NBER WORKING PAPERS SERIES

PARTICIPATION DYNAMICS: SUNSPOTS AND CYCLES

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Working Paper No. 3438

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 1990

We thank, without implicating, Jean-Michel Grandmont, Roger Guesnerie, Walter Heller, Peter Howitt and John Weinberg for helpful comments on this problem. We are grateful to the NSF, under SES-8822281, for providing financial assistance. This paper is part of NBER's research program in Economic Fluctuations. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

This paper investigates the possibility of sunspots equilibria and endogenous cycles in an overlapping generations model with strategic interactions. We consider an economy with imperfectly competitive product markets. There is a participation decision on the part of prospective firms and a strategic complementarity emerges from the interaction of firms in their entry decisions both over time and across sectors.

When these complementarities are sufficiently strong, multiple steady state equilibria will exist. Sunspot equilibria can then be constructed as randomizations in the neighborhood of these steady states. We relate the properties of our sunspot equilibria to aggregate fluctuations, with particular emphasis on the dynamics of entry and exit.

We also show that if <u>intratemporal</u> strategic interactions are sufficiently strong, then cycles may exist. Additional sunspot equilibria can be found in the neighborhood of these cycles. Finally, we show that if <u>intertemporal</u> linkages are sufficiently strong, cycles will not exist.

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I. Introduction

The extent to which beliefs alone can be important factors in generating economic fluctuations has been an important area of investigation for macroeconomists. The fact that beliefs about the future matter in determining current decisions is a property of almost all dynamic economic systems. The more difficult task is finding economic environments in which changes in beliefs about future variables influence current choices under the condition that, in equilibrium, beliefs be correct. Because the resulting fluctuations arise in the absence of variations in fundamental variables, such as preferences and technology, these are models of extrinsic uncertainty and equilibria in which beliefs matter are often termed "sunspot equilibria."

Conditions for the existence of stationary sunspot equilibria in overlapping generations economies are contained in Azariadis [1981], Cass-Shell [1983], Spear [1984] and Woodford [1984].[1] Azariadis [1981] considers an overlapping generations economy in which agents work in youth and consume in old age and finds that if consumption and leisure are gross substitutes, then a stationary sunspot equilibrium will not exist. Therefore a necessary condition for the existence of a sunspot equilibrium is that consumption and leisure be gross complements -- i.e. the income effect associated with an increase in the return to working dominates the substitution effect so that "labor supply" is downward sloping for some values of the real wage.

This condition of gross complementarities is also important for the emergence of equilibrium cycles, as in Grandmont [1985]. Azariadis-Guesnerie [1986] combine these results and find that stationary sunspot equilibria exist if and only if endogenous cycles exist.

While of considerable importance in terms of displaying internally consistent models of cycles and sunspot equilibria, these results rest on a condition of gross complementarity that is not well supported by existing empirical work on labor supply.[2] This has led to the consideration of alternative economic environments in which cycles and/or sunspot equilibria might exist. Using overlapping generations models, Reichlin [1986] generates cycles in a production economy through the interaction of capital and labor in the production process. Ravikumar [1988] discusses sunspots and cycles in an overlapping generations economy with trading externalities in product markets. Howitt-McAfee [1990] consider an infinite horizon model with transactions externalities

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[1] For conditions on the existence of non-stationary sunspot equilibria see Peck [1988].

[2] For instance, see the discussion in Blanchard-Fischer [1989].

which exhibits sunspot equilibria and cycles. Weil [1989] analyzes sunspot equilibria in a two-period model with complementarities through a non-convex storage technology. Finally, Woodford [1986] uses borrowing constraints to create stationary sunspot equilibria in an economy with infinitely lived agents.

In this paper, we consider an overlapping generations model in which there are two sectors. In each sector, there are a number of potential entrants who incur a cost to enter the economy and become a firm. A period t firm produces output in that period and consumes the output produced in the other sector in periods t and t+1.[3] Firms have market power in their output decisions.

In this economy, entry decisions are characterized by a condition of strategic complementarity: if more firms enter in one sector, there is an incentive for more firms to enter in the other sector.[4] Chatterjee-Cooper [1988] consider a static version of this economy in which firms in one sector consume the output produced by firms in the other sector while Chatterjee-Cooper [1989] consider a dynamic version of the economy in which firms producing in period t consume in period t+1. In the former setting, the complementarity reflects interactions across sectors and, in the latter, across time. In the model considered in this paper, both types of interactions are present.

When these complementarities are sufficiently strong, we find that multiple stationary equilibria may exist. Further, we find that cycles exist if the intertemporal linkages are not too strong. As in Azariadis-Guesnerie [1986], there will be sunspot equilibria in the neighborhood of these cycles. We are also able to construct sunspot equilibria as randomizations across multiple steady states. These sunspot equilibria show more persistence than those constructed near a cycle.

In contrast to Howitt-McAfee and Ravikumar, the linkages across agents in our economy is simply due to the fact that agents consume the output produced by others. In contrast to Weil and Woodford, we focus on the effects of these linkages in an overlapping generations economy which allows us to relate our findings to those stressed by Azariadis and Azariadis-Guesnerie. Our work is perhaps closest to Howitt [1990] who explores the existence of sunspot equilibria in a dynamic version of the Cooper-John [1988] model. In contrast to that

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[3] As discussed in Section II, this preference structure is intended to capture the fact that agents specialize in production and generalize in consumption.

[4] Strategic complementarity is a condition on the direction of the strategic interaction between players in a game and has been useful in a variety of contexts in industrial organization (Bulow, Geanakopolos and Klemperer [1985]) and in macroeconomics (Cooper-John [1988]).

paper, we stress a particular type of complementarity (that associated with market participation) in an overlapping generations economy.

Our model also has some interesting empirical predictions. Associated with these fluctuations are variations in the number of active firms in a market and the mark-ups of prices over cost. There is some evidence that lends support to the importance of entry and exit over the business cycle. As described in Chatterjee-Cooper [1988], net business formation (essentially entry less exit) has been used by the Commerce Department as a leading indicator of economic activity i.e., an increase in net business formation leads increases in aggregate activity. Further, Davis-Haltiwanger [1989,1990] find that in the manufacturing sector, a significant part of overall job creation and destruction is associated with the births and deaths of firms. Consistent with this evidence, the sunspot equilibria we characterize are supported by a process of entry and exit and net business formation predicts future output. Finally, in our economy, mark-ups are countercyclical which is consistent with the empirical findings of Bils [1987]. While entry and exit may certainly emerge in a competitive economy in which firms produce with U-shaped average cost curves, there would be no variations in mark-ups in such an economy.

Overall, this paper contributes to research on sunspot equilibria in two ways. First, we consider an environment in which cycles and sunspot equilibria might exist which do not rely on strong income effects. Instead, the presence of strategic complementarities in the intra- and intertemporal interactions of agents is necessary for the existence of sunspots and cycles. Second, we show that there may exist sunspot equilibria which are quite different from those stressed by Azariadis [1981], Azariadis-Guesnerie [1986] and Woodford [1984]: sunspot equilibria may be generated as randomizations between allocations in the neighborhood of two steady states.

II. The Environment

Consider an economy with two sectors where F agents are born in each sector at discrete times $t = 1,2,\ldots\infty$. They live for two periods and die at the end of the second period. At $t=1$, there is an initial generation of M old agents in each sector. Each initial old agent is endowed with one unit of fiat money.

Each agent born at $t = 1,2,\ldots\infty$ is endowed with one unit of leisure during the first period of his life which may be used to produce output in his sector on a one-for-one basis.[5] However, in order to activate this linear technology he has to forego productive, non-market activities. Assume that these opportunity costs are distributed according to a distribution function H(k), which is not sector-specific. Thus, the only difference between agents born at any time $t > 0$ is their opportunity costs. For convenience, we will index the agents in the increasing order of their opportunity costs i.e., $k_1 \leq k_2 \leq \ldots \leq k_F$.

This overlapping generations model has the virtue of creating entry and exit through the births and deaths of agents. However, this process links the time of activity to an agent's lifespan which we do not take literally. In future work we plan to explore many of these same issues in an alternative economic environment with infinitely lived agents so that entry and exit is not directly linked to the births and deaths of particular individuals. Nonetheless, this overlapping generations structure provides a vehicle for understanding, in a qualitative manner, the nature and importance of intra- and intertemporal participation complementarities in the formation of cyclic and sunspot equilibria.

Each agent born in sector i at time t derives utility from consuming the output of sector -i at time t and t+1.[6] This assumption is meant to capture the notion that the output of an agent is generally an unimportant element in that agent's consumption basket i.e., agents specialize in production but not in consumption. By restricting attention to a two-good model, the output of the other sector represents the general consumption basket of an agent. In addition, we are able to retain the tractable features of the overlapping generations model. The preferences of a representative agent born in sector i at time t are assumed to be

$$\alpha \ln c_t^{-i} + (1-\alpha) \ln c_{t+1}^{-i} - g(y_t^i) \qquad i = 1,2$$

where c_t^j is the consumption of sector j output at time t and y_t^j is the sector j labor/output at time t. To guarantee interior solutions in output, the function $g(\cdot)$ is assumed to be increasing and convex with $g'(y) \to 0$ as $y \to 0$ and $g'(y) \to \infty$ as $y \to 1$. We assume Cobb-Douglas preferences to give us a tractable formulation for

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[5] That agents produce in only one period limits the dynamic interactions in the model. In particular, variations in future demand cannot directly influence current participation decisions through a discounted profit calculation. However, the fact that agents consume when old implies that future market conditions impact on current participation decisions. It is this dynamic linkage that we exploit in this analysis.

[6] The notation "-i" refers to sectors other than sector i.

exploring the existence of the different types of stationary sunspot equilibria. This specification also enables us to discuss the relative importance of intra- and intertemporal linkages through variations in α. Results which hold for more general preference structures are indicated in Section V.

Exchange in the economy takes place, essentially, between goods of the two sectors as well as between goods and money. Output of sector i is exchanged for the output of sector -i by the young for current consumption. Output of sector i is also exchanged for fiat money held by the old in sector -i for future consumption. We assume that goods are not storable.

Any agent born at time t has two decisions to make. First, he has to decide whether to activate the production technology. If he activates the technology, then he has to decide how much to produce and how to allocate his revenues from production between consumption at time t and t+1. While making the second decision, he takes the output of other active agents in his sector and the prices of consumption goods in the other sector as given i.e.,he is assumed to be a Cournot competitor as a producer and a price taker as a buyer. This is consistent with our notion of specialist producers and non-specialist consumers.[7] If an agent chooses to be inactive, then that agent obtains utility of k_i and does not participate in existing markets.

In order to characterize the equilibria in this economy it is convenient to solve the production and consumption problem facing a representative agent in sector i before considering participation decisions. We will fix the number of active agents (firms) in each sector and characterize their utility. We will then endogenize the number of firms by comparing the maximal payoffs in a Nash equilibrium to the opportunity costs.

Any active firm in sector i at time t solves

$$\text{Max} \quad \alpha \ln c_t^{-i} + (1-\alpha) \ln c_{t+1}^{-i} - g(y_t^i)$$

subject to:

$$(1) \qquad p_t^{-i} c_t^{-i} + m_t^i = p_t^i y_t^i \qquad \text{and,}$$

$$(2) \qquad p_{t+1}^{-i} c_{t+1}^{-i} = m_t^i$$

where p_t^i is the price of sector i's output at time t quoted in fiat money units and m_t^i is the quantity of fiat money held by the agent in sector i at time t.

[7] This assumption is similar to other models of imperfect competition such as Hart [1982].

where p_t^i is the price of sector i's output at time t quoted in fiat money units and m_t^i is the quantity of fiat money held by the agent in sector i at time t.

The two budget equations may be combined to yield

$$(3)\quad p_t^{-i}c_t^{-i} + p_{t+1}^{-i}c_{t+1}^{-i} = p_t^i y_t^i$$

so that the first order conditions are

$$(4)\quad \frac{\alpha}{c_t^{-i}} = \lambda p_t^{-i}$$

$$(5)\quad \frac{1-\alpha}{c_{t+1}^{-i}} = \lambda p_{t+1}^{-i}$$

$$(6)\quad g'(y_t^i) = \lambda \left[p_t^i + y_t^i \frac{\partial p_t^i}{\partial y_t^i} \right]$$

In the above first order conditions, λ is the Lagrange multiplier associated with the budget constraint. Equations (4) and (6) relate the marginal utility of consumption today to the marginal cost of production. Equations (5) and (6) link the marginal cost of production to the marginal utility of consumption tomorrow. Since preferences are logarithmic, consumptions at time t and t+1 must be strictly positive. Output would be strictly positive given our assumptions on $g(\cdot)$, if it can be sold for a positive price. We assume that there are at least two active firms in each sector so that each agent's problem is a non-trivial one.[8]

From equations (3), (4) and (5), $\lambda = 1/(p_t^i y_t^i)$. The demand for sector -i output by an agent in sector i is

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[8] If there is a single firm, then that firm will charge an infinite price and produce ε units of output given the specification of demand in this economy.

$$(7)\qquad c_t^{-i} = \alpha\,(p_t^i y_t^i)/p_t^{-i} \qquad \text{and}$$

$$(8)\qquad c_{t+1}^{-i} = (1-\alpha)(p_t^i y_t^i)/p_{t+1}^{-i}\ .^{9}$$

Using equations analogous to (7) and (8) for sector -i agents, the aggregate (inverse) demand for sector i's output at time t is

$$p_t^i = \frac{\alpha N_t^{-i} p_t^{-i} y_t^{-i} + N_{t-1}^{-i} m_{t-1}^{-i}}{y_t^i + Y_t^i}$$

where Y_t^i is the total output of other firms in sector i at time t. The first term in the numerator is the total expenditures by the young who are active in sector -i at time t and the second term in the numerator is the total expenditures by the old in sector -i at time t. Thus,

$$\frac{\partial p_t^i}{\partial y_t^i} = \frac{-p_t^i}{y_t^i + Y_t^i}$$

so that (6) becomes

$$g'(y_t^i) = \frac{1}{y_t^i}\left[1 - \frac{y_t^i}{y_t^i + Y_t^i}\right]$$

Since all active firms in sector i at time t solve the same problem, we impose symmetry across <u>firms</u> in sector i. Hence, $y_t^i + Y_t^i = N_t^i y_t^i$ and,

$$(9)\qquad y_t^i g'(y_t^i) = 1-(1/N_t^i).$$

Equation (9) determines the level of output per firm in sector i given the number of active firms in sector i. It should be noted, however, that zero output in sector i is also a Nash equilibrium since each firm in sector i faces a unit-elastic demand curve if other firms in sector i are producing zero units of output.

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[9] The consumption of the initial old in sector i is $1/p_1^{-i}$.

Lemma 1 (i) Given the number of firms in sector i the level of output per firm is unique.

(ii) The level of output per firm in sector i increases with the number of firms in sector i.

Proof (i) The left hand side of equation (9), $yg'(y)$, is increasing in y with $yg'(y) = 0$ for $y=0$ and $yg'(y) = \infty$ for $y=1$. Hence, given $N_t^i \in [2,F]$, the solution to (9), $y(N_t^i)$ is unique.

(ii) The result follows from the fact that the right hand side of (9) is increasing in N and the left hand side is increasing in y.

QED.

The fact that output per firm in a sector actually rises when the number of firms in that sector increases is unusual relative to the more familiar results in the Cournot-Nash partial equilibrium model. This is a consequence of the fact that the demand curve facing the industry has constant elasticity so that as the number of firms increases the elasticity of the residual demand curve increases.

Notice that the output determined by equation (9) does not depend upon α. This is due to the Cobb-Douglas preference structure. Any change in α shifts the expenditure between current and future consumption leaving the total expenditure on a given sector unchanged.

Note also that the output is independent of the number of active firms in sector -i at time t. Other things remaining the same, a larger number of firms in sector -i increases the demand for sector i output which, in turn, increases the marginal revenue of each agent in sector i. Since each agent in sector i is also a supplier of labor input, the marginal revenue increase is analogous to a real wage increase. For the above preference structure, the labor supply decision is independent of real wage and hence, y_t^i is independent of N_t^{-i}. For a similar reason, y_t^i is also independent of N_{t+1}^{-i}.

The price of output in sector i is determined by

$$p_t^i = \frac{\alpha N_t^{-i} p_t^{-i} y_t^{-i} + N_{t-1}^{-i} m_{t-1}^{-i}}{y_t^i N_t^i} = \frac{A_t^{-i}}{y_t^i N_t^i}$$

Firms in sector i at time t take A_t^{-i} as given since all the terms in the numerator are outside the control of firms in sector i. Aggregate revenue in sector i is equal to A_t^{-i} and hence, the revenue to each firm in sector i declines as N_t^i increases. Since the output per firm in sector i increases with the number of active firms (Lemma 1), each firm must be worse off as N_t^i increases. Lemma 2 states the result formally.

Lemma 2 The utility to an active firm in sector i declines with N_t^i.

Proof The utility to an active firm in sector i when there are N_t^i firms participating in sector i is given by

$$\alpha \ln\left[\alpha A_t^{-i}/p_t^{-i}\right] + (1-\alpha)\ln\left[(1-\alpha)A_t^{-i}/p_{t+1}^{-i}\right] - \ln(N_t^i) - g\left[y(N_t^i)\right]$$

which is decreasing in N_t^i since sector i firms take prices and output of other sectors as given.

QED.

This is an important element of our model. Simply stated, there is a congestion effect within a sector: as the number of firms increases, each active firm is made worse off by the competition. Note too that the utility of an active firm is decreasing in sector -i prices i.e., as more firms enter sector -i (at time t and/or at t+1) and depress prices in that sector, firms in sector i are better off as buyers.

Besides making production and consumption decisions, agents decide whether or not to participate in the market as firms. Our model of entry assumes that any entrant to sector i views A_t^{-i} and the price of sector -i output at t and t+1 as constants: i.e., he does not take into account the changes his entry may have on the decisions by sector -i firms.[10] However, he does take into account any changes in output the incumbent firms in sector i may make. That is, we view the entry decision as a stage preceding the choice of output by the active firms. This follows Chatterjee-Cooper [1989].[11]

In the next two sections we will characterize the dynamics of the above economy. Section III deals with two types of perfect foresight equilibria -- steady states and cycles of period two, while Section IV deals with stationary sunspot equilibria. Our point is to use the above results to gain some insights into the possibility of cycles and sunspot equilibria. To do so, we stress two types of linkages which are important in this model. First, there is an intratemporal linkage between activities in the two sectors. This is emphasized in Chatterjee-Cooper [1988] in a static context. Second, there is an intertemporal linkage, stressed in Chatterjee-Cooper [1989], which

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[10] A formalization of this is in Hart [1982].

[11] In contrast, Chatterjee-Cooper [1988] consider the case of simultaneous entry and output decisions.

links participation decisions over time. In the next section, we indicate how the relative importance of these two linkages allows us to construct equilibrium cycles.

III. Perfect Foresight Equilibria

Since firms in sector -i possess the same technology and have symmetric preferences over consumption, we restrict our attention to symmetric Nash equilibria where

$$N_t^i = N_t^{-i} = N_t; \quad y_t^i = y_t^{-i} = y_t; \quad p_t^i = p_t^{-i} = p_t \qquad \forall t.$$

A perfect foresight symmetric Nash equilibrium is a sequence of $\{N_t\}$, $\{y_t\}$ and $\{p_t\}$ such that

(i) given the participation of N_t firms in each sector at time t, y_t satisfies

(10) $y_t g'(y_t) = 1-(1/N_t)$,

the market clearing price p_t is determined by

(11) $(1-\alpha)N_t p_t y_t = M$ and,

(ii) $W(N_t,N_{t+1}) \geq k_j$ for $j = 1,2,..N_t$ and

$W^e(N_t,N_{t+1}) < k_j$ for $j = N_t+1, N_t+2,...F.$

where $W(N_t,N_{t+1})$ is the utility of a participating firm at time t and $W^e(N_t,N_{t+1})$ is the utility to an entrant at time t.[12]

The utility of a participating firm at time t is $\alpha\ln(\alpha y_t) + (1-\alpha)\ln((1-\alpha)p_t y_t/p_{t+1}) - g(y_t)$. Using the market clearing condition (11), we can write the participant firm's utility as

$$W(N_t,N_{t+1}) = \alpha\ln\alpha + (1-\alpha)\ln(1-\alpha) + \alpha\ln(y(N_t)) + (1-\alpha)\ln(y(N_{t+1})) +$$
$$(1-\alpha)\ln(N_{t+1}/N_t) - g(y(N_t))$$

[12] $W^e(N_t,N_{t+1})$ refers to the payoffs of a single firm entering in one sector in period t and hence is not generally the same as $W(N_t+1,N_{t+1})$ which is the payoff to a participating firm if there are N_t+1 firms in period t. Only when $\alpha=0$, so that only future consumption enters into the utility function, will $W^e(N_t,N_{t+1})=W(N_t+1,N_{t+1})$

where $y(N_t)$ solves (10).[13] The level of participation N_t would be justified in equilibrium at time t only if $W(N_t,N_{t+1})$ exceeds the cost of participation to each of the N_t active firms and the remaining F-N_t agents have no incentive to enter. Thus $W^e(N_t,N_{t+1})$ is the benefit at time t to a potential entrant in one sector when there are N_t active firms in each sector at time t and N_{t+1} active firms in each sector in time t+1. Lemma 2 implies that $W(N_t,N_{t+1}) > W^e(N_t,N_{t+1})$. An increase in N_t^i, hypothesized in Lemma 2, can be viewed as the entry of one of the non-participating firms in a candidate equilibrium. The lemma states that an entrant must receive lower utility, excluding the entry cost, than an incumbent.

Note that in this equilibrium, there is a critical value of k such that all firms with $k_j<k$ participate and all others do not: i.e. equilibria are ordered. As discussed in Chatterjee-Cooper [1988] there may also exist equilibria which are not ordered.

Steady States

A steady state perfect foresight equilibrium is a perfect foresight symmetric Nash equilibrium such that $N_t=N$, $y_t=y$ and $p_t=p$ for all t. Further, let W(N) denote the utility of a participating firm if there are N active firms in steady state. The following properties of W($\cdot$) will help us characterize the entry decision and the set of steady state equilibria.

Lemma 3 W(N) is an increasing function of N.

Proof Treating N as a continuous variable, the result follows from differentiating W($\cdot$) with respect to N and observing that, because of imperfect competition, $yg'(y)$ is less than 1 and that output per firm is increasing with N. QED.

The above lemma says that each active firms is better off in an equilibrium with more firms in both sectors than in one with few firms. As a seller of output, any increase in the number of firms in sector i makes each firm in sector i worse off. This is the point of Lemma 2. However, as the number of firms in sector -i increases the price of sector -i output decreases which makes the firms in sector i better off as buyers. According

[13] Note that $W(N_t,N_{t+1})$ is continuous in α since period t output is independent of α for all t. This property will be used in the proofs of Propositions 2 and 3.

to Lemma 3, the second effect dominates. Unlike the result in Lemma 2 this is a statement about a simultaneous increase in the number of firms in both sectors.

Using Lemma 3, a steady state equilibrium for the economy can be found quite easily as Figure 1 illustrates. The increasing function W(N) is the value of participation given that N firms are active in each sector in all time periods. The second function, H(k), is the cumulative distribution function described earlier. Since there are a finite number of potential firms in each period, H(k) is a step function.

Proposition 1 If $W(2) > k_2$, then there exists at least one steady state equilibrium.

Proof Suppose that $W(F) \geq k_F$. Then there exists an equilibrium with full participation since all active firms receive a payoff exceeding their opportunity costs.

If $W(F) < k_F$, then an equilibrium with full participation will not exist. Under the hypothesis that $W(2) > k_2$, then there will exist an k^* such that $W(FH(k^*)) = k^*$. The existence of a fixed point for mappings which are upward continuous is discussed in Edwards [1965].[14]

Then $N = FH(k^*)$ will be the steady state level of participation. To see why, note that $k_{N+1} > k^* \geq k_N$ so that $W(N) \geq k_i$ for $i = 1,2,..N$. Since $W(N) > W^e(N,N)$, by Lemma 2, no inactive firm wishes to participate. QED.

It is quite straightforward to construct examples of economies with multiple equilibria. An example is given in Figure 2. Here there are multiple crossings of the W(N) and H(k) functions. The ease with which one can construct these examples comes from the independence of W(N) and H(k). In other words, the distribution of the outside opportunities and the value of participating in an economy with N firms in each sector are not interdependent. This would not be the case if firms differed in, say, their preferences or technology. That the heterogeneity in the economy stems solely from outside opportunities is quite a useful simplification.

The presence of these multiple equilibria are similar to those indicated by the work of Diamond [1982], Chatterjee [1988] and Pagano [1989]. Here the complementarities leading to the multiple equilibria stem from the effects of thick markets on mark-ups. In contrast, Diamond stresses the importance of thick markets in the probability of a match while Chatterjee and Pagano focus on the risk reduction effects of thick markets.

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[14] We are grateful to Marc Dudey for this reference.

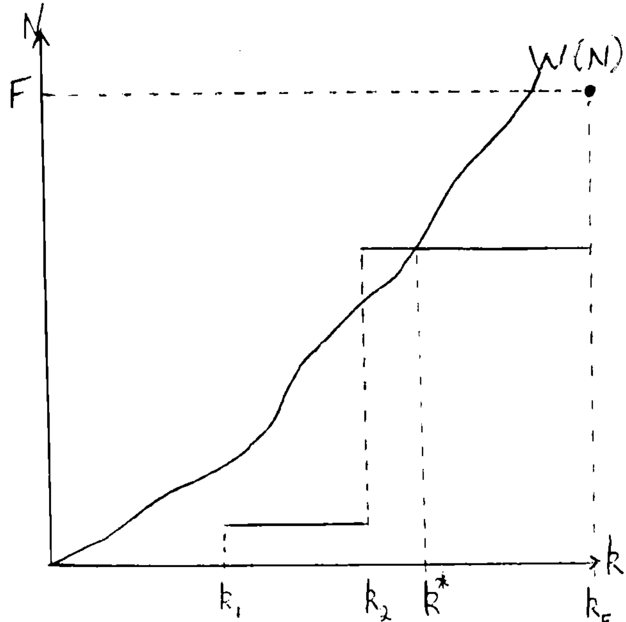

FIGURE 1

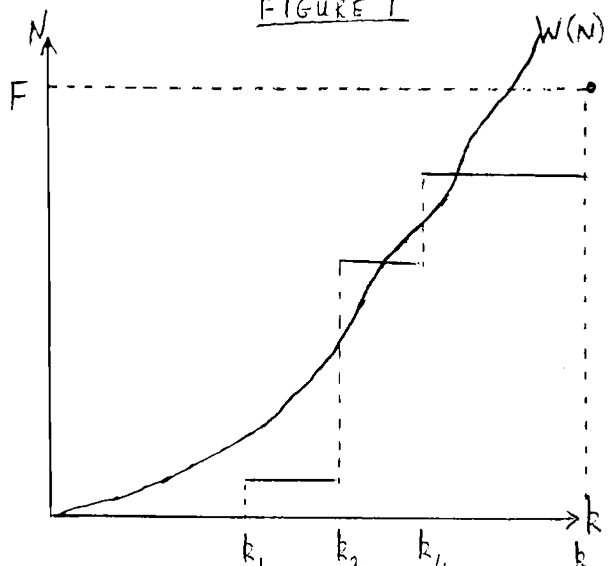

FIGURE 2

The first inequality is an application of Lemma 2. The second inequality stems from the fact that as the number of active firms in the second period increases, the firm is better off since total output next period is higher. The third inequality results from two observations. First, at $\alpha=0$, $W^e(N_2,N_2)=W(N_2+1,N_2)$ as the number of firms in the other sector during the first period of life is not relevant. Second, $N_1>N_2$ implies that $N_1 \geq N_2+1$. Hence, again using Lemma 2, $W^e(N_2,N_2) \geq W(N_1,N_2)$. The fourth inequality is the condition for participation of the N_1 firms in odd periods. The last inequality follows from $N_1>N_2$. This string of inequalities implies $W^e(N_2,N_1)>k_{N_2+1}$ so that a firm has an incentive to enter during even periods contradicting the equilibrium condition. Since $W(N_t,N_{t+1})$ is continuous in α, cycles will not exist for α close to 0. QED.

In order for cycles to occur, it must be the case that the return to becoming a firm in odd periods exceeds the return to participating in even periods. But, if thick markets are desirable as a consumer while thin markets are desired as a producer, the gains to participation will always be higher in periods with a low number of firms. Thus entry will occur and the cycle will be destroyed. Basically, this is the same result as the inability to generate cycles in overlapping generations model with perfect competition in which substitution effects dominate. Here, the lack of cycles in the presence of intertemporal linkages has the same root: a high return to participation arises when there are many firms participating in the future.[15]

To create a basis for cycles in overlapping generations models with perfect competition, it is often assumed (see Grandmont [1985] and Azariadis-Guesnerie [1986]) that income effects dominate substitution effects over some range of real returns so that dynamical offer curves are backward bending. In this paper, we do not make that assumption directly. Instead we show that sufficiently strong intratemporal linkages can create the basis for cycles.

Consider the extreme case in which there are no intertemporal links, so that $\alpha=1$. For this economy, Chatterjee-Cooper [1988] find that multiple equilibria may arise just as in the case discussed in the previous

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[15] In fact, the absence of cycles when agents only consume when old holds for more general preferences. Consider an overlapping generations model with entry and exit, as in Chatterjee-Cooper [1989], in which agents consume only in old age and produce in youth. Let preferences for generation t agents be given by $u(c_{t+1}) - g(y_t)$, where c_{t+1} is future consumption and y_t is current output, with $u(\cdot)$ strictly increasing and strictly concave and $g(\cdot)$ strictly increasing and strictly convex. Further assume that $xu'(x)$ is increasing in x so that substitution effects dominate income effects. We can prove that if $[xg'(x)/g(x)]$ is increasing in x and $[xu'(x)/u(x)]$ is decreasing in x, then equilibrium cycles will not exist.

section. Even at $\alpha=1$, W(N) is an increasing function so that multiple crossings with the H(k) function (as in Figure 2) are certainly possible. Then, we have the following result.

Proposition 3 If at $\alpha=1$ there are multiple equilibria, then for α close to 1 there exists a two-cycle.

Proof Assume first that $\alpha=1$ so that our model is an infinitely repeated game of only static interactions. Suppose that (N^*,y^*,p^*) and (N^{**},y^{**},p^{**}) are equilibria of the static economy. Then there is a trivial two-cycle in which the economy fluctuates between these static equilibria. Since $W(N_t,N_{t+1})$ is continuous in α, for α close to 1 there will exist a two-cycle, (N_1,y_1,p_1) and (N_2,y_2,p_2), where $(N_1,y_1)=(N^*,y^*)$ and $(N_2,y_2)=(N^{**},y^{**})$ and p_i satisfies market clearing for $i=1,2$. QED.

The point of the proposition is that if strategic complementarities are sufficiently strong so that multiple equilibria can arise in a static setting, then cycles can arise. At $\alpha=1$, one can show, in contrast to Proposition 2, that $N_1>N_2$ implies $W(N_1,N_2) > W(N_2,N_1)$ as agents benefit from the presence of a large number of firms in the other sector during the first period of their lives. This is trivial for the case of $\alpha=1$ but holds as long as the intratemporal effects are strong enough: i.e. α is close to 1.

IV. Stationary Sunspot Equilibria

We now introduce extrinsic uncertainty (i.e., unrelated to preferences or endowments) into the above framework. Consider two states a (sunspots) and b (no sunspots) with a stationary transition probability matrix T defined as follows:

$$T = \begin{array}{c|cc} & a & b \\ \hline a & \pi_{aa} & 1-\pi_{aa} \\ b & 1-\pi_{bb} & \pi_{bb} \end{array}$$

where π_{rs}: = probability of state r at time t+1 given that state s occurred at time t. This section is an extension of Azariadis [1981] and Azariadis-Guesnerie [1986] to the case where the goods market is not competitive and the number of firms participating in the goods market is endogenous.

Suppose all the agents in the economy believe that sunspots may affect the price of sector j output (j=1,2) in the future i.e., $p^j_{t+1} = p^j_s$ if the state at t+1 is s where s=a,b.[16] The purpose of this section is to examine if there exist equilibria in which such beliefs may be self-fulfilling. As before, we will first derive the payoffs to a participant firm and then endogenize the number of firms in a symmetric Nash equilibrium. When the state at time t is a, the optimization problem for an active firm in sector i is to maximize

$$\alpha \ln c_t^{-i} + (1-\alpha)\left[\pi_{aa} \ln \frac{m_t^i}{p_a^{-i}} + (1-\pi_{aa}) \ln \frac{m_t^i}{p_b^{-i}} \right] - g(y_t^i)$$

subject to

$$p_t^{-i} c_t^{-i} + m_t^i = p_t^i y_t^i .$$

In the above formulation, we have taken into account the constraint that future consumption must be equal to current money holdings divided by future price. Proceeding according to the steps in Section II, we can write the demand for sector -i output and money holdings by the firm as

$$c_t^{-i} = \alpha(p_t^i y_t^i)/p_t^{-i} \quad \text{and}$$

$$m_t^i = (1-\alpha)(p_t^i y_t^i)$$

Notice that the demands are independent of the transition probabilities due to the Cobb-Douglas structure. As in Section II, the demand for sector i output is unit-elastic so that the output per firm in sector i at time t must solve (in a symmetric Nash equilibrium)

$$y_t^j g'(y_t^j) = 1-(1/N_t^i).$$

We will now impose symmetry across sectors so that given the participation of N_t firms in each sector, the symmetric Nash level of output per firm at time t is uniquely determined by

$$y_t g'(y_t) = 1-(1/N_t)$$

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[16] As usual, the agents are assumed to be born after the state is realized so that any sunspot contingent contract is an agreement between agents of the same generation.

and, the market clearing price is given by

$$(1-\alpha)N_t p_t y_t = M$$

Thus the production and consumption decisions in the presence of extrinsic uncertainty are no different from those in the deterministic environment. However, as we shall see below, the participation decision will be affected by the transition probabilities.

A stationary sunspot equilibrium is an 8-tuple $(N_a, N_b, y_a, y_b, p_a, p_b, \pi_{aa}, \pi_{bb})$ such that $\pi_{aa}, \pi_{bb} \in (0,1)$, $p_a \neq p_b$ and for s=a,b,

(i) given N_s, y_s solves $y_s g'(y_s) = 1-(1/N_s)$

(ii) given N_s, the price p_s must clear the money market,

(iii) the utility payoff for active firms must exceed their cost of participation and there must be no incentive for the other firms to enter.

Clearly, any steady state would satisfy the above three conditions $\forall\ \pi_{aa}, \pi_{bb} \in [0,1]$. Though we have explicitly stated only $p_a \neq p_b$ in the definition, conditions (i) and (ii) ensure that $N_a \neq N_b$ and $y_a \neq y_b$ since the output is uniquely determined given N. Also, notice that a two-cycle is a degenerate sunspot equilibrium with $\pi_{aa} = \pi_{bb} = 0$.

The utility to a participant firm in state a is

$$V(N_a, N_b, \pi_{aa}) = \alpha\ln\alpha + (1-\alpha)\ln(1-\alpha) + \ln y_a - g(y_a) + (1-\alpha)(1-\pi_{aa})\ln(N_b y_b / N_a y_a),$$

where this payoff reflects the fact that, with probability $(1-\pi_{aa})$, $N_{t+1} = N_b$. Similarly, if the current state is b then the utility to an active firm is

$$V(N_b, N_a, \pi_{bb}) = \alpha\ln\alpha + (1-\alpha)\ln(1-\alpha) + \ln y_b - g(y_b) + (1-\alpha)(1-\pi_{bb})\ln(N_a y_a / N_b y_b).$$

At this stage, we will assume without loss of generality, $N_a > N_b$. From part (ii) of Lemma 1 we know $y_a > y_b$. By the market clearing condition, we must then have $p_a < p_b$.

Lemma 4 (i) $V(N_a,N_b,1) = W(N_a)$ and $V(N_b,N_a,1) = W(N_b)$

(ii) Given N_a and N_b, $V(N_a,N_b,\pi_{aa})$ is continuous and increasing in π_{aa}.

(iii) Given N_a and N_b, $V(N_b,N_a,\pi_{bb})$ is continuous and decreasing in π_{bb}.

Proof (i) is obvious. (ii) and (iii) hold since changes in the transition probabilities do not affect the output.

QED.

The intuition behind (ii) and (iii) is similar to Lemma 3. At time t in state a, given $N_a > N_b$ any increase in π_{aa} increases the likelihood of p_a at time t+1 so that each active firm at time t is better off as a buyer since $p_a < p_b$.

Proposition 4 If there are two steady states in the deterministic economy, then there must exist a stationary sunspot equilibrium.

Proof Let (N^*,y^*,p^*) and (N^{**},y^{**},p^{**}) be two steady states with $N^* > N^{**}$. Define $(N_a,y_a,p_a) \equiv (N^*,y^*,p^*)$ and $(N_b,y_b,p_b) \equiv (N^{**},y^{**},p^{**})$. By definition of steady state, y^* is the symmetric Nash level of output given N^* and p^* clears the market. Further, $W(N^*) > k_i$ for $i = 1,2,..N^*$ and $W^e(N^*) < k_i$ for $i = N^*+1, N^*+2,...,F$. Now, by Lemma 4, $V(N_a,N_b,1) = W(N^*)$ and by continuity, for π_{aa} close to but less than 1, the participation of N_a firms must be justified in state a. Similarly, for π_{bb} close to but less than 1 the participation of N_b firms is justified in state b. Thus, we have constructed an 8-tuple that meets the definition of a stationary sunspot equilibrium.

QED.

Notice that a stationary sunspot equilibrium will exist for all α as long as the deterministic economy possesses multiple steady states. However, the existence of cycles does depend upon α as noted in Section III. Thus there are parameter configurations for the above economy where stationary sunspot equilibria exist but two-cycles do not. In particular, suppose there exist multiple steady states when $\alpha=0$. We know from Proposition 1 that two-cycles do not exist but Proposition 4 guarantees the existence of a stationary sunspot equilibrium as a lottery between the two steady states.

As in Azariadis-Guesnerie [1986], there will also exist sunspot equilibria in economies exhibiting cycles. From Proposition 2, we know that cycles exist when α is near 1. Therefore, sunspot equilibria will exist in the neighborhood of the two-cycle in which π_{aa} and π_{bb} are both close to zero.

There is an interesting contrast in these two types of sunspot equilibria. The ones characterized in Proposition 4 represent randomizations between two steady states where the probability of the switching from one outcome to another is quite low. In contrast, the randomizations near a cycle have very high probabilities of transiting from one outcome to another.

V. Generalizations

While making the analysis tractable, Cobb-Douglas preferences imply that output decisions will be independent of the transition probabilities in the sunspot equilibria as long as participation levels do not change. Here we consider sunspot equilibria for more general preferences in which per firm output varies with the real rate of return.

Consider, as in the model above, an overlapping generations model in which agents live 2 periods. Sector i agents born in period t produce good i in that period and consume good -i in each of the two periods of their lives. Preferences are given by:

$$u(c_t^{-i}) + \beta u(c_{t+1}^{-i}) - g(y_t^i)$$

Assume that $u(\cdot)$ is strictly increasing and strictly concave and that $g(\cdot)$ is strictly increasing and strictly convex. Both functions are assumed to be twice continuously differentiable. Finally, we assume that $xu'(x)$ is an increasing function of x so that our results do not depend on backward bending offer curves.

As before, agents only consume the output of the other sector but we do not restrict attention here to Cobb-Douglas preferences. Further, the technology is linear so that $g(y)$ measures the disutility of producing y units of output. Finally, we retain the assumption that the distribution of opportunity costs is given by $H(k)$.

As the multiplicity of steady-state equilibria is key to our ability to generate sunspot equilibria, we first show that Lemma 3 holds in this more general environment. Let $c^j(N)$ for $j = 1,2$ be the steady state level of consumption for an agent in the j^{th} period of life given that N firms are active in each sector for every time period. By feasibility, the amount of employment (n(N)) or output (y(N)) must satisfy:

$$(12) \qquad c^1(N) + c^2(N) = y(N).$$

In a steady state with N active firms in each sector, the values of $c^j(N)$ for $j=1,2$ and y(N) must satisfy the first-order conditions from the optimization problem of an individual agent. These are given by

$$(13) \qquad u'(c^1(N)) = \beta u'(c^2(N)),$$

$$(14) \qquad (1 - 1/\xi) = g'(y(N))/u'(c^1(N)).$$

Here ξ is the absolute value of the elasticity of the residual demand curve facing this firm and is implicitly being evaluated at the proposed steady state allocation. Equation (13) corresponds to (4)&(5) while (14) corresponds to (4)&(6). Since, in a steady state equilibrium, consumption and output will only depend on N, ξ is only a function of N as well. Since both $u(\cdot)$ and $g(\cdot)$ are increasing functions, (14) implies that ξ must be greater than 1.

Let W(N) be the lifetime utility of an agent if there are N firms active in all sectors and all time periods. Then

$$(15) \quad W(N) = u(c^1(N)) + \beta u(c^2(N)) - g(y(N))$$

We now proceed to study conditions under which sunspot equilibria will exist. To do so, we first determine conditions under which W(N) is an increasing function. One important condition, as in Lemma 3, is that output per firm increases with N.

<u>Lemma 5</u> $\xi'(N)>0$ implies $y'(N)>0$ and $\xi'(N)<0$ implies $y'(N)<0$.

<u>Proof</u> Treating N as a continuous variable, we first show that y'(N) is positive if $\xi'(N)$ is positive. Assume not, i.e. $\xi'(N)>0$ but $y'(N)\leq 0$. Then for (14) to hold, $c^1(N)$ must increase with N. Given that y(N) is not increasing, this implies that $c^2(N)$ must fall for (12) to hold. This is inconsistent with (13). The proof of the second part

of the lemma is similar. QED.

So, the behavior of per firm output as a function of the number of firms is determined by how the elasticity of the residual demand curve behaves as N varies. Lemma 5 is similar in content to Lemma 1 (ii). Using this, we can extend Lemma 3 to this more general environment.

Proposition 5 If ξ is an increasing function of N, then W(N) is an increasing function.

Proof Using (12), (13) and (15), we know that

$$(16) \quad W'(N) = \left\{ u'(c^1(N)) - g'(y(N)) \right\} y'(N).$$

From (14), we know that the first term is positive since ξ is greater than 1. Therefore, the sign of W'(N) depends on y'(N). The result follows from Lemma 5. QED.

We now give a sufficient condition for ξ to be increasing in N. For the Cobb-Douglas economy, this condition was immediate since the elasticity of the residual demand curve was N. The fact that young agents have a savings decision that depends on the real interest rates complicates the analysis. Basically, the sufficient conditions given below restrict the elasticity of the young agent's demand for the output of other sectors so that ξ is increasing in N.

To do so, first define $R(c_t)$ as $-c_t u''(c_t)/u'(c_t)$ for t=1,2. This is the elasticity of marginal utility for the momentary utility function.

Lemma 6 If R(c) is independent of c, then ξ is an increasing function of N.

Proof Using a representative firm's inverse demand curve, one can show that $\xi = N[1+(c_1/y)\{\xi_c-1\}]$ where ξ_c is the elasticity of demand for good i from the demand of young agents in sector -i in period t. This elasticity can be calculated, following a similar derivation in Heller [1986], as $\xi_c = (1+R_2c_1/c_2)/(R_1+R_2c_1/c_2)$ where $R_t = R(c_t)$ for t=1,2.

As N varies, since we are comparing steady states, the interest rate equals zero for all N. As a consequence, R(c) independent of c implies that c_1/c_2 is independent of N and from (12), c_1/y is also independent of N. Therefore ξ_c is independent of N so that ξ must be an increasing function of N. QED.

Lemma 7 If R(c) is an increasing function and $\beta=1$, then ξ is an increasing function of N.

Proof If $\beta=1, c^1(N)=c^2(N)=c(N)$ for all N so that $\xi(N)= N[1+(1/2)\{\xi_c-1\}]$ and $\xi_c= (1+R(c(N)))/2R(c(N))$. Suppose, to the contrary, that $\xi(N)$ is a decreasing function so that, from Lemma 5, $y'(N)<0$. This implies that ξ_c must increase with N which contradicts the assertion that $\xi(N)$ is a decreasing function. QED.

If ξ is an increasing function of N so that W(N) is increasing, then, as in Section III, it is possible to construct economies having multiple steady state equilibria. As before, one is free to choose H(k) to ensure that there are multiple crossings of it with the W(N) function. At each of these crossings, the active firms obtain payoffs exceeding their opportunity costs and the inactive firms have no incentive to enter. This is because the entry of a single firm into one of the sectors creates a congestion effect, similar to that given in Lemma 2, which causes the payoffs of active firms to decrease. Thus the payoff of an entrant must be less than W(N).

Given this, Lemma 4 and Proposition 4 extend to this environment. Part (i) of Lemma 4 holds trivially and the continuity of V(·) with respect to the transition probability holds since the per capita output levels are continuous in these probabilities. Given this continuity, Proposition 4 extends as well though output and hence prices will not be independent of the transition probabilities.[17]

Empirical Implications

Finally, we describe some of the interesting implications of a sunspot equilibrium in which π_{aa} and π_{bb} are near 1. To fix notation, let (N_a,y_a,p_a) and (N_b,y_b,p_b) denote a sunspot equilibrium in which $N_a>N_b$. Under the assumption that ξ is an increasing function of N, $y_a>y_b$ and, from market clearing, $p_a<p_b$.

First, net business formation is procyclical. Moreover, high (low) net business formation in period t

[17] To prove the existence of a sunspot equilibrium, we assume that the Jacobian of the equilibrium conditions ((12),(13) and (14)) does not vanish at each of the steady states. I.e. $\pi_{aa} = \pi_{bb} =1$ is not a critical point in the parameter space.

implies high (low) output is likely in period $t+1$. Thus net business formation is a "good predictor" of future output. Note that this positive association between net business formation and future output would not be a feature of a model with endogenous cycles or with sunspot equilibria in the neighborhood of such cycles, as in Azariadis-Guesnerie [1986].

Second, fluctuations in output exhibit a high degree of persistence. Again, this would not be a feature of sunspot equilibria created in the neighborhood of cycles.

Third, markups are countercyclical. This is in accord with the evidence presented in Bils [1987].

Finally, we find that prices are countercyclical. This implies that expected real rates of return are countercyclical. This is a consequence of the fact that the action in this economy comes from the "supply-side."

VI. Conclusions

Our goal in writing this paper was to establish the existence of sunspot equilibria and cycles in an overlapping generations economy without the assumptions of gross complementarity. Instead, our analysis rests on the existence of sufficiently strong strategic complementarities in a dynamic economy with entry and exit. If these complementarities give rise to multiple stationary equilibria, then one can construct sunspot equilibria in the neighborhood of these steady states. Further, we have shown that cycles can emerge if intratemporal strategic linkages are sufficiently strong but will not exist otherwise.

It should be emphasized that the sunspot equilibria we consider are different from those constructed by Azariadis-Guesnerie [1986] and Farmer-Woodford [1984]. The sunspots exhibited by Azariadis-Guesnerie arise in the neighborhood of a two-cycle and thus have very different properties than the sunspots equilibria here in terms of the persistence of output fluctuations and the correlation between net business formation and future output. Farmer-Woodford construct sunspot equilibria in the neighborhood of a locally stable steady state equilibrium as a randomization between two paths leading to the same steady state. In this type of sunspot equilibria, large fluctuations in endogenous variables do not arise in contrast to the type of sunspot equilibria we consider.

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