

NBER WORKING PAPER SERIES

IS SOFTWARE EATING THE WORLD?

Sangmin Aum
Yongseok Shin

Working Paper 32591
<http://www.nber.org/papers/w32591>

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2024

We thank Salome Baslandze, Andrea Eisfeldt, Bart Hobijn, Matthias Kehrig, Joseba Martinez, and Ezra Oberfield for helpful comments, and Sungjoong Kim and Hyunjung Joo for their assistance at the Regional Data Center of Statistics Korea. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2024 by Sangmin Aum and Yongseok Shin. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Is Software Eating the World?
Sangmin Aum and Yongseok Shin
NBER Working Paper No. 32591
June 2024
JEL No. D22,D24,D33,E22,E25,L11

ABSTRACT

When explaining the declining labor income share in advanced economies, the macro literature finds that the elasticity of substitution between capital and labor is greater than one. However, the vast majority of micro-level estimates shows that capital and labor are complements (elasticity less than one). Using firm- and establishment-level data from Korea, we divide capital into equipment and software, as they may interact with labor in different ways. Our estimation shows that equipment and labor are complements (elasticity 0.6), consistent with other micro-level estimates, but software and labor are substitutes (1.6), a novel finding that helps reconcile the macro vs. micro-literature elasticity discord. As the quality of software improves, labor shares fall within firms because of factor substitution and endogenously rising markups. In addition, production reallocates toward firms that use software more intensively, as they become effectively more productive. Because in the data these firms have higher markups and lower labor shares, the reallocation further raises the aggregate markup and reduces the aggregate labor share. The rise of software accounts for two-thirds of the labor share decline in Korea between 1990 and 2018. The factor substitution and the markup channels are equally important. On the other hand, the falling equipment price plays a minor role, because the factor substitution and the markup channels offset each other.

Sangmin Aum
Kyung Hee University
26 Kyunghee-daero, Dongdaemun-gu
Seoul 02447
South Korea
aumsang@gmail.com

Yongseok Shin
Department of Economics
Washington University in St. Louis
One Brookings Drive
St. Louis, MO 63130
and NBER
yshin@wustl.edu

Software is eating the world.

— Marc Andreessen (The Wall Street Journal, August 20, 2011)

1 Introduction

The aggregate labor income share remained more or less constant for much of the 20th century. For example, [Keynes \(1939\)](#) wrote that “the stability of the proportion of the national dividend accruing to labour” was “one of the most surprising, yet best-established, facts.” However, the labor share started a downward trend in most advanced economies since the 1980s. With the heightened interest in economic inequality after the financial crisis of 2007–08, economists have raised a variety of explanations for the decline of the labor share, as reviewed by [Aum and Shin \(2020\)](#) and [Grossman and Oberfield \(2022\)](#).

One of the leading explanations is that capital and labor are substitutes, and the more efficient production of capital goods reduced the labor share over time ([Karabarbounis and Neiman, 2013](#)). Given the co-movement of the aggregate labor share and the price of capital goods, it seems like an obvious explanation. However, as pointed out by, for example, [Eden and Gaggl \(2019\)](#), this explanation is at odds with the majority of micro-level estimates that find capital (specifically equipment) and labor to be complements, with an elasticity of substitution less than one ([Antras, 2004](#); [Raval, 2019](#); [Oberfield and Raval, 2021](#), among others).

We address this macro-literature vs. micro-literature elasticity discord by dividing capital into equipment and software, as the two may interact with labor differently. We estimate the elasticity of substitution across software, equipment, and labor, both at the micro and the macro levels. We then quantify the distinct role of software and equipment price changes (or software/equipment-embodied technological changes) in the decline of the labor share, not only through factor substitution but also through changes in markups. To our knowledge, ours is the first paper to (i) analyze the impact of the rise of software (or intangibles more generally) on factor shares and markups at the micro and the macro levels, and (ii) explicitly reconcile the macro-literature vs. micro-literature elasticity discord.

The macro elasticity differs from the micro elasticity, because changes in factor prices not only alter the factor income shares within firms, but also reallocate resources across firms that are heterogeneous in factor intensities ([Houthakker, 1955](#); [Jones, 2005](#); [Oberfield and Raval, 2021](#)). The estimation of the macro elasticity therefore requires micro-level data capturing this cross-sectional distribution of factor

shares. More important, for the purpose of separating the elasticity of substitution between software and labor from that between equipment and labor, one needs micro-level data by capital types. Typically, software is lumped together with other intangibles, a category with immense measurement problems.

We use firm-level and plant-level data from Korea, where firms keep track of their software investment in accordance with local accounting standards, separately from other categories of intangibles that are more prone to missing values and other measurement problems.

There are reasons to think that software and equipment deserve separate investigation, beyond the fact that one is intangible capital and the other is tangible capital. For one, the evolution of the software investment share in the aggregate time series in the US aligns with the dynamics of the labor share, whereas the equipment share does not ([Aum and Shin, 2020](#)). For another, software and equipment usage varies across occupations in the US data, with software being primarily used by high-skill occupations and equipment by middle-skill ones ([Aum, 2020](#)), which suggests that the two interact with labor in different ways.

We start with two empirical patterns in the Korean data that further validate the case for investigating software separately from equipment. First, firm-level panel regressions show that firms' software intensity predicts a decrease in their labor share, especially the income share of non-production, white-collar workers. In contrast, firms' equipment intensity makes no meaningful prediction either way. Second, in a cross-section of regions, software expenditure shares and local wages (controlling for worker characteristics) are positively related, but equipment shares and local wages do not have a significant relationship.

To estimate the macro elasticity of substitution between labor and the two types of capital, we build on the approach of [Oberfield and Raval \(2021\)](#). Our contribution is to allow for three factors (labor, equipment, and software) and variable markups, which leads to two novel results. First, in terms of factor substitution, the role of reallocation depends on the covariance between factor shares across firms, in addition to the variance of factor shares. This is because, unlike in a two-factor model (where the two factor shares are perfectly negatively correlated by construction), a high software share of a firm does not necessarily imply a low labor share—it could be that its equipment share is very low and the labor share is higher than the average. Second, variable markups introduce an additional margin of adjustment in the labor share both through within-firm and between-firm adjustments in markups in response to

factor price changes.¹

Using this theoretical framework, we first estimate the micro elasticity of substitution between labor and either type of capital by instrumenting for wage variations across regions or industries. We find that the elasticity of substitution between labor and equipment is less than one (0.6), consistent with the micro-level estimates from the US and other countries. The novel finding is that the elasticity of substitution between labor and software is greater than one (1.6). We obtain very similar estimates when we use cross-sectional data on manufacturing plants with regional shift-share instruments and when we use panel data on firms in all sectors with industry-level minimum-wage instruments.

Because software and labor are substitutes at the micro level, a fall in software price reduces the labor share within a firm. The effect is amplified by the variable markups, as firms charge higher markups in response, further reducing labor shares within firms.

The firms using software more intensively benefit more from the fall in software prices. They become effectively more productive and hence larger than those using less software. Using the joint distribution of factor shares and markups from the micro data, we calculate the macro elasticity, accounting for the reallocation across firms. In the data, firms with high software shares tend to have lower-than-average labor shares. (Again, with three factors, they need not.) Hence the reallocation further reduces the aggregate labor share. Also in the data, high software share firms tend to have high markups. The implied low demand elasticity dampens the magnitude of the reallocation (relative to a constant markup environment), but the reallocation to high markup firms does raise the aggregate markup.

Quantitatively, the decrease in software prices accounts for two-thirds of the labor share decline in Korea between 1990 and 2018 (2.9 out of 4.4 percentage points). Slightly more than half of the effect comes through the markup channel (52 percent), and the rest (48 percent) through the factor substitution channel. One immediate implication is that the rise of the software or intangibles income share in the accounting sense, highlighted by [Koh et al. \(2020\)](#), will underestimate the role of software in the decline of the labor income share by more than half, as it misses the impact through the rise in markups. The effect through the markup channel can be attributed almost equally to within-firm markup growth and the between-firm reallocation. By contrast, nearly all the effect through the factor substitution channel is due to within-firm

¹[Oberfield and Raval \(2021\)](#) showed in an appendix how their theoretical derivation can be extended to a case with variable markups, although their quantitative results were for the constant markup case.

factor substitution.

By contrast, the decline in equipment prices has a negligible effect on the aggregate labor income share, because several opposing effects offset one another. The factor substitution channel pushes up the labor share because equipment and labor are complements within firms, even after factoring in the between-firm reallocation. The resulting rise in markups reduces the labor share.

The importance of markups and the between-firm reallocation for the labor share decline in Korea that we find is consistent with the findings of [Autor et al. \(2020\)](#) and [Kehrig and Vincent \(2021\)](#) for the US, although they do not specify the cause of the reallocation. Our finding suggests that software-embodied technological change may be the force driving these within-firm and between-firm empirical patterns in the US.

In summary, it is software, not equipment, that substitutes for labor and reduces the labor income share. The resulting within-firm markup growth and the reallocation toward high markup firms are quantitatively important channels through which the fall in software prices reduces the aggregate labor share. Our result helps reconcile the macro-literature vs. micro-literature elasticity discord surrounding the labor share decline: Substitution did play a role, even though equipment and labor are indeed complements. Our result also shows that the rise of software is driving the well-documented reallocation toward large firms with high markups and low labor shares.

Related literature The elasticity of substitution between labor and capital is a crucial parameter in economic theory, and numerous studies have attempted to estimate it. One complication is that the elasticity at the level of individual firms or establishments may differ from the elasticity at the level of the aggregate economy. [Oberfield and Raval \(2021\)](#) provide a framework to compute the macro elasticity from micro elasticity estimates and the distribution of factor shares across firms. Our work extends theirs to consider two distinct types of capital that can interact with labor in different ways and to allow endogenous changes in markups.

Related to our paper, [Lashkari et al. \(2023\)](#) study the interplay of capital-embodied technological change, labor share, and market concentration. Their focus is the role of information technology (IT) that has increasing returns to scale. Using French firm-level data, they find that the elasticity of substitution between IT capital (an amalgam of hardware and software) and labor is still less than one. (They assume that the elasticity of substitution between non-IT capital and labor is one.) Our paper separates software and equipment, and shows that software substitutes for labor but equipment

complements labor. It also emphasizes the role of variable markups.²

Our work also relates to the macroeconomic literature on capital-embodied technological change, for example, [Greenwood et al. \(1997\)](#), [Greenwood et al. \(2000\)](#), and [Cummins and Violante \(2002\)](#). [Krusell et al. \(2000\)](#) showed that capital-embodied technological change and the complementarity between skilled labor and capital drove up the skill premium in the US, focusing exclusively on equipment capital. Our micro data does not allow us to differentiate labor by skill, but we did run a skilled vs. unskilled labor exercise using aggregate time series. We found that both skilled labor and unskilled labor are substitutes with respect to software, but skilled labor is less substitutable. Likewise, both types of labor are complements with respect to equipment capital, but skilled labor is more complementary. To the extent that the skill premium result of [Krusell et al. \(2000\)](#) only requires that skilled labor is more complementary or less substitutable with respect to capital than is unskilled labor, our findings are consistent.

This paper also relates to the research showing that the decline in the labor share partly reflects the misclassification of labor compensation as capital income, especially when workers are involved in the production or utilization of intangible capital. [Koh et al. \(2020\)](#) raised this possibility. [Eisfeldt et al. \(2023\)](#) document a rise in equity-based compensation of high-skill workers and show that their labor income share did not fall, once their equity-based compensation is accurately classified as labor income.

Some recent studies have pointed to the role of intangible capital in technological advancement ([Corrado et al., 2009, 2022](#)). However, few asked whether and how intangible capital and tangible capital may interact with other production factors in different ways. [Aum and Shin \(2020\)](#) showed that industries with higher software intensity experienced a more rapid decline in labor share in the US. [Aum \(2019\)](#) documented that the correlation between labor and software across regions is distinct from the correlation between labor and equipment.³ Our paper substantiates and rationalizes such suggestive evidence through a structural estimation of the micro and the macro elasticities of substitution between equipment and labor and between software and labor.

The data we use in our analysis predate the ebullience surrounding artificial intelligence (AI) and its implication for the economy unleashed by OpenAI's ChatGPT in 2022. [Eisfeldt et al. \(2023\)](#) provide some evidence that Generative AI, which is an

²A related paper is [de Souza and Li \(2023\)](#). It divides equipment into robots vs. tools, and finds evidence that robots substitute for labor but tools complement labor in Brazilian data.

³Related, [Park \(2022\)](#) finds that an occupation's software intensity determines whether its employment share will grow or shrink during an investment boom.

extremely powerful class of software, is on net a substitute for labor, consistent with our finding regarding more conventional software.

The rest of the paper is structured as follows. We document relevant empirical facts in Section 2. In Section 3, we introduce our model framework. We then estimate micro elasticities and aggregate them into macro elasticities in Section 4. In Section 5, we quantify the contribution of software-embodied technological change to the decline in the aggregate labor income share through various channels. Section 6 concludes.

2 Motivating Facts

In this section, we describe the data used in our analysis and present the distinct empirical patterns exhibited by software and equipment.

Aggregate trends Before going into the micro-level data, we provide an overview of the evolution of the labor income share and the capital income shares by capital type in Korea. The labor income share is labor income divided by the sum of labor income, capital income, and profits.⁴ To calculate capital income, one needs estimates of the gross rate of return for each type of capital. We assume that the gross return R^j for capital type j satisfies the no-arbitrage condition:

$$R^j = (1 + r)p_{t-1}^j - (1 - \delta^j)p_t^j, \quad (1)$$

where r is the net rate of return, p_t^j is the price of capital j in period t , and δ^j is the depreciation rate of capital j . We use K^j , p^j , and δ^j from the Korean national accounts, and the corporate bond rate net of expected inflation to compute gross return on capital according to (1)—see Appendix B for details.

Figure 1 plots the aggregate labor income share from 1980 to 2018, which declined over time, especially in the 1990s and the 2000s.

We plot the ratio between labor income and type- j capital income in $\log$, $\log wL - \log R_j K_j$, in Figure 2. Software income grew faster than labor income (black solid line),

⁴Our measure of the labor income share is the gross labor share adjusted for proprietors' income (Gollin, 2002; Park, 2020). There have been debates about whether the decline in the net labor share in Korea is merely a measurement issue related to self-employment. Park (2020) showed that the net labor income share is sensitive to how one treats the changing share of "self-employed with employees," whose employee compensation is already included in total compensation. We follow Park (2020) and make the necessary adjustments. Our focus is on the gross labor share, because software, the production factor we are interested in, has a higher depreciation rate than other factors. The trend of the gross labor share is not affected by the proprietor income adjustment. See Appendix B.1.3 for additional details.



Fig. 1: Labor Income Share in Korea

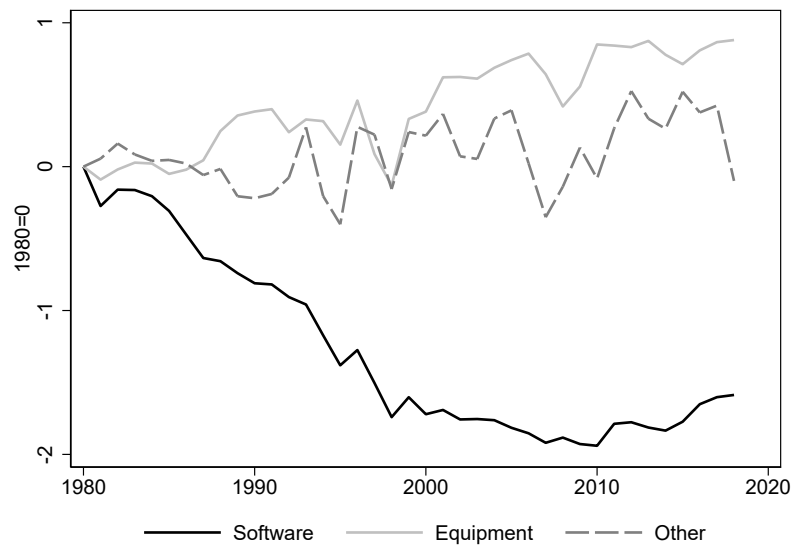


Fig. 2: Labor Income Relative to Capital Income by Capital Type

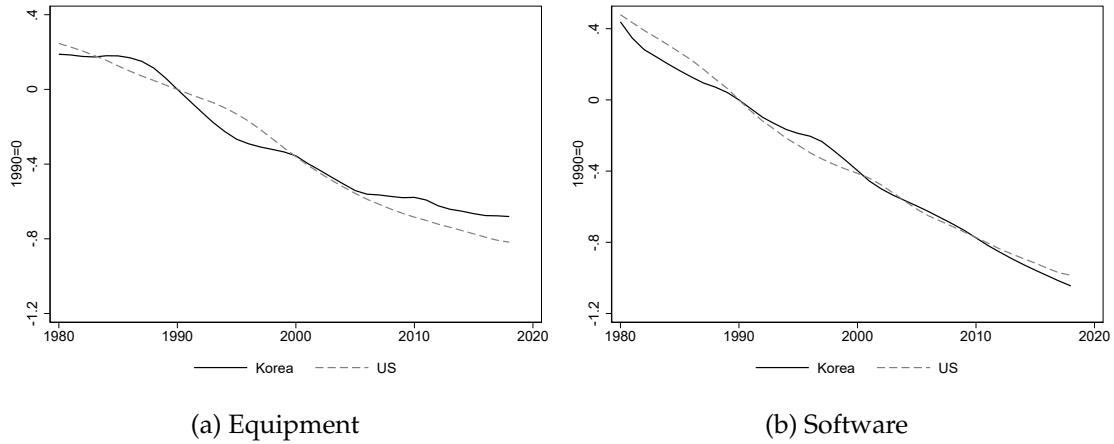


Fig. 3: Relative Price of Investment to Consumption by Capital Type

The solid lines correspond to Korea, while the dashed lines correspond to the US. We compute five-year moving averages of the relative prices and normalize them to 0 in the year 1990.

whereas equipment income, if anything, decreased relative to labor income (gray solid line). The growth of software income relative to labor income was most pronounced in the 1990s and the early 2000s, coinciding with the period of the steepest labor share decline in Figure 1.⁵ This suggests that the fall in the aggregate labor income share has more to do with software than equipment.

Researchers typically infer the technological change specific to capital goods from the declines in the relative price of investment to consumption (e.g., Cummins and Violante, 2002). Following the literature, we use five-year moving averages of the inverse of the prices of software and equipment investment relative to consumption to measure the technological change embodied in them.⁶

Figure 3 plots the relative price of investment to consumption by capital type. Both equipment (left panel) and software (right panel) have experienced substantial capital-embodied technological change, inferred from the rapid decline in their relative price to consumption. It is noteworthy that software price fell more rapidly than equipment price.

⁵The “other” category of capital (dashed line) in Figure 2 includes residential capital, structure, and R&D capital.

⁶One issue with the Korean national accounts is that the software price index may underestimate software-embodied technological change. Since 1994, the price index of software investment comes from the producer price index, which may fail to capture quality improvements. To address this concern, we adjust the software price index following Parker and Grimm (2000), as does the US Bureau of Economic Analysis (BEA). BEA makes a bias adjustment of 3.15 percent per year to the producer price index from the Bureau of Labor Statistics to compensate for the discrepancy between the hedonic method and the matched model method.

These observations are not unique to Korea. For instance, the changes in the price of equipment and software relative to consumption are remarkably similar between Korea and the US, as shown by the solid and the dashed lines in both panels of Figure 3. In addition, consistent with the patterns in Figure 2, Aum and Shin (2020) show that an industry's software intensity is the one variable that is systematically correlated with its labor share decline, based on industry-level data from the US. However, the US micro-level data do not separate software from the broader category of intangibles, which comes with immense measurement problems. For this reason, we turn to micro-level data from Korea for our analysis.

Micro-level data We use two datasets that have information on equipment and software as factors of production at the micro level. One is KISDATA, a panel covering firms in all sectors in Korea from 2003 to 2018. This dataset is a compilation of firm-level financial statements, provided by NICE Information Service. It covers firms listed in the Korea Stock Exchange, as well as those unlisted firms subject to external audit requirements.⁷ Compared with the National Accounts, our KISDATA sample accounts for 44 and 52 percent of the compensation of employees and the value-added, respectively, of the entire non-financial corporate sector in 2018.

One key advantage of KISDATA is that it reports assets by type, for example, distinguishing equipment from software. This is partly thanks to the Korea Generally Accepted Accounting Principles (K-GAAP). International Financial Reporting Standards (IFRS) are principle-based and mandate only a minimal number of items in financial statements. As a result, firm-level financial statements reported under IFRS usually do not disaggregate firms' intangible assets by type. However, K-GAAP provides standard financial statement accounts with software as a separate account item.⁸ This unique aspect of the Korean accounting principles enables us to investigate the separate roles of equipment and software. However, the software variable does not capture all types of software. In K-GAAP, software assets refer to externally purchased software with expected future benefits, which may omit software developed in-house (own account software) or software bundled with equipment that is not separately

⁷The criteria for external audit requirement vary over time. Until 2008, firms whose asset exceeded 7 billion KRW (about 6.4 million USD in 2008) had to be audited externally. Since 2009, firms with (i) assets greater than 10 billion (12 billion since 2014) KRW, or (ii) assets greater than 7 billion KRW and liability greater than 7 billion KRW, or (iii) assets greater than 7 billion KRW and more than 300 employees had to undergo external audits.

⁸Since 2010, listed firms in Korea have adopted K-IFRS and no longer follow K-GAAP. However, companies that had followed K-GAAP before 2010 continue to report detailed accounts. In fact, the number of firms reporting software as a separate asset type has steadily increased since 2010.

identifiable.⁹ We construct firm-level labor compensation, value added, and software and equipment assets to construct labor share and capital intensities at the firm level. More details on data are in Appendix B.

The other dataset is the establishment-level data from the 2015 Korean Economic Census. Conducted every five years, the census surveys all establishments with at least one employee as of December 31 of that year. For those in the manufacturing sector, the census gathers data on intangible assets by type, including externally purchased software, for all uni-establishment firms and corporate headquarters of multi-establishment firms.¹⁰ The census also contains information on annual payroll and equipment capital, among others. It also records the location of establishments, which enables us to utilize variations across regions.

Patterns in the micro-level data We begin by showing that firms’ software intensity predicts a fall in labor share, but their equipment intensity makes no meaningful prediction. The regression equation is:

$$y_{i,t} = \gamma_i + \alpha_t + \beta_s s_{i,t-1} + \beta_e e_{i,t-1} + \varepsilon_{i,t}, \quad (2)$$

where $s_{i,t-1}$ is the software intensity (software asset divided by value added) of firm i in year $t - 1$, $e_{i,t-1}$ is its equipment intensity (equipment asset divided by value added), γ_i is the firm fixed effect, and α_t is the time fixed effect.

The estimation results are in Table 1. In the KISDATA, firms’ software intensity in year $t - 1$ predicts a faster decline of their labor share (first column) and a higher sales growth (last column) between $t - 1$ and t . Firms’ equipment intensity, on the other hand, is not significantly related to changes in the labor share, although it does predict faster sales growth. These results suggest that software may be the driving force behind the empirical fact documented by Kehrig and Vincent (2021): those establishments whose labor share fell and sales increased at the same time account for most of the aggregate labor share decline in the US manufacturing sector.

⁹The Korean national accounts do not provide detailed information by software type. In contrast, the US national accounts break down aggregate software investment into three types: prepackaged, customized, and own account. Between 2003 and 2019, these types account for 40, 41, and 19 percent of total software investment, respectively. This suggests that the omission of own account software should not be a major issue for the Korean data either.

¹⁰That is, the census surveys firm-level rather than establishment-level software assets, as intangible assets at the establishment level are not well defined. In our benchmark analysis, we assume each establishment of a multi-establishment firm uses the same amount of software as its headquarters—that is, software is non-rivalrous within a firm. In a robustness check, we limit our sample to uni-establishment firms and obtain nearly the same result.

	Δ Labor Share			$\Delta \log$ Sales
	Total	Non-production	Production	
$s_{i,t-1}$	-0.287*** (0.049)	-0.195*** (0.044)	-0.102*** (0.013)	0.392*** (0.126)
$e_{i,t-1}$	-0.009 (0.007)	-0.006 (0.004)	-0.004 (0.003)	0.030** (0.015)
N	44,364	43,759	40,626	44,356
R^2	0.189	0.207	0.191	0.291

Table 1: Capital Intensities, Labor Share and Sales Change at the Firm Level

Standard errors are clustered at the firm level. *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

Another finding is that software intensity predicts a steeper fall in the income share of non-production workers than that of production workers (second and third columns). This finding suggests that different workers may be affected differently by the rise of software, but the limited dimension of worker heterogeneity in our micro-level data (essentially, production vs. non-production workers) discourages a richer analysis in this direction.

Aggregating the manufacturing establishments in the census to the region level (Si-Gun-Gu, of which there are 162), we compute the correlation between the average wage of a region and the shares of capital income by capital type in the region.¹¹ Figure 4 depicts the relationship between the log of the ratio of regional expenditures on capital (either software or equipment) to labor on the y-axis against the log of local wages on the x-axis. The ratio of software to labor expenditures tends to be higher in regions with higher local wages (solid gray line). The ratio of equipment to labor expenditures shows, if anything, a negative correlation with local wages (black dashed line). This cross-sectional pattern complements the time-series evidence in Figure 2, suggesting that the decline in the aggregate labor share has more to do with software than equipment. More specifically, the finding that regions with higher wages spend relatively more on software but not on equipment points to the possibility that software substitutes for labor more than does equipment.

To summarize, the patterns in the Korean data suggest that software may be the key to understanding the decline in the aggregate labor income share. At the aggregate level, it is software income, not equipment income, that grew at a faster rate than

¹¹Regional average wages are constructed from yet another dataset, Regional Employment Survey, controlling for various worker characteristics.

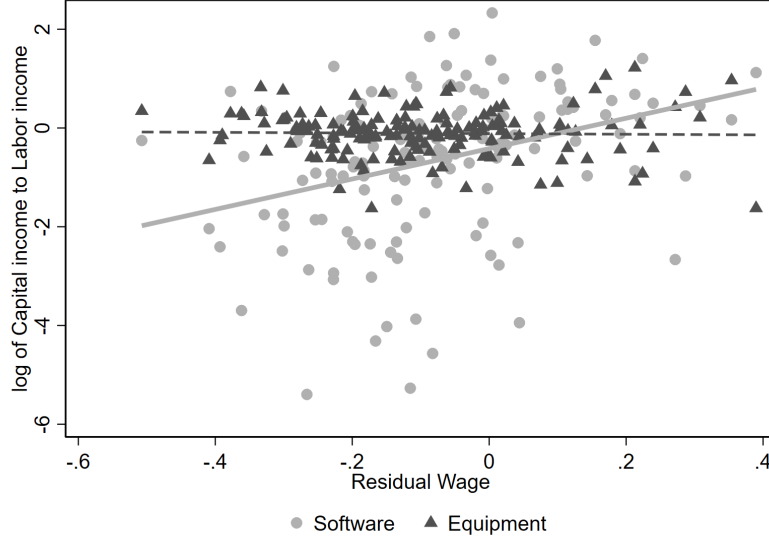


Fig. 4: Relationship between Local Wage and Factor Shares

Each dot represents a region (Si-Gun-Gu, an administrative unit in Korea). A region's average wage is from the Regional Employment Survey, controlled for workers' education, age, gender, and experience.

labor income. At the firm level, software intensity predicts a faster decline in labor income share, but equipment intensity does not. Furthermore, in regions with higher wages, establishments have higher ratios of software income to labor income, but no such relationship exists for the ratio between equipment income and labor income. Motivated by the multitudes of suggestive evidence, we now turn to a more rigorous and definitive analysis of the role of software in the decline of labor share and the rise of markups, both within firms and in the aggregate through between-firm reallocation.

3 Theory

3.1 Model

To investigate the role of software in shaping factor income shares, we consider a production function that has software and equipment as separate inputs, as well as labor and materials. We define a production of firm i as follows.

$$Y_i = \left\{ \left[\left(\alpha_i^L (A_L L_i)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_i^K (A_K K_i)^{\frac{\sigma_e - 1}{\sigma_e}} \right)^{\frac{\sigma_e (\sigma_s - 1)}{(\sigma_e - 1) \sigma_s}} + \alpha_i^S (A_S S_i)^{\frac{\sigma_s - 1}{\sigma_s}} \right]^{\frac{\sigma_s (\sigma_m - 1)}{(\sigma_s - 1) \sigma_m}} + \alpha_i^M M_i^{\frac{\sigma_m - 1}{\sigma_m}} \right\}^{\frac{\sigma_m}{\sigma_m - 1}}$$

(3)

Y_i is output, L_i is labor, K_i is tangible capital (or equipment), S_i is software, and M_i is material input. α_i^L , α_i^K , α_i^S , and α_i^M are the intensities of labor, equipment, software, and material input, respectively. All the inputs and intensities have subscript i , the firm index. A_L , A_K , and A_S represent economy-wide factor-augmenting technologies for labor, equipment, and software, respectively. For notational convenience, we also define the value added V_i and the equipment-labor bundle X_i as follows.

$$V_i \equiv \left[\left(\alpha_i^L (A_L L_i)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_i^K (A_K K_i)^{\frac{\sigma_e - 1}{\sigma_e}} \right)^{\frac{\sigma_e(\sigma_s - 1)}{(\sigma_e - 1)\sigma_s}} + \alpha_i^S (A_S S_i)^{\frac{\sigma_s - 1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s - 1}} \quad (4)$$

$$X_i \equiv \left(\alpha_i^L (A_L L_i)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_i^K (A_K K_i)^{\frac{\sigma_e - 1}{\sigma_e}} \right)^{\frac{\sigma_e}{\sigma_e - 1}}. \quad (5)$$

We note two properties of the production function in equation (3) that are relevant for our analysis. First, two parameters separately govern the elasticity of substitution between labor and equipment (σ_e) and the elasticity between labor and software (σ_s). That is, we can capture different labor share responses to technological changes embodied in different types of capital. Second, all firms are different in how intensively they use each factor, as captured by α_i 's, implying that they will be affected differently by an economy-wide factor-augmenting technological change. As a result, aggregate changes in factor income shares depend not only on within-firm adjustments but also on the reallocation of the production activity across firms. In other words, the elasticity of substitution at the aggregate level will be different from the elasticity of substitution at the firm level, as [Oberfield and Raval \(2021\)](#) showed using a two-factor production function.

To allow for the possibility that software may trigger changes in markup within firms and in the aggregate, we consider a [Kimball \(1995\)](#) aggregator. Specifically, we assume that aggregate production Y satisfies

$$\sum_i H(Y_i/Y) = 1, \quad (6)$$

where $H(\cdot)$ is an increasing and concave function and Y_i is the firm-level production in equation (3). This aggregator implies the demand for Y_i given by

$$\frac{p_i}{p} = H' \left(\frac{Y_i}{Y} \right) \text{ or } \frac{Y_i}{Y} = h \left(\frac{p_i}{p} \right), \quad (7)$$

where h is the inverse function of H' . When $H(x) = x^{\frac{\epsilon - 1}{\epsilon}}$, we obtain a standard CES demand system with the constant elasticity of substitution ϵ .

From equation (7), $d \ln(Y_i/Y) = [(h'(p_i/p)p_i/p)/h(p_i/p)] \times d \ln(p_i/p)$, which gives the demand elasticity

$$\epsilon_i = -\frac{h'(p_i/p)p_i/p}{h(p_i/p)}.$$

Firm i takes the factor prices of labor (w), equipment (r), software (q), and the price of intermediate input (v) as given. A firm's profit maximization is

$$\max_{p_i, Y_i, K_i, S_i, M_i} p_i Y_i - w L_i - r K_i - q S_i - v M_i,$$

subject to equations (3) and (7). The first-order conditions are:

$$w = p_i \alpha_i^L A_L^{\frac{\sigma_e-1}{\sigma_e}} \left(\frac{Y_i}{V_i} \right)^{\frac{1}{\sigma_m}} \left(\frac{V_i}{X_i} \right)^{\frac{1}{\sigma_s}} \left(\frac{X_i}{L_i} \right)^{\frac{1}{\sigma_e}}, \quad (8)$$

$$r = p_i \alpha_i^K A_K^{\frac{\sigma_e-1}{\sigma_e}} \left(\frac{Y_i}{V_i} \right)^{\frac{1}{\sigma_m}} \left(\frac{V_i}{X_i} \right)^{\frac{1}{\sigma_s}} \left(\frac{X_i}{K_i} \right)^{\frac{1}{\sigma_e}}, \quad (9)$$

$$q = p_i \alpha_i^S A_S^{\frac{\sigma_s-1}{\sigma_s}} \left(\frac{Y_i}{V_i} \right)^{\frac{1}{\sigma_m}} \left(\frac{V_i}{S_i} \right)^{\frac{1}{\sigma_s}}, \quad (10)$$

$$v = p_i \alpha_i^M \left(\frac{Y_i}{M_i} \right)^{\frac{1}{\sigma_m}}, \quad (11)$$

$$p_i = \mu_i(p_i/p) \left\{ \left[\phi_i^{1-\sigma_s} + \alpha_i^{S\sigma_s} \left(\frac{q}{A_S} \right)^{1-\sigma_s} \right]^{\frac{1-\sigma_m}{1-\sigma_s}} + \alpha_i^{M\sigma_m} v^{1-\sigma_m} \right\}^{\frac{1}{1-\sigma_m}}, \quad (12)$$

where $\mu_i(p_i/p)$ is the optimal markup that satisfies $\mu_i(p_i/p) = \epsilon_i(p_i/p)/(\epsilon_i(p_i/p) - 1)$. For notational convenience, we introduced ϕ_i , the price of the equipment-labor bundle X_i :

$$\phi_i \equiv \left(\alpha_{i_e}^{L\sigma} \left(\frac{w}{A_L} \right)^{1-\sigma_e} + \alpha_{i_e}^{K\sigma} \left(\frac{r}{A_K} \right)^{1-\sigma_e} \right)^{\frac{1}{1-\sigma_e}}. \quad (13)$$

The factor market clearing conditions are

$$L = \sum_i L_i, \quad K = \sum_i K_i, \quad \text{and} \quad S = \sum_i S_i. \quad (14)$$

Given L , K , and S , we can solve for the equilibrium from equations (3), (6), (7), (8), (9), (10), (12), and (14).

3.2 Elasticity of Substitution between Factors

3.2.1 Micro-level Elasticity

We now analyze the changes in factor shares in response to factor-augmenting technological changes or exogenous changes in factor prices, which are governed by the elasticity of substitution.

When there are more than two factors of production, there are multiple ways to define the elasticity of substitution between any pair of factors. For example, the elasticity of substitution between labor and equipment will vary, depending on whether we fix both output and software or fix only output and allow software to adjust (Stern, 2011). Therefore, we need to first clarify what we call the elasticity of substitution between factors.

We focus on the simplest one, the Allen-Uzawa elasticity. The Allen-Uzawa elasticity of substitution between factors x and y is defined as

$$\sigma_{x,y} = \frac{C C_{xy}}{C_x C_y},$$

where C is the cost function and C_x is its partial derivative with respect to the change in the price of input x , while all other prices are held constant. In our framework, the Allen-Uzawa elasticity of substitution between labor and equipment is simply σ_e , and the one between labor and software is σ_s in the production function in equation (3).

Proposition 1 (Micro-level elasticity of substitution) *The production function parameters σ_e and σ_s satisfy the following.*

$$\sigma_e = 1 + \frac{d \ln r K_i / w L_i}{d \ln w / r} = 1 + \frac{d \ln k_i / (1 - k_i)}{d \ln w / r}, \quad (15)$$

$$\sigma_s = 1 + \frac{d \ln q S_i / (w L_i + r K_i)}{(1 - k_i) d \ln w / q + k_i d \ln r / q} = 1 + \frac{d \ln s_i / (1 - s_i)}{d \ln \phi_i / q}. \quad (16)$$

Proof In Appendix A. ■

For convenience, we introduce new notations for the factor shares of costs within firm i :

$$\begin{aligned} k_i &\equiv \frac{r K_i}{w L_i + r K_i}, \quad \ell_i \equiv \frac{w L_i}{w L_i + r K_i + q S_i}, \\ \kappa_i &\equiv \frac{r K_i}{w L_i + r K_i + q S_i}, \quad s_i \equiv \frac{q S_i}{w L_i + r K_i + q S_i}, \quad m_i \equiv \frac{v M_i}{w L_i + r K_i + q S_i + v M_i}. \end{aligned}$$

We denote firm i 's equipment share of expenditures on equipment-labor bundle as $k_i \equiv r K_i / (w L_i + r K_i)$. Similarly, $\ell_i \equiv w L_i / (w L_i + r K_i + q S_i)$ is firm i 's labor share of non-material cost, $\kappa_i \equiv r K_i / (w L_i + r K_i + q S_i)$ is firm i 's equipment share of non-material cost, and $s_i \equiv q S_i / (w L_i + r K_i + q S_i)$ is firm i 's software share of non-material cost. Lastly, $m_i \equiv v M_i / (w L_i + r K_i + q S_i + v M_i)$ refers to firm i 's materials share of its total cost.

Proposition 1 establishes that the direction of change in factor shares in response to input price changes depends on whether σ_e and σ_s are greater than one or not.

For example, when the price of equipment (r) falls, the ratio of labor expenditure to equipment expenditure (wL_i/rK_i) falls when $\sigma_e > 1$ but rises when $\sigma_e < 1$. Similarly, when the price of software (q) falls, software expenditure relative to expenditures on labor and equipment ($qS_i/(wL_i + rK_i)$) rises when $\sigma_s > 1$ but falls when $\sigma_s < 1$.

Proposition 1 also shows that one need not identify the firm-specific factor intensity parameters, α_i 's in the production function (equation 3), to understand how factor shares respond to factor prices, even though these intensity parameters generate heterogenous factor shares across firms. Such responses are solely governed by the elasticity parameters (σ_e and σ_s), which we estimate in Section 4.

We obtain the following corollary that input price changes and factor-augmenting technological changes have equivalent effects on factor shares.

Corollary 1 *The production function parameters σ_e and σ_s satisfy the following.*

$$\sigma_e = 1 + \frac{d \ln k_i / (1 - k_i)}{d \ln A_K / A_L}, \quad (17)$$

$$\sigma_s = 1 + \frac{d \ln s_i / (1 - s_i)}{(1 - k_i) d \ln A_S / A_L + k_i d \ln A_S / A_K}. \quad (18)$$

Proof In Appendix A. ■

In other words, we cannot separately identify the impacts of factor-augmenting technological changes from the impacts of factor price changes on factor shares. One implication is that it is difficult to estimate the elasticities of substitution using aggregate time-series data on factor shares and input prices, since factor-augmenting technological changes are typically unobservable.

3.2.2 Macro-level Elasticity

Since we do not restrict how factor shares or equivalently α_i 's in equation (3) are distributed across firms, the model does not give a well-defined aggregate production function. However, as in Oberfield and Raval (2021), we can still derive the relationship between the firm-level elasticities of substitution in Proposition 1 and the changes in the aggregate factor income shares in response to factor price changes. The latter is the aggregate elasticity of substitution, defined as follows. Note that going from the usual two factors of production to three requires a new approach.

Definition 1 (Aggregate elasticity of substitution) *The aggregate elasticities of substitution $\bar{\sigma}_e^w$, $\bar{\sigma}_e^r$, $\bar{\sigma}_s^w$, $\bar{\sigma}_s^r$, and $\bar{\sigma}_s^q$ are:*

$$\bar{\sigma}_e^w \equiv 1 + \frac{d \ln rK / wL}{d \ln w}, \quad (19)$$

$$\bar{\sigma}_e^r \equiv 1 - \frac{d \ln rK/wL}{d \ln r}, \quad (20)$$

$$\bar{\sigma}_s^w \equiv 1 + \frac{d \ln qS/(wL + rK)}{wL/(wL + rK) \times d \ln w}, \quad (21)$$

$$\bar{\sigma}_s^r \equiv 1 + \frac{d \ln qS/(wL + rK)}{rK/(wL + rK) \times d \ln r}, \quad (22)$$

$$\bar{\sigma}_s^q \equiv 1 - \frac{d \ln qS/(wL + rK)}{d \ln q}, \quad (23)$$

Unlike in [Oberfield and Raval \(2021\)](#), we need to define the aggregate elasticity of substitution between factors with respect to each input price w , r , and q . That is, even for the same change in the relative factor price, w/r for example, the corresponding change in the aggregate factor shares will depend on whether the wage increased or the equipment price fell. This is because we have three factors of production (equipment, software, and labor), not just two. For example, a firm with a low labor cost share ℓ_i may also have a low equipment cost share κ_i , if its software cost share s_i is high. In this case, firms that lose less than average from a wage increase do not necessarily benefit more from a fall in equipment price. Accordingly, the reallocation across firms will depend on which factor price changes, and the change in the aggregate labor share in response to a higher wage can be different from the change in response to a lower equipment price. This is not the case when there are only two factors, because firms' labor share is equal to one minus their equipment share—that is, a perfect negative correlation between factor shares.¹²

In addition, because our model features variable markups, each firm faces a different demand elasticity. When the price of equipment falls, for example, it makes equipment-intensive firms effectively more productive than others. If more equipment-intensive firms face a lower demand elasticity (or higher markup), there is less reallocation than in the case with a constant markup. With variable markups, the magnitude of reallocation across firms depends on how factor intensities are correlated with firms' markups.¹³ Again, with three inputs, the fact that labor shares

¹²The presence of material inputs in the production function (3) does not necessarily imply asymmetry in the elasticity of substitution. This is because material inputs are a combination of factor inputs such as labor, capital, and software, and hence any change in material price is endogenously determined by the intensity of each factor input in the material goods production. We assume that factor intensities in the material goods production are the same as those in the final good production. Related, [Oberfield and Raval \(2021\)](#) has materials as another factor in the production function, but they obtain symmetry between labor and capital by assuming that changes in the relative price of materials to capital are proportional to changes in the relative price of labor to capital. Essentially, materials are produced by combining labor and capital (in a Cobb-Douglas manner), so there are only two factors of production.

¹³Another implication from variable markup is that relative changes in factor share of cost are not equal to relative changes in factor share of income. We discuss this issue in Section 3.3.

are positively correlated with markups does not necessarily mean that equipment shares are negatively correlated with markups.

Now we derive our main proposition that links firm-level elasticities of substitution to the aggregate elasticities of substitution. We introduce additional notations for convenience:

$$\begin{aligned} k &\equiv \frac{rK}{wL + rK}, \ell \equiv \frac{wL}{wL + rK + qS}, \kappa \equiv \frac{rK}{wL + rK + qS}, s \equiv \frac{qS}{wL + rK + qS}, \\ m &\equiv \frac{vM}{wL + rK + qS + vM}, \theta_i \equiv \frac{wL_i + rK_i}{wL + rK}, \omega_i \equiv \frac{wL_i + rK_i + qS_i}{wL + rK + qS}, \\ \gamma_i &\equiv \frac{wL_i + rK_i + qS_i + vM_i}{wL + rK + qS + vM}, \end{aligned}$$

where k denotes aggregate equipment share of aggregate expenditures on labor and equipment, ℓ is the aggregate labor expenditure share of aggregate non-material cost, κ is the aggregate equipment share of aggregate non-material cost, s_i is the aggregate software share of aggregate non-material cost, and m is the aggregate materials share of aggregate total cost. In addition, θ_i denotes firm i 's expenditures on labor and equipment as a fraction of aggregate expenditures on labor and equipment, ω_i is firm i 's share of aggregate expenditures on non-material inputs, and γ_i is firm i 's share of the aggregate total expenditure.

Proposition 2 (Aggregation) *The aggregate elasticities of substitution satisfy*

$$\bar{\sigma}_e^w = (1 - \chi)\sigma_e + \chi [\zeta^w\sigma_s + (1 - \zeta^w)\bar{m}_e^w\sigma_m + (1 - \zeta^w - (1 - \zeta^w)\bar{m}_e^w)\bar{\epsilon}_e^w], \quad (24)$$

$$\bar{\sigma}_e^r = (1 - \chi)\sigma_e + \chi [\zeta^r\sigma_s + (1 - \zeta^r)\bar{m}_e^r\sigma_m + (1 - \zeta^r - (1 - \zeta^r)\bar{m}_e^r)\bar{\epsilon}_e^r], \quad (25)$$

$$\bar{\sigma}_s^q = (1 - \zeta^q)\sigma_s + \zeta^q [\bar{m}_s^q\sigma_m + (1 - \bar{m}_s^q)\bar{\epsilon}_s^q], \quad (26)$$

$$\bar{\sigma}_s^w = (1 - \zeta^w)\sigma_s + \zeta^w [\bar{m}_s^w\sigma_m + (1 - \bar{m}_s^w)\bar{\epsilon}_s^w], \quad (27)$$

$$\bar{\sigma}_s^r = (1 - \zeta^r)\sigma_s + \zeta^r [\bar{m}_s^r\sigma_m + (1 - \bar{m}_s^r)\bar{\epsilon}_s^r], \quad (28)$$

where

$$\begin{aligned} \chi &\equiv \frac{\sum_i (k_i - k)^2 \theta_i}{k(1 - k)}, \zeta^w \equiv \frac{\sum_i (k_i - k)(1 - k_i)\theta_i s_i}{\sum_i (k_i - k)(1 - k_i)\theta_i}, \zeta^r \equiv \frac{\sum_i (k_i - k)k_i\theta_i s_i}{\sum_i (k_i - k)k_i\theta_i}, \\ \bar{m}_e^w &\equiv \frac{\sum_i (k_i - k)(\ell_i - \alpha_p^w)\theta_i m_i}{\sum_i (k_i - k)(\ell_i - \alpha_p^w)\theta_i}, \bar{m}_e^r \equiv \frac{\sum_i (k_i - k)(\kappa_i - \alpha_p^r)\theta_i m_i}{\sum_i (k_i - k)(\kappa_i - \alpha_p^r)\theta_i}, \\ \zeta^q &\equiv \frac{\sum_i \omega_i (s_i - s)^2}{s(1 - s)}, \zeta^w \equiv -\frac{\sum_i \omega_i (s_i - s)(\ell_i - \ell)}{s\ell}, \zeta^r \equiv -\frac{\sum_i \omega_i (s_i - s)(\kappa_i - \kappa)}{s\kappa}, \\ \bar{\epsilon}_e^w &\equiv \frac{\sum_i ((1 - k_i) - (1 - k))[\ell_i(1 - m_i) - \alpha_p^w]\theta_i \epsilon_i b_i}{\sum_i ((1 - k_i) - (1 - k))[\ell_i(1 - m_i) - \alpha_p^w]\theta_i}, \bar{\epsilon}_e^r \equiv \frac{\sum_i (k_i - k)(\kappa_i(1 - m_i) - \alpha_p^r)\theta_i \epsilon_i b_i}{\sum_i (k_i - k)(\kappa_i(1 - m_i) - \alpha_p^r)\theta_i}, \\ \bar{m}_s^q &\equiv \frac{\sum_i (s_i - s)(s_i - \alpha_p^q)\omega_i m_i}{\sum_i (s_i - s)(s_i - \alpha_p^q)\omega_i}, \bar{m}_s^w \equiv \frac{\sum_i (s_i - s)(\ell_i - \alpha_p^w)\omega_i m_i}{\sum_i (s_i - s)(\ell_i - \alpha_p^w)\omega_i}, \bar{m}_s^r \equiv \frac{\sum_i (s_i - s)(\kappa_i - \alpha_p^r)\omega_i m_i}{\sum_i (s_i - s)(\kappa_i - \alpha_p^r)\omega_i} \end{aligned}$$

$$\begin{aligned}
\bar{\epsilon}_s^q &\equiv \frac{\sum_i (s_i - s)(1 - m_i)(s_i - \alpha_p^q)\omega_i \epsilon_i b_i}{\sum_i (s_i - s)(1 - m_i)(s_i - \alpha_p^q)\omega_i}, \quad \bar{\epsilon}_s^w \equiv \frac{\sum_i (s_i - s)(1 - m_i)(\ell_i - \alpha_p^w)\omega_i \epsilon_i b_i}{\sum_i (s_i - s)(1 - m_i)(\ell_i - \alpha_p^w)\omega_i}, \\
\bar{\epsilon}_s^r &\equiv \frac{\sum_i (s_i - s)(1 - m_i)(\kappa_i - \alpha_p^r)\omega_i \epsilon_i b_i}{\sum_i (s_i - s)(1 - m_i)(\kappa_i - \alpha_p^r)\omega_i}, \quad b_i(x) \equiv \frac{1}{1 - \mu'(x)x/\mu(x)} \left(= \frac{d \ln p_i}{d \ln mc_i} \right) \\
\alpha_p^w &\equiv \frac{\sum_i p_i Y_i (1 - \epsilon_i) b_i (1 - m_i) \ell_i}{\sum_i p_i Y_i (1 - \epsilon_i) b_i (1 - m_i)}, \quad \alpha_p^r \equiv \frac{\sum_i p_i Y_i (1 - \epsilon_i) b_i (1 - m_i) \kappa_i}{\sum_i p_i Y_i (1 - \epsilon_i) b_i (1 - m_i)}, \\
\alpha_p^q &\equiv \frac{\sum_i p_i Y_i (1 - \epsilon_i) b_i (1 - m_i) s_i}{\sum_i p_i Y_i (1 - \epsilon_i) b_i (1 - m_i)}
\end{aligned}$$

Proof In Appendix A. ■

Proposition 2 is our main theoretical result. The aggregate elasticity of substitution is a weighted average of the micro elasticity of substitution and those cross-sectional moments that govern between-firm reallocation. The weights themselves are moments of the joint distribution of factor shares and markups across firms.

The aggregate elasticities of substitution between labor and equipment, $\bar{\sigma}_e^w$ and $\bar{\sigma}_e^r$ (with respect to wage and equipment price, respectively), for example, are weighted averages of the micro elasticities σ_e , σ_s , σ_m , and $\bar{\epsilon}$, with $\bar{\epsilon}$ being the weighted average of the demand elasticity across firms. The weight χ in equations (24) and (25) is proportional to the variance of equipment share of labor and equipment expenditures k_i across firms. Intuitively, when k_i 's are more dispersed, reallocation across firms becomes more important, putting more weights on the demand elasticity that governs the reallocation. If all firms had the same equipment intensity k_i , they will adjust the factor expenditure ratios by the same proportion, and the aggregate elasticity would be the same as the micro or within-firm elasticity.¹⁴ In these equations, $\bar{\epsilon}_e^w$ and $\bar{\epsilon}_e^r$ are weighted averages of $\epsilon_i b_i$ across firms, where b_i is the responsiveness of firm i 's price to a change in its marginal cost ($b_i \equiv d \ln p_i / d \ln mc_i$)—a measure of pass-through. The reallocation depends on how elastic a firm's demand is (ϵ_i) and how much markup changes when the marginal cost changes (b_i).

We now discuss the aggregate elasticities of substitution between software and the labor-equipment bundle, shown in equations (26)–(28). Again, the change in the software share of non-material cost depends on which factor price changes (software price q , wage w , or equipment price r).

Suppose that the price of equipment goes down. Then the price of the equipment-labor composite goes down, causing firms to substitute away from software if the

¹⁴In equations (24) and (25), the between-firm reallocation depends on σ_s as well as $\bar{\epsilon}$. For these elasticities between labor and equipment, what matters for reallocation is firm i 's share of the aggregate labor-equipment bundle, which can be obtained from its share of the aggregate non-material cost and the distribution of software share of non-material cost (i.e., $\theta_i = \omega_i(1 - s_i)/(1 - s)$). This is why σ_s appears.

micro elasticity σ_s is greater than one. The magnitude of the decline in the bundle price depends on how intensively a firm uses equipment, with the within-firm substitution being more prominent in firms with high κ_i (equipment share of non-material cost). It follows that the substitution away from software in the aggregate in response to a lower equipment price is larger when high κ_i firms, who become larger, tend to have a lower s_i (software share of non-material cost). This is why ζ^r is proportional to the minus of the covariance between κ_i and s_i . Again, a large κ_i does not necessarily imply a small s_i because we have three inputs and we do not restrict the factor share distribution. If the covariance between κ_i and s_i were positive, the reallocation responding to a lower equipment price will counteract the substitution away from software within firms.

Following the same logic, ζ^w in equation (27) is proportional to the minus of the covariance between ℓ_i (labor share of non-material cost) and s_i (software share of non-material cost).

Lastly, the change in software share in response to a change in software price in equation (26) depends more on between-firm reallocation when s_i 's are more dispersed (ζ^q is proportional to the variance of s_i). Given the dispersion ζ_q , the reallocation is smaller (i.e., lower $\bar{\epsilon}_s^q$) when a firm with higher software share (s_i) faces a less elastic demand (smaller ϵ_i or higher μ_i) or a less responsive markup (lower b_i).

3.3 Changes in the Aggregate Markup

When factor prices change, a firm adjusts its output price accordingly. Under the Kimball aggregator in equation (6), the magnitude of output price adjustment differs across firms, and as a result the aggregate markup changes. The following proposition summarizes how the aggregate markup changes in response to changes in factor prices.

Proposition 3 (Aggregate markup) *The change in aggregate markup in response to factor price changes is respectively given by*

$$\bar{\sigma}_\mu^w - 1 \equiv \frac{d \ln \mu}{\bar{\ell} d \ln w} = (1 - \eta^w)(\bar{b}^w - 1) + \eta^w(\bar{\epsilon}_\mu^w - 1) + (\iota^w - \eta^w)(\sigma_m - 1), \quad (29)$$

$$\bar{\sigma}_\mu^r - 1 \equiv \frac{d \ln \mu}{\bar{\kappa} d \ln r} = (1 - \eta^r)(\bar{b}^r - 1) + \eta^r(\bar{\epsilon}_\mu^r - 1) + (\iota^r - \eta^r)(\sigma_m - 1), \quad (30)$$

$$\bar{\sigma}_\mu^q - 1 \equiv \frac{d \ln \mu}{\bar{s} d \ln q} = (1 - \eta^q)(\bar{b}^q - 1) + \eta^q(\bar{\epsilon}_\mu^q - 1) + (\iota^q - \eta^q)(\sigma_m - 1), \quad (31)$$

where

$$\mu \equiv \sum_i \omega_i \mu_i, \quad \bar{\ell} \equiv \sum_i \omega_i \bar{\ell}_i, \quad \bar{\kappa} \equiv \sum_i \omega_i \bar{\kappa}_i, \quad \bar{s} \equiv \sum_i \omega_i \bar{s}_i,$$

$$\begin{aligned}
\tilde{\ell}_i &\equiv (1 - m_i)\ell_i + m_i\alpha_p^w, \quad \tilde{\kappa}_i \equiv (1 - m_i)\kappa_i + m_i\alpha_p^r, \quad \tilde{s}_i \equiv (1 - m_i)s_i + m_i\alpha_p^q, \\
\bar{b}^w &\equiv \frac{\sum_i \mu_i \tilde{\ell}_i \omega_i b_i}{\sum_i \mu_i \tilde{\ell}_i \omega_i}, \quad \bar{b}^r \equiv \frac{\sum_i \mu_i \tilde{\kappa}_i \omega_i b_i}{\sum_i \mu_i \tilde{\kappa}_i \omega_i}, \quad \bar{b}^q \equiv \frac{\sum_i \mu_i \tilde{s}_i \omega_i b_i}{\sum_i \mu_i \tilde{s}_i \omega_i}, \\
\eta^w &\equiv -\frac{\sum_i (\mu_i - \mu)(\tilde{\ell}_i - \ell)\omega_i}{\tilde{\ell}\mu}, \quad \iota^w \equiv -\frac{\sum_i (\mu_i - \mu)(\ell_i - \ell)\omega_i}{\tilde{\ell}\mu}, \\
\bar{\epsilon}_\mu^w &\equiv \frac{\sum_i (\mu_i - \mu)(\tilde{\ell}_i - \alpha_p^w)\omega_i \epsilon_i b_i}{\sum_i (\mu_i - \mu)(\tilde{\ell}_i - \alpha_p^w)\omega_i}, \\
\eta^r &\equiv -\frac{\sum_i (\mu_i - \mu)(\tilde{\kappa}_i - \kappa)\omega_i}{\tilde{\kappa}\mu}, \quad \iota^r \equiv -\frac{\sum_i (\mu_i - \mu)(\kappa_i - \kappa)\omega_i}{\tilde{\kappa}\mu}, \\
\bar{\epsilon}_\mu^r &\equiv \frac{\sum_i (\mu_i - \mu)(\tilde{\kappa}_i - \alpha_p^r)\omega_i \epsilon_i b_i}{\sum_i (\mu_i - \mu)(\tilde{\kappa}_i - \alpha_p^r)\omega_i}, \\
\eta^q &\equiv -\frac{\sum_i (\mu_i - \mu)(\tilde{s}_i - s)\omega_i}{\tilde{s}\mu}, \quad \iota^q \equiv -\frac{\sum_i (\mu_i - \mu)(s_i - s)\omega_i}{\tilde{s}\mu}, \\
\bar{\epsilon}_\mu^q &\equiv \frac{\sum_i (\mu_i - \mu)(\tilde{s}_i - \alpha_p^q)\omega_i \epsilon_i b_i}{\sum_i (\mu_i - \mu)(\tilde{s}_i - \alpha_p^q)\omega_i}
\end{aligned}$$

Proof In Appendix A. ■

Similar to the aggregate elasticities of substitution between production factors, a change in aggregate markup can be represented by a weighted sum of within-firm changes and between-firm changes. That is, a change in aggregate markup depends not only on how each firm adjusts its markup, but also on the reallocation of production activity across firms. Consider a fall in software price ($d \ln q < 0$) as an example. For each firm, a fall in software price reduces its marginal cost according to $d \ln mc_i / d \ln q = s_i(1 - m_i) + m_i\alpha_p^q$, where $s_i(1 - m_i)$ is the share of software expenditure relative to total cost and α_p^q is a share of software contents in materials (Shephard's lemma). As the responsiveness of a firm's price to its marginal cost (or pass-through) is denoted by $b_i \equiv d \ln p_i / d \ln mc_i$, the within-firm markup change is $-(b_i - 1)[s_i(1 - m_i) + m_i\alpha_p^q]$.¹⁵ The first term in the right-hand side of equation (31), $\bar{b}^q - 1$, captures a weighted average of these within-firm adjustments across firms.

Now consider the reallocation across firms, or the between-firm change. When software price falls ($d \ln q < 0$), the reallocation makes aggregate markup increase if $\eta^q < 0$ in equation (31). That is, the reallocation raises the aggregate markup when software shares and markups are positively correlated in the data (see the definition of η^q in the second-to-last line of Proposition 3), because software-intensive firms' weights rise with a fall in software price.

¹⁵For example, if $b_i < 1$ as in Klenow and Willis (2016), firms increase their markup in response to the fall in software price.

The degree of reallocation $\bar{\epsilon}_\mu^q$ in equation (31) is determined by the weighted average of the demand elasticity ϵ_i and the pass-through b_i across firms—see the last equation of Proposition 3. The product $\epsilon_i b_i$ determines how firm i 's market share changes in response to the change in software price. The weights are larger for software-intensive firms, since the marginal cost change is proportional to firms' software share s_i . Intuitively, when firms with higher software shares adjust their size more, there is more reallocation across firms in response to a fall in software price.

Note that the weight is firm i 's share of the aggregate value added (ω_i) rather than its share of the total cost (γ_i) when aggregating markup. A change in total cost share is not exactly proportional to a change in value added share, except when $\sigma_m = 1$. As a result, the adjustment term $(\iota^q - \eta^q)(\sigma_m - 1)$ appears at the end of the equation (31).

How factor shares correlate with markup is an empirical question. When a factor share is positively correlated with markup, its covariance is positive and the corresponding η is negative. The degrees of reallocation $\bar{\epsilon}_\mu$'s themselves can reinforce or counteract the within-firm markup changes depending on the joint distribution of markup and factor shares in cost.

The aggregate labor income share is the aggregate labor cost share in aggregate non-material cost divided by the aggregate markup, ℓ/μ . To see this, let firm i 's non-material cost $C_i \equiv wL_i + rK_i + qS_i$ and aggregate non-material cost $C \equiv wL + rK + qS$. Then,

$$LS = \frac{\sum_i wL_i}{\sum_i p_i Y_i} = \frac{\sum wL_i / C_i \times C_i / C}{\sum p_i Y_i / C_i \times C_i / C} = \frac{\sum \ell_i \omega_i}{\sum \mu_i \omega_i} = \frac{\ell}{\mu}. \quad (32)$$

Since a change in software price affects μ as well as ℓ , we need to consider the markup change in equation (31) together with the factor substitution in Proposition 2 when calculating the effect of software-embodied technological change on the aggregate labor income share.

4 Estimation

The micro-level elasticity of substitution can be identified from the relationship between factor income shares and factor prices (Proposition 1). However, estimating this relationship is challenging because of the simultaneous movements in factor inputs and prices driven by the factor bias of technological change, which is typically not observable. To address this challenge, one needs to either control for factor-biased technological changes or employ instruments for factor price changes that are orthogonal to factor-biased technological changes.

In the macro literature, some researchers dealt with this issue by assuming a specific form of factor-biased technological change, such as a log-linear time trend (Antras, 2004; Herrendorf et al., 2015, among others). However, it is unclear whether the assumed functional form adequately controls for all the factor bias. With more disaggregated data, more reasonable instruments become available, such as local labor market impacts of national changes in employment (Raval, 2019; Oberfield and Raval, 2021; Lashkari et al., 2023), amenities (Oberfield and Raval, 2021), and tariffs (de Souza and Li, 2023). However, the estimated elasticity from micro-level data will generally differ from the elasticity at the macro level.

We first estimate the micro-level elasticities of substitution σ_e and σ_s using micro-level data and instruments for relative factor prices. We then aggregate the micro-level elasticities into the macro-level elasticities of substitution, using our theoretical results and the data on the distribution of factor income shares.

We estimate the micro-level elasticities in two different ways, each utilizing a distinct dataset with its own strengths and weaknesses. The first approach uses the data on manufacturing establishments from the Korean Economic Census of 2015, encompassing all manufacturing establishments with at least one employee. The census provides information on the geographic location of each establishment, allowing us to use instruments with regional variations. The limitation is that it is one cross-section covering only the manufacturing sector.

The second approach uses firm-level panel data, KISDATA. It covers all sectors, but one limitation is that it only covers firms meeting specific criteria for external auditing, as discussed in Section 2. As it lacks information on firms' geographic location, we use instruments with industrial variations over time.

4.1 Cross-sectional Estimates of Micro-level Elasticities

We begin by estimating the micro-level elasticity of substitution between labor and equipment versus software, using the cross-sectional variation in the Korean Economic Census data.

Estimation strategy In estimating the relationship between relative expenditures on factors and relative factor prices, we follow Oberfield and Raval (2021), with wage differences across regions as our main explanatory variable. Local wages are obtained from the Regional Employment Survey. The Regional Employment Survey is a household-level survey reporting workers' salaries, demographics, educational attainment, and experience. We estimate a residual wage for each person in the man-

ufacturing sector, controlling for education, demographics, and experience, and then aggregate it up to the region level.¹⁶

We run the following regression across firms:

$$\log \frac{rK_i}{wL_i} = \beta_e \log w_r + \gamma_e X_i + \epsilon_{e,i}, \quad (33)$$

$$\frac{1}{1 - k_i} \log \frac{qS_i}{wL_i + rK_i} = \beta_s \log w_r + \gamma_s X_i + \epsilon_{s,i}, \quad (34)$$

where i indexes firms, w_r is the residual wage in region r that firm i belongs to, and X is a set of control variables including three-digit industry fixed effects, firm age, and multi-establishment status. Note that $\beta_e = \sigma_e - 1$ and $\beta_s = \sigma_s - 1$. This specification assumes that all firms face the same cost of capital but different wages across regions.¹⁷

When regional wage variations are used, they must be orthogonal to unobserved factor-augmented productivities, A 's in the production function (equation 3); see Corollary 1. We follow [Oberfield and Raval \(2021\)](#) and [Raval \(2019\)](#) and use [Bartik \(1991\)](#) style shift-share instrument to address this concern. Specifically, we construct a shift-share variable that captures changes in local demand for labor in the service sector and use it as an instrument for the supply of labor for manufacturing plants. The instrument is $Z_r = \sum_{i \in N_s} \omega_{r,i,0} \log(L_{i,t}/L_{i,0})$, where N_s is the set of industries in the service sector, $\omega_{r,i,0}$ is the service industry i 's share of employment in region r at time 0, and $L_{i,t}$ is the nationwide employment of industry i in time t . The employment growth is from 2010 to 2015. Because the instrument covers only service industries, we interpret this as a change in the labor supply for the manufacturing firms in the same region competing for the same pool of workers, uncorrelated with the factor-biased productivity of manufacturing firms.¹⁸

One may wonder about the plausibility of the assumption that firms in the service sector and those in the manufacturing sector hire from the same worker pool in a region. Checking for the contribution of each service industry using the Rotemberg weights as in [Goldsmith-Pinkham et al. \(2020\)](#), we find that the research and development industry and the business support service industry account for 93 percent of

¹⁶The unit of a region in our analysis is Si-Gun-Gu, which is an administrative division of South Korea. The average population size of Si-Gun-Gu is 319,200, which is a bit smaller than the average population size of a commuting zone in the US, 443,500.

¹⁷This is one reason why we focus on equipment rather than structure and equipment in the baseline estimation. It is unlikely that firms located in different regions face the same price of structure.

¹⁸[Oberfield and Raval \(2021\)](#) employs a measure of local amenities based on climate and geography as an alternative instrument. However, we cannot do the same because measures of climate and geography do not have enough variations in a small country like Korea. [Oberfield and Raval \(2021\)](#) reports that the Bartik instrument and local amenities result in similar estimates of substitution elasticity. In section 4.2, we use a different data set and a different instrumental variable.

overall weights and 80 percent of positive weights (Appendix B.4.1). Since it is likely that workers in these two industries can switch to the manufacturing sector more easily than workers in other service industries (for example, health care or education), we view the assumption of the common labor supply pool to be plausible.

As robustness checks, we also estimated a two-stage least squares (TSLS) regression and a limited-information maximum likelihood (LIML) with each industry share as instruments (Goldsmith-Pinkham et al., 2020). In addition, we estimated using an alternative instrument based on industry wage premia suggested by Beaudry et al. (2012).¹⁹ Lastly, regarding the inference of estimation with a shift-share instrument, Borusyak et al. (2021) suggests a shock (industry) level regression, when initial shares are endogenous and identification comes from exogenous shocks. We did a shock-level shift-share IV (SSIV) estimation as a robustness check.²⁰

Results Table 2 reports the estimated micro-level elasticities of substitution between labor and equipment (σ_e) and between labor and software (σ_s). We report the OLS estimates in columns 1, and IV estimates in columns 2 to 7. The benchmark result is the estimation with the shift-share instrument in column 2. The elasticity of substitution between labor and equipment mostly lies between 0.3 and 0.7, implying complementarity between labor and equipment. These estimates are in line with those in previous studies of the US data, such as Antras (2004), Herrendorf et al. (2015), Knoblauch et al. (2020), Oberfield and Raval (2021), and Raval (2019).

Our novel finding is that the elasticity of substitution between labor and software (σ_s) is greater than one, statistically significantly so in the majority of the specifications, including the benchmark (column 2). It implies that software substitutes for labor and that labor income shares would decrease within firms in response to software-embodied technological change.²¹

We next do a battery of robustness checks. We first estimate using only the establishments that already existed in 2009 or before ('Old estb.' in Table 3), since our

¹⁹Formally, the instrument is $Z_r^{\text{BGS}} = \sum_{i \in N_s} \hat{\omega}_{r,i,t} (v_{i,t} - v_{i,0})$, where N_s is the set of industries in the service sector, $\hat{\omega}_{r,i,t}$ is the predicted share of region r 's employment in industry i , and $v_{i,t}$ is the wage premium in industry i in year t . The share $\hat{\omega}_{r,i,t}$ is predicted based on national employment changes as above, and the wage premia $v_{i,t}$ are fixed effects from a regression of individual wages on industry dummies indexed by i .

²⁰See Appendix B.4.1 for more detail.

²¹The standard errors are larger for the shock-level regressions (columns 6 and 7). This specification can only utilize the variation across 34 two-digit service industries, with even smaller effective sample size. In a shock-level regression, the effective sample size is the inverse of the Herfindahl index of average industrial employment shares across regions, calculated as $1 / \sum_n s_n^2$, where $s_n = \sum_r s_{r,n}$ represents the employment share of industry n in region r (Borusyak et al., 2021). Intuitively, when regional employment is dominated by a few industries, there would not be much variation at the shock (industry) level. The effective sample size in our data is only 12.7. Detailed discussion of the shock-level regression is in Appendix B.4.1.

	OLS	Bartik	BGS	TSLS	LIML	SSIV1	SSIV2
Equipment (σ_e)	0.663 (0.084)	0.600 (0.153)	0.580 (0.222)	0.385 (0.070)	0.375 (0.098)	0.600 (1.051)	-0.164 (0.815)
Software (σ_s)	1.128 (0.118)	1.620 (0.230)	2.395 (0.357)	1.510 (0.106)	1.519 (0.134)	1.620 (0.879)	2.736 (0.928)

Table 2: Micro-Level Capital-Labor Substitution: Cross-Sectional Estimates

Column 1 is the OLS estimate. Columns 2 and 3 are IV regressions using shift-share instruments according to [Bartik \(1991\)](#) and [Beaudry et al. \(2012\)](#), respectively. Columns 4 and 5 are IV regression with two-stage least squares (TSLS) and limited-information maximum likelihood (LIML) with each industry share as an instrument. Columns 6 and 7 are the shock (industry) level shift-share IV regressions suggested by [Borusyak et al. \(2021\)](#) with complete shares and incomplete shares, respectively. Standard errors are clustered at the level of 3-digit industry and region for columns 1 to 5. Robust standard errors are reported for columns 6 and 7.

instrument has national services employment growth between 2010 and 2015. Second, in the Economic Census, a sizeable number of firms report that they do not hold software assets ($S_i = 0$), which might be measurement errors. We restrict the sample to firms with non-zero software assets ('Positive obs.'). Third, we consider an alternative ordering of nested CES structure that bundles labor and software first instead of labor and equipment.²² 'Alt. order 1' and 'Alt. order 2' are IV regressions using shift-share instruments according to [Bartik \(1991\)](#) and [Beaudry et al. \(2012\)](#), respectively, corresponding to columns 2 and 3 of Table 2. Fourth, we consider an estimation with tangible (equipment and structure) and intangible capital (software and R&D). Finally, we run the same regression using only standalone establishments, dropping multi-establishment firms ('Standalone'). The results, all using our benchmark Bartik instrument with the exception of 'Alt. order 2', are in Table 3, with the point estimates showing $\sigma_e < 1$ and $\sigma_s > 1$ in all cases.

4.2 Panel Estimates of Micro-level Elasticities

In an effort to assess the broader validity of our findings, we estimate the micro-level elasticities of substitution using a different data set and a different identification strategy. To be specific, we utilize firm-level panel data (KISDATA) and the variations in minimum wages as an instrument for labor cost.

Estimation strategy To identify the substitution elasticities with panel data, we follow the approach of [Chirinko and Mallick \(2017\)](#), focusing on relatively longer-run

²²Formally, $V_i = [\{\alpha_i^L(A_L L_i)^{\frac{\sigma_s-1}{\sigma_s}} + \alpha_i^S(A_S S_i)^{\frac{\sigma_s-1}{\sigma_s}}\}^{\frac{\sigma_s(\sigma_e-1)}{(\sigma_s-1)\sigma_e}} + \alpha_i^K(A_K K_i)^{\frac{\sigma_e-1}{\sigma_e}}]^{\frac{-\sigma_e}{\sigma_e-1}}$, instead of equation (4).

	Old Estb.	Positive Obs.	Alt. Order 1	Alt. Order 2	Tan/Intan	Standalone
Eqp. (σ_e)	0.651 (0.162)	0.674 (0.318)	0.597 (0.154)	0.572 (0.223)	0.736 (0.165)	0.595 (0.300)
Sft. (σ_s)	1.937 (0.279)	1.438 (0.404)	1.169 (0.198)	1.885 (0.318)	2.716 (0.429)	1.502 (0.387)

Table 3: Robustness Checks for Micro-Level Capital-Labor Substitution

All columns except Column 4 are estimates from IV regressions using our [Bartik \(1991\)](#) instrument. Column 1 only uses establishments established before 2010, and column 2 uses data with strictly positive S_i only. Column 3 and 4 are for the alternative CES ordering of the production function, estimated with [Bartik \(1991\)](#) and [Beaudry et al. \(2012\)](#) instruments, respectively. Column 4 uses total tangible vs. intangible capital, instead of equipment vs. software. Column 5 only includes standalone establishments. Standard errors are clustered at the level of three-digit industry and region.

variations in factor shares and prices. We first compute three-year moving averages of firms' factor shares and factor prices.²³ We then take differences between adjacent years to remove time-invariant heterogeneity across firms. The regression equations based on Proposition 1 are as follows.

$$\Delta \log(k_{i,t}^*/(1 - k_{i,t}^*)) = \alpha + (\sigma_e - 1)\Delta \log x_{1,i,t}^* + \beta \Delta \log z_{i,t}^* + \gamma_t + \varepsilon_{i,t} \quad (35)$$

$$\Delta \log(s_{i,t}^*/(1 - s_{i,t}^*)) = \alpha + (\sigma_s - 1)\Delta \log x_{2,i,t}^* + \beta \Delta \log z_{i,t}^* + \gamma_t + \varepsilon_{i,t} \quad (36)$$

With i indexing firms and j indexing industries, $k_{i,t}$ is $r_{j,t}K_{i,t}/(w_{i,t}L_{i,t})$, and $s_{i,t}$ is $q_{j,t}S_{i,t}/(W_{i,t}L_{i,t} + r_{j,t}K_{i,t})$. Also, $\log x_{1,i,t}$ is $\log w_{i,t}/r_{j,t}$, $\log x_{2,i,t}$ is $(1 - k_{i,t}) \log w_{i,t} + k_{i,t} \log r_{j,t} - \log q_{j,t}$, $\log z_{i,t}$ is log sales, and γ_t is year dummies. For any variable, the asterisk denotes its three-year moving average.²⁴

To address endogeneity concerns, we use changes in the national minimum wage as an instrument for variations in labor cost. The minimum wage is a particularly useful instrumental variable for Korea during our sample period, because the minimum wage increased drastically and unexpectedly. In 2017, the then president was impeached for a reason unrelated with the state of the economy. In the presidential election held to fill the vacancy two months later, the liberal opposition party candidate won. The new administration pursued a distinctly pro-labor policy in the labor

²³[Chirinko and Mallick \(2017\)](#) applied a low-pass filter to the industry-level variables. We are unable to apply the low-pass filter, because our firm-level data is an unbalanced panel.

²⁴To obtain the firm-level wage $w_{i,t}$, we divide the labor compensation by the number of employees. For equipment and software rental rates ($r_{j,t}$ and $q_{j,t}$) at the industry level, we use the following imputation, because the National Accounts of Korea does not disaggregate industry-level capital price by type of capital. The rental rate of equipment in industry j is $r_{j,t} = R_{j,t} \times (r_t/R_t)$, where $R_{j,t}$ is the rental rate of total capital in industry j in year t , R_t is the rental rate of total capital in the aggregate, and r_t is the rental rate of equipment in the aggregate. For the rental rate of software in industry j , replace the lower case r 's with q 's.

market, including substantial raises of the minimum wage. It rose by 16.4 percent in 2018 and 10.9 percent in 2019 in nominal terms, when the inflation rate was less than two percent.

Since Korea sets only the national minimum wage, the minimum wage does not vary across industries. To generate industrial variations, we compute two variables for each industry in each year t : (i) the share of workers between the minimum wage in year t and the minimum wage in year $t + 1$, and (ii) the share of workers reporting hourly wages below the minimum wage in year t . We then run instrumental variable regressions, using the current and one-year lagged values of these two variables as instruments for the relative factor prices ($\Delta \log x_{i,t}$). In an alternative specification, we control for firms' sales to capture changes in market demand.

Results Table 4 reports the estimation results. The elasticity of substitution between labor and equipment ranges from 0.603 to 0.627 across different specifications (columns 1 to 2), indicating complementarity between labor and equipment. On the other hand, the substitution elasticity between labor and software is invariably greater than one, confirming our previous findings from cross-sectional estimations. The patterns remain robust, when (i) we reverse the ordering of factors in the production function as in footnote 22 ('Alt. order'), (ii) we use tangible and intangible assets instead of equipment and software ('Tan/Intan'), and (iii) we replace firm-level wages with the industry average of hourly male wages controlling for education and experience ('Residual wage').

	Baseline		Alt. Order	Tan / Intan	Residual Wage
Equipment (σ_e)	0.603 (0.099)	0.627 (0.097)	0.627 (0.100)	0.047 (0.079)	0.396 (0.089)
Software (σ_s)	1.482 (0.259)	1.479 (0.259)	1.590 (0.386)	1.672 (0.177)	1.798 (0.478)
Sales Control	✓		✓	✓	✓

Table 4: Micro-Level Capital-Labor Substitution: Panel Estimates

All columns are estimates from two-stage least squared regressions using minimum wage instruments. Specifically, instrumental variables are the current and one-year lagged values of the share of workers between the minimum wage in year t and the minimum wage at time $t + 1$ by industry and the share of workers reported wage below the minimum wage by industry. Standard errors are clustered at the firm level.

From the micro-level estimates derived from two distinct datasets and identifica-

tion strategies, we conclude that software substitutes for labor but equipment complements labor.

This difference can be attributed to the fact that software enhances the productivity across a broader range of worker tasks than equipment does. While equipment primarily enhances the productivity of routine tasks, software can make more efficient cognitive and analytical tasks, as well as routine tasks (Aum, 2020). For example, better accounting software saves labor by accelerating data entry (routine) and report generation (cognitive/analytical). Similarly, progress in 3-D modeling software enables skilled architects to design more buildings in less time.

Our complete analysis will show that software indeed plays a dominant role in the decline of the aggregate labor share, both directly through factor substitution and indirectly through higher markup.

4.3 Aggregation

We now compute the aggregate elasticities of substitution, which determine the impact of factor prices on aggregate factor income shares and markup. Applying the theory in Section 3, we derive the aggregate elasticities $\bar{\sigma}_\ell$ and $\bar{\sigma}_s$ as in Proposition 2, combining our estimates of the micro-level elasticities of substitution and the moments from the joint distribution of factor shares and markups in the microdata.

Data and estimated parameters For our baseline aggregation result, we use the firm-level panel KISDATA from 2003 to 2018. To compute the reallocation parameters, we need to estimate markups (μ_i) and price pass-through (b_i) at the firm level. A firm's markup is given by the ratio of revenue to expenditure on a variable input, multiplied by the output elasticity with respect to the factor. For example, with labor as a variable input, the markup μ_i of firm i is

$$\mu_i = \frac{p_i y_i}{w L_i} \times \alpha_i, \quad (37)$$

where $p_i y_i$ is revenue and α is the output elasticity with respect to labor. Because the output elasticity is not directly observed in the data, the literature suggests several methods of estimating markup (Baqaee and Farhi, 2019; De Loecker et al., 2020; Edmond et al., 2018, among others). We use the ratio of revenue to total cost as our baseline estimate, inferring α from the firm's cost minimization condition, $\alpha_i = w L_i / (w L_i + r K_i + q S_i + v M_i)$. Further details on the cases with alternative markup measures, as well as our justification for the choice of the benchmark markup measure, are in Appendix B.2.3.

From the estimated markups, we compute each firm's price pass-through (b_i) using the relationship between markup and pass-through given by the demand aggregator in [Klenow and Willis \(2016\)](#):

$$H(Y_i/Y) = 1 + (\sigma - 1) \exp(1/\nu) \nu^{\frac{\sigma}{\nu}-1} \left[\Gamma\left(\frac{\sigma}{\nu}, \frac{1}{\nu}\right) - \Gamma\left(\frac{\sigma}{\nu}, \frac{(Y_i/Y)^{\frac{\nu}{\sigma}}}{\nu}\right) \right].$$

With this functional form, $\epsilon(x) = -\frac{h'(x)x}{h(x)} = \sigma h(x)^{-\frac{\nu}{\sigma}}$ and hence $\frac{\epsilon'(x)x}{\epsilon(x)} = -\frac{\nu}{\sigma} \frac{h'(x)x}{h(x)} = \frac{\nu}{\sigma} \epsilon(x)$. Since $\mu = \frac{\epsilon}{\epsilon-1}$, we obtain the following relationship between the markup μ_i and the price pass-through b_i :

$$b_i = \frac{1}{1 - \frac{\mu'x}{\mu}} = \frac{1}{1 + \mu_i \frac{\nu}{\sigma}}. \quad (38)$$

The value of the super-elasticity ν/σ is set to 0.11, from the empirical relationship between firms' sales shares and markups.

From the various micro-level elasticity estimates in Sections 4.1 and 4.2, we select $\sigma_s = 1.6$ and $\sigma_e = 0.6$, the Bartik instrument estimates in Table 2, which are not much different from other estimates. We also need to set the elasticity of substitution between value added and material input (σ_m). We estimate σ_m using our cross-sectional approach (Section 4.1), which results in $\sigma_m = 1$.²⁵ With $\sigma_m = 1$, the adjustment terms reflecting the discrepancy between value-added and sales disappear ($\sigma_m - 1 = 0$) in Propositions 2 and 3.²⁶

Lastly, we take five-year moving averages to smooth the reallocation parameters. Details on the construction of the reallocation parameters are in Appendix B.

Results In Table 5, we report the macro-level elasticities ($\bar{\sigma}$, first column) and decompose the total change in relative factor income shares ($\bar{\sigma} - 1$, second column) into the within-firm change ($\sigma - 1$, third column) and the remaining part that is due to reallocation ($\bar{\sigma} - \sigma$, fourth column). For example, in response to a one percent increase in labor cost, the expenditure on equipment relative to labor (rK/wL) changes by $\sigma_e - 1$ percent within firms and $\bar{\sigma}_e^w - \sigma_e$ percent due to reallocation, with a total change of $\bar{\sigma}_e^w - 1$ percent in the aggregate. As shown in Section 3.2.2, this total change in response to a change in wage is different from the one in response to a change in

²⁵The estimation equation is $(1/\ell_i) \log m_i / (1 - m_i) = \beta_m \log w_r + \gamma_m X + \epsilon_{m,i}$, which implies $\sigma_m = \hat{\beta}_m + 1$. The OLS and the IV estimation with the shift-share instrument give σ_m estimates of 0.8 and 1.3, respectively. This is similar to what [Oberfield and Raval \(2021\)](#) report in their analysis of US data.

²⁶As [Castro-Vincenzi and Kleinman \(2023\)](#) show, within a firm, a rise in material costs reduces its labor share when labor and material input are complements ($\sigma_m < 1$) and markups are positive. In our analysis, σ_m affects the labor share through reallocation across firms as well. If $\sigma_m < 1$, the impact of capital-embodied technological change is attenuated, resulting in less reallocation.

the cost of equipment, $\bar{\sigma}_e^r - 1$. In the last two columns of Table 5, we report the distributional moments that determine the magnitude of reallocation. We hold fixed the micro-level elasticities over time, but the macro-level elasticities do vary over time because the joint distribution of factor shares and markups evolves over time. In the table, we show the values for 2005 and 2015. The time series are shown in Figure 5, but we do not find a clear time trend in the macro-level elasticities.

We first note that the macro-level elasticities of substitution between equipment and labor are larger than the micro-level elasticity ($\bar{\sigma}_e > \sigma_e$). In fact, in response to changes in labor costs, equipment even substitutes for labor ($\bar{\sigma}_e^w > 1$) in the aggregate, even though the micro-level elasticity is well below one. However, with respect to changes in the price of equipment, equipment mostly complements labor ($\bar{\sigma}_e^r < 1$). This discrepancy results from the difference in the magnitude of reallocation with respect to the two factor prices ($\bar{\epsilon}_e^w > \bar{\epsilon}_e^r$). It reflects the fact that firms facing higher

	Year	Elasticity ($\bar{\sigma}$)	Total Change	Within Change	Re- allocation	Distributional Moments	
Factor Substitution (Eqp.)			$\bar{\sigma}_e - 1$	$\sigma_e - 1$	$\bar{\sigma}_e - \sigma_e$	χ	$\bar{\epsilon}_e$
$\bar{\sigma}_e^w$	2005	1.173	0.173	-0.400	0.573	0.289	4.560
	2015	1.159	0.159	-0.400	0.559	0.334	3.707
$\bar{\sigma}_e^r$	2005	0.834	-0.166	-0.400	0.234	0.289	1.925
	2015	1.036	0.036	-0.400	0.436	0.334	2.925
Factor Substitution (Sft.)			$\bar{\sigma}_s - 1$	$\sigma_s - 1$	$\bar{\sigma}_s - \sigma_s$	ζ	$\bar{\epsilon}_s$
$\bar{\sigma}_s^q$	2005	1.561	0.561	0.600	-0.039	0.068	1.048
	2015	1.599	0.599	0.600	-0.001	0.033	2.295
$\bar{\sigma}_s^r$	2005	1.979	0.979	0.600	0.379	-0.526	0.880
	2015	1.434	0.434	0.600	-0.166	0.246	0.846
$\bar{\sigma}_s^w$	2005	1.468	0.468	0.600	-0.132	-0.007	0.920
	2015	1.600	0.600	0.600	0.000	0.513	0.326
Changes in Markup			$\bar{\sigma}_\mu - 1$	$\bar{b} - 1$	$\bar{\sigma}_\mu - \bar{b}$	η	$\bar{\epsilon}_\mu$
$\bar{\sigma}_\mu^q$	2005	0.503	-0.497	-0.423	-0.074	0.198	0.637
	2015	0.677	-0.323	-0.245	-0.078	-0.003	1.430
$\bar{\sigma}_\mu^r$	2005	0.699	-0.301	-0.240	-0.062	-0.043	2.199
	2015	0.690	-0.310	-0.210	-0.100	0.017	-5.228
$\bar{\sigma}_\mu^w$	2005	0.791	-0.209	-0.223	0.014	0.013	1.831
	2015	0.804	-0.196	-0.217	0.021	-0.003	-7.319

Table 5: Macro-level Elasticities and Distributional Moments

The macro-level elasticities are based on Propositions 2 and 3 and the distributional moments from the joint distribution of k_i , ℓ_i , s_i , κ_i , m_i , θ_i , ω_i , γ_i , ϵ_i , and b_i in KISDATA. The micro-level elasticities are the Bartik IV estimates in Table 2.

demand elasticities tend to have more extreme labor expenditure shares (either very high or very low) in the data (Proposition 2).

Reallocation plays a relatively minor role in the macro-level elasticity of substitution between software and labor ($\bar{\sigma}_s$). Accordingly, this macro-level elasticity remains close to the micro-level elasticity of substitution between software and labor, no matter which factor price changes. Both the weight on the reallocation and the magnitude of the reallocation are smaller for the software-labor elasticity than for the equipment-labor elasticity ($\zeta < \chi$ and $\bar{\epsilon}_s < \bar{\epsilon}_e$). The relatively small $\bar{\epsilon}_s^\eta$ implies that firms that use software intensively does not respond much to the change in the price of software. This is because such software-intensive firms tend to have high markups (and low demand elasticities) in the data.

In fact, reallocation can even counteract the substitution between labor and software within firms. Normally, for example, an increase in the labor cost reduces the market share of labor-intensive firms, further reducing the aggregate labor share. However, when labor-intensive firms also have relatively high software shares ($\zeta^w < 0$) or when the sales of such labor-intensive firms with high software intensities respond sensitively to the shock ($\bar{\epsilon}_s^w < 1$), reallocation can result in a lower aggregate software share. As discussed in Section 3.2.2, such an outcome is not possible when there are only two factors.

Even if reallocation had a small effect in terms of factor substitution, it could still have a sizeable effect on factor shares through endogenous changes in markup.

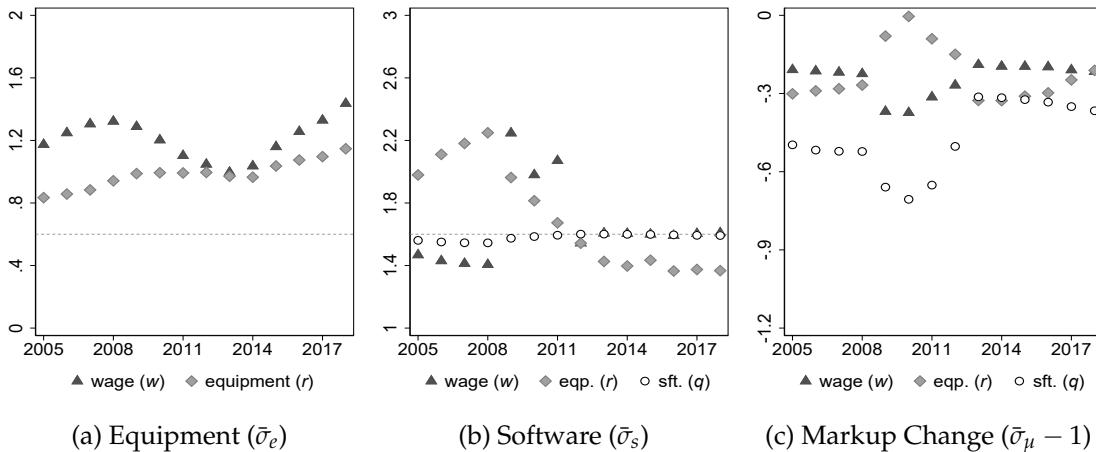


Fig. 5: Macro-level Elasticities over Time

The dotted lines in panels (a) and (b) are the micro-level elasticity estimates. The macro-level elasticities are based on Propositions 2 and 3 and the distributional moments from the joint distribution of k_i , ℓ_i , s_i , κ_i , m_i , θ_i , ω_i , γ_i , ϵ_i , and b_i in KISDATA.

Changes in factor prices affect not only within-firm markup but also the aggregate markup through reallocation, which in turn affect the aggregate income shares of all factors.

The bottom panel of Table 5 shows that the aggregate markup decreases (increases) when factor prices increase (decrease), $\bar{\sigma}_\mu - 1 < 0$. Because of the incomplete price pass-through in our specification ($b_i < 1$), on average, within-firm markup will decrease in response to higher costs ($\bar{b} < 1$, third column). Reallocation across firms can either reinforce or counteract this within-firm markup change, depending on the joint distribution of factor shares and markups across firms in the data. The weights (η , fifth column) are proportional to the correlation between factor shares and markups. In our data, markups are not significantly correlated with labor shares or equipment shares. On the other hand, they are positively correlated with software shares so that reallocation reinforces the effect of the within-firm markup changes in response to software price changes. As a result, of the three factor prices, the software price has the largest effect on the aggregate markup. This last result relates to the idea that software (or IT in general) helps firms take advantage of increasing returns (which can show up as higher markup) and scale up. Such an interpretation is in, for example, [Aghion et al. \(2023\)](#) and [Lashkari et al. \(2023\)](#).

5 Macro-level Elasticities and Labor Share

The estimated elasticities of substitution in Section 4 suggest that it is software, not equipment, whose declining price contributed more to the fall in the aggregate labor share. We now quantify the respective contribution. Proposition 4 summarizes the impact of capital-embodied (software and equipment) technological changes on the aggregate labor share.²⁷

Proposition 4 (Change in the aggregate labor share) *The impact of capital-embodied technological changes on the aggregate labor share is given by:*

$$LS_t - LS_{t-1} = \bar{LS}_t \times \left[- \left(\bar{s}_t(\bar{\sigma}_{s,t}^q - 1) - \bar{\tilde{s}}_t(\bar{\sigma}_{\mu,t}^q - 1) \right) \ln \frac{1/q_t}{1/q_{t-1}} - \left(\bar{e}_t(\bar{\sigma}_{e,t}^r - 1) - \bar{e}_t\bar{s}_t(\bar{\sigma}_{s,t}^r - 1) - \bar{\tilde{k}}_t(\bar{\sigma}_{\mu,t}^r - 1) \right) \ln \frac{1/r_t}{1/r_{t-1}} \right], \quad (39)$$

where $\bar{x}_t \equiv \frac{x_t + x_{t-1}}{2}$.

²⁷Our production function in effect assumes that other types of capital have a unitary elasticity of substitution with respect to labor and hence do not affect the labor share. See proof of Proposition 4 for details. Since the prices of equipment and software fell much more than those of other capital types in the data, our results do not hinge on this assumption. Figure 2 supports this assumption.

Proof In Appendix A. ■

Given the aggregate elasticities in Table 5, equation (39) implies that software-embodied technological change ($d \ln 1/q > 0$) reduces the aggregate labor income share through both factor substitution ($\bar{\sigma}_s^q > 1$) and markup changes ($\bar{\sigma}_\mu^q < 1$). On the other hand, equipment-embodied technological change ($d \ln 1/r$) has countervailing effects. In terms of factor substitution, it always increases the labor share within a firm because equipment complements labor ($\sigma_e < 1$). However, reallocation may or may not counteract the pattern of within-firm factor substitution ($\bar{\sigma}_e^r \gtrless 1$). The responses of markup invariably reduces the aggregate labor share ($\bar{\sigma}_\mu^r < 1$).

Data To implement the decomposition in equation (39), we construct the aggregate labor income share LS_t , aggregate equipment income share e_t , and aggregate software income share s_t from the National Accounts as described in Section 2.²⁸ Software and equipment-embodied technological change is represented by the rate of return on software and equipment, as in Section 2. The decomposition starts in 1990, when Korea's aggregate labor income share started to decline in a sustained manner. As the cross-sectional moments are available only after 2003, we impute the values between 1990 and 2002 from the 2003 values. We note that the aggregate elasticities do not have clear time trends (Figure 5). Further details are in Appendix B.

Results Table 6 and Figure 6 report by how much capital-embodied technological change affected the aggregate labor income share between 1990 and 2018 in Korea. The aggregate labor share declined by 4.4 percentage points in the data.

Using the estimated macro-level elasticities and the software price time series, we find that software-embodied technological change accounts for 2.9 p.p. or 66.9 percent of the overall decline in the labor share (first column, top panel). Of the 2.9 p.p., 1.4 p.p. (second column) is due to factor substitution and the other 1.5 p.p. (fifth column) is due to the rise in aggregate markup in response to the falling software price, a near-even split.

The decline in the labor share through factor substitution is entirely (1.5 out of 1.4 p.p.) attributed to within-firm substitution of software for labor (governed by the micro-level elasticity σ_s) and not at all to reallocation across firms (third and fourth columns). As for the markup increase in response to the falling software price, within-firm and between-firm adjustments play comparable roles (0.8 and 0.7 p.p. respec-

²⁸We use the same values for $\tilde{s}$ as for s and $\tilde{\kappa}$ as for κ . Formally, $\tilde{s} = \sum \omega_i (s_i(1 - m_i) + m_i \alpha_p^q) \approx \sum \omega_i s_i = s$ and $\tilde{\kappa} = \sum \omega_i (\kappa_i(1 - m_i) + m_i \alpha_p^r) \approx \sum \omega_i \kappa_i = \kappa$.

- Total change in labor share in the data: -0.044

• Effects of Software-embodied Technological Change ($\Delta \ln 1/q$)

	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.029	-0.014	-0.015	+0.001	-0.015	-0.008	-0.007
(% explained)	(66.9)	(31.9)	(33.0)	(-1.2)	(35.0)	(19.0)	(16.1)

• Effects of Equipment-embodied Technological Change ($\Delta \ln 1/r$)

	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.004	+0.004	+0.014	-0.010	-0.009	-0.008	-0.001
(% explained)	(10.2)	(-9.5)	(-32.8)	(23.3)	(19.8)	(17.4)	(2.3)

Table 6: Effects of Capital-Embodied Technological Change on the Labor Share

The decomposition is for the 1990–2018 period. The overall effect is calculated using the macro-level elasticities ($\bar{\sigma}$), and the within effect is computed using the micro-level elasticities (σ). The between effect is the difference between the overall and the within effects. Percent explained of the labor share decline in the data is in parentheses.

tively out of 1.5 p.p.). One may think that reallocation overall has a minor effect on the decline of the labor share, but it should be noted that the within-firm markup increase comes with a rise in market share, which entails reallocation. In fact, as in [Kehrig and Vincent \(2021\)](#), reallocation can be broken into three components: (i) large firms reducing their labor share, (ii) firms with a low labor share growing rapidly, and (iii) firms simultaneously gaining market shares and decreasing their labor shares. Their study of the US manufacturing plants found that the decline in the aggregate labor share was mostly driven by those plants that expanded rapidly while reducing their labor share—that is, (iii) above. Our decomposition is different from theirs, but the importance of the within-firm factor substitution and the within-firm markup increase in our result is consistent with firms increasing market shares while simultaneously reducing their labor shares. More important, our result suggests that software-embodied technological change may well be the cause of the pattern of reallocation observed both in the US data and in the Korean data.

The left panel of Figure 6 plots the cumulative impact of the fall in the software price on the aggregate labor share through within/between factor substitution and within/between markup increases.

The effect of equipment-embodied technological change is shown in the bottom panel of Table 6. Using the estimated macro-level elasticities and the equipment price

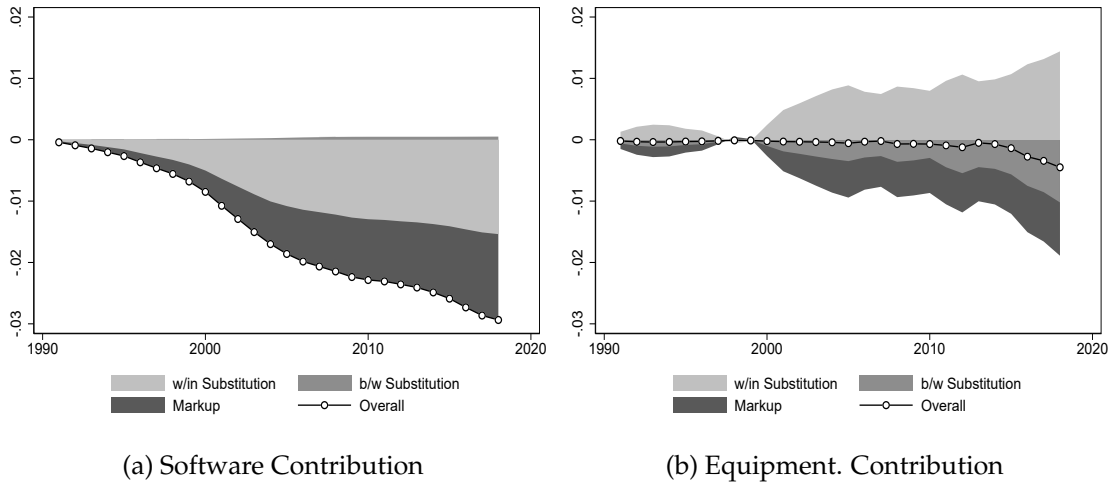


Fig. 6: Effects of Capital-Embodied Technological Change on the Labor Share

The decomposition is for the 1990–2018 period. The overall effect is calculated using the macro-level elasticities ($\bar{\sigma}$), and the within effect is computed using the micro-level elasticities (σ). The between effect is the difference between the overall and the within effects. Percent explained of the labor share decline in the data is in parentheses.

time series, we obtain a 0.4 percentage point decrease in the labor share (or 10 percent of the actual decrease) from equipment-embodied technological change. Behind this small number are several conflicting forces. Within-firm substitution pushes up the labor share by 1.4 p.p. (third column) because $\sigma_e < 1$, but the reallocation toward low labor share firms goes against it, negating 1.0 (fourth column) of the 1.4 p.p. increase. Factor substitution overall raises the labor share by 0.4 p.p. when the equipment price falls. In addition, the falling equipment price raises markups and reduces the labor share by 0.9 p.p. (fifth column), mostly through what happens within firms (sixth column). In sum, because of the heterogeneous markups, equipment-embodied technological change contributes to a small reduction in the labor share, even though equipment and labor are found to be complements at both the micro and the macro levels.

Put together, software-embodied and equipment-embodied technological change accounts for 77 percent of the decline in the aggregate labor share between 1990 and 2018 in Korea. Software-embodied technological change is the key driver (67 of the 77 percent), and variable markups are crucial for a comprehensive understanding of the overall impact of capital-embodied technological change. Although not reported here, when we simply lump together equipment and software as one capital bundle in a two-factor production function, we find this capital bundle and labor are complements: This simpler specification is a non-starter for thinking about the decline of

the labor share in the face of falling capital prices. Furthermore, even with software and equipment as two separate factors of production, When we use the standard CES aggregator with constant markup, software-embodied technological change accounts for less than one-third of the decline in the aggregate labor share.

In Appendix C, we provide additional results as robustness checks. First, we provide decomposition results using the aggregate elasticities and distributional moments from the manufacturing census data. Second, we consider alternative values of pass-through (b_i) based on the empirical relationship in Baqaee et al. (2023).²⁹ In both cases, we obtain qualitatively similar results: The equipment-labor substitution elasticity is larger at the macro level than at the micro level (but still less than one), while the macro-level software-labor substitution elasticity remains close to its micro-level counterpart. In addition, the aggregate markup is much more responsive to software prices than other factor prices.

6 Conclusion

In this paper, we establish that the capital-labor substitution elasticity is different across types of capital. The equipment-labor substitution elasticity is less than one, consistent with the micro-level findings in the literature, but the software-labor substitution elasticity is greater than one, a novel finding. This distinction is important for understanding the decline of the aggregate labor income share over time.

Our focus on software connects well with (and complements) the three leading explanations of the labor share decline. First, our results support the literature arguing that technological change embodied in capital reduced the labor share since the 1980s (Karabarbounis and Neiman, 2013). We clarify that it is software rather than equipment that substitutes for labor, both at the micro and at the macro levels. Second, our results reinforce the findings of (Koh et al., 2020) that intangible capital shares, rather than tangible capital shares, rose at the expense of labor shares in the aggregate. We not only estimate the elasticities of substitution to quantify the role of capital-embodied technological change, but also establish the importance of the heterogeneous, variable markups in pushing down the labor share as a result. In fact, the effect of the rising markups is as large as the factor substitution itself. In other words, comparing the factor income shares of labor and capital alone will underestimate the true impact of software-embodied technological change by half. Third, the literature

²⁹Baqaee et al. (2023) suggest that the Klenow-Willis specification produces too little variation in price pass-through, potentially understating the magnitude of reallocation.

has documented how reallocation across firms contributed to the decline of the aggregate labor share ([Autor et al., 2020](#); [De Loecker et al., 2020](#); [Kehrig and Vincent, 2021](#)). We find that firms with high software shares typically have low labor shares and high markups in the data. This means that software-embodied technological change can be the causal force behind the reallocation toward firms with high markups and low labor shares in the data, bringing down the aggregate labor income share.

The natural next step is to ask how such technological change affected different workers differently and hence the overall income inequality. Because of the limited scope of worker heterogeneity in our firm/plant-level data, we were not able to assess how equipment and software interact with workers of different skill levels. As previously mentioned, our calibration exercise using macro data in the spirit of [Krusell et al. \(2000\)](#) finds that equipment complements both high-skill and low-skill workers, but complements high-skill workers more (Appendix D). Software, on the other hand, substitutes for both high-skill and low-skill workers, but substitutes for low-skill workers more. This exercise shows that capital-embodied technological change can raise the skill premium and reduce the income share of low-skill workers. At a more disaggregated level, [Aum \(2020\)](#) documents that workers in middle-skill occupations tend to use equipment more, while those in high-skill occupations tend to use software more in the US O*NET data. A richer framework that allows for multiple types of capital and heterogeneous workers—for example, an enhanced version of the model in [Aum et al. \(2018\)](#)—may be a promising avenue for future research on the distributional consequences of technological changes.

References

- Aghion, P., A. Bergeaud, T. Boppart, P. J. Klenow, and H. Li (2023). A Theory of Falling Growth and Rising Rents. *The Review of Economic Studies* 90(6), 2675–2702.
- Antras, P. (2004, April). Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution. *The B.E. Journal of Macroeconomics* 4(1), 1–36.
- Aum, S. (2019). Knowledge Capital and Labor’s Income Share. KDI Policy Study 2019-06, Korea Development Institute.
- Aum, S. (2020). The Rise of Software and Skill Demand Reversal. Manuscript.
- Aum, S., S. Y. T. Lee, and Y. Shin (2018). Computerizing industries and routinizing jobs: Explaining trends in aggregate productivity. *Journal of Monetary Economics* 97, 1 – 21.
- Aum, S. and Y. Shin (2020). Why is the labor share declining? *Federal Reserve Bank of St. Louis Review* 102(4), 413–428.
- Autor, D., D. Dorn, L. F. Katz, C. Patterson, and J. Van Reenen (2020, 02). The Fall of the Labor Share and the Rise of Superstar Firms. *The Quarterly Journal of Economics* 135(2), 645–709.
- Baqaei, D. R. and E. Farhi (2019, 09). Productivity and Misallocation in General Equilibrium. *The Quarterly Journal of Economics* 135(1), 105–163.
- Baqaei, D. R., E. Farhi, and K. Sangani (2023, 06). The Darwinian Returns to Scale. *The Review of Economic Studies*, rda061.
- Bartik, T. J. (1991). *Who Benefits from State and Local Economic Development Policies?* W.E. Upjohn Institute.
- Beaudry, P., D. A. Green, and B. Sand (2012). Does industrial composition matter for wages? a test of search and bargaining theory. *Econometrica* 80(3), 1063–1104.
- Borusyak, K., P. Hull, and X. Jaravel (2021, 06). Quasi-Experimental Shift-Share Research Designs. *The Review of Economic Studies* 89(1), 181–213.
- Castro-Vincenzi, J. and B. Kleinman (2023). Intermediate Input Prices and the Labor Share. Working paper.

- Chirinko, R. S. and D. Mallick (2017, October). The substitution elasticity, factor shares, and the low-frequency panel model. *American Economic Journal: Macroeconomics* 9(4), 225–53.
- Corrado, C., J. Haskel, C. Jona-Lasinio, and M. Iommi (2022, August). Intangible capital and modern economies. *Journal of Economic Perspectives* 36(3), 3–28.
- Corrado, C., C. Hulten, and D. Sichel (2009, September). Intangible Capital And U.S. Economic Growth. *Review of Income and Wealth* 55(3), 661–685.
- Cummins, J. G. and G. L. Violante (2002). Investment-specific technical change in the united states (1947–2000): Measurement and macroeconomic consequences. *Review of Economic Dynamics* 5(2), 243–284.
- De Loecker, J., J. Eeckhout, and G. Unger (2020, 01). The Rise of Market Power and the Macroeconomic Implications. *The Quarterly Journal of Economics* 135(2), 561–644.
- de Souza, G. and H. Li (2023, November). Robots, Tools, and Jobs: Evidence from Brazilian Labor Markets. Working Paper Series WP 2023-42, Federal Reserve Bank of Chicago.
- Eden, M. and P. Gaggl (2019). Capital composition and the declining labor share. CESifo Working Paper Series 7996, CESifo.
- Edmond, C., V. Midrigan, and D. Y. Xu (2018, July). How Costly Are Markups? NBER Working Papers 24800, National Bureau of Economic Research, Inc.
- Eisfeldt, A. L., A. Falato, and M. Z. Xiaolan (2023, March). Human capitalists. In M. Eichenbaum, E. Hurst, and V. A. Ramey (Eds.), *NBER Macroeconomics Annual 2022, volume 37*, pp. 1–61. University of Chicago Press.
- Eisfeldt, A. L., G. Schubert, B. Taska, and M. B. Zhang (2023). Generative AI and firm values. Manuscript.
- Goldsmith-Pinkham, P., I. Sorkin, and H. Swift (2020, August). Bartik instruments: What, when, why, and how. *American Economic Review* 110(8), 2586–2624.
- Gollin, D. (2002). Getting income shares right. *Journal of Political Economy* 110(2), 458–474.

- Greenwood, J., Z. Hercowitz, and P. Krusell (1997, June). Long-Run Implications of Investment-Specific Technological Change. *American Economic Review* 87(3), 342–362.
- Greenwood, J., Z. Hercowitz, and P. Krusell (2000, January). The role of investment-specific technological change in the business cycle. *European Economic Review* 44(1), 91–115.
- Grossman, G. M. and E. Oberfield (2022). The Elusive Explanation for the Declining Labor Share. *Annual Review of Economics* 14.
- Herrendorf, B., C. Herrington, and A. Valentinyi (2015, October). Sectoral technology and structural transformation. *American Economic Journal: Macroeconomics* 7(4), 104–33.
- Houthakker, H. S. (1955, 04). The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis 1. *The Review of Economic Studies* 23(1), 27–31.
- Jones, C. I. (2005). The Shape of Production Functions and the Direction of Technical Change. *The Quarterly Journal of Economics* 120(2), 517–549.
- Karabarbounis, L. and B. Neiman (2013, 10). The global decline of the labor share. *Quarterly Journal of Economics* 129(1), 61–103.
- Kehrig, M. and N. Vincent (2021, 03). The Micro-Level Anatomy of the Labor Share Decline. *The Quarterly Journal of Economics* 136(2), 1031–1087.
- Keynes, J. M. (1939). Relative movements of real wages and output. *The Economic Journal* 49(193), 34–51.
- Kimball, M. S. (1995). The quantitative analytics of the basic neomonetarist model. *Journal of Money, Credit and Banking* 27(4), 1241–1277.
- Klenow, P. J. and J. L. Willis (2016). Real rigidities and nominal price changes. *Economica* 83(331), 443–472.
- Knoblauch, M., M. Roessler, and P. Zwerschke (2020). The elasticity of substitution between capital and labour in the us economy: A meta-regression analysis. *Oxford Bulletin of Economics and Statistics* 82(1), 62–82.
- Koh, D., R. Santaepulàlia-Llopis, and Y. Zheng (2020). Labor share decline and intellectual property products capital. *Econometrica* 88(6), 2609–2628.

- Krusell, P., L. E. Ohanian, J.-V. Rìos-Rull, and G. L. Violante (2000, September). Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis. *Econometrica* 68(5), 1029–1054.
- Lashkari, D., A. Bauer, and J. Boussard (2023). Information Technology and Returns to Scale. Working paper.
- Oberfield, E. and D. Raval (2021). Micro Data and Macro Technology. *Econometrica* 89(2), 703–732.
- Park, G. (2022). Investment stimulus, automation, and skill demand. Manuscript.
- Park, J. (2020). Self-Employment Sector and Functional Income Distribution of Korean Economy. *Korea Economic Forum* 12(4), 27–68.
- Parker, R. and B. Grimm (2000, May). Recognition of Business and Government Expenditures for Software as Investment: Methodology and Quantitative Impacts, 1959-98. Paper presented to BEA’s Advisory Committee May 5, 2000, Bureau of Economic Analysis.
- Raval, D. R. (2019). The micro elasticity of substitution and non-neutral technology. *The RAND Journal of Economics* 50(1), 147–167.
- Stern, D. I. (2011). Elasticities of substitution and complementarity. *Journal of Productivity Analysis* 36(1), 79–89.

Appendix A Proofs

Proof of Proposition 1 From equations (8) and (9), we have the relative price between wage and equipment price as:

$$\frac{w}{r} = \frac{\alpha_i^L}{\alpha_i^K} \left(\frac{A_L}{A_K} \right)^{\frac{\sigma_e - 1}{\sigma_e}} \left(\frac{K_i}{L_i} \right)^{\frac{1}{\sigma_e}}.$$

Taking logs, we have

$$(\sigma_e - 1) \ln \frac{w}{r} = \sigma_e \ln \frac{\alpha_i^L}{\alpha_i^K} + (\sigma_e - 1) \ln \frac{A_L}{A_K} + \ln \frac{rK_i}{wL_i}. \quad (\text{A.1})$$

Differentiating equation (A.1) holding A_L/A_K fixed and using the definition of k_i , we obtain equation (15), which is the first part of Proposition 1.

Similarly, from equations (5), (8), (9), and (13), we have $\phi_i = p_i(Y_i/V_i)^{1/\sigma_m}(V_i/X_i)^{1/\sigma_s}$.

Dividing it by (10), we obtain

$$\frac{\phi_i}{q} = \frac{1}{\alpha_i^S} \left(\frac{1}{A_S} \right)^{\frac{\sigma_s - 1}{\sigma_s}} \left(\frac{S_i}{X_i} \right)^{\frac{1}{\sigma_s}}.$$

Taking logs, we have

$$(\sigma_s - 1) \ln \frac{\phi_i}{q} = \sigma_s \ln \frac{1}{\alpha_i^S} + (\sigma_s - 1) \ln \frac{1}{A_S} + \ln \frac{qS_i}{\phi_i X_i}. \quad (\text{A.2})$$

Differentiating equation (A.2) holding A_S fixed, we derive

$$\sigma_s - 1 = \frac{d \ln \frac{qS_i}{\phi_i X_i}}{d \ln \frac{\phi_i}{q}}. \quad (\text{A.3})$$

From the definition of ϕ_i in equation (13),

$$d \ln \phi_i = \frac{1}{1 + \left(\frac{\alpha_i^K}{\alpha_i^L} \right)^{\sigma_e} \left(\frac{rA_L}{wA_K} \right)^{1-\sigma_e}} d \ln w + \frac{1}{1 + \left(\frac{\alpha_i^K}{\alpha_i^L} \right)^{\sigma_e} \left(\frac{wA_K}{rA_L} \right)^{1-\sigma_e}} d \ln r. \quad (\text{A.4})$$

Since $\left(\frac{r\alpha_i^L}{w\alpha_i^K} \right)^{\sigma_e} \left(\frac{A_L}{A_K} \right)^{\sigma_e - 1} \frac{w}{r} = \frac{wL_i}{rK_i}$ from equations (8) and (9), inserting it into equation (A.4) and using the definition of k_i , we obtain

$$d \ln \phi_i = (1 - k_i) d \ln w + k_i d \ln r.$$

Finally, we have $\phi_i X_i = wL_i + rK_i$ from equations (5), (8), (9), and (13). Equation (A.3) becomes

$$\sigma_s - 1 = \frac{d \ln \frac{qS_i}{wL_i + rK_i}}{(1 - k_i) d \ln \frac{w}{q} + k_i d \ln \frac{r}{q}},$$

which is the second part of Proposition 1. ■

Proof of Corollary 1 Totally differentiating equation (A.1) and setting $d \ln w = d \ln r = 0$, we see the first part of Corollary 1. Totally differentiating equation (A.3) and setting $d \ln q = 0$ and following the same steps as in the proof of Proposition 2, the second part of Corollary 1 follows. $\blacksquare$

Proof of Proposition 2 Let the cost function of establishment i be $c_i \equiv c(Y_i, w, r, q, v) = wL_i + rK_i + qS_i + vM_i$, and $c = \sum_i c_i$. Firstly, consider $d \ln w > 0$, $d \ln r = 0$, and $d \ln q = 0$.

Equipment-labor substitution From the definition of $\bar{\sigma}_e^w$ (Definition 1),

$$\bar{\sigma}_e^w - 1 = \frac{d \ln \frac{rK}{wL}}{d \ln w} = \frac{d \ln \frac{k}{1-k}}{d \ln w} = \frac{1}{1-k} \frac{d \ln k}{d \ln w} = \frac{1}{k(1-k)} \frac{dk}{d \ln w}. \quad (\text{A.5})$$

From the definition of $k \equiv \sum \theta_i k_i$,

$$dk = \sum_i \theta_i dk_i + \sum_i k_i d\theta_i. \quad (\text{A.6})$$

From equation (15) in Proposition 1, we have

$$dk_i = (\sigma_e - 1)k_i(1 - k_i)d \ln w. \quad (\text{A.7})$$

Denoting $\chi \equiv \frac{\sum_i (k_i - k)^2}{k(1-k)}$, we get $1 - \chi = \frac{\sum_i \theta_i (1 - k_i)k_i}{k(1-k)}$. Hence, equations (A.5)–(A.7) imply that $\frac{\sum_i \theta_i dk_i}{k(1-k)d \ln w} = (1 - \chi)(\sigma_e - 1)$ and

$$\bar{\sigma}_e^w - 1 = (1 - \chi)(\sigma_e - 1) + \frac{1}{k(1-k)} \sum_i k_i \frac{d\theta_i}{d \ln w}. \quad (\text{A.8})$$

We now turn to $\sum_i k_i \frac{d\theta_i}{d \ln w}$. As in Oberfield and Raval (2021), we repeatedly use $\sum_i x d\theta_i = 0$ for any x not varying across i . For example, $\sum_i k_i \frac{d\theta_i}{d \ln w} = \sum_i (k_i - k) \frac{d\theta_i}{d \ln w} = \sum_i (k_i - k) \theta_i \frac{d \ln \theta_i}{d \ln w}$. We also use $\sum_i x (k_i - k) \theta_i = 0$ for any x not varying across i .

Since $\theta_i = \frac{(1-s_i)(1-m_i)c_i}{(1-s)(1-m)c}$, $\theta_i d \ln \theta_i = d \ln(1 - s_i) + d \ln(1 - m_i) + d \ln c_i - d \ln(1 - s)(1 - m)c$. Note that $\sigma_m - 1 = \frac{d \ln m_i / (1 - m_i)}{d \ln \phi_i / v}$, where ϕ_i refers to the price of labor-equipment-software composite (i.e. $\phi_i V_i = wL_i + rK_i + qS_i$). By Shephard's lemma, $\frac{d \ln \phi_i}{d \ln w} = \ell_i$. Also, we denote labor content of materials as $\alpha_v^w \equiv \frac{d \ln v}{d \ln w}$, and assume that labor content of materials is the same as the labor content of the final good (i.e. $\alpha_v^w = \alpha_p^w \equiv \frac{d \ln p}{d \ln w}$). We now have $\frac{d \ln(1 - m_i)}{d \ln w} = -\frac{dm_i}{(1 - m_i)d \ln w} = m_i(\ell_i - \alpha_p^w)(1 - \sigma_m)$. Similarly, from $\frac{d \ln s_i / (1 - s_i)}{d \ln \phi_i / q}$, we have $\frac{d \ln(1 - s_i)}{d \ln q} = -\frac{ds_i}{1 - s_i} = s_i(1 - k_i)(1 - \sigma_s)$.

Combining them with the fact that $\sum_i (k_i - k) \theta_i d \ln(1 - s)(1 - m)c = 0$, we have

$$\sum_i k_i \frac{d\theta_i}{d \ln w} = \sum_i (k_i - k) \theta_i \left[\frac{d \ln(1 - s_i)}{d \ln w} + \frac{d \ln(1 - m_i)}{d \ln w} + \frac{d \ln c_i}{d \ln w} \right]$$

$$= \sum_i (k_i - k) \theta_i \left[s_i (1 - k_i) (1 - \sigma_s) + m_i (\ell_i - \alpha_p^w) (1 - \sigma_m) + \frac{d \ln c_i}{d \ln w} \right], \quad (\text{A.9})$$

where $\alpha_p^w \equiv \frac{d \ln p}{d \ln w} = \frac{d \ln v}{d \ln w}$.

Differentiating the cost function $c_i = c(Y_i, w, r, q, v)$, $d \ln c_i = \frac{c_{Y,i} Y_i}{c_i} d \ln Y_i + \frac{c_{w,i} w}{c_i} d \ln w$. From Shephard's lemma, $c_{w,i} w / c_i = \ell_i (1 - m_i) + m_i \alpha_p^w$. Constant returns to scale imply $\frac{c_{Y,i} Y_i}{c_i} = 1$. Hence,

$$\frac{d \ln c_i}{d \ln w} = \frac{d \ln Y_i}{d \ln w} + \ell_i (1 - m_i) + m_i \alpha_p^w. \quad (\text{A.10})$$

Substituting equation (A.10) into (A.9),

$$\begin{aligned} \sum_i k_i \frac{d \theta_i}{d \ln w} &= \sum_i (k_i - k) \theta_i \left[s_i (k_i - 1) \sigma_s - m_i (\ell_i - \alpha_p^w) \sigma_m + (1 - k_i) + \frac{d \ln Y_i}{d \ln w} \right] \\ &= \sum_i (k_i - k) \theta_i \left[(k_i - 1) \zeta^w \sigma_s - \bar{m}_e^w ((1 - k_i) (1 - s_i) - \alpha_p^w) \sigma_m + (1 - k_i) + \frac{d \ln Y_i}{d \ln w} \right] \\ &= \sum_i (k_i - k) \theta_i \left[(k_i - 1) \zeta^w \sigma_s - \bar{m}_e^w ((1 - k_i) (1 - \zeta^w) - \alpha_p^w) \sigma_m + (1 - k_i) + \frac{d \ln Y_i}{d \ln w} \right] \\ &= \sum_i (k_i - k) \theta_i \left[(k_i - k) (\zeta^w \sigma_s + (1 - \zeta^w) \bar{m}_e^w \sigma_m - 1) + \frac{d \ln Y_i}{d \ln w} \right] \\ &= k(1 - k) \chi (\zeta^w \sigma_s + (1 - \zeta^w) \bar{m}_e^w \sigma_m - 1) + \sum_i (k_i - k) \theta_i \frac{d \ln Y_i}{d \ln w}. \end{aligned} \quad (\text{A.11})$$

Here, we denote

$$\zeta^w \equiv \frac{\sum_i (k_i - k) (1 - k_i) \theta_i s_i}{\sum_i (k_i - k) (1 - k_i) \theta_i} \text{ and } \bar{m}_e^w \equiv \frac{\sum_i (k_i - k) (\ell_i - \alpha_p^w) \theta_i m_i}{\sum_i (k_i - k) (\ell_i - \alpha_p^w) \theta_i}.$$

Substituting equation (A.11) into (A.8), we get

$$\bar{\sigma}_e^w = (1 - \chi) \sigma_e + \chi (\zeta^w \sigma_s + (1 - \zeta^w) \bar{m}_e^w \sigma_m) + \frac{1}{k(1 - k)} \sum_i (k_i - k) \theta_i \frac{d \ln Y_i}{d \ln w}. \quad (\text{A.12})$$

Now we turn to $\frac{d \ln Y_i}{d \ln w}$. We have aggregate Y that satisfies $1 = \sum_i H(Y_i/Y)$. And cost minimization implies $p_i/p = H'(Y_i/Y)$. Denoting the inverse of H' with h , we have

$$d \ln Y_i/Y = -\epsilon_i d \ln p_i/p, \quad (\text{A.13})$$

where $\epsilon_i \equiv -\frac{h'(p_i/p) p_i/p}{h(p_i/p)}$. The optimal markup will be $\mu(p_i/p) = \frac{\epsilon(p_i/p)}{\epsilon(p_i/p) - 1}$. Optimal pricing implies $p_i = \mu(p_i/p) c_{Y,i}$.

We first define firm i 's local rate of price pass-through ($b_i \equiv \frac{d \ln p_i}{d \ln m c_i} \equiv \frac{d \ln p_i}{d \ln c_{Y,i}}$). Differentiating the optimal pricing condition, we have $d \ln p_i = \frac{\mu' \times p_i/p}{\mu} d \ln p_i + d \ln c_{Y,i}$. Rearranging terms, we get $b_i \equiv \frac{d \ln p_i}{d \ln c_{Y,i}} = \frac{1}{1 - \frac{\mu' \times p_i/p}{\mu}}$.

Note that a change in wage also affects the aggregate price ($\alpha_p^w \equiv \frac{d \ln p}{d \ln w}$). From equation (A.13), we know that $d \ln Y_i/Y$ is related to $d \ln p_i/p$. Since optimal pricing implies $p_i/p = \mu(p_i/p) c_{Y,i}/p$, differentiation gives

$$\frac{d \ln p_i/p}{d \ln w} = \frac{\mu'(p_i/p) p_i/p}{\mu(p_i/p)} \frac{d \ln p_i/p}{d \ln w} + \frac{d \ln c_{Y,i}}{d \ln w} - \frac{d \ln p}{d \ln w}.$$

Constant returns to scale and Shephard's lemma imply $\frac{d \ln c_{Y,i}}{d \ln w} = (1 - m_i) \ell_i + m_i \alpha_p^w$. (Again, we assumed $\alpha_p^w = \alpha_m^w$.) It follows that

$$\frac{d \ln p_i/p}{d \ln w} = b_i(1 - m_i)(\ell_i - \alpha_p^w) \quad (\text{A.14})$$

and

$$\frac{d \ln Y_i/Y}{d \ln w} = \epsilon_i b_i(1 - m_i)(\alpha_p^w - \ell_i). \quad (\text{A.15})$$

Now we use equation (A.15) to obtain

$$\begin{aligned} \frac{1}{k(1-k)} \sum_i (k_i - k) \theta_i \frac{d \ln Y_i}{d \ln w} &= \frac{1}{k(1-k)} \sum_i (k_i - k) \theta_i \epsilon_i b_i(1 - m_i)(\alpha_p^w - \ell_i) \\ &= \frac{1}{k(1-k)} \sum_i (k_i - k) \theta_i \bar{\epsilon}_e^w (1 - m_i)(\alpha_p^w - \ell_i) \\ &= \frac{1}{k(1-k)} \sum_i (k_i - k) \theta_i \bar{\epsilon}_e^w (1 - \bar{m}_e^w)(\alpha_p^w - \ell_i) \\ &= \frac{1}{k(1-k)} \sum_i (k_i - k) \theta_i \bar{\epsilon}_e^w (1 - \bar{m}_e^w) \left[\alpha_p^w - (1 - k_i)(1 - \zeta^w) \right] \\ &= \frac{1}{k(1-k)} \sum_i (k_i - k) \theta_i \bar{\epsilon}_e^w (1 - \bar{m}_e^w) (k_i - k)(1 - \zeta^w) \\ &= \chi(1 - \zeta^w)(1 - \bar{m}_e^w) \bar{\epsilon}_e^w. \end{aligned} \quad (\text{A.16})$$

Note that

$$\bar{\epsilon}_e^w \equiv \frac{\sum_i (k_i - k)(1 - m_i)(\alpha_p^w - \ell_i) \theta_i \epsilon_i b_i}{\sum_i (k_i - k)(1 - m_i)(\alpha_p^w - \ell_i) \theta_i}.$$

Finally, we obtain α_p from $\sum_i \frac{p_i Y_i}{p Y} \frac{d \ln p_i Y_i / p Y}{d \ln w} = 0$:

$$\begin{aligned} \sum_i \frac{p_i Y_i}{p Y} \frac{d \ln Y_i/Y}{d \ln w} &= \sum_i \frac{p_i Y_i}{p Y} (\epsilon_i - 1) b_i(1 - m_i)(\alpha_p^w - \ell_i) = 0 \\ \Rightarrow \alpha_p^w &= \frac{\sum_i p_i Y_i (\epsilon_i - 1) b_i(1 - m_i) \ell_i}{\sum_i p_i Y_i (\epsilon_i - 1) b_i(1 - m_i)}. \end{aligned}$$

Finally, from equations (A.8) and (A.16), we now have

$$\bar{\sigma}_e^w = (1 - \chi)\sigma_e + \chi [\zeta^w \sigma_s + (1 - \zeta^w) \bar{m}_e^w \sigma_m + (1 - \zeta^w - (1 - \zeta^w) \bar{m}_e^w) \bar{\epsilon}_e^w],$$

where

$$\begin{aligned}\zeta^w &\equiv \frac{\sum_i (k_i - k)(1 - k_i) \theta_i s_i}{\sum_i (k_i - k)(1 - k_i) \theta_i}, \\ \bar{m}_e^w &\equiv \frac{\sum_i (k_i - k)(\ell_i - \alpha_p^w) \theta_i m_i}{\sum_i (k_i - k)(\ell_i - \alpha_p^w) \theta_i}, \\ \bar{\epsilon}_e^w &\equiv \frac{\sum_i (k_i - k)(1 - m_i)(\alpha_p^w - \ell_i) \theta_i \epsilon_i b_i}{\sum_i (k_i - k)(1 - m_i)(\alpha_p^w - \ell_i) \theta_i}, \\ \alpha_p^w &= \frac{\sum_i p_i \gamma_i (\epsilon_i - 1) b_i (1 - m_i) \ell_i}{\sum_i p_i \gamma_i (\epsilon_i - 1) b_i (1 - m_i)}.\end{aligned}$$

Software-labor substitution Similar to the proof of the equipment-labor substitution above, we begin with

$$\bar{\sigma}_s^w - 1 = \frac{d \ln \frac{s}{1-s}}{(1-k) d \ln w} = \frac{1}{\ell s} \frac{ds}{d \ln w}. \quad (\text{A.17})$$

From the definition $s \equiv \sum \omega_i s_i$, we also have

$$ds = \sum_i \omega_i ds_i + \sum_i s_i d\omega_i. \quad (\text{A.18})$$

From Proposition 1, we know that

$$ds_i = (\sigma_s - 1) s_i (1 - s_i) (1 - k_i) d \ln w = (\sigma_s - 1) s_i \ell_i d \ln w. \quad (\text{A.19})$$

With

$$\bar{\zeta}^w \equiv -\frac{\sum_i (\ell_i - \ell)(s_i - s) \omega_i}{\ell s},$$

it is straightforward to check

$$(1 - \bar{\zeta}^w) = \frac{\sum_i \ell_i s_i \omega_i}{\ell s},$$

and $\sum_i \omega_i ds_i = \ell s (1 - \bar{\zeta}^w) (\sigma_s - 1)$ from equation (A.19). Therefore, from equations (A.17) and (A.18), we get

$$\bar{\sigma}_s^w - 1 = (1 - \bar{\zeta}^w) (\sigma_s - 1) + \frac{1}{s \ell} \sum_i s_i \frac{d\omega_i}{d \ln w}. \quad (\text{A.20})$$

Since $\omega_i = \frac{(1-m_i)c_i}{(1-m)c}$, following the steps of deriving equations (A.9) and (A.10), we have

$$\frac{d \ln \omega_i}{d \ln w} = \frac{d \ln(1 - m_i) / (1 - m)}{d \ln w} + \frac{d \ln c_i / c}{d \ln w}$$

$$= \frac{d \ln(1 - m_i)}{d \ln w} + \frac{d \ln Y_i}{d \ln w} + \ell_i(1 - m_i) + m_i \alpha_p^w - \frac{d \ln(1 - m)}{d \ln w} - \frac{d \ln c}{d \ln w}.$$

Since $\sum_i s_i \frac{d\omega_i}{d \ln w} = \sum_i (s_i - s) \frac{d\omega_i}{d \ln w} = \sum_i (s_i - s) \omega_i \frac{d \ln \omega_i}{d \ln w}$,

$$\begin{aligned} \frac{1}{s\ell} \sum_i (s_i - s) \omega_i \frac{d \ln \omega_i}{d \ln w} &= \frac{1}{s\ell} \sum_i (s_i - s) \omega_i \left[m_i(\ell_i - \alpha_p^w)(1 - \sigma_m) + (1 - m_i)(\ell_i - \alpha_p) + \frac{d \ln Y_i}{d \ln w} \right] \\ &= \frac{1}{s\ell} \sum_i (s_i - s) \omega_i \left[\bar{m}_s^w(\ell_i - \alpha_p^w)(1 - \sigma_m) + (1 - \bar{m}_s^w)(\ell_i - \alpha_p) + \frac{d \ln Y_i}{d \ln w} \right] \\ &= \frac{1}{s\ell} \sum_i (s_i - s) \omega_i \left[\ell_i(1 - \bar{m}_s^w \sigma_m) + \frac{d \ln Y_i}{d \ln w} \right] \\ &= \frac{1}{s\ell} \sum_i (s_i - s) \omega_i \left[\ell_i(1 - \bar{m}_s^w \sigma_m) + \epsilon_i b_i(1 - m_i)(\alpha_p^w - \ell_i) \right] \\ &= \frac{1}{s\ell} \sum_i (s_i - s) \omega_i \left[\ell_i(1 - \bar{m}_s^w \sigma_m) + \bar{\epsilon}_s^w(1 - \bar{m}_s^w)(\alpha_p^w - \ell_i) \right] \\ &= \frac{1}{s\ell} \sum_i (s_i - s) \omega_i \ell_i [(1 - \bar{m}_s^w \sigma_m) - \bar{\epsilon}_s^w(1 - \bar{m}_s^w)] \\ &= \frac{1}{s\ell} \sum_i -(s_i - s)(\ell_i - \ell) \omega_i [(\bar{m}_s^w \sigma_m - 1) + \bar{\epsilon}_s^w(1 - \bar{m}_s^w)] \\ &= \zeta^w (\bar{m}_s^w \sigma_m + (1 - \bar{m}_s^w) \bar{\epsilon}_s^w - 1). \end{aligned} \quad (\text{A.21})$$

With

$$\bar{m}_s^w \equiv \frac{\sum_i (s_i - s)(\ell_i - \alpha_p^w) \omega_i m_i}{\sum_i (s_i - s)(\ell_i - \alpha_p^w) \omega_i} \text{ and } \bar{\epsilon}_s^w \equiv \frac{\sum_i (s_i - s)(1 - m_i)(\alpha_p^w - \ell_i) \omega_i \epsilon_i b_i}{\sum_i (s_i - s)(1 - m_i)(\alpha_p^w - \ell_i) \omega_i},$$

inserting equation (A.21) into equation (A.20) yields

$$\bar{\sigma}_s^w = (1 - \zeta^w) \sigma_s + \zeta^w [\bar{m}_s^w \sigma_m + (1 - \bar{m}_s^w) \bar{\epsilon}_s^w], \quad (\text{A.22})$$

where

$$\begin{aligned} \zeta^w &= -\frac{\sum_i (s_i - s)(\ell_i - \ell) \omega_i}{s\ell}, \\ \bar{m}_s^w &= \frac{\sum_i (s_i - s)(\ell_i - \alpha_p^w) \omega_i m_i}{\sum_i (s_i - s)(\ell_i - \alpha_p^w) \omega_i}, \\ \bar{\epsilon}_s^w &= \frac{\sum_i (s_i - s)(1 - m_i)(\alpha_p^w - \ell_i) \omega_i \epsilon_i b_i}{\sum_i (s_i - s)(1 - m_i)(\alpha_p^w - \ell_i) \omega_i}. \end{aligned}$$

The cases with $d \ln r > 0$ and $d \ln q > 0$ are analogous. ■

Proof of Proposition 3 Consider the case with $d \ln w > 0$, $d \ln r = 0$, and $d \ln q = 0$. We define the aggregate markup as $\mu := \sum_i \mu_i \omega_i$. Note that we weight establishment i 's markup with i 's share of non-material cost, $\omega_i = (wL_i + rK_i + qS_i) / (wL + rK + qS)$.

Changes in the aggregate markup responding to a change in wage is given by

$$\frac{d \ln \mu}{d \ln w} = \frac{1}{\mu} \frac{d \mu}{d \ln w} = \frac{1}{\mu} \left[\sum_i \omega_i \frac{d \mu_i}{d \ln w} + \sum_i \mu_i \frac{d \omega_i}{d \ln w} \right]. \quad (\text{A.23})$$

Since $p_i = \mu_i \times mc_i$, from the definition of pass-through $b_i \equiv \frac{d \ln p_i}{d \ln mc_i}$ and Shephard's lemma ($\frac{d \ln mc_i}{d \ln w} = \ell_i(1 - m_i) + m_i \alpha_p^w$), we have

$$\frac{d \ln \mu_i}{d \ln w} = (b_i - 1)[\ell_i(1 - m_i) + m_i \alpha_p^w] = (b_i - 1) \tilde{\ell}_i, \quad (\text{A.24})$$

where we use $\tilde{\ell}_i \equiv \ell_i(1 - m_i) + m_i \alpha_p^w$ for notational convenience. Using equation (A.24),

$$\sum_i \omega_i \frac{d \mu_i}{d \ln w} = \sum_i \omega_i \mu_i \tilde{\ell}_i (b_i - 1) = \mu \tilde{\ell} (1 - \eta^w) (\bar{b}^w - 1), \quad (\text{A.25})$$

where

$$\eta^w := -\frac{\sum_i (\mu_i - \mu) (\tilde{\ell}_i - \tilde{\ell}) \omega_i}{\mu \tilde{\ell}} \text{ and } \bar{b}^w := \frac{\sum_i \mu_i \tilde{\ell}_i b_i \omega_i}{\sum_i \mu_i \tilde{\ell}_i \omega_i}.$$

In addition, following a similar calculation in equation (A.21),

$$\begin{aligned} \sum_i (\mu_i - \mu) \omega_i \frac{d \ln \omega_i}{d \ln w} &= \sum_i (\mu_i - \mu) \omega_i \left[m_i (\ell_i - \alpha_p^w) (1 - \sigma_m) + \tilde{\ell}_i + \frac{d \ln Y_i}{d \ln w} \right] \\ &= \sum_i (\mu_i - \mu) \omega_i \left[(\tilde{\ell}_i - \alpha_p^w) (\sigma_m - 1) - (\ell_i - \alpha_p^w) (\sigma_m - 1) + \tilde{\ell}_i - \epsilon_i b_i (\tilde{\ell}_i - \alpha_p^w) \right] \\ &= \sum_i (\mu_i - \mu) \omega_i \left[-\ell_i (\sigma_m - 1) + \tilde{\ell}_i \sigma_m - \epsilon_i b_i (\tilde{\ell}_i - \alpha_p^w) \right] \\ &= \sum_i (\mu_i - \mu) \omega_i \left[-\ell_i (\sigma_m - 1) + \tilde{\ell}_i (\sigma_m - \bar{\epsilon}_\mu^w) \right] \\ &= \sum_i (\mu_i - \mu) \omega_i (\ell_i - \ell) (1 - \sigma_m) + \sum_i (\mu_i - \mu) \omega_i (\tilde{\ell}_i - \tilde{\ell}) (\sigma_m - \bar{\epsilon}_\mu^w) \\ &= \mu \tilde{\ell} \iota^w (\sigma_m - 1) + \mu \tilde{\ell} \eta^w (\bar{\epsilon}_\mu^w - \sigma_m), \end{aligned} \quad (\text{A.26})$$

where

$$\bar{\epsilon}_\mu^w := \frac{\sum_i (\mu_i - \mu) (\tilde{\ell}_i - \alpha_p^w) \omega_i \epsilon_i b_i}{\sum_i (\mu_i - \mu) (\tilde{\ell}_i - \alpha_p^w) \omega_i} \text{ and } \iota^w := -\frac{\sum_i (\mu_i - \mu) (\ell_i - \ell) \omega_i}{\mu \tilde{\ell}}.$$

Combining equations (A.23), (A.25) and (A.26), we get

$$\frac{d \ln \mu}{\tilde{\ell} d \ln w} = (1 - \eta^w) \bar{b}^w + \eta^w \bar{\epsilon}_\mu^w - 1 + (\iota^w - \eta^w) (\sigma_m - 1),$$

with

$$\eta^w \equiv -\frac{\sum_i (\mu_i - \mu) (\tilde{\ell}_i - \tilde{\ell}) \omega_i}{\mu \tilde{\ell}},$$

$$\begin{aligned}\bar{b}^w &\equiv \frac{\sum_i \mu_i \tilde{\ell}_i b_i \omega_i}{\sum_i \mu_i \tilde{\ell}_i \omega_i}, \\ \bar{\epsilon}_\mu^w &\equiv \frac{\sum_i (\mu_i - \mu) (\tilde{\ell}_i - \alpha_p^w) \omega_i \epsilon_i b_i}{\sum_i (\mu_i - \mu) (\tilde{\ell}_i - \alpha_p^w) \omega_i}, \\ \iota^w &\equiv -\frac{\sum_i (\mu_i - \mu) (\ell_i - \ell) \omega_i}{\mu \tilde{\ell}}.\end{aligned}$$

The cases with $d \ln r > 0$ and $d \ln q > 0$ are analogous. ■

Proof of Proposition 4 Denoting firm i 's capital other than equipment and software with O_i and its price o , the aggregate labor income share is

$$\begin{aligned}LS &= \frac{\sum_i w L_i}{\sum_i p_i Y_i} = \frac{\sum_i w L_i}{\sum_i \mu_i (w L_i + r K_i + q S_i) + \sum_i \mu_i o O_i} \\ &= \frac{\sum_i \ell_i \omega_i}{\sum_i \mu_i (\omega_i + \frac{o O_i}{w L_i + r K_i + q S_i} \omega_i)} \\ &= \frac{\ell}{\sum_i \mu_i \omega_i (1 + o_i)}.\end{aligned}$$

For $\sum_i \mu_i \omega_i (1 + o_i)$,

$$d \sum_i \mu_i \omega_i (1 + o_i) = \sum_i d(\mu_i \omega_i) (1 + o_i) + \sum_i \mu_i \omega_i d(1 + o_i).$$

Assuming $\sigma_o = 1$, we have $\sum_i \mu_i \omega_i d(1 + o_i) = 0$ and hence

$$d \sum_i \mu_i \omega_i (1 + o_i) = \sum_i d(\mu_i \omega_i) (1 + o_i).$$

That is, in principle, the distribution of the income share of the other capital across firms can affect the weights for aggregating markups. To simplify the decomposition, we assume $(1 + o_i) = (1 + \bar{o})$ for all i . Then we have

$$\frac{dLS}{LS} = d \ln \ell - d \ln \mu.$$

The impact of software-embodied technological change on the aggregate labor share is given by

$$\frac{dLS}{-d \ln q} = -LS \times [s(\bar{\sigma}_s^q - 1) - \tilde{s}(\bar{\sigma}_\mu^q - 1)].$$

Similarly, the impact of equipment-embodied technological change is

$$\frac{dLS}{-d \ln r} = -LS \times [-se(\bar{\sigma}_s^r - 1) + e(\bar{\sigma}_e^r - 1) - \tilde{\kappa}(\bar{\sigma}_\mu^r - 1)].$$

Finally, an increase in wages results in

$$\frac{dLS}{d \ln w} = -LS \times [s(1 - e)(\bar{\sigma}_s^w - 1) - (1 - e)(\bar{\sigma}_e^w - 1) + \tilde{\ell}(\bar{\sigma}_\mu^w - 1)].$$

■

Appendix B Data and Estimation

B.1 National Accounts

B.1.1 Price Index

When measuring capital-embodied technological change, we rely on the price indices of investment and consumption goods in the Korean national accounts. A notable drawback is that the software price index would not fully capture software-embodied technological change. Since 1994, the software price index comes from the producer price index, potentially inadequately adjusted for quality improvements. To address this concern, we adopt the adjustment method proposed by [Parker and Grimm \(2000\)](#), which is the approach taken by the US Bureau of Economic Analysis (BEA). Specifically, BEA makes a bias adjustment of 3.15 percent per year to the producer price index from the Bureau of Labor Statistics, factoring in the discrepancy between the hedonic method and the matched model method. We apply a bias adjustment of 3.15 percent per year to the Korean software price index from 1994 onward, with the assumption that the evolution of software technology is similar between Korean and the US. Indeed, the resulting software price indices of the two countries evolve in a similar way.³⁰

B.1.2 Rate of Return on Capital

A standard practice is to assume no arbitrage: that is, the rate of return on capital for each capital type j satisfies the following condition:

$$R_{j,t} = (1 + r_t)q_{j,t} - (1 - \delta_{j,t})q_{j,t-1} ,$$

where r_t is the net rate of return, $q_{j,t}$ is the price of capital type j relative to consumption good, and $\delta_{j,t}$ is the depreciation rate of capital type j . We compute $q_{j,t}$ from the capital price index and the price of consumption in the national accounts (with a bias-adjusted software price, as described above). The depreciation rate $\delta_{j,t}$ is computed from the net capital stock and investment using the formula:

$$\delta_{j,t} = \frac{NK_{j,t-1} \times \pi_{j,t}^k - NK_{j,t} + NI_{j,t}}{NK_{j,t-1} \times \pi_{j,t}^k} ,$$

³⁰To be precise, the Bank of Korea does not distinguish between artistic originals and software in the national accounts. However, from the detailed input-output tables, we see that artistic originals constitute less than 5 percent of other intellectual property products in Korea.

where $NK_{j,t}$ is the nominal capital stock at the end of period t for capital type j , $NI_{j,t}$ is the nominal investment at time t for capital type j , and $\pi_{j,t}^k$ is the ratio of capital price index between time t and $t - 1$ for capital type j .

For the net rate of return, r_t , we consider two approaches. First, we estimate r_t from the corporate bond rate net of expected inflation, where expected inflation is measured by a three-year moving average of the CPI inflation rate. Second, we use the estimated aggregate markup and equation (B.1) to back out r_t :

$$LS_t = \frac{wL_t}{\mu_t(wL_t + \sum_j R_{j,t}K_{j,t})} , \quad (B.1)$$

where wL_t is the compensation of employees (adjusted for proprietor's income) and μ_t is the aggregate markup estimated from firm-level data. We use this latter approach in our decomposition analysis in Section 5. However, we obtain similar results with the first approach.

B.1.3 Labor Share

Total income in the national economy includes proprietors' income, which is a mix of labor and capital income. Gollin (2002) suggested several methods to estimate the labor portion of the proprietors' income. One such method assumes that the labor share in proprietors' income is the same as that of the rest of the economy (M1):

$$\begin{aligned} \text{M1 : } LS &= \frac{CE + NLS \times PI}{Y} , \\ \text{with } NLS &= \frac{CE}{Y - CFC - PI} , \end{aligned}$$

where CE represents compensation of employees, PI is proprietors' income, CFC is consumption of fixed capital, NLS is net labor share, and Y is either gross value added (without net production tax) or gross domestic product (with net production tax). To be consistent with the theory part of the paper, we compute the gross labor share, which includes the consumption of fixed capital in the denominator.

Given the prevalence of self-employment in the Korean economy, the measurement of labor share is sensitive to the treatment of proprietors' income. In particular, Park (2020) highlighted that a sizable proportion of self-employed is working with employees, whose wages are included in the compensation of employees measured in the national accounts. He proposed that one explicitly consider this fact when constructing the aggregate labor share. Specifically, he utilizes the feature in the data that the ratio of business income between those self-employed with and without employees is approximately 2.3, and that self-employed with employees hire 2.5 workers on

average. Assuming that the labor share of self-employed without employees is the same as the labor share of the rest of the economy, we get the following (M2):

$$\begin{aligned} \text{M2 : } LS &= \frac{CE + NLS \times PI_0 + NLS \times PI_e/3.5}{Y}, \\ \text{with } PI_0 &= PI \times \frac{\text{self}_0}{\text{self}_0 + 2.3 \times \text{self}_e}, \\ PI_e &= PI - PI_0, \\ NLS &= \frac{CE}{Y - CFC - PI_0}, \end{aligned}$$

where self_e and self_0 are the number of self-employed with and without employees, respectively. In the formula, PI_e and PI_0 refer to the income of self-employed with and without employees, respectively.

Yet another way is to assume that the labor share of the entire self-employed sector is the same as the labor share of the corporate sector (M4: C) or of the rest of the economy (M4: C&G). To do so, we need to subtract the labor income of workers working for those self-employed from the compensation of employees. Korean national accounts provide this information only from 2010. Instead, one can assume that the wage of workers working for those self-employed is the same as the average wage of workers working in establishments with 5 to 9 employees (M3):

$$\begin{aligned} \text{M3 : } LS &= \frac{CE \times (1 - \frac{\text{self}_0 \times 2.5}{\text{wemp}} \times wr) + NLS \times PI}{Y}, \\ \text{with } NLS &= \frac{CE \times (1 - \frac{\text{self}_0 \times 2.5}{\text{wemp}} \times wr)}{Y - CFC - PI}, \end{aligned}$$

where wemp is wage employment and wr is the ratio of the average wage of workers working in an establishment with 5 to 9 employees relative to the average wage of workers in all establishments.

Figure B.1 shows the aggregate labor share resulting from the various approaches. The magnitude of the decrease in the labor share varies depending on methods, but the patterns are generally similar especially in the case of the gross labor share. As for the net labor share, the declining trend is mitigated when adjusted for self-employment without employees (M2). Because the rate of return on capital should include depreciation, we stick to the gross labor share in our analysis. This is particularly crucial considering our focus on software, which has a much higher depreciation rate than other types of capital. Karabarbounis and Neiman (2014) showed that, during a transition when the main shock to the economy is a decline in the price of fast-depreciating capital, the gross labor share falls while the net labor share rises. In

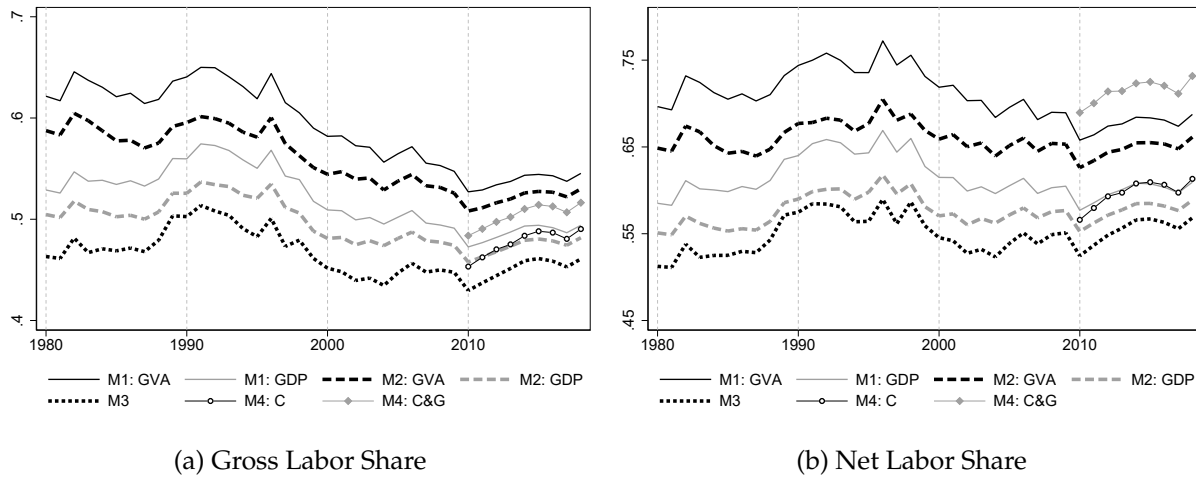


Fig. B.1: Aggregate Labor Share in Korea

M1 assumes that the labor share of self-employed income is the same as that of the rest of the economy. M2 assumes that the labor share of the income of the self-employed without employees is the same as that of the rest of the economy. M3 assumes that the average wage of employees working for those self-employed is equal to the average wage of workers in plants with 5 to 9 employees. M4 is the labor share of the corporate sector (M4-C) or the corporate and government sectors (M4-C&G). The denominator is either GDP or GVA in the case of gross labor share (left panel) and either GDP or GVA net of consumption of fixed capital in the case of net labor share (right panel).

such cases, the gross labor share serves as a better measure of the changes in welfare between workers and capital owners. In our main analysis, we use M2:GDP as the baseline measure of aggregate labor share, which is a conservative choice.

B.2 KISDATA (Panel Estimation)

KISDATA is a database of financial information for firms listed in the Korea Stock Exchange, as well as unlisted firms that are required to publish external auditing reports. The criteria for external audit requirement is as follows. Until 2008, firms whose asset exceeded 7 billion KRW (about 6.4 million USD in 2008) had to be audited externally. Since 2009, firms with (i) assets greater than 10 billion (12 billion since 2014) KRW, (ii) assets greater than 7 billion KRW and liability greater than 7 billion KRW, or (iii) assets greater than 7 billion KRW and more than 300 employees were subject to external audits. Among the firms in KISDATA, we exclude financial firms and quasi-governmental and non-profit firms from the sample. Our data runs from 2003 to 2018.

B.2.1 Labor Share of Firms

To construct firms' labor shares, we need data on labor compensation and value added. We combine employee compensation and benefits in income statements and labor costs in manufacturing cost statements to obtain firms' total labor compensation. The employee compensation and benefits in an income statement can be understood as the labor income accruing to non-production workers, and the labor cost in a manufacturing cost statement as the labor income accruing to production workers.

To compute value added, we add up operating profit, depreciation and amortization, taxes and dues, and labor compensation. The labor share at the firm level is then labor compensation divided by value added. In the regression analysis, we restrict our sample to firms with labor shares between zero and one.

B.2.2 Software and Equipment Capital

We use the variable "intangible asset - software" in firms' balance sheets to measure their software asset. According to the Korean Generally Accepted Accounting Principles (K-GAAP), a firm classifies externally purchased software with expected future benefits as software assets. A firm may have software developed in house as an intangible asset, but this component is not included in the data. It should be included in research and development in principle, but it is not separately reported. Our measure of equipment capital is the sum of machinery, transportation equipment, tools, fixtures, and furniture, reported in balance sheets.

B.2.3 Markup

We consider five different approaches to measuring firm-level markups. Our procedure follows [Baqae and Farhi \(2019\)](#) and [Edmond et al. \(2018\)](#) to a large extent.

Accounting profit (AP) For the accounting profit approach, we use the relationship

$$\text{operating profit} = \left(1 - \frac{1}{\mu^{AP}}\right) \text{sales} \quad (\text{B.2})$$

to get μ^{AP} for each firm in each year.

User cost (UC) We use

$$OS = RK + \left(1 - \frac{1}{\mu^{UC}}\right) \text{sales}, \quad (\text{B.3})$$

where OS is the operating income (with depreciation), R is the user cost of capital, and K is the stock of capital. We compute the sum of sales net of cost of goods sold and depreciation as OS , and the sum of tangible and intangible assets as K .

The user cost of capital is given by

$$R_{i,t} = (1 + r_t) - (1 - \delta_{i,t})\mathbb{E}_t p_{i,t+1}^k / p_{i,t}^k, \quad (\text{B.4})$$

where r_t is the average real rate of corporate bond, $\delta_{i,t}$ is the industry-level depreciation rate implied in the national accounts, and $\mathbb{E}_t p_{i,t+1}^k / p_{i,t}^k$ is the three-year moving average of the changes in the relative price of capital to consumption by industry. We then back out μ^{UC} from equations (B.3) and (B.4).

Production function (PF) For the production function approach, we estimate the elasticity of output with respect to variable inputs following [Baqaee and Farhi \(2019\)](#) and [De Loecker et al. \(2020\)](#). To estimate elasticity, we need an outcome variable (log sales), free variable (log cost of goods sold), state variable (log capital stock), and proxy variable (log investment). We deflate sales and cost of goods sold with gross value added deflator by industry and deflate capital expenditure with gross fixed capital formation deflator by industry. To compute capital stock, we apply the perpetual inventory method to the initial level of tangible and intangible capital and capital expenditure data. We also control for sales shares in one-digit and two-digit industries in the estimation. We exclude samples if the ratio of cost of goods sold to sales or the ratio of selling, general and administrative expense to sales is in the top or bottom 2.5 percent by year. We also exclude agriculture and the finance and insurance industry.

The elasticity is estimated as in [Olley and Pakes \(1996\)](#) with three-year rolling windows by one-digit industry. Then μ^{PF} is

$$\mu^{PF} = \frac{\partial \log F / \partial \log X}{X/Y}, \quad (\text{B.5})$$

where F is the production function, X is variable input (cost of goods sold), and Y is sales turnover.

Cost minimization (CM1 & CM2) One of the main challenges in estimating markups lies in the difficulty of estimating the output elasticity ($\partial \log F / \partial \log X$) in equation (B.5). [Bond et al. \(2021\)](#) argues that production function estimation based on revenue data (as in PF above) offers little insight into firm-level markups. [Ridder et al. \(2022\)](#) argues that revenue data can still offer information on the dispersion and trend of markups, although the level estimates are biased.

One way to circumvent this issue is to indirectly use an optimal condition derived from cost minimization. Considering labor as a variable input, for example, cost minimization under constant returns to scale implies:

$$\frac{\partial \log F_i}{\partial \log \ell_i} = \frac{w\ell_i}{w\ell_i + rK_i + qS_i + pM_i} . \quad (\text{B.6})$$

Instead of directly estimating the output elasticity, one can use equation (B.6) together with equation (B.5) to compute the markup. In practice, one can use either firm-level cost shares (CM1, [Oberfield and Raval, 2021](#)) or cost-weighted averages at the industry level (CM2, [Edmond et al., 2018](#)).

Selection of benchmark markup measure When computing the distributional moments in Proposition 2 and 3, we choose the markup estimated according to the cost minimization CM1 as our benchmark. We then compute the demand elasticity that each firm faces according to:

$$\epsilon_i = \frac{\mu_i}{\mu_i - 1} \quad (\text{B.7})$$

We trimmed observations with markup less than 1.01 (because we need $\mu_i > 1$) or greater than 10. We selected markup CM1 as our benchmark for three reasons. First, among the considered markup measures, CM1 has the fewest observations with $\hat{\mu} < 1.01$ (Table B.1). Second, CM1 has the fewest missing values among observations with positive software assets, which are crucial in our analysis. Third, CM1 is a markup measure that aligns directly with our theory. In any case, all the markup measures produce similar results in the quantification (Table 6 in the main text). The exception is CM2, which shows a much larger impact of software-embodied technological change on the aggregate markup and hence the aggregate labor share.

B.2.4 Pass-through

We compute each firm's price pass-through (b_i) using the relationship between markup and pass-through under the demand aggregator in [Klenow and Willis \(2016\)](#). It is represented by equation (38), or

$$b_i = \frac{1}{1 - \frac{\mu'x}{\mu}} = \frac{1}{1 + \mu_i \frac{\nu}{\sigma}} .$$

We follow [Edmond et al. \(2018\)](#) when setting the value of the super-elasticity or ν/σ . Under the Klenow-Willis specification, each firm's sales share and markup follow:

$$\frac{1}{\mu_{i,t}} + \ln \left(1 - \frac{1}{\mu_{i,t}} \right) = \text{constant} + \frac{\nu}{\sigma} \ln sh_{i,t} , \quad (\text{B.8})$$

where $sh_{i,t}$ is the sales share of firm i in year t . We estimate equation (B.8) controlling for firm and year fixed effects. The estimated super-elasticity ν/σ is 0.113, with a standard error (clustered at the firm-level) of 0.002.³¹

B.3 Census (Cross-section Estimation)

Our data for the cross-sectional estimation of the elasticity of substitution between labor and capital is the Korean Economic Census 2015. It surveys all establishments in the manufacturing sector with at least one employee as of December 31, 2015. We exclude branches, sole proprietorships, governmental and non-profit establishments, as they do not report intangible assets. We use annual payroll for wL_i , equipment capital (machinery and transportation equipment) for K_i , and software assets for S_i . We drop all establishments that did not report whether they have intangibles. When an establishment explicitly reports that it does not hold intangibles, we assign zero to its S_i . To compute the factor income shares, we use the rate of return on equipment and software (r and q) imputed from the national accounts. We winsorize factor shares at the first and the 99th percentiles.

B.3.1 Distribution of Factor Shares

The Census data includes the location of establishments. The unit of a region in our analysis is Si-Gun-Gu, an administrative division of Korea, comparable with commuting zones in the US in terms of the average population size. Figure B.2 shows

	Share of obs. w/		# missing μ with $sft > 0$	Decomposition results		
	$\mu < 1.01$	$\mu > 10$		Overall	Substitution	Markup
AP	0.231	0.000	8,352	-0.029	-0.020	-0.008
UC	0.089	0.000	8,701	-0.027	-0.015	-0.012
PF	0.330	0.005	16,435	-0.027	-0.015	-0.012
MC1	0.058	0.006	1,766	-0.029	-0.014	-0.015
MC2	0.196	0.027	9,425	-0.051	-0.014	-0.038

Table B.1: Alternative Markup Measures

The first and the second columns show the share of observations with estimated markups below 1.01 and above 10, respectively. The third column is the number of observations with missing or trimmed markups among those with reported software assets. The fourth to the sixth columns show the decomposition of the effects of software-embodied technological change, as in Table 6 in the main text, with alternative markup measures.

³¹Edmond et al. (2018) obtained an estimate of 0.16 using the US data.

the regional averages of establishment-level software (s_i), equipment (k_i), and labor shares (ℓ_i) in non-material costs.

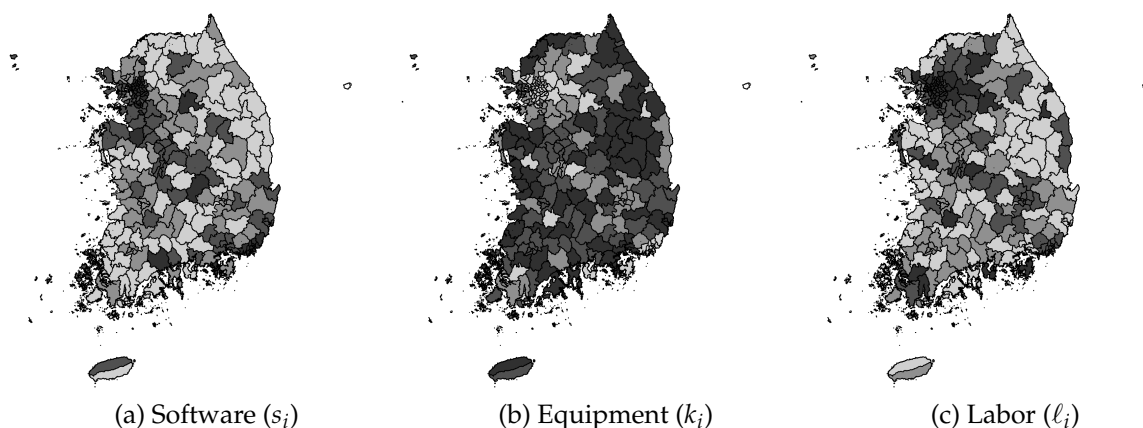


Fig. B.2: Factor Expenditure Shares by Region

The colors represent the quantiles of regions in the distribution of factor shares on non-material cost, with darker colors corresponding to higher quantiles.

Although it may not be visually clear, the covariance between software shares (s_i) and equipment shares (k_i) across regions is positive, and the covariance between software shares (s_i) and labor shares (ℓ_i) is negative. This is a reminder that factor shares are not always negatively correlated in the case of three factors. These relationships are more clear in Figure B.3.

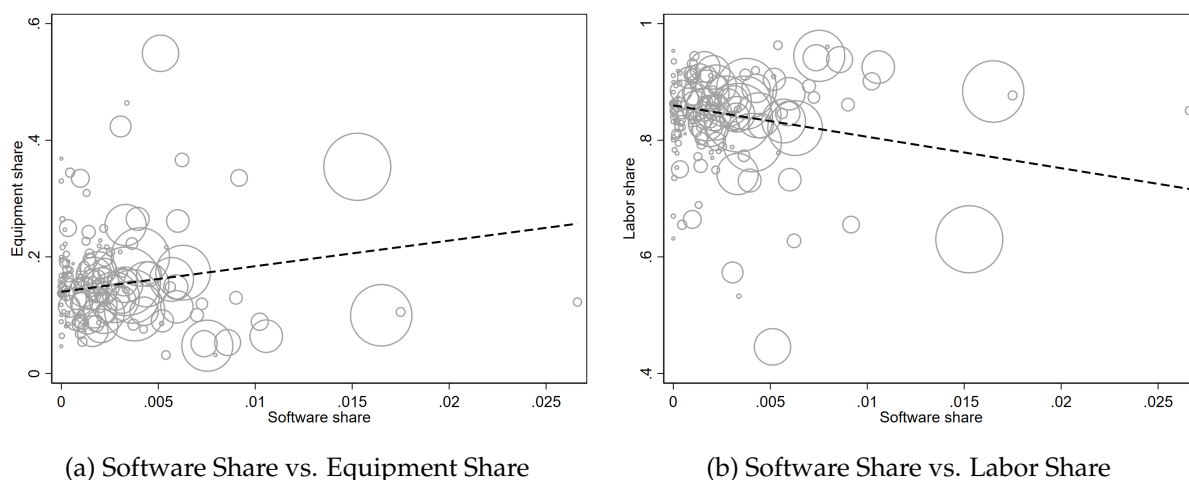


Fig. B.3: Relationship between Software Share and Labor or Equipment Share

Each circle is a region. The horizontal axis is the average software share of all firms in a region. The vertical axis is a region's equipment share in the left panel and its labor share in the right panel. The size of the circle corresponds to a region's value added.

B.4 Regional Employment Survey

We use wage differences across regions as our explanatory variable when estimating the micro elasticities of substitution from the cross-section manufacturing census data. We obtain wages from the 2015 Regional Employment Survey, which is a household-level survey of workers' salary, demographic information, educational attainment, and experience. We control for workers' observable characteristics in a Mincerian regression and then aggregate the residual wages to the region level to obtain region-level labor costs.

B.4.1 Bartik Instrument

The residualized local wages may be correlated with unobservable productivity that is not factor neutral. To address this concern, we use [Bartik \(1991\)](#) instrument. Our instrument is

$$Z_r = \sum_{i \in N_s} \omega_{r,i,0} \log(L_{i,t}/L_{i,0})$$

where N_s is the set of service industries, $\omega_{r,i,0}$ is service industry i 's share of employment in region r at time 0, and $L_{i,t}$ is the nationwide employment of service industry i at time t .

According to [Goldsmith-Pinkham et al. \(2020\)](#), the Bartik estimator $\hat{\beta}_{bartik}$ can be rewritten as the weighted sum of the just-identified estimators with one industry's regional share as instrument, $\hat{\beta}_{bartik} = \sum_i \hat{\alpha}_i \hat{\beta}_i$, where $\hat{\alpha}_i$'s are called Rotemberg weights. Table [B.2](#) is a summary of the Rotemberg weights. Panels A and C show that the two industries with the largest Rotemberg weights account for 95 percent ($=0.527+.425$) of the overall weights and 65 percent ($=0.952/1.457$) of the positive weights. They are the research and development industry and the business support services industry. The contribution of these two industries are shown by the two large circles in Figure [B.4](#).

Panel B shows that the national growth rates of service industries indexed by i ($g_i = L_{i,t}/L_{i,0}$) are positively correlated with the Rotemberg weights, but the variance of an industry's share across regions ($var(\omega_i)$) is only weakly correlated with the Rotemberg weights.

Regarding the identification with Bartik instruments, [Borusyak et al. \(2021\)](#) shows that one needs to assume either the initial shares are exogenous or the shocks are exogenous. In particular, a Bartik regression can be translated into a shock-level regression with a specific weight ($s_i = \sum_r \omega_{r,i}$), when shocks are exogenous. For an original

	Sum	Mean	Share			
Panel A. Negative and positive weights						
Negative	-0.457	-0.038	0.239			
Positive	1.457	0.066	0.761			
	$\hat{\alpha}_i$	g_i	$\hat{F}_i$	$var(\omega_i)$		
Panel B. Correlations						
$\hat{\alpha}_i$	1.000					
g_i	0.244	1.000				
$\hat{F}_i$	0.597	0.114	1.000			
$var(\omega_i)$	-0.034	-0.104	0.098	1.000		
	$\hat{\alpha}_i$	g_i	$\hat{\sigma}_e$	$\hat{\sigma}_s$	$\sum_{k=1}^i \hat{\alpha}_k$	
Panel C. Top Rotemberg weight industries						
Research & development	0.527	0.316	0.326	2.387	0.527	
Business support services	0.425	0.182	0.130	1.427	0.952	
Warehousing and transportation	0.108	0.165	0.045	0.319	1.060	
Retail trade (except motor)	0.061	-0.023	-0.647	1.904	1.122	

Table B.2: Summary of the Rotemberg Weights

Panel B reports correlations among the Rotemberg weights ($\hat{\alpha}_i$), the nationwide employment growth of service industry i ($g_i = L_{i,t}/L_{i,0}$), the first-stage F-statistic of the industry share ($\hat{F}_i$), and the variance of a given industry's employment share across regions ($var(\omega_i)$).

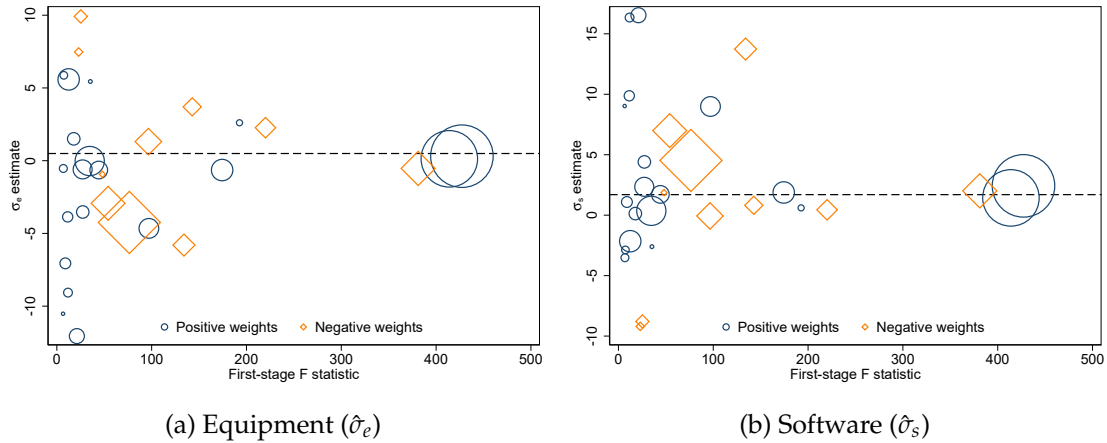


Fig. B.4: Heterogeneity of the Elasticities

The figure plots the relationship between each instruments' $\hat{\sigma}_j$, first-stage F-statistics, and the Rotemberg weights. Each point is a separate instrument's estimate. The estimated ($\hat{\sigma}_j$) for each instrument is on the y-axis and the estimated first-stage F-statistic is on the x-axis. The size of the points is scaled by the absolute value of the Rotemberg weights, with circles for positive weights and diamonds for negative weights. The dashed line is at the estimated elasticity using the Bartik instrument. Instruments with first-stage F-statistics below 5 are excluded.

IV regression $y_r = \alpha + \beta x_r + \varepsilon_r$ with Bartik instrument z_r , an equivalent shock-level regression is a s_i -weighted IV regression, using the shocks g_i as instrument:

$$\bar{y}_i = \alpha + \beta \bar{x}_i + \bar{\varepsilon}_i.$$

While the point estimates $\hat{\beta}$ are equivalent in both approaches, [Borusyak et al. \(2021\)](#) shows that the shock-level regression provides a correct inference when shocks are exogenous but initial shares are not. That is, the Bartik instrument is valid when shocks are randomly assigned (and have a common mean). Regarding the consistency of the estimator, the effective sample size of the shock-level regression is large when the expected Herfindahl index of the average shock exposures ($\sum_i s_i^2$) converges to zero as the number of regions increases.

For the shock-level regression, one needs to think about the regional weights $\omega_{r,i}$'s. When $\sum_i \omega_{r,i}$ varies across regions, it can generate non-random variations in addition to quasi-random variations of shocks. [Borusyak et al. \(2021\)](#) shows that one can address this concern by controlling for $\sum_i \omega_{r,i}$ in the regression. Because we construct our Bartik instrument using services industries, we can consider two kinds of weights: (i) construct $\omega_{r,i}$ with services employment only, so that $\sum_i \omega_{r,i} = 1$ for all r ; or (ii) construct $\omega_{r,i}$ with total employment in the region, so that $\sum_i \omega_{r,i} \leq 1$ varies across r . We consider the former as benchmark and the latter as robustness check.

	With Service Employment (1)	With Total Employment (2)
Mean g_i	0.053	0.030
Standard Deviation g_i	0.116	0.091
Interquartile Range g_i	0.100	0.005
$1/HHI$	12.70	4.78
Maximum s_i	0.155	0.429
No. of industries	34	35

Table B.3: Summary Statistics of Shocks (g_i) and Industry Weights (s_i)

Table [B.3](#) shows the summary statistics of shocks (nationwide employment growth of service industry i) $g_i = L_{i,t}/L_{i,0}$ and the inverse of the Herfindahl index of industry weights $1/\sum_i s_i^2$. In the regional employment survey, the industry classification is at the two-digit level, so we have a total of 34 service industries. The first column of Table [B.3](#) is the case with service industries only, $\sum_{i \in N_s} \omega_{r,i} = 1$ or complete share. When we construct $\omega_{r,i}$ with total regional employment, a residual industry with share $\omega_{r,0} = 1 - \sum_{i \in N_s} \omega_{r,i}$ can be thought of as one with zero growth. This case is

the second column of Table B.3. Because the non-service industry has a significant share of regional employment, the second column has a smaller interquartile range and a larger HHI, implying a smaller effective sample size. This is why we choose the service employment only case as our benchmark.

	Complete share	Incomplete share
Equipment (σ_e)	0.600 (1.051)	-0.164 (0.815)
Software (σ_s)	1.620 (0.879)	2.736 (0.928)
First stage F	7.025	7.201
Obs.	34	34

Table B.4: Shock-level Regression Result

Table B.4 reports the results from the shock-level regression with service employment only (complete share) and with total employment controlling for the initial share of non-service employment (incomplete share). The point estimates are $\sigma_e < 1$ and $\sigma_s > 1$ in both cases, but the standard errors are much larger than those clustered at the level of region and three-digit industry. (These results are reported in Table 2 as SSIV1 and SSIV2.) The standard errors are large because we have a small effective sample size, and do not utilize establishment level information here. As shown in Table B.3, we only have 34 two-digit service industries in the regional employment survey where the inverse of the HHI is only 12.7 even when we exclude non-service industries.

Appendix C Robustness of Decomposition

In this section, we redo decomposition analysis of Section 5 using alternative approaches.

C.1 Macro Elasticities from the Census Data

We check whether our decomposition results remain the same when the distributional moments and hence the macro elasticities are computed from the manufacturing census data. Note that the census data we have is a single cross-section for the year 2015, which means these macro elasticities are held constant over time, unlike in our benchmark using the KISDATA panel.

- Total changes in labor share : -0.044

- Effects of Software-embodied Technological Change ($\Delta \ln 1/q$)

	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.025	-0.016	-0.015	-0.001	-0.010	-0.007	-0.003
(% explained)	(57.5)	(35.4)	(33.0)	(2.4)	(22.0)	(15.7)	(6.4)

- Effects of Equipment-embodied Technological Change ($\Delta \ln 1/r$)

	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.000	+0.008	+0.014	-0.006	-0.009	-0.009	+0.000
(% explained)	(0.3)	(-19.2)	(-32.8)	(13.6)	(19.5)	(19.7)	(-0.2)

Table C.1: Decomposition with Macro Elasticities from Census

The decomposition is for the period between 1990 and 2018. Percent explained of the aggregate labor share decline in the data in parentheses.

The results are in Table C.1. With time-invariant macro elasticities from the census data, the aggregate markup rises less, and factor substitution plays a slightly larger role through between-firm reallocation. The overall impact of software-embodied technological change on the aggregate labor share is somewhat smaller than in our benchmark. Similarly, equipment-embodied technological change reduces the labor share even less than in our benchmark. However, the overall pattern remains the same, as can be seen in Figure C.1.

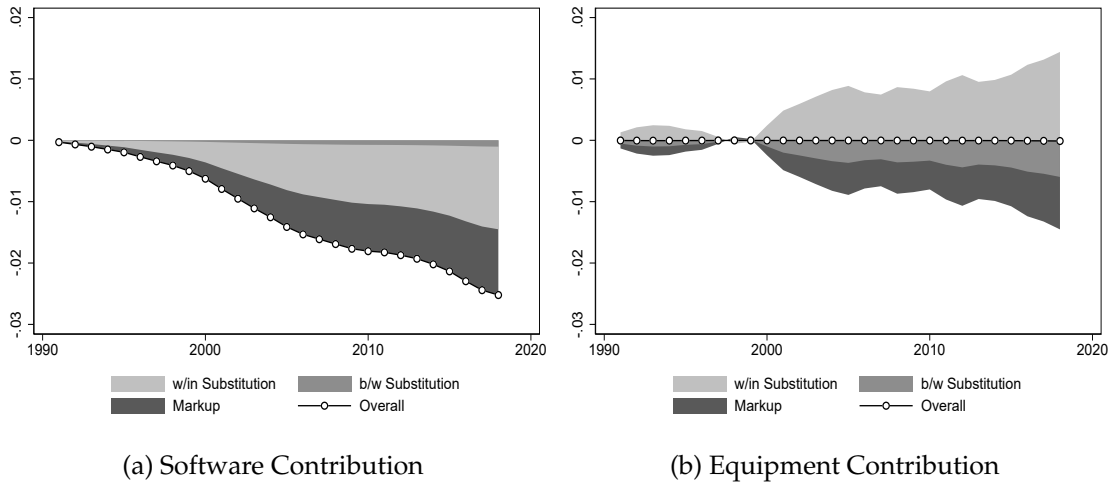


Fig. C.1: Cumulative Effect on Labor Share with Macro Elasticities from Census

C.2 Alternative Measures of Pass-through

Baqae et al. (2023) argued that the Klenow-Willis specification generates too little variation in price pass-through, compared with the estimates of the price pass-through using unique firm-level data from Belgium. To be specific, when estimating pass-through b_i from reasonably estimated markup μ_i using the Klenow-Willis specification, the resulting b_i has insufficient variation compared with the empirical b_i provided by Amiti et al. (2019).

Instead of relying on the Klenow-Willis specification, Baqae et al. (2023) use empirical pass-through and sales share in the data, along with the theoretical relationship among markup, sales share, and pass-through to back out markup (equation (C.1)). The theoretical relationship is:

$$d \log \mu_i = (\mu_i - 1) \frac{1 - b_i}{b_i} d \log \lambda_i, \text{ s.t. } E_\lambda[\mu_i^{-1}]^{-1} = \bar{\mu}, \quad (\text{C.1})$$

where μ_i is firm i 's markup, b_i is its pass-through, and λ_i is its sales share.

To assess the impact of such calculations on our decomposition, we impute price pass-through (b_i) corresponding to each firm in the Korean data using equation (C.1) and the estimated pass-through by firm size (employment) among Belgian firms in Amiti et al. (2019).³² We then redo the decomposition exercise, with the macro elasticities based on these alternative measures of firm-level pass-through.

Table C.2 reports the decomposition results with these alternative measures of pass-through. As expected, a higher dispersion in pass-through across firms amplifies the role of the markup channel. The aggregate markup rises by 0.6 percentage points more than in our benchmark (2.1 vs. 1.5 p.p.), while the factor substitution channel remains nearly the same. The stronger rise in markup is more pronounced in the case of equipment-embodied technological change. As the markup channel plays a bigger role, capital-embodied technological change (software and equipment) can account for all the decline in the aggregate labor share in the data. Figure C.2 shows the cumulative effect of capital-embodied technological change (software, left panel; equipment, right panel) through the substitution and the markup channels.

³²There are three steps. Step 1: For each employment size bin in Amiti et al. (2019), we map their estimated pass-through (b_i) to the sales share of Korean firms within the same employment size bin. This gives us the relationship between sales share (λ_i) and pass-through (b_i). Step 2: Using the relationship between λ_i and b_i derived in Step 1, we solve equation (C.1) to obtain μ_i . This gives us a unique relationship between μ_i and b_i , and hence b_i as a function of μ_i , denoted by $b(\mu)$. Step 3: Using $b(\mu)$, we impute the firm-level pass-through (b_i) corresponding to the firm-level markup (μ_i) obtained in Appendix B.2.3. We then use these imputed b_i 's to calculate the macro elasticities.

- Total changes in labor share : -0.044

- Effects of Software-embodied Technological Change ($\Delta \ln 1/q$)

	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.034	-0.013	-0.015	+0.001	-0.021	-0.018	-0.003
(% explained)	(78.0)	(30.5)	(33.0)	(-2.5)	(47.5)	(40.1)	(7.4)

- Effects of Equipment-embodied Technological Change ($\Delta \ln 1/r$)

	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.014	+0.011	+0.014	-0.004	-0.025	-0.024	-0.001
(% explained)	(31.9)	(-24.2)	(-32.8)	(8.6)	(56.0)	(54.8)	(1.2)

Table C.2: Decomposition with Alternative Measure of Pass-through

The decomposition is for the period between 1990 and 2018. Percent explained of the aggregate labor share decline in the data in parentheses.

Appendix D Elasticity of Substitution and Skill

Our estimates show that equipment complements labor and software substitutes for labor. This naturally raises the question of whether the substitutability differs across workers of different skill levels. Unfortunately, our microdata do not allow us to differentiate labor by skill. As imperfect as it may be, we estimate the elasticity of substitution by skill group and capital type using aggregate time series data from the US, in the spirit of [Krusell et al. \(2000\)](#).

We estimate the following equation:

$$\ln w_t^s / R_t^j = \alpha + (\sigma_{s,j} - 1) \ln(R_t^k k_t^j / w L_t^s) + \beta t + \epsilon_t, \quad (\text{D.1})$$

where w_t^s is the real hourly wage of skill group s , R_t^j is the rental rate of capital type j , and the time trend t captures the unobserved factor-augmenting technological change with a log-linear time trend. We compute the real wage by skill group following [Krusell et al. \(2000\)](#) and [Autor \(2014\)](#). We calculate the rental rate of capital by capital type as described in Appendix B. When computing the rental rate of capital, we adjust the price of equipment as in [Cummins and Violante \(2002\)](#).

Rather than specifying a particular functional form for the production function, we estimate equation (D.1) for a given skill-capital type pair, out of two skill levels (high h and low l) and two capital types (equipment e and software s).

Table D.1 reports the estimated elasticities of substitution. We find that software

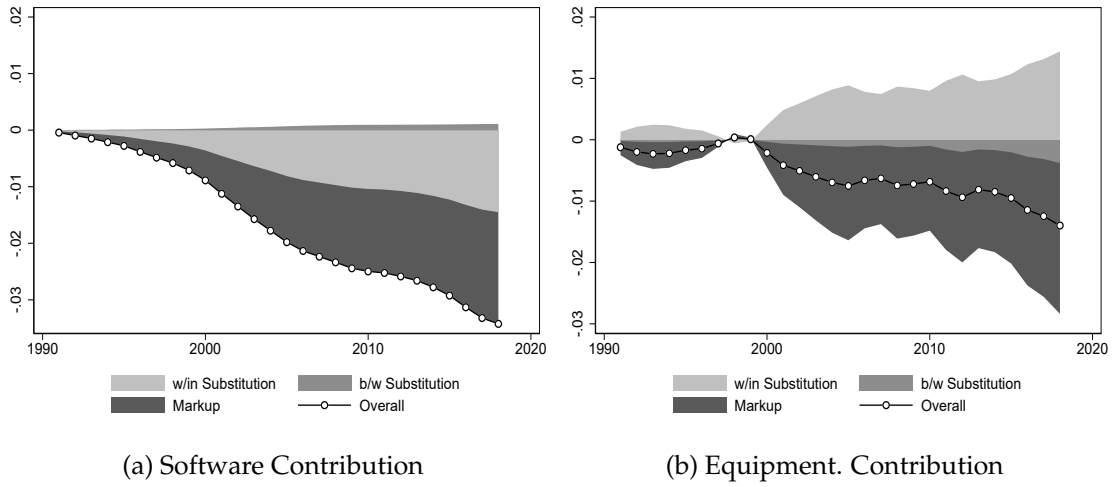


Fig. C.2: Cumulative Effect on Labor Share with Alternative Measure of Pass-through

	Equipment		Software	
	High skill	Low skill	High skill	Low skill
σ	0.579 (0.138)	0.733 (0.210)	1.592 (0.098)	1.958 (0.146)

Table D.1: Elasticity of Substitution by Skill and Capital Type

Newey-West standard error is in parenthesis.

substitutes for labor and equipment complements labor, be it high skill or low skill. This is consistent with our findings from the Korean microdata. The elasticity of substitution varies more between capital types (software vs. equipment) than between skill groups (high skill vs. low skill). That is, the type of capital matters more in determining substitutability with labor. As for the differences between skill groups, we find that the elasticity of substitution between capital (both equipment and software) and labor is smaller for high-skill labor than for low-skill labor. To the extent that one focuses on skill premia, our finding is consistent with the skill-capital complementarity in [Krusell et al. \(2000\)](#), because for the skill premium to rise, it suffices that high-skill labor is less substitutable than low-skill labor with respect to capital. It is not necessary that one elasticity be less than one and the other greater than one.

References

- Amiti, M., O. Itskhoki, and J. Konings (2019, 02). International Shocks, Variable Markups, and Domestic Prices. *The Review of Economic Studies* 86(6), 2356–2402.
- Autor, D. H. (2014). Skills, education, and the rise of earnings inequality among the “other 99 percent”. *Science* 344(6186), 843–851.
- Baqaei, D. R. and E. Farhi (2019, 09). Productivity and Misallocation in General Equilibrium. *The Quarterly Journal of Economics* 135(1), 105–163.
- Baqaei, D. R., E. Farhi, and K. Sangani (2023, 06). The Darwinian Returns to Scale. *The Review of Economic Studies*, rdad061.
- Bartik, T. J. (1991). *Who Benefits from State and Local Economic Development Policies?* W.E. Upjohn Institute.
- Bond, S., A. Hashemi, G. Kaplan, and P. Zoch (2021). Some unpleasant markup arithmetic: Production function elasticities and their estimation from production data. *Journal of Monetary Economics* 121(C), 1–14.
- Borusyak, K., P. Hull, and X. Jaravel (2021, 06). Quasi-Experimental Shift-Share Research Designs. *The Review of Economic Studies* 89(1), 181–213.
- Cummins, J. G. and G. L. Violante (2002). Investment-specific technical change in the united states (1947–2000): Measurement and macroeconomic consequences. *Review of Economic Dynamics* 5(2), 243–284.
- De Loecker, J., J. Eeckhout, and G. Unger (2020, 01). The Rise of Market Power and the Macroeconomic Implications. *The Quarterly Journal of Economics* 135(2), 561–644.
- Edmond, C., V. Midrigan, and D. Y. Xu (2018, July). How Costly Are Markups? NBER Working Papers 24800, National Bureau of Economic Research, Inc.
- Goldsmith-Pinkham, P., I. Sorkin, and H. Swift (2020, August). Bartik instruments: What, when, why, and how. *American Economic Review* 110(8), 2586–2624.
- Gollin, D. (2002). Getting income shares right. *Journal of Political Economy* 110(2), 458–474.
- Karabarbounis, L. and B. Neiman (2014, October). Capital Depreciation and Labor Shares Around the World: Measurement and Implications. (20606).

- Krusell, P., L. E. Ohanian, J.-V. Rìos-Rull, and G. L. Violante (2000, September). Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis. *Econometrica* 68(5), 1029–1054.
- Oberfield, E. and D. Raval (2021). Micro Data and Macro Technology. *Econometrica* 89(2), 703–732.
- Olley, G. S. and A. Pakes (1996). The dynamics of productivity in the telecommunications equipment industry. *Econometrica* 64(6), 1263–1297.
- Park, J. (2020). Self-Employment Sector and Functional Income Distribution of Korean Economy. *Korea Economic Forum* 12(4), 27–68.
- Ridder, M. D., B. Grassi, and G. Morzenti (2022, July). The Hitchhiker’s Guide to Markup Estimation. Discussion Papers 2210, Centre for Macroeconomics (CFM).