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AGGREGATE WELFARE WITH DISCRETE CHOICE

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ABSTRACT

Discrete-choice general-equilibrium models are widely used to study how people sort across places, sectors, and jobs, but they lack a canonical aggregate welfare statistic. This paper proposes a TFP-equivalent welfare measure: the largest uniform reduction in factor productivity such everyone can be at least as well off as in the status quo. This measures the resource surplus left after winners compensate losers, allowing prices, wages, and choices to adjust in general equilibrium. We characterize this measure using compensated supply and demand functions. A main result is a version of Hulten's theorem for discrete-choice economies: under perfect competition, the first-order welfare effect of a productivity shock to producer i is given by its sales relative to GDP, regardless of the distribution of preferences and technologies. Beyond first order, we provide approximations in terms of observable income and expenditure shares and uncompensated supply and demand elasticities. We compare this measure with common alternatives: Real output ignores amenity value; average utility depends on arbitrary cardinalizations of individual utility; and the sum of compensating variations can rise after pure redistributions. Our measure avoids these problems while preserving the logic of cost-benefit analysis in general equilibrium. In an application to the U.S. Interstate Highway System, we show that average-utility estimates vary widely across observationally equivalent representations of the same preferences, while our welfare measure is invariant to these arbitrary cardinalizations.

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A appendices is available at http://www.nber.org/data-appendix/w34703

1 Introduction

General-equilibrium models in which agents make discrete decisions—where to live, which sector to work in, or what job to take—are now standard in trade, spatial, labor, and macroeconomics. These models are popular because they capture rich heterogeneity in preferences, skills, and individual outcomes. Yet they make it surprisingly hard to answer a basic question: what is the aggregate welfare value of a change in the economy when agents are affected differently by that change? A meaningful answer is important for summarizing the overall welfare consequences of economic change and for guiding policy.

In economics, a common approach to aggregate welfare is to add up money-metric measures of individual welfare — sums of willingness-to-pay in the tradition of Harberger (1971). In their ideal form, measures constructed this way have the following attractive properties. First, they depend only on revealed preference information, and are invariant to arbitrary cardinalizations of utility. Second, they respect Pareto improvements: when every household is better off, they rise. Third, an increase in these types of measures means that there is a redistribution possible such that every household can be made better off.

In this paper, we develop a measure of this type for discrete choice economies. It is defined as the fraction of factor endowments left if winners from a change were to compensate losers. Our contribution is to characterize this general-equilibrium notion of economic surplus in terms of supply and demand curves.

We compare our measure to other common aggregate measures in discrete-choice settings: real output/consumption, average utility, and the sum of compensating variations. As we describe below, not only do these measures fail to satisfy at least one of the three properties above, they all have serious flaws.

i. **Chain-weighted real output**: This is how real output and consumption are measured in the national accounts. However, as is well-known, these measures can be an inappropriate gauge for welfare because they ignore amenity value (Roback, 1982). For example, if an agent moves from a high- to a low-wage location/job, then real output and consumption fall, even though that agent may be better off once we take into account her location preferences. Indeed, it is simple to construct examples where a shock makes every agent better off, but causes real output to decline (e.g. if productivity rises in a low-wage but high-amenity location, then real output can fall because workers move there).[1]

[1]This measure is nevertheless popular in the literature, partly because it is consistent with objects in

ii. **Average utility.** This measure is the cross-sectional average of individual utilities. A fundamental issue with this measure is that it is not invariant to monotone transformations of individual utility functions. Replacing a utility function with a monotone transformation that generates exactly the same individual choice behavior can nonetheless alter the social welfare ranking of allocations. For instance, in the spatial literature, a standard utility specification is $c\varepsilon_{hr}$, where c is consumption and ε_{hr} is h's idiosyncratic taste for choosing r.[2] This specification is observationally equivalent to $c\varepsilon_{hr}/\max_{r'}\{\varepsilon_{hr'}\}$, which normalizes each agent's highest location-specific taste to one. Although both specifications imply the same individual rankings, and hence choices, they imply different rankings of social allocations and different optimal policies. The issue is that there is no canonical basis for choosing between these normalizations. Normalizing tastes so that every agent assigns a taste of one to their favorite location is just as plausible as allowing some agents to have higher tastes in their favorite location than others. The data does not distinguish between these cardinalizations, and the resulting aggregate welfare measure hence depends on untestable and nonintuitive assumptions about the interpersonal scale of utility.[3]

iii. **Sum of compensating variations**: This measure is also called Kaldor-Hicks or cost-benefit efficiency, and has been analyzed by Small and Rosen (1981) for settings with discrete choice. It measures the amount of money left over after winners compensate the losers holding fixed post-shock equilibrium prices and wages.[4,5] While this measure does not have the issues of the previous two, it can nevertheless be unreli-

national income accounting. Some recent examples that report real output include Hsieh et al. (2019), Lamadon et al. (2022), and Bagga et al. (2025) in models with occupation choice, and Bryan and Morten (2019) and Heise and Porzio (2022) in spatial migration models.

[2]The use of average utility to measure aggregate welfare is widespread in the spatial literature. Examples include, among many others, Redding (2016), Caliendo et al. (2019), Dingel and Tintelnot (2020), Allen and Arkolakis (2022), Monte et al. (2023), and Tsivanidis (2026).

[3]See Section 6.2 for a discussion and worked-out example. Although this measure is sometimes called "expected utility," average utility is not a von Neumann–Morgenstern expected utility function. Expected utility functions represent ordinal preference relations over lotteries; they are not expectations of utility values across households with different preferences.

[4]The pioneers in the discrete choice literature, like McFadden (1981) and Anderson et al. (1992), emphasize that consumer welfare in discrete choice models should be measured using compensating or equivalent variations of the individual agents, as in standard consumer theory. Dagsvik and Karlström (2005) show how to calculate the distribution of compensating variation across agents using compensated choice probabilities. Bhattacharya (2015) and Bhattacharya (2021) study non-parametric identification of the distribution of compensating and equivalent variation.

[5]While this approach is common in the industrial organization literature, it is less common in general equilibrium spatial and occupational choice models. One exception is Kim and Vogel (2020), who study a model where workers choose among a discrete number of sectors including non-employment. They show that the elasticity of the sum of compensating variations with respect to wages is given by the income share of each sector.

able in general equilibrium. In particular, it may not be technologically feasible for winners to compensate the losers even though the sum of compensating variations is positive. The problem is that the fixed-price compensation exercise may describe a set of transfers that cannot be implemented once the transfers themselves change equilibrium prices, wages, and choices. We show, for example, that in response to pure transfers that move the economy along the Pareto-efficient frontier, the sum of compensating variations can become positive.

Our approach follows Harberger (1971) and Small and Rosen (1981) in using willingness-to-pay as the basis for welfare measurement, but not at fixed prices. Instead, following Baqaee and Burstein (2025) and inspired by Debreu (1951), we measure the increase in aggregate welfare by the fraction of primary factors that can be saved while keeping every household at least indifferent, allowing equilibrium choices and prices to adjust. The analyses in Debreu (1951) and Baqaee and Burstein (2025) do not carry over to discrete-choice economies because individual choice sets are nonconvex due to discreteness. For example, this means that the second welfare theorem may not hold in these economies.

Our definition restores the three cost-benefit properties discussed above, accounting for general equilibrium forces. First, because the statistic is defined by households' indifference conditions rather than by utility levels, it is invariant to monotone transformations of individual utility (unlike average utility). And because the answer is expressed in TFP-equivalent units, it has a simple interpretation: it is the amount of aggregate productive capacity that can be given up after compensating losers. Second, because it takes amenity into account, it respects the Pareto principle (unlike real output). Finally, by construction, it is positive only if a potential Pareto improvement is possible. In this sense, it preserves the distribution-neutral economic surplus logic of cost-benefit analysis — adding costs and benefits without taking a stance on whose gains count more — while avoiding the inconsistency of the fixed-price sum of compensating variations.

A payoff of this approach is that this measure is tractable and amenable to revealed preference reasoning. For example, in perfectly competitive economies, the first-order effect of a productivity shock to producer i is equal to producer i's sales divided by total income. Thus, despite the heterogeneity of preferences and the non-convexity created by discrete choices, aggregate welfare obeys a Hulten theorem just as in continuous-choice economies. To a first order, one does not need to solve for how households switch locations or occupations in response to the shock. Intuitively, when a household switches choices, the pecuniary gain from switching is exactly offset at the margin by the amenity loss required to make the household willing to switch.

Our baseline model abstracts from distortions and externalities (see, e.g., Fajgelbaum and Gaubert, 2020). Focusing on a perfectly competitive benchmark helps clarify how our approach differs from standard practice in the literature. We sketch how to extend our approach beyond perfectly competitive economies in Appendix G, allowing for externalities and wedges. Similarly, when calculating aggregate welfare, we focus on the benchmark case where compensations can be implemented through individual-specific lump-sum transfers, abstracting from limitations on transfers arising, for example, from private information as discussed in Schulz et al. (2023). Appendix H discusses how to adjust our measure when individual-specific lump-sum transfers are not available and compensations must instead be contingent on location choices (i.e. place-based policies).

The structure of the paper is as follows. We set up the preferences and technologies and define perfectly competitive equilibrium in Section 2. The economy is static, and agents choose consumption and the location and/or industry in which they live and work. Agents can differ flexibly in their skills and tastes over locations and occupations. Production uses different types of primary factors and intermediate inputs, and accommodates input-output networks and costly trade.

In Section 3, we define our aggregate welfare statistic and show that it can be characterized using compensated supply and demand functions. Compensated supply functions map relative wages and prices to labor supply across locations or occupations, holding each agent on a fixed indifference curve. Compensated demand maps wages and prices to total spending on different consumption goods, holding each agent on a fixed indifference curve. We show how to compute these compensated supply and demand functions using simulation methods.

In Section 4, we provide some analytical (as opposed to simulation-based) characterizations of our measure. First, we show that our measure of TFP-equivalent aggregate welfare, to a first-order approximation, obeys Hulten's theorem in perfectly competitive economies. In particular, given sales shares, one does not need to know anything about the underlying preferences or technologies. This is in contrast to average utility and real output, both of which require solving a general equilibrium model, even to a first order.

In this section, we also discuss a special case where compensated and uncompensated supply functions coincide because compensating transfers do not alter discrete choices. This allows us to nonlinearly solve for aggregate welfare using the uncompensated supply system. Under these assumptions, aggregate welfare can also be computed using the "Social Surplus Function" of McFadden (1981).

In Section 5, we investigate nonlinearities in cases where compensations do affect discrete choices. We provide a second-order approximation of changes in aggregate welfare. We show that the first-order approximation understates positive shocks and overstates negative shocks if compensated sales shares rise for producers with positive shocks. The reverse is true if compensated sales shares fall in response to positive shocks. We express the elasticities of compensated sales shares in terms of observables in the initial equilibrium: income and expenditure shares, and price elasticities of uncompensated (market-level) supply and demand systems. Given these statistics, the distribution of tastes and the functional forms of utility functions do not matter.

In Section 6, we compare our approach to the three popular alternative measures of aggregate efficiency/welfare discussed above.

i. **Chain-weighted real output**: Real output differs from our measure of TFP-equivalent aggregate welfare even to a first-order (despite the fact that the equilibrium is Pareto-efficient). Whereas our measure differs from real output, it does coincide, to a first-order approximation, with measures of multi-factor productivity growth (or the Solow residual) computed using a quality-adjusted labor input. That is, our analysis provides a theoretical justification for those statistics in efficient economies with heterogeneous tastes and discrete choice.[6]

ii. **Average utility**: This utilitarian approach does not coincide with our measure, even to a first-order. Using a model calibrated to regional U.S. data, we show how different arbitrary but equally valid monotone transformations of individual utility functions have very different implications for policy.

iii. **Sum of compensating variations**: Our measure does not generally coincide with the sum of compensating variations (Kaldor-Hicks/cost-benefit efficiency) because our measure accounts for the fact that compensating transfers can alter equilibrium prices and wages. We provide an example where lump-sum transfers strictly raise the sum of compensating variations, but a potential Pareto improvement is impossible. Intuitively, using post-transfer prices and wages, the winners can compensate losers and still come out ahead because prices fall for goods consumed principally by the losers.

In Section 7, we provide a real-world illustration using a well-known example. We revisit the aggregate welfare cost of removing the Interstate Highway System in the United States studied by Allen and Arkolakis (2014). We extend their model to allow for heterogeneity in location tastes and show that the welfare cost implied by average utility can differ substantially from that implied by our measure, depending on which observationally equivalent cardinalization of utility is used.

[6]See, e.g., chapter 3 of the OECD's manual on measuring productivity for quality-adjusted labor input.

Our paper complements the analysis of social welfare functions in spatial models by Donald et al. (2023). They decompose changes in a class of social welfare functions in response to changes in the equilibrium allocation. The most important difference between our papers is that we ask a different question. Instead of decomposing changes in the value of a social welfare function, we define and characterize a general equilibrium counterpart to cost-benefit analysis. Our analyses do not coincide even to a first order since, for example, our measure obeys Hulten's theorem whereas the class of social welfare functions they study do not.[7]

This difference is important because, as with cost-benefit analysis, our measure does not take a stance on optimal redistribution. In contrast, social welfare functions encode distributional preferences (see the discussion by Fajgelbaum and Gaubert, 2025). Therefore, policymakers that maximize a social welfare function may intervene in Pareto-efficient economies in pursuit of redistributive objectives. Which interventions are desirable (for example, whether workers should be induced to move from high-productivity to low-productivity locations) then depend on arbitrary assumptions about the monotone transformation used to represent individual utility (see Example 7 in Section 6).[8] In contrast, our measure is always maximized if the status quo is Pareto-efficient. Moreover, if a policy yields a higher value of TFP-equivalent aggregate welfare, then it must be that this policy can make every household weakly better off than in the status quo given appropriate transfers.

2 Environment and Equilibrium

In this section we define the environment and equilibrium. We consider perfectly competitive economies without externalities (e.g. positive or negative spillovers).

[7] More concretely, Donald et al. (2023) consider a model with a set of types $\theta \in \{\theta_1, \dots, \theta_S\}$. Within each type, agent h has idiosyncratic tastes ϵ_{hr}^{θ} for location r. The social surplus function for type θ is: $U^{\theta}(\boldsymbol{c}^{\theta}) = \mathbb{E}\left[\max_r g(c_r^{\theta}) + \epsilon_{hr}^{\theta}\right]$, where c_r^{θ} is consumption and g is an increasing function. They study the class of social welfare functions that can be written as: $W = \mathcal{W}\left[U^{\theta_1}(\boldsymbol{c}^{\theta_1}), \dots, U^{\theta_S}(\boldsymbol{c}^{\theta_S})\right)$. That is, whereas $\mathcal{W}$ is a flexible function, U^{θ} embeds a specific cardinal assumption about the representation of individual utility functions (within θ).

[8] Mongey and Waugh (2024) provide an alternative interpretation where average utility is interpreted as the expected utility of individuals who are ex-ante identical (before they know their tastes) and lack access to insurance markets for their idiosyncratic tastes. However, if such an ex-ante state does not exist, then the results of this type of analysis rest on an untestable cardinalizing assumption. That is, if households cannot make choices in the ex-ante stage that reveal their risk preferences about how they rank one set of taste parameters against another, then choice data identifies utility up to a monotone transformation so that the ex-ante expected utility function cannot be recovered from such data. See also the discussion of this issue by Davis and Gregory (2021).

Commodity space. There is a set C of consumption bundles, a set N of intermediates, and a set R of primary factors. Commodities are indexed by a comprehensive set of characteristics that define their uniqueness including region (e.g. haircuts in LA and New York can be treated as different commodities).[9]

Primary factors can be arbitrarily finely differentiated in the set R. For example, the set R can distinguish factors by region, occupation, skill-level, and type (e.g. high-skilled accountants in LA and commercial real estate in New York are different factors). Agents have different ability to provide the different factor services. For example, some agents, called low-skilled workers, can only supply low-skilled labor, or some agents, called landowners in region x, can only supply land to firms in that region. Each agent h makes a discrete choice $l_h \in R$ that we refer to as the *location* of h, with the understanding that it may index location in space as well as occupation, industry, and so on.[10]

For simplicity, the set of consumption bundles C has the same size as the set of primary factors R and there is a one-to-one correspondence between consumption bundles and locations. Thus, for each $r \in R$ there is a primary factor ("labor in location r") and an associated consumption bundle $c(r) \in C$ ("the consumption bundle in location r"). By allowing consumption bundles to differ by location, we allow for the possibility that some consumption goods are non-tradeable (e.g. barbers in Los Angeles consume a different bundle of goods than waiters in New York).

Households' problem. There is a set H of agents. Each agent $h \in H$ chooses a location $l_h \in R$ (e.g. being a barber in Los Angeles or a waiter in New York, etc.) and consumes a scalar quantity of a homothetic bundle c_h (composed of goods and services) in that location.

Agent h has preferences $\succeq_h$ over the location choice l_h and the quantity of the consumption bundle consumed. These preferences are represented by a well-behaved utility function $u_h(c_h, l_h)$. Almost all the results in the paper hold without putting any additional functional form requirements on $u_h(c_h, l_h)$. However, for concreteness, we suppose preferences are *ordinally additively separable* between location choices and consumption.[11] This

[9]Similarly, if there are trade costs, then traded goods are indexed by both origin and destination. For example, oranges produced in California and consumed in New York are different to oranges produced in California and consumed in Illinois.

[10]This setup is general enough to capture settings that feature commuting, as in Monte et al. (2018), where agents consume and work in different locations. In this case, we let r index pairs of locations instead.

[11]To be explicit, only Propositions 3, 5 and 13 require this ordinally additive specification of preferences. The rest of the results can be proven without relying on this assumption.

means that the utility function can be written as

$$u_h(c_h, l_h) = f_h(g(c_h) + \epsilon_{hl_h}),$$

where g is a strictly increasing function, ϵ_{hl_h} is a taste parameter for household h's preferences for location l_h, and f_h is any strictly increasing (potentially household-specific) function. Since utility is only pinned down up to monotone increasing transformations, the specific function f_h has no testable implications and can vary at the level of individual agents in arbitrary ways so long as it is increasing.

However, the shape of f_h does affect average utility, and including it allows us to contrast our approach to the utilitarian approach in Section 6.2. In contrast to f_h, the shape of the function g does have testable implications and its shape can be recovered from choice data.[12, 13]

Households' behavior. Each agent maximizes utility choosing consumption c_h and location l_h. Recall that the choice of location encompasses both spatial location and choice of occupation. The household's budget constraint is that consumption expenditures are equal to income plus transfers:

$$\sum_{r \in R} p_{c(r)} c_h \mathbf{1}[l_h = r] = \sum_{r \in R} a_{hr} Z w_r \mathbf{1}[l_h = r] + T_h,$$

where $p_{c(r)}$ is the price of the composite consumption bundle in location r, $a_{hr}Z$ is agent h's efficiency units of factor type r, w_r is the wage per efficiency unit of factor r, and T_h is a lump-sum transfer to agent h. We anticipate that firms earn zero profits in equilibrium.

[12]The shape of $g(c_h)$ has testable implications because it controls the way income and substitution effects interact with each other. In particular, $g(\cdot)$ affects how households switch choices in response to lump-sum transfers. For example, if $g(\cdot)$ is an affine function, then households do not switch their choices if we add a constant amount to their consumption in every location. By contrast, if $g(\cdot)$ is log, then households do not switch their choices if we multiply their consumption by a constant in every location. These statements are true regardless of the distribution of tastes. See Appendix B for more information on how the function $g(\cdot)$ can be identified from behavior (see also Allen and Rehbeck, 2019).

[13]Since location choices are discrete, agents' preferences are nonconvex. This means that theorems that require convexity, like the second welfare theorem, cannot be used. The nonconvexity arises because the convex combination of two location choices is undefined. To convexify preferences, we could consider lotteries over location choices (e.g. with probability π_r consumption c_r and location r is realized). If we augment the model with lotteries, and impose the von-Neumann & Morgenstern assumptions, then the function $f_h(\cdot)$ is identifiable because it disciplines h's degree of risk-aversion. However, if such lotteries are not observed, then f_h cannot be elicited from observed choices. Furthermore, if we augment the model with lotteries so that f_h is pinned down by choices, then h's utility for lottery $\boldsymbol{\pi}$ is $u_h(\boldsymbol{\pi}) = v_h(\mathbb{E}_\pi[f_h(g(c_h) + \epsilon_{hr})|h])$ where $v_h(\cdot)$ is any strictly increasing function. That is, even in models with lotteries, utilitarian social welfare functions still depend on cardinal properties of the utility function and are not invariant to monotone transformations that do not alter individual choices even though $f_h(\cdot)$ is pinned down.

The scalar Z is an aggregate factor-augmenting productivity shifter that uniformly scales the efficiency of factors in every location for all agents.[14] We use Z to define the TFP-equivalent variation later.

Each agent h chooses her location $r \in R$ to maximize $g((Za_{hr}w_r + T_h)/p_{c(r)}) + \epsilon_{hr}$. Denote the optimal *location choice* by $l_h(Z\boldsymbol{w}, \boldsymbol{p}, T_h)$. We refer to the quantity of factor type r supplied by h as *labor* supplied by h, with the understanding that this is just a shorthand and factors could in principle include non-labor inputs like land. Given this shorthand, efficiency units of labor supplied in location r (not including the aggregate shifter Z) are

$$L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) = \int a_{hr}[l_h(Z\boldsymbol{w}, \boldsymbol{p}, T_h) = r]dh.$$

We call the function $L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$ the aggregate *labor supply function*. Efficiency units of labor supplied in location r, and available to firms, are $ZL_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$. Let $\boldsymbol{\chi}$ denote the vector of final consumption spending shares in each location:

$$\chi_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) = \frac{Zw_r L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) + \int T_h[l_h(Z\boldsymbol{w}, \boldsymbol{p}, T_h) = r]dh}{\sum_{r'} Zw_{r'} L_{r'}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})}.$$

We call the function $\boldsymbol{\chi}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$ the *final demand* function. Note that these functions are partial equilibrium objects that can be evaluated for any vectors of wages, prices, and lump-sum transfers.

Producers' problem. The set of commodities is defined to be the union of consumption goods, C, and intermediate goods, N, denoted by $C + N$. Each commodity i is produced by a perfectly competitive representative firm that maximizes its profit:

$$p_i y_i - \sum_{j \in N} p_j x_{ij} - \sum_{r \in R} w_r L_{ir},$$

subject to a constant-returns technology

$$y_i = z_i F_i \left(\{x_{ij}\}_{j \in N}, \{L_{ir}\}_{r \in R} \right), \tag{1}$$

where z_i is a Hicks-neutral productivity shifter for producer i, F_i is a constant returns production technology, x_{ij} are intermediate inputs from $j \in N$, and L_{ir} are efficiency units of factor services from r.

[14] Equivalently, we can model Z as a shifter that scales value added productivity of all producers.

Resource Constraints. The resource constraint requires that consumption and intermediate input usage be equal to production. Recall that commodities are split into pure consumption goods ($i \in C$) and pure intermediates ($i \in N$). Hence, if i is a consumption bundle, then i is not used as an intermediate $x_{ji} = 0$. On the other hand, if $i \in N$, then i is not directly consumed by any household. The resource constraints are therefore:

$$\int c_h \mathbf{1}[l_h = r] dh = y_{c(r)} \text{ if } c(r) \in C \qquad \text{and} \qquad \sum_{j \in N} x_{ji} = y_i \text{ if } i \in N. \tag{2}$$

The resource constraint for factor $r \in R$ requires that total factor demand from producers is equal to total factor supply from households:

$$\sum_{i \in N} L_{ir} = Z L_r. \tag{3}$$

Finally, lump-sum transfers add up to zero:

$$\int T_h dh = 0. \tag{4}$$

Definition 1 (Equilibrium with Discrete Choice)**.** An equilibrium is a collection of consumptions, c_h, location choices, l_h, outputs, y_i, intermediate input choices, $\{x_{ij}, L_{ir}\}$, prices, p_i and wages, w_r, such that each agent chooses consumption and location to maximize utility subject to their budget constraint, producers maximize profits subject to technology taking prices as given, transfers add up to zero, and all resource constraints are satisfied.

We focus on decentralized equilibria where transfers are equal to zero: $\boldsymbol{T} = 0$.

Solving for Equilibrium. To solve for the equilibrium, we introduce some useful accounting variables. Let $\boldsymbol{\lambda}$ be the vector of Domar (1961) weights, whose ith component is i's sales divided by total income:

$$\lambda_i = \frac{p_i y_i}{\sum_r Z w_r L_r} \mathbf{1}[i \in C + N] + \frac{Z w_i L_i}{\sum_r Z w_r L_r} \mathbf{1}[i \in R].$$

Let Ω be the $|C + N + R| \times |C + N + R|$ input-output matrix. The first $|C|$ rows and columns are the consumption bundles, the next $|N|$ are the intermediates, and the final $|R|$ are the primary factors. The ijth element of Ω is the expenditure share of i on inputs from j relative to i's total sales:

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}\mathbf{1}[i \in C + N] + \frac{w_j L_{ij}}{p_i y_i}\mathbf{1}[j \in R].$$

Using this notation, we can now describe the equilibrium conditions. Perfect competition and cost-minimization by producer i implies that p_i is equal to its marginal cost:

$$p_i = z_i^{-1}\mathrm{mc}_i(\boldsymbol{p}, \boldsymbol{w}), \tag{5}$$

where mc_i is the marginal cost function and depends on input prices, factor wages, and productivity shifters z_i.

Market clearing for $i \in C + N + R$ requires that total sales of good i equal total spending on good i by households and other producers:

$$\lambda_i = \sum_{r \in R} \chi_r \Omega_{c(r)i} + \sum_{j \in N} \lambda_j \Omega_{ji}, \tag{6}$$

where $c(r)$ is the consumption bundle associated with location r, and χ_r is spending in location r. The factor market clearing condition states that spending on factor r by firms is equal to earnings by agents that supply that factor:

$$\lambda_r = \frac{Zw_r L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})}{\sum_{r'} Zw_{r'} L_{r'}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})}. \tag{7}$$

Finally, when lump-sum transfers are zero, $\boldsymbol{T} = 0$, the share of spending in each location is equal to the share of income earned in that location $\chi_r = \lambda_r$. Using this fact, and given the aggregate supply function $L_r(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$, one can solve for equilibrium prices, wages, and quantities using (5), (6), and (7).

The example below provides a concrete illustration of the structure of the model using Cobb-Douglas and logit functional forms.

Example 1 (Cobb-Douglas with Logit). Suppose each location produces a single good using labor in that location. The consumption bundle in every location is the same Cobb-Douglas aggregator with a common price. Equation (5) implies the (common) consumer price index is

$$p^c = \prod_r \left(\frac{w_r}{z_r}\right)^{\Omega_{cr}}, \tag{8}$$

where Ω_{cr} are constant parameters that do not depend on productivity.

Suppose there is a unit mass of agents with homogeneous skills ($a_{hr} = 1$ for every $h \in H$ and $r \in R$), and preferences take the form $u_h(c_h, l_h) = f_h(c_h + \epsilon_{hr}\mathbf{1}[l_h = r])$ where ϵ_{hr} are drawn from type I extreme value distribution and f_h is any strictly increasing function. Following McFadden (1973), the labor supply function is:

$$L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) = \frac{\exp(\theta Z w_r / p^c + B_r)}{\sum_{r'} \exp(\theta Z w_{r'} / p^c + B_{r'})}, \tag{9}$$

where θ and B_r are parameters of the distribution of ϵ_{hr}.

The labor market clearing condition (6) specialized to factor $r \in R$, is $\lambda_r = \sum_{r' \in R} \chi_{r'} \Omega_{c(r')r} + \sum_{j \in N} \lambda_j \Omega_{jr} = \Omega_{cr}$.[15] Equating labor supply with labor demand yields

$$\frac{w_r \exp(\theta Z w_r / p^c + B_r)}{\sum_{r'} w_{r'} \exp(\theta Z w_r' / p^c + B_r')} = \Omega_{cr}, \tag{10}$$

where Ω_{cr} is a constant. Equations (8) and (10) jointly pin down wages and the consumption price index in the decentralized equilibrium. Given prices and wages, quantities are then straightforward to calculate. Example 8 in the appendix provides a more involved example with intermediate inputs.

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3 Defining and Characterizing Aggregate Welfare

In this section, we define our measure of aggregate welfare and then characterize it. For convenience, we parameterize technologies by a scalar t and write them as $\boldsymbol{z}(t)$. We assume that $t = 0$ corresponds to an initial equilibrium, which we call the *status quo*. We calibrate the status quo to the allocation observed in the data. Counterfactual technologies of interest are indexed by $t > 0$. As t increases, productivity parameters change. We normalize Z to be constant and equal to one as a function of t.[16]

Our treatment is fairly abstract and general, so readers looking for a concrete example are encouraged to focus on the running example introduced in Example 2. Throughout the rest of the paper, we always return to this same example to illustrate the general results.

[15] The second equality follows from the fact that there are no intermediates: $\sum_{j \in N} \lambda_j \Omega_{jr} = 0$; there is a common consumption good: $\Omega_{c(r')r} = \Omega_{cr}$; and spending shares add up to one: $\sum_{r'} \chi_{r'} = 1$.

[16] To model changes in aggregate factor-augmenting productivity, we can introduce intermediaries that sell primary factor services to other producers and instead scale their productivities.

3.1 Definition of TFP-Equivalent Aggregate Welfare

We define the set of feasible allocations to be

$$\mathcal{C}(t,Z) \equiv \{\{\boldsymbol{c}_h, l_h\}_{h\in H} \text{ supported via competitive equilibrium given } \boldsymbol{z}(t),\, Z, \text{ and some transfers}\}\,.$$

Feasible here means allocations that can be implemented as competitive equilibrium allocations given some transfers (not just technologically feasible).

Inspired by Debreu (1951), we measure changes in aggregate welfare using equivalent changes in factor-augmenting productivity Z.

Definition 2 (TFP-Equivalent Aggregate Welfare)**.** Aggregate welfare in TFP-equivalent terms at t is

$$A(t) = \max\left\{Z^{-1} : \{c_h, l_h\}_{h\in H} \in \mathcal{C}(t,Z) \text{ and } (c_h, l_h) \succeq_h (c_h(0), l_h(0)) \text{ for every } h\right\}, \quad (11)$$

where $c_h(0)$ and $l_h(0)$ are the consumption and location of h in the status quo.

In words, to measure aggregate welfare, we consider a hypothetical rescaling of factor productivity by $1/A(t)$. We define $A(t)$ as the largest contraction such that every agent can be made at least as well off as in the status quo given some transfers.[17] Intuitively, if $A(t) = 1.01$, then this means that we can shrink the productivity of factors by roughly 1% (more precisely, $1 - 1.01^{-1}$) and still keep every household indifferent to the initial equilibrium. Equivalently, because Z uniformly scales the production possibility set, every household can be kept indifferent to the status quo with roughly 1% less of every good — that is, there is a 1% TFP-equivalent surplus.

Throughout, we assume that at the solution to (11), every agent can be made exactly indifferent to the status quo. This rules out pathological cases where agents are completely disconnected in goods and factor markets.[18]

[17]This is not exactly the definition in Debreu (1951) because we restrict the set of feasible allocations to be the ones that can be reached using individual-specific lump-sum transfers as opposed to any technologically feasible allocation. This distinction matters because, in this economy, the second welfare theorem need not hold. In Appendix H, we discuss how to further restrict the set of feasible allocations to be the ones that can be supported using place-based, rather than individual-specific, transfers. We show that aggregate welfare using place-based compensations is weakly less than aggregate welfare using individual-specific lump-sum transfers, since compensations using place-based policy are distortionary, but these distortions are second-order around lasseiz-faire.

[18]For example, all households are immobile across locations and only consume goods produced by their own location (i.e. autarky in both goods and factor markets). In such a case, making households exactly indifferent to the status quo can become impossible because transfers between locations are not possible. In these cases the measure is still well-defined, but some of our theorems do not go through.

Before proceeding with the analysis, we collect some remarks on how this measure can be extended.

Remark (Changes in amenities). Since t only indexes productivity parameters $\boldsymbol{z}$, our analysis focuses on the effect of productivity shocks only. However, the same definition of aggregate welfare and many of the results we present can be extended to cover changes in other parameters as well. For example, we can allow for changes in location characteristics that affect amenity values (e.g. changes in the weather in a region or in the health risk associated with particular occupations). In that case, we index the value of the relevant characteristic by t and apply Equation (11) without any other modification. If the amenity value of choosing location l_h can be written as $u_h(b_{l_h} c_h, l_h)$, where b_{l_h} is a location-specific quality shifter, then an increase in b_{l_h} has the same effect on aggregate welfare $A(t)$ as the same proportional increase in the productivity of the consumption good in that location, $z_{c(l_h)}$.

Remark (Welfare for a subset of agents). Definition (11) can also be applied to measure collective welfare for a subset of agents, such as agents of a given gender, race, or age. In that case, we contract the endowments of agents in that subset subject to indifference conditions and balanced-budget transfers within the subset. If the subset consists of a single infinitesimal agent h, then the TFP-equivalent welfare collapses to that agent's compensating variation,[19] denoted by $A_h(t)$, and defined by

$$\max_r u_h \left(\frac{a_{hr} w_r(t)}{A_h(t) p_r(t)}, r \right) = u_h^0. \tag{12}$$

The value of $A_h(t)$ depends only on household h's ordinal preferences $\succeq_h$ and is invariant to strictly increasing transformations of utility.

Remark (Imperfect competition and externalities). Although in the body of the paper we restrict attention to perfectly competitive economies, in Appendix G we show how to calculate aggregate welfare in the presence of distortions and externalities. This extension can be used to study the effects of policies (modelled as implicit or explicit tax-like wedges) or to quantify misallocation.

[19]Technically, this is a compensating variation in ratios—the proportional reduction in agent h's productivity—rather than in differences. See Appendix I for a comparison of compensating variations in ratios and in differences.

3.2 Exact Characterization of $A(t)$

Solving for $A(t)$ by brute-force is challenging. To make progress, we break the problem into pieces and characterize $A(t)$ using the aggregate *compensated* supply and demand functions.[20] As in classical consumer theory, the compensated supply and demand functions map wages and prices into labor supplied and spending in each location, holding each agent on a given indifference curve using lump-sum transfers.

3.2.1 Expenditure Function and Compensated Choices

Begin by defining the expenditure function of each consumer.

Definition 3 (Expenditure function)**.** Define $e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ to be the (net) expenditure function for agent h given wages $\boldsymbol{w}$, prices $\boldsymbol{p}$, and status quo utility $u_h^0 = u_h(c_h(0), l_h(0))$:

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min\{T_h : u_h(c_h, l_h) \geq u_h^0, \text{ and } \sum_r p_r c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h\}.$$

We call the location choice $l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ associated with this optimization problem *the compensated choice* of agent h.

In words, $e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ is the minimum lump-sum transfer agent h requires to be made at least indifferent to some reference utility level u_h^0, given prices and wages. The location choice that the agent makes, given that transfer, is what we call the compensated choice. This definition parallels the classical definition of expenditure functions and compensated demand in consumer theory.[21]

To characterize the compensated choice of each agent, we first define the consumption-equivalent variation (Lucas, 1987).

Definition 4 (Consumption-equivalent variation)**.** Define $\bar{c}_{hl_h}$ as the solution to

$$u_h(\bar{c}_{hl_h}, l_h) = u_h^0,$$

where u_h^0 is utility of h in the status quo. In words, $\bar{c}_{hl_h}$ is the consumption agent h must have in location l_h to be indifferent to the status quo.

[20] In Appendix C, we consider a special case in which agents are homogeneous in both preferences and skill levels. In this case, lump-sum transfers are not needed to evaluate the compensated supply and demand functions, since all agents are affected symmetrically by shocks. When $g(c) = \log(c)$, $A(t)$ equals the growth rate of real consumption per capita, which is common across locations.

[21] See also Small and Rosen (1981) for a related analysis. Whereas they consider discrete consumption choices, given fixed income, here the level of income depends also on the discrete choice (via the wage).

Given the consumption-equivalent variation, the following proposition shows how to compute the compensated choices and the expenditure function agent-by-agent.

Proposition 1 (Compensated Choices and Expenditure Function)**.** *The compensated choice* $l_h^{comp}(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ *satisfies*

$$l_h^{comp}(\boldsymbol{w}, \boldsymbol{p}, u_h^0) \in \arg\max_{l \in R} \left[\sum_r \left[a_{hr} w_r - p_r \bar{c}_{hr} \right] \mathbf{1} \left[l = r \right] \right].$$

The expenditure function, or transfer needed to ensure indifference to u_h^0*, is*

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = p_{l_h^{comp}} \bar{c}_{h l_h^{comp}} - a_{h l_h^{comp}} w_{l_h^{comp}}.$$

In words, $[a_{hr} w_r - p_r \bar{c}_{hr}]$ is the surplus, in dollars, agent h receives from being sent to location r. The compensated choice maximizes this surplus, and the expenditure function is equal to the negative of the surplus. Proposition 1 gives a straightforward way to calculate the compensated choice and the compensating transfer for each agent given any vector of prices $\boldsymbol{p}$ and wages $\boldsymbol{w}$.

We aggregate agent-level location choices and expenditure functions, given by Proposition 1, to get location-level variables. The compensated labor supply function, the efficiency of units of labor in location r (not including the aggregate shifter Z) given compensating transfers, is:

$$L_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0) = \int a_{hr} \mathbf{1}[l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = r] dh.$$

Similarly, the compensated spending by households that choose location r is the sum of their labor income and net transfer payments:

$$E_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0) = w_r L_r^{\text{comp}} + \int e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) \mathbf{1}[l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = r] dh,$$

where $l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ and $e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ are given by Proposition 1. We can also denote the compensated share of total spending by households in location r to be

$$\chi_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0) = \frac{E_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)}{\sum_{r'} E_{r'}^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)}.$$

By duality, compensated and uncompensated labor supply and spending are connected by the identities

$$L_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0) = L_r(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{e}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)), \qquad \text{and} \qquad E_r^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0) = E_r(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{e}(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)).$$

Proposition 1 provides a way to evaluate the compensated labor supply $\boldsymbol{L}^{\text{comp}}$ and expenditure $\boldsymbol{E}^{\text{comp}}$ functions given any arbitrary vector of wages, prices, and utility levels.

3.2.2 Calculating $A(t)$ using Compensated Choices and Spending

The next result uses the compensated supply and demand functions to solve for $A(t)$.

Theorem 1 (Aggregate Welfare using Compensated Equilibrium)**.** *TFP-equivalent aggregate welfare satisfies*

$$\sum_r E_r^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}, \boldsymbol{u}^0) = \sum_r (w_r/A(t)) L_r^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}, \boldsymbol{u}^0), \tag{13}$$

The vector of prices $\boldsymbol{p}$ equal marginal costs, (5)*, given wages $\boldsymbol{w}$ and productivities $\boldsymbol{z}(t)$. The vector of wages $\boldsymbol{w}$ satisfy market clearing conditions* (6) *and* (7)*, given compensated labor supplied $\boldsymbol{L}^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}, \boldsymbol{u}^0)$ and demand $\boldsymbol{\chi}^{comp}(\boldsymbol{w}/A(t), \boldsymbol{p}, \boldsymbol{u}^0)$. The wages, prices, labor supplies, and quantities that satisfy these conditions are not the same as the ones in the decentralized equilibrium. We call them variables in the* compensated *equilibrium instead.*

In the compensated equilibrium, every agent h receives a transfer of $e_h(\boldsymbol{w}/A(t), \boldsymbol{p}, u_h^0)$, to keep them indifferent and TFP is contracted by $A(t)$. The value of $A(t)$ is pinned down, according to Equation (13), by the condition that total spending equals income in the compensated equilibrium or, equivalently, that lump-sum transfers add up to zero.

Solving for $A(t)$ using Theorem 1 is relatively straightforward. To see the logic, consider the following iterative procedure. First, conjecture a vector of wages in the compensated equilibrium, $\boldsymbol{w}$, and aggregate TFP shifter $1/A$. (For example, $A = 1$ and $\boldsymbol{w}$ equal to its status quo values.) Use the wages and the productivities, $\boldsymbol{z}(t)$, to solve for prices by setting prices equal to marginal costs, as in (5). Then draw a population of agents with different ϵ's and apply Proposition 1 to get compensated supplies $\boldsymbol{L}^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}, \boldsymbol{u}^0)$ and spending $\boldsymbol{E}^{\text{comp}}(\boldsymbol{w}/A, \boldsymbol{p}, \boldsymbol{u}^0)$ using conjectured prices, wages, and aggregate TFP shifter. Given $\boldsymbol{L}^{\text{comp}}$ and $\boldsymbol{\chi}^{\text{comp}} \equiv \boldsymbol{E}^{\text{comp}}/\sum_r \boldsymbol{E}_r^{\text{comp}}$, use equations (6) and (7) to update wages, and given $\boldsymbol{L}^{\text{comp}}$ and $\boldsymbol{E}^{\text{comp}}$ use equation (13) to update the TFP shifter A. In a compensated equilibrium, these new wages and TFP shifter coincide with the initial conjectures.

We illustrate Theorem 1 using a simple example.

Example 2 (One-Good Economy). Suppose there is a single consumption good produced linearly from labor. The production technology in location r is

$$y_r = z_r(t)L_r = z_r(t)\int a_{hr}\mathbf{1}[l_h = r]dh,$$

where z_r is the productivity and L_r is the supply of labor in location r. Consumption goods produced in each location are perfectly substitutable and freely traded. The aggregate resource constraint is therefore total consumption equals total production:

$$\int c_h dh = \sum_r z_r(t)L_r.$$

The feasible set of equilibrium allocations is given by

$$\mathcal{C}(t, Z) = \left\{\{\boldsymbol{c}_h, \boldsymbol{l}_h\} : \int T_h dh = 0,\ c_h = Za_{hl_h}z_{l_h}(t) + T_h,\ l_h \in \arg\max u_h(c_h, l_h)\right\}.$$

The first condition is that transfers must be budget-balanced, the second are households' budget constraints, and the last states that households choose their locations to maximize utility. Applying Theorem 1 is simple in this economy since the real wage (per efficiency unit) in each location in the compensated equilibrium is equal to exogenous parameters: $z_r(t)$. Hence, (13) can be rearranged as

$$A(t) = \frac{\sum_r \int z_r(t)a_{hr}\mathbf{1}[l_h^{\text{comp}} = r]dh}{\sum_r \int \bar{c}_{hr}\mathbf{1}[l_h^{\text{comp}} = r]dh}, \tag{14}$$

which is the ratio of production (given the technologies at $\boldsymbol{z}(t)$ but compensated locations) and the amount of consumption necessary to keep everyone indifferent at those locations. By Proposition 1, the compensated choices satisfy:

$$l_h^{\text{comp}} \in \arg\max_{r\in R} \left\{z_r(t)a_{hr}/A(t) - \bar{c}_{hr}\right\}.$$

This is a simple numerical problem. For example, we first solve for l_h^{comp} agent-by-agent given a conjectured value of $A(t)$, then use (14) to update the value of $A(t)$.

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4 Analytical Results for Aggregate Welfare

In this section, we provide a more analytical characterization of aggregate welfare using compensated supply and demand curves (e.g. their elasticities and integrals). Section 4.1 provides a differential exact-hat algebra approach to computing $A(t)$. This has the virtue of being computationally simpler, since it does not require solving a system of nonlinear equations, and it also provides us with some analytical results, including a version of Hulten's theorem. Section 4.2 focuses on a special case where the uncompensated supply system coincides with the compensated one, simplifying the analysis.

Throughout, for any variable x, we use $x(t)$ and $x^{\text{comp}}(t)$ to refer to its value in the decentralized equilibrium (where there are no transfers and aggregate factor productivity $Z = 1$) and the compensated equilibrium (where there are compensating transfers and $Z = 1/A(t)$). When it is clear from context, we suppress the comp superscript to simplify the notation.

4.1 Sufficient Statistics for Aggregate Welfare

We begin with the following result showing that aggregate welfare is exactly the chained sum of microeconomic productivity shocks, weighted by Domar weights in the compensated equilibrium.

Theorem 2 (Welfare as Area Under Compensated Sales Shares)**.** *The change in TFP-equivalent aggregate welfare satisfies*

$$\log A(t) = \int_0^t \sum_i \lambda_i^{comp}(s) \frac{d \log z_i}{ds} ds, \tag{15}$$

where $\lambda_i^{comp}(s)$ are i's sales divided by GDP in the compensated equilibrium with productivities $\boldsymbol{z}(s)$. Equivalently, for any $t \geq 0$,

$$\frac{d \log A(t)}{dt} = \sum_i \lambda_i^{comp}(t) \frac{d \log z_i}{dt}. \tag{16}$$

with boundary condition $\log A(0) = 0$*.*

As shown by Baqaee and Burstein (2025), the exact same formula also holds in economies with continuous choice. In this sense, there is nothing conceptually different about measuring aggregate welfare when choices are discrete.

Theorem 2 characterizes $A(t)$ as the solution to a differential equation. Solving $A(t)$ nonlinearly requires knowledge of compensated sales shares, $\lambda_i^{\text{comp}}(s)$, for every $s \in [0, t]$.

We characterize $\lambda_i^{\text{comp}}(s)$ below, but first we make an important observation. In the status quo, $t = 0$, compensated Domar weights coincide with decentralized Domar weights $\lambda_i^{\text{comp}}(0) = \lambda_i(0)$.[22] This allows us to establish the following useful corollary of (16).

Corollary 1 (First-Order Approximation)**.** *To a first-order approximation, at $t = 0$, the change in TFP-equivalent aggregate welfare satisfies*

$$\Delta \log A \approx \sum_i \lambda_i(0) \Delta \log z_i,$$

where $\lambda_i(0)$ is the sales of i divided by GDP in the decentralized equilibrium at the status quo.

Corollary 1 is a generalization of Hulten's theorem to economies with discrete choice. Using Theorem 2 beyond first-order approximations requires knowledge of sales shares, $\lambda_i^{\text{comp}}(t)$, in the compensated equilibrium away from status quo $t > 0$. This is easy to do, using standard methods, if we know the compensated location choices, $\boldsymbol{L}^{\text{comp}}$ and compensated final demand $\boldsymbol{\chi}^{\text{comp}}$.

Given these, computing $\lambda^{\text{comp}}(s)$ follows from the results in Baqaee and Farhi (2019). For example, assume that all production and consumption functions are CES, with the ith producer in the input-output matrix Ω corresponding to a CES aggregator with elasticity of substitution θ_i. This structure is flexible enough to capture any nested-CES production and consumption functions through relabeling.[23] Then the following proposition determines sales shares in both the compensated and uncompensated equilibrium.

Proposition 2 (Prices and Expenditures in Equilibrium)**.** *In the decentralized equilibrium, given labor supplies $\boldsymbol{L}(t)$ and final demand $\boldsymbol{\chi}(t)$, the following differential equations determine the rest of the equilibrium variables:*

$$d \log \Omega_{ij} = (1 - \theta_i) \left[d \log p_j - \sum_{k \in N} \Omega_{ik} d \log p_k - \sum_{r \in R} \Omega_{ir} d \log w_r \right], \tag{17}$$

$$d \log p_i = -d \log z_i + \sum_j \Omega_{ij} d \log p_j + \sum_{r \in R} \Omega_{ir} d \log w_r, \tag{18}$$

$$d \log \lambda_r = d \log w_r + d \log L_r - \sum_{r'} \lambda_{r'} \left[d \log w_{r'} + d \log L_{r'} \right] \textit{ for } r \in R \tag{19}$$

$$d \log \lambda_i = \lambda_i^{-1} \left(\sum_r d\chi_r \Omega_{c(r)i} + \sum_r \chi_r d\Omega_{c(r)i} + \sum_{j \in N} d \left[\lambda_j \Omega_{ji} \right] \right), \tag{20}$$

[22] This is because the decentralized equilibrium is Pareto efficient (the first welfare theorem holds), and hence, $A(0) = 1$ and the decentralized equilibrium supports the compensated allocation with no transfers needed.

[23] One can extend to non-nested CES along similar lines to Baqaee and Farhi (2019).

where all the variables are indexed by t. If we swap $\boldsymbol{L}$ and $\boldsymbol{\chi}$ for $\boldsymbol{L}^{comp}$ and $\boldsymbol{\chi}^{comp}$, we get the evolution of variables in the compensated equilibrium instead. The boundary conditions are the same for both the compensated and decentralized equilibria — spending shares are equal to their values in status quo, and the initial level of prices and wages is irrelevant.

Proposition 2 can be used to calculate either the decentralized equilibrium, given $d\log\chi_r$ and $d\log L_r$, or the compensated equilibrium, given $d\log\chi_r^{\text{comp}}$ and $d\log L_r^{\text{comp}}$. The derivatives of $d\log\chi_r^{\text{comp}}$ and $d\log L_r^{\text{comp}}$ can be computed numerically using Proposition 1. This approach is useful both computationally and analytically. Computationally, it obviates the need to solve a system of nonlinear equations to compute $A(t)$ (i.e. as in Theorem 1). Instead, it requires repeatedly solving a system of linear equations and updating variables (Appendix F provides additional information). Analytically, it allows us to prove some useful results.

Before using Proposition 2, we briefly describe the intuition for equations (17)-(20). The expression in (17) is log-linearized input demand by i for input j, which depends on the elasticity of substitution, θ_i, and on how the price of j changes relative to the marginal cost of i. The expression in (18) is Shephard's lemma, relating the marginal cost of i to the expenditure-share weighted change in input prices and the productivity shock to i. Equation (19) is log-linearizing the definition of λ_r and (20) is the log-linearized versions of the market clearing conditions (6).

A simple corollary of Theorem 2 and Proposition 2 is the following exact characterization of $A(t)$.

Corollary 2 (Cobb-Douglas Economies with Common Consumption)**.** *Suppose that all production functions are Cobb-Douglas, $\theta_i = 1$ for every $i \in C + N$, and that there is a common consumption good in every location. Then, for every t,*

$$\log A(t) = \sum_i \lambda_i(0)[\log z_i(t) - \log z_i(0)],$$

regardless of the shape of the $g(c)$ function and distribution of taste and productivity shifters.

If all production functions are Cobb-Douglas and there is a common consumption good, then the right-hand sides of both (17) and (20) are zero.[24] That is, changes in location choices and spending shares do not affect the sales shares. In fact, sales shares are constant: $\lambda_i^{\text{comp}}(t) = \lambda_i^{\text{comp}}(0)$. Since compensated variables are equal to decentralized

[24]When $\theta_i = 1$ for every i, it follows from (17) that $d\Omega = 0$. From (20), we know that $d\lambda_i = \left(\sum_r d\chi_r \Omega_{c(r)i} + \sum_j d\lambda_j \Omega_{ji}\right) = \sum_j d\lambda_j \Omega_{ji}$. The last equality follows from the common consumption good assumption, which implies $\Omega_{c(r)i} = \Omega_{c(r')i}$ for every r and i. Hence, the only solution is $d\lambda = 0$.

variables in the status quo, $\lambda_i^{\text{comp}}(0) = \lambda_i(0)$, the integral in Theorem 2 can be solved easily.

4.2 Equivalence of Compensated and Uncompensated Supply

Theorem 2 shows that computing $A(t)$ nonlinearly requires knowledge of compensated Domar weights $\boldsymbol{\lambda}^{\text{comp}}$. In this section, we consider a tractable special case where compensated Domar weights are simple to solve for, because the compensated and uncompensated supply systems coincide.

Assumption 1 (Common Consumption Good and Affine g)**.** Suppose that the consumption aggregator in every location is the same as every other location, so that the consumption price index $p_{c(r)}$ is the same in every location r and can be denoted by p^c. Suppose that preferences take the form $u_h(c_h, l) = f_h(c_h + \epsilon_{hl})$ for some arbitrary increasing function f_h. (i.e. the function $g(c)$ is affine).

Under this assumption, lump-sum transfers do not change agents' location choices. To see this, note that if preferences are $f_h(c_h + \epsilon_{hl})$ then a constant increase in consumption in every location does not alter household h's location choice. Moreover, if the price of consumption is the same in every location, then a lump-sum transfer gives the same amount of consumption in every location. That is, under Assumption 1, agent h chooses the location with the highest $(Za_{hr}w_r + T_h)/p^c + \epsilon_{hr}$ — and this choice is independent of the value of T_h. This implies the following.

Proposition 3 (Characterization Using Differential Equations)**.** *If Assumption 1 holds, then the compensated supply function coincides with the uncompensated supply function:*

$$L^{comp}(\boldsymbol{w}/A, p^c, \boldsymbol{u}) = L(\boldsymbol{w}/A, p^c) = L(\boldsymbol{w}/(Ap^c), 1)$$

So, for any $t > 0$, the derivative of L^{comp} with respect to t is just:

$$dL_r^{comp} = \sum_{r'} \frac{\partial L_r}{\partial \left[w_{r'}/(Ap^c)\right]} d\left(\frac{w_{r'}}{Ap^c}\right), \tag{21}$$

where prices and wages are evaluated in the compensated equilibrium at t. Moreover, because there is a common consumption good, equation (20) *does not depend on compensated demand χ^{comp}. Hence, equations* (16) *to* (21) *fully pin down $A(t)$,*

Under Assumption 1, the compensated and uncompensated supply functions coincide. Furthermore, the distribution of spending χ^{comp} is irrelevant for determining compensated sales shares λ^{comp} because budget shares of households are the same in every location r. In particular, the distribution of spending χ^{comp} only shows up in (20) and has no effect (since $\Omega_{c(r)i} = \Omega_{ci}$ for every $r \in R$). Therefore, Proposition 3 allows us to compute $A(t)$ as the solution to a system of ordinary differential equations.

The following example solves these differential equations in the one-good economy and relates $A(t)$ to the area under compensated supply curves.

Example 3 (One-Good Economy under Assumption 1). Consider Example 2 again: there is one common and freely-traded consumption good made linearly from labor in each location. In this economy, since Domar weights always sum to one, $\sum_r \lambda_r \equiv 1$, we can rewrite (16) as

$$\sum_r \lambda_r^{\text{comp}}(t) \frac{[d \log z_r / A(t)]}{dt} = 0.$$

By definition of λ_r^{comp}, and since $w_r(t) = z_r(t)$, this is

$$\sum_r L_r^{\text{comp}}(t) \frac{d\,[z_r(t)/A(t)]}{dt} = 0.$$

Since this equation holds for every $t > 0$, we can integrate both sides to get

$$\int_0^t \sum_r L_r^{\text{comp}}(s) \frac{d\,[z_r(s)/A(s)]}{ds} ds = 0. \tag{22}$$

In words, (22) shows that $A(t)$ is the reduction in TFP such that the area under the compensated labor supply curve is zero. Suppose that Assumption 1 holds, so that we can use the uncompensated supply function in place of the compensated supply function, and hence

$$\int_0^t \sum_r L_r\left(\frac{\boldsymbol{z}(s)}{A(s)}, 1\right) \frac{d\,[z_r(s)/A(s)]}{ds} ds = 0.$$

If we know the functional form for the uncompensated supply function, then the equation above is a single nonlinear equation that pins down $A(t)$. For example, suppose that workers have homogeneous skills ($a_{hr} = 1$ for every $h \in H$ and $r \in R$), and that the uncompensated supply system is logit. In this case, the uncompensated supply is

$$L_r\left(\frac{\boldsymbol{z}(s)}{A(s)}, 1\right) = \frac{\exp(\theta \frac{z_r(s)}{A(s)} + B_r)}{\sum_{r'} \exp(\theta \frac{z_{r'}(s)}{A(s)} + B_{r'})},$$

where B_r is an exogenous location-level taste shifter or amenity. With this functional form, we can evaluate the integral in closed-form as:

$$\log \sum_r \exp(\theta \frac{z_r(t)}{A(t)} + B_r) - \log \sum_r \exp(\theta z_r(0) + B_r) = 0. \quad (23)$$

This nonlinear equation pins down $A(t)$.

The general equilibrium structure in this example is simple since real wages must equal labor productivity in each location (i.e. labor demand is infinitely elastic at a fixed real wage). Example 9 in the appendix provides a slightly richer multi-industry example where relative wages are endogenous.

Relation to social surplus function. In Appendix E we show that under Assumption 1, $A(t)$ can also be computed using the utilitarian social welfare function $U(\boldsymbol{c}) = \mathbb{E}[\max_{l_h} g(c_{l_h}) + \epsilon_{h,l_h}]$, which is called the *"Social Surplus Function"* in the discrete choice literature (see McFadden, 1981). The reason is that, by the Williams-Daly-Zachary theorem, the social surplus function is the integral of uncompensated labor supply. When Assumption 1 holds, compensated and uncompensated labor supply coincide, so the social surplus function is also the integral of compensated supply. Hence, by a similar logic to (22), $U(\boldsymbol{c})$ can be used to determine $A(t)$. However, if Assumption 1 does not hold, then the social surplus function can no longer be used to compute $A(t)$ because compensated and uncompensated labor supply are no longer the same.

5 Second-Order Approximation of Aggregate Welfare

In this section, we investigate nonlinearities in cases where transfers do affect location choices (i.e. Assumption 1 does not hold). We do this by relating derivatives of compensated labor supply and demand to derivatives of their uncompensated counterparts. This allows us to characterize changes in aggregate welfare to a second-order approximation in terms of uncompensated elasticities and expenditure shares. Throughout this section, we explicitly use the fact that preferences are ordinally additively separable. We also assume that idiosyncratic tastes $\boldsymbol{\epsilon}$ have an absolutely continuous density $f(\boldsymbol{\epsilon})$.

The previous section shows that to a first-order approximation, $A(t)$ is a Domar-weighted sum of microeconomic productivity shocks. Differentiating (16) once more and applying a second-order Taylor expansion at $t = 0$ yields the following approximation for $A(t)$.

Corollary 3 (Second-Order Approximation). *To a second-order approximation, the change in TFP-equivalent aggregate welfare satisfies*

$$\Delta \log A \approx \sum_i \lambda_i(0) \Delta \log z_i + \frac{1}{2} \sum_i \Delta \lambda_i^{comp}(0) \Delta \log z_i,$$

where $\Delta\lambda_i^{comp}$ is the (first-order) change in the compensated sales of i divided by GDP at the status quo.

The intuition is very similar to the formulas derived by Baqaee and Farhi (2019) for an economy without discrete choice. If sales shares in the compensated equilibrium rise in response to positive productivity shocks, then aggregate welfare rises more quickly than its first-order approximation would suggest. If sales shares fall, then the reverse happens.

Calculating $\Delta\lambda_i^{\text{comp}}(0)$ in Corollary 3 is a straightforward application of Proposition 2 if we know the derivatives of compensated labor supply $dL^{\text{comp}}(0)$ and demand $d\chi^{\text{comp}}(0)$. When Assumption 1 holds, this is simple to do using Proposition 3, since dL^{comp} can be computed using the uncompensated supply system and $d\chi^{\text{comp}}$ is irrelevant. In this section, we focus on characterizing dL^{comp} and $d\chi^{\text{comp}}$ at the status quo, without imposing Assumption 1, and we compare these to their decentralized uncompensated counterparts.

To simplify the analysis, we focus on the case with only preference heterogeneity and abstract from skill-heterogeneity. This implies that agents make different choices only due to differences in tastes.[25]

Assumption 2 (Homogeneous Idiosyncratic Skills). Agents have homogeneous idiosyncratic skill, which we normalize to one: for every agent $h \in H$ and location $r \in R$, we have $a_{hr} = 1$.

We now proceed to characterizing $d\chi^{\text{comp}}$ (in Proposition 4) and dL^{comp} (in Proposition 5) at the status quo. Given these derivatives, we can compute $d\lambda^{\text{comp}}$ using Proposition 2, and hence, we can approximate $\log A(t)$ to a second-order using Corollary 3.

Proposition 4 (First-Order Change in Spending Shares). *At $t = 0$, in the decentralized equilibrium, the spending share in location r satisfies*

$$d \log \chi_r = d \log w_r - \mathbb{E}_\chi [d \log w] + d \log L_r - \mathbb{E}_\chi [d \log L],$$

[25] Assumption 2 can be slightly relaxed to allow for the possibility that some agents h have skill $a_{hr} = 0$ for some locations r. This is because such an extension is isomorphic to giving those agents ϵ_{hr} values that are so low that those agents would never choose r in equilibrium. Since we do not impose strong assumptions on the distribution of ϵ's, we model such scenarios by picking an appropriate distribution of ϵ.

where the changes in wages $\boldsymbol{w}$ *and locations* $\boldsymbol{L}$ *are in the decentralized equilibrium and* $\mathbb{E}_\chi[d\log L] = \sum_r \chi_r(0) d\log L_r$. *At* $t=0$, *in the compensated equilibrium, we instead have*

$$d\log\chi_r^{comp} = d\log p_r - \mathbb{E}_\chi\left[d\log p\right] + d\log L_r^{comp} - \mathbb{E}_\chi\left[d\log L^{comp}\right],$$

where the changes in prices $\boldsymbol{p}$ *and locations* $\boldsymbol{L}^{comp}$ *are in the compensated equilibrium and* $\mathbb{E}_\chi[d\log p] = \sum_r \chi_r(0) d\log p_r$.

The intuition is the following. In the decentralized equilibrium, the share of spending by each location must move in line with the share of income earned by households in that location. This is the product of wages w_r and population shares L_r. A location's share of spending rises if wages in that location increase quickly or if households flock to that location. On the other hand, in the compensated equilibrium, the share of spending in each location depends on the rate at which the cost of consumption in location r has increased and the population shares in the compensated equilibrium. If consumption prices rise quickly in a location, or households flock to a location, in the compensated equilibrium, then that location's share of total spending rises.

The next proposition provides a general characterization of dL^{comp} in terms of the derivatives of the uncompensated labor supply function at the status quo $\partial L_r/\partial(w_r'/p_r')$. This is important because the derivatives of the uncompensated supply system are estimable using exogenous variation in the decentralized equilibrium.[26]

Proposition 5 (First-Order Changes in Location Choices)**.** *The change in the share of households in location r can be decomposed into inflows minus outflows:*

$$dL_r = \sum_{r'\neq r} dL_{r'\to r} - \sum_{r'\neq r} dL_{r\to r'}, \tag{24}$$

where $dL_{r'\to r}$ *is the share of switchers from* r' *to* r. *To a first-order, the flows satisfy:*

$$dL_{r\to r'} = \left[\frac{\partial L_{r'}}{\partial(w_r/p_r)} d(\frac{w_r}{p_r}) - \frac{\partial L_r}{\partial(w_{r'}/p_{r'})} d(\frac{w_{r'}}{p_{r'}})\right] \mathbf{1}\left[\frac{\partial L_r}{\partial(w_{r'}/p_{r'})} d(\frac{w_{r'}}{p_{r'}}) \le \frac{\partial L_{r'}}{\partial(w_r/p_r)} d(\frac{w_r}{p_r})\right], \tag{25}$$

where the changes in wages and prices are the ones in the decentralized equilibrium.

[26] Formally, in the decentralized equilibrium, where transfers are zero, the supply function $L_r(\boldsymbol{w},\boldsymbol{p},\boldsymbol{T})$ is a function only of the vector of real wages per efficiency unit in each location w_r/p_r. Hence, $L_r(\boldsymbol{w},\boldsymbol{p},\boldsymbol{0}) = L_r(\boldsymbol{w}/\boldsymbol{p},1,\boldsymbol{0})$, where with some abuse of notation, $\boldsymbol{w}/\boldsymbol{p}$ is the vector of real wages. This allows us to define $\partial L_r/\partial(w_r'/p_r')$, which can be estimated using the observed changes in market-level labor supply in response to exogenous variation in real wages.

The compensated supply function instead satisfies:

$$dL_r^{comp} = \sum_{r' \neq r} dL_{r' \to r}^{comp} - \sum_{r' \neq r} dL_{r \to r'}^{comp},$$

where $dL_{r' \to r}^{comp}$ *is the share of switches from* r' *to* r *in the compensated equilibrium. At* $t = 0$*, this satisfies:*

$$dL_{r \to r'}^{comp} = \frac{\partial L_r}{\partial \left[w_{r'} / p_{r'} \right]} \frac{1}{p_{r'}} \left(d(\frac{w_r}{A p_r}) p_r - d(\frac{w_{r'}}{A p_{r'}}) p_{r'} \right) \times \mathbf{1} \left[d(\frac{w_r}{A p_r}) p_r \leq d(\frac{w_{r'}}{A p_{r'}}) p_{r'} \right], \quad (26)$$

where the changes in wages and prices are the ones in the compensated equilibrium and $d \log A$*, at* $t = 0$*, is given by Corollary 1.*

Equation (25) states that in the decentralized equilibrium, households switch from r to r' if the change in L_r due to real wage changes in r' are outpaced by the change in $L_{r'}$ due to real wage changes in r. The rate at which households switch depends on cross-derivatives between r and r'. The overall change in the share of households choosing r, in (24), depends on the sum of all these pairwise switchings.

Equation (26) is the counterpart to (25) in the compensated equilibrium. Here, households are given individualized transfers and aggregate factor productivity is adjusted to ensure every household can be made exactly indifferent to the status quo. In this case, households switch from r to r' if the compensated real wage in dollar terms rises faster in r' than in r, and the rate at which households switch depends on the (uncompensated) cross-derivative of L_r with respect to the real wage in r'. Crucially, the partial derivatives in (26) are those of the uncompensated supply system evaluated at the status quo in the decentralized economy.

We illustrate Corollary 3 and Proposition 5 with a simple example.

Example 4 (Second-Order Approximation of Aggregate Welfare with Fréchet tastes). Consider again the one-good economy described in Example 2. Applying Corollary 3 to the one-good economy we get that, to a second-order,

$$\Delta \log A \approx \mathbb{E}_\lambda[\Delta \log z] + \frac{1}{2} \left[Var_\lambda \left(\Delta \log z \right) + Cov_\lambda (\Delta \log L^{\text{comp}}, \Delta \log z) \right], \quad (27)$$

where the variance and covariance operators use λ as the probability weights and we use the fact that $d\lambda_r^{\text{comp}} = \lambda_r (d \log(z_r L_r^{\text{comp}}) - \mathbb{E}_\lambda[d \log(z L^{\text{comp}})])$. In words, the nonlinear change in aggregate welfare depends on the variance of the shocks and the covariance of compensated migrations with the shocks. The variance term reflects the fact that, even if households are not mobile, an increase in productivity in one location raises that location's sales share, which magnifies the value of positive productivity growth in that location relative to the loglinear approximation. The covariance reflects the fact that if, in the compensated equilibrium, households move to locations experiencing more rapid productivity growth, then this further boosts the compensated sales shares of producers in that location, and raises aggregate welfare.

To solve out for $\Delta \log \boldsymbol{L}^{\text{comp}}$, suppose that $u_h(c_h, l) = \log(c_h) + \epsilon_{hl}$ and $\exp(\epsilon_{hl})$ is distributed according to a Fréchet distribution with shape parameter θ and scale parameters $\boldsymbol{B}$ (i.e. ϵ_{hl} is a type I extreme value distribution). Under this assumption, the share of households that choose location r given a vector of real wages per efficiency unit $\boldsymbol{z}$ is

$$L_r(z) = \frac{B_r z_r^\theta}{\sum_{r'} B_{r'} z_{r'}^\theta}, \tag{28}$$

for some parameters $B_r > 0$ (see, e.g. Redding and Rossi-Hansberg, 2017). Since in this one-good economy, the real wage equals productivity, the uncompensated cross-derivatives of supply are:

$$\frac{\partial L_{r'}}{\partial w_r} = -\theta L_{r'} L_r \frac{1}{z_r}.$$

For simplicity, suppose that initial productivities, $z_r(0)$, are the same in every location. Then applying Proposition 5 yields

$$\Delta \log L_r^{\text{comp}} = \theta \left(\Delta \log z_r - \mathbb{E}_\lambda \left[\Delta \log z \right] \right).$$

In this case, changes in population shares in the compensated equilibrium, $d \log L^{\text{comp}}$, are the same as the ones in the decentralized equilibrium, $d \log L$, up to a first-order approximation.

Substituting this into the approximation for $\Delta \log A$ in (27) yields:

$$\Delta \log A \approx \mathbb{E}_\lambda [\Delta \log z] + \frac{1}{2}(1+\theta) Var_\lambda \left(\Delta \log z \right).$$

Hence, the higher is θ, the more convex is aggregate welfare.[27]

[27] If instead the share function were logit (i.e. $g(c_h)$ is linear and ϵ_{hr} are type I extreme value with parameter θ^{logit}), then $\Delta \log A \approx \mathbb{E}_\lambda [\Delta \log z] + \frac{1}{2}(1 + \theta^{\text{logit}} z(0)) Var_\lambda \left(\Delta \log z \right)$, where we abuse notation and treat $z(0)$ as a scalar because productivity in every location is the same in the status quo. Calibrating the derivative of L_r with respect to real wages in r' to be the same as in the Fréchet model implies $\theta^{\text{logit}} = \theta / z(0)$. Hence, the implications for $\Delta \log A$ in the two models are the same up to a second-order approximation as long as we match the same cross-derivatives of supply.

6 Comparison to Other Aggregate Measures

In this final section, we compare our measure of aggregate welfare with three alternative aggregate measures that are popular in the literature. In Section 6.1, we relate our measure to real output (GDP) and multi-factor productivity growth as measured in the national accounts. In Section 6.2, we contrast our measure with the popular utilitarian or average utility measure. Finally, in Section 6.3, we compare our results to the sum of compensating variations (i.e. Kaldor-Hicks efficiency) previously analyzed by Small and Rosen (1981) in a discrete-choice context. Throughout the section, we quantify the differences between the different measures using the one-good economy calibrated to U.S. data.

6.1 Real Output

We contrast changes in aggregate welfare in TFP equivalent terms, defined via $A(t)$, with changes in a chain-weighted (Divisia) index of real output, as measured in the national accounts.

Definition 5 (Real Output)**.** The change in real output is the share-weighted sum of changes in final consumption quantities:

$$\log Y(t) = \int_0^t \sum_{r \in C} \frac{p_r(s) c_r(s)}{\sum_{j \in C} p_j(s) c_j(s)} \frac{d \log c_r}{ds} ds,$$

where C is the set of consumption bundles and $c_r(t) = \int c_h(t) \mathbf{1}[l_h(t) = r] dh$ is total consumption of final good r at t.

Using Hulten (1978), we can write real output as a Domar-weighted sum of technology and labor supply changes:

$$\log Y(t) = \int_0^t \left[\sum_{i \in N} \lambda_i(s) \frac{d \log z_i}{ds} + \sum_{r \in R} \lambda_r(s) \frac{d \log L_r}{ds} \right] ds.$$

Differentiating this with respect to t and evaluating at $t = 0$ yields the first order approximation:

$$\Delta \log Y \approx \sum_i \lambda_i(0) \Delta \log z_i + \sum_r \frac{w_r(0)}{\sum_{r'} w_{r'}(0) L_{r'}(0)} \Delta L_r.$$

Real output responds positively to productivity growth, and — holding productivities fixed — benefits from relocations of labor towards higher-wage locations. These labor relocations, ΔL_r, are endogenous and require solving the general equilibrium model.

Unlike $A(t)$, real output does not respect the Pareto principle. To see this, consider a positive productivity shock to a low-wage location. In this case, output can fall if the shock induces large flows out of high-wage locations into the still-low-wage location.[28] Intuitively, because agents care about amenities, they may move to a lower-wage region and this reduces production even though every agent is made (weakly) better off.[29]

In contrast, aggregate welfare, $A(t)$, does not respond to relocations (see Corollary 1). If households choose to move from high- to low-wage locations, this has no first-order effect on aggregate welfare. The change in wages experienced by movers is exactly offset by the change in the amenity value of the move. Hence, whereas real output changes due to the changes in location choices, aggregate welfare does not. We summarize this in the proposition below.

Proposition 6 (First-Order Difference between Real Output and Welfare)**.** *To a first-order approximation, the difference between real output and aggregate welfare in terms of TFP equivalents is given by the change in income caused by relocation:*

$$\Delta \log Y - \Delta \log A \approx \sum_r \frac{w_r(0)}{\sum_{r'} w_{r'}(0) L_{r'}(0)} \Delta L_r.$$

Hence, real output and aggregate welfare can differ, even in sign, to a first-order approximation. The following example quantifies these differences.

Example 5 (Quantitative Difference Between Output and Aggregate Welfare). We compute the quantitative difference between the response of aggregate welfare and real output to productivity shocks using a calibrated one-good economy with isoelastic labor supply, as in Equation (28), with elasticity parameter $\theta = 1.5$. We consider an economy

[28] In the one-good economy with isoelastic labor supply, as in Example 4, the elasticity of real output with respect to a productivity shock is

$$\frac{\Delta \log Y}{\Delta \log z_r} \approx \frac{w_r(0) L_r(0)}{\sum_k w_k(0) L_k(0)} + \theta \left[\frac{w_r(0)}{\sum_k w_k(0) L_k(0)} - 1 \right] L_r(0).$$

This elasticity can be negative if θ is high and w_r relatively low.

[29] There is a strong analogy to economies with labor-leisure choice. In such economies, real output can fall even as welfare rises because of the endogenous response of leisure. This happens because GDP does not include the value of leisure. In contrast, because of the envelope theorem, in an economy with continuous labor-leisure choice, $\Delta \log A$ still obeys Hulten's theorem (as does $\Delta \log A^{MFP}$).

with 588 locations calibrated to US commuting zones.[30] We choose productivities z_r and Fréchet scale parameters B_r, which capture amenities, to match the observed GDP share and labor-force share of each commuting zone in 2024.

Table 1: Aggregate effects of a 1 percent productivity increase in selected locations

Location of shock	$\Delta \log A / \Delta \log z_r$	Sales share λ_r	$\Delta \log Y / \Delta \log z_r$
San Francisco–Oakland, CA	0.031	0.031	0.050
New York, NY	0.057	0.057	0.091
Seattle–Tacoma, WA	0.024	0.024	0.034
Brownsville, TX	0.001	0.001	-0.0002
McAllen, TX	0.002	0.002	0.000
Daytona Beach, FL	0.001	0.001	0.000

Notes: Entries are elasticities with respect to the regional productivity increase. Sales share is λ_r in the initial equilibrium. Results use $\theta = 1.5$ and 400,000 simulated households.

Table 1 displays the change in aggregate welfare and output in response to a 1% rise in productivity in selected commuting zones.[31] The gap between output and aggregate welfare can be substantial. In response to a productivity shock in a high-productivity location such as New York, the increase in output is almost twice as large as the increase in aggregate welfare, consistent with Proposition 6. In response to a shock in a low-productivity location such as Brownsville, TX, output falls even though every agent is weakly better off in the new equilibrium.

Multi-factor productivity growth is defined as real output growth minus the growth in quality-adjusted labor:

$$\Delta \log A^{MFP} \approx \Delta \log Y - \sum_r \frac{w_r(0)}{\sum_{r'} w_{r'}(0) L_{r'}(0)} \Delta L_r,$$

[30] We define locations using the USDA Economic Research Service commuting-zone classification based on county boundaries (U.S. Department of Agriculture, Economic Research Service, 2026). The ERS crosswalk assigns each county or county-equivalent to exactly one commuting zone; commuting zones may contain multiple counties. After excluding Puerto Rico, this leaves 588 commuting zones in our sample. County GDP is from BEA's GDP by County series (U.S. Bureau of Economic Analysis, 2026), and county labor force is from the BLS Local Area Unemployment Statistics program (U.S. Bureau of Labor Statistics, 2026). For each commuting zone r, we sum GDP and labor force across all counties assigned to that commuting zone.

[31] The change in aggregate welfare is very well approximated by the shocked region's sales share, also displayed in the table, which shows that the first-order Hulten approximation in Corollary 1 is very accurate for a 1% shock. Table 5 in Appendix H reports the corresponding elasticities for a 10% shock, where nonlinearities become more visible.

Hence, Proposition 6 implies that $\Delta \log A^{MFP} \approx \Delta \log A$ to a first-order.[32] The intuition here is that if we separate changes in output due to technology shocks from changes in output due to relocation, then multi-factor productivity, as measured by a national income accountant, coincides to a first-order with our measure of aggregate welfare.

We note an important and tractable special case. Suppose there is no amenity value associated with different locations, and that there is a single consumption good. Agents can have idiosyncratic skills by location, so that a_{hr} can vary as a function of both h and r. In this case, real output, $Y(t)$, and our measure of aggregate welfare, $A(t)$, coincide. These assumptions hold in many models of occupational choice, but are less likely to hold in spatial models

Proposition 7 (Coincidence of $Y(t)$ and $A(t)$)**.** *Suppose that ϵ_{hr} does not vary by h and r (i.e. $\epsilon_{hr} = \epsilon$), and that there is a common consumption good. Then*

$$A(t) = A^{MFP}(t) = Y(t).$$

This follows from the fact that under these assumptions, $\boldsymbol{L} = \boldsymbol{L}^{\text{comp}}$, and changes in location spending shares, χ^{comp} have no effect on relative prices since all households spend income on the same consumption good (e.g. $d\chi$ drops out of equation (20)). Hence, $\lambda_i^{\text{comp}}(s) = \lambda_i(s)$. This implies that $A(t) = A^{MFP}(t)$. Furthermore, since every household chooses location to maximize nominal income, households that move from one location to another do not experience a change in their nominal wage (by the envelope theorem), which implies that $d \log Y = d \log A^{MFP}$ — since this holds for any $s > 0$, by integration, it follows that $Y(t) = A^{MFP}(t)$.

6.2 Average Utility/Utilitarian Social Welfare

In economies where households have heterogeneous preferences, by far the most common way to evaluate aggregate welfare is a utilitarian social welfare function (sometimes called "ex-ante expected utility").

Definition 6. Define a utilitarian social welfare function to be

$$W(\boldsymbol{c}(t)) = \mathbb{E}[\max_{l_h} u_h(c_{l_h}(t), l_h)], \tag{29}$$

where $\mathbb{E}$ is a population-weighted cross-sectional average and $c_{l_h}(t)$ is consumption in location l_h at t. Although the function in (29) is sometimes called "expected utility," the

[32] The nonlinear definition of $A^{MFP}(t)$ is $\log A^{MFP}(t) = \log Y(t) - \int_0^t \sum_r \lambda_r(s)(d \log L_r/ds)ds$.

expectation is over preference parameters not lotteries, so this is a misleading label.[33]

The consumption-equivalent variation for the utilitarian social welfare function W is a scalar $A^W(t)$ that solves:

$$W(\boldsymbol{c}(t)/A^W(t)) = W(\boldsymbol{c}(0)),$$

Unlike the TFP-equivalent aggregate welfare measure $A(t)$, the utilitarian welfare measure $A^W(t)$ is not invariant to increasing transformations of individual utility functions. In particular, its value is not pinned down by any observables. To see this, recall that

$$u_h(c_h, l) = f_h\big(g(c_h) + \epsilon_{hl}\big),$$

where f_h is any arbitrary strictly increasing function. The function f_h has no testable implications for households' choices: any choice of f_h represents the same underlying preference relation $\succeq_h$. Nevertheless, changing f_h has important implications for $A^W(t)$ as the following proposition demonstrates.

Proposition 8 (Arbitrariness of average utility)**.** *Hold fixed households' ordinal preferences* $\{\succeq_h\}$ *and hence their individual compensating variations* $A_h(t)$ *defined in* (12). *Then* $A^W(t)$ *can be made equal to any value in the interval*

$$\left(\min_{h\in H} A_h(t), \max_{h\in H} A_h(t)\right)$$

by varying the cardinal utility representations $\{f_h\}$.

This proposition shows that $A^W(t)$ can take any value in the range spanned by individual compensating variations, depending on the choice of $\{f_h\}$.[34]

One might imagine that if we restrict ourselves to a priori "reasonable" transformations that are symmetric across households, then $A^W(t)$ may be pinned down, but this is not true. The following example uses the popular isoelastic location choice specification to show that alternative seemingly reasonable and symmetric choices on f_h cause $A^W(t)$

[33] Expected utility is formally defined to be a representation of an ordinal preference relation over lotteries of allocations (see, e.g., chapter 6, of Mas-Colell et al., 1995, and Footnote 13 for a definition in the context of our model) not an expectation of utility values across different agents. In this paper, each household has fixed preferences u_h and there is no lottery across household tastes. This is to say, no household ever makes a choice about the parameters of their utility function, which means preferences over these parameters are undefined.

[34] Under mild conditions—satisfied, for example, in the standard Fréchet tastes case—the lower and upper bounds in Proposition 8 are the smallest and largest proportional changes in regional real wages between the equilibria at 0 and t.

to move in unpredictable ways. Of course, all choices of f_h are observationally equivalent models and consistent with the same underlying preference relations.

Example 6 (Arbitrariness of average utility under Fréchet tastes). Consider an economy like in Example 4 with a single good and homogeneous skills. Agent h has utility function

$$u_h(c_r, r) = f_h(\log(c_r) + \epsilon_{hr}),$$

where ϵ_{hr} are independent type-I extreme value random variables with parameters θ and B_r, and f_h is an arbitrarily chosen increasing function. Typically, the literature assumes that $f_h(x) = \exp(x)$ so that

$$u_h(c_r, r) = c_r \varepsilon_{hr},$$

where $\varepsilon_{hr} = \exp(\epsilon_{hr})$ is distributed according to a Fréchet distribution. Under this cardinalization, as shown by Redding and Rossi-Hansberg (2017), $A^W(t)$ can be written in closed form as

$$A^W(t) = \left[\frac{\sum_r B_r c_r^\theta(t)}{\sum_r B_r c_r^\theta(0)}\right]^{1/\theta}. \tag{30}$$

This is a widely used measure of aggregate welfare in spatial models.[35,36]

Now consider a different cardinalization of utility:

$$u_h(c_r, r) = c_r \frac{\varepsilon_{hr}}{\max_{r'} \varepsilon_{hr'}}.$$

This has no implication for choices, but normalizes each agent's vector of location-specific tastes so that its highest value is one and all other values are below one.[37] That is, each agent assigns a taste of unity to their favorite location and a value below one to every other location. One can argue that this is a more "reasonable" cardinalization of utility because it does not allow for the possibility that some agent draws high tastes in every location and some agent draws low tastes in every location. Under this cardinalization, however, $A^W(t)$ does not obey (30).

[35] An odd feature of this cardinalization is that average utility cannot be calculated when the mobility elasticity θ is less than or equal to one. In this case, $W(\boldsymbol{c})$ diverges because the Fréchet taste parameters ε_{hr} are sufficiently fat-tailed that their expectation is infinite. In contrast, our measure of welfare is well-defined for any value of θ.

[36] If one restricts the set of admissible monotone transformations, then the common Fréchet specification has an invariance property. However, these restrictions are arbitrary from the standpoint of observed choices; see Appendix D.

[37] Formally, this cardinalization sets $f_h(x) = \exp(x)/(\max_{r'} \varepsilon_{hr'})$.

To see this difference, we use the same calibrated economy as in Example 5. Table 2 quantifies the difference between A^W and A for different cardinalizations f_h. We report A^W under four observationally equivalent choices of the cardinal utility representations $\{f_h\}$. The first is the commonly used $u_h(c_r, r) = c_r \varepsilon_{hr}$, labelled as A_1^W. The second rescales utility by the maximum taste parameter $u_h(c_r, r) = c_r \varepsilon_{hr} / (\max_r \varepsilon_{hr})$, and is labelled A_2^W. The third and fourth use $u_h(c_r, r) = (c_r \varepsilon_{hr} / (\max_r \varepsilon_{hr}))^2$ and $u_h(c_r, r) = \sqrt{(c_r \varepsilon_{hr} / (\max_r \varepsilon_{hr}))}$, and are labelled A_3^W and A_4^W respectively. All these specifications are observationally equivalent but imply very different welfare gains according to the average utility metric. More generally, Proposition 8 shows that the arbitrary choice of cardinalization can make $A^W(t)$ take any value between the smallest and largest individual compensating variations.

Table 2: Welfare effects of a 1 percent productivity increase in selected commuting zones

Location of shock	$\Delta \log A$	$\Delta \log A_1^W$	$\Delta \log A_2^W$	$\Delta \log A_3^W$	$\Delta \log A_4^W$
San Francisco–Oakland, CA	0.031	0.018	0.026	0.039	0.022
New York, NY	0.057	0.034	0.049	0.071	0.041
Seattle–Tacoma, WA	0.024	0.017	0.022	0.028	0.019
Brownsville, TX	0.001	0.001	0.001	0.000	0.001
McAllen, TX	0.002	0.004	0.003	0.002	0.003
Daytona Beach, FL	0.001	0.002	0.001	0.001	0.002

Notes: Entries are elasticities with respect to the regional productivity increase. The columns $\Delta \log A_i^W$ report consumption-equivalent utilitarian welfare under observationally equivalent cardinalizations of utility. A_1^W uses $u_h(c_r, r) = c_r \varepsilon_{hr}$. The remaining specifications normalize tastes by $\max_{r'}\{\varepsilon_{hr'}\}$. A_2^W uses $u_h(c_r, r) = c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\}$, while A_3^W and A_4^W square and take the square root of this expression, respectively. All four specifications imply identical household choices and equilibrium observables. Results use $\theta = 1.5$ and 400,000 simulated households.

The following example considers optimal place-based policies chosen by a policymaker who maximizes utilitarian welfare $W(\boldsymbol{c})$. We show that the policy maker intervenes in a Pareto-efficient equilibrium and chooses Pareto-inefficient policies. Furthermore, the direction and magnitude of these policies depend on the chosen cardinalization of individual utilities.

Example 7 (Optimal policy for average utility). Consider again the one-good economy in Example 6. Suppose that the policymaker chooses place-based consumption levels to maximize a utilitarian social welfare function

$$\max_{\boldsymbol{c}} \qquad \mathbb{E}[u_h(c_{l_h}(t), l_h)]$$

subject to feasibility,

$$\sum_r L_r c_r = \sum_r L_r z_r,$$

and household location choice,

$$l_h \in \arg\max_r [\varepsilon_{hr} c_r].$$

These location-choice conditions imply implementability constraints of the form:

$$L_r = \frac{B_r c_r^\theta}{\sum_{r'} B_{r'} c_{r'}^\theta}.$$

Under the common assumption that $u_h(c_r, r) = c_r \varepsilon_{hr}$, the optimal c_r is a convex combination of productivity in region r and average consumption:

$$c_r = \frac{\theta}{\theta+1} z_r + \frac{1}{\theta+1}\left(\sum_k L_k c_k\right). \qquad (31)$$

See the appendix for the derivation. This policy redistributes consumption from high-productivity to low-productivity regions.

However, other observationally equivalent choices of the cardinal utility representations $\{f_h\}$, all of which generate the same mapping from wages to labor allocations, imply different optimal place-based policies. Using the calibrated economy from Examples 5 and 6, Table 3 displays the labor allocation induced by the optimal place-based policy under the four observationally equivalent utility representations described in Example 6. The optimal allocations differ substantially across these cardinalizations, even though all four are consistent with the same individual choice behavior.

Moreover, whereas the decentralized equilibrium in the first column is Pareto efficient, the "optimal" allocations chosen by the policymaker are Pareto inefficient. Equivalently, starting from those optimal allocations, $A(t) > 1$: there exists another feasible allocation, implemented with lump-sum transfers, that makes every household strictly better off. Thus, the utilitarian planner sacrifices aggregate efficiency to pursue redistributive goals that depend on an arbitrary cardinalization of utility.

Table 3: Labor-supply allocation implied by optimal place-based policies

Commuting zone	L_r^{initial}	$L_r^{W,1}$	$L_r^{W,2}$	$L_r^{W,3}$	$L_r^{W,4}$
San Francisco–Oakland, CA	0.018	0.013	0.016	0.024	0.014
New York, NY	0.034	0.025	0.031	0.047	0.028
Seattle–Tacoma, WA	0.017	0.014	0.016	0.022	0.014
Brownsville, TX	0.001	0.002	0.001	0.001	0.002
McAllen, TX	0.004	0.005	0.004	0.003	0.005
Daytona Beach, FL	0.002	0.003	0.002	0.001	0.002

Notes: The first column reports the initial labor share. The remaining columns report the labor shares that maximize the corresponding utilitarian social welfare functions defined in the notes to Table 2.

6.3 Sum of Compensating Variations / Kaldor-Hicks Efficiency

In partial equilibrium contexts, where there is an outside good, sometimes called "money", a common measure of aggregate welfare is the sum of compensating variations (Small and Rosen, 1981). This measure, also known as the Kaldor-Hicks measure of efficiency, can be defined, using our notation, as:

$$S(t) = -\int e_h(\boldsymbol{w}(t), \boldsymbol{p}(t), u_h^0) dh,$$

where e_h is the net expenditure function — the transfer h needs to attain u_h^0, given prices and wages. Note that if h prefers $(\boldsymbol{w}(t), \boldsymbol{p}(t))$ to status quo prices and wages, then e_h is a negative number. So, the scalar $S(t)$ measures the amount of money left, in terms of the numeraire, after winners exactly compensate the losers, holding prices and wages constant at t.[38]

We discuss this measure in more detail in Appendix I. This measure is very similar to $A(t)$ as long as prices and wages are exogenously given (e.g. as in the one-good economy example). However, $S(t)$ has undesirable properties when wages and prices are endogenous. In particular, $S(t)$ can be strictly positive in response to pure redistributions that move the economy along the Pareto frontier. Intuitively, this is because redistributions cause prices to fall for goods consumed primarily by losers relative to winners. Hence, at post-shock prices, the winners can compensate the losers and have money left over, which implies that $S(t) > 0$. See Appendix I for a worked-out example.[39]

[38]An unimportant difference between $S(t)$ and $A(t)$ is that $S(t)$ is measured in dollars, whereas $A(t)$ is unit-free, being defined as a ratio. In Appendix I we use an alternative, ratio-based version of Kaldor-Hicks efficiency — defined as total income at t relative to total income required to compensate households at those prices — and show that the conclusions of this section are unchanged.

[39]This phenomenon also occurs in general equilibrium models with continuous choice, where it is known

7 Application: Removing the Interstate Highway System

In this section, we illustrate the difference between TFP-equivalent aggregate welfare, A, and the various measures of aggregate welfare using a real-world application. We revisit the counterfactual studied in Allen and Arkolakis (2014): the aggregate welfare effect of removing the Interstate Highway System in the United States. In Allen and Arkolakis (2014), households have homogeneous preferences. We extend their homogeneous-household, perfect-mobility model to allow for heterogeneous location preferences and a finite labor-supply elasticity.[40]

Sketch of model. Each location, r, produces a differentiated good using labor with productivity z_r. Goods from r' sold to r are subject to iceberg cost $\tau_{rr'}$. Consumers in each location have CES preferences over goods from different origins with elasticity of substitution σ. The consumer price index in location r and the expenditure share in location r on goods produced in region r' are

$$p_r = \left[\sum_{r'} \left(\frac{\tau_{rr'} w_{r'}}{z_{r'}}\right)^{1-\sigma}\right]^{1/(1-\sigma)}, \qquad \lambda_{rr'} = \frac{\left(\tau_{rr'} w_{r'}/z_{r'}\right)^{1-\sigma}}{\sum_{r''}\left(\tau_{rr''} w_{r''}/z_{r''}\right)^{1-\sigma}}.$$

Productivity and amenities depend on regional population:

$$z_r = \bar{z}_r L_r^{\alpha}, \qquad b_r = \bar{b}_r L_r^{\beta},$$

where α and β govern productivity and amenity spillovers; negative values of these parameters capture congestion.[41]

We introduce heterogeneous tastes using the Fréchet preferences in Example 4 and Example 6, where θ denotes the labor-supply elasticity and $B_r = b_r^{\theta}$ is the Fréchet scale parameter associated with location r. Labor supply in the decentralized equilibrium, normalizing aggregate population to one, satisfies

$$L_r = \frac{\bar{B}_r L_r^{\theta\beta} (w_r/p_r)^{\theta}}{\sum_{r'} \bar{B}_{r'} L_{r'}^{\theta\beta} (w_{r'}/p_{r'})^{\theta}}, \qquad \bar{B}_r \equiv \bar{b}_r^{\theta}. \tag{32}$$

as the Boadway (1974) paradox. See Baqaee and Burstein (2025) for more information in a continuous choice setting.

[40]We discuss the special case where households have homogeneous preferences in Appendix C. In particular, in the original Allen and Arkolakis (2014) model, which has homogeneous preferences, $A(t) = A^W(t)$.

[41]When either α or β is nonzero, then the model features externalities and is technically not a special case of the perfectly competitive benchmark model in Section 2. However, in Appendix G we show how to extend the analysis to models with distortions and externalities.

Calibration. We follow the calibration strategy in Allen and Arkolakis (2014). The model has 3,109 counties in the continental United States ($R = 3,109$), and the Armington elasticity is $\sigma = 9$. We consider two specifications of the externality parameters: $\alpha = \beta = 0$ and the Allen–Arkolakis values, $\alpha = 0.1$ and $\beta = -0.3$. Relative to Allen and Arkolakis (2014), our model allows the labor-supply elasticity to be finite; for illustration, we set $\theta = 3$. For each specification, we calibrate $\bar{z}_r$ and the location-specific taste parameters $\bar{B}_r$ so that the model exactly reproduces the same initial equilibrium.

Quantitative exercise. We repeat the counterfactual exercise in Allen and Arkolakis (2014). We replace the baseline bilateral trade-cost matrix with a no-Interstate-highway-system version of this matrix. This matrix is constructed by removing the speed advantage associated with interstate highways while leaving the remaining road, rail, water, and air networks unchanged.

Table 4: Aggregate welfare effects of removing the Interstate Highway System

	No externalities $\alpha = \beta = 0$	With externalities $\alpha = 0.1,\ \beta = -0.3$
Panel A.		
TFP-equivalent welfare, A	-1.563	-1.612
Panel B.		
Consumption-equivalent for average utility, A^W		
For the normalized specifications, define $x_{hr} \equiv c_r \varepsilon_{hr} / \max_{r'}\{\varepsilon_{hr'}\}$.		
$u_h(c_r, r) = c_r \varepsilon_{hr}$	-1.659	-1.679
$u_h(c_r, r) = x_{hr}$	-1.424	-1.527
$u_h(c_r, r) = x_{hr}^{1/2}$	-1.544	-1.605
$u_h(c_r, r) = x_{hr}^2$	-1.150	-1.350
$u_h(c_r, r) = \exp(x_{hr})$	-0.638	-1.022
$u_h(c_r, r) = -\exp(-x_{hr})$	-1.676	-1.690
Minimum consumption-equivalent change	-11.490	-14.002
Maximum consumption-equivalent change	2.814	4.280

Notes: All entries are 100 times log changes between the baseline and the no-highway economy. All calculations use $\theta = 3$ and $\sigma = 9$.

Results. The results are shown in Table 4. Panel A reports the change in TFP-equivalent aggregate welfare, $A(t)$, which falls by around 1.6% both with and without externalities.[42] Panel B reports the consumption equivalent for average utility, A^W, under alternative cardinal representations of the same ordinal preferences. This panel shows that $A^W(t)$ can vary substantially across different cardinalizations, all of which are consistent with identical household choice behavior and observables. The first row, which has a welfare loss of around 1.7%, uses the common cardinalization $u_h(c_r, r) = c_r \varepsilon_{hr}$. The second row normalizes each household's taste vector by $\max_{r'}\{\varepsilon_{hr'}\}$, so that each household's taste for their most-preferred location is one. This yields a welfare loss between 1.4% and 1.5%. The fact that $A(t)$ lies between the values implied by these two cardinalizations is a numerical accident rather than a theoretical result. The remaining rows consider different strictly increasing transformations. The resulting aggregate welfare number moves around in erratic ways depending on the arbitrary transformation used. The panel also reports the maximum and minimum attainable values for $A^W(t)$ established by Proposition 8, showing that the bounds are quite wide, going from very negative (-11% and -14%) to positive (2.8% and 4.3%). The data provide no discipline on which of these numbers should be preferred.

Although not reported in the table, we also compute other measures for comparison. For example, real output falls by 1.2% or 1.3% depending on the externality parameters. The loss is smaller than $A(t)$ because, in response to removing the Interstate Highway System, agents tend to move to higher-wage locations. For completeness, we also compute TFP-equivalent aggregate welfare when compensation is restricted to place-based instruments, denoted by $A^{pb}(t)$ and defined in Appendix H. In this application, $A^{pb}(t)$ and $A(t)$ are very similar. The appendix shows that the two measures are first-order equivalent in the absence of externalities.

8 Conclusion

We generalize the cost-benefit approach of Harberger (1971) and Small and Rosen (1981) to measure aggregate welfare in general equilibrium environments with discrete choice. Our measure converts shocks into a welfare-equivalent change in total factor productivity.

We show that, to a first-order approximation, aggregate welfare in discrete-choice economies obeys a Hulten-type Domar-weighted formula, and we provide second-order corrections expressed in terms of uncompensated supply and demand elasticities. We contrast our welfare measure with chain-weighted real output, average utility, and the sum of compensating variations (Kaldor-Hicks efficiency), and show that these popular

[42] If we instead set $\theta = \infty$, then we recover the Allen-Arkolakis results. In this case, aggregate welfare declines by 1.1% without externalities and 1.6% with externalities.

alternatives have serious flaws. Real output can fall when every household is better off. Rankings based on average utility depend on arbitrary cardinalizations of individual utilities and can recommend Pareto-inefficient place-based policies that move the economy away from efficiency in unintuitive ways. Kaldor-Hicks efficiency can rise in response to pure redistributions that only move the economy along the Pareto frontier, attributing "efficiency gains" to changes that are purely distributional. In contrast, our measure only rises when Pareto improvements are possible, is invariant to monotone transformations of utility, and stays constant in response to pure redistributions.

Our benchmark environment is perfectly competitive, Pareto-efficient, and features lump-sum transfers that can be used for compensation. The appendices generalize this basic approach to settings with distortions, externalities, and more restricted redistributive tools. A natural next step is to use our measure of aggregate welfare A as the objective function in optimal policy problems in these settings.

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Appendix

Appendix A Proofs of propositions and theorems in the body of the paper

Proof of Proposition 1. The expenditure function is

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min_{T_h, c_h, l_h} \{T_h : u_h(c_h, l_h) \geq u_h^0, \text{ and } \sum_r p_r c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h\}.$$

Since utility is increasing in consumption, the weak inequalities above have to be equalities. The first constraint implies that:

$$u_h(c_h, l_h) = g(\bar{c}_{hl_h}) + \epsilon_{hl_h} = u_h^0$$

Hence,

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min_{T_h, l_h} \{T_h : \sum_r p_r \bar{c}_{hr} \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h\}.$$

The weak inequality above is strict at an optimum point, so substituting out for T_h yields

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = -\max_{l_h} \{\sum_r a_{hr} w_r \mathbf{1}[l_h = r] - \sum_r p_r \bar{c}_{hl_h} \mathbf{1}[l_h = r]\}.$$

The maximizer of this problem is the compensated choice $l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$. □

Proof of Theorem 1. Recall that each decentralized equilibrium is associated with a vector of lump-sum transfers to each agent, $\boldsymbol{T}$. TFP-equivalent welfare solves

$$A(t) = \max_{\boldsymbol{T}, Z} \left\{ Z^{-1} : \{c_h, l_h\}_{h \in H} \in \mathcal{C}(t, Z) \text{ and } (c_h, l_h) \succeq_h (c_h(0), l_h(0)) \text{ for every } h \right\},$$

where the choice of transfers determines the equilibrium allocation. Let $\boldsymbol{T}^*$ be a maximizer of the problem above. The indifference condition of each household implies that:

$$u_h(c_h(\boldsymbol{T}^*), l_h(\boldsymbol{T}^*)) = u_h^0.$$

Hence,

$$c_h(\boldsymbol{T}^*) = u_h^{-1}\left(u_h^0;\, l_h(\boldsymbol{T}^*)\right) \equiv \bar{c}_{hl_h(\boldsymbol{T}^*)},$$

where $u_h^{-1}(\cdot;\, l_h)$ denotes the inverse of $u_h(c, l_h)$ with respect to its first argument, holding l_h fixed.[43] In the compensated equilibrium with wages $\boldsymbol{w}$, prices $\boldsymbol{p}$, and $Z = 1/A$,

$$\frac{\frac{a_{hl_h(\boldsymbol{T}^*)} w_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)}{A} + T_h^*}{p_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)} = \bar{c}_{hl_h(\boldsymbol{T}^*)}.$$

Rearrange this to get that the optimal compensating transfers satisfies:

$$T_h^* = p_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*) \bar{c}_{hl_h(\boldsymbol{T}^*)} - \frac{a_{hl_h(\boldsymbol{T}^*)} w_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)}{A}$$

[43] If we impose that preferences are ordinally additively separable (not required for this proof),

$$g(c_h(\boldsymbol{T}^*)) + \epsilon_{hl_h(\boldsymbol{T}^*)} = u_h^0,$$

so

$$c_h(\boldsymbol{T}^*) = g^{-1}\left(u_h^0 - \epsilon_{hl_h(\boldsymbol{T}^*)}\right) \equiv \bar{c}_{hl_h(\boldsymbol{T}^*)}.$$

The sum of these transfers must add up to zero, which implies that

$$\sum_h \left[p_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)\bar{c}_{hl_h(\boldsymbol{T}^*)} - \frac{a_{hl_h(\boldsymbol{T}^*)} w_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)}{A} \right] = 0.$$

Rearranging this gives an equation that A has to satisfy given the optimal $\boldsymbol{T}^*$:

$$A = \frac{\sum_h a_{hl_h(\boldsymbol{T}^*)} w_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)}{\sum_h p_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)\bar{c}_{hl_h(\boldsymbol{T}^*)}}.$$

We know by definition that

$$l_h(\boldsymbol{T}^*) = l_h^{\text{comp}}\left(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), u_h^0\right)$$

and that

$$T_h^* = e_h(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), u_h^0).$$

Using the fact that

$$L_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0) = \sum_h a_{hr} \mathbf{1}[l_h^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), u_h^0) = r],$$

and the fact that

$$\begin{aligned}\sum_h p_{l_h(\boldsymbol{T}^*)}(\boldsymbol{T}^*)\bar{c}_{hl_h(\boldsymbol{T}^*)} &= \frac{w_r}{A} L_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0) + \sum_h e_h(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), u_h^0)\mathbf{1}\left[l_h^{\text{comp}} = r\right] \\ &= E_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0),\end{aligned}$$

we can write

$$E_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0) = \sum_r \frac{w_r(\boldsymbol{T}^*)}{A} L_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0)$$

where $\boldsymbol{w}(\boldsymbol{T}^*)$ and $\boldsymbol{p}(\boldsymbol{T}^*)$ are prices and wages in the compensated equilibrium given compensated location choices $L_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0)$ and compensated final demand $E_r^{\text{comp}}(\boldsymbol{w}(\boldsymbol{T}^*)/A, \boldsymbol{p}(\boldsymbol{T}^*), \boldsymbol{u}^0)$ and total factor productivity $1/A$. □

Proof of Theorem 2. Define the expenditure function to be

$$e_h(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h) = \min_{T_h} \left\{ T_h : \max_r g\left(\frac{\frac{w_r a_{hr}}{A} + T_h}{p_r}\right) + \epsilon_{hr} \geq u_h^0 \right\}$$
$$= T_h + \mu_h \left[\max_r \left\{ g\left(\frac{\frac{w_r a_{hr}}{A} + T_h}{p_r}\right) + \epsilon_{hr} \right\} - u_h^0 \right].$$

Hence,

$$\frac{\partial e_h}{\partial u_h^0} = -\mu_h.$$

Define the indirect utility function of the agent to be

$$v_h(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, T_h) = \max \left\{ u_h(c_h, l_h) : p_{l_h} c_h = \frac{w_{l_h} a_{hl_h}}{A} + T_h \right\}.$$

Then, by the envelope theorem, we have

$$\frac{\partial v_h}{\partial T_h} = g'\left(\frac{\frac{w_r a_{hl_h}}{A} + T_h}{p_{l_h}}\right) \frac{1}{p_{l_h}}.$$

For households that switch location, the derivative above needs to be evaluated from the appropriate direction, see Milgrom and Segal (2002). However, this issue will not play an important role in the proof, because the set of households that are exactly indifferent between two locations is measure zero. We also have the identity that

$$e_h(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, v_h(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, T_h)) = T_h.$$

Differentiating this identity gives

$$\frac{\partial e_h}{\partial u_h} \frac{\partial v_h}{\partial T_h} = 1.$$

Combining equations, we have that

$$-\mu_h = \frac{\partial e_h}{\partial u_h} = \left[\frac{\partial v_h}{\partial T_h} \right]^{-1} = \frac{p_{l_h}}{g'\left(\frac{\frac{a_{hl_h} w_{l_h}}{A} + T_h}{p_{l_h}}\right)}.$$

We know that A satisfies the feasibility condition:

$$\sum_h e_h(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0) = 0. \tag{33}$$

For households that are not on the margin of switching locations, the expenditure function is totally differentiable in t. We totally differentiate in t and write $\boldsymbol{w}$ and $\boldsymbol{p}$ for the wages and prices in the compensated equilibrium without explicitly writing the superscript comp on every variable. The reason is that in this proof, we never refer to the decentralized equilibrium — only to the compensated one. We have

$$\begin{aligned}
\frac{\partial e_h(\boldsymbol{w}/A,\boldsymbol{p},u_h)}{\partial\left(w_i/A\right)} &= a_{hl_h}\mu_h g'\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)\mathbf{1}\left[l_h\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)=i\right]\frac{1}{p_{l_h}} \\
&= -a_{hl_h}\frac{p_{l_h}}{g'\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)}\frac{1}{p_{l_h}}g'\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)\mathbf{1}\left[l_h\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)=i\right] \\
&= -a_{hl_h}\mathbf{1}\left[l_h\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)=i\right],
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial e_h(\boldsymbol{w}/A,\boldsymbol{p},u_h)}{\partial p_i} &= -\mu_h g'\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)\mathbf{1}\left[l_h\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)=i\right]\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\frac{1}{p_{l_h}} \\
&= \frac{p_{l_h}}{g'\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)}g'\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)\mathbf{1}\left[l_h\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)=i\right]\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\frac{1}{p_{l_h}} \\
&= \frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\mathbf{1}\left[l_h\left(\frac{\frac{a_{hl_h}w_{l_h}}{A}+T_h}{p_{l_h}}\right)=i\right]
\end{aligned}$$

Total differentiating (33),

$$\sum_r\sum_h\frac{\partial e_h(\frac{\boldsymbol{w}}{A},\boldsymbol{p},u_h^0)}{\partial\left[w_r/A\right]}d\left[\frac{w_r}{A}\right]+\sum_r\sum_h\frac{\partial e_h(\frac{\boldsymbol{w}}{A},\boldsymbol{p},u_h^0)}{\partial p_r}dp_r=0.$$

Substituting the expressions above,

$$
\begin{aligned}
\sum_r \sum_h a_{hl_h} \mathbf{1}\left[l_h\left(\frac{\frac{a_{hl_h} w_{l_h}}{A}+T_h}{p_{l_h}}\right)=r\right] d\left[\frac{w_r}{A}\right] &= \sum_r \sum_h \frac{\frac{a_{hl_h} w_{l_h}}{A}+T_h}{p_{l_h}} \mathbf{1}\left[l_h\left(\frac{\frac{a_{hl_h} w_{l_h}}{A}+T_h}{p_{l_h}}\right)=r\right] dp_r \\
\sum_r L_r^{\text{comp}} d\left[\frac{w_r}{A}\right] &= \sum_r \sum_h \frac{\frac{a_{hl_h} w_{l_h}}{A}+T_h}{p_{l_h}} \mathbf{1}\left[l_h\left(\frac{\frac{a_{hl_h} w_{l_h}}{A}+T_h}{p_{l_h}}\right)=r\right] dp_r \\
\sum_r L_r^{\text{comp}} d\left[\frac{w_r}{A}\right] &= \sum_r d\log p_r \sum_h \left[\frac{a_{hl_h} w_{l_h}}{A}+T_h\right] \mathbf{1}\left[l_h\left(\frac{\frac{a_{hl_h} w_{l_h}}{A}+T_h}{p_{l_h}}\right)=r\right] \\
\sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d\log\left[\frac{w_r}{A}\right] &= \sum_r E_r^{\text{comp}} d\log p_r \qquad (34) \\
\sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d\log[A] &= \sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d\log[w_r] - \sum_r E_r^{\text{comp}} d\log p_r \\
d\log A &= \sum_r \frac{L_r^{\text{comp}}\left[\frac{w_r}{A}\right]}{\sum_r L_r^{\text{comp}}\left[\frac{w_r}{A}\right]} d\log[w_r] - \sum_r \frac{E_r^{\text{comp}}}{\sum_r L_r^{\text{comp}}\left[\frac{w_r}{A}\right]} d\log p_r \\
d\log A &= \sum_r \lambda_r d\log w_r - \sum_r \chi_r d\log p_r.
\end{aligned}
$$

Shephard's lemma for producer r implies that

$$d\log p_i = -d\log z_i + \sum_j \Omega_{ij} d\log p_j + \sum_r \Omega_{ir} d\log w_r,$$

where w_r is the wage per efficiency unit of labor of type r in the compensated equilibrium. Inverting this system of equations yields

$$d\log p = -\Psi d\log z + \Psi^F d\log w,$$

where $\Psi = (I-\Omega)^{-1}$ is the Leontief inverse and Ψ^F is the matrix of factor contents. Substitute this into the expression for $d\log A$ to get

$$
\begin{aligned}
d\log A &= \sum_r \lambda_r d\log w_r - \sum_r \chi_r \left[-\Psi d\log z + \Psi^F d\log w\right] \\
&= \sum_r \lambda_r d\log w_r + \sum_r \chi_r \Psi d\log z - \sum_r \chi_r \Psi^F d\log w \\
&= \sum_r \lambda_r d\log w_r + \sum_{i\in N} \lambda_i d\log z_i - \sum_r \lambda_r d\log w_r \\
&= \sum_{i\in N} \lambda_i d\log z_i.
\end{aligned}
$$

By the fundamental theorem of calculus,

$$\log A = \int \sum_{i \in N} \lambda_i d \log z_i,$$

as needed.

□

Proof of Proposition 2. To get (17), log differentiate input demand for CES. To get (18) use Shephard's lemma, constant-returns-to-scale, and the fact that price equals marginal cost. To get (19) log differentiate the definition of λ_r for $r \in R$. To get (20), log differentiate (6) (the equation is the same regardless of whether i is a factor or a good). All these equations are equilibrium conditions that must hold in both the decentralized economy and the compensated equilibrium. □

Proof of Proposition 3. The first part follows from:

$$\begin{aligned} L_r^{\text{comp}}(\boldsymbol{w}/A, p^c, \boldsymbol{u}) &= \int a_{hr} \left[r \in \arg\max_i \left\{ a_{hi} \frac{w_i}{A p^c} + \frac{e_h(\boldsymbol{w}/A, p^c, u_h)}{p^c} + \epsilon_{hi} \right\} \right] \\ &= \int a_{hr} \left[r \in \arg\max_i \left\{ a_{hi} \frac{w_i}{A p^c} + \epsilon_{hi} \right\} \right] \\ &= L(\boldsymbol{w}/A, p^c) = L(\boldsymbol{w}/(A p^c), 1), \end{aligned}$$

showing that the compensated and uncompensated supply systems have the same functional form.

Equation (21) is just the total derivative of L_r^{comp} with respect to t. This pins down the equilibrium in conjunction with equations (16) to (20). The only unknown terms are the ones involving χ^{comp} in (20). However, since $\Omega_{ri} = \Omega_{r'i}$ for every r and r' in R (common consumption good), the terms involving χ^{comp} drop out of (20). In particular,

$$\sum_r d\chi_r \Omega_{ri} + \sum_r \chi_r d\Omega_{ri} = d\Omega_{ri},$$

since we know that $\sum_r d\chi_r = 0$ and $\sum_r \chi_r = 1$. □

Proof of Proposition 4. For the uncompensated equilibrium, the result follows easily from log-differentiating the definition:

$$\chi_r = \frac{w_r L_r}{\sum_{r'} w_{r'} L_{r'}}.$$

Now consider the change in the spending shares by location, $d\chi^{\text{comp}}$,in the compensated equilibrium. To keep the notation more manageable, we do not include the superscript comp on every variable and simply note that everything is in the compensated equilibrium.

Let $T(\epsilon, t)$ be the compensating transfer at t received by an agent with tastes ϵ. Let $B_r(t)$ denote the set of ϵ that choose location r in the compensated equilibrium. Note that it is defined by

$$B_r(t) = \left\{ \epsilon : g(\frac{\frac{w_r(t)}{A(t)} + T(\epsilon,t)}{p_r(t)}) + \epsilon_r - \max\left\{ g(\frac{\frac{w_{r'}(t)}{A(t)} + T(\epsilon,t)}{p_{r'}(t)}) + \epsilon_{r'} \right\} \leq 0 \right\} = \left\{\epsilon : \phi_r(\epsilon, t) \leq 0\right\}.$$

Households on the margin of choosing r are defined by $\phi(\epsilon, t) = 0$. Then the change in share of expenditures in each location satisfies:

$$\begin{aligned} d\log\chi_r &= d\log\int [p_r\bar{c}_{hr}]\,\mathbf{1}\,[l(h)=r]\,dh - \underbrace{d\log\sum_{r'}\int [p_r\bar{c}_{hr}]\,\mathbf{1}\,[l(h)=r']\,dh}_{\equiv X}, \\ &= \frac{1}{\lambda_r}d\int [p_r\bar{c}_{hr}]\,\mathbf{1}\,[l(h)=r]\,dh - X \\ &= \frac{1}{\lambda_r}d\left[\int_{B_r(t)} [p_r\bar{c}_r(\epsilon)]\,f(\epsilon)d\epsilon\right] - X, \end{aligned}$$

where $\bar{c}_r(\epsilon)$ is the consumption-equivalent and $f(\epsilon)$ is the density of households with tastes ϵ. Using the fact that $p_r\bar{c}_r(\epsilon) = \frac{w_r}{A} + T(\epsilon, t)$, we can write

$$\begin{aligned} d\log\chi_r &= \frac{1}{\lambda_r}d\left[\int_{B_r}\left[\frac{w_r(t)}{A(t)} + T(\epsilon,t)\right]f(\epsilon)d\epsilon\right] - X \\ &= \frac{1}{\lambda_r}\int_{B_r}\left[d\frac{w_r}{A} + \frac{\partial}{\partial t}T(\epsilon,t)\right]f(\epsilon)d\epsilon + \frac{1}{\lambda_r}\int\left[\frac{w_r(t)}{A(t)} + T(\epsilon,t)\right]\delta(\phi(z,\epsilon))f(\epsilon)\|\nabla_\epsilon\phi(z,\epsilon)\|d\epsilon - X \end{aligned}$$

where the second line uses Leibniz' rule. At the status quo, where we differentiate, we have $\frac{w_r(0)}{A(0)} + T(\epsilon, 0) = w_r(0)$. Dropping time subscripts, we can write

$$d\log\chi_r = \frac{1}{\lambda_r}\int_{B_r}\left[d\frac{w_r}{A} + \frac{\partial}{\partial t}T(\epsilon,t)\right]f(\epsilon)d\epsilon + \frac{1}{\lambda_r}\int w_r\delta(\phi(z,\epsilon))f(\epsilon)\|\nabla_\epsilon\phi(z,\epsilon)\|d\epsilon - X.$$

The first summand is the change in the compensating income of each household in location r and the second summand is the wage in location r times the mass of households that move to location r in the compensated equilibrium. Hence, we can write

$$\begin{aligned}
d\log\chi_r &= \frac{1}{\lambda_r}\left[\int_{B_r}\left[d\frac{w_r}{A}+dT(\epsilon)\right]f(\epsilon)d\epsilon\right]+\frac{1}{\lambda_r}\left[w_r dL_r\right]-X \\
&= \frac{1}{\lambda_r}\left[\frac{w_r}{p_r}dp_r\int_{B_r}f(\epsilon)d\epsilon\right]+\frac{1}{\lambda_r}\left[w_r dL_r\right]-X \\
&= \frac{1}{\lambda_r}\left[\frac{w_r}{p_r}dp_r\right]L_r+\frac{1}{\lambda_r}\left[w_r dL_r\right]-X \\
&= \frac{1}{\lambda_r}\left[\frac{L_r w_r}{p_r}dp_r\right]+\frac{1}{\lambda_r}\left[w_r dL_r\right]-X \\
&= d\log p_r+\frac{1}{\lambda_r}dL_r w_r-X \\
&= d\log p_r+d\log L_r-X.
\end{aligned}$$

By definition, we must have

$$\mathbb{E}_\chi\left[d\log\chi\right]=0,$$

hence,

$$X=\mathbb{E}_\chi\left[d\log p_r+d\log L_r\right].$$

This implies that, in the compensated equilibrium, we have

$$d\log\chi_r=\left[d\log p_r+d\log L_r\right]-\mathbb{E}_\chi\left[d\log p_r+d\log L_r\right].$$

□

Proof of Proposition 5. We start with the uncompensated equilibrium and then consider the compensated equilibrium.

Uncompensated Equilibrium. Let $B_i(t)$ denote the set of ϵ that choose i:

$$\begin{aligned}
B_i(t) &= \left\{\epsilon : g\left(\frac{w_i(t)}{p_i(t)}\right)+\epsilon_i \geq g\left(\frac{w_k(t)}{p_k(t)}\right)+\epsilon_k, \forall\, k\neq i\right\} \\
&= \left\{\epsilon : \phi_i(t,\epsilon)\leq 0\right\},
\end{aligned}$$

where

$$\phi_i(t,\epsilon)=\max_k\left\{g\left(\frac{w_k(t)}{p_k(t)}\right)+\epsilon_k-g\left(\frac{w_i(t)}{p_i(t)}\right)-\epsilon_i\right\}.$$

The set of movers from i to j is given by

$$\begin{aligned}
L_{i\to j} &= \int \mathbf{1}(\epsilon \in B_i(0))\mathbf{1}(\epsilon \in B_j(t))f(\epsilon)d\epsilon \\
&= \int_{B_j(t)} \mathbf{1}(\epsilon \in B_i(0))f(\epsilon)d\epsilon. \\
dL_{i\to j} &= d\left[\int_{B_j(t)} \mathbf{1}(\epsilon \in B_i(0))f(\epsilon)d\epsilon\right] \\
&= \left[\int_{dB_j(t)} \mathbf{1}(\epsilon \in B_i(0))f(\epsilon)d\epsilon\right] \\
&= \left[\int \mathbf{1}(\epsilon \in B_i(0))\delta(\phi_j(0,\epsilon))\|\nabla_\epsilon \phi_j(0,\epsilon)\|f(\epsilon)d\epsilon\right],
\end{aligned}$$

where δ is the Dirac delta function which means the integral is evaluated at the boundary of $B_j(0)$, that is where $\phi_j(0,\epsilon) = 0$ and $\mathbf{1}(\epsilon \in B_i(0))$. For any vector $\boldsymbol{x}$, let $h(\boldsymbol{x})$ be the density of households with $\epsilon_j - \epsilon_i = x_i - x_j$ and $\epsilon_j - \epsilon_k \geq x_k - x_j$ and $\epsilon_i - \epsilon_k \geq x_k - x_i$. This the density of households that are indifferent between i and j (and prefer i and j to all other options) given payoffs are $x_i = g(c_i)$. Define $\epsilon_{ij} = x_i - x_j$ to be the cut-off between i and j. Then we can write

$$\begin{aligned}
dL_{i\to j} &= -h(\boldsymbol{w}(0)/\boldsymbol{p}(0))d\epsilon_{ij}\mathbf{1}\left[d\epsilon_{ij} \leq 0\right] \\
&= -h(g(\boldsymbol{w}(0)/\boldsymbol{p}(0))\left[g'(\frac{w_i(0)}{p_i(0)})d\left[\frac{w_i}{p_i}\right] - g'(\frac{w_j(0)}{p_j(0)})d\left[\frac{w_j}{p_j}\right]\right]\mathbf{1}\left[d\epsilon_{ij} \leq 0\right]. \qquad (35)
\end{aligned}$$

We relate this equation to cross-derivatives of the supply function $\boldsymbol{L}(\boldsymbol{p}, \boldsymbol{w}, \boldsymbol{T})$. In particular, note that in the decentralized equilibrium, $\boldsymbol{T} = 0$, the supply function depends only on the vector of real wages $\boldsymbol{w}/\boldsymbol{p}$. Furthermore, consider a perturbation where only the real wage in location j changes (all other real wages are held constant), then from the equation above, we can write

$$\frac{\partial L_{i\to j}}{\partial(w_j/p_j)} = h(g(\boldsymbol{w}(0)/\boldsymbol{p}(0))g'(w_j(0)/p_j(0)).$$

Since

$$\frac{\partial L_i}{\partial(w_j/p_j)} = \sum_{k\neq i}\frac{\partial L_{k\to i}}{\partial(w_j/p_j)} - \sum_{k\neq i}\frac{\partial L_{i\to k}}{\partial(w_j/p_j)}$$
$$= -\frac{\partial L_{i\to j}}{\partial(w_j/p_j)}$$
$$= -h(g(\boldsymbol{w}(0)/\boldsymbol{p}(0))g'(w_j(0)/p_j(0)), \tag{36}$$

where the second line follows from equation (35). Using these equations, we can write

$$dL_{i\to j} = -h(g(\boldsymbol{w}(0)/\boldsymbol{p}(0))\left[g'(w_i(0)/p_i(0))d\left[\frac{w_i}{p_i}\right] - g'(w_j(0)/p_j(0))d\left[\frac{w_j}{p_j}\right]\right]\mathbf{1}\left[d\epsilon_{ij}\leq 0\right]$$
$$= \left[\frac{\partial L_j}{\partial(w_i/p_i)}d\left[\frac{w_i}{p_i}\right] - \frac{\partial L_i}{\partial(w_j/p_j)}d\left[\frac{w_j}{p_j}\right]\right]\mathbf{1}\left[d\epsilon_{ij}\leq 0\right]$$
$$= \left[\frac{\partial L_j}{\partial(w_i/p_i)}d\left[\frac{w_i}{p_i}\right] - \frac{\partial L_i}{\partial(w_j/p_j)}d\left[\frac{w_j}{p_j}\right]\right]\mathbf{1}\left[h(g(\boldsymbol{w}(0)/\boldsymbol{p}(0))d\epsilon_{ij}\leq 0\right]$$
$$= \left[\frac{\partial L_j}{\partial(w_i/p_i)}d\left[\frac{w_i}{p_i}\right] - \frac{\partial L_i}{\partial(w_j/p_j)}d\left[\frac{w_j}{p_j}\right]\right]\mathbf{1}\left[\left[\frac{\partial L_j}{\partial(w_i/p_i)}d\left[\frac{w_i}{p_i}\right] - \frac{\partial L_i}{\partial(w_j/p_j)}d\left[\frac{w_j}{p_j}\right]\right]\geq 0\right],$$

which is the expression in the proposition (swapping i and j for r and r').

Compensated Equilibrium. We now consider the compensated equilibrium. To show how location choices change, to a first order, let $T(\epsilon, t)$ be the compensating transfer in the compensated equilibrium for households with preferences ϵ. The set of households that are marginal movers between region i and j in the compensated equilibrium is

$$F_{ij}(t) = \left\{\epsilon : g(\frac{w_i(t)+T(\epsilon,t)}{p_i(t)}) + \epsilon_i = g(\frac{w_j(t)+T(\epsilon,t)}{p_j(t)}) + \epsilon_j \geq g(\frac{w_k(t)+T(\epsilon,t)}{p_k(t)}) + \epsilon_k, \forall\, k\notin\{j,i\}\right\}.$$

For every $\epsilon\in F_{ij}(t)$, the following equation holds:

$$\epsilon_{ij}(t) = \epsilon_{hj} - \epsilon_{hi} = g(\frac{w_i(t)/A(t)+T(\epsilon,t)}{p_i(t)}) - g(\frac{w_j(t)/A+T(\epsilon,t)}{p_j(t)}).$$

We call $\epsilon_{ij}(t)$ the cut-off values for i and j (this is different to the cut-off values in the uncompensated decentralized equilibrium because of the compensating transfers). In the status quo, transfers are zero, so the cutoff between regions i and j is

$$\epsilon_{ij}^0 = g(\frac{w_i^0}{p_i^0}) - g(\frac{w_j^0}{p_j^0}).$$

Differentiating the expression for $\epsilon_{ij}(t)$ with respect to t and suppressing dependence on t gives:

$$d\epsilon_{ij} = g'(\frac{w_i/A + T(\epsilon)}{p_i})d\left[\frac{w_i/A + T(\epsilon)}{p_i}\right] - g'(\frac{w_j/A + T(\epsilon)}{p_j})d\left[\frac{w_j/A + T(\epsilon)}{p_j}\right].$$

At the status quo, since $A = 1$ and $T(\epsilon, 0) = 0$, we have

$$d\epsilon_{ij} = g'(\frac{w_i}{p_i})\left[d\left[w_i/Ap_i\right] + \frac{1}{p_i}dT(\epsilon)\right] - g'(\frac{w_j}{p_j})d\left[d\left[w_j/Ap_j\right] + \frac{1}{p_j}dT(\epsilon)\right].$$

Note that

$$T(\epsilon, t) = e(\frac{\boldsymbol{w}(t)}{A(t)}, \boldsymbol{p}(t), u_h^0(\epsilon), \epsilon),$$

where the fourth summand captures how the expenditure function varies as a function of tastes, holding wages, prices, and utility constant. Hence, at the status quo, totally differentiating with respect to t yields:

$$dT(\epsilon) = e_1(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0, \epsilon)\cdot d\left[\frac{\boldsymbol{w}}{A}\right] + e_2(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0, \epsilon)\cdot d\boldsymbol{p} + \left[e_3(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0, \epsilon)\frac{du_h^0(\epsilon)}{d\epsilon} + e_4(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0, \epsilon)\right]d\epsilon,$$

where $\epsilon \in F_{ij}(t)$ and $d\epsilon$ is evaluated for some path that remains in $F_{ij}(z, t)$ as t changes. However, in the status quo, $T(\epsilon, 0) = 0$. Hence, moving in the cross-section of households in the status quo, we have

$$e_3(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0, \epsilon)\frac{du_h^0}{d\epsilon} + e_4(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0(\epsilon), \epsilon) = 0,$$

which means we can ignore change in the transfers due to changes in the identity of the marginal household, and the expression above simplifes to just

$$dT(\epsilon) = e_1(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0(\epsilon), \epsilon)\cdot d\left[\frac{\boldsymbol{w}}{A}\right] + e_2(\frac{\boldsymbol{w}}{A}, \boldsymbol{p}, u_h^0(\epsilon), \epsilon)\cdot d\boldsymbol{p},$$

where ϵ is evaluated for some household that's marginal between i and j. Using the envelope theorem,

$$dT(\epsilon) = \lim_{t \to 0+} \left[\sum_{i \in R} \mathbf{1} \left[l_h(\frac{\frac{w_r}{A} + T(\epsilon_{ij})}{p_r}) = i \right] \left(-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \right]$$
$$= \lim_{t \to 0+} \left[\sum_{i \in R} \mathbf{1} \left[l_h(\frac{w_r}{p_r}) = i \right] \left(-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \right],$$

where $\epsilon \in F_{ij}(t)$. In particular, at the status quo, the change in compensating transfers dT is the same for every household that is marginal between choosing i and j regardless of the other values ϵ_{hk}. If $d\epsilon_{ij} < 0$ then households move from i to j in the compensated equilibrium, else the reverse. Denote the change in compensating transfers for these marginal households by the following symbol:

$$dT_{ij} = \lim_{t \to 0+} \left[\sum_{i \in R} \mathbf{1} \left[l_h(\frac{w_r}{p_r}) = i \right] \left(-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \right]$$

Hence, we can write

$$dT_{ij} = \left(-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \mathbf{1} \left[d\epsilon_{ij} < 0 \right] + \left(-d \left[\frac{w_j}{A} \right] + \frac{w_j}{p_j} dp_j \right) \mathbf{1} \left[d\epsilon_{ij} \geq 0 \right]$$

Substitute this into the expression for $d\epsilon_{ij}$, in the case where $d\epsilon_{ij} < 0$, so households move from i to j, we get

$$d\epsilon_{ij} = g'(\frac{w_i}{p_i}) \left[d \left[w_i / Ap_i \right] + \frac{1}{p_i} \left[-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \right] -$$
$$g'(\frac{w_j}{p_j}) \left[d \left[w_j / Ap_j \right] + \frac{1}{p_j} \left(-d \left[\frac{w_i}{A} \right] + \frac{w_i}{p_i} dp_i \right) \right].$$

Use the fact that

$$d \left[w_i / \left(Ap_i \right) \right] + \left[-d \left[w_i / A \right] + \frac{w_i}{p_i} dp_i \right] / p_i = 0$$

to get

$$d\epsilon_{ij} = -g'(\frac{w_j}{p_j}) \left[d \left[w_j / Ap_j \right] - \frac{p_i}{p_j} d \left[w_i / \left(Ap_i \right) \right] \right]$$

which requires that $p_j d\left[w_j/Ap_j\right] > p_i d\left[w_i/\left(Ap_i\right)\right]$. Similarly, if $d\epsilon_{ij} > 0$, we instead get

$$d\epsilon_{ij} = g'(\frac{w_i}{p_i})\left[d\left[w_j/Ap_j\right] - \frac{p_i}{p_j}d\left[w_i/\left(Ap_i\right)\right]\right]$$

The mass of movers from i to j is then given by

$$\begin{aligned}
L_{i\to j} &= \int \mathbf{1}\left[l_h(\boldsymbol{p}(0),\boldsymbol{w}(0),\mathbf{0}) = i\right]\mathbf{1}\left[l_h(\boldsymbol{p}(t),\boldsymbol{w}(t)/A(t),T_h(t)) = j\right]dh. \\
dL_{i\to j} &= -h(\epsilon_{ij}^0)d\epsilon_{ij}\mathbf{1}\left[d\epsilon_{ij} < 0\right] \\
&= h(\epsilon_{ij}^0)g'(\frac{w_j}{p_j})\left[d\left[w_j/Ap_j\right] - \frac{p_i}{p_j}d\left[w_i/\left(Ap_i\right)\right]\right]\mathbf{1}\left[p_j d\left[w_j/Ap_j\right] > p_i d\left[w_i/\left(Ap_i\right)\right]\right],
\end{aligned}$$

where $h(\epsilon_{ij}^0)$ is the density of households for whom $\epsilon \in F_{ij}(0)$.

Note that in the uncompensated economy, in response to the perturbation $d\left[w_j/p_j\right] > 0$ — holding all other real wages fixed — (36) implies that

$$-\frac{\partial L_i}{\partial\left[w_j/p_j\right]}d\left[w_j/p_j\right] = h(\epsilon_{ij}^0)g'(w_j/p_j)d\left[w_j/p_j\right].$$

Hence, we can replace

$$-\frac{\partial L_i}{\partial\left[w_j/p_j\right]} = h(\epsilon_{ij}^0)g'(w_j/p_j). \tag{37}$$

Using this we can rewrite

$$\begin{aligned}
dL_{i\to j} &= h(\epsilon_{ij}^0)g'(\frac{w_j}{p_j})\left[d\left[w_j/Ap_j\right] - \frac{p_i}{p_j}d\left[w_i/\left(Ap_i\right)\right]\right]\mathbf{1}\left[p_j d\left[w_j/Ap_j\right] > p_i d\left[w_i/\left(Ap_i\right)\right]\right] \\
&= -\frac{\partial L_i}{\partial\left[w_j/p_j\right]}\frac{1}{p_j}\left[p_j d\left[w_j/Ap_j\right] - p_i d\left[w_i/\left(Ap_i\right)\right]\right]\mathbf{1}\left[p_j d\left[w_j/Ap_j\right] > p_i d\left[w_i/\left(Ap_i\right)\right]\right].
\end{aligned}$$

The change in the compensated labor supply of location i then follows from the equation that:

$$dL_i = \sum_{j\neq i} dL_{j\to i} - \sum_{j\neq i} dL_{i\to j}.$$

□

Proof of Proposition 6. By Hulten (1978), to first-order, we have:

$$\Delta \log Y \approx \sum_i \lambda_i(0)\Delta\log z_i + \sum_{r\in R}\lambda_r(0)\Delta\log L_r.$$

We then use the fact that to a first-order,

$$\lambda_r(0)\Delta \log L_r = \frac{w_r(0)}{\sum_{r'} w_{r'}(0) L_{r'}(0)} \Delta L_r.$$

□

Proof of Proposition 7. From Hulten's theorem, we know that

$$\log A^{\text{MFP}}(t) = \int_0^t \sum_i \lambda_i(s) \frac{d \log z_i}{ds} ds$$

whereas from Theorem 1, we know that

$$\log A(t) = \int_0^t \sum_i \lambda_i^{\text{comp}}(s) \frac{d \log z_i}{ds} ds.$$

Note that, since $\epsilon_{hr} = \bar{\epsilon}$ for every h and r, and there is a common consumption good with the law of one price, we have that

$$l_h^{\text{comp}}(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = l_h(\boldsymbol{w}, \boldsymbol{p}).$$

Furthermore, since there is only a common consumption good, Domar weights do not depend on the distribution of spending (i.e. $\chi(t)$ and $\chi^{\text{comp}}(t)$ result in the same Domar weights). Hence

$$\lambda_i(s) = \lambda_i^{\text{comp}}(s),$$

which implies that

$$A(t) = A^{MFP}(t).$$

Next, since there is only one common consumption good, real output in this economy is just the total production/consumption of that good: $Y(t) = \sum_h c_h(t) / \sum_h c_h(0)$. Since households only value the consumption good, the first welfare theorem implies that the decentralized equilibrium at t maximizes the quantity of the consumption good produced at t. By Hulten's theorem, we know that

$$\frac{d \log Y}{dt} = \sum_{i \in N} \lambda_i(t) \frac{d \log z_i}{dt} + \sum_{r \in R} \lambda_r(t) \frac{d \log L_r}{dt},$$

where the second term is a reallocation effect. However, this term must equal zero, otherwise, the equilibrium at t would not be maximizing Y. Hence,

$$\frac{d\log Y(t)}{dt} = \frac{d\log A^{MFP}(t)}{dt}$$

at every t. The result follows by integrating both sides. □

Proof of Proposition 8. Fix any b strictly between the endpoints of the cross-sectional support of $A_h(t)$, and define

$$d_h(b) \equiv \max_r u_h\left(\frac{a_{hr}w_r(t)}{bp_r(t)}, r\right) - u_h^0.$$

By the definition of $A_h(t)$ and monotonicity in consumption, $d_h(b) > 0$ if $A_h(t) > b$, $d_h(b) < 0$ if $A_h(t) < b$, and $d_h(b) = 0$ otherwise. Let

$$D_+ \equiv \mathbb{E}[d_h(b)\mathbf{1}\{A_h(t) > b\}], \qquad D_- \equiv -\mathbb{E}[d_h(b)\mathbf{1}\{A_h(t) < b\}],$$

which are both positive. Replace each function f_h with

$$\widetilde{f}_h(x) = \begin{cases} D_- f_h(x), & A_h(t) > b, \\ D_+ f_h(x), & A_h(t) < b, \\ f_h(x), & A_h(t) = b, \end{cases} \qquad \text{for every } x.$$

These positive rescalings leave preferences and $A_h(t)$ unchanged. Substituting the individual utility functions $\widetilde{f}_h(g(c_h) + \epsilon_{hl})$ into the definition of $A^W(t)$ shows that $A^W(t) = b$. Because b was arbitrarily chosen, every value in the stated interval can be attained, including values arbitrarily close to either endpoint. □

Appendix B Implications of $g(\cdot)$ function for behavior

Since utility is only pinned down up to monotone transformations, the function f_h has no observable implications. However, the shape of $g(c_h)$ has testable implications, since it controls the way income and substitution effects interact with each other. For example, if $g(c)$ is linear, then household choices are invariant to an additive increase in consumption in every location (regardless of the distribution of tastes). On the other hand, if $g(c)$ is log, then household choices are invariant to scaling consumption in every location (regardless of the distribution of tastes).

The following proposition shows that the conditional labor supply function $\boldsymbol{L}(\boldsymbol{w}|\boldsymbol{a})$, mapping vectors of real wages into labor supply in each location conditional on skills $\boldsymbol{a}$, uniquely pins down $g(c_h)$ up to an affine transformation.

Proposition 9 (Relation between L and g)**.** *Let the efficiency units of labor supplied by workers given real wages $\boldsymbol{w}$ conditional on skill type $\boldsymbol{a}$ to be:*

$$L_i(\boldsymbol{w}|\boldsymbol{a}) = \int a_i \mathbf{1}\left[g(w_i a_i) + \epsilon_{hi} \geq g(a_j w_j) + \epsilon_{hj} \quad \forall j\right] dh.$$

Given knowledge of $\boldsymbol{L}(\boldsymbol{w}|\boldsymbol{a})$, the function $g(c)$ is pinned down up to an affine transformation. A notable implication is the following. The function $\boldsymbol{L}(\boldsymbol{w}|\boldsymbol{a})$ has symmetric cross-derivatives in real wages if, and only if, $g(c)$ is affine.

That is, the functional form of $g(c_h)$ has testable implications. The linear case is noteworthy because, under this assumption, some of our calculations dramatically simplify. Intuitively, if $g(c_h)$ is linear, then a lump-sum transfer (in consumption units) to household h will not change household h's choice of location.[44]

Proof. Suppose that preferences take the form

$$u_h(c, r) = g(c) + \epsilon_{hr}.$$

Consider the set of $\epsilon' s$ marginal between i and j that also do not strictly prefer any other region:

$$F_{ij}(\boldsymbol{w}|\boldsymbol{a}) = \{\epsilon : g(a_i w_i) - g(a_j w_j) = \epsilon_{hj} - \epsilon_{hi}, g(a_i w_i) - g(a_k w_k) \geq \epsilon_{hk} - \epsilon_{hi}, \forall k \neq i\}.$$

Define the set that prefer region i by $B_i(\boldsymbol{w}|\boldsymbol{a})$. We can write

$$L_i(\boldsymbol{w}|\boldsymbol{a}) = \int a_i \mathbf{1}\left[\epsilon \in B_i(\boldsymbol{w}|\boldsymbol{a})\right] f(\epsilon) d\epsilon,$$

where $f(\epsilon)$ is the density of ϵ. A marginal change in w_j lowers the share in region i by:

$$\frac{\partial L_i(\boldsymbol{w}|\boldsymbol{a})}{\partial w_j} = -a_i f(g(a_i w_i) - g(a_j w_j)) a_j g'(a_j w_j).$$

[44]As another example, the population share function without transfers $\boldsymbol{L}(\boldsymbol{p}, \boldsymbol{w}, \mathbf{0})$ has symmetric cross semi-elasticities in real wages $\partial L_r / \partial \log w_{r'} / p_{r'} = \partial L_{r'} / \partial \log w_r / p_r$ if, and only if, $g(c_h)$ is a log function of c_h. The commonly used constant-elasticity Fréchet supply system is a special case and requires that $g(c_h)$ be log.

Similarly

$$\frac{\partial L_j(\boldsymbol{w}|\boldsymbol{a})}{\partial w_i} = -a_j f(g(a_i w_i) - g(a_j w_j)) a_i g'(a_i w_i).$$

Hence,

$$\frac{\frac{\partial L_i(\boldsymbol{w}|\boldsymbol{a})}{\partial w_j}}{\frac{\partial L_j(\boldsymbol{w}|\boldsymbol{a})}{\partial w_i}} = \frac{g'(a_j w_j)}{g'(a_i w_i)}.$$

Define the function

$$g_{ij}(w_i, w_j) = \frac{\partial L_i(\boldsymbol{w}|\boldsymbol{a})/\partial w_j}{\partial L_j(\boldsymbol{w}|\boldsymbol{a})/\partial w_i} = \frac{g'(a_j w_j)}{g'(a_i w_i)}.$$

Hence,

$$g'(a_j w_j) = g_{ij}(w_i, w_j) g'(a_i w_i).$$

By the fundamental theorem of calculus we have

$$g(w_j) - g(1) = g'(a_i w_i) \int_{1/a_j}^{w_j/a_j} g_{ij}(w_i, x) dx.$$

Specifically, at $w_i = 1/a_i$, we get

$$\begin{aligned} g(w_j) - g(1) &= g'(1) \int_{1/a_j}^{w_j/a_j} g_{ij}(1, x) dx \\ \frac{g(w_j) - g(1)}{g'(1)} &= \int_{1/a_j}^{w_j/a_j} g_{ij}(1, x) dx = G_{ij}(w_j/a_j) - G_{ij}(1/a_j), \end{aligned}$$

where $G_{ij}(x)$ is the antiderivative of the function $g_{ij}(1, x)$. Hence, the function g is pinned down up to an affine transformation. □

Appendix C Special case with homogeneous agents

In this appendix, we consider an instructive special case where agents are homogeneous in preferences and skill levels ($\succeq_h$ does not vary by h and $a_{hr} = 1$ for every h and r). Locations may nevertheless differ in their common amenity values. For concreteness, if preferences are ordinally additively separable, then we can represent every $\succeq_h$ using:

$$u_h(c_h, l_h) = f_h\left(g(c_h) + \epsilon_{l_h}\right),$$

where ϵ_r parameterizes the common amenity value of location r. Characterizing $A(t)$ in this special case is as easy as solving the decentralized equilibrium.

Proposition 10 (Aggregate Welfare with Homogeneous Agents)**.** *Suppose $\succeq_h$ does not vary by h and $a_{hr} = 1$ for every h and r. Define $c_r(\boldsymbol{z}, Z)$ and $L_r(\boldsymbol{z}, Z)$ to be consumption per capita and labor supply in location r in a decentralized equilibrium given productivity vector $\boldsymbol{z}$ and aggregate factor-augmenting productivity Z. Then $A(t)$ solves*

$$c_r(\boldsymbol{z}(0), 1) = c_r(\boldsymbol{z}(t), \frac{1}{A(t)}), \tag{38}$$

for any r with both $L_r(\boldsymbol{z}(t), 1/A(t)) > 0$ and $L_r(\boldsymbol{z}(0), 1) > 0$.

If, in addition, $g(c_h) = \log(c_h)$, then $A(t)$ is equal to the change in per capita consumption (i.e. the change in the real wage):

$$A(t) = \frac{c_r(\boldsymbol{z}(t), 1)}{c_r(\boldsymbol{z}(0), 1)}, \tag{39}$$

in any region r that is non-empty at both the status quo and at t.

Before presenting the proof, we provide some intuition. Since agents are homogeneous, in a decentralized equilibrium, agents are indifferent between different location choices (unless a location is empty). If we scale aggregate factor-augmenting productivity by $1/A(t)$, every agent is affected in the same way — there are no relative winners or losers — and lump-sum transfers are not needed for compensations. If a location r is non-empty in the status quo and in the allocation that solves $A(t)$, indifference to the status quo requires that $A(t)$ satisfies

$$f_h\big(g(c_r(\boldsymbol{z}(t), 1/A(t))) + \epsilon_r\big) = f_h\big(g(c_r(\boldsymbol{z}(0), 1)) + \epsilon_r\big).$$

The functions f_h and g and scalar ϵ_r disappear from both sides, leading to (38). In words, $A(t)$ must be such that real consumption per capita in each location r is equated to its status quo value. This shows that computing $A(t)$ is as simple as computing the decentralized equilibrium with different productivity parameters (i.e. we do not have to worry about compensating transfers when agents are homogeneous).

In addition, if $g(c) = \log(c)$, then scaling consumption in every location by the same amount leaves the location choice problem of every agent unchanged. Since agents' location choice problems do not respond to a uniform scaling of consumption, they also do not respond to a uniform scaling of factor-augmenting productivity (for more details, see the proof in the appendix). Hence, $c_r(\boldsymbol{z}(t), 1/A(t)) = c_r(\boldsymbol{z}(t), 1)/A(t)$ and (39) follows. In words, if $g(c) = \log(c)$, then $A(t)$ can be computed purely from the growth in real consumption in any location in the decentralized equilibrium: $A(t) = c_r(\boldsymbol{z}(t), 1)/c_r(\boldsymbol{z}(0), 1)$ — one does not even need to solve a counterfactual equilibrium with different factor-augmenting productivity.

One may wonder: why not always use the growth in real consumption to measure the growth in aggregate welfare? The reason is that outside of the $g(c) = \log(c)$, the growth in real consumption in each location, $c_r(\boldsymbol{z}(t), 1)/c_r(\boldsymbol{z}(0), 1)$, can be a different number, even though all agents are homogeneous in terms of welfare.[45] Our measure of aggregate welfare, $A(t)$, in contrast provides a single number.[46]

Proof of Proposition 10. Define $u_h(\boldsymbol{z}, Z)$ to be the utility of household h in a decentralized equilibrium given productivity vector $\boldsymbol{z}$ and aggregate factor augmenting productivity Z. Since in equilibrium all households have the same utility, irrespective of their chosen location, there are no lump-sum transfers in the compensated equilibrium. Hence, to calculate $A(t)$, we simply need to solve

$$u_h(\boldsymbol{z}(t), 1/A(t)) = u_h(\boldsymbol{z}(0), 1).$$

For any regions r such that $L_r(\boldsymbol{z}(t), 1/A(t)) \times L_r(\boldsymbol{z}(0), 1) > 0$ we have that

$$g(c_r(\boldsymbol{z}(t), 1/A(t))) + \epsilon_r = g(c_r(\boldsymbol{z}(0), 1)) + \epsilon_r.$$

It follows that

$$c_r(\boldsymbol{z}(t), 1/A(t)) = c_r(\boldsymbol{z}(0), 1).$$

This completes the first part of the proposition.

Consider the case where $g(c) = \log c$. We need to show that

$$c_r\big(\boldsymbol{z}(t), 1/A(t)\big) = A(t)^{-1}\, c_r\big(\boldsymbol{z}(t), 1\big).$$

Let factor-augmenting productivity Z change from 1 to $1/A(t)$ while holding $\boldsymbol{z}(t)$ fixed. Conjecture an equilibrium allocation that scales all factor inputs (in efficiency units), outputs, and intermediate inputs by $1/A(t)$, keeping prices and wages (per efficiency unit) unchanged. In this conjectured equilibrium, consumption in every location scales with

[45] To see this, note that indifference between locations implies that $g(c_r) - g(c_{r'}) = \epsilon_r - \epsilon_{r'}$ must always hold. Hence, $g(c_r(\boldsymbol{z}(t), 1)) - g(c_r(\boldsymbol{z}(0), 1)) = g(c_{r'}(\boldsymbol{z}(t), 1)) - g(c_{r'}(\boldsymbol{z}(0), 1))$. This does not, in general, imply equal growth rates for real consumption per capita across locations unless $g(c) = \log(c)$.

[46] If $g(c) = \log(c)$, then one can also calculate $A(t)$ using a specific cardinalization of utility. In particular, if we use $f_h(x) = \exp(x)$ so that $u_h(c, r) = c \exp(\epsilon_r)$, then $A(t)$ is equal to the ratio of utils in the decentralized equilibrium at t relative to status-quo. This is a common approach in spatial models with homogeneous preferences (see e.g. Allen and Arkolakis, 2014). This trick does not work if $g(c)$ is not logarithmic. However, one cannot simply assume that $g(c)$ is logarithmic without loss of generality: if ϵ_r varies by location, then the shape of $g(c)$ is determined by ordinal properties of the underlying agent-level utility functions (see Appendix B).

$1/A(t)$. With logarithmic g, if consumption in every location is scaled by a common factor, then agents do not have any incentive to switch locations. If households do not switch locations, then labor supply (in efficiency units) in each location scales by $1/A(t)$:

$$L_r(\boldsymbol{z}, 1/A(t)) = A(t)^{-1} L_r(\boldsymbol{z}, 1).$$

Scaling labor supply (in efficiency units) in every location by $1/A(t)$ makes it feasible to scale up all quantities by $1/A(t)$ (due to constant returns to scale) while keeping all relative prices constant. This confirms the existence of this conjectured equilibrium, which completes the proof.

□

Appendix D Additional results and examples

In this appendix we present multi-industry examples in Example 8 and Example 9. We derive optimal place-based policy in Example 7. We also present an invariance result about average utility with Fréchet-distributed tastes.

D.1 Multi-industry examples

We provide two examples with multiple industries.

Example 8 (Cobb-Douglas and Logit Example). Every good is produced using a Cobb-Douglas production technology, so (5) implies that the price of each good $i \in N$ can be written as

$$p_i = z_i^{-1} \prod_{j \in N} p_j^{\Omega_{ij}} \prod_{r \in R} w_r^{\Omega_{ir}},$$

where Ω_{ij} and Ω_{ir} are expenditure shares of i on j and r respectively (Cobb-Douglas implies that these expenditure shares are constant). We can solve out for λ_r in the market clearing condition, (6), to get that for every factor r:

$$\lambda_r = \sum_{r'} \chi_r \Psi_{c(r')r},$$

where $\Psi = (I - \Omega)^{-1}$ is the Leontief inverse (which, again, is constant due to the Cobb-Douglas assumption). This equation states that income of r must equal the dollar-weighted average factor content of final consumption $\sum_{r' \in R} \chi_{r'} \Psi_{c(r')r}$, where $\Psi_{c(r')r}$ is the total factor r content of consumption by agents in location r'.

Suppose that all consumers consume the same consumption good, so that the factor content Ψ_r is the same for every r'. The previous equation simplifies to

$$\lambda_r = \frac{w_r L_r}{\sum_{r''} w_{r''} L_{r''}} = \Psi_{0r}, \tag{40}$$

where $c(r') = 0$ is the index for the common consumption good and the right-hand side is a constant, depending only on the Cobb-Douglas share parameters. That is, we use the fact that $\sum_{r'} \chi_r \Psi_{c(r')r} = \sum_{r'} \chi_r \Psi_{0r} = \Psi_{0r}$.

Suppose skills are homogeneous, $a_{hr} = 1$ and normalize $Z = 1$. Utility functions are given by $u_h(c_h, l_h) = f_h(c_h + \epsilon_{hr} \mathbf{1}[l_h = r])$ where ϵ_{hr} are drawn from type I extreme value distribution and f_h is any strictly increasing function. The labor supply function is

$$L_r(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T}) = \frac{\exp(\theta w_r / p^c + B_r)}{\sum_{r'} \exp(\theta w_{r'} / p^c + B_{r'})}. \tag{41}$$

where p^c is the price of the final consumption good, and θ and B_r are parameters of the distribution of ϵ_{hr}. Given this functional form, equation (40) can be rewritten as

$$\frac{w_r \exp(\theta w_r / p^c + B_r)}{\sum_{r'} w_{r'} \exp(\theta w_{r'} / p^c + B_{r'})} = \Psi_{0r}$$

where the consumer price index satisfies

$$p^c = \prod_j z_j^{-\Psi_{0j}} \prod_r w_r^{\Psi_{0r}}.$$

These equations pin down all equilibrium prices, up to the choice of numeraire, which can then be used to pin down quantities.

Example 9 (Occupational Choice Example). Suppose that the common consumption good is a CES bundle of outputs from different industries:

$$y = \left(\sum_r \Omega_{0r}^{\frac{1}{\theta_0}} x_{0r}^{\frac{\theta_0 - 1}{\theta_0}} \right)^{\frac{\theta_0}{\theta_0 - 1}},$$

where the units of quantities are chosen so that Ω_{0r} are expenditures in the status quo. Industry r's output is

$$x_{0r} = z_r L_r.$$

Suppose that workers utility functions can be written as

$$u_h(c_h, r) = c_h + \epsilon_{hr},$$

where ϵ_{hr} is type I extreme value. In this case, $L(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$ has the standard logit functional form. This implies that

$$\frac{\partial L_r}{\partial \left[w_{r'}/p^c\right]} = \theta L_r L_{r'}.$$

Substituting this into (21) yields

$$d\log L_i^{\text{comp}} = \theta \left(d\left[w_i/\left(Ap^c\right)\right] - \mathbb{E}_{L^{\text{comp}}}\left[d\left[w_i/\left(Ap^c\right)\right]\right]\right), \tag{42}$$

where the wages and prices are evaluated in the compensated equilibrium. We now apply Proposition 2. The sales share of industry r, in the compensated equilibrium, is given by its share of the wage bill:

$$d\log \lambda_l^{\text{comp}} = d\log w_l + d\log L_l^{\text{comp}} - \mathbb{E}_{\lambda^{\text{comp}}}\left[d\log w + d\log L^{\text{comp}}\right]. \tag{43}$$

At the same time, the sales of share industry r, in the compensated equilibrium, is also given by the share of household spending on industry r:

$$d\log \lambda_l^{\text{comp}} = \left(\theta_0 - 1\right)\left[d\log z_l - d\log w_l - \mathbb{E}_{\lambda^{\text{comp}}}\left[d\log z - d\log w\right]\right], \tag{44}$$

where we use the fact that $d\log p_r = d\log w_r - d\log z_r$. Shephard's lemma implies that the consumer price index in the compensated equilibrium is

$$d\log p^c = \sum_l \lambda_l^{\text{comp}}\left[d\log w_l - d\log z_l\right]. \tag{45}$$

Finally, from Theorem 2, the change in TFP-equivalent aggregate welfare is:

$$d\log A = \sum_i \lambda_i^{\text{comp}} d\log z_i. \tag{46}$$

Equations (42)-(46) form a system of ordinary differential equations that can be solved to obtain $A(t)$ without simulation methods. The boundary conditions are that at $t = 0$, expenditure and population shares coincide with the status quo, and $A(0) = 1$. Solving the system is simple: discretize the productivity shocks, and iterate on the linear system, updating variables each time.

D.2 Deriving optimal place-based policy in Example 7

In this example, there is a measure–1 continuum of agents, R locations, no differences in idiosyncratic skills, and a single good produced with labor in each location. Agent h has utility

$$u_h(c) = \max_{r \in R} \{c_r \bar{\varepsilon}_h \varepsilon_{hr}\},$$

where $\varepsilon_{hr} > 0$ are independent Fréchet random variables with common shape parameter θ and location-specific scale parameters $B_r > 0$. Given a vector of consumption by location, $\{c_r\}$, the supply of labor in location r is

$$L_r = \frac{B_r c_r^{\theta}}{\sum_{r'} B_{r'} c_{r'}^{\theta}}.$$

Under the cardinalization $\bar{\varepsilon}_h = 1$ for all h, the utilitarian social welfare function is

$$W = \int u_h(c)\, dh = \kappa \left[\sum_r B_r c_r^{\theta} \right]^{1/\theta},$$

where $\kappa = \Gamma(1 - 1/\theta)$.

Derivation of (31). Consider the problem of choosing $\{c_r^0\}$ to maximize W, subject to individual rationality, which implies the labor-supply system and the resource constraint

$$\sum_r L_r^0 c_r^0 = \sum_r L_r^0 z_r.$$

Using the labor-supply function, the resource constraint can be written as

$$\sum_r B_r (c_r^0)^{\theta+1} = \sum_r B_r (c_r^0)^{\theta} z_r.$$

Since W is increasing in $\sum_r B_r c_r^\theta$, the Lagrangian is

$$\mathcal{L}(c,\lambda) = \sum_r B_r c_r^\theta + \lambda \left(\sum_r B_r c_r^{\theta+1} - \sum_r B_r c_r^\theta z_r \right).$$

The FOC for c_r is

$$\theta B_r c_r^{\theta-1} + \lambda B_r \left[(\theta+1) c_r^\theta - \theta c_r^{\theta-1} z_r \right] = 0.$$

Dividing by $\theta B_r c_r^{\theta-1} > 0$ gives

$$1 + \frac{\lambda}{\theta} \left[\frac{\theta+1}{\theta} c_r - z_r \right] = 0.$$

Hence $(\theta+1)c_r - \theta z_r$ is constant across locations. Denote this constant by C:

$$c_r^0 = \frac{\theta}{\theta+1} z_r + \frac{C}{\theta+1}.$$

Multiplying by L_r^0 and summing over r, using $\sum_r L_r^0 = 1$ and the resource constraint, gives

$$C = \sum_r L_r^0 c_r^0.$$

Therefore, we obtain (31).

The following proposition shows that, starting at the "optimal" allocation (31), there exists a schedule of lump-sum transfers such that every agent can be made better off. For simplicity, we consider the version of the model with two regions, but the same argument applies in an economy with R regions.

Proposition 11. *Suppose $R = 2$ and $z_1 \neq z_2$. If the status quo is the "optimal" allocation implied by* (31)*, then this allocation is Pareto inefficient and $A > 1$.*

Proof. Let $l_h^0 \in \arg\max_{r=1,2} c_r^0 \varepsilon_{hr}$ denote the initial location choice. If household h is assigned to location r, the consumption required to keep her indifferent to the status quo is

$$\bar{c}_{hr} = c_{l_h^0}^0 \frac{\varepsilon_{hl_h^0}}{\varepsilon_{hr}}.$$

Applying Theorem 1 and using $w_r = z_r$ and $p_r = 1$, aggregate welfare can be written as

$$A = \max_l \frac{\sum_r \int z_r \mathbf{1}[l_h = r] \, dh}{\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h = r] \, dh}.$$

By Dinkelbach's theorem, the solution A satisfies $F(A) = 0$, where

$$F(x) = \int \max_{r=1,2} \{z_r - x\bar{c}_{hr}\}\, dh.$$

We show that $F(1) > 0$, which implies $A > 1$. Define

$$\phi_h(r) = z_r - \bar{c}_{hr}.$$

Under the initial location choice,

$$\phi_h(l_h^0) = z_{l_h^0} - c^0_{l_h^0}.$$

Therefore, using the resource constraint,

$$\int \phi_h(l_h^0)\, dh = \sum_r L_r^0(z_r - c_r^0) = 0.$$

Since $\max_r \phi_h(r) \geq \phi_h(l_h^0)$ for every h, we have $F(1) \geq 0$. It remains to show that the inequality is strict. Without loss of generality, suppose $z_1 > z_2$. From (31),

$$c_1^0 - c_2^0 = \frac{\theta}{\theta+1}(z_1 - z_2) > 0.$$

Consider agents initially in location 2. Initial choice of location 2 over location 1 requires

$$\frac{\varepsilon_{h2}}{\varepsilon_{h1}} \geq \frac{c_1^0}{c_2^0}.$$

For such agents,

$$\phi_h(2) = z_2 - c_2^0, \qquad \phi_h(1) = z_1 - c_2^0\frac{\varepsilon_{h2}}{\varepsilon_{h1}}.$$

Thus $\phi_h(1) > \phi_h(2)$ if and only if

$$\frac{\varepsilon_{h2}}{\varepsilon_{h1}} < 1 + \frac{z_1 - z_2}{c_2^0}.$$

The interval

$$\left[\frac{c_1^0}{c_2^0},\, 1 + \frac{z_1 - z_2}{c_2^0}\right)$$

is nonempty because

$$1+\frac{z_1-z_2}{c_2^0}-\frac{c_1^0}{c_2^0}=\frac{z_1-z_2-(c_1^0-c_2^0)}{c_2^0}=\frac{z_1-z_2}{(\theta+1)c_2^0}>0.$$

Since $(\varepsilon_{h1},\varepsilon_{h2})$ has a continuous positive density, there is a positive-measure set of agents with

$$\frac{\varepsilon_{h2}}{\varepsilon_{h1}}\in\left[\frac{c_1^0}{c_2^0},\,1+\frac{z_1-z_2}{c_2^0}\right).$$

For these agents, $l_h^0=2$ and $\phi_h(1)>\phi_h(2)=\phi_h(l_h^0)$. For all other agents, $\max_r\phi_h(r)\geq\phi_h(l_h^0)$. Therefore,

$$F(1)=\int\max_r\phi_h(r)\,dh>\int\phi_h(l_h^0)\,dh=0.$$

Since $F(x)$ is decreasing in x and satisfies $F(A)=0$, it follows that $A>1$. Hence, starting from the allocation in (31), there exists another feasible allocation, implemented with lump-sum transfers, that makes every household strictly better off. The case $z_2>z_1$ is symmetric. □

D.3 Invariance Property when Tastes are Fréchet

The Fréchet specification of utility does have an invariance property, stated below in Proposition 12. Specifically, when $\{\varepsilon_{hr}\}$ are assumed to be independently Fréchet distributed and $f_h=f$ for every h, then $A^W(t)$ is invariant to the choice of f. This result, however, follows from arbitrarily restricting the set of allowable monotone transformations of utility. For example, under the restrictions in Proposition 12, differences in the overall level of tastes across households are left unnormalized: some households have high values of ε_{hr} in every location, while others have low values in every location. For example, the cardinalizations discussed in Example 6 and Example 7 fall outside this restricted class.

Proposition 12 (Invariance to Restricted Monotone Transformations)**.** *Suppose that*

$$u_h(c_r,r)=f\big(\log c_r+\epsilon_{hr}\big),$$

where the same strictly increasing function f applies to every household and $\varepsilon_{hr}\equiv\exp(\epsilon_{hr})$ are independent Fréchet random variables satisfying

$$\Pr(\varepsilon_{hr}\leq z)=\exp\left(-B_r z^{-\theta}\right),\qquad z>0.$$

For any f for which average utility is finite,

$$A^W(t) = \left[\frac{\sum_r B_r c_r^\theta(t)}{\sum_r B_r c_r^\theta(0)}\right]^{1/\theta},$$

and hence $A^W(t)$ is invariant to the choice of f.

Proof. Let

$$v_h(\boldsymbol{c}) = \max_r\{\log c_r + \epsilon_{hr}\}.$$

Its distribution is

$$\Pr\big(v_h(\boldsymbol{c}) \le v\big) = \exp\left(-e^{-\theta v}\sum_r B_r c_r^\theta\right).$$

Hence,

$$\Pr\left(v_h\left(\frac{\boldsymbol{c}(t)}{A}\right) \le v\right) = \exp\left(-e^{-\theta v}A^{-\theta}\sum_r B_r c_r^\theta(t)\right).$$

This distribution is identical to that of $v_h(\boldsymbol{c}(0))$ when

$$A^{-\theta}\sum_r B_r c_r^\theta(t) = \sum_r B_r c_r^\theta(0).$$

The corresponding average utilities are therefore equal. Since average utility is strictly decreasing in A, this solution is unique. Solving for A gives

$$A^W(t) = \left[\frac{\sum_r B_r c_r^\theta(t)}{\sum_r B_r c_r^\theta(0)}\right]^{1/\theta}.$$

□

Appendix E Relation to the social surplus function

Consider the cardinalization of utility functions $f_h(x) = x$, which results in the following utilitarian social welfare function

$$U(\boldsymbol{c}) = \mathbb{E}[\max_{l_h} g(c_{l_h}) + \epsilon_{h,l_h}].$$

This social welfare function is called the "*Social Surplus Function*" in the discrete choice literature (see McFadden, 1981).[47]

[47] The social surplus function plays an important role in the literature in part because of the Williams-Daly-Zachary theorem (Williams, 1977; Daly and Zachary, 1978), which gives conditions under which the

The proposition below shows that under Assumption 1, the social surplus function can be used to calculate $A(t)$.

Proposition 13 (Using Social Surplus to Calculate A)**.** *Suppose Assumption 1 holds. Let $U(\boldsymbol{w}/p^c) = \mathbb{E}[\max_r a_{hr}w_r/p^c + \epsilon_{hr}]$ be the social surplus function evaluated at real wages $\boldsymbol{w}/p^c$. Then, aggregate welfare in terms of TFP-equivalents, $A(t)$, satisfies the following equation:*

$$U(\boldsymbol{w}(t)/(A(t)p^c(t))) = U(\boldsymbol{w}(0)/p^c(0)),$$

where $\boldsymbol{w}(t)/(A(t)p^c(t))$ are real wages in a decentralized equilibrium with productivities $\boldsymbol{z}(t)$ and aggregate factor-augmenting productivity $1/A(t)$ and $\boldsymbol{w}(0)/p^c(0)$ are real wages in the status quo. If, in addition, real wages do not depend on labor supplied in each region (e.g. as in the one-good economy), then we also have that $A(t) = A^U(t)$, where $A^U(t)$ is the consumption-equivalent variation defined via $U(\boldsymbol{c}(t)/A^U(t)) = U(\boldsymbol{c}(0))$.

The proof is at the end of this subsection. In words, $A(t)$ can be computed as the reduction in labor productivity in every location such that the social surplus function, with productivity shocks $\boldsymbol{z}(t)$, is equal to social surplus under the status quo (without the need to explicitly compute any compensating transfers). Assumption 1 is critical and if it does not hold, then the social surplus function $U(\boldsymbol{c})$ cannot be used to compute $A(t)$ (see Example 11 below).

Notably, even if Assumption 1 holds, $A(t)$ may still differ from the consumption-equivalent under the social surplus function, $A^U(t)$. This is because real wages with productivities $(\boldsymbol{z}(t), 1/A(t))$ are not the same as real wages with productivities $\boldsymbol{z}(t)$ divided by $A(t)$. Intuitively, a reduction in aggregate factor productivity can cause agents to switch locations, and if real wages are a function of labor supply, then this can cause real wages to change and $A(t)$ to be different than $A^U(t)$. The last part of the proposition points out that if, in addition to Assumption 1, real wages do not respond to changes in labor supplies, as in the one-good economy where they are pinned down by productivity parameters, then $A(t)$ coincides with $A^U(t)$.

Proposition 13 relates to a well-known property of discrete choice models where consumers choose among a discrete set of options given fixed prices and incomes. In that literature, it is known that if the indirect utility function is linear in the price of the good, then the social surplus function can be used to calculate the sum of compensating variations (Small and Rosen, 1981).[48]

social surplus function acts like a potential function (i.e. its derivatives are equal to aggregate location choices).

[48] In the absence of linearity, calculating the sum of compensating variations typically requires resorting

The example below illustrates Proposition 13 using a simple example.

Example 10 (Using Social Surplus to Calculate Aggregate Welfare). Consider the one-good economy example from Example 3 again. There is a single freely-traded consumption good produced linearly from labor and productivity in each location r is z_r. We impose Assumptions 1 and 2. The real wage per efficiency unit in each location is simply $w_r = z_r$. Hence, by Proposition 13, we have

$$U(\boldsymbol{z}(t)/A(t)) = \mathbb{E}[\max_r \frac{z_r(t)}{A(t)} + \epsilon_{hr}] = \mathbb{E}[\max_r z_r(0) + \epsilon_{hr}] = U(\boldsymbol{z}(0)).$$

If we assume that ϵ are drawn from a Type I extreme value distribution, then we get

$$U(\boldsymbol{z}(t)/A(t)) = \sum_r \exp(\theta z_r(t)/A(t) + B_r) = \sum_r \exp(\theta z_r(0) + B_r) = U(\boldsymbol{z}(0)),$$

which coincides with equation (23). Furthermore, in this one-good economy, real wages in each location are equal to productivity and independent of labor supply. Hence, scaling all productivities by $1/A(t)$ is observationally equivalent to dividing all real wages by $A(t)$. This makes $A(t) = A^U(t)$.

Proposition 13 breaks down if Assumption 1 does not hold: i.e. if either $g(c)$ is nonlinear or if consumption prices vary by location. Intuitively, when Assumption 1 holds, we have a transferable utility model where the sum of $g(c_h)$ across households is interpretable as an aggregate resource constraint and can be split across households freely using lump-sum transfers. However, if either $g(c)$ is nonlinear or agents in different locations consume different consumption goods, then utility is not transferable and the sum of $g(c_h)$ across households cannot be interpreted as an aggregate resource constraint.

The following example demonstrates that when Assumption 1 does not hold, as in the commonly studied Fréchet case, then $A^U(t)$ and $A(t)$ need not agree even to a first order approximation (and can even have different signs).

to simulation methods. See Hortaçsu and Joo (2023), and the references therein, for a recent textbook discussion. As discussed by Hortaçsu and Joo (2023), many recent studies in the industrial organization literature ignore individual compensating variations and directly use the social surplus function as the starting point of their analysis, even if the quasi-linear justification for this approach does not hold. In this case, the social surplus function is interpreted like a social welfare function. Dubé et al. (2025) analyze the welfare properties of this social surplus function for a broad class of demand systems.

Example 11 (When Proposition 13 Fails). Consider Example 6 again. There is a common freely-traded consumption good produced linearly from labor, with productivity z_r in location r. Assumption 1 does not hold because $g(c)$ is logarithmic. Assume that $\exp(\epsilon)$ is distributed according to a Fréchet distribution. Utilitarian welfare, as measured by $A^U(t)$, is the solution to

$$U(\boldsymbol{z}(t)/A^U(t)) = \mathbb{E}[\max_r \log(z_r(t)/A^U(t)) + \epsilon_{hr}] = \mathbb{E}[\max_r \log(z_r(0)) + \epsilon_{hr}] = U(\boldsymbol{z}(0)).$$

One can show that $A^U(t) = \left(\sum_r z_r^\theta(t)/\sum_r z_r^\theta(0)\right)^{\frac{1}{\theta}}$.[49] To a first-order approximation, changes in $\log A^U$ are a population-weighted sum of changes in productivities:

$$\Delta \log A^U(t) \approx \sum_r L_r(0)\Delta \log z_r,$$

where $L_r(0)$ is the share of the population choosing location r at the status quo. This does not coincide, even to a first-order, with changes in aggregate welfare as measured by A, because $\Delta \log A \approx \sum_r \lambda_r(0)\Delta \log z_r$, where the Domar weights in this example are $\lambda_r(0) = L_r(0)z_r(0)/(\sum_{r'} L_{r'}(0)z_{r'}(0))$. It can easily be the case that $\Delta \log A > 0$, so that winners can compensate losers and have resources left over, and yet $\Delta \log A^U < 0$.

Proof of Proposition 13. Without loss of generality, let the common consumption good price p^c be the numeraire. The social surplus function is

$$U(\boldsymbol{w}) = \mathbb{E}\left[\max_r \{a_{hi}w_r + \epsilon_{hr}\}\right],$$

[49] Compare this with the right hand side of (30), which calculates $A^W(t)$ under the $f_h(x) = \exp(x)$ cardinalization. The fact that the consumption-equivalent variation of the social welfare function $U(\boldsymbol{c}) = \mathbb{E}[\max_{l_h} \log(c_{l_h}(t)) + \epsilon_{hl_h}]$ is equal to that of the social welfare function $W(\boldsymbol{c}) = \mathbb{E}[\max_{l_h} c_{l_h}(t)\exp(\epsilon_{hl_h})]$ is a special implication from the assumption that $\exp(\epsilon)$ is distributed according to a Fréchet distribution. See Proposition 12 in Appendix D for more details.

where w_r is now the real wage in location r. Then,

$$\begin{aligned}
\frac{\partial U}{\partial w_r} &= \frac{\partial}{\partial w_r} \int \max_i \{a_{hi} w_i + \epsilon_{hi}\} f(\epsilon) d\epsilon \\
&= \int \frac{\partial}{\partial w_r} \left[\max_i \{a_{hi} w_i + \epsilon_{hi}\} \right] f(\epsilon) d\epsilon \\
&= \int a_{hr} \mathbf{1} \{a_{hr} w_r + \epsilon_{hr} \geq a_{hj} w_j + \epsilon_{hj} \forall j\} f(\epsilon) d\epsilon \\
&= L_r(\boldsymbol{w}).
\end{aligned}$$

Equation (34) in the proof of Theorem 2 states that with a common consumption good

$$\sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d \log \left[\frac{w_r}{A}\right] = \sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d \log p^c,$$

where the wages and prices are the ones in the compensated equilibrium. Since the consumer price index is the numeraire, this is the same as:

$$\sum_r L_r^{\text{comp}} \left[\frac{w_r}{A}\right] d \log \left[\frac{w_r}{A}\right] = 0.$$

Equivalently,

$$\sum_r L_r^{\text{comp}} d \left[\frac{w_r}{A}\right] = 0.$$

Integrating between 0 and t gives:

$$\int_{w(0)/p^c(0)}^{w(t)/(A(t))} \sum_r L_r^{\text{comp}}(\boldsymbol{x}) d\boldsymbol{x} = 0.$$

By Proposition 3, the compensated and uncompensated labor supply functions are the same. Furthermore, given productivities $\boldsymbol{z}(t)$ and aggregate factor-augmenting productivity $1/A(t)$, the compensated and uncompensated equilibrium real wages are the same. Hence, we can write

$$\int_{w(0)}^{w(t)/(A(t))} \sum_r L_r(\boldsymbol{x}) d\boldsymbol{x} = 0,$$

substituting in $\frac{\partial U}{\partial w_r}$ from above gives

$$\int_{w(0)}^{w(t)/(A(t))} \sum_r \frac{\partial U}{\partial w_r}(\boldsymbol{x}) d\boldsymbol{x} = 0,$$

which is equal to

$$U(w(t)/(A(t))) - U(w(0)) = 0,$$

by the fundamental theorem of calculus.

□

Appendix F Calculating $A(t)$ via system of linear differential equations

In this appendix we describe how to calculate $A(t)$ using a system of linear differential equations implied by Theorem 2 and Proposition 2, as discussed in Section 5. We present the log-linearized system and then give a step-by-step algorithm.

System of equations

TFP-equivalent welfare

$$\frac{d\log A(t)}{dt} = \sum_i \lambda_i^{\text{comp}}(t)\,\frac{d\log z_i(t)}{dt}, \qquad A(1) = 1. \tag{47}$$

Input-output and prices

$$d\log \Omega_{ij} = (1-\theta_i)\left[d\log p_j - \sum_{k\in N}\Omega_{ik}\,d\log p_k - \sum_{r\in R}\Omega_{ir}\,d\log w_r\right]. \tag{48}$$

$$d\log \boldsymbol{p} = -\Psi\,d\log \boldsymbol{z} + \Psi^F\,d\log \boldsymbol{w}, \tag{49}$$

where $\Psi = (I-\Omega)^{-1}$ is the Leontief inverse and Ψ^F the matrix of factor contents.

Compensated supply and demand derivatives Let

$$L_r^{\text{comp}} = L_r^{\text{comp}}\left(\tfrac{\boldsymbol{w}}{A}, \boldsymbol{p}\right), \qquad \chi_r^{\text{comp}} = \chi_r^{\text{comp}}\left(\tfrac{\boldsymbol{w}}{A}, \boldsymbol{p}\right),$$

and define

$$\eta_{r,s}^{L} = \frac{\partial \log L_r^{\text{comp}}}{\partial \log(w_s/A)}, \quad \eta_{r,i}^{L,p} = \frac{\partial \log L_r^{\text{comp}}}{\partial \log p_i},$$

$$\eta^{\chi}_{r,s} = \frac{\partial \chi_r^{\text{comp}}}{\partial \log(w_s/A)}, \quad \eta^{\chi,p}_{r,i} = \frac{\partial \chi_r^{\text{comp}}}{\partial \log p_i}.$$

Then

$$d \log L_r^{\text{comp}} = \sum_s \eta^L_{r,s} \left(d \log w_s - d \log A \right) + \sum_i \eta^{L,p}_{r,i} \, d \log p_i, \tag{50}$$

$$d\chi_r^{\text{comp}} = \sum_s \eta^{\chi}_{r,s} \left(d \log w_s - d \log A \right) + \sum_i \eta^{\chi,p}_{r,i} \, d \log p_i. \tag{51}$$

Factor Domar weights

$$d \log \lambda_r^{\text{comp}} = d \log w_r + d \log L_r^{\text{comp}} - \sum_{r'} \lambda_{r'}^{\text{comp}} \left[d \log w_{r'} + d \log L_{r'}^{\text{comp}} \right]. \tag{52}$$

Using (50), this can be written in terms of $d \log w_s$, $d \log p_i$, and $d \log A$.

Goods/intermediate Domar weights

$$d \log \lambda_i^{\text{comp}} = \left(\lambda_i^{\text{comp}} \right)^{-1} \left(\sum_r d\chi_r^{\text{comp}} \, \Omega_{c(r)i} + \sum_r \chi_r^{\text{comp}} \, d\Omega_{c(r)i} + \sum_j d\left[\lambda_j^{\text{comp}} \, \Omega_{ji} \right] \right), \tag{53}$$

with

$$d\left[\lambda_j^{\text{comp}} \, \Omega_{ji} \right] = \lambda_j^{\text{comp}} \Omega_{ji} \, d \log \lambda_j^{\text{comp}} + \lambda_j^{\text{comp}} \Omega_{ji} \, d \log \Omega_{ji}.$$

Here $d\chi_r^{\text{comp}}$ comes from (51), $d\Omega$ from (48), and $d \log \lambda_j^{\text{comp}}$ from (52)–(53). Given $d \log A$ from (47) and $d \log z_i$, the system (48)–(49)–(50)–(51)–(52)–(53) is linear in

$$d \log p_i, \quad d \log w_r, \quad d \log \Omega_{ij}, \quad d \log \lambda_i^{\text{comp}}.$$

Algorithm

Step 0 (initialization, $t = 1$)

i. Solve the decentralized equilibrium at $t = 1$ to obtain

$$\boldsymbol{p}(1), \; \boldsymbol{w}(1), \; \boldsymbol{\Omega}(1), \; \boldsymbol{\lambda}(1).$$

ii. Set compensated objects at $t = 1$:

$$\lambda_i^{\text{comp}}(1) = \lambda_i(1),$$

and use the same $\boldsymbol{p}(1)$, $\boldsymbol{w}(1)$, $\boldsymbol{\Omega}(1)$.

iii. Set $A(1) = 1$.

Step 1 (shocks and grid)

i. Specify a path $\boldsymbol{z}(t)$ and compute $d \log z_i(t)/dt$.

ii. Choose a time grid $1 = t_0 < \cdots < t_K$ with $\Delta t_k = t_{k+1} - t_k$.

Step 2 (for each $k = 0, \ldots, K-1$) Given

$$\boldsymbol{p}(t_k),\ \boldsymbol{w}(t_k),\ \boldsymbol{\Omega}(t_k),\ \boldsymbol{\lambda}^{\text{comp}}(t_k),\ A(t_k),$$

proceed as follows.

2.1 Hulten step

Compute

$$\left.\frac{d \log A}{dt}\right|_{t_k} = \sum_i \lambda_i^{\text{comp}}(t_k)\ \left.\frac{d \log z_i}{dt}\right|_{t_k}.$$

2.2 Compensated demand and supply

i. Evaluate $L_r^{\text{comp}}(t_k)$ and $\chi_r^{\text{comp}}(t_k)$ at

$$\left(\frac{\boldsymbol{w}(t_k)}{A(t_k)}, \boldsymbol{p}(t_k)\right)$$

using the compensated choice problem.

ii. Compute

$$\eta_{r,s}^{L},\ \eta_{r,i}^{L,p},\ \eta_{r,s}^{\chi},\ \eta_{r,i}^{\chi,p}$$

at that point (analytically or by finite differences).

iii. Use (50) and (51) to express $d \log L_r^{\text{comp}}$ and $d\chi_r^{\text{comp}}$ as linear functions of $d \log w_s$, $d \log p_i$ and the known $d \log A\big|_{t_k}$.

2.3 General equilibrium

i. Plug (50), (51), (48), and (49) into (52) and (53).

ii. This yields a linear system for

$$\frac{d\log p_i}{dt}, \quad \frac{d\log w_r}{dt}, \quad \frac{d\log \Omega_{ij}}{dt}, \quad \left.\frac{d\log \lambda_i^{\text{comp}}}{dt}\right|_{t_k},$$

where $\left(d\log A/dt\right)\big|_{t_k}$ and $\left(d\log z_i/dt\right)\big|_{t_k}$ are known. Solve this system.

2.4 Time update

For any variable $x \in \{p_i, w_r, \Omega_{ij}, \lambda_i^{\text{comp}}, A\}$,

$$\log x(t_{k+1}) = \log x(t_k) + \left.\frac{d\log x}{dt}\right|_{t_k} \Delta t_k.$$

Repeat until t_K.

Appendix G Aggregate welfare with distortions and externalities

In the body of the paper, we consider perfectly competitive discrete choice models. In this appendix we discuss how Theorem 1 extends beyond the perfectly competitive baseline. We first introduce wedge distortions and then agglomeration externalities.

Wedge distortions

The household block of the model is unchanged. Each agent chooses consumption and location to maximize utility given prices, wages, and transfers, where transfers now include revenues from wedges (e.g. tax revenues). Aggregating individual choices gives rise to labor supply and final demand functions, $L_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$ and $\chi_r(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{T})$.

Producers choose quantities to minimize costs, taking prices as given. The price of good i is now given by

$$p_i = \mu_i z_i^{-1} \text{mc}_i(\boldsymbol{p}, \boldsymbol{w}), \tag{54}$$

where $\mu_i > 0$ is a tax, markup, or any other wedge between price and marginal cost for producer of good i.[50] Lump sum transfers across households must equal total revenues

[50] Even though we assume that wedges are on gross output only, this is without loss of generality. This is because we can recreate buyer-seller wedges by treating firm i's purchases of an input from j as a distinct good (made linearly using j's output). A wedge on this good is then isomorphic to a buyer-seller wedge.

generated by the wedges,

$$\int T_h dh = \sum_i p_i y_i (1 - \frac{1}{\mu_i}). \tag{55}$$

In the decentralized equilibrium, transfers are distributed across households according to some rule.

To solve for a decentralized equilibrium with wedges, the market clearing conditions (6) and (7) are unchanged, but when defining the ratio of sales, factor payments, and spending to GDP ,

$$\lambda_i = \frac{p_i y_i}{GDP} \mathbf{1}[i \in C + N] + \frac{Z w_i L_i}{GDP} \mathbf{1}[i \in R], \qquad \text{and} \qquad \chi_r = \frac{Z w_r L_r + \int T_h \mathbf{1}[l_h = r] dh}{GDP},$$

we use the fact that GDP (or aggregate income) is given by

$$GDP = \sum_r Z w_r L_r + \sum_i p_i y_i (1 - \frac{1}{\mu_i}).$$

Given wedges, some transfer rule, and the aggregate supply and demand functions, equilibrium prices, wages, and quantities satisfy (6), (7), and (54).

We parameterize technologies and wedges at t by $\boldsymbol{z}(t)$ and $\boldsymbol{\mu}(t)$. Define the set of feasible allocations given productivity shifters $\boldsymbol{z}(t)$, wedges $\boldsymbol{\mu}(t)$, and aggregate factor-augmenting productivity Z to be

$$\mathcal{C}(t, Z) \equiv \{\{\boldsymbol{c}_h, l_h\}_h \text{ supported via equilibrium with wedges given } \boldsymbol{z}(t), \boldsymbol{\mu}(t), Z, \text{and some transfers.}\}$$

The change in productivity $A(t)$ is defined exactly as in Equation (11). In response to a change in technologies and wedges, we consider a hypothetical rescaling of Z by a factor $1/A(t)$, and define $A(t)$ as the largest contraction such that every agent can be made at least as well off as in the initial equilibrium. Misallocation due to distortions in the status quo can be quantified by setting $\boldsymbol{\mu}(t) = 1$ and $\boldsymbol{z}(t) = \boldsymbol{z}(0)$.

We can calculate $A(t)$ using a compensated equilibrium with wedges, extending the results in Section 3.2. In the compensated equilibrium with wedges, every agent receives a transfer that keeps them indifferent, the sum of transfers equals the sum of wedge revenues, and TFP is contracted by $A(t)$. Compensated choices, $l_h^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ and the expenditure function, $e_h(Z\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ are obtained using Proposition 1, and these individual choices are aggregated to get location-level variables, $L_r^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)$, $E_r^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)$, and $\chi_r^{\text{comp}}(Z\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{u}^0)$, exactly as in the baseline model without wedges. The generalization of Theorem 1 is:

Theorem 3 (Aggregate Welfare)**.** *TFP-equivalent aggregate welfare satisfies*

$$\sum_r E_r^{comp}(\boldsymbol{w}/A(t),\boldsymbol{p},\boldsymbol{u}^0) = \frac{1}{1-\sum_i \lambda_i(1-\frac{1}{\mu_i})} \sum_r (w_r/A(t)) L_r^{comp}(\boldsymbol{w}/A(t),\boldsymbol{p},\boldsymbol{u}^0). \quad (56)$$

Given compensated labor supply $\boldsymbol{L}^{comp}(\boldsymbol{w}/A(t),\boldsymbol{p},\boldsymbol{u}^0)$ and demand $\boldsymbol{\chi}^{comp}(\boldsymbol{w}/A(t),\boldsymbol{p},\boldsymbol{u}^0)$, the vector of prices $\boldsymbol{p}$ in the compensated equilibrium satisfies (54)*, and the vector of wages $\boldsymbol{w}$ and sales shares $\boldsymbol{\lambda}$ satisfy market clearing conditions* (6) *and* (7)*.*

To prove this result, we follow the same steps in the proof of Theorem 1, except for condition (55) that the sum of transfers is not equal to zero but equal to total revenues from wedges.

Introducing agglomeration externalities

We now briefly discuss how to account for agglomeration externalities. For simplicity, we only consider local externalities determined by the workers located in the location where the good is produced, but it is straightforward to also allow for spillovers between regions. Suppose that the production function of good i produced in region r is given by

$$y_i = \tilde{z}_i F_i\left(\{x_{ij}\}_{j\in N}, \{L_{ir}\}_{r\in R}\right), \quad (57)$$

where the Hicks-neutral productivity shifter is

$$\tilde{z}_i = z_i L_r^{\gamma_i}.$$

Producers take $\tilde{z}_i$ as given when minimizing costs. If $\gamma_i > 0$, then there is a positive externality from a larger mass of agents, weighted by their skills exclusive of the aggregate TFP shifter Z, in location r on the production of good i. If $\gamma_i < 0$, then there is a congestion externality from a larger mass of agents in location r on the production of good i.

For example, suppose each location produces output using labor only according to

$$y_r = z_r L_r^{1+\gamma_r} = \tilde{z}_r L_r.$$

The equilibrium wage is equal to the private marginal product of labor,

$$w_r = p_r z_r L_r^{\gamma_r} = p_r \tilde{z}_r,$$

where producers take prices and $\tilde{z}_r$ as given. When solving for equilibrium and for $A(t)$, we must take into account how changes in labor supply affect productivities $\tilde{\boldsymbol{z}}$.

Theorem 1 remains unchanged, with $\tilde{\boldsymbol{z}}$ in place of $\boldsymbol{z}$, and with the additional constraint

$$\tilde{z}_i = z_i \left(L_r^{\text{comp}}\right)^{\gamma_i}. \tag{58}$$

If there are multiple compensated equilibria, choose the one that implies the highest value of $A(t)$. The solution algorithm described after Theorem 1 remains unchanged, with $\tilde{\boldsymbol{z}}$ in place of $\boldsymbol{z}$ and where each iteration must also impose condition (58).

We illustrate for the one-good economy of Example 2. TFP-equivalent aggregate welfare $A(t)$ is given by a modified version of (14):

$$A(t) = \frac{\sum_r \int \tilde{z}_r a_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh}{\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh},$$

where compensated location choices satisfy:

$$l_h^{\text{comp}} \in \arg\max_{r \in R} \left\{ \tilde{z}_r a_{hr} / A(t) - \bar{c}_{hr} \right\},$$

and

$$\tilde{z}_r = z_r(t) \left(L_r^{\text{comp}}\right)^{\gamma_r}, \qquad \text{where} \qquad L_r^{\text{comp}} = \int a_{hr} \mathbf{1}[l_h^{\text{comp}} = r] dh.$$

Appendix H Aggregate welfare with place-based compensation

In the body of the paper, we measure aggregate welfare by the amount of factor endowments that can be saved after winners compensate losers using lump-sum transfers. In this appendix, we briefly discuss how to extend this definition of aggregate welfare to cases where individual-level lump-sum transfers are not available, but place-based redistributive policies are.

Suppose that instead of individual-specific transfers, location-level consumption taxes, denoted by τ_r, can be levied on households that choose location r. In this case, the budget constraint of agent h is now:

$$\sum_r (1 + \tau_r) p_r c_h \mathbf{1}\left[l_h = r\right] = Z \sum_r w_r a_{hr} \mathbf{1}\left[l_h = r\right] + T,$$

where T is a uniform lump-sum rebate (the same for all households). Budget balance requires that

$$T = \int \sum_r \tau_r p_r c_h \mathbf{1}\left[l_h = r\right] dh. \tag{59}$$

All other conditions defining equilibrium are the same as in Section 2. We can now define the feasible set of allocations that can be supported using these place-based tax instruments:

$$\mathcal{C}^{pb}(t, Z) \equiv \left\{\{\boldsymbol{c}_h, l_h\}_{h\in H} \text{ supported via equilibrium given } \boldsymbol{z}(t),\, Z, \text{ and some consumption taxes}\right\}.$$

Given this feasible set, the definition of TFP-equivalent aggregate welfare, given place-based compensations, is the same as before replacing $\mathcal{C}(t, Z)$ with $\mathcal{C}^{pb}(t, Z)$.

Definition 7. TFP-equivalent aggregate welfare with place-based compensation at t is

$$A^{pb}(t) = \max\left\{Z^{-1} : \{c_h, l_h\}_{h\in H} \in \mathcal{C}^{pb}(t, Z) \text{ and } (c_h, l_h) \succeq_h (c_h(0), l_h(0)) \text{ for every } h\right\}. \tag{60}$$

where $c_h(0)$ and $l_h(0)$ are the consumption and location of h in the status quo.

To see how this works in practice, consider the one-good economy of Example 2. We focus on the one-good economy for simplicity, but the first-order result in Proposition 14 extends to economies with multiple consumption goods.

Example 12 (One-Good Economy with Place-Based Policies). Consider the one-good economy again: there is a single freely traded consumption good produced linearly from labor, and productivity in each location r is $z_r(t)$. Production in location r is

$$y_r = Zz_r(t)L_r = Zz_r(t)\int a_{hr}\mathbf{1}[l_h = r]\, dh.$$

The following shows that, in the one-good economy, $A^{pb}(t)$ is bounded above by $A(t)$ but that, to a first-order approximation, the two are the same.

Proposition 14 (Hulten's Theorem for One-Good Economy). *In the one-good economy, we have*

$$A^{pb}(t) \leq A(t) \quad \textit{for all } t.$$

Moreover, around the status quo $t = 0$, the two measures coincide to a first-order approximation and both obey Hulten's theorem:

$$\Delta \log A^{pb} \approx \Delta \log A \approx \sum_r \lambda_r(0)\, \Delta \log z_r.$$

Before providing a formal proof (below), we provide a heuristic discussion of the non-linear problem. For this discussion, we assume homogeneous skills, $a_{hr} = 1$ for every h and r, so that all households choosing a given location have the same consumption. To calculate A^{pb}, rearrange the budget-balance condition, (59), as

$$Z^{-1} = \frac{\sum_r w_r L_r}{\sum_r p_r c_r L_r} = \frac{\sum_r z_r L_r}{\sum_r c_r L_r}, \tag{61}$$

where c_r denotes per-capita consumption in location r, and we use the fact that the real wage in location r is equal to $z_r(t)$ in this one-good economy. Using equation (60) and (61), we can write

$$A^{pb}(t) = \max_{\boldsymbol{c}} \left\{ \frac{\sum_r z_r(t) L_r(\boldsymbol{c})}{\sum_r c_r L_r(\boldsymbol{c})} : u_h(c_r, l_h(\boldsymbol{c})) \geq u_h^0 \text{ for every } h \right\},$$

where $L_r(\boldsymbol{c})$ is labor supply in location r as a function of the vector of location-level per capita consumption $\boldsymbol{c}$.

If, in the solution to the problem above, for every location $r \in R$, some agent $h \in H$ stays in the same location as in the status-quo and all locations are non-empty, then compensating the stayers requires ensuring that consumption per capita in each location is at least as high as in the status quo. Hence, we can replace the H inequalities above with just R inequalities:

$$A^{pb}(t) = \max_{\boldsymbol{c}} \left\{ \frac{\sum_r z_r(t) L_r(\boldsymbol{c})}{\sum_r c_r L_r(\boldsymbol{c})} : c_r \geq c_r^0 \text{ for every } r \right\}.$$

This is a relatively simple constrained optimization maximization problem given the supply function.

To make it more concrete, suppose that $\boldsymbol{L}(\boldsymbol{c})$ is the iso-elastic labor supply function:

$$L_r(\boldsymbol{c}) = \frac{B_r c_r^\theta}{\sum_{r'} B_{r'} c_{r'}^\theta}.$$

Then, it is straightforward to show that the solution to $A^{pb}(t)$ satisfies:

$$c_r^{pb}(t) = z_r(0) \mathbf{1} \left[\frac{z_r(t)/z_r(0)}{A^{pb}(t)} \leq \frac{\theta+1}{\theta} \right] + \frac{\theta}{\theta+1} \frac{z_r(t)}{A^{pb}(t)} \mathbf{1} \left[\frac{z_r(t)/z_r(0)}{A^{pb}(t)} > \frac{\theta+1}{\theta} \right].$$

If the shocks are sufficiently small, $\frac{z_r(t)/z_r(0)}{A^{pb}(t)} \leq \frac{\theta+1}{\theta}$ for every r, then the solution sets consumption in each location equal to its status quo value $c_r^{pb}(t) = z_r(0)$. This is achieved by scaling aggregate factor productivity by $1/A^{pb}(t)$ and setting consumption taxes appropriately. Since consumption is equal to its status-quo value in every location, no agent switches locations, and hence every agent is exactly indifferent to the status-quo.

However, if region r experiences much faster productivity growth than average — in the sense that $\frac{z_r(t)/z_r(0)}{A^{pb}(t)} > \frac{\theta+1}{\theta}$ — then consumption in region r must exceed its status-quo level to incentivize additional workers to move there. In this case, stayers in region r are strictly better off than in the status quo. Given the optimal choices $\boldsymbol{c}^{pb}(t)$ and the implied labor supplies $L_r(\boldsymbol{c}^{pb}(t))$, the place-based aggregate welfare measure $A^{pb}(t)$ is obtained from (61).

Table 5 reports A^{pb} for the same calibrated economy as in Example 5. For a 1% productivity increase, A, A^{pb}, and the shocked location's sales share are nearly identical, consistent with the first-order result in Proposition 14. For a larger 10% increase, nonlinearities become more visible: the elasticity of A exceeds the sales share, as discussed in Example 4. Moreover, A^{pb} is slightly lower than A, as implied by the inequality in Proposition 14.

Table 5: Aggregate welfare with lump-sum versus place-based compensations

		1 percent increase		10 percent increase	
Location of shock	Sales share λ_r	$\Delta \log A$	$\Delta \log A^{pb}$	$\Delta \log A$	$\Delta \log A^{pb}$
San Francisco–Oakland, CA	0.031	0.031	0.031	0.035	0.032
New York, NY	0.057	0.057	0.057	0.064	0.059
Seattle–Tacoma, WA	0.024	0.024	0.024	0.027	0.025
Brownsville, TX	0.001	0.001	0.001	0.001	0.001
McAllen, TX	0.002	0.002	0.002	0.003	0.002
Daytona Beach, FL	0.001	0.001	0.001	0.001	0.001

Notes: Entries in the $\Delta \log A$ and $\Delta \log A^{pb}$ columns are elasticities, computed as the log change in the welfare measure divided by the size of the productivity shock, 0.01 or 0.10. A^{pb} restricts compensation to place-based taxes and a common rebate. Sales share is λ_r in the initial equilibrium. Results use $\theta = 1.5$, $\gamma = 1$, and 400,000 simulated households.

The proof of Proposition 14 makes use of the following lemma:

Lemma 1. *In the one-good economy, for any time-invariant skills a_{hr} and preferences $u_h(c_h, l) = f_h(g(c_h) + \epsilon_{hl})$, the problem where aggregate welfare is calculated by choosing feasible locations agent by agent,*

$$\tilde{A} = \max_{\boldsymbol{l}} \frac{\sum_r \int z_r a_{hr} \mathbf{1}[l_h = r]\, dh}{\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h = r]\, dh}, \tag{62}$$

yields the same value as aggregate welfare with household-specific lump-sum transfers: $\tilde{A} = A$.

Proof of Lemma 1. Let $\tilde{A}$ be the value of the maximization problem in (62), and denote by $\tilde{\boldsymbol{l}}$ the corresponding optimal location profile. By Dinkelbach (1967), $\tilde{\boldsymbol{l}}$ also solves the alternative problem

$$\max_{\boldsymbol{l}} \left[\sum_r \int \frac{z_r a_{hr}}{\tilde{A}} \mathbf{1}[l_h = r]\, dh - \sum_r \int \bar{c}_{hr} \mathbf{1}[l_h = r]\, dh \right],$$

or equivalently

$$\max_{\boldsymbol{l}} \sum_r \int \left(\frac{z_r a_{hr}}{\tilde{A}} - \bar{c}_{hr} \right) \mathbf{1}[l_h = r]\; dh.$$

Given $\tilde{A}$, this problem is separable across h, so it can be solved household by household:

$$\tilde{l}_h \in \arg\max_{r \in R} \left\{ \frac{z_r a_{hr}}{\tilde{A}} - \bar{c}_{hr} \right\}.$$

By Proposition 1, the compensated location choice of household h when aggregate factor productivity is $1/\tilde{A}$ is

$$l_h^{\text{comp}} \in \arg\max_{r \in R} \left\{ \frac{z_r a_{hr}}{\tilde{A}} - \bar{c}_{hr} \right\},$$

so $\tilde{l}_h = l_h^{\text{comp}}$ for all h. By definition, the value of A that supports this compensated allocation is precisely $A = \tilde{A}$, as claimed. □

Proof of Proposition 14. The first part is a consequence of Lemma 1. Consider any place-based compensated equilibrium with aggregate factor productivity $Z = 1/A^{pb}(t)$ and location profile $\boldsymbol{l}$. Since every household is at least as well off as in the status quo, its consumption in its chosen location is at least $\bar{c}_{hr}$. Resource feasibility therefore implies

$$\sum_r \int \bar{c}_{hr} \mathbf{1}[l_h = r]\, dh \le \frac{1}{A^{pb}(t)} \sum_r \int z_r(t) a_{hr} \mathbf{1}[l_h = r]\, dh.$$

Hence, $A^{pb}(t)$ cannot exceed the value of the feasible agent-by-agent location problem in (62). By Lemma 1, that value is $A(t)$. Therefore,

$$A^{pb}(t) \le A(t) \quad \text{for all } t.$$

For the second part, we construct an explicit feasible place-based policy that yields a lower bound for $A^{pb}(t)$ and then apply a squeezing argument. Let

$$L_r^0 = \int a_{hr} \mathbf{1}[l_h(0) = r]\, dh$$

denote the efficiency units of labor supplied in location r in the status quo. Define

$$A^{lb}(t) \equiv \frac{\sum_r z_r(t) L_r^0}{\sum_r z_r(0) L_r^0}. \tag{63}$$

Consider the following policy. Set the common rebate to zero, scale aggregate factor productivity by $1/A^{lb}(t)$, and choose the consumption tax in each location so that

$$1 + \tau_r^{lb} = \frac{z_r(t)/A^{lb}(t)}{z_r(0)} \quad \text{for every } r. \tag{64}$$

Under this policy, the consumption of household h in location r is

$$c_{hr}^{lb}(t) = \frac{a_{hr} z_r(t)/A^{lb}(t)}{1+\tau_r^{lb}} = a_{hr} z_r(0) = c_{hr}(0).$$

Thus, every household faces exactly its status-quo consumption in every location. The household's entire menu is unchanged, so every household is indifferent to the status quo and location choices remain at their status-quo values. Moreover, aggregate tax revenue is

$$\sum_r \tau_r^{lb} z_r(0) L_r^0 = \frac{\sum_r z_r(t) L_r^0}{A^{lb}(t)} - \sum_r z_r(0) L_r^0 = 0,$$

where the last equality follows from (63). Hence, the policy satisfies budget balance with a zero common rebate.

Since this policy keeps everyone indifferent to the status quo and is implementable with place-based consumption taxes, it is a feasible solution for the problem that defines $A^{pb}(t)$. Therefore,

$$A^{pb}(t) \geq A^{lb}(t) \quad \text{for all } t.$$

For the no-reallocation allocation, the efficiency units $\{L_r^0\}$ are fixed, so the lower bound is the standard aggregate productivity index $A^{lb}(t) = \sum_r z_r(t) L_r^0 / \sum_r z_r(0) L_r^0$. Its derivative at the status quo satisfies

$$\frac{d}{dt} \log A^{lb}(t)\Big|_{t=0} = \sum_r \lambda_r(0) \frac{d}{dt} \log z_r(t)\Big|_{t=0}, \tag{65}$$

where $\lambda_r(0) = z_r(0) L_r^0 / \sum_{r'} z_{r'}(0) L_{r'}^0$. On the other hand, the full-transfer measure $A(t)$ also satisfies Hulten's theorem at the status quo (by Corollary 1). So,

$$\frac{d}{dt} \log A(t)\Big|_{t=0} = \sum_r \lambda_r(0) \frac{d}{dt} \log z_r(t)\Big|_{t=0}. \tag{66}$$

Combining $A^{lb}(t) \leq A^{pb}(t) \leq A(t)$ for all t with $A^{lb}(0) = A^{pb}(0) = A(0) = 1$, we have for $t > 0$,

$$\frac{\log A^{lb}(t) - \log A^{lb}(0)}{t} \leq \frac{\log A^{pb}(t) - \log A^{pb}(0)}{t} \leq \frac{\log A(t) - \log A(0)}{t}.$$

Using (65) and (66), the left and right terms converge to the same limit as $t \downarrow 0$. By the squeeze theorem, the middle term must converge to the same limit. Applying the same argument for $t < 0$, with the inequalities reversed after division by t, establishes the corresponding left derivative. Hence $\log A^{pb}(t)$ is differentiable at $t = 0$ and

$$\frac{d}{dt} \log A^{pb}(t)\Big|_{t=0} = \sum_r \lambda_r(0) \frac{d}{dt} \log z_r(t)\Big|_{t=0}.$$

Rewriting this in terms of finite changes yields the first-order approximation in the statement of the proposition. □

Appendix I Further Discussion of Sum of Compensating Variations and Kaldor-Hicks efficiency

In this appendix, we discuss aggregate welfare as measured by the sum of compensating variations. First, we provide a worked-out example showing that the sum of compensating variations can be strictly positive in response to pure transfers. Second, we discuss an alternative but very closely related definition of Kaldor-Hicks efficiency that coincides with $A(t)$ when prices and wages do not respond to transfers.

I.1 Sum of Compensating Variations Rises in Response to Transfers

The following example demonstrates that a pure transfer, which moves the allocations along the Pareto frontier, can nevertheless cause $S(t)$ to rise.

Example 13 (Redistribution raises Kaldor-Hicks efficiency). Assume that consumption in each location is a Cobb-Douglas aggregator of inputs from different locations:

$$y_{c(r)} = \int c_h \mathbf{1}[l_h = r] dh = x_{rr}^{\alpha} \prod_{r' \in R} x_{rr'}^{(1-\alpha)/R},$$

where $\alpha \geq 0$ controls the degree of home bias in consumption. Output in location r is produced one-for-one from labor:

$$\sum_{r'} x_{r'r} = L_r.$$

Workers' utility functions are

$$u_h(c_h, l_h) = f_h(c_h + \epsilon_{hl_h}),$$

where f_h is any monotone increasing function. Consider an economy with two symmetric locations and suppose that $\epsilon_{h1} = 1$ and ϵ_{h2} is an i.i.d. uniform random variable in the interval $[1-d, 1+d]$, where d controls the elasticity of location choices to changes in real consumption. This elasticity falls to zero as d rises because as d rises, taste dispersion grows, and the measure of households that are close enough to indifferent between locations for a given wage change goes to zero.

The status quo is a symmetric equilibrium without transfers. Consider a lump-sum transfer from households in location 2 to location 1. That is, agents that chose location 1 in the status quo, $l_h(0) = 1$, receive a lump-sum transfer of T_1. The transfer is financed by a lump-sum tax on agents that chose location two in the status quo, $l_h(0) = 2$. Budget balance requires the lump-sum tax T_2 to be $T_2 = L_1(0)/L_2(0)T_1$.

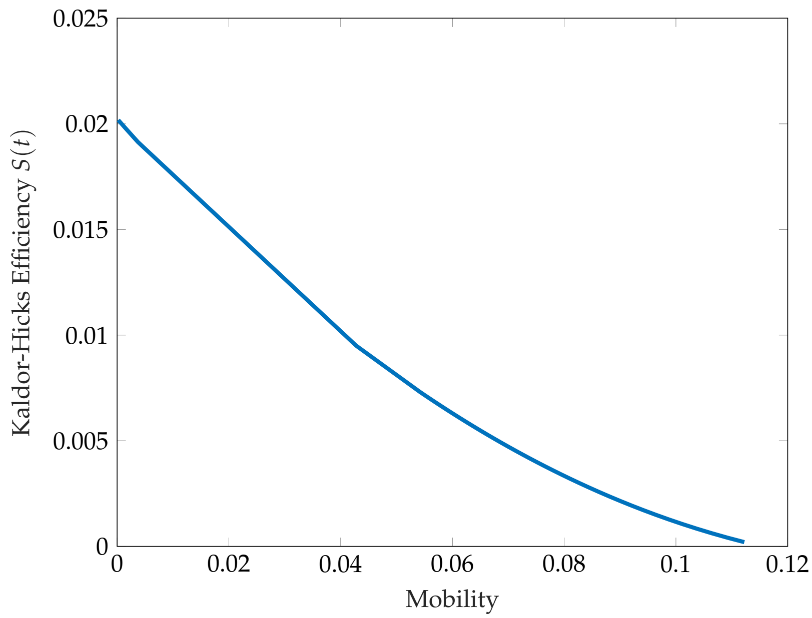
We solve for the post-shock equilibrium at t and calculate the Kaldor-Hicks efficiency measure, $S(t)$. Figure 1 reports $S(t)$ (with nominal GDP as the numeraire) for different levels of worker mobility (by varying the parameter d). We measure mobility by the mass of agents that start in location 2 (for whom $l_h(0) = 2$) and move to location 1 after the redistribution (for whom $l_h(t) = 1$).[51]

Figure 1 shows that Kaldor-Hicks efficiency, $S(t)$, rises in response to a pure redistribution. In contrast, our measure of aggregate welfare in terms of TFP equivalents is unchanged, $A(t) = A(0) = 1$ for every value of mobility and preference parameters. Pure transfers move the economy along the Pareto frontier; by definition of $A(t)$, they do not change the factor-saving amount needed to keep everyone indifferent.

The reason $S(t)$ is non-zero for pure redistributions is because redistributions change relative wages and prices in general equilibrium. Since $S(t)$ holds wages and prices con-

[51] In our numerical example, we set the transfer to be 10% of wages in the status quo $T_1 = 0.1w_1(0)$.

Figure 1: Kaldor-Hicks Efficiency as a function of mobility for a pure transfer



stant in the compensation, undoing the transfers does not necessarily result in $S(t) = 0$. The magnitude of $S(t)$ depends on parameters — two important parameters are the extent of mobility and home bias. For example, if households do not have heterogeneous tastes, then real wages are equated in the two locations by arbitrage. Since productivities are symmetric, this implies that relative wages and prices are also equated in equilibrium. In this limit, the transfer does not alter relative wages and prices, and so $S(t)$ tends to 0 as we increase mobility.

Similarly, in the absence of home bias ($\alpha = 0$), then $S(t) = 0$. The utility agent h gets in location r is $(w_r + T_h)/p_r$. Without home-bias, the price of consumption in the two locations is the same: $p_1 = p_2$. Hence, because Assumption 1 holds, the transfer T_h does not change h's location choice. That is, $\boldsymbol{L}(t) = \boldsymbol{L}(0)$. Furthermore, since there is no home-bias, agents in both locations spend their income in the same proportion, so that $w_i L_i = 1/2$ (recall that GDP is the numeraire). Since location choices do not respond to the transfers, wages also do not respond to the transfer. Since wages do not respond to the transfers, prices do not respond to the transfers. Since prices and wages do not respond to the redistribution, $S(t) = 0$ in this limit because undoing the redistribution at the new prices is equivalent to undoing it at the old prices. Hence, in this limit, the compensating-variation based $S(t)$ correctly detects that a pure transfer does not alter efficiency.

In Example 13, the pure redistribution is due to a lump-sum transfers. However, the same logic applies when redistributions are caused by productivity shocks.[52] These examples illustrate that Kaldor-Hicks efficiency is a poor measure of efficiency in environments where prices and wages are endogenous to the distribution of income and expenditures.

I.2 Alternative Definition Kaldor-Hicks Efficiency

In Section 6.3 we defined Kaldor-Hicks efficiency as

$$S(t) \equiv -\int_h e_h(\boldsymbol{w}(t), \boldsymbol{p}(t), u_h^0)\, dh$$

where

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min \left\{ T_h : u_h(c_h, l_h) \geq u_h^0, \sum_r p_r c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h \right\}.$$

The following lemma shows that $S(t)$ is equal to the maximum difference between total income and the sum of compensating incomes (minimum income to attain indifference with the status quo), evaluated at t equilibrium wages and prices.

Lemma 2 (Equivalent definition of $S(t)$)**.**

$$S(t) = \max_{\{(c_h, l_h)\}_{h \in H}} \left\{ \int_h \left[w_{l_h}(t)\, a_{l_h} - p_{l_h}(t)\, c_h \right] dh : \; u_h(c_h, l_h) = u_h^0 \; \forall h \right\}.$$

Proof. By local non-satiation in c_h, the minimizer in the definition of $e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0)$ must satisfy $u_h(c_h, l_h) = u_h^0$. Thus

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min_{\substack{c_h, l_h \\ u_h(c_h, l_h) = u_h^0}} \left\{ T_h : \sum_r p_r c_h \mathbf{1}[l_h = r] \leq \sum_r a_{hr} w_r \mathbf{1}[l_h = r] + T_h \right\}.$$

For any (c_h, l_h), the budget constraint implies $T_h \geq p_{l_h} c_h - w_{l_h} a_{hl_h}$, so at the minimum

[52] For example, to generate a pure redistribution using productivity shocks, suppose productivity rises for some agents and falls for others and the labor of these agents are perfect substitutes in production. Hence, the consumption possibility set of the economy does not change, but some agents gain and some agents lose. In this case, if agents consume different goods, the productivity shocks will alter relative prices and the sum of compensating variations is positive. This is because the winners can compensate the losers at post-shock prices and have money left over. Intuitively, relative prices fall for goods more valued by losers, so they can be compensated more cheaply using post-shock prices.

$T_h = p_{l_h} c_h - w_{l_h} a_{hl_h}$. Hence

$$e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \min_{\substack{c_h, l_h \\ u_h(c_h, l_h) = u_h^0}} \{p_{l_h} c_h - w_{l_h} a_{hl_h}\},$$

and therefore

$$-e_h(\boldsymbol{w}, \boldsymbol{p}, u_h^0) = \max_{\substack{c_h, l_h \\ u_h(c_h, l_h) = u_h^0}} \{w_{l_h} a_{hl_h} - p_{l_h} c_h\}.$$

Summing over h,

$$S(t) = \int_h \max_{u_h(c_h, l_h) = u_h^0} \left\{ w_{l_h}(t) a_{l_h} - p_{l_h}(t) c_h \right\} dh.$$

Because choices and payoffs are additively separable across households, maximizing the integral over all feasible profiles $\{(c_h, l_h)\}_{h \in H}$ (i.e. those that satisfy $u_h(c_h, l_h) = u_h^0 \ \forall h$) is equivalent to choosing, for each h, a maximizer of the integrand, giving the stated result. □

Consider now an alternative definition of Kaldor-Hicks efficiency as the maximum *ratio* — rather than the difference in terms of the numeraire — between aggregate income and the sum of compensating incomes, at t wages and prices:

$$A^{KH}(t) \equiv \max_{\{(c_h, l_h)\}_{h \in H}} \left\{ \frac{\int_h w_{l_h}(t)\, a_{l_h}\, dh}{\int_h p_{l_h}(t)\, c_h\, dh} : \ u_h(c_h, l_h) = u_h^0 \ \forall h \right\}.$$

Note that, in general, the allocations that maximize $S(t)$ do not coincide with those that maximize $A^{KH}(t)$. However, the following proposition shows that Kaldor-Hicks efficiency rises (i.e. $S(t) > 0$) if and only if $A^{KH}(t) > 1$. Therefore, $A^{KH}(t)$ inherits the same undesirable properties in general equilibrium as $S(t)$. For example, as discussed in Section 6.3, pure transfers cause Kaldor-Hicks efficiency to rise.

Proposition 15 (Equivalence in sign between $S(t)$ and $A^{KH}(t)$.)**.**

$$\operatorname{sign}\big(A^{KH}(t) - 1\big) = \operatorname{sign}\big(S(t)\big).$$

Proof. We can write

$$\begin{aligned} \int_h \left[w_{l_h}(t)\, a_{l_h} - p_{l_h}(t)\, c_h \right] dh &= \int_h w_{l_h}(t)\, a_{l_h}\, dh - \int_h p_{l_h}(t)\, c_h\, dh \\ &= \left(\int_h p_{l_h}(t)\, c_h\, dh \right) \left(\frac{\int_h w_{l_h}(t)\, a_{l_h}\, dh}{\int_h p_{l_h}(t)\, c_h\, dh} - 1 \right). \end{aligned}$$

Since in equilibrium $\int_h p_{l_h}(t)\, c_h\, dh > 0$, the sign of the left-hand side is the same as the sign of the term in parentheses. Thus, for every feasible profile, the sign of

$$\int_h \left[w_{l_h}(t)\, a_{l_h} - p_{l_h}(t)\, c_h \right] dh$$

coincides with the sign of

$$\frac{\int_h w_{l_h}(t)\, a_{l_h}\, dh}{\int_h p_{l_h}(t)\, c_h\, dh} - 1.$$

By Lemma 2, $S(t)$ is the maximum of the former over all feasible allocations, while $A^{KH}(t)$ is the maximum of the latter over all feasible allocations. Since the sign equivalence holds allocation by allocation and both $S(t)$ and $A^{KH}(t)$ are maxima over the same feasible set, it follows that

$$S(t) > 0 \iff A^{KH}(t) > 1, \quad S(t) = 0 \iff A^{KH}(t) = 1, \quad S(t) < 0 \iff A^{KH}(t) < 1,$$

which is equivalent to $\text{sign}(A^{KH}(t) - 1) = \text{sign}(S(t))$. □

The virtue of defining Kaldor-Hicks as a ratio, rather than a difference, is that when real wages are pinned by technologies — i.e., they do not depend on labor supply $\boldsymbol{L}$ or on demand $\boldsymbol{\chi}$ — then $A^{KH}(t) = A(t)$. The assumption that real wages are independent of $\boldsymbol{L}$ and $\boldsymbol{\chi}$, that holds in the one-good economy of Example 2, is very strong. It implies that the real marginal product of labor is constant, which is to say that labor in each location can be converted in consumption goods in that location using a linear technology.

Proposition 16 (Equivalence between $A(t)$ and $A^{KH}(t)$)**.** *Suppose that real wages and relative prices in the decentralized equilibrium are independent of labor supply $\{L_r\}$ and demand $\{\chi_r\}$. Then $A^{KH}(t) = A(t)$.*

Proof. We use the representation

$$A^{KH}(t) = \frac{\sum_r w_r(t)\, L_r^{\text{comp}}\Big(\frac{\boldsymbol{w}(t)}{A^{KH}(t)}, \boldsymbol{p}(t), \boldsymbol{u}^0\Big)}{\sum_r E_r^{\text{comp}}\Big(\frac{\boldsymbol{w}(t)}{A^{KH}(t)}, \boldsymbol{p}(t), \boldsymbol{u}^0\Big)}. \tag{67}$$

This is a special case of Equation (13) in Theorem 1: there, $A(t)$ is characterized as the maximum proportional reduction in aggregate factor-neutral productivity such that winners can compensate losers, with wages and prices evaluated in the compensated equilibrium. Here, $A^{KH}(t)$ is the maximum proportional reduction in wages in all locations such that it is possible for winners to compensate losers (based on prices and wages at t) and there is no money left over (sum of net transfers equals zero).

By assumption, equilibrium real wages and relative prices do not depend on labor supply or demand. Hence the wages and prices in the compensated equilibrium coincide with those in the decentralized equilibrium without transfers. Under this condition, Equation (13) reduces exactly to (67). Since (67) characterizes $A^{KH}(t)$ and Equation (13) characterizes $A(t)$, it follows that $A^{KH}(t) = A(t)$. $\square$