\left[W_t^{\star}\left(j,\hat{\psi},U\right) - W_t^{\star}\left(j,\hat{\psi},s\right)\right] \\
&+ \delta_j^{\star}\left(k\left(s\right)\right) W_t^{\star}\left(j,\hat{\psi},s\right) + \hat{\psi}\hat{\delta}_j^{\star}\left(k\left(s\right)\right)\frac{\partial}{\partial\hat{\psi}} W_t^{\star}\left(j,\hat{\psi},s\right) \\
&+ \frac{\partial}{\partial j} W_t^{\star}\left(j,\hat{\psi},s\right) + \frac{\partial}{\partial t} W_t^{\star}\left(j,\hat{\psi},s\right)
\end{aligned} \tag{B.26}$$

with $\rho^{\star} \equiv \rho - g$. These normalized and de-trended values depend on time only through the de-trended prices of labor services $\mathcal{P}_t^k$ for the reasons discussed in Appendix B.3. Proceeding similarly for the values (B.5)–(B.6) also confirms that $W_t\left(j,\psi,\hat{\psi},O\right)$ and $W_t\left(j,\psi,\hat{\psi},U\right)$ scale with $\psi \exp\left(gt\right)$ and yields the laws of motion for the normalized values, which we spell out explicitly in Appendix C.1.

Turning to firm's values (B.8), the same steps confirm that $V_t\left(j,\psi,\hat{\psi},s\right)$ can indeed be expressed as (B.23), where the normalized value evolves as

$$
\begin{aligned}
r^\star V_t^\star\left(j,\hat{\psi},s\right) = & \left(\mathcal{P}_t^{k(s)} - w\left(s\right)\right) h^\star\left(\hat{\psi},k\left(s\right)\right) \\
& - \lambda^E \sum_{s'} p_t^\star\left(s';j,\hat{\psi},s\right) f\left(\theta_t^\star\left(j,\hat{\psi},s'\right)\right) V_t^\star\left(j,\hat{\psi},s\right) \\
& - \chi V_t^\star\left(j,\hat{\psi},s\right) \\
& + \delta_j^\star\left(k\left(s\right)\right) V_t^\star\left(j,\hat{\psi},s\right) + \hat{\psi}\hat{\delta}_j^\star\left(k\left(s\right)\right) \frac{\partial}{\partial\hat{\psi}} V_t^\star\left(j,\hat{\psi},s\right) \\
& + \frac{\partial}{\partial j} V_t^\star\left(j,\hat{\psi},s\right) + \frac{\partial}{\partial t} V_t^\star\left(j,\hat{\psi},s\right),
\end{aligned} \tag{B.27}
$$

where $r^\star \equiv r - g$ with $r = \rho$ in equilibrium. Again, these normalized and detrended values depend on time only through $\mathcal{P}_t^k$.

Having verified the properties (B.22)–(B.23), we now show that the properties (B.24) also hold. Using the free entry condition (B.9) and the property (B.23), we obtain $\theta_t\left(j,\psi,\hat{\psi},s\right) = \theta_t^\star\left(j,\hat{\psi},s\right)$ for all ψ, where

$$
q\left(\theta_t^\star\left(j,\hat{\psi},s\right)\right) V_t^\star\left(j,\hat{\psi},s\right) \leq \nu \tag{B.28}
$$

with the usual complementary slackness condition, which confirms that $\theta_t^\star\left(j,\hat{\psi},s\right)$ does not depend on the reference skill ψ and depends on time only through changes in $\mathcal{P}_t^k$. Finally, using the search probabilities (B.7) and the property (B.22), we obtain $p_t\left(s';j,\psi,\hat{\psi},s\right) = p_t^\star\left(s';j,\hat{\psi},s\right)$ for all ψ, where

$$
p_t^\star\left(s';\tilde{\mathbf{x}},s\right) \equiv \frac{\exp\left(f\left(\theta_t^\star\left(\tilde{\mathbf{x}},s'\right)\right)\left[W_t^\star\left(\tilde{\mathbf{x}},s'\right) - W_t^\star\left(\tilde{\mathbf{x}},s\right)\right]/\zeta\right)}{\sum_{\hat{s}} \exp\left(f\left(\theta_t^\star\left(\tilde{\mathbf{x}},\hat{s}\right)\right)\left[W_t^\star\left(\tilde{\mathbf{x}},\hat{s}\right) - W_t^\star\left(\tilde{\mathbf{x}},s\right)\right]/\zeta\right)} \tag{B.29}
$$

with $\tilde{\mathbf{x}} \equiv \left(j,\hat{\psi}\right)$, which confirms that $p_t^\star\left(s';j,\hat{\psi},s\right)$ does not depend on the reference skill ψ and depends on time only through $\mathcal{P}_t^k$.

The homogeneity property (B.22) implies that we can economize one state variable when computing workers' and firms' values or iterating on the distribution of workers' states (Appendix C). In particular, none of the moments that we target in Section 5.2 depend on the level of the reference skill ψ. In Section 6.3, we compute workers' earnings losses under the status quo transition relative to no transition at all (Figure 6.2). In this case, the level of ψ and its evolution are relevant and we use stochastic simulations to track the evolution of workers' states over time.

B.5 Balanced-Growth Path

This appendix characterizes the balanced-growth path (BGP) of the economy. We assume for now that such a BGP exists and we later state the conditions under which this is the case. In the following, we treat the government subsidies τ_t^k and ς_t^k and the final good production technology $F_t\left(X_t^D, X_t^C\right)$ as constant over time.

Lemma 3. *Along a BGP, sectoral TFPs a_t^k, sectoral outputs X_t^k and aggregate output Y_t grow at the same rate $g > 0$. Sectoral labor supplies n_t^k, the prices of sectoral outputs P_t^k and the de-trended prices of labor services $\mathcal{P}_t^k$ are constant.*

Proof. We guess and verify that quantities and prices grow at these rates. Appendix B.4 showed that firms' vacancy posting and workers' labor supplies are characterized by a closed system that is parametrized by the de-trended prices of labor services $\mathcal{P}_t^k$. If these prices are constant along a balanced-growth path, then sectoral labor supplies n_t^k given by expression (B.13) are constant as well.

Turning to intermediate goods production, if sectoral TFPs a_t^k grow at rate g and sectoral labor supplies n_t^k are constant, then the technologies (B.12) imply that the sectoral outputs X_t^k indeed grow at rate g. Furthermore, if a_t^k grow at rate g while P_t^k and n_t^k are constant, then the labor demands (B.14) imply that $\mathcal{P}_t^k$ are indeed constant.

Considering final good production, if the output of intermediaries X_t^k grow at rate g, then the technology (B.10) implies that aggregate output Y_t indeed grows at rate g. Furthermore, the demands for intermediaries (B.11) imply that P_t^k are indeed constant since F_t has constant returns so the first derivatives of F_t are homogeneous of degree 0.

Finally, we turn to the innovation choices. Suppose that the prices of the final goods P_t^k and sectoral labor supplies n_t^k are constant, while both the investment $I_t^{k'}$ and the sectoral TFP $a_t^{k'}$ grow at rate g in the other sector $k' \neq k$. Then, the optimality conditions (B.17) and (B.19)–(B.20) imply that I_t^k and a_t^k grow at rate g and the multipliers ϑ_t^k are constant over time.

We now explain why the spillovers in the innovation process (B.20) ensure that the two sectors must grow at the same rate. Let

$$\frac{da_t^D}{a_t^D} = z_t^D dt \quad \text{where} \quad z_t^D \equiv \sum_k \beta_k^D \times \iota\left(i^k\right) \frac{a_t^k}{a_t^D} - \delta_a^D \tag{B.30}$$

where $i^k \equiv I_t^k / a_t^k$ is the investment rate in each sector along the BGP. Suppose, by contradiction, that the growing sector grows at a higher rate asymptotically than the declining sector, i.e., $g^C > g^D$. Then

$$z_t^D \to \beta_C^D \times \iota\left(i^C\right) \frac{a_t^C}{a_t^D} \to \infty \text{ as } t \to +\infty \tag{B.31}$$

so $g^D > g^C$, which yields the desired contradiction. The argument is symmetric if the declining sector were to grow faster asymptotically. Therefore, the sectoral TFPs must eventually grow at the same rate g. $\square$

In summary, we have established the rates at which quantities and prices must grow along a BGP. We now discuss the conditions under which the BGP is unique. The model features strategic complementarity in innovation due to spillovers across sectors: higher TFP in one sector raises the incentives to innovate in the other. We show below that this complementarity, in itself, does not lead to multiple BGPs or equilibrium paths in our model.

Lemma 4. *Fix a value of $P^k y^k \left(n^k\right)$. Then, the sectoral investment rates I_t^k / a_t^k jump immediately to their BGP levels i^k, while sectoral TFPs a_t^k converge gradually to their BGP.*

Proof. Along a BGP, the optimality condition (B.17) implies that

$$\vartheta^k \beta_k^k \iota'\left(i^k\right) = 1 - \varsigma^k \tag{B.32}$$

where ϑ^k is the constant multiplier. Equation (B.19) then implies

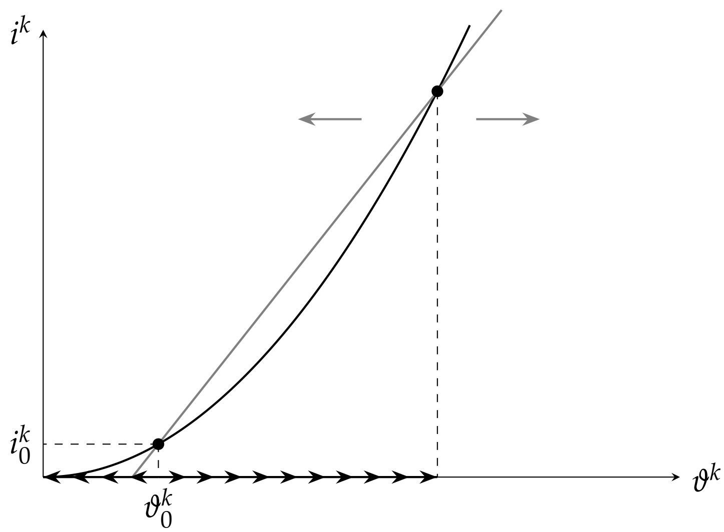
$$\left(1+\tau^k\right) P^k y^k \left(n^k\right) + \left(\frac{1}{\omega_1} - 1\right)\left(1-\varsigma^k\right) i^k = \left(r + \delta_a^k\right) \vartheta^k \tag{B.33}$$

and equation (B.20) becomes, for each $k = D, C$

$$g = \sum_{k'} \beta_{k'}^k \times \iota\left(i^{k'}\right) \frac{a_t^{k'}}{a_t^k} - \delta_a^k. \tag{B.34}$$

Note that this system is separable. The first two sets of expressions pin down the multiplier ϑ^k and the investment rate i^k, separately for each sector k. In particular,

Figure B.1: Phase diagram (R&D investment)



these objects do *not* depend on innovation spillovers $\beta^k_{k'}$ across different sectors $k' \neq k$. The last set of expressions is a set of two equations in two endogenous variables consisting of the common growth rate g and the relative TFP a_t^C/a_t^D.

Note that the expression (B.32) defines an increasing and convex schedule that starts at $(0,0)$ in the space $\left(\vartheta^k, i^k\right)$, as illustrated by the black curve in Figure B.1. In turn, equation (B.33) defines an increasing and linear schedule that starts at $\left(\tilde{\vartheta}^k, 0\right)$ for some $\tilde{\vartheta}^k > 0$, as illustrated by the gray line. A BGP exists whenever these schedules intersect, which we assume in the rest of this section. This is the case when the innovation technology $\iota\left(\cdot\right)$ is sufficiently productive and does not exhibit excessive decreasing returns to scale, i.e., the black curve is sufficiently to the right and the gray curve is sufficiently steep. Note that, whenever one intersection exists, there must be two, as illustrated by Figure B.1. These candidates for the BGP are ranked in that both ϑ^k and i^k are larger in the second BGP compared to the first one.

The expression (B.17) implies that ϑ_t^k and i_t^k must be on the convex schedule (in black) at all times. In turn, the expression (B.19) implies that, whenever i_t^k is above the linear schedule, ϑ_t^k goes down as indicated by a gray arrow. The opposite happens when i_t^k is below the linear schedule. Therefore, a single combination of jump variables ϑ_0^k and i_0^k in period $t = 0$ is consistent with the first candidate. In contrast, many combinations of ϑ_0^k and i_0^k converge to the second one.

Which of these two candidates does the economy converge to in the long-run?

To answer this question, consider the entrepreneurs' problem (B.16). An increase in revenues $P^k y^k\left(n^k\right)$ raises the marginal benefit of increasing sectoral TFP, and should therefore lead to a higher TFP $\exp\left(-gt\right) a_t^k$ and hence a higher investment rate ι^k using the law of motion of TFP (B.20). Graphically, an increase in revenues shifts the gray linear schedule down in Figure B.1. The fact that the long-run investment rate ι^k increases *at optimum* implies that the relevant BGP is the first one. Formally, the transversality condition for a_t^k implies that the economy should converge to the first BGP, not the second one. Therefore, the two jump variables ϑ_0^k and i_0^k are uniquely pinned down in $t = 0$ by the first candidate. They jump immediately to the unique values that do not violate the transversality condition.[53]

Over time, sectoral TFPs a_t^k move until the growth rates (B.34) converge to one another

$$\frac{\beta_C^C \times \iota\left(i^C\right) + \beta_D^C \times \iota\left(i^D\right)\hat{a}_t - \delta_a^C}{\beta_C^D \times \iota\left(i^C\right)/\hat{a}_t + \beta_D^D \times \iota\left(i^D\right) - \delta_a^D} \to 1, \tag{B.35}$$

which pins down uniquely the long-run relative TFP $\hat{a}_t \equiv a_t^C / a_t^D$. Note that the initial TFPs $\hat{a}_0$ need not satisfy (B.35) so sectoral TFPs adjust slowly over time. □

We have shown so far that the investment rate is uniquely pinned down when revenues $P^k y^k\left(n^k\right)$ are fixed and taken as given in each sector. The only other potential source of strategic complementarity in our model comes from the interaction between labor supply n^k and investment ι^k. A higher investment rate in a sector raises wages and hence labor supply there. At the same time, a higher labor supply in a sector raises the incentives to innovate there. We explain below the conditions under which this complementarity is sufficiently strong that it can induce multiple BGPs. The presence of labor market frictions complicates this explanation. For this reason, we momentarily consider the frictionless limit of our economy where there are no vacancy-posting costs $\nu \to 0$ and we abstract from skill accumulation, as in Sections 2–4.

Lemma 5. *Depending on the elasticity of substitution in the aggregate production function F_t, the strength of decreasing returns at the sectoral level $y^{k,\prime}\left(\cdot\right)$, and the decreasing returns in the investment technology $1/\omega_1$, the economy can potentially admit multiple BGPs. However, there is a unique equilibrium path given initial conditions a_0^k.*

Proof. Consider a particular sector k. We are interested in characterizing the effect

[53] This restriction is akin to the one pinning down consumption in the neoclassical growth model.

of a larger labor supply n^k on the price of labor services in that sector

$$\mathcal{P}^k \equiv \frac{\varepsilon^k - 1}{\varepsilon^k} \left(1 + \tau^k\right) \exp\left(-gt\right) a_t^k P^k y^{k,\prime} \left(n^k\right), \tag{B.36}$$

since, in turn, we know that $\mathcal{P}^k$ determines workers' labor supplies (Appendix B.3).[54] An increase in labor supply n^k has a direct and an indirect effect on the price of labor services $\mathcal{P}^k$. The direct effect leads to a lower $\mathcal{P}^k$, both due to decreasing returns $y^{k,\prime}\left(n^k\right)$ and to the fact that the price of the intermediate good P_t^k decreases by expressions (B.11)–(B.12). The indirect effect operates through a change in sectoral TFP a_t^k. The investment rate i^k and hence $\exp\left(-gt\right) a_t^k$ increase with sectoral revenues $P^k y^k \left(n^k\right)$. These revenues increase with the labor supply n^k if the aggregate production function F has an elasticity of substitution larger than 1, i.e., intermediate goods are substitutes in production, and decrease otherwise. Depending on this elasticity, $\exp\left(-gt\right) a_t^k$ can therefore increase or decrease with the labor supply n_t^k. As a result, the schedule $n^k \mapsto \mathcal{P}^k$ can be increasing, decreasing, or even non-monotonic in principle. The direct effect is more likely to dominate, and hence the relation $n^k \mapsto \mathcal{P}^k$ is more likely to decrease monotonically, when: the elasticity of substitution in production is close to 1; the marginal $y^{k,\prime}\left(n^k\right)$ falls quickly; and the decreasing returns in the innovation technology $1/\omega_1$ are large.

Appendix B.3 already established that workers' labor supply n^k (B.13) is given by a second and increasing schedule $\mathcal{P}^k \mapsto n^k$. The economy must satisfy this relationships at all time since labor reallocation is immediate in the frictionless benchmark studied in Sections 2–4. If the first schedule $n^k \mapsto \mathcal{P}^k$ described previously is decreasing, the BGP is unique. If, on the contrary, it is increasing or non-monotonic, then a higher labor supply n^k raises the sectoral TFP sufficiently that it attracts even more workers. There can be multiple BGPs in this case.

Finally, the equilibrium *path* is always unique given initial conditions a_0^k, even if there happen to be multiple BGPs. To see that, note that the labor supplies n_t^k are jump variables in the frictionless benchmark studied here. In contrast, the productivities $\exp\left(-gt\right) a_t^k$ are pre-determined state variables. Given $\exp\left(-gt\right) a_t^k$, the labor supplies are pinned down by the static system defined by expression (B.36), the equilibrium prices (B.11)–(B.12), and the labor supplies $n_t^C = \int \mathbf{1}_{\left\{\hat{\psi} \geq A_t^D / A_t^C\right\}} dG\left(\hat{\psi}\right)$

[54] The new growth rate that the economy jumps to might, in theory, be different from the initial growth rate, i.e., $g' \neq g$. We therefore deflate $\mathcal{P}^k$ by the new growth rate g'. We elaborate on this point at the end of this appendix.

and $n_t^D = 1 - n_t^C$. This system admits a unique solution, since there is no other source of strategic complementarity given $\exp(-gt)\, a_t^k$. By construction, there is a single equilibrium in every period, and the evolution of a_t^k is then given by the optimality conditions (B.17) and (B.19)–(B.20). □

Two further points of discussion are warranted. First, we assume that the government intervenes in the short-run to accelerate the transition to a given BGP. It is worth noting that all of our qualitative results would go through if, instead of speeding up the convergence to the *same* BGP, the government were to 1) intervene in the long-run to change this BGP or 2) if there are multiple BGPs, lift the economy to a different BGP where a_t^C / a_t^D is larger. In all these scenarios, the incumbent workers that we focus on experience a larger shifts in MRPLs over their lifetime.

Second, the growth rate g could, in theory, be different across the initial and final BGPs, i.e., before the transition begins and after it ends. As discussed in Section 3, we model sectoral shifts as transitions that affect the composition of economic activity across sectors, but not the long-term growth rate, in the spirit of Kongsamut et al. (2001) and Ngai and Pissarides (2007). This exercise implicitly places restrictions on the parameters governing the innovation process. Along the initial BGP,

$$g^{\text{init}} = \beta_D^C \iota\left(i^{D,\text{init}}\right) / \hat{a}^{\text{init}} - \delta_a^C \tag{B.37}$$

$$g^{\text{init}} = \beta_D^D \iota\left(i^{D,\text{init}}\right) - \delta_a^D, \tag{B.38}$$

where $\hat{a}^{\text{init}}$ is relative TFP defined as in expression (B.35). The two conditions above use the fact that $\iota^{C,\text{init}} = 0$ along the initial BGP since there is no demand for the good produced in that sector so $P^c = 0$.[55] In turn, along the final BGP,

$$g^{\text{fin}} = \beta_C^C \iota\left(i^{C,\text{fin}}\right) + \beta_D^C \iota\left(i^{D,\text{fin}}\right) / \hat{a}^{\text{fin}} - \delta_a^C \tag{B.39}$$

$$g^{\text{fin}} = \beta_C^D \iota\left(i^{C,\text{fin}}\right) \hat{a}^{\text{fin}} + \beta_D^D \iota\left(i^{D,\text{fin}}\right) - \delta_a^D \tag{B.40}$$

where $i^{C,\text{fin}} > i^{C,\text{init}} = 0$ and $i^{D,\text{fin}} < i^{D,\text{init}}$ once the sectoral shift is completed.

Note that the spillover parameters β_C^C and β_C^D are irrelevant for the initial BGP (B.37)–(B.38). However, they determine the long-run growth rate and the relative

[55] Knowledge spillovers across sectors imply that $a_t^C > 0$ along the initial BGP, even if $i^{C,\text{init}} = 0$.

TFP $\hat{a}^{\text{fin}}$ in the final BGP (B.39)–(B.40). We now characterize the set of values for β_C^C and β_C^D that ensures $g^{\text{init}} = g^{\text{fin}} = g$ so that the economy grows at the same rate along its initial and final BGPs.

Fix two arbitrary relative productivities $\hat{a}^{\text{init}}$ and $\hat{a}^{\text{fin}}$ along the initial and final BGPs with $\hat{a}^{\text{fin}} > \hat{a}^{\text{init}}$. These relative productivities pin down sectoral prices and sectoral labor supplies. By Lemma 4, we can treat the investment rates i^C and i^D as fixed conditional on $\hat{a}$. Expressions (B.37)–(B.38) allow us to solve for values of β_D^C and β_D^D that implement $\hat{a}^{\text{init}}$. Equating expressions (B.39)–(B.40) to either (B.37) or (B.38) yields two more equations which can be used to solve for the values of β_C^C and β_C^D that implement $\hat{a}^{\text{fin}}$ and ensure that $g^{\text{init}} = g^{\text{fin}} = g$. Varying the values $\hat{a}^{\text{init}}$ and $\hat{a}^{\text{fin}}$ allows us to span the entire set of $\beta_{k'}^k$ that ensure that ensure that the economy grows at the same rate along the initial and final BGPs.

B.6 Proof of Proposition 1 in the Full Model

In order to prove Proposition 1, we first establish the following result.

Lemma 6. *De-trended sectoral TFPs* $\exp(-gt)\, a_t^k$ *and the prices of intermediate goods* P_t^k *depend on the rest of the economy exclusively through the marginal revenue products of labor* A_t^k *and the innovation subsidies* ς_t^k.

Proof. We have already shown in Appendix B.3 that workers' outcomes and, in particular, sectoral labor supplies n_t^k (B.13), depend exclusively on the paths of A_t^k. In turn, the entrepreneurs' innovation problem (B.16) can be expressed as

$$\max_{\{I_t^k, a_t^k\}} \int_0^{+\infty} \exp(-rt) \left[\frac{\varepsilon^k}{\varepsilon^k - 1} \exp(gt)\, A_t^k \frac{y^k\left(n_t^k(\mathbf{A})\right)}{y^{k,\prime}\left(n_t^k(\mathbf{A})\right)} - \left(1 - \varsigma_t^k\right) I_t^k \right] dt \tag{B.41}$$
$$\text{s.t. } da_t^k = \left(\sum_{k'} \beta_{k'}^k \times \iota\left(\frac{I_t^{k'}}{a_t^{k'}}\right) a_t^{k'} - \delta_a^k a_t^k \right) dt,$$

where we have used the definition of MRPLs A_t^k (3.1) and made the dependence of labor supplies n_t^k on the sequence $\mathbf{A} \equiv \left\{A_t^k\right\}$ explicit. This system is therefore exclusively indexed by the sequence of $\mathbf{A}$ and the sequence of government subsidies to R&D $\boldsymbol{\varsigma} \equiv \left\{\varsigma_t^k\right\}$, so the equilibrium choice of $\mathbf{a} \equiv \left\{a_t^k\right\}$ only depends on these two objects.

Finally, note that the demands for intermediaries (B.11) and the sectoral technologies (B.12) imply that the price of intermediate goods $\mathbf{P} \equiv \left\{P_t^k\right\}$ can be expressed exclusively in terms of MRPLs $\mathbf{A}$ and R&D subsidies ς. □

We are now ready to prove Proposition 1 in our full model.

Proof of Proposition 1. Suppose that the government seeks to implement arbitrary paths of marginal revenue products of labor $\mathbf{A} \equiv \left\{A_t^k\right\}$. Fix the paths of subsidies on R&D $\boldsymbol{\varsigma} \equiv \left\{\varsigma_t^k\right\}$. By Lemma 6, sectoral productivities a_t^k and P_t^k can be expressed directly as a function of these two sequences $\mathbf{A}$ and $\boldsymbol{\varsigma}$. Furthermore, we already know that labor supplies n_t^k can also be expressed exclusively in terms of these two objects. By definition (3.1), the following sequence of subsidies on purchases of intermediate goods

$$\tau_t^k = \frac{\varepsilon^k}{\varepsilon^k - 1} \frac{A_t^k}{\exp\left(-gt\right) a_t^k\left(\mathbf{A}, \boldsymbol{\varsigma}\right) P_t^k\left(\mathbf{A}, \boldsymbol{\varsigma}\right) y^{k,\prime}\left(n_t^k\left(\mathbf{A}, \boldsymbol{\varsigma}\right)\right)} - 1 \tag{B.42}$$

implements the $\mathbf{A}$ path in equilibrium, where we have made the dependence of the denominator on $\mathbf{A}$ and $\boldsymbol{\varsigma}$ explicit. Therefore, the government can indeed implement any path of marginal revenue products $\mathbf{A} \equiv \left\{A_t^k\right\}$ it desires. Given those, we have already established in Appendix B.5 that the equilibrium path is unique and so are the subsidies τ_t^k. Finally, the government chooses lump sum taxes to balance its budget (B.21). As discussed in Appendix B.3, these taxes are non-distortionary and hence do not affect workers' labor supplies or equilibrium quantities or prices. □

B.7 Status Quo Transition and Counterfactuals

In this appendix, we explain why our model generates paths of sectoral MRPLs that tend to be S-shaped over time. This type of S-curves arise naturally in the context of innovation and technology adoption (Griliches, 1958; Shepherd et al., 2012). We then clarify the nature of the policy interventions that underlie the counterfactuals that we introduce in Section 3.2.

Status quo transition. Suppose that the government subsidies τ_t^k and ς_t^k are small along the status quo transition, for each $k = P, C$ and all t.[56] The economy initially rests on a BGP where sectoral TFPs grow at rate g. The transition begins with a change in the aggregate production function F_t in favor of the good produced in the growing sector C. As we discussed in Appendix B.5, the economy converges to a new BGP where the investment rate is permanently higher in one sector $i^{C,\text{fin}} > i^{C,\text{init}} = 0$ and lower in the other $i^{D,\text{fin}} < i^{D,\text{init}}$, the relative productivity in sector C is permanently higher $\hat{a}^{\text{fin}} > \hat{a}^{\text{init}}$, and sectoral TFPs again grow at rate g.

An S-shaped transition could arise for three reasons in the model. First, the path of F_t might itself generate an S-shape. This would occur, for instance, if information about the climate transition disseminates slowly, or if cultural preferences for clean and dirty output shift slowly over time.

Second, the innovation process contributes to$S$-shaped transitions. For the sake of argument, suppose for now that investment rates $i_t^k \equiv I_t^k / a_t^k$ jump to their new BGP immediately when the transition starts. In this case, the laws of motion (B.20) imply that, even with i^k fixed, the growth rate in sector C is larger over the transition than it is in the long-run, as the relative TFPs $\hat{a}_t \equiv a_t^C / a_t^D$ adjust progressively. Therefore, the sectoral TFP a_t^C grows at rate g along the initial BGP, then at a faster rate $g_t^C > g$ over the transition, then at a rate that eventually converges back to its previous level $g_t^C \to g$ along the final BGP. Therefore, the *de-trended* sectoral TFP $\exp(-gt)\, a_t^C$ features a convex segment over time at first, followed by an inflection point and a concave segment, before converging to a permanently higher level. The opposite happens in the sector D.

Finally, movements in the investment rates i_t^k themselves can also contribute to S-shaped transitions. As we established in Lemma 4 in Appendix B.5, the investment rate in sector k rises with labor supply n^k if sectors C and D are substitutes in production. In this case, labor reallocation across sectors causes i_t^C to rise initially as more and more workers move to that sector, before settling to its long-run level.

As discussed in Section 3.2, we suppose that the economy reaches its final BGP along the status quo in period T_0 (as opposed to asymptotically). This assumption gives our counterfactuals a tangible interpretation and allows us to map our

[56] These government subsidies could, in principle, be non-zero along the status quo. As we established in the previous appendix, the government can implement any arbitrary paths of MRPLs and hence undo the natural S-shapes that the model generates. We assume that the subsidies are sufficiently small along the status quo that this is not the case.

discussion to actual policy goals in our climate application, i.e., achieving zero net carbon by 2060, 2050, etc. The government can ensure that the economy settles on its long-run BGP in period T_0 by intervening from $t = T_0$ onward using its instruments to stabilize the investment rate i^k to their long-run values. This intervention is arbitrarily small when T_0 is sufficiently large. None of our conceptual results hinge on the fact that the de-trended MRPLs reach their final levels *exactly* in period T_0, i.e., they could be arbitrarily close to these levels in T_0.

Counterfactuals. As discussed in Section 3, the government can speed up the transition using *short-run* subsidies and taxes on the sales of the two intermediate goods or on innovation in the sectors producing them. As a result, the de-trended MRPLs converge faster to their long-run levels. As the government phases out its intervention and the sectoral TFPs (B.20) inch closer to their new BGP, the investment rate in sector C becomes lower than it would have been in the same period absent the intervention and the opposite happens in sector D, so the de-trended MRPLs flatten earlier. The government intervenes until these MRPLs reach the long-run levels that they would have attained under the status quo transition. At that point, the investment rates stay at their new permanent levels and the economy settles on its new BGP growing at rate g. In Appendix B.5, we also discussed the case where the government intervenes in the long-run to affect the final BGP and explained why our conceptual insights carry through in this case.

C Computational Appendix

We state workers' and firm's values recursively in Appendix C.1 and introduce some useful notation. We discretize the resulting system of differential equations in Appendix C.2 and describe how we solve it numerically. We then explain how we iterate on the distribution of workers' idiosyncratic states in Appendix C.3. Finally, we provide details about our calibration strategy in Appendix C.4.

C.1 Workers' and Firm's Values

We now formulate workers' and firms' values recursively. As shown in Appendix B.4, we do not need to include workers' reference skill ψ in our state space. Relative to that appendix, we state workers' values in all employment states, i.e., whether they are employed, unemployed, or claim the outside option. To save on notation, we let $W_t(j, \hat{\psi}, s) \equiv W_t^\star(j, \hat{\psi}, s)$ denote workers' normalized values defined in Appendix B.4. We define the functions V_t, θ_t, p_t, δ_j, $\hat{\delta}_j$ and h and the constant ρ similarly, dropping the $\star$ superscript relative to Appendices B.1 and B.4.

Workers. Workers are indexed by their age j; their relative skill $\hat{\psi}$ in the clean sector; and their employment state s. The value of an employed worker satisfies

$$\begin{aligned}
\rho W_t(j, \hat{\psi}, s) = {} & w(s)\, h(\hat{\psi}, k(s)) \\
& + \lambda^E \sum_{s'} p_t(s'; j, \hat{\psi}, s)\, f(\theta_t(j, \hat{\psi}, s')) \left[W_t(j, \hat{\psi}, s') - W_t(j, \hat{\psi}, s)\right] \\
& + \chi \left[W_t(j, \hat{\psi}, U) - W_t(j, \hat{\psi}, s)\right] \\
& + \delta_j(k(s))\, W_t(j, \hat{\psi}, s) + \hat{\psi}\hat{\delta}_j(k(s)) \frac{\partial}{\partial \hat{\psi}} W_t(j, \hat{\psi}, s) \\
& + \frac{\partial}{\partial j} W_t(j, \hat{\psi}, s) + \frac{\partial}{\partial t} W_t(j, \hat{\psi}, s)
\end{aligned} \tag{C.1}$$

for all ages before retirement $j \in [0, J)$, relative productivities $\hat{\psi}$ in the clean sector, and markets $s \equiv (k, w)$. As explained above, we have removed $\star$'s for notational convenience so that, for instance, $\delta_j^\star(k)$ from equation (B.1) becomes $\delta_j(k)$. The probability that a worker searches in market s' when they receive a search opportunity and are employed in sector s is

$$p_t(s'; j, \hat{\psi}, s) \equiv \frac{\exp\left(f(\theta_t(j, \hat{\psi}, s')) \left[W_t(j, \hat{\psi}, s') - W_t(j, \hat{\psi}, s)\right] / \zeta\right)}{\sum_{\hat{s}} \exp\left(f(\theta_t(j, \hat{\psi}, \hat{s})) \left[W_t(j, \hat{\psi}, \hat{s}) - W_t(j, \hat{\psi}, s)\right] / \zeta\right)} \tag{C.2}$$

In turn, the value of a worker who claims the outside option is

$$\begin{aligned}
\rho W_t(j, \hat{\psi}, O) = {} & h(\hat{\psi}, O) \\
& + \lambda^E \sum_{s} p_t(s; j, \hat{\psi}, O)\, f(\theta_t(j, \hat{\psi}, s)) \left[W_t(j, \hat{\psi}, s) - W_t(j, \hat{\psi}, O)\right] \\
& + \chi \left[W_t(j, \hat{\psi}, U) - W_t(j, \hat{\psi}, O)\right] \\
& + \delta_j(O)\, W_t(j, \hat{\psi}, O) + \frac{\partial}{\partial j} W_t(j, \hat{\psi}, O) + \frac{\partial}{\partial t} W_t(j, \hat{\psi}, O),
\end{aligned} \tag{C.3}$$

Finally, the value of an unemployed worker satisfies

$$\begin{aligned}
\rho W_t\left(j,\hat{\psi},U\right) = {} & h\left(\hat{\psi},U\right) \\
& + \lambda^U \sum_s p_t\left(s;j,\hat{\psi},U\right) f\left(\theta_t\left(j,\hat{\psi},s\right)\right)\left[W_t\left(j,\hat{\psi},s\right) - W_t\left(j,\hat{\psi},U\right)\right] \\
& + \delta_j\left(U\right) W_t\left(j,\hat{\psi},U\right) + \frac{\partial}{\partial j} W_t\left(j,\hat{\psi},U\right) + \frac{\partial}{\partial t} W_t\left(j,\hat{\psi},U\right)
\end{aligned} \tag{C.4}$$

The boundary conditions are

$$W_t\left(J,\hat{\psi},s\right) = W_t\left(J,\hat{\psi},O\right) = W_t\left(J,\hat{\psi},U\right) = 0 \tag{C.5}$$

when they retire at age $j = J$, for all relative productivities $\hat{\psi}$ and markets s.

Firms. The value of a firm satisfies

$$\begin{aligned}
rV_t\left(j,\hat{\psi},s\right) = {} & \left(A_t^{k(s)} - w\left(s\right)\right) h\left(\hat{\psi},k\left(s\right)\right) \\
& - \lambda^E \sum_{s'} p_t\left(s';j,\hat{\psi},s\right) f\left(\theta_t\left(j,\hat{\psi},s'\right)\right) V_t\left(j,\hat{\psi},s\right) \\
& - \chi V_t\left(j,\hat{\psi},s\right) \\
& + \delta_j\left(k\left(s\right)\right) V_t\left(j,\hat{\psi},s\right) + \hat{\psi}\hat{\delta}_j\left(k\left(s\right)\right) \frac{\partial}{\partial \hat{\psi}} V_t\left(j,\hat{\psi},s\right) \\
& + \frac{\partial}{\partial j} V_t\left(j,\hat{\psi},s\right) + \frac{\partial}{\partial t} V_t\left(j,\hat{\psi},s\right),
\end{aligned} \tag{C.6}$$

for all ages before retirement $j \in [0, J)$, relative productivities $\hat{\psi}$ in the clean sector, and markets $s \equiv (k, w)$, where $r \equiv \rho$ is the interest rate. The expression above uses the fact that $\mathcal{P}_t^k = A_t^k$, as shown in Appendix B.2. The boundary condition is

$$V_t\left(J,\hat{\psi},s\right) = 0 \tag{C.7}$$

when workers retire at age $j = J$, for all relative productivities $\hat{\psi}$ and markets s.

Free entry. The equilibrium tightnesses $\theta\left(j,\hat{\psi},s\right)$ are pinned down by the firms' free entry condition

$$q\left(\theta_t\left(j,\hat{\psi},s\right)\right) V_t\left(j,\hat{\psi},s\right) = \nu \tag{C.8}$$

if this equation has a solution given $q\left(\cdot\right) \leq 1$, and $\theta_t\left(j,\hat{\psi},s\right) = 0$ otherwise.

C.2 Solving for Workers' and Firms' Values

Discretized system. We start by discretizing the differential equations of Appendix C.1. We let $\Delta > 0$ denote the time step between consecutive periods t or ages j, and $\hat{\Delta} > 0$ denote the distance between grid points for the relative skill $\hat{\psi}$. We let $\Delta \equiv 1/4$ so each period corresponds to a quarter. Our finite difference scheme is implicit in markets s, i.e., we evaluate the continuation values associated to changes in s in period t. As usual, the coefficients in front of these continuation values are evaluated in $t + \Delta$ to obtain a system that is linear in the current values. On the contrary, our scheme is explicit in the relative skills $\hat{\psi}$, which allows us to parallelize computations over $\hat{\psi}$ to speed up our code. As usual, the scheme is explicit in age j and time t, i.e., we evaluate the continuation values associated to aging or the passing of time in period $t + \Delta$.

After discretization, the value of being employed becomes

$$\begin{aligned}
\rho W_t\left(j, \hat{\psi}, s\right) = {} & w\left(s\right) h\left(\hat{\psi}, k\left(s\right)\right) \\
& + \lambda^E \sum_{s'} p_{t+\Delta}\left(s'; j+\Delta, \hat{\psi}, s\right) f\left(\theta_{t+\Delta}\left(j+\Delta, \hat{\psi}, s'\right)\right) \\
& \qquad \times \left[W_t\left(j, \hat{\psi}, s'\right) - W_t\left(j, \hat{\psi}, s\right)\right] \\
& + \chi\left[W_t\left(j, \hat{\psi}, U\right) - W_t\left(j, \hat{\psi}, s\right)\right] \\
& + \delta_j\left(k\left(s\right)\right) W_t\left(j, \hat{\psi}, s\right) \\
& + \hat{\psi}\hat{\delta}_j\left(k\left(s\right)\right) \frac{W_{t+\Delta}\left(j+\Delta, \hat{\psi} + \tilde{\Delta}\left(k\left(s\right)\right), s\right) - W_{t+\Delta}\left(j+\Delta, \hat{\psi}, s\right)}{\tilde{\Delta}\left(k\left(s\right)\right)} \\
& + \frac{W_{t+\Delta}\left(j+\Delta, \hat{\psi}, s\right) - W_t\left(j, \hat{\psi}, s\right)}{\Delta}
\end{aligned} \tag{C.9}$$

for all markets $s \equiv (k, w)$, where $\Delta > 0$ denotes the time step. In turn, the step $\hat{\Delta}$ for the relative skill $\hat{\psi}$ depends on which sector the worker is employed in. We let $\tilde{\Delta}\left(k\left(s\right)\right) \equiv \hat{\Delta} > 0$ when $k\left(s\right) = C$ so $\hat{\psi}$ increases and $\tilde{\Delta}\left(k\left(s\right)\right) \equiv -\hat{\Delta} < 0$ when $k\left(s\right) = P$ so $\hat{\psi}$ decreases.

In turn, the value of claiming the outside option satisfies

$$\begin{aligned}\rho W_t\left(j,\hat{\psi},O\right) = {} & h\left(\hat{\psi},O\right) \\ & + \lambda^E \sum_s p_{t+\Delta}\left(s; j+\Delta,\hat{\psi},O\right) f\left(\theta_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)\right) \\ & \qquad \times \left[W_t\left(j,\hat{\psi},s\right) - W_t\left(j,\hat{\psi},O\right)\right] \\ & + \chi\left[W_t\left(j,\hat{\psi},U\right) - W_t\left(j,\hat{\psi},O\right)\right] \\ & + \delta_j\left(O\right) W_t\left(j,\hat{\psi},O\right) + \frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi},O\right) - W_t\left(j,\hat{\psi},O\right)}{\Delta}\end{aligned} \tag{C.10}$$

and the value of being unemployed satisfies

$$\begin{aligned}\rho W_t\left(j,\hat{\psi},U\right) = {} & h\left(\hat{\psi},U\right) \\ & + \lambda^U \sum_s p_{t+\Delta}\left(s; j+\Delta,\hat{\psi},U\right) f\left(\theta_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)\right) \\ & \qquad \times \left[W_t\left(j,\hat{\psi},s\right) - W_t\left(j,\hat{\psi},U\right)\right] \\ & + \delta_j\left(U\right) W_t\left(j,\hat{\psi},U\right) + \frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi},U\right) - W_t\left(j,\hat{\psi},U\right)}{\Delta}\end{aligned} \tag{C.11}$$

Finally, the value of a firm becomes

$$\begin{aligned}rV_t\left(j,\hat{\psi},s\right) = {} & \left(A_t^{k(s)} - w\left(s\right)\right) h\left(\hat{\psi},k\left(s\right)\right) \\ & - \lambda^E \sum_{s'} p_{t+\Delta}\left(s'; j+\Delta,\hat{\psi},s\right) f\left(\theta_{t+\Delta}\left(j+\Delta,\hat{\psi},s'\right)\right) V_t\left(j,\hat{\psi},s\right) \\ & - \chi V_t\left(j,\hat{\psi},s\right) \\ & + \delta_j\left(k\left(s\right)\right) V_t\left(j,\hat{\psi},s\right) \\ & + \hat{\psi}\hat{\delta}_j\left(k\left(s\right)\right) \frac{V_{t+\Delta}\left(j+\Delta,\hat{\psi}+\tilde{\Delta}\left(k\left(s\right)\right),s\right) - V_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)}{\tilde{\Delta}\left(k\left(s\right)\right)} \\ & + \frac{V_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right) - V_t\left(j,\hat{\psi},s\right)}{\Delta},\end{aligned} \tag{C.12}$$

for all markets $s \equiv (k, w)$, where $\tilde{\Delta}(k)$ was defined above.

Workers' and firms' values are characterized by the system (C.9)–(C.12) together with the boundary conditions (C.5) and (C.7), the search probabilities (C.2), and the free entry condition (C.8). Note that this system is independent of the distribution of workers' states given paths of A_t^k — this is the block-recursive property of directed search models (Menzio and Shi, 2011).

Numerical solution. We explain below how we solve for workers' and firms' values.

Algorithm. Solve for workers' and firms' values.

Step 0. Let the terminal values for workers $W_T\left(J,\hat{\psi},s\right)$, $W_T\left(J,\hat{\psi},O\right)$ and $W_T\left(J,\hat{\psi},U\right)$ and firms $V_T\left(J,\hat{\psi},s\right)$ satisfy the boundary conditions (C.5) and (C.7) at retirement at age J. Initiate the recursion by setting the time index to $t=T-\Delta$ and the age index to $j=J-\Delta$.

Step 1. Compute the search probabilities $p_{t+\Delta}\left(s';j+\Delta,\hat{\psi},s\right)$ using (C.2), and the tightnesses $\theta_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)$ using (C.8).

Step 2. Solve for workers' values:

Solve for the values

$$\mathbf{W}_t\left(j,\hat{\psi}\right)=\left[\mathbf{\Lambda}\left(j\right)-\mathbf{M}_{t+\Delta}\left(j+\Delta,\hat{\psi}\right)\right]^{-1}\mathbf{B}_t\left(j,\hat{\psi}\right), \tag{C.13}$$

where the vector $\mathbf{W}_t\left(j,\hat{\psi}\right)$ contains workers' values, the matrix $\mathbf{M}_{t+\Delta}\left(j+\Delta,\hat{\psi}\right)$ captures workers' transitions across markets and employment statuses which are part of the implicit scheme, the vector $\mathbf{B}_t\left(j,\hat{\psi}\right)$ captures workers' flow earnings and the evolution of the relative skill $\hat{\psi}$, time t and age j, and the matrix $\mathbf{\Lambda}\left(j\right)$ captures discounting and the evolution of the reference skill ψ. Formally, workers' values are

$$\mathbf{W}_t\left(j,\hat{\psi}\right)_{[s,1]}\equiv W_t\left(j,\hat{\psi},s\right), \tag{C.14}$$

the transition matrix is

$$\mathbf{M}_{t+\Delta}\left(j,\hat{\psi}\right)\equiv\begin{bmatrix}\mathbf{M}_{1,1,t+\Delta}\left(j,\hat{\psi}\right) & \mathbf{M}_{1,2,t+\Delta}\left(j,\hat{\psi}\right) & \mathbf{M}_{1,3,t+\Delta}\left(j,\hat{\psi}\right)\\ \mathbf{M}_{2,1,t+\Delta}\left(j,\hat{\psi}\right) & M_{2,2,t+\Delta}\left(j,\hat{\psi}\right) & M_{2,3,t+\Delta}\left(j,\hat{\psi}\right)\\ \mathbf{M}_{3,1,t+\Delta}\left(j,\hat{\psi}\right) & M_{3,2,t+\Delta}\left(j,\hat{\psi}\right) & M_{3,3,t+\Delta}\left(j,\hat{\psi}\right)\end{bmatrix}, \tag{C.15}$$

and the flows and discounting are

$$\mathbf{B}_t\left(j,\hat{\psi}\right)\equiv\begin{bmatrix}\mathbf{B}_{1,t}\left(j,\hat{\psi}\right)\\ B_{2,t}\left(j,\hat{\psi}\right)\\ B_{3,t}\left(j,\hat{\psi}\right)\end{bmatrix}\quad,\quad\mathbf{\Lambda}\left(j\right)\equiv\begin{bmatrix}\mathbf{\Lambda}_1\left(j\right) & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \Lambda_2\left(j\right) & 0\\ \mathbf{0} & 0 & \Lambda_3\left(j\right)\end{bmatrix} \tag{C.16}$$

We start by describing the terms involving employed workers. We define the square matrix

$$\mathbf{M}_{1,1,t+\Delta}\left(j,\hat{\psi}\right)_{[s,s']} \equiv \lambda^{E} p_{t+\Delta}\left(s';j+\Delta,\hat{\psi},s\right) f\left(\theta_{t+\Delta}\left(j+\Delta,\hat{\psi},s'\right)\right) + \mathbf{N}_{t+\Delta}\left(j,\hat{\psi}\right)_{[s,s']} \tag{C.17}$$

for each pair of markets $s=(k,w)$ and $s'=(k',w')$, with

$$\mathbf{N}_{t+\Delta}\left(j,\hat{\psi}\right)_{[s,s']} \equiv \begin{cases} -\lambda^{E}\sum_{\hat{s}} p_{t+\Delta}\left(\hat{s};j+\Delta,\hat{\psi},s\right)\times \quad f\left(\theta_{t+\Delta}\left(j+\Delta,\hat{\psi},\hat{s}\right)\right)-\chi & \text{if } s'=s \\ 0 & \text{otherwise} \end{cases}, \tag{C.18}$$

and the column vectors

$$\mathbf{M}_{1,2,t+\Delta}\left(j,\hat{\psi}\right)_{[s,1]} = \lambda^{E} p_{t+\Delta}\left(O;j+\Delta,\hat{\psi},s\right) \tag{C.19}$$

$$\mathbf{M}_{1,3,t+\Delta}\left(j,\hat{\psi}\right)_{[s,1]} = \lambda^{E} p_{t+\Delta}\left(U;j+\Delta,\hat{\psi},s\right)+\chi \tag{C.20}$$

where we have used the fact that workers can transition instantaneously to the outside option or unemployment if they are given the opportunity to search, i.e., $f^{O}=f^{U}=1$. We also define the column vectors

$$\begin{aligned}\mathbf{B}_{1,t}\left(j,\hat{\psi}\right)_{[s,1]} \equiv\ & w\left(s\right)h\left(\hat{\psi},k\left(s\right)\right) \\ & + \hat{\psi}\hat{\delta}_{j}\left(k\left(s\right)\right)\frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi}+\tilde{\Delta}\left(k\left(s\right)\right),s\right)-W_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)}{\tilde{\Delta}\left(k\left(s\right)\right)} \\ & + \frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)}{\Delta},\end{aligned} \tag{C.21}$$

and the diagonal matrix

$$\mathbf{\Lambda}_{1}\left(j\right)_{[s,s']} = \begin{cases} \rho+\frac{1}{\Delta}-\delta_{j}\left(k\left(s\right)\right) & \text{if } s'=s \\ 0 & \text{otherwise} \end{cases} \tag{C.22}$$

We now turn to workers claiming the outside option. We define the row vector

$$\mathbf{M}_{2,1,t+\Delta}\left(j,\hat{\psi}\right)_{[1,s]} \equiv \lambda^{E} p_{t+\Delta}\left(s; j+\Delta, \hat{\psi}, O\right) f\left(\theta_{t+\Delta}\left(j+\Delta, \hat{\psi}, s\right)\right), \tag{C.23}$$

for each market $s=(k,w)$, the scalar

$$\begin{aligned} M_{2,2,t+\Delta}\left(j,\hat{\psi}\right) = {} & \lambda^{E} p_{t+\Delta}\left(O; j+\Delta, \hat{\psi}, O\right) \\ & - \lambda^{E} \sum_{s} p_{t+\Delta}\left(s; j+\Delta, \hat{\psi}, O\right) f\left(\theta_{t+\Delta}\left(j+\Delta, \hat{\psi}, s\right)\right) \\ & - \chi, \end{aligned} \tag{C.24}$$

and the scalar

$$M_{2,3,t+\Delta}\left(j,\hat{\psi}\right) \equiv \lambda^{E} p_{t+\Delta}\left(U; j+\Delta, \hat{\psi}, O\right) + \chi. \tag{C.25}$$

We also define the scalars

$$B_{2,t}\left(j,\hat{\psi}\right) \equiv h\left(\hat{\psi}, O\right) + \frac{W_{t+\Delta}\left(j+\Delta, \hat{\psi}, O\right)}{\Delta} \tag{C.26}$$

and

$$\Lambda_{2}\left(j\right) \equiv \rho + \frac{1}{\Delta} - \delta_{j}\left(O\right) \tag{C.27}$$

Finally, we consider unemployed workers. We define the row vector

$$\mathbf{M}_{3,1,t+\Delta}\left(j,\hat{\psi}\right)_{[1,s]} \equiv \lambda^{U} p_{t+\Delta}\left(s; j+\Delta, \hat{\psi}, U\right) f\left(\theta_{t+\Delta}\left(j+\Delta, \hat{\psi}, s\right)\right), \tag{C.28}$$

for each market $s=(k,w)$, the scalar

$$M_{3,2,t+\Delta}\left(j,\hat{\psi}\right) = \lambda^{U} p_{t+\Delta}\left(O; j+\Delta, \hat{\psi}, U\right), \tag{C.29}$$

and the scalar

$$\begin{aligned} M_{3,3,t+\Delta}\left(j,\hat{\psi}\right) \equiv {} & \lambda^{U} p_{t+\Delta}\left(U; j+\Delta, \hat{\psi}, U\right) \\ & - \lambda^{U} \sum_{s} p_{t+\Delta}\left(s; j+\Delta, \hat{\psi}, U\right) f\left(\theta_{t+\Delta}\left(j+\Delta, \hat{\psi}, s\right)\right). \end{aligned} \tag{C.30}$$

We also define the scalars

$$B_{3,t}\left(j,\hat{\psi}\right) \equiv h\left(\hat{\psi},U\right) + \frac{W_{t+\Delta}\left(j+\Delta,\hat{\psi},U\right)}{\Delta} \tag{C.31}$$

and

$$\Lambda_3\left(j\right) \equiv \rho + \frac{1}{\Delta} - \delta_j\left(U\right). \tag{C.32}$$

Step 3. Solve for the firms' values:

Solve for the values

$$\mathbf{V}_t\left(j,\hat{\psi}\right) = \left[\mathbf{C}\left(j\right) - \mathbf{P}_{t+\Delta}\left(j,\hat{\psi}\right)\right]^{-1}\mathbf{D}_t\left(j,\hat{\psi}\right), \tag{C.33}$$

where the vector $\mathbf{V}_t\left(j,\hat{\psi}\right)$ contains firms' values, the matrix $\mathbf{P}_{t+\Delta}\left(j+\Delta,\hat{\psi}\right)$ captures workers' transitions across markets and employment statuses which are part of the implicit scheme, the vector $\mathbf{D}_t\left(j,\hat{\psi}\right)$ contains firms' flow profits and and the evolution of the relative skill $\hat{\psi}$, time t and age j, the matrix $\mathbf{C}\left(j\right)$ captures discounting and the evolution of the reference skill ψ. Formally, the firms' values are

$$\mathbf{V}_t\left(j,\hat{\psi}\right)_{[s,1]} \equiv V_t\left(j,\hat{\psi},s\right), \tag{C.34}$$

with the square matrix

$$\mathbf{P}_{t+\Delta}\left(j,\hat{\psi}\right)_{[s,s']} \equiv \begin{cases} -\lambda^E \sum_{\hat{s}} p_{t+\Delta}\left(\hat{s};j+\Delta,\hat{\psi},s\right) \times & \text{if } s'=s \\ \qquad f\left(\theta_{t+\Delta}\left(j+\Delta,\hat{\psi},\hat{s}\right)\right) - \chi & \\ 0 & \text{otherwise} \end{cases}, \tag{C.35}$$

the column vector

$$\begin{aligned}\mathbf{D}_t\left(j,\hat{\psi}\right)_{[s,1]} = &\left(A_t^{k(s)} - w\left(s\right)\right) h\left(\hat{\psi},k\left(s\right)\right) \\ &+ \hat{\psi}\hat{\delta}_j\left(k\left(s\right)\right)\frac{V_{t+\Delta}\left(j+\Delta,\hat{\psi}+\tilde{\Delta}\left(k\left(s\right)\right),s\right) - V_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)}{\tilde{\Delta}\left(k\left(s\right)\right)} \\ &+ \frac{V_{t+\Delta}\left(j+\Delta,\hat{\psi},s\right)}{\Delta},\end{aligned} \tag{C.36}$$

and the square matrix

$$C(j)_{[s,s']} = \begin{cases} r + \frac{1}{\Delta} - \delta_j(k(s)) & \text{if } s' = s \\ 0 & \text{otherwise} \end{cases}. \tag{C.37}$$

for all markets $s = (k, w)$ and employment statuses.

Step 4. Set the age $j' = j - \Delta$, repeat Steps 1–3, and continue until $j = 0$.

Step 5. When solving for transitions, set $t' = t - \Delta$, repeat Steps 1–4 in that period, and continue until $t = 0$. This last step is redundant when solving for stationary values along a BGP.

C.3 Distribution of Workers' States

We denote the density of workers' states by $\mu_t(j, \hat{\psi}, s)$. As in Appendix C.1, we do not keep track of a worker's reference skill ψ when iterating over the distribution.

As usual, the law of motion for the distribution is closely related to the recursive formulation of workers' values (Appendix C.2). In fact, the matrix $\mathbf{M}_t(j, \omega)$ encodes the information required to iterate on the distribution. This is intuitive: this matrix captures the outflows from the current state and the inflows into subsequent ones. The only flows that are not captured by $\mathbf{M}_t(j, \omega)$ are the changes in age j and the relative productivity in the clean sector $\hat{\psi}$ that are included in $\mathbf{B}_t(j, \hat{\psi})$ as part of the explicit scheme.

Abstracting momentarily from changes in $\hat{\psi}$, the law of motion for the distribution is

$$\frac{d}{d\epsilon}\boldsymbol{\mu}_{t+\epsilon}(j+\epsilon, \hat{\psi}) = \mathbf{M}_t(j, \omega)' \boldsymbol{\mu}_t(j, \hat{\psi}) \tag{C.38}$$

for all ages $j \in [0, J - \Delta)$, where $\mathbf{M}_t(j, \omega)'$ is the matrix transpose of $\mathbf{M}_t(j, \omega)$ and

$$\begin{gathered} \boldsymbol{\mu}_t(j, \hat{\psi})_{[s,1]} \equiv \mu_t(j, \hat{\psi}, s) \qquad \boldsymbol{\mu}_t(j, \hat{\psi})_{[\text{end-1},1]} \equiv \mu_t(j, \hat{\psi}, O) \\ \boldsymbol{\mu}_t(j, \hat{\psi})_{[\text{end},1]} \equiv \mu_t(j, \hat{\psi}, U) \end{gathered} \tag{C.39}$$

Integrating (C.38) over a short time interval Δ,

$$\boldsymbol{\mu}_{t+\Delta}(j+\Delta, \hat{\psi}) = \boldsymbol{\mu}_t(j, \hat{\psi}) \exp\left(\mathbf{M}_t(j, \hat{\psi})' \Delta\right), \tag{C.40}$$

where $\exp(X) \equiv \sum_{k=0}^{\infty} 1/k! X^k$ is the matrix exponential. We iterate over this law of motion over intervals over duration Δ, starting in period $t = 0$. The non-linear formulation (C.40) is computationally more intensive than a simple linear approximation of expression (C.38). However, it allows us to compute workers' values over relative long time intervals, e.g., a quarter $\Delta \equiv 1/4$, while still accounting for multiple, high-frequency changes in employment status that occur over shorter periods, e.g., a month. Doing so is important to ensure that our model-simulated moments (such as monthly job flows) are consistent with their empirical counterparts measured in the CPS (Appendix C.4).

As mentioned, the law of motion above does not account for changes in the relative productivity $\hat{\psi}$ captured by our explicit scheme. We account for these transitions by computing

$$\boldsymbol{\mu}_{t+\Delta}\left(j+\Delta,\hat{\psi}\right) = \sum_{\tilde{\psi}} \mathbf{Q}\left(\hat{\psi},\tilde{\psi}\right) \boldsymbol{\mu}_t\left(j,\tilde{\psi}\right) \exp\left(\mathbf{M}_t\left(j,\tilde{\psi}\right)' \Delta\right), \tag{C.41}$$

where the sum is over the initial relative productivities $\tilde{\psi}$. The matrices $\mathbf{Q}\left(\hat{\psi},\tilde{\psi}\right)_{[s,s']}$ are diagonal and their elements $[s,s]$ indicate the probability that $\tilde{\psi}$ switches to $\hat{\psi}$ depending on workers' current employment status s. The relative productivity evolves deterministically by $\hat{\delta}\left(k\left(s\right)\right)\Delta$ after a time step Δ if the worker is employed i.e., $k\left(s\right) = P, C$. When this new value does not fall on a grid point for $\hat{\psi}$, we allocate the corresponding mass to the neighboring points following the non-stochastic simulation method of Young (2010). Accordingly, we set the diagonal elements to

$$\mathbf{Q}\left(\hat{\psi},\tilde{\psi}\right)_{[s,s]} \equiv \begin{cases} 1 - q\left(\hat{\psi}_0\left(\tilde{\psi},s\right),\tilde{\psi},s\right) & \text{if } \hat{\psi} = \hat{\psi}_0\left(\tilde{\psi},s\right) \\ q\left(\hat{\psi}_0\left(\tilde{\psi},s\right),\tilde{\psi},s\right) & \text{if } \hat{\psi} = \hat{\psi}_1\left(\tilde{\psi},s\right), \\ 0 & \text{otherwise} \end{cases} \tag{C.42}$$

where $\hat{\psi}_0\left(\tilde{\psi},s\right)$ is the largest point on the grid for relative productivities that is smaller or equal to $\tilde{\psi} + \hat{\delta}\left(k\left(s\right)\right)\Delta$ and $\hat{\psi}_1\left(\tilde{\psi},s\right) \equiv \hat{\psi}_0\left(\tilde{\psi},s\right) + \hat{\Delta}$ is the smallest point on the grid that is strictly larger than $\tilde{\psi} + \hat{\delta}\left(k\left(s\right)\right)\Delta$, and

$$q\left(\hat{\psi}_0,\tilde{\psi},s\right) \equiv \left(\tilde{\psi} + \hat{\delta}\left(k\left(s\right)\right)\Delta - \hat{\psi}_0\right)/\hat{\Delta} \tag{C.43}$$

allows us to assign masses in inverse proportion to the distance between $\tilde{\psi} + \hat{\delta}(k(s))\,\Delta$ and the closest grid points. All the off-diagonal elements are zero.

An alternative approach would be to extend the law of motion (C.40) by stacking the vectors $\boldsymbol{\mu}_t\left(j, \hat{\psi}\right)$ for all values of $\hat{\psi}$, and define a larger transition matrix that also accounts for flows between values of $\hat{\psi}$. This approach turns out to be computationally demanding since this new transition matrix is very large and prevents numerical parallelization. The only difference between our approach (C.41) and this alternative one is that we suppose that the state $\hat{\psi}$ evolves only once at the end of the time interval Δ, whereas the alternative approach accounts for potentially multiple changes over this time interval.

Finally, the law of motion (C.41) has a boundary at age $j = J$ when workers retire. The flow of retirees, and hence newborns, is

$$\mu_t^R = \int\int \mu_{t-\Delta}\left(J, d\hat{\psi}, ds\right) \tag{C.44}$$

Newborn workers are endowed with a reference skill $\psi = 1$ (which we do not need to keep track of) and a relative skill $\hat{\psi} \sim G$. Accordingly, the distribution of workers' states among newborn workers is

$$\mu_t\left(0, d\hat{\psi}, s\right) = \begin{cases} \mu_t^R \times G\left(d\hat{\psi}\right) & \text{if } s = U \\ 0 & \text{otherwise} \end{cases} \tag{C.45}$$

since newborns start unemployed.

Stationary distribution and transition. As explained in Section 3, the de-trended economy starts from a stationary distribution where sector C is absent, i.e., $A^C \equiv 0$. We normalize the stationary distribution so it has a unit mass.

C.4 Calibration

We discretize our continuous time model into short periods corresponding to a quarter, i.e., $\Delta \equiv 1/4$. We explain below how we compute the moments that we use in our steady state calibration (Section 5.2).

State space reduction along the initial BGP. The fact that the good produced in sector C is not valued along the initial BGP means that $P^c = 0$ so there are no vacancies posted in that sector and no workers supply labor there (Sections 3.1 and 5.1). Therefore, our initial BGP does not depend on the distribution of relative skill $\hat{\psi}$ and, in particular, on its endowment distribution G. Formally, one can guess and verify that the de-trended values satisfy $W_t\left(j, \hat{\psi}, s\right) = W_t\left(j, s\right)$ and $V_t\left(j, \hat{\psi}, s\right) = V_t\left(j, s\right)$ for any relative productivities $\hat{\psi}$ along the initial BGP.[57] Furthermore, this initial BGP does not depend on $\hat{\delta}_j$ itself, only on $\delta_j^+ + \hat{\delta}_j$ which we directly measure in the data (Section 5.2). Without loss of generality, we can therefore normalize $\hat{\psi} = 1$ and set $\hat{\delta} = 0$ for our steady state calibration. This property allows us to discipline internally the seven parameters in Table 5.2 to match steady state moments *before* structurally estimating the distribution G and the rate $\hat{\delta}$.

Of course, different distributions of skill endowments G and skill accumulation rates $\hat{\delta}$ imply different stationary distributions of relative skill $\hat{\psi}$, which will matter for the economy's behavior during the transition. The structural estimation thus involves simulating the initial stationary distribution of $\hat{\psi}$ given G and $\hat{\delta}$ — accounting for the endogenous evolution of $\hat{\psi}$ — and then simulating workers forward over the transition.

Employment flows. We compute the EE, EU and UE probabilities over one month, i.e., $\Delta^m \equiv 1/12$. When doing so, we account for the possibility that workers might experience multiple job transitions within the same month. For instance, a worker who first experiences an EU transition followed by an UE one during the same month will be counted in the EE probability, which ensures that the moment we compute in the model is comparable to the EE probability measured using the CPS.

To compute the EU probability, we use the law of motion (C.40) since we do not account for changes in $\hat{\psi}$ within a given quarter (Appendix C.3). We make two changes compared to this law of motion. First, we initialize it using the distribution of states conditional on employment $\boldsymbol{\mu}_t^E$ instead of the unconditional distribution $\boldsymbol{\mu}_t$. Second, we set the time step to $\Delta^m = 1/12$ instead of $\Delta = 1/4$. We iterate on this new law of motion for all ages j and relative productivities $\hat{\psi}$. The EU probability corresponds to the mass of workers with state $s = U$ after a month. We proceed analogously to compute the UE probability.

[57] When doing so, we use the fact that: $h\left(\hat{\psi}, s\right) = h\left(s\right)$ does not depend on $\hat{\psi}$ whenever the worker is employed in sector D, claims the outside option, or is unemployed; $\frac{\partial}{\partial \hat{\psi}} W_t\left(j, \hat{\psi}, s\right) = 0$ for all employment states since $W_t\left(j, \hat{\psi}, s\right) = W_t\left(j, s\right)$ under our guess; and $\theta_t\left(j, s\right) = 0$ in all markets s in sector C since $V_t\left(j, s\right) \leq 0$ so that $f\left(\theta_t\left(j, s\right)\right) = 0$.

Table C.1: Internal calibration

Parameter	Description	Frictionless	+ Costly	+ Gross	+ Ladders	+ Outside
Search						
λ^U	Search intens. (U)	—	9.332	8.492	8.280	5.973
λ^E	Search intens. (E)	—	0	0	1.569	1.185
ζ	Search directedn.	—	0	0.023	0.023	0.035
Job creation & destruction						
ν	Vacancy post. cost	—	0.932	0.879	0.382	0.437
χ	Job destruction	—	0.072	0.073	0.071	0.070
Benefits						
b^O	Benefit in O	—	0	0	0	0.642
b^U	Benefit in U	—	0.354	0.354	0.351	0.343

In turn, we combine two moments to compute the EE probability over a month: the probability that an employed worker is again employed in a month in *any* market; and the probability that the worker stays in their current job over that period.[58] The EE probability is the difference between the these two quantities. The first moment is computed in a way similar to the one described above. To compute the second moment, we use the same law of motion as before, except with an alternative matrix $\widetilde{\mathbf{M}}_t\left(j, \hat{\psi}\right)$ in which all the *inflows*, i.e., the positive terms, are all zero. This matrix lets workers "flow out" of the system as they move out of their current job. The remaining mass of workers after a month corresponds to the probability that a worker stays in their current job.

Calibrated parameters. Table C.1 describes the calibrated parameters across the various models that we consider in our decomposition in Section 5.3. In particular, "Costly," "Gross," "Ladders," and "Outside" correspond to the models where we introduce costly vacancy creation, gross flows, job ladders, and the outside option, respectively. The rightmost column corresponds to our full model.

[58] A worker who leaves their current job for unemployment and subsequently moves back to the same market (i.e., to a different job) is counted in the EE probability, consistently with the data.

D Details on the Climate Application

D.1 Empirics

Sector definitions and S-curves. To define polluting industries (D), we use data on emissions intensity — measured as the kilograms of CO2-equivalent gas emissions per dollar of output — by 6-digit NAICS industry codes from the Supply Chain Greenhouse Gas Emission Factors published by the Environmental Protection Agency (EPA). An industry is deemed polluting if it is in the top decile of emissions intensities, weighted by employment.[59] These emissions data are missing for some key sectors, such as the energy sector. Some industries are not polluting themselves but produce or sell a polluting output, e.g., gas stations, or are related to fossil fuel extraction and power generation, e.g., fossil fuel electric power generation or pipelines for oil and gas. We handcode these industries as polluting.

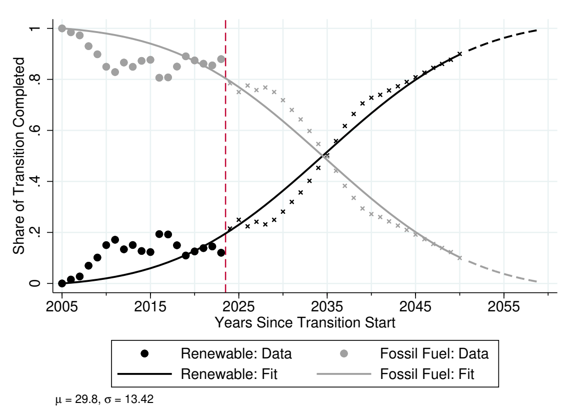
To define clean industries (C), we combine two approaches. First, we follow the output-based approach of Bontadini and Vona (2023), who find that 95% of the production of "green goods" is concentrated in just 13 out of 119 4-digit European NACE industries and these industries do not produce polluting goods.[60] We classify these 13 sectors as clean. We supplement this output-based approach with the task-based approach of the US Energy Employment Report (Granholm, 2024). We use data from the 2012 Green Goods and Services (GGS) survey produced by US Bureau of Labor Statistics (BLS) and define a sector as clean if at least 50% of their employees work on green tasks. Finally, we handcode battery manufacturing as green, since this is a crucial input to electrification of transport, and is not included by any of the two classifications above. Industries that are neither clean nor dirty correspond to the outside option (O).

The largest polluting sectors are gasoline stations (840,000), various sectors associated with freight or trucking (1.3 million combined), air travel (447,500) and plastic product manufacturing (307,000). The largest clean sectors are semiconductor manufacturing (a key input to solar panels), other motor vehicle parts manufacturing (e.g. car batteries), and recyclable material wholesalers. Around 11 million

[59] Employment information comes from the Quarterly Census of Employment and Wages (QCEW).

[60] The list of green goods employed by Bontadini and Vona (2023) is based on the Combined List of Environmental Goods (CLEG) of the OECD and considers goods which are useful for the green transition. For instance, solar panels and wind turbines are classified as "green goods" even if their production is emission-intensive, since the goods themselves are green by nature.

Figure D.1: Estimated *S*-curve of energy generation transmission



Notes: This figure plots the estimated shape $\Phi(t)$ of the *S*-curve for the sectoral MRPLs, which is assumed to take the form of a truncated normal CDF. The data comes from the US Energy Information Administration (EIA). Dashed lines indicate projections through 2050.

jobs remained in the polluting sectors as of 2025, suggesting a substantial remaining scope for job displacement.

To calibrate the shape $\Phi(t)$ of the S-curve for the MRPLs, we use data from the US Energy Information Authority (US EIA), which provides information and projections on energy production from 2005 to 2050. We calculate the share of energy produced by renewable sources in every year. We then normalize the share of the transition completed as of 2005 to be 0 (corresponding to the start of the transition for workers initially attached to polluting sectors), and assume that the projected share in 2050 reaches 90% of its final level since the transition might not be fully complete until 2060 (Section 3.2). This yields the data points in Figure D.1. A normal CDF, truncated between 2005 and 2060, fits the data well and suggests that the transition should accelerate after 2025 and half of it should be completed by 2035.

Current Population Survey – Cleaning and targets. The source of many of the moments used in our calibration is the Current Population Survey (CPS), which is a large, nationally-representative survey of U.S. households. The CPS has a rotating panel structure whereby households enter the sample for a four-month rotation, before leaving for 8 months and returning for a subsequent four months. Respondents are asked additional questions about their usual weekly earnings and hourly wage in the final month of each rotation (month 4 and 8 in the sample). All wages are deflated by the CPI-U. We use the IPUMS sample of Flood et al. (2024), restricting attention to those aged 25 to 65. We use the survey to measure employment in clean and polluting sectors, and to discipline the value of the outside option. We use the provided sample weights throughout out analysis.

To construct our empirical moments, we must merge our sector definitions with the industries in the CPS. The CPS uses Census industry codings, which are generally at a coarser level of aggregation than the 6-digit NAICS codes that we classify as polluting, clean, or other. We thus adopt an iterative approach to assign CPS industry codes to polluting versus clean sectors. We first merge our CPS data with Census industry to NAICS crosswalks provided by the Census, available here. We then merge these coarser CPS-NAICS codings with our finer clean versus polluting 6-digit NAICS classifications at progressively coarser levels beginning with 6-digit NAICS. At each level, we assign clean vs polluting vs other sectors according to the classification of the majority employment in lower-level NAICS industries.

We label a worker as "polluting-attached" in the pre-transition period (1995–2004) if they are employed in the polluting sector for four consecutive months during the first rotation. Among workers employed in the polluting sector in the final month of their first CPS rotation, 87.9% were polluting-attached before 2005.

The CPS does not allow us to track these polluting-attached workers over long periods. Instead, we weight individuals in subsequent years (2005–2025) by the likelihood that they were polluting-attached in 2005 based on their current observables. In effect, we study how the transition affects workers who, based on observable characteristics, would likely have been working in the polluting sector if the transition had not occurred. We use information on workers' education, race and marital status, as well as geographic information which the CPS partially provides down to the county level. The Census Bureau has changed confidentiality rules regarding individuals living in small geographic areas over time. As a result, some counties and metro areas (MSAs) appear in some waves of the CPS but not others. In order to have a consistent probability of being polluting-attached, we require consistent geographic codings over time; we therefore focus on county and MSA codes that are present in every year from 2004 to 2025. This yields 194 counties and 209 MSAs. State information is included for all years.

We start by computing the probability that an individual i observed in year t was polluting-attached in 2005. To do so, we use the pre-transition period (1995–2004) to regress a dummy variable $P_{i,t}^{\text{attached}}$ indicating whether a worker is polluting-attached on a vector of observables $X_{i,t}$ (education $\times$ race $\times$ marital status $\times$ sex $\times$ geography). For later years (2005-2025), we use the estimated fixed effects in this regression to obtain a predicted probability $\hat{P}_{i,t}^{\text{attached}}$ that each worker was polluting-attached in 2005 based on their current observables $X_{i,t}$. Since some combinations of observables do not exist in all years, we run the regressions $P_{i,t}^{\text{attached}}$ at various geographic levels and use the fixed effects corresponding to the finest geographic level available for each individual i in period t (county $\rightarrow$ MSA $\rightarrow$ state $\rightarrow$ national) to compute $\hat{P}_{i,t}^{\text{attached}}$. The individuals most likely to be polluting-attached given their observables are less-educated men in traditional mining communities (e.g. parts of Texas and West Virginia). About 25% of individuals are assigned a zero likelihood of being polluting-attached and are thus dropped.

We use this information to calculate the share of workers attached to polluting industries in 2005 who subsequently supply labor in clean or polluting industries in every period. For instance, we compute $C_t = \sum_{i \in C} \hat{P}_{i,t}^{\text{attached}} / \sum_{i \in \{C,D\}} \hat{P}_{i,2005}^{\text{attached}}$ for clean industries, where the numerator is the number of workers in the CPS currently in sector C weighted by their individual probabilities of having been polluting-attached in 2005, and the denominator is the predicted number of attached workers in 2005. By definition, all polluting-attached workers supply labor to sector D in 2005, yet the measure C_t could still be positive since some workers working in C in 2005 are assigned a positive $\hat{P}_{i,t}^{\text{attached}}$. We therefore include such workers in the denominator above, and adjust our measure of reallocation to sector C by computing $z_t^{\text{data}} = (C_t - C_{2005}) / (1 - C_{2005})$, so it starts at zero in 2005 and is 1 if all workers in the CPS reported working in clean industries ($C_t = 1$).

To correct for misspecification from overly coarse industry classifications and within-industry shifts in manufacturing, we ascribe a fraction of polluting manufacturing employment D_t to clean industries C_t. For instance, the industry classifications associated with motor vehicle manufacturing do not distinguish between ICEV and EV vehicles. We therefore allocate employment in motor vehicle manufacturing to clean and polluting sectors based on the share of cars sold that run on alternative fuel (hybrid, plug-in hybrid, electric, or biofuel) — which proxies for clean technology adoption.[61] We proceed in the same way for other industries

[61] To calculate these shares, we use data from the US Bureau of Transportation on total car sales by year, available from here, and on alternative energy vehicle sales, available here (both accessed June 3, 2025). Total car sales data go through 2021 while alternative-fuel sales go to 2024. We therefore extrapolate total car sales assuming a constant log-linear trend.

within manufacturing, using the same shares as we use for car manufacturing.[62] We then extract the trend component of C_t, computed as a moving average over 12 quarters before and after a target date, applying a correction to the end of the sample which accounts for the fact that there are fewer observations available. The resulting series for C_t is the one that we use to compute z_t^{data} using the formula above. Figure 6.1 plots z_t^{data} over 2005–2025. Finally, we construct the same series for different cohorts to obtain the $z_{\tau,t}^{\text{data}}$ series that we use in our estimation (6.1).

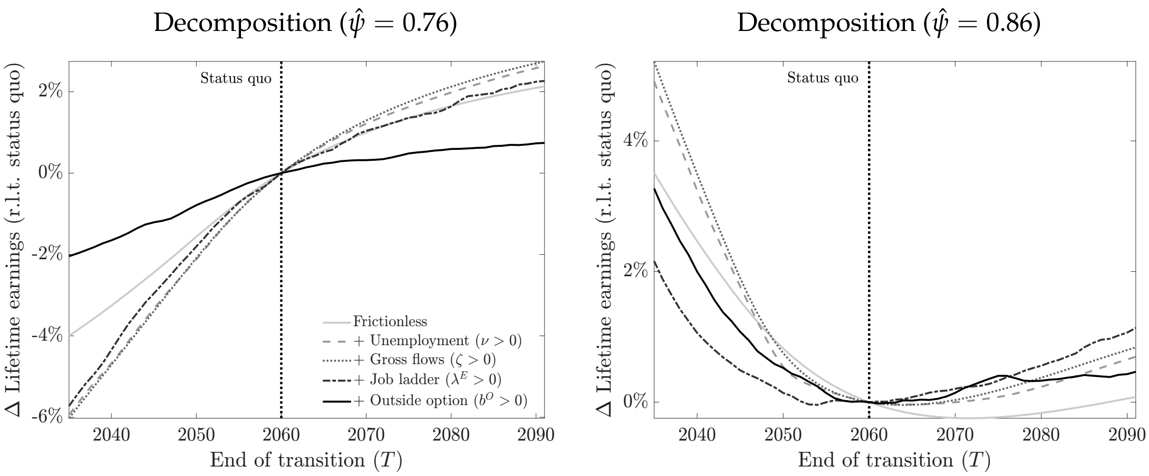
D.2 Amenity Values

One possible concern for our estimation is that our model assumes that workers reallocate only on the basis of their relative productivity $\hat{\psi}$ and the sectoral MRPLs A_t^k, and not because they derive some intrinsic utility from working in cleaner sectors. We argue that this distinction is unimportant for our exercise. To understand why, consider a version of the frictionless model of Section 4, where the flow payoff from working in the clean sector now includes both monetary and amenity values $\hat{\psi}\xi A_t^C \Lambda_t$, where ξ is the idiosyncratic component of the relative amenity and Λ_t is the sectoral component. Workers supply labor to the clean sector if and only if $\hat{\psi}\xi A_t^C \Lambda_t \geq A_t^D$. This condition is isomorphic to the one in Section 4, when setting $\hat{\psi}' \equiv \hat{\psi}\xi$ and $A_t^{C\prime} \equiv A_t^C \Lambda_t$. Assuming that the amenity value co-moves with the monetary value of supplying labor to clean industries, our estimation backs out the distribution of $\hat{\psi}\xi$ which determines workers' incentives to reallocate and their values. The same idea applies to our full model: if workers prefer to work in the clean sector, they accept lower wages, leading to higher firm values — exactly as if they were more skilled in that sector. Thus, the distinction between amenities and productivity is not crucial to understand the labor market implications of the climate transition, as long as we are willing to interpret our estimates of Section 6.3 as a broader measure of worker well-being inclusive of amenities.

[62] We maintain the following four manufacturing sectors as polluting over the full transition: Petroleum products, Petrochemical manufacturing, Oil and gas field machinery and equipment, and Motor vehicle gasoline engine and parts manufacturing.

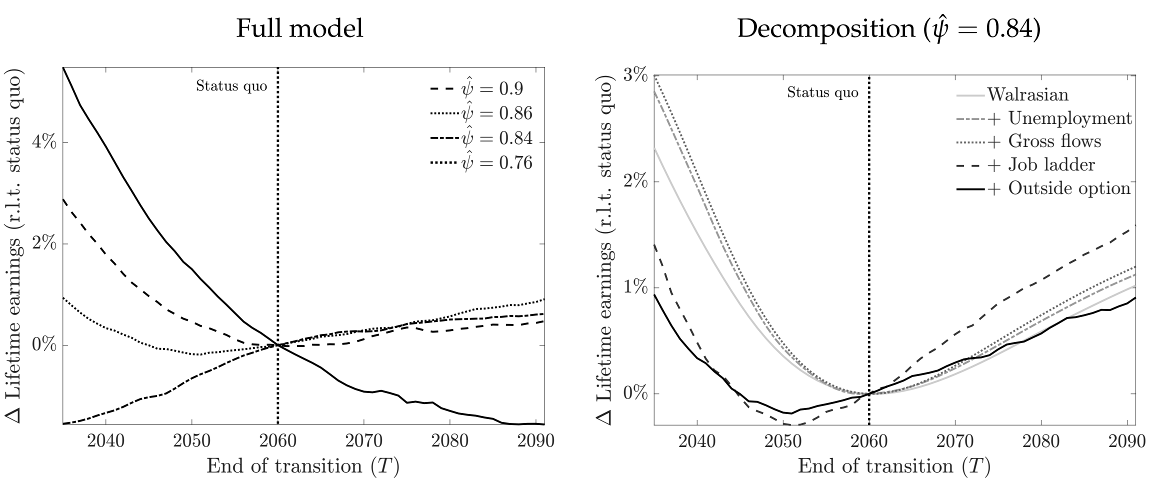
E Additional Quantitative Results

Figure E.1: Counterfactual lifetime earnings changes



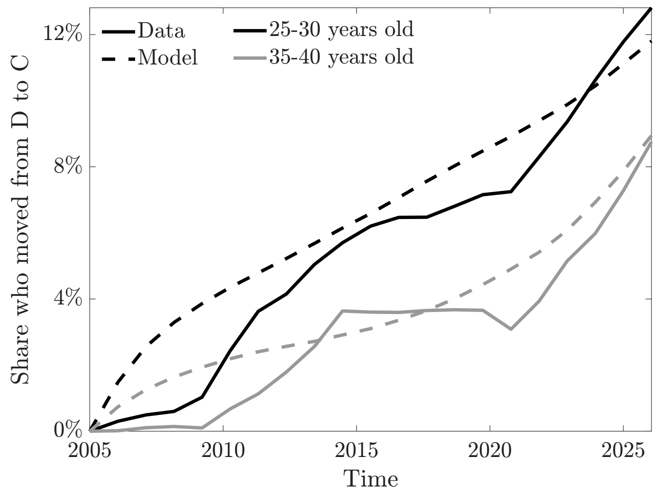
Notes: This figure plots workers' lifetime earnings changes in 2025 as a function of the transition horizon, for the various models considered in Section 5.3. Earnings changes are relative to the status quo transition ending in 2060, and transitions are parameterized as in equation (3.3). All workers have 20 years remaining in their careers. The two panels correspond to workers with different relative productivities $\hat{\psi}$.

Figure E.2: Counterfactual lifetime earnings changes



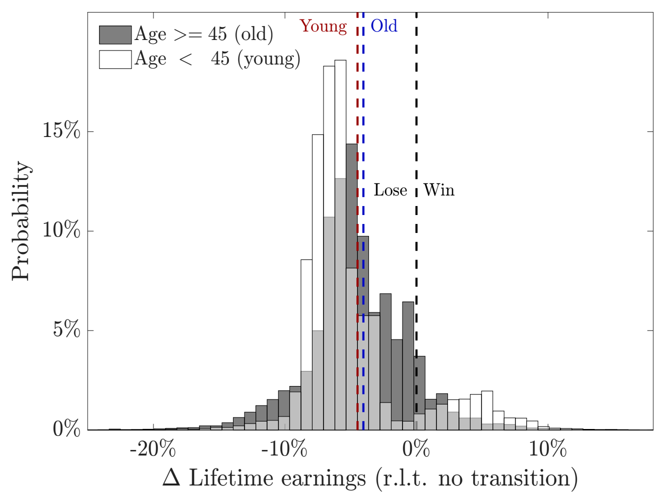
Notes: This plot reproduces Figure 5.1 in the main text, this time averaging across employment states in 2025 in our climate application, instead of conditioning on unemployment. The left panel plots workers' lifetime earnings changes in 2025 as a function of the transition horizon, for various values of the relative productivity $\hat{\psi}$. All changes are relative to the status quo transition ending in 2060, and the counterfactuals are parameterized as in equation (3.3). Workers have 20 years remaining in their careers (median age). The right panel plots the same earnings changes for a worker with relative productivity $\hat{\psi} = 0.84$ and for the various models considered in Section 5.3.

Figure E.3: Labor reallocation by cohort



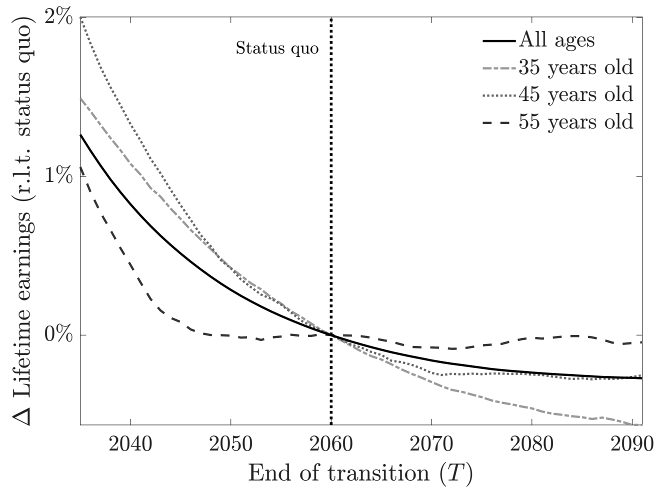
Notes: This figure plots the share of workers initially attached to polluting industries who moved to clean industries, for the two cohorts used in our estimation (aged 25-30 and 35-40 in 2005). The solid curve is the data from the CPS (see Appendix D). Workers are weighted by the probability that they are attached to the polluting sector based on observables in the data. The dashed curves correspond to the model-generated reallocation when using the estimated distribution of $\hat{\psi}^{\text{endow}}$ and the rate of specific skill accumulation $\hat{\delta}$.

Figure E.4: Distribution of lifetime earnings changes by age



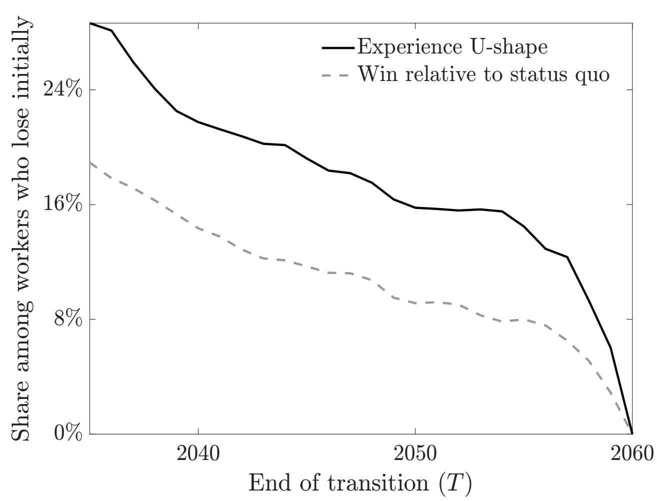
Notes: This figure plots the distribution of lifetime earnings losses in 2025, relative to a scenario without climate transition, for young and old workers. Young workers are defined as those who are less than 45 years old (i.e., $j < 20$) in 2025, while old workers make up the rest.

Figure E.5: Counterfactual lifetime earnings changes by age



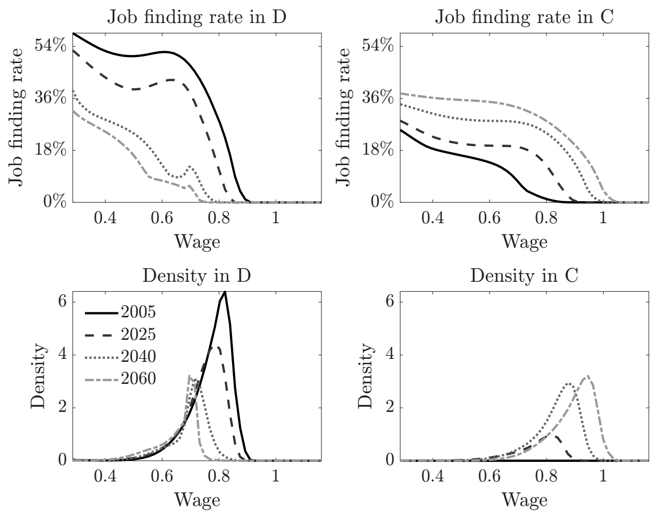
Notes: This figure plots average lifetime earnings changes in 2025 in our full model as a function of the transition horizon, for various ages. All changes are relative to the status quo transition ending in 2060, and the counterfactuals are parameterized as in equation (3.3). We average workers within each age using the distribution that prevails in 2025 in our climate application.

Figure E.6: Share of workers who experience non-monotonicities



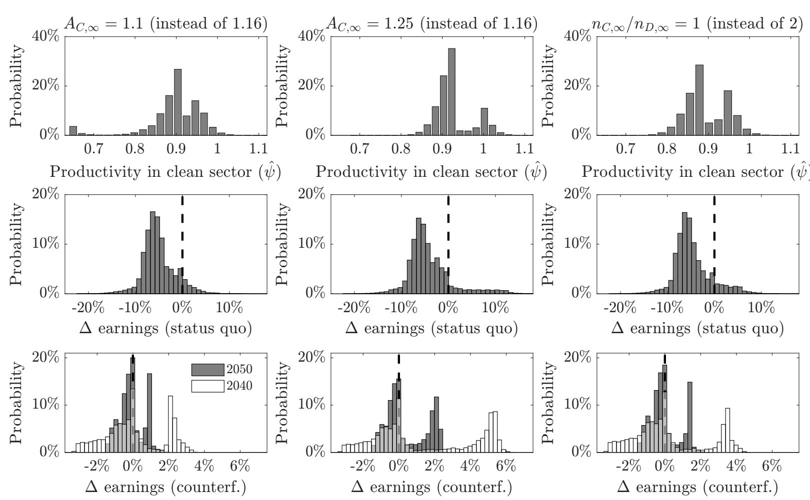
Notes: This figure plots the share of workers who experience non-monotonic changes in lifetime earnings as a function of the transition horizon. This share is expressed relative to workers who lose at first when accelerating the transition, starting from the status quo one ending in 2060. The solid black curve indicates the share of workers who cross the trough of their *U*-shape at some point as the transition accelerates, i.e., who gain *at the margin*, and the dashed gray curve indicates the share of workers who gain relative to the status quo transition, i.e., who gain *in level* compared to the 2060 transition.

Figure E.7: Job finding rates and the wage distribution



Notes: This plot reproduces Figure 5.2 in the main text, this time averaging across relative productivities $\hat{\psi}$ at birth using the distribution G estimated in our application. The first row plots the job finding rate as a function of the wage posted by firms in various periods under the status quo transition. The second row plots the equilibrium wage distribution, for the same periods. The first column corresponds to the polluting sector, and the second to the clean sector.

Figure E.8: Sensitivity analyses



Notes: The first column corresponds to the case where the final MRPL in the clean sector is $A_{C,\infty} = 1.1$ instead of 1.16 in our baseline scenario. The second column corresponds to $A_{C,\infty} = 1.25$. The third column corresponds to the case where we target a 1:1 ratio of labor supplies in sector C relative to D in the long-run, instead of 2:1 in our baseline. In turn, the first row plots the distribution of workers' relative productivities $\hat{\psi}$ in 2025. The second row plots the distribution of earnings losses in 2025 from the status quo transition ending in 2060, relative to no transition at all. The third row plots the distribution of lifetime earnings changes in 2025 from two transitions ending earlier.

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