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ABSTRACT

I discuss private and central-bank-issued digital currencies, summarizing my prior research. I argue that prices of private digital currencies such as bitcoin follow random walks or, more generally, risk-adjusted martingales. For central bank digital currencies, I argue that they enhance the "CBDC trilemma" facing a central bank: out of the three objectives, price stability, efficiency, and monetary trust, it can achieve at most two.

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1 Introduction

Digital currencies have been on the rise. Once a niche phenomenon for enthusiasts, they have become mainstream, as evidenced by the introduction of bitcoin exchange-traded funds, the build-up of FinTech, rising interest in the possibilities of blockchain technologies, regulatory frameworks such as the markets in crypto assets (MICA) regulation in the EU, and the possible introduction of central bank digital currencies or CBDCs. There has been an explosion of literature on the topic from the academic side, and there still is much to understand. The literature is too vast to survey here, so I shall point to some surveys written by colleagues. Allen et al. (2021) provide a "Survey of Fintech Research and Policy Discussion". Regarding CBDCs, Chiu et al. (2023) provide a literature survey, while Kosse and Mattei (2023) summarize the developments at central banks worldwide. Prassad (2021) is a wonderful book on the "Future of Money," which I had a chance to review in Uhlig (2023). Thus, discussing digital currencies in great breadth here would be futile.

Instead, I will summarize and report on my research to address two questions in particular. First, how should we think about pricing cryptocurrencies? I will address this in section 2, reporting on our findings in Schilling-Uhlig (2019c). Second, what does introducing CBDCs imply for the stability of central banks? This will be the topic in section 3, summarizing the key insights in Schilling, Fernández-Villaverde, and Uhlig (2024). The technical details of cryptocurrencies, such as the inner workings of blockchains and cryptography, will play no role. My interest here is in monetary economics.

In the interest of space, I shall refrain from reporting on my research on the monetary policy consequences of competing against global currencies, see Benigno et al. (2019, 2022), and of pricing in parallel digital currencies, analyzed in Uhlig and Xie (2021), from expositing my analysis of the Luna stable coin crash, see Uhlig (2022) and from examining issues of privacy in Uhlig et al (2023). These issues are important, though, and I hope interested readers will feel inclined to find and read these contributions, complementing the analysis reported here.

The audience for this paper is not just fellow researchers but also educated lay readers and policymakers. With this summary, I am hoping to help inform the public debate. I concede that a level of technicality will make it tricky for the latter audience to appreciate everything here. Still, hopefully, the rest (and the occasional consultation with an academic economist at hand) will suffice.

2 A Theory of Cryptocurrency Pricing

Bitcoin, the first widely circulating and anonymous digital currency or cryptocurrency, alongside its blockchain technology, was created by Nakamoto (2008). The quantity of bitcoin rises according to an essentially deterministic schedule and converges to an upper limit; see the left panel of Figure 1. The first time bitcoin was used for a good purchase was on "Pizza Day" on May 22, 2010, when programmer Laszlo Hanyecz purchased two large pizzas for 10,000 bitcoin. As of February 7, 2024, those 10,000 bitcoins would be worth around 440 million U.S. dollars: quite a hefty price for two large pizzas. Could one have foreseen that bitcoin would rise in price, see the left panel of Figure 1? How should one think about pricing intrinsically worthless cryptocurrencies such as bitcoin? Can cryptocurrencies such as bitcoin serve as a medium of exchange despite their price volatility? What are the consequences for monetary policy and its government-issued currency, such as the dollar? These are the questions we address in Schilling-Uhlig (2019c). Schilling-Uhlig (2019b) summarizes that paper, while Schilling-Uhlig (2019a) provides an extension, allowing for transaction costs.

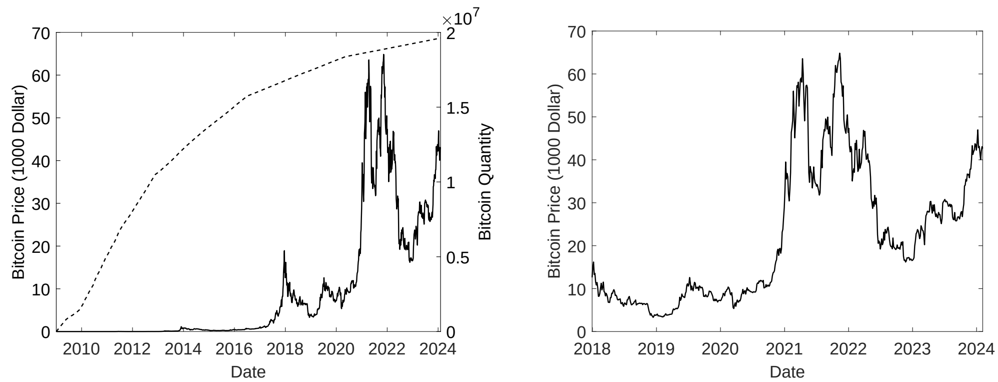

Figure 1: *Left panel: bitcoin quantity and price since 2009. Right panel: bitcoin price since 2018. Source: https://www.blockchain.com/charts/total-bitcoins.*

Our analysis builds on and echoes the ground-breaking "exchange rate indeterminacy" insight by Kareken and Wallace (1981) and the stochastic extension in Manuelli and Peck (1990). We show that there is a "fundamental pricing equation" for pricing bitcoin and similar cryptocurrencies. As a special case, the bitcoin price is a martingale or random walk: the expected future price is the current price. Put differently, when Laszlo Hanyecz purchased the two pizzas in 2010, it was rational for him to expect that his bitcoin would not rise in value: spending them rather than keeping them is then the wiser choice. This may seem at odds with the price development shown in the left panel of Figure 1. However, it was that rise in the bitcoin price that gave rise to academics like us to study bitcoin in the first place.[1] We started thinking about it and drafting the first version of the paper as of early 2018. The right panel of Figure 1 shows the price developments since then, and it appears rather martingale-like to us, vindicating our analysis.[2] We show a "no speculation" theorem: bitcoin are not "hodled" for

[1]This is akin to an anthropic principle.

[2]Obviously, there is an overall increase in price since 2018 until the time of writing,

speculative purposes, but rather they are spent.[3] Understanding "hodling" must, therefore, still await future research. We show that the price volatility does not invalidate the medium-of-exchange function of bitcoin. As for monetary policy, we show that the bitcoin block rewards are not a tax on bitcoin "hodlers"; rather, they are financed by a dollar tax or by a reduction of the seignorage earned by the central bank.

To demonstrate these insights, we provide a novel model of an endowment economy with two intrinsically worthless currencies, the dollar and bitcoin, as a medium of exchange. We seek to allow for the possibility of holding the currencies for speculative purposes and, therefore, need to go beyond the two-period overlapping generations model utilized by Kareken and Wallace (1981) as well as Manuelli and Peck (1990). The details are stated in Schilling-Uhlig (2019c); a birds-eye description shall suffice here. Time is discrete and infinite $t = 0, 1, 2, \ldots$. Some random variable θ_t is drawn at the beginning of each period, generating the history θ^t. All variables are assumed to be functions of θ^t. There is one perishable consumption good per period. There are two types of agents: "red" and "green". "Red" agents consume in odd periods, enjoying the consumption per $u(c_t)$, and have endowments in even periods. Things are the other way around for "green" agents. Agents discount the future at rate $0 < \beta < 1$. There are two monies: bitcoins in total quantity B_t and dollars in total quantity D_t. A central bank steers the quantity of dollars per lump sum transfers, $D_{t+1} = D_t + \tau_{t+1},\ \tau_{t+1} \in \mathbb{R}$. The central bank aims to implement some exogenously given price path P_t. An example would be

February 2024. It may be harder to decide whether that price increase is due to chance or an upward drift in the bitcoin price.

[3]The term "hodler" rather than "holder" was introduced by a typo on https://bitcointalk.org/index.php?topic=375643.0 and is now firmly part of the vocabulary, describing individuals holding on to a cryptocurrency rather than spending or exchanging it.

a constant inflation rate $P_t/P_{t-1} \equiv \bar{\pi}$.[4] We assume that the bitcoin quantity is deterministic, $B_{t+1} = B_t + A_t$, $A_t \geq 0$. The increase A_t is exogenously given, reflecting the behavior of bitcoin quantity shown in the left panel of Figure 1. In the baseline version of our model, we assume that A_t is freely given at the beginning of the period to the agents who are about to consume. We show that it makes no difference if we assume that agents must spend mining efforts instead to receive A_t.

There is a large amount of literature investigating why agents would use money rather than some contractual arrangements. We do not have anything new to contribute here and, therefore, simply impose that goods are traded for money. However, agents do not need to spend all their money, i.e., they can become "hodlers" instead. Likewise, they do not need to accept either money as sellers. We impose assumptions so that all dollars are always spent. We are interested in whether and when this is true for bitcoin and understanding the bitcoin price or exchange rate $Q_t = Q(\theta^t)$ in terms of dollars.

In Schilling-Uhlig (2019c), we state and prove the following result.

Proposition 1 *Assume agents use both dollars and bitcoins to buy goods at t and $t+1$. Then*

$$\mathbb{E}_t \left[u'(c_{t+1}) \frac{P_t}{P_{t+1}} \right] = \mathbb{E}_t \left[\left(u'(c_{t+1}) \frac{P_t}{P_{t+1}} \right) \frac{Q_{t+1}}{Q_t} \right] \tag{1}$$

If production (consumption) is constant at $t+1$ or if agents are risk-neutral, and if further Q_{t+1} and $1/\pi_{t+1}$ are conditionally uncorrelated, then the bitcoin price Q_t in dollar is a martingale,

$$Q_t = E_t[Q_{t+1}] \tag{2}$$

[4]There is a large literature on the details of monetary policy implementation, and they are not the focus of our paper. We simply assume that the central bank can achieve this price path exactly.

We call (1) the fundamental pricing equation. Without any randomness, we obtain the Kareken-Wallace (1981) result that $Q_{t+1} = Q_t$. Equation (1) is akin to the result in Manuelli-Peck (1990), generalizing the exchange rate indeterminacy result in Kareken-Wallace (1981). One can rewrite (1) in more conventional asset pricing terms. Define the nominal stochastic discount factor

$$\mathcal{M}_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{P_t}{P_{t+1}} \tag{3}$$

The nominal return on holding bitcoin is $R_{t+1} = Q_{t+1}/Q_t$. With that, equation (1) can be equivalently restated as

$$1 = \frac{\mathbb{E}_t \left[\mathcal{M}_{t+1} R_{t+1}\right]}{\mathbb{E}_t \left[\mathcal{M}_{t+1}\right]} \tag{4}$$

Note that this is not the usual asset pricing formula $1 = \mathbb{E}_t \left[\mathcal{M}_{t+1} R_{t+1}\right]$. This difference highlights the distinction between an asset pricing approach and a monetary economics approach to pricing cryptocurrencies. In essence, there is no interest on holding bitcoin. It shares that property with the other currency, i.e., the dollar. Holding a dollar yields the nominal return $R_{t+1} \equiv 1$, obviously satisfying (4). Currencies do not earn interest since they are useful as a medium of exchange.[5] On average and in risk-adjusted terms, both have the same return.

Define risk-adjusted expectations $\tilde{\mathbb{E}}_t[X_{t+1}]$ for any random variable per $\tilde{\mathbb{E}}_t[X_{t+1}] = \mathbb{E}_t \left[\mathcal{M}_{t+1} X_{t+1}\right] / \mathbb{E}_t \left[\mathcal{M}_{t+1}\right]$. Equation (4) shows that Q_t is a risk-adjusted martingale,

$$Q_t = \tilde{\mathbb{E}}_t[Q_{t+1}]. \tag{5}$$

The difference to (2) is only that expectations and probabilities are adjusted for risk in (5). We use this formulation in Benigno et al. (2022).

[5]From an asset pricing perspective, one can think of liquidity services as the dividend of money. One still needs to be careful when applying an asset pricing approach since doubling the price of bitcoin also doubles its liquidity services.

Under mild conditions on preferences, our " no speculation theorem" arises: all bitcoins are spent every period. This is a consequence of the feature that currencies do not pay interest but that agents discount the future. It is then better to spend all currency in every period than to hold on to it. In the online appendix to Schilling-Uhlig (2019c), available at https://ars.els-cdn.com/content/image/1-s2.0-S0304393219301199-mmc1.pdf, we show that any price process Q_t, which satisfies (1) or, equivalently, (4) and some mild boundedness condition, can arise as the bitcoin price in equilibrium.[6] In other words, the fundamental pricing equation is all one can say about the bitcoin price. In particular, there are no restrictions on the bitcoin price volatility beyond this pricing equation and some mild boundedness condition.

As a direct consequence of these results, we obtain that volatility does not invalidate the medium-of-exchange function of bitcoin. All bitcoins are always used in transactions, regardless of the volatility of the underlying price process. The economic reason for this result is the fundamental pricing equation (1): buyers and sellers are compensated for the bitcoin risk. To see this, examine equation (4). Consider a buyer and a seller facing each other at date t, knowing the stochastic nature of the price Q_{t+1} in the future. The expression on the right-hand side of (5) is how sellers value bitcoin once the appropriate risk compensation is taken into account.[7] If Q_t was higher than the right-hand side of (5), then sellers would insist on only receiving dollars since bitcoin is too expensive. Thus, buyers seeking to pay with bitcoin must accept a lower value: the price would be bid down until it reaches the value expressed in (5). Conversely, if Q_t were lower than the right-hand side of

[6]The boundedness condition arises from the feature that the total value of bitcoin cannot exceed the total value of all goods when all bitcoins are always spent.

[7]Note that the seller becomes a buyer in $t+1$, i.e., it is the utility function of the current seller on the right-hand side of (5). The risk compensation can also take the form of an "insurance premium", i.e., there are generally price processes Q_t so that the expected return on bitcoin is below unity.

(5), sellers would insist on only receiving bitcoin, and the price would be bid up. In equilibrium, equation (5) holds, and bitcoins are used freely as a medium of exchange, despite their price volatility, since agents are exactly compensated for the risk that they entail.

Finally, consider two economies that differ in the growth paths for the bitcoin quantity but where the central bank achieves the same path for prices. To make things more specific, imagine that the first economy has a constant supply of bitcoin while the quantity is growing for the second economy, as shown in the left panel of Figure 1. Due to the no-speculation theorem, a version of the quantity theory equation has to hold in both economies,

$$P_t y_t = D_t + Q_t B_t \tag{6}$$

As argued above, there is no restriction on the price path Q_t. Therefore, the same price path may arise in both economies, provided some mild boundedness condition is satisfied. The equation immediately implies that more bitcoins B_t means less dollars D_t, keeping everything else the same. The dollar quantity in the second economy grows more slowly (or falls faster) than the dollar quantity in the first economy precisely by the value of the bitcoin increases in the second economy. The central bank pays for the bitcoin block rewards. The economic reason is that the central bank keeps the price path P_t in check. It, therefore, has to live with the consequences of that policy choice.

3 Central Bank Digital Currency and the Stability of Central Banks

Central banks worldwide are considering the introduction of central bank digital currencies or CBDCs, see Kosse and Mattei (2023). What are the consequences? In particular, could they lead to instability of the central banks themselves? Diamond-Dybvig (1983) provides the classic framework to examine the stability of banks. That paper has spawned a huge literature, but it has rarely, if ever, been applied to the issue of the stability of the monetary system and the central bank itself. We do so in Schilling, Fernández-Villaverde, and Uhlig (2024), considerably extending Fernández-Villaverde et al. (2021), addressing the key question. It turns out that our analysis applies to central banks more broadly, even absent CBDCs. However, our analysis becomes particularly relevant for a widely used CBDC, given the speed at which private citizens can then execute electronic transactions in central bank money.

The essence of our model is this: while a central bank can obviously always make good on its nominal obligations, agents may nonetheless be concerned that these nominal obligations, i.e., that money, will be worthless in the future. Agents will then seek to spend their money as quickly as possible, even though they do not need to consume urgently. Usually, a "bank run" is a run away from deposits into cash. Here, the run on the central bank is a run away from money into goods. They both share that it is a run away from what the bank can issue to something it cannot. A run on the central bank is a spending run. In a spending run, money, whether in traditional form or in the form of a CBDC, loses its "store of value" function. The trust in the monetary system and CBDC evaporates, and monetary instability results. This phenomenon should be familiar from hyperinflations or currency crises. More recently, we have seen versions of this in the form of temporary pandemic stockouts.

The baseline version of our model is highly stylized and "stripped-down" to the bare bones essence: there are only households and a central bank, and the central bank is running the country. Obviously, this is not the case in practice, and we do not mean to imply that this is so. An extension in the paper describes a decentralization with firms and private banks. Nothing of essence changes.

We imagine a central bank that interacts with households by providing them with money, perhaps in the form of a CBDC, to purchase goods. We build on Diamond-Dybvig (1983) and the nominal versions in Skeie (2008) and Allen et al. (2014). Indeed, the real portion is just a minor variation of Diamond-Dybvid (1983). There are three periods, $t = 0, 1, 2$ and a continuum $[0, 1]$ of agents. These agents are endowed with one unit of a real good in $t = 0$. In $t = 1$, their type is revealed. Agents may be "impatient" with probability λ and "patient" with probability $1-\lambda$. Probabilities are fractions, using the law of large numbers for a continuum; see Uhlig (1996). Impatient agents must consume in $t = 1$, while patient agents may consume in either period. The utility function $u(\cdot)$ is strictly increasing, concave, and has a relative risk aversion exceeding unity, $-x \cdot u''(x)/u'(x) > 1$. An investment technology turns 1 unit of the good in $t = 0$ into 1 unit of the good in $t = 1$ or R units of the good in $t = 2$. The problem of a social planner who maximizes ex-ante welfare is

$$\max \lambda u(x_1) + (1-\lambda)u(x_2) \quad \text{s.t. } \lambda x_1 + (1-\lambda)\frac{x_2}{R} = 1 \tag{7}$$

The unique solution satisfies $u'(x_1^*) = Ru'(x_2^*)$ and $x_1^* > 1$. Consequently, $y^* = \lambda x_1^*$ are the total resources liquidated in period $t = 1$.

We now add a nominal layer. In period $t = 0$, the agents sell their goods to the central bank for M units of money or CBDC. The central bank invests the real goods, using the investment technology. In $t = 1$, the impatient agents spend their balances M, while patient agents may. Let n be the fraction of spending agents, with $\lambda \leq n \leq 1$. The central bank observes λ. This is particularly appropriate in the CBDC context when the central bank can observe private monetary transactions in real time. The central bank then liquidates a fraction $y = y(n) \in [0, 1]$ of the invested goods and sells them. The market clearing price is P_1. In $t = 2$, the remaining agents receive an interest payment $i(n)$ on their balances: again, this is particularly easy to do with a CBDC. They then spend $(1 + i(n))M$. The central bank sells the remaining invested goods $R(1 - y)$. The market clearing price is P_2. Note that we have assumed that the central bank only issues CBDCs. Nothing hinges on that formally. One can interpret the model equally well as a central bank issuing cash or extending the environment, allowing for both cash and a CBDC: details are in the paper.[8]

There are several choices for the central bank to make. We assume that it announces them in $t = 0$ and that agents expect the central bank to stick to it. In Schilling, Fernández-Villaverde, and Uhlig (2024), we define a central bank policy as follows.

Definition 1 *A* **central bank policy** *is a triple* $(M, y(\cdot), i(\cdot))$*, where* $y : [0, 1] \to [0, 1]$ *is the central bank's liquidation policy for every observed fraction* n *of spending agents, and* $i : [0, 1] \to [-1, \infty)$ *is the nominal interest rate policy.*

A boring example of a central policy is shown in Figure 2. How should the central bank choose? We assume that there are three competing objectives:

1. **Price Stability:** the traditional objective of a commitment to stable prices. Here, we mean that the price $P_1(n)$ remains the same, regardless of circumstances n.

2. **Efficiency:** here, we mean that the central bank implements the socially optimal risk sharing. It is achieved, if $y(\lambda) = y^* = \lambda x_1^*$.

3. **Monetary Trust:** avoid spending runs and monetary instability.

In Schilling, Fernández-Villaverde, and Uhlig (2024), and as I shall outline below, we show that the central bank can only achieve at most two of these

[8]With cash and a CBDC, there is a "boring" run where agents exchange CBDC for cash or vice versa. This is a rather benign scenario and not our focus.

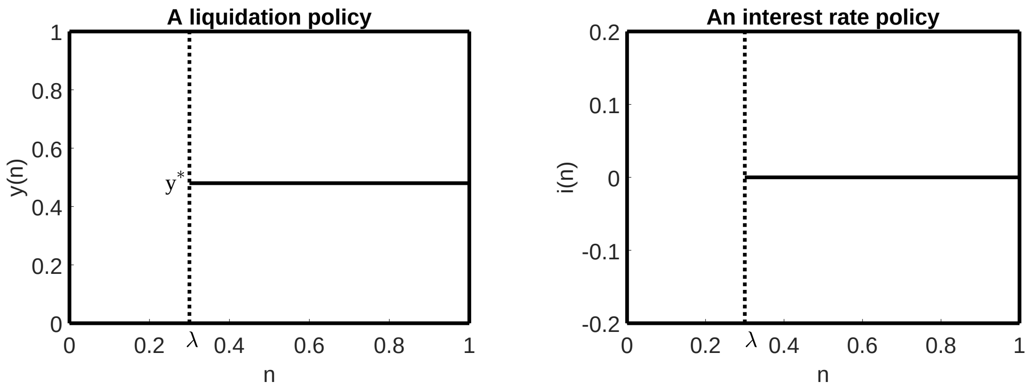

Figure 2: *A boring example for a central bank policy. Set M so that $P_1 = 1$ clears the market, if $n = \lambda$ agents spend in $t = 1$. Set liquidation $y(n)$ always equal to $y^* = \lambda x_1^*$; see the left panel. Set $i(n) = 0$ for all n; see the right panel.*

three objectives. This result applies regardless of whether the central bank uses traditional forms of cash, CBDC, or both. Still, the remarks above show it is particularly relevant for a widely used CBDC. We, therefore, call this result the CBDC trilemma; see Figure 3. To that end, let us take a closer look.

Market Clearing yields

$$nM = P_1 y(n) \tag{8}$$

$$(1-n)(1+i(n))M = P_2 R(1-y(n)) \tag{9}$$

The central bank policy and n, therefore, pins down the price levels

$$P_1(n) = \frac{nM}{y(n)} \text{ and } P_2(n) = \frac{(1-n)(1+i(n))M}{R(1-y(n))} \tag{10}$$

Note that $P_2(n)$ can be "anything" per $i(n)$, but the interest policy $i(n)$ does not affect $P_1(n)$. While perhaps obvious in terms of the equations, there is a deeper economic meaning here. In typical Keynesian-style models, economic

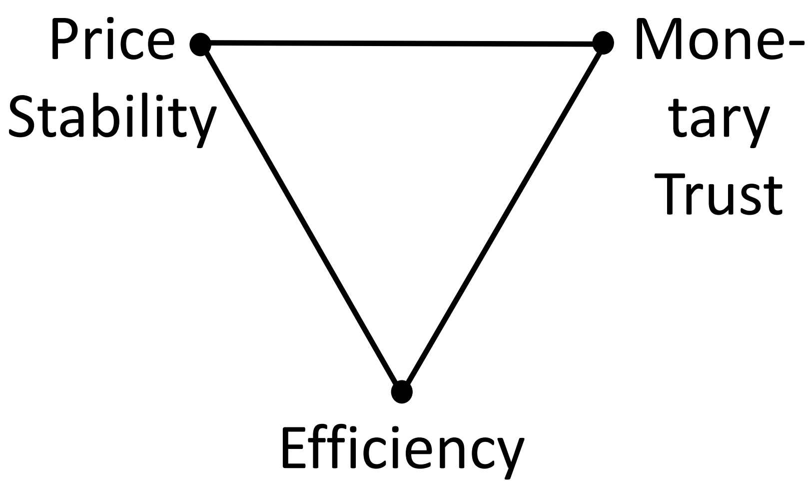

Figure 3: *The CBDC Trilemma: out of three competing objectives, a central bank can achieve at most two. While that tradeoff applies to all central banks, the trilemma is particularly relevant when issuing a widely used CBDC.*

activity can be subdued by raising interest rates. Applied to the model here, this would lead one to think that excessive spending $n > \lambda$ in period $t = 1$ can be combated with raising interest rates $i(n)$. However, this has no effect. Interest rates here are Fisherian, not Keynesian: a rise in the nominal interest rate raises the inflation from $t = 1$ to $t = 2$ and does not affect prices in $t = 1$. Therefore, the real allocation and the decision of patient agents to spend or not in period $t = 1$ depends only on the liquidation policy and the fraction n of spending agents. Agents will consume $x_1(n)$, if they spend in $t = 1$ and $x_2(n)$, if they spend in $t = 2$, where

$$x_1(n) = \frac{M}{P_1} = \frac{y(n)}{n} \text{ and } x_2(n) = \frac{(1 + i\,(n))M}{P_2} = \frac{1 - y(n)}{1 - n} R \qquad (11)$$

Given n and the central bank policy, patient agents spend in $t = 1$ or "run", iff $x_1(n) \geq x_2(n)$. A run on the central bank is a spending run, visualized in Figure 4:

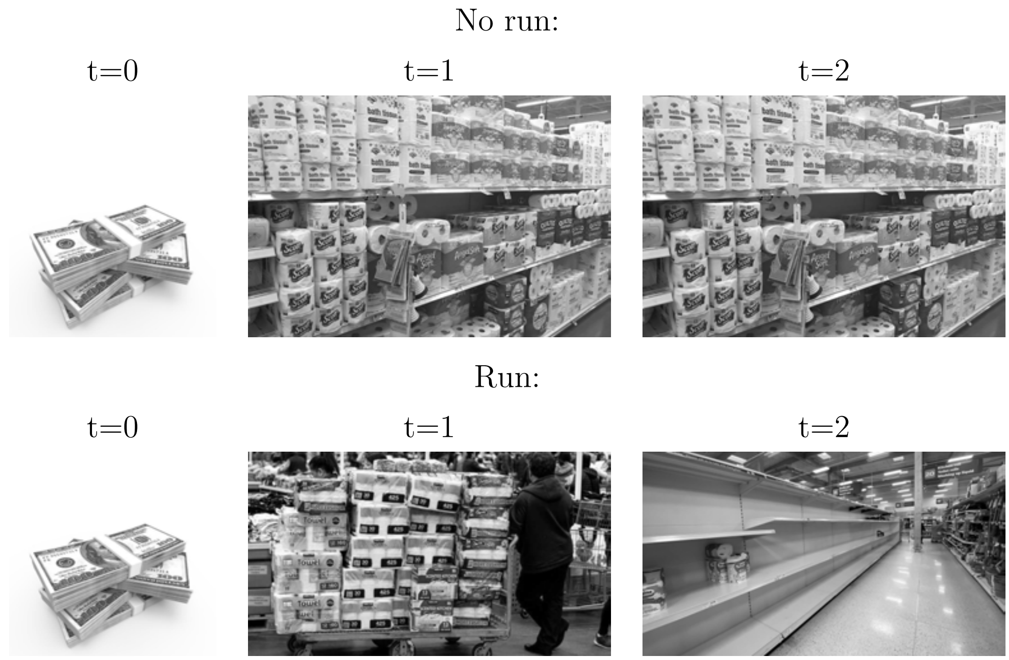

Figure 4: *Visualization of a run. In period $t = 0$, agents receive money for their goods. If there is no run, some agents will spend in $t = 1$, and some will spend in $t = 2$. In a run, agents fear that too few goods are available for purchase in $t = 2$, therefore purchasing goods in greater quantity than usual in $t = 1$.*

Definition 2 *A* **run** *occurs if $n > \lambda$: patient agents also spend.*

To achieve the third objective, the central bank must choose a policy such that a run cannot occur in equilibrium. A policy is "run-proof", if $n \neq \lambda$ is "off equilibrium", i.e. if $x_1(n) < x_2(n)$ for all n. For the aggregate liquidation, this means that the central bank must choose $y(n) < \bar{y}(n) = nR/(1 + n(R - 1))$. The left panel of Figure 5 shows this limit to liquidation and compares it to the boring policy of Figure 2: the boring policy satisfies it.

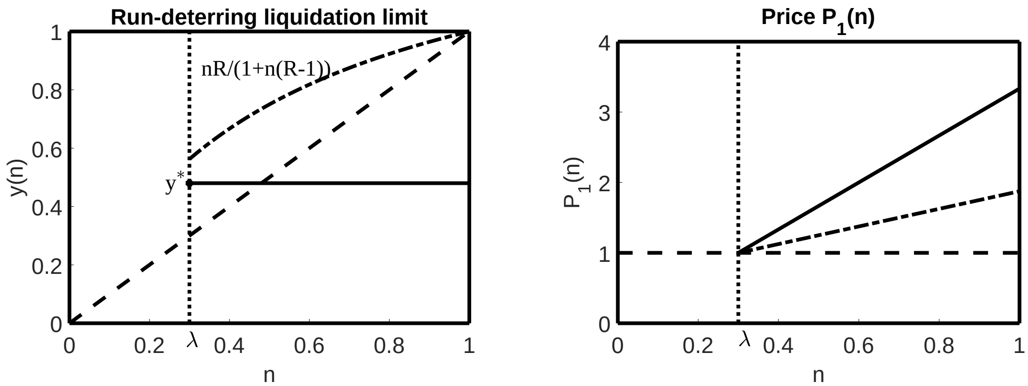

Figure 5: *Run-proof policies must have liquidation strategies not exceeding the limit shown in the left panel. For example, the boring policy shown in Figure 2 is run-proof. However, run-proof policies induce rising prices off equilibrium, as the right panel shows. Parameters are* $\lambda = 0.3, R = 3$ *and* $u(c)$ *is CRRA with* $\eta = 3.3$.

A central bank should, however, be concerned about the price level $P_1(n)$, should a run $n > \lambda$ nonetheless occur. For the boring policy $y(n) \equiv y^*$, this price increases according to $P_1(n) = n\frac{M}{y^*}$, shown in the right panel of Figure 5. That panel also shows even a policy at the run-proof liquidation limit leads to increasing prices for $n > \lambda$. While these price rises are off equilibrium, questions about time consistency arise, as famously pointed out in a different context by Kydland and Prescott (1977). Suppose that $n > \lambda$ agents spend their balances in $t = 1$. Would the central bank stick to its announced policy, thereby giving up on maintaining price stability? While perhaps optimal in the stripped-down version of a model as described here, price stability is often seen as the foremost objective of a central bank. Schilling, Fernández-Villaverde, and Uhlig (2024) discuss how such an objective can arise from an extended planner objective or concerns about price stickiness. More broadly, an extensive literature argues why price stability is an essential objective for monetary policy. For the discussion here, we take it as granted that it is.

Given the central bank's dedication to price stability, the threat of raising prices "off equilibrium" is empty; it is not sub-game perfect. Let us examine the consequences if a central bank pursues that first objective, i.e., if policy choices are restricted to achieving price stability for all n as well as possible. We consider two versions.[9] The central bank is committed to price stability at all costs for the first version. For the second version, the central bank seeks to maintain price stability for as long as it is capable of doing it, i.e., as long as it can satisfy existing demand at that price level. Schilling, Fernández-Villaverde, and Uhlig (2024) make this precise with the following definition.

Definition 3 *1. A central bank policy is **fully price stable**, if it achieves $P_1(n) \equiv \overline{P}$ for all n.*

*2. A central bank policy is **partially price stable**, if it achieves either $P_1(n) = \overline{P}$ or there is full liquidation, $y(n) = 1$, for all n.*

What are the consequences of such policies? Recall market clearing (10), i.e.

$$P_1(n) = \frac{nM}{y(n)} \tag{12}$$

Therefore, the liquidation policies must satisfy

$$y(n) = \begin{cases} \frac{nM}{\overline{P}} & \text{if "fully price stable"} \\ \min\{\frac{nM}{\overline{P}}\} & \text{if "partially price stable"} \end{cases} \tag{13}$$

These policies are shown in Figure 6 and compared to the run-proof liquidation limit. The top solid line shows a liquidation policy that satisfies partial price stability and efficiency. The line crosses the run-proof liquidation limit. This shows that partial price stability can achieve the second objective of

[9]The paper also includes a discussion of the price level P_2. Briefly, note that any price level P_2 can be achieved by the appropriate choice of the interest rate policy $i(n)$.

efficiency at $n = \lambda$ but permits runs, violating the third objective. The third objective can be maintained with full price stability, as shown by the solid line at the bottom. However, efficiency at $n = \lambda$ is no longer feasible, violating the second objective. In sum, the CBDC Trilemma of Figure 3 arises.

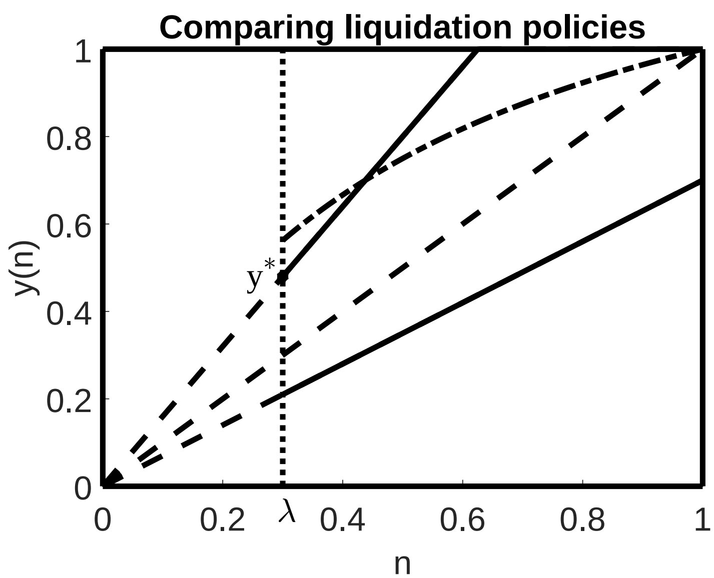

Figure 6: *Comparing the liquidation policies for partial price stability (top solid line) and full price stability (bottom solid line) to the run-proof liquidation limit (dash-dotted line). Partial price stability can achieve the second objective of efficiency at $n = \lambda$ but crosses the run-proof limit, i.e., permits runs, violating the third objective. The third objective can be maintained with full price stability and the bottom line, but then efficiency at $n = \lambda$ is no longer feasible, violating the second objective.*

We discuss several variations and considerations in Schilling, Fernández-Villaverde, and Uhlig (2024). One might wonder, for example, whether the central bank could address the trilemma by changing the money supply in

$t = 1$. It could do so by making money balances $M(n)$ held by agents contingent on the fraction n of agents wishing to spend. As an example, suppose $y(n) \equiv y^*$. To maintain price stability at some $\bar{P}$, the state-contingent money balances would then need to be

$$nM(n) = \bar{P}y^* = \lambda M(\lambda) \tag{14}$$

As a result, the total money spent in $t = 1$ is constant, regardless of n, while the money available and spent per agent decreases with n. In essence, the central bank commits itself to reducing the quantity of money in response to a demand shock and a spending spree. With a CBDC, the central bank could achieve this directly per its control of the amounts held on the private accounts at the central bank. The central bank could also impose a "suspension of spending" (similar to "suspension of convertibility" in the bank run literature) and allow that only a fraction of money can be spent in period $t = 1$, where that fraction depends on n. Alternatively, one can consider state-dependent taxes so that agents end up with $M(n)$ money.

While feasible in principle, such policies should be treated with considerable caution. It may well be that changing the money in the hands of people and its anticipation may undermine the trust in the monetary system in the first place. It remains worth emphasizing that interest rate policies or open market operations have no effect since $i(n)$ does not impact the real allocation or the price level in $t = 1$ and since only non-spending agents would ever wish to participate in trading bonds with the central bank. Indeed, whether a run occurs depends entirely on the liquidation policy $y(n)$. Money is neutral here. Conversely, the state contingency of money balances may be an intriguing possibility for the design of a CBDC, which is not easily implementable in a cash-based system. Such a CBDC design feature may offer hope to resolve the trilemma if implemented with great care.

4 Conclusions

The topic of "digital currencies" is broad. Discussing it here in great breadth would have been futile.

Instead, I addressed two questions in particular, summarizing and reporting on the findings in two of my co-authored papers. In Schilling-Uhlig (2019c), we provide a theory for pricing cryptocurrencies and demonstrate that their prices should follow risk-adjusted martingales, see proposition 1 and equation (4). In Schilling, Fernández-Villaverde and Uhlig (2024), we highlight that central banks can be subject to "spending runs" and that this phenomenon is particularly relevant when central banks issue widely used central bank digital currencies or CBDCs. This results in a CBDC trilemma, see Figure 3: out of the three objectives, price stability, efficiency, and monetary trust, the central bank can achieve at most two.

Obviously, a lot more can and should be done. There is a wide-open canvas for researchers to make their mark and influence important policy discussions. By providing the essence of these two papers in my exposition here, I hope I helped contribute to that latter goal.

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