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A DEFENSE OF THE EFFICIENCIES OFFENSE

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ABSTRACT

This paper uses an aggregative games framework to predict consumer welfare when market structure is endogenously determined. Specifically, we characterize mergers whose synergies reduce consumer welfare by inducing rivals to exit. Our results apply to quantity competition among homogeneous products as well as price competition among multiproduct firms facing multinomial logit demand. Based on models calibrated to commonly used parameter values, we find that the synergies required to maintain consumer surplus can be much higher than those implied by traditional merger analysis. We also find that neither follow-on mergers nor subsequent entry necessarily eliminate the problem.

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1 Introduction

Horizontal mergers create efficiencies through economies of scale and scope, improved coordination, and the exchange of "best practices," which reduce costs. All else equal, as savings are passed through, they put downward pressure on prices. For these reasons, merger review traditionally treats efficiency gains as an unambiguously pro-competitive factor, pitted against upward pressure applied by increased internalization of business-stealing externalities (Williamson, 1968; Farrell and Shapiro, 1990). Such treatment, however, implicitly assumes that market structure is unaffected by the merger. Relaxing the assumption can dramatically change how certain mergers are evaluated, as this paper shows.

The change occurs because merger synergies can cause rivals to exit the market, further weakening the competitive process. In such cases, efficiencies could constitute an "offense" rather than "defense" under antitrust laws. Courts and agencies have sharply disagreed about the validity of this claim over time. In the US, the argument was almost uniformly accepted under the Warren Court of the 1960s only to be rejected out-of-hand by the Chicago School, which has mostly dominated judicial thinking since the 1980s. For example, in 1967, the Federal Trade Commission successfully argued against Procter & Gamble's acquisition of Clorox on the grounds that the scale of the merged entity would deter competitors from challenging it.[1] Yet, in 2000, the same basic force that worked against Procter & Gamble worked in favor of General Electric when it proposed its acquisition of Honeywell to the Department of Justice (Majoras, 2001).

We use an aggregative games framework to predict merger-related changes in consumer welfare when market structure is endogenously determined. The analysis applies recent advances by Nocke and Schutz (2018, 2024) to work by Motta and Vasconcelos (2005), who provide an early, insightful treatment of the efficiencies offense.[2] In their model, four symmetric firms facing unit linear inverse demand compete à la Cournot, and mergers reduce variable costs by 50%. In contrast, we permit any number of asymmetric firms facing any well-behaved demand, competing on quantity or price, and realizing any merger synergies.

The paper makes three contributions. First, by generalizing the model, we can determine the exact conditions that produce the efficiencies offense, providing researchers and policymakers a clearer understanding of precisely when it occurs. For example, one learns that the insights derived from quantity competition in homogeneous product markets extend to price competition between multiproduct firms facing multinomial-logit demand. This is an important lesson because, as Nocke and Schutz (2024) point out, "almost all mergers involve multiproduct firms selling differentiated products." Second, we show the problem arises under a wide range of parameter values, meaning we reach a qualitatively different conclusion than earlier work, which ultimately found no rationale for the efficiencies offense (Motta and Vasconcelos, 2005).[3] Third, our findings support influential, policy-relevant claims by Nocke and Whinston (2022), who argue that the concentration thresholds in the 1982-2010 *Horizontal Merger Guidelines* were likely too lax. (In fact, in many cases, we suggest even stricter screens.) Likewise, we align with Ciliberto, Murry, and Tamer (2021), whose detailed estimates imply that airline mergers raise price by inducing exit (but who never mention the efficiencies offense).

[1] *FTC v. Procter & Gamble Co.*, 386 US 568 (1968).

[2] Motta (2004) provides an alternative formulation (pp. 252-3) alongside a concise synthesis of competitive effects, efficiency gains, changes in the level and division of surplus, and merger-induced exit by nonmerging parties (pp. 253-63). The text points to Motta and Vasconcelos (2005) for an analysis that includes follow-on mergers, so we make comparisons to the latter work as well.

[3] Our results reveal that many of their insights apply very broadly. For example, the problem arises under Bertrand as well as Cournot competition, under asymmetric costs, under a range of synergies, etc. In other areas, we are more cautious. For instance, whereas follow-on mergers solve the problem in their model, we find that this corrective mechanism is limited. More precisely, we find that the follow-on merger may need to be almost as efficient as the initial one, which requires the potentially strong assumption that all firms can access the same "merger technology."

Our main focus is merger-induced exit by nonmerging competitors, which we model using a two-stage perfect information game. In the first stage, each firm decides whether to stay in the market or exit. If the firm stays, it pays a fixed cost, and if not, it pays zero. In the second stage, firms set prices or quantities and earn gross profits, while consumers demand the products. Our approach parallels ones commonly used in modern empirical work (e.g., Mazzeo (2002); Ciliberto and Tamer (2009); Fan (2013); Eizenberg (2014); Wollmann (2018); Fan and Yang (2020)).[4] It also draws from and even extends work by Caradonna, Miller, and Sheu (2024), who demonstrate the limits of postmerger entry but abstract away from the efficiencies offense. Keeping with convention, we evaluate hypothetical mergers by solving each game twice. First, we consider a "premerger" environment in which each firm is independently owned. Second, we consider a "postmerger" environment in which two firms are combined just prior to the start of the game.[5]

We start by introducing the primitives that govern supply and demand, describing equilibrium strategies, and characterizing the efficiencies offense. By definition, the latter occurs when a merger (a) induces exit, (b) reduces CS, but (c) would raise CS were exit not to occur. We show that two inequalities must be satisfied. First, merger $\mathscr{M}$'s synergies must be large enough to push some nonmerging firm i's profit below its fixed cost. This condition corresponds to (a) but turns out to also ensure (c). Second, $\mathscr{M}$'s synergies must not be so large as to restore CS in the absence of i. The inequalities we obtain make it easy to depict the contours of the efficiencies offense in terms of interpretable objects, such as compensating marginal cost reductions and premerger primitives like fixed costs.

Next, we study two special but important cases. First, we assume that i exactly breaks even prior to $\mathscr{M}$. (Situations like this may arise in many economically important markets; see, e.g., the discussion at the end of this section.) Here, we find that the synergies required for $\mathscr{M}$ to be CS-positive with i in the market are *exactly the same* as the synergies that induce i to leave the market. That is, once $\mathscr{M}$ reaches the point that traditional merger analysis predicts it will raise CS, we show that exit occurs, "ratcheting" CS back down. As a result, much larger synergies are typically required to safeguard CS—under symmetry, they are at least twice the level reported in prior literature. Second, we relax the break-even assumption and instead assume that there exists a potential entrant capable of replicating the operations of the least profitable firm in the market. This allows us to bound the fixed costs of the least profitable firm and, in turn, show that there always exist synergies for which an efficiencies offense occurs.

Finally, we address other market responses that might mitigate the problems we identified. As one example, we consider subsequent entry. All else equal, exit makes the market more attractive to a potential entrant. If entry occurs and the incoming firm is more efficient than the departing one, then the combination of the merger, exit, and subsequent entry might raise CS even when the combination of the merger and exit alone would reduce it. However, we find that only a narrow range of parameter values permit this to occur. The limiting factor is the entrant, which must be efficient enough to profitably enter and offset harm but not so efficient that it would have already entered prior to the merger.

As another example, we consider follow-on mergers. A firm that becomes unprofitable because its rivals have combined to become more efficient may itself merge (and generate its own synergies). In the presence of endogenous exit, two mergers that occur together can be CS-positive even when each of them, in isolation, is CS-negative (Motta and Vasconcelos, 2005). Yet, we find that synergistic follow-on mergers do not necessarily eliminate efficiencies offenses. Moreover, we argue that practical reliance on

[4]It relates to analogous games with incomplete information, which typically mitigate complications from multiplicity but increase computational demands (Seim, 2006). It also relates to dynamic games with entry, exit, repositioning, and/or investment (see, e.g., (Bajari et al., 2007; Ryan, 2012; Sweeting, 2013)).

[5]We agree with Farrell and Shapiro (2000) that "absent-merger" and "with-merger" are more accurate terms, but we opt, as they have, for standard descriptors.

follow-on mergers requires caution. The likelihood that a subsequent, synergistic transaction emerges depends critically on what "merger technologies" firms possess. At the present time, such capabilities are not well understood (Demirer and Karaduman, 2022).

To be clear, this paper does not advocate against crediting synergies—or against any particular policy, for that matter. Instead, it relaxes a standard assumption in a general framework and explores the economic consequences. If one juxtaposes our propositions with the common refrain that merger review must be tailored to each transaction's diverse circumstances (i.e., "every merger is different" (Benkard, 2010)), then, in our view, it seems hard to claim that "an allegation of an efficiencies offense has no actual basis in [antitrust] law" (Schmitz, 2002).[6] Furthermore, even one who worries about the *unintended consequences* of scrutinizing synergies would still want to carefully understand the *expected consequences* of doing so.

We should also clarify our framework's objective function. If one maintains our assumptions, then our findings imply that antitrust authorities should account for mergers' effects on rivals' profits. Advocates of a "New Brandeisian" approach to enforcement make the same point,[7] albeit for different reasons. They either place positive welfare weights on competitors' profits or assume that spillovers accrue through nonmarket mechanisms (e.g., market concentration may facilitate lobbying and capture, which raise entry barriers or undermine democratic processes in unmodeled future periods, as in Teachout and Khan (2014) or Teachout (2020)). Instead, we employ the consumer welfare standard.[8] Questions about which standard is correct are beyond the scope of this paper.

Additionally, while measuring the frequency at which these situations arise is also outside the scope of this paper, anecdotes abound in trade press and business media that market structure endogenously responds to mergers among rivals. For example, when the FCC relaxed ownership restrictions on radio broadcasting, the industry rapidly consolidated. Following the initial wave of mergers, firms wishing to remain independent allegedly struggled to survive. An industry analyst quoted in the *New York Times* summarized the concern: "Mom and pop broadcasters will be forced to sell" (Adelson, 1993). As another example, similar pressures have been faced by wholesalers of products ranging from building materials and industrial gases to electronics and food. As "distribution consolidators . . . rapidly acquired former competitors, . . . smaller, regional wholesalers . . . were forced to sell out or dissolve" (Fein and Jap, 1999).

This is not to say that merger-induced exit only arises in fragmented markets with niche producers. Concentrated markets populated by firms with billions of dollars in revenue may pose risks. For example, when Canadian Pacific proposed to buy Norfolk Southern in 2015, industry sources indicated that the merger could "trigger further industry consolidation," with one former railroad executive indicating, "companies like CSX and [Canadian National] would also explore a merger to remain competitive" (Dummett and Mickle, 2015). Nearly a decade earlier, the chief executive of Continental Airlines used similar language following a bid by US Airways to takeover Delta, telling employees that their company "wants to remain independent" but "would consider a merger to remain competitive if the industry continues

[6]The quote references European Commission law in particular (the omitted word is "EC"). However, the surrounding text states that the EC is a comparatively favorable jurisdiction for this type of argument, implying that the statement applies broadly.

[7]Justice Brandeis was an early, influential critic of not just market power but also "bigness." His ideas shaped competition policy through much of the 20th century. Many continue to influence legal scholarship (Khan, 2016; Wu, 2018; Asil, 2024). In the absence of formal models, it is often hard to know precisely where he and his ideological opponents disagree. In any event, our framework supports one of his controversial, counterintuitive claims. Brandeis criticized "destructive competition," especially when it is "cutthroat" in nature, as it paves the "road to monopoly" (see, e.g., his dissent in *New State Ice Co. v. Liebmann*, 285 US 262, 311 (1932) or Brandeis (1913)). Nearly all modern analysis, such as Greve (2001), ridicules this view, ruling out the mere possibility. However, as we show, it may be entirely reasonable to expect cost savings to raise prices by eliminating competition.

[8]This is the substantive legal standard employed in nearly all US merger analysis (Farrell and Shapiro, 2010). See, e.g., *FTC v. Elders Grain*, 868 F.2d 901, 904 (1989), which affirms the standard.

to consolidate" (Carpenter, 2006).[9] By the same token, when Comcast proposed to buy Time Warner Cable, analysts "speculat[ed] . . . that DirecTV and Dish Network Corp., the second and third biggest pay TV providers, respectively, could seek to merge to gain scale and remain competitive" (Ramachandran, 2014).

The remainder of the paper is organized as follows. Section 2 studies the efficiencies offense under Cournot competition. Section 3 conducts similar analysis under Bertrand competition. Section 4 models and evaluates potential defenses such as subsequent entry and follow-on mergers. Section 5 concludes.

2 Quantity competition with homogeneous products

2.1 Primitives and equilibrium

Consider a market with a set $\mathscr{F}$ of firms. In the first stage, each firm $f \in \mathscr{F}$ decides to stay or leave. If f stays, then it pays a fixed cost ϕ_f and moves to the second stage. If not, then it receives a payoff normalized to zero and exits. In the second stage, each firm that stays in the market produces a homogeneous product with constant returns to scale. f sets a quantity q_f and receives gross profit equal to $(P(Q) - c_f)q_f$, where $P(Q)$, Q, and c_f denote inverse demand, aggregate quantity, and per-unit variable cost, respectively.

For simplicity, we assume that $\phi_f > 0$, $c_f > 0$, and $\phi_f < \phi_{f'}$ if and only if $c_f < c_{f'}$. For expositional ease, firms are sorted in decreasing order of their costs from 1 to N.[10] Also, to ensure a unique equilibrium exists, we assume that firms make first stage decisions in reverse order—the lowest cost firm decides first. For the same reason, we make the standard assumptions on inverse demand. $P'(Q) < 0$ and $P'(Q) + QP''(Q) < 0$ for all Q satisfying $P(Q) > 0$, and $\lim_{Q\to\infty} P(Q) = 0$.

Firms maximize profits. Equilibrium strategies may be obtained by backwards induction. In the second stage, given all $f < J$ exit, each $f \geq J$ sets the first order condition of its gross profit equal to zero. It chooses

$$q_f = \max\left\{\frac{P(Q) - c_f}{-P'(Q)}, 0\right\}, \tag{1}$$

where Q is understood to equal $\sum_{f' \geq J} q_{f'}$. Assuming $q_f > 0$, it earns

$$\pi_f(J) = \frac{(P(Q) - c_f)^2}{-P'(Q)}. \tag{2}$$

In the first stage, given the order in which firms make decisions,[11] each firm f stays if and only if

$$\pi_f(f) \geq \phi_f. \tag{3}$$

We study the merger of two firms, m and n. Under Cournot competition, it is natural to assume that the transaction consolidates production into a single entity and gives rise to synergies, meaning it reduces variable costs. We denote the merger by $\mathscr{M}$ and the merged firm by M. To rule out uninteresting cases, we

[9] As we alluded earlier, these sentiments square with the results of counterfactual simulations run by Ciliberto, Murry, and Tamer (2021). See, e.g., their Table 11 and surrounding discussion in their Section VI.

[10] If there exist f and f' such that $c_f = c_{f'}$ and $\phi_f = \phi_{f'}$, then the firms can be arbitrarily ordered without loss of generality.

[11] If f exits, then its fixed cost exceeds its variable profit. Given the indexing of firms, $f' < f$ has weakly smaller variable profit and weakly larger fixed cost, so f' also exits. By analogous logic, if f stays, then any $f' > f$ stays.

restrict attention to $c_M \leq \min\{c_m, c_n\}$, $\phi_M \leq \min\{\phi_m, \phi_n\}$, and $\min\{m, n\} > 1$ (i.e., neither m nor n is the least profitable firm prior to the merger),[12] and we impose that all firms produce positive quantity after the merger.[13] Finally, we denote premerger equilibrium objects using an asterisk and, unless otherwise stated, make the standard assumption that all firms $f \in \mathscr{F}$ earn positive profit prior to the merger (i.e., $\pi_1^\star(1) \geq \phi_1$).

Our formulation of the problem generalizes Motta and Vasconcelos (2005). In their model, prior to any merger, four firms face demand given by $P(Q) = 1 - Q$, per-unit variable cost equal to α/k, and total fixed cost equal to fk, with $k = 1/4$ to start. Merging two firms into one doubles the value of k—mergers cut per-unit variable costs in half and double total fixed costs. Our model relaxes four restrictions: it accepts any well-behaved demand; it permits any variable or fixed costs that rationalize premerger market structure, ensure positive production, and order the firms similarly; and it considers any synergies. We revisit these differences at the end of the section.

2.2 Zero-rival-profit and consumer-welfare-neutrality curves

Borrowing nomenclature from Caradonna et al. (2024), who analyze entry induced by increased postmerger market power, we introduce two sets of equilibrium objects. First, we define zero-rival-profit (ZRP) curves. ZRP_i will represent the synergies required for firm i to break even, assuming all $i' < i$ exit. We obtain

$$\phi_i = \left(P(Q^i) - c_i\right) q_i = \frac{\left(P(Q^i) - c_i\right)^2}{-P'(Q^i)}, \tag{4}$$

where Q^i is the aggregate quantity at which i earns exactly zero profit. Equation 4 shows that Q^i can be written as a function of ϕ_i. If we write aggregate quantity as a sum of individual quantities and rearrange terms, then we have

$$\tilde{c}_M^i = (N-i)P(Q^i(\phi_i)) + P'(Q^i(\phi_i))Q^i(\phi_i) - \sum_{h \in \mathscr{H}_i} c_h, \tag{5}$$

where $\mathscr{H}_i = \{\{i, i+1, ..., N\} \backslash \{m, n\}\}$ and $\tilde{c}_M^i$ denotes the value of c_M associated with ZRP_i. Equation 5 shows that $\tilde{c}_M^i$ can be written as a function of ϕ_i. To facilitate comparisons, we make normalizations. Specifically, we replace $\tilde{c}_M^i$ with $(c - \tilde{c}_M^i)/c$, its fractional change from some variable cost c, and replace ϕ_i with $\Phi_i \equiv \phi_i/\pi_i^\star$, its ratio to premerger equilibrium gross profit. ZRP_i is given by

$$\frac{c - \tilde{c}_M^i}{c} = \frac{1}{c}\left(c - (N-i)P(Q^i(\Phi_i)) - P'(Q^i(\Phi_i))Q^i(\Phi_i) + \sum_{h \in \mathscr{H}_i} c_h\right). \tag{6}$$

Second, we define CS-neutrality (CSN) curves. CSN_i will represent the synergies required to make

[12] We impose the first condition because a court that uses a consumer welfare standard will block any merger that raises variable cost. We impose the second condition for two reasons. One is that, in this framework, fixed costs are unlikely to increase due to the merger. The other is to guarantee that firms do not merge and immediately exit. Such cases complicate exposition but are unlikely to ever occur as they are never profitable. As for the third condition, we make frequent references to the "least profitable firm prior to the merger" (i.e., $f = 1$) as it often exits following the merger of m and n. Without this restriction, such references would be erroneously directed were the merging firm to be the least profitable. Rather than repeatedly restate this, we rule it out here.

[13] In terms of the model's primitives, this requires $c_1 < P(Q^{\star\star})$, where $Q^{\star\star} = \sum_{h \in \mathscr{H}} -(P(Q^{\star\star}) - c_h)/P'(Q^{\star\star})$ and $\mathscr{H} \equiv \{\{\mathscr{F} \backslash \{m, n\}\} \cup M\}$. Doing so rules out cases where a nonmerging firm exits because its variable cost exceeds the equilibrium price charged when the firm stays in the market. An efficiencies offense can still arise in such cases, although they are much less interesting, and if we do not rule them out, then they complicate exposition.

the merger CS-neutral, given all $i' \leq i$ exit. Since the second stage is an aggregative game, a CS-neutral merger must leave aggregate quantity unchanged from its premerger level (Nocke and Whinston, 2022). This implies

$$Q^\star = \sum_{f \in \mathcal{F}} \frac{P(Q^\star) - c_f}{-P'(Q^\star)} = \sum_{g \in \mathcal{G}_i} \frac{P(Q^\star) - c_g}{-P'(Q^\star)} + \frac{P(Q^\star) - \bar{c}_M^i}{-P'(Q^\star)}, \tag{7}$$

where $Q^\star$ is the equilibrium aggregate premerger quantity, $\mathcal{G}_i = \{\{i+1, ..., N\} \backslash \{m, n\}\}$ and $\bar{c}_M^i$ denotes the value of c_M associated with CSN_i. The first equality follows from the definition of $Q^\star$, while the second follows from the definition of $\bar{c}_M^i$, which guarantees that the set of firms $\{\{i+1, ..., N\} \backslash \{m, n\} \cup M\}$ produce $Q^\star$ in aggregate. Equation 7 implies

$$\bar{c}_M^i = (N-i-1)P(Q^\star) + P'(Q^\star)Q^\star - \sum_{g \in \mathcal{G}_i} c_g, \tag{8}$$

Following analogous normalizations to the ones described above, CSN_i is given by

$$\frac{c - \bar{c}_M^i}{c} = \frac{1}{c}\left(c - (N-i-1)P(Q^\star) - P'(Q^\star)Q^\star + \sum_{g \in \mathcal{G}_i} c_g\right). \tag{9}$$

We can extend the indexing of the CSN curves to construct CSN_0, which measures the synergies required for a CS-neutral merger when no firm exits. As this particular case is isomorphic to a model in which market structure is held fixed, it coincides with traditional merger analysis. Specifically, if we replace c with the share-weighted average of premerger variable costs of the merging firms, then CSN_0 corresponds to what is typically called the "compensating marginal cost reduction" (or simply "CMCR," as in Miller and Sheu (2020)). To avoid confusion, we refer to it for the remainder of this paper as the "fixed-market-structure CMCR."

2.3 Efficiencies offense

Our first proposition concisely characterizes the efficiencies offense in markets where firms choose quantities. The stated conditions ensure three criteria are met—the merger induces exit, reduces CS, but would raise CS were market structure fixed. These conditions are also easy to visualize, as we show below.

Proposition 1. *If $\bar{c}_M^i < c_M < \tilde{c}_M^i$ for any $i \in \{\mathcal{F} \backslash \{m, n\}\}$, then an efficiencies offense arises under Cournot competition.*

Proof. Since $c_M < \tilde{c}_M^i$, the aggregator exceeds the level required for i to break even, so all $i' \leq i$ exit. Conditional on all $i' < i$ exiting, Since $c_M > \bar{c}_M^i$, the aggregator falls short of $Q^\star$, which is required for CS-neutrality, so $\mathscr{M}$ is CS-negative. Finally, since $c_M < \tilde{c}_M^i$ and $\tilde{c}_M^i < \bar{c}_M^{i-1}$, $\mathscr{M}$ would be CS-positive were i not to exit. ■

Proposition 1 implies that the efficiencies offense arises whenever synergies lie on any of the nonoverlapping open intervals bounded by the ZRP and CSN curves. By extension, if we vary synergies and fixed costs, the efficiencies offense corresponds to the area between the curves.

Figure I depicts the contours of the problem by shading regions where it occurs. For comparability, our parameter choices are the same as the ones Nocke and Whinston (2022) use in their leading example. Fourteen firms with symmetric variable costs c prior to the merger face linear demand with a price elasticity equal to 1.5.

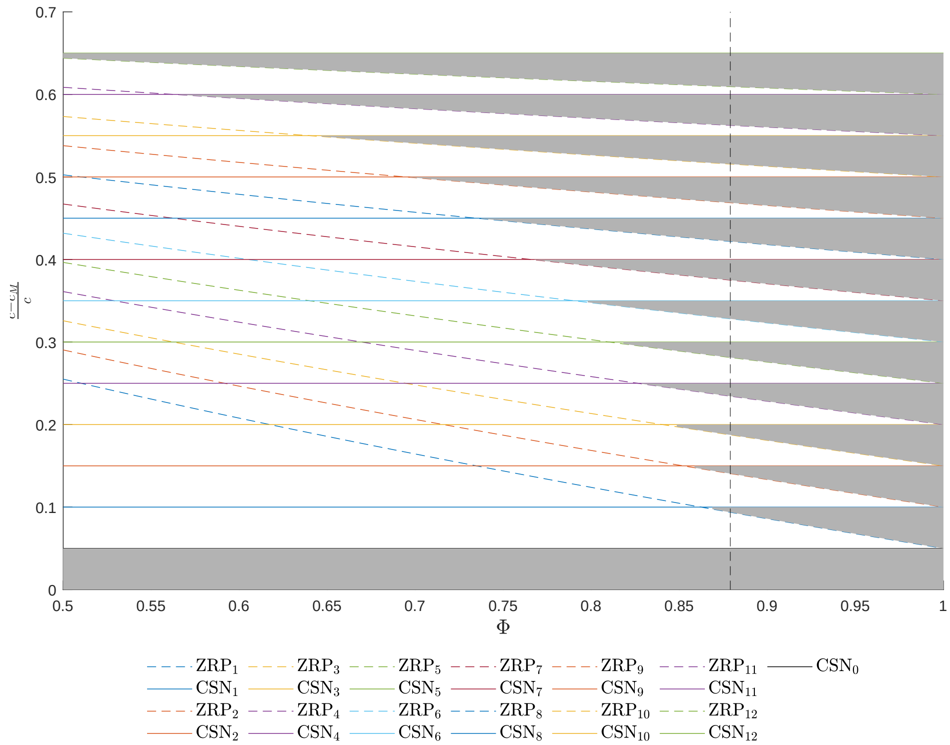

Figure I: *Efficiencies offenses in the Cournot model*

This figure plots normalized fixed costs, Φ_f, on the horizontal axis and normalized merger synergies, $(c - c_M)/c$, on the vertical axis. Dashed lines represent ZRP curves, while solid lines represented CSN curves. The height of CSN_0 corresponds to the fixed-market-structure CMCR—the benchmark against which synergies are compared in traditional merger analysis. To improve legibility, the axes values' range from 0.5 to 1 and from 0 to 0.7, respectively.

At first, turn attention to CSN_0. As we mentioned above, its height corresponds to the synergies that would be required for a CS-neutral merger under the assumption that market structure is held fixed. As such, it reflects one of the key takeaways in the paper from which our parameters are drawn: even in a market like this one where several firms compete for price-sensitive consumers, synergies of 5% are required to prevent consumer harm.[14] However, as the shaded regions above CSN_0 indicate, when we allow market structure to equilibrate, 5% no longer suffices—much higher synergies can reduce CS.

This is especially true when we restrict attention to "reasonable" fixed costs—ones that rationalize premerger market structure. Prior to the merger, exactly N firms are present in the market. Presumably, though, the pool of potential competitors was larger, and the number of actual competitors reflected market clearing. Algebraically, ϕ must be greater than $\pi_{N+1}(\cdot)$, or else an $(N+1)^{\text{th}}$ firm would be present. By the same logic, ϕ must be less than or equal to $\pi_N(\cdot)$, or else the N^{th} firm would not be present. If we apply

[14]For the first instance, see the abstract of Nocke and Whinston (2022).

this logic to Figure I, Φ must lie on the half-open interval $(0.88, 1]$.[15] In this region of the graph, which lies to the right of the vertical dashed line, most of the area is shaded—*the majority of mergers here reduce consumer welfare by inducing exit.*

Notably, each CSN curve intersects its subsequently indexed ZRP curve. To see the importance, assume that the least profitable nonmerging firm breaks even prior to the merger, i.e., $\Phi = 1$. Under this assumption, comparative statics on synergies equate to movements along the right-hand side vertical axis. Now, starting at zero, gradually increase synergies so that they cross CSN_0 and ZRP_1. By definition of CSN_0, this is the point at which, were market structure held fixed, a merger turns CS-positive. Yet, by definition of ZRP_1, this is also the point at which the least profitable nonmerging firm exits, resulting in a discontinuous decline in CS. In other words, precisely when traditional merger analysis predicts synergies are large enough to reduce price *below* its premerger level, we predict that price rises sharply *above* its premerger level.

Figure II depicts mergers for which our framework yields different enforcement decisions than traditional merger analysis. As in the previous exercise, N firms with symmetric marginal costs c prior to the merger face linear demand with a price elasticity equal to 1.5. Here, however, we no longer restrict attention to 14 firms. Instead, we let N vary, which manifests as differences in ΔHHI. We evaluate each transaction twice: once ignoring merger-induced exit and once allowing for it. To start, we assume the least profitable nonmerging firm has "sensible" market-clearing fixed costs equal to $(1 + \pi_{N+1}(\cdot))/2$ (see the discussion above regarding the dashed vertical line in Figure I for what we mean by "sensible").

The shaded region of the graph represents mergers for which an antitrust authority that applies our framework would take different actions than one who applies traditional merger analysis. Specifically, both approaches suggest blocking all mergers in region A and allowing all mergers in regions B and D, but in region C, they differ, as efficiencies offenses arise. That is, region C mergers are CS-positive when one ignores merger-induced exit but CS-negative when one allows for it. To illustrate, suppose $N \approx 14$ so that $\Delta HHI \approx 100$. Existing work suggests approving mergers whose variable cost reductions reach 5%, whereas ours implies a threshold that is twice as high.

2.4 Implications

We now turn to special cases. First, we consider a market in which, prior to the merger, the least profitable firm breaks even—knife-edge conditions that provide intuition. The proposition states that any merger which raises CS when market structure is held fixed will cause the least profitable firm to exit. This implies that our earlier observation—CSN_0 intersects ZRP_1 at $\Phi_1 = 1$—always holds. Neither symmetry, a specific elasticity, nor a particular number of firms are required.

Proposition 2. *In the Cournot model, assume that the least profitable firm breaks even prior to the merger. If $\mathscr{M}$ would be CS-positive were market structure held fixed, then $\mathscr{M}$ causes the least profitable firm to exit.*

Proof. Define $\pi_1(Q)$ equal to the gross profit that the least profitable firm earns given that Q is produced.

[15] When firms have symmetric constant marginal costs and face linear inverse demand, individual gross profit takes the form $(P(0) - c)^2/((N+1)^2 P')$, where $P(0)$, c, and P' are constants (wherever P' is well-defined). Market clearing implies $\phi > (P(0) - c)^2/(16^2 P')$. Normalizing by premerger equilibrium gross profit, which equals $\Phi = \phi/[(P(0) - c)^2/(15^2 P')]$, we have $\Phi > [(P(0) - c)^2/(16^2 P')]/[(P(0) - c)^2/(15^2 P')] = 15^2/16^2 \approx 0.88$. Likewise, $\Phi \leq 15^2/15^2 = 1$.

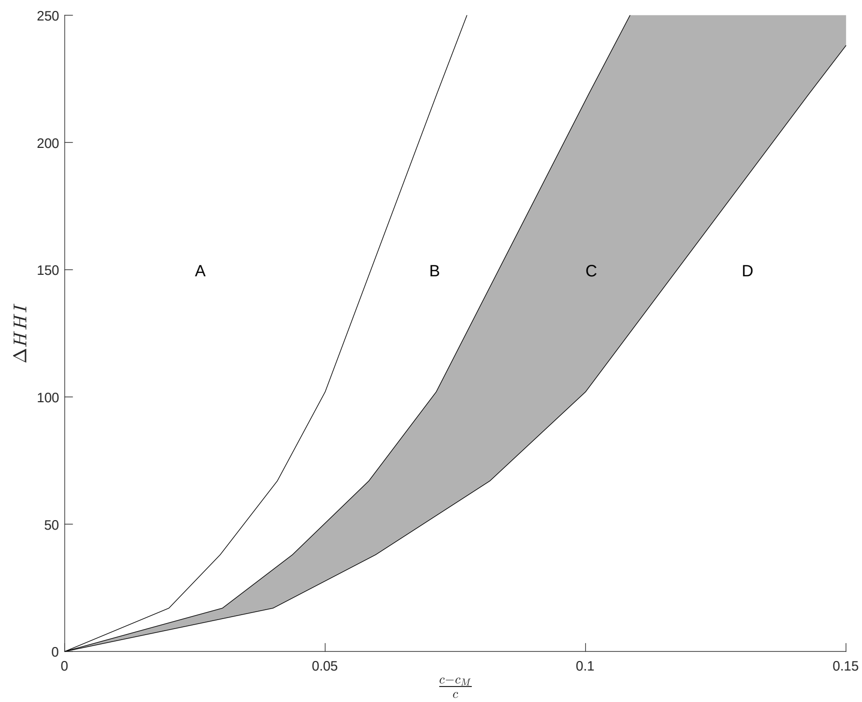

Figure II: *Contours of the efficiencies offense in the Cournot model*

This figure plots ΔHHI against $(c - c_M)/c$. That is, the vertical axis measures equilibrium concentration changes (ignoring endogenous exit), while the horizontal axis measures normalized merger synergies. In regions A, B, and D, traditional merger analysis and our framework imply the same enforcement actions, but in region C, they conflict. To improve legibility, the axes' values range from 0 to 0.15 and from 0 to 250, respectively.

We have

$$\begin{aligned}\frac{\partial \pi_1(\cdot)}{\partial Q} &= \frac{-2(P(Q) - c_1)(P'(Q))^2 + (P(Q) - c_1)^2 P''(Q)}{P'(Q)^2} \\ &= 2\frac{P(Q) - c_1}{-P'(Q)} P'(Q) + \left(\frac{P(Q) - c_1}{-P'(Q)}\right)^2 P''(Q) \\ &\propto 2P'(Q) + q_1 P''(Q) \\ &< 0.\end{aligned}$$

To arrive at the first equality, differentiate, and to arrive at the second one, simplify the resulting expression. To obtain the proportionality relationship, substitute q_1 for $-[P(Q) - c_1]/P'(Q)$, and factor q_1 out. To arrive at the inequality, recall that $P'(Q) < 0$ and $P'(Q) + QP''(Q) \leq 0$. If $P''(Q) \leq 0$, then $2P'(Q) + q_1 P''(Q) \leq 2P'(Q) < 0$. Alternatively, if $P''(Q) > 0$, then $2P'(Q) + q_1 P''(Q) < 2P'(Q) + QP''(Q) < P'(Q) + QP''(Q) < 0$. In either case, $2P'(Q) + q_1 P''(Q) < 0$.

Let $\tilde{Q}$ denote postmerger aggregate quantity if no firms exited. Since CS is strictly increasing in Q, and given that $\mathscr{M}$ would be CS-positive were market structure held fixed, $\tilde{Q} > Q^\star$. Given that $\partial \pi_1(\cdot)/\partial Q < 0$ and $\tilde{Q} > Q^\star$, $\pi_1(\tilde{Q}) < \pi_1(Q^\star)$. Since $\pi_1(Q^\star) = \phi_1$, $\pi_1(\tilde{Q}) < \phi_1$, so the least profitable firm exits after the merger. ■

Under the same conditions, we can produce a closed-form solution to the synergies required for a merger to increase CS.

Corollary 1. *In the Cournot model, assume that the least profitable firm breaks even prior to the merger. Denote the price elasticity of demand by* $\epsilon = -\partial log(Q)/\partial log(P(Q))$*, and denote the sum of the premerger shares of the merging firms and the least profitable firm by* $\check{s} = s_1^\star + s_m^\star + s_n^\star$*. Also, denote their average share-weighted premerger variable cost by* $\check{c} = (s_1^\star c_1 + s_m^\star c_m + s_n^\star c_n)/\check{s}$*. Finally, denote their within-merger concentration by* $HHI_M = (s_1^{\star 2} + s_m^{\star\,2} + s_n^{\star 2})/\check{s}^2$ *and their within-merger concentration change by* $\Delta HHI_M = 2(s_m^\star s_n^\star + s_m^\star s_1^\star + s_n^\star s_1^\star)$*. If* $\mathscr{M}$ *is CS-positive, then*

$$\frac{\check{c} - c_M}{\check{c}} > \frac{\Delta HHI_M}{\check{s}\,(\epsilon - \check{s} HHI_M)}. \tag{10}$$

Proof. See the appendix.

To put Corollary 1 in perspective, consider the case where all firms are symmetric. If market structure is held fixed, then the synergies required for a CS-positive merger must exceed $s^\star/(\epsilon - s^\star)$, as shown by Nocke and Whinston (2022). However, if market structure equilibrates, then synergies must exceed $\Delta HHI_M/3s^\star(\epsilon - 3s^\star HHI_M)$, which is exactly twice the previous figure.[16]

Proposition 2 and Corollary 1 require a firm to exactly break even prior to the merger. We now relax that restriction. Instead, we make an assumption about the pool of potential entrants. We assume that there is one potential entrant with variable and fixed costs equal to those of the least profitable firm.[17] In words, it would be technically possible to replicate the operations of the least profitable firm, but since we do not observe this, it must be because fixed costs make it unprofitable.[18] As our next proposition shows, under this assumption, synergies always exist that create an efficiencies offense.

Proposition 3. *In the Cournot model, assume the market has one additional firm whose variable and fixed costs equal those of the least profitable firm. Then, there exist synergies that create an efficiencies offense.*

Proof. See the appendix.

Recall Motta and Vasconcelos (2005). Prior to any merger, four symmetric firms face unit linear inverse demand, $P(Q) = 1 - Q$, produce at a per-unit variable cost equal to 4α, and pay total fixed cost equal to $f/4$. Mergers cut variable cost in half but double total fixed cost. The authors determine that an efficiencies offense can occur under these conditions. Specifically, the problem arises whenever $1/9 < \alpha < 1/6$ and $(1 - 6\alpha)^2/36 < f < (1 - 6\alpha)^2/9$. This section shows that their insight applies much more broadly—it does not rely on, e.g., synergies that cut variable cost in half, symmetric firms, or linear demand. The next section shows that it also applies under price competition. Later in the paper, however, our conclusions diverge from theirs.

[16]If we substitute in for ΔHHI_M and HHI_M, then the numerator equals $6s^{\star 2}$ and denominator equals $3s^\star(\epsilon - 3s^\star[3s^{\star 2}/9s^{\star 2}]) = 3s^\star(\epsilon - s^\star)$. The fraction simplifies to $2 \times s^\star/(\epsilon - s^\star)$.

[17]This model nests one with symmetric fixed costs and one with asymmetric fixed costs in which the disturbances are expectational errors (i.e., ν_2 in the notation of Pakes et al. (2015), which has been widely adopted in empirical literature).

[18]The added firm did not enter prior to the merger, so we can bound its fixed cost, which are also the least profitable firm's fixed costs. For this reason, the added firm provides information about when the least profitable firm will exit.

3 Price competition with differentiated products

3.1 Primitives and equilibrium

Now, consider a game that is similar to the one described above with one exception. In the prior section, firms set quantities for homogeneous products. In this one, firms set prices for differentiated products that face multinomial logit demand.

Just prior to the start of the game, each firm $f \in \mathscr{F}$ is assigned a set $\mathscr{K}_f$ of products. This set determines the firm's type, T_f, which is defined as $\sum_{k \in \mathscr{K}_j}(v_k - \alpha c_k)$. Here, k indexes products, v_k denotes the mean utility of consumption, c_k denotes the variable cost of production, and α measures sensitivity to price. Firms are sorted in increasing order of their types from 1 to N. The first stage proceeds as it did earlier—firms decide to stay or exit. In the second stage, each firm j that stays in the market sets a price p_k for each $k \in \mathscr{K}_j$ and earns gross profit equal to $\sum_{k \in \mathscr{K}_j} (p_k - c_k)\, s_k(p)$, where s_k denotes market share. Given multinomial logit demand, k's market share takes the form

$$s_k(p) = \frac{e^{v_k - \alpha p_k}}{1 + \sum_{k' \in \mathscr{K}} e^{v_{k'} - \alpha p_{k'}}}, \tag{11}$$

where $\mathscr{K} = \cup_j \mathscr{K}_j$.

Recall that firms maximize profits and that the production technology provides constant returns to scale. In the second stage, given all firms $f < J$ exit, the price that firm $f \geq J$ charges for k satisfies

$$s_k(p) + \sum_{k' \in \mathscr{K}_f / k} (p_{k'} - c_{k'})\, \alpha s_{k'}(p) s_k(p) = (p_k - c_k)\, \alpha s_k(p)\, (1 - s_k(p)), \tag{12}$$

which implies

$$\alpha(p_k - c_k) = 1 + \alpha \sum_{k' \in \mathscr{K}_f} (p_{k'} - c_{k'})\, s_{k'}(p). \tag{13}$$

Nocke and Schutz (2018) prove that not only is the pricing game described here aggregative but also each firm's strategy can be neatly summarized as a scalar. Borrowing their notation, we denote the aggregator by H, which equals $1 + \sum_{k \in \mathscr{K}} e^{v_k - \alpha p_k}$. We also denote the aforementioned scalars by μ_f and refer to them as ι-markups. Each is given by the left-hand side of equation 13.

The authors show that equilibrium is characterized by three conditions that derive from profit maximization, demand, and an adding-up constraint. Given all $f < J$ exit, the conditions provide

$$\mu_f = \frac{1}{1 - s_f}, \tag{14}$$

$$s_f = \frac{T_f}{H} e^{-\mu_f}, \tag{15}$$

and

$$\frac{1}{H} + \sum_{f \geq J} s_f = 1, \tag{16}$$

respectively. Each firm f earns gross profit equal to

$$\pi_f(J) = \frac{1}{\alpha}(\mu_f - 1). \tag{17}$$

In the first stage, each firm f stays in if and only if $\pi_f(f) \geq \phi_f$.

Merger $\mathscr{M}$ combines m and n into M and creates synergies by increasing types such that $T_M \geq T_m + T_n$. As in Section 2, we denote premerger equilibrium objects using an asterisk and, unless otherwise stated, make the standard assumption that all firms earn positive profit prior to the merger.

3.2 Zero-rival-profit and consumer-welfare-neutrality curves

We start with the i^{th} ZRP curve. Since i breaks even,

$$\phi_i = \frac{\mu_f - 1}{\alpha}. \tag{18}$$

The aggregator required to ensure this occurs equals

$$H^i(\phi_i) = \frac{T_i(1+\alpha\phi_i)}{\alpha\phi_i e^{1+\alpha\phi_i}}. \tag{19}$$

The type of merged firm that yields such an aggregator, provided all $i' < i$ exit, equals

$$\tilde{T}^i_M = H^i(\phi_i) S^{-1}\left(1 - \frac{1}{H^i(\phi_i)} - \sum_{g\in\mathscr{G}} S\left(\frac{T_g}{H^i(\phi_i)}\right)\right), \tag{20}$$

where $\mathscr{G} = \{\{i, ..., N\}\backslash\{m,n\}\}$ and $S(T_f/H)$ represents f's function mapping H to its market share.[19] In the Bertrand model, it is most natural to normalize synergies by dividing through by the sum of the types of the merging firms, since M can obtain $T_m + T_n$ through mere joint ownership. As Nocke and Whinston (2022) show, if $\mathscr{M}$ reduces the variable costs of products offered by m and n by the same amount Δc, then the log of the type ratio is directly proportional to the variable cost reduction:

$$\begin{aligned}
\log\left(\frac{T^M}{T^m + T^n}\right) &= \log\left(\frac{\sum_{j\in\{m,n\}}\exp(v_j - \alpha c_j + \alpha\Delta c)}{\exp(v_m - \alpha c_m) + \exp(v_n - \alpha c_n)}\right) \\
&= \log\left(\frac{\exp(\alpha\Delta c)\sum_{j\in\{m,n\}}\exp(v_j - \alpha c_j)}{\exp(v_m - \alpha c_m) + \exp(v_n - \alpha c_n)}\right) \\
&= \alpha\Delta c.
\end{aligned}$$

[19] Nocke and Schutz (2018) show that μ_f and s_f are the unique solutions to equations 14 and 15, provided H satisfies equation 16. One can make substitutions and obtain $s_f = (T_f/H)\exp(-(1-s_f)^{-1})$. $S(T_f/H)$ is f's "fitting-in" function that maps H to its share.

Substituting $\Phi_i \equiv \phi_i/\pi_i^\star$ for ϕ_i and $\tilde{T}_M^i/(T_m+T_n)$ for $\tilde{T}_M^i$, we obtain ZRP$_i$:

$$\frac{\tilde{T}_M^i}{T_m+T_n} = \frac{1}{T_m+T_n} H^i(\phi_i) S^{-1}\left(1 - \frac{1}{H^i(\phi_i)} - \sum_{g\in\mathscr{G}} S\left(\frac{T_g}{H^i(\phi_i)}\right)\right). \tag{21}$$

The type of merged firm required to make $\mathscr{M}$ CS-neutral, provided all $i' \leq i$ exit, equals

$$\bar{T}_M^i = H^\star S^{-1}\left(S\left(\frac{T_m}{H^\star}\right) + S\left(\frac{T_n}{H^\star}\right) + \sum_{f<i} S\left(\frac{T_f}{H^\star}\right)\right). \tag{22}$$

Making analogous substitutions to the ones described above, we obtain CSN$_i$:

$$\frac{\bar{T}_M^i}{T_m+T_n} = \frac{1}{T_m+T_n} H^\star S^{-1}\left(S\left(\frac{T_m}{H^\star}\right) + S\left(\frac{T_n}{H^\star}\right) + \sum_{f<i} S\left(\frac{T_f}{H^\star}\right)\right). \tag{23}$$

As in the previous section, extending the indexing of the CSN curves lets us construct CSN$_0$ by computing $\bar{c}_M^i$ when $i=0$.

3.3 Efficiencies offense

Our fourth proposition characterizes the efficiencies offense in markets where firms sets prices:

Proposition 4. *If $\tilde{T}_M^i < T_M < \bar{T}_M^i$ for any $i \in \{\mathscr{F}\backslash\{m,n\}\}$, then an efficiencies offense arises under Bertrand competition.*

Proof. Since $T_M > \tilde{T}_M^i$, the aggregator exceeds the level required for i to break even, so all $i' \leq i$ exit. Conditional on all $i' \leq i$ exiting, since $T_M < \bar{T}_M^i$, the aggregator falls short of $H^\star$, which is required for CS-neutrality, so $\mathscr{M}$ is CS-negative. Finally, since $T_M > \tilde{T}_M^i$ and $\tilde{T}_M^i > \bar{T}_M^{i-1}$, $\mathscr{M}$ would be CS-positive were i not to exit. ■

We can also now visualize the contours of the efficiencies offense under price competition. We plot the curves defined by the equations 21 and 23. Borrowing parameters from the leading example in Nocke and Whinston (2022), we assume symmetric firms, each of which have a 10% market share prior to the merger and face an own-price elasticity of 5.

Figure III reports the result. Its basic features closely resemble those of Figure I. Evidenced by the shaded portions of the graph above CSN$_0$, many mergers that would be CS-positive were market structure held fixed are CS-negative when market structure is allowed to endogenously adjust. This is especially true when we restrict attention to fixed costs that rationalize premerger market structure—the area to the right of the vertical, dashed line—suggesting that efficiencies offense may commonly arise in "real world" situations. Third, each CSN curve intersects its subsequently indexed ZRP curve on the unit interval. Thus, in markets where one or more firms are only marginally profitable, the efficiencies offense may be the rule rather than exception.

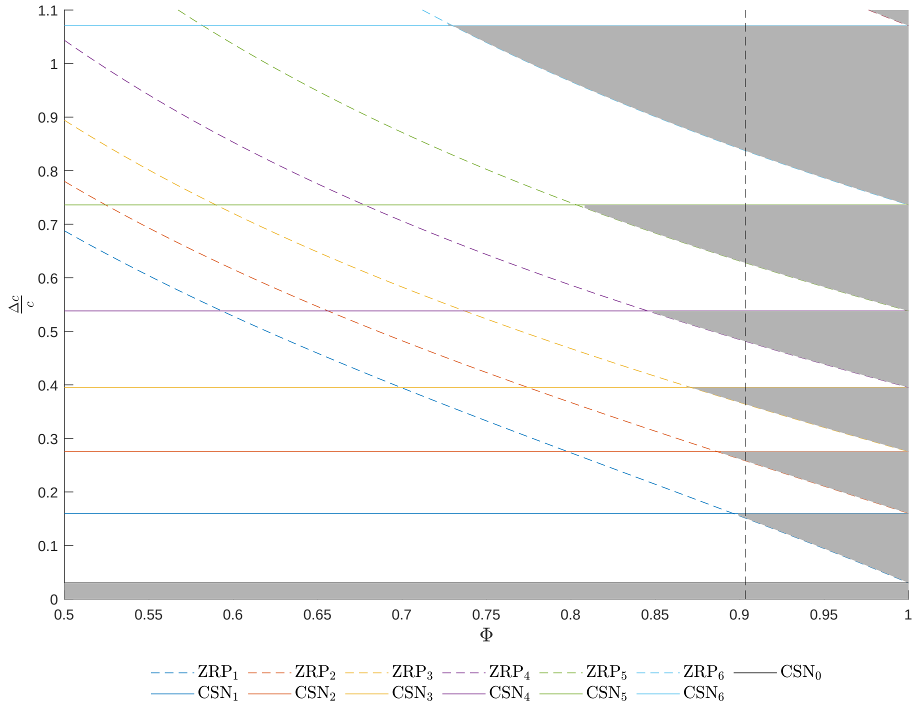

Figure III: *Efficiencies offenses in the Bertrand model*

This figure plots normalized fixed costs, Φ_f, on the horizontal axis and normalized merger synergies, expressed as $\Delta c/c$, on the vertical axis. Dashed lines represent ZRP curves, while solid lines represented CSN curves. The height of CSN_0 corresponds to the fixed-market-structure CMCR—the benchmark against which synergies are compared in traditional merger analysis. To improve legibility, the axes' values range from 0.5 to 1 and from 0 to 1.1, respectively.

3.4 Implications

The Bertrand framework we analyze bears important similarities to the one in which firms compete à la Cournot. For instance, their first stages are qualitatively the same, while their second stages are both aggregative. As a consequence, the broad lessons we took away from Section 2.4 apply here. We begin with Proposition 5, which is analogous to Proposition 2—knife-edge conditions that provide intuition about the efficiencies offense.

Proposition 5. *In the Bertrand model, assume that the least profitable firm breaks even prior to the merger. If $\mathscr{M}$ would be CS-positive were market structure held fixed, then $\mathscr{M}$ causes the least profitable firm to exit.*

Proof. See the appendix.

We can also produce a closed-form solution for the type increase required for a CS-positive merger. It is identical to what one obtains when market structure is held fixed but for one difference, which is that s_1 also appears in the numerator.

Corollary 2. *In the Bertrand model, assume that the least profitable firm breaks even prior to the merger. Let* $\check{s} = s_1^\star + s_m^\star + s_n^\star$ *denote the sum of the premerger shares of the merging firms and the least profitable firm. If* $\mathscr{M}$ *is CS-positive, then*

$$\frac{T_M}{T_m + T_n} > \frac{\check{s}\exp\left(\frac{1}{1-\check{s}}\right)}{s_m^\star \exp\left(\frac{1}{1-s_m^\star}\right) + s_n^\star \exp\left(\frac{1}{1-s_n^\star}\right)}.$$

Proof. See the appendix.

Finally, we can relax the restriction that at least one firm breaks even prior to the merger and instead assume that there exists a potential entrant with type and fixed costs equal to those of the least profitable firm. As in the prior section, under these conditions, synergies always exist that create an efficiencies offense.

Proposition 6. *In the Bertrand model, assume the market has one additional firm whose type and fixed cost equal those of the least profitable firm. Then, there exist synergies that create an efficiencies offense.*

Proof. See the appendix.

4 Potential defenses

In this section, we address other market responses that might mitigate the efficiencies offense.

4.1 Follow-on mergers

Managers that prefer to run their firms independently (for any number of business or personal reasons) will be forced reconsider when faced with exit. As a result, firms that turn unprofitable because their rivals have merged will themselves be more likely to merge. If this occurs and the follow-on merger generates synergies, then the two mergers together might be CS-neutral, even though the first merger (alongside ensuing exit) would reduce CS (Motta and Vasconcelos, 2005). Here, we determine precisely which follow-on mergers eliminate the efficiencies offense.

Our characterization assumes Cournot competition. Let $\mathscr{M}'$ denote a merger that combines the least profitable firm with another firm in $\mathscr{F}$, indexed by m', to form M'. To rule out less interesting cases, assume $\mathscr{M}'$ is "disjoint" in the sense that $m' \neq M'$, and assume synergies satisfy $c_{M'} < \min\{c_1, c_{m'}\}$. Define $\mathscr{G} = \{\mathscr{F} \backslash \{1, m, m', n\}\}$ and $\mathscr{H} = \{\mathscr{F} \backslash \{1, m'\}\}$. Compute the sum of the first order conditions and then solve for aggregate quantity, which remains unchanged at $Q^\star$, as $\mathscr{M}$ and $\mathscr{M}'$ are together CS-neutral:

$$Q^\star = \sum_{g \in \mathscr{G}} \frac{P(Q^\star) - c_g}{-P'(Q^\star)} + \frac{P(Q^\star) - c_M}{-P'(Q^\star)} + \frac{P(Q^\star) - c_{M'}}{-P'(Q^\star)}. \tag{24}$$

Equation 24 is deceptively informative. To make this clear, multiply through by $P'(Q^\star)$, add and subtract

$2P(Q^\star) - c_m - c_n$ to and from the right-hand side of the equation, solve for $c_{M'}$, and obtain

$$c_{M'} = \left[(N-1)P(Q^\star) + P'(Q^\star)Q^\star + \sum_{h \in \mathscr{H}} (P(Q^\star) - c_h)\right] + \left[c_m + c_n - P(Q^\star) - c_M\right], \tag{25}$$

where the first bracketed term's value is $\bar{c}^0_{M'}$, the variable cost required to make $\mathscr{M}'$ CS-neutral were it to occur in isolation, while the second bracketed term's value can be inferred from $\mathscr{M}$ in isolation.

Equation 25 proves it is broadly true that fewer synergies are required to make $\mathscr{M}'$ CS-neutral when it occurs alongside $\mathscr{M}$ than when it occurs in isolation. (To see this, recall from earlier that $\mathscr{M}$ must be CS-positive to induce a rival's exit, and recall from Farrell and Shapiro (1990) or Nocke and Whinston (2010) that if $\mathscr{M}$ is to be CS-positive, then $P(Q^\star) - c_M > 2P(Q^\star) - c_m - c_n$, i.e., $c_m + c_n - P(Q^\star) - c_M > 0$. Thus, $c_{M'}$ need not be as low as $\bar{c}^0_{M'}$.) Extending this logic, if all mergers reduce per-unit variable cost by the same amount, then follow-on mergers eliminate the efficiencies offense. Motta and Vasconcelos (2005) consider two symmetric mergers among four firms, so they reach this conclusion—there is no rationale for the efficiencies offense.

Yet, equation 25 also clarifies the limitations of follow-on mergers. Namely, as c_M approaches $\bar{c}^0_M$, $P(Q^\star) - c_M$ approaches $2P(Q^\star) - c_m - c_n$, so $c_m + c_n - P(Q^\star) - c_M$ approaches zero, meaning $c_{M'}$ approaches $\bar{c}^0_{M'}$. Conceptually, in isolation, if the initial merger is "barely" CS-positive, then the follow-on merger must be "almost" CS-neutral. In other words, *the second pair of merging partners will need to possess nearly the same "merger technology" as the first*. For instance, in the 14-symmetric-firm case we studied earlier, if $(c - c_M)/c \approx 5\%$, then we require $(c - c_{M'})/c \approx 5\%$ as well.

Finally, one must check whether the follow-on merger—in concert with the initial one—create their own efficiencies offense. Under the assumption that $\mathscr{M}$ and $\mathscr{M}'$ are jointly CS-neutral, this never occurs. However, if we relax this restriction, then nothing ensures the second-least-profitable firm prior to the mergers will stay active once they are completed. Were this to occur, one would need further assumptions about the synergies produced by yet another merger.

These findings highlight that practical reliance on follow-on mergers depends critically on what merger technologies the firms possess. At the present moment, little is known about these capabilities. Few papers cleanly identify merger synergies; even fewer use firm or market characteristics to broadly predict how mergers will reduce cost or improve quality. In a narrow sense, it depends how "special" m and n are. On the one hand, business conditions might permit any pair of firms m' and n' to consolidate their operations as adroitly as m and n, mitigating the concerns we raise. On the other hand, m and n might represent a rare opportunity—one that has been carefully selected by highly experienced, skilled, and motivated bankers, consultants, and managers for its match value—in which case any follow-on merger $\mathscr{M}'$ will yield few improvements in terms of quality or cost.

Our subjective belief is that the latter view more closely conforms with reality. One reason is that, as Farrell and Shapiro (1990) state, "mergers differ enormously in the extent to which productive assets can usefully be recombined." Another reason is that firms commonly express skepticism about their own prospects to generate synergies. As just one example, consider remarks made by American Vanguard, a fertilizing producer, in its recent report to investors. It first states that "industry consolidation may threaten the Company's position in various markets," that "many of the Company's competitors have grown or are expected to grow through mergers and acquisitions," and that "consequently, the Company may find it more difficult to compete in various markets." Most interestingly, it then states that "while such merger activity may generate acquisition opportunities for the Company, there is no guarantee that the Company will benefit from such opportunities."[20]

4.2 Subsequent entry

All else equal, exit makes markets more attractive to potential entrants, so a merger that causes exit is more likely to induce subsequent entry. If this occurs and the entrant has an especially low variable cost (or high type), then the merger, exit, and subsequent entry together might raise CS, even though the merger and exit alone would reduce CS. However, this requires a special sort of entrant: it must be efficient enough to offset harm caused by the loss of two competitors, efficient enough to profitably enter after this has occurred, but not so efficient as to enter before this has occurred. Here, we determine exactly which entrants meet these criteria.

Once again, our characterization assumes Cournot competition.[21] Let e index the new entrant, and denote its variable and fixed costs by c_e and ϕ_e, respectively. Define $\mathscr{H} = \{\mathscr{F} \backslash \{m, n, 1\}\}$, $\mathscr{I} = \{\{\mathscr{F} \backslash \{m, n, 1\}\} \cup \{M, e\}\}$, and $\mathscr{J} = \{\mathscr{F} \cup e\}$, and let $\hat{Q}$ and $\hat{\hat{Q}}$ denote the aggregate quantities produced when $\mathscr{I}$ and $\mathscr{J}$ stay in the market, respectively. Before the merger occurs, e does not enter, so we can bound ϕ_e from below in relation to c_e. Similarly, after the merger occurs and the least profitable firm exits, e enters, so we can bound ϕ_e from above in relation to c_e. That is, we can write

$$\underline{\phi}_e \equiv \frac{(P(\hat{\hat{Q}}) - c_e)^2}{-P'(\hat{\hat{Q}})} < \phi_e \leq \frac{(P(\hat{Q}) - c_e)^2}{-P'(\hat{Q})} \equiv \overline{\phi}_e. \tag{26}$$

If the merger, exit, and subsequent entry are together CS-positive, we can also bound c_e in a way that does not depend on ϕ_e. We compute the sum of the first order conditions, solve for $Q^\star$ and $\hat{Q}$, compare their values, and rearrange terms to arrive at

$$c_e < P(\hat{Q}) + \sum_{h \in \mathscr{H}} (P(\hat{Q}) - c_h) + P(\hat{Q}) - c_M - \frac{P'(\hat{Q})}{P'(Q^\star)} \sum_{f \in \mathscr{F}} (P(Q^\star) - c_f). \tag{27}$$

To visualize the set of mergers that satisfy these inequalities, we plot the set of permissible fixed and variable costs. To preserve comparability to earlier results, we impose the same assumptions used to construct Figure I. Likewise, we normalize the entrant's fixed costs by dividing them through by premerger gross profit, and we express the entrant's variable costs as a fractional change relative to the incumbents' variable costs. Specifically, we set $\underline{\Phi}_e = \underline{\phi}_e / \pi_1^\star$, $\overline{\Phi}_e = \overline{\phi}_e / \pi_1^\star$, and replace c_e with $(c_e - c)/c$. In this example, we assume that $\mathscr{M}$ reduces variable cost by 7%. This is 2% points greater than the fixed-market-structure CMCR (see, e.g., Figure I), so $\mathscr{M}$ reduces CS by causing the least profitable firm to exit.

Figure IV reports the result. Two features are especially salient. One is the narrowness of the shaded area. Only a thin "slice" of the parameter space is permitted by the model, which is consistent with our claim that entrants cannot be widely relied upon to remedy the efficiencies offense. The other is that the

[20] See the firm's December 31, 2022 10-K filing, which is available at https://www.sec.gov/Archives/edgar/data/5981/000095017023008438/avd-20221231.htm.

[21] Caradonna et al. (2024) characterize entry induced by postmerger profit increases, concisely enumerating its limits (e.g., if a merger attracts entrants whose products strongly appeal to consumers, then the merger will never be proposed in the first place, as it will be unprofitable). As we consider the joint effect of entry *and* postmerger exit on postmerger entry, we view this section as extending part of their results to reflect the efficiencies offense.

shaded area extends to the right of zero. Even inefficient entrants—ones whose variable costs exceed those of incumbents—can restore CS. Of course, such firms must have very low fixed costs to profitably enter. For example, an entrant with 1% higher variable costs requires 37% lower fixed costs.

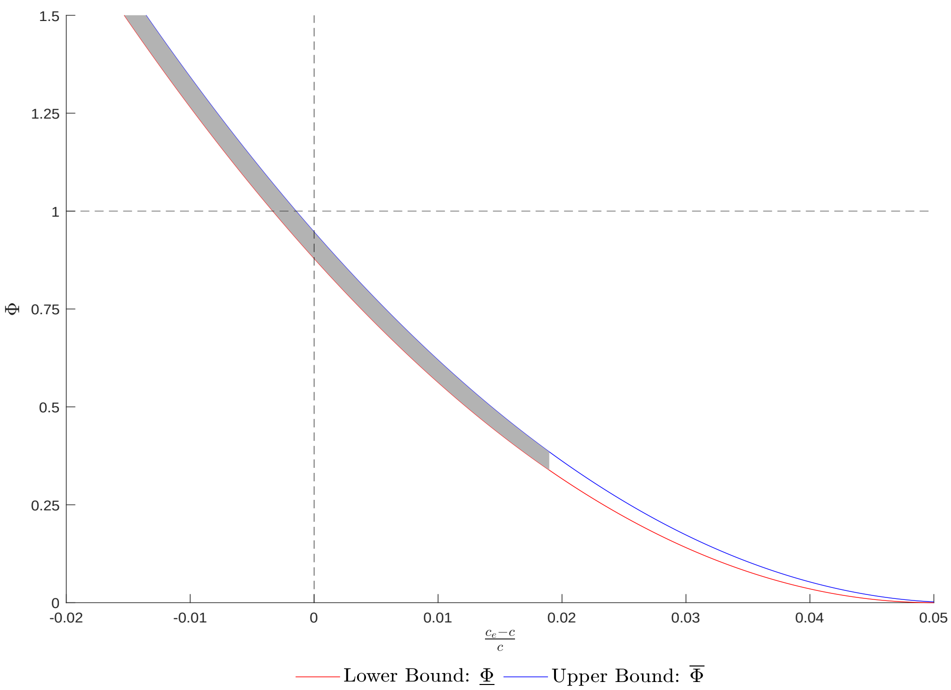

Figure IV: *Types of entrants that remedy an efficiencies offense*

This figure plots normalized differences between the entrant's and incumbents' variable costs, $(c_e - c)/c$, on the horizontal axis against normalized fixed costs of the entrant, Φ_e, on the vertical axis. The red and blue lines correspond to lower and upper bounds provided by inequalities 26, respectively. Synergies are also bounded from above in absolute terms by inequality 27. The shaded area corresponds to parameter values that remedy the efficiencies offense.

4.3 Other considerations

At least three other issues may arise in practice. Katz (2002) crisply summarizes them as follows.[22] First, he argues that authorities must "balance post-exit harms against the consumer gains that would arise during any period in which the merged firm enjoyed the efficiencies and its rivals still were viable competitors." This is undoubtedly an important consideration. Even though a comprehensive analysis of dynamic situations in which viability depreciates over time is beyond the scope of this paper, we can capture its essential features by modifying our framework.

For the purposes of this illustration, we assume Cournot competition calibrated to the parameters used to produce Figure I. Given the point at issue, we also assume the least profitable firm breaks even prior to

[22] Katz (2002) also questions "how likely it is that merged firm would have incentives to price so low that it would deter existing or new rivals from making investments needed to remain or become viable competitors." Our analysis reveals that only static profit maximization is necessary to produce an efficiencies offense. In short, such incentives may be very likely. To be clear, though, we are making generalizations, whereas the author's statements should be interpreted in the context of a particular case—General Electric's proposed acquisition of Honeywell—and its individual circumstances. Moreover, given the author's knowledge of the transaction and cogent analysis of its expected effects, we defer to his assessment.

the merger, which would be CS-positive were market structure held fixed. We denote the discount factor δ and the number of years it would take an unprofitable firm to exit by y. We define $\mathscr{H} = \{\mathscr{F} \backslash \{m, n\} \cup M\}$ and $\mathscr{I} = \{\mathscr{F} \backslash \{1, m, n\} \cup M\}$, define $\hat{Q}$ and $\hat{\hat{Q}}$ as the aggregate quantities produced when $\mathscr{H}$ and $\mathscr{I}$ stay in the market.

The present value of the consumer welfare change caused by $\mathscr{M}$ is given by

$$\text{CS Present Value} = \frac{\delta(1-\delta^y)}{1-\delta}\left(\int_0^{\hat{Q}}(P(x)-P(\hat{Q}))dx - \int_0^{Q^\star}(P(x)-P(Q^\star))dx\right) \tag{28}$$

$$+\frac{\delta^{y+1}}{1-\delta}\left(\int_0^{\hat{\hat{Q}}}(P(x)-P(\hat{\hat{Q}}))dx - \int_0^{Q^\star}(P(x)-P(Q^\star))dx\right). \tag{29}$$

The first term is an annuity comprised of consumer gains, which accrue while the least profitable firm remains in the market. The second term is a perpetuity comprised of CS losses, which begin accruing when the least profitable firm exits and must be discounted to the time of the merger. To put these tradeoffs in perspective, if $\delta = 0.9$, then approval turns on whether the least profitable firm would exit in under 6.1 years. If consumers are more or less patient, then the period is longer or shorter. For example, if $\delta = 0.925$, then approval turns on whether the least profitable firm will sustain losses for 8.4 years. Our subjective view is that most firms would exit long before this time has elapsed.

Second, one may question "the abilities of enforcement authorities to reliably predict this type of harm even after a detailed investigation of market conditions" (Katz, 2002). Our analysis highlights that, at least under the stated assumptions, the only inputs that are required to study the efficiencies offense but absent from traditional merger analysis are fixed costs; demand, marginal costs, and synergies are required irrespective of whether the agencies and courts wish to consider the postmerger viability of rivals. Further, at the risk of oversimplifying their measurement, fixed costs can often be read directly off the firms' income statements, making them much cheaper to obtain than any of the data required for second stage estimation.

Moreover, were a preliminary investigation to suggest that one or more firms is only marginally profitable prior to the merger, our analysis suggests that uncertainty about synergies works *against* the merger rather than *for* it. (Recall, for example, the right-hand sides of Figures I or III, where the majorities of the graphs are shaded.) Alternatively, of course, were a preliminary investigation to reveal operationally and financial sound competitors, then one can presumably set these concerns aside. As an example, in the case referenced by Katz (2002), which involved General Electric's proposed acquisition of Honeywell, the competitive set comprised Rolls-Royce and Pratt & Whitney. At the time, both rivals boasted "growing revenues and profits" as well as plans to "heavily [invest] in the development of their next-generation [products]." Facts such these mitigate or even eliminate the risk of merger-induced exit.

Third, one should "consider the broader or longer-term implications of a policy that attacks mergers suspected of having these effects." In markets comprised mainly of firms that would survive against a more efficient rival, exhaustively investigating synergistic mergers amounts to a tax on transactions that help rather than harm consumers. However, in markets where relevant rivals are clearly at risk, one could incorporate an efficiencies offense without deterring CS-positive mergers. Other unintended consequences are harder to sign. For instance, foreseeing an efficiencies offense, merging parties may delay CS-positive investments until their transaction is complete but may also opt to internally develop capabilities that they would otherwise have obtained through acquisition. In the latter case, even if the least profitable firm exits, two rather than one efficient firms may remain.

5 Conclusions

This paper studies the efficiencies offense, which occurs when a merger's synergies reduce consumer welfare by inducing rivals to exit. Mid-twentieth century jurisprudence broadly accepted this theory of harm, but as Chicago School perspectives swept through US courts, the idea was unconditionally rejected. The consumer welfare standard, coupled with formalism, were assumed to have ruled out the possibility (see the introduction and cites therein).

We revisit this important question and early, insightful formulations of it (Motta, 2004; Motta and Vasconcelos, 2005) using recent technical advances in aggregative games (Nocke and Schutz, 2018, 2024). Our formulation determines the precise conditions under which the efficiencies offense arises and when exactly corrective mechanisms mitigate it. Our findings indicate that the problem may arise more frequently than previously thought. Markets can self-correct but only under special circumstances (e.g., follow-on mergers offer a solution but must typically generate their own large synergies). Policy-wise, our framework would "screen in" more transactions than traditional merger analysis. This supports recent claims that US enforcement guidelines published over the last four decades were too lax (Nocke and Whinston, 2022).

New questions arise that are beyond this paper's scope. How often does synergy-induced exit by nonmerging firms occur? Is it rare, or, like many other economic phenomena, simply hard to measure? If it is rare, then which of our assumptions are most commonly violated? If it is not, then are even large firms in economically important industries at risk, as anecdotes presented at the outset of the paper indicate. Is the mechanism obscured because firms are slow to exit, reflecting underlying transaction costs, contracting problems, option value in the face of uncertainty, or even mistakes by boundedly rational managers who learn about the viability of their enterprises over time? Conversely, are firms so slow to exit that the short term consumer benefits outweigh the long term harm? Further, does the mechanism simply manifest as follow-on mergers, precipitated by investors who are "forced" to sell?

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Appendix

Proof of Corollary 1.
We have

$$
\begin{aligned}
\frac{\check{c}-c_M}{\check{c}} &> \frac{\check{c}-\bar{c}_M^1}{\check{c}} \\
&= \frac{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1 - \check{s}\bar{c}_M^1}{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1} \\
&= \frac{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1 - \check{s}\left[c_m + c_n + c_1 - 2P\left(Q^*\right)\right]}{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1} \\
&= \frac{s_m^\star\left[P\left(Q^*\right)-c_n\right] + s_m^\star\left[P\left(Q^*\right)-c_1\right] + s_n^\star\left[P\left(Q^*\right)-c_m\right]}{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1} \\
&\quad + \frac{s_n^\star\left[P\left(Q^*\right)-c_1\right] + s_1^\star\left[P\left(Q^*\right)-c_m\right] + s_1^\star\left[P\left(Q^*\right)-c_n\right]}{s_m^\star c_m + s_n^\star c_n + s_1^\star c_1} \\
&= \frac{s_m^\star P\left(Q^*\right)\frac{s_n^\star}{\epsilon} + s_m^\star P\left(Q^*\right)\frac{s_1^\star}{\epsilon} + s_n^\star P\left(Q^*\right)\frac{s_m^\star}{\epsilon}}{s_m^\star P\left(Q^*\right)\left[1-\frac{s_m^\star}{\epsilon}\right] + s_n^\star P\left(Q^*\right)\left[1-\frac{s_n^\star}{\epsilon}\right] + s_1^\star P\left(Q^*\right)\left[1-\frac{s_1^\star}{\epsilon}\right]} \\
&\quad + \frac{s_n^\star P\left(Q^*\right)\frac{s_1^\star}{\epsilon} + s_1^\star P\left(Q^*\right)\frac{s_m^\star}{\epsilon} + s_1 P\left(Q^*\right)\frac{s_n^\star}{\epsilon}}{s_m^\star P\left(Q^*\right)\left[1-\frac{s_m^\star}{\epsilon}\right] + s_n^\star P\left(Q^*\right)\left[1-\frac{s_n^\star}{\epsilon}\right] + s_1^\star P\left(Q^*\right)\left[1-\frac{s_1^\star}{\epsilon}\right]} \\
&= \frac{\frac{2s_m^\star s_n^\star + 2s_m^\star s_1^\star + 2s_n^\star s_1^\star}{\epsilon}}{\check{s}\left[1-\frac{s_m^{\star\,2} + s_n^{\star\,2} + s_1^{\star\,2}}{\check{s}\epsilon}\right]} \\
&= \frac{\Delta HHI_M}{\check{s}\left[\epsilon - \check{s}HHI_M\right]}.
\end{aligned}
$$

If $\mathscr{M}$ is CS positive, then $c_M < \bar{c}_M^1$, which implies the inequality. To obtain the first equality, substitute $(s_1^\star c_1 + s_m^\star c_m + s_n^\star c_n)/\check{s}$ for $\check{c}$. By definition, $\bar{c}_M^1$ leaves CS unchanged, so it leaves aggregate quantity unchanged. Since $\mathscr{M}$ does not affect the variable costs of nonmerging firms, and since each firm's equilibrium quantity depends only on its variable cost and aggregate quantity, the equilibrium quantities produced by the nonmerging firms do not change as a result of the merger. Thus, the postmerger quantity produced by M must equal the sum of the premerger quantities produced by m, n, and the least profitable firm. Since each firm f produces $-(P(Q^\star) - c_f)/P'(Q^\star)$, $P(Q^\star) - \bar{c}_M^1 = 3P(Q^\star) - c_m - c_n - c_1$, so $\bar{c}_M^1 = c_m + c_n + c_1 - 2P(Q^\star)$. Make this substitution to obtain the second equality. Rearrange terms to obtain the third equality, and simplify the expression to obtain the fourth equality. Substitute in for ϵ to obtain the fifth equality, and substitute in with ΔHHI_M, HHI_M, and $\check{s}$ to obtain the sixth equality. ■

Proof of Proposition 3.
It is helpful to begin by establishing permissible bounds on fixed costs. Recall that f's gross profit takes the form $-(P(Q) - c_f)^2/P'(Q)$. Since exactly N firms stay prior to $\mathscr{M}$, $-(P(Q^\star) - c_1)^2/P'(Q^\star) \geq \phi_1 \equiv \overline{\phi}$, where $\overline{\phi}$ represents a (weak) upper bound. By the same logic, $-(P(\hat{\hat{Q}}) - c_1)^2/P'(\hat{\hat{Q}}) < \phi_1 = \underline{\phi}$, where $\hat{\hat{Q}}$ denotes the quantity produced if all $N+1$ firms stay in the market and $\underline{\phi}$ represents a (weak) lower bound.

Next, recall that $\tilde{c}_M^1$ is given by $(N-1)P(Q^1(\phi_1)) + P'(Q^1(\phi_1))\underline{Q}^1(\phi_1) - \sum_{h\in\mathscr{H}_1} c_h$, where $\mathscr{H}_i = \{\mathscr{F}\backslash\{m,n\}\}$, and recall that $\bar{c}_M^1$ is given by $(N-2)P(Q^\star) + P'(Q^\star)Q^\star - \sum_{g \in \mathscr{G}_1} c_g$, where $\mathscr{G}_1 = \{\{2,3,...,N\} \setminus \{m,n\}\}$. Let $\phi^\star$ denote the value of ϕ_1 that sets these expressions equal. Since $\tilde{c}_M^1$ is increasing in ϕ_1 while $\bar{c}_M^1$ does not depend on it, $\tilde{c}_M^1 > \bar{c}_M^1$ for all $\phi_1 > \phi^\star$. Hence, so long as $\phi_1 > \phi^\star$, there exists a c_M that satisfies $\bar{c}_M^1 < c_M < \tilde{c}_M^1$. Thus, it suffices to prove that $\underline{\phi} > \phi^\star$.

Let $\mathscr{A}$ denote a hypothetical market with a set $\mathscr{I}$ of firms, where $\mathscr{I} = \{\mathscr{F} \setminus \{1,m,n\} \cup M\}$ and $c_M = \tilde{c}_M^1 = \bar{c}_M^1$. Let $\mathscr{A}'$ denote $\mathscr{A}$ with one additional firm that has variable cost c_1. Let $\mathscr{B}$ denote a hypothetical market with the set $\mathscr{F}$ of firms, and let $\mathscr{B}'$ denote $\mathscr{B}$ with one additional firm that has variable cost c_1. Additionally, let $\hat{Q}$ denote the aggregate quantity produced in $\mathscr{A}'$. Notice that $Q^\star$ is produced in $\mathscr{A}$ and $\mathscr{B}$ and that $\hat{\hat{Q}}$ is produced in $\mathscr{B}'$.

By definition, when the variable cost M is $\tilde{c}_M^1$ and the set of firms that stay in the market is given by $\mathscr{H}_1$, the least profitable firm breaks even. $\mathscr{A}'$ corresponds exactly to these conditions, so the least profitable firm breaks even. Also, by definition, when $\tilde{c}_M^1 = \bar{c}_M^1$, $\phi_1 = \phi^\star$. Hence, the gross profit of the least profitable firm in $\mathscr{A}'$ equals $\phi^\star$. Separately, notice that the gross profit of the least profitable firm in $\mathscr{B}'$ equals $\underline{\phi}$. Thus, it suffices to prove that the gross profit of the least profitable firm in $\mathscr{A}'$ is less than the gross profit of either of the least profitable firms in $\mathscr{B}'$.

Take the first order condition of gross profit with respect to individual quantity and sum over firms. In $\mathscr{A}$, arrive at

$$Q^\star P'(Q^\star) + (N-2)P(Q^\star) - \sum_{i \in \mathscr{I}} c_i - c_M = 0, \tag{30}$$

and in $\mathscr{B}$, arrive at

$$Q^\star P'(Q^\star) + NP(Q^\star) - \sum_{f \in \mathscr{F}} c_f = 0. \tag{31}$$

In both $\mathscr{A}$ and $\mathscr{B}$, aggregate quantity equals $Q^\star$. Solve the system of equations and obtain $c_M = c_1 + c_m + c_n - 2P(Q^\star)$.

In $\mathscr{A}'$, sum the first order conditions and arrive at

$$\hat{Q}P'(\hat{Q}) + (N-1)P(\hat{Q}) - \sum_{i \in \mathscr{I}} c_i - c_1 - c_M = 0, \tag{32}$$

substitute $c_1 + c_m + c_n - 2P(Q^\star)$ for c_M, and rearrange terms to obtain

$$\hat{Q}P'(\hat{Q}) + (N+1)P(\hat{Q}) - \sum_{f \in \mathscr{F}} c_f - c_1 + 2(P(Q^\star) - P(\hat{Q})) = 0. \tag{33}$$

Finally, in $\mathscr{B}'$, obtain the same sum and arrive at

$$\hat{\hat{Q}}P'(\hat{\hat{Q}}) + (N+1)P(\hat{\hat{Q}}) - \sum_{f \in \mathscr{F}} c_f - c_1 = 0. \tag{34}$$

Set the left-hand sides of equations 33 and 34 equal and rearrange terms so that

$$[\hat{Q}P'(\hat{Q}) - \hat{\hat{Q}}P'(\hat{\hat{Q}})] + N[P(\hat{Q}) - P(\hat{\hat{Q}})] - 2[P(\hat{Q}) - P(Q^\star)] = 0. \tag{35}$$

All else equal, adding a firm increases aggregate quantity, so $\hat{Q} > Q^\star$. Since $P'(Q) < 0$, $P(\hat{Q}) < P(Q^\star)$, which implies $P(\hat{Q}) - P(Q^\star) < 0$, which in turn implies $-2(P(\hat{Q}) - P(Q^\star)) > 0$. To balance the equation, $\hat{\hat{Q}}P'(\hat{\hat{Q}}) + NP(\hat{\hat{Q}})$ must exceed $\hat{Q}P'(\hat{Q}) + NP(\hat{Q})$. Since $P'(Q) + QP''(Q) < 0$, the first set of terms are decreasing in Q, and since $P'(Q) < 0$ and $N > 0$, the second set of terms are decreasing in Q, so $\hat{Q}$ must exceed $\hat{\hat{Q}}$. Hence, aggregate quantity is greater in $\mathscr{A}'$ than $\mathscr{B}'$. Since the gross profit of the least profitable firm is declining in aggregate quantity, the gross profit of the least profitable firm in $\mathscr{A}'$ is less than the gross profit of one of the least profitable firms in $\mathscr{B}'$. ■

Proof of Proposition 5.

Let $H^{\star\star}$, $\mu_1^{\star\star}$, $\pi_1^{\star\star}$ denote postmerger equilibrium values of H, μ_1, and π_1, respectively. The least profitable firm breaks even prior to the merger, meaning that $\pi_1^\star = \phi_1$. If $\pi_1^{\star\star} < \pi_1^\star$, then $\pi_1^{\star\star} < \phi_1$, which means that the least profitable firm exits following $\mathscr{M}$. Hence, it suffices to prove that $\pi_1^{\star\star} < \pi_1^\star$. $\mathscr{M}$ would be CS-positive were market structure held fixed, and CS is strictly increasing in H, so $H^{\star\star} > H^\star$. Thus, it suffices to prove that variable profit is strictly decreasing in H.

Combine equations 14 and 15 and then arrange terms to arrive at $\mu_f(1 - (T_f/H)\exp(-\mu_f)) = 1$. Based on this relationship, let $m(T_f/H)$ denote the implicit function that maps T_f/H to μ_f. Proposition 6 of Nocke and Schutz (2018) states that $m' > 0$, implying that μ_f is strictly decreasing in H. Per equation 17, variable profit is given by $(\mu_f - 1)/\alpha$. α is positive, so this expression is strictly increasing in μ_f. Since variable profit is strictly increasing in μ_f, which is strictly decreasing in H, variable profit is strictly decreasing in H. ■

Proof of Corollary 2.

We have

$$\begin{aligned}
\frac{T_M}{T_m + T_n} &> \frac{\bar{T}_M^1}{T_m + T_n} \\
&= \frac{\bar{T}_M^1/H^\star}{T_m/H^\star + T_n/H^\star} \\
&= \frac{\bar{T}_M^1/H^\star}{s_m^\star \exp(\frac{1}{1-s_m^\star}) + s_n^\star \exp(\frac{1}{1-s_n^\star})} \\
&= \frac{S(\bar{T}_M^1/H^\star)\exp\left(\frac{1}{1-S(\bar{T}_M^1/H^\star)}\right)}{s_m^\star \exp(\frac{1}{1-s_m^\star}) + s_n^\star \exp(\frac{1}{1-s_n^\star})} \\
&= \frac{(s_m^\star + s_n^\star + s_1^\star)\exp(\frac{1}{1-s_m^\star-s_n^\star-s_1^\star})}{s_m^\star \exp(\frac{1}{1-s_m^\star}) + s_n^\star \exp(\frac{1}{1-s_n^\star})} \\
&= \frac{\check{s}\exp(\frac{1}{1-\check{s}})}{s_m^\star \exp(\frac{1}{1-s_m^\star}) + s_n^\star \exp(\frac{1}{1-s_n^\star})}.
\end{aligned}$$

By Proposition 6 of Nocke and Schutz (2018), consumer surplus is increasing in T_f for every f. Hence, if $\mathscr{M}$ is CS-positive, then T_M must exceed $\bar{T}_M^1$, which implies the inequality. Multiply the numerator and denominator by $1/H^\star$ to obtain the first equality. Combine equations 14 and 15 to obtain

$$\frac{T_f}{H} = s_f \exp\left(\frac{1}{1-s_f}\right) \tag{36}$$

for every f. Substitute in for $T_m/H^\star$ and $T_n/H^\star$ using equation 36 to obtain the second equality. Equation 36 shows that each firm's market share depends only on its type and the aggregator, so write

$$s_f = S(T_f/H). \tag{37}$$

Combine equations 36 and 37, and substitute in for $\bar{T}^1_M/H^\star$ to obtain the third equality. Rearrange terms in equation 16 to write

$$1 - \frac{1}{H^\star} = \sum_{f\in\mathscr{F}} S(T_f/H^\star) = S(\bar{T}^1_M/H^\star) + \sum_{h\in\mathscr{H}} S(T_h/H^\star), \tag{38}$$

where $\mathscr{H} = \{\{\mathscr{F}\backslash\{1,m,n\}\}$, which implies that $S(\bar{T}^1_M/H^\star) = S(T_m/H^\star) + S(T_n/H^\star) + S(T_1/H^\star) = s^\star_1 + s^\star_m + s^\star_n$. Substitute in for $S(\bar{T}^1_M/H^\star)$ to obtain the fourth equality. Finally, substitute in with $\check{s}$ to obtain the fifth equality. ■

Proof of Proposition 6.
It is helpful to begin by establishing permissible bounds on fixed costs. Recall that f's gross profit takes the form $(\mu_f - 1)/\alpha$. Since exactly N firms stay prior to $(\mu^\star - 1)/\alpha \geq \phi_1 \equiv \overline{\phi}_1$, where $\overline{\phi}$ represents a (weak) lower bound. By the same logic, $(\hat{\mu}_i - 1)/\alpha \leq \phi_1 \equiv \underline{\phi}_1$, where $\hat{\mu}_i$ denotes i's ι-markup if all of the $N+1$ firms stay and $\underline{\phi}$ represents a (weak) upper bound.

Let $\phi^\star$ be the value of ϕ_1 at which $\tilde{T}^1_M = \bar{T}^1_M$. $\tilde{T}^1_M$ is strictly increasing in ϕ_1, while $\bar{T}^1_M$ does not depend on ϕ_1, so for any $\phi > \phi^\star$, there exists a T_M such that $\tilde{T}^1_M < T_M < \bar{T}^1_M$. Therefore, it suffices to prove $\phi^\star < \underline{\phi}$.

Let $\mathscr{A}$ denote a hypothetical market with a set $\mathscr{H}$ of firms, where $\mathscr{H} = \{\mathscr{F}\backslash\{1,m,n\} \cup M\}$ and $T_M = \tilde{T}^i_M = \bar{T}^i_M$. Let $\mathscr{A}'$ denote $\mathscr{A}$ but with one additional firm that has type T_1. Let $\mathscr{B}'$ denote a hypothetical market with the set $\mathscr{F}$ of firms, and let $\mathscr{B}'$ denote $\mathscr{B}$ with one additional firm that has type T_1. Notice that $\phi^\star$ equals the variable profit that the least profitable firm earns in $\mathscr{A}'$. Also, notice that $\underline{\phi}$ equals the variable profit that either of the least profitable firms earn in in $\mathscr{B}'$. Hence, it suffices to prove that the variable profit of the least profitable firm in $\mathscr{A}'$ is less than the variable profit of either of the least profitable firms in $\mathscr{B}'$.

In both $\mathscr{A}$ and $\mathscr{B}$, the aggregator equals $H^\star$. Thus, we can use the adding-up constraints to obtain

$$\frac{1}{H^\star} + \sum_{f\in\mathscr{H}} s^\star_f + s^\star_M = 1. \tag{39}$$

and

$$\frac{1}{H^\star} + \sum_{f\in\mathscr{F}} s^\star_f = 1. \tag{40}$$

We can then solve the system of equations and obtain $s^\star_M = s^\star_1 + s^\star_m + s^\star_n$. The adding-up constraint also provides

$$\frac{1}{\hat{H}} + \sum_{f \in \mathscr{H}} \hat{s}_f + \hat{s}_1 + \hat{s}_M = 1 \tag{41}$$

where $\hat{H}$ denotes the aggregator and $\hat{s}_i$ denote i's share in $\mathscr{A}'$, which can be rewritten as

$$\frac{1}{\hat{H}} + \sum_{f \in \mathscr{F}} \hat{s}_f + \hat{s}_1 + \hat{s}_M - (\hat{s}_1 + \hat{s}_m + \hat{s}_n) = 1. \tag{42}$$

Again, the adding-up constraint provides

$$\frac{1}{\hat{\hat{H}}} + \sum_{f \in \mathscr{F}} \hat{\hat{s}}_f + \hat{\hat{s}}_1 = 1, \tag{43}$$

where $\hat{\hat{H}}_i$ denotes the aggregator and $\hat{\hat{s}}_i$ denotes i's share in $\mathscr{B}'$. Set the left-hand sides of equations 42 and 43 equal and rearrange terms to obtain

$$\left[\frac{1}{\hat{H}} - \frac{1}{\hat{\hat{H}}_i}\right] + \sum_{f \in \mathscr{F}} \left[\hat{s}_f - \hat{\hat{s}}_f\right] + \left[\hat{s}_1 - \hat{\hat{s}}_1\right] + \left[\hat{s}_M - \hat{s}_1 - \hat{s}_m - \hat{s}_n\right] = 0. \tag{44}$$

We will now prove that $\hat{s}_M - \hat{s}_1 - \hat{s}_m - \hat{s}_n > 0$. We combine equations 14 and 15 to obtain

$$\frac{T_f}{H} = s_f \exp\left(\frac{1}{1 - s_f}\right) \tag{45}$$

for every f, which implies that each firm's market share depends only on its type and the aggregator, which, in turn, implies that we can write $s_f = S(T_f/H)$. We differentiate $S(T_M/H) - S(T_1/H) - S(T_m/H) - S(T_n/H)$ with respect to H and evaluate the resulting expression at $H^\star$ to obtain

$$\begin{aligned}
&-\tfrac{T_M}{H^\star} S'\left(\tfrac{T_M}{H^\star}\right) + \tfrac{T_1}{H^\star} S'\left(\tfrac{T_1}{H^\star}\right) + \tfrac{T_m}{H^\star} S'\left(\tfrac{T_m}{H^\star}\right) + \tfrac{T_n}{H^\star} S'\left(\tfrac{T_n}{H^\star}\right) \\
&\qquad = -\epsilon\left(\tfrac{T_M}{H^\star}\right) S\left(\tfrac{T_M}{H^\star}\right) + \epsilon\left(\tfrac{T_1}{H^\star}\right) S\left(\tfrac{T_1}{H^\star}\right) + \epsilon\left(\tfrac{T_m}{H^\star}\right) S\left(\tfrac{T_m}{H^\star}\right) + \epsilon\left(\tfrac{T_n}{H^\star}\right) S\left(\tfrac{T_n}{H^\star}\right) \\
&\qquad = -\epsilon\left(\tfrac{T_M}{H^\star}\right)\left[S\left(\tfrac{T_1}{H^\star}\right) + S\left(\tfrac{T_m}{H^\star}\right) + S\left(\tfrac{T_n}{H^\star}\right)\right] + \epsilon\left(\tfrac{T_1}{H^\star}\right) S\left(\tfrac{T_1}{H^\star}\right) + \epsilon\left(\tfrac{T_m}{H^\star}\right) S\left(\tfrac{T_m}{H^\star}\right) + \epsilon\left(\tfrac{T_n}{H^\star}\right) S\left(\tfrac{T_n}{H^\star}\right) \\
&\qquad = \left[\epsilon\left(\tfrac{T_1}{H^\star}\right) - \epsilon\left(\tfrac{T_M}{H^\star}\right)\right] S\left(\tfrac{T_1}{H^\star}\right) + \left[\epsilon\left(\tfrac{T_m}{H^\star}\right) - \epsilon\left(\tfrac{T_M}{H^\star}\right)\right] S\left(\tfrac{T_m}{H^\star}\right) + \left[\epsilon\left(\tfrac{T_n}{H^\star}\right) - \epsilon\left(\tfrac{T_M}{H^\star}\right)\right] S\left(\tfrac{T_n}{H^\star}\right) \\
&\qquad > 0.
\end{aligned}$$

To arrive at the second line, use Lemma 5 of Nocke and Schutz (2024), which states that the elasticity of S is given by $\epsilon(x) = xS'/S(x)$, and replace $xS'(x)$ with $\epsilon(x)S(x)$, where $x \in \{T_M/H^\star, T_m/H^\star, T_n/H^\star, T_1/H^\star\}$. (The derivation of Lemma 5 of Nocke and Schutz (2024) appears in Section XXXi.3 of the Online Appendix of Nocke and Schutz (2018).) To arrive at the third line, replace $S(T_M/H^\star)$ with $s^\star_m + s^\star_n + s^\star_1$, and to arrive at the fourth line, rearrange terms. To arrive at the fifth line, use Lemma XXV of Nocke and Schutz (2018), which states that $\epsilon(x)$ is strictly decreasing in x, and the fact that $T_M > \max\{T_1, T_m, T_n\}$. Together, they imply that $\epsilon(T_\ell/H^\star) - \epsilon(T_M/H^\star) > 0$. Since $S(T_M/H) - S(T_1/H) - S(T_m/H) - S(T_n/H)$ is increasing in H, and since$\tilde{H} > H^{\star}$, $\tilde{s}_M - \tilde{s}_m - \tilde{s}_n - \tilde{s}_1 > \hat{s}_M - \hat{s}_1 - \hat{s}_m - \hat{s}_n$. Moreover, since $\hat{s}_M - \hat{s}_1 - \hat{s}_m - \hat{s}_n = 0$, $\tilde{s}_M - \tilde{s}_m - \tilde{s}_n - \tilde{s}_1 > 0$.

To balance both sides of equation 44, given $\hat{s}_M - \hat{s}_1 - \hat{s}_m - \hat{s}_n > 0$, we must have

$$\left[\frac{1}{\hat{H}} - \frac{1}{\hat{\hat{H}}_i}\right] + \sum_{f \in \mathscr{F}} \left[\hat{s}_f - \hat{\hat{s}}_f\right] + \left[\hat{s}_1 - \hat{\hat{s}}_1\right] < 0. \tag{46}$$

$1/\hat{H} - 1/\hat{\hat{H}}_i < 0$ if and only if $\hat{H} > \hat{\hat{H}}$. Also, shares are strictly decreasing in the aggregator, so $\hat{s}_f < \hat{\hat{s}}_f$ if and only if $\hat{H} > \hat{\hat{H}}$. Thus, inequality 46 implies $\hat{H} > \hat{\hat{H}}$, which implies $\hat{\mu}_1 < \hat{\hat{\mu}}_1$, which implies that the variable profit of the least profitable firm in $\mathscr{A}'$ is less than the variable profit of either of the least profitable firms in $\mathscr{B}'$. ■