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Inflation Dynamics with a Generalized Lucas Phillips Curve
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ABSTRACT

I investigate a generalized form of the Lucas (1972) Phillips curve, in which firms take varying amounts of time to learn aggregate demand, in standard textbook new-Keynesian models. This Phillips curve helps to reconcile the sharp divergence between the standard model on the one hand and the beliefs of most policy analysts and the available evidence on the other. In the standard model, inflation and output rise after interest rates rise, with only the possibility of a one-time downward jump. With the generalized Lucas Phillips curve, inflation is initially unstable, so a small initial disinflation builds up before turning around. The model preserves the desirable long-run stability and neutrality of the standard model.

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A data appendix is available at http://www.nber.org/data-appendix/w33888
A Author webpage, always hosting the up to date draft and computer program. is available at https://www.johnh

1 Introduction

This paper investigates a generalization of the Lucas (1972) Phillips curve, in which some firms take longer than others to figure out whether a price change represents aggregate or relative demand. The output gap x_t follows

$$\kappa x_t = p_t - \int_{j=0}^{\infty} \alpha_j E_{t-j} p_t dj \tag{1}$$

where p_t is the log price level, and α_j is the fraction of firms that take j periods to learn the aggregate state of the economy. I place this Phillips curve in the standard new-Keynesian framework, i.e. an intertemporal IS curve and an interest rate target. The resulting response of inflation to shocks is hump-shaped. Inflation is initially unstable, building up over time. Later on, inflation dynamics become stable, and inflation goes away.

Specifically, I start by uniting the Phillips curve (1) with the standard intertemporal substitution IS curve,

$$\frac{E_t(dx_t)}{dt} = \sigma(i_t - \pi_t), \tag{2}$$

and I compute the response to a shock consisting of an AR(1) interest rate and an -0.5% initial inflation. Figure 1 presents the resulting response of inflation and output, using exponential weights $\alpha_j = ae^{-aj}$. Disinflation grows following the initial shock, in a hump-shaped pattern, before subsiding. That is the central result. (The reason for the initial inflation assumption is explained in detail below. The same responses occur under the plotted disturbance $\{u_t\}$ to a Taylor rule $i_t = \phi\pi_t + u_t$, also discussed below.)

Figure 2 presents the response of inflation and output to the same shock, using a hump-shaped pattern of weights generated by the difference between two exponentials. (See (6) and Figure 4.) The major difference is that inflation does not jump downwards on impact. Avoiding that feature is a main motivation (aside realism) for examining hump-shaped weights.

This is an easy computation. With exponential weights $\alpha_j = ae^{-aj}$, the output and price level responses to a time-0 shock in (1) are related by

$$\kappa x_t = e^{-at} p_t. \tag{3}$$

At time t, output is proportional to the fraction $\int_{j=t}^{\infty} ae^{-aj} dj = e^{-at}$ of firms that have not gotten the news, times the price surprise. The IS curve (2) says that the output, interest rate, and

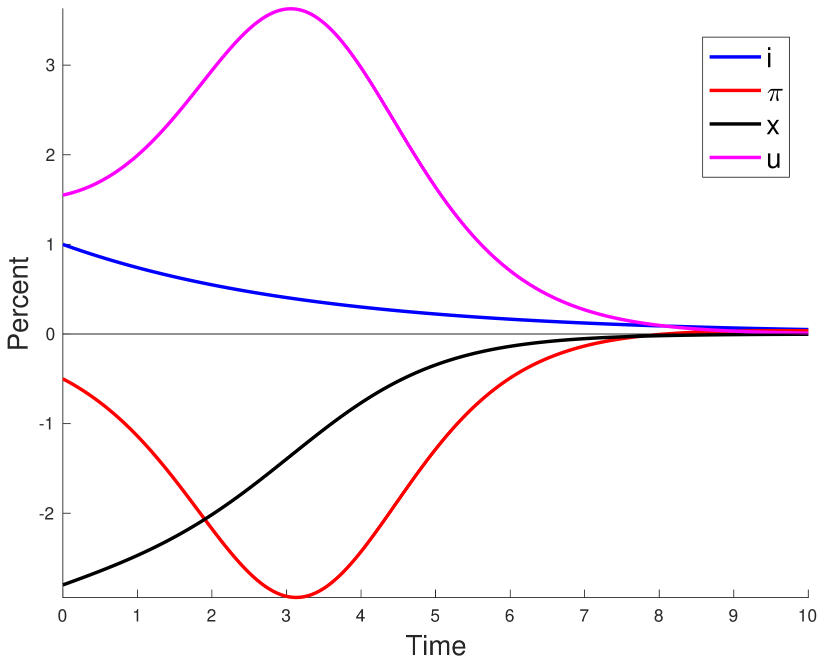

Figure 1: Responses to AR(1) interest rate and $\pi_0 = -0.5\%$ with exponential weights $\alpha_j = ae^{-aj}$. Dynamic IS and a generalized Lucas Phillips curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $b = 0$, $\eta_i = 0.3$. The u line plots u_t in $i_t = \phi\pi_t + u_t$ with $\phi = 1.1$. With $r = 2\%$ and short-term debt, the inflation path requires a fiscal tightening $\tilde{s}/r$ equal to 13.7% of the value of debt.

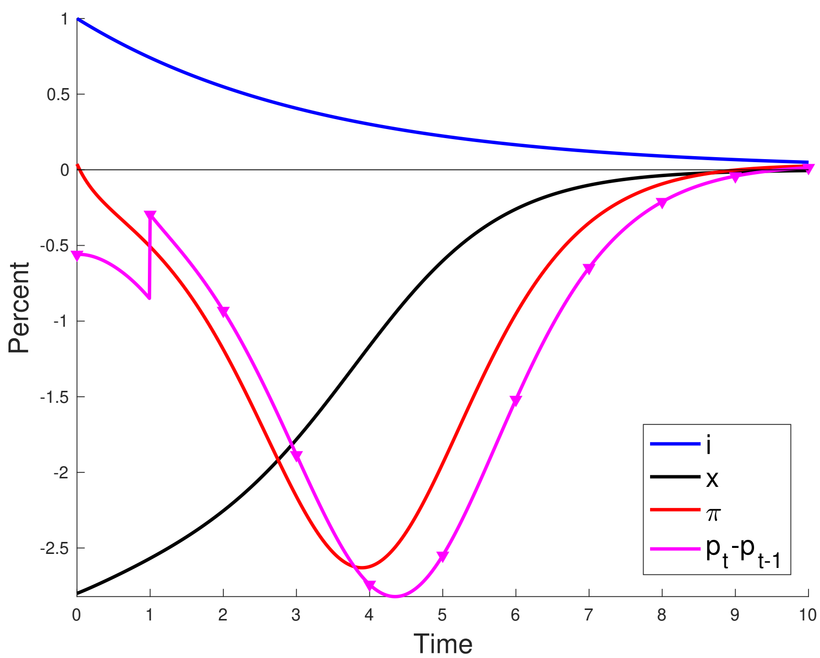

Figure 2: Responses to AR(1) interest rate and $p_0 = -0.56\%$ with double-exponential weights. Dynamic IS and a generalized Lucas Phillips curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $b = 2$, $\eta_i = 0.3$.

inflation responses are related by

$$\frac{dx_t}{dt} = \sigma(i_t - \pi_t). \tag{4}$$

Take the derivative of (3), eliminate output growth from it and (4), and solve the resulting ordinary differential equation linking $dp/dt = \pi_t$, p_t, and i_t.

1.1 Motivation

The generalized Lucas Phillips curve makes progress towards solving one of the two great problems with the standard new-Keynesian model and a bit of progress on the other. To illustrate, Figure 3 presents the response function of the standard textbook new-Keynesian model. The model consists of an intertemporal IS curve, a forward-looking Phillips curve, and a Taylor rule $i_t = \phi\pi_t + u_t$. I plot the response to an AR(1) shock to the disturbance u_t. (I present the model and calculation below.)

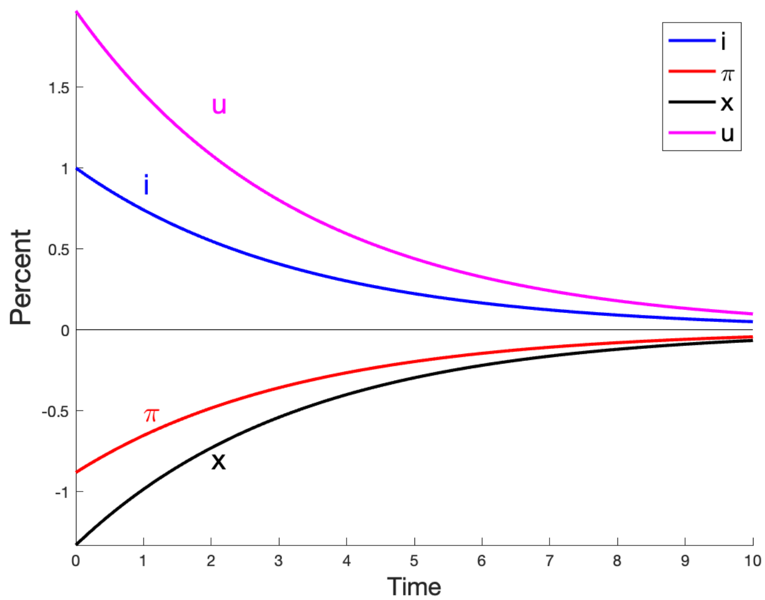

Figure 3: Response of the standard three-equation new-Keynesian model to an AR(1) Taylor-rule policy disturbance u_t. Parameters $\eta_i = 0.3$, $\phi = 1.1$, $r = 2\%$, $\sigma = 0.21$, $\kappa = 0.21$, $u_0 = 1.97\%$. The passive fiscal tightening to pay interest costs on the debt is $\tilde{s}_0/r = 5.9\%$ of the value of debt.

In this model, inflation and output *rise* in response to higher interest rates, after an initial downward jump. In the standard view among policy makers, commentators, and many economists, as well as the results of many VARs, higher interest rates lead inflation and output to steadily *decline* over a period of months to years, roughly as plotted for inflation π_t in Figure 2.

That higher interest rates raise inflation and output going forward is robustly embedded in standard-forward looking models. If real interest rates rise, consumption and consumption growth must rise, via the standard first order condition, at least relative to their levels a minute after the interest rate rise. Similarly, with flexible prices and therefore a constant real rate, the nominal interest rate equals the real rate plus expected inflation, so higher nominal rates raise expected inflation. Sticky prices do not overturn that result, which would require a real interest rate that rises more than the nominal rate. In the standard forward-looking Phillips curve, lower output means lower inflation relative to future inflation, i.e. that inflation rises over time (Ball, 1994), again at least relative to its value a minute after the shock.

These results fundamentally come from the underlying stability and determinacy of the economy. In the traditional policy view, driven by adaptive expectations, inflation is *unstable* under an interest rate peg, or when interest rates react less than one for one to inflation, as described by Friedman (1968). With rational expectations in the standard new-Keynesian model, inflation is stable, and eventually converges to the nominal interest rate.

Stability, and consequent long-run neutrality – higher interest rates eventually lead to higher inflation – are lovely properties for a basic textbook economic model. But those properties make it harder to construct a model in which higher interest rates temporarily lower inflation, consistent with policy beliefs. This is a major point of Cochrane (2024).

In that context, the generalized Lucas Phillips curve produces short-run instability while retaining long-run stability. Roughly speaking, it generates dynamics $d\pi/dt = \gamma(t)\pi_t + \delta(t)i_t$, in which $\gamma(t) > 0$ for small t, but $\gamma(t) < 0$ for larger t. By generating short-run instability, the generalized Lucas Phillips curve produces an inflation response closer to standard beliefs and VARs, but maintaining the long-run stability and neutrality of the standard rational expectations model.

Put another way, the standard policy beliefs want backward-looking dynamics. Rational expectations and optimization want forward-looking dynamics. With menu costs or Calvo pricing, firms set prices with an eye to next year's prices. In the Lucas Phillips curve, there is no reason for firms to anticipate future price increases. Many empirical models add some backward-looking dynamics in a more or less ad-hoc fashion. For many applications, such a fudge is reasonable. But it leaves open the question, is there *no* simple, textbook level, rational economic model that delivers the basic sign and operation of monetary policy? My goal is to see if such an object exists rather than to construct a more complex pastiche that fits a particular data sample.

Mixing this generalized Lucas Phillips curve with a standard intertemporal IS equilibrium condition, however, output still rises going forward, with at most an initial downward jump as can be seen in Figures 1 and 2. Using an IS curve that features an external habit or one that downweights future output produces response functions in which output also falls before turning around, more quickly than inflation.

The downward jump in Figure 3 is the second puzzle of the standard model. Since output and inflation rise over time, this downward jump is the only way for the model to produce lower output or inflation at all coincident with a rise in interest rates. Where does it come from?

In this model as in the model with a generalized Lucas Phillips curve, the nominal interest rate path only determines a family of solutions. One must also pick one member of that family, for instance by specifying initial inflation. I show this below via analytic solutions (14) and (23) relating inflation to interest rates. For both models, inflation is a weighted average of the interest rate, plus a transient which we can index by initial inflation. Since the weights of inflation on the interest rate are positive throughout, the only way to produce lower inflation with higher interest rates is via a lower initial value.

So how do we pick initial inflation, or otherwise select among the family of inflation paths consistent with a given nominal interest rate path? The standard New-Keynesian approach adds an equilibrium-selection policy, supported by "passive" fiscal accommodation. That policy consists of a choice of the monetary policy disturbance, among the set that produce the same interest rate, or by a direct equilibrium-selection threat. Fiscal policy "passively" cooperates, raising taxes or cutting spending to pay for interest costs on the debt and greater real payments to bondholders. The fiscal theory of the price level specifies the fiscal policy contraction directly, and infers initial inflation from that choice. Thus any ability of the standard model to lower inflation at all also depends on a downward equilibrium-selection/fiscal shock coincident with higher nominal interest rates. The generalized Lucas Phillips curve does not change this fact. It does ameliorate it a bit however. With it, a large negative inflation can be set off by a small downward jump.

To produce a response in which higher interest rates result in lower inflation without a simultaneous fiscal contraction (either active or passive), or equivalently to produce a model in which higher interest rates and a negative equilibrium/selection-fiscal shock are tied together in a more believable way, giving the central bank any reason to raise interest rates in the first place, I merge the generalized Lucas Phillips curve with long-term debt. This combination produces the Sims (2011) “stepping on a rake” or “unpleasant interest-rate arithmetic” mechanism (see also Cochrane, 2017, 2024). With this mechanism and the standard Phillips curve, inflation still jumps down and then reverts over time. It just gives a reason for the jump. The generalized rational-expectations Phillips curve turns that jump into a long period of disinflation, similar to standard policy beliefs. Thus, the generalized Lucas Phillips curve brings the stepping on a rake mechanism closer to standard beliefs.

Though the generalized Lucas Phillips curve produces responses closer to standard policy beliefs, the mechanism remains fundamentally different, as it is in the standard new-Keynesian model. An initial downward inflation jump $\pi_0 < 0$ is amplified for a while, but a higher nominal interest rate just drags inflation up. The standard view wants higher real interest rates to push inflation down, something like $d\pi/dt = -\gamma(t)(i_t - \pi_t)$ with $\gamma(t) > 0$ for small t, rather than $d\pi/dt = \gamma(t)\pi_t + \delta(t)i_t$. Mechanically adaptive expectations produce such responses, but our goal since the 1980s has been to avoid such non-economic and hence likely ephemeral and non-policy-invariant ingredients.

The standard story that high real interest rates depress output and lower output pushes down inflation (especially going forward) simply does not apply to either the standard model or the generalized Lucas variant. That similar responses can be produced in a flexible price version of the model with constant real rates (Woodford, 2003, Ch. 2) is a good indicator that real interest rates are not central to the story. Indeed, the central bank can lower inflation with no change in interest rates, or with lower interest rates, just as well or more effectively as it can with higher interest rates. The best that anyone has achieved so far is that the rational expectations new-Keynesian model somewhat mimics standard beliefs about the effects of monetary policy — the impulse response function — but not the standard story for its mechanism. This paper is not different in this respect.

It is of course possible that the standard beliefs are false and the standard new-Keynesian model is right: inflation and output do rise in response to higher interest rates, potentially after a one-time negative jump induced by, or accommodated by, tighter fiscal policy. The VAR evidence is ephemeral and the subject of a long specification search to produce the “right” answer (Ramey, 2016). Maybe the price puzzle was trying to tell us something all these years. Maybe monetary policy acts much more quickly than the standard belief supposes (Buda et al., 2023). It’s hard to measure instantaneous inflation, and the change from a year ago will always appear sluggish. Maybe fiscal policy does quietly tighten, in hard to measure present-value terms, alongside monetary policy, to pay the higher interest costs on the debt and a windfall to bondholders. Most successful disinflations have included fiscal reforms. No VAR has tried to orthogonalize monetary policy shocks to present-value fiscal shocks. Policy beliefs may come from playing with ISLM models more than the actual accumulated weight of history and institutional experience. And that experience always has difficulty separating causation from correlation, and separating the influence of multiple policies and economic shocks. But in our exploration, it is useful to know if there *is* a textbook model with somewhat respectable intertemporal forward-looking economics that can express something like the standard belief about the effects of policy if not the mechanism. At least the widespread cognitive dissonance between beliefs and equations need not continue.

I suspect that many economists and policy makers square the circle between the downward jump of simple models and the long decline of intuition by alluding to “long and variable lags,” some unstated process of adjustment to equilibrium. Yes, models say inflation and output jump down immediately, but we interpret that process as taking place over a few years. The term originates from Milton Friedman (1948) (p. 254) and must represent one of the most durable of economic dictums. Dupor (2023) has a nice review, and Nelson (2020) (p. 141) discusses the term’s coinage. But modern dynamic economic theory does not allow hand-waving dynamics. Each price and quantity must be part of an intertemporal equilibrium. The whole point of intertemporal macroeconomic modeling is that we must understand and model the long and variable lags.

The basic rational expectations model incorporating the Lucas Phillips curve lost favor from the impression that it could not give persistent output responses. The output response only lasts one “period” until all firms learn to distinguish relative vs. aggregate demand. That already leaves open how long a “period” is. While in 1972 and with $MV = PY$ in mind it might have seemed reasonable that everyone knew aggregate demand perfectly the minute money stock numbers were published, that is less compelling today. Economists are still debating if the 2021 inflation surge came from aggregate (deficits) vs. relative (supply chain) shocks, and politicians seem never to understand the distinction. That average businesses don’t know relative from aggregate demand for quite some time is not unreasonable. The distribution of such information lags gives rise to pretty dynamics in place of a simple iid process.

1.2 Literature

This Phillips curve is similar to the "sticky information" Phillips curve of Mankiw and Reis (2002, 2007), Mankiw and Reis start from a firm that wishes to set its price by

$$p_t^* = p_t + \alpha x_t$$

where x_t is log output, and sets its price based on information that is j periods old. I start instead from a Lucas firm that, using information from j periods ago, sets output x_t according to

$$\kappa x_t = p_t - E_{t-j} p_t.$$

Averaging across firms leads to (1). This small difference leads to a Phillips curve that does not have output growth on the right hand side, which was an unfortunate complication of Mankiw and Reis' formulation. It also leads to models in which it is easy to compute impulse-response functions under an interest rate target. Mankiw and Reis (2007) is difficult to compute, which perhaps explains why that model has not been widely adopted.

Mankiw and Reis (2002) (Figure I, bottom panel) show a pattern similar to that of Figure 1 in response to a one-time decline in the money stock. Inflation jumps down in the first period, and then keeps going, before turning around and returning to zero. One might call that "locally explosive" as I do, or "inertial" as Mankiw and Reis do. That behavior contrasts with the forward-looking Phillips curve, in which inflation jumps down and then reverts in an AR(1) pattern.

Some calibrated medium-scale models produce a response somewhat in line with VARs and policy beliefs, in which inflation and output decline going forward after a shock. Christiano, Eichenbaum, and Trabandt (2016, 2018) are notable examples. (Many others, such as the the famous Smets and Wouters, 2007 model do not produce such a response. Inflation jumps down with the shock.) However, to produce that result they perform major surgery on all of the model's equations. They include prices and wages fixed one period in advance, a strong one-period habit in utility, sticky wages driven by labor monopoly power, firms must borrow to pay the next period's wage bill, capital with adjustment costs that depend on investment growth rather than the investment to capital ratio, wage and price indexation, and more.

The complexity of these efforts outline the challenge: How can we get a forward-looking rational model to exhibit the appearance of backward-looking dynamics, at least temporarily? So far, all of these ingredients are *necessary* to deliver the basic desired sign of monetary policy effects. If we stop with such complex models to embody the policy beliefs about the effects of monetary policy, if simpler models fail to generate the basic signs, we cannot honestly say the more complex model just adds bells and whistles to a basic explainable structure such as the classic aggregate demand story. We need all of the machinery of the simplest model that does the trick.

Thus, simplicity is also a goal of this paper, and should be a goal of the larger research program. I ask not what is a *sufficient* set of extensions, trying to match details of specific estimates, but I ask what is a minimal and interpretable set of changes that deivers the basic sign of monetary policy, while retaining as much as possible the rational and micro-founded flavor that distinguishes the post-Lucas new-Keynesian DSGE effort from simply writing down some relations among aggregates that seem to fit the data and calling it a model. Somewhat surprisingly, after 30+ years, we still do not have a basic textbook economic model that can reproduce the standard and widely reiterated beliefs about the operation of monetary policy.

The Appendix includes a more detailed literature review.

2 A Generalized Lucas Model

Again, I start with a generalization of the Lucas Phillips curve,

$$\kappa x_t = p_t - \int_{j=0}^{\infty} \alpha_j E_{t-j} p_t dj.$$

One can do all the computations in discrete time, starting with

$$\kappa x_t = p_t - \sum_{j=1}^{\infty} \alpha_j E_{t-j} p_t.$$

Continuous time forces us to think a bit harder about which variables can jump or diffuse in response to shocks and which must move differentiably. The formulas and pictures are a bit more elegant as well.

The weights $\{\alpha_j\}$ are positive and satisfy $\int_{j=0}^{\infty} \alpha_j dj = 1$. I use two examples for calculations, an exponential

$$\alpha_j = ae^{-aj} \tag{5}$$

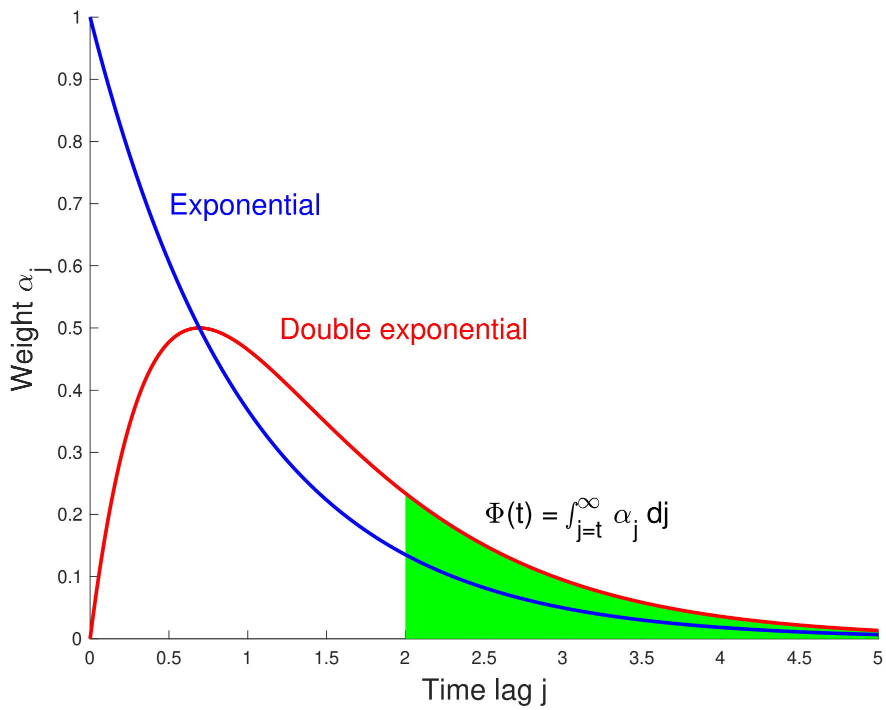

Figure 4: Weights in the generalize Lucas Phillips curve. The plot shows weights α_j for an exponential $\alpha_j = ae^{-aj}$ and double exponential $\alpha_j = (e^{-aj} - e^{-bj})/(a-b)$ examples. $a = 1$ and $b = 2$. The shaded area illustrates $\Phi_t \equiv \int_{j=t}^{\infty} \alpha_j dj$ for $t = 2$ in the double exponential case

and a double exponential,

$$\alpha_j = \frac{e^{-aj} - e^{-bj}}{\frac{1}{a} - \frac{1}{b}}. \tag{6}$$

The simple exponential leads to simple calculations and captures most of the message. The double exponential has a hump-shaped pattern, with $\alpha_0 = 0$, graphed in Figure 4. Unlike the exponential case, no firms figure out instantly whether a disturbance is relative or aggregate. It adds a few simplifications and refinements to the results.

I characterize the model solutions by impulse-response functions. Since the models are linear, the response function—how $(E_{t+1} - E_t)y_{t+j}$ changes when there is a shock at time $t+1$, for any variable y_t—is the same as the response to an "MIT shock" at time 0, in which all variables and expectations start at zero, and then jump unexpectedly to new values. I use unadorned time-series notation y_t to denote such impulse-response functions.

From the Phillips curve, we can compute how the price level or inflation responses to any shock are related to the output response. For any $j < 0$, $E_j p_t = 0$, while $E_j p_t = p_t$ for any $j \geq 0$.

Thus, the price and output responses are related by

$$\kappa x_t = p_t - \int_{j=0}^{t} \alpha_j dj \; p_t$$

$$\kappa x_t = \left(\int_{j=t}^{\infty} \alpha_j dj \right) \; p_t = \Phi(t) p_t. \tag{7}$$

Output moves in proportion to the fraction of firms that have not yet learned aggregate information. Equation (7) also gives us the output response once we solve for the price level response below.

The last equality in (7) introduces notation $\Phi(t)$ for the fraction of firms that have not yet learned aggregate demand; also the right tail of $\{\alpha_j\}$. It obeys

$$\Phi(0) = 1; \; \Phi(\infty) = 0; \; \frac{d\Phi(t)}{dt} = -\alpha_t.$$

For exponential weights (5)

$$\Phi(t) = e^{-at},$$

and for double exponential weights (6)

$$\Phi(t) = \frac{\frac{1}{a} e^{-aj} - \frac{1}{b} e^{-bj}}{\frac{1}{a} - \frac{1}{b}}.$$

We will shortly need the response of output growth. Taking the derivative of (7),

$$\kappa \frac{dx_t}{dt} = -\alpha_t p_t + \Phi(t) \frac{dp_t}{dt}. \tag{8}$$

One can derive a Phillips curve linking output and inflation, in the standard tradition, rather than output and the price level. However, it turns out to be algebraically simpler to solve the model in terms of the price level and then take a derivative.

In the standard Calvo pricing model, only a fraction λdt of firms can change prices between time t and $t + dt$, so the price level cannot have a jump or diffusion, though the rate of inflation can do so. In this generalized Lucas Phillips curve prices and output can diffuse or jump. If that result seems counterintuitive, remember that individual firm prices and output flows do jump and all our statistics are time averaged. One could add small costs of output or price movement, but let's first see how the model behaves without them.

3 Model Response With a Dynamic IS curve

How the model behaves depends, of course, on both IS and Phillips curves. The (MIT shock) response of the standard intertemporal IS curve is, for $t \geq 0$

$$\frac{dx}{dt} = \sigma(i_t - \pi_t). \tag{9}$$

This is the continuous time equivalent to the familiar

$$x_t = x_{t+1} - \sigma(i_t - \pi_{t+1}).$$

Figures 1 and 2 plot the response of the model consisting of the generalized Lucas Phillips curve and the IS curve (9) to the combination of an AR(1) interest rate path and a $p_0 = -0.5\%$ price level shock.

To compute these responses, I eliminate output growth from (8) and (9). Rearranging, the price level then follows the differential equation

$$[\Phi(t) + \sigma\kappa]\ \frac{dp}{dt} - \alpha_t p_t = \sigma\kappa i_t \tag{10}$$

All of the derivatives are forward-looking and so interpreted hold at time zero as well.

Since $\Phi'(t) = \alpha_t$ the left hand side is the total differential of $[\Phi(t) + \sigma\kappa]\, p_t$. We quickly have the solution

$$p_t = \frac{1 + \sigma\kappa}{\Phi(t) + \sigma\kappa} p_0 + \frac{\sigma\kappa}{\Phi(t) + \sigma\kappa} \int_{j=0}^{t} i_j dj. \tag{11}$$

For exponential weights,

$$p_t = \frac{1 + \sigma\kappa}{e^{-at} + \sigma\kappa} p_0 + \frac{\sigma\kappa}{e^{-at} + \sigma\kappa} \int_{j=0}^{t} i_j dj. \tag{12}$$

The coefficient in front of p_0 is the central story. The variables $\Phi(t)$ and e^{-at} in particular start at 1 and then decrease. Price grows over time from an initial p_0, but then stabilizes at $(1 + \sigma\kappa)/(\sigma\kappa)$ of that original value. Also in the differential equation (10), the coefficient of dp/dt on p_t is positive, so the model exhibits explosive dynamics, amplifying an initial negative shock over time. However $\alpha_t \to 0$ and $\Phi(t) \to 0$ as t grows, so the price level eventually converges.

I use an AR(1) for the interest rate

$$i_t = i_0 e^{-\eta_i t}$$

so

$$\int_{j=0}^{t} i_j dj = \frac{1-e^{-\eta_i t}}{\eta_i}.$$

This completes the calculation of the price level path.

To find the inflation response, we can differentiate the price response, yielding

$$\pi_t = \frac{(1+\sigma\kappa)\,\alpha_t}{[\Phi(t)+\sigma\kappa]^2} p_0 + \frac{\sigma\kappa\alpha_t}{[\Phi(t)+\sigma\kappa]^2} \int_{j=0}^{t} i_j dj + \frac{\sigma\kappa}{\Phi(t)+\sigma\kappa} i_t. \tag{13}$$

In the single exponential case,

$$\pi_t = \frac{(1+\sigma\kappa)\,ae^{-at}}{[e^{-at}+\sigma\kappa]^2} p_0 + \frac{\sigma\kappa a e^{-at}}{[e^{-at}+\sigma\kappa]^2} \int_{j=0}^{t} i_j dj + \frac{\sigma\kappa}{e^{-at}+\sigma\kappa} i_t. \tag{14}$$

Equations (11) and (13) show that we must characterize the policy input by both an interest rate path and an initial price level, which solves the mystery of why I did so. Given a path of interest rates, there are multiple equilibria, indexed by p_0. Thus, and anticipating later discussion, we can call the choice of p_0 the equilibrium-selection policy. Each choice of p_0 also carries different fiscal implications.

Holding constant the initial price level p_0, higher nominal interest rates *raise* inflation. Thus, any disinflationary response must come by amplifying a negative initial price level or negative inflation. For this reason, I plot responses to an AR(1) interest rate path and an initial $\pi_0 = -0.5\%$ or $p_0 = -0.56\%$ price decline. I show below that the model with a standard forward-looking Phillips curve behaves the same way; this behavior is not a particularity of the generalized Lucas Phillips curve. Since $p_0 < 0$ does all the disinflationary work, and given p_0 raising interest rates only raises inflation, it seems almost dishonest to plot the response to such a joint policy and call it the result of monetary policy that raises interest rates. I only do so reflecting a long tradition that does exactly that. I discuss this joint policy specification and ways of tying the interest rate path and initial price level below.

To understand the behavior at time zero and how it differs across the two computations, consider the initial conditions. We can read these directly off the differential equation (10) at t=0, or in the inflation response (13) at $t = 0$:

$$\left(1+\sigma\kappa\right)\pi_0 - \alpha_0 p_0 = \sigma\kappa i_0. \tag{15}$$

When $\alpha_0 > 0$ as it is for the exponential weights, then the initial price level and inflation rate are tied together. I specify $\pi_0 = -0.5\%$ and find the implied $p_0 = -0.56\%$. One can in this case also solve (15) as

$$p_0 = \frac{1}{\alpha_0}\left[\left(1+\sigma\kappa\right)\pi_0 - \sigma\kappa i_0\right], \tag{16}$$

and express inflation in (14) as a function of initial inflation rather than the initial price level.

When $\alpha_0 = 0$ as in the double exponential weights, initial inflation has no connection to the initial price level. Initial inflation must be positive when the initial interest rate is positive, a brief "price puzzle." I specify $p_0 = -0.56\%$ so the calculation is comparable to the exponential weight case.

Both Figure 1 and Figure 2 plot the instantaneous inflation rate, ignoring the price level jump that gets the whole thing going. To display that phenomenon, Figure 2 also plots inflation measured as change from the previous period $p_t - p_{t-1}$. Here we see inflation starting at -0.56%, the initial price level jump less its value (0) one period previously. Measured inflation jumps up as the initial price level drop enters the moving average. That seems unrealistic, and one might add small frictions to smooth out the model. However, the triangles remind us that we measure inflation at discrete intervals, however, so such jumps are not obviously counterfactual. In reality, the price level is also time-averaged to monthly or quarterly intervals.

I plot the simple exponential response as well as the double exponential response to emphasize that the hump-shaped response does not come from hump-shaped weights. The central story is initially unstable inflation, followed by a return to stability as time increases.

To see this behavior, we can write the differential equation for inflation by differentiating the price differential equation (10), and then using it again to eliminate the price level from the differential equation in favor of inflation. For exponential weights, the result is

$$\frac{d\pi_t}{dt} = \frac{a(e^{-at} - \sigma\kappa)}{(e^{-at} + \sigma\kappa)}\pi_t + \frac{1}{e^{-at} + \sigma\kappa}\left[a\sigma\kappa i_t + \sigma\kappa\frac{di_t}{dt}\right] \tag{17}$$

For $\sigma\kappa < 1$, inflation is initially unstable, with a positive coefficient on inflation. As t rises and e^{-at} falls, the sign changes and inflation becomes stable in the long run, with a negative coefficient on π_t. For this reason, I calculate the response in Figure 1 with σ and κ relatively small, at 0.2 each. A smaller value of σ and κ allows a generous period of instability before e^{-at} falls below $\sigma\kappa$ and inflation becomes stable. The parameter $a = 1$ means the typical firm learns the aggregate price level in about a year. Smaller values and slower learning slow down the dynamics.

Output x_t follows from the price level (11) and (7),

$$\kappa x_t = \left(\int_{j=t}^{\infty} \alpha_j dj \right) \ p_t = \Phi(t) p_t. \tag{18}$$

Whether output rises or falls after an initial jump $\kappa x_0 = p_0$ depends whether the reduction in $\Phi(t)$ outweighs the growth in p_t. Using (11)

$$\kappa x_t = \frac{\Phi(t) + \Phi(t)\sigma\kappa}{\Phi(t) + \sigma\kappa} \kappa x_0 + \frac{\Phi(t)\sigma\kappa}{\Phi(t) + \sigma\kappa} \int_{j=0}^{t} i_j dj. \tag{19}$$

We see that for $p_0 < 0$ and $x_0 < 0$, the leading coefficient is less than one, and output always reverts to trend, as shown in Figures 1 and 2. Fiddling with parameters will not help. If we want to modify this behavior we have to modify the model, which I do below.

We can see the basic mechanism for inflation dynamics with a static IS curve and the Lucas Phillips curve,

$$x_t = -\sigma \left(i_t - E_t \pi_{t+1} \right)$$

$$\kappa x_t = \pi_t - E_{t-1} \pi_t.$$

Eliminating x_t, we obtain the equilibrium condition

$$E_t \pi_{t+1} = i_t + \frac{1}{\sigma\kappa} (\pi_t - E_{t-1} \pi_t)$$

Denote the impulse response function $\pi_0, \ \pi_1$, etc. following a shock at time 0. Given an interest rate path $\{i_t\}$ and first period inflation π_0, inflation follows

$$\pi_0; \ \pi_1 = i_1 + \frac{1}{\sigma\kappa} \pi_0; \ \pi_2 = i_1, , ... \pi_t = i_{t-1}.$$

If $\sigma\kappa < 1$, the period 2 response can amplify the period 1 response. We have initially unstable dynamics, before turning around and becoming stable and neutral.

This simple case helps to see the intuition: If $\pi_0 > 0$, then we have a positive output response x_0 via the Lucas Phillips curve. In this model, higher output x_0 requires a lower real interest rate. So $i_0 - E_1\pi_1$ must fall. With a fixed nominal rate i_0, that rise requires a rise in π_1 relative to i_0. The rise in π_1 is, when time 1 rolls around, a rise in expected inflation — $\pi_1 = E_0\pi_1$ in the impulse response function — so has no further output effect. Lucas (1972) is famously a model without persistence. But that dictum is restricted to output. Even Lucas' Phillips curve generates inflation persistence when married to an interest rate target and an IS condition.

4 Policy Specification

It is a little unconventional that I specify the policy environment by an interest rate path and also by an equilibrium-selection/fiscal choice parameterized by initial inflation or the price level. It also seems unusual that equilibrium-selection choice does all the work of lowering inflation, and that higher nominal interest rates on their own raise inflation. This section explains.

The issues are the same for the standard rational-expectations model, and not a pathology of the Lucas Phillips curve. Stating policy in this way clarifies how both models work — especially in this case since we have been playing with the model for 35 years and these points are still unconventional. It also lets us examine the identification assumptions used to tie the interest rate path to the equilibrium-selection shock.

4.1 Flexible Prices

The basic issue occurs even in the flexible price model with no Phillips curve at all. The flexible price model reduces to

$$i_t = E_t\pi_{t+1}$$

so the response function is

$$\pi_{t+1} = i_t;\ t \geq 0. \tag{20}$$

The interest rate at time 0 (the time of the shock) determines π_1, but interest rates do not pin down π_0. To determine how the full path of inflation follows an interest rate path, we need an additional assumption to tie down π_0. With sticky prices, the assumption about π_0 bleeds forward to following inflation. Without a change in π_0, higher interest rates raise inflation at all other dates. Sticky prices turn out to smooth out, but not overturn, these properties.

4.2 Calvo Pricing

The standard model with a dynamic IS and a forward-looking Calvo-style Phillips curve is

$$E_t dx_t = \sigma(i_t - E_t dp_t)$$
$$E_t d\pi_t = r\pi_t - \kappa x_t.$$

though the results are similar with either representation. This is the continuous time version of the familiar

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1})$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$

Again, one can do the calculations either way.

The response to a time-0 shock satisfies

$$\frac{dx_t}{dt} = \sigma\left(i_t - \frac{dp_t}{dt}\right) \tag{21}$$
$$\frac{d\pi_t}{dt} = r\pi_t - \kappa x_t \tag{22}$$

The price level may not jump, so with short term debt the value of debt may also not jump. We have $p_0 = 0$ among our initial conditions. π_0, x_0, i_0 may all jump at time zero.

Again we can eliminate output and find a differential equation linking the inflation response to the interest rate path,

$$\frac{d^2\pi_t}{dt^2} = r\frac{d\pi_t}{dt} - \sigma\kappa\left(i_t - \pi_t\right).$$

Solving that equation, the inflation response is

$$\pi_t = \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2}}\left[Ce^{-\lambda_1 t} + \int_{s=0}^{t} e^{-\lambda_1(t-s)} i_s ds + \int_{s=t}^{\infty} e^{-\lambda_2(s-t)} i_s ds\right] \tag{23}$$

where C is an initial condition, related to π_0 by

$$\pi_0 = \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2}}\left[C + \int_{s=0}^{\infty} e^{-\lambda_2 s} i_s ds\right]$$

and

$$\lambda_1 = \frac{-r + \sqrt{r^2 + 4\sigma\kappa}}{2} > 0$$

$$\lambda_2 = \frac{r + \sqrt{r^2 + 4\sigma\kappa}}{2} > 0.$$

(Algebra in the Appendix.) Inflation is a two-sided moving average of the interest rate, plus an exponentially decaying transient which can be indexed by initial inflation. Given the interest rate, there are multiple possible equilibria, so the choice of initial inflation can again be called an "equilibrium-selection" choice. The weights of inflation on the interest rate are all positive, so for a given equilibrium-selection choice higher interest rates raise inflation.

Compare (23) to the inflation response (14) of the model with a generalized Lucas Phillips curve, expressed as a function of initial inflation with (16). The form is the same: Inflation depends on the interest rate path plus an equilibrium-selection choice indexed by initial inflation. In both cases, the weights are positive so given the initial inflation choice, higher interest rates raise inflation. In both cases, the only way to produce a decline in inflation coincident with higher interest rates is to imagine a contemporaneous negative shock to initial inflation.

Inflation in the model with Calvo pricing (14) responds to future interest rates, while only past interest rates enter in model with a generalized Lucas Phillips curve (23). This is a selling point as conventional beliefs see more backward-looking dynamics, yet the Lucas curve maintains rational expectations. The response to the initial inflation shock in (14) is a pure exponential decay, which is why the Calvo model produces a rise in inflation after an initial downward jump. The first unstable then stable dynamics of the generalized Lucas model (23) are by contrast its central innovation.

4.3 Three Characterizations of Equilibrium Choice

How does the government select equilibria, and what if any connection is there between higher interest rates and lower inflation? How does the interest rate plus initial inflation specification relate to more traditional policy specifications?

There are three observationally equivalent characterizations of the policy environment. First, the traditional approach is to specify a policy rule such as

$$i_t = \phi\pi_t + u_t,$$

and then calculate the response to a shock to the disturbance $\{u_t\}$. This approach seems to require only one input, the monetary policy disturbance. That impression is mistaken however.

Even this traditional framing incudes a separate interest rate and equilibrium selection specification, and no believable tie between the two.

We want to specify monetary policy by the interest rate path $\{i_t\}$. That is, after all, what the central bank actually controls, what we observe, and the data that a model matches. To match $\{i_t\}$, we find a disturbance $\{u_t\}$ that generates the desired interest rate path. But it turns out that there are many possible paths $\{u_t\}$ that generate any given interest rate path $\{i_t\}$. Each of those choices for $\{u_t\}$ implies a different path $\{\pi_t\}$, which can be indexed by initial inflation π_0. So, after choosing an interest rate path such the above AR(1), one still must also make an equilibrium selection choice π_0, in this case implicitly by choosing from the set of paths for $\{u_t\}$ that generate the same path $\{i_t\}$.

This statement is easy to demonstrate. Fix an interest rate path $\{i_t\}$. Choose any consistent inflation path, any path (23) of the Calvo model, (13) of the generalized Lucas model, or (20) of the flexible price model, generated by the interest rate path $\{i_t\}$ and an initial value π_0, p_0, or C. Choose any $\phi > 1$. Then construct $u_t = i_t - \phi\pi_t$. The $\{u_t\}$ so constructed will generate the chosen $\{i_t\}$ and $\{\pi_t\}$ as the unique locally bounded equilibrium following the usual solution method. Each different π_0 and thus $\{\pi_t\}$ correspond to a *different* $\{u_t\}$. So by choice of the stochastic process for $\{u_t\}$ among the set that generate a given $\{i_t\}$ is equivalent to the choice of equilibrium, or of initial inflation π_0.

Researchers also often specify additionally that u_t follows a given stochastic process, typically an AR(1). Adding the restriction that $\{u_t\}$ must be an AR(1), as well as generate the desired interest rate path, amounts then to an equilibrium selection choice for π_0. But there is no particular reason that the Fed should follow an AR(1). The Taylor rule disturbance is not observable, so there is no way to independently measure it. Different values of ϕ produce different disturbances as well, with no change in the observables.

Specifically, the conventional solution of the Calvo pricing model with a Taylor rule and AR(1) disturbance u_t, as plotted in Figure 3, is an instance of the general formula (23) for one specific value of initial inflation π_0 or equivalently of the transient C,[1]

$$C = -\frac{1}{\eta_i - \lambda_1} i_0 \tag{25}$$

But one can use the identical interest rate path and different values of initial inflation. They won't imply AR(1) disturbances.

Second, we can write the policy rule in the equivalent representation suggested by King (2000),

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*). \tag{26}$$

Here i_t^* is the central bank's desired equilibrium interest rate path, and π_t^* is the equilibrium the central bank wishes to choose, among the set consistent with $\{i_t^*\}$.

The two representations are equivalent. Given $\{\pi_t^*\},\ \{i_t^*\}$ in (26), construct $u_t = i_t^* - \phi\pi_t^*$. Given $\{u_t\}$, compute the equilibrium and use that to define π_t^* and i_t^*. Again, by following the rule (26), the central bank induces explosive dynamics for any $\pi_t \neq \pi_t^*$.

King's rule shows equilibrium selection more directly and transparently. The central bank picks an interest rate target $\{i_t^*\}$, but that leaves open multiple equilibria. The central bank adds an equilibrium-selection policy to pick π_0 directly, by inducing explosive behavior for any other value. King's rule shows that ϕ is a parameter describing off-equilibrium threats used to select equilibrium, and not observed in equilibrium where $\pi_t = \pi_t^*$.

Third, the choice of equilibrium also has fiscal consequences, or underpinnings, depending on how you look at it. We can select, or at least index, multiple equilibria by their fiscal underpinnings. The linearized government debt identity implies

$$\int_{j=0}^{\infty} e^{-rj}\tilde{s}_j dj = \int_{j=0}^{\infty} e^{-rj}(i_j - \pi_j)dj - p_0 + q_0. \tag{27}$$

where $\tilde{s}$ is the real primary surplus divided by the initial (steady state) value of debt. (See Cochrane,

[1] Using $i_t = i_0 e^{-\eta_i t}$ in (23),

$$\pi_t = \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2}} \left[\left(C + i_0 \frac{1}{\eta_i - \lambda_1} \right) e^{-\lambda_1 t} + i_0 \frac{\lambda_1 - \lambda_2}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)} e^{-\eta_i t} \right] \tag{24}$$

(Algebra in the Appendix). Thus inflation is an AR(1) with coeffiient η_1 for one particular value

$$C = -i_0 \frac{1}{\eta_i - \lambda_1}.$$

If inflation is an AR(1) with coefficient η_i and so is the interest rate, then $u_t = i_t - \phi\pi_t$ is also such an AR(1). Thus the AR(1) restriction picks this one particular value of C.

2023.)

A period of nominal interest rates above inflation, which is what we wish the models to produce, means higher interest costs on the debt. Primary surpluses must rise to pay those interest costs. A negative price level jump $p_0 < 0$ or a higher nominal price $q_0 > 0$ of the portfolio of long-term government bonds raises the value of nominal government debt, and thus requires higher real primary surpluses. Different equilibria, indexed by different values of initial inflation or price level, thus have different fiscal requirements. Inflation is always and everywhere determined by fiscal and monetary policy jointly, there are only different visions of how this coordination comes about.

In the new-Keynesian reading, of either the Taylor or King rule, these fiscal underpinnings of monetary policy happen "passively." The central bank follows an equilibrium-selection policy separate from its interest rate policy, consisting of threats of what it might do for undesired values of inflation. Congress always changes taxes and spending to produce primary surpluses that will repay the debt at the central-bank chosen price level. But those changes happen, and must happen. They are not "passive" in the sense of happening automatically without human intervention. If fiscal policy does not tighten to pay interest costs on the debt and a bondholder windfall, then the corresponding equilibrium $\{\pi_t\}$ cannot occur.

Fiscal theory of the price level specifies that the change (or its absence) in fiscal surpluses $\tilde{s}$ pins down the left side of (27) and thereby which $\{\pi_t\}$ can be an equilibrium. In this interpretation it is even more natural to separate policy into "monetary policy" which chooses interest rates and "fiscal policy" which selects equilibria. Those policies may be correlated or coordinated, of course.

The three stories are observationally equivalent. The bottom line: the policy setting is always characterized by the interest rate path *and* the equilibrium choice, and that equilibrium choice has fiscal consequences.

To illustrate these foundations, for each response I compute the fiscal underpinnings or determinants, the left-hand side of (27). If the surplus rise is permanent, then it is equal to $\tilde{s}_0/r$. I use that notation. In several cases I compute the Taylor rule disturbance $u_t = i_t - \phi\pi_t$, mostly to emphasize that there always is one – that one can generate any of these equilibria with Taylor rule disturbance. The King rule interpretation is evident by the direct choice of initial inflation.

Figure 1 includes all three interpretations. In all cases, I specify the indicated AR(1) interest rate path. Most directly, I *also* specify $\pi_0 = -0.5\%$, needed for the inflation response (14)-(16), which we can consider an instance of the King rule equilibrium selection policy plus passive fiscal tightening. I compute a Taylor rule disturbance $u_t = i_t - \phi\pi_t$ with $\phi = 1.1$. While the hump-shaped disturbance may seem unusual, if you want an AR(1) interest rate and a hump-shaped inflation, you will have to have a hump-shaped disturbance. I also compute the change in fiscal surplus, the left-hand side of the identity (27). With short-term debt ($q_0 = 0$), this solution requires a fiscal tightening equal to 13.7% of the value of debt. You can see the long period of interest rates larger than inflation, generating interest costs.

So, we can describe the policy experiment of Figure 1 as a response to the plotted Taylor rule disturbance $\{u_t\}$ supported by a "passive" 13.7% fiscal contraction; we can describe it as a response to the plotted interest path and a $\pi_0 = -0.5\%$ equilibrium-selection shock, again with a "passive" 13.7% fiscal contraction, or we can describe it as the plotted interest rate path plus an "active" 13.7% fiscal contraction which directly selects the plotted inflation path. Obviously, I find the latter interpretation most compelling, but the reader may choose any interpretation.

4.4 Interest Rates vs. Equilibrium Selection

In case staring at the solutions (14) and (23) does not drive the point home enough, I now examine how the initial inflation or price level assumption vs. the interest rate path assumption affect inflation.

Figure 5 presents the inflation response for the same generalized Lucas and exponential weight model and interest rate path as Figure 1, but different values of initial inflation π_0. The solid line repeats the case $\pi_0 = -0.5\%$ from Figure 1. The dashed lines present choices $\pi_0 = 0$, $\pi_0 = -0.25\%$, and $\pi_0 = -1\%$.

Clearly, the initial equilibrium-selection / fiscal shock has a major influence on the path of inflation. With $\pi_0 = 0$, inflation rises throughout the episode. The disinflation episode depends crucially on a negative initial inflation shock.

Figure 6 presents instead the inflation response for different values of the interest rate path, holding constant initial inflation π_0. The figure shows that the initial disinflation is nearly entirely driven by the selection/fiscal shock π_0, as it is nearly the same for all values of the interest rate. Moreover, higher interest rates *raise* inflation throughout.

I include a response with a permanent interest rate rise. Inflation rises to meets this inter-

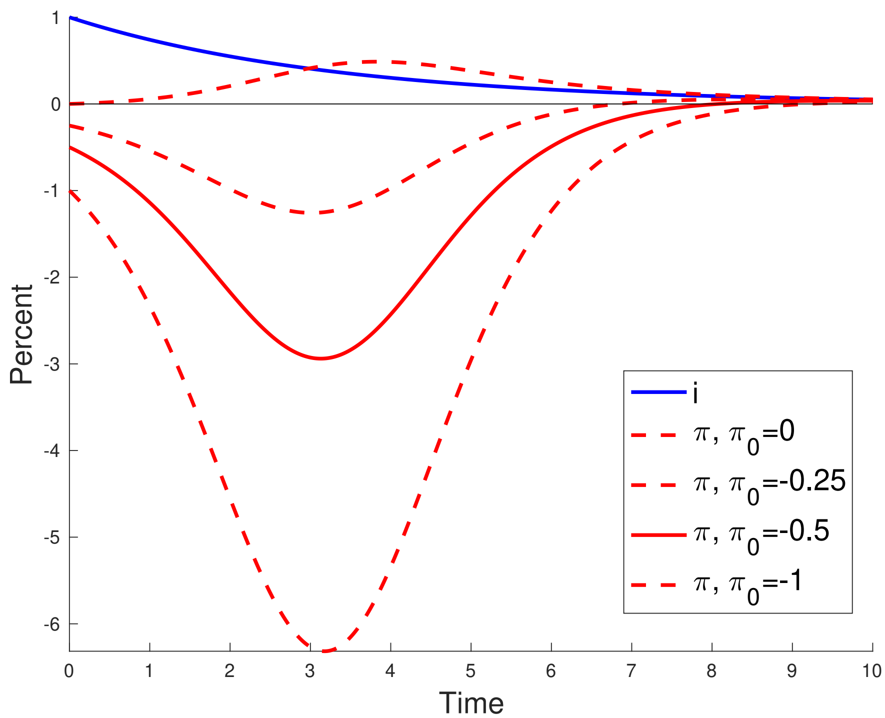

Figure 5: Inflation responses for various values of initial inflation, and the same interest rate. Continuous-time model with a standard dynamic IS curve and a generalized Lucas Phillips curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $\eta_i = 0.3$. The solid line uses the original value $\pi_0 = -0.5\%$. Dashed lines give the inflation response for alternative choices $\pi_0 = 0$, $\pi_0 = -0.25\%$, and $\pi_0 = -1\%$. With $r = 2\%$, and short term debt, a permanent fiscal surplus must rise by 1.06%, 7.4%, 13.7%, and 26.4% of the debt respectively.

est rate in the long run. This response emphasizes an unconventional but desirable property of rational expectations models: Inflation is stable and neutral in the long run. But stability and long-run neutrality imply that higher interest rates must eventually raise inflation, just as stability and long-run neutrality imply that higher money growth must eventually raise inflation in standard monetarist analysis. In this response, however, inflation declines in the short run. So common experience, which sees a lot more short run than long run, could well see that negative sign but not the long run positive relationship.

I include a response with no interest rate movement at all. This is sometimes called an "open-mouth" operation. In a new-Keynesian view, it is a pure equilibrium-selection shock. The central bank can announces a changed inflation target π^*, or a calibrated sequence of Taylor-rule disturbances $\{u_t\}$, that generate a change of inflation with no movement in the interest rate. In fiscal theory, this is the response to a fiscal shock with no change in monetary policy, roughly the opposite of what happened in 2021-2023. It generates a hump-shaped disinflation, just as

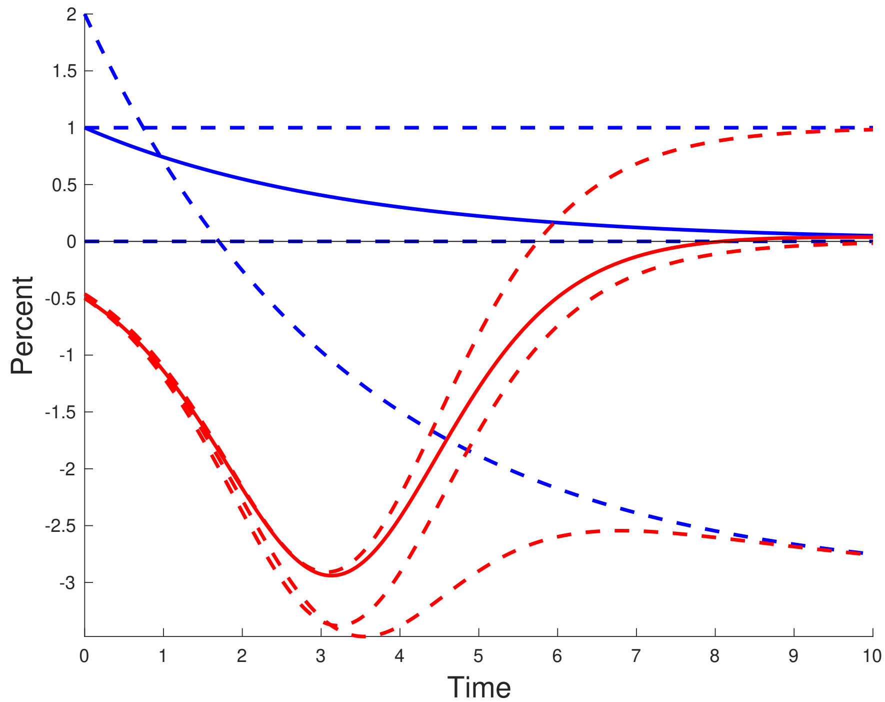

Figure 6: Inflation for various values of the interest rate path, and the same initial inflation. Continuous-time model with a standard dynamic IS curve and a generalized Lucas Phillips curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $\pi_0 = -0.5\%$. The solid line uses the original value $i_0 = 1\%$. Other interest rate lines use $i_0 = 0$ and $i_0 = 2\%$ as indicated. The permanent shock uses $\eta_i = 0$, other lines use $\eta_i = -0.3$. With $r = 2\%$, and short term debt, a permanent fiscal surplus must rise by 12.7%, 13.7%, 15.9%, and 11.0% of the debt, from top to bottom.

occurred in 2021-2023. This response is almost exactly the same as the response with an AR(1) interest rate shock, and again the response to the AR(1) shock is *higher*. Interest rates, on their own, raise inflation uniformly.

I also include an interest rate path that rises temporarily and then declines rather than the standard AR(1). This path is reminiscent of what happened in the 1980s. A high interest rate was followed by lower interest rates as inflation came down. Inflation and interest rates decline together over a long period of time, as they did in the 1980s. Cause and effect are reversed from the usual interpretation, of course. Here the interest rates drag down inflation, not the other way around. And the disinflation would have been quicker had the central bank not raised interest rates at all, and simply followed the negative selection/fiscal shock with lower nominal interest rates. Still, we should plot monetary policy responses that look something like successful disinflations not just AR(1)s that leave no mark. And it is comforting that this model can produce something that *looks* like the 1980s, though with a mechanism utterly different from the standard one.

Figure 6 shows that monetary policy does completely control long-run inflation, no matter what the equilibrium-selection/fiscal shock does. However the direction is Fisherian: inflation follows interest rates up or down.

The same analysis holds for the standard Calvo pricing model. Again, Figure 3 plots a response of this model to the standard AR(1) shock to the Taylor rule. The downward jump has often been misinterpreted as this model's ability to generate lower inflation by higher interest rates. Separating the policy instrument to the interest rate plus equilibrium selection/fiscal choice of initial inflation shows that *any* decline in inflation occurs because of the latter.

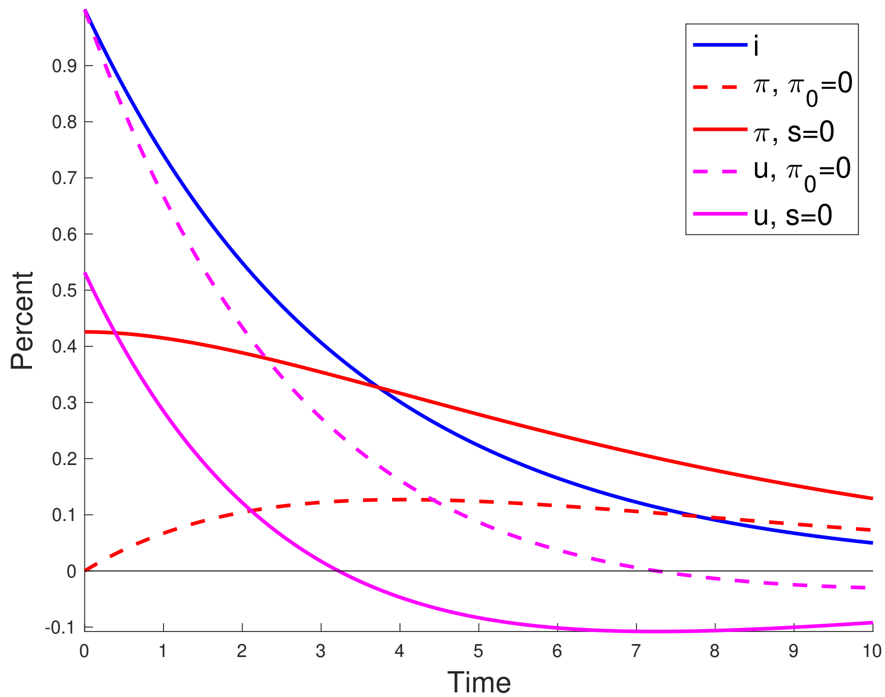

Figure 7: Response of the standard new-Keynesian model to an AR(1) interest rate plus equilibrium selection that sets initial inflation π_0 to zero, and that sets surpluses to zero so net interest costs are zero. Parameters $\eta_i = 0.3$, $\phi = 1.1$, $r = 2\%$, $\sigma = 0.21$, $\kappa = 0.21$, $u_0 = 1.97\%$. In the $\pi_0 = 0$ equilibrium, fiscal tightening to pay interest costs on the debt is $\tilde{s}_0/r = 2.0\%$ of the value of debt.

To see this point, Figure 7 plots the response of the same model as Figure 3 (the standard three-equation new-Keynesian model), with same parameters, and the identical interest rate path (the vertical scale is different), but instead specifying $\pi_0 = 0$. Now inflation rises uniformly. The apparent ability of the standard model to lower inflation comes entirely from the π_0 equilibrium selection/fiscal choice, and has nothing to do with the interest rate path. The figure includes a Taylor rule disturbance $u_t = i_t - \phi\pi_t$ reverse-engineered to produce the $\pi_0 = 0$ equilibrium and the same interest rate path. One can regard the change from the original AR(1) to this disturbance as producing positive inflation. The disturbance is visually similar to an AR(1).

The $\pi_0 = 0$ equilibrium still requires a fiscal tightening, 2% of the value of debt not 5.8% as in Figure 3. The interest rate is persistently above the inflation rate, so average interest cost $\int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt$ is greater than zero. Figure 7 also plots an equilibrium chosen so that no fiscal policy tightening is needed. The average interest costs are zero; times when the interest rate exceeds inflation are balanced by times when the interest rate is lower than inflation. This equilibrium has still higher inflation starting immediately.

The downward equilibrium-selection jump in Figure 3 is large, indeed all of the downward movement of inflation. By contrast the responses with the generalized Lucas Phillips curve can amplify a small downward jump to create a much larger disinflation. Thus, while the generalized Lucas Phillips curve does not change the essential nature of the model — lower inflation all comes from the initial equilibrium-selection policy — it does help quantitatively, since a much smaller shock to initial inflation or the initial price level can set off a large disinflation.

5 A Coordinating Mechanism

It seems that monetary policy relies on a coincident fiscal contraction for all of any disinflationary power of higher interest rates. That isn't altogether unreasonable as a reading of the data. The same events that provoke a central bank to tighten also provoke fiscal authorities to tighten. But if the central bank's interest rate increases are per-se counterproductive, that's revolutionary news to central banks.

The Taylor rule with AR(1) disturbance offers a vision linking higher interest rates to the fiscal tightening that does all the work. By pledging to follow that policy, not any of the other disturbance paths that produce the same interest rate path, the central bank ties higher interest rates to contractionary equilibrium selection.

This conventional story has deficiencies as well. The coefficient ϕ and disturbance are not identified nor directly visible to economic agents (Cochrane, 2011). No central bank's forward guidance has ever breathed a word about such an object. And as we have seen the central bank could issue different pledges to produce the same disinflation with no change in interest rates at all. The King reformulation of the rule $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ makes clear just how any connection between equilibrium-selection and interest rate parts of the rule is arbitrary, and how equilibrium selection relies on unobservable quantities.

Here I pursue the one coordination mechanism I know of in which higher interest rates lower initial inflation π_0, and thus subsequent inflation, *without* requiring any change in fiscal policy. This is the “stepping on a rake” (Sims, 2011; Cochrane, 2017) or “unpleasant interest rate arithmetic” mechanism (Cochrane, 2023). When the central bank raises the nominal interest rate persistently, it raises long-term inflation. That lowers long-term bond prices. If the present value of surpluses does not change, then the nominal market value of government debt declines against an unchanged real present value. That difference is resolved by an unexpected deflation, raising the real value of debt back to its present value.

With this mechanism, the central bank has a reason to raise interest rates when it wishes to fight a spurt of inflation. Conversely, lowering interest rates, which would otherwise lower inflation, would temporarily raise inflation and so dissuade the central bank from that otherwise (in these models) attractive option.

With a standard forward-looking Phillips curve, however, this mechanism still generates inflation that jumps down and then rises going forward. The generalized Lucas Phillips curve produces disinflation that increases going forward even with this mechanism. The dynamics are no different than previous cases, but the fiscal underpinnings mean that the central bank, acting on its own, can produce a negative initial inflation π_0 without any change in fiscal surpluses and without any unobserved equilibrium-selection threats, by raising interest rates.

Again, the fiscal identity (27) reads

$$\int_{t=0}^{\infty} e^{-rt}\tilde{s}_t = \int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt - p_0 + q_0$$

where q_0 is the nominal price of the government bond portfolio. I assume a geometric maturity structure in which the face value at maturity j is proportional to $\omega e^{-\omega j}$ and the expectations hypothesis, so

$$q_0 = -\int_{t=0}^{\infty} e^{-(r+\omega)t} i_t dt.$$

Thus, in place of (27), surpluses must change by

$$\int_{t=0}^{\infty} e^{-rt}\tilde{s}_t = \int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt - p_0 - \int_{t=0}^{\infty} e^{-(r+\omega)t} i_t dt. \tag{28}$$

Suppose there is no surplus and no price level jump. With short-term debt, $\omega = \infty$

$$0 = \int_{t=0}^{\infty} e^{-rt}(\pi_t - i_t)dt.$$

This fiscal criterion picks the equilibrium with no average interest costs and no average real rate. This criterion makes it hard for higher nominal rates to lower inflation.

With perpetual debt $\omega = 0$, however,

$$0 = \int_{t=0}^{\infty} e^{-rt}\pi_t dt,$$

average *inflation* must be zero, not the average real rate. Intuitively, expected inflation can devalue long-term bonds by devaluing nominal payments when they come due. Now, if persistently higher interest rates create a period of inflation in the long run, that can be matched by a period of lower inflation in the short run. I call the mechanism "unpleasant interest rate arithmetic" because, as in the classic Sargent and Wallace (1981), the central bank can only lower inflation now by raising inflation later. But it is interest rate arithmetic not monetarist arithmetic, since central bank sets an interest rate target rather than control inflation via the money supply.

Figure 8 presents a calculation. This is the same model and parameters as in Figure 1, except that I specify a more persistent interest rate movement with $\eta_i = 0.05$ rather than $\eta_i = 0.30$. To generate disinflation early on, there must be inflation later on, as one sees in Figure 8. In Figure 1, the interest rate essentially returns to zero before inflation has a chance to follow interest rates upward. This mechanism requires persistent interest rates to generate lower inflation.

	0	0.0026	0.1	0.3	∞
s_0/r, $\eta = 0.05$ (Figure 8)	-0.50%	0	7.9%	12.9%	13.8%
s_0/r, $\eta = 0.30$ (Figure 1)	10.6%	10.6%	11.4%	12.1%	13.7%

Table 1: Surplus required for different values of the maturity structure of debt ω.

Table 1 presents the change in surplus required for the solution plotted in Figure 8, for different values of the maturity structure of debt ω, along with the same calculation for the solution plotted in Figure 1. This is the left side of equation (28). If debt has maturity structure characterized by $\omega = 0.026$, this equilibrium requires exactly no change in surplus. This is the central case whose possibility the calculation emphasizes.

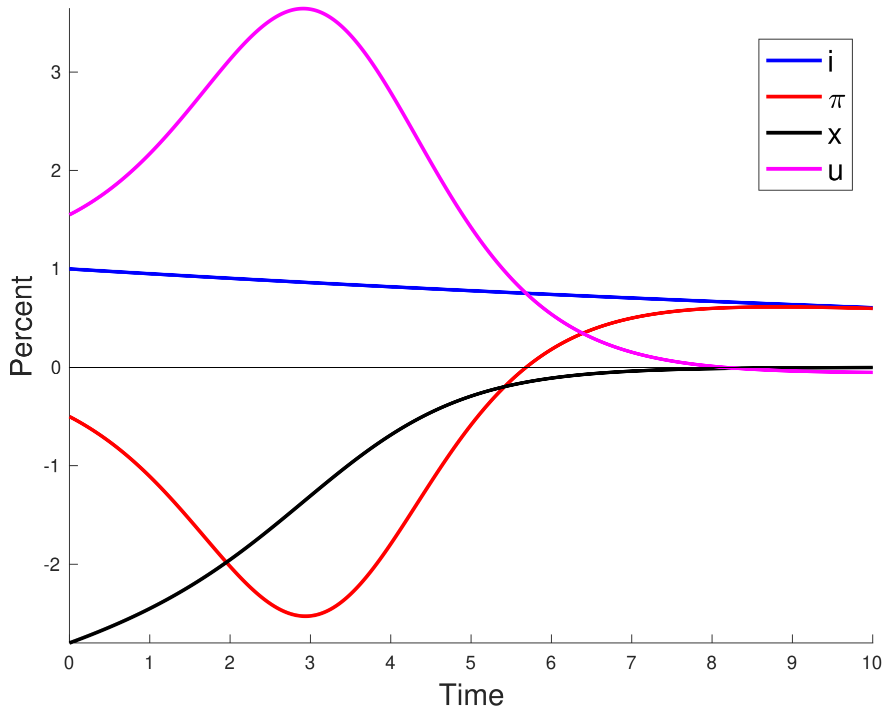

Figure 8: Higher interest rates can lower inflation with no fiscal tightening. Response of inflation and output to the indicated AR(1) interest rate and initial $\pi_0 = -0.5\%$ disinflation. Continuous-time model with a standard dynamic IS curve and a generalized Lucas Phillips curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $\eta_i = 0.05$. The u line plots a Taylor-rule disturbance u_t in $i_t = \phi\pi_t + u_t$ with $\phi = 1.1$ that generates the inflation and output paths.

If the maturity structure had been longer, then the same interest rate path would have had a greater short-run deflationary impact, producing even greater disinflation. Equivalently, as shown in the top left corner of Table 1, the plotted inflation would occur even with additional primary deficits totaling 0.5% of debt. If the maturity structure had been shorter, the initial disinflation would have been less. Equivalently, this equilibrium, with the more persistent interest rate and with $\pi_0 = -0.5\%$, would have required a rise in primary surplus equal to 13.8% of debt.

The second row shows that with the less persistent interest rate path of Figure 1, even with perpetual debt $\omega = 0$, the inflation path starting at $\pi_0 = -0.5\%$ would need a 10.6% rise in surplus to debt. The interest rate has converged back to zero before it can cause higher future inflation. Equivalently, for any given maturity structure, the less persistent interest rate movement produces less disinflation.

The mechanism seems to require a very persistent interest rate movement and very long maturity structure to produce disinflation. However, I chose σ, κ, and a to produce a likely unrealistically large (relative to the interest rate rise) and persistent (years) disinflation. My goal is to illustrate model dynamics, not to match specific estimates. Model parameters that give rise to smaller and less persistent disinflation will allow a less persistent interest rate path and lower debt maturity to raise long-term inflation and thus produce short-term disinflation.

Of course, this mechanism also has nothing to do with the standard story that high real interest rates depress aggregate demand which pushes down inflation via the Phillips curve. Neither does the standard new-Keynesian model have anything to do with that story, as we have seen. But at least it gives a result with much the same appearance as the standard beliefs.

I emphasize the response with no change in current and future primary surpluses to illustrate its possibility, and to give a reason by which the central bank acting on its own might wish to raise interest rates in order to lower inflation, even temporarily. Other coordinated monetary-fiscal responses are interesting and important for actual policy and historical episodes. It is not necessary in fiscal theory to hold surpluses constant, or to specify them exogenously, or to specify that they do not react to central bank action or to macroeconomic variables. Matching episodes or realistic policy evaluation should include how surpluses react to inflation, recession, interest costs, and debt, and consider coincident and coordinated fiscal policy shocks.

6 Down-Weighted Expectations

In order to produce output dynamics more consistent with standard beliefs, I mix the generalized Lucas Phillips curve with the two additional IS curves. Here, I consider

$$m\frac{dx_t}{dt} - (1-m)x_t = \sigma\left(i_t - \frac{dp_t}{dt}\right) \tag{29}$$

This specification allows an intermediate case between the previous dynamic IS curve, with $m = 1$, and a static IS curve in which the level of output is directly affected by the real interest rate, with $m = 0$. The specification (9) is the continuous time equivalent of downweighting future output by a factor m in discrete time,

$$x_t = mx_{t+1} - \sigma(i - \pi),$$

as in Gabaix (2020).

Figure (9) presents the results, contrasting the previous case ($m = 1$, dashed lines) and a

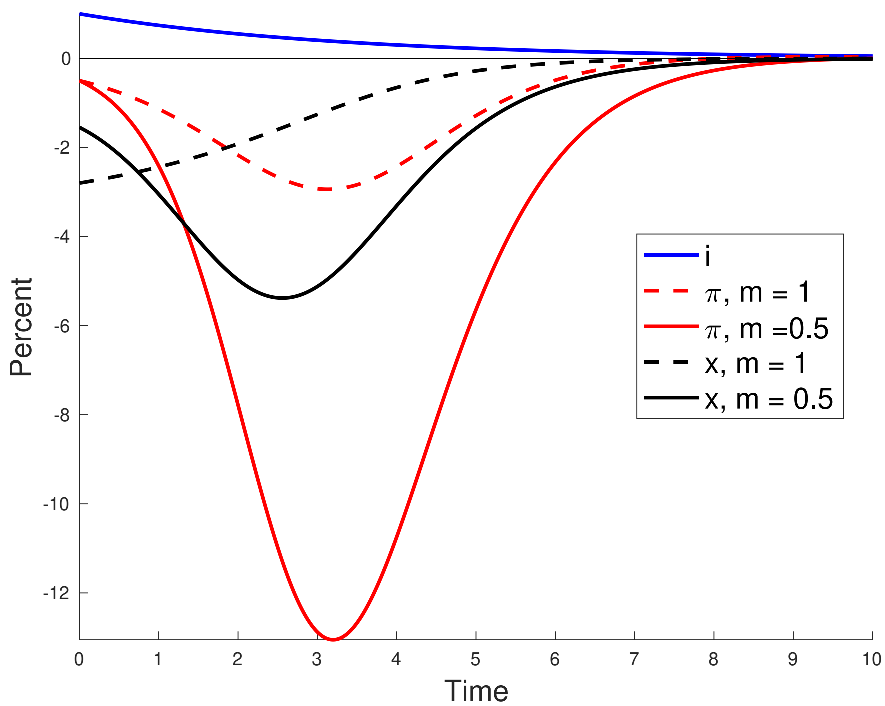

Figure 9: Inflation and output with a generalized Lucas Phillips curve and an IS curve with down weighted expectations. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $\eta_i = 0.3$, $i0 = 1\%$, $\pi_0 = -0.5\%$, $r = 2\%$. The surplus rises by $\tilde{s}/r = 13\%$ and 43% of the value of debt in the two cases.

downweighted case ($m = 0.5$, solid lines). Otherwise I use the same parameters. I include the AR(1) interest rate path for comparability, even though we saw above that higher interest rates just uniformly raise inflation and the initial inflation does all the work.

The specification enhances the power of the Lucas Phillips curve to generate lower inflation. It also leads to output that declines before rising again, bringing that response closer to common policy beliefs.

7 Habits

I also mix the generalized Lucas Phillips curve with an IS curve that features external habit persistence. This specification maintains the purity of rational expectations. It is a well known ingredient to produce hump-shaped output dynamics.

Marginal utility depends on consumption relative to a moving average of past consumption.

The IS curve becomes

$$E_t d\left(x_t - \theta b \int_{s=0}^{\infty} e^{-bs} x_{t-s} ds\right) = \sigma[i_t dt - E_t(dp_t)]. \tag{30}$$

(In discrete time one-period habits are common, $x_t - \theta x_{t-1}$, but a cost to a second derivative of consumption is awkward in continuous time. The AR(1) formulation keeps a lag in place while shrinking the time period.)

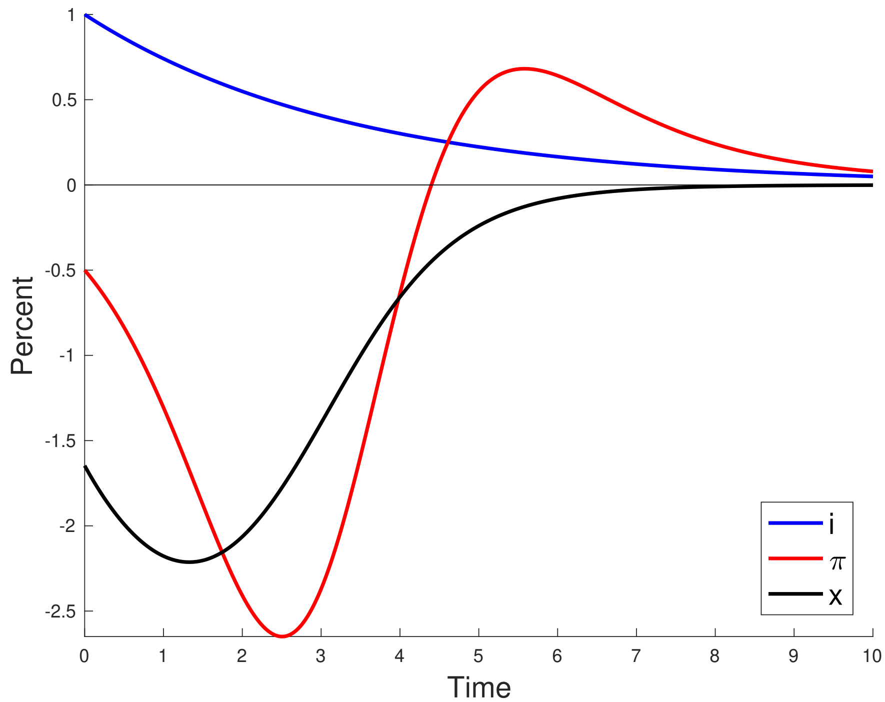

Figure 10: Inflation and Output Response with Autoregressive Habit and Generalized Lucas Phillips Curve. Parameters $\sigma = 0.2$, $\kappa = 0.2$, $a = 1$, $\eta_i = 0.3$, $i_0 = 1\%$, $\pi_0 = -0.5\%$, $r = 2\%$, $\theta = 0.7$, $b = 1$. The surplus rises by $\tilde{s}/r = 8.23\%$ of the value of debt.

Figure 10 presents the results, adding habit with $b = 1$ and $\theta = 0.7$. Habits induce a hump-shaped output response, as hoped.

To derive the inflation response to interest rates, define the external habit as

$$X_t = b\int_{s=0}^{t} e^{-b(t-s)} x_s ds$$

which solves

$$\frac{dX_t}{dt} = b(x_t - X_t).$$

Then (30) becomes

$$\frac{dx}{dt} - \theta b(x_t - X_t) = \sigma \left(i_t - \frac{dp}{dt} \right).$$

I eliminate output and its derivative from the generalized Lucas Phillips curve with exponential weights (7) and its derivative (8), yielding

$$(e^{-at} + \sigma\kappa)\frac{dp}{dt} - (a + \theta b)e^{-at}p_t + \theta b\kappa X_t = \sigma\kappa i_t.$$

$$\frac{d\kappa X_t}{dt} = b(\kappa e^{-at}p_t - \kappa X_t).$$

I solve this pair of differential equations numerically, starting with the initial condition

$$(1 + \sigma\kappa)\pi_0 - (a + \theta b)p_0 = \sigma\kappa i_0$$

and $X_0 = 0$.

7.1 Habits and Calvo Pricing

One may wonder if habits alone are enough to induce hump-shaped responses in both output and inflation, using the traditional forward-looking Phillips curve. To that end I solve a model with habit IS and standard Phillips curve,

The response solves

$$\frac{dX}{dt} = b(x_t - X_t)$$

$$\frac{dx_t}{dt} - \theta b(x_t - X_t) = \sigma(i_t - \pi_t)$$

$$\frac{d\pi_t}{dt} = r\pi_t - \kappa x_t$$

$$\frac{di}{dt} = -\eta_i i_t$$

The initial condition is given by $X_0 = 0$, π_0, and i_0. I use the one forward-looking eigenvalue to find x_0. I solve the model by the standard matrix method, reviewed in the Appendix .

Figure 11 presents the resuts. No, at least at the parameter values we have been using so far, habits alone together with the standard forward-looking Phillips curve do not generate a hump-shaped inflation response.

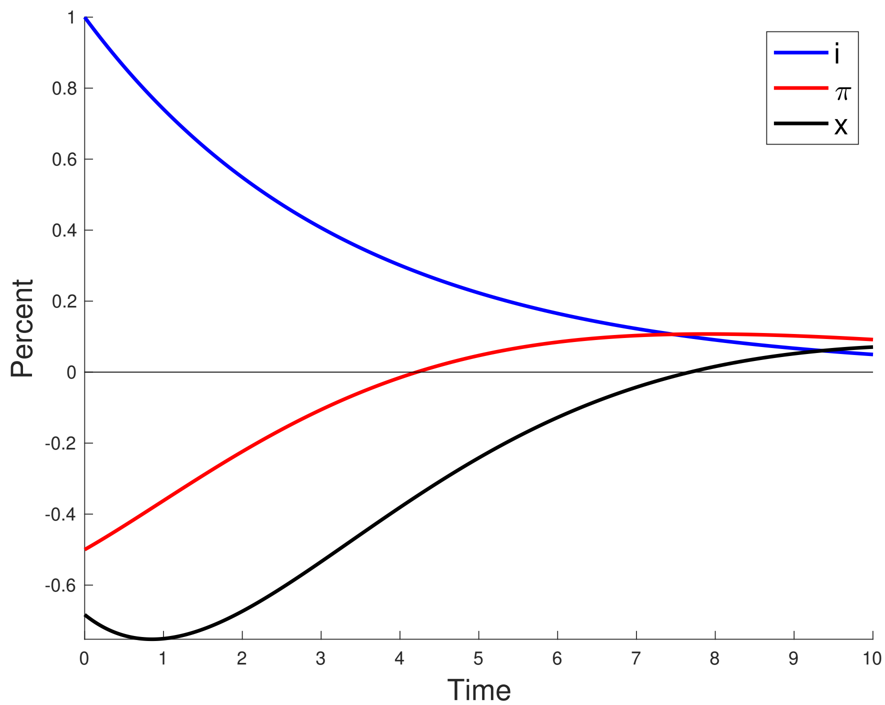

Figure 11: Autogressive Habits and Calvo Phillips Curve. Parameters $\theta = 0.7$, $b = 1$, $\pi_0 = -0.5\%$, $\eta_i = 0.3$, $\phi = 1.1$, $r = 2\%$, $\kappa = 0.2$, $\sigma = 0.2$, $i_0 = 1\%$.

8 Concluding Comments

This paper investigates a generalization of the classic Lucas Phillips curve. It works towards a model of textbook simplicity that can start to bridge the yawning gap between the effects of monetary policy most economists and central bank officials talk about and the effects of monetary policy in rational expectations/DSGE/new-Keynesian models that have dominated academic work for the last three decades. In the former, higher nominal interest rates mean higher real interest rates that lower aggregate demand, lower output, and then via a Phillips curve lower inflation, all pushing inflation and output downward in the future, with long and perhaps variable lags. That view is captured by 1970s era adaptive expectations models, but not by models with forward-looking expectations (Cochrane, 2024). In the latter, higher nominal rates raise inflation going forward. The only mechanism for lower inflation is a coincident downward equilibrium-selection jump. In both views, the disinflation must be accompanied by a fiscal contraction to pay higher interest costs and greater real payments to government bondholders.

The generalized Lucas Phillips curve turns a small downward inflation or price level jump into a larger disinflation, coincident with the basic sign of standard beliefs. The long term debt mechanism allows higher nominal interest rates to set off such a small downward jump without needing a coincident fiscal contraction, and it gives central banks a reason to raise interest rates if they want to lower inflation.

The resulting responses look a good deal like those of common beliefs. They continue to act, however, by a completely different mechanism, as do those of all other rational expectations models. Whether that is a feature or a bug needing fixing remains to be seen.

References

Ball, Laurence. 1994. "Credible Disinflation with Staggered Price-Setting." *The American Economic Review* 84 (1):282–289.

Buda, Gergely, Vasco Carvalho, Giancarlo Corsetti, Jo ao B. Duarte, Stephen Hansen, Alfonso S. Moura, Álvaro Ortiz, Tomasa Rodrigo, José V Rodríguez Mora, and Guilherme Alves da Silva. 2023. "The Short Lags of Monetary Policy." *CEPR Discussion Paper DP18022* URL https://cepr.org/publications/dp18022.

Calvo, Guillermo A. 1983. "Staggered prices in a utility-maximizing framework." *Journal of Monetary Economics* 12 (3):383–398.

Campbell, John Y. and John H. Cochrane. 1999. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107:205–251.

Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. 2016. "Unemployment and Business Cycles." *Econometrica* 84 (4):1523–1569.

———. 2018. "On DSGE Models." *Journal of Economic Perspectives* 32 (3):113–40.

Cochrane, John H. 2011. "Determinacy and Identification with Taylor Rules." *Journal of Political Economy* 119:565–615.

———. 2016. "Comments on 'A Behavioral New-Keynesian Model' by Xavier Gabaix." Manuscript.

———. 2017. "Stepping on a Rake: The Fiscal Theory of Monetary Policy." *European Economic Review* 101:354–375.

———. 2023. *The Fiscal Theory of the Price Level*. Princeton NJ: Princeton University Press.

———. 2024. "Expectations and the Neutrality of Interest Rates." *Review of Economic Dynamics* 53:194–223. URL https://www.sciencedirect.com/science/article/pii/S1094202524000140.

Coibion, Olivier. 2010. "Testing the Sticky Information Phillips Curve." *The Review of Economics and Statistics* 92 (1):87–101. URL https://doi.org/10.1162/rest.2009.11148.

Dupor, Bill. 2023. "Examining Long and Variable Lags in Monetary Policy." *Regional Economist* URL https://www.stlouisfed.org/publications/regional-economist/2023/may/examining-long-variable-lags-monetary-policy. May.

Dupor, Bill, Tomiyuki Kitamura, and Takayuki Tsuruga. 2010. "Integrating Sticky Prices and Sticky Information." *The Review of Economics and Statistics* 92 (3):657–669. URL https://doi.org/10.1162/REST_a_00017.

Friedman, Milton. 1948. "A Monetary and Fiscal Framework for Economic Stability." *The American Economic Review* 38 (3):245–264.

———. 1968. "The Role of Monetary Policy." *The American Economic Review* 58:1–17.

Gabaix, Xavier. 2020. "A Behavioral New Keynesian Model." *American Economic Review* 110:2271–2327.

García-Schmidt, Mariana and Michael Woodford. 2019. "Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis." *American Economic Review* 109:86–120.

Kiley, Michael T. 2007. "A Quantitative Comparison of Sticky-Price and Sticky-Information Models of Price Setting." *Journal of Money, Credit and Banking* 39 (s1):101–125. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1538-4616.2007.00017.x.

King, Robert G. 2000. "The New IS-LM Model: Language, Logic, and Limits." *Federal Reserve Bank of Richmond Economic Quarterly* 86 (3):45–104.

Lucas, Robert E. 1972. "Expectations and the Neutrality of Money." *Journal of Economic Theory* 4:103–124.

Mankiw, N. Gregory and Ricardo Reis. 2002. "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *Quarterly Journal of Economics* 117:1295–1328.

———. 2007. "Sticky Information in General Equilibrium." *Journal of the European Economic Association* 5 (2-3):603–613.

Nelson, Edward. 2020. *Milton Friedman and Economic Debate in the United States, 1932–1972, Volume 1.* Chicago: University of Chicago Press.

Ramey, Valerie. 2016. "Macroeconomic Shocks and Their Propagation." In *Handbook of Macroeconomics Vol. 2,* edited by John B. Taylor and Harald Uhlig. Amsterdam: Elsevier, 71–162.

Rotemberg, Julio J. 1982. "Sticky Prices in the United States." *Journal of Political Economy* 90 (6):1187–1211.

Sargent, Thomas J. and Neil Wallace. 1981. "Some Unpleasant Monetarist Arithmetic." *Federal Reserve Bank of Minneapolis Quarterly Review* 5:1–17.

Sims, Christopher A. 2011. "Stepping on a Rake: The Role of Fiscal Policy in the Inflation of the 1970s." *European Economic Review* 55:48–56.

Smets, Frank and Raf Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97:586–606.

Woodford, Michael. 2003. *Interest and Prices.* Princeton: Princeton University Press.

Appendices

A Literature and its Lessons

Addressing this sharp divergence between models versus beliefs and VAR evidence has led to deep surgery in the standard model. It's not easy. Standard policy beliefs reflect adaptive, backward-looking expectations and unstable dynamics, as described by Friedman (1968). Inflation spirals away under an interest rate peg. Higher interest rates provoke such a downward spiral, until the central bank lowers interest rates to stop that spiral. New-Keynesian models are founded on rational, forward-looking expectations, which lead to stable dynamics. Higher interest rates must eventually lead to higher inflation, a natural long-run stability and neutrality result. A model in which stable forward looking dynamics temporarily look like unstable backward looking dynamics is hard. A simple, interpretable model that a Fed chair might not be embarrassed to mention in public or that might be taught to undergraduates is harder still.

Thus, Gabaix (2020), García-Schmidt and Woodford (2019)) essentially revert to adaptive expectations, with sophisticated underlying models. That works, but throws out the simplicity and the economics. Is there *no* simple, fully-rational supply and demand theory that produces the standard policy belief? Are steadfastly adaptive expectations *necessary* to deliver the basic sign of monetary policy effects?

Why is the generalized rational expectations Phillips curve not already widely known and standard? Mankiw and Reis (2002) motivate their Phillips curve by "sticky information," and arrive at a slightly different form (p. 1300),

$$\pi_t = \left[\frac{\alpha\lambda}{1-\lambda}\right] y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j} \left(\pi_t + \alpha \Delta y_t\right) \tag{31}$$

where y is output. The presence of the extra Δy_t on the right hand side is a bit of a nuisance. More importantly, Mankiw and Reis (2002) only draw some illustrative responses to money supply shocks. The central challenge of monetary theory, met by the standard new-Keynesian model, is to describe monetary policy in terms of interest rates, not money supplies, because our central banks control interest rates and not money supplies.

Mankiw and Reis (2007) take on the latter challenge, embedding their sticky information Phillips curve in a model with an interest rate target. Their aggregate log-linearized Phillips curve is (p.605)

$$p_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j} \left[p_t + \frac{\beta(w_t - p_t) + (1-\beta)y_t - a_t}{\beta + \nu(1-\beta)} - \frac{\beta \nu_t}{(\nu - 1)[\beta + \nu(1-\beta)]} \right]$$

where beyond standard symbols, β measures returns to scale, a_t is productivity, and ν_t is a time-varying substitution elasticity between varieties. With wages in the Phillips curve, we also need a wage adjustment equation, (p. 606)

$$w_t = \omega \sum_{j=0}^{\infty} (1-\omega)^j E_{t-j} \left[p_t + \frac{\gamma(w_t - p_t)}{\gamma + \psi} + \frac{l_t}{\gamma + \psi} + \frac{\psi(y^n_{\infty} - \theta R_t)}{\theta(\gamma + \psi)} - \frac{\psi \gamma_t}{(\gamma + \psi)(\gamma - 1)} \right]$$

where beyond standard symbols, l_t is labor, ψ is the Frisch elasticity of labor supply, θ is the intertemporal elasticity of substitution, and ω is the fraction of workers who become informed each period. Mankiw and Reis complete the model with a standard production function $y_t = a_t + \beta l_t$ and IS curve. They plot impulse-response functions to shocks (p.608) sadly omitting the interest rate however.

I provide this level of detail so we can answer the puzzle, and learn appropriate lessons. Mankiw and Reis (2002) has, as of February 18 2025, an impressive 3508 Google scholar citations. Mankiw and Reis (2007) though more directly useful to the construction of a standard NK-DSGE model with an interest rate target, has only 309 citations. And of these, I have not yet found a single follower who has *used* the Mankiw-Reis model. After hallucinating three plausible but false citations, Claude.ai reports "What I can say with confidence is that while the Mankiw-Reis model has been influential in theoretical discussions, empirical applications using their exact framework have been less common than papers that cite or discuss their work conceptually."

The Mankiw-Reis Phillips curve is apparently not quite ready to replace the standard Phillips curve in textbooks. In part, it "raises several thorny technical issues" (p. 603). "Ready-to-use solution algorithms" are not "particularly useful to solve the sticky-information model. The model involves both an infinite number of past expectations of the present through sticky information, as well as present expectations of variables at an infinite number of future dates through intertemporal smoothing. This double infinity implies that the state space of the model has an infinite dimension, which current algorithms cannot handle." (p. 604.) Mankiw and Reis provide an alternative solution algorithm, but apparently nobody else has wanted to follow them in that path. Infinity of the state space seems solveable by replacing the AR information revelation structure by a MA structure, in which everyone learns the aggregate after k periods. In any case, one can read part of this paper's contribution as simplifying slightly the Mankiw-Reis structure so that it can be easily solved either by state-space methods or analytically.

Lucas (1972) has been more influential, of course, with 8416 Google Scholar cites (curiously, not including Mankiw and Reis). Lucas' actual model is much harder. But Lucas leads to a very simple and useable aggregate Phillips curve,

$$\kappa x_t = \pi_t - E_{t-1}\pi_t. \tag{32}$$

This form, rather than the full microfounded islands plus overlapping generations model, took off.

Calvo (1983) (13454 Google scholar citations) is similarly influential, despite its obviously counterfactual microfoundations, and the rather tortured path one must take to arrive at its familiar Phillips curve (Woodford, 2003, Ch. 3). But that Phillips curve is both intuitive and easy to incorporate into other models. Rotemberg (1982) menu costs are a similar story.

There are hundreds of other papers that investigate various microfounded models of sticky prices, many designed to replicate the equally large microeconomic empirical work on price stickiness. And none seem to stick, become part of the larger project, or make their way to the standard textbook model. Apparently, we are all the drunk of the joke, who looks for his car keys in the light not across the street where he dropped them. Well, looking in the light has some wisdom.

The lesson seems clear: To be sticky, a model must produce an easily *useable* aggregate Phillips curve, or a similarly simple and tractable description of price-level dynamics. That principle guides this paper.

Mankiw and Reis (2007) may also have erred in jumping to the construction of a "medium-scale" model without a simpler "textbook" model. Smets and Wouters (2007), Christiano, Eichenbaum, and Trabandt (2016), and Christiano, Eichenbaum, and Trabandt (2018) all built on top of the above simple two-equation model that gave people a lot of intuition. I focus on that textbook model here, with an invitation for a more realistic medium scale elaboration to follow.

Lucas (1972) was famously abandoned, first in favor of real business cycles, and later in favor of Calvo-Rotemberg pricing, because it appears unable to produce output persistence from monetary policy shocks. Equation (32) implies that output x_t follows an iid process in levels. I find it curious that the rational expectations school gave up so easily. Output is independent over time only because Lucas assumed that the aggregate money supply and price level are published after one "period," so everybody can perfectly distinguish relative price movements from aggregate demand. It would seem natural to query just how long a "period" is anyway, and whether publication of the money stock aggregates fully resolves the state of aggregate demand. As I write in 2025, economists and the Fed are still hotly debating in inflation broke out in 2021 due to fiscal and monetary shocks or whether it reflected relative-price "supply" and "relative demand" shocks. If it takes us this long to sort it out, it might make sense that agents take a while to sort it out as well. Hence, specifying that it takes some people longer to separate relative prices from price level shocks in (1) seems natural, even if it has taken more than 50 years to spy the possibility.

Kiley (2007) and Coibion (2010) test sticky-price vs. sticky-information Phillips curves, finding the latter performs better (or less badly). Their technique is essentially regressions of inflation on measures of inflation expectations and, for Kiley unit labor costs, and for Coibon the output gap. While well executed, however, these findings are not necessarily bad news for our purposes: Which view can produce a textbook-size economically based general equilibrium model consistent with the standard view of monetary policy? Regression fits, with necessary identification assumptions to control for endogenous right-hand variables, are not necessarily good guides to an ingredient's contribution to policy analysis in general equilibrium models.

Dupor, Kitamura, and Tsuruga (2010) propose a model with both sticky prices and sticky information. The result is to add a backward-looking inflation term to a Phillips curve that otherwise resembles Mankiw and Reis'. Such backward looking terms help to fit the data. Alas, as with Mankiw and Reis, 'it is generally impossible to derive the closed form solution to the dual stickiness model, due to infinitely many lagged expectations terms."

B Algebra

B.1 Downweighted expectations

The IS is (29)

$$m\frac{dx_t}{dt} - (1-m)x_t = \sigma\left(i_t - \frac{dp_t}{dt}\right)$$

We use the Phillips curve (7) and its derivative (8)

$$\kappa x_t = \Phi(t) p_t.$$

$$\kappa \frac{dx_t}{dt} = -\alpha_t p_t + \Phi(t) \frac{dp_t}{dt}.$$

Eliminating x_t and dx/dt, the price response function solves the differential equation

$$-m\alpha_t p_t + m\Phi(t)\frac{dp_t}{dt} - (1-m)\Phi(t)p_t = \sigma\kappa\left(i_t - \frac{dp_t}{dt}\right)$$

$$[m\Phi(t) + \sigma\kappa]\frac{dp_t}{dt} - [(1-m)\Phi(t) + m\alpha_t]p_t = \sigma\kappa i_t.$$

For the exponential case,

$$\left(me^{-at} + \sigma\kappa\right)\frac{dp}{dt} - \left[1 - m(1-a)\right]e^{-at}p_t = \sigma\kappa i_t.$$

The initial condition linking inflation, price level and interest rate is

$$p_0 = \frac{m\pi_0 + \sigma\kappa(\pi_0 - i_0)}{1 - m(1-a)}$$

The integrating factor is

$$[me^{-at} + \sigma\kappa]^{\frac{1-m(1-a)}{ma}} = [me^{-at} + \sigma\kappa]^{1+\frac{1-m}{ma}}.$$

Check:

$$\frac{dp}{dt} - \frac{\left[1 - m(1-a)\right]e^{-at}}{(me^{-at} + \sigma\kappa)}p_t = \frac{\sigma\kappa}{(me^{-at} + \sigma\kappa)}i_t$$

$$[me^{-at}+\sigma\kappa]^{1+\frac{1-m}{ma}}\frac{dp}{dt} - [me^{-at}+\sigma\kappa]^{1+\frac{1-m}{ma}}\frac{\left[1 - m(1-a)\right]e^{-at}}{(me^{-at} + \sigma\kappa)}p_t = [me^{-at}+\sigma\kappa]^{1+\frac{1-m}{ma}}\frac{\sigma\kappa}{(me^{-at} + \sigma\kappa)}i_t$$

$$[me^{-at} + \sigma\kappa]^{1+\frac{1-m}{ma}}\frac{dp}{dt} - [me^{-at} + \sigma\kappa]^{\frac{1-m}{ma}}\left[1 - m(1-a)\right]e^{-at}p_t = [me^{-at} + \sigma\kappa]^{\frac{1-m}{ma}}\sigma\kappa i_t$$

$$d\left[(me^{-at} + \sigma\kappa)^{\frac{1-m(1-a)}{ma}}p_t\right] = [me^{-at} + \sigma\kappa]^{\frac{1-m}{ma}}\sigma\kappa i_t$$

The solution is

$$(me^{-at} + \sigma\kappa)^{\frac{1-m(1-a)}{ma}}p_t - (m + \sigma\kappa)^{\frac{1-m(1-a)}{ma}}p_0 = \sigma\kappa\int_{s=0}^{t}[me^{-aj} + \sigma\kappa]^{\frac{1-m}{ma}}i_s ds$$

$$p_t = \left(\frac{m + \sigma\kappa}{me^{-at} + \sigma\kappa} \right)^{\frac{1-m(1-a)}{am}} p_0 + \frac{\sigma\kappa}{me^{-at} + \sigma\kappa} \int_{s=0}^{t} \left(\frac{me^{-as} + \sigma\kappa}{me^{-at} + \sigma\kappa} \right)^{\frac{1-m}{am}} i_s ds.$$

Inflation follows

$$\pi_t = e^{-at} \left(\frac{m + \sigma\kappa}{me^{-at} + \sigma\kappa} \right)^{\frac{1-m}{am}+2} \frac{m\pi_0 + \sigma\kappa(\pi_0 - i_0)}{m + \sigma\kappa} +$$

$$+[1 - m(1-a)] \frac{\sigma\kappa e^{-at}}{(me^{-at} + \sigma\kappa)^2} \int_{s=0}^{t} \left(\frac{me^{-as} + \sigma\kappa}{me^{-at} + \sigma\kappa} \right)^{\frac{1-m}{am}} i_s ds$$

$$+ \frac{\sigma\kappa}{me^{-at} + \sigma\kappa} i_t.$$

B.2 Calvo Pricing Algebra

The response is

$$\frac{dx_t}{dt} = \sigma(i_t - \frac{dp_t}{dt})$$

$$\frac{d\pi_t}{dt} = r\pi_t - \kappa x_t$$

To solve analytically, eliminate output, yielding

$$\frac{d^2\pi_t}{dt^2} = r\frac{d\pi_t}{dt} - \kappa\frac{dx_t}{dt}$$

$$\frac{d^2\pi_t}{dt^2} - r\frac{d\pi_t}{dt} - \sigma\kappa\pi_t = -\sigma\kappa i_t$$

$$(D^2 - rD - \sigma\kappa)\pi_t = -\sigma\kappa i_t$$

$$(D + \lambda_1)(D - \lambda_2)\pi_t = -\sigma\kappa i_t$$

$$\lambda_1\lambda_2 = \sigma\kappa$$

$$\lambda_1 - \lambda_2 = -r$$

$$\lambda_1 - \frac{\sigma\kappa}{\lambda_1} = -r$$

$$\lambda_1^2 + r\lambda_1 - \sigma\kappa = 0$$

$$\lambda_1 = \frac{-r + \sqrt{r^2 + 4\sigma\kappa}}{2} > 0$$

$$\lambda_2 = \frac{r + \sqrt{r^2 + 4\sigma\kappa}}{2} > 0.$$

Now

$$\pi_t = -\frac{1}{(D+\lambda_1)(D-\lambda_2)}\sigma\kappa i_t$$

$$\pi_t = \frac{1}{\lambda_1+\lambda_2}\left[\frac{1}{(D+\lambda_1)} - \frac{1}{(D-\lambda_2)}\right]\sigma\kappa i_t.$$

To solve each piece, note

$$(D+\lambda_1)x_t = y_t \rightarrow x_t = x_0 e^{-\lambda_1 t} + \int_{s=0}^{t} e^{-\lambda_1(t-s)} y_s ds$$

$$(D-\lambda_2)x_t = y_t \rightarrow x_t = -\int_{s=t}^{\infty} e^{-\lambda_2(s-t)} y_s ds.$$

Thus

$$\pi_t = \frac{\sigma\kappa}{\lambda_1+\lambda_2}\left[Ce^{-\lambda_1 t} + \int_{s=0}^{t} e^{-\lambda_1(t-s)} i_s ds + \int_{s=t}^{\infty} e^{-\lambda_2(s-t)} i_s ds\right]$$

$$\pi_t = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}\left[Ce^{-\lambda_1 t} + \int_{s=0}^{t} e^{-\lambda_1(t-s)} i_s ds + \int_{s=t}^{\infty} e^{-\lambda_2(s-t)} i_s ds\right]$$

$$\pi_t = \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left[Ce^{-\lambda_1 t} + \int_{s=0}^{t} e^{-\lambda_1(t-s)} i_s ds + \int_{s=t}^{\infty} e^{-\lambda_2(s-t)} i_s ds\right]$$

We can relate the constant C to initial inflation π_0 with

$$\pi_0 = \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}C + \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\int_{s=0}^{\infty} e^{-\lambda_2 s} i_s ds$$

An analytic solution for AR(1) interest rate is

$$\pi_t = \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left[Ce^{-\lambda_1 t} + \int_{s=0}^{t} e^{-\lambda_1(t-s)} i_s ds + \int_{s=t}^{\infty} e^{-\lambda_2(s-t)} i_s ds\right]$$

$$\pi_t = \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left[Ce^{-\lambda_1 t} + \int_{s=0}^{t} e^{-\lambda_1(t-s)} i_0 e^{-\eta_i s} ds + \int_{s=t}^{\infty} e^{-\lambda_2(s-t)} i_0 e^{-\eta_i s} ds\right]$$

$$\pi_t = \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left[Ce^{-\lambda_1 t} + i_0 e^{-\lambda_1 t}\int_{s=0}^{t} e^{-(\eta_i-\lambda_1)s} ds + i_0 e^{\lambda_2 t}\int_{s=t}^{\infty} e^{-(\lambda_2+\eta_i)s} ds\right]$$

$$\pi_t = \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left[Ce^{-\lambda_1 t} + i_0 e^{-\lambda_1 t}\frac{1-e^{-(\eta_i-\lambda_1)t}}{\eta_i-\lambda_1} + i_0 e^{\lambda_2 t}\frac{e^{-(\lambda_2+\eta_i)t}}{\lambda_2+\eta_i}\right]$$

$$\pi_t = \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left[Ce^{-\lambda_1 t} + i_0\frac{e^{-\lambda_1 t}-e^{-\eta_i t}}{\eta_i-\lambda_1} + i_0\frac{e^{-\eta_i t}}{\lambda_2+\eta_i}\right]$$

$$\pi_t = \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left[\left(C + i_0\frac{1}{\eta_i-\lambda_1}\right)e^{-\lambda_1 t} + i_0\left(\frac{-1}{\eta_i-\lambda_1} + i_0\frac{1}{\eta_i+\lambda_2}\right)e^{-\eta_i t}\right]$$

$$\pi_t = \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left[\left(C + i_0\frac{1}{\eta_i-\lambda_1}\right)e^{-\lambda_1 t} + i_0\frac{\lambda_1-\lambda_2}{(\eta_i-\lambda_1)(\eta_i+\lambda_2)}e^{-\eta_i t}\right].$$

Thus inflation is an AR(1) with coeffiicient η_i for one particular value

$$C = -i_0\frac{1}{\eta_i-\lambda_1}.$$

If inflation is an AR(1) with coefficient η_i and so is the interest rate, then $u_t = i_t - \phi\pi_t$ is also such an AR(1). Thus the AR(1) restriction picks this one particular value of C.

We can express the answer in terms of initial inflation by

$$\pi_0 = \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left[C + i_0\frac{1}{\lambda_2+\eta_i}\right]$$

$$\frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}C = \pi_0 - \frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}i_0\frac{1}{\lambda_2+\eta_i}$$

$$\pi_t = \pi_0 e^{-\lambda_1 t} + \frac{i_0}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}\left[\frac{e^{-\lambda_1 t}-e^{-\eta_i t}}{\eta_i-\lambda_1} - \frac{e^{-\lambda_1 t}-e^{-\eta_i t}}{\lambda_2+\eta_i}\right]$$

$$\pi_t = \pi_0 e^{-\lambda_1 t} + \frac{i_0}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}(e^{-\lambda_1 t}-e^{-\eta_i t})\frac{\lambda_2+\lambda_1}{(\eta_i-\lambda_1)(\eta_i+\lambda_2)}$$

$$\pi_t = \pi_0 e^{-\lambda_1 t} + i_0\frac{\lambda_1\lambda_2}{(\eta_i-\lambda_1)(\eta_i+\lambda_2)}(e^{-\lambda_1 t}-e^{-\eta_i t})$$

We can find output from

$$x_t = \frac{1}{\kappa}\left(\rho\pi_t - \frac{d\pi}{dt}\right)$$

$$\frac{d\pi_t}{dt} = -\lambda_1\pi_0 e^{-\lambda_1 t} + i_0\frac{\lambda_1\lambda_2}{(\eta_i-\lambda_1)(\eta_i+\lambda_2)}(-\lambda_1 e^{-\lambda_1 t} + \eta_i e^{-\eta_i t})$$

$$\rho\pi_t - \frac{d\pi}{dt} = [\rho+\lambda_1]\pi_0 e^{-\lambda_1 t} + i_0\frac{\lambda_1\lambda_2}{(\eta_i-\lambda_1)(\eta_i+\lambda_2)}[\rho(e^{-\lambda_1 t}-e^{-\eta_i t}) - (-\lambda_1 e^{-\lambda_1 t} + \eta_i e^{-\eta_i t})]$$

$$\rho\pi_t - \frac{d\pi}{dt} = [\rho+\lambda_1]\pi_0 e^{-\lambda_1 t} + i_0\frac{\lambda_1\lambda_2}{(\eta_i-\lambda_1)(\eta_i+\lambda_2)}[(\rho+\lambda_1)e^{-\lambda_1 t} - (\rho+\eta_i)e^{-\eta_i t}]$$

To find the zero surplus solution

$$0 = \int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt.$$

$$\pi_t = \pi_0 e^{-\lambda_1 t} + i_0 \frac{\lambda_1\lambda_2}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)}(e^{-\lambda_1 t} - e^{-\eta_i t})$$

$$\int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt = -\pi_0 \frac{1}{r+\lambda_1} + i_0 \int_{t=0}^{\infty} e^{-rt}\left[e^{-\eta_i}t - \frac{\lambda_1\lambda_2}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)}(e^{-\lambda_1 t} - e^{-\eta_i t})\right]dt$$

$$\int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt = -\pi_0 \frac{1}{r+\lambda_1} + i_0 \int_{t=0}^{\infty} e^{-rt}\left[\left(1 + \frac{\lambda_1\lambda_2}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)}\right)e^{-\eta_i t} - \frac{\lambda_1\lambda_2}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)}e^{-\lambda_1 t}\right]dt$$

$$\int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt = -\pi_0 \frac{1}{r+\lambda_1} + i_0 \frac{1}{(r+\eta_i)}\left(1 + \frac{\lambda_1\lambda_2}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)}\right) - \frac{1}{(r+\lambda_1)}\frac{\lambda_1\lambda_2}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)}$$

$$\int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt = -\pi_0 \frac{1}{r+\lambda_1} + i_0 \frac{\eta_i}{(r+\eta_i)}\frac{\eta_i + (\lambda_2 - \lambda_1)}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)} - i_0 \frac{1}{(r+\lambda_1)}\frac{\lambda_1\lambda_2}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)}$$

$$\int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt = -\pi_0 \frac{1}{r+\lambda_1} + i_0 \frac{1}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)}\left[\frac{\eta_i(\eta_i + (\lambda_2 - \lambda_1))}{(r+\eta_i)} - \frac{\lambda_1\lambda_2}{(r+\lambda_1)}\right]$$

so the zero surplus solution is

$$\pi_0 \frac{1}{r+\lambda_1} = i_0 \frac{1}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)}\left[\frac{\eta_i(\eta_i + (\lambda_2 - \lambda_1))}{(r+\eta_i)} - \frac{\lambda_1\lambda_2}{(r+\lambda_1)}\right]$$

$$\pi_0 = i_0 \frac{1}{(\eta_i - \lambda_1)(\eta_i + \lambda_2)}\left[\frac{(r+\lambda_1)\eta_i(\eta_i + (\lambda_2 - \lambda_1))}{(r+\eta_i)} - \lambda_1\lambda_2\right].$$

Alternatively, we can set up the matrix solution method with AR(1) interest rate

$$\frac{d}{dt}\begin{bmatrix} x_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} 0 & -\sigma & \sigma \\ -\kappa & r & 0 \\ 0 & 0 & -\eta \end{bmatrix}\begin{bmatrix} x_t \\ \pi_t \\ i_t \end{bmatrix}$$

Eigenvalues

$$(-\eta - \lambda)[-\lambda(r - \lambda) - \sigma\kappa)] = 0$$

$$(-\eta - \lambda)[\lambda^2 - r\lambda - \sigma\kappa)] = 0$$

$\lambda = -\eta$ and

$$\lambda = \frac{r \pm \sqrt{r^2 + 4\sigma\kappa}}{2}$$

B.3 Taylor rule disturbances

To specify the conventional Taylor rule with an AR(1) shock, substitute

$$i_t = \phi\pi_t + u_t$$

$$\frac{du_t}{dt} = -\eta u_t$$

so the response function solves

$$\frac{dx_t}{dt} = \sigma[(\phi - 1)\pi_t + u_t]$$

$$\frac{d\pi_t}{dt} = r\pi_t - \kappa x_t$$

To solve analytically, eliminate output,

$$\frac{d^2\pi_t}{dt^2} = r\frac{d\pi_t}{dt} - \kappa\frac{dx_t}{dt}$$

$$\frac{d^2\pi_t}{dt^2} - r\frac{d\pi_t}{dt} + \sigma\kappa(\phi - 1)\pi_t = -\sigma\kappa u_t$$

$$[D^2 - rD + \sigma\kappa(\phi - 1)]\pi_t = -\sigma\kappa u_t$$

$$(D - \lambda_1)(D - \lambda_2)\pi_t = -\sigma\kappa u_t$$

$$\lambda_1\lambda_2 = \sigma\kappa(\phi - 1)$$

$$\lambda_1 + \lambda_2 = r$$

$$\lambda_1 + \frac{\sigma\kappa(\phi - 1)}{\lambda_1} = r$$

$$\lambda_1^2 - r\lambda_1 + \sigma\kappa(\phi - 1) = 0$$

$$\lambda_1 = \frac{r + \sqrt{r^2 - 4\sigma\kappa(\phi - 1)}}{2}$$

$$\lambda_2 = \frac{r - \sqrt{r^2 - 4\sigma\kappa(\phi - 1)}}{2}$$

$$\pi_t = -\frac{1}{(D - \lambda_1)(D - \lambda_2)}\sigma\kappa u_t$$

$$\pi_t = \frac{1}{\lambda_2 - \lambda_1}\left[\frac{1}{(D - \lambda_1)} - \frac{1}{(D - \lambda_2)}\right]\sigma\kappa u_t$$

$$(D - \lambda_2)x_t = y_t \rightarrow x_t = -\int_{s=t}^{\infty} e^{-\lambda_2(s-t)}y_s ds$$

$$\pi_t = \frac{\sigma\kappa}{\lambda_1 - \lambda_2}\left[\int_{s=t}^{\infty}\left[e^{-\lambda_1(s-t)} - e^{-\lambda_2(s-t)}\right]u_s ds\right]$$

For an AR(1)

$$u_s = u_0 e^{-\eta s}$$

we have

$$\pi_t = u_t\frac{\sigma\kappa}{\lambda_1 - \lambda_2}\left[\int_{s=t}^{\infty}\left[e^{-\lambda_1(s-t)} - e^{-\lambda_2(s-t)}\right]e^{-\eta(s-t)}ds\right]$$

$$\pi_t = u_t\frac{\sigma\kappa}{\lambda_1 - \lambda_2}\left[\frac{1}{\lambda_1+\eta} - \frac{1}{\lambda_2+\eta}\right]$$

$$\pi_t = -\frac{\sigma\kappa}{(\lambda_1+\eta)(\lambda_2+\eta)}u_t$$

$$\pi_t = -\frac{\sigma\kappa}{(\lambda_1\lambda_2 + \eta(\lambda_1+\lambda_2)) + \eta^2)}u_t$$

$$\pi_t = -\frac{\sigma\kappa}{\sigma\kappa(\phi-1) + \eta r + \eta^2}u_t$$

$$\pi_t = -\frac{\sigma\kappa}{\sigma\kappa(\phi-1) + \eta(r+\eta)}u_t$$

The interest rate follows

$$i_t = \left[1 - \frac{\sigma\kappa\phi}{\sigma\kappa(\phi-1) + \eta r + \eta^2}\right]u_t$$

$$i_t = \left[\frac{\sigma\kappa(\phi-1) + \eta r + \eta^2 - \sigma\kappa\phi}{\sigma\kappa(\phi-1) + \eta r + \eta^2}\right]u_t$$

$$i_t = \left[\frac{-\sigma\kappa + \eta r + \eta^2}{\sigma\kappa(\phi-1) + \eta r + \eta^2}\right]u_t$$

$$i_t = \left[\frac{\eta(r+\eta) - \sigma\kappa}{\eta(r+\eta) + \sigma\kappa(\phi-1)}\right]u_t$$

We can relate the interest rate to inflation by

$$\pi_t = -\frac{\sigma\kappa}{\eta(r+\eta) + \sigma\kappa(\phi-1)}u_t$$

$$i_t = \left[\frac{\eta(r+\eta) - \sigma\kappa}{\eta(r+\eta) + \sigma\kappa(\phi-1)}\right]u_t$$

$$i_t = \left[\frac{\sigma\kappa - \eta(r+\eta)}{\sigma\kappa}\right]\pi_t$$

$$i_t = \left[1 - \frac{\eta(r+\eta)}{\sigma\kappa}\right]\pi_t \tag{33}$$

For typical parameter values, $\sigma \approx 1$, $\kappa \approx 1$ and small r and η, the interest rate is of the same sign as inflation. The Fisherian force is hard to avoid. It takes small σ and κ and large η and r for higher interest rates to exist with lower inflation. I choose $\sigma\kappa = 1/2\eta^2$ to force inflation to be about the negative of the interest rate. This forces small values of σ and κ.

Inflation is always the opposite sign of the monetary policy shock. The interest rate can go either way, depending on parameters. To get inflation to go in the opposite direction of the interest rate, we need

$$\eta(r+\eta) > \sigma\kappa$$

Large η, larger r and small $\sigma\kappa$ help.

A fun case happens if

$$\eta(r+\eta) = \sigma\kappa$$

Then inflation moves, with no movement at all in the interest rate. I call it an "open mouth operation." This is a good case to remind ourselves that equilibrium selection, not interest rates, are behind inflation in this model.

To find output

$$\frac{d\pi_t}{dt} = r\pi_t - \kappa x_t$$

$$\kappa x_t = \frac{\sigma\kappa}{\sigma\kappa(\phi-1)+\eta r+\eta^2}[-ru_t + \frac{du_t}{dt}]$$

$$\kappa x_t = -\frac{\sigma\kappa}{\sigma\kappa(\phi-1)+\eta r+\eta^2}[r+\eta]u_t$$

$$\pi_t = -\frac{\sigma\kappa}{\sigma\kappa(\phi-1)+\eta r+\eta^2}u_t$$

$$x_t = \frac{r+\eta}{\kappa}\pi_t$$

The "passive" fiscal policy behind this solution is

$$s_0 = (r+\eta_s)\int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt.$$

Using (33),

$$i_t = \left[1 - \frac{\eta(r+\eta)}{\sigma\kappa}\right]\pi_t$$

$$s_0 = (r+\eta_s)\int_{t=0}^{\infty} e^{-rt}\left[-\frac{\eta(r+\eta)}{\sigma\kappa}\right]\pi_t dt.$$

Using

$$\pi_t = -\frac{\sigma\kappa}{\eta(r+\eta)+\sigma\kappa(\phi-1)}u_t$$

$$s_0 = (r+\eta_s)\left[\frac{\eta(r+\eta)}{\sigma\kappa}\right]\frac{\sigma\kappa}{\eta(r+\eta)+\sigma\kappa(\phi-1)}\int_{t=0}^{\infty} e^{-rt}u_t dt$$

$$s_0 = (r+\eta_s)\frac{\eta(r+\eta)}{\eta(r+\eta)+\sigma\kappa(\phi-1)}\int_{t=0}^{\infty} e^{-rt}u_0 e^{-\eta t}dt$$

$$s_0 = (r+\eta_s)\frac{\eta(r+\eta)}{\eta(r+\eta)+\sigma\kappa(\phi-1)}\frac{1}{(r+\eta)}u_0$$

$$s_0 = \frac{\eta(r+\eta_s)}{\eta(r+\eta)+\sigma\kappa(\phi-1)}u_0$$

we can also write

$$\pi_0 = -\frac{\sigma\kappa}{\eta(r+\eta)+\sigma\kappa(\phi-1)}u_0$$

so

$$s_0 = -\frac{\eta(r+\eta_s)}{\sigma\kappa}\pi_0$$

The classic matrix solution of this model proceeds from :

$$\frac{d}{dt}\begin{bmatrix} x_t \\ \pi_t \\ u_t \end{bmatrix} = \begin{bmatrix} 0 & \sigma(\phi-1) & \sigma \\ -\kappa & r & 0 \\ 0 & 0 & -\eta \end{bmatrix}\begin{bmatrix} x_t \\ \pi_t \\ u_t \end{bmatrix}$$

B.4 Matrix solution in continuous time

The general Blanchard-Kahn matrix solution method for continuous time is

$$\frac{dx}{dt} = Ax_t = Q\Lambda Q^{-1}x_t$$

$$\frac{dQ^{-1}x}{dt} = \Lambda Q^{-1}x_t$$

$$\frac{dz}{dt} = \Lambda z_t.$$

To avoid explosive solutions, we set to zero the z_t corresponding to positive eigenvalues. This is a restriction on the initial conditions x_0, and implies the same restriction on subsequent x_t.

Partitioning z_t, the blocks of Λ, the columns of Q and the rows of Q^{-1} into those corresponding to positive and non-positive eigenvalues,

$$z_0^+ = Q_+^{-1} x_0 = 0.$$

This is a linear restriction, which lets us solve for some x_0 (output) in terms of others (inflation). Partitioning x_0 into the known and unknown values and likewise the columns of Q_-^{-1},

$$Q_{+,u}^{-1} x_{0,u} + Q_{+,k}^{-1} x_{0,k} = 0$$

$$x_{0,u} = -[Q_{+,u}^{-1}]^{-1} Q_{+,k}^{-1} x_{0,k}$$

Reassembling x_0,

$$z_0^- = Q_-^{-1} x_0$$

$$z_t^- = e^{\Lambda_- t} z_0^-$$

and

$$x_t = Q_- z_t^- = Q_- e^{\Lambda_- t} Q_+^{-1} x_0$$

is our solution.

B.5 Calvo pricing with autoregressive habits

Define

$$dX_t = b(x_t - X_t)dt$$

The first order condition is

$$E_t dx_t - \theta b(x_t - X_t)dt = \sigma[i_t dt - E_t(dp_t)]$$

The impulse response is

$$\frac{dX}{dt} = bx_t - bX_t$$

$$\frac{dx_t}{dt} - \theta b(x_t - X_t) = \sigma(i_t - \pi_t)$$

$$\frac{d\pi_t}{dt} = r\pi_t - \kappa x_t$$

Write it in matrix form

$$\frac{d}{dt}\begin{bmatrix} X_t \\ x_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} -b & b & 0 & 0 \\ -\theta b & \theta b & -\sigma & \sigma \\ 0 & -\kappa & r & 0 \\ 0 & 0 & 0 & -\eta_i \end{bmatrix}\begin{bmatrix} X_t \\ x_t \\ \pi_t \\ i_t \end{bmatrix}$$

There is one forward-looking (positive real part) eigenvalue and two jump variables, π and x. I specify π_0 and use the explosive root to find x_0.