NBER WORKING PAPER SERIES

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Joshua S. Gans

Working Paper 34669
http://www.nber.org/papers/w34669

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 2026

Thanks to Kevin Bryan, Yongmin Chen, Heski Bar-Isaac and Manuel Trajtenberg for helpful discussions. Research assistance from Refine.ink, Claude Opus 4.5, ChatGPT 5.2 and Gemini 3 are acknowledged. Responsibility for all errors remains my own. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 34669
January 2026
JEL No. D11, D42, L12

ABSTRACT

This paper shows that income effects create an endogenous barrier to arbitrage, allowing price discrimination to survive costless resale. A monopolist sells an indivisible good to consumers with heterogeneous incomes who can freely resell. When the good is strictly normal, a consumer's reservation price to resell increases as the purchase price decreases—lower prices leave buyers wealthier and raise their valuation of the good. The monopolist exploits this by subsidising low-income consumers to raise their reservation prices to a target that high-income consumers must also pay. The optimal schedule increases dollar-for-dollar with income in the subsidised segment, weakly dominates uniform pricing, and achieves the first-best allocation when the entire market is served. We show the mechanism extends beyond income effects: low substitutability with market alternatives generates large reservation-price responses even when income sensitivity is modest. Sustaining discrimination requires market power at the individual level—consumer-specific quantity limits—not merely aggregate output restrictions. Extensions examine multiple monopolists and endogenous privacy choices.

Joshua S. Gans
University of Toronto
Rotman School of Management
and NBER
joshua.gans@rotman.utoronto.ca

1 Introduction

The classical theory of (first and third-degree) price discrimination rests on three pillars: market power, the ability to segment consumers, and limits on arbitrage. Remove any single pillar, and it is believed that price discrimination is unsustainable. The focus of this paper is on the third pillar of costly arbitrage or resale. Textbooks unambiguously claim that costless resale collapses discriminatory pricing to a uniform price (Varian, 1989; Tirole, 1988). The claim in the present paper is that there exists an economically plausible exception to this theoretical presumption. It is shown that under certain preference conditions, a monopoly provider of a product can engage and find it profitable to engage in price discrimination even when resale amongst its customers is possible and costless. This is done using conventional economic tools that have existed since Samuelson (1947).[1] The result here is purely theoretical, although arguably of potential empirical relevance.

This mechanism stands in contrast to the mechanisms emphasised in the previous literature on price discrimination under resale. Much of the literature has treated resale as an overriding threat to discrimination and has sought mechanisms under which resale does *not* fully arbitrage price differences. Existing explanations typically rely on frictions or institutional constraints outside consumers' underlying valuations (thin secondary markets, transaction and search costs, contractual or technological resale restrictions), or on special features of the seller's revenue environment (e.g., revenue non-concavities) that make randomisation or rationing optimal.

Several strands of work illustrate these approaches. Anderson and Ginsburgh (1994) show how durability and the induced thickness of second-hand markets can be used as a screening instrument. Alger (1999) studies menus when intermediaries can make multiple or joint purchases and shows how partial arbitrage can sharpen, rather than eliminate, discrimination. Gans and King (2007) demonstrate that even with zero transaction costs, uncertainty about matching in thin resale markets can sustain price differences. Calzolari and Pavan (2006) analyse stochastic allocations and strategic information disclosure that soften the bite of resale. Loertscher and Muir (2022) show that when the revenue function under market-clearing pricing is non-concave, the optimal mechanism may involve randomisation and rationing, with resale no longer collapsing prices. In all of these cases, discriminatory pricing survives resale because of additional frictions, institutional constraints, or special features of the seller's revenue environment.

The present paper starts by placing income effects front and centre. Most papers in the

[1] Hence, this is a paper in an older tradition than recent explorations of price discrimination using mechanism and information design; e.g., Bergemann et al. (2015).

field assume income effects away by imposing quasilinear preferences. In canonical models of quality and quantity discrimination (e.g. Mussa and Rosen, 1978; Maskin and Riley, 1984; Stole, 2007), heterogeneity is modelled as differences in tastes and budget constraints do not bind differentially across consumers, so income effects are shut down by assumption. More recent contributions that take income heterogeneity seriously still do not use it to support discrimination with resale. Nie et al. (2021) study durable goods with secondary markets and discrete wealth types, showing how wealth distributions shape new and used prices. Galera et al. (2019) analyse non-quasilinear preferences and show that discriminatory pricing can increase aggregate utility even when total output is fixed. These papers illustrate that income effects can overturn standard welfare conclusions, but they do not generate a mechanism through which *costless resale* coexists with systematic price discrimination.

This paper takes a different route. We show that *income effects alone* are sufficient to sustain discriminatory pricing under frictionless resale. We consider a monopolist selling an indivisible good to consumers with heterogeneous incomes who can freely resell the good on a competitive secondary market. Departing from the quasilinear benchmark, we assume consumers have general preferences over the monopoly good and a numeraire. When the good is strictly normal, the reservation price to give it up is increasing in disposable income and hence depends on the purchase price. A subsidy that lowers the purchase price leaves the buyer wealthier and increases their utility from the good, raising their reservation value. In other words, a lower price makes resale *less* attractive for that buyer, not more. The opportunity cost of resale becomes endogenous to the initial price, and this endogenous reservation price acts as a "firewall" against arbitrage that requires no external enforcement.

The monopolist exploits this by strategically subsidising low-income consumers to raise their reservation price to a common target level. High-income consumers are then charged exactly that price, while low-income consumers pay lower type-specific prices that leave their reservation price equal to the targeted reservation price. In the subsidised region, the optimal price schedule increases one-for-one with income: keeping the reservation price constant requires keeping disposable income constant. Relative to the existing literature on resale, our mechanism (i) does not rely on thin markets, transaction costs, or legal restrictions as in Anderson and Ginsburgh (1994); Gans and King (2007); (ii) does not require non-concavity of the revenue function or randomisation, in contrast to Loertscher and Muir (2022); and (iii) operates through standard income effects in otherwise frictionless secondary markets. Relative to the quasilinear screening literature, we show that once income effects are admitted, the usual conclusion that costless resale forces uniform pricing need not hold.

The paper proceeds as follows. A two-income example (Section 2) introduces the core mechanism and delivers the first main result. The monopolist "invests" in low-income consumers—charging them below their willingness to pay—to raise their reservation prices, which then caps the price that can be charged to the rich. This investment pays off when the low-income share is not too large: discrimination then weakly dominates uniform pricing. An interesting comparative-static finding is that as the rich become wealthier, their willingness to pay rises, and the monopolist must *deepen* the subsidy to the poor to keep the arbitrage constraint satisfied—so greater inequality can actually benefit low-income consumers.

The general model (Section 3) allows for a continuum of income types and uncovers a remarkable structure in the optimal price schedule. In the subsidised region, prices rise exactly one-for-one with income, because maintaining a constant reservation price across consumers requires maintaining constant disposable income. Above a threshold, all consumers pay a uniform target price. This schedule achieves a second main result: under strong income effects and a thin left tail of very poor consumers, the monopolist serves the entire market and attains the first-best allocation, eliminating the exclusionary distortion usually associated with monopoly.

A key theoretical contribution (Section 4) is to show that income effects are sufficient but not necessary for the mechanism. Drawing on Hanemann (1991)'s analysis of willingness-to-accept versus willingness-to-pay gaps in the environmental value literature, we derive a sufficient statistic—a "firewall index"—that measures how much a one-dollar subsidy raises the reservation price. This index can be large for two distinct reasons: high income sensitivity of demand, *or* low substitutability with private-market alternatives. The latter channel is especially important for ecosystem goods, life-saving medicines, and entitlements with few close substitutes, where willingness to accept compensation can far exceed willingness to pay even when income effects are modest. Four empirical applications (Section 5) illustrate the mechanism's reach: international pharmaceutical pricing, live-event ticketing, inclusionary housing, and public food distribution. These cases show when the mechanism sustains price differences—limited substitutes and effective quantity enforcement—and when it fails, as in food rationing with high substitutability and weak enforcement.

The robustness of key assumptions is explored in Section 6. The main model assumes perfect observability of consumer incomes. With noisy income signals, the mechanism survives moderate leakage: strong income effects make discrimination not only feasible but resilient to imperfect information. A key assumption in the model is that the monopolist can restrict sales to individual consumers (specifically, low-income ones). The treatment of quantity limits yields a conceptually important insight. Without consumer-specific purchase restrictions, the secondary market unravels: the consumer receiving the deepest subsidy can acquire unlimited units and become the market's residual supplier, collapsing prices to the bottom of the schedule. Sustaining discrimination, therefore, requires that the monopolist restrict *each subsidised consumer* to a single unit. This reframes the role of market power. Conventionally, market power means quantity restrictions at the aggregate level. Here, market power exercised at the individual level, which is feasible when the seller can identify consumers, emerges as the essential foundation for discrimination under costless resale.

The next two sections consider extensions to other environments of interest. Section 7 examines an economy with multiple monopolists and shows that the burden of subsidisation is shared across firms: the slope of the price schedule in the subsidised region falls as more products enter. Whether this raises or lowers welfare depends on whether the goods are complements or substitutes. Section 8 endogenises information acquisition and shows that low-income consumers must disclose data to access subsidised prices, and their disclosure decisions generate pecuniary externalities across income groups. In contrast to settings where competition drives excessive data disclosure, we find that equilibrium data sharing is *inefficiently low*: the marginal discloser ignores the profit externality their participation confers on the monopolist, which would have enabled lower prices throughout the market. Section 9 concludes.

2 A Two-Income Example

To build intuition for the general model, consider an economy with two income types. A measure $\alpha \in (0,1)$ of consumers have low income I_L, and a measure $1-\alpha$ have high income $I_H > I_L$. All agents in the economy know the income of each consumer, which is their only source of heterogeneity. The monopolist sells a single indivisible good at zero marginal cost. Preferences are CES over the monopoly good $q_M \in \{0,1\}$ (for which a consumer with income I_i pays a price $P(I_i)$) and a composite good q_C with price normalised to one:

$$U(q_M, q_C) \;=\; \big(\beta\, q_M^{\rho} + (1-\beta)\, q_C^{\rho}\big)^{1/\rho}. \tag{1}$$

We assume $0 < \beta < 1$ and $0 < \rho < 1$. The elasticity of substitution between q_M and q_C is $\sigma = \frac{1}{1-\rho}$, which is greater than 1 given our assumptions on ρ. Because $\sigma = 1/(1-\rho)$, larger σ (equivalently larger ρ) corresponds to weaker income effects in this CES example.

For a consumer of type $i \in \{L, H\}$, willingness to pay $W(I_i)$ is defined by $U(1, I_i - W(I_i)) = U(0, I_i)$. Substituting from the budget constraint that holds with equality at the optimum, the WTP simplifies to

$$W(I_i) = I_i - \Big(I_i^{\rho} - \tfrac{\beta}{1-\beta}\Big)^{1/\rho}. \tag{2}$$

We assume incomes are large enough such that $W(I_i)$ is well-defined (i.e., $< I_i$ requiring $I_i^\rho > \beta/(1-\beta)$). Under $0 < \rho < 1$, the good is strictly normal, so $W(I_H) > W(I_L)$.

A consumer with income I_i has a reservation price, which is the price they are willing to accept to sell the product to another consumer rather than consume it themselves. The reservation price $R(I_i, P(I_i))$ solves $U(1, I_i - P(I_i)) = U\big(0, I_i - P(I_i) + R(I_i, P(I_i))\big)$ and is given by

$$R(I_i, P(I_i)) = \Big((I_i - P(I_i))^\rho + \tfrac{\beta}{1-\beta}\Big)^{1/\rho} - \big(I_i - P(I_i)\big). \tag{3}$$

The core insight of the paper is that this reservation price depends on the purchase price, $P(I_i)$. Differentiating (3) with respect to $P(I_i)$ yields:

$$\frac{\partial R(I_i, P)}{\partial P(I_i)} = 1 - \left[\tfrac{(I_i - P(I_i))^\rho}{(I_i - P(I_i))^\rho + \beta/(1-\beta)}\right]^{\frac{\rho-1}{\rho}}. \tag{4}$$

The mechanism relies on $\frac{\partial R}{\partial P(I_i)} < 0$: a lower price (a subsidy) increases disposable income. Since the monopoly good is normal, this discount increases the consumer's reservation price (i.e., the minimum price they require to give up the good). In the CES example, under our maintained restriction $0 < \rho < 1$ (equivalently $\sigma > 1$), this sign condition holds throughout: the exponent $(\rho-1)/\rho$ is negative, so $\frac{\partial R(I_i,P)}{\partial P(I_i)} < 0$. At the boundary $\rho \to 1$ (the linear/perfect-substitutes limit in this CES family), income effects vanish and $\frac{\partial R}{\partial P} \to 0$, so the mechanism breaks down.[2]

2.1 Monopoly pricing outcomes

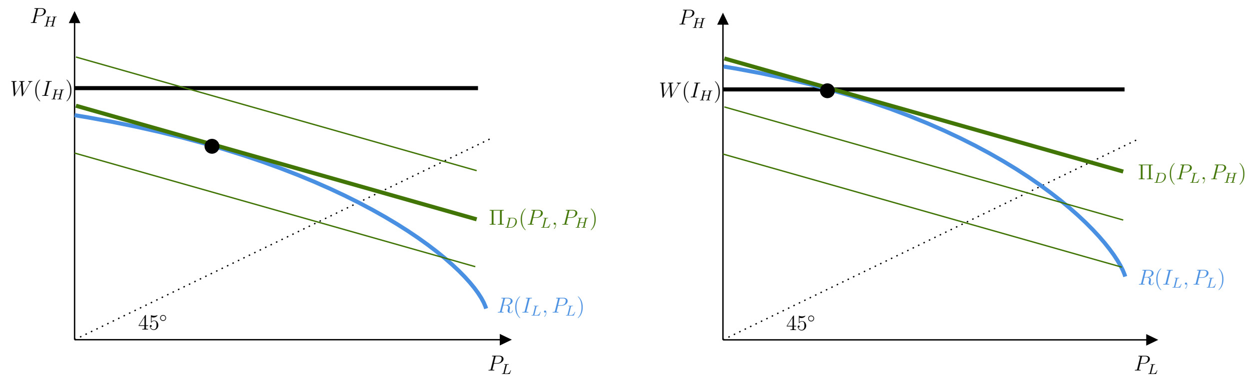
The monopolist can either set a single uniform price P_U or discriminate by income. Under uniform pricing, the optimal profit is standard: $\Pi_U^* = \max\big\{W(I_L),\, (1-\alpha)W(I_H)\big\}$.

Under discriminatory pricing, the monopolist chooses prices (P_L, P_H) to maximise profit subject to the following constraints:

$$\begin{aligned} P_H &\le R(I_L, P_L) \quad \text{(No-Arbitrage (Resale) Constraint)} \\ P_H &\le W(I_H) \quad \text{(High-income Participation Constraint)} \\ P_L &\le W(I_L) \quad \text{(Low-income Participation Constraint)} \\ 0 &\le P_L, P_H \quad \text{(Non-negativity)} \end{aligned}$$

The high-income price is, therefore, bounded by $P_H \le \min\{R(I_L, P_L),\, W(I_H)\}$. The low-

[2]For $0 < \rho < 1$, the derivative in (4) can be written $1 - f^{(\rho-1)/\rho}$, where f is the fraction inside the brackets and satisfies $0 < f < 1$. Since the base is less than one, $f^{(\rho-1)/\rho} > 1$ holds exactly when the exponent $(\rho-1)/\rho$ is negative; hence $\frac{\partial R}{\partial P} < 0$ for all $0 < \rho < 1$.



(a) No-Arbitrage Constraint Binding

(b) Participation Constraint Binding

Figure 1: Two-Income Equilibrium Types

income participation constraint makes explicit that we are studying discriminatory outcomes in which low-income consumers actually purchase; if it fails, the problem collapses to exclusionary uniform pricing. These constraints embed an assumption that the monopolist *can restrict sales to at most one unit per consumer*. The assumption of consumer-specific quantity restrictions is maintained throughout the paper, and the impact of relaxing it is explored in Section 6.

The monopolist sets P_H to bind the effective ceiling and, thus, chooses P_L to maximise:

$$\Pi_D(P_L) = \alpha\, P_L \;+\; (1-\alpha)\min\bigl\{R(I_L, P_L),\, W(I_H)\bigr\}. \tag{5}$$

The optimal solution depends on which constraint binds.

Figure 1 depicts these constraints in (P_L, P_H). Two cases are depicted. In (a), only the no-arbitrage constraint binds for all P_L. Note that it is decreasing in P_L but also concave with:

$$\frac{\partial^2 R(I_i, P)}{\partial P(I_i)^2} = \frac{(\rho-1)\beta}{1-\beta} \cdot \frac{1}{(I_i - P)^{2-\rho}} \cdot \frac{1}{\left[(I_i - P)^{\rho} + \frac{\beta}{1-\beta}\right]^{\frac{2\rho-1}{\rho}}} < 0. \tag{6}$$

where the inequality follows as $\rho < 1$. As ρ increases (equivalently, as σ increases and income effects weaken), $R(I_i, P)$ falls and its slope becomes flatter. In (b), where ρ is relatively low (i.e., there are strong income effects), the participation constraint binds for low P_L. Also depicted in each is the iso-profit lines whose slope depends on α (with higher α implying steeper iso-profit lines).

Figure 1a shows a situation where income effects are moderate or inequality is not extreme. In this case, the resale constraint binds before the participation constraint ($R(I_L, P_L) <$

$W(I_H)$). The monopolist sets $P_H = R(I_L, P_L)$ and the first-order condition is:

$$\frac{d\Pi_D}{dP_L} = \alpha + (1-\alpha)\frac{\partial R}{\partial P_L}\Big(I_L, P_L\Big) = 0. \tag{7}$$

If this equilibrium exists, the optimal low price P_L^* is given by:

$$P_L^* = \max\{I_L - \left[\tfrac{\beta}{1-\beta}\left(\tfrac{1}{(1-\alpha)^{1-\sigma}-1}\right)\right]^{\frac{\sigma}{\sigma-1}}, 0\}. \tag{8}$$

Substituting this back into the reservation price function yields the optimal high price:

$$P_H^* = \max\{R(I_L, P_L^*), P_L^*\} = \max\{\left[\tfrac{\beta}{1-\beta}\left(\tfrac{1}{(1-\alpha)^{1-\sigma}-1}\right)\right]^{\frac{\sigma}{\sigma-1}}\left((1-\alpha)^{-\sigma}-1\right), P_L^*\}.^{3} \tag{9}$$

This solution involves *strict* discriminatory pricing (i.e., $P_H^* > P_L^*$) if and only if $P_H^* \in (P_L^*, W(I_H)]$, which requires $\alpha < \bar{\alpha}$. The boundary case $\alpha = \bar{\alpha}$ yields $P_H^* = P_L^*$ (uniform pricing) and defines $\bar{\alpha}$ implicitly by:

$$\frac{\beta}{1-\beta} = I_L^{\frac{\sigma-1}{\sigma}}(1-\bar{\alpha})^{\sigma-1}\left[(1-\bar{\alpha})^{1-\sigma}-1\right]$$

which is derived by setting $P_H^* = P_L^*$ and simplifying.[4]. Note that $P_L^* > 0$ if:

$$\alpha > \underline{\alpha} = 1 - \left[\frac{(1-\beta)I_L^{\frac{\sigma-1}{\sigma}} + \beta}{(1-\beta)I_L^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{1-\sigma}}.$$

If $\alpha < \underline{\alpha}$, then $P_L^* = 0$ and $P_H^* = R(I_L, 0) = \left(I_L^{\frac{\sigma-1}{\sigma}} + \frac{\beta}{1-\beta}\right)^{\frac{\sigma}{\sigma-1}} - I_L$ so the outcome still involves price discrimination.[5] Therefore, when the no-arbitrage constraint binds (i.e., $R(I_L, P_L) \leq W(I_H)$ for all P_L), discriminatory pricing is feasible so long as $\alpha < \bar{\alpha}$ (with $\alpha = \bar{\alpha}$ delivering uniform pricing).

By contrast, if income effects are very strong or inequality is high (as in Figure 1b), the interior solution P_H^* derived in (9) may exceed $W(I_H)$. Since the monopolist cannot charge more than the high types' valuation, the price is capped at $P_H = W(I_H)$.

[3]In the interior resale-constrained regime we have $R(I_L, P_L^*) > P_L^*$, so (9) reduces to $P_H^* = R(I_L, P_L^*)$. The $\max\{\cdot\}$ operator only matters at the boundary $\alpha = \bar{\alpha}$, where $R(I_L, P_L^*) = P_L^*$ and discriminatory pricing collapses to uniform pricing.

[4]Note that this implicit condition is equivalent to the closed-form threshold $\bar{\alpha}$ provided in Proposition 1 using $\rho = 1 - 1/\sigma$) below

[5]Throughout, α denotes the population share of low-income consumers. The symbols $\bar{\alpha}$ (defined below) and $\underline{\alpha}$ are parameter-dependent cutoffs: $\bar{\alpha}$ is the profitability threshold at which discrimination collapses to inclusive uniform pricing, and $\underline{\alpha}$ is the non-negativity threshold below which the optimal low-income price hits $P_L^* = 0$.

In this regime, the monopolist re-optimises P_L. Since the high price is fixed at $W(I_H)$, there is no marginal benefit to lowering P_L further to raise the reservation price. Instead, the monopolist should *raise* P_L (reduce the subsidy) to capture more revenue from low types, up to the point where the resale constraint exactly matches the high income willingness to pay. The optimal low price $\tilde{P}_L$ solves:

$$R(I_L, \tilde{P}_L) = W(I_H). \tag{10}$$

Because $\partial R/\partial P < 0$, we have $\tilde{P}_L > P_L^*$. The monopolist achieves perfect surplus extraction from the high types while extracting the maximum feasible revenue from the low types, consistent with segmentation.

2.2 Equilibrium Characterisation

The equilibrium outcome is determined by the interplay between the profitability of discrimination, the constraints imposed by resale, the consumers' willingness to pay (Participation Constraints), and the non-negativity constraint on prices (particularly, $P_L \geq 0$). The monopolist chooses the strategy that maximises global profit, comparing discriminatory pricing with uniform pricing (inclusionary or exclusionary).

We first define key terms that characterise the unconstrained Resale-Constrained (RC) optimum, which solves the first-order condition (7), ignoring the participation constraint $W(I_H)$ and the non-negativity constraint $P_L \geq 0$. Let $\mathcal{D}(\alpha)$ denote the optimal disposable income of low types in this theoretical case:

$$\mathcal{D}(\alpha) \equiv \left[\frac{\beta}{1-\beta}\frac{1}{(1-\alpha)^{1-\sigma}-1}\right]^{\frac{\sigma}{\sigma-1}}. \tag{11}$$

The corresponding prices are $P_L^{RC} = I_L - \mathcal{D}(\alpha)$ and $P_H^{RC} = R(I_L, P_L^{RC})$. The resulting profit is Π_D^{RC}.

We also define the maximum achievable reservation price when the non-negativity constraint binds:

$$\mathcal{R}_0(I_L) \equiv R(I_L, 0) = \left(I_L^{\frac{\sigma-1}{\sigma}} + \tfrac{\beta}{1-\beta}\right)^{\frac{\sigma}{\sigma-1}} - I_L. \tag{12}$$

This is the highest price the monopolist can sustain if they are restricted to $P_L \geq 0$.

We can now state the necessary and sufficient conditions for discriminatory pricing to be the unique globally optimal strategy.

Proposition 1 (Characterisation of Optimal Discriminatory Pricing)**.** *The unique global equilibrium involves discriminatory pricing ($P_H^* > P_L^*$) if and only if the following two con-*

ditions on model primitives hold:

1. *Price discrimination is profitable (α small relative to substitution):*

$$\alpha < \bar{\alpha} \equiv 1 - \left[1 - \frac{\beta}{(1-\beta)\, I_L^{\frac{\sigma-1}{\sigma}}}\right]^{\frac{1}{\sigma-1}}. \tag{13}$$

2. *Price discrimination is dominant (Strictly preferred to exclusion): One of the following must hold:*

   (a) *Moderate Inequality (Reachable WTP): The high-income WTP is strictly reachable with positive prices.*

$$W(I_H) < \mathcal{R}_0(I_L). \tag{14}$$

   (b) *High Inequality (Dominant RC Equilibrium): The high-income WTP is not strictly reachable ($W(I_H) \geq \mathcal{R}_0(I_L)$), but the interior RC equilibrium is feasible for serving low types (i.e., $0 < P_L^{RC} \leq W(I_L)$) and yields strictly higher profits than exclusion.*

$$\Pi_D^{RC} > (1-\alpha)W(I_H). \tag{15}$$

The proofs of all results are in the appendix. The two conditions in Proposition 1 capture distinct economic forces. The Profitability Condition governs the choice between discrimination and serving everyone at a low uniform price. The Dominance Condition governs the choice between discrimination and excluding the poor entirely. Together, they define the parameter region where the subsidy mechanism is both effective and worthwhile.

The subsidy mechanism works by transferring surplus from low-income to high-income transactions. The monopolist "invests" in the poor—charging them below their willingness to pay—in order to raise their reservation price, which determines what the rich must pay. This investment is profitable only if the return (higher revenue from the rich) exceeds the cost (foregone revenue from the poor).

Condition (13) formalises this trade-off. When α is large, the subsidy cost is high (many consumers receive below-WTP prices) while the benefit is limited (few consumers pay the elevated price). As α approaches the threshold $\bar{\alpha}$, the marginal cost of subsidising one more poor consumer exactly equals the marginal benefit of raising the price to rich consumers, and the mechanism loses its profitability. Beyond this threshold, the monopolist prefers to abandon discrimination entirely and serve everyone at the inclusionary uniform price $W(I_L)$.

The threshold $\bar{\alpha}$ increases with the strength of income effects (lower σ). Intuitively, stronger income effects mean that each dollar of subsidy produces a larger increase in the reservation price, improving the return on the monopolist's investment. In the limit as $\sigma \to \infty$ (quasi-linear preferences), $\bar{\alpha} \to 0$: income effects vanish, the subsidy mechanism becomes ineffective, and discrimination is never profitable.

Even when discrimination is profitable relative to inclusionary pricing, the monopolist might prefer to exclude the poor and charge a high uniform price $W(I_H)$. Condition 2 ensures this alternative is dominated. The economic logic differs depending on the severity of inequality.

Moderate Inequality (Case 2a). When $W(I_H) < \mathcal{R}_0(I_L)$, the income-effect "firewall" is strong enough to fully protect the high price. The monopolist can raise the reservation price of low-income consumers all the way to $W(I_H)$ while still charging them a strictly positive price. This guarantees that discrimination dominates exclusion: the monopolist collects the same $(1-\alpha)W(I_H)$ from high-income consumers as under exclusion, plus additional revenue $\alpha P_L^* > 0$ from low-income consumers. (Even if the unconstrained optimum involves a lower high price, profit is strictly higher because the Participation-Constrained solution is always feasible). The only remaining question is whether the equilibrium is Participation-Constrained (the monopolist extracts full surplus from the rich, $P_H^* = W(I_H)$) or Resale-Constrained (the monopolist optimally sets $P_H^* < W(I_H)$ because the marginal cost of deeper subsidies exceeds the marginal benefit).

High Inequality (Case 2b). When $W(I_H) \geq \mathcal{R}_0(I_L)$, the firewall has a ceiling. Even if the monopolist gives the good away free to the poor ($P_L = 0$), their reservation price cannot reach $W(I_H)$. Full surplus extraction from the rich is impossible under discrimination. The monopolist faces a genuine trade-off: exclusion sacrifices all revenue from the poor but extracts full surplus from the rich; discrimination serves both groups but requires accepting a lower price from the rich.

Discrimination dominates in this high-inequality case only if two conditions hold. First, the unconstrained optimum must involve a positive low price ($P_L^{RC} > 0$); otherwise, the best discriminatory outcome yields profit $(1-\alpha)\mathcal{R}_0(I_L) < (1-\alpha)W(I_H)$, which is dominated by exclusion. Second, the discriminatory profit Π_D^{RC} must strictly exceed the exclusionary profit $(1-\alpha)W(I_H)$. This comparison depends on parameters: when α is very small, the revenue from the poor is negligible and the price concession to the rich is costly, favouring exclusion. When α is moderate (but below $\bar{\alpha}$), the inclusion gains outweigh the extraction losses, favouring discrimination.

Limits of the Income-Effect Mechanism. The income-effect mechanism that enables discrimination has inherent limits. As I_H rises holding I_L fixed, $W(I_H)$ increases while $\mathcal{R}_0(I_L)$ remains constant. Eventually, $W(I_H)$ exceeds the maximum reservation price the monopolist can induce from low-income consumers, even with maximal subsidisation ($P_L = 0$).

At this corner, discrimination yields zero revenue from the poor and $(1-\alpha)\mathcal{R}_0(I_L)$ from the rich. Exclusion yields $(1-\alpha)W(I_H)$. Since $W(I_H) > \mathcal{R}_0(I_L)$, exclusion strictly dominates this corner outcome. The monopolist might still prefer an interior solution with $P_L^{RC} > 0$, but as inequality widens, the foregone extraction from the rich increasingly outweighs the inclusion gains from the poor.

Thus, while income effects create a "firewall" against arbitrage, this firewall has a ceiling determined by I_L and the strength of income effects. Extreme inequality pushes $W(I_H)$ above this ceiling, causing the mechanism to break down and forcing the monopolist back to exclusionary uniform pricing.

The focus on income effects is specific to the CES example explored in this section. Section 3 generalises the mechanism and isolates the sufficient statistic $\Gamma(I, P)$ under preference conditions that imply $\partial R/\partial P < 0$. Section 4 then shows how $\Gamma(I, P) > 1$ can arise from broader primitives (e.g., limited substitutability, complementarities, and asymmetric fallback options), even when "large income effects" are not the most natural description of demand.

The equilibrium transitions smoothly across regimes as parameters vary:

- *Increasing α (more poor consumers):* For small α, the equilibrium is Participation-Constrained—the monopolist extracts full surplus from the rich. As α rises, the equilibrium transitions to Resale-Constrained—the subsidy cost induces a lower target price. At $\alpha = \bar{\alpha}$, the mechanism collapses to inclusionary uniform pricing.

- *Increasing inequality (I_H rising, I_L fixed):* For moderate inequality, the equilibrium may be PC or RC depending on α. As I_H rises past the threshold where $W(I_H) = \mathcal{R}_0(I_L)$, the PC regime disappears entirely. Further increases in I_H shrink the region where discrimination dominates exclusion, until eventually only exclusionary uniform pricing survives.

- *Strengthening income effects (lower σ):* Stronger income effects expand the discrimination region on both margins—raising $\bar{\alpha}$ (favouring discrimination over inclusion) and raising $\mathcal{R}_0(I_L)$ relative to $W(I_H)$ (favouring discrimination over exclusion).

These transitions highlight the subtle nature of the subsidy mechanism. Its effectiveness depends on a delicate balance: income effects must be strong enough to create a meaningful firewall, inequality must be moderate enough that the firewall can reach the target, and the poor must be scarce enough that subsidising them is worthwhile. When any of these conditions fail, the mechanism unravels and uniform pricing prevails. That said, in Section 4 it is demonstrated that in a more general framework, the centrality of strong income effects is not required to sustain price discrimination.

2.3 The Degree of Price Discrimination

We now analyse how the extent of price discrimination varies with the underlying parameters. We define the *Degree of Price Discrimination* (DPD) as the spread between the prices paid by the high- and low-income consumers:

$$\Delta P^* \equiv P_H^* - P_L^*. \tag{16}$$

A higher ΔP^* implies a more aggressive use of income effects to segment the market. The behaviour of ΔP^* depends critically on whether the equilibrium is in the Resale-Constrained regime (interior solution) or the Participation-Constrained regime (corner solution).

In the CES case, the sign of $\partial \Delta P^*/\partial \sigma$ can flip in a knife-edge region where the baseline composite-good term is very small. To avoid carrying unilluminating case distinctions in the example, we state the comparative static for the parameter region in which

$$(1-\beta) I_L^{\frac{\sigma-1}{\sigma}} > 1. \tag{17}$$

For any σ satisfying (17), weaker income effects (higher σ) compress the sustainable price gap in the interior regime, as stated in Proposition 2. This restriction is only used for the CES comparative statics; the general-model (Section 3) results do not rely on it.

Case 1: The Interior Regime ($P_H^* < W(I_H)$). When inequality is moderate, or the low-income share α is significant, the monopolist optimises against the resale constraint without hitting the high-income participation cap.

Proposition 2 (Discrimination in the Interior Regime)**.** *In the Resale-Constrained equilibrium, the Degree of Price Discrimination ΔP^*:*

1. *Decreases in the population share of low-income consumers (α).*
2. *Decreases in the elasticity of substitution (σ) under condition* (17)*.*
3. *Is independent of high income (I_H).*

In this regime, the "target" reservation price R^* is determined solely by the cost of subsidising the low-income group. As α rises, subsidisation becomes more expensive; the monopolist reduces the subsidy (raising P_L), which lowers the achievable reservation price P_H. Consequently, the price spread compresses. Similarly, as σ increases (approaching quasi-linearity), income effects weaken, rendering the subsidy mechanism less effective and compressing prices toward uniformity.

Case 2: The Corner Regime ($P_H^* = W(I_H)$). When inequality is high or income effects are very strong, the mechanism is so effective that the endogenous reservation price would exceed the high-income WTP. The monopolist is capped at $P_H = W(I_H)$.

Proposition 3 (Discrimination in the Corner Regime)**.** *In the participation-constrained equilibrium, the equilibrium prices are:*

$$P_H^* = W(I_H), \quad P_L^* = \max\{P^*(I_L, W(I_H)), 0\}.$$

Consequently:

1. $P_H^* = W(I_H)$ *is increasing in* I_H*. Under CES preferences it is strictly decreasing in* σ *(i.e., it increases with the strength of income effects).*

2. *If* $P_L^* > 0$ *(non-negativity does not bind), then*

$$P_L^* = W(I_H) - (I_H - I_L) \quad \textit{and} \quad \Delta P^* \equiv P_H^* - P_L^* = I_H - I_L,$$

   which is independent of σ*. If instead* $P_L^* = 0$ *(non-negativity binds), then* $\Delta P^* = W(I_H)$*, and under CES it is strictly decreasing in* σ*.*

In the corner solution, the high price is fixed at $W(I_H)$. To sustain this price against arbitrage, the monopolist must ensure $R(I_L, P_L) = W(I_H)$. As I_H increases, $W(I_H)$ rises. To match this higher resale value, the low-income consumer must be made wealthier (and thus more attached to the good). The monopolist achieves this by *lowering* P_L (increasing the subsidy). Thus, as the rich get richer, the poor pay less, and the price gap ΔP^* widens.

2.4 Welfare Analysis

We evaluate the welfare implications of the subsidy mechanism relative to standard uniform pricing. We consider Total Surplus (TS) and Consumer Surplus (CS) for both the high-income (CS_H) and low-income (CS_L) groups.

2.4.1 Total Surplus and Efficiency

Because the good is indivisible and each consumer demands at most one unit, and because marginal cost is zero, total surplus equals the sum of valuations of consumers who are served. In the standard uniform pricing model, the monopolist chooses between an *Inclusive Uniform Price* ($P = W(I_L)$, serving all) and an *Exclusive Uniform Price* ($P = W(I_H)$, serving only $1 - \alpha$).

Proposition 4 (Efficiency of Discriminatory Pricing). *In the discriminatory equilibrium, total surplus*

1. *is the first best ($TS_D = TS_{FB}$).*
2. *strictly dominates Exclusive Uniform pricing in Total Surplus ($TS_D > TS_U^{Exc}$).*
3. *is welfare-equivalent to Inclusive Uniform pricing ($TS_D = TS_U^{Inc}$), though with different distribution.*

The subsidy mechanism eliminates the deadweight loss associated with standard monopoly exclusion. By decoupling the prices, the monopolist can extract high value from the rich without pricing out the poor.

2.4.2 Distributional Welfare and Consumer Surplus

While Total Surplus is maximised, the distribution of welfare varies sharply by regime. First, in the interior solution, the monopolist charges $P_H^* < W(I_H)$. High-income consumers enjoy a positive surplus ($CS_H > 0$) because the presence of resale prevents the monopolist from extracting their full valuation. Low-income consumers receive a subsidy $P_L^* < W(I_L)$, ensuring $CS_L > 0$. Paradoxically, the high-income consumers benefit from the existence of the low-income group: the "threat" of resale by the poor forces the monopolist to keep the high price below $W(I_H)$.

Second, in the corner solution, $P_H^* = W(I_H)$, so $CS_H = 0$. The monopolist extracts all surplus from the high-income group. However, to sustain this high price, the monopolist must deepen the subsidy to the low-income group relative to the interior resale-constrained outcome. Thus, moving from the interior to the corner regime (e.g., via rising inequality) transfers welfare from high-income consumers to the monopolist and potentially to low-income consumers.

Corollary 1 (The Inequality Paradox). *In the Participation-Constrained regime, an increase in I_H (holding I_L constant) increases the Consumer Surplus of the low-income group (CS_L).*

As the rich become wealthier, their WTP rises. To prevent arbitrage, the monopolist must raise the reservation price of the poor to match this new level. This requires increasing the subsidy (lowering P_L), thereby making the low-income group better off.

3 The General Model

We now generalise the analysis to a continuum of income types. Let consumers' incomes be drawn from a distribution $F(I)$ on $[\underline{I}, \bar{I}]$, with density $f(I) > 0$ on $(\underline{I}, \bar{I})$. We assume F has a continuous density $f(I) > 0$ on a compact support $[\underline{I}, \bar{I}]$ with $0 < \underline{I} < \bar{I} < \infty$. We assume standard regularity conditions (e.g., monotone hazard rate or log-concavity of $1 - F(I)$) hold to ensure quasi-concavity of the profit function. The monopolist sells the good at zero marginal cost and can offer a discriminatory schedule $P(I)$, subject to the constraint that $P(I) \geq 0$ for all I.

This generalisation, while of independent interest, provides a set of results that involve interior price discrimination but, more interestingly, also demonstrate that exclusion can occur under price discrimination, adding richer welfare implications that do not arise in the two-income case. Importantly, it allows us to derive a sufficient statistic for the mechanism here that, in turn, greatly enriches the possible mechanisms and preference conditions beyond strong income effects that are the initial focus; see Section 4 below.

3.1 Model setup

We use the indirect utility function $V(q_M, I')$ to represent the maximum utility a consumer can achieve with consumption of the monopoly good $q_M \in \{0, 1\}$ and disposable income I'. We impose the following regularity conditions.

Assumption 1 (Regularity, normality, and marginal utility)**.** The indirect utility function $V(q_M, I')$ is twice continuously differentiable and strictly increasing in I'.

1. *Strict normality.* The monopoly good is strictly normal: $I' \mapsto V(1, I') - V(0, I')$ is strictly increasing. Hence, wealthier consumers value the good more highly.

2. *Non-increasing marginal utility of income.* $V(q_M, \cdot)$ is weakly concave in I' for each q_M (equivalently, $\frac{\partial^2 V}{\partial I'^2} \leq 0$).

Remark 1 (CES as a boundary case)**.** The CES utility function (1) satisfies Assumption 1 as a boundary case. When $q_M = 1$, the indirect utility $V(1, I') = (\beta + (1-\beta)(I')^\rho)^{1/\rho}$ is strictly concave in I' for $\rho < 1$. When $q_M = 0$, the indirect utility is $V(0, I') = (1-\beta)^{1/\rho} I'$, which is linear in I'—satisfying weak concavity with equality. The key property $\Gamma(I, P) > 1$ (established in Lemma 1) relies on the combination of strict normality and the fact that $\lambda(0, \cdot)$ is non-increasing. In the CES case, $\lambda(0, I') = (1-\beta)^{1/\rho}$ is constant, so the inequality $\lambda(0, I - P + R) \leq \lambda(0, I - P)$ holds with equality. The mechanism remains effective because strict normality alone ensures $\lambda(1, I - P) > \lambda(0, I - P)$.

Strict normality implies both willingness to pay (WTP) and the reservation price are increasing in income. The assumption of non-increasing marginal utility of income (weak concavity) is sufficient, combined with strict normality, to ensure that income effects are present and the subsidy mechanism is potentially effective. These assumptions hold for a broad class of preferences and rule out quasi-linear utility where the mechanism is ineffective.

3.2 WTP, reservation price, and the subsidy mechanism

Define the consumer's WTP and reservation price as follows.

Definition 1 (WTP and Reservation Price)**.** For any income $I \in [\underline{I}, \bar{I}]$, $W(I)$ satisfies $V(1, I - W(I)) = V(0, I)$. For any price $P \geq 0$ and income I for which the consumer buys the good, their reservation price is $R(I, P)$ solving $V\big(1, I - P\big) = V\big(0, I - P + R(I, P)\big)$.

Assumption 1 implies $W(I)$ is strictly increasing in I. Let $\lambda(q_M, I') = \partial V / \partial I'$ denote the marginal indirect utility of income. The key to the subsidy mechanism is how the reservation price responds to the price paid:

Lemma 1. *Under Assumption 1, the reservation price $R(I, P)$ is strictly positive for all $P < W(I)$. Moreover,*

$$\frac{\partial R}{\partial P}(I, P) = 1 - \frac{\lambda(1, I - P)}{\lambda(0, I - P + R(I, P))} = 1 - \Gamma(I, P), \tag{18}$$

where $\Gamma(I, P) := \lambda(1, I - P)/\lambda(0, I - P + R(I, P))$. Furthermore, $\frac{\partial R}{\partial P} < 0$ and $\frac{\partial R}{\partial I} > 0$.

The term $\Gamma(I, P)$ measures the ratio of the marginal utility of money when the consumer owns the good relative to when they do not. Strict normality ensures $\Gamma(I, P) > 1$, so $\frac{\partial R}{\partial P} < 0$: lowering the price P (a subsidy) increases disposable income, which increases the reservation price. Thus, the monopolist can raise low-income consumers' reservation price by offering subsidies. As will be shown in Section 4, $\Gamma(I, P)$ permits a much richer set of conditions on preferences that can sustain price discrimination.

3.3 Why not use quantity-quality menus?

Before proceeding, we address a natural question: if the monopolist can observe income, why use this subsidy mechanism rather than standard screening instruments such as quantity discounts or quality differentiation?

The answer lies in the nature of the good and the market structure. The subsidy mechanism is particularly relevant when the good is indivisible—as with pharmaceutical doses, concert tickets, and housing units—so quantity-based screening via discounts is infeasible when $q_M \in \{0, 1\}$. It also applies when quality is fixed or non-contractible, either because the good is effectively homogeneous (e.g., generic drugs or standardised event seating) or because the seller wishes to provide uniform quality for equity reasons.[6] Finally, it is suited to settings in which resale cannot be legally prohibited; when resale restrictions are unenforceable (as in secondary ticket markets or international drug arbitrage), the endogenous reservation price provides a natural barrier that legal restrictions cannot. Under these conditions, our mechanism is not an alternative to standard screening—it is the only available instrument. The contribution of this paper is to show that income effects make it effective even when resale is costless.

3.4 The monopolist's problem

The monopolist chooses a pricing policy to maximise profit. A policy consists of a served set $S \subseteq [\underline{I}, \bar{I}]$ and a schedule $P(I)$ for $I \in S$ such that $P(I) \geq 0$. We assume the following:

Assumption 2 (Consumer-specific purchase limits)**.** Each consumer can purchase at most one unit directly from the monopolist at their quoted price. The monopolist can enforce this identity-linked quantity limit.

We maintain Assumption 2 throughout unless stated otherwise.[7]

After direct purchases, a frictionless resale market opens. Because resale is frictionless, any schedule that would allow a low-price buyer to profitably resell to a high-price buyer cannot be sustained. Within S, the schedule must satisfy the endogenous no-arbitrage condition (ENAC):

Definition 2 (Endogenous No-Arbitrage Condition or ENAC)**.** A pricing schedule $P(I)$ on S satisfies the ENAC if for all $I, I' \in S$,

$$P(I') \;\leq\; R\big(I, P(I)\big). \tag{19}$$

Equivalently, letting $\underline{R}(P) \equiv \min_{I \in S} R\big(I, P(I)\big)$, the ENAC can be written as $\max_{I \in S} P(I) \leq \underline{R}(P)$. In particular, it is sufficient to check the highest price against the lowest reservation price. If a consumer is indifferent between consuming the good and reselling it at the prevailing resale price, they do not resell. (Zero-margin resale trades, when they exist, are, therefore, ignored.)

The ENAC states that no buyer pays more than the minimum reservation price of any seller in the market. To maximise revenue given ENAC, the monopolist will choose a target

[6] The mechanism is especially natural when the seller has social objectives: pharmaceutical companies, governments, and non-profits may want to serve low-income consumers while maximising revenue from those who can pay, without introducing separate product lines or degraded versions for the poor.

[7] Section 6.2 studies what happens when it fails.

reservation price $R > 0$ such that the reservation price of all served consumers is at least R. This involves subsidising low-income consumers until their reservation price equals R. To formalise this, define the price function $P^*(I, R)$ that induces a consumer with income I to have a reservation price exactly equal to R.

Lemma 2 (Optimality of Direct Sales)**.** *Any profit-maximising pricing strategy can be implemented using only direct sales to final consumers: given the price schedule, any* potential *resale opportunity is at most zero-margin, and under our tie-breaking convention such zero-margin resale does not occur in equilibrium. Consequently, the optimal pricing strategy must satisfy the ENAC: for any two consumers I and I' in the served set S, $P(I') \leq R(I, P(I))$.*

The ENAC implies that the monopolist effectively crowds out the secondary market. By setting the price schedule such that the purchase price $P(I)$ leaves the consumer with a reservation price $R(I, P(I))$ exactly equal to the market resale price R, the monopolist extracts the maximum feasible revenue. Any third-party intermediary attempting to buy and resell would face zero margin: it must pay at least the seller's reservation value $R(I, P(I))$ and, by the ENAC, cannot resell above R.

Lemma 3. *Fix $R > 0$. There exists a unique function $P^*(I, R)$ defined implicitly by $R\big(I, P^*(I, R)\big) = R$. This function has the following properties:*

1. *$P^*(I, R) \leq R$ if and only if $W(I) \leq R$.*

2. *$P^*(I, R)$ is strictly decreasing in R.*

3. *$P^*(I, R)$ is strictly increasing in I, with derivative $\frac{\partial P^*}{\partial I} = 1$.*

The result that $\partial P^*/\partial I = 1$ is crucial. It means that in the region where the monopolist subsidises consumers to maintain a constant reservation price R, the price schedule increases exactly one-for-one with income. This occurs because maintaining a constant reservation price requires maintaining a constant level of disposable income $(I - P^*(I, R))$.

Since $P^*(I, R)$ increases in I with slope 1, there exists a unique income threshold $I_-(R)$ solving $P^*\big(I_-(R), R\big) = 0$. For incomes below $I_-(R)$, achieving reservation price R requires a negative price. In what follows, we constrain prices to be non-negative.

3.5 Optimal discriminatory schedule with exclusion

Given the non-negativity constraint, the monopolist's problem is to choose a lower cutoff $\hat{I}$ and a target reservation price R to maximise profit, ensuring $\hat{I} \geq I_-(R)$. The served set is $S = [\hat{I}, \bar{I}]$.

Let $I^*(R)$ solve $W(I^*(R)) = R$. Consumers with $I \geq I^*(R)$ are willing to pay at least R. Since the monopolist cannot charge more than the minimum reservation price in the market (which is R), these consumers pay exactly R. Consumers with $\hat{I} \leq I < I^*(R)$ pay the subsidised price $P^*(I, R)$. The price schedule is:

$$P(I) \;=\; \min\bigl\{R, P^*(I, R)\bigr\}, \qquad I \in S. \tag{20}$$

(Note that $P^*(I, R) \geq 0$ for $I \in S$ since $\hat{I} \geq I_-(R)$).

Define the monopolist's profit from serving $[\hat{I}, \bar{I}]$ at target reservation price R as

$$\Pi(\hat{I}, R) = \int_{\hat{I}}^{I^*(R)} P^*(I, R)\, dF(I) \;+\; R\bigl[1 - F(I^*(R))\bigr]. \tag{21}$$

The monopolist solves

$$\max_{R>0,\, \hat{I}} \; \Pi(\hat{I}, R), \tag{22}$$

subject to the constraints $\hat{I} \geq I_-(R)$ and $\hat{I} \leq I^*(R)$ (which implies $R \geq W(\hat{I})$). The following theorem characterises the solution.

Theorem 1 (Optimal discriminatory schedule)**.** *Under Assumption 1 and the assumed regularity conditions on F, the monopolist's optimal strategy is to choose a cutoff $\hat{I}^* \geq I_-(R^*)$ and a target reservation price R^* solving*

$$(\hat{I}^*, R^*) \in \operatorname*{arg\,max}_{\hat{I} \geq I_-(R),\, R \geq W(\hat{I})} \Pi(\hat{I}, R).$$

In the optimal schedule:

1. *Consumers with $I < \hat{I}^*$ are excluded.*
2. *Consumers with $\hat{I}^* \leq I < I^*(R^*)$ pay $P^*(I, R^*) \geq 0$ (with equality at $I = \hat{I}^*$ when the non-negativity constraint binds) and have their reservation price raised to R^*.*
3. *Consumers with $I \geq I^*(R^*)$ pay the uniform price $P(I) = R^*$.*

If $P^*(\underline{I}, R^*) \geq 0$ (equivalently, $\underline{I} \geq I_-(R^*)$, so the non-negativity constraint does not bind at the bottom of the distribution) and $\hat{I}^* = \underline{I}$, full coverage occurs. Otherwise $\hat{I}^* = I_-(R^*) > \underline{I}$, and exclusion occurs.

In the discriminatory equilibrium (Figure 2), the monopolist sets a target reservation price R^* and subsidises low-income consumers until their reservation price reaches R^*. Consumers in the subsidised region pay a price that increases exactly one-for-one with income (Lemma 3); those with incomes above $I^*(R^*)$ pay R^*.

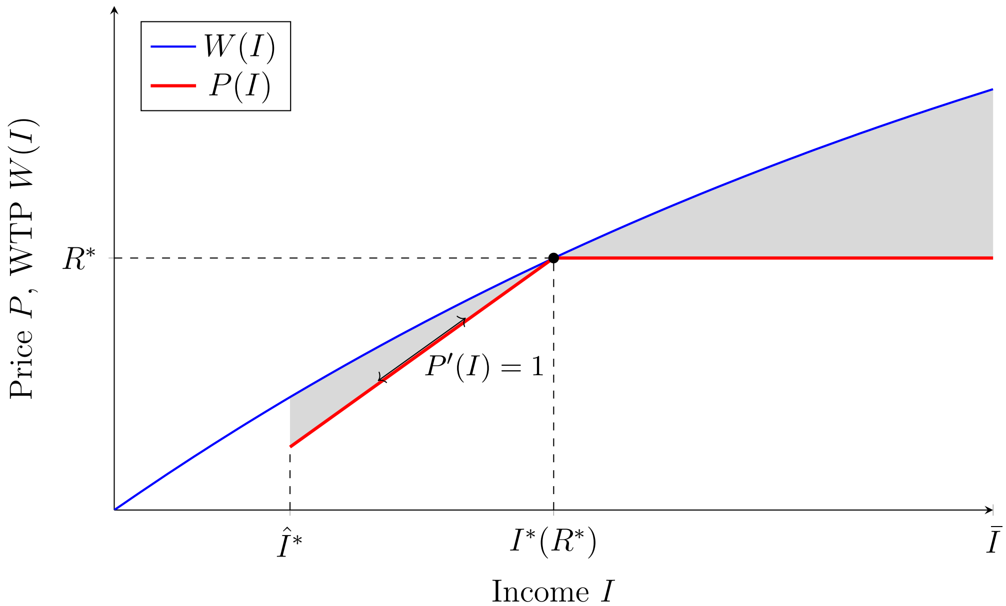


Figure 2: Optimal Discriminatory Pricing Schedule $P(I)$ (red) and Willingness to Pay $W(I)$ (blue). The curves meet exactly at $I^*(R^*)$, where $W(I^*) = R^*$. In the subsidised region $[\hat{I}^*, I^*(R^*)]$, the price increases one-for-one with income ($P'(I) = 1$) and is strictly below $W(I)$. This gap is the strategic subsidy required to raise the reservation price to R^*. The shaded area represents consumer surplus (CS).

3.6 Comparison with uniform monopoly pricing

A uniform pricing (UMP) strategy sets a single price P, serving those with $W(I) \geq P$. Let $I(P)$ solve $W(I(P)) = P$. The monopolist chooses P to maximise $\Pi_U(P) = P\,[1 - F(I(P))]$. Let P_U^* denote the profit-maximising uniform price.

Proposition 5 (Dominance of Discrimination)**.** *Let* $\Pi_D^* = \Pi(\hat{I}^*, R^*)$ *denote the optimal discriminatory profit and let* Π_U^* *denote the optimal uniform profit. The monopolist always weakly prefers the discriminatory strategy:* $\Pi_D^* \geq \Pi_U^*$*.*

Discrimination is strictly preferred if the optimal strategy involves a non-empty subsidised segment ($\hat{I}^* < I^*(R^*)$) and this yields strictly higher profits than P_U^*. This occurs when income effects are sufficiently strong (Γ significantly greater than 1) such that the benefit of raising R^* (by subsidising low-income consumers) outweighs the cost of the subsidy.

The decision to serve all consumers ($\hat{I}^* = \underline{I}$) or exclude the poorest depends on the income distribution and the strength of income effects. Exclusion ($\hat{I}^* > \underline{I}$) occurs if and only if the non-negativity constraint binds at the bottom, i.e. $P^*(\underline{I}, R^*) < 0$ (equivalently, $I_-(R^*) > \underline{I}$). In that case the optimal cutoff is $\hat{I}^* = I_-(R^*)$, so the lowest served type pays zero: $P(\hat{I}^*) = 0$. Exclusion is therefore more likely when income effects are weak (making subsidies less effective) or the income distribution is bottom-heavy (making the subsidy-intensive region large).

3.7 Welfare analysis

We evaluate welfare assuming zero marginal costs. Total Surplus (TS) is the sum of Consumer Surplus (CS) and Profit (Π). With zero marginal costs, TS equals the total utility (WTP) of the consumers served. Under the optimal discriminatory strategy $(\hat{I}^*, R^*)$:

$$\mathrm{TS}_D^* = \int_{\hat{I}^*}^{\bar{I}} W(I)dF(I). \tag{23}$$

Under the optimal Uniform Monopoly Pricing (UMP) P_U^*, the marginal consumer is $I^U = I(P_U^*)$:

$$\mathrm{TS}_U^* = \int_{I^U}^{\bar{I}} W(I)dF(I). \tag{24}$$

The first-best surplus is $TS^{FB} = \int_{\underline{I}}^{\bar{I}} W(I)dF(I)$.

Proposition 6 (Welfare under discrimination)**.** *Let $(\hat{I}^*, R^*)$ be the monopolist's optimal discriminatory strategy.*

1. Efficiency. *If $\hat{I}^* = \underline{I}$ (full coverage), the discriminatory strategy achieves the first-best outcome ($TS_D^* = TS^{FB}$). If $\hat{I}^* > \underline{I}$ (exclusion), there is a deadweight loss equal to $\int_{\underline{I}}^{\hat{I}^*} W(I)dF(I)$.*
2. Comparison with uniform pricing.
   - *Total surplus under discrimination (TS_D^*) is higher than under UMP (TS_U^*) if and only if the discriminatory scheme serves more consumers, i.e., $\hat{I}^* < I^U$.*
   - *The welfare ranking depends on the distribution of income and the strength of income effects.*

We now provide sufficient conditions for welfare improvement.

Proposition 7 (A sufficient condition for strict welfare improvement)**.** *If the optimal discriminatory strategy achieves full coverage ($\hat{I}^* = \underline{I}$) and the optimal uniform monopoly price excludes some consumers ($I^U > \underline{I}$), then total surplus is strictly higher under discrimination:*

$$TS_D^* > TS_U^*.$$

Equivalently, the condition can be stated as $P^*(\underline{I}, R^*) \geq 0$ together with $P_U^* > W(\underline{I})$.

Efficiency among served consumers is guaranteed in both strategies because the good has zero marginal cost and is allocated based on WTP. The welfare comparison hinges entirely on coverage. When income effects are strong (so subsidies raise reservation price sharply) and the mass of very poor consumers is small, the discriminatory scheme achieves higher coverage ($\hat{I}^* < I^U$) and, thus, higher total surplus.

4 Beyond Income Effects

Section 3 shows that with costless resale, the monopolist's ability to sustain systematic price differences hinges on a single preference-based object: how a consumer's reservation price responds to the price they paid. Formally, recall Lemma 1:

$$\frac{\partial R}{\partial P}(I,P) = 1 - \Gamma(I,P), \qquad \Gamma(I,P) := \frac{\lambda(1, I-P)}{\lambda\big(0, I-P+R(I,P)\big)}. \tag{25}$$

Thus, the *required condition* for an endogenous barrier to arbitrage is

$$\frac{\partial R}{\partial P}(I,P) < 0 \quad \Longleftrightarrow \quad \Gamma(I,P) > 1. \tag{26}$$

In words, a marginal subsidy (a reduction in P) must *raise* the consumer's willingness to accept compensation to part with the good (their resale reservation value). This is the sense in which the "firewall" in our model is preference-based: it arises when the marginal value of an extra dollar differs across the ownership and non-ownership states.

A convenient scalar summary is the local *firewall index*

$$\phi(I,P) := -\frac{\partial R}{\partial P}(I,P) = \Gamma(I,P) - 1 > 0. \tag{27}$$

A one-dollar subsidy ($dP = -1$) raises the reservation price by approximately $\phi(I,P)$ dollars. All of the results on sustainable discrimination with frictionless resale can be read through this lens: the larger is ϕ, the cheaper it is for the monopolist to "buy" a high market-clearing resale price by subsidising the relevant marginal group.

The baseline interpretation in this paper is that $\Gamma(I,P) > 1$ is generated by income effects for a strictly normal indivisible good. That interpretation is sufficient, but it is not required. The point of this section is that $\Gamma(I,P) > 1$ can be generated by other primitives that shift the relative marginal utility of income across states, and that Hanemann (1991) provides a unifying way to see how those primitives enter.

4.1 The Hanemann Framework

Because $q_M \in \{0,1\}$, acquiring the monopoly good is a discrete *quantity/quality* change. The relevant welfare objects are therefore the compensating and equivalent variations. Hanemann (1991) studies the classic disparity between willingness to pay (WTP) and willingness to accept (WTA) for such changes and shows that, for quantity/quality changes, the wedge reflects *both* income effects and substitution possibilities. While his motivation was the valuation of environmental/public goods, this is precisely the distinction that matters here, because the no-arbitrage constraint in resale markets is governed by WTA (reservation values), not WTP.

Fix disposable income I'. Define compensating and equivalent variations for the change from $q_M = 0$ to $q_M = 1$ by

$$C(I') \text{ s.t. } V(1, I' - C(I')) = V(0, I'), \tag{28}$$

$$E(I') \text{ s.t. } V(1, I') = V(0, I' + E(I')). \tag{29}$$

Thus $C(I) = W(I)$ coincides with our WTP function in Definition 1, while $E(I')$ is the WTA to give up the good when it is already owned.

For a consumer with income I who paid price P, disposable income is $\mathcal{D} = I - P$. The reservation price in Definition 1 is exactly WTA evaluated at post-purchase wealth:

$$R(I, P) = E(I - P). \tag{30}$$

Differentiating (29) and recalling $\lambda(q_M, I') = \partial V(q_M, I')/\partial I'$ yields

$$E'(I') = \frac{\lambda(1, I')}{\lambda\big(0, I' + E(I')\big)} - 1, \qquad \Rightarrow \qquad -\frac{\partial R}{\partial P}(I, P) = E'(I - P) = \Gamma(I, P) - 1. \tag{31}$$

Equation (31) makes the connection to Lemma 1 explicit: the resale "firewall" is the statement that WTA rises with wealth, $E'(I') > 0$, so that lowering P (a subsidy) raises reservation values and weakens arbitrage pressure.

A common inference from the standard price-change case (e.g., Willig (1976)-type approximations) is that large WTA–WTP gaps require unusually large income effects. Hanemann (1991) shows that for *quantity/quality changes* this inference is incomplete: substitution possibilities are equally fundamental.

To express this using our reduced form, suppose $V(q_M, I')$ is induced by an underlying problem with private goods $x \in \mathbb{R}^N_+$ at prices p and a nonmarket attribute $q \in \{0,1\}$ (here $q = q_M$), so that $V(q, I') = v(p, q, I')$ holding p fixed. Hanemann's analysis highlights three facts.

Proposition 8 (Hanemann (1991))**.** *Consider a change in the nonmarket attribute from* $q = 0$ *to* $q = 1$*.*

1. *(Perfect substitutes) If* q *is a perfect substitute for at least one private market good, then* $E(I') = C(I')$ *for all* I' *(Hanemann's Proposition 1).*

2. *(No substitutes) If* q *has (locally) zero substitutability with each private good, it can occur that* $C(I')$ *is finite while* $E(I')$ *is unbounded (Hanemann's Proposition 2).*

3. *(Decomposition) Let* η_q *denote the income elasticity of demand for* q *in the (hypothetical) market in which* q *could be purchased, and let* σ_0 *denote an aggregate Allen–Uzawa elasticity of substitution between* q *and the Hicksian composite of private goods. The* price flexibility of income ξ *(the income elasticity of the inverse demand for* q*) satisfies the following decomposition (notation adapted from Hanemann (1991)):*

$$\xi = \frac{\eta_q}{\sigma_0}. \tag{32}$$

(Notation is adapted: σ_0 *denotes Hanemann's aggregate substitution elasticity between* q *and the Hicksian composite of private goods.)*

The interpretation for resale is immediate. Equation (32) says that the relevant inverse demand under quantity constraints is more income-sensitive when substitution is more difficult.[8] Translating into our sufficient statistic, a large firewall index $\phi(I, P) = \Gamma(I, P) - 1$ can be generated by at least two distinct forces:

1. **High income sensitivity of the constrained attribute** (large η_q), which is the channel emphasised by the strict-normality interpretation in the baseline model; and/or

2. **Low effective substitutability** (small σ_0), meaning that maintaining utility when q is lost requires disproportionately large compensation. This shows why a "high income effects" narrative is only one possibility: even moderate income effects can generate large WTA responses when substitutes are scarce.

This is the conceptual nesting: in our model, $\Gamma(I, P) > 1$ is the sufficient statistic; Hanemann identifies the microeconomic primitives that make $\Gamma(I, P)$ (and hence ϕ) large, namely the joint configuration of income sensitivity and substitution possibilities.

[8] In our notation, the relevant local monetary object is $E'(I')$, and (31) shows $\phi(I, P) = E'(I - P)$. Hanemann's price-flexibility-of-income term ξ is the corresponding income-elasticity object (up to a budget-share normalisation), so (32) can be read as a decomposition of the drivers of ϕ.

4.2 Mechanism I: complementarities and ecosystem goods

Hanemann's substitution channel naturally points to settings where the monopoly good is difficult to replace, not necessarily because it is "income elastic" in the narrow sense, but because it changes what other expenditure can accomplish. Many modern markets feature *ecosystem* goods: owning the good increases the productivity of spending on other consumption (apps and data for smartphones; software for consoles; services and accessories for devices; usage-driven complements in platforms).

A reduced-form way to capture this is a state-dependent marginal utility of income. Suppose that for some observable one-dimensional characteristic θ (usage intensity / complementarity strength),

$$V(1, D; \theta) = \theta\, u(D), \qquad V(0, D; \theta) = u(D), \tag{33}$$

with u strictly increasing and strictly concave and $\theta > 1$.[9]

The reservation price $R(\theta, P)$ solves

$$\theta\, u(I - P) = u(I - P + R(\theta, P)).$$

Differentiating gives

$$\frac{\partial R}{\partial P}(\theta, P) = 1 - \theta\, \frac{u'(I - P)}{u'(I - P + R(\theta, P))}. \tag{34}$$

Since $R(\theta, P) > 0$ and u' is decreasing, $u'(I - P + R) < u'(I - P)$, so the fraction in (34) exceeds one whenever $\theta > 1$. Hence $\partial R/\partial P < 0$ without requiring income heterogeneity: subsidies raise reservation values because they finance complementary spending in the ownership state, making money more valuable at the margin when the good is held.

[9]Three examples illustrate this setup. First, console manufacturers sell hardware whose value depends on games and online services; consumers who purchase many games (high θ) have higher reservation prices for the console itself. Second, mobile devices exhibit strong complementarities with apps and data services; carriers subsidise handsets in exchange for service commitments from high-usage consumers. Third, the classic razor-and-blades model (printers and ink) captures usage intensity through θ: heavy users have higher reservation prices for the printer. These examples are intended to illustrate *demand-side* complementarities that make the ownership state valuable, not to claim that firms literally implement our two-price schedule or rely on below-cost pricing. In many settings (e.g., consoles, smartphones, printers) discounts are broadly available rather than finely targeted on observables. Moreover, contractual service commitments (common in handset markets) are a separate anti-arbitrage device; our analysis isolates the preference-based channel whereby higher complementarity (higher θ) raises reservation values and can make subsidisation profitable even when resale is frictionless. In the canonical razor-and-blades (or handset-plus-service) story, the firm also earns margin on the complements, so discounts are often aimed at high-usage (high-θ) consumers; here we abstract from complement-side profit and isolate the preference-based ENAC channel, under which the marginal subsidy falls on low-θ types. Importantly, in our mechanism the subsidy that relaxes the ENAC applies at the margin to *low*-θ buyers.

In Hanemann's terms, complementarities operate like a reduction in the effective substitution elasticity σ_0: losing q_M reduces the set of private goods that can replicate the same utility. In our terms, complementarities raise $\lambda(1, \cdot)$ relative to $\lambda(0, \cdot)$, thereby raising Γ. Appendix B.1 provides a worked example (including closed forms under power utility) and shows how the ENAC logic yields a discriminatory schedule when types differ in θ.

4.3 Mechanism II: asymmetric substitution and fallback consumption

Substitutes are often treated as fatal to this mechanism: if close substitutes exist, a subsidy can be reallocated toward those substitutes, potentially lowering attachment to the monopoly good. The key caveat is that many substitutes are *asymmetric fallback options*: they are used only when the monopoly good is absent, and are redundant once it is owned.[10]

Asymmetric fallback consumption makes the marginal utility of income state-dependent in the opposite direction from the complementarity channel. When the consumer owns the good, they do *not* spend on the fallback; when they sell it, they must divert some of the resale proceeds to the fallback. If the fallback absorbs *less* than the resale proceeds, then the resale state is relatively cash-rich, lowering the marginal utility of income there and making a subsidy more valuable in the ownership state. This yields $\Gamma > 1$ and hence $\partial R/\partial P < 0$.

Appendix B.2 develops a two-good worked example of this mechanism and derives an explicit sign condition:

$$\frac{dR}{dP} < 0 \quad \Longleftrightarrow \quad q_S^*(\cdot) < R,$$

i.e., the optimal fallback expenditure after resale is strictly less than the resale revenue. The appendix also introduces a natural one-dimensional heterogeneity parameter (the quality of the fallback option) that generates type-dependent WTP and reservation values, and hence supports discriminatory schedules under the ENAC.

4.4 What this section adds to the baseline interpretation

The "income-effects firewall" is one coherent microfoundation for $\Gamma(I, P) > 1$, but it is not the only one. Hanemann's result clarifies why: the wealth-sensitivity of WTA for quantity/quality changes is governed by both income sensitivity and substitution opportunities.

[10] For instance, consider car ownership versus public transit or ride-sharing. Once a consumer owns a car, public transit provides little additional value for most trips; without a car, transit is essential. Second, owning a home eliminates the need for rental payments; without ownership, renting is the necessary fallback. Third, a smartphone user rarely needs payphones; without a smartphone, such alternatives become essential. Finally, consumers with good kitchens cook at home; those without rely on prepared food.

Table 1: Comparison of discrimination mechanisms

	Income effects (Sections 2–3)	**Complementarity** (Section 4.2)	**Fallback substitutes** (This section)
Source of $\partial R/\partial P < 0$	Wealth raises value of normal good	Budget for complements raises bundle value	Selling creates cash surplus
Heterogeneity	Income I	Complementarity θ	Fallback quality γ
Who is subsidised?	Low-income consumers	Low-θ consumers	High-γ consumers
Observable signal	Wealth, demographics	Usage intensity, purchase history	Geography, infrastructure access
Key condition	$\sigma > 1$	$\theta > 1$	$q_S^* < R$

Complementarities and asymmetric fallback substitution are two economically common ways of making substitution difficult in the relevant Hicksian sense, thereby raising Γ and strengthening the endogenous barrier to arbitrage. This expands the domain of markets in which the mechanism in Sections 2–3 may be expected to apply, even when "large income effects" are not the most salient feature of demand.

All three mechanisms generate the key property $\partial R/\partial P < 0$, but through distinct channels (see Table 1):

1. **Income effects:** A subsidy makes the consumer wealthier, increasing their valuation of a normal good.

2. **Complementarities:** A subsidy frees budget for complements, increasing the value of the consumption bundle.

3. **Fallback substitutes:** A subsidy raises utility in the ownership state, making fallback (second-best) options more costly at the margin. When resale leaves the consumer *net* cash-rich after financing fallback spending (as in the condition $q_S^* < R$), marginal utility is lower in the post-resale state, implying $\partial R/\partial P < 0$.

The unifying insight is that costless resale does not automatically unravel price discrimination. Whenever preferences exhibit sufficient curvature—whether through income effects, complementarities, or asymmetric substitutability—the monopolist can exploit endogenous reservation prices to sustain discriminatory pricing without recourse to transaction costs, legal restrictions, or market frictions.

5 Empirical Relevance and Applications

The central result of this paper is that systematic price discrimination can be sustained even under costless resale when the resale constraint is endogenously relaxed by preferences: subsidising a consumer raises their reservation value, so that

$$\frac{\partial R}{\partial P} < 0 \quad \Longleftrightarrow \quad \Gamma > 1 \quad \Longleftrightarrow \quad \phi := -\frac{\partial R}{\partial P} > 0,$$

as characterised in Lemma 1 and synthesised in Section 4. While strict normality ("income effects") is one route to $\phi > 0$, Section 4 emphasises broader foundations: for quantity/quality changes, Hanemann (1991) shows that WTA–WTP gaps (and hence high reservation values) are governed by substitution possibilities as well as income sensitivity; and Sections 4.2–4.3 show how complementarities and asymmetric fallback substitution can generate $\Gamma > 1$ even when an "income effects" narrative is not the most natural description of demand. We discuss four applications that illustrate the mechanism's reach and the consequences of its failure.

5.1 International pharmaceuticals

The global pharmaceutical market is a paradigmatic case of persistent international price disparities for identical medicines (e.g., antiretrovirals, insulin, Hepatitis C treatments) (Danzon and Towse, 2003). Although regulatory barriers to parallel trade exist, they are often porous; the relevant economic question is why large-scale diversion does not mechanically erase price gaps.

In this context, the key mechanism is the wedge between willingness-to-pay and willingness-to-accept for a discrete, survival-relevant quantity/quality change. In our notation, the resale reservation value $R(I, P)$ is a WTA (an equivalent-variation object), not a WTP. As Section 4.1 emphasises, Hanemann (1991) shows there is no presumption that WTA and WTP are close for quantity/quality changes: limited substitutability can generate very large WTA even when WTP is bounded by liquidity. Life-saving medicines are precisely the case where substitutes are scarce in the relevant Hicksian sense, so the consumer's compensation required to forgo treatment can far exceed their ability to pay.

This maps directly into $\Gamma > 1$. A subsidised price P_L raises disposable income $I - P_L$ for the treated patient, but more importantly it places the consumer in an ownership/health state in which marginal resources are exceptionally valuable for maintaining utility. In the language of Section 4, both (i) high income sensitivity of the health state and (ii) extremely low effective substitutability (few private substitutes for health) push up the firewall index

$\phi = \Gamma - 1$. Consequently, the reservation value to resell the drug can plausibly be at or above the high-income market price, producing compliance and consumption rather than mass patient-level resale even when black-market premiums exist (Danzon, 2018).

5.2 Live events and WTA–WTP gaps

The market for high-demand live events (concerts and sports) provides a clean illustration of why sustainable price differences need not rest solely on "income effects." Artists and venues often wish to price inclusively (P_L), while scalpers attempt to arbitrage toward high-WTP consumers (P_H). A large empirical literature documents that WTA for certain discrete entitlements can substantially exceed WTP, a pattern often labelled an "endowment effect" (Kahneman et al., 1990).

In our framework, what matters is not the label but the structural implication: when $\phi > 0$ (i.e., when subsidies raise reservation values), acquiring a ticket at P_L can push a consumer's reservation value $R(I, P_L)$ above the market-clearing resale price, limiting arbitrage. Section 4 provides two complementary reasons this can occur beyond a narrow "income effects" story. First, the ticket is a discrete quantity/quality change with limited substitutes: there are few private goods that can replicate the same experience at the same time and place. Hanemann's logic implies that limited substitutability alone can generate large WTA–WTP wedges for such changes (Hanemann, 1991). Second, attendance often unlocks complements (travel, accommodation, coordination with friends, complementary consumption at the venue), making the marginal value of money higher in the ownership state, as in the complementarity mechanism of Section 4.2. Both forces raise Γ.

Empirical studies in ticketing markets find that recipients of tickets through lotteries or controlled allocations often exhibit WTA substantially above their initial WTP, reducing resale even when secondary-market prices surge (Leslie and Sorensen, 2014). Industry practices such as "Verified Fan" technologies represent technological implementation of the personal quantity limit ($Q_{\max} = 1$) that prevents the unravelling described in Section 6; the theory here clarifies that, conditional on quantity enforcement, the extent of resale then depends on the magnitude of ϕ (and hence on substitutability and complementarities), not only on income effects.

5.3 Inclusionary zoning and housing lock-in

Housing markets frequently feature dual-pricing structures through inclusionary zoning or rent control, where observationally similar units in the same building are rented or sold at vastly different prices based on eligibility. Standard theory predicts misallocation and mobility distortions (Glaeser and Luttmer, 2003). Our model points to an additional, preference-based force behind the empirically salient "lock-in" phenomenon: the endogeneity of reservation values.

A household that secures a unit at a sub-market price P_L experiences a discrete increase in housing quality and security. Their reservation value R for giving up that entitlement reflects not merely the arbitrage profit from subletting but the compensation required to restore utility in the non-ownership state. Section 4 highlights why this compensation can be large even if one does not frame housing purely as a high-income-elasticity good. Substitution opportunities are limited in a location-specific sense (schools, commute, neighbourhood networks), so Hanemann's quantity/quality logic implies WTA can substantially exceed WTP when close substitutes are scarce (Hanemann, 1991). Moreover, the relevant outside option often involves asymmetric fallback expenditure: losing the unit forces spending on an inferior or more costly alternative (market rent elsewhere, longer commutes, temporary accommodation), which is precisely the "fallback consumption" channel in Section 4.3. When fallback expenditure absorbs less than the implicit value of the entitlement, the marginal utility of income can be lower in the post-resale state, pushing $\Gamma > 1$ and increasing lock-in.

This helps rationalise extremely low turnover and weak incentives to arbitrage through illegal subletting: the consumer is not maximising cash wealth but maximising utility under state-dependent marginal values of income, where the reservation value of housing security can be very high.

5.4 The failure case: food rationing and leakage

The necessity of quantity limits (Section 6) is illustrated by public food distribution systems. In India's Public Distribution System (PDS), essential grains are sold to the poor at deep discounts. Historically, quantity limits have been difficult to enforce due to corruption and monitoring constraints, leading to "leakage" estimates between 40% and 50% (Khera, 2011).

This case also highlights a distinct reason the mechanism can fail: even with enforcement, the preference-based firewall may be weak when substitutability is high. Staple grains are relatively easy to substitute across time and across close commodity categories, so the quantity/quality change is less likely to generate large WTA–WTP wedges in the sense emphasised by Hanemann (1991). In our notation, Γ and hence ϕ may be small: subsidising recipients does not greatly increase their reservation value to resell, making arbitrage pressure intrinsically strong. When this interacts with weak enforcement of $Q_{\max}$, low-income recipients can acquire surplus units ($q > 1$) and become secondary-market suppliers; as predicted in Section 6, this excess supply depresses the resale price and undermines the dual-pricing scheme.

Recent digitisation efforts (e.g., biometric identification) that strengthen personal quantity enforcement have been associated with reduced leakage (Puri and Pingali, 2024). Our framework clarifies the broader logic: quantity enforcement is necessary to prevent unravelling, but the extent to which resale and diversion persist conditional on enforcement depends on the magnitude of the firewall index ϕ, which is shaped by substitution opportunities and other mechanisms in Section 4, not solely by "income effects."

6 Robustness

The above analysis makes two core assumptions: (i) that income can be perfectly observed; and (ii) that the monopolist can impose customer-specific restrictions on purchases (in this case, to "one per customer"). In this section, we explore the implications of relaxing each of these assumptions in turn. It is demonstrated that while complete relaxation would eliminate price discrimination, the main result of the paper is robust to some relaxation of each assumption.

6.1 Imperfect observability and signal noise

The preceding analysis assumes the monopolist can perfectly verify consumer income. In many practical settings, firms must rely on imperfect signals of income—such as location, purchase history, or participation in time-intensive activities (e.g., queuing or coupon collection)—to segment the market. We now relax the perfect information assumption to examine whether the subsidy mechanism survives when the signal is noisy.

6.1.1 Signal structure and leakage

We adopt the two-type framework of Section 2 with incomes I_L and I_H in proportions α and $1-\alpha$. Instead of observing income directly, the monopolist observes a signal $S \in \{S_L, S_H\}$.

- **Low-income consumers** always generate the low signal S_L.
- **High-income consumers** generate the low signal S_L with probability $\psi \in [0, 1)$ and the high signal S_H with probability $1-\psi$.

The parameter ψ represents the "noisiness" of the signal or the extent of "leakage." A fraction ψ of the high-income population successfully disguises themselves as low-income (or inadvertently possesses the low-income characteristic). The total mass of consumers in the S_L segment is $\alpha + \psi(1-\alpha)$, while the mass in the S_H segment is $(1-\psi)(1-\alpha)$.[11]

The monopolist sets prices $P(S_L)$ and $P(S_H)$. Crucially, the Endogenous No-Arbitrage Condition (ENAC) is determined by the *minimum* reservation price within the subsidised group. The segment S_L contains both types. However, strict normality (Assumption 1) implies that for any subsidised price P_L, the high-income consumers have a strictly higher reservation price than the low-income consumers:

$$R(I_H, P_L) > R(I_L, P_L). \tag{35}$$

Consequently, the "rich in disguise" do not bind the arbitrage constraint. They value the good at its subsidised price more than the poor do, and thus have no incentive to resell at the target price R if the poor do not. The monopolist need only ensure that the true low-income agents do not resell. The pricing constraints remain identical to the perfect information case: $P(S_H) \leq R(I_L, P(S_L))$.

6.1.2 Re-optimisation and the collapse of discrimination

The noise ψ does not break the mechanism's feasibility, but it alters the optimisation by imposing a *leakage cost*. The monopolist must re-optimise the prices P_L and $R = R(I_L, P_L)$ to account for the fact that the subsidy P_L is now also paid to the disguised high-income consumers.

The monopolist maximises the expected profit under noise:

$$\Pi_D(P_L; \psi) = \underbrace{[\alpha + \psi(1-\alpha)]P_L}_{\text{Revenue from } S_L} + \underbrace{(1-\psi)(1-\alpha)R(I_L, P_L)}_{\text{Revenue from } S_H}. \tag{36}$$

The first-order condition defining the optimal subsidised price $P_L^*(\psi)$ is:

$$\frac{d\Pi_D}{dP_L} = [\alpha + \psi(1-\alpha)] + (1-\psi)(1-\alpha)\frac{\partial R}{\partial P_L} = 0. \tag{37}$$

Compared to the perfect-information case ($\psi = 0$), misclassification increases the effective marginal cost of subsidising the low-signal group. In the CES case the objective in (36) is strictly concave in P_L; Appendix A.12 proves that the maximiser $P_L^*(\psi)$ is unique and

[11] Note that we do not model the reverse case of low-income consumers generating a high-income signal. Since the mechanism is incentive-compatible, low-income consumers would never voluntarily mimic high-income types to pay a higher price $R > P_L$. Thus, relevant signal leakage is strictly one-sided.

satisfies

$$\frac{dP_L^*(\psi)}{d\psi} > 0.$$

Since $\partial R(I_L, P_L)/\partial P_L < 0$, the associated target price $R^*(\psi) \equiv R(I_L, P_L^*(\psi))$ is strictly decreasing in ψ.

Let $\Pi_{\text{sep}}^*(\psi)$ denote the maximal profit among separating two-price policies under noise. The monopolist sustains price discrimination if and only if $\Pi_{\text{sep}}^*(\psi)$ exceeds the optimal uniform profit Π_U^*.

Proposition 9 (Noise in the income signal)**.** *Let $\Pi_{\text{sep}}^*(\psi)$ denote the monopolist's maximal profit among* separating *two-price policies (P_L, P_H) with $P_H > P_L$ subject to the resale constraint $P_H \leq R(I_L, P_L)$. Then $\Pi_{\text{sep}}^*(\psi)$ is continuous and strictly decreasing in ψ on $[0, 1)$. Hence there exists at most one threshold $\psi^* \in [0, 1)$ such that $\Pi_{\text{sep}}^*(\psi^*) = \Pi_U^*$. Moreover, if $\Pi_{\text{sep}}^*(0) > \Pi_U^*$, then there exists a unique $\psi^* \in (0, 1)$ and the monopolist sustains separation if and only if $\psi < \psi^*$ (otherwise it optimally uses uniform pricing).*

The proof reveals that the marginal impact of noise on profit equals the marginal leakage cost: the mass of high-income consumers $(1 - \alpha)$ times the subsidy gap $(R^*(\psi) - P_L^*(\psi))$.

The robustness of the mechanism depends critically on the strength of income effects. Strong income effects (e.g., low σ in the CES case) mean the mechanism is highly effective, generating substantial profit gains over uniform pricing. This allows the mechanism to tolerate higher levels of noise (ψ^* is higher).

Conversely, as income effects weaken (large σ), the gains from discrimination diminish, and the mechanism becomes fragile. In the limit of quasi-linear preferences ($\sigma \to \infty$), income effects vanish, discrimination collapses even without noise (as $\bar{\alpha} \to 0$), and thus the tolerance for noise also vanishes ($\psi^* \to 0$).

6.2 Quantity limits and market unravelling

The preceding analysis rests on the assumption that the monopoly good is indivisible and consumers demand at most one unit ($q_M \in \{0, 1\}$). We now relax this assumption to consider the scenario where consumers can purchase multiple units. This extension reveals a fragility in the discriminatory mechanism: without personalised quantity restrictions, the secondary market generates a "race to the bottom" that collapses the price schedule.

6.2.1 The unraveling principle

Consider the general model with a continuum of income types $I \in [\underline{I}, \bar{I}]$. In the optimal discriminatory equilibrium, the monopolist serves a range of consumers $[\hat{I}, \bar{I}]$ with a strictly increasing price schedule $P^*(I, R^*)$ for incomes $I < I^*(R^*)$, and a uniform price R^* thereafter.

If consumers are not restricted to a single unit, they acquire the ability to act as suppliers in the resale market. A consumer with income I can purchase unlimited units at cost $P^*(I, R^*)$ and resell them (note: for additional units bought for resale, the opportunity cost is the purchase price, not the consumption reservation price). In a frictionless secondary market, the prevailing resale price is determined by the lowest available marginal cost among suppliers.

Because the optimal price schedule is strictly increasing in income ($\partial P^*/\partial I = 1$), the consumer with the lowest income in the served set, $\hat{I}$, faces the lowest price, $P_{min} = P^*(\hat{I}, R^*)$. If the quantity constraint is relaxed for the consumer at $\hat{I}$, they can profitably undercut any other potential reseller, supplying the entire high-income market at a price $P_{resale} = P_{min} + \epsilon$.

This creates an unravelling effect. High-income consumers, who would otherwise pay R^*, can instead purchase from the $\hat{I}$-type agents at a price significantly below R^*. The monopolist's attempt to segment the market fails because the "weakest link"—the consumer with the deepest subsidy—sets the global price.

6.2.2 Complementary personalised quantity restrictions

To sustain the discriminatory profit Π_D^* and the pricing outcome derived in Theorem 1, the monopolist must implement a schedule of quantity restrictions, $Q_{max}(I)$, alongside the price schedule $P(I)$.

The subsidised segment ($\hat{I} \leq I < I^*$). For these consumers, the optimal price is strictly less than the target reservation price: $P(I) < R^*$. To preserve the equilibrium, the monopolist must strictly ration these consumers:

$$Q_{max}(I) = 1, \quad \forall I \in [\hat{I}, I^*). \tag{38}$$

The uniform segment ($I \geq I^*$). For these consumers, the price is set at the target reservation price: $P(I) = R^*$. The arbitrage profit is zero, so the quantity constraint is non-binding:

$$Q_{max}(I) \geq 1, \quad \forall I \geq I^*. \tag{39}$$

The analysis yields a stark conclusion: there is *no* relaxation of quantities beyond unit demand for the subsidised segment that can achieve the same profit and pricing outcome as the strict unit quantity restriction. The "one unit" assumption is binding and essential for the low-income recipients of the subsidy. However, the assumption is superfluous for the high-income segment.

What this shows, however, is that it is market power that is central to price discrimination more than frictions in resale markets. Usually, we consider market power as the exercise of quantity restrictions at the market level. But what has been shown here is that market power exercised at the individual consumer level (something that is feasible when the monopolist can identify individual consumers and their attributes) is the important foundation for price discrimination. As noted, however, because resale markets do constrain the effects of monopoly power the welfare consequences of such individual-level quantity restrictions are more subtle than their aggregate counterpart.

7 Multiple Monopolists

We now generalise the model to the case where $N \geq 2$ monopolists sell distinct goods M_j ($j = 1, \ldots, N$) alongside the competitive numeraire. Each good is strictly normal. We analyse the symmetric case with non-negativity constraints.

7.1 Model setup and reservation price interdependence

We extend the model to a setting where $N \geq 2$ monopoly firms, indexed by $j = 1, \ldots, N$, each sell a distinct good M_j at zero marginal cost. Consumers have income I drawn from a distribution $F(I)$ on $[\underline{I}, \bar{I}]$. We assume a symmetric equilibrium where each firm sets the same price schedule $P(I)$. The disposable income after purchasing all N goods at prices $P_j(I)$ is $I' = I - \sum_{j=1}^{N} P_j(I)$. In a symmetric equilibrium, $P_j(I) = P(I)$ for all j, so $I' = I - NP(I)$.

Let $V(N, I')$ denote the indirect utility from consuming all N monopoly goods with disposable income I'. We maintain the assumption that the bundle of monopoly goods is strictly normal: $I' \mapsto V(N, I') - V(0, I')$ is strictly increasing.

The reservation price for any single good j, denoted $R_j(I')$, is defined by $V(N, I') = V(N-1, I' + R_j)$. This is the price at which a consumer is indifferent between keeping the full bundle and selling good j. By symmetry, $R_j(I') = R_k(I')$ for all j, k. We assume the reservation price for a single good is increasing in disposable income ($dR_j/dI' > 0$).

7.2 Equilibrium characterisation

We focus on symmetric pricing solutions in which each product is priced according to the same income-contingent schedule $P_j(I) = P(I)$, as would arise under a multiproduct monopolist (or, equivalently, under full coordination across the N monopoly products). The ENAC must hold for each good separately.

We define the bundle willingness-to-pay $W_{\text{Bundle}}(I, N)$ as the solution to $V(N, I - W_{\text{Bundle}}) = V(0, I)$.

Assumption 3 (Bundle demand)**.** Consumers either purchase the entire bundle of monopoly goods or purchase none. Formally, each $q_j \in \{0, 1\}$ and feasible allocations satisfy $(q_1, \ldots, q_N) \in \{(0, \ldots, 0), (1, \ldots, 1)\}$.

Proposition 10 (Symmetric pricing with N monopoly products under bundle demand)**.** *Suppose Assumption 3 holds and the N monopoly products are priced symmetrically according to a common schedule $P(\cdot)$ (e.g., by a multiproduct monopolist or under full coordination). Then any optimal symmetric schedule serves an upper interval $[\hat{I}, \bar{I}]$ and has the form:*

$$P(I) = \begin{cases} \frac{I - \hat{I}}{N}, & \textit{if } I \in [\hat{I}, I^*], \\ R, & \textit{if } I \geq I^*, \end{cases}$$

where consumers with $I < \hat{I}$ are excluded, and $(\hat{I}, I^*, R)$ satisfy $R = \frac{I^* - \hat{I}}{N}$ and $W_{bundle}(I^*, N) = NR$. In particular, relative to the single-product case, the income-indexed subsidy in the intermediate region is shared across products, so the per-product schedule has slope $1/N$.

Proposition 10 provides a tractable symmetric equilibrium in the restricted class of continuous, non-decreasing schedules. Establishing global uniqueness (allowing arbitrary nonlinear deviations) would require a full best-response analysis, which we do not pursue here.

7.2.1 General comparative statics and welfare

The slope of the price schedule, $1/N$, unambiguously decreases as N increases. This reflects the shared subsidy burden: as more firms participate, the cost of preventing arbitrage via subsidisation is distributed. The impact on the equilibrium levels of R, I^*, and $\hat{I}$ depends critically on how preferences change with N, reflecting the degree of substitutability or complementarity.

Impact on prices. The equilibrium price R is determined by the firms' optimisation process, which depends on how the reservation price for individual goods changes as N increases.

- **Substitutes:** If goods are substitutes, the marginal value of adding a good diminishes as N increases. This puts downward pressure on the equilibrium reservation price, R.
- **Complements:** If goods are complements, the marginal value increases with N, putting upward pressure on R.

- **Independent Goods:** If the marginal value is independent of N, R tends to be stable.

Impact on welfare and exclusion. Total welfare depends on coverage, i.e. on the exclusion threshold $\hat{I}$. Under the symmetric schedule, $\hat{I} = I^* - L$ with $L \equiv I^* - \hat{I} = NR$, so changes in N affect coverage through both $R(N)$ and the marginal-buyer condition $W_{\text{bundle}}(I^*(N), N) = NR(N)$. The derivative of L,

$$\frac{dL}{dN} = R + N\frac{dR}{dN},$$

is therefore a useful diagnostic for how the size of the subsidised region moves with N (holding fixed the movement in I^*), but a full welfare comparison requires tracking how I^* shifts as well. The interval expands ($dL/dN > 0$) if and only if the elasticity of the per-good price with respect to N is less than one (in magnitude):

$$\eta_{R,N} \triangleq -\frac{N}{R}\frac{dR}{dN} < 1. \tag{40}$$

This condition is thus an informative measure of how the *subsidised region* responds to N, capturing the trade-off between the shared subsidy effect (which tends to expand L) and the valuation effect (which affects R).

7.2.2 CES formulation and closed-form solutions

To derive precise comparative statics and analyse the interaction between substitution patterns and income effects, we employ a nested CES utility structure. Assume utility is $U(Q_M, q_C) = (\beta Q_M^\rho + (1-\beta)q_C^\rho)^{1/\rho}$, where $Q_M = (\sum q_j^\gamma)^{1/\gamma}$ is an aggregator of the N monopoly goods. We require $\rho \in (0,1)$ to ensure the income effect is strong enough (elasticity of substitution between Q_M and q_C is $\sigma = 1/(1-\rho) > 1$). We assume $\gamma \in (0,1]$.

In a symmetric equilibrium where the consumer buys the bundle, the utility achieved with disposable income I' is $V(N, I') = (\beta N^\kappa + (1-\beta)(I')^\rho)^{1/\rho}$. The parameter $\kappa = \rho/\gamma$ measures the effective degree of complementarity.

The crucial element for the N-monopolist equilibrium is the reservation price for a single good j, $R_j(I')$, as this governs the arbitrage constraint. It is defined by the utility equivalence $V(N, I') = V(N-1, I' + R_j)$. Solving this yields:

$$R_j(I') = \left((I')^\rho + \frac{\beta}{1-\beta}\Delta(N)\right)^{1/\rho} - I', \tag{41}$$

where $\Delta(N) = N^\kappa - (N-1)^\kappa$ is the marginal contribution of the Nth good to the utility index. The behavior of $\Delta(N)$ characterizes the relationship between the goods:

- $\kappa = 1$ ($\rho = \gamma$): $\Delta(N) = 1$ (Independent goods).
- $\kappa < 1$ ($\rho < \gamma$): $\Delta(N)$ is decreasing in N (Substitutes; diminishing marginal value).
- $\kappa > 1$ ($\rho > \gamma$): $\Delta(N)$ is increasing in N (Complements; increasing marginal value).

Here "complements" refers to an *increasing marginal contribution* $\Delta(N)$ in the nested aggregator (an effective complementarity in N), even though the q_j remain CES-substitutes within Q_M under $\gamma \in (0, 1]$. The structure of the reservation price function (41) is analogous to the single-good CES case (Section 2), with the preference weight scaled by $\Delta(N)$. In the CES formulation, the key object governing the resale constraint is $\Delta(N)$ in (41): for any fixed residual income I', the per-good reservation price $R_j(I')$ is strictly increasing in $\Delta(N)$. Thus the direction in which the resale constraint relaxes or tightens with N is determined by whether $\Delta(N)$ falls, is constant, or rises with N.

Proposition 11 (Comparative statics of the resale constraint in the CES-N-good model)**.** *In the nested CES model, define $\kappa \equiv \rho/\gamma$ and $\Delta(N) \equiv N^\kappa - (N-1)^\kappa$. Then:*

1. *If $\kappa < 1$, the sequence $\Delta(N)$ is strictly decreasing in N.*
2. *If $\kappa = 1$, $\Delta(N) \equiv 1$ for all N.*
3. *If $\kappa > 1$, the sequence $\Delta(N)$ is strictly increasing in N.*

Moreover, for any fixed $I' \geq 0$, the per-good reservation price

$$R_j(I') = \left((I')^\rho + \tfrac{\beta}{1-\beta}\Delta(N)\right)^{1/\rho} - I'$$

is strictly increasing in $\Delta(N)$ and therefore inherits the same monotonicity in N.

These results rigorously confirm the general intuition. We can apply the welfare condition $\eta_{R,N} < 1$ to determine when increasing the number of goods improves welfare. For substitutes ($\kappa < 1$), R decreases in N and the shared subsidy effect is negative. To connect (40) to the CES formulation, note from (41) that $R_j(I')$ depends on N only through $\Delta(N) = N^\kappa - (N-1)^\kappa$. Differentiating (treating N as continuous) gives

$$\frac{dR_j(I')}{dN} = \frac{1}{\rho}\Big((I')^\rho + \tfrac{\beta}{1-\beta}\Delta(N)\Big)^{\frac{1}{\rho}-1} \cdot \tfrac{\beta}{1-\beta}\Delta'(N),$$

so the (absolute) elasticity is

$$\eta_{R,N} = -\frac{N}{R_j(I')}\frac{dR_j(I')}{dN} = \frac{1}{\rho}\Big(-\frac{N\Delta'(N)}{\Delta(N)}\Big)\cdot\frac{\frac{\beta}{1-\beta}\Delta(N)}{(I')^{\rho}+\frac{\beta}{1-\beta}\Delta(N)}.$$

When $\kappa < 1$, $\Delta(N)$ is decreasing and the second factor lies in $(0, 1]$, while $-\frac{N\Delta'(N)}{\Delta(N)}$ is approximately $1-\kappa$ (and equals $1-\kappa$ under the continuous approximation $\Delta(N) \approx \kappa N^{\kappa-1}$). Hence a simple sufficient condition for (40) to hold is $1-\kappa < \rho$.

8 Endogenous Information and Privacy Choice

We now endogenise the information structure and investigate how privacy choices interact with the subsidy mechanism. We return to the two-income model (I_H, I_L) with population shares $1-\alpha$ and α. Each consumer has an idiosyncratic privacy cost $\tau \geq 0$, drawn from type-specific distributions T_L and T_H. We assume $\tau > 0$ for all consumers. We interpret τ as a disclosure cost in numeraire units: if a consumer discloses ($d_i = 1$), her disposable income is reduced by τ; remaining anonymous ($d_i = 0$) entails no privacy cost.

8.1 Model setup

Consumers choose whether to disclose their income data. Let $d_i \in \{0, 1\}$ denote individual i's disclosure decision and let s_L, s_H denote the resulting disclosure rates (fractions) for low- and high-income consumers. The monopolist sets personalised prices P_L, P_H and an anonymous price P_A. The pricing must respect the global Endogenous No-Arbitrage Condition (ENAC): the maximum price charged to any consumer cannot exceed the minimum reservation price of any consumer who purchases the good.

The timing of the game is as follows (see Figure 3):

8.2 Optimal anonymous pricing

Given (s_L, σ_H), the monopolist maximises revenue from the anonymous market. The monopolist compares inclusive pricing ($P_A = W(I_L)$) with exclusionary pricing ($P_A = W(I_H)$).

Lemma 4 (Stage-2 pricing given disclosure)**.** *Fix disclosure rates* (s_L, s_H)*. In any subgame-perfect equilibrium,* $s_H = 0$*. Given* $s_H = 0$*, the monopolist's stage-2 problem reduces to choosing* (P_A, P_L) *to maximise profit subject to participation and ENAC. Moreover, any stage-2 optimum is of one of the following two forms:*

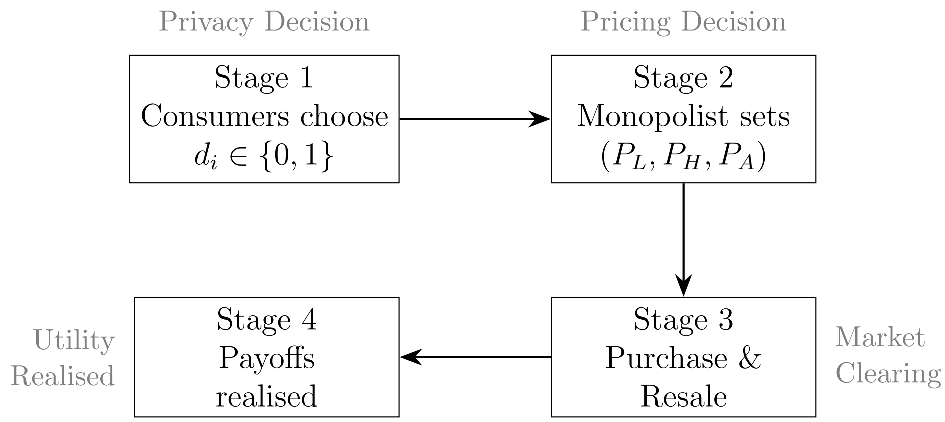


Figure 3: Timeline of the privacy and pricing game. Consumers first make disclosure decisions based on privacy costs τ_i, followed by monopolist pricing and market clearing.

1. Inclusive uniform pricing: $P_A = P_L = W(I_L)$, *in which case all consumers buy and disclosure is strictly dominated because privacy costs are strictly positive.*

2. Exclusionary resale-constrained pricing: $P_A > W(I_L)$ *so anonymous low-income consumers do not buy,* $P_A \leq W(I_H)$ *so high-income consumers buy anonymously, and ENAC binds so that* $P_A = R(I_L, P_L)$. *The monopolist then chooses* $P_L \in [0, W(I_L)]$ *to maximise*

$$\Pi(P_L; s_L) = s_L \alpha \, P_L + (1 - \alpha) \, R(I_L, P_L),$$

and sets $P_A = P_H = R(I_L, P_L)$.

This objective makes the key trade-off transparent: raising P_L increases revenue from disclosing low-income consumers (the term $s_L \alpha P_L$) but lowers the resale cap $R(I_L, P_L)$ and therefore reduces revenue from anonymous high-income consumers (the term $(1 - \alpha) R(I_L, P_L)$). The unique interior optimum satisfies

$$s_L \alpha + (1 - \alpha) \frac{\partial R}{\partial P}(I_L, P_L) = 0,$$

where $\frac{\partial R}{\partial P} < 0$.

8.3 Privacy equilibrium

We now analyse the full equilibrium, determining the optimal pricing strategy (P_L, P_H, P_A) and the resulting sharing decisions (s_L^*, σ_H^*).

Proposition 12 (Equilibrium with disclosure costs)**.** *Suppose privacy costs are strictly positive and* T_L *is strictly increasing with* $\inf \operatorname{supp}(T_L) = 0$ *and unbounded upper support. Then* *there exists a subgame-perfect equilibrium, and it takes one of two forms:*

1. Inclusive equilibrium. *If* $W(I_L) \geq (1-\alpha)\,W(I_H)$*, then there is an equilibrium with* $s_L^* = s_H^* = 0$ *and*
$$P_A^* = P_L^* = P_H^* = W(I_L).$$

2. Exclusionary equilibrium. *If* $W(I_L) < (1-\alpha)\,W(I_H)$*, then there is an equilibrium with* $s_H^* = 0$*,* $P_A^* = P_H^* = R^* > W(I_L)$*, and*
$$P_L^* \in \arg\max_{P_L \in [0, W(I_L)]} \Big\{ s_L^* \alpha\, P_L + (1-\alpha)\, R(I_L, P_L) \Big\}, \qquad R^* = R(I_L, P_L^*),$$
where the low-income disclosure rate $s_L^* \in (0,1)$ *solves the fixed point*
$$s_L^* = T_L\big(W(I_L) - P_L^*\big).$$

Proposition 12 illustrates how the resale mechanism interacts with recent literature on privacy choice. As noted by Rhodes and Zhou (2025), privacy choices are driven by the expected market outcome. Here, if the low-income group is large, the monopolist abandons data collection for a uniform price. If the low-income group is small, the monopolist induces sharing to facilitate the subsidy mechanism. The high anonymous price $P_A = R^*$ serves as a stick to force low-income consumers to reveal their type, while high-income consumers remain anonymous to avoid paying their full valuation, consistent with the "purchase history" dynamics discussed in Rhodes and Zhou (2024).

8.4 Externalities and Welfare

In the Exclusionary Regime, the decision of a low-income consumer to share data generates significant pecuniary externalities. Let $\Pi_{Ex}(R; s_L)$ denote the monopolist's total profit. Assuming $\sigma_H = 0$, and $P_A = R$:

$$\Pi_{Ex}(R; s_L) = s_L \alpha P^*(I_L, R) + (1-\alpha) R. \tag{42}$$

An increase in s_L means a larger mass $s_L\alpha$ of low-income consumers receive the subsidised contract $P^*(I_L, R)$. This shifts the monopolist's trade-off: a higher R extracts more from anonymous high-income buyers but requires deeper subsidies for each disclosing low-income buyer. Under concavity, the optimal target $R^*(s_L)$ therefore decreases with s_L.

This creates a divergence between private and social incentives. Rhodes and Zhou (2025) highlight that when sharing imposes negative externalities on others (e.g., by raising prices), equilibrium sharing is often inefficiently high. In our setting, the externality is distributional: a lower R^* benefits high-income consumers (who pay less) but harms other low-income consumers (who receive a smaller subsidy). However, the dominant effect here is the *positive* fiscal externality on the monopolist. A consumer who shares data allows the monopolist to extract surplus more efficiently, a benefit the consumer does not internalise.

Proposition 13 (Efficiency of data sharing)**.** *In the Exclusionary Regime, assuming non-negative prices ($P_L^* \geq 0$) and an interior optimum for R^*, the equilibrium level of data sharing by low-income consumers (s_L^*) is weakly inefficiently low relative to the socially optimal level.*

Consequently, unlike models where competition drives excessive data disclosure (Rhodes and Zhou, 2025), the monopoly subsidy mechanism leads to under-provision of data. The marginal consumer hides their data to avoid privacy costs, ignoring the positive *profit externality* their disclosure creates: the planner compares $W(I_L)$ to the privacy cost, while the consumer compares $W(I_L) - P_L^*$.

9 Conclusion

Costless resale does not eliminate the scope for price discrimination once income effects are acknowledged. When the monopoly good is strictly normal, the monopolist can use strategic subsidisation to raise consumers' reservation prices and thereby neutralise arbitrage. The resulting price schedule increases dollar-for-dollar with income in a subsidised region and is flat above a threshold—a structure that arises because maintaining a constant reservation price requires maintaining constant disposable income. This mechanism weakly dominates uniform pricing and, when the entire market is served, achieves the first-best allocation.

The mechanism extends beyond income effects narrowly construed. Drawing on Hanemann (1991), we showed that low substitutability with private-market alternatives can generate large reservation-price responses to subsidies even when income sensitivity is modest—a channel relevant for ecosystem goods, life-saving medicines, and entitlements with few close substitutes.

The analysis also reframes the role of market power in price discrimination. Without consumer-specific quantity restrictions, the subsidised consumer can become a residual supplier and unravel the price schedule. Sustaining discrimination, therefore, requires market power exercised at the individual level—the ability to restrict each consumer's purchases—rather than merely at the aggregate level. This distinction has practical implications for markets where resale is feasible but identity-linked purchase limits are enforceable.

Extensions to multiple monopolists show that the subsidisation burden is shared, with the slope of each price schedule falling as more products enter. Endogenising privacy choices reveals that equilibrium data sharing is inefficiently low: the marginal discloser ignores the profit externality their participation confers on the monopolist, a result that contrasts with settings where competition drives excessive disclosure.

Several questions remain open. How does the mechanism interact with dynamic pricing when consumers can time their purchases? Can the firewall index, ϕ, be estimated empirically, and does it help explain observed price dispersion in markets with active resale? These extensions would further clarify the boundary between settings where costless resale disciplines prices and those where preference-based arbitrage barriers allow discrimination to persist.

A Appendix: Proofs

A.1 Proof of Proposition 1

The monopolist maximises global profit. The alternatives are Uniform Pricing (UP) and Discriminatory Pricing (DP). The optimal uniform profit is $\Pi_U^* = \max\{W(I_L), (1-\alpha)W(I_H)\}$. The optimal discriminatory profit Π_D^* is the maximum profit achievable subject to the ENAC, the Participation Constraint (PC), and non-negative prices ($P_L \geq 0$).

Discriminatory pricing is the unique global equilibrium if and only if $\Pi_D^* > \Pi_U^*$. This requires two conditions: (1) $\Pi_D^* > W(I_L)$. (2) $\Pi_D^* > (1-\alpha)W(I_H)$.

1. Dominance over Inclusionary UP (Profitability Condition). We know that $\Pi_D^* \geq W(I_L)$ since setting $P_L = P_H = W(I_L)$ is feasible. Strict dominance holds if it is locally profitable to reduce P_L below $W(I_L)$. Assuming $W(I_H)$ does not bind locally (since $W(I_H) > W(I_L)$), this requires the derivative of the profit function (under the resale constraint) to be negative at $P_L = W(I_L)$.

$$\left.\frac{d\Pi_D}{dP_L}\right|_{P_L=W(I_L)} = \alpha + (1-\alpha)\left.\frac{\partial R}{\partial P_L}\right|_{P_L=W(I_L)} < 0.$$

As derived in the analysis preceding the original Proposition 1, this condition yields the threshold $\bar{\alpha}$ defined in Condition (13). Thus, $\alpha < \bar{\alpha}$ is necessary and sufficient for $\Pi_D^* > W(I_L)$.

2. Dominance over Exclusionary UP (Dominance Condition). We require $\Pi_D^* > (1-\alpha)W(I_H)$. We analyze this by comparing $W(I_H)$ with the maximum sustainable price under non-negativity, $\mathcal{R}_0(I_L) = R(I_L, 0)$.

Case 2(a): Moderate Inequality $W(I_H) < \mathcal{R}_0(I_L)$. In this case, $R(I_L, 0) > W(I_H)$. Since $R(I_L, P_L)$ is continuous and strictly decreasing in P_L, there exists a unique price P_L^{PC} such that $R(I_L, P_L^{PC}) = W(I_H)$. Furthermore, since $R(I_L, 0) > R(I_L, P_L^{PC})$, it must be that $P_L^{PC} > 0$.

Consider the feasible discriminatory strategy $(P_L^{PC}, W(I_H))$. This yields profit:

$$\Pi_D^{PC} = \alpha P_L^{PC} + (1-\alpha)W(I_H).$$

Since $P_L^{PC} > 0$ and we assume $\alpha > 0$, $\Pi_D^{PC} > (1-\alpha)W(I_H)$. The optimal discriminatory profit Π_D^* is at least Π_D^{PC}. Thus, strict dominance holds. This establishes Condition 2(a).

Case 2(b): High Inequality $W(I_H) \geq \mathcal{R}_0(I_L)$. In this case, $R(I_L, 0) \leq W(I_H)$. The monopolist cannot sustain $P_H = W(I_H)$ while maintaining $P_L > 0$. The maximum price achievable under $P_L \geq 0$ is $\mathcal{R}_0(I_L)$ (at $P_L = 0$).

The optimal discriminatory strategy is the Resale-Constrained optimum, subject to $P_L \geq 0$. Let P_L^{RC} be the unconstrained RC optimum.

If $P_L^{RC} > 0$. The optimum is interior. $\Pi_D^* = \Pi_D^{RC}$. Note that $P_H^{RC} = R(I_L, P_L^{RC}) < R(I_L, 0) \leq W(I_H)$. Discrimination is strictly optimal if $\Pi_D^{RC} > (1-\alpha)W(I_H)$. This is Condition (15).

If $P_L^{RC} \leq 0$. The profit function under the resale constraint is concave, so it is weakly decreasing for all $P_L \geq P_L^{RC}$. Since the feasible set under non-negativity is $[0, \infty) \subseteq [P_L^{RC}, \infty)$, the constrained optimum is attained at the left boundary $P_L^* = 0$. The corresponding price is $P_H^* = \mathcal{R}_0(I_L)$. The profit is $\Pi_D^* = (1-\alpha)\mathcal{R}_0(I_L)$. Since $\mathcal{R}_0(I_L) \leq W(I_H)$, we have $\Pi_D^* \leq (1-\alpha)W(I_H)$. Strict dominance fails.

Therefore, when $W(I_H) \geq \mathcal{R}_0(I_L)$, discrimination is strictly optimal if and only if $P_L^{RC} > 0$ and Condition (15) holds. This establishes Condition 2(b).

A.2 Proof of Proposition 2

Throughout this proof, we work in the *interior* resale-constrained regime, i.e., the resale constraint binds and the participation constraint for the high-income type does not bind:

$$P_H^* = R(I_L, P_L^*) < W(I_H), \qquad 0 < P_L^* < W(I_L).$$

Step 1: Closed-form expression for ΔP^*. Under CES preferences, in the interior resale-constrained regime the non-negativity constraint is slack and the $\max\{\cdot\}$ operators in (8)–(9) select their first arguments (so $P_L^* > 0$ and $P_H^* = R(I_L, P_L^*) > P_L^*$). Let $\rho \equiv 1 - \frac{1}{\sigma} \in (0,1)$ and $K \equiv \frac{\beta}{1-\beta} > 0$. Define

$$A(\alpha, \sigma) \equiv (1-\alpha)^{1-\sigma} > 1.$$

Then (8) implies

$$P_L^* = I_L - \Big(\frac{K}{A-1}\Big)^{1/\rho}.$$

Because the resale constraint binds, $P_H^* = R(I_L, P_L^*)$. Using the CES formula for $R(\cdot)$ and the identity

$$(I_L - P_L^*)^\rho = \frac{K}{A-1},$$

we obtain

$$P_H^* = \Big(\frac{KA}{A-1}\Big)^{1/\rho} - \Big(\frac{K}{A-1}\Big)^{1/\rho}.$$

Therefore, the equilibrium price gap is

$$\Delta P^* \equiv P_H^* - P_L^* = \Big(\frac{KA}{A-1}\Big)^{1/\rho} - I_L.$$

In particular, ΔP^* does not depend on I_H.

Step 2: ΔP^* decreases in α. Write

$$\Delta P^* + I_L = \Big(\frac{KA}{A-1}\Big)^{1/\rho} = \Big(\frac{K}{1-(1-\alpha)^{\sigma-1}}\Big)^{1/\rho}.$$

Because $(1-\alpha)^{\sigma-1}$ is strictly decreasing in α, the denominator $1-(1-\alpha)^{\sigma-1}$ is strictly increasing in α, hence the fraction inside parentheses is strictly decreasing in α. Since $1/\rho > 0$, it follows that $\Delta P^* + I_L$ is strictly decreasing in α, and therefore ΔP^* is strictly decreasing in α.

Step 3: ΔP^* decreases in σ under (17). Let

$$B(\alpha,\beta,\sigma) \equiv \frac{K}{1-(1-\alpha)^{\sigma-1}}, \qquad \text{so that} \qquad \Delta P^* + I_L = B^{1/\rho}.$$

We compute the derivative of $\ln(\Delta P^* + I_L)$:

$$\frac{d}{d\sigma}\ln(\Delta P^* + I_L) = \frac{d}{d\sigma}\Big(\frac{1}{\rho}\ln B\Big) = -\frac{\rho'}{\rho^2}\ln B + \frac{1}{\rho}\frac{B'}{B},$$

where $\rho' = \frac{d\rho}{d\sigma} = \frac{1}{\sigma^2} > 0$. Since $0 < (1-\alpha) < 1$, the term $(1-\alpha)^{\sigma-1}$ is strictly decreasing in σ, implying that $1-(1-\alpha)^{\sigma-1}$ is strictly increasing in σ, hence B is strictly *decreasing* in σ and thus $B'/B < 0$.

The sign ambiguity arises from the first term, which depends on $\ln B$. In the interior discriminatory regime $\Delta P^* > 0$, so $\Delta P^* + I_L > I_L$. Raising both sides to the power $\rho \in (0,1)$ preserves the inequality and yields

$$B = (\Delta P^* + I_L)^\rho > I_L^\rho.$$

Under condition the condition that $(1-\beta)I_L^\rho > 1$, i.e.,

$$I_L^\rho > \frac{1}{1-\beta} = 1 + \frac{\beta}{1-\beta} = 1 + K > 1.$$

Therefore $B > I_L^\rho > 1$, implying $\ln B > 0$. Hence both terms in

$$-\frac{\rho'}{\rho^2}\ln B + \frac{1}{\rho}\frac{B'}{B}$$

are strictly negative, and we conclude that

$$\frac{d}{d\sigma}\ln(\Delta P^* + I_L) < 0 \quad \Rightarrow \quad \frac{d}{d\sigma}(\Delta P^* + I_L) < 0 \quad \Rightarrow \quad \frac{d}{d\sigma}\Delta P^* < 0.$$

Step 4: ΔP^* does not depend on I_H. This follows directly from the closed-form expression for ΔP^* in Step 1, which depends only on $(\alpha, \beta, \sigma, I_L)$.

A.3 Proof of Proposition 3

In the corner (participation-constrained) regime, the monopolist sets

$$P_H^* = W(I_H).$$

Given this P_H^*, resale-freeness requires that any low-income buyer who pays P_L has reservation price at least P_H^*:

$$R(I_L, P_L) \geq P_H^*.$$

Since $R(I_L, P_L)$ is strictly decreasing in P_L, the revenue-maximising feasible low-income price is

$$P_L^* = \max\{P^*(I_L, P_H^*), 0\} = \max\{P^*(I_L, W(I_H)), 0\}.$$

Claim 1: If $P_L^* > 0$, then $\Delta P^* = I_H - I_L$. Lemma 3 (property (iii)) implies that for fixed target R,

$$\frac{\partial P^*(I, R)}{\partial I} = 1,$$

hence $P^*(I, R) = I - c(R)$ for some function $c(R)$. Evaluating at $I = I_H$ and $R = W(I_H)$, note that at price $P = W(I_H)$ the high-income type is indifferent to buying, so her reservation price equals $W(I_H)$, i.e., $R(I_H, W(I_H)) = W(I_H)$, which implies $P^*(I_H, W(I_H)) = W(I_H)$. Therefore

$$P^*(I_L, W(I_H)) = P^*(I_H, W(I_H)) - (I_H - I_L) = W(I_H) - (I_H - I_L).$$

If this is positive, non-negativity does not bind and

$$P_L^* = W(I_H) - (I_H - I_L), \qquad \Delta P^* = P_H^* - P_L^* = W(I_H) - \big(W(I_H) - (I_H - I_L)\big) = I_H - I_L.$$

This expression is independent of σ.

Claim 2: If $P_L^* = 0$, then $\Delta P^* = W(I_H)$. If $P^*(I_L, W(I_H)) \leq 0$, the revenue-maximising feasible low-income price under the non-negativity constraint is $P_L^* = 0$, so

$$\Delta P^* = P_H^* - P_L^* = W(I_H).$$

Monotonicity. The claim that $P_H^* = W(I_H)$ is increasing in I_H follows from standard monotonicity of willingness-to-pay in income. Under CES preferences, $W(I)$ is given by (2); direct differentiation shows $W(I)$ is strictly decreasing in σ, hence $P_H^* = W(I_H)$ is strictly decreasing in σ. In the $P_L^* = 0$ subcase, $\Delta P^* = W(I_H)$ inherits the same monotonicity in σ.

A.4 Proof of Proposition 4

Total Surplus is $TS = \int_S W(I)dF(I)$ where S is the set of served consumers.

1. *First Best:* $TS_{FB} = \alpha W(I_L) + (1-\alpha)W(I_H)$.

2. *Discriminatory (TS_D):* Provided $P_L^* \leq W(I_L)$ and prices are non-negative, both types purchase. $TS_D = \alpha W(I_L) + (1-\alpha)W(I_H) = TS_{FB}$.

3. *Uniform Exclusive (TS_U^{Exc}):* Only H-types buy. $TS_U^{Exc} = (1-\alpha)W(I_H)$. Comparing TS_D and TS_U^{Exc}:
$$TS_D - TS_U^{Exc} = \alpha W(I_L) > 0.$$

Thus, discrimination strictly dominates exclusive pricing in allocative efficiency.

A.5 Proof of Lemma 1

Positivity of $R(I,P)$: By Definition 1, $R(I,P)$ solves $V(1, I-P) = V(0, I-P+R)$. If $P < W(I)$, then by monotonicity of V in I', $V(1, I-P) > V(1, I-W(I)) = V(0, I)$. For the equation to hold, we need $V(0, I-P+R) > V(0, I)$, which requires $I - P + R > I$, hence $R > P > 0$ when $P > 0$. When $P = 0$, $R(I, 0) > 0$ by strict normality (noting that generally $R(I, 0) \neq W(I)$ outside the quasi-linear case).

Derivatives: The reservation price $R(I,P)$ is defined by $V(1, I-P) = V(0, I-P+R(I,P))$. Differentiate with respect to P:

$$-\lambda(1, I-P) = \lambda(0, I-P+R)\left(-1 + \frac{\partial R}{\partial P}\right).$$

Rearranging yields

$$\frac{\partial R}{\partial P} = 1 - \frac{\lambda(1, I-P)}{\lambda(0, I-P+R)} = 1 - \Gamma(I, P).$$

Differentiate with respect to I:

$$\lambda(1, I-P) = \lambda(0, I-P+R)\left(1 + \frac{\partial R}{\partial I}\right).$$

Rearranging yields

$$\frac{\partial R}{\partial I} = \frac{\lambda(1, I-P)}{\lambda(0, I-P+R)} - 1 = \Gamma(I, P) - 1.$$

Signs: To establish the signs, we must show $\Gamma(I, P) > 1$, i.e., $\lambda(1, I-P) > \lambda(0, I-P+R)$.

1. *Step 1:* By Assumption 1.1 (Strict Normality), $V(1, I') - V(0, I')$ is strictly increasing in I'. Taking the derivative yields:

   $$\lambda(1, I') - \lambda(0, I') > 0.$$

   Thus, $\lambda(1, I') > \lambda(0, I')$ for any I'.

2. *Step 2:* Since $R(I, P) > 0$ (established above), we have $I - P + R > I - P$. By Assumption 1.2 (Non-increasing Marginal Utility), $\lambda(0, \cdot)$ is non-increasing. Therefore:

   $$\lambda(0, I-P+R) \leq \lambda(0, I-P).$$

3. *Step 3:* Combining Steps 1 and 2:

   $$\lambda(1, I-P) > \lambda(0, I-P) \geq \lambda(0, I-P+R).$$

This establishes $\Gamma(I, P) > 1$. Therefore, $\frac{\partial R}{\partial P} = 1 - \Gamma(I, P) < 0$ and $\frac{\partial R}{\partial I} = \Gamma(I, P) - 1 > 0$.

A.6 Proof of Lemma 2

We first prove that any strategy involving resale (intermediation) is weakly dominated by a strategy involving only direct sales. We then show that a strategy involving only direct sales is sustainable in equilibrium if and only if it satisfies the ENAC.

Part 1: Optimality of Direct Sales. Consider an equilibrium outcome where resale occurs. This implies there is at least one unit sold by the monopolist to Consumer A (the intermediary, income I_A) at price P_A, which is subsequently resold to Consumer B (the final consumer, income I_B) at a resale price P_R. Due to the unit demand assumption and the restriction that the monopolist sells at most one unit to any individual, if Consumer A resells the unit, they do not consume the good ($q_M = 0$). Consumer A acts purely as an intermediary.

For this sequence of transactions to be voluntary (individually rational), the following conditions must hold:

1. **Intermediary Participation (Consumer A):** Consumer A must weakly prefer intermediating over not participating. Since V is strictly increasing in disposable income (Assumption 1), this requires the net profit to be non-negative:

$$P_R \geq P_A \quad \text{(non-negative monetary profit).} \tag{43}$$

   In addition, for Consumer A to prefer reselling rather than consuming the unit themselves, we require

$$P_R \geq R(I_A, P_A). \tag{44}$$

2. **Final Consumer Participation (Consumer B):** Consumer B must weakly prefer purchasing at P_R over not consuming the good. By the definition of willingness to pay $W(I_B)$, this implies:

$$P_R \leq W(I_B). \tag{45}$$

Combining (43) and (45) yields $P_A \leq P_R \leq W(I_B)$. If resale occurs in equilibrium, then (44) also holds. In this scenario, the monopolist's revenue from this specific unit is P_A.

Now, consider an alternative strategy. Since the monopolist has perfect information, they know I_B. Suppose the monopolist sells the unit directly to Consumer B at a price $P'_B = P_R$, instead of selling it to A.

Consumer B will accept this direct offer, as their participation constraint remains satisfied by (45). The monopolist's revenue from this direct sale is $P'_B = P_R \geq P_A$. The monopolist's revenue is weakly greater when selling directly to B.

This logic applies to every unit sold through an intermediary. By replacing all intermediated sales with direct sales to the final consumers at the prevailing resale price, the monopolist weakly increases total profit. Therefore, an optimal strategy exists in which only direct sales occur (no resale).

Part 2: Necessity of the ENAC. If the monopolist's strategy is optimal, by Part 1, we can assume it involves only direct sales. This means the intended allocation and prices must constitute an equilibrium where no resale occurs.

Suppose the monopolist intends to serve the set S with the schedule $P(I)$. If the ENAC is violated, there exist two consumers $I, I' \in S$ such that $P(I') > R(I, P(I))$.

In this case, an arbitrage opportunity exists. Consumer I can purchase at $P(I)$ and offer to resell to Consumer I' at a price P_{trade} such that $R(I, P(I)) < P_{trade} < P(I')$. Both parties strictly prefer this trade to the monopolist's intended outcome, contradicting the equilibrium. Therefore, an optimal strategy must satisfy the ENAC.

A.7 Proof of Lemma 3

Fix a target resale price $R > 0$. For any disposable income level $y \geq 0$, define the (one-unit) reservation price $\widetilde{R}(y)$ as the unique number satisfying

$$V(1, y) = V(0, y + \widetilde{R}(y)). \tag{46}$$

Existence and uniqueness of $\widetilde{R}(y)$ follow from: (i) $V(0, \cdot)$ is strictly increasing and continuous in income, and (ii) for each fixed y, the map $r \mapsto V(0, y + r)$ is strictly increasing and continuous, so (46) has a unique solution.

By the definition of $R(I, P)$ in the main text, when a consumer of income I purchases at price P her disposable income is $y = I - P$, and the resale price that makes her indifferent between keeping and selling the good solves

$$V(1, I - P) = V(0, I - P + R(I, P)).$$

Comparing with (46) shows that

$$R(I, P) = \widetilde{R}(I - P),$$

i.e., the reservation price depends on (I, P) only through disposable income $I - P$.

Existence and uniqueness of $P^*(I, R)$. For fixed R, define $y^*(R)$ as the unique solution to $\widetilde{R}(y) = R$. Since $\widetilde{R}(\cdot)$ is strictly increasing (proved below), $y^*(R)$ exists and is unique whenever R lies in the range of $\widetilde{R}$. Define

$$P^*(I, R) \equiv I - y^*(R).$$

Then $I - P^*(I, R) = y^*(R)$ and thus

$$R(I, P^*(I, R)) = \widetilde{R}(I - P^*(I, R)) = \widetilde{R}(y^*(R)) = R,$$

which proves existence. Uniqueness follows because $P \mapsto R(I,P) = \widetilde{R}(I-P)$ is strictly decreasing in P (since $\widetilde{R}$ is strictly increasing in disposable income).

Property (i). Consider the price $P = R$. If $W(I) \le R$, then the consumer weakly prefers not buying at price R:

$$V(1, I-R) \le V(0, I) = V(0, I-R+R).$$

Because $r \mapsto V(0, I-R+r)$ is strictly increasing, the unique r satisfying $V(1, I-R) = V(0, I-R+r)$ must satisfy $r \le R$, i.e., $R(I,R) = \widetilde{R}(I-R) \le R$. Since $P \mapsto R(I,P)$ is strictly decreasing, the unique solution to $R(I,P) = R$ must satisfy $P \le R$, i.e., $P^*(I,R) \le R$.

Conversely, if $W(I) > R$, then $V(1, I-R) > V(0, I-R+R)$. By strict monotonicity in r, the unique r satisfying $V(1, I-R) = V(0, I-R+r)$ must satisfy $r > R$, i.e., $R(I,R) > R$. Since $R(I,P)$ is strictly decreasing in P, the unique solution to $R(I,P) = R$ must satisfy $P > R$, i.e., $P^*(I,R) > R$. Thus $P^*(I,R) \le R$ if and only if $W(I) \le R$.

Monotonicity of $\widetilde{R}$ and property (ii). Differentiate (46) with respect to y. Let $\lambda(q,y) \equiv \partial V(q,y)/\partial y$ denote marginal indirect utility. We obtain

$$\lambda(1, y) = \lambda(0, y + \widetilde{R}(y)) \cdot (1 + \widetilde{R}'(y)),$$

hence

$$\widetilde{R}'(y) = \frac{\lambda(1,y)}{\lambda(0, y+\widetilde{R}(y))} - 1.$$

Lemma 1 implies $\lambda(1,y) > \lambda(0, y+\widetilde{R}(y))$ (equivalently $\partial R(I,P)/\partial P < 0$), so the fraction exceeds one and $\widetilde{R}'(y) > 0$. Therefore $\widetilde{R}$ is strictly increasing, so its inverse $y^*(R)$ is strictly increasing, implying

$$\frac{\partial P^*(I,R)}{\partial R} = -\frac{dy^*(R)}{dR} < 0,$$

which is property (ii).

Property (iii). Since $y^*(R)$ depends only on R, not on I,

$$P^*(I,R) = I - y^*(R) \quad \Rightarrow \quad \frac{\partial P^*(I,R)}{\partial I} = 1.$$

A.8 Proof of Theorem 1

Let $P(\cdot)$ be any feasible schedule with non-negative prices, and let the served set be

$$\mathcal{I}(P) \equiv \{I \in [\underline{I}, \bar{I}] : P(I) \le W(I)\}.$$

If $\mathcal{I}(P)$ is empty, profit is zero ,and the theorem is trivial, so assume $\mathcal{I}(P) \neq \emptyset$.

Define the maximal price among buyers

$$R \equiv \sup_{I \in \mathcal{I}(P)} P(I).$$

Because every buyer pays at most R, ENAC (19) implies that for every buyer $I \in \mathcal{I}(P)$,

$$R \leq R(I, P(I)). \tag{47}$$

Step 1: For fixed $(\mathcal{I}, R)$, revenue is pointwise maximised by $P(I) = \min\{R, P^*(I, R)\}$. Fix a nonempty served set $\mathcal{I} \subseteq [\underline{I}, \bar{I}]$ and a value $R > 0$ interpreted as the maximal price among buyers (as defined above). For any $I \in \mathcal{I}$, feasibility requires $P(I) \leq R$ (by definition of R) and $R \leq R(I, P(I))$ (by (47)). Because $P \mapsto R(I, P)$ is strictly decreasing, the constraint $R \leq R(I, P(I))$ is equivalent to $P(I) \leq P^*(I, R)$. Therefore, any feasible schedule must satisfy

$$P(I) \leq \min\{R, P^*(I, R)\} \qquad \forall I \in \mathcal{I}.$$

Since prices are non-negative and marginal cost is zero, increasing $P(I)$ weakly increases profit pointwise. Hence, holding $(\mathcal{I}, R)$ fixed, revenue is maximised by setting

$$P(I) = \min\{R, P^*(I, R)\} \quad \text{for all } I \in \mathcal{I},$$

and setting $P(I)$ arbitrarily (e.g., $P(I) = +\infty$) for $I \notin \mathcal{I}$ to ensure non-purchase.

Step 2: The served set is an upper interval. Let $P(\cdot)$ be an optimal feasible schedule with a nonempty served set. We claim $\mathcal{I}(P)$ is of the form $[\hat{I}, \bar{I}]$ for some $\hat{I}$. Suppose not. Then there exist $\underline{I} \leq I_1 < I_2 \leq \bar{I}$ such that $I_1 \in \mathcal{I}(P)$ but $I_2 \notin \mathcal{I}(P)$. Let R be the maximal price among buyers under P. Consider a modified schedule $\widetilde{P}$ that coincides with P on $\mathcal{I}(P)$ and additionally serves type I_2 at price

$$\widetilde{P}(I_2) = \min\{R, P^*(I_2, R)\}.$$

Participation for type I_2 holds because $\widetilde{P}(I_2) = \min\{R, P^*(I_2, R)\} \leq W(I_2)$. Indeed, if $R \leq W(I_2)$ then $\widetilde{P}(I_2) \leq R \leq W(I_2)$. If instead $R > W(I_2)$, then $R(I_2, W(I_2)) = W(I_2) < R$ and $P \mapsto R(I_2, P)$ is strictly decreasing, so the price $P^*(I_2, R)$ that solves $R(I_2, P) = R$ must satisfy $P^*(I_2, R) < W(I_2)$, implying $\widetilde{P}(I_2) \leq P^*(I_2, R) < W(I_2)$. Moreover, ENAC continues to hold: the maximal buyer price remains R, and $R \leq R(I_2, \widetilde{P}(I_2))$ holds by construction. Thus $\widetilde{P}$ is feasible and yields strictly higher profit (since it adds a positive-revenue buyer), contradicting optimality. Therefore, $\mathcal{I}(P)$ must be an upper interval $[\hat{I}, \bar{I}]$.

Step 3: Characterisation for a given R**.** Fix R and let $I^*(R)$ solve $W(I^*(R)) = R$ (with the convention that $I^*(R) = \bar{I}$ if $R \leq W(\bar{I})$ and $I^*(R) = \underline{I}$ if $R \geq W(\underline{I})$). By Lemma 3 (property (i)), for incomes with $W(I) \leq R$ we have $P^*(I, R) \leq R$, and for incomes with $W(I) \geq R$ we have $P^*(I, R) \geq R$. Therefore, the pointwise-maximising schedule in Step 1 can be written as

$$P(I) = \begin{cases} P^*(I, R), & I \in [\hat{I}, I^*(R)], \\ R, & I \in [I^*(R), \bar{I}], \end{cases}$$

for some lower cutoff $\hat{I}$.

Non-negativity requires $P^*(I, R) \geq 0$ on the subsidised region. Define $I_-(R)$ by $P^*(I_-(R), R) = 0$ (equivalently $R(I_-(R), 0) = R$). Then, the feasibility under non-negative prices requires $\hat{I} \geq I_-(R)$.

Step 4: Reduction to a two-dimensional maximisation and existence. Combining the previous steps, any optimal non-negative-price schedule is characterised by a pair $(\hat{I}, R)$ satisfying

$$\hat{I} \geq I_-(R), \qquad R \geq W(\hat{I}),$$

and takes the form stated in the theorem. Note that $R \geq W(\hat{I})$ is equivalent to $\hat{I} \leq I^*(R)$; hence for all $I \in [\hat{I}, I^*(R)]$ we have $W(I) \leq R$ and therefore $P^*(I, R) \leq W(I) \leq R$ (since $\partial R/\partial P < 0$ and $R(I, W(I)) = W(I)$), so participation and the cap $P(I) \leq R$ are automatic on the subsidised segment. For such a pair, profit is exactly

$$\Pi(\hat{I}, R) = \int_{\hat{I}}^{I^*(R)} P^*(I, R)\, dF(I) + R\, [1 - F(I^*(R))].$$

The feasible set

$$\{(\hat{I}, R) : \hat{I} \in [\underline{I}, \bar{I}],\ R \in [0, W(\bar{I})],\ \hat{I} \geq I_-(R),\ R \geq W(\hat{I})\}$$

is compact under the maintained continuity assumptions, and $\Pi(\hat{I}, R)$ is continuous in $(\hat{I}, R)$. Therefore, an optimal pair $(\hat{I}^*, R^*)$ exists by the Weierstrass theorem, and the corresponding schedule is optimal.

A.9 Proof of Proposition 5

The optimisation problem solved in Theorem 1 maximises profit over the space of all pricing schedules satisfying the ENAC (Definition 2) and non-negativity. A uniform price P_U^*

satisfies ENAC, as $R(I, P_U^*) \geq R(I, W(I)) = W(I) \geq P_U^*$ (using $\partial R/\partial P < 0$) for all served consumers. Therefore, the optimal uniform pricing strategy is a feasible solution to the general maximisation problem. The optimised discriminatory profit must therefore be at least as large as the optimised uniform profit. Thus, $\Pi_D^* \geq \Pi_U^*$.

A.10 Proof of Proposition 6

Part (1) follows directly from the definition of Total Surplus.

For (2), we compare TS_D^* and TS_U^*. Since $W(I) > 0$, $TS_D^* \geq TS_U^*$ if and only if $\hat{I}^* \leq I^U$. The comparison between $\hat{I}^*$ and I^U is generally ambiguous without further assumptions on the distribution and utility functions, as the optimisation problems are distinct and subject to different constraints (ENAC and non-negativity for discrimination).

A.11 Proof of Proposition 7

If discrimination achieves full coverage, then

$$\mathrm{TS}_D^* = \int_{\underline{I}}^{\bar{I}} W(I)\, dF(I) = TS^{FB}.$$

If the optimal uniform monopoly price excludes some consumers, then $I^U > \underline{I}$ and hence $(\underline{I}, I^U)$ has positive measure. Therefore,

$$\mathrm{TS}_U^* = \int_{I^U}^{\bar{I}} W(I)\, dF(I) < \int_{\underline{I}}^{\bar{I}} W(I)\, dF(I) = \mathrm{TS}_D^*,$$

where the inequality is strict because $W(I) > 0$ and the set of excluded types has positive measure.

A.12 Concavity and monotonicity in the noisy-signal extension

Recall the noisy-signal objective (36):

$$\Pi_D(P_L; \psi) = [\alpha + \psi(1-\alpha)]\, P_L + (1-\psi)(1-\alpha)\, R(I_L, P_L),$$

where $R(I_L, P_L)$ is the low-income reservation price induced by a subsidised low-signal price P_L.

Lemma 5. *Under CES preferences, for each $\psi \in [0,1)$ the function $P_L \mapsto \Pi_D(P_L;\psi)$ is strictly concave on $[0, W(I_L)]$ and admits a unique maximiser $P_L^*(\psi)$. Moreover, $P_L^*(\psi)$ is strictly increasing in ψ, and $R^*(\psi) \equiv R(I_L, P_L^*(\psi))$ is strictly decreasing in ψ.*

Proof. Fix $\psi \in [0,1)$. Under CES preferences, $R(I_L, P_L)$ is strictly concave in P_L on $[0, W(I_L)]$, i.e., $\partial^2 R(I_L, P_L)/\partial P_L^2 < 0$. Since $\Pi_D(P_L;\psi)$ is an affine function of P_L plus the term $(1-\psi)(1-\alpha)R(I_L, P_L)$ with strictly positive weight $(1-\psi)(1-\alpha) > 0$, it follows that $\Pi_D(\cdot;\psi)$ is strictly concave. Therefore the maximiser $P_L^*(\psi)$ exists and is unique on the compact set $[0, W(I_L)]$.

The first-order condition characterising the unique maximiser is (37):

$$0 = \frac{\partial \Pi_D(P_L;\psi)}{\partial P_L} = [\alpha + \psi(1-\alpha)] + (1-\psi)(1-\alpha)\,\frac{\partial R(I_L, P_L)}{\partial P_L} \quad \text{evaluated at } P_L = P_L^*(\psi).$$

Let $R_P(P_L) \equiv \partial R(I_L, P_L)/\partial P_L$ and $R_{PP}(P_L) \equiv \partial^2 R(I_L, P_L)/\partial P_L^2 < 0$. Define

$$F(P_L, \psi) \equiv [\alpha + \psi(1-\alpha)] + (1-\psi)(1-\alpha)\, R_P(P_L).$$

Then $F(P_L^*(\psi), \psi) = 0$. By strict concavity, F is strictly decreasing in P_L:

$$\frac{\partial F}{\partial P_L}(P_L, \psi) = (1-\psi)(1-\alpha)\, R_{PP}(P_L) < 0.$$

Thus the implicit function theorem applies and yields

$$\frac{dP_L^*(\psi)}{d\psi} = -\frac{\partial F/\partial \psi}{\partial F/\partial P_L} = -\frac{(1-\alpha)\,[1 - R_P(P_L^*(\psi))]}{(1-\psi)(1-\alpha)\, R_{PP}(P_L^*(\psi))} = -\frac{1 - R_P(P_L^*(\psi))}{(1-\psi)\, R_{PP}(P_L^*(\psi))}.$$

Because $R_P(P_L) < 0$, we have $1 - R_P(P_L) > 0$, and because $R_{PP}(P_L) < 0$, the fraction above is strictly negative; the leading minus sign therefore implies $dP_L^*(\psi)/d\psi > 0$.

Finally, by the chain rule,

$$\frac{dR^*(\psi)}{d\psi} = \frac{\partial R(I_L, P_L)}{\partial P_L}\Big|_{P_L = P_L^*(\psi)} \cdot \frac{dP_L^*(\psi)}{d\psi} = R_P(P_L^*(\psi)) \cdot \frac{dP_L^*(\psi)}{d\psi} < 0,$$

since $R_P(P_L^*(\psi)) < 0$ and $dP_L^*(\psi)/d\psi > 0$. □

A.13 Proof of Proposition 9

By Lemma 5, for each $\psi \in [0,1)$ there exists a unique maximiser $P_L^*(\psi)$ of $\Pi_D(P_L;\psi)$, and the resulting separating profit

$$\Pi^*_{\text{sep}}(\psi) \equiv \Pi_D(P_L^*(\psi);\psi)$$

is well-defined and continuous in ψ.

Moreover, the envelope theorem gives, for $\psi \in [0,1)$,

$$\frac{d\Pi^*_{\text{sep}}(\psi)}{d\psi} = \frac{\partial \Pi_D(P_L;\psi)}{\partial \psi}\Big|_{P_L=P_L^*(\psi)} = (1-\alpha)\,P_L^*(\psi) - (1-\alpha)\,R(I_L, P_L^*(\psi)) = -(1-\alpha)\,[R^*(\psi) - P_L^*(\psi)].$$

In a separating outcome we have $R^*(\psi) = P_H^*(\psi) > P_L^*(\psi)$, hence $R^*(\psi) - P_L^*(\psi) > 0$ and therefore $d\Pi^*_{\text{sep}}(\psi)/d\psi < 0$. This proves that $\Pi^*_{\text{sep}}(\psi)$ is strictly decreasing on $[0,1)$.

Since Π^*_{sep} is strictly decreasing, the equation $\Pi^*_{\text{sep}}(\psi) = \Pi^*_U$ has at most one solution $\psi^* \in [0,1)$. If additionally $\Pi^*_{\text{sep}}(0) > \Pi^*_U$, continuity implies the existence of a unique $\psi^* \in (0,1)$ such that $\Pi^*_{\text{sep}}(\psi^*) = \Pi^*_U$, and separation is optimal if and only if $\psi < \psi^*$.

A.14 Proof of Proposition 10

Under Assumption 3, a type-I consumer either purchases the entire bundle of N monopoly goods or purchases none. Under symmetric pricing $P_j(I) = P(I)$, a buyer of type I pays $NP(I)$ and has residual income

$$I'(I) \equiv I - NP(I).$$

Total (industry) profit under a symmetric schedule is $N \int_{I\in S} P(I)\, dF(I)$, so (since N is a constant) it suffices to characterise the per-product revenue maximiser

$$\max_{S,\, P(\cdot)} \int_{I\in S} P(I)\, dF(I) \tag{48}$$

subject to:

1. **Monotonicity/regularity:** $P(\cdot)$ is continuous and non-decreasing on S.
2. **Non-negativity:** $P(I) \geq 0$ for all $I \in S$.
3. **Participation (bundle demand):** for all $I \in S$,

$$NP(I) \leq W_{\text{Bundle}}(I, N), \tag{49}$$

where $W_{\text{Bundle}}(I,N)$ solves $V\big(N, I - W_{\text{Bundle}}(I,N)\big) = V(0,I)$.

4. **ENAC for each good:** for all $I, I' \in S$,

$$P(I') \leq R_j\big(I - NP(I)\big), \tag{50}$$

where $R_j(I')$ is the reservation price for a single good j defined by $V(N,I') = V(N-1, I' + R_j)$ and is strictly increasing in I'.

Step 1: ENAC collapses to a single cap. Let

$$\bar{P} \equiv \sup_{I \in S} P(I) \qquad \text{and} \qquad \underline{I}' \equiv \inf_{I \in S}\big(I - NP(I)\big)$$

denote (respectively) the highest per-good price charged among served types and the smallest residual income among served types. Since $R_j(\cdot)$ is strictly increasing, (50) is equivalent to the single inequality

$$\bar{P} \leq R_j(\underline{I}'). \tag{51}$$

Indeed, if (51) holds then for any $I \in S$ we have $I - NP(I) \geq \underline{I}'$ so $R_j\big(I - NP(I)\big) \geq R_j(\underline{I}') \geq \bar{P} \geq P(I')$, which implies (50). Conversely, if (50) holds, taking I' arbitrarily close to an argmax of P and I arbitrarily close to an argmin of $I - NP(I)$ yields (51).

Define the *target (cap) price* as

$$R \equiv \bar{P}.$$

In any optimum, (51) binds, so $R = R_j(\underline{I}')$. If it were slack, then (by continuity of W_{Bundle} and monotonicity of P) one could raise $P(I)$ slightly for the highest-income served types while preserving (49) and (51), increasing profit in (48).

Step 2: Subsidised types must equalise residual income. Consider any served income $I \in S$ for which $P(I) < R$ (a subsidised buyer). Suppose, toward a contradiction, that this buyer has residual income strictly above the minimum:

$$I - NP(I) > \underline{I}'.$$

Then $R_j\big(I - NP(I)\big) > R_j(\underline{I}') = R$. Because both R_j and W_{Bundle} are continuous and $P(I) < R$, there exists $\varepsilon > 0$ such that increasing $P(I)$ to $P(I) + \varepsilon$ preserves (49) and still satisfies $P(I) + \varepsilon \leq R_j\big(I - N(P(I) + \varepsilon)\big)$, so ENAC remains satisfied. This strictly increases profit, contradicting optimality.

Hence, for every subsidised served type (every $I \in S$ with $P(I) < R$), we must have

$$I - NP(I) = \underline{I}'.$$

Let $\hat{I} \equiv \underline{I}'$. Then on the subsidised region,

$$P(I) = \frac{I - \hat{I}}{N}. \tag{52}$$

In particular, the intermediate segment has slope $1/N$.

Step 3: Non-negativity implies exclusion below $\hat{I}$. Equation (52) implies $P(I) < 0$ for $I < \hat{I}$. Since feasible schedules must satisfy $P(I) \geq 0$, types with $I < \hat{I}$ cannot be served while maintaining the target cap R. Thus the optimal served set takes the threshold form $S = [\hat{I}, \bar{I}]$, and the schedule can be written as $P(I) = 0$ for $I < \hat{I}$ (exclusion), with $P(\cdot)$ given by (52) or R on $[\hat{I}, \bar{I}]$.

Step 4: The uniform-pricing region and the cutoff I^*. Since $W_{\text{Bundle}}(I, N)$ is strictly increasing in I, there is a unique income level I^* at which a buyer is just willing to purchase the bundle when each good is priced at the cap R:

$$W_{\text{Bundle}}(I^*, N) = NR. \tag{53}$$

For any $I \geq I^*$, participation is satisfied at per-good price R. Because $P(I) \leq R$ by definition of R and profit is increasing in $P(I)$ pointwise, optimality requires $P(I) = R$ for all $I \geq I^*$. For $I < I^*$, participation would fail at $P(I) = R$, so any served type below I^* must lie on the subsidised segment (52).

Step 5: Continuity pins down the stated relationships. Continuity at the kink implies

$$R = P(I^*) = \frac{I^* - \hat{I}}{N},$$

and together with (53) this yields exactly the two relationships stated in Proposition 10.

Combining the exclusion region $I < \hat{I}$, the linear subsidised region $\hat{I} \leq I \leq I^*$, and the uniform region $I \geq I^*$ proves the claimed piecewise form. Finally, the pointwise improvement arguments in Steps 1–4 also provide the missing best-response verification: any admissible deviation that raises prices violates either participation or ENAC, while any deviation that lowers prices weakly reduces profit because it relaxes no binding constraint at the optimum.

A.15 Proof of Proposition 11

Consider $f(x) = x^\kappa$ on $(0, \infty)$. If $\kappa < 1$, then f is strictly concave, so discrete increments $f(N) - f(N-1)$ are strictly decreasing in N; hence $\Delta(N)$ decreases. If $\kappa = 1$, then $f(x) = x$ and $\Delta(N) = 1$ for all N. If $\kappa > 1$, f is strictly convex, so increments are strictly increasing; hence $\Delta(N)$ increases.

For fixed I', differentiate $R_j(I')$ with respect to Δ:

$$\frac{\partial R_j(I')}{\partial \Delta} = \frac{1}{\rho}\Big((I')^\rho + \tfrac{\beta}{1-\beta}\Delta\Big)^{\frac{1}{\rho}-1} \cdot \frac{\beta}{1-\beta} > 0,$$

so $R_j(I')$ is strictly increasing in $\Delta(N)$, and the stated monotonicity in N follows from the monotonicity of $\Delta(N)$.

A.16 Proof of Lemma 4

Consider a high-income consumer in stage 1. If they disclose, they are identified and can be charged P_H; if they do not disclose, they pay the anonymous price P_A. Because privacy costs are strictly positive, a necessary condition for any high-income consumer to disclose with positive probability is that disclosure weakly reduces the purchase price, i.e., $P_H \leq P_A$. But in stage 2, the monopolist can always weakly increase profit by setting $P_H \geq P_A$ (it can raise P_H up to the binding constraint without affecting anonymous demand), hence no stage-2 optimum satisfies $P_H < P_A$. Therefore, in any subgame-perfect equilibrium, we must have $s_H = 0$.

Now fix $s_H = 0$. If $P_A \leq W(I_L)$, then anonymous low-income consumers buy. Since some low-income consumers buy at price P_A, ENAC implies that no other buyer can be charged more than their reservation price at P_A, i.e., the maximal price is at most $R(I_L, P_A)$. Raising P_A weakly increases revenue from the anonymous group; the highest anonymous price consistent with keeping low types buying is $P_A = W(I_L)$. At $P_A = W(I_L)$, the low-income type is indifferent and has reservation $R(I_L, W(I_L)) = W(I_L)$, so ENAC implies every other price is at most $W(I_L)$. Thus, the profit-maximising choice in the inclusive case is $P_A = P_L = P_H = W(I_L)$.

If instead $P_A > W(I_L)$, anonymous low-income consumers do not buy. Then the anonymous demand is only for high-income consumers, so participation requires $P_A \leq W(I_H)$. If the monopolist sets $P_A > P_L$, then low-income consumers who buy at P_L could resell to high-income anonymous consumers at price P_A unless ENAC holds, which requires $P_A \leq R(I_L, P_L)$. Given P_L, profit is increasing in P_A subject to this constraint and $P_A \leq W(I_H)$, so the optimum sets $P_A = \min\{W(I_H), R(I_L, P_L)\}$. In the resale-constrained exclusionary regime, $R(I_L, P_L) \leq W(I_H)$ and hence $P_A = R(I_L, P_L)$. Substituting this into profit yields the stated objective $\Pi(P_L; s_L)$.

A.17 Proof of Proposition 12

Inclusive equilibrium. Suppose $W(I_L) \geq (1-\alpha)W(I_H)$. Consider the candidate profile with $s_L = s_H = 0$ and $P_A = P_L = P_H = W(I_L)$. Given $s_L = s_H = 0$, the monopolist's stage-2 choice is between (i) pricing inclusively at $W(I_L)$ (profit $W(I_L)$) and (ii) excluding low types and pricing at $W(I_H)$ (profit $(1-\alpha)W(I_H)$). The assumed inequality implies the monopolist prefers inclusive pricing, hence $(P_A, P_L, P_H) = (W(I_L), W(I_L), W(I_L))$ is a best response. Given these prices and strictly positive privacy costs, no consumer strictly benefits from disclosing (disclosure does not lower the purchase price), so $s_L = s_H = 0$ is optimal for both types. Thus, this is a subgame-perfect equilibrium.

Exclusionary equilibrium. Suppose instead $W(I_L) < (1-\alpha)W(I_H)$. In the subgame with $s_L = s_H = 0$, the monopolist strictly prefers exclusionary pricing at $P_A = W(I_H)$. Therefore no-sharing with inclusive pricing cannot be an equilibrium. By Lemma 4, in any subgame-perfect equilibrium we must have $s_H^* = 0$ and the monopolist's stage-2 problem is to choose P_L to maximise

$$\Pi(P_L; s_L) = s_L \alpha \, P_L + (1-\alpha) \, R(I_L, P_L),$$

setting $P_A = P_H = R(I_L, P_L)$. Under CES (and, more generally, under the concavity condition used in the paper), $P_L \mapsto R(I_L, P_L)$ is strictly concave, hence $\Pi(\cdot; s_L)$ is strictly concave and admits a unique maximiser $P_L^*(s_L)$ for each $s_L \in (0,1)$. The associated anonymous/high price is $R^*(s_L) \equiv R(I_L, P_L^*(s_L))$.

Given prices $(P_A, P_L) = (R^*(s_L), P_L^*(s_L))$ with $R^*(s_L) > W(I_L)$, an anonymous low-income consumer does not buy. A low-income consumer who discloses buys at price P_L and incurs privacy cost τ (in numeraire units), so she discloses if and only if $\tau \leq W(I_L) - P_L$. Thus, best-response disclosure implies

$$s_L = T_L(W(I_L) - P_L^*(s_L)).$$

Define the mapping

$$\mathcal{B}(s) \equiv T_L(W(I_L) - P_L^*(s)).$$

By strict concavity of $\Pi(\cdot; s)$, the maximiser $P_L^*(s)$ is continuous and strictly increasing in s (from the implicit-function argument applied to the first-order condition $s\alpha + (1 -$

$\alpha)\partial R/\partial P_L = 0$ and $\partial^2 R/\partial P_L^2 < 0$). Since T_L is strictly increasing, $\mathcal{B}(s)$ is continuous and strictly decreasing. Therefore, the fixed point equation $s = \mathcal{B}(s)$ has at most one solution.

Existence follows from boundary behaviour of $h(s) \equiv \mathcal{B}(s) - s$. As $s \downarrow 0$, the monopolist places vanishing weight on low-income revenue and thus chooses the maximal subsidy, so $P_L^*(s) \downarrow 0$ and hence $h(0) = \mathcal{B}(0) > 0$. As $s \uparrow 1$, $P_L^*(s)$ converges to $P_L^*(1)$, the resale-constrained optimum when all low-income consumers disclose. In general $P_L^*(1) < W(I_L)$, so $\mathcal{B}(1) = T_L(W(I_L) - P_L^*(1)) \in (0,1)$ (since T_L has unbounded upper support) and therefore $h(1) < 0$. By continuity, there exists a (unique) fixed point $s_L^* \in (0,1)$. Continuity implies there is a unique fixed point $s_L^* \in (0,1)$. Setting $P_L^* = P_L^*(s_L^*)$ and $R^* = R^*(s_L^*)$ yields the stated equilibrium.

A.18 Proof of Proposition 13

We analyse the derivative of Total Surplus $TS(s_L)$ with respect to s_L in the Exclusionary Regime. Note that Total Surplus is $TS(s_L) = s_L \alpha W(I_L) + (1-\alpha)W(I_H) - C(s_L)$, where $C(s_L)$ is total privacy cost. TS does not depend directly on R, so $\frac{\partial TS}{\partial R^*} = 0$ (prices are transfers).

We analyse the direct effect. The marginal cost of privacy is $C'(s_L) = \alpha \tau_L(s_L)$.

$$\frac{dTS}{ds_L} = \alpha W(I_L) - \alpha \tau_L(s_L).$$

Because τ enters as a monetary (numeraire) cost that reduces disposable income one-for-one, indifference between disclosing-and-buying and remaining anonymous implies $V(1, I_L - P_L^* - \tau) = V(0, I_L)$, which by definition of $W(I_L)$ is equivalent to $\tau = W(I_L) - P_L^*$. In equilibrium s_L^*, the marginal sharer is indifferent: $\tau_L(s_L^*) = W(I_L) - P_L^*$. Substituting this into the derivative:

$$\left.\frac{dTS}{ds_L}\right|_{s_L^*} = \alpha W(I_L) - \alpha(W(I_L) - P_L^*) = \alpha P_L^*.$$

If prices are non-negative ($P_L^* \geq 0$), then $\frac{dTS}{ds_L} \geq 0$. The equilibrium level of data sharing is weakly inefficiently low because the marginal consumer ignores the profit P_L^* that their participation generates.

B Worked examples from Section 4

B.1 Complementary goods

This appendix provides formal details for the complementarity mechanism discussed in Section 4.2. The key idea is that ownership of the monopoly good changes the productivity of spending on other consumption, generating $\Gamma > 1$ even absent income heterogeneity.

A monopolist sells an indivisible good $q_M \in \{0, 1\}$ at price P and zero marginal cost. Consumers have common income $I > 0$ but differ in a complementarity parameter $\theta \in [\underline{\theta}, \bar{\theta}]$ with $\underline{\theta} > 1$. Disposable income is $D = I - P$ when the good is purchased.

Assumption 4 (Multiplicative complementarities)**.** Preferences admit an indirect-utility representation

$$V(q_M, D; \theta) = \begin{cases} \theta\, u(D) & \text{if } q_M = 1, \\ u(D) & \text{if } q_M = 0, \end{cases} \tag{54}$$

where u is strictly increasing and strictly concave.

B.1.1 WTP, reservation price, and the firewall index

Definition 3 (WTP and reservation price under complementarities)**.** For a type-θ consumer:

1. $W(\theta)$ solves $V(1, I - W(\theta); \theta) = V(0, I; \theta)$.
2. $R(\theta, P)$ solves $V(1, I - P; \theta) = V(0, I - P + R(\theta, P); \theta)$.

Lemma 6 (General characterisation and sign of $\partial R/\partial P$)**.** *Under Assumption 4, $R(\theta, P) > 0$ for all $P < W(\theta)$ and*

$$\frac{\partial R}{\partial P}(\theta, P) = 1 - \theta\, \frac{u'(I - P)}{u'(I - P + R(\theta, P))} < 0. \tag{55}$$

Equivalently, $\Gamma(\theta, P) > 1$ and $\phi(\theta, P) = \Gamma(\theta, P) - 1 > 0$.

Proof. The reservation price solves $\theta u(I - P) = u(I - P + R)$; since $\theta > 1$ and u is increasing, it follows that $R > 0$. Differentiating gives

$$-\theta u'(I - P) = u'(I - P + R) \left(-1 + \frac{\partial R}{\partial P} \right),$$

which rearranges to (55). Since $R > 0$ and u' is decreasing, $u'(I - P + R) < u'(I - P)$ and thus the term subtracted from 1 exceeds $\theta > 1$, implying $\partial R/\partial P < 0$. □

B.1.2 Closed forms under power utility

For a transparent closed-form illustration, let $u(D) = D^\rho$ with $\rho \in (0, 1)$.

Lemma 7 (Closed forms under power utility)**.** *If* $u(D) = D^\rho$ *with* $\rho \in (0, 1)$*, then*

$$W(\theta) = I\left(1 - \theta^{-1/\rho}\right), \tag{56}$$

$$R(\theta, P) = \left(\theta^{1/\rho} - 1\right)(I - P), \tag{57}$$

$$-\frac{\partial R}{\partial P}(\theta, P) = \theta^{1/\rho} - 1 > 0. \tag{58}$$

Proof. For $W(\theta)$, solve $\theta(I - W)^\rho = I^\rho$. For $R(\theta, P)$, solve $\theta(I - P)^\rho = (I - P + R)^\rho$. Differentiating (57) yields (58). □

B.1.3 Mapping to ENAC-based discrimination

When θ is observable, the ENAC logic applies with θ as the type index: for a target resale price $R > 0$, define $P^*(\theta, R)$ by $R(\theta, P^*(\theta, R)) = R$. Under (57),

$$P^*(\theta, R) = I - \frac{R}{\theta^{1/\rho} - 1}, \tag{59}$$

which is strictly increasing in θ. Thus lower-θ consumers (weaker complementarities) are the natural "weak link" in arbitrage terms: they have lower reservation values at a given price and are the group that must be subsidised to raise the market-clearing reservation price to R.

B.2 Asymmetric substitution and fallback consumption

This appendix provides formal details for the asymmetric "fallback substitute" mechanism discussed in Section 4.3. The mechanism operates even when the monopoly good has a substitute, provided substitutability is state-dependent: the fallback is valuable only when the monopoly good is absent.

A monopolist sells an indivisible good M (denote $q_M \in \{0, 1\}$) at price P and zero marginal cost. Consumers have income $I > 0$ and access to a fallback option S at unit price 1 and a numeraire C.

Assumption 5 (Asymmetric fallback substitution)**.** Preferences take the form

$$U(q_M, q_S, q_C; \gamma) = \phi(q_M) + (1 - q_M)\,\gamma\, g(q_S) + h(q_C), \tag{60}$$

where $\phi(1) = \bar{\phi} > 0$ and $\phi(0) = 0$, $\gamma \in (0, 1]$ indexes the effectiveness of the fallback option, g is strictly increasing and strictly concave with Inada-type conditions ensuring an interior choice of q_S when $q_M = 0$, and h is strictly increasing and strictly concave.

B.2.1 Indirect utilities with and without the monopoly good

Let D denote disposable income in a given state.

With M owned. When $q_M = 1$, the fallback term drops out and

$$U_1(D) = \bar{\phi} + h(D). \tag{61}$$

Without M. When $q_M = 0$, the consumer solves $\max_{q_S \geq 0} \ \gamma g(q_S) + h(D - q_S)$. Let $q_S^*(\gamma, D)$ denote the optimal fallback choice, characterised by the FOC

$$\gamma g'\big(q_S^*(\gamma, D)\big) = h'\big(D - q_S^*(\gamma, D)\big), \tag{62}$$

and define

$$U_0(\gamma, D) = \gamma g\big(q_S^*(\gamma, D)\big) + h\big(D - q_S^*(\gamma, D)\big). \tag{63}$$

B.2.2 WTP, reservation price, and the sign condition

Definition 4 (WTP and reservation price under fallback consumption)**.** For type γ:

1. $W(\gamma)$ solves $U_1(I - W(\gamma)) = U_0(\gamma, I)$.
2. $R(\gamma, P)$ solves $U_1(I - P) = U_0(\gamma, I - P + R(\gamma, P))$.

Proposition 14 (Derivative of reservation price and a sharp sign condition)**.** *Under Assumption 5,*

$$\frac{\partial R}{\partial P}(\gamma, P) = 1 - \frac{h'(I - P)}{h'\big(I - P + R(\gamma, P) - q_S^*(\gamma,\ I - P + R(\gamma, P))\big)}. \tag{64}$$

Moreover,

$$\frac{\partial R}{\partial P}(\gamma, P) < 0 \quad \Longleftrightarrow \quad q_S^*\big(\gamma,\ I - P + R(\gamma, P)\big) < R(\gamma, P). \tag{65}$$

Proof. Write the reservation indifference as

$$\bar{\phi} + h(I - P) = U_0\big(\gamma,\ I - P + R(\gamma, P)\big),$$

and differentiate with respect to P. Envelope arguments combined with the FOC (62) imply

$$\frac{\partial U_0}{\partial D}(\gamma, D) = h'\big(D - q_S^*(\gamma, D)\big).$$

Letting $D_1 = I - P$ and $D_0 = I - P + R(\gamma, P)$, differentiation gives

$$-h'(D_1) = h'\big(D_0 - q_S^*(\gamma, D_0)\big)\left(-1 + \frac{\partial R}{\partial P}\right),$$

which rearranges to (64). Since h' is strictly decreasing, $\partial R/\partial P < 0$ holds if and only if

$$h'(D_1) > h'\big(D_0 - q_S^*(\gamma, D_0)\big) \iff D_1 < D_0 - q_S^*(\gamma, D_0) \iff q_S^*(\gamma, D_0) < R(\gamma, P),$$

proving (65). □

Interpretation. Condition (65) says that resale frees up net cash even after financing the fallback:

$$R(\gamma, P) - q_S^*\big(\gamma,\, I - P + R(\gamma, P)\big) > 0.$$

Equivalently, the consumer's disposable income after resale and fallback spending, $(I - P + R) - q_S^*$, is higher than the ownership-state disposable income $I - P$. The post-resale state is therefore relatively cash-rich, lowering the marginal utility of income there and making a subsidy more valuable in the ownership state. This is exactly $\Gamma(\gamma, P) > 1$.

B.2.3 Heterogeneity and discrimination

Type γ governs the quality of the fallback option. Better fallback (higher γ) raises $U_0(\gamma, \cdot)$, lowering both WTP and WTA for the monopoly good. Hence, under mild regularity conditions, $W(\gamma)$ and $R(\gamma, P)$ are decreasing in γ, and the ENAC logic applies with γ as the screening index: the group with the *best* outside option (high γ) is the natural weak link for arbitrage and is the group that must be subsidised to sustain a high market-clearing reservation price.

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