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RE-INTERPRETING THE FAILURE OF FOREIGN EXCHANGE MARKET EFFICIENCY TESTS: SMALL TRANSACTION COSTS, BIG HYSTERESIS BANDS

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ABSTRACT

Small transaction costs and uncertainty imply that optimal cross-currency interest rate speculation is marked by a first-order hysteresis band. Consequently uncovered interest parity does not hold and market efficiency tests based on it are misspecified. Indeed measured prediction errors are a combination of true prediction errors and a wedge that consists of the "option value" of being in foreign currency and either plus or minus the transaction cost. Due to the nature of this wedge, we should expect measured prediction errors to be serially correlated, correlated with the current forward rate and perhaps have a non-sero mean, if the interest differential itself is serially correlated. The existence of the wedge helps account both for the failure of market efficiency tests and the difficulties in finding an empirically successful model of the risk premium.

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If the foreign exchange market is efficient, traders risk neutral and transactions costs negligible, the difference between the forward rate and the realised future spot rate should be white noise. Empirical research has resoundingly rejected this joint hypothesis. Hansen and Hodrick (1980), Meese and Singleton (1980), Levich (1980), Frenkel (1981), Cumby and Obstfeld (1981), Hsieh (1984) and Frankel and Froot (1987), inter alia, find that prediction errors sometimes have a non-sero mean, are serially correlated and correlated with lagged variables including the forward rate itself. Meese and Rogoff (1983) find that the current spot and forward rate are equally good predictors of the future spot. Hodrick (1987) and Levich (1985) provide excellent analytic surveys of this vast literature.

Hodrick (1987) states that there are three interpretations of this evidence. The first claims that the data are not ergodic (due to regime changes) and attributes the failures to severe small sample bias. The second views the failures as evidence of foreign exchange market inefficiency (e.g., Bilson 1981, Krugman 1989). The third views the failures as evidence of a risk premium in the foreign exchange market. Roll and Solnik (1977), Meese and Singleton (1980), Frankel (1982, 1986), Hansen and Hodrick (1983), Fama (1984), Hodrick and Srivastava (1984), Rogoff (1984), Domowits and Hakkio (1985), Giovannini and Jorion (1987), Campell and Clarida (1987) and Lewis (1988) consider various models of the risk premium, however, a theoretical model of the risk premium that performs well empirically has proved elusive.

This paper contributes a fourth (not mutually exclusive) interpretation to this list. We argue that very small transaction costs can help account for the failure of these market efficiency tests, and may help explain why an empirical model of the risk premium has proved elusive. Foreign exchange market efficiency tests assume that uncovered arbitrage equalises expected rates of return on all currencies. With this assumption, unobservable expectations can be deduced from observed interest differentials and spot rates. An entirely unrelated line of research shows that sunk entry and/or exit costs create a range of inactivity in firms' dynamic entry-exit strategies, and that this band permits hysteresis.¹ Dixit (1989c) provides an analytic approximation which shows that even third-order small sunk costs produces a first-order hysteresis band. Since transaction costs are sunk costs, cross-currency interest rate arbitrage should be marked by a first-order hysteresis band even if trading costs are only third-order small. In other words, even third-order small transaction costs may prevent arbitrage from equalizing expected rates of return and so invalidate uncovered interest parity. The importance of this observation is that the prediction errors used in market efficiency tests are actually a combination of true prediction errors and a first—order wedge that consists of the "option value" of being in foreign currency and either plus or minus the transaction cost. As it turns out the wedge is positive for low interest rate differentials and negative for high differentials.

This suggest an alternative interpretation of market efficiency tests. Namely, the failure of measured prediction errors to be white noise may be due to the wedge. Even if the true prediction errors were white noise, if the interest differential itself is serially correlated, the measured prediction errors will be serially correlated, correlated with the forward rate and other lagged variables, and may have a non-sero mean due to the nature of the wedge. We also show that white noise tests should fail more strongly during prolonged appreciations and depreciations of the exchange rate. Furthermore, since empirical models of the risk premium use measured prediction errors as data, their lack of success may be due in part to the presence of the wedge. More constructively, any attempt to identify a risk premium must first separate out the wedge.

It has long been known that transaction costs lead to a band within which deviations from covered interest rate parity may persist because they cannot be profitably exploited. The literature on transaction cost has tended to focus on these deviations as a direct measure of the cost of exchange rate uncertainty. Levich (1979), Frenkel and Levich (1975, 1977) and McCormick (1979) find these costs vary considerably across currencies and have increased under floating rates. Fieleke (1975) finds that they are positively related to uncertainty.

Aiyagari and Gertler (1989) and Constantinides (1986) consider the impact of differential transaction costs on capital market equilibrium in a closed economy. These models require numerical solutions and focus on the liquidity premium when trading stocks involves a transaction cost while trading bonds does not. Constantinides (1986) finds that transaction costs have only a second—order effect on equity prices. Aiyagari and Gertler (1989) include uninsurable income risk which forces a trading volume large enough to make transaction costs a significant determinant of equity prices. We focus instead on the hysteresis band and its implications for foreign exchange market efficiency tests. Note that unlike market efficiency tests in international economics, differences between expected rates of return in domestic markets are not interpreted as tests of market efficiency.

II. An Illustrative Example

That tiny transaction costs may account for the failure of market efficiency tests is implausible at first glance. To address such priors, this section studies an illustrative example. The example employs the regulated Brownian motion approach. Although this approach is not appropriate for the principle purpose of this paper, its renown and analytic convenience makes it well-suited to addressing priors. In particular we wish to demonstrate that even minuscule transaction costs together with uncertainty significantly alter the basis of market efficiency tests. Consider a risk neutral trader choosing between holding his wealth in dollar denominated certificates of deposits (CDs) and pound denominated CDs. The return on dollar CDs, i, is constant. The trader's expectation of the dollar rate of return on sterling, R⁺, obeys:

$$(2.1) dR^*/R^* = \mu dt + \sigma ds,$$

where ds is the increment of a Wiener process. A small transaction cost, κ , is incurred in moving between dollar CDs and sterling CDs. The problem is to find the arbitrage strategy that maximises discounted expected cash flow measured in dollars.

This problem can be reduced to one previously solved. Consider an equivalent problem: the trader receives a flow of i whether he is in dollars or pounds; when in pounds he receives i plus R^* —i. Consider a third problem: He receives zero when in dollars and R^* —i when in pounds. The objective functions of the second and third problems different only by a constant, so they have identical optimal strategies. The third problem is isomorphic to Dixit (1989a). Assign i to Dixit's w, R^* to his P, and K to his k and l. Dixit (1989a) is well known, so the solution is relegated to the appendix.

2.1 How Wide is the Hysteresis Band ?

Analytic solutions for the hysteresis band are impossible with Brownian motion uncertainty due to non-integer powers in the value function. Numerical solutions are readily had. Dixit (1989a,b), Bertola (1989), Constantinides (1986), Brennan and Schwarts (1985) and Bentolila and Bertola (1988) find the band is remarkably wide for reasonable parameters in applications ranging from hysteresis in trade to natural resource management. Dixit (1989c) provides an analytic approximation to the band which helps us understand why these bands are so wide. Dixit's results imply that for small κ :

(2.2)
$$\alpha^{i} - \alpha^{\circ} \cong 2\left[(3/4) \frac{\sigma^{2} 2\kappa}{i}\right]^{1/3}$$

where α^{1} is the R* at which a dollar holder switches to pounds and α^{0} is where a sterling holder

switches to dollars. The key feature of this relationship is the 1/3 power. Even if the value of the expression inside the brackets is only 0.000001 the band width will be two percentage points.

Suppose σ is 0.02 (i.e., there is a 95 percent chance that annual changes in the differential are less than plus or minus 4 percent), i is 5 percent and the transaction cost is 10 basis points. (Frenkel and Levich 1977 estimate these to be 50 basis points in the early 1970s. McCormick 1979 estimate them to be 9 to 18 basis points in 1976. Currently bid—ask spreads on spot transactions are about 5 to 10 basis points.) <u>The resulting band width is 5.7 percentage points</u>. Suppose the transaction costs are only one basis point, and σ is 0.02. The resulting band is still 2.7 percentage points wide. Finally, supposing σ is only 0.01 and κ is only one basis point the band is still 1.3 percentage points wide. The thrust of all this is clear — uncertainty and even minuscule transaction costs require a significant modification of optimal uncovered interest arbitrage, if indeed the dollar return on sterling is close to Brownian motion. Dixit (1989c) shows that the band is centered on i for small κ .

2.1.1 Band Width and the Measured Forward Rate Prediction Broor

If κ is zero, the trader sells pounds whenever \mathbb{R}^* is less than i and buys pounds when ever \mathbb{R}^* is greater than i. In a more complete model with many such agents, the price of pounds would adjust to maintain parity between expected rates of return at all instants when $\kappa=0$. If covered interest parity also holds, the difference between the forward and future spot exactly equals the unobservable expectation errors. If the market is efficient these errors should be white noise. However, if κ is not zero no arbitrage occurs when \mathbb{R}^* falls in the hysteresis band. Consequently, uncovered interest parity need not hold, so the forward-future spot gap does not equal pure expectations errors. Consequently measured prediction errors need not be white noise. A key question is how is the band width related to the difference between measured and true predication errors. Clearly, \mathbb{R}^* can differ from i by plus or minus $(\alpha^i - \alpha^o)/2$ without inducing arbitrage. Stepping outside the continuous time setup, suppose \mathbb{R}^* equals $i_t^* + s_{t+1}^e - s_t$, where s_{t+1}^e is the

expected future spot. The band implies that s_{t+1}^e need not equal $i-i_t^*+s_t^*$, rather it equals this plus a wedge which is bounded by plus or minus $(\alpha^i - \alpha^o)/2$. That is even with risk neutrality and efficient information processing, the expected difference between the forward and future spot might be a couple percentage points on an annual basis without inducing arbitrage.

2.1.2 Chartists as Fundamentalists

An informal check on the plausibility of the assertion that transaction costs are important is to examine real world trading strategies. The data necessary to do this formally are not easily had, however, the optimal trading strategy implied by the hysteresis band could be implemented with commonly used technical or chartists methods. One such strategy draws a line across the top of recent quotes and a line across the bottom of recent quotes. The trader buys sterling when R^* passes through the top line; he sells when it passes through the bottom line. The chart is redraw after major market movements. In our example the trader should draw lines a chart of R^* —i, not the spot rate chart as foreign exchange traders are wont to do. However if the spot is serially correlated and the interest differential relatively constant, the two are equivalent. Note that the band width is related to some measure of spot rate volatility. By contrast, arbitrage strategies which would be neccessary to enforce uncovered interest parity condition would not lead to an inactivity range in traders' charts.

2.2 Limitations of the Regulated Brownian Motion Approach

The virtues of the regulated Brownian motion approach are abundant and well-appreciated. However, it is of limited use in analysing the behavior of rational agents when the forcing variable is an endogenously determined price since such prices tend to revert to a steady-state value or trend. In the present situation it is plain that R^{*} cannot literally be a random walk with drift. Almost any sensible macro model would imply that it is mean-reverting, or at least trend-reverting. Furthermore, the market data and efficiencies tests are explicitly stated in a discrete time framework. Of course, if Brownian motion closely approximates the mean reverting process, the regulated Brownian motion solution may closely approximate the true solution. Krugman (1988), Miller and Weller (1988) and Svensson (1989) discuss how to stretch the regulated Brownian motion framework to address such problems.

Lucas and Prescott (1974), Caplin and Krishna (1986), Rafael (1989), Baldwin and Lyons (1989) and Baldwin (1989) take a different approach. Applying discrete time dynamic programming techniques, these papers show that the sunk cost hysteresis band exists when the forcing variable follows almost any process. Baldwin (1989) develops techniques that allow the characterisation of the hysteresis band for a general Markov process. In Baldwin and Lyons (1989) the number firms is determined by free entry. We shall see that the analytics for determining the spot rate are quite similar to those determining the number of firms in Baldwin and Lyons (1989). The next section applies this analysis to exchange rate dynamics.

III. Basic Model

Consider a world with two investments possibilities, one period dollar CDs and one period sterling CDs, populated by atomistic, risk neutral foreign exchange traders who face a small transaction cost, κ , of moving between dollars and sterling. The dollar return on dollar CDs is constant. The pound return on sterling CDs is stochastic. The log of the spot rate, s, is endogenously determined by the traders' behavior. The timing is as follows: all traders observe i^{*} at the beginning of every period and then trade simultaneously. We assume the interest differential follows the simple Markov process:

 $(3.1) \qquad i_t^*-i = \rho(i_{t-1}^*-i) + \epsilon_t, \text{ where } 0 \le \rho < 1, E\{\epsilon_t\} = 0, E\{\epsilon_t, \epsilon_j\} = 0, \text{ all } j, t \in \{0, 1\}$

P[·] is ϵ 's density function, G[·, i_{t-1}^{\bullet}] is i_t^{\bullet} 's conditional density and the real line is their support.

The state variables are i_t^* , s_{t-1} and A_t (A_t is a binary variable equal to one when the trader was in pounds last period and equal to zero otherwise). The first two are occasionally grouped together in the vector x_t for notational convenience. The control variable is U_t ($U_t = 1$ if

the investor chooses to be in pounds, $U_t = 0$ otherwise). The laws of motion are:

(3.2)
$$A_{t+1} = U_t, \quad i_{t+1}^* - i = \rho(i_t^* - i) + \epsilon_{t+1}, \quad s_t = S[i_t^*, s_{t-1}]$$

The typical trader chooses a trading strategy, $\mu_t = \mu[x_t, A_t]$, to maximize his discounted

cash flow:

(3.3)
$$V_{\mathbf{\varphi}}[\mathbf{x}_{0}] = \mathbb{E}\left\{\sum_{t=0}^{\infty} \delta^{t} \mathbf{g}\left[\mathbf{x}_{t}, \mathbf{A}_{t}, \boldsymbol{\mu}[\mathbf{x}_{t}]\right], \text{ where:} \\ \mathbf{g}\left[\mathbf{x}_{t}, \mathbf{A}_{t}, \boldsymbol{\mu}[\mathbf{x}_{t}]\right] = \left\{\begin{array}{l} \mathbf{i}_{t}^{*} + \mathbb{E}\left\{\mathbf{e}_{t+1} \mid \mathbf{x}_{t}\right\} - \mathbf{S}[\mathbf{x}_{t}] - (1 - \mathbf{A}_{t})\kappa, \text{ if } \boldsymbol{\mu}_{t} = 1 \\ \mathbf{i} - \mathbf{A}_{t}\kappa, & \text{ if } \boldsymbol{\mu}_{t} = 0 \end{array}\right\}$$

Here we have made use of the standard log approximation of the dollar rate of return on sterling, and δ is $(1+r)^{-1}$ where r is the constant discount rate (r $\geq i$). Expectations are over all ϵ .

Our two primary tasks are to characterise the optimal arbitrage strategy and the law of motion for the spot rate, $S[\cdot, \cdot]$. To this end we employ discrete time dynamic programming techniques. That is, we suppose the existence of a value function, $V_t = V[x_t, A_t]$, use it to characterise the optimal strategy and then use the optimal strategy to characterise the value function. Given the value function, we characterize the optimal trading strategy by characterising the much simpler problem of choosing U_t to maximise $g[x_t, A_t, U_t]$ plus $\delta E\{V_{t+1} | x_t, U_t\}$, where $E\{V_{t+1} | x_t, U_t\}$ is the expectation of V_{t+1} conditioned on x_t, U_t .

A holder of a dollar's worth of pounds has two options: move into dollars or stay in pounds. If he stays in pounds the expected value of his cash flow today will be:

$$\mathbf{i}_{t}^{*} + \mathbb{E}\{\mathbf{s}_{t+1} | \mathbf{x}_{t}\} - \mathbb{S}[\mathbf{i}_{t}^{*}, \mathbf{s}_{t-1}] + \delta \mathbb{E}\{\mathbb{V}_{t+1} | \mathbf{x}_{t}, 1\},\$$

If he moves out of sterling he pays κ so the expected value of his cash flow would be:

$$\mathbf{i} + \delta \mathbf{E} \{ \mathbf{V}_{t+1} | \mathbf{x}_{t}, \mathbf{0} \} - \kappa.$$

Likewise a dollar holder who stays in dollars is worth i plus $\delta E\{V_{t+1} | x_t, 0\}$; in pounds he is worth $i_t^* + E\{s_{t+1} | x_t\} - S[i_t^*, s_{t-1}]$ plus $\delta E\{V_{t+1} | x_t, 1\}$ less K. Since a trader may switch between

being a pound holder and a dollar holder, the value function is:

 $V_{t} = \operatorname{Max}\left[i_{t}^{*} + E\{s_{t+1} \mid x_{t}\} - S[x_{t}] + \delta E\{V_{t+1} \mid x_{t}, 1\} - (1 - A_{t})\kappa; i + \delta E\{V_{t+1} \mid x_{t}, 0\} - A_{t}\kappa\right].$ Plainly, a trader's current decision can be summarised by two numbers, α_{t}^{o} and α_{t}^{i} . If he was holding pounds and i_{t}^{*} turns out to be below α_{t}^{o} , he goes out of pounds; otherwise he stays put. If he was holding dollars and i_{t}^{*} turns out to be above α_{t}^{i} , he goes into pounds; otherwise he stays put. If he was holding dollars and i_{t}^{*} turns out to be above α_{t}^{i} , he goes into pounds; otherwise he stays put. α_{t}^{o} and α_{t}^{i} are the i_{t}^{*} 's at which he is indifferent between the alternatives (for clarity the elements of x_{t} are written out), that is:

$$(3.4) \qquad \alpha_{t}^{o} + \mathbb{E}\{\mathbf{s}_{t+1} | \alpha_{t}^{o}, \mathbf{s}_{t-1}\} - \mathbb{S}[\alpha_{t}^{o}, \mathbf{s}_{t-1}] + \delta \mathbb{E}\{\mathbb{V}_{t+1} | \alpha_{t}^{o}, \mathbf{s}_{t-1}, 1\} \\ = \mathbf{i} + \delta \mathbb{E}\{\mathbb{V}_{t+1} | \alpha_{t}^{o}, \mathbf{s}_{t-1}, 0\} - \kappa.$$

$$(3.5) \qquad \alpha_{t}^{i} + \mathbb{E}\{\mathbf{s}_{t+1} | \alpha_{t}^{i}, \mathbf{s}_{t-1}\} - \mathbb{S}[\alpha_{t}^{i}, \mathbf{s}_{t-1}] + \delta \mathbb{E}\{\mathbb{V}_{t+1} | \alpha_{t}^{i}, \mathbf{s}_{t-1}, 1\} - \kappa$$

$$= \mathbf{i} + \delta \mathbb{E}\{\mathbb{V}_{t+1} | \alpha_{t}^{i}, \mathbf{s}_{t-1}, 0\}.$$

(3.4) and (3.5) are the value-matching conditions. Note that the traders' indifference implies $S[\alpha_{t}^{i},s_{t-1}]$ and $S[\alpha_{t}^{o},s_{t-1}]$ both equal s_{t-1} .

3.2 Determination of the Spot Rate

The actual spot rate will be determined by the actions of traders. Traders' strategies and the α 's are depicted in Figure 3.1. The $\pi\pi$ curve plots the difference between the current value of being in sterling and dollars when the spot rate is unchanged, vis. $i_t^*-i+E\{s_{t+1} | x_t\}-s_{t-1}$ plus $\delta E\{V_{t+1} | x_t, 1\}-\delta E\{V_{t+1} | x_t, 0\}$. The two horisontal lines plot plus and minus κ . For i_t^* to the left of α_t^0 pound holders would like to sell pounds; to the right they would do nothing. For i_t^* to the right of α_t^i , dollar holders would like to buy pounds; to the left they would do nothing. With homogeneous traders there would be sero activity in the foreign exchange market for realisations of i_t^* between α_t^0 and α_t^i . This is the sunk cost hysteresis band. Consequently for such i_t^* , the dollar price of pounds posted at the close of the previous period would still be the posted price at the end of the current period. That is, for i_t^* in the hysteresis band, s_t equals s_{t-1} . This is one segment of the S[\cdot, \cdot] function. For i_t^* outside the hysteresis band, the market is entirely one-sided. Below α_t^o there would be no pound buyers, only pound sellers. To eliminate excess supply the price of pounds, s_t , must fall to the point where pound sellers are indifferent between dollars and pounds:

(3.6)
$$\mathbf{i}_{t}^{*} + \mathbf{E}\{\mathbf{s}_{t+1} | \mathbf{i}_{t}^{*}, \mathbf{s}_{t-1}\} - \mathbf{s}_{t} + \delta \mathbf{E}\{\mathbf{V}_{t+1} | \mathbf{i}_{t}^{*}, \mathbf{s}_{t-1}, 1\} = \mathbf{i} + \delta \mathbf{E}\{\mathbf{V} | \mathbf{i}_{t}^{*}, \mathbf{s}_{t-1}, 0\} - \kappa$$

Since all traders are aware of this, the posted spot would jump down immediately to this level upon announcement on i_t^* . Likewise, if i_t^* turned out greater than α_t^i , there will be only pound buyers so the spot jumps up to:

(3.7)
$$\mathbf{i}_{t}^{*} + \mathbb{E}\{\mathbf{s}_{t+1} | \mathbf{i}_{t}^{*}, \mathbf{s}_{t-1}\} - \mathbf{s}_{t} + \delta \mathbb{E}\{\mathbf{V}_{t+1} | \mathbf{i}_{t}^{*}, \mathbf{s}_{t-1}, 1\} - \kappa = \mathbf{i} + \delta \mathbb{E}\{\mathbf{V} | \mathbf{i}_{t}^{*}, \mathbf{s}_{t-1}, 0\},$$

Equations (3.6) and (3.7) implicitly define the rest of the $S[\cdot, \cdot]$ function. Formally:

(3.8)
$$S[i^{*}_{t}, s_{t-1}] = \begin{cases} i^{*}_{t} - i + E\{s_{t+1} \mid x_{t}\} + \delta \Psi[i^{*}_{t}, s_{t-1}] + \kappa, & i^{*}_{t} \leq \alpha^{o}_{t} \\ s_{t-1}, & \alpha^{o}_{t} < i^{*}_{t} < \alpha^{i}_{t} \\ i^{*}_{t} - i + E\{s_{t+1} \mid x_{t}\} + \delta \Psi[i^{*}_{t}, s_{t-1}] - \kappa, & i^{*}_{t} \geq \alpha^{i}_{t} \end{cases}$$

where we have defined $\delta E\{V_{t+1} | x_t, 1\}$ minus $\delta E\{V_{t+1} | x_t, 0\}$ as $\Psi[i_t^*, s_{t-1}]$. The implied relationship between s_t , i_t^* and s_{t-1} is shown in Figure 3.2 as the solid line SS. The Properties of the Law of Motion of s_t .

It is clear from Figure 3.2 that the current dollar price of pounds is non-decreasing in i_t^* . The dependence of s_t on s_{t-1} is slightly more complicated. Inside the hysteresis band, it is clear that s_t is increasing in s_{t-1} . However, outside the band s_{t-1} has no effect on s_t since outside the band s_t is determined by forward-looking behavior.

Of course the α 's and V's must be determined simultaneously by solving the value matching conditions and definition of V together with the laws of motion for s and i. Except for a number of simple distributions of ϵ , it is not possible to solve analytically for the α 's. This lack of an explicit solution is the price we pay for being able to consider general i* processes. Nevertheless such a solution is not really necessary since we can characterise the band position and width.

3.3 Conditional Expectation of the Future Spot

Figure 3.2 will be similar for t+1. We can therefore use it to calculate the conditional expectation of s_{t+1} . We integrate over SS weighted by the distribution of i_{t+1}^* conditional on i_t^* . Formally, the conditional expectation of s_{t+1} is:

$$(3.9) \quad - \omega^{\int_{-\infty}^{\omega} \int_{-\infty}^{\omega} \left[s - i + E\{s_{t+2} \mid s, S[i_{t}^{*}, s_{t-1}]\} + \delta \Psi[s, S[i_{t}^{*}, s_{t-1}]] + \kappa \right] dG[s, i_{t}^{*}] \\ + \alpha_{t+1}^{\circ} \int_{+\infty}^{\omega} \left[s - i + E\{s_{t+2} \mid s, S[i_{t}^{*}, s_{t-1}]\} + \delta \Psi[s, S[i_{t}^{*}, s_{t-1}]] - \kappa \right] dG[s, i_{t}^{*}].$$

The properties of the conditional expectation are easily established. Since s_{t+1} is non-decreasing in i_{t+1}^{\bullet} and i^{\bullet} displays positive persistence, $E\{s_{t+1} | i_t^{\bullet}, s_{t-1}\}$ is increasing in i_t^{\bullet} . That is a higher realisation of i_t^{\bullet} shifts the distribution of i_{t+1}^{\bullet} (conditioned in i_t^{\bullet}) to the right, shifting weight from low values of s_{t+1} to high values. Also the conditional expectation is non-decreasing in s_{t-1} . Specifically, for i_t^{\bullet} is less than α_t° , or greater than α_t^{i} , it is unaffected by s_{t-1} . This should be intuitively obvious since when forward-looking arbitrage is taking place lagged values of s are irrelevant. Also for i_t^{\bullet} in the band, $E\{s_{t+1} | i_t^{\bullet}, s_{t-1}\}$ is increasing in s_{t-1} .

3.4 Expected Value and Option Value of Being in Pounds

The actual value of a dollar holder's and a pound holder's wealth next period depend on the realisation of i_{t+1}^* . If $i_{t+1}^* < \alpha_{t+1}^i$ the dollar holder stays in dollars so the value of his wealth is $i+\delta E\{V_{t+2} | x_{t}, 0\}$, otherwise all he and all other dollar holders attempt to sell dollars forcing the spot up to the point where they are indifferent to moving. Thus whether he actually moves into pounds or not, the value of his cash flow is $i+\delta E\{V_{t+2} | x_t, 0\}$. This obviously equals (i/r). If $i_{t+1}^* < \alpha_{t+1}^\circ$ pound holders' desire to buy dollars would force the spot down to the point where they are indifferent between the two assets. In this region a dollar's worth of pounds is worth $i_{t+1}^* + E\{s_{t+2} | x_t\} - s_{t+1}^* + \delta E\{V_{t+2} | x_t, 1\}$ which is equal to $(i/r) - \kappa$. If $i_{t+1}^* > \alpha_{t+1}^i$ dollar holders' desires to buy sterling forces \mathbf{s}_t up to the point where $\mathbf{i}_{t+1}^* + \mathbf{E}\{\mathbf{s}_{t+2} | \mathbf{x}_t\} - \mathbf{s}_{t+1} + \delta \mathbf{E}\{\mathbf{V}_{t+2} | \mathbf{x}_t, 1\}$ equals $(\mathbf{i}/\mathbf{r}) + \kappa$. If \mathbf{i}_{t+1}^* turns out to be in the hysteresis band, the value of sterling increases with \mathbf{i}_t^* according to $\mathbf{i}_{t+1}^* + \mathbf{E}\{\mathbf{s}_{t+2} | \mathbf{x}_t\} - \mathbf{s}_{t+1} + \delta \mathbf{E}\{\mathbf{V}_{t+2} | \mathbf{x}_t, 1\}$. These relationships are plotted in Figure 3.3.

A concept that plays an important part in the analysis is the difference between the conditional expectation of V_{t+1} when U_t equals zero and one. We refer to this as the option value of being in sterling as opposed to dollars (or the incumbency premium for short). The function that relates Ψ_{t+1} to x_t , $\Psi[i_t^*, s_{t-1}]$, is defined as:

(3.10)
$$\begin{array}{c} & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & &$$

The properties of Ψ are also simple to establish. First Ψ is increasing in i_t^* . To see this note that the realized difference between being in pounds and dollars depends on what i_{t+1}^* turns out to be. By inspection of Figure 3.3, this difference in non-decreasing in i_{t+1}^* . Since the i^{*} process displays positive persistence, a higher i_t^* shifts the conditional distribution of i_{t+1}^* to the right, giving more weight to higher values of the difference. Obviously, then the conditional expectation of the difference is increasing in i_t^* . By similar reasoning, Ψ_{t+1} is bounded between $-\kappa$ and κ .

3.5 Characterizing the Width of the Hysteresis Band

Re-arranging the value matching conditions we have:

$$(3.11) \quad \alpha_{t}^{i} - \alpha_{t}^{o} = 2\kappa - (\mathbb{E}\{\mathbf{s}_{t+1} \mid \alpha_{t}^{i}, \mathbf{s}_{t-1}\} - \mathbb{E}\{\mathbf{s}_{t+1} \mid \alpha_{t}^{o}, \mathbf{s}_{t-1}\}) - \delta(\Psi[\alpha_{t}^{i}, \mathbf{s}_{t-1}] - \Psi[\alpha_{t}^{o}, \mathbf{s}_{t-1}]).$$

3.5.1 Persistence and Band Width

Consider two polar cases of the i_t^* process: perfect persistence and no persistence. When i_t^* is iid there is no persistence in i_t^* . In this case both the expected future spot and Ψ_{t+1} are

independent of i_t^* , so the band width is 2κ regardless of how volatile the iid process is. Basically when i_t^* is iid the current realisation of i_t^* tells the traders nothing about future returns to holding pounds, so a dollar holder will require the full κ to be covered by this period's interest differential before moving. Likewise, a pound holder would suffer a negative differential up to κ before moving. Thus greater uncertainty does not always widen the band. The other extreme is of perfect persistence. That is, any change in i_t^* is expected to persist forever (i.e., all future ℓ 's are sero, and $\rho = 1$). In this case the current i_t^* is a perfect signal about future i*'s. In this case the band is much narrower, namely, $r2\kappa$. Comparing the two polar cases we see that the band is wider with some uncertainty than with sero uncertainty, but increasing the amount of uncertainty does not always widen the band. A heuristic way of thinking about this is to examine the effect of more uncertainty on the signal quality of the current i_t^* .

3.5.2 Period Length and Annualized Interest Rate Differentials

The link between transaction costs and the band width measured in terms of annualised rates of return depends crucially on the period length since the band end points in (3.11) are in terms of interest rates quoted on the basis of the period length. To illustrate this point assume that i_t^* is iid and κ is small. In this case the band is exactly 2κ wide and approximately centered on i. Take κ equal to 0.0005 and i equal to a 5 percent annual rate and contrast the case of trading only once a year with the case of daily trading. In the first case arbitrage forces the expected dollar return on pounds to be between 0.0495 and 0.0505, so uncovered interest parity comes pretty close to holding. Consequently the misspecification of the market efficiency test is minimal. However with daily trading, arbitrage keeps the daily dollar return on pounds between 0.000637 and -0.000363 (5 percent on a daily basis is 0.000137). On an annualised basis the upper end point of the band is 25.44 percent and the lower end point is -12.12 percent. Consequently the measured prediction error may be an extremely poor estimate of the true prediction errors. The formula used to generate the end points on the period length basis is $1+\alpha^i = (1.05)^{1/N} + \kappa$ and $1+\alpha^o = (1.05)^{1/N} - \kappa$, where N is the number of periods per year. (We ignore the constant Ψ_{t+1} since κ is small. This implies the band is approximately centered on i) Expressing the α 's in terms of annual rates, we have $\alpha^i = ((1.05)^{1/N} + \kappa)^N - 1$ and $\alpha^o = ((1.05)^{1/N} - \kappa)^N - 1$. Using a log approximation, for large N and small κ we have : $\ln[\alpha^i] - \ln[\alpha^o] \cong N2\kappa$. This dovetails with Dixit's result that in continuous time the derivative of the band width with respect to κ is infinite at $\kappa = 0$. In the modern foreign exchange market a trade only takes a few seconds, so even a tiny band would translate to an enormous annualised interest rate differential, if i_t^* were indeed iid.

3.5.3 Maturity of CDs and the term structure of the hysteresis band

The hysteresis band described above applies to CDs of a one period maturity. In this subsection we show that it can be applied to CDs of longer maturities with little modification. If the English capital market is efficient (maintaining the assumption of risk neutral investors) sterling CDs of various maturities will be priced such that investors are indifferent to holding the various CDs during the next period. Thus whether regardless of a CD's maturity, the optimal cross-currency arbitrage can be deduced from the $\alpha_t^i - \alpha_t^o$ band described above. Of course, in deciding when to trade long CDs, it is not sufficient to look at the interest rates; expected price changes of the long CDs must also be included.

It is simple to restate the band in terms of the interest rates on sterling of any maturity. For instance suppose the period length is one day and traders have the choice between one day and two day sterling. We know that incipient cross-currency arbitrage will be triggered when the daily sterling rate of return is greater that α_t^i or less than $\alpha_t^{o,2}$. If the English capital market is efficient, the expected return on holding a two day CD for one day equals the return on a one day CDs. That is, $E\{(1+i_{2,t}^*)/(1+i_{t+1}^*)\}$ equals $(1+i_t^*)$, where $i_{2,t}^*$ is the two day sterling rate. Thus the high side of the band stated in terms of 2 day interest rates is defined implicitly by:

$$(1+\alpha_{t}^{i}) = \mathbb{E}\{\frac{1+\omega_{t}^{i}}{1+i\frac{s}{t+1}}\} = \mathbb{E}\{(1+\omega_{t}^{i})/(1+i+\rho(\alpha_{t}^{i}-i)+\epsilon_{t+1})\},\$$

where ω_t^i denotes the critical value in terms of 2 day interest rates. To get a handle on this expression, we take a log approximation:

$$(\omega_{t}^{i}-2i) = (\alpha_{t}^{i}-i)(1+\rho)-\gamma$$

where γ is the correction from Jensen's inequality and i is on a daily basis.

Two cases highlight the implications of this formula. On one hand, suppose ρ is near one and γ is negligible. In this case the high end of the band for two day sterling rates is twice as far from the 2 day dollar rate as α_t^i is from the one day dollar rate. In other words the band in terms of two day rates is roughly twice the width of the band in terms of one day rates. On the other hand, if ρ is zero (vis. the interest differential is iid) and γ is negligible, the band width is roughly invariant to maturities. Thus the band should be insignificant for long term interest rates. In this case, we also have a strong testable implication. White noise tests on measured prediction errors should fail much more strongly on short term interest rates than on long term interest rates.

3.5.4 Band Width and Unexploited Profit Opportunities

We just showed that the band in terms of long sterling rates can be more that 2κ wide, if the interest differential is close to a random walk. At first glance this seems impossible. That is, it would seem that if the expected dollar rate of return between dollar CDs and say one year sterling CDs differed by more than twice the transaction costs there would be unexploited profit opportunities. Dixit's seminal work on costly entry and exit under uncertainty explains why this is possible. The easiest way to explain this is to first lay out the incorrect argument that the rates of return cannot get more than 2κ out of line, and then directly show what this argument is leaving out. To be concrete, suppose the expected future spot equals the current spot and the one year sterling rate is 2κ plus epsilon (epsilon is an arbitrarily small positive number) above the dollar rate.

Here is the (incorrect) argument that this cannot be a equilibrium situation: An investor could borrow at the dollar rate and invest in a one year sterling and thereby expect to earn enough to more than cover the cost of the round trip in and out of sterling. Since a very large number of dollar holders would want to do this the spot rate is not an equilibrium. Now we argue that it would not be optimal for any investors to undertake the arbitrage just described.³ To be concrete, suppose the expected dollar rate of return differential (including both the interest and exchange rate change components) is close to a random walk; each period there is about 50 percent chance that the differential is x percent higher than last period and about 50 percent chance that it is x percent lower. If an investor borrows dollars and invests in one year sterling when the differential is 2K plus epsilon, his expected profit would be epsilon per dollar. Suppose instead he waited till next period. If the expected rate of return rises and then he buys into one year sterling, his expected profit would be x plus epsilon per dollar. If he waits and the expected return falls, he need not move into sterling so his expected profit is worth no less than sero. Clearly, if the discount factor is not too large then the expected value of waiting till next period exceeds the expected value of investing in sterling today. Consequently, no investor would move to take advantage of the 2κ plus epsilon differential. They would require more than 2κ plus epsilon in order to find the invest-now alternative more attractive than the wait-and-see alternative.

The basic fault with the incorrect argument is that it looks at the problem as if the investor's choice is between dollars today and sterling today. In fact the true problem is whether to move into pounds today, or wait and see whether to move tomorrow. As Dixit (1989) describes it, a dollar holder owns a dollar and a call option to buy into the randomly fluctuating rate of return differential. Due to transaction costs and uncertainty, the value of this call option is not sero. When he moves into pounds he gives up the option on pounds and gets a call option on dollars. Consequently, he will not move until the expected return to being in pounds exceeds the

expected return to being in dollars by at least 2κ plus the difference in the values of the options. As the maturity of the sterling CDs increases, the volatility of the daily rate of return on holding them increases (as long as i_t^* is not iid), so the value of the call option on them increases. Consequently, the investor will require a greater interest rate differential in order to find it optimal to exercise the call option.

3.5.5 Upper and Lower Bounds on the Band Width

To get a rough idea of the band width for one period CDs, we establish lower and upper bounds. Certainly the band is wider than r2 κ , since there is less than perfect persistence. Taking $\kappa = 0.0001$ and r = i = 0.05 on an annual basis and consider daily trading. We have that α_t^i -i equals 0.000005 on a daily basis. Annualised, this is 18.7 basis points. Thus a lower bound on the band width is approximately 3.7 tenth of one percentage point. For the upper bound we take the iid case, where half the band is equal to κ . Employing daily trade, the same i but taking $\kappa = 0.0005$, we get a band which is 37.56 percentage points wide on an annualised basis. Clearly these upper and lower limits are a long way from good estimates. The interest differential is quite volatile so the lower limit is too low, however, it displays significant persistence, so the upper bound case is too high.

3.5 Characterizing the Position of the Band

From the value matching conditions the band position depends upon s_{t-1} . Figure 3.4 illustrates this relationship. LL plots (3.6) for $i_t^* < \alpha_t^o$, and HH plots (3.7) for $i_t^* > \alpha_t^i$. Since the current spot does not depend on s_{t-1} when i_t^* falls outside the band, LL and HH are in exactly the same position in both the top panel (which depicts the period t situation) and the bottom panel (which depicts the period t situation) and the bottom panel (which depicts the period t+1 situation) of the diagram. If i_t^* turns out to be greater than α_t^i , say i^* , then s_t will be \bar{s}_t which is higher than s_{t-1} . This implies that in the bottom panel, the band will be shifted to the right as shown. Note that the band in t+1 is such that α_{t+1}^i equals \bar{i}^* . This should be obvious since the equation that determines s_t looks exactly like the value matching

condition in t+1. If ρ is near unity and ϵ has a single-peaked symmetric distribution then the likelihood that i_{t+1}^* will fall near the high side of the band is large. Basically i_t^* displays positive persistence and the band tends to follow it around.

4. A Number of Testable Implications

If the hysteresis band has a significant effect on the foreign exchange market it should be evident in the data. This section discusses several empirically testable propositions.

4.1 The Hysteresis Band and the Forward Rate Prediction Brrors

If uncovered and covered interest parity hold, the forward rate is an unbiased predictor of the future spot. The hysteresis band invalidates uncovered interest parity.

Proposition 1: The forward rate may not be an unbiased predictor of the future spot.

The spot is determined by $i_t^* + E\{s_{t+1} | x_t\} - s_t + \delta \Psi[i_t^*, s_{t-1}] = i - \kappa$ in the sell-pounds region of Figure 3.2. Clearly, in this region the expected value of the measured prediction error is:

(4.1)
$$\left(f_t - \mathbb{E} \{ \mathbf{s}_{t+1} | \mathbf{x}_t \} \right) = \delta \Psi [\mathbf{i}_t^*, \mathbf{s}_{t-1}] + \kappa > 0,$$

defining f_i to be $i_i - i_i^* + s_i$. Similar reasoning implies that in the buy-pounds region:

(4.2)
$$\left(f_t - E\{s_{t+1} | x_t\}\right) = \delta \Psi[i_t^*, s_{t-1}] - \kappa < 0$$

Lastly in the hysteresis band:

(4.3)
$$0 < \delta \Psi[\alpha_{t}^{i}, \mathbf{s}_{t-1}] + \kappa \ge \left(\mathbf{f}_{t} - \mathbb{E}\{\mathbf{s}_{t+1} | \mathbf{x}_{t}\} \right) \ge \delta \Psi[\alpha_{t}^{o}, \mathbf{s}_{t-1}] - \kappa < 0,$$

where $\mathbf{s}_t = \mathbf{s}_{t-1}$. Inspection of Figure 3.4 shows that $\Psi[i_t^*, \mathbf{s}_{t-1}]$ is increasing in i^* , is strictly less than κ and strictly greater than $-\kappa$. Plainly, then (4.3) implies that there is a unique realisation of i^* for which $(f_t - E\{\mathbf{s}_{t+1} | \mathbf{x}_t\}) = 0$.

In summary, equations (4.1)—(4.3) imply that the measured difference between the forward rate and the actual future spot should not be expected to be zero. Measured prediction errors are not pure expectation errors even when traders are risk neutral. They are expectation errors plus the incumbency premium and either κ or $-\kappa$. We continue to refer to

 $(f_t - E\{s_{t+1} | x_t\})$ as the measured predictions errors to suggest comparisons with existing empirical literature. Proposition 4 considers the magnitude of the wedge.

Proposition 2: The prediction errors should display positive serial correlation

Using equations (4.1)-(4.3) and the fact that $\Psi[i_{t}^*, s_{t-1}]$ is increasing in i_{t}^* , we plot $(f_t - E\{s_{t+1} | x_t\})$ as a function of i* in Figure 4.1. Clearly, the errors are positive for low i*, and negative for high i*. If i* itself is positively serially correlated, then the errors should be positively serially correlated as well.

<u>Proposition 3</u>: The prediction errors should be positively correlated with the current forward rate and negatively correlated with i^{*}

Inspection of Figure 4.1 indicates that $(f_t - E\{s_{t+1} | x_t\})$ decreases as i* increases. Since the forward rate moves negatively with the interest differential, $(f_t - E\{s_{t+1} | x_t\})$ moves positively with f_t .

<u>Proposition 4</u>: The absolute value of $(f_t - E\{s_{t+1} | x_t\})$ should be greatest at the ends of the hysteresis band. The difference between the largest $(f_t - E\{s_{t+1} | x_t\})$ and the smallest $(f_t - E\{s_{t+1} | x_t\})$ is greater than the width of the hysteresis band.

Consider Figure 4.2 which is similar to Figure 3.1. The locus $i^* + E\{s_{t+1} | x_t\} - s_t$, plotted as $\pi\pi$, is convex in i^* since $E\{s_{t+1} | x_t\}$ is increasing in i^* . Given (4.1) and (4.2), the difference between the maximum and minimum $(f_t - E\{s_{t+1} | x_t\})$ equals the distance between $\kappa - \delta \Psi[\alpha_t^i, s_{t-1}]$ and $-\kappa - \delta \Psi[\alpha_t^o, s_{t-1}]$. The dashed line is what $\pi\pi$ would look like if $E\{s_{t+1} | x_t\}$ were constant. It is obvious that the distance between $\kappa - \delta \Psi[\alpha_t^i, s_{t-1}]$ and $-\kappa - \delta \Psi[\alpha_t^o, s_{t-1}]$ is greater than the distance between $\kappa - \delta \Psi[\alpha_t^o, s_{t-1}]$ and X. This latter distance obviously equals $\alpha_t^i - \alpha_t^o$. Thus the maximum wedge is of the same order of magnitude as one half of the band width.

<u>Proposition 5:</u> Measured prediction errors should be especially large and positive during prolonged dollar appreciations, and especially large and negative during prolonged dollar

depreciations.

As we showed in Section 3, a realisation of i* outside the band shifts the nearest edge of the band to the realisation. Thus if by chance we observed that for a number of sequential periods the interest rate differential decreased, lowering the spot rate, we know that the band was shifting to the left. Thus we know that period after period the i* realisations are typically falling near the lower end of band where the wedge is largest. Similarly, sequential increases in the spot lead us to expect that the measure prediction errors will be quite negative.

4.2 The Size of the Bid-Ash Spread, the Degree of Persistence and Failures of White Noise Tests of the Forward Prediction Brrors

The hysteresis band widens as κ increases, and as the i^{*} process gets less persistent. Since Proposition 4 shows that the size of $(f_t - E\{s_{t+1} | x_t\})$ increases with the size of the hysteresis band, greater κ and a less persistent i^{*}_t should magnify the effects predicted in Propositions 1 through 3. Consequently we should expect Propositions 1 through 3 should look better, the larger are the bid—ask spreads on the spot rate, and the less persistent is i^{*}_t. This suggests that comparisons of tests on different time periods and different currencies would provide evidence on the importance of the transaction costs.

5. Summary and Concluding Remarks

It seems quite plausible that foreign exchange traders are risk averse. This paper assumes they are risk neutral and rational in order to apply the basic intuition of the irreversible investment literature to the foreign exchange market. We showed that very small transaction costs together with uncertainty imply that uncovered interest speculation should be marked by a non-negligible hysteresis band. Thus the most general point in this paper is that measured prediction errors include a wedge in addition to true prediction errors. We show that the empirically observed properties of measured prediction errors are consistent with the nature of the wedge. More generally, we show that even tiny transaction costs should not be ignored in models of exchange rate determination. Specifically, any empirical effort to identify a risk premium must somehow separate the wedge from the risk premium.

Additional Implications

Uncovered interest parity is one of the linchpins of modern exchange rate theory. Since small transaction cost require a modification of this condition, the analysis in this paper suggests a number of interesting extensions. Uncovered interest parity implies that fixed exchange rates require $i^* = i$ exactly at all times. We just showed that the presence of transaction costs equal to one hundredth of one percent implies that central banks might not face pressure on its reserves as long as the interest differential was less than something like 1 to 4 percentage points. Thus exchange rate variability together with tiny transaction costs would allow a moderate degree on monetary independence in a fixed exchange rate regime.

Additionally removing all uncertainty on cross interest arbitrage, by maintaining sero margins around parity, might lead to greatly increased arbitrage on the interest rate differential. This would argue that the as the European Monetary System attempts to reduce margins to zero it might encounter difficulties. Namely, unless EC interest rate are actually pegged to each other on a minute-to-minute basis substantial cross-border interest rate arbitrage could strain reserve heldings.

The hysteresis band also suggests an approach to calculating optimal margins for fixed exchange rate systems. Introducing margins around parity introduces uncertainry, and so would lead to non-negligible bands. The bands would permit the central bank to use the interest rate to stabilise some minor domestic shocks. This minor gain in stabilisation is traded off against the costs of slightly increased exchange rate uncertainty.

Standard exchange rate dynamics are based on uncovered interest parity. Since we showed that uncovered interest parity is not in general supported by cross-currency interest

arbitrage, the standard dynamics may be incorrect or incomplete.

Lastly, note that there is absolutely nothing international about the point. The bands would exists even for arbitrage between assets denominated in the same currency. This suggests that assets prices even in domestic market can get quite out of line without leading to equalising arbitrage. Unlike market efficiency test in international economics, differences between expected rates of return in domestic markets are not interpreted as tests of market efficiency. Moreover, it suggests that even a small Tobin tax on asset trade could allow rates of return on various assets to get fairly far out of line. This clearly has a deleterious effect on economic efficiency.

APPENDIX 1: Smooth Pasting and the Sunk Cost Model

Dixit (1989a) is well-known so we only briefly sketch the derivation of the optimal strategy. (See Dixit 1988 for an intuitive exposition of the smooth pasting approach.) The strategy is summarised by two time-invariant numbers, α^{0} and α^{i} . If \mathbb{R}^{*} is greater than α^{i} , he goes into pounds (if he was in dollars); if \mathbb{R}^{*} is less than α^{0} be goes out of pounds (if he was in pounds).

Four necessary conditions help us determine α^{o} and α^{i} and the value functions. Clearly, at the α 's the trader must be indifferent to acting, so we have the value-matching conditions:

(A1.1)
$$V^*[\alpha^i] - \kappa = V[\alpha^i], \text{ and } V^*[\alpha^o] = V[\alpha^o] - \kappa$$

Because the decision making is ceaseless, and R⁺ is Brownian motion, we get another set of necessary conditions, the so--called smooth--pasting conditions :

(A1.2)
$$V^{*}[\alpha^{\circ}] = V'[\alpha^{\circ}], \text{ and } V^{*}[\alpha^{i}] = V'[\alpha^{i}]$$

By Ito's lemma and option pricing techniques the value function V and V* obey:

(A1.3)
$$rV^{*}[R^{*}] = (R^{*}-i) + (\sigma^{2}/2)(R^{*})^{2}V^{*}[R^{*}] + \mu R^{*}V^{*}[R^{*}], \text{ and}$$

(A1.4)
$$rV[R^*] = (\sigma^2/2) (R^*)^2 V''[R^*] + \mu R^* V'[R^*]$$

The general solutions for these second order differential equations are:

(A1.5)
$$V^*[R^*] = Ae^{-\alpha} + R^*/(r-\mu) - i/r$$
, and $V[R^*] = Be^{\beta}$,

where $-\alpha$ and β are the roots of the associated quadratic equation:

(A1.6)
$$(\sigma^2/2)\xi(\xi-1) + \mu\xi - r = 0$$

Using these definitions of V and V^{*} with the two value-match and two smooth-pasting conditions, we have four necessary conditions to solve for the four unknowns: α^{i} , α^{o} , A and B.

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Footnotes

1. Hysteresis is the failure of an effect to reverse itself as its underlying cause is reversed.

2. Here we have ignored the positive covariance between the future spot and the future short term sterling rate. This is a problem even in the absence of transaction costs. For instance, if uncovered interest parity holds for one period deposits, then $(1+i) = (1+i_t^*)(s_{t+1}^e/s_t)$, where i_t^* is the one period sterling rate. But if the English market is efficient and investors risk neutral, then $(1+i_t^*)$ equals $E\{(1+i_{2,t}^*)/(1+i_{t+1}^*)\}$, where $i_{2,t}^*$ is the two day sterling rate. Now since i_{t+1}^* and s_{t+1}^e have positive covariance, it is clear that the expected dollar rate of return on holding two day sterling CDs for one day, which equals $E\{s_{t+1}(1+i_{2,t}^*)/(1+i_{t+1}^*)s_t\}$, is greater than the dollar rate of return on dollar CDs. To put it differently, if the spot adjusts to equate the expected rate of return on one period deposits, the expected rates of return on long deposits cannot be equalised. 3. Here we assume that each investor can buy only a finite amount of sterling. (The finite

amount need not be small; limiting them to, say, 100 billion times world GNP apiece would suffice). The problem is that if he can borrow an infinite amount at i and invest it in sterling at i plus epsilon his expected profit would be infinite. Thus although he could expect to earn more per dollar by waiting, this involves the comparison of two infinite values. We could, however, allow an infinite supply of investors, each of which has a finite investment potential.

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Figure 3.2











Figure 4.2

