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# A MODERN LOOK AT ASSET PRICING AND SHORT-TERM INTEREST RATES

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## ABSTRACT

This paper uses modern asset pricing theory to examine the behavior of short-term nominal interest rates over the past 25 years. The analysis investigates whether variation in the stochastic behavior of output and inflation can explain movements in the rate of interest. Our results reveal that much of the month to month movement in nominal interest rates reflects changes in the real rate and the risk premia rather than inflationary expectations.

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## INTRODUCTION

The so-called Fisher equation is probably the oldest and best known equilibrium asset pricing model. Much to the dismay of generations of economists, the majority of empirical tests of the model fail to provide support for it. In this paper, modern asset pricing theory is used to develop a generalization of the Fisher relationship. We find that an econometric specification which allows for changing attitudes towards risk through coefficients that evolve over time provides substantial insights into the movement of interest rates in the last 25 years. Our results provide an informative framework for evaluating the influence of movements in expected inflation, the real rate of interest and the risk premium.

The Fisher model for interest rates, like the expectations theory of the term structure, is an element of economic folklore which is repeatedly used despite the mounting empirical evidence that refutes it.[1] There are other elements which determine nominal interest rates which need to be explored in order to understand interest rate movements. The consumption based Capital Asset Pricing Model developed by Lucas (1978) and Breeden (1979) is used in the next section to derive a generalized ex ante Fisher model that includes specifications for the real rate and for the risk premium.

The model uses intertemporal expected utility maximization to introduce two equilibrium conditions which together determine the nominal interest rate. The approach, which is similar to that found in Benninga and Protopapadakis (1983) and in Shome, Smith and Pinkerton (1988), relates the risk premium to the conditional covariance of inflation and consumption growth and also relates the

[1] For evidence that the there is little support in the data for a simple Fisher model of short-term interest rates see Summers (1983) and Barsky (1987). At the very best the relationship is highly unstable and the hypothesized coefficient of unity for the full Fisher effect of inflation on interest rates is resoundingly rejected.

real rate to the conditional expectation and variance of real consumption. The theory shows us that the proximate determinants of nominal interest rates are, in addition to the expected inflation rate, the conditional covariance of inflation and consumption growth, and the conditional expectation and variance of real consumption growth.

There is an empirical as well as a theoretical motivation for our interest in these proximate determinants of interest rates. Discussions of the behavior of interest rates since the start of the Federal Reserve's flirtation with monetary aggregates targeting in October 1979 often emphasize the purported increase in the risk premium in interest rates. This emphasis is due in part to the puzzle created by the failure of nominal interest rates to fall during the disinflation of the 1980's even when monetary policy appeared to be relatively loose. Typical of such discussions are Bodie, Kane and McDonald (1984) and Mankiw (1986). The thrust of this empirical literature is that the nominal interest rates cannot simply be partitioned into components for the constant expected real rate and the expected inflation rate, but that there are important additional considerations, the varying real rate and risk premium.

The proximate determinants of nominal interest rates - the conditional moments of consumption growth and inflation - are not directly observable. In the second section, we describe the data and use modern time series techniques to develop appropriate measures for each of the independent variables in the model. The specification chosen is an autoregressive model with time varying parameters and an ARCH specification for the residuals.

In the third section of the paper we investigate the ability of both the simple Fisher equation and the generalized model to explain monthly movements in the one-month Treasury Bill rate for the period 1964 to 1987. The data reject the simple Fisher equation suggesting that the assumptions of a constant real rate and a constant risk premium are untenable. In addition, the generalized model with constant coefficients provides only limited support for the hypotheses that variations in the risk premium and the real rate effect the nominal interest rate. However, we are able to provide an adequate explanation of interest rate movements with an expanded version of the generalized model which allows for heterogeneity among consumers which induces variation in the aggregate or average attitude towards risk.

The interpretation of interest rate movements based on the joint maximum likelihood estimation of the times series model and the generalized Fisher equation with time varying parameters is found in the last section of the paper. Among our important empirical findings are:

* the real rate of interest has varied considerably over the last 25 years.
* the risk premium is usually small, although the annual average exceeds 50 basis points in three years.
* there is little correlation between the monthly changes in the nominal interest rate and expected inflation (the so-called Fisher effect), while annual changes in interest rates are more strongly correlated with changes in expected inflation.
* the increase in interest rates in the mid-1970's was largely due to higher expected inflation, while the further increases between 1978 and 1981 were due to increases in the real rate of interest.
* the decline in interest rates the 1980's was due to a large decline in the expected inflation rate until 1986-87 when expected inflation rose and the real rate of interest declined.

## ASSET PRICING THEORY

In this section we present the asset pricing equilibrium conditions and derive the generalized Fisher equation. The first element of the model is a general equilibrium condition that relates the nominal rate of interest to the real rate. Once that is developed, we will turn to the second condition which provides a formal model for the real rate of interest.

Interest Rate Equilibrium Let $R_t$ and $I_t$ be the known real and nominal rates of interest in period t. Thus one good in period t can be exchanged for $(1 + R_t)$ goods in period t+1 in the real bond market, while the purchase of a nominal bond for \$1 at t entitles the holder to $\$(1+I_t)$ in period t+1. When bond portfolios are optimally selected, expected returns on nominal and real bonds must be equal when measured in terms of an individual's expected utility. This implies that in a simple representative agent economy, the equilibrium condition for asset pricing equates expected future marginal utility evaluated at the real rate with that evaluated at the nominal rate corrected for change in the price level. Formally, this condition can be written as:

$$E_t[\ U'(C_{t+1})\ (1 + I_t)\ P_t / P_{t+1}] \quad = \quad E_t[\ U'(C_{t+1})\ (1 + R_t)\ ] \tag{1}$$

where $U'(C_t)$ is the representative agent's marginal utility of consumption $C_t$. $E_t$ denotes expectations conditional on information available in period t.

In order to provide a tractable form, we assume that utility is isoelastic. Thus, marginal utility can be written as $U'(C_t) = C_t^{-\gamma}$, where $\gamma$ may be interpreted as the coefficient of relative risk aversion. With the additional assumption that prices and consumption are jointly log normally distributed, the equilibrium asset pricing condition can be re-written by substituting for U'(.) in (1) and simplifying to obtain:

$$\begin{aligned} \log(1 + R_t) &= \log(1 + I_t) - (E_t[\log P_{t+1} - \log P_t]) \\ &\quad + (1/2)\mathrm{Var}_t(\log P_{t+1} - \log P_t) - \gamma \mathrm{Cov}_t(\log C_{t+1}, \log P_{t+1} - \log P_t) \end{aligned} \tag{2}$$

where $\mathrm{Var}_t(.)$ and $\mathrm{Cov}_t(.)$ denote variances and covariances conditional on information available at t. This expression can be simplified if we use small letters for the natural logs of variables:

$$i_t = r_t + E_t\Delta p_{t+1} - (1/2)Var_t(\Delta p_{t+1}) - \gamma Cov_t(\Delta c_{t+1},\Delta p_{t+1}) \tag{3}$$

where $\Delta p_{t+1} = \log P_{t+1} - \log P_t \qquad \Delta c_{t+1} = \log C_{t+1} - \log C_t.$

The first term on the right hand side of equation (3) is the real interest rate. The next two terms are the expected rate of inflation, which is familiar from the simple Fisher model, and the variance of the price level. Both terms appear because our theory indicates that the Fisher equation should be specified as the relationship between nominal interest rates and the expected rate of depreciation of money. Individuals are concerned with the expected change in the purchasing power of money [i.e. $E_t(P_t/P_{t+1})$] rather than the expected rate of inflation when deciding between nominal and real bonds.[2] In our model these terms are related by

$$\exp[-E_t\Delta p_{t+1} + \tfrac{1}{2} Var_t(\Delta p_{t+1})] = E_t(P_t/P_{t+1}). \tag{4}$$

Thus, either an increase in expected inflation <u>or</u> a fall in the variance of future prices implies a decline in expected future purchasing power [i.e. a fall in $E_t(P_t/P_{t+1})$]. Under these circumstances nominal bonds become less attractive so that the nominal interest rate must rise to clear markets. Hence, a fall in the variance of future prices induces a rise in $i_t$ as shown in equation (3).

The last term in equation (3) identifies the effect of risk aversion on nominal interest rates. Ceteris paribus, risk averse investors will find nominal bonds more attractive in situations where [ex post] unexpectedly high real returns coincide with high marginal utility. Since the real return on a nominal bond is inversely related to next period's price level and marginal utility is decreasing in consumption, this means that a rise in $Cov_t(\Delta c_{t+1},\Delta p_{t+1})$ should

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[2] See Fama (1976) for a discussion. The importance of price level uncertainty on the Fisher equation was shown by Amihud and Barnea (1977); Shome, Smith and Pinkerton (1988) use survey data to emphasize the empirical importance of this often overlooked variable.

increase the demand for nominal bonds and lower the equilibrium nominal interest rate. $-\gamma cov_t(\Delta c_{t+1}, \Delta p_{t+1})$ can thus be interpreted as the risk premium on nominal bonds.[3]

Real Rate Model In equilibrium, the expected marginal rate of substitution between consumption at t and t+1 must be equal to the return on real bonds, $(1 + R_t)$. If we continue to employ the isoelastic utility function, this condition can be written as

$$C_t^{-\gamma} = \frac{1}{1+\delta} E_t \left[C_{t+1}^{-\gamma} \, [1+R_t]\right] \tag{5}$$

where $\delta$ is the discount rate.

Under uncertainty the expected marginal rate of substitution depends upon expected consumption growth and the variability of future consumption. This can be easily seen when consumption is log normally distributed as (5) becomes

$$-\gamma \log C_t = \log(1+R_t) - \log(1+\delta) - \gamma E_t \log C_{t+1} + (1/2)\gamma^2 Var_t(\log C_{t+1}). \tag{6}$$

Our model for the real rate is shown by rearranging (6) and simplifying as before:

$$r_t = \gamma E_t \Delta c_{t+1} - (1/2)\gamma^2 Var_t(\Delta c_{t+1}) + \delta \tag{7}$$

where $\log(1+\delta)$ is approximated by $\delta$. Clearly an increase in the variance of future consumption raises the right hand side necessitating a fall in the equilibrium real rate.

Our generalized model for the nominal interest rate is obtained by substituting for the real rate from (7) into (3):

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[3] This is simply an example of Breeden's (1979) result showing that the covariance of any asset return with consumption is sufficient to describe the risk premium.

$$i_t = E_t\Delta p_{t+1} - (1/2)Var_t(\Delta p_{t+1}) - \gamma Cov_t(\Delta c_{t+1}, \Delta p_{t+1}) + \gamma E_t\Delta c_{t+1} - (1/2)\gamma^2 Var_t(\Delta c_{t+1}) + \delta. \qquad (8)$$

The coefficient $\gamma$ appears in three places in equation (8), a restricted form of our model. Since the coefficient is subject to different interpretations, we will also estimate an unrestricted form which allows the coefficients to be estimated freely. The unrestricted form of the model is:

$$i_t = E_t\Delta p_{t+1} - (1/2)Var_t(\Delta p_{t+1}) - \gamma_1 Cov_t(\Delta c_{t+1}, \Delta p_{t+1}) + \gamma_2 E_t\Delta c_{t+1} - (1/2)\gamma_3 Var_t(\Delta c_{t+1}) + \delta. \qquad (9)$$

The motivation for considering the unrestricted form is that the coefficient $\gamma$ is not only the coefficient of relative risk aversion but that it can also be interpreted as the inverse of the elasticity of substitution. This can be seen by examining the equilibrium condition for the real rate when there is no uncertainty about future consumption. It implies that

$$d[\log C_{t+1} - \log C_t]/d\log(1 + R_t) = (1/\gamma) \qquad (10)$$

so that $(1/\gamma)$ can be interpreted as the elasticity of substitution. Notice however, that our choice of utility function implies that a low elasticity of substitution must be associated with a high degree of risk aversion and vice versa. This correspondence between the elasticity of substitution and coefficient of relative risk aversion is simply an artifice of the isoelastic functional form and not of any economic consequence. Therefore, estimates of both the restricted (8) and unrestricted forms (9) of the generalized model will be examined.

Estimates of the generalized model will enable us to decompose movements in the nominal interest rates into components which represent contributions of the expected inflation rate, the risk premium and nominal interest rates. In order to do so we must first construct measures for the unobservable explanatory variables, the expectations and underlying variances and covariance.

**DATA AND TIME SERIES MODEL**

In this section we first describe the data and then develop the time series model that is used to derive estimates of the conditional moments that are needed to estimate our generalized model.

Data Definitions Our interest in this paper is the short-run nominal interest rate and the series used is the one month Treasury Bill returns from the CRSP date tape (see Ibbotson and Sinquefeld, 1982) which is available monthly from 1964 to 1987.

The price series used is the Consumer Price Index for all urban consumers (CPI–U). Since 1983 the index utilizes a rental equivalence calculation for the owner occupied housing component which eliminates the undue weighting of mortgage interest costs which was a source of severe criticism of the CPI in the 1970's. However, the Bureau of Labor Statistics (BLS) does not revise the historical data when revisions to the CPI are introduced. We use a consistent series for the whole period which uses the rental equivalence calculation.[4]

The specification of the risk premium and of the real rate of interest require that we have a measure of real consumption growth. Although the Commerce Department now prepares monthly data on real consumption and real personal disposable income, both seasonally adjusted series exhibit irregularities which seem to be related to the timing of tax payments and refunds which are not picked up by the seasonal adjustment procedures. Thus, the monthly growth rates exhibit a large number of abnormal outliers.[5] We use instead an output series, real per

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[4] The CPI based on the rental equivalence measure for the period 1967-83 has been prepared by the BLS and is called CPIX. The CBO has worked that series back and their data for 1964–67 were obtained from Frederic Mishkin.

[5] The annualized rate of growth in the seasonally adjusted monthly data for real consumption was over 20% in 19 instances between 1964 and 1987.

capita personal income seasonally adjusted. Both the inflation rate and the output growth rates are defined as the change in the natural logs of the levels.

Two corrections to the raw data were used for the estimation of the time series model. The inflation data were corrected for the influence of the Nixon price controls and the output growth (income) series was corrected for the influence of the 1975 income tax rebate. These corrections are made so that these events would not influence the estimates of the times series structure of inflation and output growth which are used to generate the conditional moments of inflation and output growth. In each case the series was regressed on a dummy variable for the events in question (without a constant term) and the residuals from these regressions are inflation and output growth purged of the controls and rebate effects respectively. For income growth the dummy was simply equal to one in the rebate month (May 1975) and zero elsewhere; no other tax changes resulted in large outliers. For inflation the dummy scheme varies from zero to one based on the proportion of the CPI which was covered by price controls.[6] For the estimation of the interest rate equations the effects of the price controls and tax rebate were added back in to derive the appropriatre series for the expected inflation and real output growth rates.[7]

Time Series Model for Inflation and Output The model used to estimate expected output growth, expected inflation and the variances of output and inflation and their covariance is an autoregressive structure for inflation and output growth with time varying parameters and an ARCH specification of the covariance matrix. The ARCH model as a means of estimating time varying second moments was introduced by Engle (1982). However, the ARCH model assumes that the time series structure for the first moments is constant, an assumption which

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[6] These proportions were estimated by Blinder (1979, p.125) and were non-zero for 33 months from 1971 to 1974.

[7] This was done, for example, by adding the controls effect on $\Delta p_t$ (i.e. the predicted values from the dummy variable regression for $\Delta p_t$) to the predicted value of $\Delta p_t$ from the time series model.

we view as untenable.[8] We allow for variation in the time series processes by introducing time varying parameter estimates of the autoregressive model.

Before we present our estimates of the time series model we will provide a brief formal presentation of the specification. Consider the following stylized first order autoregressive model for some pxl vector of variables $X_t$:

$$X_t = A(t) X_{t-1} + \epsilon_t \qquad E_{t-1} \epsilon_t \epsilon_t' = \Sigma(t) \tag{11}$$

where $\epsilon_t$ is a pxl vector of serially uncorrelated disturbances with conditional covariance matrix $\Sigma(t)$.

There are two types of structural variation in equation (11), both in the spirit of the Lucas critque, which affect the behavior of $X_t$. The first source of structural variation is variation in the coefficient matrix A(t). These aggregate behavioral parameters reflect the optimizing decisions of individuals and optimal behavior changes as shocks occur. Thus, the aggregate parameters are subject to variation.

The second source of structural variation in equation (11) is variation in the conditional covariance matrix $\Sigma(t)$. The matrix can be viewed as a reduced form which reflects both the true structural shocks and their impact on $X_t$. Thus, variation in $\Sigma(t)$ can result from two distinct phenomena. First, changes in behavior affect the short run susceptibility of $X_t$ to structural shocks, such as money, productivity and price shocks. Second, it is unlikely that perceptions of the frequency with which these structural disturbances occur remain constant. For example, the variance of monetary shocks is likely to be high during periods of greater uncertainty about the future course of monetary policy. Similarly, the perceived variance of price shocks probably rises around discrete events such

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[8] The Lucas critique suggests that the time series structure or the degree of persistence in the processes generating inflation and output growth is likely to vary.

as an OPEC meeting.[9]

In order to estimate the model given by (11) for inflation and output growth we assume that the coefficients, A(t), follow a random walk and a first order ARCH model is used for the conditional covariance matrix $\Sigma(t)$. We further assume that the innovations $\epsilon_t$ are jointly normally distributed so that the model can be estimated by maximum likelihood in conjunction with a modified version of the Kalman Filter. The estimation process is described in the Technical Appendix and the estimated model is shown in Table 1. Checks for serial correlation in both the output and inflation residuals, also shown in Table 1, reveal that this specification captures all the statistically significant persistence in both processes.

The smoothed model estimates of expected inflation and output growth (the predicted values of $\Delta p$ and $\Delta y$) are shown in Figures 1a and 1b respectively.[10] There is much more short-run volatility in the expected inflation rate than in expected output growth. The expected inflation rate climbs through the mid-1960's to an average of 5.2% in 1971. It reaches a peak in excess of 13% after each of the two oil shocks in the 1970's. After 1981 the expected inflation rate fell very rapidly and it averaged under 2% in 1986 before increasing to a 4.1% average in 1987. Expected output growth climbed at the start of the sample period to a peak in 1966 when it averaged 3.9%. It fell after that and the average for 1972 was 2.2%. It has since hovered around 2% except for the major recessions in 1973-75 and 1979-82.

The estimates of $\alpha_0$ and $\beta_0$ are the trend components of the inflation rate and real output growth respectively. Movements in the trend inflation and output

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9 We shall not attempt to isolate or specify these different sources of variation in $\Sigma(t)$ when estimating the model although this can be done using the method developed in Evans (1989).

10 Smoothed estimates are obtained from the Kalman smoother. Unlike the Kalman filter which uses information available at time t to estimate the parameter value for that time period, the smoothed estimates utilize data from the entire sample period.

Table 1

Estimates of the Output Inflation Model

$$\Delta p_t = \alpha_0(t) + \alpha_1(t)\,\Delta p_{t-1} + \epsilon_{pt} \qquad E_{t-1}\epsilon^2_{pt} = \underset{(8.471)}{5.668} + \underset{(2.119)}{0.189}\,\epsilon^2_{pt-1}$$

$$\Delta y_t = \beta_0(t) + \beta_1(t)\,\Delta y_{t-1} + \epsilon_{yt} \qquad E_{t-1}\epsilon^2_{yt} = \underset{(12.001)}{24.754} + \underset{(2.325)}{0.261}\,\epsilon^2_{yt-1}$$

$$E_{t-1}[\epsilon_{pt}\epsilon_{yt}] = \underset{(0.055)}{0.052} - \underset{(0.925)}{0.098}\,[\epsilon_{pt-1}\epsilon_{yt-1}]$$

$$\alpha_0(t+1) = \alpha_0(t) + v_{1t} \qquad \sigma_{v1} = \underset{(3.753)}{0.458}$$

$$\alpha_1(t+1) = \alpha_1(t) + v_{2t} \qquad \sigma_{v2} = \underset{(1.638)}{0.024}$$

$$\beta_0(t+1) = \beta_0(t) + v_{3t} \qquad \sigma_{v3} = \underset{(3.034)}{0.333}$$

$$\beta_1(t+1) = \beta_1(t) + v_{4t} \qquad \sigma_{v4} = \underset{(0.521)}{0.004}$$

Residual Diagnostics

| Equation | $\rho_1$ | $\rho_2$ | $\rho_3$ | $\rho_4$ | $\rho_5$ | $\rho_6$ | $\chi^2(6)$ | $\chi^2(12)$ |
|---|---|---|---|---|---|---|---|---|
| Output | -0.021 | 0.052 | 0.013 | -0.098 | -0.005 | -0.078 | 7.257 | 18.565 |
| | 0.066 | 0.013 | -0.116 | -0.060 | -0.039 | 0.003 | 8.038 | 12.888 |
| Inflation | 0.032 | -0.018 | 0.008 | 0.016 | -0.042 | 0.001 | 5.547 | 11.186 |
| | 0.014 | -0.036 | -0.008 | -0.032 | 0.107 | 0.172 | 6.659 | 14.133 |

Notes: Asymptotic t-statistics are reported in parenthesis. Autocorrelations are denoted by $\rho_i$ i = 1,2,...6 for $\epsilon_t/\sigma(t)$ [first line] and for $(\epsilon_t)^2/\sigma^2(t)$ [second line] where $\epsilon_t$ and $\sigma^2(t)$ are the estimated residuals and associated conditional variance from each of the model's equations. The $\chi^2$ statistics report on two Conditional Moment tests [Newey (1985)] for 6'th and 12'th order serial correlation in each set of standardized residuals. The 5% critical values are 12.59 for $\chi^2(6)$ and 21.02 for $\chi^2(12)$. The form of the test is described in the technical appendix.

growth rates are similar to those in expected inflation and output. The trend inflation rate was over 6% in the early 1980's and has now levelled off at about 3%. The trend rate of output growth declined over the 1960's from 3.6% to less than 0.5% in the two major recessions. It rose in the expansion of the 1980's but only to about 2.0%.

The estimates of $\alpha_1$ and $\beta_1$ show the changes over time in the degree of persistence of inflation and output growth respectively. If the persistence coefficient is zero then the variable follows a random walk.[11] Output persistence increases for much of the period, but it is always very small (never more than .03) so output appears to follow a random walk.[12] Inflation persistence rose from near zero (suggesting a random walk) in the early 1960's to about 0.4 in 1979.[13]

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[11] To see this, consider the autoregressive equation for some variable $x_t$:

$$x_t = a(t) + b(t)\ x_{t-1} + e_t$$

where the coefficients follow random walks:

$$a(t) = a(t-1) + u_a \text{ and } b(t) = b(t-1) + u_b.$$

Take the difference in x(t) and substitute for the parameters:

$$\Delta x_t = u_a + u_b\ x_{t-1} + b(t)\ \Delta x_{t-1} + \Delta e_t$$

Hence $x_t$ follows a random walk when $b(t) = 0$.

[12] In addition we can see from (7) that when output growth follows a random walk and if the variation in the variance of output growth is inconsequential then the real rate follows a random walk as well. Earlier estimates of the Fisher equation with time varying parameters, Garbade and Wachtel (1978) and Antoncic (1986), maintained that assumption.

[13] Barsky (1987) shows that there would be no ex post Fisher effect (relationship between inflation and nominal interest rates) when inflation follows a random walk. Our results are consistent with this observation. The correlation between inflation and interest rates is weakest in the late 1960's when the persistence of inflation, $\alpha_1$, is small.

## THE MODERN FISHER EQUATION

In this section we examine the ex ante Fisher equation with the data on expectations and the variance-covariance structure estimated in the previous section. We begin by presenting the least squares estimates with constant coefficients of the simple and generalized Fisher models. These results suggest that the risk premium term (the covariance) and to a lesser extent the real rate terms do influence the nominal interest rate. However, the inclusion of these additional terms do little to improve the statistical performance of the simple ex ante Fisher equation.

We also estimate an extended version of the generalized model which allows for time-varying coefficients. The motivation for this approach is twofold. First, the apparent instability of the coefficients noted above suggests that time variation will improve the model's ability to explain movements in nominal interest rates. Second, a principle assumption of the theory underlying the generalized model is that the world can be adequately characterized by a single representative agent model. When this assumption is relaxed and agents are assumed to be heterogeneous, the model implies that nominal interest rates are determined by the same variables but that the coefficients can vary as the distribution of agents changes.

The extended model with time varying coefficients is much more successful in capturing the major movements in nominal interest rates. These estimates will be used to provide a taxonomy with which to analyze interest rate movements over the past 25 years.

<u>Constant Coefficient Estimates</u> In Table 2, we show least squares estimates of five models - the simple ex ante Fisher equation with the coefficient on expected inflation unrestricted and restricted to its theoretical value of unity, the Fisher equation augmented to allow for variation in the risk premium and two variants of the Fisher equation augmented to allow for variation in both the risk premium and the real rate. Each model is estimated over the whole sample and

Table 2

Estimates of Fisher Equation Models

| $\delta$ | $E_t\Delta p_{t+1}$ | $var_t(\Delta p_{t+1})$ | $cov_t(\Delta y_{t+1},\Delta p_{t+1})$ | $E_t\Delta y_{t+1}$ | $var_t(\Delta y_{t+1})$ | SEE | DW |
|---|---|---|---|---|---|---|---|
| | | | 1964:2 - 1987:12 | | | | |
| 3.64<br>(7.5) | 0.56<br>(5.7) | | | | | 2.23 | 0.25 |
| 1.32<br>(4.4) | 1.0 | | | | | 2.54 | 0.35 |
| 5.42<br>(17.8) | 1.0 | -0.5 | -0.1184<br>(0.9) | | | 2.72 | 0.57 |
| 5.60<br>(9.1) | 1.0 | -0.5 | -0.1171<br>(0.8) | -0.0233<br>(0.1) | -0.0038<br>(0.4) | 2.72 | 0.58 |
| 5.38<br>(21.5) | 1.0 | -0.5 | -0.0532<br>(0.8) | * | * | 2.74 | 0.56 |
| | | | 1964:2 - 1979:9 | | | | |
| 2.98<br>(10.7) | 0.47<br>(8.3) | | | | | 1.09 | 0.39 |
| 0.25<br>(0.9) | 1.0 | | | | | 1.76 | 0.40 |
| 4.35<br>(16.7) | 1.0 | -0.5 | -0.0056<br>(0.02) | | | 2.07 | 0.87 |
| 3.52<br>(5.0) | 1.0 | -0.5 | 0.0075<br>(0.04) | 0.3970<br>(1.8) | -0.0009<br>(0.1) | 1.99 | 0.94 |
| 4.37<br>(18.5) | 1.0 | -0.5 | -0.1318<br>(2.8) | * | * | 2.05 | 0.93 |

Notes: t-statistics reported in parentheses are corrected for heteroskedasticity and serial correlation using the Newey-West procedure. All regressors are generated from the output-inflation model in Table 1. * indicates that the coefficients are constrained and the equation estimated by NLLS:

$$-\gamma\ cov_t(\Delta p_{t+1},\Delta y_{t+1}) + \gamma E_t\Delta y_{t+1} - (\tfrac{1}{2})\ \gamma^2\ var_t(\Delta y_{t+1})$$

where the estimate of $-\gamma$ is shown above.

Table 2 continued

| $\delta$ | $E_t\Delta p_{t+1}$ | $var_t(\Delta p_{t+1})$ | $cov_t(\Delta y_{t+1},\Delta p_{t+1})$ | $E_t\Delta y_{t+1}$ | $var_t(\Delta y_{t+1})$ | SEE | DW |
|---|---|---|---|---|---|---|---|
| | | | 1979:10 - 1987:12 | | | | |
| 5.26 | 0.65 | | | | | 2.30 | 0.55 |
| (7.8) | (4.9) | | | | | | |
| 3.38 | 1.0 | | | | | 2.44 | 0.76 |
| (7.4) | | | | | | | |
| 7.45 | 1.0 | -0.5 | -0.2227 | | | 2.31 | 0.70 |
| (15.5) | | | (1.6) | | | | |
| 7.08 | 1.0 | -0.5 | -0.2441 | 0.2208 | 0.0033 | 2.49 | 0.64 |
| (6.6) | | | (1.8) | (0.5) | (0.1) | | |
| 7.66 | 1.0 | -0.5 | -0.1483 | * | * | 2.68 | 0.70 |
| (16.6) | | | (1.3) | | | | |

Notes: See above.

over two sub-periods with the sample split at October 1979 when the Federal Reserve changed the operating procedure used to conduct monetary policy. The results do not provide very strong support for the model; the Durbin-Watson statistics indicate that the errors are serially correlated and the coefficients differ between the sub-periods.[14] Like most other estimates of ex ante Fisher equations, the coefficient on the expected inflation variable in the unconstrained equations is significantly less than one. Adding the additional variables which explain the risk premium and the real interest rate does not change the coefficient on expected inflation. This implies that the failure of the simple Fisher model (i.e., the result that the coefficient on expected inflation is less than one) is not due to any omitted variables bias.

[14] Our theory of the generalized Fisher model theory should hold true throughout the sample period. Any changes in the relationship between nominal interest rates and expected inflation induced by changes in the monetary policy regime must, according to the model, be captured by the movements in expected output growth and the variance-covariance structure.

Some support for the generalized model is found moving down the Table to the third and fourth equation in each panel. In these equations the coefficient restrictions implied by the model are imposed; the coefficient on expected inflation is constrained to unity and the coefficient on the variance of prices is constrained to its theoretical value of -0.5. With these constraints the covariance term, reflecting the risk premium, has the expected negative sign and is almost twice its standard error in the second sub-period. When variation in the real rate is allowed for as well, the estimate of $\gamma_2$, the coefficient on expected output growth, $E_t\Delta y_{t+1}$, is positive and almost twice its estimated standard error in the first sub-period.

The last equation in each panel shows the results from estimating the fully constrained model by non-linear least squares. It maintains the theoretical constraint that the coefficient on the covariance and the estimate of the inverse of the elasticity of substitution from the real rate model are equal. Under these conditions the estimates of $\gamma$ are about the same in both sub-periods and almost three times its estimated standard error in the first period.

These results must be viewed as something of a blow against the theoretical framework of the generalized Fisher model. First, the Durbin Watson statistics suggest that there is insufficient serial correlation in the explanatory variables to account for all of the observed persistence of nominal interest rates. Thus, simply adding variables to the simple Fisher equation to reflect the other determinants of interest rates does not lead to an adequate representation of the time-series behavior of nominal interest rates. Second, there is only weak evidence that the poor performance of the simple Fisher equation is due to its neglect of the variations in the real rate and the risk premia. Although these terms have some impact, the coefficients are unstable. These empirical results suggest that we amend our framework by examining the assumption of a single representative individual and its implication that the model coefficients are constants.

Varying Coefficient Estimates There are both empirical and theoretical reasons for estimating the generalized Fisher model with time varying coefficients. The first reason is the weaknesses of the constant coefficient estimates just discussed. The second is that time varying coefficient estimates are indicated when we relax the assumption of a single representative individual. In this section we will show how the assumption can be relaxed and present estimates of the generalized model with time varying coefficients.

The constant coefficient estimates rely heavily on the representative individual assumption since each of the coefficients relates to that individual's preferences. In an economy with heterogeneous agents with different coefficients of relative risk aversion and different discount rates, equation (8) continues to describe the equilibrium relationship between nominal rates, real rate and expected inflation except that the parameters $\delta$ and $\gamma$ now represent the economy-wide averages of individual preferences.[15] Thus, as the composition of the population varies, so too may the parameters $\delta$ and $\gamma$. We will show here that such changes are an important in the determination of nominal interest rates.

The model that we estimate is the unrestricted form of the generalized model, equation (9). It fixes the coefficients on the expected inflation rate and the variance of prices at their theoretical values of 1 and $-0.5$ respectively. Only the risk aversion parameter and discount rate, $\gamma_1(t)$ and $\delta(t)$, were allowed to vary over time because we were unable to reject this specification. The model was also estimated with a first order ARCH process in the error term in order to completely capture the skewness in their conditional distribution. The interest rate model and the time series model used to generate

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[15] The Grossman and Shiller (1982) model can be used to show that the aggregate equation for the nominal interest rate has the same regressors as before. The coefficient on the covariance term is the geometric mean of the individual coefficients of relative risk aversion with relative consumption weights and the discount parameter is a function of the individual discount rates and the parameters of the cross-sectional distribution of individuals.

the regressors were jointly estimated by maximum likelihood.[16] The time varying coefficient estimates of the generalized Fisher equation and the diagnostic tests on the residuals are shown in Table 3.

Table 3

Varying Coefficient Estimates of the Generalized Fisher Equation

1964:2 - 1987:12

$$i_t = E_t\Delta p_{t+1} - (1/2)Var_t(\Delta p_{t+1}) - \gamma_1(t)Cov_t(\Delta y_{t+1},\Delta p_{t+1}) + \delta(t) + \underset{(0.444)}{0.055}\, E_t\Delta y_{t+1} - (1/2)\underset{(0.244)}{0.0049}\, Var_t(\Delta y_{t+1}) + u_t$$

$$E\, u_t^2 = \underset{(7.217)}{0.479} + \underset{(5.644)}{1.514}u_{t-1}^2$$

$$\delta(t) = \delta(t-1) + u_{1t} \qquad \sigma_{u1} = \underset{(12.662)}{0.793}$$

$$\gamma_1(t) = \gamma_1(t-1) + u_{2t} \qquad \sigma_{u2} = \underset{(9.738)}{0.398}$$

Residual Diagnostics

| $\rho_1$ | $\rho_2$ | $\rho_3$ | $\rho_4$ | $\rho_5$ | $\rho_6$ | $\chi^2(6)$ | $\chi^2(12)$ |
|---|---|---|---|---|---|---|---|
| 0.165 | 0.001 | -0.112 | -0.028 | -0.084 | -0.176 | 18.708 | 40.860 |
| 0.070 | 0.042 | 0.043 | 0.034 | 0.016 | 0.112 | 10.049 | 18.599 |

Notes: All t-statistics are corrected for the presence of "generated regressors" [i.e. $E_t\Delta p_{t+1}$, $E_t\Delta y_t$, $Var_t(\Delta p_{t+1})$, $Cov_t(\Delta y_{t+1},\Delta p_{t+1})$ and $Var_t(\Delta y_{t+1})$] using the procedure described in the technical appendix. Autocorelations are denoted by $\rho_i$ $i = 1,2,\ldots 6$ for $u_t/\sigma(t)$ [first line] and for $(u_t)^2/\sigma^2(t)$ [second line] where $u_t$ and $\sigma^2(t)$ are the estimated residuals and associated conditional variance. The $\chi^2$ statistics report on two CM tests for 6'th and 12'th order serial correlation in each set of standardized residuals. The 5% critical values are 33.924 for $\chi^2(6)$ and 41.337 for $\chi^2(12)$.

[16] Details of the estimation procedure are described in the Technical Appendix.

With the exception of the coefficients on the real rate terms, $\gamma_1$ and $\gamma_2$, all the parameter estimates are highly significant. Moreover, the diagnostic tests on the estimated residuals reveal little evidence of serial correlation. This suggests that our model captures the main features of the time-series process for nominal interest rates.

The coefficient on the covariance term, $\gamma_1(t)$, can be interpreted as the aggregate (or economy-wide average) coefficient of relative risk aversion. Changes in $\gamma_1(t)$ indicate that some of the movements in nominal interest rate can be attributed not just to the variations in the covariance between output and inflation, but also to changes in the average attitude towards risk. This can be seen in Figure 2 which shows the smoothed estimates of $\gamma_1(t)$. The mean value of the estimated parameter for the whole sample period is –0.05 and its standard deviation is 0.69. The negative mean is largely the consequence of the values estimated around the time of the first oil shock. The average estimate of the coefficient of relative risk aversion for the period 1975–87 is 0.27, a small positive number which is consistent with the priors of most researchers.

The coefficient $\gamma_1(t)$ shown in Figure 2 also represents the response of nominal interest rates to movements in the covariance between output and inflation. It varies considerably and the highest levels of $\gamma_1(t)$ roughly coincide with the Federal Reserve's changes in operating procedure in 1979 and 1982. One might reasonably conjecture that the observed volatility of interest rates during this period made individuals more aware of the risks associated with holding nominal bonds. In any event, the marked rise in $\gamma_1(t)$ implies that movements in the covariance between output and inflation contributed more to the short-term volatility of nominal interest rates than in any other period barring OPEC 1.

The coefficients $\gamma_2$ and $\gamma_3$ can be used to derive estimates of the elasticity of substitution between consumption today and tomorrow. The implied elasticities are very large, 18.2 and 14.3 respectively. However, more reasonable values are within two standard deviations of our estimate.[17]

INTERPRETING INTEREST RATE MOVEMENTS

In this section we will use the model estimates in Table 3 to examine some of the stylized facts about interest rates and to decompose movements in the nominal interest rate into its components. Our model indicates that the nominal interest rate can be decomposed into terms that relate to expected price change, the risk premium and the real rate of interest. In terms of the variables and parameters of the model structure, we can write:

| | |
|---|---|
| expected inflation rate | $E_t\Delta p_{t+1}$ |
| expected rate of depreciation of money | $-E_t\Delta p_{t+1} + \frac{1}{2}\, Var_t(\Delta p_{t+1})$ |
| risk premium | $\gamma_1(t)\ Cov_t(\Delta y_{t+1}, \Delta p_{t+1})$ |
| real rate | $\delta(t) + \gamma_2 E_t\Delta y_{t+1} - \frac{1}{2}\,\gamma_3\ Var_t(\Delta y_{t+1})$ |

The asset pricing model indicates that the nominal rate should depend on the expected rate of depreciation of money [i.e. $-E_{t-1}(P_{t-1}/P_t)$] while most popular specifications of Fisher relationships relate interest rates to the expected rate of inflation. We can see above that the expected rate of inflation will be a poor measure of the expected rate of depreciation in money when prices are highly variable. This turns out to be true during the 1960's and 70's when the sample correlation between the changes in expected inflation and the depreciation of money was -0.22. During the 1980's however, the correlation rises to -0.81. This suggests that little can be gained from reformulating the Fisher equation in terms of the expected depreciation of money if we want to understand the recent movements in nominal interest rates.

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[17] When two standard deviations are added to our estimates of $\gamma_2$ and $\gamma_3$, the elasticities are 1.1 and 1.4. These are somewhat larger than the estimate found in Friend and Blume (1975) [near 0.5] and in Hansen and Singleton (1983) [approximately one].

The mean risk premium is rather small, 0.18% or 18 basis points and its standard deviation is 1.14%. The annual average of the risk premium varies substantially from year to year and it exceeds 50 basis points in only three years:

1973 -- 0.66

1974 -- 0.89

1986 -- 0.70

The annual average of the risk premium tends to be large around episodes where economic uncertainty was widespread. It was greater than its overall average around the credit crunch in 1966-67, during the price controls in 1971, after the OPEC crisis in 1973-74, after the Federal Reserve's dramatic change in operating procedures (1980-81) and again in 1986.

Finally, the mean real rate is 5.24% and its standard deviation is 2.20. The smoothed version of the real rate is shown in Figure 3. The following statistics suggest that the real rate was high and stable in the 1960's, low and variable in the 1970's and high and variable in the 1980's:

| | Mean | Standard deviation |
|---|---|---|
| 1964-70 | 5.21 | 0.47 |
| 1971-80 | 3.68 | 1.39 |
| 1981-87 | 7.72 | 1.65 |

Thus, the stylized fact of real rate constancy is not supported by our results.[18] The second stylized fact is that the real rate and the expected inflation rate are negatively correlated. The correlations shown in Table 4 support this statement.

[18] Mishkin (1988) provides a taxonomy of stylized facts concerning the relationship of the real interest rate and the expected inflation rate to nominal interest rates in recent U.S. experience.

Table 4

Correlations of Changes

| | Real Rate and Expected Inflation | Nominal Rate and Expected Inflation | Nominal Rate and Real Rate | Nominal Rate and Risk Premium |
|---|---|---|---|---|
| Monthly | | | | |
| 65:1 to 87:12 | -0.388 | -0.028 | 0.397 | 0.204 |
| 65:1 to 79:9 | -0.320 | 0.017 | 0.308 | 0.106 |
| 79:10 to 82:10 | -0.385 | -0.158 | 0.575 | 0.418 |
| 82:11 to 87:12 | -0.523 | 0.144 | 0.248 | 0.184 |
| Annually | | | | |
| 1965 to 1987 | -0.431 | 0.619 | 0.396 | 0.323 |

The correlations of expected inflation and the nominal interest rates are a simple and informative way of looking at the strength of the Fisher effect. The second column in Table 4 shows that the correlation of the changes is always very weak with high frequency (monthly) data.[19] Quite a different picture emerges when we look at changes between annual averages; in the long run there is strong support for the existence of the Fisher effect.

These results have at least one fairly striking policy implication. Since high frequency movements in nominal interest rates are usually uninformative about the changes in expected inflation, the Federal Reserve should concentrate on the annual variation in nominal interest rates when pursuing an anti-inflationary policy since they are more likely to signal a rise in

[19] The correlations between the monthly levels of interest rates and the expected inflation rate are higher. For the three sub-periods shown in Table 4, they are 0.74, 0.12 and 0.27 respectively. Thus, the level data supports the existence of a short run Fisher effect prior to 1979 which is consistent with prior research.

inflationary expectations. A monthly rise in nominal rates, on the other hand, probably signals a rise in the real rate suggesting the need for more liquidity if a recession is to be avoided. Unfortunately interest rate movements are interpreted quite differently in practice. More often than not the observation of a monthly rise in nominal interest rates is thought to be a signal of rising inflationary expectations which requires a tightened monetary policy. Our results suggest that policy changes based on this interpretation are likely to lead to excessive (and probably undesirable) fluctuations in the real rate.

The final stylized fact that can be gleaned from the correlations in Table 4 is that short-run movements in nominal interest rates are more closely related to real rate variation than variation in expected inflation.

An informative way of looking at the components of the nominal interest rate is to decompose changes in the rate into parts attributable to inflation expectations, the risk premium and the real rate changes. In Table 5, we show changes in annual averages of the nominal rate and its components between various years which represent the major cyclical fluctuations in the economy.

Table 5

Decomposition of Interest Rate Changes

| | Nominal Rate | Expected Inflation | Real Rate | Risk Premium |
|---|---|---|---|---|
| 1965-69 | 2.48 | 2.59 | 0.11 | -0.26 |
| 1969-72 | -2.53 | 0.28 | -2.91 | 0.09 |
| 1972-78 | 3.16 | 2.48 | 0.78 | -0.04 |
| 1978-81 | 6.75 | 1.48 | 5.45 | 0.33 |
| 1981-86 | -8.03 | -6.91 | -1.23 | 0.32 |
| 1986-87 | -0.67 | 2.29 | -2.99 | -0.58 |

Here we see that the run up of nominal rates in the late 1960's (1965 to 1969) was matched almost exactly by the increase in the expected inflation rate. The ensuing fall in rates which brought the interest rate in 1972 back to its 1965 level was entirely due to a fall in the real rate; the expected inflation rate and the risk premium changed very little. The increase in rates during the expansion of the mid-1970's (1972 to 1978) was due to an increase in the expected inflation rate and a small increase in the real rate. The large increase in the nominal interest rate between 1978 and 1981 was due to a very large increase in the real rate and a small increase in the expected inflation rate.

Nominal interest rates declined by 803 basis points during the disinflation of the 1980's (1981-86). The decline was largely due to a fall in the expected inflation rate, but the real rate declined as well. This period also saw a significant rise in the risk premium. It was only in 1987 that the real rate returned to a level that might be viewed as closer to historical norms. The nominal rate only declined slightly because the expected inflation rate rose.

## CONCLUSIONS

Modern asset pricing theory shows that the simple Fisher relationship between interest rates and expected inflation is only one part of the equilibrium relationship that provides a structural model for interest rates. In this paper, we show that the inadequacies of the simple model are due to the omission of an appropriate specification for the real rate, the risk premium and the expected rate of depreciation of money. Our estimates of the generalized model extended to allow for heterogeneity among agents provides an adequate explanation of the movements in short-term interest rates.

Our preferred estimates of the generalized Fisher model has time varying parameters and ARCH residuals. We find that there is significant variation in the coefficient of relative risk aversion. Furthermore, the contribution of expected inflation, the real rate and, to a lesser extent, the risk premium to movements in the nominal interest rates vary over time. The technique and results in this paper provide for the first time a way of linking the structural modelling of interest rate equilibrium to an understanding of the wide movements in interest rates observed in recent years.

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TECHNICAL APPENDIX

This appendix describes the algorithm used to estimate both the output inflation model and the time varying version of our generalized Fisher equation and the diagnostic tests performed on both models.

Estimation Consider the standard time varying parameter model:

$$\begin{aligned} y_t &= x_t B_t + \epsilon_t \qquad & E_{t-1}[\epsilon_t \epsilon_t'] &= R(t) \\ B_{t+1} &= B_t + V_t \qquad & E_{t-1}[V_t V_t'] &= Q \end{aligned} \tag{12}$$

where B is the kx1 vector of time varying parameters with covariance matrix Q. Our specification for the output inflation model can be represented in the form of (12) by letting

$$y_t = [\Delta p_t, \Delta y_t]' \qquad B_t = [\alpha_0(t), \alpha_1(t), \beta_0(t), \beta_1(t)]'$$

and

$$x_t = \begin{bmatrix} 1, & \Delta p_{t-1} & 0, & 0, \\ 0, & 0, & 1, & \Delta y_{t-1} \end{bmatrix}.$$

The standard Kalman Filter equations for this model are simply

$$y_t = x_t B_{t/t-1} + \eta_t \tag{13}$$

$$H_t = x_t P_{t/t-1} x_t' + R(t) \tag{14}$$

$$B_{t+1/t} = B_{t/t-1} + K_{t/t-1}\eta_t \tag{15}$$

$$P_{t+1/t} = [I_k - K_{t/t-1} x_t] P_{t/t-1} + Q \tag{16}$$

$$K_{t/t-1} = P_{t/t-1} x_t' H_t^{-1} \tag{17}$$

The measurement equation (13) shows how the estimates of the coefficient vector $B_t$ based on information available at t-1, $B_{t/t-1}$ is combined with the observations on $y_t$ and $x_t$ to calculate the innovations $\eta_t$. $P_{t/t-1}$ is the covariance matrix of $B_t$ given information available at t-1. Equation (14) shows how uncertainty about $B_t$ contributes with $R_t$ to the innovation covariance matrix $H_t$.

Our model for R(t) requires the estimates of the past forecasts errors. Since $B_{t/t} = B_{t/t-1} + K_{t/t-1}\eta_{t/t-1}$, these can be obtained from

$$\hat{\epsilon}_t = y_t - x_t B_{t/t} = [I - x_t K_{t/t-1}]\eta_t \tag{18}$$

Estimates of Q, and the ARCH parameters in R(t) are obtained by maximizing

Estimates of Q, and the ARCH parameters in R(t) are obtained by maximizing the likelihood

$$\sum_{t=1}^{T} \log l(z_t,\theta) = \sum_{t=1}^{T} -\log(2\pi) + \log|H_t| - ½(\eta_t' H_t^{-1} \eta_t) \quad (19)$$

where $\theta$ is the vector of parameters and $\eta_t$ and $H_t$ are derive from (13) – (18).

Once the likelihood has been maximized we can re-apply the filtering equations (13) – (18) to obtain the time paths for the parameter vector $B_{t/t-1}$. Estimates of the $B_t$ using the complete sample [i.e. $B_{t/T}$] are obtained from the smoothing equations. For this model these are given by,

$$B_{t/T} = B_{t/t} + P_t^*[B_{t+1/T} - B_{t/t}] \quad (20)$$

$$P_{t/T} = P_{t/t} + P_t^*[P_{t+1/T} - P_{t+1/1}]P_t^* \quad (21)$$

$$P_t^* = P_{t/t} [P_{t+1/t}]^{-1}. \quad (22)$$

Computational considerations make it necessary to estimate the varying coefficient version of the Fisher Equation by limited rather than full information maximum likelihood. Specifically, we treat the estimates of expected output growth and inflation together with their conditional covariance matrix as data when estimating our generalized version of the Fisher model. The estimated equation can then be written in the form of (12) if we redefine $y_t$, $x_t$ and $B_t$ above as

$$y_t = [i_t - \pi_t - \gamma_2 E_t \Delta y_{t+1} + (1/2)\gamma_3 \text{ var}_t(\Delta y_{t+1})]$$
$$x_t = [1, \ -\text{cov}_t(\Delta y_{t+1}, \Delta p_{t+1})] \qquad B_t = [\ \delta(t),\ \gamma_1(t)\ ]'. \quad (23)$$

Using these definitions, the likelihood (19) can formed using (13) – (18) and maximized in the usual way.

The T –statistics reported in Table 3 are derived from the score vector for the *complete* model [i.e. a model that combines the output, inflation and interest rate equations and imposes the full set of cross equation restrictions] evaluated at the parameter estimates reported in Tables 1 and 3. This procedure avoids the inference problems caused by presence of "generated regressors" in the interest rate equation.

<u>Testing</u> We use a series of Conditional Moment CM tests [Newey (1985)] to test the adequacy of our specifications. Specifically, let $\theta_0$ be the population value of the parameter vector so that the probability density for the data z is $l(z,\theta_0)$. CM tests are derived from a r x 1 vector of functions $m(z,\theta)$ that satisfy the moment conditions

$$0 = E[m(z,\theta_0)] = \int m(z,\theta_0) l(z,\theta_0) d\mu(z) \quad (24)$$

where E[.] denotes the expectation taken at $l(z,\theta_0)$. If this condition is satisfied, the sample moment $m_T(\theta^*) = (1/T)\Sigma_t\, m(z_t,\theta^*)$ evaluated at the maximum likelihood estimates $\theta^*$ should be close to zero. To test for this we calculate $TxR^2$ from a regression of a Tx1 vector of ones on the set of r x 1 vectors $m(z_t,\theta^*)$ and the vector of scores $s(z_t,\theta^*)$. Newey shows that under the null of a correctly specified model this test statistic is $\chi^2$ with r degrees of freedom.

We consider the following moment conditions in our tests:

$$m_j(z_t,\theta) = \left[\eta_{it}/H_{ii}(t)\right]\left[\eta_{it-j}\ /H_{ii}(t-k)\right] \qquad j \geq 1. \qquad (25)$$

$$m_k(z_t,\theta) = \left[\eta_t\eta_t' H(t)^{-1} - I\right]\left[\eta_{t-k}\eta_{t-k}' H(t-k)^{-1} - I\right] \qquad k \geq 1. \qquad (26)$$

Equation (25) shows how moments are constructed to test for serial correlation in the standardized innovations. The moment's in equation (26) are used to test for the adequacy of our ARCH process for R(t). If the model specification is correct, the discrepancy between the standardized cross products of the innovations and the identity matrix should be orthogonal to information available at t-1 and thus uncorrelated with pervious discrepancies.

The diagnostics reported in Tables 1 and 3 are as follows: In Table 1 statistics shown at the end of the first line for each of the equations tests the hypothesis that $m_j(z_t,\theta) = 0$ for j = 1,2,..6 and j = 1,2,..12, those in the second line second test $m_k(z_t,\theta) = 0$ for k = 1,2,..6 and k = 1,2,..12. Evaluating the interest rate model is complicated by the presence of the "generated regressors" which tend to distort the "size" of test statistics based solely on the second-stage estimates. To correct this problem we extend the set of moment restrictions under consideration to include the score vector for the *complete* model [say $S(z_t,\theta)$] evaluated at our parameter estimates $\theta^*$. [Note that $(1/T)\Sigma_t S(z_t,\theta^*)$ should be close to zero if the model is correctly specified]. Under these conditions, only those deviations in $m_j(z_t,\theta)$ or $m_k(z_t,\theta)$ that are uncorrelated with $S(z_t,\theta^*)$ - and hence independent of variations in the parameters of the output and inflation model - contribute to a rejection of the model. The statistics shown in Table 3 therefore test the joint hypothesis that $(1/T)\Sigma_t S(z_t,\theta^*) = 0$ with $m_j(z_t,\theta) = 0$ for j = 1,2,..6 and j = 1,2,..12 [first line], and $(1/T)\Sigma_t S(z_t,\theta^*) = 0$ with $m_k(z_t,\theta) = 0$ for k = 1,2,..6 and k = 1,2,..12 [second line].

Figure 1a
Expected Inflation

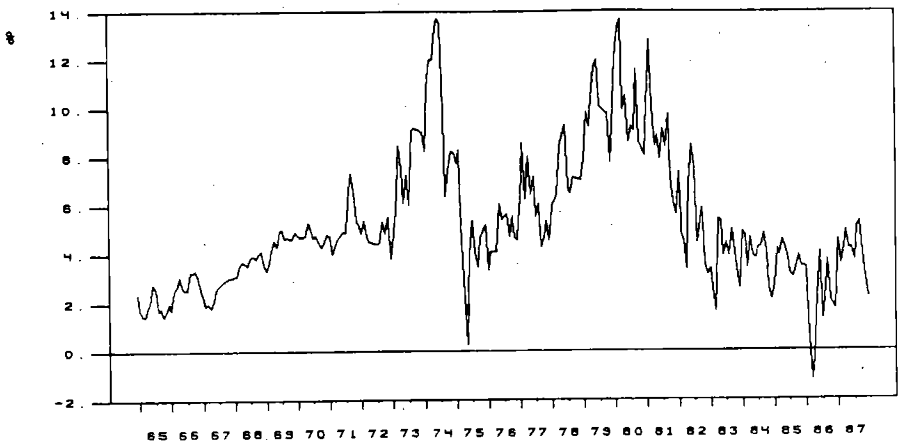


Figure 1b
Expected Output Growth

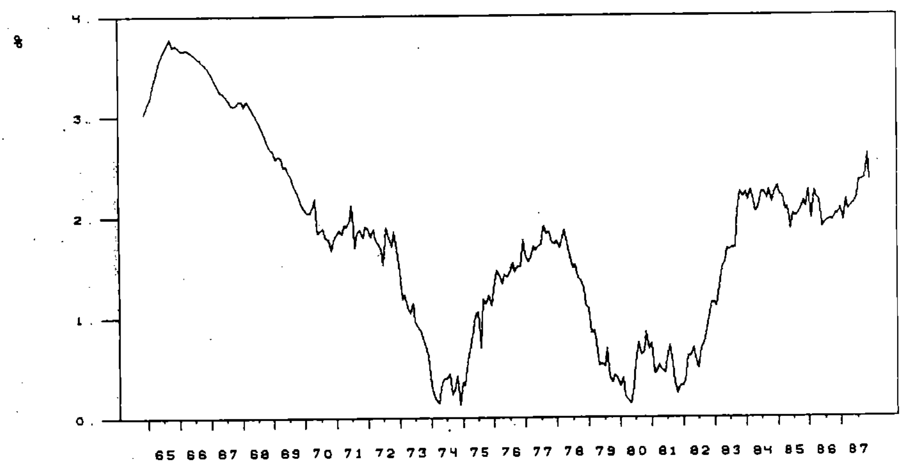

Figure 2
Aggregate Coefficient of Relative Risk Aversion

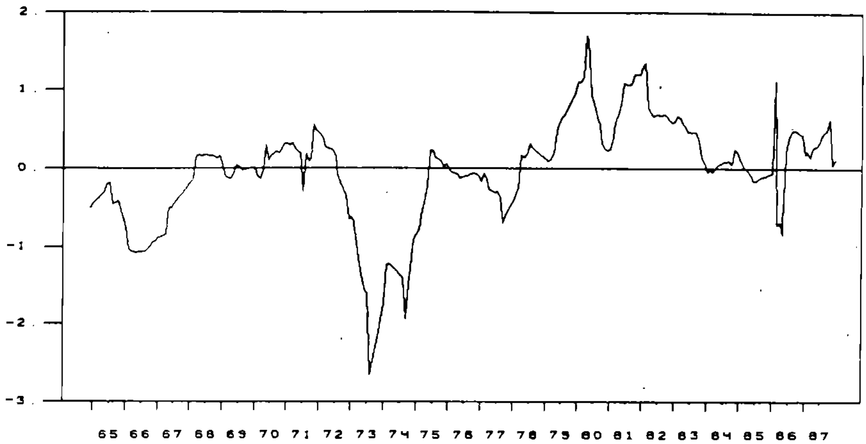


Figure 3
The Real Rate (%)