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ABSTRACT

How does the presence of risk sharing affect sorting patterns on productive attributes when there are complementarities among partners' skills in match output? We develop a matching model in which risk-averse agents, who differ in skills, match pairwise for productive purposes. Match output has stochastic returns and matched partners efficiently share this risk. We find that under plausible assumptions the risk-sharing benefit of marriage tends to push toward negative sorting on partners' skills. To obtain the prediction of positive skill sorting—a robust empirical feature of marriage markets—this force needs to be counteracted by sufficiently strong skill complementarities in match output. We provide a novel inequality that characterizes monotone (positive or negative) equilibrium sorting, balancing out skill complementarities and risk-sharing considerations in the right way. Several classes of primitives (utility and match output functions) render monotone sorting optimal. We then highlight a new implication of positive sorting on exogenous skills for matching patterns on endogenous differences in risk aversion: Positive sorting on skills translates into positive sorting on risk aversion—in line with the evidence from marriage markets.

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1 Introduction

How a group of individuals optimally shares risk is a widely studied problem in economics (Wilson, 1968) with numerous applications, ranging from project diversification by firms to risk-sharing arrangements within partnerships and households. This problem is well understood when one group shares risks in isolation. But, less is known about the matching problem of heterogeneous agents in a market environment for the purpose of risk sharing and production, despite having multiple applications, such as to the formation of professional or marriage partnerships.

This paper analyzes a canonical equilibrium model in which agents first decide whom to match with and then engage in risk sharing with the chosen partner. Our motivation is that in practice many risk-sharing problems are preceded by a matching problem. There is a market in which risk-averse individuals who differ in skills look for partners to engage in joint production or consumption that is subject to complementarities in their skills and stochastic returns. A prominent example is that of singles in the marriage market who seek a partner to undertake some risky household production or would like to share their income risks. Indeed, risk sharing has been deemed one of the main economic benefits from marriage.

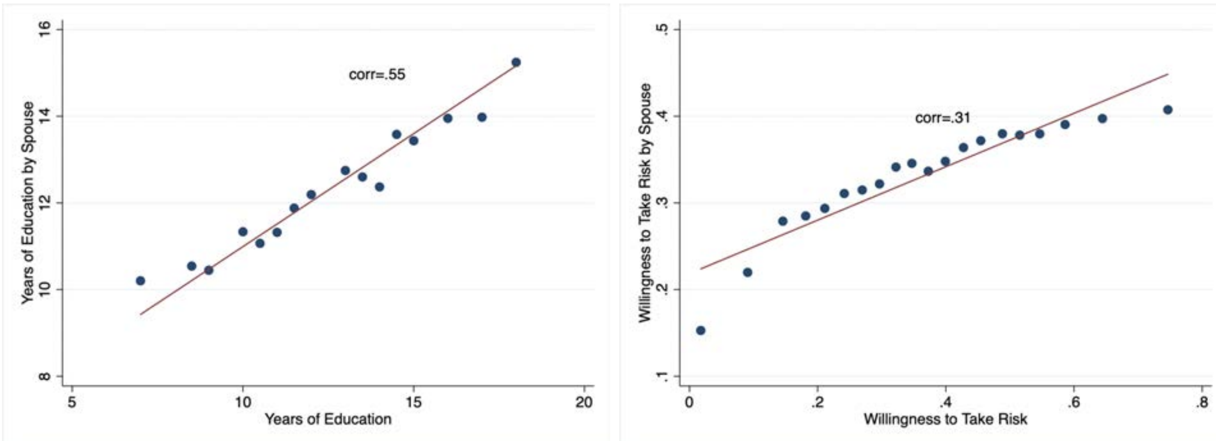
Few insights are known about how risk sharing and matching interact in this type of setting and the implications for equilibrium sorting of partners. Existing theoretical work has focused on heterogeneity in risk aversion as the only sorting-relevant variable, which renders negative sorting optimal as it maximizes risk-sharing benefits within couples.[1] It disregards, however, the diversity in skills or education of partners, which is the central differentiating factor in real-world marriage markets. As such, this literature has not shed light on the extensively documented phenomenon of positive assortative matching on skills in this setting—see, for instance, Figure 1 (left) for evidence from Germany, which shows a strong positive correlation of partners' years of education.[2] Little is known about whether risk sharing reinforces or limits this type of positive sorting on *skills*. This is where our paper comes in.

We focus on ex ante heterogeneity in skills (as opposed to risk aversion) and ask whether, also in this setting in which agents sort on *skills*, risk-sharing considerations push toward negative sorting; and, if so, how this feature can be overcome to generate predictions in line with the empirical finding of positive marriage sorting on skills. Indeed, we find that under plausible assumptions the risk-sharing benefit of marriage tends to push toward negative sorting in partners' skill types. To counteract this force and yield positive sorting patterns, sufficiently strong skill complementarities in match output are needed. We derive an economically interpretable condition that characterizes

[1] See Schulhofer-Wohl (2006), Legros and Newman (2007), and the general analysis in Chiappori and Reny (2016) for this theoretical insight. See also Serra-Garcia (2021) for an empirical application. For a detailed discussion, see our literature review below.

[2] Positive marriage sorting on skills/education is a robust feature of marriage markets also in other countries (see, for instance, Browning, Chiappori, and Weiss, 2014; Greenwood, Guner, Kocharkov, and Santos, 2016; Greenwood, Guner, and Vandenbroucke, 2017; Eika, Mogstad, and Zafar, 2019).

Figure 1: Marriage Market Sorting on Education (left) and Risk Attitude (right)



Notes: The left panel shows a bin scatter plot between years of education of the spouse and years of education of the household head with linear fitted line, based on cohabiting and married couples. The right panel shows a bin scatter between the willingness to take risk by the spouse and the willingness to take risk by the household head with linear fitted line, based on cohabiting and married couples. The willingness to take risk is constructed using principal component analysis based on three risk-attitude measures: general willingness to take risk, willingness to take risks in financial matters and willingness to take risks in occupational choices. For details and robustness, see Appendix B. Source: German Socio Economic Panel.

positive sorting, which balances out skill complementarities and risk-sharing considerations in the right way. This condition is easy to tame in several classes of primitives—utility and match output functions—that are commonly used in applications.

A natural question is how positive sorting on exogenous skills impacts sorting on risk aversion, which is *endogenously* heterogeneous in our setting as differences in skills across couples trigger differences in consumption and thus in their coefficient of absolute risk aversion. We provide an answer in a canonical setting in which we can derive the equilibrium in closed form: Positive sorting on skills begets *positive* sorting on risk aversion, a phenomenon that has garnered empirical support in the marriage market context and we replicate in Figure 1 (right).[3] It highlights the fact that spouses' attitudes toward risk—here measured by the first principal component of their general willingness to take risk, their willingness to take risks in financial matters and their willingness to take risks in occupational choices—are positively correlated in the German data.[4]

We now describe our model and main insights in more detail. There are two populations of risk-averse agents, which we will denote as women and men. Each side is heterogeneous in a payoff-relevant attribute—for example, skill or education—and each agent seeks a match partner from the other side of the market. Once matched, a couple produces output that depends on both partner's skills. Importantly, match output is subject to risk. Either there is a single shock that affects match output or each partner faces a different shock that hits their respective contributions to match output. Since agents are risk averse, this risk is shared efficiently within each couple.

[3]See Dohmen, Falk, Huffman, and Sunde (2011) and Serra-Garcia (2021) for further evidence.
[4]Several robustness checks are discussed in Appendix B.

Risk aversion implies that, in general, matching exhibits imperfectly transferable utility (Legros and Newman, 2007), and thus sorting takes into account both complementarities in production and how difficult it is to transfer utility between the partners in a match. An equilibrium consists of a matching and utility schedules (which are functions of either male or female skills) such that agents behave optimally in the matching market and the appropriate stability conditions hold.

After deriving some properties of the optimal risk sharing within a given couple, we turn to the conditions that generate positive sorting. We show that when individuals choose a partner in the marriage market, they face a potential trade-off between productive complementarities among partners' skills in match output and risk-sharing opportunities. On the one hand, skill complementarities in the match output function push towards positive sorting. On the other hand, this motive for positive sorting can be counteracted if the scope for risk sharing is reduced in couples of similar types. This happens when the covariance between the partners' incremental contributions to match output weighted by their risk aversion (both functions of the shock) is positive. That is, partners' risk-sharing opportunities are shaped by two factors: the co-movement of the marginal contributions of their skills to match output; and the co-movement and strength of their risk aversions. We show that under a broad set of assumptions, these co-movements are positive and thus undesirable from a risk-sharing perspective. This demonstrates that risk-sharing considerations make positive sorting on skills costly and push toward negative sorting instead.

Our main theorem provides a novel characterization of the conditions for positive sorting in this setting and states that skill complementarities in match output need to outweigh the risk-sharing motive in order for positive sorting to materialize. We then demonstrate the usefulness of this characterization: It allows us to easily peel off commonly used primitives (match output and utility functions) under which equilibrium sorting on skills is positive.

What is the source of these attribute complementarities in marriages? We discuss several micro-foundations for skill complementarities in match output that appear in the literature, such as income complementarities in couples or complementarities of partners' labor hours that render complementarities of their skills in individuals' wages. We also consider a natural extension of our model that introduces additional complementarities in the couple's problem—in home production, leisure, or love—to counteract the risk-sharing force.

We end by deriving the equilibrium of this model in closed form under preferences that feature constant relative risk aversion (CRRA), a class that has found considerable empirical support, and a match-output function that is multiplicatively separable in the shock, a common assumption in economic applications. The closed form allows us to obtain several new equilibrium properties.

First, we analyze the implications of positive sorting on (exogenous) skills for sorting on (endogenous) risk aversion. Indeed, an important implication of positive sorting on skills is *positive sorting* on risk aversion, measured by the covariance of matched-partners' coefficients of absolute risk aversion. High-skilled women match with high-skilled men, which implies their contributions to match output are also high and so are their individual consumption shares, when compared with less skilled couples. High consumption shares dampen *both* partners' absolute risk aversion. As a result, there is little variation in risk aversion within compared to across couples. We see couples in which either both partners have low risk aversion—the high-skilled matches—or both have high risk aversion pertaining to low-skilled couples. Risk aversion is therefore *positively* correlated among partners in the marriage market. Importantly, positive sorting on risk aversion is an implication of positive sorting on skills and occurs despite the fact that in our setting there is no exogenous variation in risk aversion across individuals.

Second, we provide new comparative statics of risk, with focus on how changes in risks—stochastically better, more variable, or more correlated risks among spouses—affect risk-sharing arrangements and inequality within households. For instance, given that agents are risk averse, a stronger dependence of their income shocks limits risk-sharing opportunities within the couple, reflected by an inward shift of the Pareto frontier, and decreases the output shares and thus utilities of both partners in equilibrium.

It is important to note that even though we apply our model to the marriage market, it more generally fits any partnership model that involves risk sharing after matching. Consider, for instance, business partners who match to launch a joint project that reaps the complementarities from their skills but also to share the risk associated with uncertain project returns; or the two sides could be lawyers or physicians with different specializations that match to form partnerships that augment their returns but also insure them. In each of these cases, there is not only a matching problem but also uncertainty about the performance of the partnership, which depends on the partners' skills and some randomness that leads to a risk-sharing problem. All it takes to fit these applications is to re-label our agents and re-interpret the attribute complementarities.

Our model is thus fairly general and widely applicable. To our knowledge, our theoretical analysis is the first to highlight how risk sharing and type complementarities interact in essential ways, and thereby shape the sorting patterns on partners' skills that arise in equilibrium.

Related Literature. Our work primarily relates to the scant theoretical literature on matching with risk sharing. The seminal paper by Chiappori and Reny (2016) assumes that agents differ (in a precise sense) in their coefficient of absolute risk aversion, and sort into marriage based on this variable. A robust result from their analysis is that any equilibrium exhibits *negative* sorting on absolute risk aversion. The intuition is that more risk-averse agents have a stronger desire for insurance and thus a high willingness to pay for it. In turn, agents with lower risk aversion are well equipped to provide it. For similar results in more specialized environments, see Schulhofer-Wohl (2006) and Legros and Newman (2007).

Our work complements these papers, as we do not allow for (exogenous) heterogeneity in risk aversion but focus on differences in productive attributes as the primary sorting-relevant trait. Nevertheless, risk aversion plays a key role in determining the conditions for monotone sorting in our setting. Moreover, we show a novel connection between sorting on skills and sorting on risk aversion, as differences in agents' risk aversion arise endogenously given the variability in consumption across couples if preferences feature decreasing absolute risk aversion. Indeed, a natural implication of positive sorting on skills in our setting is *positive* sorting on partners' absolute risk aversion—a new prediction of our model that finds support in the data: There is growing evidence for the tendency of spouses to align positively in terms of their risk aversion (Dohmen, Falk, Huffman, and Sunde, 2011 and Serra-Garcia, 2021), which is a prediction that is difficult to obtain when agents sort on risk aversion only.

Also related are Li, Sun, Wang, and Yu (2016), who focus on matching to share heterogeneous risks that partners bring to the table; and Chade and Eeckhout (2018), who analyze risk sharing and optimal sorting in the case in which agents have some ex ante attributes on which sorting is based, but the payoff-relevant attributes are realized after matching. In contrast to our work, neither paper emphasizes the role of complementarities in production of the sorting-relevant attributes or the trade-off between reaping productive complementarities and risk-sharing considerations in couples. Moreover, both papers focus on the special class of problems that is transferable-utility representable. This is in contrast to our characterization of monotone sorting, which provides insights into how skill complementarities and risk sharing interact in general.

2 The Model

For concreteness and in light of our main economic application, we cast the model in terms of a marriage market, but with notational changes it would apply to many other settings of partnership formation as well. Also, we make some simplifying assumptions to derive the main insights in the most transparent way, and leave the analysis of important variations for a later section.

There is a unit measure of women and of men. Each woman has an attribute $x \in [0, 1]$, distributed according to a cumulative distribution function (cdf) H with strictly positive density h, while each man has an attribute $y \in [0, 1]$ distributed according to a cdf G with strictly positive density g.[5] Women are risk averse with von Neumann-Morgenstern utility function for income given by $u : \mathbb{R}_+ \to \mathbb{R}$, which is continuous on $\mathbb{R}_+$ and three times continuously differentiable on $\mathbb{R}_{++}$, with $u' > 0$, $u'' \leq 0$, and $u''' \geq 0$.[6] Similarly, men are risk averse with von Neumann-Morgenstern utility function for income given by $v : \mathbb{R}_+ \to \mathbb{R}$, with analogous properties. We will denote the coefficients of absolute risk aversion of women and men by $R = -u''/u'$ and $r = -v''/v'$, respectively. Only for the purpose of ensuring an interior solution to the risk-sharing

[5]We will use the terms attribute and skill interchangeably in this paper.

[6]We use the following conventions. Given $z : X \to \mathbb{R}$, $X \subset \mathbb{R}^n$, we write $z \geq 0$ ($z \leq 0$) or $z > 0$ ($z < 0$) if it holds for all $x \in X$. We use increasing, decreasing, concave, convex, positive, etc., in the weak sense, adding "strictly" if needed. For a multivariate function z, we will denote by z_x the partial derivative with respect to x, and by $(z)_x$ the total derivative when x appears in more than one argument. When there is no confusion, we suppress the arguments of functions and limits of integrals.

problem, we assume that $\lim_{c\to 0} u'(c) = \lim_{c\to 0} v'(c) = \infty$.[7]

Women and men match pairwise in a competitive marriage market, for both production and risk-sharing purposes. Each matched couple produces some positive output, determined by function f that depends on the partners' attributes and on the realization of a production shock $\alpha \in [0,1]$, drawn from a continuous cdf L. This shock can be an aggregate shock to match output or an idiosyncratic shock to the pair's match output. Hence, if a woman with attribute x matches with a man with attribute y, and the shock realization is α, they produce $f(x,y,\alpha)$. The function f is increasing and supermodular, three times continuously differentiable with positive derivatives f_x, f_y, f_α, f_{xy}, $f_{x\alpha}$, and $f_{y\alpha}$, strictly so on $(0,1]^3$. Also, $f(0,y,\alpha) = f(x,0,\alpha) = 0$ for all α, and so a pair with the lowest attributes is unproductive. The output of unmatched agents is zero, an assumption we will relax below.

The shock is realized after matching and is contractible. Thus, before matching takes place a couple can flexibly decide how to share this risk. For example, if a woman with attribute x matches with a man with attribute y who has to be given a utility level of at least v_0, then the woman solves the following risk-sharing problem: $\phi(x,y,v_0) = \max_w \int u(w(\alpha))dL(\alpha)$ subject to $\int v(f(x,y,\alpha) - w(\alpha))dL(\alpha) \geq v_0$. Similarly, if the woman has to be given at least u_0, then the man solves $\psi(y,x,u_0) = \max_s \int v(s(\alpha))dL(\alpha)$ subject to $\int u(f(x,y,\alpha) - s(\alpha))dL(\alpha) \geq u_0$.

We will focus on monotone sorting in the matching market. To this end, let $\mu : [0,1] \to [0,1]$ be a measurable and measure-preserving one-to-one matching function that assigns each woman with attribute x to a man with attribute $y = \mu(x)$. Then μ is given either by $G(\mu(x)) = H(x)$ for all x (positive sorting), or by $G(\mu(x)) = 1 - H(x)$ for all x (negative sorting).

Let $\bar{u} : [0,1] \to \mathbb{R}$ and $\bar{v} : [0,1] \to \mathbb{R}$. An *equilibrium* consists of a triple $(\bar{u}, \bar{v}, \mu)$ that satisfies the following properties: *(a)* $\mu(x) \in \text{argmax}_y \phi(x,y,\bar{v}(y))$ and $\bar{u}(x) = \max_y \phi(x,y,\bar{v}(y)) \geq 0$ for all x; *(b)* $\mu^{-1}(y) \in \text{argmax}_x \psi(y,x,\bar{u}(x))$ and $\bar{v}(y) = \max_x \psi(y,x,\bar{u}(x)) \geq 0$ for all y; *(c)* μ is either $\mu(x) = G^{-1}(H(x))$ or $\mu(x) = G^{-1}(1 - H(x))$ for all x. In words, given the utility schedules and efficient risk sharing, each agent optimally chooses a partner with an attribute that coincides with the one allocated to them by the matching function μ, and their utility is bigger than their reservation utility, which equals zero.[8]

Remark 1 We have assumed that the shock, α, is a scalar but it is straightforward to relax this assumption and instead allow α to be a vector (α_1, α_2) with cdf $L(\alpha_1, \alpha_2)$, where within each match, α_1 affects the woman and α_2 the man (for example, these shocks can affect how much each can contribute to match output). The only new feature is that there can be complementarities between the shocks in production, and also that their stochastic dependence can affect how much

[7] These boundary conditions rule out constant absolute risk aversion utility (CARA), but as we will see below, our main results will apply to this case as well.

[8] Proving existence is beyond the scope of this paper. But if we instead define u and v from $\mathbb{R}$ to $\mathbb{R}$ with full range (recall that the boundary conditions are only to ensure interiority), then $\phi(x,y,\cdot)$ will have full range for each (x,y), and existence then follows from Proposition 6 in Nöldeke and Samuelson (2018).

each partner obtains in equilibrium (see Section 5). Things are more complex if we additionally relax the assumption that the reservation utility is equal to zero for every agent, and allow for attribute-dependent reservation utilities instead. This case is especially relevant in the marriage market when the attribute of each partner (along with a shock) determines their income, and agents can consume their own income if unmatched. We flesh out this extension in Appendix A.7, and show that our main insights extend to this case.

3 Main Results

In this section we will derive necessary and sufficient conditions for monotone sorting. Two-sided risk aversion and efficient risk sharing turns this problem into one of imperfectly transferable utility, and so the appropriate condition for positive sorting is Legros-Newman's generalized increasing difference condition (Legros and Newman, 2007). Our main result is a novel inequality that characterizes monotone sorting and has a clear economic intuition that balances complementarities in match output with risk-sharing considerations within the couple. We derive several classes of primitives commonly used in applications under which the inequality holds, and thus any equilibrium exhibits positive assortative matching along partners' productive attributes. We complement this analysis with a characterization of negative sorting.

3.1 Efficient Risk Sharing

Consider first the risk-sharing problem solved by a couple that consists of a woman with attribute x and a man with y who matched in the marriage market. The man has to receive a utility level of at least $v_0 \in [v(0), v_{max}(x, y)]$, where $v_{max}(x, y) \equiv \psi(y, x, 0) = \int_0^1 v(f(x, y, \alpha))dL(\alpha)$ and so the woman chooses the sharing rule of match output that maximizes her expected utility subject to the utility constraint plus some feasibility constraints:

$$\begin{aligned} \phi(x, y, v_0) = \max_{w} & \int_0^1 u(w(\alpha))dL(\alpha) \\ s.t. & \int_0^1 v(f(x, y, \alpha) - w(\alpha))dL(\alpha) \geq v_0 \\ & 0 \leq w(\alpha) \leq f(x, y, \alpha), \ \ \forall \alpha. \end{aligned} \tag{1}$$

It is clear that the first constraint always binds at the optimum. Also, if $v_0 = v(0)$ then the solution is $w(\alpha) = f(x, y, \alpha)$ for all α, and if $v_0 = v_{max}(x, y)$, then the solution is $w(\alpha) = 0$ for all α. In what follows we therefore focus on $v_0 \in (v(0), v_{max}(x, y))$.

Let λ be the multiplier of the first constraint, and ignore the second constraint since the solution is interior under our boundary conditions on u and v. A standard optimal control or perturbation argument yields the following well-known condition for optimal risk sharing:

$$\frac{u'(w(\alpha))}{v'(f(x,y,\alpha)-w(\alpha))}=\lambda \quad \forall\alpha. \tag{2}$$

From (2) one obtains a unique function $\hat{w}$, which depends on (α, x, y, λ), and differentiation and manipulation of (2) reveals that $\hat{w}$ increases in x, y, and α, and strictly decreases in λ:

$$\hat{w}_i=\frac{r}{R+r}f_i\geq 0,\;\; i=x,y,\alpha, \qquad \hat{w}_\lambda=-\frac{1}{\lambda(R+r)}<0. \tag{3}$$

Intuitively, given λ, an increase in x, y, or α increases f and thus reduces the man's marginal utility of v, which requires an increase in the share of output transferred to the woman, $\hat{w}$, to satisfy (2). In turn, an increase in λ means that it is harder to satisfy the problem's expected-utility constraint, and hence this effect reduces the share of match output accruing to the female partner for any given α.

Inserting $\hat{w}$ into the constraint, we obtain

$$\int_0^1 v(f(x,y,\alpha)-\hat{w}(\alpha,x,y,\lambda))dL(\alpha)=v_0. \tag{4}$$

Using the monotonicity properties of v and $\hat{w}$ in λ, it is easy to show that there is a unique $\lambda^*>0$ that solves (4). Moreover, λ^* is strictly increasing in the male partner's market utility, v_0, and strictly decreasing in the productive attributes, x and y. To see this, differentiate (4) and use the expressions for the derivatives in (3) to obtain

$$\lambda_i^*=-\lambda^*\frac{\int v'\frac{R}{R+r}f_i dL}{\int v'\frac{1}{R+r}dL}<0,\;\; i=x,y, \qquad \lambda_{v_0}^*=\lambda^*\frac{1}{\int v'\frac{1}{R+r}dL}>0. \tag{5}$$

Intuitively, if x or y increase, then output f increases and it is easier to meet the expected-utility constraint and hence its shadow price λ^* goes down. The opposite is true if v_0 increases, which tightens the expected-utility constraint.

Since $\hat{w}$ and λ^* are unique, there exists a unique sharing rule w^*, given by $w^*(\alpha,x,y,v_0)=\hat{w}(\alpha,x,y,\lambda^*(x,y,v_0))$ for all α. Thus, the value of the risk-sharing problem is $\phi(x,y,v_0)=\int_0^1 u(w^*(\alpha,x,y,v_0))dL(\alpha)$, which is the maximum utility that a woman with attribute x achieves when sharing risk with a man with attribute y who receives v_0. The following lemma summarizes intuitive properties of the solution to the risk-sharing problem, given by (w^*,ϕ), which we will use in the analysis of equilibrium sorting below.

Lemma 1 (Properties of w^* and ϕ) *The solution to the risk-sharing problem, given by (w^*,ϕ), has the following properties:*

(i) Derivatives of w^*: $w_i^*=\hat{w}_i+\hat{w}_\lambda\lambda_i^*>0$, $i=x,y$, $w_\alpha^*=\hat{w}_\alpha\geq 0$, and $w_{v_0}^*=\hat{w}_\lambda\lambda_{v_0}^*<0$;

(ii) Derivatives of ϕ: $\phi_i = \int u' f_i dL > 0$, $i = x, y$, and $\phi_{v_0} = -\lambda^* < 0$.

The derivatives of sharing rule w^* follow from the chain rule and the properties of $\hat{w}$ and λ^*. Intuitively, an increase in the couple's productive traits x or y increases match output f and thus the share of a woman with attribute x, and similarly for a better shock realization α. In addition, an increase in utility v_0 received by the male partner y leads to a decrease in the share obtained by x. In turn, the derivatives of the value of the risk-sharing problem, ϕ, follow from the Envelope Theorem. The intuition is that an increase in x or y shifts the Pareto frontier of the problem outwards, while an increase in v_0 reduces the maximum utility a woman with attribute x can obtain by an amount equal to the shadow price of the constraint.

Remark 2 Equivalently, we could solve the problem of a man with attribute y who shares risk with a woman that is characterized by x and must receive at least $u_0 \in [u(0), u_{max}(x, y)]$, where $u_{max}(x, y) \equiv \int_0^1 u(f(x, y, \alpha))dL(\alpha)$. Formally, his risk-sharing problem is $\psi(y, x, u_0) = \max_s \int_0^1 v(f(x, y, \alpha) - s(\alpha))dL(\alpha)$ subject to $\int_0^1 u(s(\alpha))dL(\alpha) \geq u_0$ and $0 \leq s(\alpha) \leq f(x, y, \alpha)$, $\forall\alpha$. Letting ξ be the multiplier of the expected-utility constraint and denoting by $\hat{s}$ the solution to the first-order condition of the risk-sharing problem, we obtain optimal sharing rule s^* as $s^*(\alpha, x, y, u_0) = \hat{s}(\alpha, x, y, \xi^*(x, y, u_0))$, and the problem's value, $\psi(y, x, u_0) = \int_0^1 v(f(x, y, \alpha) - s^*(\alpha, x, y, u_0))dL(\alpha)$. Since ϕ and ψ are inverse functions to each other, i.e., $u_0 = \phi(x, y, \psi(y, x, u_0))$ and $v_0 = \psi(y, x, \phi(x, y, v_0))$, the maximizers of the two problems are related by $w^*(\alpha, x, y, v_0) = s^*(\alpha, x, y, \phi(x, y, v_0))$ and $s^*(\alpha, x, y, u_0) = w^*(\alpha, x, y, \psi(y, x, u_0))$. Moreover, for all (x, y, v_0), the multipliers satisfy $\lambda^*(x, y, v_0) = 1/\xi^*(x, y, \phi(x, y, v_0))$.

Remark 3 Although the constant absolute utility function (CARA) does not satisfy our assumptions on u and v (its domain is the entire real line and it does not satisfy the assumed boundary conditions), one can show that all the results hold in this case as well. We provide the details in Appendix A.1 and proceed below under the assumption that CARA is subsumed by our analysis.

3.2 Characterization of Equilibrium Sorting

We now ask under which conditions our model can generate the robust empirical feature of *positive sorting* on skills in the marriage market, as illustrated in the left panel of Figure 1 based on German data. Using the value of the risk-sharing problem, ϕ (or ψ), we can characterize positive sorting using a differential version of the generalized increasing differences condition of Legros and Newman (2007) (see Chade, Eeckhout, and Smith (2017) for the derivation). This condition asserts that positive sorting obtains in equilibrium if and only if for all x, y, and feasible v_0, the (negative of the) marginal rate of substitution, $-\phi_y/\phi_{v_0}$, increases in x. More precisely, under this condition, any equilibrium in the marriage market is payoff equivalent to an equilibrium with positive sorting. And if it strictly increases in x, then *any* equilibrium exhibits positive sorting. That is, wlog, equilibrium matching is given by matching function μ with $\mu(x) = G^{-1}(H(x))$ for all x.[9]

The interpretation is that women with higher x are willing to transfer more utils v_0 per increase in male attribute y, i.e., they have a higher willingness to pay for more productive men. An equivalent way of expressing that $-\phi_y/\phi_{v_0}$ is increasing in x is

$$\frac{\phi_{xy}}{\phi_y} \geq \frac{\phi_{xv_0}}{\phi_{v_0}}, \tag{6}$$

which neatly reveals that equilibrium positive sorting obtains if, for each x, the attribute complementarities of ϕ in (x, y) per unit of marginal utility of y, given by the left-hand side (lhs) of (6), are larger than the attribute-transfer complementarities per unit of marginal utility of the partner, given by the right-hand side (rhs) of (6). Intuitively, if it is increasingly difficult for high-x women to transfer utility to the male partner (captured by $\phi_{xv_0}/\phi_{v_0} > 0$), then stronger complementarities in partners' attributes ϕ_{xy}/ϕ_y are needed to guarantee positive sorting. In turn, if it becomes easier for high-x women to transfer utility to the male partner (captured by $\phi_{xv_0}/\phi_{v_0} < 0$), positive sorting ensues if $\phi_{xy} \geq 0$ (recall $\phi_y > 0$), which resembles the well-known result from the case of perfectly transferable utility. Clearly, if $-\phi_y/\phi_{v_0}$ decreases in x, or equivalently, if we reverse the sign of (6), we obtain a characterization of *negative sorting*.

To highlight the difficulty of ensuring monotone sorting, we focus on positive sorting and discuss each side of (6). Consider the right side, which we claim is always negative: Note that $\phi_{v_0} = -\lambda^* < 0$, which reflects the intuitive property that transferring utility to y is costly for x. In addition, $\phi_{xv_0} = -\lambda^*_x > 0$ since $\lambda^*_x < 0$: An increase in x increases output f and makes it easier to satisfy the constraint, $\int v(f - w^*)dL \geq v_0$, so its shadow price goes down. In words, the higher is female attribute x, the *easier* it is for her to transfer utility to the male partner ($\phi_{xv_0} = -\lambda^*_x > 0$ along with $\phi_{v_0} < 0$ renders the rhs of (6) negative).

Positive sorting therefore emerges if $\phi_{xy} \geq 0$. But, unlike the risk-neutral case, here complementarities in f, the classical force toward positive sorting, face an opposing force that stems from risk aversion, which leads to an ambiguous sign of ϕ_{xy}. Formally, from Lemma 1 (*ii*), $\phi_x = \int u' f_x dL$. If u' was a constant (risk neutrality), then well-known attribute complementarities, $f_{xy} \geq 0$, would yield (6). When agents are strictly risk averse, however, we have

$$\phi_{xy} = \int u' f_{xy} dL + \int u'' w^*_y f_x dL.$$

The first term is positive under our assumption of supermodular f and, despite the presence of

[9]The Legros-Newman condition is a weak single-crossing property and equivalent to the Spence-Mirrlees condition we use. And if the Spence-Mirrlees condition is strict, then the single-crossing property is strict as well. See Edlin and Shannon (1998), Theorem 2.1; their assumed regularity condition is automatically satisfied in our case given the continuous differentiability of ϕ (and ψ) and the strict sign of its derivatives.

u' and the integral with respect to α, it reflects the standard attribute complementarity in match output that is also present in the risk-neutral case. The second term, however, is new and due to risk aversion. It is negative and can be interpreted as an income effect: The marginal utility of x from being matched with a higher y is lower since higher y implies more income for x, which at the margin is less valuable due to risk aversion. The sign of ϕ_{xy} is a priori ambiguous, and we will see below that it is not hard to construct examples in which the second term is strong enough to render ϕ_{xy} negative and yield negative sorting.

It follows that in order to ensure positive sorting, we need to exploit the slack available on the rhs of (6). We will use the expressions for the derivatives of ϕ from Lemma 1 to provide an insightful inequality that characterizes positive sorting and allows for the derivation of classes of primitives f, u, and v, under which any equilibrium exhibits positive sorting.

To this end, define the "marginal utility-adjusted" density of shock α given (x, y, v_0):

$$m(\alpha, x, y, v_0) = \frac{\frac{u'(w^*(\alpha,x,y,v_0))}{R(w^*(\alpha,x,y,v_0))+r(f(x,y,\alpha)-w^*(\alpha,x,y,v_0))} l(\alpha)}{\int \frac{u'(w^*(\alpha,x,y,v_0))}{R(w^*(\alpha,x,y,v_0)))+r(f(x,y,\alpha)-w^*(\alpha,x,y,v_0))} dL(\alpha)}, \tag{7}$$

which can also be written using v' instead of u' by using the first-order condition for efficient risk sharing, $u' = \lambda v'$. Any expectation that uses this density will be indicated by a subscript m to distinguish it from an expectation that uses the original shock density, l. We are now ready to state our main result:

Theorem 1 (Sorting and Risk Sharing) *Positive sorting emerges if and only if for all* $(x, y, v_0) \in [0,1] \times [0,1] \times [v(0), v_{max}(x, y)]$,

$$\mathbb{E}_m[f_{xy}(R + r)] \geq cov_m(f_x R, f_y r). \tag{8}$$

If the inequality is reversed, then it characterizes negative sorting.

The proof is in Appendix A.2 but the idea is simple and based on showing that $-\phi_y/\phi_{v_0}$ is increasing in x. From Lemma 1 and equation (2), we have $-\phi_y/\phi_{v_0} = \int u' f_y dL/\lambda^* = \int v' f_y dL$. This is intuitive: A woman with attribute x is willing to pay up to $\int v' f_y$ more utils for a man with a marginally better attribute y, which amounts to transfer the entire marginal gain from a better match to the man. The rest of the proof manipulates the derivative of $\int v' f_y dL$ with respect to x to obtain (8).

When individuals choose a partner in the marriage market, they balance a (potential) trade-off between productive complementarities in match output and risk-sharing opportunities. The lhs of (8) is a modified version of the pure attribute-complementarity effect, while the covariance on the rhs combines the income effect of the attribute-complementarity with the utility-transferability effect, which together—as we will argue—shape the potential to diversify risk within a couple. If this covariance is negative, then (8) clearly holds and positive sorting in (x, y) materializes; if not, one needs to ensure that the pure attribute complementarity is strong enough.

Our novel characterization of positive sorting has an intuitive interpretation. Complementarities in the match output function always push toward positive sorting. However, this motive for positive sorting can be counteracted if the scope for risk-sharing is reduced in couples of similar attributes. This happens when the covariance of spouses' (weighted) marginal products—that is, of partners' weighted output changes as we increase their skills—on the rhs of (8) is positive, where the weights are given by their coefficients of absolute risk aversion. This shows that spouses' risk-sharing opportunities are shaped by two factors: the co-movement of partners' income changes as we move up their skill distributions as well as the co-movement and strength of their risk aversions. For pedagogical reasons, we first discuss them separately, as if only one of them was at play.

When, in the face of risk α, spouses' match-output contributions co-move positively as we increase both of their skills (which is the case under our assumptions on $f_{x\alpha}$ and $f_{y\alpha}$, pushing toward a positive covariance), the scope for risk diversification within the couple is limited. This drives the matching toward negative sorting whereby a woman with higher x prefers a man with lower y. Positive sorting then only arises if the pure attribute complementarity in match output is strong enough. In contrast, a negative relation between spouses' marginal contributions to match output (pushing toward a negative covariance) improves insurance opportunities of positively assorted couples, so that both productive complementarities and risk-sharing considerations push in the direction of positive sorting.

In turn, if income shock α induces a positive co-movement of partners' risk aversion—which depend on each spouses' level of consumption and thus on (x, y, α)—this also hampers insurance opportunities in marriage. This is because risk-sharing benefits are larger when partners' differences in risk aversion are sizeable. Men and women then seek partners of opposite skill levels, which pushes toward negative sorting. This drives a wedge between their consumption levels and—for the large class of utility functions in which risk aversion is monotone in consumption—between their coefficients of absolute risk aversion, and thus augments the risk-sharing benefits of marriage.

Ultimately, *both* of these factors—marginal contributions of both partners to match output and risk aversion—shape risk-sharing opportunities in couples, captured by the covariance of partners' marginal products in skills, *scaled* by their coefficients of absolute risk aversion.

3.3 Equilibrium Sorting Based on Primitives

Clearly, inequality (8) is stated in terms of endogenous objects since u', R, and r have the optimal risk-sharing rule, w^*, as an argument. Hence, it is not directly applicable as a condition for sorting. Our next task is to derive conditions from primitives such that (8) holds.

Recall that constant absolute risk aversion utility function (CARA) is given by $(1 - e^{-\rho c})/\rho$, where $\rho > 0$ is the coefficient of absolute risk aversion; and the constant relative risk aversion utility function (CRRA) is given by $(c^{1-\sigma}-1)/(1-\sigma)$, $\sigma>0$, whose coefficient of absolute risk aversion is σ/c. Finally, the hyperbolic absolute risk aversion utility function (HARA) is given by $\sigma/(1-\sigma)(((ac/\sigma)+b)^{1-\sigma}-1)$, where $a>0$ and $(ac/\sigma)+b>0$ for all levels of c. This utility function subsumes as special cases CRRA (set $a=\sigma$ and $b=0$) and logarithmic utility (set $a=\sigma$ and $b=0$ and take the limit as $\sigma\to 1$). The coefficient of absolute risk aversion for HARA is given by $1/((c/\sigma)+(b/a))=\sigma a/(ac+\sigma b)$.

Our next result provides several cases in which we can sign (8) using conditions on primitives u, v, and f. To ensure that positive sorting emerges in *any* equilibrium we will focus on primitives that ensure (8) with strict inequality. We will need one definition: We say that f is a *separable class* if $f(x,y,\alpha)=\eta(\alpha)z(x,y)$ with η and z positive, increasing, and twice-continuously differentiable, and z is supermodular, strictly so on a set of values of (x,y,α) of positive measure; it is a *log-separable class* if "supermodular" is replaced by "log-supermodular".

Proposition 1 (Positive Sorting) *Positive sorting materializes in the following cases:*

(i) Either women or men are risk neutral;

(ii) Women and men have CRRA utility functions with risk aversion parameters $\sigma_1>0$ *and* $\sigma_2>0$*, respectively, and* f *is a separable class;*

(iii) Women and men have identical HARA utility functions with $b\geq 0$ *and* $\sigma\in(0,1]$*, and* $f(\cdot,\cdot,\alpha)$ *is log-supermodular in* (x,y) *for each* $\alpha\in[0,1]$*;*

(iv) Women and men have CARA utility functions with $R(w)=R>0$ *and* $r(w)=r>0$ *for all* w*, respectively,* f *is a log-separable class with* $\eta(\alpha)=\alpha$ *for all* α*,* $\mathbb{E}_m[\alpha]>\beta var_m(\alpha)\ \forall\beta>0$*, where* $m=e^{-\beta\alpha}l/\int e^{-\beta\alpha}dL$ *and* $\beta\equiv(Rr/(R+r))z$*, and* β *is sufficiently small.*

The proof is in Appendix A.3. What these cases have in common is that partners' risk-sharing motive does not interfere with the positive sorting pattern that skill complementarities call for. We accomplish this by either eliminating the importance of risk sharing in couples or allowing one party to absorb all the risk (part (i)), which effectively reduces the problem to the standard one with transferable utility, and so a strictly supermodular f suffices for positive sorting; or by making the covariance of spouses' income changes on the rhs of (8) sufficiently small (part (iii)) or negative (part (ii)), so that risk is adequately diversified within positively assorted couples.[10]

Regarding part (iv), the condition for positive sorting holds for small β—meaning for small risk aversion of either husband or wife—which essentially follows from continuity since it holds for $\beta=0$. It is easy, however, to construct examples in which positive sorting holds for *all* values of β. In Appendix A.4 we show that $\mathbb{E}_m[\alpha]>\beta\text{var}_m(\alpha)$ holds for all β when (a) α is distributed

[10]Part (ii) is somewhat surprising since this case cannot be reduced to a transferable-utility representable case, and absolute risk aversion is not constant in income, which makes the income effect described above hard to tame. Note that in the special case in which $\sigma_1=\sigma_2$ the covariance is zero. Regarding part (iii), we impose the restriction of a common utility function on both sides, which pays off by subsuming some classes of problems that are not contained in (i)–(ii), since it allows for f non-separable and some additional utility functions in the HARA class.

on $[0, \infty)$ according to a gamma distribution with shape and rate parameters $\xi > 0$ and γ, which subsumes the exponential and chi-squared densities, among others; and (b) when α is uniformly distributed on $[0, 1]$.[11] But, we also provide an example with a binary shock in which the result robustly fails.[12]

Our model assumes that f exhibits complementarities in (x, y, α), which makes the left side of (8) (strictly) positive. Without these assumptions, it is straightforward to provide classes of primitives for which equilibrium exhibits negative sorting instead, or for which there are no clear (i.e., monotone) sorting patterns at all.

Proposition 2 (Negative Sorting and No Sorting Pattern)

(i) Assume that f is twice continuously differentiable, strictly increasing, submodular in (x, y) for each α, and strictly supermodular in (x, α) for each y and in (y, α) for each x. Also assume that women and men are strictly risk averse with utility functions that exhibit increasing absolute risk aversion (IARA). Then any equilibrium entails negative sorting;

(ii) Assume that f is given by $f(x, y, \alpha) = ax + by + c\alpha$, $a > 0$, $b > 0$, $c > 0$. Also assume that both women and men have utility functions that exhibit strictly monotone absolute risk aversion in the same direction. Then any equilibrium entails negative sorting;

(iii) Assume that f is given by $f(x, y, \alpha) = ax + by + c\alpha$, $a > 0$, $b > 0$, $c > 0$. If either women or men have CARA utility functions with $R > 0$ and $r > 0$, respectively, then sorting is neither positive nor negative.

The proof is in Appendix A.5. Cases (i)–(ii) in the proposition reverse the inequality in (8). They both assume that match output is either additively separable in the contributions of the matched partners, or substitutable (submodular) in their contributions, which renders $f_{xy} \leq 0$ on the lhs of (8). In turn, the covariance term on the rhs of (8) is positive due to either IARA plus our assumptions that make the marginal contributions of the partners to match output, f_x and f_y, co-move in the face of risk; or due to strictly monotone absolute risk aversion in the same direction (either strictly decreasing—strict DARA—or strictly increasing—strict IARA) and additively separable f. This implies that positive sorting on skills (x, y) would reduce risk-sharing opportunities in couples—a clear force toward matching in a negatively assortative way. Note that (i) subsumes the CARA case and (ii) subsumes the case in which the only reason for matching is to share risks, without any attribute complementarities.[13] Finally, in (iii) partners' attributes are neither complementary nor substitutable in match output, nor do they affect risk-sharing opportunities. That is, (8) holds with equality, and thus any sorting pattern is optimal.

[11] We have also numerically checked other examples with log-normally, Frechet, Beta, and Pareto distributed α.

[12] For another example with CARA in which (8) fails, let $f(x, y, \alpha) = \alpha(a_1 x + a_2 y + a_3 xy)$, $a_i > 0$, $i = 1, 2, 3$. Then the inequality in (iv) is $a_3 \mathbb{E}_m[\alpha] > (Rr/(R + r))(a_1 + a_3)(a_2 + a_3 x)\text{var}_m(\alpha)$, which fails for a_3 sufficiently small.

[13] An example of f that satisfies the assumptions in (i) is $f(x, y, \alpha) = (x + y)^{\delta}\alpha$, $\delta \in (0, 1)$.

4 What Drives Positive Marriage Market Sorting on Skills?

Our analysis highlights the fact that when men and women form couples to share risk, this is a natural force toward negative sorting on skills. The matching of parters with unequal skills tends to improve spouses' risk-sharing opportunities. Therefore, sufficiently strong skill complementarities in the marriage value are needed in order to generate the positive sorting patterns on skills/education that have been widely documented in the data (see, e.g., Figure 1). A crucial question revolves around the sources of skill complementarities within the context of marriage.

In this section, we discuss several microfoundations for skill complementarities in match output f as well as extensions that are guided by the literature and introduce additional complementarities in the couple's problem to counteract the risk-sharing force.

Microfoundation for Skill Complementarities in Match Output f. We flesh out a number of sources of skill complementarities in f.

First, suppose the couple's problem is based on Calvo, Lindenlaub, and Reynoso (2023), whose model features public good production in the household, whereby spouses' time inputs in home production are complementary, along with firms that value workers' labor hours. This setting gives rise to individual wages that depend on labor hours and, through this channel, not only on own but also the partner's attribute. We consider an extension of this model that introduces income risk α. The resulting individual wages for women and men $\omega_i(x, y, \alpha), i \in \{1, 2\}$ are complementary in skills (x, y) for each α, $\partial^2\omega_i(x, y, \alpha)/\partial x\partial y > 0$. Match output f, which we think of as the sum spouses' wages $f(x, y, \alpha) = \omega_1(x, y, \alpha) + \omega_2(x, y, \alpha)$, thus features skill complementarities $f_{xy} > 0$.

Second, complementarities in match output can stem from spouses' complementary career investments. Suppose again that match output is a function of spouses' labor market incomes. Having a richer and thus more successful partner allows not only for targeted career investment of the other partner that increases their income; but it also increases the other partner's informal referral networks and improves the flow of information about job opportunities, which increases their earnings potential (Luke, Munshi, and Rosenzweig, 2004; Luke and Munshi, 2006; Wang, 2013). Moreover, a high-earning spouse has more occupational resources and social capital that facilitate the other partner's success in the labor market (Bernasco, de Graaf, and Ultee, 1998; Bernardi, 1999). These examples can be captured by a match output function that is complementary in spouses' wages, $f(x, y, \alpha) = \omega_1(y, \alpha) + \omega_2(x, \alpha) + \omega_1(y, \alpha)\omega_2(x, \alpha)$, where we assume that each individual wage is increasing in their skill, which renders $f_{xy} > 0$.

Finally, an interesting potential avenue to micro-found "complementarities" in f is via a risky portfolio choice problem faced by couples after they match. Assume that each partner's wage is equal to their skill, so that a couple with skills x and y has joint income $x + y$. This income is allocated between a safe asset with return of one and a risky asset with return $\alpha \in [a, b]$, $a \geq 0$ and

$b > 1$, the proceeds of which are consumed by the couple. We can write the couple's problem as

$$
\begin{aligned}
\phi(x, y, v_0) = &\max_{\kappa, s} \int_a^b u(x + y - (1-\alpha)\kappa - s(\alpha)) dL(\alpha) \\
&s.t. \int_a^b v(s(\alpha)) dL(\alpha) \geq v_0 \\
&\quad 0 \leq s(\alpha) \leq x + y - (1-\alpha)\kappa, \;\; \forall \alpha \\
&\quad 0 \leq \kappa \leq x + y,
\end{aligned}
$$

where κ is the amount of $x + y$ invested in the risky asset (and the rest in the safe asset), and s is the share of total ex-post income $x + y - (1-\alpha)\kappa$ allocated to the male partner. The optimal solution is a pair of functions, $s^*(\alpha, x + y, v_0)$ and $\kappa^*(x + y, v_0)$; and the analog to our match output function f is $x + y - (1-\alpha)\kappa^*(x + y, v_0)$ in this case. Although f is not supermodular in (x, y) for each α, since $1 - \alpha$ can be positive or negative, a variation of our sorting condition (8) still characterizes positive sorting.

Complementarities Beyond Match Output f. A natural extension of our model is to augment the utility function to allow for an additional source of well-being, either stemming from home production, leisure or love. Assume the utility functions for men and women are

$$
\begin{aligned}
&u(c_1) + h(x, y) \\
&v(c_2) + h(x, y),
\end{aligned}
$$

where u and v are functions of private consumption (c_1, c_2) and have the same properties as before; and h—capturing home production, leisure or love—is strictly increasing in both arguments and strictly supermodular. These skill complementarities can capture that home production is more productive if spouses of similar skills match (Gayle and Shephard, 2019; Calvo, 2022); the fact that it is more pleasant to spend leisure with someone who has similar interests based on comparable education levels (Mansour and McKinnish, 2014); or homophily preferences in matching (e.g., Chiappori, Dias, and Meghir, 2018 and Reynoso, 2023).

We show in Appendix A.8 that the condition for positive sorting in Theorem 1 becomes

$$
\mathbb{E}_m[f_{xy}(R + r)] + T(h_{xy}, h_x, h_y) \geq cov_m[f_x R, f_y r],
$$

where the new term, T, is a function that strictly increases in all of its arguments (see Appendix A.8 for the exact expression). Thus, introducing home production, leisure or love into our problem facilitates positive sorting: $h_{xy} \geq 0$ strengthens the skill complementarities in the match value (which can be generated by one of the mechanisms highlighted above), while $h_x > 0$ and $h_y > 0$ make it easier to transfer utility to high skills—both pushing toward positive sorting.

5 Equilibrium Implications for Risk Sharing

To derive our characterization of monotone sorting when agents are risk averse and match output is subject to risk and complementarities in partners' attributes, we used a differential version of the generalized increasing/decreasing difference condition (Legros and Newman, 2007). Pinning down optimal sorting patterns did not require the derivation of the full equilibrium. Although a full equilibrium analysis is beyond the scope of the paper, in this section we solve for the unique equilibrium when men and women have identical CRRA utility functions, a widely used form of preferences that has found considerable empirical support (e.g., Brunnermeier and Nagel, 2008 and Chiappori and Paiella, 2011). Under CRRA, we can derive this equilibrium of the marriage market in closed form, which allows us to obtain several new implications for risk sharing among spouses that go beyond the results on sorting from the last sections. Not only do we analyze the implications of positive sorting on skills for sorting on (endogenous) risk aversion. We also provide new comparative statics of risk, with focus on how changes in risks—stochastically better, more variable or more correlated risks among spouses—affect risk-sharing arrangements within the household and intra-household inequality.

Environment. Men and women have identical CRRA utility functions with parameter $\sigma \in (0,1)$, so that u is given by $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$.[14] Further assume that f is a separable class, $f(x, y, \alpha) = \eta(\alpha) z(x, y)$. Appendix A.6 contains the omitted technical details from this section.

Equilibrium. We first construct the equilibrium of the market. Under the imposed assumptions, Proposition 1 (*iii*) shows that any equilibrium exhibits positive sorting and hence $\mu(\cdot) = G^{-1}(H(\cdot))$. Toward obtaining the sharing rule, consider the risk-sharing problem of a woman with x and a man with y who must be given expected utility of at least v_0. Following the same steps as in Section 3, we obtain

$$w^*(\alpha, x, y, v_0) = \left(1 - \left(\frac{v_0(1-\sigma)+1}{\int (f(x,y,\alpha))^{1-\sigma} dL(\alpha)}\right)^{\frac{1}{1-\sigma}}\right) f(x, y, \alpha). \tag{9}$$

This sharing rule is intuitive and owes to the assumption that men and women have the same CRRA utility function. Under optimal risk sharing, a woman with attribute x receives a fraction of output f that is independent of α, and similarly for her spouse y. The fraction that x receives is decreasing in v_0 (the more utility needs to be given to her partner y, the smaller is the share x receives), and increasing in x and y (since f increases and thus there is more to share while v_0 is fixed).

Inserting (9) into the problem's value, $\int u(w^*(\alpha, x, y, v_0) dL(\alpha)$, yields $\phi(x, y, v_0)$, the maximum expected utility that a woman with x obtains when matched with a man of type y who requires

[14] All the results go through for $\sigma > 0$ with minor modifications. For simplicity, we will focus on the case $\sigma \in (0, 1)$.

utility v_0. A straightforward monotone transformation allows us to represent her preferences as $\hat{\phi} = (1 + (1 - \sigma\phi)^{1/(1-\sigma)}$, given by

$$\hat{\phi}(x, y, \pi_0) = F(x, y) - \pi_0, \tag{10}$$

where we have set $F \equiv (\int f^{1-\sigma} dL)^{1/(1-\sigma)} = z(x, y)(\mathbb{E}[\eta^{1-\sigma}])^{\frac{1}{1-\sigma}}$ and $\pi_0 = (v_0(1 - \sigma) + 1)^{1/(1-\sigma)}$. The problem now exhibits transferable utility, where $\hat{\phi}$ is the share of "output" F that x receives if y is given a share π_0. So we can think of the equilibrium in this market in terms of an equivalent problem where matched pairs share F and have quasilinear payoffs. Note that F is strictly supermodular in (x, y) since z is, which confirms that any equilibrium exhibits positive sorting.

Given the transferable-utility representation of the model, we can focus on the equilibrium shares of output F and then recover the equilibrium utilities from it. Assume that $\pi(y)$ is the share of F that a man with attribute y receives in equilibrium and $\gamma(x)$ is the corresponding share of a woman with x. These shares are pinned down by the woman's and man's optimization problems in the matching market:

$$\pi(y) = \int_0^y F_y(\mu^{-1}(s), s)ds = \left(\mathbb{E}[\eta^{1-\sigma}]\right)^{\frac{1}{1-\sigma}} \int_0^y z_y(\mu^{-1}(s), s)ds,$$
$$\gamma(x) = \int_0^x F_x(s, \mu(s))ds = \left(\mathbb{E}[\eta^{1-\sigma}]\right)^{\frac{1}{1-\sigma}} \int_0^y z_x(s, \mu(s))ds.$$

It is now routine to show that (μ, π, γ) is the unique equilibrium in the marriage market, given by the positive sorting matching function $\mu(\cdot) = G^{-1}(H(\cdot))$, and shares of output F for all women and men in couples given by functions γ and π, respectively.

Sorting on Risk Aversion. Although, in terms of exogenous heterogeneity, the agents only differ in skills (x, y), our model makes predictions on sorting based on (endogenous) heterogeneity in risk aversion as well. The reason is that differences in skills across couples cause differences in match output, which translate into differences in output shares and consumption of partners. This channel causes risk aversion to be *endogenously* different across individuals ex post—i.e., after the realization of their output contributions and consumption shares—if their utility function exhibits decreasing absolute risk aversion, as CRRA does. We will now show that this setting, which predicts marital sorting on skills (x, y), also has clear-cut implications for sorting on risk aversion.

As mentioned in the Introduction, an established result is that if, from an ex ante point of view, individuals only differ in risk aversion—i.e., their absolute risk aversion differs for *the same* level of consumption—then equilibrium sorting on risk attitudes is negative (Schulhofer-Wohl, 2006; Legros and Newman, 2007; Chiappori and Reny, 2016). The reason is that risk-sharing opportunities within couples can be better reaped when risk aversion among spouses differ. This is because the less risk-averse partner is happy to provide insurance to the more risk-averse one, in exchange for a high price (transfer) which the more risk-averse spouse is happy to pay.

Whereas existing theoretical work focuses on sorting on exogenously heterogeneous risk preferences, we complement these results by highlighting the implications of skill sorting for matching patterns on endogenous differences in risk aversion. Indeed, an important implication of positive sorting on skills is *positive sorting* on risk aversion, as measured by the covariance of the coefficient of absolute risk aversion of matched partners. Define the coefficients of absolute risk aversion for husband and wife in this setting by $\tilde{R}(x,\alpha) \equiv \sigma/w^*(x,\mu(x),\alpha,\pi(\mu(x)))$ and $\tilde{r}(x,\alpha) \equiv \sigma/(f(x,\mu(x),\alpha) - w^*(x,\mu(x),\alpha,\pi(\mu(x))))$.

Proposition 3 (Sorting on Absolute Risk Aversion) *If men and women have the same CRRA utility function with $\sigma \in (0,1)$ and f is a separable class, then positive sorting on skills (x,y) with $\mu'(x) > 0$ implies positive sorting on risk aversion in the sense that cov$(\tilde{R}(x,\alpha),\tilde{r}(x,\alpha)) > 0$.*

The intuition is clear. Under positive sorting on skills, high-x women match with high-y men, which implies that their contributions to match output are also high and so are their individual consumption shares $f - w^*$ and w^*, when compared with less skilled couples. High consumption shares dampen *both* partners' absolute risk aversion. As a result, there is little variation in risk aversion within couples but lots of variation across couples. We see couples in which either both partners have low risk aversion—the high-skilled matches—or both have high risk aversion pertaining to low-skilled couples. Risk aversion is therefore *positively* correlated among partners that match in the marriage market. Importantly, positive sorting on risk aversion is an implication of positive sorting on skills and occurs despite the fact that in our setting there is no exogenous variation in risk aversion across individuals.[15]

It is notoriously difficult to empirically measure marriage market sorting on risk aversion because most surveys lack information on respondents' risk attitudes. But the scant evidence that has emerged indeed supports the notion of positive sorting on spouses' risk aversion (see the right panel of Figure 1 and also Dohmen, Falk, Huffman, and Sunde, 2011 and Serra-Garcia, 2021), in line with our Proposition 3. This empirical evidence is based on survey data from the German Socio Economic Panel, which asks questions about respondents' risk attitudes. Naturally, the survey answers reflect a mix of individuals' preference heterogeneity (outside of our model) and wealth/consumption heterogeneity (present in our setting), which impacts risk aversion unless the underlying utility function features constant absolute risk aversion. But as long as the reported risk attitudes are affected by individuals' wealth and consumption, we view this evidence as supportive of our theoretical prediction.

Comparative Statics of Risk. The closed-form solution for the unique marriage market equilibrium affords interesting comparative statics of risk. We analyze how changes in the shock

[15] To make this statement on the lack of exogenous variation in risk aversion precise, note that in our CRRA setting there is no heterogeneity in risk aversion across agents with *the same* level of consumption.

distributions affect risk-sharing opportunities within each household, as well as consumption inequality among partners. Given CRRA and a separable class f, the shock only affects payoffs in a multiplicative way, so the analysis of changes in the shock distribution under different stochastic shifts is quite tractable. We will consider three shifts in the distribution of risk.

We first consider a first-order stochastic dominance (FOSD) shift in the distribution of the shock, L. Since $\eta^{1-\sigma}$ is increasing in α, any such change increases $\mathbb{E}[\eta^{1-\sigma}]$ and thus increases the shares γ and π for every x and y. Intuitively, a stochastically better shock leads to higher levels of expected utility for both men and women of each attribute x and y, respectively.

Next, we analyze a change in the variability of spouses' shocks. Assume that the shock becomes larger in the convex order (that is, riskier in the Rothschild-Stiglitz sense; for example, L undergoes a mean-preserving spread). Then $\mathbb{E}[\eta^{1-\sigma}]$ goes up if $\eta^{1-\sigma}$ is convex in α, which reduces to $\eta\eta'' - \sigma\eta'^2 \geq 0$, a simple condition to impose on primitives, which holds if η is log-convex in α. As a result, the shares of F going to the female and male partner, given by γ and π, increase for every matched pair. In contrast, if $\eta^{1-\sigma}$ is concave in α (for which a concave η is a sufficient condition), then $\mathbb{E}[\eta^{1-\sigma}]$ goes down, and both partners receive less in response.

Finally, we turn to the case in which the shock is a vector and we analyze an increase in the dependence of spouses' shocks. Assume that the joint density of shocks is given by $\ell(\alpha|t) = \ell(\alpha_1, \alpha_2|\tau)$, where α_1 is the shock pertaining to the woman's income or match output contribution and α_2 to the man's. In turn, τ is a scalar that indexes the degree of dependence between these shocks. Higher τ means stronger dependence in the sense of the supermodular order.[16] Now η is assumed to be a positive and increasing function of both shocks. It follows that $\mathbb{E}[\eta^{1-\sigma}]$ decreases as τ increases if $\eta^{1-\sigma}$ is submodular in (α_1, α_2). If so, F goes down in τ, and hence for each value of π_0 the Pareto frontier shifts inward, and both consumption shares γ and π decrease as well.

The intuition is clear. When agents are risk averse, a stronger dependence of their income shocks limits risk-sharing opportunities within the couple, reflected in an inward shift of the Pareto frontier, and in equilibrium the shares that both partners receive decrease.

When is $\eta^{1-\sigma}$ submodular in (α_1, α_2)? The sign of the cross-partial derivative of $\eta^{1-\sigma}$ is equal to the sign of $\eta\eta_{\alpha_1\alpha_2} - \sigma\eta_{\alpha_1}\eta_{\alpha_2}$, which is negative if η is submodular in (α_1, α_2). If instead η is log-supermodular in (α_1, α_2), then $\eta\eta_{\alpha_1\alpha_2} - \eta_{\alpha_1}\eta_{\alpha_2} \geq 0$, and this implies $\eta\eta_{\alpha_1\alpha_2} - \sigma\eta_{\alpha_1}\eta_{\alpha_2} > 0$. Hence, in this case, a stronger dependence of spouses' shocks yields a net gain for the couple since the benefit stemming from the complementarity of the shocks in match output outweighs the cost of diminished risk sharing.

We summarize these results in the following proposition.

[16] A random vector $X = (X_1, X_2)$ is larger than a random vector $Y = (Y_1, Y_2)$ in the supermodular order if $\mathbb{E}[k(X)] \geq \mathbb{E}[k(Y)]$ for all supermodular functions k for which the expectations exist. In particular, this implies that $\text{cov}(X_1, X_2) \geq \text{cov}(Y_1, Y_2)$. See Muller and Stoyan (2002) and Meyer and Strulovici (2013) for a detailed discussion of this stochastic order.

Proposition 4 (Comparative Statics of Risk) *If men and women have the same CRRA utility function with $\sigma \in (0,1)$ and f is a separable class, then:*

(i) A stochastically larger shock (FOSD) increases the equilibrium shares of every agent;

(ii) A stochastically more variable shock (an increase in risk) increases the equilibrium utility of each agent if $\eta^{1-\sigma}$ is convex, and decreases it if $\eta^{1-\sigma}$ is concave;

(iii) If $\alpha = (\alpha_1, \alpha_2)$, then an increase in the dependence of the shocks (in the sense of the supermodular order) decreases the equilibrium utility of each agent if η is submodular in (α_1, α_2), and increases it if η is log-supermodular.

Under the three stochastic shifts, these are the only effects of the comparative statics of risk, since positive sorting or the efficient risk-sharing rule (9) are not affected.[17]

6 Concluding Remarks

How a group of individuals optimally shares risk is a well-trodden problem in economics with numerous applications, ranging from project diversification by firms to risk-sharing arrangements within households. We embed this problem in a marriage market setting and ask what, in theory, generates the empirically robust prediction of positive sorting on partners' skills in the presence of risk sharing within households and productive complementarities in match output.

Little is known about how risk sharing and matching interact in this type of setting and shape sorting patterns among spouses. Existing theoretical work focuses on heterogeneity in risk aversion as the only sorting-relevant variable, which renders negative sorting optimal as it maximizes risk-sharing benefits within couples. It disregards, however, the diversity in skills or education of partners, which constitutes the central differentiating factor in real-world marriage markets. As such, it is silent about the extensively documented phenomenon of positive assortative matching on skills in the marriage context.

We develop a matching model in which risk-averse agents differ in skills and match pairwise for risk-sharing *and* productive purposes. Match output has stochastic returns and matched couples efficiently share this risk. We find that under plausible assumptions the risk-sharing benefit of marriage tends to push toward negative sorting on partners' skills, a force that needs to be counteracted by sufficiently strong skill complementarities in match output in order to predict the empirically documented positive skill sorting. We derive a novel inequality that characterizes monotone sorting in equilibrium, balancing out skill complementarities and risk-sharing considerations in the right way, and provide several common classes of primitives that satisfy this inequality. Specializing the model to a canonical setting allows us also to highlight a new impli-

[17] To see this, it suffices to note that one can use the optimality condition and the constraint to show that the term in parenthesis in (9) is equal to $\lambda^{\frac{1}{\sigma}}/(1+\lambda^{\frac{1}{\sigma}})$, and if one replaces $((1-\sigma)v_0+1)^{\frac{1}{1-\sigma}}$ by $\pi(y)$ and also uses $\mu(\cdot)$, then after simplification one obtains that the ratio in λ's is, in equilibrium, independent of the distribution of α.

cation of positive sorting on exogenous skills for matching patterns on *endogenous* differences in risk aversion: Positive sorting on skills translates into *positive* sorting on risk aversion—in line with the evidence from marriage markets. Moreover, it delivers sharp comparative statics of risk with focus on how changes in risks—stochastically better, more variable or more correlated risks among spouses—affect risk-sharing arrangements and inequality within households.

Our analysis fills a gap in the literature, which is to understand from a theoretical point of view how the interaction between two prominent motives for marriage—partners' complementarities in match output and risk sharing—shape sorting patterns. As such, this paper sheds new light on the mechanisms that are at work in most quantitative marriage market models that feature these elements, and can serve as a building block for structural modeling in future work.

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Appendix

A Proofs

A.1 Details of the CARA Case

Let u and v be CARA utility functions given by $u(c) = (1 - e^{-Rc})/R$ and $v(c) = (1 - e^{-rc})/r$, respectively. Below we will omit the arguments of the functions whenever possible. Let $v_0 \in [0, \int_0^1((1-e^{-rf})/r)dL]$. Recall the optimality condition of the risk-sharing problem: for all values of α, we have $u'(w)/v'(f-w) = \lambda$. Since $u'(w) = e^{-Rw}$ and $v'(f-w) = e^{-r(f-w)}$, we obtain

$$\hat{w} = \frac{r}{R+r} f - \frac{1}{R+r} \log \lambda.$$

Inserting this into the constraint $\int((1 - e^{-r(f-\hat{w})})/r)dL = v_0$ we obtain, after some algebra

$$\lambda^* = \left(\frac{\int e^{-\frac{rR}{R+r}f} dL}{1 - rv_0} \right)^{\frac{R}{r}+1},$$

and hence

$$w^* = \frac{r}{R+r} f - \frac{1}{r} \log \left(\frac{\int e^{-\frac{rR}{R+r}f} dL}{1 - rv_0} \right).$$

Finally, inserting the expression for c into the objective function $\int((1 - e^{-Rc})/R)dL$, we obtain after some algebra

$$\phi(x, y, v_0) = \frac{1}{R} \left(1 - \frac{\left(\int e^{-\frac{rR}{R+r}f} dL\right)^{\frac{R}{r}+1}}{(1 - rv_0)^{\frac{R}{r}}} \right). \tag{11}$$

One can easily verify that w^* and ϕ satisfy all the properties in Lemma 1, which are the only ones used in the derivation of our main result on the characterization of monotone sorting.

A.2 Proof of Theorem 1

We will show that (8) is equivalent to $(-\phi_y/\phi_{v_0})_x \geq 0$ for all (x, y, v_0). From Lemma 1,

$$-\frac{\phi_y}{\phi_{v_0}} = \frac{\int u' f_y dL}{\lambda^*} = \int v' f_y dL,$$

where the second equality uses $u' = \lambda v'$ under optimal risk sharing.

It follows that $-\phi_y/\phi_{v_0}$ increasing in x is tantamount to $\int v' f_y dL$ increasing in x. Now,

$$
\begin{aligned}
\left(\int v' f_y dL\right)_x &= \int v' f_{xy} dL + \int v''(f_x - \hat{w}_x - \hat{w}_\lambda \lambda_x^*) f_y dL \\
&= \int v' f_{xy} dL + \int v'' \left(f_x - \frac{r}{R+r} f_x - \frac{1}{\lambda^*(R+r)} \lambda^* \frac{\int v' \frac{R}{R+r} f_x dL}{\int v' \frac{1}{R+r} dL} \right) f_y dL \\
&= \left(\int \frac{v'}{R+r} dL\right) \frac{\int f_{xy}(R+r) \frac{v'}{R+r} dL}{\int \frac{v'}{R+r} dL} + \int \frac{v''}{v'} \left(R f_x f_y - f_y \frac{\int R f_x \frac{v'}{R+r} dL}{\int v' \frac{1}{R+r} dL} \right) \frac{v'}{R+r} dL \\
&= \left(\int \frac{v'}{R+r} dL\right) \mathbb{E}_m[f_{xy}(R+r)] - \int r \left(R f_x f_y - f_y \mathbb{E}_m[R f_x] \right) \frac{v'}{R+r} dL \\
&= \left(\int \frac{v'}{R+r} dL\right) \Big(\mathbb{E}_m[f_{xy}(R+r)] - (\mathbb{E}_m[r R f_x f_y] - \mathbb{E}_m[r f_y] \mathbb{E}_m[R f_x]) \Big) \\
&= \left(\int \frac{v'}{R+r} dL\right) \Big(\mathbb{E}_m[f_{xy}(R+r)] - \mathrm{cov}_m[R f_x, r f_y] \Big),
\end{aligned}
$$

where the first equality follows by differentiation; the second one follows by replacing $\hat{w}_x$, $\hat{w}_\lambda$, and λ_x^* with the expressions derived above. The third equality follows by multiplying and dividing the first term by $\int v'/(R+r)dL$ and the integrand f_{xy} by $R+r$; and the second term by v' and some simple algebra. The fourth equality follows from using the definition of density m in the first term, and from replacing v''/v' by r and from using m in the second term; the fifth equality follows from once again using m; and the last equality follows from the definition of covariance. It follows that $-\phi_y/\phi_{v_0}$ is increasing in x if and only if (8) holds. $\square$

A.3 Proof of Proposition 1

(i) If R or r is zero, the right side of (8) is zero, and $f_{xy} > 0$ for almost all α implies positive sorting. And if both sides are risk neutral, it is immediate from problem (1) that $\phi(x, y, v_0) = \int f dL - v_0$, and thus positive sorting ensues given the assumed properties of f.

(ii) The coefficient of absolute risk aversion of women is $R = \sigma_1/w^*$, and of men is $r = \sigma_2/(f - w^*)$. The left side of (8) is strictly positive by assumption, so consider the covariance on the right side, which is given by

$$
\mathrm{cov}_m(f_x R, f_y r) = z_x z_y \sigma_1 \sigma_2 \,\mathrm{cov}_m \left(\frac{\eta}{w^*}, \frac{\eta}{\eta z - w^*} \right) = z_x z_y \sigma_1 \sigma_2 \,\mathrm{cov}_m \left(\frac{1}{\frac{w^*}{\eta}}, \frac{1}{z - \frac{w^*}{\eta}} \right).
$$

It suffices to show that w^*/η either increases or decreases in α, for then the first term inside the covariance either decreases in α while the second term increases in α, or vice versa (the first term increases while the second term decreases). Either case implies that the covariance is negative.

Now, $w^*_\alpha = (r/(R+r))f_\alpha$ and $f_\alpha = \eta' z$, and so the derivative of w^*/η with respect to α is

$$\begin{aligned}
\frac{\partial}{\partial \alpha}\left(\frac{w^*}{\eta}\right) &= \frac{1}{\eta^2}\left(w^*_\alpha \eta - w^* \eta'\right) \\
&= \frac{1}{\eta^2}\left(\frac{r}{R+r}\eta' \eta z - w^* \eta'\right) \\
&= \frac{\eta'}{\eta^2}\left(\frac{r}{R+r}\eta z - w^*\right) \\
&= \frac{\eta'}{\eta^2}\left(\frac{\sigma_2 w^*}{\sigma_1(\eta z - w^*) + \sigma_2 w^*}\eta z - w^*\right) \\
&= \frac{\eta'}{\eta^2} w^* \left(\frac{\sigma_2 \eta z - (\sigma_1(\eta z - w^*) + \sigma_2 w^*)}{\sigma_1(\eta z - w^*) + \sigma_2 w^*}\right) \\
&= \frac{\eta'}{\eta^2} w^* \left(\frac{(\sigma_2 - \sigma_1)(\eta z - w^*)}{\sigma_1(\eta z - w^*) + \sigma_2 w^*}\right),
\end{aligned}$$

where the second equality follows by replacing w^*_α, the fourth by replacing r by $\sigma_2/(\eta z - w^*)$ and R by σ_1/w^*, and the last one by rearrangement. Since $\eta z - w^* > 0$ for (almost) all α, w^*/α is increasing in α if and only if $\sigma_2 \geq \sigma_1$, and decreases otherwise. In either case, it is monotone and thus the covariance is negative.

(iii) Consider the left side of (8). Using f log-supermodular in (x, y) and the expressions for risk aversion $R = \sigma a/(aw^* + \sigma b)$ and $r = \sigma a/(a(f - w^*) + \sigma b)$ we obtain

$$\begin{aligned}
\mathbb{E}_m[f_{xy}(R+r)] &\geq \mathbb{E}_m\left[\frac{f_x f_y}{f}\left(\frac{\sigma a}{aw^* + \sigma b} + \frac{\sigma a}{a(f - w^*) + \sigma b}\right)\right] \\
&= a\sigma \mathbb{E}_m\left[\frac{f_x f_y}{f}\left(\frac{af + 2\sigma b}{(aw^* + \sigma b)(a(f - w^*) + \sigma b)}\right)\right] \\
&= a\sigma \mathbb{E}_m\left[\frac{f_x}{(aw^* + \sigma b)}\frac{f_y}{(a(f - w^*) + \sigma b)}\left(a + \frac{2\sigma b}{f}\right)\right] \\
&= a^2\sigma \mathbb{E}_m\left[\frac{f_x}{(aw^* + \sigma b)}\frac{f_y}{(a(f - w^*) + \sigma b)}\right] + 2a\sigma^2 b \mathbb{E}_m\left[\frac{f_x}{(aw^* + \sigma b)}\frac{f_y}{(a(f - w^*) + \sigma b)}\frac{1}{f}\right] \\
&\geq a^2\sigma \mathbb{E}_m\left[\frac{f_x}{(aw^* + \sigma b)}\frac{f_y}{(a(f - w^*) + \sigma b)}\right], \qquad (12)
\end{aligned}$$

where the first inequality uses $ff_{xy} - f_x f_y \geq 0$ and the expressions for R and r, the next three equalities follow from simple algebra; and the last inequality follows from $b \geq 0$, which implies that the last term is positive.

Consider now the right side of (8). Using the definition of covariance and the expressions for R and r, we obtain

$$\begin{aligned}
\mathrm{cov}_m(f_x R, f_y r) &= \mathrm{cov}_m\left(\frac{f_x a\sigma}{(aw^* + \sigma b)}, \frac{f_y a\sigma}{(a(f - w^*) + \sigma b)}\right) \\
&= a^2\sigma^2 \mathrm{cov}_m\left(\frac{f_x}{(aw^* + \sigma b)}, \frac{f_y}{(a(f - w^*) + \sigma b)}\right) \\
&= a^2\sigma^2\left(\mathbb{E}_m\left[\frac{f_x}{(aw^* + \sigma b)}\frac{f_y}{(a(f - w^*) + \sigma b)}\right] - \mathbb{E}_m\left[\frac{f_x}{(aw^* + \sigma b)}\right]\mathbb{E}_m\left[\frac{f_y}{(a(f - w^*) + \sigma b)}\right]\right) \\
&< a^2\sigma^2\mathbb{E}_m\left[\frac{f_x}{(aw^* + \sigma b)}\frac{f_y}{(a(f - w^*) + \sigma b)}\right], \qquad (13)
\end{aligned}$$

where the strict inequality follows from $b \geq 0$, which makes the second term inside the parenthesis (the product of expectations) strictly positive.

Comparing (12) and (13) reveals that since $\sigma \in (0, 1]$, the first is bigger than the second, and hence (8) holds with strict inequality.

(iv) Under CARA, $u(c) = (1 - e^{-Rc})/R$ and $v(c) = (1 - e^{-rc})/r$. Hence, $R + r$ can be pulled out of the expectation in (8) and Rr, z_x, and z_y out of the covariance, and thus (8) becomes

$$\mathbb{E}_m[\alpha] \geq \beta\frac{z_x z_y}{z z_{xy}}\mathrm{var}_m(\alpha).$$

But then, by log-supermodularity of z, we have that $z_x z_y/(z z_{xy}) \leq 1$, and it suffices to show that $\mathbb{E}_m[\alpha] \geq \beta\mathrm{var}_m(\alpha)$. To show that this inequality holds if β is sufficiently small, note that since z is bounded and $\mathbb{E}_m[\alpha] > 0$, there is a threshold for β (uniform in (x, y)), such that for all β less than the threshold the inequality holds. $\square$

A.4 Examples with CARA

We asserted in the text that (1) holds for all $\beta > 0$ when the shock is distributed according to a gamma or a uniform density. Here are the details:

Gamma Distribution. Assume that α is distributed on $[0, \infty)$ according to a gamma distribution with shape parameter $\xi > 0$ and rate parameter γ, so that l is given by $l(\alpha) = \gamma^\xi\alpha^{\xi-1}e^{-\gamma\alpha}/\Gamma(\xi)$. Then

$$m(\alpha) = \frac{\frac{e^{-\beta\alpha}\gamma^\xi\alpha^{\xi-1}e^{-\gamma\alpha}}{\Gamma(\xi)}}{\int\frac{e^{-\beta\alpha}\gamma^\xi\alpha^{\xi-1}e^{-\gamma\alpha}}{\Gamma(\xi)}d\alpha} = \frac{\frac{\gamma^\xi\alpha^{\xi-1}e^{-(\beta+\gamma)\alpha}}{\Gamma(\xi)}}{\left(\frac{\gamma}{\beta+\gamma}\right)^\xi\int\frac{(\beta+\gamma)^\xi\alpha^{\xi-1}e^{-(\beta+\gamma)\alpha}}{\Gamma(\xi)}d\alpha} = \frac{(\beta+\gamma)^\xi\alpha^{\xi-1}e^{-(\beta+\gamma)\alpha}}{\Gamma(\xi)},$$

where the last equality follows from the integral being equal to one since it is the integral of a gamma density with parameters ξ and $\beta + \gamma$. Thus, m is gamma with parameters ξ and $\beta + \gamma$.

But then, since its mean is $\xi/(\beta+\gamma)$ and its variance is $\xi/(\beta+\gamma)^2$, it follows that

$$\frac{\mathbb{E}_m[\alpha]}{var_m(\alpha)} - \beta = \beta + \gamma - \beta = \gamma > 0,$$

and thus positive sorting obtains under gamma distributed shocks.

Uniform Distribution. Assume that α is uniformly distributed on $[0, 1]$, so that $l = 1$ on that interval. Then, $m(\alpha) = e^{-\beta\alpha}/\int_0^1 e^{-\beta\alpha}d\alpha = \beta e^{-\beta\alpha}/(1-e^{-\beta})$. Simple but tedious algebra reveals that $\mathbb{E}_m[\alpha] = (1/\beta) + (1/(1-e^{-\beta})$ and $var_m(\alpha) = (1/\beta^2) + (1/(2(1-\cosh\beta))$, where $\cosh\beta = (e^{\beta} + e^{-\beta})/2$. Writing the sufficient condition as $\mathbb{E}_m[\alpha] - \beta var_m(\alpha) > 0$, we obtain after some algebra

$$\mathbb{E}_m[\alpha] - \beta var_m(\alpha) = \frac{1-(1-\beta)e^{\beta}}{(1-e^{\beta})^2} \geq 0,$$

where the inequality follows since it holds for $\beta > 1$, and for $\beta \in [0, 1]$, the maximum of $(1-\beta)e^{\beta}$ obtains at $\beta = 0$ and thus is 1. Hence, positive sorting obtains in this case. In fact, one can generalize this example to the class of densities l given by $l(\alpha) = a\alpha^{a-1}$, $\alpha \in [0, 1]$, $a > 0$. Computing this example with Mathematica reveals that $\mathbb{E}_m[\alpha] - \beta var_m(\alpha) > 0$ for all $\beta > 0$.

We also asserted that with a binary shock the condition can fail. Here are the details:

Binary Distribution. Assume that $\alpha \in \{0, 1\}$ with equal probability. Then $\mathbb{E}_m[\alpha] = e^{-\beta}/(e^{-\beta}+1)$ and $var_m(\alpha) = e^{-\beta}/(e^{-\beta}+1)^2$, and so $\mathbb{E}_m[\alpha] - \beta var_m(\alpha) > 0$ reduces to $e^{-\beta} + 1 - \beta > 0$, which holds if and only if β is less than approximately 1.27. This example shows that the sufficient condition *does not* always hold.

A.5 Proof of Proposition 2

(i) In this case, $\mathbb{E}_m[f_{xy}(R + r)] \leq 0$ since $f_{xy} \leq 0$. In turn, the rhs of (1) is strictly positive since f_x and f_y are strictly increasing in α, R and r are increasing in their arguments, and their arguments are increasing in α. Hence, any equilibrium exhibits negative sorting.

(ii) In this case, $\mathbb{E}_m[f_{xy}(R + r)] = 0$ since $f_{xy} = 0$. But, $\text{cov}_m(f_x R, f_y r) = a\, b\, \text{cov}_m(R, r) > 0$, since a and b are strictly positive, R and r are both either strictly increasing or decreasing, and their arguments are strictly increasing in α. Hence, any equilibrium exhibits negative sorting.

(iii) In this case, $(R + r)\mathbb{E}_m[f_{xy}] = 0$. And since f_x and f_y are strictly positive constants, the sorting pattern depends on $\text{cov}_m(R, r)$. But, since either R or r is constant, $\text{cov}_m(R, r) = 0$. It follows that equilibrium exhibits no clear sorting pattern. □

A.6 CRRA Preferences and Separable Class f

In this section we will provide the derivations omitted from the text in Section 5.

Derivation of Equilibrium. Consider the risk-sharing problem of a woman with x and a man with y who must be given expected utility of at least v_0. Following the same steps as in Section 3, we can use the optimality condition and the problem's constraint

$$\frac{\hat{w}^{-\sigma}}{(f-\hat{w})^{-\sigma}} = \lambda \quad \forall \alpha, \quad \int_0^1 \frac{(f-\hat{w})^{1-\sigma}-1}{1-\sigma} dL = v_0,$$

to obtain, after long algebra,

$$w^*(\alpha, x, y, v_0) = \left(1 - \left(\frac{v_0(1-\sigma)+1}{\int (f(x,y,\alpha))^{1-\sigma} dL(\alpha)}\right)^{\frac{1}{1-\sigma}}\right) f(x,y,\alpha). \tag{14}$$

Inserting (14) into $\int u(w^*(\alpha, x, y, v_0) dL(\alpha)$ yields

$$\phi(x,y,v_0) = \frac{1}{1-\sigma}\left(\left(\left(\int (f(x,y,\alpha))^{1-\sigma} dL\right)^{\frac{1}{1-\sigma}} - (v_0(1-\sigma)+1)^{\frac{1}{1-\sigma}}\right)^{1-\sigma} - 1\right).$$

Multiplying ϕ by the positive constant $(1-\sigma)$ and adding 1 does not alter the preferences of a woman with attribute x. Similarly, we can raise the resulting expression to the power $1/(1-\sigma)$, and this strictly increasing transformation leaves her preferences unaltered. Thus, we can focus on the equivalent representation of her preferences $\hat{\phi} = (1+(1-\sigma)\phi)^{1/(1-\sigma)}$ and write

$$\hat{\phi}(x,y,\pi_0) = F(x,y) - \pi_0, \tag{15}$$

where $F \equiv (\int f^{1-\sigma} dL)^{1/(1-\sigma)} = z(x,y)\left(\mathbb{E}[\eta^{1-\sigma}]\right)^{\frac{1}{1-\sigma}}$ and $\pi_0 = (v_0(1-\sigma)+1)^{1/(1-\sigma)}$.

We now derive the share of F that each man and woman obtains in equilibrium. The problem of a woman with attribute x is $\max_y F(x,y) - \pi(y)$, which yields $F_y(x,y) - \pi_y(y) = 0$. In the equilibrium of the marriage market, $y = \mu(x)$ or $x = \mu^{-1}(y)$, with $\mu(\cdot) = G^{-1}(H(\cdot))$, and thus $\pi_y(y) = F(\mu^{-1}(y), y)$. From here, we obtain

$$\pi(y) = \int_0^y F_y(\mu^{-1}(s), s) ds = \left(\mathbb{E}[\eta^{1-\sigma}]\right)^{\frac{1}{1-\sigma}} \int_0^y z_y(\mu^{-1}(s), s) ds, \tag{16}$$

where we have used the properties of f, which imply that $\pi(0) = 0$, and the expression for F. It is straightforward to show that positive sorting and (16) imply that choosing $y = \mu(x)$ is optimal for every woman with attribute x.

Let $\gamma(x) = \max_y \hat{\phi}(x, y, \pi(y))$. The Envelope Theorem reveals that

$$\gamma(x) = \int_0^x F_x(s, \mu(s)) ds = \left(\mathbb{E}[\eta^{1-\sigma}]\right)^{\frac{1}{1-\sigma}} \int_0^x z_x(s, \mu(s)) ds, \tag{17}$$

and it is easy to show that, given $\gamma(\cdot)$, it is optimal for each man of attribute y to choose $x = \mu^{-1}(y)$. Hence, $\pi(y) = \max_x \hat{\psi}(y, x, \gamma(x))$ for every y.[18]

This completes the construction of the unique equilibrium of the marriage market.

Positive Sorting on Risk Aversion: Proof of Proposition 3. Denote the coefficient of absolute risk aversion in equilibrium for a woman x matched with $y = \mu(x)$ by $\tilde{R}(x, \alpha) \equiv \sigma / w^*(x, \mu(x), \alpha, \pi(\mu(x)))$ and for a man y matched with $x = \mu^{-1}(y)$ by $\tilde{r}(x, \alpha) \equiv \sigma / (f(x, \mu(x), \alpha) - w^*(x, \mu(x), \alpha, \pi(\mu(x))))$. We aim to show that under positive sorting on skills $\mu'(x) > 0$, there is positive sorting on risk aversion, i.e., $cov(\tilde{R}(x, \alpha), \tilde{r}(x, \alpha)) > 0$. We proceed in two steps. We first show that $\tilde{R}$ and $\tilde{r}$ are decreasing in both arguments (x, α). We then apply Theorem 2.3 in Karlin and Rinott (1980) to prove the claim.

Under CRRA and f from the separable class and when using our notation from the text, the optimal sharing rule can be expressed as

$$\tilde{w}^*(x, \alpha) = \left(1 - \frac{\pi(\mu(x))}{F(x, \mu(x))}\right) \eta(\alpha) z(x, \mu(x)),$$

where

$$\begin{aligned} F(x, \mu(x)) &= z(x, \mu(x)) \left(\mathbb{E}[\eta^{1-\sigma}]\right)^{\frac{1}{1-\sigma}} \\ \pi(\mu(x)) &= \left(\mathbb{E}[\eta^{1-\sigma}]\right)^{\frac{1}{1-\sigma}} \int_0^{\mu(x)} z_y(s, \mu(s)) ds. \end{aligned}$$

After some algebra, we obtain the following derivatives of the coefficients of absolute risk aversion:

$$\begin{aligned} \tilde{R}_\alpha &= \frac{\sigma}{\tilde{w}^2}(-\tilde{w}_\alpha) < 0 \\ \tilde{R}_x &= \frac{\sigma}{\tilde{w}^2}(-\tilde{w}_x) = \left(-\frac{\sigma}{\tilde{w}^2}\right) \left((z_x + z_y \mu')\eta \left(1 - \frac{\pi}{F}\right) + z\eta\left(-\frac{1}{F^2}(\pi_y \mu' F - \pi(F_x + F_y \mu'))\right)\right) = -\frac{\sigma}{\tilde{w}^2}\eta z_x < 0 \\ \tilde{r}_\alpha &= \frac{\sigma}{(f - \tilde{w})^2}(-(f_\alpha - \tilde{w}_\alpha)) < 0 \\ \tilde{r}_x &= -\frac{\sigma}{(f - \tilde{w})^2}(f_x + f_y \mu' - \tilde{w}_x) = -\frac{\sigma}{(f - \tilde{w})^2}\eta z_y \mu' < 0 \end{aligned}$$

According to Theorem 2.3 in Karlin and Rinott (1980), the covariance of two functions of vectors of random variables is positive if (i) the joint density function is log-supermodular, which is the case in our setting since the joint density of (x, α) is $h(x)l(\alpha)$; and (ii) both functions are monotone in the same direction in both arguments, which we just showed is also the case when $\mu' > 0$. Hence, $cov(\tilde{R}(x, \alpha), \tilde{r}(x, \alpha)) > 0$. □

[18] A routine change of variable reveals that $\gamma(x) + \pi(\mu(x)) = F(x, \mu(x))$ for all x, as it should be in equilibrium.

A.7 Idiosyncratic Shocks and Skill-Dependent Outside Options

In Remark 1 we asserted that our model can be extended to two shocks and skill-dependent payoffs for unmatched men and women. In this section we flesh out this extension. We will highlight the main differences with the model and analysis in the text.

Model. We introduce the following modifications. Regarding the utility functions, now $u : \mathbb{R} \to \mathbb{R}$, is three times continuously differentiable, with $u' > 0$, $u'' \leq 0$, and $u''' \geq 0$; similarly, for $v : \mathbb{R} \to \mathbb{R}$. We no longer impose the boundary conditions at 0 from the baseline model.

Match output is given by a positive function f that depends on the partners' attributes and on the realization of a production shock of each partner. We denote the shocks by the vector of random variables, $\alpha = (\alpha_1, \alpha_2) \in [0,1]^2$, where α_1 is the shock of the woman and α_2 that of the man, and they are drawn from a continuous joint cdf L with support $[0,1]^2$ and joint density function l.[19] Hence, if a woman with attribute x matches with a man with attribute y, and the shock realizations are α_1 and α_2, the pair produces $f(x, y, \alpha_1, \alpha_2)$, which—unless there is risk of confusion—we will write using the vector notation, $f(x, y, \alpha)$. The function f is increasing and three times continuously differentiable with positive derivatives f_x, f_y, f_{xy}, f_{α_i}, $f_{x\alpha_i}$, and $f_{y\alpha_i}$, $i = 1, 2$, strictly so on $(0,1]^3$.

An unmatched woman of attribute x and shock realization α_1 now produces and consumes $p(x, \alpha_1)$, while an unmatched man with attribute y and shock α_2 produces and consumes $q(y, \alpha_2)$, where p and q are strictly increasing in each of their arguments. We will assume that

$$f(x, y, \alpha_1, \alpha_2) \geq p(x, \alpha_1) + q(y, \alpha_2), \quad \forall (x, y, \alpha_1, \alpha_2). \tag{18}$$

That is, the amount produced by a couple with attributes x and y when the shock realizations are α_1 and α_2 exceeds the sum of outputs when they remain unmatched. For example, this holds with equality if $x + \alpha_1$ and $y + \alpha_2$ (or if $\alpha_1 x$ and $\alpha_2 y$) were the partners' individual stochastic incomes and the couple's income f was the sum of the partners' incomes. The importance of this assumption is that everyone will be matched in equilibrium since with a suitable division of f, they can always leave each other as well off as in autarky, and they can do strictly better under efficient risk sharing.

The equilibrium definition requires the following modifications. Let $\bar{v} : [0,1] \to \mathbb{R}$, and let $u_a(x) \equiv \mathbb{E}[u(p(x, \alpha_1))]$ for all x and $v_a(y) \equiv \mathbb{E}[v(q(y, \alpha_2))]$ for all y be the expected utility of an unmatched woman with attribute x and an unmatched man with attribute y, respectively. An *equilibrium* consists of a triple $(\bar{u}, \bar{v}, \mu)$ that satisfies the following properties: (a) $\mu(x) \in \text{argmax}_y \phi(x, y, \bar{v}(y))$ and $\bar{u}(x) = \max_y \phi(x, y, \bar{v}(y)) \geq u_a(x)$ for all x; (b) $\mu^{-1}(y) \in$

[19] Note that these assumptions rule out the case of extreme "positive dependence" in which $L(\alpha_1, \alpha_2) = \min\{L_1(\alpha_1), L_2(\alpha_2)\}$, where L_i is the marginal cdf with respect to α_i, $i = 1, 2$. This implies that α_2 is a deterministic increasing function of α_1, and thus the support of L is concentrated on the graph of this function, which renders risk sharing irrelevant.

argmax$_x \psi(y, x, \bar{u}(x))$ and $\bar{v}(y) = \max_x \psi(y, x, \bar{u}(x)) \geq v_a(y)$ for all y; (c) μ is either $\mu(x) = G^{-1}(H(x))$ or $\mu(x) = G^{-1}(1 - H(x))$ for all x. In words, given utility schedules $\bar{u}$ and $\bar{v}$ and efficient risk sharing, each agent optimally chooses a partner with a trait that coincides with the one allocated by μ, and their utility is bigger than the utility they obtain when remaining unmatched.

Efficient Risk Sharing. In the risk-sharing problem, the man now has to receive at least a utility level $v_0 \in [v_a(y), v_{max}(x, y)]$, where $v_{max}(x, y) \equiv \psi(y, x, u_a(x))$. Thus, recalling that now α is a vector, we have

$$\begin{aligned} \phi(x, y, v_0) = &\max_w \int_0^1 \int_0^1 u(w(\alpha)) dL(\alpha) \\ &s.t. \int_0^1 \int_0^1 v(f(x, y, \alpha) - w(\alpha)) dL(\alpha) \geq v_0 \\ &0 \leq w(\alpha) \leq f(x, y, \alpha), \;\; \forall \alpha. \end{aligned} \tag{19}$$

The first constraint always binds at the optimum; if not, then increase $w(\alpha)$ by a small amount for every α at which $f - w > 0$, improving the objective function. Also, given condition (18), one feasible solution when $v_0 = v_a(y)$ is for the woman to set $w(\alpha) = f(x, y, \alpha) - q(y, \alpha_2)$ for all α, but she can do strictly better by sharing risk efficiently since the man is strictly risk averse.

Let λ be the multiplier of the first constraint. Then at an interior solution we have

$$\frac{u'(w(\alpha))}{v'(f(x, y, \alpha) - w(\alpha))} = \lambda \;\;\; \forall \alpha. \tag{20}$$

This condition and the properties of $f(x, y, \cdot, \cdot)$ also imply that w will be increasing in α_1 and α_2 and thus the nonnegativity constraint $w(\alpha) \geq 0$ for all α can bind, if at all, only at a "low region of values of (α_1, α_2)," that is values with either low α_1 or low α_2, while the opposite holds for the constraint $w(\alpha) \leq f(x, y, \alpha)$ for all α.[20] Henceforth we will ignore these constraints and assume that the solution is interior, in which case (20) holds for all values of (α_1, α_2).

The rest of the analysis of the risk-sharing problem is as before. Denote the solution to (20) by $\hat{w}$, which depends on (α, x, y, λ) and is continuously differentiable in each argument. Then

$$\hat{w}_i = \frac{r}{R + r} f_i \geq 0, \;\; i = x, y, \alpha_1, \alpha_2, \;\; \hat{w}_\lambda = -\frac{1}{\lambda(R + r)} < 0.$$

Inserting $\hat{w}$ into the constraint, we obtain a unique $\lambda^* > 0$, which is continuously differentiable,

[20] For a simple proof of the well-known fact that w is increasing, assume wlog that, for a given α_2, w is strictly decreasing in α_1 on some subinterval $(\underline{\alpha}_1, \bar{\alpha}_1) \subset [0, 1]$. Then it must be that $w(\alpha_1, \alpha_2) \in (0, f(x, y, \alpha_1, \alpha_2))$ for all $\alpha_1 \in (\underline{\alpha}_1, \bar{\alpha}_1)$, and thus (20) holds on that interval. Take $\alpha_1'' > \alpha_1'$ with α_1'' and α_1' both in $(\underline{\alpha}_1, \bar{\alpha}_1)$; then $w(\alpha_1'', \alpha_2) < w(\alpha_1', \alpha_2)$ and thus $f(x, y, \alpha_1'', \alpha_2) - w(\alpha_1'', \alpha_2) > f(x, y, \alpha_1', \alpha_2) - w(\alpha_1', \alpha_2)$. But then, (20) implies that $u'(w(\alpha_1', \alpha_2))/u'(w(\alpha_1'', \alpha_2)) = v'(f(x, y, \alpha_1', \alpha_2) - w(\alpha_1', \alpha_2))/v'(f(x, y, \alpha_1'', \alpha_2) - w(\alpha_1'', \alpha_2))$, which is impossible since the left-hand side is strictly less than one while the right-hand side is strictly greater than one; a contradiction. Hence, w is increasing.

with derivatives

$$\lambda_i^* = -\lambda^* \frac{\int\int v' \frac{R}{R+r} f_i dL}{\int\int v' \frac{1}{R+r} dL} < 0, \;\; i = x, y, \;\; \lambda_{v_0}^* = \lambda^* \frac{1}{\int\int v' \frac{1}{R+r} dL} > 0.$$

Since $\hat{w}$ and λ^* are unique, there is a unique w^*, given by $w^*(\alpha, x, y, v_0) = \hat{w}(\alpha, x, y, \lambda^*(x, y, v_0))$ for all α. Thus, the value of the risk-sharing problem is $\phi(x, y, v_0) = \int_0^1 \int_0^1 u(w^*(\alpha, x, y, v_0))dL(\alpha)$. Lemma 1 holds without any changes except for a double integral in ϕ_i.

Equilibrium Sorting. The characterization of monotone sorting extends without any changes except for the presence of a double integral due to the two shocks. The density, m, is now

$$m(\alpha, x, y, v_0) = \frac{\frac{u'(w^*(\alpha,x,y,v_0))}{R(w^*(\alpha,x,y,v_0))+r(f(x,y,\alpha)-w^*(\alpha,x,y,v_0))} l(\alpha)}{\int_0^1 \int_0^1 \frac{u'(w^*(\alpha,x,y,v_0))}{R(w^*(\alpha,x,y,v_0)))+r(f(x,y,\alpha)-w^*(\alpha,x,y,v_0))} dL(\alpha)}, \tag{21}$$

which can also be written using v' instead of u' when taking into account $u' = \lambda v'$.

Our main result goes through as well; we include it here for completeness.

Theorem 1′ (Sorting and Risk Sharing) *Positive sorting emerges if and only if for all* $(x, y, v_0) \in [0, 1] \times [0, 1] \times [v_a(y), v_{max}(x, y)]$,

$$\mathbb{E}_m[f_{xy}(R + r)] \geq cov_m(f_x R, f_y r). \tag{22}$$

If the inequality is reversed, then it characterizes negative sorting.

Proof. As before, $-\phi_y/\phi_{v_0}$ increasing in x is tantamount to $\int\int v' f_y dL$ increasing in x. Now,

$$\begin{aligned}
\left(\int\int v' f_y dL\right)_x &= \int\int v' f_{xy} dL + \int\int v''(f_x - \hat{w}_x - \hat{w}_\lambda \lambda_x^*) f_y dL \\
&= \int\int v' f_{xy} dL + \int\int v'' \left(f_x - \frac{r}{R+r} f_x - \frac{1}{\lambda^*(R+r)} \lambda^* \frac{\int\int v' \frac{R}{R+r} f_x dL}{\int\int v' \frac{1}{R+r} dL}\right) f_y dL \\
&= \left(\int\int \frac{v'}{R+r} dL\right) \frac{\int\int f_{xy}(R+r) \frac{v'}{R+r} dL}{\int\int \frac{v'}{R+r} dL} + \int\int \frac{v''}{v'} \left(R f_x f_y - f_y \frac{\int\int R f_x \frac{v'}{R+r} dL}{\int\int v' \frac{1}{R+r} dL}\right) \frac{v'}{R+r} dL \\
&= \left(\int\int \frac{v'}{R+r} dL\right) \mathbb{E}_m[f_{xy}(R+r)] - \int\int r \left(R f_x f_y - f_y \mathbb{E}_m[R f_x]\right) \frac{v'}{R+r} dL \\
&= \left(\int\int \frac{v'}{R+r} dL\right) \Big(\mathbb{E}_m[f_{xy}(R+r)] - (\mathbb{E}_m[r R f_x f_y] - \mathbb{E}_m[r f_y]\mathbb{E}_m[R f_x])\Big) \\
&= \left(\int\int \frac{v'}{R+r} dL\right) \Big(\mathbb{E}_m[f_{xy}(R+r)] - \mathrm{cov}_m[R f_x, r f_y]\Big),
\end{aligned}$$

where the justification of each equality is as in the proof of Theorem 1. □

Sorting Based on Primitives. Regarding Proposition 1, which derives primitives such that (22) holds, most of it extends to the current setting with minor modifications. First, part (*i*) goes through as is, and the same goes for part (*iii*) since it depends on bounds on each side of (22) but not on whether the expectation and covariance is taken with respect to the distribution of a single shock or two shocks. Assuming that η is a strictly increasing function of α_1 and α_2, part (*ii*) holds if $\sigma_1 = \sigma_2$, since in this case w^*/η is independent of (α_1, α_2). Finally, part (*iv*) goes through with minor modifications for $f = \eta z$, with η now strictly increasing in each shock.[21]

CRRA Case. We now turn to the CRRA closed-form derivation in Section 5. The separable class $f = \eta z$ now has η given by $\eta(\alpha_1, \alpha_2)$ for each realization of α_1 and α_2, with η strictly increasing in each argument. The risk-sharing rule is the same as in (9) but with $\int\int f^{1-\sigma} dL$ instead of $\int f^{1-\sigma} dL$ and with v_0 now restricted to $[v_a(y), v_{\max}(x, y)]$. Regarding equilibrium, now $F \equiv (\int\int f^{1-\sigma} dL)^{1/(1-\sigma)} = z(x, y)(\mathbb{E}[\eta^{1-\sigma}])^{\frac{1}{1-\sigma}}$, with the expectation taken with respect to the joint distribution of shocks, and $\pi_0 = (v_0(1-\sigma) + 1)^{1/(1-\sigma)}$, with $\pi_0 \in [\pi_a(y), \pi_{\max}(x, y)]$, where $\pi_a(y) = (v_a(y)(1-\sigma) + 1)^{1/(1-\sigma)}$ and similarly for $\pi_{\max}(x, y)$. Since F is strictly supermodular, any equilibrium exhibits positive sorting as before.

To derive equilibrium utility $\pi(y)$ of each man of skill y, we proceed as before, with the exception that we cannot assert that $\pi(0) = 0$ given the skill-dependent outside options. Thus,

$$\pi(y) = \pi(0) + \int_0^y F_y(\mu^{-1}(s), s) ds,$$

with $\pi(0) \in [\pi_a(0), \pi_{\max}(0, 0)]$. It is well-known that when individuals with the lowest attributes do not obtain zero payoff, equilibrium is not unique since there is leeway on how to set the constant, $\pi(0)$. On top of that, here we need to ensure that $\pi(y) \in [\pi_a(y), \pi_{\max}(\mu^{-1}(y), y)]$ for all y. We will derive a sufficient condition that will guarantee that $\pi(y)$ stays within those bounds for all y. Consider $\pi(y) \geq \pi_a(y)$ for all y. Note that we can write

$$\pi_a(y) = \pi_a(0) + \int_0^y \pi_a'(s) ds.$$

Comparing this expression with the one for $\pi(y)$, we see that a sufficient condition for $\pi(y) \geq \pi_a(y)$ is that the constant $\pi(0)$ be chosen such that $\pi(0) \geq \pi_a(0)$, and that $F_y(\mu^{-1}(y), y) \geq \pi_a'(y)$ for all y. Since $\mu^{-1}(y) = H^{-1}(G(y))$ and $\pi_a'(y)$ is pinned down by $v_a(y)$ and its derivative, it follows that this sufficient condition, although cumbersome, depends only on primitives.

Consider now $\pi(y) \leq \pi_{\max}(\mu^{-1}(y), y)$ for all y. Following the same steps as with the previous boundary condition, we obtain that $\pi(0) \leq \pi_{\max}(0, 0)$ and $F_y(\mu^{-1}(y), y) \leq (\pi_{\max}(\mu^{-1}(y), y))_y$ constitute sufficient conditions that only depend on primitives. We can simplify this further: note

[21] To see this, if one follows the proof of part (*iv*) of Proposition 1, it reduces to showing that $\mathbb{E}_m[\eta] \geq \beta \text{var}_m(\eta)$, and this holds for small enough β.

that $\pi_{\max}(\mu^{-1}(y), y) = F(\mu^{-1}(y), y) - \gamma_a(\mu^{-1}(y))$, because the maximum equilibrium payoff that y can obtain when matching with $x = \mu^{-1}(y)$ is attained when x receives her outside option. Thus, $(\pi_{\max}(\mu^{-1}(y), y))_y = F_x(\mu^{-1})' + F_y - \gamma_a'(\mu^{-1}(y))(\mu^{-1})'$, and $F_y(\mu^{-1}(y), y) \leq (\pi_{\max}(\mu^{-1}(y), y))_y$ reduces to $F_x(x, \mu(x)) \geq \gamma_a'(x)$ for all x.

In short, any $\pi(0) \in [\pi_a(0), \pi_{\max}(\mu^{-1}(0), 0)]$, along with the conditions $F_y(\mu^{-1}(y), y) \geq \pi_a'(y)$ for all y and $F_x(x, \mu(x)) \geq \gamma_a'(x)$ for all x guarantee that the equilibrium utility of each agent is greater than their skill-dependent reservation utility, completing the equilibrium construction.

Regarding Proposition 3, it goes through with minor changes since now $\tilde{R}$ and $\tilde{r}$ are decreasing in each α_1 and α_2. Finally, the comparative statics results with respect to an increase in dependence, namely, Proposition 4 extends without change to this case in which outside options are type dependent.

A.8 Model Extension: Skill Complementarities Beyond Match Output f

We now derive the characterization of positive sorting in the extended model that we laid out in Section 4. Assume the utility functions for men and women are given by

$$u(c_m) + h(x, y)$$
$$v(c_w) + h(x, y),$$

where u and v are functions of private consumption and have the same properties as before and h is strictly increasing in both arguments and strictly supermodular. In addition, we assume that f has the same properties as in the baseline model and can feature complementarities, $f_{xy} > 0$. The baseline model is nested when $h = 0$.

Risk-Sharing Stage. The problem is given by

$$\begin{aligned} \phi(x, y, v_0) = &\max_w \int_0^1 u(w(\alpha))dL(\alpha) + h(x, y) \\ &s.t. \int_0^1 v(f(x, y, \alpha) - w(\alpha))dL(\alpha) + h(x, y) \geq v_0 \\ &\quad 0 \leq w(\alpha) \leq f(x, y, \alpha), \ \ \forall \alpha. \end{aligned}$$

We obtain the same condition for optimal risk sharing as in our baseline case:

$$\frac{u'(w(\alpha))}{v'(f(x, y, \alpha) - w(\alpha))} = \lambda \quad \forall \alpha.$$

From this rule, we can again obtain a unique function $\hat{w}$, which depends on (α, x, y, λ), and differentiation and manipulation reveals that

$$\hat{w}_i = \frac{r}{R+r} f_i \geq 0, \ \ i = x, y, \alpha, \ \ \hat{w}_\lambda = -\frac{1}{\lambda(R+r)} < 0.$$

We then again use the problem's constraint,

$$\int_0^1 v(f(x,y,\alpha) - \hat{w}(\alpha, x, y, \lambda))dL(\alpha) + h(x,y) = v_0,$$

to obtain the derivatives of $\lambda(x, y, v_0)$:

$$\begin{aligned}
\lambda_x &= (-\lambda)\frac{\int v' f_x \frac{R}{r+R} dL + h_x}{\int v' \frac{1}{r+R} dL} \\
\lambda_y &= (-\lambda)\frac{\int v' f_y \frac{R}{r+R} dL + h_y}{\int v' \frac{1}{r+R} dL} \\
\lambda_{v_0} &= \lambda \frac{1}{\int v' \frac{1}{r+R} dL}.
\end{aligned}$$

We then use the value of the problem $\phi = \int u(\hat{w}(x, y, \alpha, \lambda(x, y, v_0)))dL + h(x, y)$ to obtain the derivatives of ϕ as:

$$\begin{aligned}
\phi_x &= \int u'(\hat{w}_x + \hat{w}_\lambda \lambda_x)dL + h_x \\
&= \int u' \left(\frac{r}{r+R} f_x + \left(-\frac{1}{\lambda}\right) \frac{1}{r+R} \left((-\lambda) \frac{\int v' \frac{R}{r+R} f_x dL + h_x}{\int v' \frac{1}{r+R} dL} \right) \right) + h_x \\
&= \int u' f_x dL + h_x \frac{\int u' \frac{1}{r+R} dL}{\int \frac{u'}{\lambda} \frac{1}{r+R} dL} + h_x \\
&= \int u' f_x dL + h_x(1+\lambda), \\
\phi_y &= \int u' f_y dL + h_y(1+\lambda), \\
\phi_{v_0} &= -\lambda.
\end{aligned}$$

Matching Stage. Sorting is positive when $-\phi_y/\phi_{v_0}$ is increasing in x. Following the proof of Theorem 1, we have

$$-\frac{\phi_y}{\phi_{v_0}} = \frac{\int u' f_y dL + h_y(1+\lambda)}{\lambda} = \int v' f_y dL + h_y \frac{1+\lambda}{\lambda},$$

where the second equality uses $u' = \lambda v'$ under optimal risk sharing (see above).

As in the baseline model, $-\phi_y/\phi_{v_0}$ increasing in x is tantamount to $\int v' f_y dL + h_y(1+\lambda)/\lambda$ increasing in x. We have

$$
\begin{aligned}
\left(\int v' f_y dL + h_y \frac{1+\lambda}{\lambda}\right)_x &= \int v' f_{xy} dL + \int v''(f_x - \hat{w}_x - \hat{w}_\lambda \lambda_x^*) f_y dL + \frac{(h_{xy}(1+\lambda) + h_y \lambda_x)\lambda - \lambda_x h_y (1+\lambda)}{\lambda^2} \\
&= \int v' f_{xy} dL + \int v'' \left(f_x - \frac{r}{R+r} f_x - \frac{1}{\lambda(R+r)} \lambda \frac{\int v' \frac{R}{R+r} f_x dL + h_x}{\int v' \frac{1}{R+r} dL} \right) f_y dL \\
&\quad + \frac{h_{xy}(1+\lambda)\lambda - \lambda_x h_y}{\lambda^2} \\
&= \left(\int \frac{v'}{R+r} dL \right) \frac{\int f_{xy}(R+r) \frac{v'}{R+r} dL}{\int \frac{v'}{R+r} dL} + \int \frac{v''}{v'} \left(R f_x f_y - f_y \frac{\int R f_x \frac{v'}{R+r} dL}{\int v' \frac{1}{R+r} dL} \right) \frac{v'}{R+r} dL \\
&\quad + h_x \frac{\int \frac{v''}{v'} (-f_y) \frac{v'}{R+r} dL}{\int v' \frac{1}{R+r} dL} + h_{yx} \frac{1+\lambda}{\lambda} + \frac{1}{\lambda} \frac{\int v' f_x \frac{R}{r+R} dL}{\int v' \frac{R}{r+R} dL} h_y + \frac{1}{\lambda} \frac{h_x h_y}{\int v' \frac{1}{r+R} dL} \\
&= \left(\int \frac{v'}{R+r} dL \right) \mathbb{E}_m[f_{xy}(R+r)] - \int r \left(R f_x f_y - f_y \mathbb{E}_m[R f_x] \right) \frac{v'}{R+r} dL \\
&\quad + \mathbb{E}_m[f_y r] h_x + h_{yx} \frac{1+\lambda}{\lambda} + \frac{1}{\lambda} \mathbb{E}_m[f_x R] h_y + \frac{1}{\lambda} \frac{h_x h_y}{\int v' \frac{1}{r+R} dL} \\
&= \left(\int \frac{v'}{R+r} dL \right) \Big(\mathbb{E}_m[f_{xy}(R+r)] - \left(\mathbb{E}_m[r R f_x f_y] - \mathbb{E}_m[r f_y] \mathbb{E}_m[R f_x] \right) \Big) \\
&\quad + \mathbb{E}_m[f_y r] h_x + h_{yx} \frac{1+\lambda}{\lambda} + \frac{1}{\lambda} \mathbb{E}_m[f_x R] h_y + \frac{1}{\lambda} \frac{h_x h_y}{\int v' \frac{1}{r+R} dL} \\
&= \left(\int \frac{v'}{R+r} dL \right) \Big(\mathbb{E}_m[f_{xy}(R+r)] - \mathrm{cov}_m[R f_x, r f_y] \Big), \\
&\quad + \mathbb{E}_m[f_y r] h_x + h_{yx} \frac{1+\lambda}{\lambda} + \frac{1}{\lambda} \mathbb{E}_m[f_x R] h_y + \frac{1}{\lambda} \frac{h_x h_y}{\int v' \frac{1}{r+R} dL}.
\end{aligned}
$$

After rearrangement, it follows that the condition for positive sorting is:

$$
\underbrace{\frac{1}{\lambda} \frac{(1+\lambda) h_{xy} + \lambda \mathbb{E}_m[f_y r] h_x + \mathbb{E}_m[f_x R] h_y + \frac{h_x h_y}{\int v' \frac{1}{r+R} dL}}{\int v' \frac{1}{r+R} dL}}_{\equiv T(h_{xy}, h_x, h_y) > 0} + \mathbb{E}_m[f_{xy}(R+r)] \geq cov_m[f_x R, f_y r],
$$

which is the inequality asserted in the text. $\square$

B Data Appendix

In this appendix, we describe the data source, variable construction and robustness exercises relating to Figure 1.

Data. To construct Figure 1, we use data from the German Socio Economic Panel (GSOEP, 2019), a household survey conducted by the German Institute of Economic Research (Deutsches Institut fuer Wirtschaftsforschung) starting in 1984. The core study of the GSOEP surveys about 25,000 individuals living in 15,000 households each year. All individuals aged 16 and older respond to the individual questionnaire. The head of household additionally answers a household questionnaire. This survey is longitudinal in nature, and collects rich information on demographics (such as marital status, education, fertility, family background, etc.), labor market variables, and—for some years—risk attitudes. Important for the analysis of marriage sorting, the GSOEP contains the same information for both the head of household and their partner (whether married or cohabiting).

When analyzing marriage sorting on both education and risk aversion, we focus on the period 2004-2015, as it covers the years for which our main risk attitude measures are available (2004, 2009, 2014). We focus on couples (either married or cohabiting), in which both partners are between 18 and 65 years old. In our baseline analysis, we use a pooled panel (for sorting on education: 2004-2015; for sorting on risk aversion: pooled years 2004, 2009, 2014).

Variable Construction. When it comes to marriage sorting on skills, we proxy skills by *years of education* and use it to capture our sorting-relevant variables (x, y). When it comes to marriage sorting on risk attitude, we construct our baseline measure of risk attitude as the first principal component of three variables that are related to risk-taking in monetary matters, which are available for the survey years 2004, 2009 and 2014: *Willingness to take risk in general*, *Willingness to take risk in financial matters*, and *Willingness to take risk in your occupation*. These three measures are highly correlated (the pair-wise correlation coefficient ranges between 0.43 and 0.82). The loadings of all three variables on the first principal component are positive and sizeable.

Robustness. We assess the robustness of the facts presented in Figure 1 in various ways. For sorting on education, the large positive correlation between spouses' years of education remains intact when we focus on (i) the repeated cross-section; (ii) when also controlling for other sorting-relevant variables, such as risk aversion. For sorting on risk attitudes, the sizeable positive correlation found in Figure 1 is robust to (i) focusing on the individual years 2004, 2009, 2014 instead of pooling them; (ii) controlling for other sorting-relevant variables, such as years of education; (iii) using the three risk measures that enter the principal component analysis (PCA) individually instead of using the first principal component; (iv) using alternative measures for risk attitude such as the *Willingness to take risks after winning the lottery* (which is however on a different scale, which is why we did not include it in the PCA).