NBER WORKING PAPER SERIES

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Working Paper 32999
http://www.nber.org/papers/w32999

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2024

Support from the National Science Foundation (SES-2343948) is gratefully acknowledged. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 32999
September 2024
JEL No. C78, D71, D72, H41

ABSTRACT

We study the volunteer’s dilemma in environments with heterogeneous preferences and private information. We characterize the efficiency properties of equilibrium, which is a departure from all the previous literature that focuses only on the probability of group success. While the probability of success may be non-monotonic in the size of the group, we show that per-capita welfare is always increasing for all types, strictly for sufficiently high types. As group size increases, the expected utility of every type converges to the expected utility of the type with the lowest possible cost, which is the same expected utility when there is no free rider problem, i.e., when there is only a single player in the game and that player has the lowest possible cost.

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1 Introduction

One of the simplest collective action problems is known as "the volunteer's dilemma" (henceforth, VoD). A group requires at least one of its members (a "volunteer") to undertake a costly action in order to accomplish a task or goal that is valuable by all members of the group. Introduced by Diekmann [1985] as a model of social conflict and public good production, it has proven to be an important paradigm to study a wide range of problems including, among others, problems in evolutionary biology (Archetti and Schauring [2012], Chen et al. [2013], Szathmáry [2011]); animal interactions (Schneider et al. [2012], Heifetz et al. [2021]); the invasion of cells by bacteria (Patel et al. [2018]); opens source and other communication protocols in computer science (Roberto et al. [2011], Gasparini et al. [2020]); and voting behavior in political science (Goeree and Holt [2005]).

The VoD is a dilemma because of the tension between the collective need for a volunteer and the incentive to free ride. Each individual must trade off the private cost of volunteering against the collective benefit of accomplishing the group goal ("success"). Even if the volunteering cost is less than the benefit for everyone involved, it's possible nobody volunteers ("failure") because they hope that others will volunteer. The existing literature has studied the probability of success in the VoD, modelling it as a game, either assuming that the cost of volunteering is the same for all or almost all individuals and common knowledge (Diekmann [1984, 1993]); or allowing for heterogeneous costs of volunteering (Weesie [1994], Bergstrom [2017]). The focus has been on studying how the probability of success, that is the probability that at least one individual in the group volunteers, changes with group size. Two paradoxical properties of equilibrium have been identified. First, the group's probability of success may be *strictly decreasing* as the size of the group increases. Second, the probability of success may be bounded strictly below one even in the limiting case of an infinite population.

Here we study the VoD as a Bayesian game. We depart from the existing analysis because we don't limit attention to the probability of success, but consider more generally the overall efficiency of the allocation; i.e., the expected value of achieving the common goal minus the expected cost of volunteering. We pose and answer three basic questions here: (1) Can efficiency, like the probability of success, decrease in the size of the population? (2) Is there a sense in which inefficiency persists for arbitrarily large populations? (3) Are the answers to the first two questions robust to behavioral models of equilibrium that relax the full rationality assumption? Considering overall efficiency is key in social science applications, since economists and other social scientists are concerned with individual and group welfare. Individual and group welfare are also essential considerations in evolutionary biology and, more generally, applications in which the probability of replication of a strategy or trait depends on the impact on fitness of individuals associated with it (Archetti [2009]).

We show that in terms of efficiency, the dilemma label needs to be significantly qualified. This follows from the three main results of the paper. First, we show that although the probability of success may be decreasing in the number of players, efficiency is always strictly increasing in the size of the group, in a very strong sense. Not only does the ex ante expected payoff to each member of the group increase, but this payoff monotonicity property holds for *every possible cost realization* (strictly for sufficiently high cost members). That is, there is nothing paradoxical in how welfare changes with n.

The second set of results concerns welfare with large populations. The limiting properties of the probability of success and welfare superficially appear to depend on the support of types: if the lower bound of the support of types is not restricted to be strictly positive, then for arbitrarily large groups, the probability of success converges to one and the expected fraction of the group that volunteers converges to zero, so average welfare per player converges to the fully efficient level. If the lower bound of volunteering costs is strictly positive, then the limit probability of success is strictly lower than one and equal to one minus the lower bound of the support of costs, and again the expected fraction of the group that volunteers is zero. While this may appear at first glance to be a large loss, in fact it is the same welfare that is achieved in the baseline case with no free-rider problem, i.e., where the group size equals one and single individual has the *lowest possible cost*. It follows that in terms of per-capita efficiency, there is no welfare loss in large groups relative to the benchmark of the no free-rider problem case with group size equal to one, regardless of the distribution of costs.

In both cases, however, the total welfare loss (i.e., n times the per capita loss) is strictly bounded away from zero and diverges at infinity as the number of players increases. In this sense, the volunteer's paradox persists in the limit, no matter what the distribution of types is. Moreover, contrary to the case with finite group sizes, it can be 100% attributed, in an unambiguous way that we will specify below, to a suboptimal probability of success, rather than to redundant cost of volunteering in the limit as $n \to \infty$.

The third set of results addresses the robustness question. We consider a standard generalization of Nash equilibrium, the *Quantal Response Equilibrium* (McKelvey and Palfrey [1995]), that relaxes the rationality of the players allowing for small payoff-dependent errors. We show that for *any* degree of rationality, even arbitrarily high, the probability of success converges to 1 and all members contribute in the limit with strictly positive probability in the VoD as the number of players increases. Moreover, this result is true for any distribution of costs, even if the lower bound of costs is greater than 0. For any degree of rationality, therefore, the probability of success converges to one as $n \to \infty$, but per capita inefficiency does not, due to excessive volunteering. However, this excessive volunteering decreases with the degree of rationality. Thus, in a quantal response equilibrium, the dilemma is turned on its head: there are too many volunteers rather than too low a probability of success.

2 The Volunteer's dilemma game

2.1 Model and equilibrium

The VoD with private information is a game, where each individual in a group of $n > 1$ members simultaneously and independently decides whether to take a costly action ("volunteer") or not ("free ride"). If at least one member of the group volunteers, everyone in the group receives a public benefit of v. The private cost of volunteering, c_i, is private information and each member's private cost is an independent draw from a cost distribution, $F(c)$. We assume $F(c)$ has a strictly positive continuous density function, $f(c)$ with support on $[\underline{c}, \overline{c}]$ with $\underline{c} < v$.[1]

The *unique* symmetric Bayesian equilibrium of the game is characterized by a cut-point c_n that satisfies the indifference condition:

$$\begin{aligned} v - c_n &= v\left[1 - (1 - F(c_n))^{n-1}\right] \\ &\Leftrightarrow \quad c_n = v(1 - F(c_n))^{n-1} \end{aligned} \tag{1}$$

where, on the first line, the LHS is the net benefit of volunteering and the RHS is the expected benefit of not volunteering, which equals the public benefit, v, times the probability at least one other member volunteers. A member with cost below c_n strictly prefers to volunteer,

[1] The upper bound on the support of costs could be ∞ since all types with cost $c > v$ have a dominant strategy to free ride.

while a member with cost above c_n strictly prefers to free ride, assuming all other members are using this equilibrium cutpoint strategy.

The equilibrium has a number of properties that have been identified in the literature (Weesie 1994, Bergstrom 2017, Bergstrom and Leo 2020). First the cutpoint is unique, with $c_n \in (\underline{c}, v)$ since c/v is an increasing function on that interval, ranging from $\underline{c}/v$ to 1, and $(1 - F(c))^{n-1}$ is a decreasing function on that interval, ranging from 1 to 0. Second, c_n is decreasing in n since c/v does not depend on n, and $(1 - F(c))^{n-1}$ shifts down for $c > \underline{c}$ as n increases. Third, $\lim_{n\to\infty} c_n = \underline{c}$ since $\lim_{n\to\infty}(1 - F(c))^{n-1} = 0$ for all $c > \underline{c}$.

The key metric of performance on which the literature has focused is the probability of success,

$$P_n = 1 - (1 - F(c_n))^n, \tag{2}$$

and how it changes with n. According to this measure, group performance is better if P_n is higher. Two main findings emerge. First, since one can rewrite the probability of success as $P_n = 1 - \left(\frac{c_n}{v}\right)^{\frac{n}{n-1}}$, and since $c_n \to \underline{c}$ it follows that

$$\lim_{n\to\infty} P_n = 1 - \frac{\underline{c}}{v}, \tag{3}$$

which is precisely equal to 1 if $\underline{c} = 0$, and is bounded strictly below 1 if $\underline{c} > 0$. Second, P_n is not necessarily monotone increasing in n (part of the dilemma), as first shown by Bergstrom (2017). For example, suppose $\underline{c} > 0$ and F is Pareto distributed with parameter $\theta > 0$, $F(c) = 1 - \left(\frac{\underline{c}}{c}\right)^{\theta}$. Then P_n is monotonically increasing in n if $\theta > 1$ and monotonically decreasing in n if $\theta < 1$.[2]

While the probability of success is one benchmark of performance, it fails to measure the well-being of the players in equilibrium because it ignores the volunteering costs that are incurred in the achievement of success. For example, according to this measure one would rate a group's performance as *perfect* if everyone in the group volunteered. However, in this case the total expected benefit to the group, *net of costs,* equals $n[v - E(c)]$ where $E(c) = \int_{\underline{c}}^{\overline{c}} c dF(c)$ which could actually be negative, so the group would actually be better off if nobody volunteered. In the remaining sections we instead consider expected per capita efficiency and the expected total efficiency which will be formally defined below.

[2] With complete information and symmetric costs as in Diekmann [1985], the probability of success is always strictly decreasing in n and it always converges strictly below 1 as $n \to \infty$, which led him to call it a dilemma.

2.2 Group Size and Efficiency

2.2.1 Per capita efficiency

To account for the costs of volunteering as well as the benefits of success, the start of our analysis is on how *expected equilibrium welfare* changes with n, which depends on much more than whether the probability of success is increasing or decreasing in n. This more involved equilibrium welfare analysis depends on both *the probability of success* as well as *the expected costs* borne by volunteers. Expected equilibrium welfare will also vary across individuals according to their cost-type, since high cost types will not bear any volunteer costs, and the actual costs borne by volunteers varies.

We define the expected utility of a member who has cost equal to c when the group size is n by:

$$V_n(c) = \max\{v - c, v\left[1 - (1 - F(c_n))^{n-1}\right]\}$$

so $V_n(c) = v - c$ for $c \leq c_n$ and $V_n(c) = v\left[1 - (1 - F(c_n))^{n-1}\right]$ for $c \geq c_n$. Hence the *expected group welfare*, normalized by the number of members ("per capita welfare") is given by:

$$W_n = \int_{\underline{c}}^{\overline{c}} V_n(c)dF(c) = vP_n - \int_{\underline{c}}^{c_n} cdF(c) \tag{4}$$

The following result shows that welfare is always monotonically increasing in a very strong sense: not only in expectation, but for every type. We have:

Proposition 1. *For any $F, n > 1$, and $\underline{c}$, the expected equilibrium utility of every member of the group is increasing in n: that is, $V_n(c) \leq V_{n+1}(c)$ for all $c \geq \underline{c}$. Furthermore $V_n(c) < V_{n+1}(c)$ for all $c > c_n$. It follows that expected per capita group efficiency is strictly increasing in n. That is, $EV_n < EV_{n+1}$.*

Proof. Recall that, regardless of the prior cost distribution, $c_n < c_{n-1}$. Hence $V_n(c) = v - c = V_{n-1}(c)$ if $c \leq c_n$. If $c \in (c_n, c_{n-1})$, $V_n(c) = v\left[1 - (1 - F(c_n))^{n-1}\right] > v - c = V_{n-1}(c)$, because such a cost-type prefers to free ride in a group with n members. Consider now $c > c_{n-1}$, so this type is a free rider for both group sizes $n-1$ and n. We have $V_n(c) = v\left[1 - (1 - F(c_n))^{n-1}\right]$ for $c \geq c_n$ and $V_{n-1}(c) = v\left[1 - (1 - F(c_{n-1}))^{n-2}\right]$ for $c \geq c_{n-1}$ For all $c \geq c_{n-1}$, we have:

$$\begin{aligned} V_n(c) &= V_n(c_{n-1}) > v - c_{n-1} = V_{n-1}(c_{n-1}) \\ &= v\left[1 - (1 - F(c_{n-1}))^{n-2}\right] = V_{n-1}(c) \end{aligned}$$

where the inequality follows from the fact that $c_{n-1} > c_n$, so with n players a type c_{n-1} strictly prefers not to volunteer. We conclude that $V_n(c) \geq V_{n-1}(c)$ for all c, strictly for $c > c_n$. The third statement in the proposition is that expected per capita efficiency *to the group* is strictly monotonically increasing in n. This follows immediately from the definition: $EV_n = \int_{\underline{c}}^{\bar{c}} V_n(c)dF(c)$. ■

This result may appear counterintuitive. Recall that P_n may be decreasing in n for some distributions $F(c)$. At first blush, it would seem that very high cost members, who would free ride regardless of how many other members there are, would be *worse* off in larger groups if P_n is decreasing in n, which can easily happen for some prior cost distributions. However, this is not the case, because $P_n = 1 - (1 - F(c_n))^n$ but $V_n(c) = v\left[1 - (1 - F(c_n))^{n-1}\right]$, and $(1 - F(c_n))^n \neq (1 - F(c_n))^{n-1}$ for any n! It follows that while P_n may be decreasing in n for some distributions, Proposition 1 shows that $V_n(c)$ *is always non-decreasing in* n *for all* c, and strictly increasing in n for $c > c_n$.

Since by Proposition 1, per capita welfare is monotonically increasing in n and bounded, it must converge to an upperbound as $n \to \infty$. The following result uses the equilibrium condition (3) to establish the limit:[3]

Proposition 2. As $n \to \infty$, expected welfare converges to the utility of the lowest possible type in autarchy: $\lim_{n\to\infty} EV_n = v - \underline{c}$.

Proof. Note that the second term in (4) converges to zero since $c_n \to \underline{c}$ as $n \to \infty$. The result follows from the fact that by (3), $vP_n \to v - \underline{c}$. ■

Propositions 1 and 2 show that there is little that is paradoxical in the volunteer's dilemma, when it is looked in terms of welfare. All types benefit and expected welfare improves when the size of the group increases. The ex post optimal per capita welfare level is $v - \frac{1}{n}E\left[c_n^1\right]$ where c_n^1 is the smallest cost realization n in a sample of n trials. As $n \to \infty$, it converges to v. Proposition 2 shows that this limit is not attainable when the support of the costs is bounded above zero, but it is attainable when the lower bound of the support is $\underline{c} = 0$. In this case the volunteer's dilemma vanishes in the limit.

Should we conclude that the volunteers' dilemma is not really a dilemma, at least for large groups? More than the theoretical limit, however, what matters for welfare in large but finite economies is the speed with which welfare converges to the minimum. A limit

[3] Versions of this result are also presented by Diekmann [1985], Bergstrom [2017] and others.

efficiency result has little value if it is reached only for unreasonable values of n. We address this question in the next section, characterizing the speed at which the inefficiency declines in n.

2.2.2 Rate of convergence and total group efficiency

To study the rate at which the per capita inefficiency converges to zero, and therefore the size of the total inefficiency as $n \to \infty$, let us define the first best expected level of efficiency as:

$$W_n^* = nv - E\left(c_n^1\right),$$

which is the theoretical expected level of efficiency achieved when success is achieved with probability one and the players with the lowest cost volunteers. Define, moreover, the equilibrium level of efficiency:

$$\begin{aligned} W_n &= nEV_n & (5) \\ &= nvP_n - nF\left(c_n\right)E\{c|c \leq c_n\} & (6) \end{aligned}$$

where c_n is characterized in (1), the first term in (5) is the expected total benefits of success weighted by the probability of success, and the second term is the expected total costs of volunteers. We can define the efficiency loss for the group the difference:

$$\Delta_n = W_n^* - W_n = nv\left(1 - P_n\right) + \left[nF\left(c_n\right)E\{c|c \leq c_n\} - E\left(c_n^1\right)\right] \quad (7)$$

Note that $W_n^*/n \to v$ and $W_n/n \to v - \underline{c}$, so when $\underline{c} > 0$ the total inefficiency grows linearly in n and diverges at infinity. In the following, we therefore focus on the more interesting case in which $\underline{c} = 0$. In this case, the first term of Δ_n, $nv\left(1 - P_n\right)$, is the loss due to the failure to succeed with probability one, and this probability of failure becomes infinitesimal as $n \to \infty$ since $P_n \to 1$; but this loss affects all players, so it is multiplied by n to evaluate total efficiency loss. The second term is the loss due to the fact that in equilibrium many players, typically with types larger than c_n^1, contribute in equilibrium. Two questions seem important: first, what is the limit of Δ_n if $\underline{c} = 0$? Is it zero, positive but finite, or infinite? The second question is more qualitative: is the distortion due to a too low probability of success, or to excessive participation (i.e. multiple volunteers)? When n is finite, both events with excessive and suboptimal participation have positive probability.

To evaluate these two quantities, we need to evaluate how fast c_n converges to zero, and consequentially how large is participation in equilibrium. The following lemma shows that c_n converges to zero sufficiently slow, so that the expected number of participants $nF(c_n)$ becomes arbitrarily large in equilibrium.

Lemma 1. *If* $\underline{c} = 0$ *then the expected number of volunteers diverges to infinity. That is,* $\lim_{n\to\infty} nF(c_n) = \infty$.

Proof. First note that for large n we have $F(c_n) \approx f(0)c_n$ and we know that $\lim_{n\to\infty} c_n = v \lim_{n\to\infty}(1 - F(c_n))^{n-1}$. Consider the limit of $(1 - F(c_n))^{n-1}$. We have:

$$\begin{aligned}
\lim_{n\to\infty} \log\left(1 - F\left(c_n\right)\right)^{(n-1)} &= \lim_{n\to\infty} \left[f(0)c_n\left(n-1\right)\log\left(\left(1 - f(0)c_n\right)^{\frac{1}{f(0)c_n}}\right)\right] \\
&= \lim_{n\to\infty} \left[f(0)c_n\left(n-1\right)\log\frac{1}{e}\right] = \lim_{n\to\infty} \log\left(\frac{1}{e}\right)^{(n-1)F(c_n)} \\
&\Leftrightarrow \lim_{n\to\infty} \left(1 - F\left(c_n\right)\right)^{n-1} = \lim_{n\to\infty} e^{-(n-1)F(c_n)}
\end{aligned}$$

So we can write:

$$\begin{aligned}
0 &= \lim_{n\to\infty} c_n = v \lim_{n\to\infty}\left(1 - F\left(c_n\right)\right)^{n-1} &(8)\\
&= ve^{F(c_n)} \cdot \lim_{n\to\infty} e^{-nF(c_n)} = v \cdot \lim_{n\to\infty} e^{-nF(c_n)} &(9)
\end{aligned}$$

This implies that $nF\left(c_n\right) \to \infty$, else the right hand side would be bounded away from zero. ■

We can now evaluate Δ_n and assess whether this cost is generated by too little success, or an excess of volunteers with types in $[0, c_n]$. Because the distribution is approximately uniform in the neighborhood of 0 and $\lim_{n\to\infty} c_n = 0$, we have $E\{c|c \leq c_n\} \approx c_n/2$ for large n. Hence, for large n we have:

$$W_n \approx nvP_n - nF(c_n)\, c_n/2$$

Next, we substitute $P_n = 1 - (1 - F(c_n))^n$ and $c_n/2 = \frac{v}{2}(1 - F(c_n))^{n-1}$ to get:

$$\begin{aligned} \Delta_n &\approx nv(1 - F(c_n))^n + n\frac{v}{2}F(c_n)(1 - F(c_n))^{n-1} - E\left(c_n^1\right) \\ &= nc_n^*\left[1 - \frac{1}{2}F(c_n)\right] + o(n) \end{aligned} \tag{10}$$

where $o(n) \to 0$ as $n \to \infty$. Denote the first term of (7) by $G_n^* = nv(1 - P_n)$, that is the cost due to underproduction of the public good. Using (7) we can prove:

Proposition 3. *The total efficiency loss becomes arbitrarily large as $n \to \infty$; moreover we have that loss is entirely due to underprovision in the limit: $G_n^*/\Delta_n \to 1$ as $n \to \infty$.*

Proof. The argument is in two steps. From (10), we have:

$$\Delta_n = nc_n^*\left[1 - \frac{1}{2}F(c_n)\right] \approx nc_n^* \approx \frac{nF(c_n)}{f(0)} \to \infty$$

where the equivalence $\approx$ means that the two terms have the same limit: $nc_n^* \approx \frac{nF(c_n)}{f(0)}$ since F is approximately uniform in the neighborhood of 0; and $\frac{nF(c_n)}{f(0)} \to \infty$ follows from Lemma 1. To see that $G_n^*/\Delta_n \to 1$, we note that we can write $G_n^* = nv(1 - P_n) = nc_n^*[1 - F(c_n)]$, so

$$\frac{G_n^*}{\Delta_n} = \frac{1 - F(c_n)}{1 - \frac{1}{2}F(c_n)} \to 1$$

which completes the argument. ∎

If Propositions 1 and 2 questioned whether the volunteers' paradox is indeed a paradox, Proposition 3 vindicates it, at least in part. For any distribution F the total inefficiency always diverges at infinity as $n \to \infty$.

Figure 1 illustrates the rates of convergence for the case of $v = 1$, and F is uniform in $[0, 1]$. The left panel of Figure 1 illustrates the equilibrium cutpoint c_n, the probability of success P_n, and the per capita welfare EV_n as functions of n, for group sizes of n up to 100. The convergence of all these variables is very fast. In contrast, total welfare loss, Δ_n, increases in n extremely slowly, reaching approximately 8 when $n = 25,000$.[4]

[4]We conjecture that Δ_n increases at the rate of $\log n$.

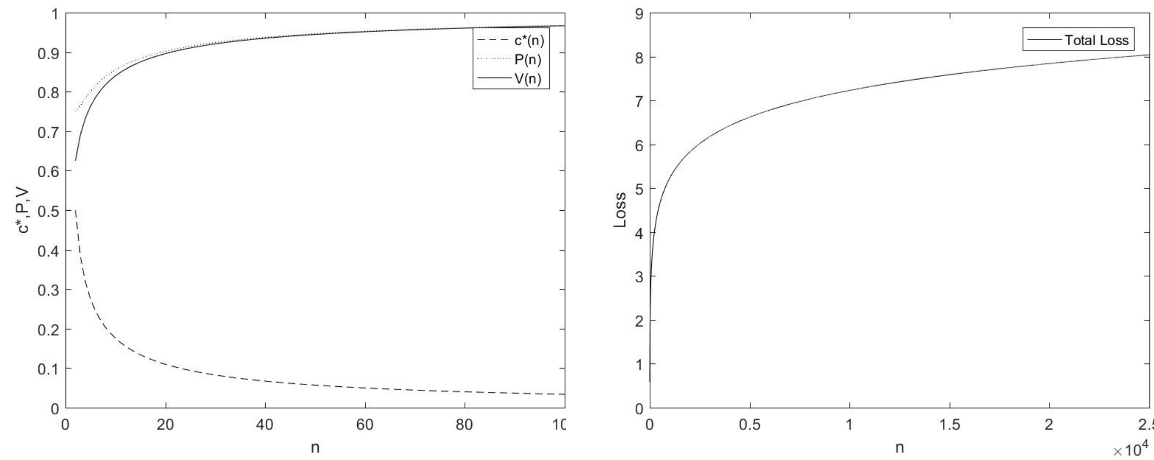

Figure 1: The key equilibrium variables in two examples in which F is Uniform. Per capita efficiency and probability of success converge to first best quickly (left panel) Total welfare loss increases very slowly, roughly on the order of log(n) (right panel).

3 Robustness

Quantal Response Equilibrium (QRE) is a stochastic generalization of Nash equilibrium that relaxes the assumption of exact best responses with "better responses" in which higher expected payoff actions are chosen with higher probability than actions with lower expected payoffs . The exact best response property of Nash equilibrium is the limiting case of QRE where the probability of choosing any lower payoff actions equals zero. QRE maintains the Nash assumption of "rational expectations": the expected payoff of each action of each player is endogenous and derived from the equilibrium strategies of all the players.

A symmetric QRE in the volunteer's dilemma is characterized as follows. Because there are only two actions available to each player (volunteer or not), the quantal response function of a player specifies a probability of volunteering as a function of c, denoted $p(c)$, where $p(c)$ is a decreasing function of c, since the expected benefit of not volunteering is constant in c while the expected benefit (net of cost) is decreasing in c. Given any strategy $p(c)$, the *expected* probability a player volunteers is given by:

$$\pi = \int_0^{\bar{c}} p(c)f(c)dc \tag{11}$$

Hence the expected benefit of not volunteering is equal to $v[1-(1-\pi)^{n-1}]$ and the expected (net) benefit of volunteering is equal to $v-c$. As is standard in QRE analysis, to simplify the equilibrium characterization we take the quantal response function to be logistic.[5] That is:

$$\begin{aligned} p(c) &= \frac{e^{\lambda(v-c)}}{e^{v[1-(1-\pi)^{n-1}]}+e^{\lambda(v-c)}} = \frac{1}{1+e^{\lambda[v[1-(1-\pi)^{n-1}]-(v-c)]}} & (12) \\ &= \frac{1}{1+e^{\lambda[c-(1-\pi)^{n-1}]}} & (13) \end{aligned}$$

This can also be expressed in terms of the log odds of volunteer vs. free-riding, which is proportional to the difference in expected payoffs from volunteering or not, with the proportionality factor, λ:

$$\ln\left[\frac{p(c)}{1-p(c)}\right] = \lambda\left[(1-\pi)^{n-1}-c\right] \text{ for all } c \in [0,1] \tag{14}$$

The proportionality factor is interpreted as a responsiveness or "rationality" parameter in the sense that for very low values of λ a player's choice is essentially random and almost unresponsive to c, while for very high values of λ a player's choice is an approximate best response. The symmetric (logit) QRE is a function $p_n^*(c;\lambda)$ that simultaneously satisfies equations (11) and (14) and we can denote the equilibrium probability of success by:

$$P_n^*(\lambda) = 1-(1-\pi_n^*(\lambda))^n = 1-\left(1-\int_0^{\bar{c}} p_n^*(c;\lambda)f(c)dc\right)^n.$$

For any fixed values of λ and n the RHS of 12 is bounded below by $\frac{1}{1+e^{\lambda[\bar{c}-(1-\pi)^{n-1}]}} > 0$ for all c. Hence $\pi_n^*(\lambda) \geq \frac{1}{1+e^{\lambda[\bar{c}-(1-\pi)^{n-1}]}}$, implying that for any given λ, we have $\lim_{n\to\infty}(1-\pi_n^*(\lambda))^{n-1} = 0$, which immediately implies that $P_\infty^*(\lambda) \equiv \lim_{n\to\infty} P_n^*(\lambda) = 1$ and $p_\infty^*(c;\lambda) \equiv \lim_{n\to\infty} p_n^*(c;\lambda) = \frac{1}{1+e^{\lambda c}}$. Hence, in any logit equilibrium of the volunteer's dilemma, the probability of success is approximately 1 in very large groups. We can see these results as a

[5] The results in this section hold more generally than the logit specification of QRE, for any continuous strictly increasing quantal response function Q such that $Q(0) = 1/2$, $Q(x) = 1-Q(-x)$, $\lim_{x\to\infty} Q(x) = 1$, and $p(c) = Q\left[\lambda((1-\pi)^{n-1}-c)\right]$.

providing a robustness analysis of the findings of the previous section if we consider the limit as $\lambda \to \infty$ (imperfect optimization with very small errors). When λ is high, the prediction of nearly 100% probability of success is similar to the Nash equilibrium discussed in the previous section under the assumption that the support of types is full, the lower bound is $\underline{c} = 0$. Of additional interest here is that the limiting QRE result holds *regardless of the distribution of types*; i.e., for all $\underline{c} < 1$.

The analysis of efficiency is more complicated, since $p_n^*(c; \lambda) > 0$ for all c implies that there will be significant overcontribution, even if decision errors are very small ($\lambda \to \infty$). The expected value to a member with cost c is equal to $V_n(c; \lambda) = vP_n^*(\lambda) - cp_n^*(c; \lambda)$. From the above limiting results we have:

$$\begin{aligned} V_\infty(c; \lambda) &\equiv \lim_{n \to \infty} V_n(c; \lambda) = \lim_{n \to \infty} [vP_n^*(\lambda) - cp_n^*(c; \lambda)] \\ &= v \lim_{n \to \infty} P_n^*(\lambda) - c \lim_{n \to \infty} p_n^*(c; \lambda) = v - c\frac{1}{1 + e^{\lambda c}} \end{aligned}$$

Similarly, we can define $EV_n(\lambda) = \int_{\underline{c}}^{\overline{c}} V_n(c; \lambda) dF(c)$, so we have:

$$EV_\infty(\lambda) = \int_{\underline{c}}^{\overline{c}} V_\infty(c; \lambda) dF(c) = v - \int_{\underline{c}}^{\overline{c}} c\frac{1}{1 + e^{\lambda c}} dF(c).$$

However, it is easy to see that the last term, $c\frac{1}{1+e^{\lambda c}}$ vanishes for all c for large λ, so $V_\infty(c; \lambda) = v$ and $EV_\infty \equiv \lim_{\lambda \to \infty} EV_\infty(\lambda) = v$. Thus, in the limit, as the rationality parameter approaches ∞ (i.e., full rationality), the efficiency in a QRE approaches the full information first best (v) in large groups, for *any* distribution of costs such that. Once again, this confirms the prediction of the Nash equilibrium model with full support discussed in the previous section, in which we have full per capita efficiency as $n \to \infty$.

The QRE model also confirms that the total efficiency loss is unboundedly large as $n \to \infty$, a feature present both in the model with $\underline{c} = 0$ and $\underline{c} > 0$. The results with the QRE sit in the middle of the findings with Nash and $\underline{c} = 0$ and $\underline{c} > 0$. As in the case with $\underline{c} > 0$, for any λ, the total distortion grows linearly in n, at a rate approximately equal to $(1 - 1/n) \int_{\underline{c}}^{\overline{c}} c\frac{1}{1+e^{\lambda c}} dF(c)$; but as in the case with $\underline{c} = 0$, this distortion grows very slowly when λ is large.

4 Conclusions

There is a large literature on the volunteer's dilemma, that has studied how a group's probability of success changes with its size. This literature has highlighted the fact that the probability of success may be non monotonic in n, and bounded away from 1 even for arbitrarily large groups.

In this paper we depart from the existing literature by focusing the analysis on welfare, which includes the probability of success but also the expected cost of achieving it, both at the individual and aggregate level. With a simple revealed preferences argument, we have shown that per capita welfare is always increasing in a strong sense in the size of the group: not only it is strictly increasing in expectation; it is weakly increasing for all possible cost types c, strictly for c sufficiently high. We have also shown that per capita welfare for a type c converges to the value that would be achieved by a player with the lowest cost in autharchy, i.e. if left alone. Convergence however, is slower than the convergence to zero of $1/n$, so the total inefficiency diverges at infinity for any distribution of costs $F(c)$, with lower bound of the support bounded away from zero, or equal to zero.

Finally, we have studied the robustness of the results to the equilibrium concept, by studying the symmetric Quantal Response Equilibria (QRE) of the game. In all QRE, the probability of success of the group, converges to zero as $n \to \infty$, but the distortion is bounded always from zero at the per capita level, and diverges to infinity linearly in n, thus generating results that are qualitatively similar the the Bayesian Nash equilibrium.

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