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ABSTRACT

We provide a general formula for optimal unilateral policies in multi-sector, general-equilibrium Ricardian models with various widely adopted labor market specifications. Sector-specific tariffs are summarized by a matrix of partial supply elasticities and the share of Home's import in foreign incomes, reflecting Home's import market power. Sector-specific export taxes depend on trade elasticities and Home's market share in foreign consumption, reflecting Home's export market power. Home imposes higher tariffs or export taxes on sectors with larger market powers. We apply the general formula to specific cases: perfectly mobile labor, imperfectly mobile labor across sectors, Ricardian-Roy models, and inefficient labor markets.

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1 Introduction

A large number of quantitative models analyze the impact of trade on labor markets. A natural question is: what are the policy implications of such models? In this paper, we study optimal policies in the extended multi-sector Ricardian trade models, encompassing various labor market specifications that are widely used in quantitative works. The main contribution of our paper is to develop a general formula for optimal policies, regardless of the labor market conditions. We then apply this formula to a range of labor market specifications, whether they are efficient or inefficient, and prescribe the specific optimal policies for each scenario.

We study the unilateral policies of a Home country in the extended multi-sector Ricardian model under different labor market specifications. Home government chooses policies, consumption, production, trade, and prices subject to technical feasibility and resource constraints. The economic objective of the policy is classic: to exploit terms of trade benefit and to correct distortions. We use Lagrange multiplier methods to derive Home's optimal domestic and trade policies, similar to Dixit (1985) and Costinot, Donaldson, Vogel, and Werning (2015). Our first finding is that optimal policies are tied to the Lagrange multipliers on the resource constraints, regardless of the specific labor market or production technology assumptions. The multipliers on foreign-goods-market constraint reflect the impact of foreign goods supply on Home's utility and how Home government internalizes such impact and uses policies to exploit Home's market power. The multipliers on domestic markets reflect how Home internalizes the impact of domestic supply on Home utility and uses policies to correct domestic distortions.

We further unpack these multipliers and connect optimal policies with Home's market powers and domestic labor distortions. First, we show that Home's optimal import tariff in a sector depends on the entire matrix of foreign partial supply elasticity and the proportion of Home purchases in foreign income within that sector. Second, given other taxes, export tax in a sector depends on trade elasticities and the significance of foreign demand for Home goods. Lastly, domestic taxes are directly associated with Home's own partial supply elasticities, which reflects the Bhagwati-Johnson principle of targeting. All these Foreign and Home elasticities and trade shares are jointly determined in the general equilibrium and are affected by optimal policies. The optimal tax formula is general and independent of different labor market specifications. However, these specifications, whether they are efficient or inefficient, can affect labor allocation across sectors and supply elasticities, and thus the market power of Home, thereby influencing Home optimal policies.

Moreover, we introduce the *CES supply system* under Ricardian models. A labor market specification generates a CES supply system with a constant elasticity κ if a sector's partial supply elasticities with respect to wages in any two other sectors are the same, though the two elasticities may not be constant, and the supply elasticity with respect to the sector's own wage relative to the elasticity to any other sector's wages is constant, given by κ. It turns out that many widely studied labor market specifications belong to this CES supply system, such as a specification with perfectly mobile labor across sectors ($\kappa \to \infty$), a specification with sector-specific labor ($\kappa = 1$), or a Ricardian-Roy model with one factor.

Under a CES supply system, the interlinks across sectors break down because Home's tariffs of any two sectors have an equal impact on a third sector's income in the foreign country. As a result, we can further simplify the general optimal tax formula and obtain the following insights. First, a sector j's tariff relative to a sector s's tariff can be summarized by two sufficient statistics: i) the inverse of κ and ii) Home imports over foreign income in sector j relative to that in sector s. Together, the two reflect Home's import market power in sector j relative to sector s. In sum, Home tends to impose higher tariffs in a sector when Home is a large buyer of foreign goods in that sector, and the tariff gap across sectors decreases with κ. Second, given other taxes, a sector's export tax depends on i) the inverse of trade elasticity and ii) Home's market shares in foreign consumption in that sector, together reflecting Home's export market power. Home uses high export taxes on sectors where Home is a large seller to Foreign. Lastly, optimal domestic taxes are zero. This implies that labor markets that generate the CES supply system must be efficient.

We apply the general formula for optimal policies to specific cases of labor markets. The first one we consider is the widely adopted model with perfectly mobile labor across sectors. This labor market generates a CES supply system with κ approaching infinity. In this case, Home would not use sector-specific tariffs and can further set all tariffs to zero. This is because, with labor freely mobile across sectors, foreign supply is completely elastic. Thus, Home has no market power to change foreign supply prices, resulting in zero tariffs. This result is consistent with Costinot et al. (2015). In contrast, with imperfectly substitutable labor across sectors, imposing a tariff decreases Home's demand for foreign goods and further depresses foreign wages and prices in that sector. The larger Home's purchase share in the foreign sector, the greater the impact of tariffs on foreign prices, and the higher the tariff. We also apply the general formula to imperfectly substitutable labor with finite labor market elasticity of substitution κ.

We further study two examples of labor market specifications that do not generate a CES supply system. One is the Ricadian-Roy model with multiple types of labor, and the other involves inefficient labor markets similar to Waugh (2023) and Donald, Fuku, and Miyauchi (2023), where workers select into sectors based on both productivity and worker preferences. Optimal policies still follow the general formula. Under the Ricadian-Roy model, there are no labor market frictions, and Home does not use any domestic tax. In contrast, in the model with an inefficient labor market, Home chooses domestic taxes that are negatively related to sectors' wages to correct domestic inefficiency. Given that the supply system is not CES, optimal tariffs consider the impacts of each sector's tariff on all other sectors. Thus, the optimal tariffs are determined jointly and depend on the whole supply elasticity matrix and the vector of net import shares in the foreign country.

Our approach of providing a general formula for optimal trade policies in general-equilibrium models can be traced back to the classic work of Dixit (1985), which derives optimal conditions under a foreign offer curve. We consider the workhorse trade models under various labor market specifications and configure the associated foreign offer curves. Moreover, we provide formulae for sector-specific import tariffs and export taxes in both general and specific cases.

Our paper is related to the literature on optimal trade policies. As pointed out by Caliendo and Parro (2021), characterizing optimal trade policy in current trade modes is challenging, but the literature has introduced new methods and new insights. For example, optimal trade policies in recent general equilibrium trade models are theoretically derived in: Costinot et al. (2015), which studies trade policy in the canonical Ricardian model Dornbusch, Fischer, and Samuelson (1977). Costinot, Rodríguez-Clare, and Werning (2020) characterizes optimal firm-level trade policy in a two-country-single-sector Melitz model. Lashkaripour and Lugovskyy (2023) studies policies in multi-industry, multi-country trade models where misallocation occurs due to scale economies. Bai, Jin, and Lu (2023) studies optimal dynamic policies with endogenous technology through innovation. See Caliendo and Parro (2021) for a review of optimal trade policies in general equilibrium models.

Our paper focuses on the implications of labor market specifications on supply curves. Fajgelbaum, Goldberg, Kennedy, Khandelwal, and Taglioni (2024) shows that inelastic supply curves can be micro-founded by both returns to scale in production and elasticity of factor mobility across products and sectors. An example of optimal trade policies under production-driven supply curves is Bai, Jin, and Lu (2023) (BJL hereafter), whose finding is consistent with the current paper: optimal import tariffs are related to foreign export supply elasticities. However, the current paper has an upward-sloping curve under a fixed technology and imperfectly substitutable labor across sectors, but BJL has a downward-sloping foreign supply curve under endogenous technology and perfectly mobile labor. Thus, the two papers have different implications on the sign of tariffs but follow the same formula.

Furthermore, BJL is another example of the CES supply system. Similarly, the multi-sector Krugman and multi-sector Melitz-Pareto models are the CES supply system as well because they are isomorphic to BJL at the steady state. We can apply the optimal policy formula in BJL to these two models (see Appendix I). In summary, both imperfectly substitutable labor and endogenous technology can provide an inelastic foreign supply, and our general formula applies.

The paper proceeds as follows. Section 2 extends the two-country, multi-sector Ricardian framework to incorporate general labor market specifications and details the model environments. Section 3 derives optimal policies for general labor market specifications and, in particular, if the supply system satisfies CES. Section 4 gives a list of labor market examples and explains the specific optimal policies for each scenario. Section 5 concludes.

2 Theoretical Framework

We study optimal trade and domestic policies in a multi-sector Ricardian model with various labor market specifications. Within each sector, the technology and preference assumptions can be either Eaton-Kortum or Armington. Both yield the same gravity equations.

The world has $N = 2$ countries and J sectors. Country n has a measure $\bar{L}_n$ of labor. Let country 1 be Home, and country 2 be Foreign. All sectors have a linear production function and use labor to produce. Let T_{nj} denote the technology level of sector j at country n, and θ governs the trade elasticity.

Consumers in all countries have the same utility over final goods C given by $u(C)$. Final goods is a Cobb-Douglas function across the consumption of different sector $j \in J$ goods, $C_n = \Pi_{j\in J} \left(C_{nj}\right)^{\beta_j}$, where the constant β_j captures the share of sector j. Within a sector, there is a continuum of varieties of consumption goods, which are aggregated with a Cobb-Douglas function $C_{nj} = \exp \int_0^1 \ln c_{nj}(\omega) d\omega$. All goods are tradable under an iceberg trade cost d_{ni} between country n and i.

We consider a variety of labor market specifications that have been extensively studied in the trade and labor literature. The labor market specification determines wages and labor in equilibrium. Let w_{nj} and L_{nj} be the wage and labor in sector j of country n, and let Ω represent the equilibrium labor conditions. For example, when labor is perfectly mobile across sectors, we can define $\Omega = \{(w_{nj}, L_{nj}) : \sum_j L_{nj} = \bar{L}_n,\ w_{nj} = w_n\}$. Section 4 provides specific examples of Ω and the associated optimal policies.

Suppose Home imposes unilateral trade and domestic policies, but Foreign is passive and has zero taxes. Specifically, Home government imposes sector-specific import tariffs τ_j^m, export taxes τ_j^x, and domestic sales tax τ_j^d on its imports, exports, and domestic sales, respectively. Let P_n and x_n be country n's final goods price and expenditures, respectively, with $x_n = P_n C_n$.[1] Below, we define the world market equilibrium under Home's policies.

Definition 1 (World Market Equilibrium)**.** *The world market equilibrium consists of an allocation of labor and consumption $\{L_{nj}, C_{nj}\}$, expenditures $\{x_n\}$, prices $\{P_n\}$, and wages $\{w_{nj}\}$ such that consumers maximize utility, firms maximize profits, and the market clearing conditions hold, taking*

[1]This indicates transfers are allowed if there are different factors.

as given Home government's policies $\left\{\tau_j^m, \tau_j^x, \tau_j^d\right\}$:

1. *Expenditures are given by*

$$x_1 = \sum_{j=1}^{J} w_{1j} L_{1j} + \sum_{j=1}^{J} \beta_j \left[\frac{\tau_j^x}{1+\tau_j^x} \pi_{21,j} x_2 + \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1 \right] \tag{1}$$

$$x_2 = \sum_{j=1}^{J} w_{2j} L_{2j} \tag{2}$$

2. *Trade shares satisfy, for each sector j*

$$\pi_{11,j} = \frac{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}, \tag{3}$$

$$\pi_{12,j} = \frac{T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}, \tag{4}$$

$$\pi_{21,j} = \frac{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}w_{2j}^{-\theta}}, \tag{5}$$

$$\pi_{22,j} = \frac{T_{2j}(w_{2j})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}w_{2j}^{-\theta}}. \tag{6}$$

3. *Consumer prices are given by*

$$P_1 = \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta} \right]^{-\frac{\beta_j}{\theta}}, \tag{7}$$

$$P_2 = \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}w_{2j}^{-\theta} \right]^{-\frac{\beta_j}{\theta}}. \tag{8}$$

4. *Goods market clearing conditions, for each j*

$$w_{1j} L_{1j} = \beta_j \left[\frac{1}{1+\tau_j^d} \pi_{11,j} x_1 + \frac{1}{1+\tau_j^x} \pi_{21,j} x_2 \right], \tag{9}$$

$$w_{2j} L_{2j} = \beta_j \left[\frac{1}{1+\tau_j^m} \pi_{12,j} x_1 + \pi_{22,j} x_2 \right]. \tag{10}$$

5. *The labor market clearing conditions $\Omega(\{w_{nj}, L_{nj}\})$ hold.*

Without loss of generality, we normalize Home's final goods price $P_1 = 1$. Note that in the world market equilibrium, the market clearing conditions and definition of expenditures imply that the balanced trade conditions hold.

3 General Formula of Optimal Policy

This section presents a general formula for optimal policy that is applicable to various labor market conditions. Section 4 examines specific labor market assumptions and their corresponding optimal policies. It is worth noting that under the multi-sector trade framework (Eaton-Kortum and Armington), sector-specific export taxes, tariffs, and sales taxes are the finest policy instruments as they adequately address the externalities arising from terms of trade and potential labor market frictions.

Definition 2. *Home government's problem is to choose sector-specific* $\{\tau_j^d, \tau_j^x, \tau_j^m\}$ *to maximize domestic consumers' utility,* $\max u(x_1/P_1)$, *subject to the world market equilibrium given by Definition 1.*

We use Lagrange multiplier methods to derive the general formula for optimal policies, similar to Dixit (1985). Let γ_{1j} and γ_{2j} be the Lagrange multipliers on the goods market clearing constraint of Home (9) and Foreign (10).

Lemma 1 (Optimal Policies and Multipliers). *Irrespective of labor market specifications, optimal policies in sector j take the form of*

$$\tau_j^m = -\frac{\gamma_{2j}}{u_c}, \quad \tau_j^d = -\frac{\gamma_{1j}}{u_c}, \quad 1 + \tau_j^x = \frac{1 + \tau_j^d}{1 + \tau_j^m} \left(1 + \frac{1}{\theta \pi_{22,j}}\right). \tag{11}$$

Proof. Appendix A.

There are two potential reasons why Home government would like to utilize taxes: to manipulate terms of trade and to address domestic distortions in labor markets. First, Home, as a whole, is a monopoly on the goods it exports and may have buyer powers in the goods it imports. Individual agents do not consider the impact of their imports or exports on the prices they pay, but Home government internalizes this and aims to use trade policies to manipulate the terms of trade. γ_{2j} reflects how foreign goods supply affects Home utility, hence Home government's consideration of exploiting buyer's power. When γ_{2j} is positive, Home government would like to subsidize imports of sector j goods with $\tau_j^m < 0$. Second, when there are domestic distortions, Home government seeks to use domestic policies to correct these distortions. Domestic sales taxes τ_j^d depend on multiplier γ_{1j} or how Home goods supply matters for Home utility. It reflects the Home government's consideration of correcting domestic distortions. When γ_{1j} is positive, Home government subsidizes sector j with $\tau_j^d < 0$.

In general, the terms-of-trade gains of trade policy come from Home's ability to change import and export prices by affecting the demand Foreign face and the supply of Home goods to Foreign. In other words, Home's gain from manipulating terms of trade depends on its buyer power in Foreign goods and its monopoly power in exporting to Foreign. Furthermore, these Home powers are reflected in Foreign export supply and import demand elasticities, respectively. In the general equilibrium models, these elasticities are endogenous as goods supplies are interlinked across sectors. Below, we show that the matrix of partial supply elasticities and Home's net import shares in Foreign sector are sufficient for import tariffs, and trade elasticity and Home's export shares in Foreign consumption determine export taxes.

Let Y_{nj} denote the income of sector j in country n. We can define the *partial supply elasticity* between sector i and j as $\frac{\partial \ln Y_{ni}}{\partial \ln w_{nj}} \frac{Y_{ni}}{Y_{nj}}$, which is the product of the partial elasticity of a sector's income with respect to another sector's wage, multiplied by the relative income share of these two sectors. These partial supply elasticities show how changes in one sector's wage affect the relative supply of other sectors, assuming equilibrium wages are exogenous. Moreover, these elasticities are determined by labor market specifications given the preferences, technology, and market structure assumptions. Let matrix Λ_n consist of these elasticities in country n, with its jth-row-ith-column entry as $\frac{\partial \ln Y_{ni}}{\partial \ln w_{nj}} \frac{Y_{ni}}{Y_{nj}}$.

We now present the other determinant of Foreign export supply: Home imports as a share of Foreign, for example, $\frac{\beta_j x_1 - (1+\tau_{1j}^d) Y_{1j}}{Y_{2j}}$ for sector j. Here, $\beta_j x_1$ and $(1 + \tau_{1j}^d) Y_{1j}$ are Home's total expenditure and its expenditure on own goods in sector j, respectively, and the difference between the two is the Home's net spending on Foreign goods in sector j. Let the vector Ψ contain all these shares, with the jth entry of Ψ as $\frac{\beta_j x_1 - (1+\tau_{1j}^d) Y_{1j}}{Y_{2j}}$. These shares and the partial supply elasticities capture Home's buyer powers across sectors.

Definition 3. *Matrix Λ_n consist of partial suppy elasticities in country n, with its jth-row-ith-column entry as $\frac{\partial \ln Y_{ni}}{\partial \ln w_{nj}} \frac{Y_{ni}}{Y_{nj}}$. Vector Ψ contains Home's net import spending in Foreign income with the jth entry of $\frac{\beta_j x_1 - (1+\tau_{1j}^d) Y_{1j}}{Y_{2j}}$.*

Different labor market specifications, whether elastic or inelastic, efficient or inefficient, can affect labor allocations and buyer power of Home, showing up in the partial supply elasticities and Home's import shares in Foreign income. However, it is shown below that the optimal tax *formula* is independent of these specifications.

Proposition 1 (Optimal Policy Formula). *Home government's unilateral optimal policy satisfies,*

$$\begin{aligned}
&\textit{Domestic tax vector:} && (\Lambda_1 - I)(1+\tau^d) && = 0, && (12)\\
&\textit{Import tariff vector:} && \tau^m && = \Lambda_2^{-1}\Psi - \frac{\zeta}{u_c}, && (13)\\
&\textit{Export tax:} && 1+\tau_j^x && = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\theta \pi_{22,j}}\right), \quad \forall j, && (14)
\end{aligned}$$

where τ^d and τ^m are vectors of domestic taxes and import tariffs across sectors, respectively. I is an identity matrix, ζ is endogenous, but the same across sectors[2]*, and u_c is Home's marginal utility.*

Proof. Appendix B.

First, domestic taxes are directly related to Home's partial supply elasticities Λ_1, which depends on Home's own labor market efficiency. Second, as we discussed above, import tariffs are linked to Foreign export supply elasticities, which in turn depends on Foreign's partial supply elasticity matrix Λ_2 and how important is Home purchase in Foreign, Ψ. The level of import tariffs also depends on a term common across sectors, ζ/u_c. Lastly, given other taxes, export tax is higher if the demand elasticity θ is smaller, i.e., demand is less elastic, and if $\pi_{22,j}$ is lower, which implies a greater Foreign demand for Home goods. The schedule of export taxes is consistent with Costinot et al. (2015), which shows that export taxes reflect the Home government's incentive to manipulate relative prices in its favor by

[2] $\zeta = \sum_{k=1}^{J}(\gamma_x \beta_k \frac{\tau_k^x}{1+\tau_k^x}\pi_{21,k} + \gamma_{1k}\beta_k \frac{1}{1+\tau_k^x}\pi_{21,k} + \gamma_{2k}\beta_k \pi_{22,k}) - \gamma_x + u_c$ is the same across sectors. See details in Appendix B.

exploiting its export market power. In equilibrium, all taxes are jointly determined.

Definition 4 (**CES Supply System**). *Let κ be a constant. A CES supply system with elasticity κ satisfies*

$$\frac{\partial \ln Y_s}{\partial \ln w_j}\frac{Y_s}{Y_j} - \frac{\partial \ln Y_s}{\partial \ln w_i}\frac{Y_s}{Y_i} = \begin{cases} 0 & \text{for } s \neq i, j \\ \kappa & \text{for } s = j. \end{cases} \tag{15}$$

Similar to the logic of "CES demand system" in Arkolakis, Costinot, and Rodríguez-Clare (2012), we define a supply system satisfying Definition 4 as a CES supply system. In extended multi-sector Ricardian models, examples of CES supply systems include the standard case of perfectly mobile labor and the case with totally immobile labor across sectors, corresponding to $\kappa \to \infty$ and $\kappa = 1$, respectively. In the next section, we discuss a variety of labor market structures that generate different supply systems and their associated optimal policies.

Proposition 2 (**Optimal Policy under CES Supply System**). *Under a CES supply system with elasticity κ, Home government's unilateral optimal policy satisfies, for any sectors j, i,*

$$\text{Domestic tax:} \quad \tau_j^d = \bar{\tau}^d, \tag{16}$$

$$\text{Import tariff:} \quad \tau_j^m - \tau_i^m = \frac{1}{\kappa}\left(\frac{\beta_j x_1 - (1+\bar{\tau}^d)Y_{1j}}{Y_{2j}} - \frac{\beta_i x_1 - (1+\bar{\tau}^d)Y_{1i}}{Y_{2i}}\right), \tag{17}$$

$$\text{Export tax:} \quad 1+\tau_j^x = \frac{1+\bar{\tau}^d}{1+\tau_j^m}\left(1+\frac{1}{\theta\pi_{22,j}}\right), \tag{18}$$

where $\bar{\tau}_d$ is endogenous and independent of sectors.

Proof. Appendix C.

Proposition 2 demonstrates that under a CES supply system, Home would not use sector-specific domestic tax. In Appendix D-G, we further establish that if the labor market condition Ω is homogenous of degree zero on wages, tax neutrality holds. In this case, one of the domestic taxes can be normalized to zero, and since Home optimally will not use sector-specific domestic tax, all domestic taxes are zero with a CES supply system.

Moreover, under a CES supply system, the partial elasticities of a sector's income with respect to wages in any other two sectors (weighted by relative income shares) are the same, and the partial elasticities of a sector's income with respect to its own wage relative to that with respect to the other sector's wage is constant at κ, i.e., $\frac{\partial \ln Y_j}{\partial \ln w_j} - \frac{\partial \ln Y_j}{\partial \ln w_i}\frac{Y_j}{Y_i} = \kappa$. Under this property, the difference of any two rows in the general import tariff vector (13) in Proposition 1 yields the formula (17).

Thus, the relative import tariffs between any two sectors only depend on the relative Home purchases of the two sectors' goods. A larger Home net import spending relative to Foreign income, $\frac{\beta_j x_1 - (1+\bar{\tau}^d)Y_{1j}}{Y_{2j}}$, results in greater Home buyer power and thus a greater import tariff τ_j^m. Importantly, the relative import tariffs $\tau_j^m - \tau_i^m$ does not depend on Home's import share in other sectors. Furthermore, instead of depending on the whole supply elasticity matrix, the relative import tariffs only depend on the constant elasticity κ. Lower κ leads to higher incentives to impose tariffs on sectors with larger Home buyer powers.

4 Examples

In this section, we explore the labor market specifications that are widely used in the literature. We show how the labor market conditions Ω is determined in each case and how Proposition 1 and 2 on optimal policies apply to each case.

We consider three labor market specifications, all of which maintain the assumption that labor cannot move across countries. The first specification features a constant elasticity of labor substitution across sectors. We call these models *CES labor market*. This specification nests the models with perfect mobile labor across sectors and models with sector-specific labor. The second specification is a *Ricadian-Roy model*. The third one involves *inefficient labor markets*, where worker selection into sectors depends on both productivity and worker preferences.

4.1 CES Labor Market

Here, total labor is a CES aggregator over labor in different sectors, $L_n = [\sum_{j=1}^{J} \alpha_j^{-\frac{1}{\kappa-1}} L_{nj}^{\frac{\kappa}{\kappa-1}}]^{\frac{\kappa-1}{\kappa}}$, where $\{\alpha_j\}$ are constants and $\kappa - 1 \geq 0$ captures the elasticity of substitution across labor in different sectors. This implies a labor supply curve as

$$L_{nj} = \alpha_j \left(\frac{w_{nj}}{W_n} \right)^{\kappa - 1} \bar{L}_n, \quad \text{with } W_n = \left[\sum_{j=1}^{J} \alpha_j {w_{nj}}^{\kappa} \right]^{\frac{1}{\kappa}}. \tag{19}$$

When $\kappa \to \infty$, $L_n = \sum_{j=1}^{J} L_{nj}$ and labor is perfectly substitutable across sectors, which is widely adopted in the literature. For example, Costinot, Donaldson, and Komunjer (2012) extends Eaton and Kortum (2002) to multi-sectors. In this case, the equilibrium wages are equalized across sectors, leading to $\Omega = \{(w_{nj}, L_{nj}) : \sum_j L_{nj} = \bar{L}_n,\ w_{nj} = w_n\}$. When $\kappa = 1$, the model is the same as the specific-factor model with fixed labor in each sector, and thus $\Omega = \{(w_{nj}, L_{nj}) : L_{nj} = \alpha_j \bar{L}_n\}$.

In general, we can rewrite the labor supply curve (19) and obtain the equilibrium labor market condition as

$$\Omega = \left\{ (w_{nj}, L_{nj}) : \frac{w_{nj} L_{nj}}{W_n \bar{L}_n} = \alpha_j \left(\frac{w_{nj}}{W_n} \right)^{\kappa}, \ W_n = [\sum_{j=1}^{J} \alpha_j {w_{nj}}^{\kappa}]^{\frac{1}{\kappa}} \right\}. \tag{20}$$

We define the income in sector j, $Y_{nj} = w_{nj} L_{nj}$. Labor market condition (20) further implies a supply system,

$$Y_{nj} = \alpha_j \left(\frac{w_{nj}}{W_n} \right)^{\kappa} Y_n.$$

Let the share of sector j income in country n as $m_{nj} = Y_{nj}/Y_n$. We can show that the partial-supply-elasticity matrix Λ_n takes the form of

$$\Lambda_n = \begin{pmatrix} \kappa - (\kappa-1)m_{n1} & \dots & -(\kappa-1)m_{nj} & \dots & -(\kappa-1)m_{nJ} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa-1)m_{n1} & \dots & \kappa - (\kappa-1)m_{nj} & \dots & -(\kappa-1)m_{nJ} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa-1)m_{n1} & \dots & -(\kappa-1)m_{nj} & \dots & \kappa - (\kappa-1)m_{nJ} \end{pmatrix}. \tag{21}$$

Take the difference between any two rows of Λ_n, say i and j, and call the resulting vector $d\Lambda_{n,ij}$. The condition (15) of the definition of CES supply system requires all entries of $d\Lambda_{n,ij}$ to be zero, except for the ith or jth entry, meaning that any two sectors' tariffs must have a symmetric impact on the third sector. It is clear that Λ_n matrix (21) satisfies this requirement. Thus, the model with constant labor elasticity is a CES supply system. We can then apply the Proposition 2 for optimal policies. See Appendix F for details. Furthermore, $\tau_j^d = \bar{\tau}^d$, in addition, Ω in (20) has the property of homogenous of degree zero in wages. Hence, tax neutrality holds, and we can set $\bar{\tau}^d = 0$. The optimal domestic taxes are zeroes for all sectors. Intuitively, without domestic frictions in the model, Home government has no incentives to use domestic taxes.

The optimal tariff formula equation (17) nests the case with perfectly mobile labor across sectors ($\kappa \to \infty$). In such case, the optimal import tariffs can be set to zero, $\tau_j^m = 0$ for any j. This is because under completely elastic Foreign supply, Home has no market power to change foreign supply prices and, thus, no incentives to use tariffs. In contrast, with imperfect substitute labor, including sector-specific labor model, where $\kappa < \infty$, imposing a tariff decreases the demand that Foreign faces and depresses the sector wage and price. The larger Home's share in the Foreign sector, the greater the impact of tariffs on Foreign prices, and the higher the tariff.

4.2 Ricardian-Roy models

Ricardian-Roy models, such as those in Costinot and Vogel (2010) and Costinot and Vogel (2015), extend the Ricardian trade model with more than one factor, as in the Roy model. Here we adopt Galle, Rodríguez-Clare, and Yi (2023), which combines multi-sector EK Costinot, Donaldson, and Komunjer (2012) and Roy model as in Lagakos and Waugh (2013) and Hsieh, Hurst, Jones, and Klenow (2019).

There are G groups of workers in each country.[3] The total number of group-g workers in country n is fixed at $\bar{L}_{ng}$. A worker in group g of country n draws an efficiency unit z_j in sector j from a Fréchet distribution F_{njg} with shape parameter κ and scale parameter A_{njg}. A worker with the vector $\mathbf{z} = (z_1, z_2, \ldots, z_J)$ chooses to work in a sector that gives her the highest income based on her productivity and wage in that sector. Let Ξ_{nj} be the set of workers choosing sector j, $\Xi_{nj} \equiv \{\mathbf{z}\colon w_{nj}z_j \geq w_{nk}z_k \text{ for all } k\}$ and $F_{ng}(\mathbf{z})$ be the joint

[3]For example, there could be two groups of workers: skilled and unskilled groups.

probability distribution of $\mathbf{z}$ for group-g workers in country n.

In equilibrium, the share of workers in group g that apply to sector j in country n is given by

$$\lambda_{njg} = \frac{A_{njg}(w_{nj})^{\kappa}}{\sum_{s=1}^{J} A_{nsg}(w_{ns})^{\kappa}} = \frac{A_{njg}(w_{nj})^{\kappa}}{(W_{ng})^{\kappa}}, \text{ with } W_{ng} = \left[\sum_{s=1}^{J} A_{nsg}(w_{ns})^{\kappa}\right]^{\frac{1}{\kappa}}.$$

The sum of efficiency units supplied to sector j in country n is $L_{njg} \equiv \bar{L}_{ng} \int_{\Xi_{nj}} z_j dF_{ng}(\mathbf{z}) = \xi \frac{W_{ng}}{w_{nj}} \lambda_{njg} \bar{L}_{ng}$, where $\xi \equiv \Gamma(1-1/\kappa)$. Let $L_{nj} = \sum_g L_{njg}$ be the total labor worked in sector j. Hence, Ω satisfies

$$\Omega = \left\{ (w_{nj}, L_{nj}) : w_{nj} L_{nj} = \sum_g \left[\frac{A_{njg}(w_{nj})^{\kappa}}{(W_{ng})^{\kappa}} W_{ng} \bar{L}_{ng} \right], \; W_{ng} = \left[\sum_{s=1}^{J} A_{nsg}(w_{ns})^{\kappa}\right]^{\frac{1}{\kappa}} \right\}.$$

When there is only one group, the equilibrium condition becomes $\Omega = \left\{ (w_{nj}, L_{nj}) : \frac{w_{nj} L_{nj}}{W_n \bar{L}_n} = A_{nj} \left(\frac{w_{nj}}{W_n}\right)^{\kappa} \right\}$, which is the same as the condition (20) under imperfect labor substitution with $\alpha_j = A_{nj}$. Thus, the one-group Ricardian-Roy model is a CES supply system.

Generally, the Ricardian-Roy model does not satisfy the conditions of the CES supply system. In particular, the matrix of partial supply elasticity of Foreign country takes the form of

$$\Lambda_2 = \begin{pmatrix} \kappa - (\kappa-1)\frac{\sum_g m_{21g} m_{21g} W_{2g} \bar{L}_{2g}}{Y_{21}} & \ldots & -(\kappa-1)\frac{\sum_g m_{21g} m_{2jg} W_{2g} \bar{L}_{2g}}{Y_{21}} & \ldots & -(\kappa-1)\frac{\sum_g m_{21g} m_{2Jg} W_{2g} \bar{L}_{2g}}{Y_{21}} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -(\kappa-1)\frac{\sum_g m_{2jg} m_{21g} W_{2g} \bar{L}_{2g}}{Y_{2j}} & \ldots & \kappa - (\kappa-1)\frac{\sum_g m_{2jg} m_{2jg} W_{2g} \bar{L}_{2g}}{Y_{2j}} & \ldots & -(\kappa-1)\frac{\sum_g m_{2jg} m_{2Jg} W_{2g} \bar{L}_{2g}}{Y_{2j}} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -(\kappa-1)\frac{\sum_g m_{2Jg} m_{21g} W_{2g} \bar{L}_{2g}}{Y_{2J}} & \ldots & -(\kappa-1)\frac{\sum_g m_{2Jg} m_{2jg} W_{2g} \bar{L}_{2g}}{Y_{2J}} & \ldots & \kappa - (\kappa-1)\frac{\sum_g m_{2Jg} m_{2Jg} W_{2g} \bar{L}_{2g}}{Y_{2J}} \end{pmatrix},$$

where $m_{2jg} = Y_{2jg}/Y_2$ is the share of income in sector j produced by group g workers in Foreign. Take the difference between any two rows of Λ_n, say i and j, and call the resulting vector $d\Lambda_{n,ij}$. Condition (15) of the CES supply system requires all entries of $d\Lambda_{n,ij}$ to be zero, except for the ith or jth entry. Here, none of the entries of $d\Lambda_{n,ij}$ are zero. Thus, the general Ricardian-Roy model fails to satisfy the CES supply requirement.

Specifically, the elasticity of sector i's income on sector j's tariff is given by $-(\kappa - 1)\frac{\sum_g m_{2jg} m_{2ig} W_{2g} \bar{L}_{2g}}{Y_{2j}}$, which depends on the covariance between the two income shares m_{2jg} and m_{2ig} or how likely a group g working in sector j apply to sector i. If the covariance is big, the elasticity is high, and the impact of sector j's tariff on sector i's income is large since workers are more likely to move between the two sectors in response to the tariff in j. This covariance is ultimately determined by the correlation of productivities $\{A_{2jg}\}$. In general, the covariance between sector j and i can differ from that between sector s and i. Thus, the impact of tariffs of sector j and sector s on the third sector i are different, and thus, Ricardian-Roy is not a CES supply system.

We can apply Proposition 1 for optimal taxes. Moreover, even if the condition for the CES supply system is not met, we can still prove that the Home government does not use any domestic taxes, i.e., $\tau_j^d = 0$ for any sector j. The reason is that the Ricardian-Roy model has no labor market frictions. See Appendix G for details.

4.3 Model with inefficient labor markets

Each inefficient labor market model calls for different policies. Here, we study a labor sectorial choice model with preference shock that is widely used in spatial models. Mongey and Waugh (2024) and Donald, Fuku, and Miyauchi (2023) have established the inefficiency of this type of models.

Workers have preferences over different sectors and choose to work in a sector that maximizes their utility, depending both on their preference and the wage in the sector. A country n's worker draws a preference ϵ_{nj} for working in sector j from a Fréchet distribution with κ governing the degree of dispersion across individuals. The law of large numbers ensures the share of workers in sector j given by $\lambda_{nj} = \Pr\left(\epsilon_{nj} w_{nj} \geq \max_s \{\epsilon_{ns} w_{ns}\}\right)$. Under the Fréchet distribution, the labor share becomes $\lambda_{nj} = \frac{(w_{nj})^\kappa}{\sum_{s=1}^J (w_{ns})^\kappa}$ and total employment in sector j of country n is $L_{nj} = \lambda_{nj} \bar{L}_n$. Hence, the equilibrium condition is given by

$$\Omega = \left\{ (w_{nj}, L_{nj}) : \frac{Y_{nj}}{Y_n} = \frac{w_{nj} L_{nj}}{W_n L_n} = \frac{(w_{nj})^{\kappa+1}}{\sum_{s=1}^J (w_{ns})^{\kappa+1}},\ W_n = \frac{\sum_{s=1}^J (w_{ns})^{\kappa+1}}{\sum_{s=1}^J (w_{ns})^\kappa} \right\}. \tag{22}$$

In this case, a typical entry for Λ_2 is $\frac{\partial \ln Y_{2s}}{\partial \ln w_{2j}} \frac{Y_{2s}}{Y_{2j}} = -\kappa m_{2s} \frac{W_2}{w_{2j}}$ for $s \neq j$, and the model does not satisfy the requirement for the CES supply system in Definition 4. Sector-specific tariffs are interlinked. For domestic taxes, with inefficiency in labor markets, Home government has incentives to fix domestic distortions. Suppose Foreign country has zero taxes, including its domestic taxes. The following Corollary shows Home's optimal policies in this environment.

Corollary 1. *Under the inefficient labor market model, Home government's unilateral optimal policy satisfies, for any sectors j, i,*

$$\textit{Domestic tax:} \quad \frac{1+\tau_j^d}{1+\tau_i^d} = \frac{w_{1i}}{w_{1j}} \tag{23}$$

$$\textit{Import tariff:} \quad \tau_j^m - \tau_i^m = \frac{1}{\kappa+1}\left(\frac{\beta_j x_1 - (1+\tau_j^d)Y_{1j}}{Y_{2j}} - \frac{\beta_i x_1 - (1+\tau_i^d)Y_{1i}}{Y_{2i}}\right) \tag{24}$$

$$+\kappa\left(\frac{1}{w_{2j}} - \frac{1}{w_{2i}}\right)\sum_{s=1}^{J}\left[\left(\tau_s^m + \frac{\zeta}{u_c}\right) m_{2s} W_2\right],$$

where ζ is endogenous and the same across sectors, u_c Home's marginal utility.

Proof. Appendix H.

This model differs from Ricardian-Roy, where labor is allocated to sectors according to their revenue productivity, and expected income per worker is equalized, even if wages per efficiency unit of labor differ across sectors. Home government cares about workers' total consumption, which reaches the optimum when workers are allocated among sectors based on their productivity. However, in the market equilibrium, workers also relocate based on their sector preferences. Therefore, as shown in (23), Home government subsidizes sectors with higher wages that result from better technologies in those sectors.

The relative import tariffs look similar to the formula (17) in Proposition 2 under CES supply system. It is still the case that relative Home import shares, $\frac{\beta_j x_1 - (1+\tau_j^d)Y_{1j}}{Y_{2j}} - \frac{\beta_i x_1 - (1+\tau_i^d)Y_{1i}}{Y_{2i}}$, matter for the relative import tariffs across sectors. However, in this case, there is an extra term, which depends on Foreign country's relative wages in the two sectors, $\frac{1}{w_{2j}} - \frac{1}{w_{2i}}$. The reason is that inefficiencies are still present in Foreign without Foreign taxes. These inefficiencies affect Foreign supply elasticities, which in turn shapes Home government's incentives to use import tariffs.

We further consider a case in which Foreign uses domestic taxes to fix its domestic inefficiencies, but Foreign still does not invoke any trade policies. In this case, Foreign's domestic taxes satisfy $\frac{1+\tau_{2j}^d}{1+\tau_{2i}^d} = \frac{w_{2i}}{w_{2j}}$, and the second term at Home's relative import tariffs become zero. Now, the optimal import tariffs satisfy

$$\tau_j^m - \tau_i^m = \frac{1}{\kappa+1}\left(\frac{\beta_j x_1 - (1+\tau_j^d)Y_{1j}}{(1+\tau_{2j}^d)Y_{2j}} - \frac{\beta_i x_1 - (1+\tau_i^d)Y_{1i}}{(1+\tau_{2i}^d)Y_{2i}}\right),$$

which is the same as the efficient case under the CES supply system. See Appendix H.4 for details. Hence, trade policies depend on the Foreign labor market, while domestic policies depend on the domestic labor market.

5 Conclusion

We study optimal policies in a set of Ricardian models with different labor market specifications, which are widely used for quantitative investigations of the labor market impact of trade. We provide a general formula for optimal policies in models with efficient or inefficient labor markets and a specific one for the CES supply system.

We demonstrate that sector-specific optimal tariffs are tied to the matrix of foreign export supply elasticities, which in turn depends on the foreign partial supply elasticities and the share of Home's net import in foreign income, together reflecting Home's import market power. The sector-specific export taxes depend on goods market elasticities and Home's market share in foreign consumption, together reflecting Home's export market power. Home tends to impose higher tariffs on large importing sectors and higher export taxes on its comparative advantage sector, though all taxes are jointly determined.

Other factors, in addition to assumptions on production technology and preferences, can also influence trade and optimal trade policies. These factors include market structure and political and strategic considerations. While extensive literature has examined some of these factors, our paper specifically concentrates on examining the implications of the labor market for trade policy. Using a simple unified setup, we aim to provide a clear and explicit understanding of how different labor market assumptions matter in optimal taxation.

References

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare. 2012. "New trade models, same old gains?" *American Economic Review* 102 (1):94–130.

Bai, Yan, Keyu Jin, and Dan Lu. 2023. "Technological Rivalry and Optimal Dynamic Policy in an Open Economy." Working Paper 31703, National Bureau of Economic Research. URL `http://www.nber.org/papers/w31703`.

Caliendo, Lorenzo and Fernando Parro. 2021. "Trade policy." .

Costinot, Arnaud, Dave Donaldson, and Ivana Komunjer. 2012. "What goods do countries trade? A quantitative exploration of Ricardo's ideas." *The Review of economic studies* 79 (2):581–608.

Costinot, Arnaud, Dave Donaldson, Jonathan Vogel, and Iván Werning. 2015. "Comparative advantage and optimal trade policy." *The Quarterly Journal of Economics* 130 (2):659–702.

Costinot, Arnaud, Andrés Rodríguez-Clare, and Iván Werning. 2020. "Micro to macro: Optimal trade policy with firm heterogeneity." *Econometrica* 88 (6):2739–2776.

Costinot, Arnaud and Jonathan Vogel. 2010. "Matching and inequality in the world economy." *Journal of Political Economy* 118 (4):747–786.

———. 2015. "Beyond Ricardo: Assignment models in international trade." *Annual Review of Economics* 7 (1):31–62.

Dixit, Avinash. 1985. "Tax policy in open economies." In *Handbook of public economics*, vol. 1. Elsevier, 313–374.

Donald, Eric, M Fuku, and Yuhei Miyauchi. 2023. "Unpacking Aggregate Welfare in a Spatial Economy." Tech. rep.

Dornbusch, Rudiger, Stanley Fischer, and Paul Anthony Samuelson. 1977. "Comparative advantage, trade, and payments in a Ricardian model with a continuum of goods." *The American Economic Review* 67 (5):823–839.

Eaton, Jonathan and Samuel Kortum. 2002. “Technology, geography, and trade.” *Econometrica* 70 (5):1741–1779.

Fajgelbaum, Pablo, Pinelopi Goldberg, Patrick Kennedy, Amit Khandelwal, and Daria Taglioni. 2024. “The US-China trade war and global reallocations.” *American Economic Review: Insights* 6 (2):295–312.

Galle, Simon, Andrés Rodríguez-Clare, and Moises Yi. 2023. “Slicing the pie: Quantifying the aggregate and distributional effects of trade.” *The Review of Economic Studies* 90 (1):331–375.

Hsieh, Chang-Tai, Erik Hurst, Charles I Jones, and Peter J Klenow. 2019. “The allocation of talent and us economic growth.” *Econometrica* 87 (5):1439–1474.

Lagakos, David and Michael E Waugh. 2013. “Selection, agriculture, and cross-country productivity differences.” *American Economic Review* 103 (2):948–980.

Lashkaripour, Ahmad and Volodymyr Lugovskyy. 2023. “Profits, scale economies, and the gains from trade and industrial policy.” *American Economic Review* 113 (10):2759–2808.

Mongey, Simon and Michael E Waugh. 2024. “Discrete choice, complete markets, and equilibrium.” Tech. rep., National Bureau of Economic Research.

Waugh, Michael E. 2023. “Heterogeneous Agent Trade.” Tech. rep., National Bureau of Economic Research.

Online Appendix to "Optimal Trade Policies and Labor Markets "

by Yan Bai, Dan Lu, and Hanxi Wang

This appendix is organized as follows.

A Proof for Lemma 1: policy and multipliers

Home government chooses $\{\tau_j^d, \tau_j^x, \tau_j^m, x_1, L_{1j}, L_{2j}, w_{1j}, w_{2j}\}$ to solve the following problem:

$$\max \ u\left(\frac{x_1}{P_1}\right),$$

subject to world market equilibrium characterized by the following constraints:

$$x_1 = \sum_j^J w_{1j}L_{1j} + \sum_{j=1}^J \beta_j \frac{\tau_j^x}{1+\tau_j^x}\pi_{21,j}x_2 + \sum_{j=1}^J \beta_j \frac{\tau_j^m}{1+\tau_j^m}\pi_{12,j}x_1 + \sum_{j=1}^J \beta_j \frac{\tau_j^d}{1+\tau_j^d}\pi_{11,j}x_1, \quad (\gamma_x)$$

$$w_{1j}L_{1j} = \beta_j\left[\frac{1}{1+\tau_j^d}\pi_{11,j}x_1 + \frac{1}{1+\tau_j^x}\pi_{21,j}x_2\right], \quad (\gamma_{1j}, \quad J)$$

$$w_{2j}L_{2j} = \beta_j\left[\frac{1}{1+\tau_j^m}\pi_{12,j}x_1 + \pi_{22,j}x_2\right], \quad (\gamma_{2j}, \quad J-1)$$

and the labor market clearing conditions $\Omega(\{w_{nj}, L_{nj}\}$ determined by the labor market specification, where $\{x_2, P_1, P_2\}$ and trade shares are given by

$$
\begin{aligned}
x_2 &= \sum_{j=1}^{J} w_{2j} L_{2j}, \\
P_1 &= \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = 1, \\
P_2 &= \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j} w_{2j}^{-\theta} \right]^{-\frac{\beta_j}{\theta}}, \\
\pi_{11,j} &= \frac{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}, \\
\pi_{12,j} &= \frac{T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}, \\
\pi_{21,j} &= \frac{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}(w_{2j})^{-\theta}}, \\
\pi_{22,j} &= \frac{T_{2j}(w_{2j})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}(w_{2j})^{-\theta}}.
\end{aligned}
$$

Optimal conditions:

We use first-order conditions (FOC) to characterize the Home government's problem and link the optimal policies with multipliers on goods market clearing conditions.

FOC over x_1

$$
u_c + \sum_{j=1}^{J} \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\pi_{11,j} + \sum_{j=1}^{J} \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\pi_{12,j} - \gamma_x \left(1 - \sum_{j=1}^{J} \beta_j \frac{\tau_j^m}{1+\tau_j^m}\pi_{12,j} - \sum_{j=1}^{J} \beta_j \frac{\tau_j^d}{1+\tau_j^d}\pi_{11,j} \right) = 0
$$

FOC over export tax τ_j^x

$$
\begin{aligned}
&\gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial \tau_j^x} x_1 - \gamma_{1j}\beta_j \frac{1}{(1+\tau_j^x)^2}\pi_{21,j}x_2 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial \tau_j^x} x_2 \\
&+ \gamma_{2j}\beta_j \left[\frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial \tau_j^x} x_1 + \frac{\partial \pi_{22,j}}{\partial \tau_j^x} x_2 \right] + \gamma_x \beta_j \frac{1}{(1+\tau_j^x)^2}\pi_{21,j}x_2 + \gamma_x \beta_j \frac{\tau_j^x}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial \tau_j^x} x_2 = 0
\end{aligned}
$$

after plugging in derivatives and simplifying, the FOC over τ_j^x becomes

$$
1+\tau_j^x = \frac{(\gamma_x - \gamma_{1j})(\theta \pi_{22,j} + 1)}{(\gamma_x - \gamma_{2j})\,\theta \pi_{22,j}}. \tag{A.1}
$$

FOC over import tariff τ_j^m

$$
\begin{aligned}
& -u_c x_1 \beta_j \frac{\pi_{12,j}}{1+\tau_j^m} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial \tau_j^m} x_1 - \gamma_{2j}\beta_j \frac{\pi_{12,j}}{\left(1+\tau_j^m\right)^2} x_1 + \gamma_{2j}\frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial \tau_{n,j}^m} x_1 \\
& + \gamma_x \beta_j \frac{1}{\left(1+\tau_j^m\right)^2}\pi_{12,j} x_1 + \gamma_x \beta_j \frac{\tau_j^m}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial \tau_j^m} x_1 + \gamma_x \beta_j \frac{\tau_j^d}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial \tau_j^m} x_1 = 0
\end{aligned}
$$

after plugging in derivatives and simplifying, the FOC over τ_j^m becomes

$$
1+\tau_j^m = \frac{\left(\gamma_x - \gamma_{2j}\right)\left(1+\theta\pi_{11,j}\right)}{\left(\gamma_x - \gamma_{1j}\right)\frac{1}{1+\tau_j^d}\theta\pi_{11,j} + u_c}. \tag{A.2}
$$

FOC over domestic tax τ_j^d

$$
\begin{aligned}
& -u_c x_1 \beta_j \frac{\pi_{11,j}}{1+\tau_j^d} - \gamma_{1j}\frac{1}{\left(1+\tau_j^d\right)^2}\beta_j \pi_{11,j} x_1 + \gamma_{1j}\frac{1}{1+\tau_j^d}\beta_j \frac{\partial \pi_{11,j}}{\partial \tau_j^d} x_1 + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial \tau_j^d} x_1 \\
& + \gamma_x \beta_j \frac{\tau_j^m}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial \tau_j^d} x_1 + \gamma_x \beta_j \frac{1}{\left(1+\tau_j^d\right)^2}\pi_{11,j} x_1 + \gamma_x \beta_j \frac{\tau_j^d}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial \tau_j^d} x_1 = 0
\end{aligned}
$$

after plugging in derivatives and simplifying, the FOC over τ_j^d becomes

$$
1+\tau_j^d = \frac{\left(\gamma_x - \gamma_{1j}\right)\left(1+\theta\pi_{12,j}\right)}{\left(\gamma_x - \gamma_{2j}\right)\frac{1}{1+\tau_j^m}\theta\pi_{12,j} + u_c}. \tag{A.3}
$$

Combining the optimal tariff (A.2) and domestic tax (A.3), we can get

$$
1+\tau_j^m = \frac{\gamma_x - \gamma_{2j}}{u_c}, \qquad 1+\tau_j^d = \frac{\gamma_x - \gamma_{1j}}{u_c}. \tag{A.4}
$$

Combining the above two equations with equation (A.1), we get

$$
1+\tau_j^x = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\theta\pi_{22,j}}\right). \tag{A.5}
$$

We show later that tax neutrality holds. Thus, we can set $\gamma_x = u_c$, which implies $\tau_j^m = -\gamma_{2j}/u_c$ and $\tau_j^d = -\gamma_{1j}/u_c$. $\square$

B Proof for Proposition 1: general formula

FOC over w_{1j}

$$
\begin{aligned}
&-u_c x_1 \beta_j \frac{\pi_{11,j}}{w_{1j}} - \gamma_{1j} L_{1j} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{1j}} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial w_{1j}} x_2 \\
&+ \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{1j}} x_1 + \gamma_{2j}\beta_j \frac{\partial \pi_{22,j}}{\partial w_{1j}} x_2 + \gamma_x L_{1j} + \gamma_x \beta_j \frac{\tau_j^x}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial w_{1j}} x_2 + \gamma_x \beta_j \frac{\tau_j^m}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{1j}} x_1 \\
&+ \gamma_x \beta_j \frac{\tau_j^d}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{1j}} x_1 + A_j / w_{1j} = 0
\end{aligned}
\tag{A.6}
$$

Where $A_j = \sum_{k=1}^J (\gamma_x - \gamma_{1k}) w_{1k} \frac{\partial L_{1k}}{\partial w_{1j}} w_{1j} = \sum_{k=1}^J (\gamma_x - \gamma_{1k}) \frac{\partial w_{1k} L_{1k}}{\partial w_{1j}} w_{1j} - (\gamma_x - \gamma_{1j}) w_{1j} L_{1j}$.

FOC over w_{2j}

$$
\begin{aligned}
&-u_c x_1 \beta_j \frac{\pi_{12,j}}{w_{2j}} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{2j}} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial w_{2j}} x_2 + \sum_{k=1}^J \gamma_{1k}\beta_k \frac{1}{1+\tau_k^x}\pi_{21,k} L_{2j} \\
&- \gamma_{2j} L_{2j} + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{2j}} x_1 + \gamma_{2j}\beta_j \frac{\partial \pi_{22,j}}{\partial w_{2j}} x_2 + \sum_{k=1}^J \gamma_{2k}\beta_k \pi_{22,k} L_{2j} \\
&+ \gamma_x \beta_j \frac{\tau_j^x}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial w_{2j}} x_2 + \gamma_x \sum_{k=1}^J \beta_k \frac{\tau_k^x}{1+\tau_{x2}^k}\pi_{21,k} L_{2j} + \gamma_x \beta_j \frac{\tau_j^m}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{2j}} x_1 + \gamma_x \beta_j \frac{\tau_j^d}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{2j}} x_1 + B_j / w_{2j} = 0
\end{aligned}
\tag{A.7}
$$

Where

$$
B_j = \sum_{k=1}^J (\eta - \gamma_{2k}) w_{2k} \frac{\partial L_{2k}}{\partial w_{2j}} w_{2j} = \sum_{k=1}^J (\eta - \gamma_{2k}) \frac{\partial w_{2k} L_{2k}}{\partial w_{2j}} w_{2j} - (\eta - \gamma_{2j}) w_{2j} L_{2j},
$$

with $\eta = \sum_{k=1}^J (\gamma_x \beta_k \frac{\tau_k^x}{1+\tau_k^x}\pi_{21,k} + \gamma_{1k}\beta_k \frac{1}{1+\tau_k^x}\pi_{21,k} + \gamma_{2k}\beta_k \pi_{22,k})$.

B.1 Proof of $A_j = 0$ and optimal domestic policies

Combining FOCs over w_{1j} (A.6), τ_j^x(A.5), and τ_j^m (A.4), we get

$$
-u_c x_1 \beta_j + (\gamma_x - \gamma_{1j}) w_{1j} L_{1j} + (\gamma_x - \gamma_{2j})\beta_j \frac{1}{1+\tau_j^m}\pi_{12,j} x_1 - (\gamma_x - \gamma_{1j})\beta_j \frac{1}{1+\tau_j^x}\pi_{21}^j x_2 + A_j = 0
$$

hence,

$$
-u_c x_1 \beta_j + (\gamma_x - \gamma_{2j})\beta_j \frac{1}{1+\tau_j^m}\pi_{12,j} x_1 + (\gamma_x - \gamma_{1j})\beta_j \frac{1}{1+\tau_j^d}\pi_{11,j} x_1 + A_j = 0.
$$

Using the optimal policies $1+\tau_j^m = \frac{\gamma_x - \gamma_{2j}}{u_c}$ and $1+\tau_j^d = \frac{\gamma_x - \gamma_{1j}}{u_c}$, we arrive at

$$-u_c x_1 \beta_j \pi_{12,j} + u_c x_1 \beta_j \pi_{12,j} + A_j = 0.$$

which implies $A_j = 0$. Now let us revisit the definition of A_j:

$$\sum_{k=1}^{J} (\gamma_x - \gamma_{1k}) \frac{\partial w_{1k} L_{1k}}{\partial w_{1j}} w_{1j} - (\gamma_x - \gamma_{1j}) w_{1j} L_{1j} = 0.$$

Plugging the optimal formula for τ_j^d in (A.4) to the above equation, we can further show that

$$\sum_{k=1}^{J} (1+\tau_k^d) \frac{\partial \ln(w_{1k} L_{1k})}{\partial \ln(w_{1j})} \frac{w_{1k} L_{1k}}{w_{1j} L_{1j}} - (1+\tau_j^d) = 0.$$

Hence, domestic taxes $\tau^d = [\tau_1^d, ..., \tau_j^d, ..., \tau_J^d]'$ for J sectors satisfy

$$(\Lambda_1 - I) \cdot (1+\tau^d) = 0,$$

where element of matrix Λ_n at row j and column k is given by $\frac{\partial \ln(Y_{nk})}{\partial \ln(w_{nj})} \frac{Y_{nk}}{Y_{nj}}$, and

$$\Lambda_1 - I = \begin{pmatrix} \frac{\partial ln(w_{11}L_{11})}{\partial ln(w_{11})} \frac{w_{11}L_{11}}{w_{11}L_{11}} - 1 & ... & \frac{\partial ln(w_{1j}L_{1j})}{\partial ln(w_{11})} \frac{w_{1j}L_{1j}}{w_{11}L_{11}} & ... & \frac{\partial ln(w_{1J}L_{1J})}{\partial ln(w_{11})} \frac{w_{1J}L_{1J}}{w_{11}L_{11}} \\ ... & ... & ... & ... & ... \\ \frac{\partial ln(w_{11}L_{11})}{\partial ln(w_{1j})} \frac{w_{11}L_{11}}{w_{1j}L_{1j}} & ... & \frac{\partial ln(w_{1j}L_{1j})}{\partial ln(w_{1j})} \frac{w_{1j}L_{1j}}{w_{1j}L_{1j}} - 1 & ... & \frac{\partial ln(w_{1J}L_{1J})}{\partial ln(w_{1j})} \frac{w_{1J}L_{1J}}{w_{1j}L_{1j}} \\ ... & ... & ... & ... & ... \\ \frac{\partial ln(w_{11}L_{11})}{\partial ln(w_{1J})} \frac{w_{11}L_{11}}{w_{1J}L_{1J}} & ... & \frac{\partial ln(w_{1j}L_{1j})}{\partial ln(w_{1J})} \frac{w_{1j}L_{1j}}{w_{1J}L_{1J}} & ... & \frac{\partial ln(w_{1J}L_{1J})}{\partial ln(w_{1J})} \frac{w_{1J}L_{1J}}{w_{1J}L_{1J}} - 1 \end{pmatrix}.$$

B.2 Proof of optimal import tariffs

Combining FOCs over w_{1j} (A.6) and w_{2j} (A.7), we get

$$-u_c x_1 \beta_j + (\gamma_x - \gamma_{1j}) w_{1j} L_{1j} + (\eta - \gamma_{2j}) w_{2j} L_{2j} + B_j = 0.$$

Plugging in the formula of B_j and combining terms, we get

$$\sum_{k=1}^{J} (\frac{\zeta}{u_c} + \tau_k^m) \frac{\partial \ln(w_{2k} L_{2k})}{\partial \ln(w_{2j})} \frac{w_{2k} L_{2k}}{w_{2j} L_{2j}} = \frac{x_1 \beta_j - (1+\tau_j^d) w_{1j} L_{1j}}{w_{2j} L_{2j}},$$

where $\zeta = \eta - \gamma_x + u_c$ is common across all the sectors.

Hence, import tariffs $\tau^m = [\tau_1^m, ..., \tau_j^m, ..., \tau_J^m]'$ for J sectors satisfy

$$\Lambda_2(\frac{\zeta}{u_c} + \tau^m) = \begin{pmatrix} \frac{x_1\beta_1 - (1+\tau_1^d)w_{11}L_{11}}{w_{21}L_{21}} \\ ... \\ \frac{x_1\beta_j - (1+\tau_j^d)w_{1j}L_{1j}}{w_{2j}L_{2j}} \\ ... \\ \frac{x_1\beta_J - (1+\tau_J^d)w_{1,J}L_{1,J}}{w_{2J}L_{2J}} \end{pmatrix} = \Psi,$$

where the element of matrix Λ_2 at row j and column i is given by $\frac{\partial \ln(w_{2i}L_{2i})}{\partial \ln(w_{2j})}\frac{w_{2i}L_{2i}}{w_{2j}L_{2j}}$, i.e.,

$$\Lambda_2 = \begin{pmatrix} \frac{\partial ln(w_{21}L_{21})}{\partial ln(w_{21})}\frac{w_{21}L_{21}}{w_{21}L_{21}} & ... & \frac{\partial ln(w_{2i}L_{2i})}{\partial ln(w_{21})}\frac{w_{2i}L_{2i}}{w_{21}L_{21}} & ... & \frac{\partial ln(w_{2J}L_{2J})}{\partial ln(w_{21})}\frac{w_{2J}L_{2J}}{w_{21}L_{21}} \\ ... & ... & ... & ... & ... \\ \frac{\partial ln(w_{21}L_{21})}{\partial ln(w_{2j})}\frac{w_{21}L_{21}}{w_{2j}L_{2j}} & ... & \frac{\partial ln(w_{2i}L_{2i})}{\partial ln(w_{2j})}\frac{w_{2i}L_{2i}}{w_{2j}L_{2j}} & ... & \frac{\partial ln(w_{2J}L_{2J})}{\partial ln(w_{2j})}\frac{w_{2J}L_{2J}}{w_{2j}L_{2j}} \\ ... & ... & ... & ... & ... \\ \frac{\partial ln(w_{21}L_{21})}{\partial ln(w_{2J})}\frac{w_{21}L_{21}}{w_{2J}L_{2J}} & ... & \frac{\partial ln(w_{2i}L_{2i})}{\partial ln(w_{2J})}\frac{w_{2i}L_{2i}}{w_{2J}L_{2J}} & ... & \frac{\partial ln(w_{2J}L_{2J})}{\partial ln(w_{2J})}\frac{w_{2J}L_{2J}}{w_{2J}L_{2J}} \end{pmatrix}.$$

□

C Proof of Proposition 2: optimal policy under CES supply system

Proof of domestic policies As shown in Proposition 1, $\forall k, j$,

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{1s}}{\partial \ln w_{1j}} \frac{Y_{1s}}{Y_{1j}} (1 + \tau_s^d) = (1 + \tau_j^d), \tag{A.8}$$

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{1s}}{\partial \ln w_{1k}} \frac{Y_{1s}}{Y_{1k}} (1 + \tau_s^d) = (1 + \tau_k^d). \tag{A.9}$$

Subtracting (A.9) from (A.8) and using Definition 4, we have

$$0 + \left(\frac{\partial \ln Y_j}{\partial \ln w_j} - \frac{\partial \ln Y_j}{\partial \ln w_k}\frac{Y_j}{Y_k}\right)(\tau_j^d) + \left(\frac{\partial \ln Y_k}{\partial \ln w_j}\frac{Y_k}{Y_j} - \frac{\partial \ln Y_k}{\partial \ln w_k}\right)(\tau_k^d) = \tau_j^d - \tau_k^d.$$

Thus, $\tau_j^d = \tau_k^d$, and Home does not use differential domestic taxes across sectors.

Proof of optimal tariffs As shown in Proposition 1, $\forall k, j$, import tariffs satisfy

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{2s}}{\partial \ln w_{2j}} \frac{Y_{2s}}{Y_{2j}} \left(\frac{\zeta}{u_c} + \tau_s^m \right) = \Psi_j, \tag{A.10}$$

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{2s}}{\partial \ln w_{2k}} \frac{Y_{2s}}{Y_{2k}} \left(\frac{\zeta}{u_c} + \tau_s^m \right) = \Psi_k. \tag{A.11}$$

Subtracting (A.11) from (A.10) and using Definition 4, we have

$$0 + \left(\frac{\partial \ln Y_{2j}}{\partial \ln w_{2j}} - \frac{\partial \ln Y_{2j}}{\partial \ln w_{2k}} \frac{Y_{2j}}{Y_{2k}} \right) \left(\frac{\zeta}{u_c} + \tau_j^m \right) + \left(\frac{\partial \ln Y_{2k}}{\partial \ln w_{2j}} \frac{Y_{2k}}{Y_{2j}} - \frac{\partial \ln Y_{2k}}{\partial \ln w_{2k}} \right) \left(\frac{\zeta}{u_c} + \tau_k^m \right) = \Psi_j - \Psi_k,$$

where Ψ_j and Ψ_k are the jth and kth row of vector Ψ. This implies $\tau_j^m - \tau_k^m = \frac{\Psi_j - \Psi_k}{\kappa}$. $\square$

D Optimal policy with perfectly mobile labor across sectors

With mobile labor, we can sum over labor markets across sectors. Now, the market clearing conditions become:

$$w_1 L_1 = \sum_{j=1}^{J} \beta_j \left[\frac{1}{1+\tau_j^d} \pi_{11,j} x_1 + \frac{1}{1+\tau_j^x} \pi_{21,j} x_2 \right], \quad (\gamma_1)$$

$$w_2 L_2 = \sum_{j=1}^{J} \beta_j \left[\frac{1}{1+\tau_j^m} \pi_{12,j} x_1 + \pi_{22,j} x_2 \right], \quad (\gamma_2)$$

$$x_1 = w_1 L_1 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^x}{1+\tau_j^x} \pi_{21,j} x_2 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1, \quad (\gamma_x)$$

where $x_2 = w_2 L_2$.

Since the multipliers γ_1 and γ_2 are not sector-specific, the optimal policy following equations (A.4) becomes $1 + \tau_j^m = \frac{\gamma_x - \gamma_2}{u_c} = 1 + \tau^m$ and $1 + \tau_j^d = \frac{\gamma_x - \gamma_1}{u_c} = 1 + \tau^d$. Thus, domestic tax or tariffs are also not independent of sectors. Furthermore, the following subsection shows tax neutrality, and both the uniform tariff and domestic tax can be normalized to zero.

D.1 Proof of tax neutrality

Given $\Gamma = \{(\tau^m+1, \tau_j^x+1, \tau^d+1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}^m+1, \check{\tau}_j^x+1, \check{\tau}^d+1 : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_n}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1+\check{\tau}^m = \lambda(1+\tau^m)$, $1+\check{\tau}_j^x = \frac{1+\tau_j^x}{\mu}$ and $1+\check{\tau}^d = \lambda\frac{1+\tau^d}{\mu}$, for any constants $\mu > 0$ and $\lambda > 0$ for any j. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_n}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_1 = \mu\frac{w_1}{\lambda}$, $\check{w}_n = \frac{w_n}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{w}_1 L_1 = \sum_{j=1}^{J} \beta_j \left[\frac{\mu}{\lambda(1+\tau^d)}\pi_{11,j}x_1 + \frac{\mu}{1+\tau_j^x}\pi_{21,j}\frac{x_2}{\lambda}\right] = \frac{\mu}{\lambda}w_1 L_1 \tag{A.12}$$

$$\check{w}_2 L_2 = \sum_{j=1}^{J} \beta_j \left[\frac{1}{\lambda(1+\tau^m)}\pi_{12,j}x_1 + \pi_{22,j}\frac{x_2}{\lambda}\right] = \frac{w_2}{\lambda}L_2 \tag{A.13}$$

$$\begin{aligned}
\check{x}_1 &= \mu\frac{w_1}{\lambda}L_1 + \sum_{j=1}^{J}\beta_j(1-\frac{\mu}{1+\tau_j^x})\pi_{21,j}\frac{x_2}{\lambda} + \sum_{j=1}^{J}\beta_j(1-\frac{1}{\lambda(1+\tau^m)})\pi_{12,j}x_1 + \sum_{j=1}^{J}\beta_j(1-\frac{\mu}{\lambda(1+\tau^d)})\pi_{11,j}x_1 \\
&= \sum_{j=1}^{J}\frac{\mu}{\lambda}\beta_j\left[\frac{1}{1+\tau^d}\pi_{11,j}x_1 + \frac{1}{1+\tau_j^x}\pi_{21,j}x_2\right] + \sum_{j=1}^{J}\beta_j(1-\frac{\mu}{1+\tau_j^x})\pi_{21,j}\frac{x_2}{\lambda} + \sum_{j=1}^{J}\beta_j(1-\frac{1}{\lambda(1+\tau^m)})\pi_{12,j}x_1 \\
&+ \sum_{j=1}^{J}\beta_j(1-\frac{\mu}{\lambda(1+\tau^d)})\pi_{11,j}x_1 \\
&= \sum_{j=1}^{J}\beta_j\pi_{21,j}\frac{x_2}{\lambda} + \sum_{j=1}^{J}\beta_j(1-\frac{1}{\lambda(1+\tau^m)})\pi_{12,j}x_1 + \sum_{j=1}^{J}\beta_j\pi_{11,j}x_1 = \sum_{j=1}^{J}\beta_j\pi_{12,j}x_1 + \sum_{j=1}^{J}\beta_j\pi_{11,j}x_1 = x_1
\end{aligned}$$

$$\check{x}_2 = \frac{w_2}{\lambda} L_2 = \frac{x_2}{\lambda} \tag{A.14}$$

$$\check{P}_1 = \Pi_j \left[T_{1j} (\mu \frac{w_1}{\lambda} \lambda \frac{1+\tau^d}{\mu})^{-\theta} + T_{2j} (\frac{w_2}{\lambda} \lambda (1+\tau^m) d_{12})^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = P_1 = 1 \tag{A.15}$$

$$\check{P}_2 = \Pi_j \left[T_{1j} (\mu \frac{w_1}{\lambda} \frac{1+\tau_j^x}{\mu} d_{21})^{-\theta} + T_{2j} (\frac{w_2}{\lambda})^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = \frac{P_2}{\lambda} \tag{A.16}$$

$$\check{\pi}_{11,j} = \frac{T_{1j} (\mu \frac{w_1}{\lambda} \lambda \frac{1+\tau^d}{\mu})^{-\theta}}{T_{1j} (\mu \frac{w_1}{\lambda} \lambda \frac{1+\tau^d}{\mu})^{-\theta} + T_{2j} (\frac{w_2}{\lambda} \lambda (1+\tau^m) d_{12})^{-\theta}} = \pi_{11,j} \tag{A.17}$$

$$\check{\pi}_{21,j} = \frac{T_{1j} (\mu \frac{w_1}{\lambda} \frac{1+\tau_j^x}{\mu} d_{21})^{-\theta}}{T_{1j} (\mu \frac{w_1}{\lambda} \frac{1+\tau_j^x}{\mu} d_{21})^{-\theta} + T_{2j} (\frac{w_2}{\lambda})^{-\theta}} = \pi_{21,j} \tag{A.18}$$

The same allocations satisfy the equilibrium conditions under $\{\check{\tau}^m + 1, \check{\tau}_j^x + 1, \check{\tau}^d + 1\}$, hence we proved the allocations and welfare are the same under $\{\tau^m + 1, \tau_j^x + 1, \tau^d + 1\}$ and $\{\check{\tau}^m + 1, \check{\tau}_j^x + 1, \check{\tau}^d + 1\}$. For any policy Γ, we can set $\frac{\mu}{\lambda} = 1 + \tau^d$ and obtain $\check{\tau}^d = 0$. Then $\gamma_x - u_c = \gamma_1$. Second, by further adjusting λ, we can get $\gamma_1 = 0$, $\gamma_x = u_c$ and $\tau^m = 0$. From equation A.1, $\tau_j^x = \frac{1}{\theta \pi_{22}}$. □

E Optimal policies with immobile/sector-specific labor

In this case, labor is fixed in each sector, and goods market clearing conditions are sector-specific:

$$\begin{aligned}
w_{1j} \bar{L}_{1j} &= \beta_j \left[\frac{1}{1+\tau_j^d} \pi_{11,j} x_1 + \frac{1}{1+\tau_j^x} \pi_{21,j} x_2 \right], \quad (\gamma_{1j}, \quad J) \\
w_{2j} \bar{L}_{2j} &= \beta_j \left[\frac{1}{1+\tau_j^m} \pi_{12,j} x_1 + \pi_{22,j} x_2 \right], \quad (\gamma_{2j}, \quad J-1)
\end{aligned}$$

where

$$\begin{aligned}
x_1 &= \sum_{j=1}^J w_{1j} \bar{L}_{1j} + \sum_{j=1}^J \beta_j \frac{\tau_j^x}{1+\tau_j^x} \pi_{21,j} x_2 + \sum_{j=1}^J \beta_j \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \sum_{j=1}^J \beta_j \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1 \quad (\gamma_x), \\
x_2 &= \sum_{j=1}^J w_{2j} \bar{L}_{2j}
\end{aligned}$$

and trade shares and prices satisfy equations (3) - (8).

E.1 Proof of tax neutrality and $\tau_j^d = 0$

Given $\Gamma = \{(\tau_j^m+1, \tau_j^x+1, \tau_j^d+1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1) : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_{nj}}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We define that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. Hence tax neutrality reflects the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1+\check{\tau}_j^m = \lambda(1+\tau_j^m)$, $1+\check{\tau}_j^x = \frac{1+\tau_j^x}{\mu_j}$ and $1+\check{\tau}_j^d = \lambda\frac{1+\tau_j^d}{\mu_j}$, for any constants $\mu_j > 0$ and $\lambda > 0$ for any j. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_{nj}}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_{1j} = \mu_j\frac{w_{1j}}{\lambda}$, $\check{w}_{nj} = \frac{w_{nj}}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{w}_{1j}\bar{L}_{1j} = \beta_j\left[\frac{\mu_j}{\lambda(1+\tau_j^d)}\pi_{11,j}x_1 + \frac{\mu_j}{1+\tau_j^x}\pi_{21,j}\frac{x_2}{\lambda}\right] = \frac{\mu_j}{\lambda}w_{1j}\bar{L}_{1j}$$
$$\check{w}_{2j}\bar{L}_{2j} = \beta_j\left[\frac{1}{\lambda(1+\tau_j^m)}\pi_{12,j}x_1 + \pi_{22,j}\frac{x_2}{\lambda}\right] = \frac{w_{2j}}{\lambda}\bar{L}_{2j}$$

$$\check{x}_2 = \sum_{j=1}^J\frac{w_{2j}}{\lambda}\bar{L}_{2j} = \frac{x_2}{\lambda} \tag{A.19}$$
$$\begin{aligned}
\check{x}_1 &= \sum_{j=1}^J\mu_j\frac{w_{1j}}{\lambda}\bar{L}_{1j} + \sum_{j=1}^J\beta_j(1-\frac{\mu_j}{1+\tau_j^x})\pi_{21,j}\frac{x_2}{\lambda} + \sum_{j=1}^J\beta_j(1-\frac{1}{\lambda(1+\tau_j^m)})\pi_{12,j}x_1 + \sum_{j=1}^J\beta_j(1-\frac{\mu_j}{\lambda(1+\tau_j^d)})\pi_{11,j}x_1 \\
&= \sum_{j=1}^J\frac{\mu_j}{\lambda}\beta_j\left[\frac{1}{1+\tau_j^d}\pi_{11,j}x_1 + \frac{1}{1+\tau_j^x}\pi_{21,j}x_2\right] + \sum_{j=1}^J\beta_j(1-\frac{\mu_j}{1+\tau_j^x})\pi_{21,j}\frac{x_2}{\lambda} + \sum_{j=1}^J\beta_j(1-\frac{1}{\lambda(1+\tau_j^m)})\pi_{12,j}x_1 \\
&+ \sum_{j=1}^J\beta_j(1-\frac{\mu_j}{\lambda(1+\tau_j^d)})\pi_{11,j}x_1 \\
&= \sum_{j=1}^J\beta_j\pi_{21,j}\frac{x_2}{\lambda} + \sum_{j=1}^J\beta_j(1-\frac{1}{\lambda(1+\tau_j^m)})\pi_{12,j}x_1 + \sum_{j=1}^J\beta_j\pi_{11,j}x_1 = \sum_{j=1}^J\beta_j\pi_{12,j}x_1 + \sum_{j=1}^J\beta_j\pi_{11,j}x_1 = x_1,
\end{aligned}$$

where the second to last equation used balanced trade condition. We can further prove the following equalities:

$$\check{P}_1 = \Pi_j \left[T_{1j}(\mu_j \frac{w_{1j}}{\lambda} \lambda \frac{1+\tau_j^d}{\mu_j})^{-\theta} + T_{2j}(\frac{w_{2j}}{\lambda} \lambda (1+\tau_j^m) d_{12})^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = P_1 = 1 \tag{A.20}$$

$$P_2 = \Pi_j \left[T_{1j}(\mu_j \frac{w_{1j}}{\lambda} \frac{1+\tau_j^x}{\mu_j} d_{21})^{-\theta} + T_{2j}(\frac{w_{2j}}{\lambda})^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = \frac{P_2}{\lambda} \tag{A.21}$$

$$\check{\pi}_{11,j} = \frac{T_{1j}(\mu_j \frac{w_1}{\lambda} \lambda \frac{1+\tau_j^d}{\mu_j})^{-\theta}}{T_{1j}(\mu_j \frac{w_1}{\lambda} \lambda \frac{1+\tau_j^d}{\mu_j})^{-\theta} + T_{2j}(\frac{w_{2j}}{\lambda} \lambda (1+\tau_j^m) d_{12})^{-\theta}} = \pi_{11,j} \tag{A.22}$$

$$\check{\pi}_{21,j} = \frac{T_{1j}(\mu_j \frac{w_1}{\lambda} \frac{1+\tau_j^x}{\mu_j} d_{21})^{-\theta}}{T_{1j}(\mu_j \frac{w_1}{\lambda} \frac{1+\tau_j^x}{\mu_j} d_{21})^{-\theta} + T_{2j}(\frac{w_{2j}}{\lambda})^{-\theta}} = \pi_{21,j}. \tag{A.23}$$

The same allocations satisfy the equilibrium conditions under $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$, hence we proved the allocations and welfare are the same under $\{\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1\}$ and $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$. For any policy Γ, we can set $\mu_j = \lambda(1+\tau_j^d)$ and obtain $\check{\tau}_j^d = 0$. Home can also scale up τ_j^d and τ_j^x to any level; this only leads to change in w_{1j} and has no real impacts.

E.2 Proof of optimal tariffs

In this case, $\Lambda_2 = I$ becomes the identity matrix. We subtract row k from row j in matrix Λ_2, assuming $k < j$. The resulting row j is: $\begin{pmatrix} 0 & ... & -1 & ... & 1 & ... & 0 \end{pmatrix}$. For any sector j and k, we can show

$$\begin{aligned} \tau_j^m - \tau_k^m = & \frac{\beta_j x_1 - (1+\tau_j^d) w_{1j} \bar{L}_{1j}}{w_{2j} \bar{L}_{2j}} - \frac{\beta_k x_1 - (1+\tau_k^d) w_{1k} \bar{L}_{1k}}{w_{2k} \bar{L}_{2k}} \\ = & \frac{\beta_j x_1 - (1+\tau_j^d) Y_{1j}}{Y_{2j}} - \frac{\beta_k x_1 - (1+\tau_k^d) Y_{1k}}{Y_{2k}}. \end{aligned}$$

Thus, Home's import tariff imposed on good j increases with the share of Home net imports in Foreign production, relative to sector k. □

F Optimal policies with imperfect labor substituion

In this case, labor across sectors are aggregated with a CES function, i.e.,

$$L = [\sum_{j=1}^{J} \alpha_j{}^{-\frac{1}{\kappa-1}} L_j{}^{\frac{\kappa}{\kappa-1}}]^{\frac{\kappa-1}{\kappa}}, \quad 0 < \kappa - 1 < \infty.$$

In addition, labor market clearing conditions are (9) and (10), and the labor supply satisfies

$$L_{1j} = \alpha_j (\frac{w_{1j}}{W_1})^{\kappa-1} L_1, \tag{A.24}$$

$$L_{2j} = \alpha_j (\frac{w_{2j}}{W_2})^{\kappa-1} L_2, \tag{A.25}$$

where $W_n = [\sum_{j=1}^{J} \alpha_j w_{nj}{}^{\kappa}]^{\frac{1}{\kappa}}$. Furthermore, trade shares and prices satisfy equations (3) - (8).

F.1 Proof of tax neutrality

Given $\Gamma = \{(\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1) : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_{nj}}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1 + \check{\tau}_j^m = \lambda(1 + \tau_j^m)$, $1 + \check{\tau}_j^x = \frac{1+\tau_j^x}{\mu}$ and $1 + \check{\tau}_j^d = \lambda \frac{1+\tau_j^d}{\mu}$, for any constants $\mu > 0$ and $\lambda > 0$. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_{nj}}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_{1j} = \mu \frac{w_{1j}}{\lambda}$, $\check{w}_{nj} = \frac{w_{nj}}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{L}_{1j} = \alpha_j (\frac{\frac{\mu}{\lambda} w_{1j}}{\frac{\mu}{\lambda} W_1})^{\kappa-1} L_1 = L_{1j}, \qquad \check{L}_{2j} = \alpha_j (\frac{\frac{1}{\lambda} w_{2j}}{\frac{1}{\lambda} W_2})^{\kappa-1} L_2 = L_{2j}.$$

The market clearing conditions, expenditures, prices, and trade shares equations are the same as A.12-A.18.

The same allocations satisfy the equilibrium conditions under $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$. Hence, we proved the allocations and welfare are the same under $\{\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1\}$ and $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$. For any policy Γ, we can set $\frac{\mu}{\lambda} = 1 + \tau_1^d$ and obtain $\check{\tau}_1^d = 0$. This differs from the case with immobile labor, from the proof of tax neutrality, only one domestic tax is redundant. Then $\gamma_x - u_c = \gamma_{1,1}$. By further adjusting λ, we can eliminate one tariff. Alternatively, we can set $\gamma_{1,1} = 0$ and get $\gamma_x = u_c$ and $\tau_j^m = -\frac{\gamma_{2j}}{u_c}$.

F.2 Proof of optimal domestic tax and tariffs

In this case, Λ_n satisfies Definition 4 and becomes

$$\Lambda_1 - I = \begin{pmatrix} \kappa-(\kappa-1)\frac{w_{11}L_{11}}{W_1\bar{L}_1}-1 & ... & -(\kappa-1)\frac{w_{1j}L_{1j}}{W_1\bar{L}_1} & ... & -(\kappa-1)\frac{w_{1J}L_{1J}}{W_1\bar{L}_1} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\frac{w_{11}L_{11}}{W_1\bar{L}_1} & ... & \kappa-(\kappa-1)\frac{w_{1j}L_{1j}}{W_1\bar{L}_1}-1 & ... & -(\kappa-1)\frac{w_{1J}L_{1J}}{W_1\bar{L}_1} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\frac{w_{11}L_{11}}{W_1\bar{L}_1} & ... & -(\kappa-1)\frac{w_{1j}L_{1j}}{W_1\bar{L}_1} & ... & \kappa-(\kappa-1)\frac{w_{1J}L_{1J}}{W_1\bar{L}_1}-1 \end{pmatrix}.$$

We subtract row k from row j in matrix $\Lambda_1 - I$, assuming $k < j$. The resulting row j is:

$$\begin{pmatrix} 0 & ... & -\kappa+1 & ... & \kappa-1 & ... & 0 \end{pmatrix}.$$

This indicates $(\kappa-1)(\tau_j^d - \tau_k^d) = 0$ for any k, j. Thus, the domestic tax is uniform across all sectors. All can be normalized to zero.

Let us revision Λ_2:

$$\Lambda_2 = \begin{pmatrix} \kappa-(\kappa-1)\frac{w_{21}L_{21}}{W_2\bar{L}_2} & ... & -(\kappa-1)\frac{w_{2j}L_{2j}}{W_2\bar{L}_2} & ... & -(\kappa-1)\frac{w_{2J}L_{2J}}{W_2\bar{L}_2} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\frac{w_{21}L_{21}}{W_2\bar{L}_2} & ... & \kappa-(\kappa-1)\frac{w_{2j}L_{2j}}{W_2\bar{L}_2} & ... & -(\kappa-1)\frac{w_{2J}L_{2J}}{W_2\bar{L}_2} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\frac{w_{21}L_{21}}{W_2\bar{L}_2} & ... & -(\kappa-1)\frac{w_{2j}L_{2j}}{W_2\bar{L}_2} & ... & \kappa-(\kappa-1)\frac{w_{2J}L_{2J}}{W_2\bar{L}_2} \end{pmatrix}.$$

We subtract row k from row j in matrix Λ_2, assuming $k < j$. The resulting row j is: $\begin{pmatrix} 0 & ... & -\kappa & ... & \kappa & ... & 0 \end{pmatrix}$. Thus, for any sector j and k,

$$\tau_j^m - \tau_k^m = \frac{1}{\kappa}\left(\frac{x_1\beta_j - (1+\tau_j^d)w_{1j}L_{1j}}{w_{2j}L_{2j}} - \frac{x_1\beta_k - (1+\tau_k^d)w_{1k}L_{1k}}{w_{2k}L_{2k}}\right).$$

□

G Ricardian-Roy models

In this case, labor market clearing conditions are given by (9) and (10), and labor supply satisfies

$$w_{1j}L_{1j} = \sum_g \frac{A_{1jg}w_{1j}^\kappa}{W_{1g}^\kappa}W_{1g}\bar{L}_{1g}, \quad w_{2j}L_{2j} = \sum_g \frac{A_{2jg}w_{2j}^\kappa}{W_{2g}^\kappa}W_{2g}\bar{L}_{ng},$$

and trade shares and prices satisfy equations (3) - (8).

G.1 Proof of tax neutrality

Given $\Gamma = \{(\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1 : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_{nj}}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1 + \check{\tau}_j^m = \lambda(1 + \tau_j^m)$, $1 + \check{\tau}_j^x = \frac{1+\tau_j^x}{\mu}$ and $1 + \check{\tau}_j^d = \lambda\frac{1+\tau_j^d}{\mu}$, for any constants $\mu > 0$ and $\lambda > 0$. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_{nj}}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_{1j} = \mu\frac{w_{1j}}{\lambda}$, $\check{w}_{nj} = \frac{w_{nj}}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{w}_{1j}L_{1j} = \sum_g \frac{A_{1jg}(\frac{\mu}{\lambda}w_{1j})^\kappa}{(\frac{\mu}{\lambda}W_{1g})^\kappa}\frac{\mu}{\lambda}W_{1g}\bar{L}_{1g} = \frac{\mu}{\lambda}w_{1j}L_{1j}$$
$$\check{w}_{2j}L_{2j} = \sum_g \frac{A_{2jg}(\frac{1}{\lambda}w_{2j})^\kappa}{(\frac{1}{\lambda}W_{2g})^\kappa}\frac{1}{\lambda}W_{2g}\bar{L}_{ng} = \frac{1}{\lambda}w_{2j}L_{2j}$$

The market clearing, expenditures, prices, and trade shares equations are the same as A.12-A.18. The same allocations satisfy the equilibrium conditions under $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$, hence we proved the allocations and welfare are the same under $\{\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1\}$ and $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$. For any policy Γ, we can set $\frac{\mu}{\lambda} = 1 + \tau_1^d$ and obtain $\check{\tau}_1^d = 0$. Then $\gamma_x - u_c = \gamma_{1,1}$. By further adjusting λ, we can eliminate one tariff. Alternatively, we can set $\gamma_{1,1} = 0$ and get $\gamma_x = u_c$ and $\tau_j^m = -\frac{\gamma_{2j}}{u_c}$.

G.2 Proof of optimal domestic tax

With one factor:

$$\Lambda_1 - I = \begin{pmatrix} \kappa - (\kappa-1)\frac{m_{11}m_{11}W_1\bar{L}_1}{w_{11}L_{11}} - 1 & \dots & -(\kappa-1)\frac{m_{11}m_{1j}W_1\bar{L}_1}{w_{11}L_{11}} & \dots & -(\kappa-1)\frac{m_{11}m_{1J}W_1\bar{L}_1}{w_{11}L_{11}} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa-1)\frac{m_{1j}m_{11}W_1\bar{L}_1}{w_{1j}L_{1j}} & \dots & \kappa-(\kappa-1)\frac{m_{1j}m_{1j}W_1\bar{L}_1}{w_{1j}L_{1j}} - 1 & \dots & -(\kappa-1)\frac{m_{1j}m_{1J}W_1\bar{L}_1}{w_{1j}L_{1j}} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa-1)\frac{m_{1J}m_{11}W_1\bar{L}_1}{w_{1J}L_{1J}} & \dots & -(\kappa-1)\frac{m_{1J}m_{1j}W_1\bar{L}_1}{w_{1J}L_{1J}} & \dots & \kappa-(\kappa-1)\frac{m_{1J}m_{1J}W_1\bar{L}_1}{w_{1J}L_{1J}} - 1 \end{pmatrix}.$$

We subtract row k from row j in matrix $\Lambda_1 - I$, assuming $k < j$. The resulting row j is:

$$\begin{pmatrix} 0 & \dots & -\kappa+1 & \dots & \kappa-1 & \dots & 0 \end{pmatrix}.$$

This indicates $(\kappa-1)(\tau_j^d - \tau_k^d) = 0$ for any k, j. Thus, the domestic tax is uniform across all sectors.

With multiple factors:

$$\Lambda_1 - I = (\kappa-1)\begin{pmatrix} 1 - \frac{\sum_g m_{11g}m_{11g}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} & \dots & -\frac{\sum_g m_{11g}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} & \dots & -\frac{\sum_g m_{11g}m_{1Jg}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g m_{1jg}m_{11g}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} & \dots & 1 - \frac{\sum_g m_{1jg}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} & \dots & -\frac{\sum_g m_{1jg}m_{1Jg}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g m_{1Jg}m_{11g}W_{1g}\bar{L}_{1g}}{w_{1J}L_{1J}} & \dots & -\frac{\sum_g m_{1Jg}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{1J}L_{1J}} & \dots & 1 - \frac{\sum_g m_{1Jg}m_{1Jg}W_{1g}\bar{L}_{1g}}{w_{1J}L_{1J}} \end{pmatrix}. \quad \text{(A.26)}$$

Due to tax neutrality, the domestic tax can only be determined at a relative level. We define a matrix E that satisfies

$$E = (\kappa-1)\begin{pmatrix} 1 - \frac{\sum_g m_{11g}m_{11g}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} & \dots & -\frac{\sum_g m_{11g}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} & \dots & -\frac{\sum_g m_{11g}m_{1,J-1,g}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g m_{1jg}m_{11g}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} & \dots & 1 - \frac{\sum_g m_{1jg}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} & \dots & -\frac{\sum_g m_{1jg}m_{1,J-1,g}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g m_{1,J-1,g}m_{11g}W_{1g}\bar{L}_{1g}}{w_{1,J-1}L_{1,J-1}} & \dots & -\frac{\sum_g m_{1,J-1,g}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{1,J-1}L_{1,J-1}} & \dots & 1 - \frac{\sum_g m_{1,J-1,g}m_{1,J-1,g}W_{1g}\bar{L}_{1g}}{w_{1,J-1}L_{1,J-1}} \end{pmatrix}.$$

Using this matrix, we can rewrite the system of equations (A.26) as

$$E \cdot \begin{pmatrix} \tau_1^d - \tau_J^d \\ \dots \\ \tau_j^d - \tau_J^d \\ \dots \\ \tau_{J-1}^d - \tau_J^d \end{pmatrix} = \begin{pmatrix} 0 \\ \dots \\ 0 \\ \dots \\ 0 \end{pmatrix}.$$

As $|E| \neq 0$, $\tau_j^d - \tau_J^d = 0$, and thus domestic taxes are uniform across all sectors. □

G.3 Proof of optimal tariffs

With one factor,

$$\Lambda_2 = \begin{pmatrix} \kappa - (\kappa-1)\frac{m_{21}m_{21}W_2\bar{L}_2}{w_{21}L_{21}} & \dots & -(\kappa-1)\frac{m_{21}m_{2j}W_2\bar{L}_2}{w_{21}L_{21}} & \dots & -(\kappa-1)\frac{m_{21}m_{2J}W_2\bar{L}_2}{w_{21}L_{21}} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa-1)\frac{m_{2j}m_{21}W_2\bar{L}_2}{w_{2j}L_{2j}} & \dots & \kappa - (\kappa-1)\frac{m_{22}m_{2j}W_2\bar{L}_2}{w_{2j}L_{2j}} & \dots & -(\kappa-1)\frac{m_{2j}m_{2J}W_2\bar{L}_2}{w_{2j}L_{2j}} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa-1)\frac{m_{2J}m_{21}W_2\bar{L}_2}{w_{2J}L_{2J}} & \dots & -(\kappa-1)\frac{m_{2J}m_{2j}W_2\bar{L}_2}{w_{2J}L_{2J}} & \dots & \kappa - (\kappa-1)\frac{m_{2J}m_{2J}W_2\bar{L}_2}{w_{2J}L_{2J}} \end{pmatrix}.$$

We subtract row k from row j in matrix Λ_2, assuming $k < j$. The resulting row j is:

$$\begin{pmatrix} 0 & \dots & -\kappa & \dots & \kappa & \dots & 0 \end{pmatrix}.$$

Thus, for any sector j and k

$$\tau_j^m - \tau_k^m = \frac{1}{\kappa}\left(\frac{x_1\beta_j - (1+\tau_j^d)w_{1j}L_{1j}}{w_{2j}L_{2j}} - \frac{x_1\beta_k - (1+\tau_k^d)w_{1k}L_{1k}}{w_{2k}L_{2k}}\right).$$

With multiple factors:

$$\Lambda_2 = \begin{pmatrix} \kappa-(\kappa-1)\frac{\sum_g m_{21g}m_{21g}W_{2g}\bar{L}_{2g}}{w_{21}L_{21}} & ... & -(\kappa-1)\frac{\sum_g m_{21g}m_{2jg}W_{2g}\bar{L}_{2g}}{w_{21}L_{21}} & ... & -(\kappa-1)\frac{\sum_g m_{21g}m_{2Jg}W_{2g}\bar{L}_{2g}}{w_{21}L_{21}} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\frac{\sum_g m_{2jg}m_{21g}W_{2g}\bar{L}_{2g}}{w_{2j}L_{2j}} & ... & \kappa-(\kappa-1)\frac{\sum_g m_{2jg}m_{2jg}W_{2g}\bar{L}_{2g}}{w_{2j}L_{2j}} & ... & -(\kappa-1)\frac{\sum_g m_{2jg}m_{2Jg}W_{2g}\bar{L}_{2g}}{w_{2j}L_{2j}} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\frac{\sum_g m_{2Jg}m_{21g}W_{2g}\bar{L}_{2g}}{w_{2J}L_{2J}} & ... & -(\kappa-1)\frac{\sum_g m_{2Jg}m_{2jg}W_{2g}\bar{L}_{2g}}{w_{2J}L_{2J}} & ... & \kappa-(\kappa-1)\frac{\sum_g m_{2Jg}m_{2Jg}W_{2g}\bar{L}_{2g}}{w_{2J}L_{2J}} \end{pmatrix}$$

The supply system doesn't satisfy Definition 4; hence, optimal tariffs can not be written as Proposition 2. Optimal tariffs satisfy the general formula as in Proposition 1.

□

H Optimal policy under inefficient labor markets

In addition to wages, workers have preferences over different sectors. Workers then choose a sector to work in to maximize their utility. Let λ_{nj} denote the share of workers in sector j relative to the total number of workers in country n. The law of large numbers ensures the proportion of these workers who work in sector j is $\lambda_{nj} = \Pr\left(\epsilon_{nj}w_{nj} \geq \max_s \{\epsilon_{ns}w_{ns}\}\right)$. Assume that preferences ϵ_{nj} over sectors follow a Frechet distribution where κ governs the degree of dispersion across individuals. Then the share of workers in sector j relative to the total number of workers in country n is

$$\lambda_{nj} = \frac{(w_{nj})^{\kappa}}{\sum_{k=1}^{J}(w_{nk})^{\kappa}}$$

and total employment in sector j of country n is $L_{nj} = \lambda_{nj}\bar{L}_n$. Hence, the equilibrium requires

$$\Omega = \left\{(w_{nj}, L_{nj}) : \frac{Y_{nj}}{Y_n} = \frac{w_{nj}L_{nj}}{W_nL_n} = \frac{w_{nj}^{\kappa+1}}{\sum_{s=1}^{J} w_{ns}^{\kappa+1}},\ W_n = \frac{\sum_{s=1}^{J} w_{ns}^{\kappa+1}}{\sum_{s=1}^{J} w_{ns}^{\kappa}}\right\}. \tag{A.27}$$

In this case, labor market clearing conditions are (9) and (10), and labor supply satisfy

$$L_{1j} = \lambda_{1j}\bar{L}_1 = \frac{(w_{1j})^{\kappa}}{\sum_{k=1}^{J}(w_{1k})^{\kappa}}\bar{L}_1, \quad L_{2j} = \lambda_{2j}\bar{L}_2 = \frac{(w_{2j})^{\kappa}}{\sum_{k=1}^{J}(w_{2k})^{\kappa}}\bar{L}_2,$$

and trade shares and prices satisfy equations (3) - (8).

H.1 Proof of tax neutrality

Given $\Gamma = \{(\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1 : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_{nj}}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1 + \check{\tau}_j^m = \lambda(1 + \tau_j^m)$, $1 + \check{\tau}_j^x = \frac{1+\tau_j^x}{\mu}$ and $1 + \check{\tau}_j^d = \lambda \frac{1+\tau_j^d}{\mu}$, for any constants $\mu > 0$ and $\lambda > 0$. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_{nj}}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_{1j} = \mu \frac{w_{1j}}{\lambda}$, $\check{w}_{nj} = \frac{w_{nj}}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{L}_{1j} = \frac{\left(\frac{\mu}{\lambda} w_{1j}\right)^\kappa}{\sum_{k=1}^J \left(\frac{\mu}{\lambda} w_{1k}\right)^\kappa} \bar{L}_1 = L_{1j}, \quad \check{L}_{2j} = \frac{\left(\frac{1}{\lambda} w_{2j}\right)^\kappa}{\sum_{k=1}^J \left(\frac{1}{\lambda} w_{2k}\right)^\kappa} \bar{L}_2 = L_{2j}.$$

The market clearing, expenditures, prices, and trade shares equations are the same as A.12-A.18. The same allocations satisfy the equilibrium conditions under $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$, hence we proved the allocations and welfare are the same under $\{\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1\}$ and $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$. For any policy Γ, we can set $\frac{\mu}{\lambda} = 1 + \tau_1^d$ and obtain $\check{\tau}_1^d = 0$. Then $\gamma_x - u_c = \gamma_{1,1}$. By further adjusting λ, by using tax neutrality, we can only eliminate one tariff. Or we can set $\gamma_{1,1} = 0$, and then we get $\gamma_x = u_c$ and $\tau_j^m = -\frac{\gamma_{2j}}{u_c}$.

H.2 Proof of optimal domestic tax

$$\Lambda_1 - I = \begin{pmatrix} \kappa - \kappa m_{11} \frac{W_1}{w_{11}} & \dots & -\kappa m_{1j} \frac{W_1}{w_{11}} & \dots & -\kappa m_{1J} \frac{W_1}{w_{11}} \\ \dots & \dots & \dots & \dots & \dots \\ -\kappa m_{11} \frac{W_1}{w_{1j}} & \dots & \kappa - \kappa m_{1j} \frac{W_1}{w_{1j}} & \dots & -\kappa m_{1J} \frac{W_1}{w_{1j}} \\ \dots & \dots & \dots & \dots & \dots \\ -\kappa m_{11} \frac{W_1}{w_{1J}} & \dots & -\kappa m_{1j} \frac{W_1}{w_{1J}} & \dots & \kappa - \kappa m_{1J} \frac{W_1}{w_{nJ}}. \end{pmatrix}$$

where the jth and kth rows are given by

$$-\kappa \frac{1}{w_{1j}} \sum_{s=1}^J m_{1s} W_1 (1 + \tau_s^d) + \kappa(1 + \tau_j^d) = 0,$$
$$-\kappa \frac{1}{w_{1k}} \sum_{s=1}^J m_{1s} W_1 (1 + \tau_s^d) + \kappa(1 + \tau_k^d) = 0,$$

respectively. Taking the differences of the two and rearranging terms, we get

$$\frac{1+\tau_j^d}{1+\tau_k^d} = \frac{w_{1k}}{w_{1j}}.$$

This indicates that the domestic tax imposed on a high-wage sector is lower than that on the low-wage sector. $\square$

H.3 Proof of optimal import tariff

In this case, Λ_2 is given by

$$\Lambda_2 = \begin{pmatrix} \kappa+1-\kappa m_{21}\frac{W_2}{w_{21}} & \dots & -\kappa m_{2j}\frac{W_2}{w_{21}} & \dots & -\kappa m_{2J}\frac{W_2}{w_{21}} \\ \dots & \dots & \dots & \dots & \dots \\ -\kappa m_{21}\frac{W_2}{w_{2j}} & \dots & \kappa+1-\kappa m_{2j}\frac{W_2}{w_{2j}} & \dots & -\kappa m_{2J}\frac{W_2}{w_{2j}} \\ \dots & \dots & \dots & \dots & \dots \\ -\kappa m_{21}\frac{W_2}{w_{2J}} & \dots & -\kappa m_{2j}\frac{W_2}{w_{2J}} & \dots & \kappa+1-\kappa m_{2J}\frac{W_2}{w_{2J}} \end{pmatrix}.$$

Again, consider two rows, jth and kth,

$$\begin{aligned} -\kappa\frac{1}{w_{2j}}\sum_{s=1}^{J} m_{2s}W_2(\frac{\zeta}{u_c}+\tau_s^m) + (\kappa+1)(\frac{\zeta}{u_c}+\tau_j^m) &= \Psi_j \\ -\kappa\frac{1}{w_{2k}}\sum_{s=1}^{J} m_{2s}W_2(\frac{\zeta}{u_c}+\tau_s^m) + (\kappa+1)(\frac{\zeta}{u_c}+\tau_k^m) &= \Psi_k. \end{aligned}$$

The difference of the two implies

$$(\kappa+1)(\tau_j^m-\tau_k^m) - \kappa(\frac{1}{w_{2j}}-\frac{1}{w_{2k}})\sum_{s=1}^{J} m_{2s}W_2(\frac{\zeta}{u_c}+\tau_s^m) = \Psi_j-\Psi_k.$$

Thus, for any sector j and k, the following holds

$$\tau_j^m-\tau_k^m = \frac{1}{\kappa+1}(\frac{x_1\beta_j-(1+\tau_j^d)w_{1j}L_{1j}}{w_{2j}L_{2j}} - \frac{x_1\beta_k-(1+\tau_k^d)w_{1k}L_{1k}}{w_{2k}L_{2k}} + \kappa\sum_{s=1}^{J}(\tau_s^m+\frac{\zeta}{u_c})(\frac{1}{w_{2j}}-\frac{1}{w_{2k}})m_{2s}W_2)$$

The additional term reflects the inefficiency in the foreign labor market and non-CES supply system.

H.4 Optimal tariff when foreign correct distortions

Now, suppose Foreign imposes domestic taxes that satisfy

$$\frac{1+\tau_{2j}^{d}}{1+\tau_{2k}^{d}} = \frac{w_{2k}}{w_{2j}}, \quad \forall j,k.$$

Home planner chooses $w_{nj}, x_n, \tau_j^x, \tau_j^m, \tau_j^d, L_{nj}$ to maximizes domestic welfare $\max u(x_1/P_1)$ subject to the world market equilibrium characterized by

$$\begin{aligned}
&w_{1j}L_{1j} = \beta_j \left[\frac{1}{1+\tau_j^d}\pi_{11,j}x_1 + \frac{1}{1+\tau_j^x}\pi_{21,j}x_2\right] \quad (\gamma_{1j}, \quad J)\\
&(1+\tau_{2j}^d)w_{2j}L_{2j} = \beta_j \left[\frac{1}{1+\tau_j^m}\pi_{12,j}x_1 + \pi_{22,j}x_2\right] \quad (\gamma_{2j}, \quad J-1)\\
&x_1 = \sum_{j=1}^{J} w_{1j}L_{1j} + \sum_{j=1}^{J}\beta_j\frac{\tau_j^x}{1+\tau_j^x}\pi_{21,j}x_2 + \sum_{j=1}^{J}\beta_j\frac{\tau_j^m}{1+\tau_j^m}\pi_{12,j}x_1 + \sum_{j=1}^{J}\beta_j\frac{\tau_j^d}{1+\tau_j^d}\pi_{11,j}x_1 \quad (\gamma_x)
\end{aligned}$$

where

$$\begin{aligned}
&L_{1j} = \lambda_{1j}\bar{L}_1 = \frac{(w_{1j})^{\kappa}}{\sum_{k=1}^{J}(w_{1k})^{\kappa}}\bar{L}_1, \quad L_{2j} = \lambda_{2j}\bar{L}_2 = \frac{(w_{2j})^{\kappa}}{\sum_{k=1}^{J}(w_{2k})^{\kappa}}\bar{L}_2\\
&x_2 = \sum_{j=1}^{J} w_{2j}L_{2j} + \sum_{j=1}^{J}\beta_j\frac{\tau_{2j}^d}{1+\tau_{2j}^d}\left[\frac{1}{1+\tau_j^m}\pi_{12,j}x_1 + \pi_{22,j}x_2\right] = \sum_{j=1}^{J}(1+\tau_{2j}^d)w_{2j}L_{2j}\\
&P_1 = \Pi_j\left[T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_{2j}^d)d_{12})^{-\theta}\right]^{-\frac{\beta_j}{\theta}} = 1\\
&P_2 = \Pi_j\left[T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_{2j}^d))^{-\theta}\right]^{-\frac{\beta_j}{\theta}}\\
&\pi_{11,j} = \frac{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_{2j}^d)d_{12})^{-\theta}}\\
&\pi_{12,j} = \frac{T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_{2j}^d)d_{12})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_{2j}^d)d_{12})^{-\theta}},
\end{aligned}$$

$$\begin{aligned}
\pi_{21,j} &= \frac{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_{2j}^d))^{-\theta}},\\
\pi_{22,j} &= \frac{T_{2j}(w_{2j}(1+\tau_{2j}^d))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_{2j}^d))^{-\theta}}.
\end{aligned}$$

H.5 Proof of tax neutrality

Given $\Gamma = \{(\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1, \tau_{2j}^d + 1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1, \check{\tau}_{2j}^d + 1 : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_{nj}}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1 + \check{\tau}_j^m = \lambda(1 + \tau_j^m)$, $1 + \check{\tau}_j^x = \frac{1+\tau_j^x}{\mu}$, $1 + \check{\tau}_j^d = \lambda \frac{1+\tau_j^d}{\mu}$ and $1 + \check{\tau}_{2j}^d = 1 + \tau_{2j}^d$, where $i \neq n$, for any constants $\mu > 0$ and $\lambda > 0$. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_{nj}}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_{1j} = \mu \frac{w_{1j}}{\lambda}$, $\check{w}_{nj} = \frac{w_{nj}}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{w}_{1j} L_{1j} = \beta_j \left[\frac{\mu}{\lambda(1+\tau_j^d)} \pi_{11,j} x_1 + \frac{\mu}{1+\tau_j^x} \pi_{21,j} \frac{x_2}{\lambda} \right] = \frac{\mu}{\lambda} w_{1j} L_{1j}$$

$$(1 + \check{\tau}_{2j}^d) \check{w}_{2j} L_{2j} = \beta_j \left[\frac{1}{\lambda(1+\tau_j^m)} \pi_{12,j} x_1 + \pi_{22,j} \frac{x_2}{\lambda} \right] = (1 + \tau_{2j}^d) \frac{w_{2j}}{\lambda} L_{2j}$$

$$\begin{aligned}
\check{x}_1 &= \sum_{j=1}^{J} \mu \frac{w_{1j}}{\lambda} L_{1j} + \sum_{j=1}^{J} \beta_j (1 - \frac{\mu}{1+\tau_j^x}) \pi_{21,j} \frac{x_2}{\lambda} + \sum_{j=1}^{J} \beta_j (1 - \frac{1}{\lambda(1+\tau_j^m)}) \pi_{12,j} x_1 + \sum_{j=1}^{J} \beta_j (1 - \frac{\mu}{\lambda(1+\tau_j^d)}) \pi_{11,j} x_1 \\
&= \sum_{j=1}^{J} \frac{\mu}{\lambda} \beta_j \left[\frac{1}{1+\tau_j^d} \pi_{11,j} x_1 + \frac{1}{1+\tau_j^x} \pi_{21,j} x_2 \right] + \sum_{j=1}^{J} \beta_j (1 - \frac{\mu}{1+\tau_j^x}) \pi_{21,j} \frac{x_2}{\lambda} + \sum_{j=1}^{J} \beta_j (1 - \frac{1}{\lambda(1+\tau_j^m)}) \pi_{12,j} x_1 \\
&+ \sum_{j=1}^{J} \beta_j (1 - \frac{\mu}{\lambda(1+\tau_j^d)}) \pi_{11,j} x_1 \\
&= \sum_{j=1}^{J} \beta_j \pi_{21,j} \frac{x_2}{\lambda} + \sum_{j=1}^{J} \beta_j (1 - \frac{1}{\lambda(1+\tau_j^m)}) \pi_{12,j} x_1 + \sum_{j=1}^{J} \beta_j \pi_{11,j} x_1 = \sum_{j=1}^{J} \beta_j \pi_{12,j} x_1 + \sum_{j=1}^{J} \beta_j \pi_{11,j} x_1 = x_1
\end{aligned}$$

$$\check{x}_2 = \sum_{j=1}^{J}(1+\tau_{2j}^d)\frac{w_{2j}}{\lambda}L_{2j} = \frac{x_2}{\lambda}$$

$$\check{P}_1 = \Pi_j\left[T_{1j}(\mu\frac{w_{1j}}{\lambda}\lambda\frac{1+\tau_j^d}{\mu})^{-\theta} + T_{2j}(\frac{w_2}{\lambda}\lambda(1+\tau_j^m)(1+\tau_{2j}^d)d_{12})^{-\theta}\right]^{-\frac{\beta_j}{\theta}} = P_1 = 1$$

$$\check{P}_2 = \Pi_j\left[T_{1j}(\mu\frac{w_{1j}}{\lambda}\frac{1+\tau_j^x}{\mu}d_{21})^{-\theta} + T_{2j}(\frac{w_{2j}}{\lambda}(1+\tau_{2j}^d))^{-\theta}\right]^{-\frac{\beta_j}{\theta}} = \frac{P_2}{\lambda}$$

$$\check{\pi}_{11,j} = \frac{T_{1j}(\mu\frac{w_1}{\lambda}\lambda\frac{1+\tau_j^d}{\mu})^{-\theta}}{T_{1j}(\mu\frac{w_1}{\lambda}\lambda\frac{1+\tau_j^d}{\mu})^{-\theta} + T_{2j}(\frac{w_{2j}}{\lambda}\lambda(1+\tau_j^m)(1+\tau_{2j}^d)d_{12})^{-\theta}} = \pi_{11,j}$$

$$\check{\pi}_{21,j} = \frac{T_{1j}(\mu\frac{w_1}{\lambda}\frac{1+\tau_j^x}{\mu}d_{21})^{-\theta}}{T_{1j}(\mu\frac{w_1}{\lambda}\frac{1+\tau_j^x}{\mu}d_{21})^{-\theta} + T_{2j}(\frac{w_{2j}}{\lambda}(1+\tau_{2j}^d))^{-\theta}} = \pi_{21,j}$$

$$\check{L}_{1j} = \frac{\left(\frac{\mu}{\lambda}w_{1j}\right)^{\kappa}}{\sum_{k=1}^{J}\left(\frac{\mu}{\lambda}w_{1k}\right)^{\kappa}}\bar{L}_1 = L_{1j}$$

$$\check{L}_{2j} = \frac{\left(\frac{1}{\lambda}w_{2j}\right)^{\kappa}}{\sum_{k=1}^{J}\left(\frac{1}{\lambda}w_{2k}\right)^{\kappa}}\bar{L}_2 = L_{2j}$$

The same allocations satisfy the equilibrium conditions under $\{\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1, \check{\tau}_{2j}^d+1\}$, hence we proved the allocations and welfare are the same under $\{\tau_j^m+1, \tau_j^x+1, \tau_j^d+1, \tau_{2j}^d+1\}$ and $\{\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1, \check{\tau}_{2j}^d+1\}$. For any policy Γ, we can set $\frac{\mu}{\lambda} = 1+\tau_1^d$ and obtain $\check{\tau}_1^d = 0$. Then $\gamma_x - u_c = \gamma_{1,1}$. By further adjusting λ, we can eliminate one tariff. Or we can set $\gamma_{1,1} = 0$, and then we get $\gamma_x = u_c$ and $\tau_j^m = -\frac{\gamma_{2j}}{u_c}$.

H.6 Proof of optimal domestic tax

In this case, Λ_1 is given by

$$\Lambda_1 - I = \begin{pmatrix} \kappa-\kappa m_{11}\frac{W_1}{w_{11}} & ... & -\kappa m_{1j}\frac{W_1}{w_{11}} & ... & -\kappa m_{1J}\frac{W_1}{w_{11}} \\ ... & ... & ... & ... & ... \\ -\kappa m_{11}\frac{W_1}{w_{1j}} & ... & \kappa-\kappa m_{1j}\frac{W_1}{w_{1j}} & ... & -\kappa m_{1J}\frac{W_1}{w_{1j}} \\ ... & ... & ... & ... & ... \\ -\kappa m_{11}\frac{W_1}{w_{1J}} & ... & -\kappa m_{1j}\frac{W_1}{w_{1J}} & ... & \kappa-\kappa m_{1J}\frac{W_1}{w_{nJ}} \end{pmatrix}$$

. Consider two rows, j and k

$$
\begin{aligned}
-\kappa\frac{1}{w_{1j}}\sum_{s=1}^{J} m_{1s}W_1(1+\tau_s^d)+\kappa(1+\tau_j^d)=0,\\
-\kappa\frac{1}{w_{1k}}\sum_{s=1}^{J} m_{1s}W_1(1+\tau_s^d)+\kappa(1+\tau_k^d)=0.
\end{aligned}
$$

Taking the difference of the two and reorganizing, we can show

$$
\frac{1+\tau_j^d}{1+\tau_k^d}=\frac{w_{1k}}{w_{1j}}.
$$

$\square$

H.7 Proof of optimal import tariff

In this case,

$$
\Lambda_2=\begin{pmatrix}
\kappa+1-\kappa\frac{w_{21}L_{21}}{(1+\tau_{21}^d)w_{21}\bar{L}_2} & \dots & -\kappa\frac{w_{2j}L_{2j}}{(1+\tau_{21}^d)w_{21}\bar{L}_2} & \dots & -\kappa\frac{w_{2J}L_{2J}}{(1+\tau_{21}^d)w_{21}\bar{L}_2}\\
\dots & \dots & \dots & \dots & \dots\\
-\kappa\frac{w_{21}L_{21}}{(1+\tau_{2j}^d)w_{2j}\bar{L}_2} & \dots & \kappa+1-\kappa\frac{w_{2j}L_{2j}}{(1+\tau_{2j}^d)w_{2j}\bar{L}_2} & \dots & -\kappa\frac{w_{2J}L_{2J}}{(1+\tau_{2j}^d)w_{2j}\bar{L}_2}\\
\dots & \dots & \dots & \dots & \dots\\
-\kappa\frac{w_{21}L_{21}}{(1+\tau_{2J}^d)w_{2J}\bar{L}_2} & \dots & -\kappa\frac{w_{2j}L_{2j}}{(1+\tau_{2J}^d)w_{2J}\bar{L}_2} & \dots & \kappa+1-\kappa\frac{w_{2J}L_{2J}}{(1+\tau_{2J}^d)w_{2J}\bar{L}_2}
\end{pmatrix}
$$

Consider two rows:

$$
\begin{aligned}
-\kappa\frac{1}{(1+\tau_{2j}^d)w_{2j}}\sum_{s=1}^{J} m_{2s}W_2(\frac{\zeta}{u_c}+\tau_s^m)+(\kappa+1)(\frac{\zeta}{u_c}+\tau_j^m)=\Psi_j\\
-\kappa\frac{1}{(1+\tau_{2k}^d)w_{2k}}\sum_{s=1}^{J} m_{2s}W_2(\frac{\zeta}{u_c}+\tau_s^m)+(\kappa+1)(\frac{\zeta}{u_c}+\tau_k^m)=\Psi_k
\end{aligned}
$$

Taking the difference of the two and using Foreign condition $\frac{1}{(1+\tau_{2j}^d)w_{2j}}=\frac{1}{(1+\tau_{2k}^d)w_{2k}}$, we have

$$
(\kappa+1)(\tau_j^m-\tau_k^m)=\Psi_j-\Psi_k.
$$

Thus, for any sector j and k.

$$\begin{aligned}\tau_j^m - \tau_k^m &= \frac{1}{\kappa+1}\left(\frac{x_1\beta_j - (1+\tau_j^d)w_{1j}L_{1j}}{(1+\tau_{2j}^d)w_{2j}L_{2j}} - \frac{x_1\beta_k - (1+\tau_k^d)w_{1k}L_{1k}}{(1+\tau_{2k}^d)w_{2k}L_{2k}}\right)\\ &= \frac{1}{\kappa+1}\left(\frac{x_1\beta_j - (1+\tau_j^d)Y_{1j}}{(1+\tau_{2j}^d)Y_{2j}} - \frac{x_1\beta_k - (1+\tau_k^d)Y_{1k}}{(1+\tau_{2k}^d)Y_{2k}}\right).\end{aligned}$$

□

I Revisit optimal policy in Bai, Jin, and Lu (2023)

We revisit the steady-state optimal policies in Bai, Jin, and Lu (2023) with endogenous technology accumulation. We can also view this model as a multi-sector Krugman or multi-sector Melitz-Pareto. We show that this model has the property of CES supply system. In this case, labor is perfectly mobile and technology endogenously depends on labor. Let T_{nj} be the technology of sector j in country n, endogenously depending on an efficiency parameter ν_{nj} and labor in this sector L_{nj}, i.e.,

$$T_{nj} = \nu_{nj}L_{nj}.$$

Now, labor market clearing conditions are

$$\begin{aligned}\frac{1+\theta}{\theta}w_1L_{1j} &= \beta_j\left[\frac{1}{1+\tau_j^d}\pi_{11,j}x_1 + \frac{1}{1+\tau_j^x}\pi_{21,j}x_2\right], \quad (\gamma_{1j}, \quad J)\\ \frac{1+\theta}{\theta}w_2L_{2j} &= \beta_j\left[\frac{1}{1+\tau_j^m}\pi_{12,j}x_1 + \pi_{22,j}x_2\right] \quad (\gamma_{2j} \quad J)\end{aligned}$$

where $(1+\theta)/\theta$ captures firms' markup. The total expenditures are given by

$$\begin{aligned}x_1 &= \frac{1+\theta}{\theta}\sum_{j=1}^{J}w_1L_{1j} + \sum_{j=1}^{J}\beta_j\frac{\tau_j^x}{1+\tau_j^x}\pi_{21,j}x_2 + \sum_{j=1}^{J}\beta_j\frac{\tau_j^m}{1+\tau_j^m}\pi_{12,j}x_1 + \sum_{j=1}^{J}\beta_j\frac{\tau_j^d}{1+\tau_j^d}\pi_{11,j}x_1, \quad (\gamma_x)\\ x_2 &= \frac{1+\theta}{\theta}\sum_{j=1}^{J}w_2L_{2j}\end{aligned}$$

Labor market clearing condition implies,

$$\sum_{j=1}^{J}L_{1j} = \bar{L}_1, \quad (\gamma_{L1}), \quad \sum_{j=1}^{J}L_{2j} = \bar{L}_2. \quad (\gamma_{L2})$$

Lastly, trade shares and prices satisfy equations (3) - (8).

I.1 Proof of optimal domestic tax and import tariff

FOC over L_{1j}

$$[u_c x_1 \beta_j \frac{\pi_{11,j}}{\theta T_{1j}} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial T_{1j}} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial T_{1j}} x_2$$
$$+ \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial T_{1j}} x_1 + \gamma_{2j}\beta_j \frac{\partial \pi_{22,j}}{\partial T_{1j}} x_2 + \gamma_x \beta_j \frac{\tau_j^x}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial T_{1j}} x_2 + \gamma_x \beta_j \frac{\tau_j^m}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial T_{1j}} x_1$$
$$+ \gamma_x \beta_j \frac{\tau_j^d}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial T_{1j}} x_1]\nu_{1j} + (\gamma_x - \gamma_{1j})\frac{1+\theta}{\theta} w_1 - \gamma_{L1} = 0$$

Combine FOCs over L_{1j}, τ_j^x and τ_j^m, we get

$$-u_c x_1 \beta_j - (\gamma_x - \gamma_{1j})\frac{1+\theta}{\theta} w_1 L_{1j}\theta + (\gamma_x - \gamma_{2j})\beta_j \frac{1}{1+\tau_j^m}\pi_{12,j} x_1 - (\gamma_x - \gamma_{1j})\beta_j \frac{1}{1+\tau_j^x}\pi_{21}^j x_2 + \gamma_{L1}\theta L_{1j} = 0.$$

Using the optimal tax formua $1+\tau_j^m = \frac{\gamma_x - \gamma_{2j}}{u_c}$ and $1+\tau_j^d = \frac{\gamma_x - \gamma_{1j}}{u_c}$, we have

$$(\gamma_x - \gamma_{1j})\frac{(1+\theta)^2}{\theta} w_1 = \gamma_{L1}\theta.$$

Thus, $1+\tau_j^d = \frac{\gamma_{L1}}{u_c}\frac{\theta^2}{(1+\theta)^2}\frac{1}{w_1}$, which is uniform across sectors.

FOC over L_{2j}

$$[u_c x_1 \beta_j \frac{\pi_{12,j}}{\theta T_{2j}} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial T_{2j}} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial T_{2j}} x_2 + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial T_{2j}} x_1 + \gamma_{2j}\beta_j \frac{\partial \pi_{22,j}}{\partial T_{2j}} x_2$$
$$+ \gamma_x \beta_j \frac{\tau_j^x}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial T_{2j}} x_2 + \gamma_x \beta_j \frac{\tau_j^m}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial T_{2j}} x_1 + \gamma_x \beta_j \frac{\tau_j^d}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial T_{2j}} x_1]\nu_{2j} + (\eta - \gamma_{2j})\frac{1+\theta}{\theta} w_2 - \gamma_{L2} = 0$$

where $\eta = \sum_k^J (\gamma_x \beta_k \frac{\tau_k^x}{1+\tau_k^x}\pi_{21,k} + \gamma_{1k}\beta_k \frac{1}{1+\tau_k^x}\pi_{21,k} + \gamma_{2k}\beta_k \pi_{22,k})$.

Combine FOCs over L_{1j} and L_{2j}, we get

$$
\begin{aligned}
& u_c x_1 \beta_j \frac{1}{\theta} + (\gamma_x - \gamma_{1j}) \frac{1+\theta}{\theta} w_1 L_{1j} + (\eta - \gamma_{2j}) \frac{1+\theta}{\theta} w_2 L_{2j} - \gamma_{L1} L_{1j} - \gamma_{L2} L_{2j} = 0 \\
(\Rightarrow) \quad & u_c x_1 \beta_j \frac{1}{\theta} - (\gamma_x - \gamma_{1j}) \frac{1+\theta}{\theta^2} w_1 L_{1j} + (\eta - \gamma_{2j}) \frac{1+\theta}{\theta} w_2 L_{2j} - \gamma_{L2} L_{2j} = 0 \\
(\Rightarrow) \quad & \frac{1}{\theta}[x_1 \beta_j - (1+\tau_j^d) \frac{1+\theta}{\theta} w_1 L_{1j}] + (\frac{\zeta}{u_c} + \tau_j^m) \frac{1+\theta}{\theta} w_2 L_{2j} - \frac{\gamma_{L2}}{u_c} L_{2j} = 0 \\
(\Rightarrow) \quad & -\theta[\tau_j^m + \frac{\zeta}{u_c} - \frac{\gamma_{L2}\theta}{u_c(1+\theta) w_2}] = \frac{x_1 \beta_j - (1+\tau_j^d) \frac{1+\theta}{\theta} w_1 L_{1j}}{\frac{1+\theta}{\theta} w_2 L_{2j}} \\
(\Rightarrow) \quad & -\theta[\tau_j^m + \frac{\zeta}{u_c}] = \frac{x_1 \beta_j - (1+\tau_j^d) \frac{1+\theta}{\theta} w_1 L_{1j}}{\frac{1+\theta}{\theta} w_2 L_{2j}}
\end{aligned}
$$

where $\zeta = \eta - \gamma_x + u_c - \frac{\gamma_{L2}\theta}{(1+\theta) w_2}$. □

I.2 The supply system is CES

Below, we show that the supply system is CES. Thus, we can apply the general formula.

Define the cost in sector j of country n as $c_{nj}^{-\theta} = T_{nj} w_n^{-\theta}$. The element of matrix Λ_n at row j and column k is given by

$$
\begin{aligned}
\frac{\partial \ln(Y_{nk})}{\partial \ln(c_{nj})} \frac{Y_{nk}}{Y_{nj}} &= \frac{\partial \ln(Y_{nk})}{\partial \ln(T_{nj}^{-1/\theta} w_n)} \frac{Y_{nk}}{Y_{nj}} = -\theta \frac{\partial \ln(Y_{nk})}{\partial \ln(T_{nj})} \frac{Y_{nk}}{Y_{nj}} \\
&= -\theta \frac{\partial \ln(Y_{nk})}{\partial \ln(L_{nj})} \frac{Y_{nk}}{Y_{nj}} = \begin{cases} -\theta & \text{if } k = j, \\ 0 & \text{if } k \neq j. \end{cases}
\end{aligned}
$$

Thus, $\Lambda_n = -\theta I$ where I is the identity matrix. This indicates the domestic tax is uniform across sectors. All can be normalized to zero. Furthermore, we can show

$$
\begin{aligned}
\tau_j^m - \tau_k^m &= -\frac{1}{\theta}(\frac{x_1 \beta_j - (1+\tau_j^d) \frac{1+\theta}{\theta} Y_{1j}}{\frac{1+\theta}{\theta} Y_{2j}} - \frac{x_1 \beta_k - (1+\tau_k^d) \frac{1+\theta}{\theta} Y_{1k}}{\frac{1+\theta}{\theta} Y_{2k}}) \\
&= -\frac{1}{\theta}(\frac{x_1 \beta_j - \frac{1+\theta}{\theta} Y_{1j}}{\frac{1+\theta}{\theta} Y_{2j}} - \frac{x_1 \beta_k - \frac{1+\theta}{\theta} Y_{1k}}{\frac{1+\theta}{\theta} Y_{2k}}).
\end{aligned}
$$

□