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AMERICAN DISCLOSURE OPTIONS

Kerry Back
Bruce I. Carlin
Seyed Mohammad Kazempour
Chloe L. Xie

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ABSTRACT

We study strategic disclosure timing by correlated firms in the presence of risk-averse investors. Firms delay disclosures in the hope that positively correlated firms will announce especially good news and lift their own price. Risk premia rise before disclosures, drop when disclosures occur, and then rise again. Conditional risk premia can be much larger than unconditional risk premia. Disclosures are always good news, but disclosures that are only moderately good news induce clustering of disclosures by other positively correlated firms. We present evidence of strategic behavior in earnings announcement timing as predicted by the model.

Kerry Back
Jones Graduate School of Business
Rice University
P.O. Box 2932
Houston, TX 77252
kerry.e.back@rice.edu

Bruce I. Carlin
Jones Graduate School of Business
Rice University
1900 Rice Boulevard
Houston, TX 77005
and NBER
carlin@rice.edu

Seyed Mohammad Kazempour
E.J. Ourso College of Business
Louisiana State University
501 South Quad Drive
Baton Rouge, LA 70803
skazempour@lsu.edu

Chloe L. Xie
Sloan School of Management
Massachusetts Institute of Technology
100 Main St
Cambridge, MA 02142
chloexie@mit.edu

We investigate how companies choose when to make information public and study the asset pricing implications of discretion over disclosure timing. Discretion gives rise to an American disclosure option, which firms optimally exercise based on strategic actions of peer firms. So, the model we study is both dynamic and game-theoretic, which distinguishes it from the previous literature, and yields novel implications that we test in the setting of corporate earnings announcements.

We show that the presence of disclosure options causes risk premia to rise prior to disclosures, and conditional risk premia can be much higher than unconditional risk premia. The pattern of returns is similar to that documented by Kapadia and Zekhnini (2019), who empirically show that firms' returns are earned almost entirely on positive jump days and that stocks experience negative drifts prior to positive jumps. Indeed, in our model, strategic disclosure induces the same negative drift and positive jump pattern (the positive jumps occur even when the firms' news is bad from an ex ante perspective). Additionally, the jump risks have positive risk premia, because firms' information has both systematic and idiosyncratic components (Savor and Wilson, 2016; Ben-Rephael et al., 2020).

When a firm discloses its information, it exchanges one price stream (its valuation by uninformed risk-averse investors) for another (its valuation by informed investors). The option to disclose is an American exchange option, in which the price streams play the role of dividend streams in the usual set-up. An important aspect of our model is that the pre-disclosure price of a firm is not static. Instead, it fluctuates based on the disclosures made by other firms that are correlated with it. Therefore, each firm's optimal timing for disclosure is influenced by the disclosure strategies of other firms. This dynamic interaction forms a specific type of option game, which has not been studied before.

Equilibrium policies are threshold policies with stochastic thresholds, meaning that at each point in time there is a minimum signal value (threshold), depending on prior disclosures or nondisclosures by other firms, such that it is optimal to disclose if and only if one's signal is above the threshold. Like other American options, the option to disclose must be sufficiently far in the money before it is optimal to exercise. This means that firms continue to delay disclosing (up to a point) even when they could increase their price by disclosing.

Dye and Hughes (2018) – hereafter DH – add risk aversion to a static, single-firm model that is usually studied under risk neutrality (Grossman, 1981; Milgrom, 1981; Dye, 1985; Jung and Kwon, 1988). DH show that nondisclosure is variance increasing when investors are risk averse and also variance increasing for positively correlated uninformed firms. In our model, risk premia depend on covariances with the stochastic discount factor, so our result that nondisclosure increases risk premia means that nondisclosure increases covariances (in absolute value). As DH note, it is somewhat surprising that nondisclosure is variance increasing – and it is equally surprising that it increases covariances – because nondisclosure is itself information, and information "should" reduce variances and covariances.

Acharya, DeMarzo, and Kremer (2011) – herafter ADK – introduce the concept of disclosure as an American option but with only a single firm and with risk neutrality. In their model, the option value arises from an exogenous public announcement. They show that there is a clustering of disclosures following a negative announcement, in the sense that a range of firm types would disclose immediately following such an announcement. A key question in our model is whether there can be *endogenous* announcements that are sufficiently negative as to induce clustering. A firm only discloses when it is in its own interest to do so, and in fact a firm that discloses always has a strictly positive return. Thus, endogenous announcements are always good news for the disclosing firm, and they are also good news for positively correlated firms. Nevertheless, we show, for the case of two firms, that disclosures sufficiently close to the threshold are sufficiently negative that they induce a range of types of the other firm to disclose immediately. This is because a disclosure from a correlated firm both changes the underlying of the disclosure option and changes the amount of uncertainty remaining. The latter effect can more than offset the former and induce immediate optimal exercise of the disclosure option.

Our theory predicts that the value of the disclosure option is hump-shaped in the correlation of the firms. The fact that it must be zero at correlations of 0 or 1 is noted by ADK, who discuss but do not solve the risk-neutral model with multiple firms. Additionally, our model has implications regarding the announcement returns of early versus late announcers. Announcement returns are usually higher for early announcers, because firms select to announce early when they have especially good news. However, the fact that risk premia rise in the absence of disclosures can induce large average returns for late announcers. This effect dominates when the correlation between firm values is low but is less important when the correlation is higher, because more uncertainty is resolved by the first announcement and consequently subsequent risk premia are lower when the firms are more correlated.

We test the model's implications by analyzing strategic behavior that firms exhibit when they schedule earnings announcements. The model is more general, but this setting is convenient in that we have data from Wall Street Horizons on market forecasts of when firms will announce earnings and their actual announcement dates. Johnson and So (2018) use this data to show that firms with more negative news delay the release of information. Our focus is on whether firms schedule announcement dates strategically based on peer disclosures. For example, consider Uber's response to Lyft's earnings announcement in May, 2022. According to the Wall Street Journal (May 4, 2022), "Lyft's commentary was so bad, Uber Technologies moved up its earnings release and conference call after watching its own shares trade off sharply in sympathy."

Our cross-sectional tests provide more general evidence that firms do in fact use their disclosure options strategically. Defining peers as firms within the same industry according to Fama-French 12 industry or 4-digit GICS classifications, we document a positive correlation between the dispersion of announcement dates and dispersion in returns within earnings cycles. Further, we demonstrate a statistically significant pattern of firms announcing later, relative to Wall Street Horizons forecasts, when their peers release more positive news.

We present our model in Section 1 and discuss the disclosure option in Section 2. We study the model with a single strategic firm in Section 3 and with two strategic firms in Section 4. Implications of the two-firm model are described in Section 5. Some results on an n-firm model are presented in Section 6. Empirical results are in Section 7, and Section 8 concludes the paper. All proofs are in the appendices.

1. Model

In most of the paper, we study a model with two firms. We will describe it first. The firms learn their values $\tilde{x}_i$ at independent uniformly distributed times $\tilde{\theta}_i \in [0, 1]$. After a firm discloses, its price is $\tilde{x}_i$ until date 1. Before it discloses, pricing is by a representative investor with constant absolute risk aversion α and date-1 wealth $\tilde{w}$. The distribution of $(\tilde{x}_1, \tilde{x}_2, \tilde{w})$ is assumed to be normal and symmetric in the $\tilde{x}_i$. Assume the information arrival times $\tilde{\theta}_i$ are independent of $\tilde{w}$.

We assume the correlation between the two firms' values is positive. Thus, a positive disclosure by one firm has a positive spillover effect on the estimated value of the other firm. This is consistent with Savor and Wilson (2016). A positive correlation naturally arises from both firms being positively exposed to market risk. We do not constrain the correlation of the idiosyncratic shocks of the firms, provided only that the overall correlation is positive. A positive idiosyncratic shock for one firm might signal that the firm's product market has grown, which would be good news for peer firms, or it might signal that the firm has taken market share from competitors, which would obviously be bad news for peer firms. Our model accommodates both possibilities.

Assume the interest rate r is constant. For convenience, normalize it to zero. The stochastic discount factor (SDF) at date 0 for pricing payoffs at date 1 is proportional to the representative investor's marginal utility, which is proportional to $\mathrm{e}^{-\alpha\tilde{w}}$. Because the interest rate is zero, the SDF is

$$\tilde{m} = \frac{\mathrm{e}^{-\alpha\tilde{w}}}{\mathsf{E}[\mathrm{e}^{-\alpha\tilde{w}}]} . \tag{1.1}$$

The date–t price of firm i is

$$S_{it} := \frac{\mathsf{E}_t[\tilde{m}\tilde{x}_i]}{\mathsf{E}_t[\tilde{m}]} = \mathsf{E}_t\left[\mathrm{e}^{-\alpha\tilde{w}}\tilde{x}_i\right] \Big/ \mathsf{E}_t\left[\mathrm{e}^{-\alpha\tilde{w}}\right] . \tag{1.2}$$

Regress $\tilde{w}$ on the $\tilde{x}_i$ as

$$\tilde{w} = a + b(\tilde{x}_1 + \tilde{x}_2) + \tilde{z} . \tag{1.3}$$

Because of joint normality, the residual $\tilde{z}$ is independent of $\tilde{x}_1$ and $\tilde{x}_2$. Thus, the date–t price of firm i is

$$S_{it} = E_t\left[\mathrm{e}^{-\alpha b(\tilde{x}_1+\tilde{x}_2)}\tilde{x}_i\right] \Big/ E_t\left[\mathrm{e}^{-\alpha b(\tilde{x}_1+\tilde{x}_2)}\right] . \tag{1.4}$$

Set $M_1 = \mathrm{e}^{-\alpha b(\tilde{x}_1+\tilde{x}_2)}$ and $M_t = \mathsf{E}_t[M_1]$, so we can write (1.4) as

$$S_{it} = \frac{\mathsf{E}_t[M_1\tilde{x}_i]}{M_t} . \tag{1.5}$$

To focus on the role of disclosure, we assume no value-relevant information about $\tilde{x}_1$ and $\tilde{x}_2$ arrives during $[0, 1]$ except through disclosures. However, as (1.4) shows, the residual $\tilde{z}$ in (1.3) is not value relevant, so, for pricing the firms, it does not matter when uncertainty about it is resolved. As an example, we could have $\tilde{z} = Z_1$, where Z is a Brownian motion observable by investors that has zero drift and variance σ_z^2. Then, the date–t value of the market portfolio would be

$$W_t := a + Z_t - (1-t)\alpha\sigma_z^2 + b(S_{1t} + S_{2t})\,. \tag{1.6}$$

The term $(1-t)\alpha\sigma_z^2$ is the risk premium for the $\tilde{z}$ risk. We emphasize that assuming b is nonnegligible does not mean that $\tilde{x}_1$ and $\tilde{x}_2$ constitute a significant part of market wealth. The coefficient b includes the signaling effects of $\tilde{x}_1$ and $\tilde{x}_2$ regarding the values of other correlated stocks. That is, it reflects the systematic risk in $\tilde{x}_1$ and $\tilde{x}_2$.

The prices S_i evolve as

$$\frac{\mathrm{d}S_{it}}{S_{it}} = A_{it}\,\mathrm{d}t + \mathrm{d}Y_{it} \tag{1.7}$$

where Y_i is a martingale (a compensated sum of jumps), and A_{it} is the risk premium. The risk premium of firm i is

$$A_{it}\,\mathrm{d}t = -\mathsf{E}_t\left[\frac{\Delta M_t}{M_t}\Delta Y_{it}\right] \tag{1.8}$$

where Δ denotes the jump in a process and the expectation takes into account the arrival rate of jumps, which is of order $\mathrm{d}t$.[1] Jumps occur only when firms make disclosures.

In lieu of (1.2), we can use risk-neutral pricing. The risk-neutral probability of any event A is $\mathsf{E}[\tilde{m}1_A]$, where 1_A denotes the zero-one indicator function of the event. The risk-neutral conditional expectation of any random variable $\tilde{y}$ is

$$\mathsf{E}_t^*[\tilde{y}] = \frac{\mathsf{E}_t[\tilde{m}\tilde{y}]}{\mathsf{E}_t[\tilde{m}]}\,, \tag{1.9}$$

so (1.2) can be expressed as

$$S_{it} = \mathsf{E}_t^*[\tilde{x}_i]\,. \tag{1.10}$$

Lemma 1.1 below states that the risk-neutral and physical distributions

[1] More rigorously, the risk premium is the negative of the differential of the sharp bracket process, that is, $A_{it}\,\mathrm{d}t = -\mathrm{d}\langle M, Y_i\rangle_t / M_t$. This follows from the fact that M and MS_i are martingales. (Back, 1991, 2017).

differ only with regard to means, which is a standard result in this CARA–normal setting.

Let μ denote the mean of $\tilde{x}_i$, let σ^2 denote the variance of $\tilde{x}_i$, and let ρ denote the correlation between $\tilde{x}_1$ and $\tilde{x}_2$. The risk premium $\alpha\,\mathrm{cov}(\tilde{x}_i, \tilde{w})$ in Lemma 1.1 equals risk aversion multiplied by the covariance of the firm value $\tilde{x}_i$ with the representative investor's wealth $\tilde{w}$, as in the CAPM.

Lemma 1.1. *Under the risk-neutral probability, the firm values $\tilde{x}_i$ are joint normally distributed with means $\mu^* = \mu - \alpha\,\mathrm{cov}(\tilde{x}_i, \tilde{w})$ and with the same standard deviations and correlation as under the physical probability. Furthermore, they are independent of the information arrival times θ_i, which are independent and uniformly distributed, as under the physical probability.*

Assume a firm's objective is to maximize

$$\mathsf{E}\left[\tilde{m}\int_0^1 S_{it}\,\mathrm{d}t\right] = \mathsf{E}^*\left[\int_0^1 S_{it}\,\mathrm{d}t\right]. \tag{1.11}$$

The firm's control variable is its disclosure date, which we denote by τ_i. For any time t before $\tilde{\theta}_i$, the firm has no choice but to remain silent. However, for $t \geq \tilde{\theta}_i$, the firm optimally chooses τ_i to maximize its price from t onward, understanding that its price is determined by representative investor pricing before disclosure and is equal to $\tilde{x}_i$ after disclosure. The disclosure date is chosen based on all information before that date, including the firm's value and whether and what the other firm has disclosed. Our choice of the objective function (1.11) is motivated by the assumption that the firm or its managers benefit from having a higher share price over the course of time until date 1. For simplicity, we assume the benefit is additive in time and is valued according to the market's SDF, producing the objective (1.11).

Let P_t denote the price at date t of all firms that have not disclosed. This is the pool price – the price of all firms that have not separated by disclosing. The price process of each firm i is

$$S_{it} = \begin{cases} P_t & \text{for } t < \tau_i \\ \tilde{x}_i & \text{for } t \geq \tau_i \,. \end{cases} \tag{1.12}$$

In equilibrium, the value of the objective function (1.11) must be $\mathsf{E}^*[\tilde{x}_i]$ because the price process is a risk-neutral martingale in equilibrium. However, out of equilibrium, the objective function can take values larger or smaller than $\mathsf{E}^*[\tilde{x}_i]$ because the price need not be a risk-neutral martingale when the market's beliefs and firms' strategies are inconsistent. As a leading example, if the market assumes firms will disclose as soon as they receive their signals, then firms can achieve values higher than $\mathsf{E}^*[\tilde{x}_i]$ by delaying disclosure; hence, such beliefs are inconsistent with equilibrium. On the other hand, if firms could commit to disclosing as soon as they receive their signals, and the market understood the commitments, then firms would achieve $\mathsf{E}^*[\tilde{x}_i]$, the same as in our equilibrium without commitment. We assume that firms cannot commit, so they choose ex-post optimal strategies given the market's beliefs and given other firms' strategies.

We must work with the conditional distributions arising as firms disclose their values. We describe these here for the n–firm model studied in Section 6. It is convenient to define them inductively. Put subscripts n on the unconditional parameters μ, μ^*, σ, and ρ. This indicates that there are n firms that have not yet disclosed. Order the firms in the reverse order of disclosure, so firm n is the firm that discloses first, etc. For any $1 < k \leq n$, define

$$\mu_{k-1}(\tilde{x}_k, \dots \tilde{x}_n) = \rho_k \tilde{x}_k + (1 - \rho_k)\mu_k(\tilde{x}_{k+1}, \dots \tilde{x}_n) \tag{1.13a}$$

$$\mu^*_{k-1}(\tilde{x}_k, \dots \tilde{x}_n) = \rho_k \tilde{x}_k + (1 - \rho_k)\mu^*_k(\tilde{x}_{k+1}, \dots \tilde{x}_n) \tag{1.13b}$$

$$\sigma_{k-1} = \sigma_k \sqrt{1 - \rho_k^2} \tag{1.13c}$$

$$\rho_{k-1} = \frac{\rho_k}{1 + \rho_k} \,. \tag{1.13d}$$

For the case of two firms, the following lemma states that the distribution of $\tilde{x}_i$ conditional on $\tilde{x}_j$ is normal with standard deviation $\sigma\sqrt{1-\rho^2}$ and with mean equal to

$$\mu_1(\tilde{x}_j) = \rho\tilde{x}_j + (1-\rho)\mu \tag{1.14a}$$

under the physical probability and equal to

$$\mu_1^*(\tilde{x}_j) = \rho\tilde{x}_j + (1-\rho)\mu^* \tag{1.14b}$$

under the risk-neutral probability.

Lemma 1.2. *Conditional on $\tilde{x}_{k+1}, \ldots, \tilde{x}_n$, the random variables $\tilde{x}_1, \ldots, \tilde{x}_k$ are joint normal under the physical probability with means $\mu_k(\tilde{x}_{k+1}, \ldots \tilde{x}_n)$, standard deviations σ_k and correlation ρ_k. The distribution is the same under the risk-neutral probability except that the means are $\mu_k^*(\tilde{x}_{k+1}, \ldots \tilde{x}_n)$.*

2. The Disclosure Option

We consider 'threshold strategies', meaning that at each date t, there is a random variable B_t, depending on information at all dates $s < t$, such that each firm i that has received its signal and has not disclosed at t will disclose at t if and only if $\tilde{x}_i \geq B_t$. This is equivalent to assuming that if a firm with signal x chooses to disclose at t, then all firms with signals $y > x$ will also disclose. There is greater urgency for firms with high values to disclose, because they lose more by trading at prices below their values, so the assumption that higher types disclose first is very natural.

We assume that the threshold or boundary B_t is a continuous decreasing function of time between disclosure dates. The assumption that it is decreasing is motivated by the fact that the price will be decreasing between disclosure dates, because as time passes it becomes more likely that firms have received their signals, and nondisclosure then indicates that the signals are low. The boundary can jump up or down when a firm discloses, depending on the content of the disclosure. Let B_{t+} denote the disclosure boundary for the remaining firms after a disclosure at t, with $B_{t+} = B_t$ if there is no disclosure at t.

We analyze the option to disclose like any other American option. The optimal disclosure threshold is determined by a differential equation in the inaction region, in conjunction with value matching and smooth pasting conditions at the boundary of the region. At each date t after a firm learns its value $\tilde{x}_i$, define the value function

$$J_t \;=\; \sup_{\tau} \; \mathsf{E}_t^* \left[\int_t^{\tau} P_u \, \mathrm{d}u + (1-\tau)\tilde{x}_i \mid \tilde{x}_i \right] . \tag{2.1}$$

The supremum in (2.1) is taken over disclosure times that can depend on all prior information, and the value (2.1) depends on all prior disclosures and on the firm's value $\tilde{x}_i$.

We can alternatively write the value function (2.1) as

$$\mathsf{E}_t^* \left[\int_t^1 P_u \, \mathrm{d}u \mid \tilde{x}_i \right] + \sup_{\tau} \; \mathsf{E}^* \left[\int_{\tau}^1 (\tilde{x}_i - P_u) \, \mathrm{d}u \mid \tilde{x}_i \right] \tag{2.2}$$

or as

$$(1-t)\tilde{x}_i + \sup_{\tau} \; \mathsf{E}^* \left[\int_t^{\tau} (P_u - \tilde{x}_i) \, \mathrm{d}u \mid \tilde{x}_i \right] . \tag{2.3}$$

In both (2.2) and (2.3), the second term is the value of an American exchange option. In (2.2), we view the firm as being endowed with the price process P and exchanging it for $\tilde{x}_i$ when it exercises the option to disclose. In (2.3), the first term is the value the firm would attain if disclosure were mandatory, and the second term is the value of having discretion over when to disclose.

Returning to (2.1), Bellman's Principle of Optimality implies that the stochastic process

$$\int_0^t P_u \, \mathrm{d}u + J_t \tag{2.4}$$

is a martingale under the risk-neutral probability. Taking the differential of (2.4), we see that the martingale property of (2.4) can be stated as

$$P \, \mathrm{d}t + \mathsf{E}^*[\mathrm{d}J] \;=\; 0 \, . \tag{2.5}$$

The value J_t is determined by equation (2.5) in conjunction with value matching and smooth pasting. The value matching condition is that, at the optimal disclosure time $t = \tau$,

$$J_t = (1 - \tau)\tilde{x}_i \,. \tag{2.6}$$

The smooth pasting condition at the boundary is that J paste together smoothly (in x) with the value on the right-hand side of (2.6).

We show in Appendix A that equation (2.5), value matching, and smooth pasting are equivalent to a simple marginal condition for the exercise boundary. Consider a firm with value $\tilde{x}_i$ such that $\tilde{x}_i > P_t$. In other words, the firm would trade at a higher price if it disclosed. The cost of delaying disclosure for an instant $\mathrm{d}t$ is $(\tilde{x}_i - P_t)\,\mathrm{d}t$. The benefit of delaying disclosure is that another firm may disclose a high value during the instant $\mathrm{d}t$, lifting the price of firms that have not disclosed and producing a positive jump ΔJ_t in J_t.[2] The marginal condition is that the costs and benefits of delaying disclosure must be equal for a firm at the boundary. Therefore, the equilibrium exercise boundary at date t is the number B_t such that the probability another firm discloses at t times the expected value of ΔJ_t conditional on a disclosure equals the cost $(B_t - P_t)\,\mathrm{d}t$ of delaying disclosure.

3. A Single Strategic Firm

When there is only one strategic firm, the description of the equilibrium is a standard result. It can be derived from a static model by interpreting t as the probability of being informed rather than as time. See Dye and Hughes (2018) for example.

[2]For a firm at the optimal exercise boundary, downward jumps in J_t are not possible, because a downward jump in the boundary does not affect the optimal policy of a firm that is already at the boundary – the firm should still disclose, so the value of J is still the right-hand side of (2.6) after such a jump in the boundary. On the other hand, an upward jump in the boundary means that it is optimal to delay disclosure a while longer, so the value of J becomes larger than the right-hand side of (2.6), that is, there is a positive jump ΔJ_t.

Proposition 3.1. *For each date t, consider the fixed-point problem:*

$$z = -\frac{t\phi(z)}{1 - t + t\Phi(z)}. \tag{3.1}$$

There is a unique solution $z(t)$, and it is decreasing in t. Furthermore, at each date t prior to disclosure, the equilibrium price is $P_t = \mu^* + \sigma z(t)$.

Because the changes in the price and disclosure boundary induced by a disclosure by another firm are so important in this model, we state here a result for the simple case in which there is one strategic firm and another firm that is known to be uninformed and is therefore nonstrategic. Consider the same signal structure as described before except that one firm is known to be uninformed.

Proposition 3.2. *The price of the firm that is known to be uninformed is equal to $\rho P_t + (1-\rho)\mu^*$ at all dates t prior to disclosure by the strategic firm, where P_t is defined in Proposition 3.1, and it is equal to $\rho\tilde{x}_1 + (1-\rho)\mu^*$ after disclosure by the strategic firm, where $\tilde{x}_1$ is the signal disclosed by the strategic firm.*

The proposition shows that the disclosure-induced change in price of the uninformed firm is $\rho(\tilde{x}_1 - P_t)$, so the uninformed firm always shares in any jump in the price of the strategic firm. Thus, voluntary disclosures by a strategic firm are always good news for a positively correlated nonstrategic firm.

4. Two Strategic Firms

We will see that voluntary disclosures are also good news for a positively correlated *strategic* firm. Nevertheless, voluntary disclosures can be sufficiently negative that they hasten disclosures by other firms. In fact, there is a nonzero probability of another firm disclosing immediately afterwards. This occurs when the second firm's disclosure option was in the money before the first firm's disclosure and remains in the money after the disclosure, and the value of keeping the option alive diminishes sufficiently when the first firm discloses that the option is in the exercise region post-disclosure.

4.1. The Last Firm

To solve the model with two firms, we work backwards starting with the last firm to disclose. The equilibrium for the last firm is very similar to that for a single firm. The only difference is that the equilibrium boundary for the firm might not have been strictly decreasing in the past—it might have jumped up when the other firm disclosed—so at any date t when there is only one firm remaining it is possible that the boundary had been lower at some date $s < t$ than it is at date t. This adds a new wrinkle to the fixed point condition for the equilibrium.

Proposition 4.1. *Suppose firm 2 is the first firm to disclose and discloses x at date t when the boundary is $B_t \leq x$. Define*

$$\beta = \frac{B_t - \mu_1^*(x)}{\sigma_1}\,.$$

For each $u > t$, consider the fixed point problem:

$$y = -\frac{t\phi(\beta) + (u-t)\phi(y)}{1 - u + t\Phi(\beta) + (u-t)\Phi(y)}\,. \tag{4.1}$$

There is a unique solution $y(u, \beta)$, and it is decreasing in u. Let $z(\cdot)$ denote the function defined in Proposition 3.1. Then, for each $u > t$, $z(u) \leq \beta$ if and only if $y(u, \beta) \leq \beta$. The price of firm 1 at each $u > t$ prior to firm 1's disclosure is

$$P_u = \begin{cases} \mu_1^*(x) + \sigma_1 y(u, \beta) & \text{if } z(u) > \beta \\ \mu_1^*(x) + \sigma_1 z(u) & \text{otherwise}\,. \end{cases} \tag{4.2}$$

Proposition 4.1 implies that if $P_u > B_t$, then $P_u = \mu_1^*(x) + \sigma_1 y(u, \beta)$. The price will drop as u increases and eventually fall to B_t, at which time the price formula switches (continuously) to $P_u = \mu_1^*(x) + \sigma_1 z(u)$, which is the pricing formula from a one-firm model. We will see next that if the disclosure x is sufficiently low, then we will have $P_u < B_t$ for u immediately after t, so the price will be given by the formula $P_u = \mu_1^*(x) + \sigma_1 z(u)$ for all $u > t$.

4.2. Clustering

The next proposition states that there can be clustering following endogenous announcements by strategic firms. In fact, it implies extreme clustering: disclosures can occur on top of each other with no time elapsing between them, when the first disclosure is sufficiently low. More generally, we will see that lower disclosures always hasten subsequent disclosures.

Proposition 4.2. *At each t, there exists $\epsilon > 0$ such that if the first disclosure by either firm is less than $B_t + \epsilon$, then the post-disclosure price P_{t+} of the other firm is strictly less than B_t. The other firm will immediately disclose if it knows its value and its value is in the nonempty interval $[P_{t+}, B_t]$.*

Proposition 4.2 is of interest in its own right, and it is also useful for computing the equilibrium. To find the boundary prior to any disclosure, we solve the marginal condition described in Section 2. Suppose firm 1 knows its value $\tilde{x}_1$, which is at the boundary B_t. The benefit of delaying disclosure is that firm 2 might make a disclosure that is beneficial to firm 1. There are two reasons why firm 2 might disclose at t: (i) it might have learned its value $\tilde{x}_2$ prior to t and $\tilde{x}_2 = B_t$, or (ii) it might learn its value at t and $\tilde{x}_2 \geq B_t$. Proposition 4.2 implies that disclosures by firm 2 due to reason (i) do not benefit firm 1 – neither do they harm firm 1 – because it remains optimal for firm 1 to disclose immediately when firm 2 makes such a disclosure. Thus, to compute the benefit to firm 1 of delaying disclosure, we only need to calculate the benefit of disclosures by firm 2 that are due to reason (ii).[3]

[3] Notice that the arrival rate of disclosures due to reason (i) depends on the slope of the boundary at t: The probability of a disclosure due to reason (i) is higher in any time interval around t when the

4.3. Pre-Disclosure Boundary

To compute the equilibrium pre-disclosure boundary B_t, we equate the expected benefit of delaying disclosure to the cost $B_t - P_t$ of delaying disclosure for a firm at the boundary. Calculating P_t given B_t is a straightforward application of Bayes' rule. With the aid of Proposition 4.2, it is also straightforward to calculate the expected benefit of delaying disclosure. We end up with the messy but numerically very tractable equation (4.3).

Proposition 4.3. *Let z be the function defined in Proposition 3.1, and let y be the function defined in Proposition 4.1. At each date t prior to any disclosure, the normalized disclosure boundary $(B_t - \mu^*)/\sigma$ is the solution b of*

$$\frac{t(1+\rho)\phi(b)\left[1-t+t\Phi\left(b\sqrt{\frac{1-\rho}{1+\rho}}\right)\right]}{\left[1-t+t\Phi(b)\right]^2+t^2\left[\Gamma(b,b,\rho)-\Phi(b)^2\right]}+b \;=\; \frac{\sqrt{1-\rho^2}}{\rho}\times\frac{\int_{-\infty}^{z(t)}\int_t^{z^{-1}(\beta)}\left[y(u,\beta)-\beta\right]\phi\left(\frac{\beta-b\sqrt{1-\rho^2}}{\rho}\right)\mathrm{d}u\,\mathrm{d}\beta}{1-t+t\Phi\left(b\sqrt{\frac{1-\rho}{1+\rho}}\right)}, \tag{4.3}$$

where $\Gamma(\cdot,\cdot,\rho)$ denotes the bivariate distribution function for normal random variables with zero means, unit standard deviations, and correlation equal to ρ. The equilibrium price P_t prior to any disclosure satisfies

$$\frac{P_t-\mu^*}{\sigma} = -\frac{t(1+\rho)\phi(b)\left[1-t+t\Phi\left(b\sqrt{\frac{1-\rho}{1+\rho}}\right)\right]}{\left[1-t+t\Phi(b)\right]^2+t^2\left[\Gamma(b,b,\rho)-\Phi(b)^2\right]}\,. \tag{4.4}$$

Figure 4.1 shows possible paths of the game. Prior to the first disclosure, the blue curves are the equilibrium boundary, and the orange curves are the price for both firms. The red dots indicate disclosures. In both panels, the first disclosure occurs at $t = 0.3$. In Panel (a), the firm that discloses at $t = 0.3$ learned its value prior to $t = 0.3$ and discloses when the equilibrium

graph of B is steeper, because the range of values traversed by B during the time interval is larger; hence, the arrival rate is higher. Thus, if the benefit of such disclosures were nonzero, the marginal condition would depend both on B and its derivative at each point in time.

disclosure boundary first falls to its value. The disclosure induces a jump in its price from the orange curve at $t = 0.3$ to the red dot. This price increment was the cost of delaying disclosure, which was just balanced in equilibrium by the expected benefit from the other firm disclosing a high value. The first firm's price stays at the red dot level, while the second firm's price equals its disclosure boundary, which is the green curve. The disclosure induced an increase in the second firm's price (from the orange curve to the green curve) because the firms have positively correlated values ($\rho = 0.5$). In Panel (b), the firm that discloses at $t = 0.3$ first learns its value at $t = 0.3$ and discloses immediately, because its value is above the equilibrium disclosure boundary. This induces a greater jump in the second firm's price than in Panel (a), and the disclosure by the second firm occurs later than in Panel (a). In Panel (a), the second firm strategically moves up its disclosure date, relative to Panel (b), because of the relatively disappointing disclosure by the first firm in Panel (a), as in the Lyft/Uber example cited in the introduction.

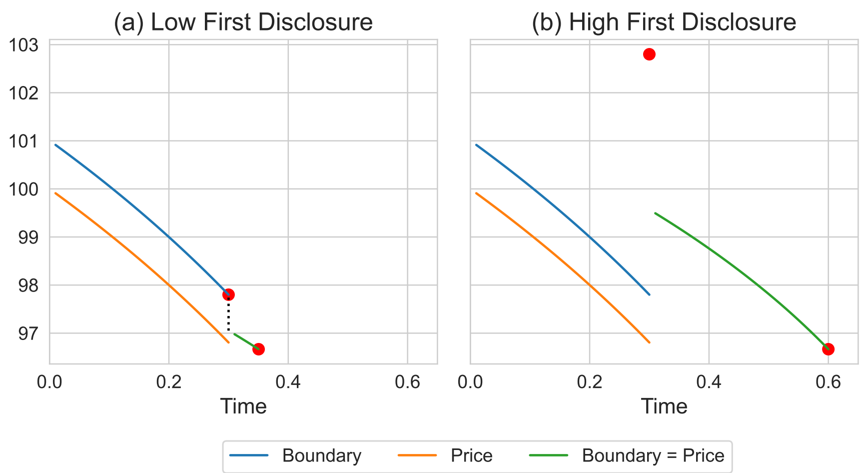

Figure 4.1: The first disclosure occurs at $t = 0.3$ and disclosures are represented by red dots. The equilibrium boundary and price prior to the first disclosure are the blue and orange curves. The post-disclosure price (which is the same as the post-disclosure boundary) is shown as the green curve. In Panel (a), the first disclosure occurs at the boundary. In Panel (b), the first disclosure is higher. In both panels, the second red dot represents the second disclosure. The dotted line in Panel (a) is the range of values that would induce a second disclosure immediately following the first. The parameter values are $\mu = 105$, $\mu^* = 100$, $\sigma = 15$, and $\rho = 0.5$.

The dotted vertical line in Panel (a) of Figure 4.1 represents clustering, as described in Proposition 4.2. All firm types in that range would disclose immediately following the first disclosure. There is no range of that type in Panel (b), because the disclosure boundary jumps up following the relatively high disclosure of the first firm in Panel (b). More generally, the figure illustrates the fact that lower disclosures induce a lower post-disclosure boundary, which hastens the subsequent disclosure.

5. Implications of the Model

The equilibrium disclosure boundary prior to any disclosures is given by Proposition 4.3. The parameters of the model that determine the boundary are the risk-neutral mean μ^*, the standard deviation σ, and the correlation ρ. Proposition 4.3 shows that the boundary translates with μ^* and scales with σ, so the most interesting parameter is the correlation ρ. In fact, ρ is the only parameter that appears in Equation (4.3) in Proposition 4.3. To illustrate the implications of the model, we set μ^* (which is the initial price of both firms) equal to 100, and we set $\sigma = 15$. As discussed in Section 1, the relation between the risk-neutral mean μ^* and the physical mean μ is that $\mu^* = \mu - \alpha \operatorname{cov}(\tilde{x}_i, \tilde{w})$, where α is aggregate absolute risk aversion and $\tilde{w}$ is market wealth. We set the unconditional risk premium $(\mu - \mu^*)/\mu^*$ to be 5%, so $\alpha \operatorname{cov}(\tilde{x}_i, \tilde{w}) = 0.05\mu^*$.

The correlation between the firm values is of course the same as the correlation between their realized gross returns $\tilde{x}_i/\mu^*$. Consider the market model regressions

$$\frac{\tilde{x}_i}{\mu^*} = \gamma + \beta \tilde{R} + \tilde{\varepsilon}_i$$

where $\tilde{R}$ denotes the market return $\tilde{w}/\mathsf{E}[\tilde{m}\tilde{w}]$. Assume the firms have unit market betas. It is natural for the idiosyncratic risks $\tilde{\varepsilon}_i$ to be correlated if the firms are in the same industry. Let ρ_ε denote their correlation. Then, the correlation between the $\tilde{x}_i$ is

$$\rho = \frac{1+\rho_\varepsilon \kappa^2}{1+\kappa^2}\,,$$

where κ denotes the ratio of the standard deviation of the idiosyncratic risk $\tilde{\varepsilon}_i$ to the standard deviation of the market return $\tilde{R}$. For illustration, we consider two sets of parameter combinations: $\kappa = 2$ and $\rho_\varepsilon = -1/8$, which imply $\rho = 0.1$, and $\kappa = 2$ and $\rho_\varepsilon = 3/8$, which imply $\rho = 0.5$.

The correlation ρ affects the regression coefficient b in (1.3), because it affects the variance of $\tilde{x}_1 + \tilde{x}_2$. The regression coefficient is

$$b = \frac{\operatorname{cov}(\tilde{w}, \tilde{x}_i)}{(1+\rho)\sigma^2}\,. \tag{5.1}$$

The assumption of a 5% unconditional risk premium implies $\operatorname{cov}(\tilde{w}, \tilde{x}_i) = 0.05\mu^*/\alpha$ as remarked above, so with $\mu^* = 100$ and $\sigma = 15$, we have

$$\alpha b = \frac{1}{45(1+\rho)}\,. \tag{5.2}$$

We use this to calculate the jump risk premium (1.8).

Figure 5.1 shows how the conditional distribution of the asset value evolves over time when there are no disclosures. Nondisclosure increases the variance as shown by Dye and Hughes (2018) in the single-firm model. The effect is greater when the correlation is larger, because the inference from two positively correlated firms not disclosing is stronger than the inference from nondisclosure by a single firm or from two less correlated firms.

5.1. Cost of Strategic Behavior

A firm would have a higher average price if it could commit to disclosing as soon as it learned its information. Figure 5.2 depicts the average equilibrium price at each date (averaging over firms and averaging over simulations under the physical distribution) relative to what the average price would be if firms disclosed as soon as they learned their values. At

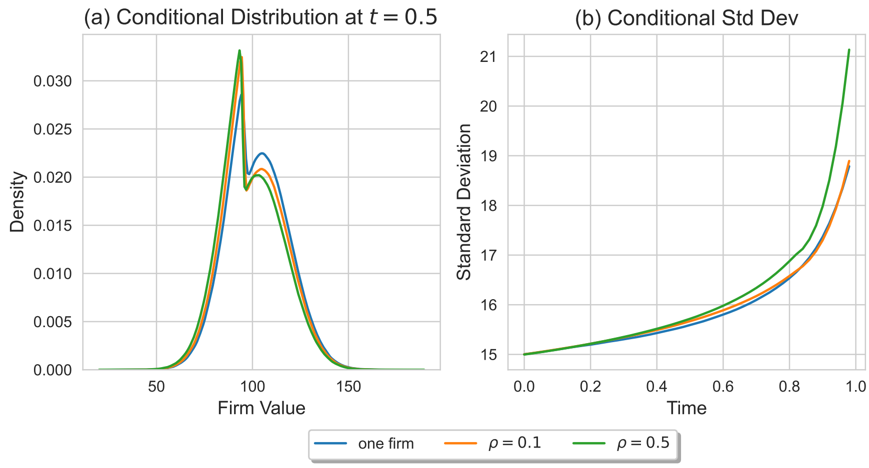

Figure 5.1: Panel (a) depicts the distribution of the firm value conditional on no disclosure by $t = 0.5$ in the single-firm model and in the two-firm model with $\rho = 0.1$ and $\rho = 0.5$. Panel (b) depicts how the standard deviation conditional on no disclosure evolves over time. The other parameter values are $\mu = 105$, $\mu^* = 100$, and $\sigma = 15$.

all dates $t \in (0, 1)$, the policy of disclosing immediately would generate a higher average price. This is a consequence of risk aversion and the slower resolution of risk when firms engage in strategic delay.

5.2. Risk Premia

Figure 5.3 shows that nondisclosure causes the conditional risk premium (1.8) to rise. The risk premium rises prior to the first disclosure, drops upon disclosure due to the resolution of risk by the disclosure, and then rises again until the second disclosure. The conditional risk premium is sometimes much higher than the unconditional risk premium (5% in the example depicted). Prior to the first disclosure, the risk premium rises faster when the correlation between the firms is larger, due to the stronger inference made when two correlated firms fail to disclose. However, the drop following the first disclosure is larger and the risk premium is subsequently smaller when the correlation is larger, due to the greater resolution of risk when a more correlated firm discloses.

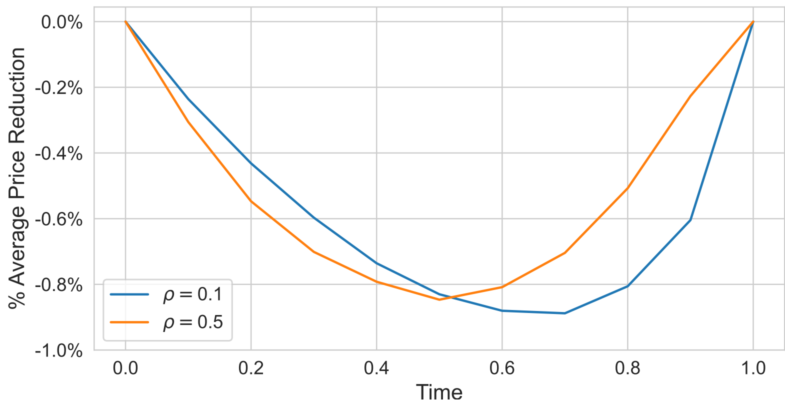

Figure 5.2: The figure depicts the difference between the average price and what the average price would be with immediate disclosure as a percent of what the average price would be with immediate disclosure, averaging over the the two firms and averaging over 100,000 simulations. The parameter values are $\mu = 105$, $\mu^* = 100$, and $\sigma = 15$.

5.3. Signals versus Returns

A firm's announcement return is not a perfect measure of the quality of its news. The market has rational expectations, so it discounts firms that are late to announce. Furthermore, the inferences from nondisclosure change the conditional distribution of news in such a way that risk premia rise

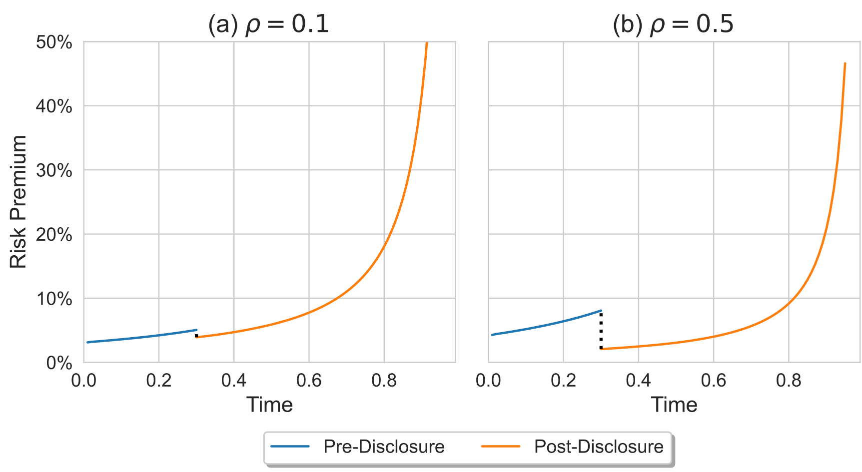

Figure 5.3: The instantaneous risk premium (1.8) is shown, for an example in which the first disclosure occurs at $t = 0.3$. The parameter values are $\mu = 105$, $\mu^* = 100$, $\sigma = 15$, and $\rho = 0.1$ (Panel a) or $\rho = 0.5$ (Panel b). The figure illustrates the general principle that risk premia rise before the first disclosure, drop upon disclosure, and then rise again until the next disclosure.

over time, as shown in the previous subsection. This means that there is additional discounting of firms that are late to announce. Hence, it is possible for late-announcing firms to have high announcement returns even if their news is relatively poor from an ex-ante (date 0) perspective. However, our model does imply a positive correlation between signals and announcement returns. The correlation is shown in Figure 5.4 for a simulation of the model.

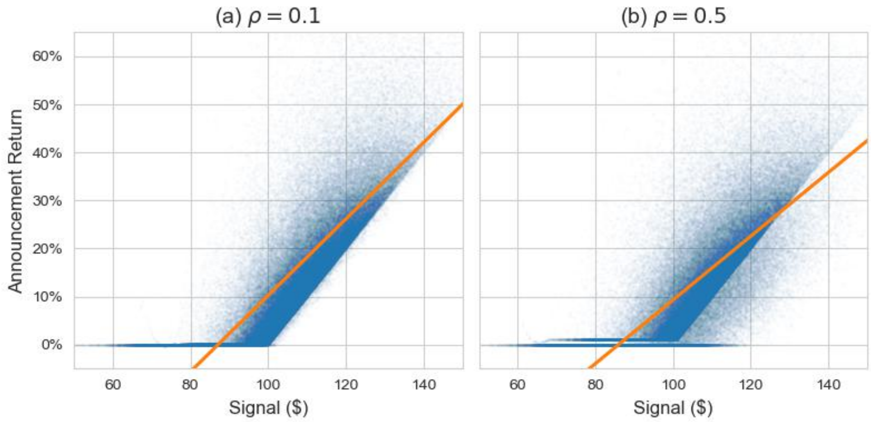

Figure 5.4: The model is simulated 100,000 times with parameter values $\mu = 105$, $\mu^* = 100$, $\sigma = 15$, and $\rho = 0.1$ (Panel a) or $\rho = 0.5$ (Panel b). Each plot shows the 200,000 announcement returns (two firms in each of 100,000 simulations) and signals. The tan lines are regression lines.

5.4. Dispersions of Announcement Times and Returns

If the two firms in the model have very different signals, then they are likely to announce at times that are far apart – the firm with a high signal will announce as soon as it receives its signal and the other firm will announce late, free riding in the meantime on the bump it gets from the first firm's disclosure. Thus, we expect dispersion in signals to be positively correlated with dispersion in announcement times across different realizations of the model. As explained in the previous section, dispersion in signals is not the same as dispersion in returns. Nevertheless, because signals and returns are positively correlated, it is reasonable that dispersion in returns should also be positively correlated with dispersion in announcement times. This is indeed the case. In Figure 5.5, we simulate the model, form deciles based on the dispersion in announcement dates, and compute mean signal dispersion and mean announcement return dispersion in each decile. Signal dispersion is much more tightly linked to date dispersion than is return dispersion, but there is still a positive correlation between date dispersion and return dispersion. This latter correlation is more empirically relevant because the signal in corporate earnings announcements is more difficult to empirically measure than the directly observable returns. We will see that this correlation between date dispersion and return dispersion also exists in the data (Figure 7.6).

5.5. Announcement Returns of Early and Late Announcers

Whether early or late announcers have higher average announcement returns depends on the correlation between the firm values. This is shown in Figure 5.6. We measure "early" or "late" by whether the firm is the first or second to disclose. There is of course a selection effect in that early announcers tend to be firms with especially good news, which induces higher announcement returns for early announcers. However, there is also a risk premium effect in that risk premia rise prior to disclosures, inducing higher average announcement returns for late announcers. The first effect usually dominates, but the second effect dominates when the correlation between firm values is low, causing late announcers to have higher average returns. This is consistent with the fact that the risk premium for the second announcer is higher and rises faster when the correlation between firm values is low, as shown in Figure 5.3.

5.6. Value of the Disclosure Option

When there are multiple firms that have not yet disclosed, there is a value to keeping the disclosure option alive. In other words, it must be sufficiently far in the money before it is optimal to exercise. This required "moneyness" is the difference between the exercise boundary and

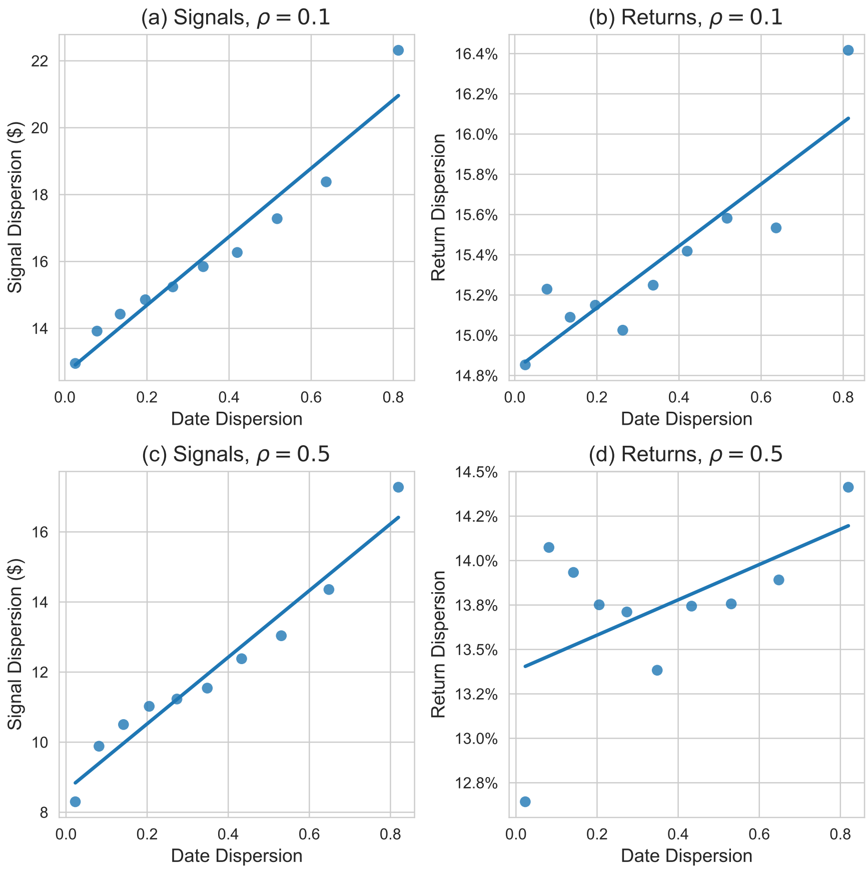

Figure 5.5: The model is simulated 100,000 times with parameter values $\mu = 105$, $\mu^* = 100$, $\sigma = 15$, and $\rho = 0.1$ (Panels a and b) or $\rho = 0.5$ (Panels c and d). In each simulation, date dispersion is calculated as the absolute difference between the two firms' announcement times, signal dispersion is calculated as the absolute difference between the two firms' signals, and return dispersion is calculated as the absolute difference between the two firms' percentage returns. The 100,000 simulations are grouped into deciles based on the date dispersion. Mean date dispersion, signal dispersion, and return dispersion are calculated in each decile. The figure shows the decile means.

the equilibrium price. The boundary and price are plotted in Figure 5.7. Figure 5.8 provides more information about how the gap between the boundary and price depends on the correlation. The dependence on the correlation is an inverse U-shape, and the gap is zero at the boundaries. The fact that it must be zero at the boundaries was first noted by Acharya, DeMarzo, and Kremer (2011). If the correlation is 0, then the two-firm model reduces to a pair of independent single-firm models. If the correlation is 1, then a firm that knows its value faces no additional uncertainty, so its disclosure option has zero value in that case also.

6. n Firms

Suppose now that there are n firms with symmetrically distributed values $\tilde{x}_i$. Let $\rho > 0$ denote the correlation of $\tilde{x}_i$ with $\tilde{x}_j$ for $i \neq j$. We can generalize Proposition 4.1 to describe the equilibrium price for the last firm to disclose. For each date t and each $s < t$, define

$$C_{st} = \min_{s \le u \le t} B_u .$$

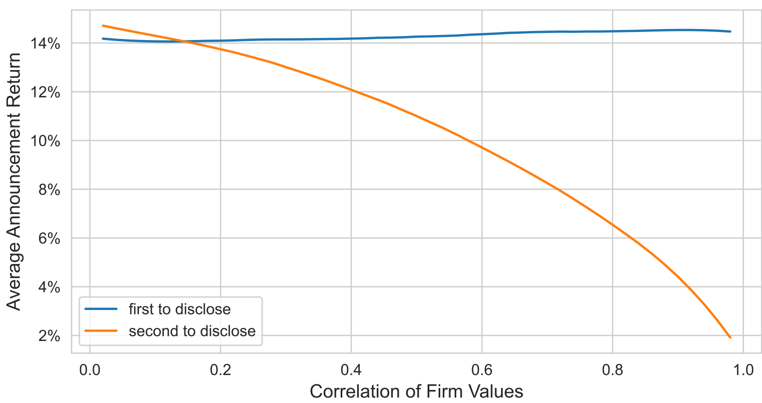

Figure 5.6: The model is simulated 100,000 times for each correlation with the other parameter values being $\mu = 105$, $\mu^* = 100$, and $\sigma = 15$. Average announcement returns are shown for the first firm to disclose and the second to disclose.

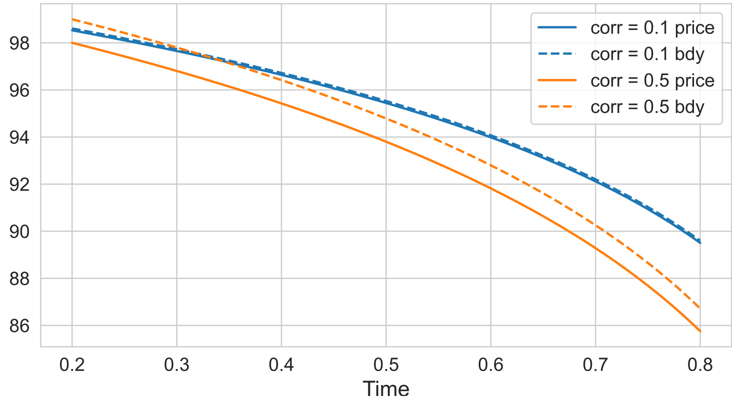

Figure 5.7: This figure shows the price (P_t) and the boundary (B_t) for various correlations, assuming neither firm has disclosed. The parameter values are $\mu = 105$, $\mu^* = 100$, and $\sigma = 15$.

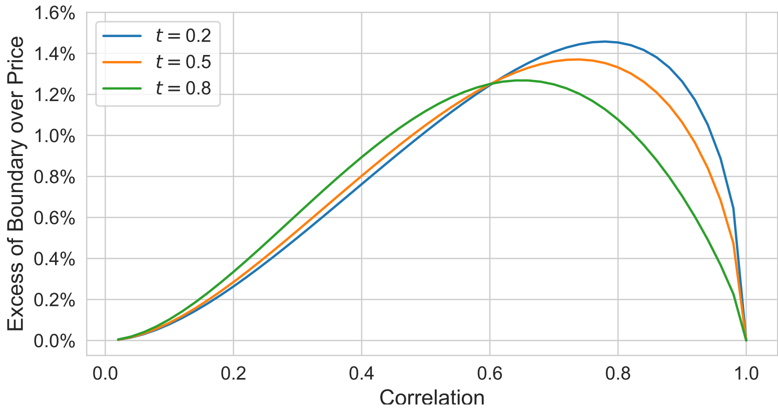

Figure 5.8: When a firm is at the disclosure boundary, the flow loss $B_t - P_t$ matches the expected benefit from the other firm disclosing a high value and lifting the price. This shows the flow loss or equivalently the expected benefit as a percentage of the price at three different dates for various correlations, assuming neither firm has disclosed by the date. The parameter values are $\mu = 105$, $\mu^* = 100$, and $\sigma = 15$.

This is the 'forward looking' minimum of the boundary at date s, looking forward through t. It matters for pricing because the market knows at t that some firm(s) may have received its signal before t – say, at $s < t$ – and has not yet disclosed because its signal was below the threshold at all dates between s and t. This means that its signal is below C_{st}. Because the market does not know the date s at which the firm received its signal (or even if it has received a signal at all) it has to integrate over s. These calculations produce the following result for the price of the last firm to disclose.

Proposition 6.1. *Let t denote the date at which the penultimate firm discloses. Let $c_1 < \cdots < c_j$ denote the distinct values of C_{st} for $s < t$, let $\Delta_1, \ldots, \Delta_j$ denote the lengths of the intervals on which $C_{st} = c_i$ for $i = 1, \ldots, j$. Set $\mu_1^* = \mu_1^*(\tilde{x}_2, \ldots, \tilde{x}_N)$, and set $\beta_i = (c_i - \mu_1^*)/\sigma_1$ for $i = 1, \ldots, j$. For each $u \geq t$ and for $k = 1, \ldots, j+1$, consider the following equation in z_k:*

$$z_k \;=\; -\frac{\sum_{i=1}^{k-1} \phi(\beta_i)\Delta_i + \phi(z_k)\left[u - \sum_{i=1}^{k-1} \Delta_i\right]}{1 - u + \sum_{i=1}^{k-1} \Phi(\beta_i)\Delta_i + \Phi(z_k)\left[u - \sum_{i=1}^{k-1} \Delta_i\right]} \tag{6.1}$$

There is a unique solution z_k. It is decreasing in u for each k. The equilibrium price is:

$$P_u = \begin{cases} \mu_1^* + \sigma_1 z_{j+1} & \textit{if } z_j > \beta_j \\ \mu_1^* + \sigma_1 z_j & \textit{otherwise, if } z_{j-1} > \beta_{j-1} \\ \mu_1^* + \sigma_1 z_{j-1} & \textit{otherwise, if } z_{j-2} > \beta_{j-2} \\ \cdots & \cdots \\ \mu_1^* + \sigma_1 z_1 & \textit{otherwise}. \end{cases} \tag{6.2}$$

It should in principle be possible to start with the last firm and work backwards to compute the equilibrium with n firms, as we did for two firms. What needs to be shown, which we have not done, is that the clustering result Proposition 4.2 extends to $n > 2$. That result is instrumental in deriving the two-firm equilibrium characterization in Proposition 4.3.

7. Empirics

Even though our model has implications for discretionary disclosures of all types, we examine information disclosure by firms in the context of earnings announcements. This setting is convenient, because it is easy to identify peer firms making disclosures in the same well-defined period (an earnings season). However, in some respects, it is not ideal for testing our model. There are at least two issues. For one, firms make earnings announcements repeatedly, after each fiscal quarter ends. Repeated games have many equilibria other than just repeating the equilibrium of the stage game, so the earnings disclosure policies of firms may be inconsistent with our one-shot equilibrium. For example, firms may develop a reputation of commitment to non-strategic disclosure policies. As shown in Section 5.1, such commitment can produce higher average prices, though it is inconsistent with equilibrium in a single play of the game. This biases against finding support for our theory in earnings announcements. To mitigate the bias somewhat, we will filter out firms that follow very predictable earnings announcement policies.

The second issue is that firms tend to schedule their earnings announcements in advance of the announcement date. The median announcement in our sample is scheduled 15 days in advance, and scheduled announcement dates are rarely revised. At first blush, scheduling announcements in advance seems at odds with strategic delay. However, the scheduling is not fully predictable at the beginning of the earnings cycle. Although firms give up the option of last minute adjustments, firms may still strategically schedule announcement dates based on information about their own and peer performance. For example, DeHaan, Shevlin, and Thornock (2015) show that firms with poor earnings news tend to schedule announcements when markets are less attentive (such as on Friday afternoons). Scheduling is an economic force that also biases against finding evidence of strategic timing of earnings disclosures.

7.1. Data

Our initial sample consists of all CRSP securities with share codes 10, 11, 12, or 31. This includes ADRs and companies registered outside the U.S. that trade on U.S. exchanges. Our main results are robust to filtering to only U.S. stocks (share codes 10 and 11). We keep only firm-quarter observations for which the fiscal quarter matches a calendar quarter – i.e., the fiscal quarter ends on March 31, June 30, September 31, or December 31. The fiscal quarters run from the first quarter of 2007 through the third quarter of 2021. We exclude firm-quarters for which the share price is below \$5 or the market cap is below \$50 million at the end of the fiscal quarter. We compute three-day announcement returns centered on the announcement date using CRSP return data running from the second quarter of 2007 through the first quarter of 2022.

We construct two measures to capture shifts in earnings announcement dates that could possibly be strategically motivated as in our model. First, we compare announcement dates to forecasts of announcement dates provided by Wall Street Horizons (WSH) at the beginning of each earnings season. We have forecasts for 147,376 announcements and 5,314 firms. We compute forecast errors defined as the difference in calendar days between the actual announcement date and the WSH forecast. For each announcement for which we have a WSH forecast, we also compute the year-on-year change in the announcement date, as another measure of the difference between the actual date and what might have been anticipated. We winsorize both measures at 1%. The winsorized distributions are shown in Figure 7.1. The correlation between the two measures is 80%. As can be seen from the figure, the distributions have fat tails, and the 1st and 99th percentiles exceed 10 days in absolute value.

7.2. Predictable Firms

Following Noh, So, and Verdi (2021), we look for calendar date patterns in earnings announcement dates. The typical firm makes most of its announcements on some particular (firm-specific) day of the week. The average across firms of the fraction of announcements made on the firm's most favored day is 60%. This frequency is significantly higher than by chance.[4] Across all firms, the most popular announcement day is Thursday, on which 34% of earnings announcements are made. Firms with unpredictable announcement dates have a lesser tendency to favor a particular day of the week for their announcements. Firms with above-median average absolute forecast errors are 6.9 percent less likely to announce on their most favored day compared to below-median firms, and the difference is statistically significant.

[4]To assess the significance of this frequency, we performed a Monte Carlo analysis. For each firm-quarter in our panel, we drew a random day of the week based on the frequency of each day in the full sample. We then computed for each firm the most favored day in the simulation and the frequency of announcing on that day, and then we averaged the frequencies across firms. We repeated the simulation 100 times. The Monte Carlo analysis indicates that our statistic that has a value of 60% in the data has a mean of 43% under the null of random choice with a standard error of only 0.12%.

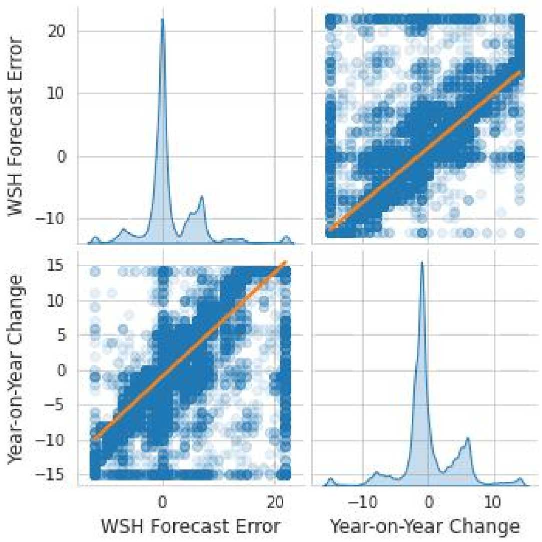

Figure 7.1: The firm-quarter panels of forecast errors and year-on-year changes in earnings dates are winsorized at 1%. The figure shows the empirical densities and scatter plots of the winsorized variables. The tan lines are regression lines.

In our subsequent analysis, we focus on firms that tend to have unpredictable earnings announcements. We exclude predictable firms. Predictable firms are the ones that are most likely to be playing a repeated game as described above rather than the one-shot game in our model. We define predictable firms as firms for which WSH exactly forecasts the announcement date for more than half of the firm's announcements or which announce on the same day of the week for more than 90 percent of their announcements.

Figure 7.2 shows the fraction of firms that we retain in our sample, sorted by industry. The fraction varies from 65 to 85 percent. Figure 7.3 shows the market cap distributions of predictable and unpredictable firms across all firm-quarters. There is a slight tendency for predictable firms to be larger. Figure 7.4 shows the distribution of observations across industries and over time after excluding predictable firms. The sample of unpredictable firms consists of 4,068 firms and 115,030 stock-quarter observations.

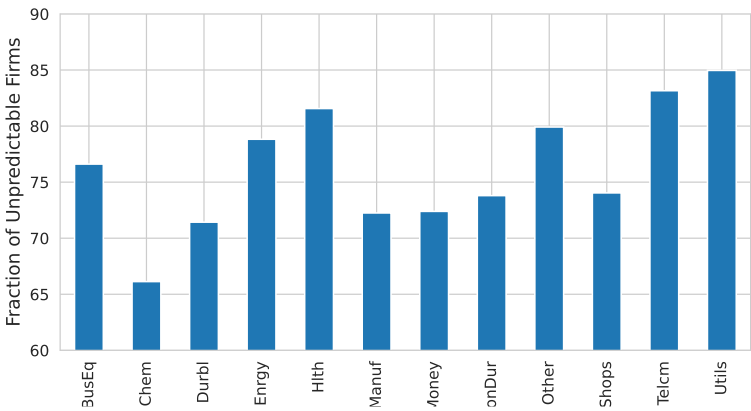

Figure 7.2: A firm is classified as predictable if WSH exactly forecasts its announcement date for more than half of our sample or it announces on the same day of the week for more than 90% of our sample. The figure shows the fraction of unpredictable firms in our sample for each of the Fama-French 12 industries.

This cross-sectional variation in earnings announcement date predictability implies that some firms exercise their option to strategically delay disclosure, while other firms do not. Table 1 shows that for a size-matched sample of firms, firms with unpredictable earnings dates tend to have significantly lower earnings response coefficients (ERC) and worse analyst forecasts (larger earnings and revenue surprise and disagreement among analysts). This suggests that when the information environment is less precise, there may be more to gain from strategically timing disclosure.

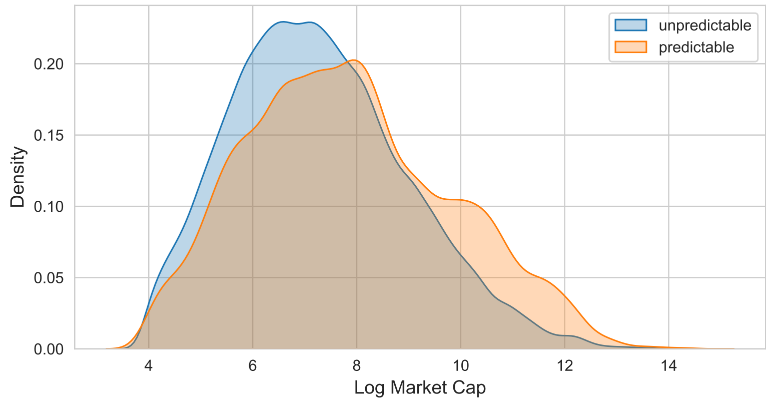

Figure 7.3: A firm is classified as predictable if WSH exactly forecasts its announcement date for more than half of our sample or it announces on the same day of the week for more than 90% of our sample. The figure shows the distributions of market capitalization across firm-quarters for both predictable and unpredictable firms.

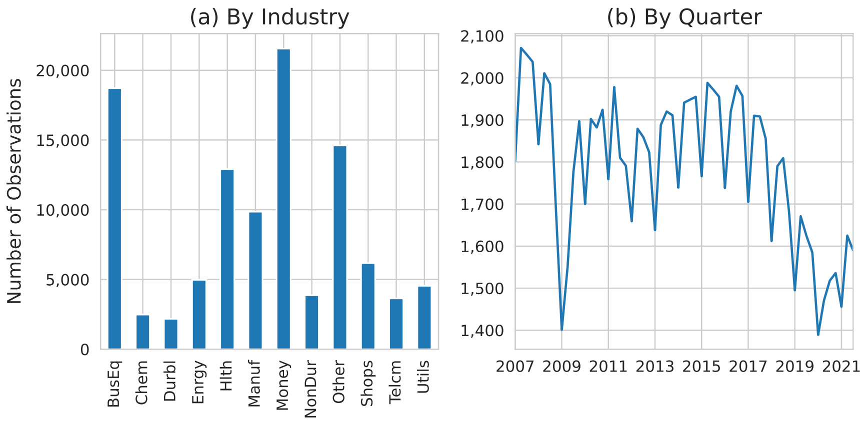

Figure 7.4: A firm is classified as predictable if WSH exactly forecasts its announcement date for more than half of our sample or it announces on the same day of the week for more than 90% of our sample. We retain the unpredictable firms in our sample. The figure shows the number of firm-quarters in our sample by Fama-French industry and the number of firms in our sample by quarter.

7.3. Dispersion of Announcement Dates and Returns within Earnings Cycles

In the model, there is a linkage between announcement date dispersion and return dispersion (Figure 5.5). To assess this in the data, we examine the dispersion of announcement dates and announcement returns within each industry quarter. For this task, we measure industries at the 6-digit GICS level. There are 73 industries represented in our panel, and there are 4,139 industry-quarters. The dispersion of announcement dates is measured as the standard deviation of calendar dates within an industry-quarter. Figure 7.5 shows the distribution of announcement date dispersion across our panel. The dispersion varies from 5 days at the 10th percentile to 15 days at the 90th percentile. Only approximately half of the variation in dispersion is explained by industry and quarter fixed effects.

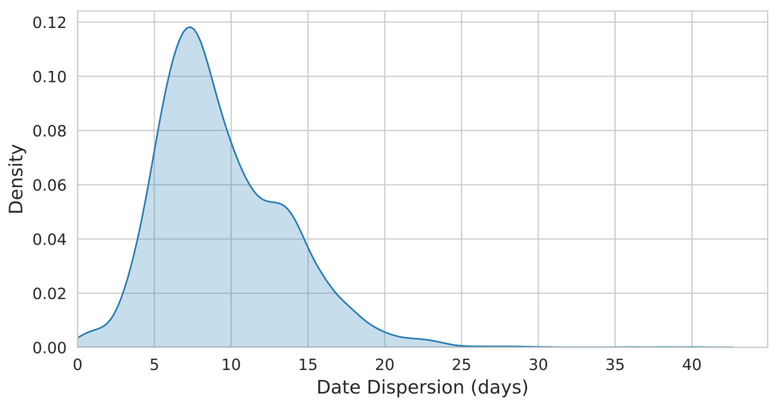

Figure 7.5: In each GICS 6 industry and in each quarter, we calculate the standard deviation of earnings announcement dates (date dispersion) for the firms in our sample. The sample is the sample of unpredictable firms. The distribution of date dispersions across our panel of industry quarters is shown.

We bin the industry-quarter panel into deciles based on announcement date dispersion and then compute mean announcement date dispersion and mean announcement return dispersion in each decile. Figure 7.6 shows that earnings seasons with high announcement date dispersion tend to also have high announcement return dispersion. This figure is similar to the corresponding figure generated from the model (Figure 5.5).

7.4. Strategic Scheduling of Announcement Dates

We test the core economic mechanism of our model: strategic timing in response to peer disclosures. The model predicts that firms delay their disclosures when peer firms disclose positive news and move their disclosures forward when peer firms disclose relatively negative news. We define peer firms as firms within the same industry according to Fama-French 12 and 4-digit GICS classifications. We also look at prior disclosures of other firms regardless of industry. For each announcement, we look at disclosures of peer firms and of all firms in a three-day window preceding the announcement.

We identify announcements as early, on-time, or late relative to Wall Street Horizons' forecasts. For early and on-time announcements, we consider the three-day window immediately preceding the announcement. Late announcements were possibly postponed due to peer announcements. If so, the peer announcements must have occurred prior to the postponement. Therefore, for late announcements, we look at the three-day window immediately preceding the Wall Street Horizons' forecast date. Figure

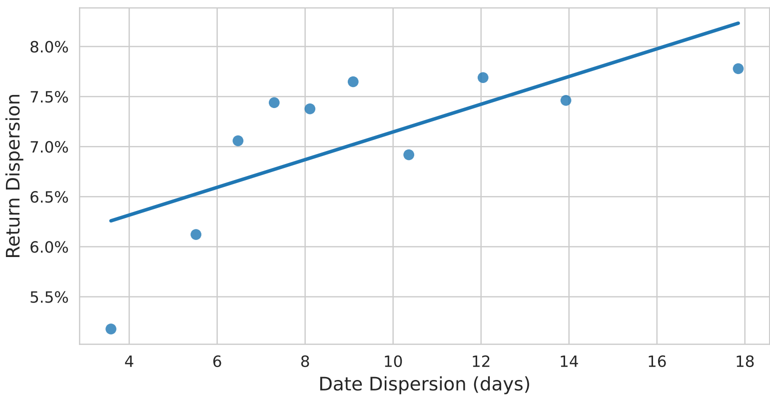

Figure 7.6: In each GICS 6 industry and in each quarter, we calculate the standard deviation of earnings announcement dates (date dispersion) and the standard deviation of 3-day announcement returns (return distribution). The panel of industry quarters is grouped into deciles based on date dispersion. The sample is the sample of unpredictable firms. The figure shows the mean return dispersion and the mean date dispersion in each decile.

7.7 illustrates how the windows are constructed. Figure 7.8 shows the frequency of late announcements for each of the Fama-French 12 industries and by quarter.

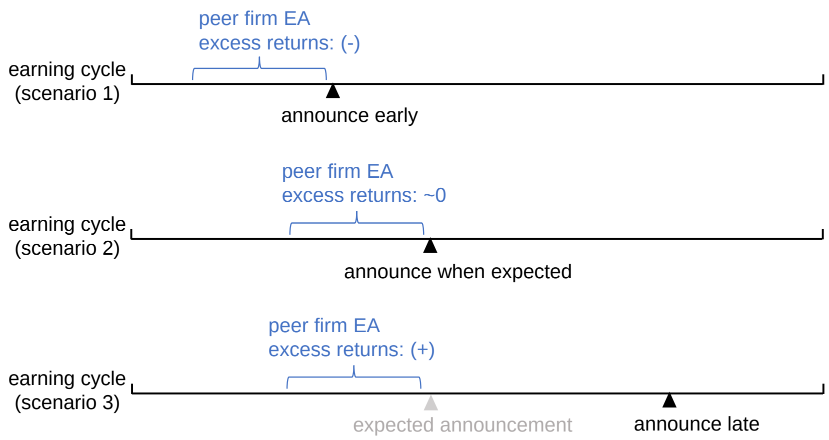

Figure 7.7: We classify a firm's announcement date as early, on time, or late relative to the WSH forecast. We calculate value-weighted announcement returns of peer firms and of all other firms who announce during a three-day window preceding the firm's announcement. The figure shows how the windows are defined for early, on-time, and late firms.

We seek to identify whether news disclosed by announcing firms during the three-day window was positive or negative. We do that using the three-day announcement returns of disclosing firms. We adjust the announcement returns for market, size, and value returns in order to get more precise estimates of the news conveyed by announcements. We adjust for market, size, and value returns by computing three-factor Fama-French excess returns (over the three-day window centered on the announcement date for each announcing firm). We value weight the three-factor excess returns for (i) firms within the same industry that announce within the window and (ii) all firms that announce within the window. We value weight using Compustat market caps at the end of the fiscal period for which the announcement is being made. We denote the value-weighted

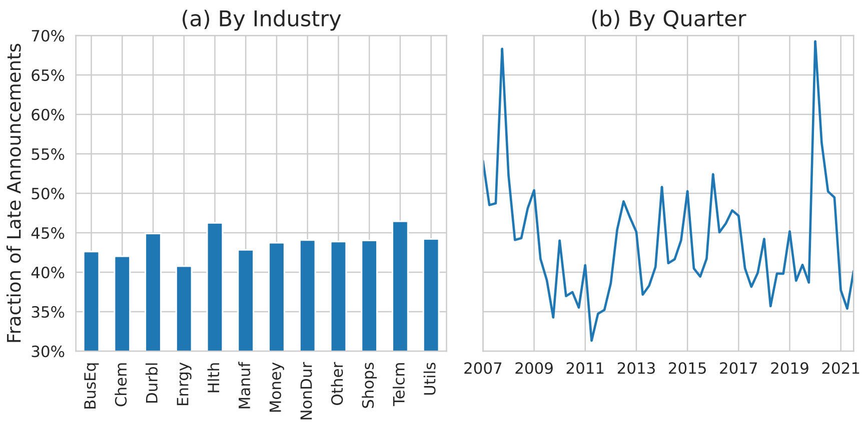

Figure 7.8: We classify a firm's announcement date as early, on time, or late relative to the WSH forecast. The figure shows the fraction of earnings-quarters that are late by Fama-French 12 industry and the fraction of firms that announce late by quarter. The sample is the sample of unpredictable firms. The peaks in Panel (b) occur at 2007Q4 and 2020Q1.

excess return (i) by $\mathrm{R}^{\mathrm{ind}}$ and the value-weighted excess return (ii) by $\mathrm{R}^{\mathrm{agg}}$.

It is important to point out that for early and on-time firms, and some late firms, the variables $\mathrm{R}^{\mathrm{ind}}$ and $\mathrm{R}^{\mathrm{agg}}$ are not observable prior to the firm scheduling its announcement date (which, as remarked earlier, typically occurs around 15 days prior to the announcement date). However, it seems reasonable that firms have some anticipation of the news that will be released by peers and take this into account when scheduling their own announcements. Our hypothesis is that firms that anticipate poor earnings to be released by peers schedule their own announcements as on time or early, and that firms that anticipate good earnings to be released by peers delay their own announcements.

Our first test of this hypothesis is to simply check whether peer announcement returns are higher in the three-day window for late announcers than for early and on-time announcers. We do this by regressing $\mathrm{R}^{\mathrm{ind}}$ on a dummy variable for late announcements. Table 2 reports the results. There is a higher average return in the three-day window for late announcers as hypothesized, though it is only marginally significant when quarter fixed effects are included. The coefficients are in basis points in Table 2. A 6 basis point return over a three-day announcement window corresponds to a 5 percent annual return.

Our second set of tests takes advantage of the fact that we have measures of how much an announcement was delayed or moved forward. We look both at the WSH forecast error and year-on-year calendar changes. Figure 7.9 shows the distributions of those variables in our sample after winsorization.

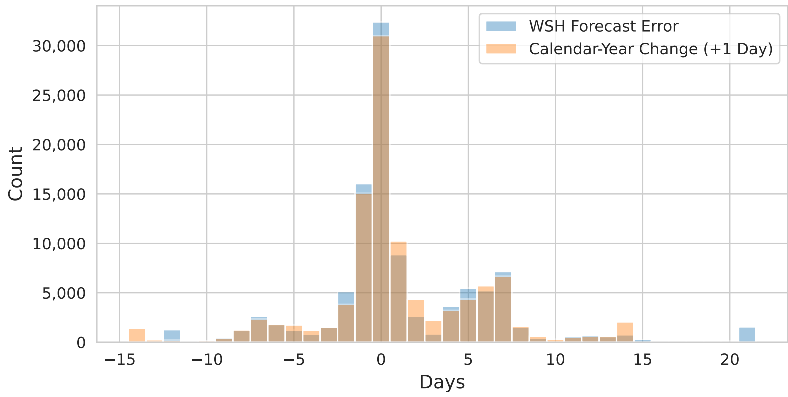

Figure 7.9: The figure shows the distributions of WSH forecast errors and year-on-year announcement date changes for our sample of unpredictable firms. The year-on-year changes are adjusted for the fact that firms frequently announce on the same day of the week, and a non-leap year consists of 52 weeks plus 1 day. The variables are winsorized at 1% within the full sample of predictable and unpredictable firms.

To test whether firms strategically time their earnings disclosures in response to announcement returns of peer and other firms, we estimate the following by OLS:

$$\Delta\tau_{ij} = \alpha_i + \delta_{t_{ij}} + \beta \mathrm{R}_{ij}^{\mathrm{ind}} + \gamma \mathrm{R}_{ij}^{\mathrm{agg}} + \epsilon_{ij} \tag{7.1}$$

where $\Delta\tau_{ij}$ denotes either the Wall Street Horizons forecast error or the year-to-year change of the announcement date of firm i in quarter j, α_i is a firm fixed effect, t_{ij} is the announcement day of firm i in quarter j, δ_t is a day fixed effect, $\mathrm{R}_{ij}^{\mathrm{ind}}$ is the industry excess return in the relevant three-day window for firm i in quarter j, and $\mathrm{R}_{ij}^{\mathrm{agg}}$ is the aggregate excess return in the relevant three-day window for firm i in quarter j. If firms strategically delay in response to positive peer news and move their disclosures forward in response to relatively negative peer news, we expect to find positive β (industry-specific peers) and γ (all potential peers).

Table 4 reports estimates of (7.1) when the dependent variable is the WSH forecast error. We consider the Fama-French 12 industry and the 4-digit GICS classifications for defining peer returns. The Fama-French-12 peer return is positive as expected and is significant in all cases. The GICS-4 peer return is positive in all cases but is only marginally significant when day fixed effects are included and becomes insignificant when the aggregate return is included in the specification. The aggregate return is positive and significant in all cases. However, the R^2's (untabulated) are less than 1%, and the point estimates are small. Table 5 reports the analogous estimates when the dependent variable is the year-on-year change in the announcement date. The results are very similar, as Figure 7.9 suggests they should be.

8. Conclusion

When there is a possibility but not a certainty that a firm is informed, then it may fail to voluntarily disclose bad news. Investors make inferences from the absence of disclosures, so prices fall during periods of nondisclosure. Because the absence of disclosures is itself information, one might expect risk premia to also fall during periods of nondisclosure, but we show that the opposite is true. Risk premia rise before disclosures, fall for non-disclosing firms upon a correlated firm's disclosure, and then rise again.

Voluntary disclosures are always good news and are also good news for positively correlated firms. Nevertheless, disclosures can hasten disclosures by other correlated firms because the information revealed by disclosures reduces the value of keeping disclosure options alive. Thus, clustering of disclosures sometimes occurs, even when information arrival times are not clustered.

We find empirical support for the model's implications regarding the timing of disclosures. The model also has implications for announcement returns, which we have not tested. To analyze those implications in the context of earnings announcements, we would have to take into account the advance scheduling of earnings announcements. The returns in our model conjoin two phenomena: the fact that the firm has chosen to disclose and the actual content of the disclosure. Earnings announcement dates, once scheduled, are rarely revised. So, the news from choosing to disclose should be reflected in the price of the firm after scheduling. This news is always positive according to the model. The return on the announcement date, which depends on the content of the announcement, can be positive or negative according to the model – it has a zero risk-neutral mean. However, there is evidence of high average returns around announcement dates that do not seem to be risk-based, which would be inconsistent with our model. For example, Frazzini and Lamont (2022) document abnormal returns from holding stocks over announcement months. It would be useful in future research to examine the extent to which the model's implications regarding announcement returns are consistent with the evidence.

Appendices

A. Marginal condition for Optimal Stopping

Consider a firm that has learned its value x. If it has disclosed by t, its remaining value at t is $(1-t)x$. Before disclosing, denote its value function by $J(t, x, \ldots)$, where the omitted data is the history of disclosures before t. The value matching condition is $J = (1-t)x$ when $B_t = x$. The smooth pasting condition is $J_x = 1-t$ when $B_t = x$, where we use subscripts to denote partial derivatives. We regularize J to be left continuous and denote right limits by J^+. Set $\Delta J = J^+ - J$. The process ΔJ is zero at all dates other than (possibly) those dates at which other firms disclose, and on a disclosure date J denotes the pre-disclosure value, J^+ denotes the post-disclosure value, and $\Delta J = J^+ - J$ is the (possible) jump. The stochastic differential of J is $\mathrm{d}J = J_t\,\mathrm{d}t + \Delta J$. The Bellman condition for optimality is that $P\,\mathrm{d}t + \mathsf{E}^*[\mathrm{d}J] = 0$.

Let $\lambda(t, x, \ldots)$ denote the arrival rate of disclosures that create jumps, conditional on the firm's signal x. Then,

$$\mathsf{E}^*[\Delta J] \;=\; \Big\{\mathsf{E}^*[J^+ \mid \text{jump}] - J\Big\}\,\lambda\,\mathrm{d}t\,.$$

Substituting this into the Bellman condition, we have

$$P + J_t + \lambda \mathsf{E}^*[J^+ \mid \text{jump}] - \lambda J = 0\,. \tag{A.1}$$

We will solve (A.1) for J in terms of J^+, P, and λ. Fix any history of disclosures through any date s. Consider any firm that learns its value x at s with $x < B_s$. Assume there are no subsequent disclosures. Because the boundary is decreasing between disclosures, there is a unique $t(x) > s$ at which $B_{t(x)} = x$. We will solve (A.1) on the time interval $(s, t(x))$. Set $\omega = \lambda \mathsf{E}^*[J^+ \mid \text{jump}]$. Because we are holding past disclosures fixed, everything evolves deterministically on $(s, t(x))$. We can write everything as a function of time and x: $J(\tau, x)$, $\omega(\tau, x)$, $P(\tau)$, and $\lambda(\tau, x)$ for $\tau \in$

$(s, t(x))$. The value matching condition $J(t(x), x) = (1 - t(x))x$ is a terminal condition at $t(x)$ for the linear ordinary differential equation (A.1) for J. Solving the equation subject to the terminal condition gives, for $\tau \in (s, t(x))$,

$$
\begin{aligned}
J(\tau, x) &= (1 - t(x))x \exp\left(-\int_\tau^{t(x)} \lambda(u, x)\,\mathrm{d}u\right) \\
&\quad + \int_\tau^{t(x)} \exp\left(-\int_\tau^u \lambda(a, x)\,\mathrm{d}a\right)[P(u) + \omega(u, x)]\,\mathrm{d}u\,. \qquad \text{(A.2)}
\end{aligned}
$$

To apply the smooth pasting condition, we differentiate with respect to x, evaluate the derivative at $\tau = t(x)$ and set it equal to $1 - t(x)$. The derivative with respect to x evaluated at $\tau = t(x)$ is

$$
1 - t(x) - xt'(x) - \lambda(t(x))[1 - t(x)]xt'(x) + [P(t(x) + \omega(t(x), x)]t'(x)\,.
$$

Equating this to $1 - t(x)$ and cancelling the $t'(x)$ factor gives

$$
x - P(t) = [\mathsf{E}^*[J^+(t) \mid \text{jump}] - (1 - t)x]\lambda(t)\,, \qquad \text{(A.3)}
$$

where we substitute $\omega = \lambda\mathsf{E}^*[J^+(t) \mid \text{jump}]$. The right-hand side of (A.3) is the expected jump in J conditional on a jump multiplied by the arrival rate of jumps. The equation states that the instantaneous loss $x - P$ for a firm whose price is below its true value equals the gain that the firm expects to realize from another firm disclosing, when the firm is at the disclosure boundary.

B. Proofs

Proof of Lemma 1.1. Set $\tilde{x} = (\tilde{x}_1 \cdots \tilde{x}_n)'$. The moment generating function of $\tilde{x}$ under the risk-neutral probability is the function of $\lambda \in \mathbb{R}^n$ defined by

$$
\mathsf{E}^*\left[\mathrm{e}^{\lambda'\tilde{x}}\right] = \mathsf{E}\left[\tilde{m}\mathrm{e}^{\lambda'\tilde{x}}\right] = \mathsf{E}\left[\mathrm{e}^{-\alpha\tilde{w} + \lambda'\tilde{x}}\right] \Big/ \mathsf{E}\left[\mathrm{e}^{-\alpha\tilde{w}}\right].
$$

Using the joint normality, it is straightforward to calculate that this equals

$$\mathrm{e}^{\mu^* \sum_{i=1}^n \lambda_i + \frac{1}{2}\lambda' \Sigma \lambda},$$

where $\mu^* = \mu - \alpha \operatorname{cov}(\tilde{x}_i, \tilde{w})$ and Σ is the physical covariance matrix of the $\tilde{x}_i$. This proves the claim about the risk-neutral distribution of the $\tilde{x}_i$. The last statement of the lemma follows from the fact that $\tilde{m}$ and the information arrival times are independent. □

Proof of Lemma 1.2. The conditional distributions are a standard Bayes rule application. The distributions under the risk-neutral probability follow as in Lemma 1.1. □

Proof of Proposition 3.1. Set $z = (P_t - \mu^*)/\sigma$. Given that P is a decreasing function of time by assumption, the firm fails to disclose by t if and only if it either did not receive its signal by t, which happens with risk-neutral probability $1 - t$, or it received its signal which was less than P_t, which happens with risk-neutral probability $t\Phi(z)$. Furthermore, by the standard formula for the mean of a truncated normal, $\mathsf{E}^*[\tilde{x} \mid \tilde{x} \leq P_t] = \mu^* - \sigma\phi(z)/\Phi(z)$. Therefore,

$$\mathsf{E}^*[\tilde{x} \mid \text{no disclosure}] = \mu^* - \sigma \frac{t\phi(z)}{t\Phi(z) + 1 - t}.$$

Because $\mu^* + \sigma z = P_t = \mathsf{E}^*[\tilde{x} \mid \text{no disclosure}]$, we have

$$z = -\frac{t\phi(z)}{t\Phi(z) + 1 - t}.$$

Equivalently, $F(t, z) = 0$, where

$$F(t, z) := t\phi(z) + z[t\Phi(z) + 1 - t].$$

We have $\partial F/\partial z = t\Phi(z) + 1 - t > 0$, $F(0) = t\phi(0) > 0$, and $\lim_{z \to -\infty} F(t, z) = -\infty$. Therefore, there is a unique z such that $F(t, z) =$

0 and $z < 0$. Furthermore, at this root, by the implicit function theorem,

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -\frac{\partial F}{\partial t}\Bigg/\frac{\partial F}{\partial z} = \frac{-\phi(z) - z\Phi(z) + z}{t\Phi(z) + 1 - t} < 0\,.$$

To see the inequality in the previous line, recall that the denominator is positive and observe that the numerator is increasing in z and negative at $z = 0$, hence negative for all $z \leq 0$. □

Proof of Proposition 3.2. Let $\tilde{x}_1$ denote the value of the strategic firm and $\tilde{x}_2$ the value of the nonstrategic firm. By joint normality and the fact that $\tilde{x}_1$ and $\tilde{x}_2$ are identically distributed, $\mathsf{E}^*[\tilde{x}_2 \mid \tilde{x}_1] = \rho\tilde{x}_1 + (1-\rho)\mu^*$. This is the post-disclosure price of firm 2, and, by iterated expectations, the pre-disclosure price is

$$\begin{aligned}&\mathsf{E}^*[\tilde{x}_2 \mid \text{nondisclosure by firm 1}]\\ &\qquad = \rho\mathsf{E}^*[\tilde{x}_1 \mid \text{nondisclosure by firm 1}] + (1-\rho)\mu^* \equiv \rho P_t + (1-\rho)\mu^*\,.\end{aligned}$$

□

Proof of Proposition 4.1. When firm 2 discloses at date t, the equilibrium price for $u \geq t$ is the solution P_u of the fixed-point condition

$$\begin{aligned}P_u \;=\;& \\ \mathsf{E}^*&\left[\tilde{x}_1 \,\middle|\, \tilde{x}_2, \left\{\tilde{\theta}_1 > u\right\} \cup \left\{\tilde{\theta}_1 \leq t, \tilde{x}_1 < B_t \wedge P_u\right\} \cup \left\{t \leq \tilde{\theta}_1 \leq u, \tilde{x}_1 < P_u\right\}\right].\end{aligned} \tag{B.1}$$

Considering the two possible cases $P_u \leq B_t$ and $P_u > B_t$, we see that (B.1) holds if and only if $P_u = p$ where either

$$p = \mathsf{E}^*\left[\tilde{x}_1 \,\middle|\, \tilde{x}_2, \left\{\tilde{\theta}_1 > u\right\} \cup \left\{\tilde{\theta}_1 \leq u, \tilde{x}_1 < p\right\}\right] \leq B_t \tag{B.2}$$

or

$$p = \mathsf{E}^*\left[\tilde{x}_1 \,\middle|\, \tilde{x}_2, \left\{\tilde{\theta}_1 > u\right\} \cup \left\{\tilde{\theta}_1 \le t, \tilde{x}_1 < B_t\right\} \cup \left\{t \le \tilde{\theta}_1 \le u, \tilde{x}_1 < p\right\}\right] > B_t\,. \quad \text{(B.3)}$$

From the proof of Proposition 3.1, there is a unique p satisfying the equality in (B.2) and it is $p = \mu_1^* + \sigma_1 z$ where z is the unique root of the function $F(u, \cdot)$ defined in the proof of Proposition 3.1. By similar reasoning, we can see that there is also a unique p satisfying the equality in (B.3), and it is $p = \mu_1^* + \sigma_1 y$ where y is the unique root of $G(u, \cdot)$, with

$$G(u, y) = t\phi(\beta) + (u - t)\phi(y) + y[t\Phi(\beta) + (u - t)\Phi(y) + 1 - u]\,.$$

We have $F(u, \beta) = G(u, \beta)$ and both F and G are increasing in their second arguments, so $z \le \beta$ if and only if $y \le \beta$. The solution $p = \mu_1^* + \sigma_1 z$ of the equality in (B.3) satisfies $p \le B_t$ if and only if $z \le \beta$, and the solution $p = \mu_1^* + \sigma_1 y$ of the equality in (B.2) satisfies $p \le B_t$ if and only if $y \le \beta$. Thus, there is a solution of one and only one of (B.2) and (B.3), and the solution coincides with the formula (4.2). □

To prove Proposition 4.2, we will use the following lemmas.

Lemma B.1. *The cost $B_t - P_t$ of waiting to disclose for a firm at the boundary can be expressed in terms of the standardized boundary $b = (B_t - \mu^*)/\sigma$ as*

$$\sigma \;\times\; \left[b + \frac{t(1+\rho)\phi\left(b\right)\left[1 - t + t\Phi\left(b\sqrt{\frac{1-\rho}{1+\rho}}\right)\right]}{\left[1 - t + t\Phi\left(b\right)\right]^2 + t^2\left[\Gamma\left(b, b, \rho\right) - \Phi(b)^2\right]}\right]\,. \qquad \text{(B.4)}$$

Proof of Lemma B.1. If there has been no disclosure prior to date t, then either (i) $\tilde{\theta}_1 > t$ and $\tilde{\theta}_2 > t$, or (ii) $\tilde{\theta}_1 > t$, $\tilde{\theta}_2 \le t$, and $\tilde{x}_2 < B_t$, or (iii) $\tilde{\theta}_1 \le t$, $\tilde{x}_1 < B_t$, and $\tilde{\theta}_2 > t$, or (iv) $\tilde{\theta}_1 \le t$, $\tilde{x}_1 < B_t$, $\tilde{\theta}_2 \le t$, and $\tilde{x}_2 < B_t$.

The risk-neutral probabilities of events (i)–(iv) are, respectively,

$$\begin{aligned}
p_1 &= (1-t)^2 \\
p_2 &= t(1-t)\Phi(b) \\
p_3 &= t(1-t)\Phi(b) \\
p_4 &= t^2\Gamma(b,b,\rho)\,.
\end{aligned}$$

The risk-neutral expectation of $\tilde{x}_1$ conditional on each of the four events is

$$\begin{aligned}
a_1 &= \mu^* \\
a_2 &= \mu^* - \sigma\phi(b)/\Phi(b)\,, \\
a_3 &= \mu^* - \rho\sigma\phi(b/\Phi(b) \\
a_4 &= \mu^* - \frac{\sigma(1+\rho)\phi(b)\Phi\left((1-\rho)b/\sqrt{1-\rho^2}\right)}{\Gamma(b,b,\rho)}\,.
\end{aligned}$$

We use equation (1) in Rosenbaum (1961) for a_4. The equilibrium price is

$$\sum_{i=1}^{4} p_i a_i \bigg/ \sum_{i=1}^{4} p_i\,,$$

which yields

$$P_t = \mu^* - \sigma \times \frac{t(1+\rho)\phi(b)\left[1-t+t\Phi\left(b\sqrt{\frac{1-\rho}{1+\rho}}\right)\right]}{\left[1-t+t\Phi(b)\right]^2 + t^2\left[\Gamma\left(b,b,,\rho\right)-\Phi(b)^2\right]}\,.$$

This implies (B.4). □

Lemma B.2. *For $b<d<0$, define*

$$\rho(b,d) = \frac{b^2-d^2}{b^2+d^2}\,. \tag{B.5}$$

Let $\Gamma(\cdot,\cdot,\rho)$ denote the bivariate distribution function for normal random variables with zero means, unit standard deviations, and correlation equal *to ρ. Then*

$$\frac{\partial}{\partial b}\Gamma(b,b,\rho(b,d)) \;=\; \phi(b)\left[2\Phi(d)+\frac{\phi(d)\sqrt{1-\rho(b,d)^2}}{b}\right]. \tag{B.6}$$

Proof of Lemma B.2. Consider two standard normals with correlation ρ defined as $\tilde{x}_1 = \tilde{\varepsilon}_1$ and $\tilde{x}_2 = \rho\tilde{\varepsilon}_1 + \sqrt{1-\rho^2}\tilde{\varepsilon}_2$ where $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are independent standard normals. We have

$$\begin{aligned}\Gamma(s_1,s_2,\rho) &= \text{prob}\left(\tilde{\varepsilon}_1 \le s_1, \tilde{\varepsilon}_2 \le \frac{s_2-\rho\tilde{\varepsilon}_1}{\sqrt{1-\rho^2}}\right)\\ &= \int_{-\infty}^{s_1}\int_{-\infty}^{(s_2-\rho\varepsilon_1)/\sqrt{1-\rho^2}} \phi(\varepsilon_2)\,\mathrm{d}\varepsilon_2\,\phi(\varepsilon_1)\,\mathrm{d}\varepsilon_1\\ &= \int_{-\infty}^{s_1}\Phi\left(\frac{s_2-\rho\varepsilon_1}{\sqrt{1-\rho^2}}\right)\phi(\varepsilon_1)\,\mathrm{d}\varepsilon_1\end{aligned}$$

This implies

$$\frac{\partial}{\partial s_1}\Gamma(s_1,s_2,\rho) = \Phi\left(\frac{s_2-\rho s_1}{\sqrt{1-\rho^2}}\right)\phi(s_1) \tag{B.7}$$

and by symmetry

$$\frac{\partial}{\partial s_2}\Gamma(s_1,s_2,\rho) = \Phi\left(\frac{s_1-\rho s_2}{\sqrt{1-\rho^2}}\right)\phi(s_2)\,. \tag{B.8}$$

Furthermore, using the fact that

$$\frac{\partial}{\partial\rho}\frac{s_2-\rho\varepsilon_1}{\sqrt{1-\rho^2}} = (1-\rho^2)^{-3/2}(\rho s_2-\varepsilon_1)\,,$$

we obtain

$$\frac{\partial\Gamma(s_1,s_2,\rho)}{\partial\rho} = (1-\rho^2)^{-3/2}\int_{-\infty}^{s_1}\phi\left(\frac{s_2-\rho\varepsilon_1}{\sqrt{1-\rho^2}}\right)(\rho s_2-\varepsilon_1)\phi(\varepsilon_1)\,\mathrm{d}\varepsilon_1\,.$$

Also,

$$\phi\left(\frac{s_2-\rho\varepsilon_1}{\sqrt{1-\rho^2}}\right)\phi(\varepsilon_1)=\frac{1}{2\pi}\exp\left(-\frac{1}{2}\left[\frac{(s_2-\rho\varepsilon_1)^2}{1-\rho^2}+\varepsilon_1^2\right]\right)$$

and

$$\frac{(s_2-\rho\varepsilon_1)^2}{1-\rho^2}+\varepsilon_1^2=s_2^2+\frac{(\varepsilon_1-s_2\rho)^2}{1-\rho^2}$$

so

$$\phi\left(\frac{s_2-\rho\varepsilon_1}{\sqrt{1-\rho^2}}\right)\phi(\varepsilon_1)=\phi(s_2)\phi\left(\frac{\varepsilon_1-s_2\rho}{\sqrt{1-\rho^2}}\right).$$

Hence,

$$\frac{\partial\Gamma(s_1,s_2,\rho)}{\partial\rho}=(1-\rho^2)^{-3/2}\phi(s_2)\int_{-\infty}^{s_1}(\rho s_2-\varepsilon_1)\phi\left(\frac{\varepsilon_1-s_2\rho}{\sqrt{1-\rho^2}}\right)\,\mathrm{d}\varepsilon_1\,.$$

Making the change of variables $u=(\varepsilon_1-s_2\rho)/\sqrt{1-\rho^2}$, we obtain

$$\begin{aligned}\int_{-\infty}^{s_1}(\rho s_2-\varepsilon_1)\phi\left(\frac{\varepsilon_1-s_2\rho}{\sqrt{1-\rho^2}}\right)\,\mathrm{d}\varepsilon_1&=(1-\rho^2)\int_{-\infty}^{(s_1-s_2\rho)/\sqrt{1-\rho^2}}-u\phi(u)\,\mathrm{d}u\\&=(1-\rho^2)\phi\left(\frac{s_1-s_2\rho}{\sqrt{1-\rho^2}}\right).\end{aligned}$$

Therefore,

$$\frac{\partial\Gamma(s_1,s_2,\rho)}{\partial\rho}=\frac{1}{\sqrt{1-\rho^2}}\cdot\phi(s_2)\phi\left(\frac{s_1-s_2\rho}{\sqrt{1-\rho^2}}\right).$$

Also,

$$\frac{\partial\rho(b,d)}{\partial b}=\frac{4bd^2}{(b^2+d^2)^2}\,,$$

so

$$\frac{\partial\Gamma(s_1,s_2,\rho(b,d))}{\partial b}=\frac{1}{\sqrt{1-\rho^2}}\cdot\phi(s_2)\phi\left(\frac{s_1-s_2\rho}{\sqrt{1-\rho^2}}\right)\cdot\frac{4bd^2}{(b^2+d^2)^2}\,. \tag{B.9}$$

Combining (B.7), (B.8), and (B.9) yields

$$\frac{\partial \Gamma(b,b,\rho(b,d))}{\partial b} = 2\Phi\left(\frac{(1-\rho)b}{\sqrt{1-\rho^2}}\right)\phi(b) + \frac{1}{\sqrt{1-\rho^2}}\cdot\phi(b)\phi\left(\frac{(1-\rho)b}{\sqrt{1-\rho^2}}\right)\cdot\frac{4bd^2}{(b^2+d^2)^2}\,.$$

This simplifies to (B.6). □

Lemma B.3. *For all $b<0$ and $x \in (0,1)$,*

$$-2b\Phi(b)\left(b^2+\frac{1-x^2}{1+x^2}\right)+\frac{2xb^2\phi(b)}{1+x^2}-b^2\phi(b)\geq 0\,. \tag{B.10}$$

Proof of Lemma B.3. Set $h(b) = -2b\Phi(b) - \phi(b)$. It is easy to verify that h is positive, increasing, and convex on $(-\infty,-\sqrt{3})$ and concave on $(-\sqrt{3},0)$. Set $b^* = \inf\{b<0 \mid h(b)<0\}$. Then b^* is in the region in which h is concave and, because h is larger at some $b<b^*$ than at b^*, h must be decreasing at b^* and for all $b>b^*$. Hence, $h(b)<0$ for $b>b^*$.

The inequality (B.10) holds at all b for which $h(b)\geq 0$. Thus, it holds on the interval $(-\infty,b^*]$. It remains to show that it holds on $(b^*,0)$. Multiplying (B.10) by $-(1+x^2)/b>0$, we can see that (B.10) is equivalent to

$$[-bh(b)-2\Phi(b)]x^2-2b\phi(b)x+2\Phi(b)-bh(b)\geq 0\,. \tag{B.11}$$

On the region $(b^*,0)$, $h(b)<0$, so (B.11) is concave in x. It is equal to $4b^2\Phi(b)>0$ at $x=1$, and it is equal to $k(b) := 2\Phi(b)-bh(b)$ at $x=0$. A concave function on $(0,1)$ that is positive at 0 and at 1 must be positive everywhere on $(0,1)$, so it suffices now to show that $k(b)>0$ for $b\in(b^*,0)$. The function k is increasing on $(b^*,0)$, because its derivative equals $(1+b^2)\phi(b)-2h(b)$. Furthermore, $h(b^*)=0$, so $k(b^*)=2\Phi(b^*)>0$. Hence, k is positive on $(b^*,0)$. □

Lemma B.4. *At each date t prior to either firm's disclosure, it must be* *true that*

$$B_t > \mu^* + \sigma\sqrt{\frac{1+\rho}{1-\rho}}z\,, \tag{B.12}$$

where z is the root of the function $F(t,\cdot)$ defined in the proof of Proposition 3.1.

Proof of Lemma B.4. Set $b = (B_t - \mu^*)/\sigma$ and

$$d = b\sqrt{\frac{1-\rho}{1+\rho}}\,. \tag{B.13}$$

Inequality (B.12) is equivalent to $d \geq z$. Because $F(t,\cdot)$ is monotone and $F(t,z) = 0$, this is equivalent to $F(t,d) \geq 0$. So, what we need to show is that

$$d \;>\; -\frac{t\phi(d)}{1-t+t\Phi(d)}\,. \tag{B.14}$$

Clearly, this is true if $d \geq 0$, so we can assume $d < 0$, which implies $b < d$.

The nonnegativity of $B_t - P_t$ in (B.4) implies that

$$b \;\geq\; -\frac{t(1+\rho)\phi\left(b\right)\left[1-t+t\Phi(d)\right]}{\left[1-t+t\Phi\left(b\right)\right]^2 + t^2\left[\Gamma\left(b,b,\rho\right) - \Phi(b)^2\right]}\,.$$

Multiplying by $\sqrt{(1-\rho)/(1+\rho)}$ yields

$$d \;\geq\; -\frac{t\sqrt{1-\rho^2}\phi\left(b\right)\left[1-t+t\Phi(d)\right]}{\left[1-t+t\Phi\left(b\right)\right]^2 + t^2\left[\Gamma\left(b,b,\rho\right) - \Phi(b)^2\right]}\,.$$

Therefore, to establish (B.14), it suffices to show that

$$\frac{\sqrt{1-\rho^2}\phi\left(b\right)\left[1-t+t\Phi(d)\right]}{\left[1-t+t\Phi\left(b\right)\right]^2 + t^2\left[\Gamma\left(b,b,\rho\right) - \Phi(b)^2\right]} \;<\; \frac{\phi(d)}{t\Phi(d)+1-t}\,. \tag{B.15}$$

The definition (B.13) of d is equivalent to $\rho = \rho(b,d)$ as in Lemma B.2. Substitute $\rho = \rho(b,d)$ into the left-hand side of (B.15) to write it as $H(b,d)$. The right-hand side of (B.15) is then $H(d,d)$. Because $b < d$, it suffices to show that H is strictly increasing in its first argument. This is equivalent to

$$\begin{aligned}
&\left(\left[1-t+t\Phi\left(b\right)\right]^2+t^2\left[\Gamma\left(b,b,\rho\right)-\Phi(b)^2\right]\right)\\
&\qquad\qquad\times\frac{\partial}{\partial b}\left(\sqrt{1-\rho^2}\phi\left(b\right)\left[1-t+t\Phi(d)\right]\right)\\
&\qquad\qquad>\left(\sqrt{1-\rho^2}\phi\left(b\right)\left[1-t+t\Phi(d)\right]\right)\\
&\qquad\times\frac{\partial}{\partial b}\left(\left[1-t+t\Phi\left(b\right)\right]^2+t^2\left[\Gamma\left(b,b,\rho\right)-\Phi(b)^2\right]\right). \qquad \text{(B.16)}
\end{aligned}$$

With the aid of Lemma B.2 and the fact that

$$\sqrt{1-\rho(b,d)^2}=2bd/(b^2+d^2)\,,$$

we can write (B.16) as

$$\begin{aligned}
&\left(\left[1-t+t\Phi\left(b\right)\right]^2+t^2\left[\Gamma\left(b,b,\rho\right)-\Phi(b)^2\right]\right)\left(-b-\frac{\rho}{b}\right)\\
&\qquad>2t\phi(b)-2t^2\phi(b)\left(1-\Phi(d)-\frac{\phi(d)\sqrt{1-\rho^2}}{2b}\right). \qquad \text{(B.17)}
\end{aligned}$$

Because $\Gamma(b,b,\rho)>\Phi(b)^2$ for $\rho>0$, a sufficient condition for (B.17) is

$$\begin{aligned}
&\left[1-t+t\Phi\left(b\right)\right]^2\left(-b-\frac{\rho}{b}\right)-2t\phi(b)\\
&\qquad+2t^2\phi(b)\left(1-\Phi(d)-\frac{\phi(d)\sqrt{1-\rho^2}}{2b}\right)\geq 0\,. \qquad \text{(B.18)}
\end{aligned}$$

The left-hand side of (B.18) is quadratic in t with a unique minimum value over $t\geq 0$ of

$$2\left[\Phi(b)+\Phi(d)-\frac{\phi(d)\sqrt{1-\rho^2}}{2b}\right]\left(-b-\frac{\rho}{b}\right)-\phi(b)\,. \qquad \text{(B.19)}$$

Thus, a sufficient condition for (B.18) to hold for all $t\in[0,1]$ is that (B.19) be nonnegative. Multiplying (B.19) by b^2, we see that it is nonnegative if and only if

$$2\left[b\Phi(b)+b\Phi(d)-\frac{1}{2}\phi(d)\sqrt{1-\rho^2}\right]\left(-b^2-\rho\right)-b^2\phi(b)\geq 0\,. \tag{B.20}$$

Set $x=\sqrt{(1-\rho)/(1+\rho)}$ so $0<x<1$, $d=xb$, and $\rho=(1-x^2)/(1+x^2)$. The inequality (B.20) is equivalent to

$$2\left[-b\Phi(b)-b\Phi(xb)+\frac{x\phi(xb)}{1+x^2}\right]\left(b^2+\frac{1-x^2}{1+x^2}\right)-b^2\phi(b)\geq 0\,. \tag{B.21}$$

Using the fact that $-b\Phi(xb)>0$ and $\phi(xb)\geq\phi(b)$, we see that the left-hand side of (B.21) is larger than

$$\begin{aligned}2\left[-b\Phi(b)+\frac{x\phi(xb)}{1+x^2}\right]&\left(b^2+\frac{1-x^2}{1+x^2}\right)-b^2\phi(b)\\ &\geq -2b\Phi(b)\left(b^2+\frac{1-x^2}{1+x^2}\right)+\frac{2xb^2\phi(b)}{1+x^2}-b^2\phi(b)\,.\end{aligned}$$

Therefore, (B.21) is implied by Lemma B.3. □

Proof of Proposition 4.2. It suffices to show that if the first disclosure is equal to B_t, then the post-disclosure price of the other firm is strictly less than B_t. The existence of ϵ as claimed in the proposition then follows from continuity. It follows from Proposition 4.1 that $P_{t+}<B_t$ if $z<\beta$, where $\beta:=(B_t-\mu_1^*(x))/\sigma_1$ and z is the root of $F(t,\cdot)$, with F as defined in the proof of Proposition 3.1. When $\tilde{x}_2=B_t$, we have, using $\sigma_1=\sigma\sqrt{1-\rho^2}$ and Lemma B.3,

$$\beta=\frac{B_t-\rho\tilde{x}_2-(1-\rho)\mu^*}{\sigma\sqrt{1-\rho^2}}=\frac{1-\rho}{\sqrt{1-\rho^2}}\cdot\frac{B_t-\mu^*}{\sigma}>\frac{1-\rho}{\sqrt{1-\rho^2}}\cdot\sqrt{\frac{1+\rho}{1-\rho}}z=z\,.$$

□

Proof of Proposition 4.3. The left-hand side of the marginal condition (A.3) is given in Lemma B.1. We need to calculate the right-hand side of (A.3). Suppose firm 1 is at the exercise boundary at t, that is, $\tilde{x}_1=B_t$. We need to calculate the arrival rate of disclosures by firm 2 that are value enhancing for firm 2, and we need to compute the expected change in value, conditional on such a disclosure. Define a function β by $\beta(x) = (B_t - \mu_1^*(x))/\sigma_1$. Proposition 4.1 implies that, if firm 1 discloses x at date t, then the post-disclosure boundary $P_{t+} = B_{t+}$ is higher than the pre-disclosure boundary B_t if and only if $z(t) > \beta(x)$ where z is defined in Proposition 3.1. When this condition holds, the price remains above B_t until the date u such that $z(u) = \beta(x)$, that is, $u = z^{-1}(\beta(x))$. Proposition 4.1 shows that, for $u \in \big[t, z^{-1}(\beta(x))\big]$,

$$P_u \;=\; \mu_1^*(x) + \sigma_1 y(u, \beta(x)) = B_t - \sigma_1\beta(x) + \sigma_1 y(u, \beta(x))\,. \tag{B.22}$$

Combining these facts, we conclude that the risk-neutral expected jump in firm 1's value conditional on a jump occurring and conditional on firm 1 being at the boundary at t is

$$\sigma_1\,\mathsf{E}^*\left[\int_t^{z^{-1}(\beta(\tilde{x}_2))} \big\{y(u, \beta(\tilde{x}_2)) - \beta(\tilde{x}_2)\big\}\,\mathrm{d}u \,\middle|\, \tilde{x}_1 = B_t, \beta(\tilde{x}_2) < z(t)\right]. \tag{B.23}$$

The distribution of $\tilde{x}_2$ conditional on $\tilde{x}_1 = B_t$ is normal with mean $\rho B_t + (1-\rho)\mu^*$ and standard deviation σ_1, which implies that the distribution of $\beta(\tilde{x}_2)$ conditional on $\tilde{x}_1 = B_t$ is normal with mean

$$\frac{(1-\rho)(B_t - \mu^*)}{\sigma_1} = b\sqrt{\frac{1-\rho}{1+\rho}}$$

and standard deviation ρ, where b is the normalized boundary $(B_t - \mu^*)/\sigma$. The distribution of $\beta(\tilde{x}_2)$ conditional on $\tilde{x}_1 = B_t$ and conditional on $\beta(\tilde{x}_2) < z(t)$ is truncated normal with density

$$\frac{1}{\rho}\,\phi\left(\frac{\beta - b\sqrt{1-\rho^2}}{\rho}\right)\bigg/\;\Phi\left(\frac{z(t) - b\sqrt{1-\rho^2}}{\rho}\right). \tag{B.24}$$

The last piece we need is the arrival rate of disclosures by firm 2 having $\beta(\tilde{x}_2) < z(t)$. For any $\epsilon > 0$, the probability that firm 2 learns its value at some $u \in (t, t+\epsilon)$ and $\beta(\tilde{x}_2) < z(t)$, conditional on $\tilde{x}_1 = B_t$, is

$$\int_t^{t+\epsilon} \Phi\left(\frac{z(t) - b\sqrt{1-\rho^2}}{\rho}\right) \mathrm{d}u \,.$$

The probability conditional on $\tilde{x}_1 = B_t$ and conditional on firm 1 not having disclosed prior to t is

$$\int_t^{t+\epsilon} \Phi\left(\frac{z(t) - b\sqrt{1-\rho^2}}{\rho}\right) \mathrm{d}u \Bigg/ \left[1 - t + t\Phi\left(\sqrt{b\frac{1-\rho}{1+\rho}}\right)\right] .$$

The arrival rate is the derivative of this ratio with respect to ϵ, evaluated at $\epsilon = 0$, so it is

$$\Phi\left(\frac{z(t) - b\sqrt{1-\rho^2}}{\rho}\right) \Bigg/ \left[1 - t + t\Phi\left(\sqrt{b\frac{1-\rho}{1+\rho}}\right)\right] . \tag{B.25}$$

Putting these things together, we arrive at the following formula for the right-hand side of the marginal condition (A.3):

$$\left\{\frac{\sigma\sqrt{1-\rho^2}}{\rho} \Bigg/ \left[1 - t + t\Phi\left(b\sqrt{\frac{1-\rho}{1+\rho}}\right)\right]\right\}$$
$$\times \int_{-\infty}^{z(t)} \int_t^{z^{-1}(\beta)} [y(u,\beta) - \beta]\, \phi\left(\frac{\beta - b\sqrt{1-\rho^2}}{\rho}\right) \mathrm{d}u \, \mathrm{d}\beta \,. \tag{B.26}$$

Equating this to (B.4) produces the formula in the proposition. □

Proof of Proposition 6.1. Let $S_i = \{s \le t \mid C_{st} = c_i\}$ for $i = 1, \ldots j$, The equilibrium price is the expected value of $\tilde{x}_1$ conditional on $\tilde{x}_2, \ldots, \tilde{x}_n$ and conditional on the union of the following events:

$$\{u < \tilde{\theta}_1\}, \{t < \tilde{\theta}_1 < u, \tilde{x}_1 < c_j \wedge P_u\}, \{t - \Delta_j < \tilde{\theta}_1 < t, \tilde{x}_1 < c_{j-1} \wedge P_u\}$$
$$\ldots, \{\Delta_1 < \tilde{\theta}_1 < \Delta_1 + \Delta_2, \tilde{x}_1 < c_2 \wedge P_u\}, \{0 < \tilde{\theta}_1 < \Delta_1, \tilde{x}_1 < c_1 \wedge P_u\} \,.$$

It follows that $P_u = p$ where either

$$p = p_1 = \mathsf{E}^* \left[\tilde{x}_1 \middle| \tilde{x}_2, \ldots, \tilde{x}_n, \left\{\tilde{\theta}_1 > u\right\} \cup \left\{\tilde{\theta}_1 \le u, \tilde{x}_1 < p_1\right\} \right] \le c_1 \quad \text{(B.27a)}$$

or

$$p = p_2 = \mathsf{E}^* \left[\tilde{x}_1 \middle| \tilde{x}_2, \ldots, \tilde{x}_n, \left\{\tilde{\theta}_1 > u\right\} \cup \left\{\tilde{\theta}_1 \in S_1, \tilde{x}_1 < c_1\right\} \right. \\ \left. \cup \left\{\tilde{\theta}_1 \le u, \tilde{\theta}_1 \notin S_1, \tilde{x}_1 < p_2\right\} \right] \in (c_1, c_2]\,. \quad \text{(B.27b)}$$

or

$$p = p_3 = \\ \mathsf{E}^* \left[\tilde{x}_1 \middle| \tilde{x}_2, \ldots, \tilde{x}_n, \left\{\tilde{\theta}_1 > u\right\} \cup \left\{\tilde{\theta}_1 \in S_1, \tilde{x}_1 < c_1\right\} \cup \left\{\tilde{\theta}_1 \in S_2, \tilde{x}_1 < c_2\right\} \right. \\ \left. \cup \left\{\tilde{\theta}_1 \le u, \tilde{\theta}_1 \notin S_1 \cup S_2, \tilde{x}_1 < p_3\right\} \right] \in (c_2, c_3]\,. \quad \text{(B.27c)}$$

or ... or

$$p = p_{j+1} = \\ \mathsf{E}^* \left[\tilde{x}_1 \middle| \tilde{x}_2, \ldots, \tilde{x}_n, \left\{\tilde{\theta}_1 > u\right\} \cup \left\{\tilde{\theta}_1 \in S_1, \tilde{x}_1 < c_1\right\} \cup \left\{\tilde{\theta}_1 \in S_2, \tilde{x}_1 < c_2\right\} \right. \\ \left. \ldots \cup \left\{\tilde{\theta}_1 \in S_j, \tilde{x}_1 < c_j\right\} \cup \left\{t < \tilde{\theta}_1 \le u, \tilde{x}_1 < p_{j+1}\right\} \right] > c_j\,. \quad \text{(B.27d)}$$

We want to show that the system (B.27) determines a unique p, which satisfies (6.2). First, we observe that each of the equalities in (B.27) has a unique solution, and then we show that one and only one of the inequality conditions is satisfied. Proposition 3.1 shows that the unique solution of the equality in (B.27a) is $p_1 = \mu_1^* + \sigma_1 z_1$. Also, Proposition 4.1 shows that the unique solution of the equality (B.27b) is $p_2 = \mu_1^* + \sigma_1 z_2$. By similar reasoning, each equality has a unique solution, and it is $p_k = \mu_1^* + \sigma_1 z_k$. It remains to show that exactly one of the inequality conditions is satisfied. The proof of Proposition 4.1 shows that $z_1 \le \beta_1$ if and only if $z_2 \le \beta_1$ and each of these conditions is equivalent to $p_1 \leq c_1$ and $p_2 \leq c_1$. By similar reasoning, we can show for each k that

$$p_k \leq c_k \quad \Leftrightarrow \quad z_k \leq \beta_k \quad \Leftrightarrow \quad z_{k+1} \leq \beta_k \quad \Leftrightarrow \quad p_{k+1} \leq c_k \,. \tag{B.28}$$

If $p_{j+1} > c_j$, then we have, because of (B.28) and the fact that the β's are increasing in k,

$$\begin{aligned} z_{j+1} > \beta_j \;\Rightarrow\; z_j > \beta_j \;\Rightarrow\; z_j > \beta_{j-1} \;\Rightarrow\; z_{j-1} > \beta_{j-1} \\ \Rightarrow\; \cdots \;\Rightarrow\; z_1 > \beta_1 \,. \end{aligned} \tag{B.29}$$

Therefore $p_{j+1} > c_j$ implies $p_i > c_i$ for $i = 1, \ldots, j$. Hence, in this case, (B.27d) is the unique inequality satisfied. Suppose on the other hand that $p_{j+1} \leq c_j$. Let $k = \min\{i \mid p_{i+1} \leq c_i\}$. Then, $p_k > c_{k-1}$ and by the same reasoning as in (B.29) we have $p_i > c_i$ for $i = 1, \ldots, k-1$. Hence, $p_i \notin (c_{i-1}, c_i]$ for $i \leq k-1$. But, $p_{k+1} \leq c_k$ implies $p_k \leq c_k$, so $p_k \in (c_{k-1}, c_k]$. It remains only to show that $p_i \notin (c_{i-1}, c_i]$ for $i > k$. This follows from the fact that

$$\begin{aligned} p_k \leq c_k \;\Rightarrow\; p_{k+1} \leq c_k \;\Rightarrow\; p_{k+1} \leq c_{k+1} \;\Rightarrow\; p_{k+2} \leq c_{k+2} \\ \Rightarrow\; \cdots \;\Rightarrow\; p_{j+1} \leq c_j \,. \end{aligned}$$

□

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Table 1: Sized Matched Summary Statistics for Firms with Unpredictable versus Predictable Earnings Dates

	N	Mean	SD	Diff	TStat	Q1	Med	Q2
TA	1103	6.920	1.978			5.331	6.800	8.330
	1103	6.887	2.037	-0.033	-0.386	5.403	6.820	8.142
Equity / TA	1103	0.586	0.367			0.357	0.571	0.800
	1103	0.572	0.276	-0.014	-0.986	0.347	0.569	0.822
ROA	1103	-0.035	0.170			-0.030	0.010	0.043
	1103	-0.017	0.178	0.017	2.319	-0.006	0.018	0.064
Accrual SD	1028	2.039	3.102			0.178	1.071	2.473
	912	1.930	2.849	-0.110	-0.807	0.199	1.120	2.375
Earnings Persistence	1034	0.290	0.652			0.015	0.332	0.590
	922	0.356	0.670	0.066	2.205	0.070	0.369	0.612
ERC EPS	910	8.953	20.575			0.808	3.995	11.203
	831	12.567	27.776	3.614	3.102	1.189	6.082	17.593
ERC Revenue	901	0.448	1.078			0.008	0.240	0.734
	832	0.531	1.311	0.083	1.451	0.004	0.278	0.847
| Surprise EPS |	948	0.880	1.622			0.180	0.364	0.815
	895	0.726	1.698	-0.154	-1.988	0.125	0.261	0.573
| Surprise Rev |	939	8.201	9.860			2.637	4.533	8.981
	894	6.295	7.500	-1.906	-4.641	2.423	3.938	7.134
Disagreement EPS	900	0.521	1.140			0.094	0.186	0.402
	854	0.383	1.022	-0.138	-2.658	0.063	0.131	0.296
Disagreement REV	869	6.817	10.082			1.582	2.783	6.543
	832	5.087	7.999	-1.729	-3.908	1.459	2.486	5.286
Analyst	947	1.667	0.932			1.209	1.771	2.280
	900	1.701	0.927	0.035	0.799	1.207	1.797	2.343
CAPM Beta	1103	0.928	0.355			0.728	0.955	1.150
	1100	0.875	0.369	-0.052	-3.390	0.647	0.900	1.106

Notes: This table presents size-matched summary statistics by whether the firm's earnings announcements are predictable. We define predictable firms as firms for which WSH exactly forecasts the announcement date for more than half of the firm's announcements or which announce on the same day of the week for more than 90 percent of their announcements. The first row for each variable is the unpredictable earnings date group, and the second row is the predictable group. We sort predictable firms into deciles based on total assets. For each decile, we compute the average log total assets. We then select the same number of unpredictable firms by choosing the unpredictable firms that have the closest log total assets to the predictable decile average. Each observation is one firm. TA is the log of a firm's average total assets over our sample. Equity/TA is a measure of a firm's leverage. ROA is return on assets. Accrual SD is the the standard deviation of a firm's accruals. Earnings persistence is the yearly autocorrelation of a firm's earnings. ERC EPS (Revenue) is the earnings response coefficient to earnings per share (revenue). | Surprise | is the average absolute value of EPS surprise. Disagreement EPS is the standard deviation of analyst forecast disagreement. Analyst Coverage is the average number of analysts covering the firm. CAPM Beta is the stock's beta to the market estimated over 1-year of daily returns.

	R^{FF12}	R^{FF12}	R^{FF12}	R^{GCIS4}	R^{GCIS4}	R^{GCIS4}
Late Indicator	6.57***	6.88***	4.47*	5.82**	6.13*	3.75
	(1.74)	(2.39)	(2.38)	(2.27)	(3.24)	(2.95)
Industry FE	N	Y	Y	N	Y	Y
Quarter FE	N	N	Y	N	N	Y
Num Obs	105,066	105,066	105,066	104,973	104,973	104,973

$^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table 2: Peer Returns for Late Announcers Relative to Early and On-Time Announcers. Value-weighted three-day announcement returns for peers that announce in the three-day window described in Figure 7.7 are regressed on a dummy variable for whether the announcement is later than forecast by WSH (with an intercept). Peers are defined either by the Fama-French 12-industry classification or the GICS 4-digit classification. The coefficients are in basis points. Standard errors are in parentheses and are clustered by each fixed effect. In constructing peer returns, we use our full sample, including predictable firms and including firm-quarters for which we do not have WSH announcement date forecasts.

	count	mean	std	25%	50%	75%
WSH Error	105,530	1.303	5.089	-1	0	4
Year-on-Year Change	105,530	-0.032	4.679	-2	-1	2
R^{FF12}	105,066	0.070	2.739	-1.228	0.037	1.332
R^{GICS4}	104,973	0.073	3.578	-1.624	0.037	1.749
R^{agg}	105,332	0.051	1.157	-0.461	0.059	0.559

Table 3: Summary Statistics
Observations are firm-quarters for which the firm is classified as unpredictable. Peer and aggregate returns are calculated using our full sample, including predictable firms and firm-quarters for which WSH did not forecast the announcement date. Returns are in percent. For example, the mean value of the Fama-French 12-industry peer return R^{FF12} is 7 basis points.

	(1)	(2)	(3)	(4)	(5)	(6)
R^{FF12}	0.07***	0.07***			0.04**	
	(0.02)	(0.02)			(0.02)	
R^{GICS4}			0.05***	0.04*		0.03
			(0.02)	(0.02)		(0.02)
R^{agg}					0.20**	0.19**
					(0.09)	(0.10)
Firm FE	Y	Y	Y	Y	Y	Y
Day FE	N	Y	N	Y	Y	Y
Num Obs	105,066	105,066	104,973	104,973	105,065	104,972

$^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table 4: WSH Forecast Error Regressions

We estimate equation (7.1) with the dependent variable being the WSH forecast error. All exogenous variables are standardized to have unit standard deviations. The unit of the dependent variable is days. Standard errors are clustered by each fixed effect and shown in parentheses.

	(1)	(2)	(3)	(4)	(5)	(6)
R^{FF12}	0.05***	0.06***			0.05**	
	(0.02)	(0.02)			(0.02)	
R^{GICS4}			0.04**	0.04**		0.03
			(0.02)	(0.02)		(0.02)
R^{agg}					0.17**	0.19**
					(0.08)	(0.10)
Firm FE	Y	Y	Y	Y	Y	Y
Day FE	N	Y	N	Y	Y	Y
Num Obs	105,066	105,066	104,973	104,973	105,065	104,972

$^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

Table 5: Year-on-Year Change Regressions

We estimate equation (7.1) with the dependent variable being the year-on-year change in the announcement date. All exogenous variables are standardized to have unit standard deviations. The unit of the dependent variable is days. Standard errors are clustered by each fixed effect and shown in parentheses.