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THE ELUSIVE QUEST FOR DISARMED PEACE:
CONTEST GAMES AND INTERNATIONAL RELATIONS

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ABSTRACT

This paper extends the canonical security competition one-shot game by developing a dynamic framework where states can invest in a technology to eliminate their rival. The model includes a settlement stage for negotiation, stochastic contestable resources, and it assumes diplomacy never fails–so payoff-dominated equilibria are not played. Within this setting, we fully characterize equilibrium behavior for all discount factors, comparing cases with high and low elimination costs. The dynamic structure reveals why, even when states coordinate on the best possible equilibrium outcome, long-lasting disarmed peace is rare. By combining the escalation of military capabilities with the constraining implications of effective diplomacy, the model rationalizes the persistent cycles of peace, arms races, and conflict observed in history. Our approach identifies strategic mechanisms that restrict the sustainability of disarmament and clarifies the conditions under which arms races or conflict are inevitable. These findings offer a deeper understanding of the recurrent nature of international conflict under repeated interaction.

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1 Introduction

In the economics of conflict and international relations literature, the standard game-theoretic model of conflict is a one-shot contest game, in which states simultaneously and independently make investments (e.g., weapons) to increase their probability of winning a prize (e.g., a disputable resource).[1] In the Nash equilibrium, even when partial resource destruction is modeled as a deterrent to conflict, there are positive levels of arming and open conflict, and part of the resource is destroyed in the fight, leading to an inefficient outcome. All states could be better off if they chose not to arm and peacefully split the resource.

If the game is augmented with a bargaining and settlement stage after arming decisions, the destruction associated with open conflict can be mitigated, but unless such destruction is unrealistically high, inefficient arming persists. In other words, the equilibrium that is robust to changes in the destruction of resources associated with open conflict consists of armed peace and negotiated settlement, and only when open conflict destroys the vast majority of the resources can efficient equilibria be sustained.

In summary, in realistic scenarios, there is positive arming, and the model has the structure of a prisoner's dilemma (often referred to as a security dilemma in international relations), which could open the door for more cooperative outcomes to be sustained if repeated interactions are allowed.[2]

Repeated interactions among states are a reasonable environment to explore for at least four reasons. First, in many cases, there are few relevant states or blocs of states involved in an international dispute. Drawing an analogy with firms' behavior, collusion tends to be easier to sustain with a smaller number of firms involved. Second, states are long-lived entities, which suggests the importance of considering dynamic interactions. Third, states tend to have powerful incentives to plan and act through diplomacy and coordination, given that mis-

[1]See Hirshleifer (1989) for an important seminal work and Garfinkel and Skaperdas (2007) for a comprehensive survey of the literature in the economics of conflict. This is also the standard rent-seeking model (Tullock, 1980; Nitzan, 1994).

[2]Although the interpretation of the model can be extended to potential conflict over disputable resources in a general sense, the model's structure best fits disputes over resources or territories that are both highly valuable and insecurely held–situations where formal ownership or control is not universally recognized and remains subject to challenge. Historical examples include the Korean Peninsula, where both North and South claim legitimacy over the entire territory, the Falkland Islands dispute between the United Kingdom and Argentina, and oil-rich borderlands such as the Iran–Iraq frontier. These contests are sustained by the inability to institutionalize a permanent settlement and by the perception that control of the prize yields long-term strategic or economic advantage. Similar dynamics also appear in access disputes over strategic maritime routes like the Suez Canal or the South China Sea, where shifts in commercial and military importance keep the stakes alive.

takes could lead to serious consequences (e.g., war). Finally, states usually have the capabilities to make contingent plans and act strategically (e.g., diplomats and military experts). Thus, international relations should be an excellent candidate for applying the folk theorem and related results for infinitely repeated games to sustain more cooperative outcomes (see, for example, Keohane, 1986; Oye, 1986; Kydd, 2015).[3]

Indeed, if the standard model of conflict is repeated infinitely and if states are patient enough, disarmed peace can be sustained as a subgame perfect equilibrium, presenting us with a conundrum. In a one-shot contest game model of conflict, the equilibrium is either open conflict (when a negotiated settlement is not possible) or armed peace (when a negotiated settlement is possible). If conflict is modeled as a repeated contest game, the set of equilibrium payoffs can range from the efficient outcome of disarmed peace (assuming states are patient) to the same highly inefficient equilibria of the one-shot game. There is, therefore, a wider range of more cooperative equilibria in repeated interactions. In international relations, however, diplomacy is often viewed as the key mechanism to navigate these equilibria. Through diplomatic channels, states are expected to be capable of eliminating dominated equilibria and converging toward the most favorable equilibrium payoffs. Given these premises, one would anticipate that this dynamic version of the standard one-shot contest game would naturally lead to a state of disarmed peace, provided states are patient enough.

The empirical reality, however, presents a stark contrast to these theoretical expectations. Despite the theoretical possibility of achieving unarmed peace through repeated interactions and diplomatic efforts, the historical record is replete with instances of ongoing arming, arms races, and even outright war. If conflict is modeled as an infinite repetition of the standard one-shot contest game, the only way to rationalize these events is to attribute them to diplomatic failures. For example, disarmed peace in odd periods and open conflict in even periods can be sustained as a subgame perfect equilibrium, which can be interpreted as diplomacy failing in even periods. We do not deny the possibility of historical instances of diplomatic failures, but we think that the discrepancy between theory and reality suggests a more fundamental gap in the existing models of conflict. Therefore, in our analysis we give diplomacy the benefit of the doubt and assume that states always find a way to avoid payoff-dominated equilibria. Alternatively, our analysis can be seen as an effort to rationalize some of the historical record on conflict using an infinite horizon dynamic contest game, even when diplomacy fully exhausts all the possibilities of cooperation.

[3] One might even argue that if these results do not apply to interactions among states, there should be little hope that they will apply to other strategic interactions, such as firms trying to organize collusion or commoners solving the tragedy of the commons.

The paper introduces a comprehensive model that provides a nuanced explanation for the persistent occurrence of arming and conflict despite the theoretical predictions of disarmament and peace under infinitely repeated interactions and fully effective diplomacy. Our model incorporates the foundational aspects of the standard contest game and extends it by introducing dynamic elements that more accurately reflect the complexities of international relations. First, we allow states to interact repeatedly over time. Second, after observing each other's military power, states can agree on a settlement rule such that each country gets a fraction of the resource. If no settlement is reached, open conflict occurs, and a portion of the resource is destroyed. Third, there is an exogenously and randomly determined opportunity that the victor of an open conflict can eliminate the other state at an additional fixed cost.[4] Moreover, we distinguish between two cases, whether this elimination of the rival is permanent or temporary.[5]

With these technologies, we obtain several novel results. First, we establish an impossibility result: if the elimination of the rival is permanent, greater patience (high δ) makes unarmed peace harder to sustain, and in the limit ($\delta \to 1$), open conflict with extremely high arming becomes unavoidable. Second, the cost of elimination plays a central role: there are threshold values–dependent on other parameters–above which moderate levels of patience can sustain temporary or permanent cooperation, particularly when the elimination technology is only temporary. In this temporary elimination case, cooperation becomes cyclical, producing recurring patterns of peace, conflict, and, at times, heavy arming

[4]Elimination technologies vary in form but share the property of decisively removing a rival's ability to contest the resource in the near future. At the extreme, the nuclear bombings of Hiroshima and Nagasaki illustrate near-permanent elimination, while other cases include post-World War II forced deportations or the dismantling of industrial capacity in occupied territories. More commonly, elimination is temporary, as in the destruction of Iraq's military capacity in 1991 or the temporary incapacitation of Argentina's navy after the Falklands War. More recently, digital and AI-enabled technologies suggest new forms of elimination operating through informational and command-and-control channels rather than physical destruction, such as autonomous weapons, cyber capabilities, or AI-driven surveillance systems that irreversibly degrade an adversary's ability to mobilize, coordinate, or recover credible deterrence.

[5]Temporary elimination occurs when the defeated side eventually recovers its strength and re-enters the contest. This was evident after Napoleon's exile to Elba in 1814, followed by his return during the Hundred Days; after the initial North Korean retreat in late 1950, which was reversed by Chinese intervention and a renewed offensive; and after the Iran–Iraq War's alternating phases of exhaustion and renewed fighting in the 1980s. In these cases, the resource–whether territorial control, political authority, or strategic advantage–remains contestable, and the defeated side invests in rearmament or strategic alliances to mount a comeback, producing cycles of peace and conflict rather than a single, decisive end to hostilities. An extended version of this paper analyzes a randomly determined disputable resource; that is, the resource's value is stochastic and varies from period to period. This extension is available in Online Appendix B.

followed by temporary elimination. Finally, when the value of the contestable resource fluctuates, the model also generates arms races (e.g., a cold war) as part of the broader cycle of peace and conflict.

While most elements in our model have been individually examined in the literature, they have not previously been integrated into a single framework. Our contribution is to systematically combine repeated interactions, strategic settlement negotiations, and the possibility of a decisive elimination, yielding a more robust and multi-dimensional understanding of conflict dynamics. This unified approach recovers standard results as special cases while clarifying the mechanisms behind a variety of outcomes. For example, cycles of escalation and de-escalation with temporary elimination emerge when the discount factor is moderately high. Even without the elimination technology, the interaction between repeated play and random resource values can generate cycles of armed and unarmed peace without culminating in open conflict. As noted by Powell (1993), Skaperdas and Syropoulos (1996), and McBride and Skaperdas (2006), shifts in future relative power–including the prospect of substantially weakening a rival–can generate incentives for open conflict. Our model extends this insight by introducing explicit adversary-elimination technologies in a fully dynamic, infinite-horizon environment. With permanent elimination, greater patience makes conflict more attractive and leads to a single open conflict followed by perpetual unarmed peace, whereas with non-permanent elimination, the model produces recurrent cycles of armed peace, conflict, temporary defeat, and eventual resurgence.

The paper contributes to two bodies of literature in two subjects of study: game-theoretic models of conflict in economics and security competition in international relations.

There are several papers that have formalized the idea that open conflict differs from settlement under the shadow of conflict because it changes the future bargaining power of the parties (see Fearon, 1995, for a seminal formalization of this idea). Settlement is rationalized by the idea that open conflict is destructive, so parties are better off not engaging in it. For instance, the notion that the victor of an open conflict can eliminate its enemy is one possible such mechanism. Skaperdas and Syropoulos (1996) and Garfinkel and Skaperdas (2000) introduce elimination opportunities but only consider a two-period model. Powell (1993) and McBride and Skaperdas (2006) explore infinite-horizon dynamic games but restrict attention to Markov perfect equilibria, which rules out history-dependent strategies such as grim-trigger punishments. Our first contribution to this literature is, then, to explore an environment in which the classical folk theorem has a serious chance, in the sense that without introducing the elimination technology, states can sustain unarmed peace equilibria. Our second contribution is to address one potentially unrealistic prediction of these models (including our version with permanent elimination), namely, that after one country eliminates its enemy, unarmed peace persists forever. McBride and Skaperdas (2006) consider that several battles might be necessary to finally eliminate an enemy, which explains arming after open conflict. Still, once the enemy has been finished off, unarmed peace persists. Our model with temporary elimination, in contrast, allows for recurrent periods of peace and conflict.

Another strand of the literature has studied infinite-horizon dynamic models of conflict. Yared (2010) develops an asymmetric repeated game in which an aggressive country seeks concessions from a non-aggressive country. Temporary wars might occur after a period of escalating demands to incentivize the non-aggressive country to make concessions. Asymmetric information plays a key role because when concessions are not made, the aggressive country does not know if the non-aggressive country is able to make them. Instead, we focus on large symmetric conflicts, i.e., when both countries have the capacity to attack and even annihilate each other, and consider environments with complete information.[6] Acemoglu et al. (2012) study an infinite horizon dynamic model of resource war in which a resource-poor country might consider attacking a resource-rich country. The model can generate open conflict (i.e., war) but only in period $t = 0$. Acemoglu and Wolitzky (2014) generate an equilibrium with cycles of conflict but use a very different modeling approach. They consider an overlapping-generations model with incomplete information in which two groups play a coordination game, and each group does not know if the other consists of good or bad types. Cooperative actions might be wrongly misperceived, starting a cycle of conflict until agents realize that the conflict began by mistake and give cooperation a renewed opportunity. In contrast, in our model, there is complete information, and we assume that diplomacy can eliminate any misperception or coordination failure.

Our study is also related to international relations and the well-known debate between realists and liberals. A simple way of describing the realist school in international relations is that, for realists, international politics is predominantly a prisoner's dilemma among countries (e.g., Mearsheimer, 2001). From this perspective, the standard game-theoretic model of conflict as a one-shot contest game can be interpreted as a formalization of the fundamental realist view of international relations as a security dilemma. The problem, of course, is that one can start with a one-shot contest game/security dilemma and, through repeated interactions, end up with disarmed peace sustained, for example, by grim-trigger strategies (e.g., Keohane, 1986). In other words, the folk theorem provides a the-

[6] Although states often face incomplete information regarding an adversary's military capabilities or resolve, advanced intelligence apparatuses generally provide a reliable assessment of a rival's overall power.

oretical link between a realist world and highly liberal cooperative outcomes. The conundrum is that this extreme liberal prediction is rarely observed in practice. Why?

Our model explores several answers to this question. Bargaining, even in a one-shot game, can prevent open conflict. Nevertheless, arming is typically part of the equilibrium. In fact, only extremely high destructive costs from open conflict could rationalize an unarmed equilibrium. In a repeated game, the destruction level required for disarmed peace interacts with the degree of patience, implying that war and inefficient arming can only be prevented when states are highly patient.

When the elimination technology is permanent, high patience makes long-standing disarmed peace unsustainable. Some cooperation is still possible in states of the world where elimination is not feasible, but as soon as the opportunity to eliminate arises, open conflict becomes inevitable for patient states. A drawback of this equilibrium is that, after the victor eliminates its enemy, disarmed peace persists forever. When elimination is only temporary, however, this no longer holds as $\delta \to 1$. In equilibrium, open conflict grants the victor only temporary control over the disputed resource. Eventually, the defeated country recovers, and the resource becomes contestable again. Thus, there is room for both temporary and permanent cooperation. Nevertheless, some arming is often necessary, meaning that security competition and war cannot be fully eradicated from the international system.

The rest of the paper is organized as follows. Section 2 introduces the stochastic model. Section 3 solves a simpler version of the model, without the elimination technology. We break down the study of the elimination technology in two main cases: we analyze permanent elimination in Section 4 and temporary elimination in Section 5. Finally, Section 6 presents our conclusions.

2 A Dynamic Model of Conflict with Destruction

We depart from the standard model of conflict by adding several elements that will help explain more realistic features observed historically, especially in the last century. We pay special attention to an exogenous randomly determined opportunity that the victor of an open conflict can eliminate the other player at an additional fixed cost; moreover, we distinguish between two cases, whether elimination is permanent or not. Due to the random realization of states that affect the payoffs and actions, we study a stochastic game.[7]

[7]This is a slight extension of a repeated game. Although there is repeated interaction, there is no unique stage game, since payoffs and actions depend on different realizations of state

In each period, countries $i = 1, 2$ seek access to a disputable resource R. These countries choose a level of military power denoted by $G_i \geq 0$ (for guns) that fully depreciates by next period. Given a profile of military powers (G_1, G_2), the probability of winning a conflict is:

$$\pi_i(G_i, G_j) = \begin{cases} \frac{G_i}{G_i+G_j}, & G_i + G_j > 0 \\ \frac{1}{2}, & G_i = G_j = 0 \end{cases}$$

The perishable resource will be replaced by a new stock R in each period. After countries observe the profile of military power (G_1, G_2), they decide whether to settle ($S_i \in \{0, 1\}$). If countries settle ($S = S_1 S_2 = 1$), they receive $\pi_i(G_i, G_j)R$ of the resource.[8] If countries do not settle ($S = 0$), this necessarily starts an open conflict, a fraction $1 - \theta$ of the resource is destroyed, and country i obtains R with probability $\pi_i(G_i, G_j)$ and nothing with probability $1 - \pi_i(G_i, G_j)$.[9]

At the beginning of each period, before deciding G_i, a random state is realized which gives the victor of an open conflict, say i, the opportunity to pay an additional cost X to eliminate the loser (i.e., country j). This exogenous opportunity arrives with a probability ϕ, and it is common knowledge. The opportunity to eliminate is realized before the decision to spend on military power G_i, but the decision to eliminate is made after winning the conflict. Since the decision to eliminate may or may not be taken, we need to be explicit about this action being taken. We use a random variable Z to denote whether an opportunity to eliminate has arrived ($Z = 1$) or not ($Z = 0$). Moreover, we denote by $W_i = 1$ if a country decides to eliminate and $W_i = 0$ otherwise.

After an elimination occurs, the next period, the game transitions to a state in which the previous conflict's victor can claim all the resources at zero cost. More formally, when an elimination occurs, the game transitions to an uncontestable state ($C = 0$). Periods in which the resource can be contested will be labeled contestable states and denoted by $C = 1$. At the end of each period in an uncontestable state, with probability γ, the game will transition back to a contestable state and the loser of the previous conflict will be able to contest the resource again. With $1 - \gamma$, the game will remain in the uncontestable state next period.

variables.

[8] Although there are other allocation rules, this particular one is the most natural. Moreover, we assume that the settlement rule S is decided independently and simultaneously and that it is fully enforceable. That is, we do not model the possible advantages of a unilateral unexpected aggression after settlement.

[9] Note that, conditional on R, payoffs are deterministic under settlement and stochastic under open conflict. Expected payoffs, nevertheless, look similar due to two assumptions. First, the probability of winning a war equals the share obtained during settlement. Second, players are assumed risk neutral.

Let $a_i = (G_i, S_i, W_i)$ be the actions taken by player i in contestable states and $\psi = (Z, C)$ the profile of states. Then, each country's instant payoff when $C = 1$ is:

$$u_i(a_i, a_j, \psi) = \pi_i(G_i, G_j)\big[SR + (1-S)\theta R - (1-S)ZW_iX\big] - G_i, \qquad (1)$$

for i and $j = 1, 2$. After the opportunity to eliminate is realized and used, as long as $C = 0$ remains, all subsequent payoffs will be $u_i(a_i, a_j, \psi) = R$ for the victor of the most recent open conflict and $u_i(a_i, a_j, \psi) = 0$ for the vanquished nation. After the state returns to $C = 1$, the payoff will be given by (1) once again.

Countries discount future payoffs by a factor $0 \leq \delta < 1$. Given a stream of actions $a_i^t = (G_i^t, S_i^t, W_i^t)$ from each country at each period, $\{a_i^t\}_{t=0}^{\infty}$, and states $\psi^t = (Z^t, C^t)$, $\{\psi^t\}_{t=0}^{\infty}$, the (normalized) payoff of the supergame is:

$$(1-\delta)\sum_{t=0}^{\infty} \delta^t E\left[u_i(a_i^t, a_j^t, \psi^t)\right],$$

where the expectation is taken over states and actions. In this supergame, a history at the beginning of period t is a sequence of all action profiles and states up to period t:[10]

$$h^t = \left\{(a_1^0, a_2^0, \psi^0), (a_1^1, a_2^1, \psi^1), \ldots, (a_1^{t-1}, a_2^{t-1}, \psi^{t-1})\right\},$$

for $t \geq 1$ and $h^0 = \emptyset$ in the initial period, and we assume that the game starts in a contestable state. Since all information is public in this game, the equilibrium concept is subgame perfect equilibrium (SPE). We restrict attention to symmetric equilibria.[11]

The following tables summarize the states of world and endogenous variables of the model (Table 1) and the parameters of the model (Table 2).

[10] To simplify notation, we can say that even in state $C = 0$, countries can take actions. However, such actions do not affect the payoffs.

[11] Since there are effectively no actions being taken in uncontestable states, we consider that as part of a "symmetric" equilibrium, as long as actions and payoffs are ex-ante symmetric in contestable states. Moreover, there could be equilibria in which a country, say i, obtains a larger share of the resources. As long as the other country, j, gets a payoff higher than its minmax, that can be an equilibrium. We ignore such equilibria because they do not add relevance or insight to the study, but greatly increase the space of possible equilibria.

Table 1: Summary of Model States of the World and Endogenous Variables

Variable	Description
$C \in \{0, 1\}$	Resource contestable ($C = 0$, no, $C = 1$, yes)
$Z \in \{0, 1\}$	Opportunity to eliminate the enemy ($Z = 0$, no, $Z = 1$, yes)
$G_i \geq 0$	Guns choice
$S_i \in \{0, 1\}$	Settlement choice ($S = S_1 S_2 = 0$ no, $S = S_1 S_2 = 1$ yes)
$W_i \in \{0, 1\}$	Elimination choice ($W_i = 0$ no, $W_i = 1$ yes)

Table 2: Summary of Model Parameters

Parameter	Description
R	Resource per period
θ	Share of R that survives open conflict
δ	Discount factor
ϕ	Probability that the elimination technology is available
γ	Probability that loser comes back
X	Cost of using the elimination technology
$\overline{X}(\gamma)$	Threshold value for X
$R^E(\gamma)$	Discounted value of the resource accessible to the winner following an elimination

To better understand the dynamics of the game, we build up in complexity by solving simpler scenarios first. Table 3 summarizes all the scenarios. In all cases, the approach to sustain high-payoff equilibria is to use low-continuation-payoff equilibria as a credible threat. To refer to these incentive compatible conditions, we denote the difference between the payoff from a cooperative equilibrium and the payoff from a deviation followed by the non-cooperative equilibrium with functions $F(\cdot)$, which will be clear in context within each subsection.

Table 3: Summary of Scenarios

Scenario	Description	Section	Sustainability
One-Shot Game	$C = 1$, $Z = 0$ and $\delta = 0$	3.1	Equation (2)
Repeated Game	$\delta > 0$ and $\phi = 0$	3.2	Equation (3)
Permanent elimination	$\delta > 0$, $\phi > 0$, and $\gamma = 0$	4	Equations (4) and (5)
Temporary elimination	$\delta > 0$, $\phi > 0$, and $\gamma > 0$	5	Equations (6) and (7)

3 Equilibrium with No Elimination

In this section, we start by solving the one-shot game and then consider the repeated game without stochastic states. In both cases, we do not allow the winner of an open conflict to eliminate its rival.

3.1 One-shot Game

There are no actions to be taken in uncontestable states. Thus, we refer to the case when $\delta = 0$, the current state is contestable, and there is no opportunity to eliminate as the one-shot game ($C = 1$ and $Z = 0$). Note that even in the one-shot game, we still need to use subgame perfection as the equilibrium concept, as the level of guns (G_1, G_2) is first decided and observed and then the settlement stage follows (S_1, S_2), which leads to either peace or conflict (i.e., no settlement necessarily translates into open conflict). This, in turn, means that interesting equilibria can arise even in this simple case. In order to understand possible equilibria, we relate how countries react to observing anticipated or unanticipated arming by the opponent. These reactions have strong implications on the type of equilibria that can arise. After stating the results, we will also have a discussion of implications and interpretations.

First, we note that for any profile of guns choices (G_1, G_2), settlement (i.e., $S_1S_2 = 1$) and no settlement (i.e., $S_1S_2 = 0$) can both be Nash equilibrium outcomes. Moreover, settlement always Pareto dominates no settlement. In this sense, the post-arming subgame has the structure of a coordination problem. Two reasonable selection criteria are the following norms. Under the *non-cooperative norm*, $S_1S_2 = 0$ for all (G_1, G_2). Under the *cooperative norm*, $S_1S_2 = 1$ for all (G_1, G_2). That is, under the non-cooperative norm, there is always open conflict. In contrast, under the cooperative norm, there is never an open conflict as the countries always reach an agreement.

Regardless of whether the non-cooperative or cooperative norm prevails for the settlement stage, guns choices have the structure of a prisoner's dilemma. In equilibrium, both countries invest in guns, but their investments neutralize each other, and each country obtains the resource with equal probability.

Finally, it is possible to have arming-dependent selection criteria. In particular, under the *conditional cooperative norm*, $S_1S_2 = 1$ for all (G_1, G_2) such that $G_i \leq G$ and $S_1S_2 = 0$ otherwise. This opens the door to more cooperative outcomes. To see this, suppose that both countries choose a relatively low arming G, which will induce settlement. If such actions are not observed during the arming stage, deviations are punished by not settling. Let $g^D(G, R)$ be the best response function that maximizes (1) given a rival's G and effective resources R.

From Equation (1) evaluated at $S_1S_2 = 0$ and $Z = 0$, the best response would be $g^D(G, \theta R) = \sqrt{G\theta R} - G$. Introducing the best response into Equation (1) evaluated at $S_1S_2 = 0$ and $Z = 0$, we obtain the highest deviation payoff $u_i^D(G)$. In contrast, if both countries play G, each would obtain $u_i^C(G) = R/2 - G$. Thus, a country has no incentive to deviate as long as $u_i^C(G) \geq u_i^D(G)$, which is satisfied if and only if

$$F(G) := -2G + 2\sqrt{G\theta R} + \left(\frac{1-2\theta}{2}\right) R \geq 0. \tag{2}$$

Given this incentive compatibility constraint for an equilibrium under the conditional cooperative norm, the following proposition summarizes the subgame perfect Nash equilibrium gun levels under each selection criterion.

Proposition 1. *Assume that* $\delta = 0$.

1. *Under the non-cooperative norm (i.e., players coordinate in* $S_1S_2 = 0$ *for all* (G_1, G_2)*). Then, the equilibrium level of arming is* $G^{NN} = \theta R/4$ *and the associated payoff is* $V^{NN} = \theta R/4$.

2. *Under the cooperative norm (i.e., players coordinate in* $S_1S_2 = 1$ *for all* (G_1, G_2)*). Then, the equilibrium level of arming in this one-shot game is* $G^{CN} = R/4$ *and the associated payoff is* $V^{CN} = R/4$.

3. *Under the conditional cooperative norm:*

   (a) *If* $\theta \leq 1/2$*, then the most cooperative equilibrium level of arming is* $G^{FB} = 0$ *and the associated payoff is* $V^{FB} = R/2$.

   (b) *If* $\theta > 1/2$*, then the most cooperative equilibrium level of arming is* $G^{SB} \in (0, \theta R/4]$*, where* G^{SB} *is the unique solution to* $F(G) = 0$*, and the associated payoff is* $V^{SB} = R/2 - G^{SB}$.

Proof. See the Appendix A.1. □

The equilibrium described in Proposition 1.1 is identical to the Nash equilibrium of the standard one-shot contest game when no settlement stage is included. The non-cooperative norm can be interpreted as the total lack of trust as players expect that they will never be able to coordinate in the Pareto-superior settlement equilibrium, regardless of their guns choices. This makes the settlement stage completely irrelevant. For international relations, one possible interpretation is that this is a situation in which countries do not have a diplomatic channel that can be used to reach a negotiated settlement.

The equilibrium described in Proposition 1.2 is the equilibrium usually stressed when the standard one-shot contest game is augmented with a settlement stage.

The idea is that regardless of gun choices, countries can always use diplomacy and peaceful negotiations to settle the dispute, divide the disputed resource according to the winning probabilities, avoid the destruction associated with open conflict, and enforce the agreement with guns. In other words, players will always coordinate in the Pareto-superior settlement equilibrium. Thus, armed peace can always be achieved.[12] Note that in the particular case of $\theta = 1$, the payoffs from the equilibria in Propositions 1.1 and 1.2 are equivalent.

The equilibrium described in Proposition 1.3 is often overlooked in the literature.[13] However, it is interesting because it anticipates the opportunities for cooperation if the game is repeated infinitely and also suggests that diplomacy can be a double-edged sword. To see the opportunities for cooperation, note that the equilibrium in Proposition 1.3 involves a lower level of arming and a higher payoff than the equilibrium in Propositions 1.1 or 1.2. The reason is that the conditional cooperative norm leverages the multiplicity of equilibrium in the settlement stage to enact more cooperative behavior in gun choices. Specifically, the conditional cooperative norm "punishes" high levels of arming with the Pareto-inferior no settlement equilibrium and "rewards" low levels of arming with the Pareto-superior settlement equilibrium. When the destruction associated with open conflict is high (formally, $\theta \leq 1/2$), this is enough to induce disarmed peace. When the destruction associated with open conflict is low (formally, $\theta > 1/2$), fully disarmed peace is not an equilibrium, but a lower level of arming than in Proposition 1.1 can still be sustained.

To see the double-edged sword nature of diplomacy, note that the conditional cooperative norm can be interpreted as "offering" a diplomatic solution when arming is low but also "threatening" with open conflict when arming is high.

[12]Note that one prediction of the one-shot rent-seeking game is that the equilibrium level of arming is lower under open conflict ($G^{NN} = \theta R/4$) than under settlement ($G^{CN} = R/4$). One possibility to avoid such predictions would be to replace the rent-seeking model (1) by the following guns vs. butter model:

$$u_i(a_i, a_j, \psi) = \pi_i(G_i, G_j)\left[S + (1 - S)\theta\right](B_1 + B_2),$$

where $B_i = R_i - G_i$ for $i = 1, 2$. Under this specification and assuming that $1/3 < R_1/R_2 < 3$, the equilibrium level of arming is $G = R/4$, where $R = R_1 + R_2$, which does not depend on S. The associated equilibrium payoffs ($V = [S + (1 - S)\theta]R/4$) are identical for both models. It is perfectly possible to redo all our analysis, replacing the rent-seeking model for this guns vs. butter model. Analogous results will remain.

[13]There are notable exceptions. For example, Beviá and Corchón (2010) study a one-period game in which players first decide whether to declare war or not, if both abstain from declaring war, peace results, and only if under war, they have the chance of arming. Garfinkel and Syropoulos (2021) study unarmed peace in a static setting. In their model, players simultaneously choose whether to declare war and guns.

From this perspective, the problem of the non-cooperative norm in Proposition 1.1 is the total lack of diplomacy (no carrots), while the problem of the cooperative norm in Proposition 1.2 is an excessive attachment to diplomacy (no sticks).

Finally, note that when $\theta = 1$ (that is, if there is no destruction associated with open conflict), all the norms induce the same equilibrium, that is, $G = R/4$. Thus, when there is no advantage of a settled agreement over open conflict because conflict is not destructive, the conditional cooperative norm cannot use the destruction of open conflict as a threat to reduce arming.

3.2 Repeated Game

Now we consider a standard repeated environment (i.e., without stochastic states). Thus, the opportunity to eliminate never arrives ($\phi = 0$). In addition, an open conflict destroys a proportion $1-\theta$ of the resources; as a consequence, since there is no opportunity to eliminate, a settlement equilibrium always dominates an open conflict equilibrium. Thus, we ignore equilibria without settlement on the equilibrium path.

Under the previous considerations, we can use a standard folk theorem to characterize the best attainable equilibrium. The mechanism that allows for such cooperative equilibrium is also standard: countries behave nicely to each other under the credible threat that if anyone ever deviates from such cooperative equilibrium, the next period both countries will revert to an inefficient one-period equilibrium allocation and remain there forever (i.e., grim-trigger strategies).

From Proposition 1, there are three possible equilibria to consider as punishment for a deviation. The most severe is the equilibrium described in Proposition 1.1. Under a non-cooperative norm, both countries play $G^{NN} = \theta R/4$ and $S_1 = S_2 = 0$, which gives instant payoffs of $V^{NN} = \theta R/4$. The equilibrium described in Proposition 1.2 is slightly less severe as players continue to avoid open conflict on the punishment path. Specifically, under a cooperative norm (i.e., with settlement), both countries play $G^{CN} = R/4$ and $S_1 = S_2 = 1$, which gives instant payoffs of $V^{CN} = R/4$. It is also worth noting that Proposition 1.1 includes Proposition 1.2 (setting $\theta = 1$). Finally, the equilibrium described in Proposition 1.3 is the least severe and, hence, the least effective in inducing cooperation. Consequently, we will not consider it as a punishment path. In conclusion, we will employ the non-cooperative norm equilibrium in Proposition 1.1 to punish deviations starting in the next period.

To further induce cooperation, a deviation can also be immediately punished by open conflict, which yields a payoff of θR. In summary, a deviation triggers not only a permanent reversion to the worst one-period non-cooperative equilibrium starting next period, but also an immediate switch to the non-cooperative norm.[14]

The best attainable allocation is the efficient allocation $G^{FB} = 0$. The payoff from remaining in this efficient equilibrium is $R/2$, as nothing is wasted in guns and the resource is equally split. However, given that both countries have zero military power, an arbitrarily small deviation by one country would greatly improve its "bargaining power" at virtually zero cost. The other country will immediately react switching to open conflict. Thus, the one-shot deviation payoff would be θR. Finally, as prescribed by the equilibrium, starting in the next period, both countries will play the equilibrium in Proposition 1.1, which gives instant payoff of $\theta R/4$. Thus, to sustain the efficient equilibrium in a repeated game, the following condition must be satisfied:

$$\frac{R}{2} \geq (1-\delta)\theta R + \delta\frac{\theta R}{4}$$

What if this condition is not satisfied? A level of arming $G \in (0, \theta R/4)$ can still be sustained in the repeated game. The reason is that when countries chose $G > 0$, the potential deviation would be less profitable. Following this idea, the payoff $u_i^C(G) = R/2 - G$ can be sustained in equilibrium as long as it is greater than a one-shot deviation and then a reversion to the equilibrium in Proposition 1.1 forever, which yields maximum deviation payoff

$$v_i^D(G) = (1-\delta)\left(\frac{g^D(G,\theta R)}{G + g^D(G,\theta R)}\theta R - g^D(G,\theta R)\right) + \delta\frac{\theta R}{4}$$

where $g^D(G,\theta R) = \sqrt{\theta R G} - G$ is the static best-response to G given resources θR.[15] Thus, a country has no incentive to deviate as long as $u_i^C(G) \geq v_i^D(G)$ or, equivalently,

$$F(G) := -(2-\delta)G + 2(1-\delta)\sqrt{G\theta R} + \left[\frac{2-(4-3\delta)\theta}{4}\right]R \geq 0 \qquad (3)$$

Given the incentive compatibility constraint (3), the following proposition characterizes the best possible equilibrium that countries can sustain.

Proposition 2. *Assume that $\delta \geq 0$.*

1. *If $\theta \leq \frac{2}{4-3\delta}$, then the first-best payoff $V^{FB} = R/2$ characterized by unarmed peace $G^P = 0$ can be attained as an equilibrium of the repeated game.*

[14] This immediate switch to the non-cooperative norm selects the worst possible Nash equilibrium, i.e., $S_1 = S_2 = 0$, in the settlement stage.

[15] Formally, the solution to $\max_g \pi_i(g,G)\theta R - g$ is $g^D(G,\theta R) = \sqrt{\theta R G} - G$.

2. *If $\theta > \frac{2}{4-3\delta}$, then the second-best payoff $V^{SB} = R/2 - G^{SB} > R/4$ with military power $G^{SB} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ is sustainable as an equilibrium of the repeated game. Moreover, G^{SB} is the unique solution to $F(G) = 0$ and $\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R \leq \frac{\theta R}{4}$ (with strict inequality for $\delta > 0$).*

Proof. See the Appendix A.2. □

It is interesting to compare Propositions 1 and 2. Not surprisingly, repeated interactions can support more cooperation than in the one-shot game when in the latter we consider either a non-cooperative norm (as in Proposition 1.1) or a fully cooperative norm (as in Proposition 1.2). More interesting is that repeated interactions also open the door for more cooperation than the one-shot game under the conditional cooperative norm. To illustrate this, assume that an open conflict is not destructive (i.e. $\theta = 1$). Then, if $\delta \geq 2/3$, unarmed peace can be sustained as an equilibrium of the repeated game, while the conditional cooperative norm in Proposition 1.3 cannot improve in $V^{CN} = R/4$.

A more systematic way to compare Propositions 1.3 and 2 is to observe that the higher the value of δ, the more likely the first-best payoff can be sustained as an equilibrium, and the higher the second-best payoff when the first-best cannot be sustained. Moreover, Proposition 2 is identical to Proposition 1.3 when $\delta = 0$. Thus, the equilibrium in Proposition 2 always Pareto dominates the equilibrium in Proposition 1.3 for $\delta > 0$.

Summarizing, depending on the parameters, in the best equilibrium, there is either eternal unarmed peace or a repeated settlement with small military spending. Thus, repeated interactions open the door for substantial international cooperation and even the possibility of a Utopian fully peaceful world. This hardly corresponds to the historical record. In reality, we observe cycles of peace, arms races, and serious instances of open conflict.[16] So far, the only possibility to obtain open conflict is to shut down repeated interactions and either assume that conflict is not destructive or that countries follow a strict non-cooperative norm, which are not reasonable assumptions. Nevertheless, under those assumptions, we obtain an unrealistic result, that is, open conflict always prevails.

[16] In Appendix B we show that adding stochastic states can accommodate arms races within the best equilibrium path, which might help explain the rise of tensions in a cold war equilibrium, but not instances of open conflict.

4 Equilibrium with Permanent Elimination

In this section, we study the main feature of the model: the possibility of eliminating the enemy. Specifically, with probability $\phi > 0$, if countries go to open conflict, the winner has the opportunity to eliminate the loser at a cost X, after which the loser will not be able to contest the resource. This elimination opportunity greatly increases the complexity of the model. To better understand the basic interactions, in this section we consider the case in which the elimination is permanent ($\gamma = 0$).

We consider two types of cooperative equilibria: *permanent* and *temporary* cooperation. In an equilibrium with permanent cooperation, countries always choose settlement, regardless of whether open conflict allows the victor to eliminate the loser or not. In an equilibrium with temporary cooperation, settlement is sustained only when the victor of an open conflict will not be able to eliminate the loser.

For both types of equilibria, we characterize the best attainable equilibrium. The mechanism that allows for such cooperative equilibrium is related but not identical to the grim-trigger strategy. Similarly to the grim-trigger strategy, countries stick to the cooperative path under the credible threat that if anyone ever deviates from such cooperative equilibrium, the next period both countries will revert to the worst equilibrium with open conflict. Moreover, as we stipulated in the repeated game, we consider that a deviation is immediately punished by open conflict. In contrast to the grim-trigger strategy, when the victor can eliminate the loser, not every deviation can be effectively punished. In particular, if a country deviates from $S_i = 1$ to $S_i = 0$ when $Z = 1$, it might be the case that one of the countries is eliminated and hence there is no possible future punishment. Nevertheless, it is useful to start the analysis by characterizing the most severe equilibrium punishment (i.e., the worst possible equilibrium under open conflict) that countries can threaten to revert to starting next period if the resource is still contestable.

4.1 Punishment Equilibria

To determine the lowest possible equilibrium payoffs under the possibility of open conflict, we note that the non-cooperative norm should be used, as it leads to the lowest possible payoffs. Moreover, we also note that there is no unilateral profitable deviation from $S_1 = S_2 = 0$ because settlement requires that both players agree $S_1 = S_2 = 1$. Thus, under the non-cooperative norm, in contestable states (no player has been eliminated yet), there will be open conflict.

The following lemma characterizes the worst possible equilibrium under the non-cooperative norm. When $Z = 0$, open conflict will be used in this equilibrium. In contrast, when $Z = 1$, depending on the parameters, sometimes the lowest equilibrium payoff is attained by using the elimination option after an open conflict and sometimes the lowest equilibrium payoff is attained by not using the elimination option. Since elimination is an absorbing state, we compute the most severe credible punishment starting in a contestable state.

Lemma 1. *Under the non-cooperative norm.*[17] *Let* $\overline{X} = \frac{[(4-\theta)(1-\delta)+3\phi\delta]\delta R}{(1-\delta)[4(1-\delta)+3\phi\delta]}$ *be an elimination-cost threshold and* $R^E = \theta R - X + \frac{\delta R}{1-\delta}$ *be the discounted value of the resource accessible to the winner following an elimination.*

1. *Suppose that* $X > \overline{X}$. *Then, the worst equilibrium is* $G^{NN} = \theta R/4$ *for* $Z = 0$ *and* $G^{NN} = \theta R/4$ *and no elimination for* $Z = 1$. *The associated punishment expected payoff is* $V^{PU} = V^{NN} = \frac{\theta R}{4}$.

2. *Suppose that* $X \leq \overline{X}$. *Then, the worst equilibrium is* $G^{NN} = \theta R/4$ *for* $Z = 0$ *and* $G^E = R^E/4$ *and elimination of the loser for* $Z = 1$. *The associated punishment expected payoff is* $V^{PU} = V^E = \frac{(1-\delta)\left[(1-\phi)\theta R+\phi R^E\right]}{4(1-\delta+\phi\delta)}$.

Proof. See the Appendix A.3. □

In words, there are two possible equilibria to consider, but the one that is chosen as punishment to keep incentives aligned is the equilibrium with the lowest payoff. In one equilibrium, countries never use elimination. This equilibrium is just the eternal repetition of the equilibrium in Proposition 1.1 or, giving the same payoff, the punishment used in Proposition 2 to sustain cooperation. In the second equilibrium, countries use the elimination option whenever it is available (that is, when $Z = 1$). In this equilibrium, the first time that $Z = 1$ the victor eliminates the loser, accounting for an instant resource gain plus an eternal uncontested resource R^E, since the game permanently transitions to an uncontestable state in which the victor takes all the resources in every period. Moreover, there is a unique (and very high) Nash equilibrium level of arming. To see this, assume that countries expect to use the elimination option. Then, country i's expected payoff is given by:

$$u_i\left(G_i, G_{-i}\right) = \pi_i\left(G_i, G_{-i}\right)\left[(1-\delta)\left(\theta R - X\right) + \delta R\right] - (1-\delta)\, G_i.$$

Thus, the unique Nash equilibrium level of arming is $G_i = G^E = R^E/4 = \left(\theta R - X + \frac{\delta R}{1-\delta}\right)/4$ for $i = 1, 2$.[18]

[17] That is, assume that in every contestable state, $S_1 S_2 = 0$ for all (G_1, G_2) and Z.

[18] $X \leq \overline{X}$ ensures that $-X + \frac{\delta R}{1-\delta} > 0$ and, hence, $G^E > G^{NN} = \theta R/4 > 0$. See the Appendix A.3 for more details.

Depending on the parameter values, only one or both of these equilibria exist. In particular, for X greater than a threshold, only the first equilibrium exists; for intermediate values of X both equilibria exist; and for X below a threshold only the second equilibrium exists. Moreover, we show that whenever both equilibria exist, the one that generates lower payoffs is the equilibrium in which countries use the elimination option.

The intuition behind the proof of Lemma 1 is simple. When the cost of using the elimination technology is high (formally $X > \overline{X}$), countries are not interested in using it, and, hence, the worst possible equilibrium is identical to the equilibrium punishment in Proposition 2. In contrast, when the cost of the elimination technology is low (formally $X \leq \overline{X}$), both countries are willing to use it if they expect the other country to do the same. Moreover, in such circumstances, the lower-payoff equilibrium is always the one in which the victor eliminates the loser.[19]

4.2 Cooperation Strategies and Payoffs

Next, we explore cooperative equilibria in contrast to those described in Lemma 1. We start with permanent cooperation, i.e., when countries choose to settle regardless of whether the elimination option is available or not. In particular, consider the following strategy to sustain permanent cooperation.

Definition 1. ***Permanent cooperation strategy (PC)***.

- *Cooperation phase: Suppose that no deviation has occurred. If $Z = 0$, play $G(0)$ and $S = 1$; if $Z = 1$, play $G(1)$ and $S = 1$.*
- *Punishment phase for $Z = 0$: Any deviation from $G(0)$ and $S = 1$ triggers, starting in the next period, the worst equilibrium in Lemma 1. In addition, a deviation from $G(0)$ triggers an open conflict immediately.*
- *Punishment phase for $Z = 1$:*
    - *Deviation from $S = 1$: If this leads to an elimination, the deviation cannot be punished. If it does not lead to an elimination, it triggers, starting in the next period, the worst equilibrium in Lemma 1.*
    - *Deviation from $G(1)$: This is immediately punished with $S = 0$ and elimination of the loser if elimination is an optimal choice for the*

[19] Note that the threshold $\overline{X}$ determines which equilibrium has the lowest payoff, not which equilibria exist. As noted above, for a range of X, elimination and non-elimination are both equilibria.

victor; otherwise, starting in the next period, the worst equilibrium in Lemma 1 is played.

– *To determine whether elimination is optimal, the victor assumes that, starting in the next period, the worst equilibrium in Lemma 1 will be played.*

Suppose that countries follow the permanent cooperation strategy with $G(0) \in [0, \theta R/4)$ and $G(1) \in [0, \theta R/4)$ and that no country deviates. Then:

$$V^{PC}(0) = (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta\left[\phi V^{PC}(1) + (1-\phi)V^{PC}(0)\right],$$
$$V^{PC}(1) = (1-\delta)\left(\frac{R}{2} - G(1)\right) + \delta\left[\phi V^{PC}(1) + (1-\phi)V^{PC}(0)\right].$$

Solving:

$$V^{PC}(0) = \frac{R}{2} - (1-\delta\phi)\, G(0) - \delta\phi\, G(1),$$
$$V^{PC}(1) = \frac{R}{2} - (1-\delta+\delta\phi)\, G(1) + \delta(1-\phi)\, G(0).$$

For some parameters, permanent cooperation cannot be sustained, but countries can still cooperate when the victor will *not* be able to eliminate the loser. Consider:

Definition 2. ***Temporary cooperation strategy (TC).***

- *Cooperation phase for $Z = 0$: If no deviation has occurred, play $G(0)$ and $S = 1$.*

- *Punishment phase for $Z = 0$: Any deviation from $G(0)$ and $S = 1$ triggers, starting in the next period, the worst equilibrium in Lemma 1. A deviation from $G(0)$ also triggers immediate open conflict.*

- *No cooperation for $Z = 1$: Play G^E, set $S = 0$, and eliminate.*

Suppose that countries follow the temporary cooperation strategy with $G(0) \in [0, \theta R/4)$ and that no country deviates. Then:

$$V^{TC}(0) = (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta\left[\phi V^{TC}(1) + (1-\phi)V^{TC}(0)\right],$$
$$V^{TC}(1) = \frac{1}{2}\Big[(1-\delta)(\theta R - X) + \delta R\Big] - (1-\delta)G^E.$$

Solving:

$$V^{TC}(0) = \frac{(1-\delta)\left(\frac{R}{2} - G(0) + \frac{\delta\phi R^E}{4}\right)}{1-\delta+\delta\phi},$$
$$V^{TC}(1) = \frac{(1-\delta)R^E}{4}.$$

4.3 Optimal Deviations and Sustainability Conditions

The following lemma characterizes the optimal deviation and the associated deviation rewards for a country that chooses to stop following the cooperation strategies.

Lemma 2. *Suppose that countries are following either the permanent cooperation strategy with $G(0) \in [0, \theta R/4)$ and $G(1) \in [0, \theta R/4)$, or the temporary cooperation strategy with $G(0) \in [0, \theta R/4)$.*

1. *Assume that $Z = 0$. Then, the most profitable deviation is*
$$g^D(G(0), \theta R) = \sqrt{\theta R\, G(0)} - G(0),$$
and the associated deviation payoff is
$$V^D(0) = (1-\delta)\, u^D(G(0), \theta R) + \delta V^{PU},$$
where
$$u^D(G(0), \theta R) = \frac{g^D(G(0), \theta R)}{G(0) + g^D(G(0), \theta R)}\, \theta R - g^D(G(0), \theta R),$$
and V^{PU} is given in Lemma 1.

2. *Assume that $Z = 1$. Let*
$$\overline{X} = \frac{\big[(4-\theta)(1-\delta) + 3\phi\delta\big]\delta R}{(1-\delta)\big[4(1-\delta)+3\phi\delta\big]}, \qquad R^E = \theta R - X + \frac{\delta R}{1-\delta}.$$

   (a) *If $X > \overline{X}$, then the most profitable deviation is*
$$g^D(G(1), \theta R) = \sqrt{\theta R\, G(1)} - G(1),$$
which induces open conflict without elimination. The associated deviation payoff is
$$V^D(1) = (1-\delta)\, u^D(G(1), \theta R) + \delta\, \frac{\theta R}{4},$$
where $u^D(\cdot)$ is defined as above.

(b) If $X \leq \overline{X}$, then the most profitable deviation is

$$g^D\big(G(1), R^E\big) = \sqrt{R^E G(1)} - G(1),$$

which induces open conflict followed by elimination. The associated deviation payoff is

$$V^D(1) = (1-\delta)\, u^D\big(G(1), R^E\big)\,,$$

where

$$u^D\big(G(1), R^E\big) = \frac{g^D(G(1), R^E)}{G(1) + g^D(G(1), R^E)}\, R^E - g^D(G(1), R^E).$$

Proof. See the Appendix A.3. □

Depending on the cost X, the most profitable deviation may or may not involve the use of the elimination technology. As X increases, deviating from a more cooperative equilibrium will not lead to elimination in the current period nor in the punishment periods that follow.

Combining Lemmas 1 and 2, we obtain sustainability conditions for cooperative equilibria. To sustain the permanent cooperation strategy with $G(0), G(1) \in [0, \theta R/4)$, it must be that

$$F^{PC,Z}(G(0), G(1)) := V^{PC}(Z) - V^D(Z) \geq 0, \tag{4}$$

for $Z = 0, 1$.

To sustain the temporary cooperation strategy with $G(0) \in [0, \theta R/4)$, it must be that

$$F^{TC}(G(0)) := V^{TC}(0) - V^D(0) \geq 0. \tag{5}$$

4.4 Impossibility of Avoiding Open Conflict for $\delta \to 1$

Before we fully characterize cooperative equilibria, it is imperative to explore what happens to the sustainability conditions for cooperative equilibria when $\delta \to 1$. After all, as we have shown in Proposition 2, when there is no technology that allows the elimination of the enemy, unarmed peace can be sustained for $\delta \geq \frac{2}{3}$. The following proposition provides the answer.

Proposition 3. *Suppose that $\delta \to 1$. Then, it is impossible to sustain a permanent cooperative equilibrium, but it is always possible to sustain a temporary cooperative equilibrium. Moreover, for any δ, a temporary cooperative equilibrium is not Pareto efficient.*

Proof. See the Appendix A.3. □

The intuition behind Proposition 3 is as follows. When $\delta \to 1$, it is impossible to deter a deviation when $Z = 1$. The reason is that the elimination technology behaves like an investment whose attractiveness increases with patience, since the effective value of the resource satisfies $R^E \to \infty$ as $\delta \to 1$. At the same time, the certainty that cooperation will break down when there is a chance of eliminating the rival does not prevent cooperation while $Z = 0$. The reason is that there is no advantage in switching to open conflict when it is not possible for the victor to eliminate the loser.

It is worth recalling the path implied by a temporary cooperative equilibrium. Countries cooperate, avoiding open conflict and possibly reducing arming levels while they cannot eliminate their enemy; but as soon as the elimination opportunity arrives, an instance of open conflict is inevitable. Thereafter, the victor takes full control of the disputed resource, and disarmed peace prevails forever. In other words, what might appear to be patient players cooperating forever (à la folk theorem) reveals only temporary cooperation that suddenly switches to full-fledged war to permanently settle the dispute.

Finally, to see that a temporary cooperative equilibrium is not Pareto efficient, note that in any such equilibrium, whenever $Z = 0$ countries invest $G(0) \geq 0$; the first time that $Z = 1$ they invest $G^E > 0$; and thereafter, guns are never used again. Thus, even if a temporary cooperative equilibrium with $G(0) = 0$ can be sustained, there is still a large positive investment in guns the first time $Z = 1$. In contrast, efficiency requires zero investment in guns along the entire equilibrium path. For $\delta \to 1$, we have

$$\lim_{\delta \to 1} V^{TC}(0) = \lim_{\delta \to 1} V^{TC}(1) = \frac{R}{4},$$

while efficiency would give each country $V^{FB} = R/2$. Therefore, temporary cooperation is not efficient.

4.5 Full Characterization of Cooperative Equilibria

Beyond analyzing the limit case as $\delta \to 1$, arguably the most interesting and counterintuitive scenario, we also fully characterize the set of equilibria for all discount factors. To do so, we categorize elimination costs into two groups: high and low elimination costs. In addition, when multiple equilibria arise, we compare them using the ex-ante expected payoff induced by each equilibrium as our evaluation criterion. That is, we assume diplomacy is fully effective in the sense that, for any form of cooperation (whether permanent or temporary), it identifies and implements the best possible outcome that can be sustained as an equilibrium. However, even under this optimistic assumption, diplomacy may not always suffice to achieve ideal outcomes.

Formally, given the probability of elimination availability ϕ, in the case of permanent cooperation we focus on the cooperative equilibrium that maximizes

$$V^{PC} = \max_{G(0),G(1)} \{\phi V^{PC}(1) + (1-\phi)V^{PC}(0)\}$$

subject to the sustainability constraints $F^{PC,Z}(G(0), G(1)) \geq 0$ for $Z = 0, 1$, that is, equation (4).

For temporary cooperation, we look for the equilibrium that maximizes

$$V^{TC} = \max_{G(0)} \{\phi V^{TC}(1) + (1-\phi)V^{TC}(0)\}$$

subject to the sustainability constraint $F^{TC}(G(0)) \geq 0$, that is, equation (5).

We now differentiate between cases of high and low elimination costs. Proposition 4 specifically examines the high-cost regime.

Proposition 4. *Suppose that $X > \overline{X}$.*

1. *If $\theta \leq \frac{2}{4-3\delta}$, then permanent complete cooperation, that is, disarmed peace with $G(Z) = 0$ for $Z = 0, 1$, can be sustained.*

2. *If $\theta > \frac{2}{4-3\delta}$, then the best sustainable cooperative equilibrium is permanent partial cooperation (armed peace) with*

$$G^{PC} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$$

for $Z = 0, 1$, where G^{PC} is the unique solution to

$$F^{PC,Z}(G, G) = F(G) = -(2-\delta)G + 2(1-\delta)\sqrt{G\theta R} + \left[\frac{2-(4-3\delta)\theta}{4}\right] R = 0,$$

and $\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R \leq \frac{\theta R}{4}$ (with strict inequality for $\delta > 0$).

Proof. See the Appendix A.3. □

Proposition 4 is nearly identical to Proposition 2.[20] Not surprisingly, if the elimination cost is high enough, countries are not willing to use the elimination

[20] Note that when $X > \overline{X}$, countries never use elimination to punish deviations, so the sustainability constraints $F^{PC,Z}(G(0), G(1)) \geq 0$ for $Z = 0, 1$ (equation (4)) are identical for $Z = 0, 1$ and coincide with the sustainability constraint for repeated game $F(G) \geq 0$ (equation (3)).

technology, and hence the most severe punishment available to deter a deviation from cooperation is to revert forever to the one-shot equilibrium under the non-cooperative norm. In other words, the logic of the folk theorem is reinstated. There is, however, a subtle difference: Proposition 4 requires the condition $X > \overline{X}$, and crucially, this condition fails when δ is sufficiently large, in particular, it never holds when $\delta \to 1$.

Next, we study the more interesting case in which the elimination cost is low enough so that the punishment path involves elimination. We define upper bounds for the levels of arming $\overline{G}(Z)$ that can arise in equilibrium, as well as a lower bound $\hat{G}(1)$ sufficient to deter a deviation leading to elimination when $Z = 1$. The following proposition summarizes these results. We also define two cost thresholds satisfying $\overline{X}_{low} < \overline{X}_{high} < \overline{X}$ such that: if $X \geq \overline{X}_{high}$ (and θ satisfies appropriate conditions), permanent disarmed peace can still be sustained; and if $\overline{X}_{low} < X < \overline{X}_{high}$, permanent but armed peace can be sustained.

Proposition 5. *Suppose that $X \leq \overline{X}$ and define*

$$\overline{X}_{low} = \theta R + \frac{\left[2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right] R}{2(1-\delta)}, \qquad \overline{X}_{high} = \theta R + \frac{(2\delta-1)}{2(1-\delta)} R.$$

1. *Suppose that either $\theta \leq \frac{1-2\delta}{2(1-\delta)}$ or*

$$\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}, \quad \textit{and} \quad X \geq \overline{X}_{high}.$$

   Then permanent complete cooperation (disarmed peace with $G(Z) = 0$ for $Z = 0, 1$) can be sustained.

2. *Suppose that*

$$\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)} \quad \textit{and} \quad \overline{X}_{low} < X < \overline{X}_{high}.$$

   Let $\overline{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^2 \theta R < G^{NN}$, $\overline{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 R^E < G^E$, and let $\hat{G}(1) \in (0, \overline{G}(1))$ be the unique solution to $F^{PC,1}(0, \hat{G}(1)) = 0$.

   (a) *If $\hat{G}(1) \leq F^{PC,0}(0,0)/(\delta\phi)$, then the best permanent cooperative equilibrium is partial cooperation with $G(0) = 0$ and $G(1) = \hat{G}(1)$.*

   (b) *If $\hat{G}(1) > F^{PC,0}(0,0)/(\delta\phi)$, then the best permanent cooperative equilibrium is partial cooperation with*

$$G(0) \in (0, \overline{G}(0)), \qquad G(1) \in (\hat{G}(1), \overline{G}(1)),$$

given by the unique solution to $F^{PC,0}(G(0), G(1)) = F^{PC,1}(G(0), G(1)) = 0$ *satisfying*

$$\frac{\partial F^{PC,0}(G(0), 0)}{\partial G(0)} \cdot \frac{\partial F^{PC,1}(0, G(1))}{\partial G(1)} > \delta^2(1-\phi)\phi.$$

3. *If* $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$*, then the best temporary cooperative equilibrium has* $G(0) = 0$*.*

4. *If* $\theta > \frac{2}{4-3\delta+3\phi\delta}$*, then the best temporary cooperative equilibrium satisfies*

$$G(0) \in \left(0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R\right),$$

where $G(0)$ *solves* $F^{TC}(G(0)) = 0$*. Moreover,*

$$\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R < G^{NN} = \frac{\theta R}{4}.$$

Proof. See the Appendix A.3. □

Proposition 5.1 shows that even when $X \leq \overline{X}$ (from Lemma 1), it may still be possible to sustain the first-best allocation $G(Z) = 0$ for $Z = 0, 1$. Why is a country not willing to deviate, initiate open conflict, win with probability one, and eliminate the loser? There are two reasons. First, during open conflict a fraction $1-\theta$ of the resource is destroyed. Second, the elimination cost X may still be high enough to deter deviation, even if it is not high enough to avoid elimination on the worst possible non-cooperative equilibrium. That is, when $\theta \leq \frac{1-2\delta}{2(1-\delta)}$, the destruction from conflict is severe enough to deter deviation even if elimination were costless.[21] When $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$, costless elimination would always induce deviation, but sufficiently high X still blocks deviation.

If complete permanent cooperation cannot be sustained, Proposition 5.2 shows that a second-best allocation with permanent but partial cooperation can be sustained, provided that elimination is not too cheap and open conflict is sufficiently destructive. These equilibria feature fluctuating arming choices. For example, when $\hat{G}(1) \leq F^{PC,0}(0,0)/(\delta\phi)$, the best permanent cooperative equilibrium involves disarmed peace when $Z = 0$ but positive arming when $Z = 1$. Moreover, it

[21] Under complete permanent cooperation a country receives $R/4$, while deviating to open conflict and eliminating its rival yields $(\theta R - X) + \delta R$. Thus, for $\theta \leq \frac{1-2\delta}{2(1-\delta)}$, deviation does not pay.

is never optimal to have $G(0) > 0$ and $G(1) = 0$; some arming is always required to prevent open conflict when elimination is possible.

From Proposition 3, we know that when $\delta \to 1$, even these second-best equilibria cannot be sustained. More precisely, the following corollary characterizes when open conflict is unavoidable.

Corollary 1. *Open conflict is unavoidable if and only if*

$$X \leq \begin{cases} \overline{X} & \text{when } \theta > \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}, \\ \overline{X}_{low} & \text{when } \frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}. \end{cases}$$

Moreover:

1. *The lower the cost of elimination (X) and the higher the probability of its availability (ϕ), the harder it is to avoid open conflict.*

2. *The destructiveness of conflict ($1-\theta$) has a non-monotonic effect: as destructiveness declines (higher θ), open conflict initially becomes more likely, but eventually becomes easier to avoid.*

Proof. See the Appendix A.3. □

Note that the thresholds $\overline{X}$ and $\overline{X}_{low}$ depend on θ. When the fraction of the resource not destroyed by open conflict θ is high enough, the threshold value of X that makes open conflict unavoidable is also high. When θ is lower (but still above another threshold), the relevant cutoff becomes $\overline{X}_{low} < \overline{X}$.

Importantly, the fact that open conflict is unavoidable for some parameter values does *not* mean that cooperation disappears entirely. Rather, it means that cooperation *cannot survive when elimination becomes possible*. In these cases, temporary cooperation still exists: countries cooperate when $Z = 0$, avoiding conflict and saving on arming, but once $Z = 1$ arrives they switch to open conflict, after which the victor eliminates the loser and disarmed peace prevails thereafter. In fact, Propositions 5.3 and 5.4 fully characterize the best temporary cooperative equilibrium in which countries only cooperate when $Z = 0$. Moreover, these equilibria exist for all parameter values. Also note that the level of cooperation when $Z = 0$ is indeed affected by the expectation that cooperation will not survive when $Z = 1$. Specifically, as the probability that the elimination technology will be available increases (a higher ϕ), less cooperation can be sustained when such technology is not available.[22]

[22] Formally, in Proposition 5.3, the condition $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$ is less likely to hold the higher the value of ϕ. Also, in Proposition 5.4, $G(0)$ is strictly increasing in ϕ. This also implies that gun choices in Proposition 4 are always lower than gun choices in Propositions 5.3 and 5.4.

Finally, Proposition 5 highlights the richness of the model. Cooperation may involve falling and rising arming levels, producing sequences of "calm" and "hot" periods that do not necessarily foreshadow war, but help sustain cooperation. At the same time, when elimination is sufficiently attractive, open conflict becomes inevitable: countries cooperate while elimination is impossible, then fight a decisive war once it becomes possible, and thereafter settle into perpetual disarmed peace.

4.6 Permanent versus Temporary Cooperation

Proposition 5 fully characterizes the best possible permanent and temporary cooperative equilibria, but does not compare them with each other. There are three situations to consider. First, when complete permanent cooperation can be sustained, no comparison is needed: complete permanent cooperation always induces a higher expected payoff than any other equilibrium. Second, when neither complete nor partial permanent cooperation can be sustained, there is again nothing to compare. The best possible outcome is the temporary cooperative equilibrium described in Propositions 5.3 and 5.4. Finally, the only non-trivial case occurs when partial permanent cooperation is feasible, but complete permanent cooperation is not. In this scenario, a comparison becomes meaningful as temporary cooperation remains a possibility. Proposition 6 formalizes this comparison.

Proposition 6. *Suppose that*

$$\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)} \qquad \textit{and} \qquad \overline{X}_{low} < X < \overline{X}_{high}.$$

Then partial permanent cooperation induces a higher ex-ante expected payoff than temporary cooperation if and only if

$$\begin{aligned}\Delta =& F^{PC,0}(0,0) - \delta\phi\, G^{PC}(1) - \delta(1-\phi)\, G^{PC}(0) \\ &- \frac{1-\delta}{1-\delta+\delta\phi}\left[F^{TC}(0) - \delta(1-\phi)\, G^{TC}(0)\right] \geq 0,\end{aligned}$$

where

$$\begin{aligned}F^{PC,0}(0,0) &= \frac{R}{2} - (1-\delta)\theta R - \frac{(1-\delta)\delta\left[\theta R + \phi\left(-X + \frac{\delta R}{1-\delta}\right)\right]}{4(1-\delta+\phi\delta)}, \\ F^{TC}(0) &= \frac{2-\theta(4-3\delta+3\phi\delta)}{4}R,\end{aligned}$$

and $G^{PC}(0)$, $G^{PC}(1)$ are given by Proposition 5.2, while $G^{TC}(0)$ is given by Propositions 5.3 and 5.4.

Moreover, if $\theta > \frac{2}{4-3\delta+3\phi\delta}$ *and* $\hat{G}(1) \leq F^{PC,0}(0,0)/(\delta\phi)$*, then* $\Delta \geq 0$*; while if* $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$ *and* $\hat{G}(1) > F^{PC,0}(0,0)/(\delta\phi)$*, then* $\Delta < 0$.

Proof. See the Appendix A.3. □

The most interesting result in Proposition 6 is that partial permanent cooperation does not always dominate temporary cooperation. In fact, when $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$ and $F^{PC,0}(0,0) < 0$, it is strictly better to cooperate only when $Z = 0$. The reason is that inducing cooperation both when $Z = 0$ and when $Z = 1$ may require very high levels of arming compared to those required to sustain cooperation only when $Z = 0$. Thus, avoiding open conflict–even when sustainable–is not necessarily the best option.

To summarize, the elimination technology substantially alters the standard folk-theorem logic. First, open conflict and the associated destruction become unavoidable as $\delta \to 1$. Second, avoiding open conflict may require fluctuating arming choices along the cooperative path. Third, even if countries *can* avoid open conflict, they may choose not to do so because the long-run benefits of post-war, uncontested peace outweigh the additional arming costs required to prevent war.

There is, however, one unrealistic feature of the temporary cooperative equilibrium described in Propositions 5 and 6. Because elimination is an absorbing state, the first instance of open conflict permanently settles the dispute: after elimination, the resource is no longer contested and disarmed peace prevails forever. In the next section, we relax this assumption of permanent elimination to obtain more realistic dynamics. Since we are still interested in equilibria involving open conflict, we will focus on situations in which the elimination cost is low.

5 Equilibrium with Temporary Elimination

Suppose that $\gamma > 0$. Then, there is a chance that countries return to contestable states. In other words, elimination is temporary and the vanquished nation can return to contest the resource with probability γ. This does not affect choices in uncontestable states, where the victor still collects R without investing anything in guns, but makes the elimination technology less attractive. Fortunately, to characterize the best possible equilibrium when $\gamma > 0$, we can employ the same approach as in Section 4.

In this section, we will see that the same equilibrium classes are valid: permanent complete cooperation and permanent partial cooperation, as well as temporary cooperation. One key difference, however, is that actions following an elimination matter in a temporary cooperation equilibrium. In such equilibria, if countries have been playing a temporary cooperation strategy and an elimination happens on-path, then, when the game returns to a contestable state, countries resume the equilibrium strategy prescribed by temporary cooperation. If, by contrast, an elimination has occurred off-path, when the game returns to a contestable state, countries move to the equilibrium strategy prescribed by the punishment equilibrium (see Lemmas 3 and 4).

As before, we begin the analysis by characterizing the most severe equilibrium punishment (i.e., the worst possible equilibrium under open conflict) that countries can threaten to revert to in order to punish a deviation from the cooperative path. For this purpose, we extend the threshold that previously determined which actions lead to the most severe punishment equilibrium payoffs to now depend on γ, as $\overline{X}(\gamma)$. Similarly, we extend the discounted value of the resource accessible to the winner following an elimination $R^E(\gamma)$ and its associated arming $G^E(\gamma)$ and payoff $V^E(\gamma)$ as follows:

Lemma 3. *Under the non-cooperative norm.*[23] *Let*

$$\overline{X}(\gamma) = \frac{[(4-\theta)(1-\delta+\delta\gamma)+3\delta\phi]\,\delta R}{(1-\delta+\delta\gamma)[4(1-\delta+\delta\gamma)+3\delta\phi]}$$

and

$$R^E(\gamma) = \theta R - X + \frac{\delta R}{1-\delta+\delta\gamma}.$$

1. *Suppose that $X > \overline{X}(\gamma)$. Then the worst equilibrium is $G^N = \frac{\theta R}{4}$ for $Z = 0$ and $G^N = \frac{\theta R}{4}$ and no elimination for $Z = 1$. The associated punishment expected payoff is $V^{PU} = V^N = \frac{\theta R}{4}$.*

2. *Suppose that $X \leq \overline{X}(\gamma)$. Then the worst equilibrium is $G^N = \frac{\theta R}{4}$ for $Z = 0$ and $G^E(\gamma) = \frac{R^E(\gamma)}{4}$ and elimination for $Z = 1$. The associated punishment expected payoff is*

$$V^{PU} = V^E(\gamma) = \frac{(1-\delta+\delta\gamma)\left[\phi R^E(\gamma) + (1-\phi)\,\theta R\right]}{4[1-\delta(1-\phi-\gamma)]}.$$

Proof. See the Appendix A.4. □

Lemma 3 is a generalization of Lemma 1.[24] When the cost of using the elimination technology is greater than $\overline{X}(\gamma)$, the most severe punishment is the static Nash equilibrium under no settlement. In contrast, when the cost of using the

[23] That is, assume that in every contestable state, $S_1S_2 = 0$ for all (G_1, G_2) and Z.

[24] Formally, for $\gamma = 0$, we have $\overline{X}(0) = \overline{X}$ and $R^E(0) = R^E$.

elimination technology is less than $\overline{X}(\gamma)$, the most severe punishment includes the use of the elimination technology whenever it is available.

As in Section 4, better equilibria can be sustained than those in Lemma 3. It is straightforward to adapt the permanent and temporary cooperation strategies (Definitions 1 and 2, respectively) to deal with the possibility that $\gamma > 0$. All we need to do is to use Lemma 3 instead of Lemma 1 in the punishment phases.

Computing expected payoffs under these cooperation strategies is also straightforward. Suppose that countries are following the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Since under this strategy the game never transitions to the uncontestable state, the expected payoffs for each contestable state are identical to those computed for $\gamma = 0$ in Section 4. Thus:

$$\begin{aligned} V^{PC}(1,0) &= \frac{R}{2} - (1-\delta\phi)\, G(0) - \delta\phi G(1) \\ V^{PC}(1,1) &= \frac{R}{2} - (1-\delta+\delta\phi)\, G(1) + \delta(1-\phi)\, G(0). \end{aligned}$$

Intuitively, if countries are cooperating in each contestable state, the elimination technology is never used, and hence no country is ever temporarily eliminated, which implies that the probability that a defeated country comes back has no effect on expected payoffs.

In contrast, when countries are following a temporary cooperation strategy, the game will eventually transition to the uncontestable state, then move back again to the contestable state, and so on. Therefore, for $\gamma > 0$, the payoffs we computed in Section 4 do not apply anymore. Suppose that countries are following the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$, and that no country deviates. Let $V^{TC}(1, Z)$ denote the payoff in a contestable state $C = 1$ with $Z = 0, 1$, and let $V^{TC}(0, w)$ and $V^{TC}(0, l)$ denote the payoffs in the uncontestable state $C = 0$ for the winner and the loser, respectively. Then:

$$\begin{aligned} V^{TC}(1,0) &= (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta V^{TC}(1), \\ V^{TC}(1) &= \phi V^{TC}(1,1) + (1-\phi)\, V^{TC}(1,0), \\ V^{TC}(1,1) &= \frac{(1-\delta)(\theta R - X) + \delta V^{TC}(0,w)}{2} + \frac{\delta V^{TC}(0,l)}{2} - (1-\delta)\, G^{E}(\gamma), \\ V^{TC}(0,w) &= (1-\delta) R + \delta\left[\gamma V^{TC}(1) + (1-\gamma)\, V^{TC}(0,w)\right], \\ V^{TC}(0,l) &= \delta\left[\gamma V^{TC}(1) + (1-\gamma)\, V^{TC}(0,l)\right]. \end{aligned}$$

Thus, when the resource is contestable and $Z = 0$, each country obtains $R/2 - G(0)$ and in the next period the resource will also be contestable. In particular, with probability ϕ the state will be $(C = 1, Z = 1)$ and with probability $1 - \phi$ it will be $(C = 1, Z = 0)$. When the resource is contestable and $Z = 1$, countries fight an open conflict, the winner obtains θR and pays the elimination cost X in the present, and in the next period the game transitions to the uncontestable state. The loser gets nothing in the present. Both countries invest $G^E(\gamma)$ in guns. If the resource is uncontestable, the winner gets R at no cost in the present period, and in the next period the game switches to the contestable state with probability γ. The loser gets nothing in the present.

Solving for the above equations, the expected payoffs for $C = 1$ and $Z = 0, 1$ are given by:

$$V^{TC}(1,0) = \frac{\left(1-\delta+\delta\gamma-\phi\delta^2\gamma\right)\left(\frac{R}{2}-G(0)\right)+\frac{\delta\phi R^E(\gamma)}{4}}{1-\delta(1-\gamma-\phi)},$$

$$V^{TC}(1,1) = \frac{\frac{(1-\delta+\delta\gamma)(1-\delta)R^E(\gamma)}{4}+(1-\phi)\,\delta^2\gamma V^{TC}(1,0)}{1-\delta+\delta\gamma-\delta^2\gamma\phi}.$$

The following lemma characterizes optimal deviations from cooperation strategies.

Lemma 4. *Suppose that countries are following either the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ or the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$. Assume that $X \leq \overline{X}(\gamma)$.*

1. *Assume that $Z = 0$. Then the most profitable deviation is*

$$g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0).$$

The associated deviation payoff is given by:

$$V^D(0) = (1-\delta)\, u^D(G(0), \theta R) + \delta V^E(\gamma),$$

where

$$u^D(G(0), \theta R) = \frac{g^D(G(0), \theta R)}{G(0) + g^D(G(0), \theta R)}\theta R - g^D(G(0), \theta R).$$

2. *Assume that $Z = 1$. Then the most profitable deviation is*

$$g^D\left(G(1), R^E(\gamma)\right) = \sqrt{G(1)R^E(\gamma)} - G(1),$$

which induces an open conflict, after which the victor eliminates the loser. The associated deviation payoff is given by:

$$V^D(1) = (1-\delta)\, u^D\left(G(1), R^E(\gamma)\right) + \frac{\gamma\delta^2 V^E(\gamma)}{1-\delta+\delta\gamma},$$

where

$$u^D\left(G(1), R^E(\gamma)\right) = \frac{g^D\left(G(1), R^E(\gamma)\right)}{G(1) + g^D\left(G(1), R^E(\gamma)\right)} R^E(\gamma) - g^D\left(G(1), R^E(\gamma)\right).$$

Proof. See the Appendix A.4. □

Lemma 4 is a generalization of Lemma 2 for the case $X \leq \overline{X}(\gamma)$. That is, we focus on the more interesting case in which the most severe punishment uses the elimination technology.

Combining Lemmas 3 and 4, we can generalize sustainability constraints (4) for permanent cooperation and (5) for temporary cooperation. In particular, to sustain as an equilibrium the permanent cooperation strategy with $G(0), G(1) \in \left[0, \frac{\theta R}{4}\right)$ it must be the case that:

$$F_\gamma^{PC,Z}(G(0), G(1)) = V^{PC}(1, Z) - V^D(Z)) \geq 0 \tag{6}$$

for $Z = 0, 1$. To sustain as an equilibrium the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ it must be the case that:

$$F_\gamma^{TC}(G(0)) = V^{TC}(1, 0) - V^D(0) \geq 0. \tag{7}$$

The following proposition characterizes sustainability constraints (6) when $\delta \to 1$.

Proposition 7. *Suppose that $\gamma > 0$ and $\delta \to 1$. Then permanent complete cooperation, i.e., disarmed peace with $G(Z) = 0$ for $Z = 0, 1$, can be sustained.*

Proof. See the Appendix A.4. □

Note the difference with Proposition 3. While for $\gamma = 0$ the limit $\delta \to 1$ makes disarmed peace unsustainable, for $\gamma > 0$ we recover sustainability of the Pareto-efficient allocation. The reason behind this difference is that for $\gamma = 0$, if countries play $G(1) = 0$, it is extremely tempting to deviate. A defector will win the open conflict with probability 1 and permanently eliminate its enemy at virtually no cost, obtaining R in perpetuity. In contrast, when $\gamma > 0$, the defector will collect R only until the game moves back to a contestable state, at which point the defector will be punished with the equilibrium described in Lemma 3.2. Moreover, this punishment equilibrium can be very severe, given that for $Z = 1$, the victor of the open conflict always chooses to use the elimination technology.

However, Proposition 7 does not imply that for $\gamma > 0$ it is always possible to avoid open conflict and the use of the elimination technology. In fact, we obtain a generalization of Proposition 5, which implies that although temporary cooperation can always be sustained, permanent cooperation might not be sustainable. Moreover, we also obtain a generalization of Proposition 6, which implies that even when partial permanent cooperation is sustainable, countries might obtain a higher expected payoff with temporary cooperation. In what follows, we introduce cost thresholds analogous to the analysis in Proposition 5: $\overline{X}_{low}(\gamma) < \overline{X}_{high}(\gamma)$ such that, if $X \geq \overline{X}_{high}(\gamma)$ (and θ satisfies some conditions), permanent disarmed peace (complete cooperation) can still be sustained; and if $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$, permanent but armed peace (partial cooperation) can be sustained. These thresholds are associated with the share of resources that remain after open conflict, $\overline{\theta}^{PC}_{low}$ and $\overline{\theta}^{PC}_{high}$. In addition, we also characterize temporary cooperative equilibria as a function of another threshold regarding the share of the resources not destroyed, $\overline{\theta}^{TC}$.

Proposition 8. *Suppose that* $\gamma \geq 0$ *and* $X \leq \overline{X}(\gamma) = \frac{[(4-\theta)(1-\delta+\delta\gamma)+3\delta\phi]\delta R}{(1-\delta+\delta\gamma)[4(1-\delta+\delta\gamma)+3\delta\phi]}$. *There are thresholds* $\overline{\theta}^{PC}_{low} < \overline{\theta}^{PC}_{high}$, $\overline{\theta}^{TC}$, *and* $\overline{X}_{low}(\gamma) < \overline{X}_{high}(\gamma)$ *such that:*

1. *Suppose that* $\overline{\theta}^{PC}_{low} < \theta \leq \overline{\theta}^{PC}_{high}$ *and* $X \geq \overline{X}_{high}(\gamma)$. *Then permanent complete cooperation, i.e., disarmed peace with* $G(Z) = 0$ *for* $Z = 0, 1$, *can be sustained as an equilibrium.*

2. *Suppose that* $\overline{\theta}^{PC}_{low} < \theta \leq \overline{\theta}^{PC}_{high}$ *and* $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$. *Let* $\overline{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^2 \theta R < G^N$, $\overline{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 R^E(\gamma) < G^E(\gamma)$, *and* $\hat{G}(1) \in \left(0, \overline{G}(1)\right)$ *be the unique solution to* $F^{PC,1}_{\gamma}(0, G(1)) = 0$.

   (a) *If* $\hat{G}(1) \leq F^{PC,0}_{\gamma}(0,0)/(\delta\phi)$, *then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with* $G(0) = 0$ *and* $G(1) = \hat{G}(1)$.

   (b) *If* $\hat{G}(1) > F^{PC,0}_{\gamma}(0,0)/\delta\phi$, *then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with* $G(0) \in \left(0, \overline{G}(0)\right)$ *and* $G(1) \in \left(\hat{G}(1), \overline{G}(1)\right)$ *given by the unique solution to* $F^{PC,0}_{\gamma}(G(0), G(1)) = F^{PC,1}_{\gamma}(G(0), G(1)) = 0$ *that satisfies:*

$$\frac{\partial F^{PC,0}_{\gamma}(G(0), 0)}{\partial G(0)} \frac{\partial F^{PC,1}_{\gamma}(0, G(1))}{\partial G(1)} > \delta^2 (1-\phi)\phi.$$

3. *If* $\theta \leq \overline{\theta}^{TC}$, *then the best temporary cooperative equilibrium that can be sustained is* $G(0) = 0$.

4. *If $\theta > \overline{\theta}^{TC}$, then the best temporary cooperative equilibrium that can be sustained is $G(0) > 0$ given by the unique solution to $F_{\gamma}^{TC}(G(0)) = 0$. Moreover, $G(0) < G^N = \frac{\theta R}{4}$.*

Proof. See the Appendix A.4. □

As previously mentioned, Proposition 8 is a generalization of Proposition 5 for any γ. Crucially, when $\gamma > 0$ it is still the case that for some parameter values it is not possible to sustain permanent cooperation, while it is always possible to sustain temporary cooperation. In other words, sometimes open conflict is unavoidable, but countries can always benefit from temporary cooperation, at least while they have not yet been able to use the elimination technology.[25] More precisely, we have the following corollary.

Corollary 2. *Suppose that $\gamma > 0$. Open conflict is unavoidable if and only if*

$$X \leq \begin{cases} \overline{X}(\gamma) & \text{when} \quad \theta > \overline{\theta}_{high}^{PC}, \\ \overline{X}_{low}(\gamma) & \text{when} \quad \overline{\theta}_{low}^{PC} < \theta \leq \overline{\theta}_{high}^{PC}. \end{cases}$$

Moreover:

1. *The lower the cost of the elimination technology (X lower), the less likely it is that open conflict can be avoided.*

2. *The destructiveness of open conflict has a non-monotonic effect on the likelihood of open conflict. Initially, as destructiveness declines (higher θ), open conflict becomes more likely, but eventually further reductions in destructiveness make open conflict easier to avoid.*

3. *If $\theta > \overline{\theta}_{high}^{PC}$, an increase in γ makes open conflict less likely.*

Proposition 8 does not compare permanent versus temporary cooperative equilibria. There are three situations to consider. When $\theta \leq \overline{\theta}_{low}^{PC}$, or $\overline{\theta}_{low}^{PC} < \theta \leq \overline{\theta}_{high}^{PC}$ and $X \geq \overline{X}_{high}(\gamma)$, disarmed peace can be sustained (Proposition 8.1), and this is clearly the best possible cooperative equilibrium. When $\overline{\theta}_{low}^{PC} < \theta \leq \overline{\theta}_{high}^{PC}$ and $X \leq \overline{X}_{low}(\gamma)$ or $\theta > \overline{\theta}_{high}^{PC}$, neither complete nor partial permanent cooperation can be sustained and, therefore, the best possible equilibrium is the temporary cooperative equilibrium described in Propositions 8.3 and 8.4. Finally, when

[25] Of course, while for $\gamma = 0$, the region of the parameter space for which disarmed peace can be sustained never covers the case $\delta \to 1$, for $\gamma > 0$ it always includes it.

$\overline{\theta}^{PC}_{low} < \theta \leq \overline{\theta}^{PC}_{high}$ and $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$, only partial but not complete permanent cooperation can be sustained (Proposition 8.2). Temporary cooperation can always be sustained (Propositions 8.3 and 8.4). The following proposition characterizes this comparison.

Proposition 9. *Suppose that $\overline{\theta}^{PC}_{low} < \theta \leq \overline{\theta}^{PC}_{high}$ and $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$. Then partial permanent cooperation induces a higher ex-ante expected payoff than temporary cooperation if and only if*

$$\Delta = \begin{bmatrix} F^{PC,0}_{\gamma}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi) G^{PC}(0) \\ -\dfrac{1-\delta+\delta\gamma}{1-\delta(1-\gamma-\phi)} \left[F^{TC}_{\gamma}(0) - \delta(1-\phi) G^{TC}(0)\right] \end{bmatrix} \geq 0,$$

where

$$F^{PC,0}_{\gamma}(0,0) = \frac{R}{2} - (1-\delta)\theta R - \frac{\delta(1-\delta+\delta\gamma)\left[\phi R^{E}(\gamma) + (1-\phi)\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]},$$

$$F^{TC}_{\gamma}(0) = \left[\frac{1-\delta(1-\gamma)-\phi\delta^{2}\gamma}{1-\delta+\delta\gamma}\right]\frac{R}{2} - \left[\frac{(1-\delta+\delta\gamma)\delta(1-\phi) + 4(1-\delta)\left[1-\delta(1-\phi-\gamma)\right]}{4(1-\delta+\delta\gamma)}\right]\theta R.$$

Here $G^{PC}(0)$ and $G^{PC}(1)$ are given by Proposition 8.2 and $G^{TC}(0)$ by Propositions 8.3 and 8.4. Moreover, if $\theta > \overline{\theta}^{TC}$ and $\hat{G}(1) \leq F^{PC,0}_{\gamma}(0,0)/(\delta\phi)$, then $\Delta \geq 0$, while if $\theta \leq \overline{\theta}^{TC}$ and $\hat{G}(1) > F^{PC,0}_{\gamma}(0,0)/(\delta\phi)$, then $\Delta < 0$.

Proof. See the Appendix A.4. □

Proposition 9 is a generalization of Proposition 6 for any γ. In particular, even when $\gamma > 0$ (which reduces the incentives to use the elimination technology) and permanent cooperation can be sustained, countries might obtain a higher expected payoff by engaging in open conflict and only restricting cooperation to states with $Z = 0$.

To summarize, once we introduce the possibility that a country can only temporarily eliminate its rival, the model can generate several instances of open conflict without compromising any of the key results in Section 4. There are instances in which open conflict is unavoidable. When avoidable, countries may need to modify arming choices to discourage deviations from the cooperative equilibrium. Finally, even when diplomacy is fully effective and countries coordinate on the best possible cooperative equilibrium, and it is possible to avert open conflict, countries may choose to periodically engage in open conflict and restrict cooperation to states in which the elimination technology is not available.

6 Conclusions

Departing from a standard static security dilemma game, we develop an infinite-horizon stochastic game that focuses on a technology that can eliminate adversaries at some cost. Depending on the parameters, the equilibrium can sustain more cooperative outcomes than the static Nash equilibrium. Paradoxically, for some parameters $\delta \to 1$ makes it impossible to avoid costly arming and, more importantly, open conflict.

Since international relations—particularly through diplomacy—are often viewed as a setting where the folk theorem should apply, one might expect that sufficiently patient actors could sustain optimal outcomes. However, this logic leads to an overly optimistic and historically unrealistic prediction: that eternal unarmed peace can be supported as an equilibrium whenever players are patient enough. To obtain predictions more consistent with observed patterns of peace and conflict, we incorporate a randomly available elimination technology. This modification generates equilibria that can exhibit alternating periods of unarmed peace, armed peace with settlement, and open conflict. Importantly, although the elimination technology reduces the scope for permanent cooperation, opportunities for cooperation never disappear entirely: even when eventual conflict is unavoidable, countries can still benefit from temporary cooperation while elimination is not yet available.

Although there remains scope to enrich how interactions unfold during non-contestable periods and how the game transitions back to contestable states, those extensions would not add much to the resulting equilibria. We have assumed that during non-contestable states, the victor exercises full control and obtains R at no cost, while the transition back to a contestable state occurs randomly. Under these assumptions, neither the victor nor the loser can influence payoffs during non-contestable periods or the chance that the loser will return. Relaxing the first assumption is straightforward. For example, the loser may face a higher cost of arming, which would reduce its share of the resource. This modification would complicate the computation of expected payoffs in non-contestable states but would not alter the qualitative logic of the model. Relaxing the second assumption is more involved, but one natural extension is to allow the victor and/or the loser to influence the probability of return—that is, to make γ endogenous through investments in arming. Provided these investments do not eliminate the temptation to use the elimination technology, the main comparative statics and equilibrium logic remain intact. In short, these extensions would not materially alter our main results and would introduce an additional, and arguably realistic, channel: arming decisions during non-contestable states.

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Online Appendix: “The Elusive Quest for Disarmed Peace: Contest Games and International Relations”

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Abstract

Appendix A presents the proofs of all lemmas and propositions in “The Elusive Quest for Disarmed Peace: Contest Games and International Relations”. Appendix B develops an extension with stochastic resource values, which allows us to distinguish between nonaggressive and aggressive escalations.

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A Proofs

A.1 One-shot Game

This Appendix presents the proof of Proposition 1.

Proposition 1. *Assume that $\delta = 0$.*

1. *Under the non-cooperative norm (i.e., players coordinate on $S_1S_2 = 0$ for all (G_1, G_2)), the equilibrium level of arming is $G^{NN} = \theta R/4$ and the associated payoff is $V^{NN} = \theta R/4$.*
2. *Under the cooperative norm (i.e., players coordinate on $S_1S_2 = 1$ for all (G_1, G_2)), the equilibrium level of arming is $G^{CN} = R/4$ and the associated payoff is $V^{CN} = R/4$.*
3. *Under the conditional cooperative norm:*

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(a) If $\theta \leq 1/2$, then the most cooperative equilibrium level of arming is $G^{FB} = 0$ and the associated payoff is $V^{FB} = R/2$.

(b) If $\theta > 1/2$, then the most cooperative equilibrium level of arming is $G^{SB} \in (0, \theta R/4]$, where G^{SB} is the unique solution to

$$F(G) = -2G + 2\sqrt{G\theta R} + \left(\frac{1-2\theta}{2}\right) R = 0,$$

and the associated payoff is $V^{SB} = R/2 - G^{SB}$.

Proof. For

$$u_i(G_1, G_2, S_1, S_2) = \pi_i(G_1, G_2)\,[S_1 S_2 R + (1 - S_1 S_2)\theta R] - G_i,$$

we analyze the two stages.

Settlement stage. Fix (G_1, G_2).

If $\theta = 1$, settlement is irrelevant, since

$$u_i(G_1, G_2, S_1, S_2) = \pi_i(G_1, G_2) R - G_i \quad \forall (S_1, S_2).$$

Assume $\theta < 1$. Consider each candidate equilibrium:

(i) $S_1 = S_2 = 0$. A unilateral deviation to $S_i = 1$ leaves $S_1 S_2 = 0$, so payoffs are unchanged. Hence it is a Nash equilibrium.

(ii) $S_1 = S_2 = 1$. If player i deviates to $S_i = 0$, then $S_1 S_2 = 0$ and the payoff becomes $\pi_i(G_1, G_2)\theta R - G_i$, which is strictly lower than $\pi_i(G_1, G_2) R - G_i$ since $\theta < 1$. Thus $S_1 = S_2 = 1$ is also a Nash equilibrium.

(iii) $S_i = 0$, $S_{-i} = 1$. If $-i$ deviates to $S_{-i} = 0$, the outcome moves from settlement to no settlement, and

$$\pi_{-i}(G_1, G_2) R - G_{-i} > \pi_{-i}(G_1, G_2)\theta R - G_{-i},$$

so, this cannot be an equilibrium.

Thus, there are exactly two Nash equilibria: full settlement and no settlement.

Guns stage. Depending on the cooperation norm, there are three possible cases to consider:

Case 1: Non-cooperative norm: Suppose that $S_1 = S_2 = 0$ for all (G_1, G_2). Then, each player maximizes:

$$\max_{G_i \geq 0} \frac{G_i}{G_i + G_{-i}} \theta R - G_i.$$

Thus, the unique Nash equilibrium profile of guns is $G_1 = G_2 = G^{NN} = \theta R/4$. The associated equilibrium payoff is $V^{NN} = R/4$.

Case 2: Cooperative norm: Suppose that $S_1 = S_2 = 1$ for all (G_1, G_2). Then, each player maximizes:

$$\max_{G_i \geq 0} \frac{G_i}{G_i + G_{-i}} R - G_i,$$

Thus, the unique Nash equilibrium profile of guns is $G_1 = G_2 = G^{CN} = R/4$. The associated equilibrium payoff is $V^{CN} = R/4$.

Case 3: Conditional cooperation. Suppose the cooperative norm prescribes settlement ($S_1S_2 = 1$) whenever $(G_1, G_2) \in [0, G] \times [0, G]$, and no settlement otherwise. If both choose $G_1 = G_2 = G$, then the on-path payoff is

$$u_i^C = \frac{R}{2} - G.$$

If player i deviates upward to $G_i > G$, the outcome becomes no settlement. The optimal deviation solves:

$$\max_{G_i > G} \left\{ \frac{G_i}{G_i + G} \theta R - G_i \right\}.$$

This gives the deviation choice:

$$g^D(G, \theta R) = \sqrt{G\theta R} - G,$$

and deviation payoff

$$u_i^D = \frac{g^D(G, \theta R)}{g^D(G, \theta R) + G} \theta R - g^D(G, \theta R).$$

Deviation is unprofitable if and only if $u_i^C \geq u_i^D$ or, equivalently

$$F(G) := -2G + 2\sqrt{G\theta R} + \left(\frac{1 - 2\theta}{2} \right) R \geq 0.$$

To obtain the most cooperative equilibrium, choose G to maximize $R/2 - G$ subject to $F(G) \geq 0$. There are two cases to consider:

Case 3.a: Suppose that $\theta \leq 1/2$. Then, $F(0) \geq 0$ and, hence, the solution to the above optimization problem is $G^{FB} = 0$.

Case 3.b: Suppose that $\theta > 1/2$. Then, $F(0) < 0$, $\lim_{G \to \theta R/4} F(G) = (1 - \theta) R/2 > 0$ and $F'(G) = -2 + \sqrt{\theta R/G} > 0$. Thus, there is a unique $G^{SB} \in [0, \theta R/4)$ such that $F(0) < 0$ for all $G \in (0, G^{SB})$, $F(G^{SB}) = 0$, and $F(G) > 0$ for all $G \in (G^{SB}, \theta R/4)$. Hence, G^{SB} is the least arming level consistent with cooperation and the unique solution to max $R/2 - G$ subject to $F(G) \geq 0$.

This completes the proof. ■

A.2 Repeated Game

This section presents the proof of Proposition 2.

Proposition 2. *Assume that $\delta \geq 0$.*

1. *If $\theta \leq \frac{2}{4-3\delta}$, then the first-best payoff $V^{FB} = \frac{R}{2}$, characterized by unarmed peace $G^{FB} = 0$, can be attained as an equilibrium of the repeated game.*

2. *If $\theta > \frac{2}{4-3\delta}$, then the second-best payoff $V^{SB} = \frac{R}{2} - G^{SB} > \frac{R}{4}$ with military power $G^{SB} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ is sustainable as an equilibrium of the repeated game. Moreover, G^{SB} is the unique solution to*

$$F(G) = -(2 - \delta)G + 2(1 - \delta)\sqrt{G\theta R} + \left[\frac{2 - (4 - 3\delta)\theta}{4} \right] R = 0$$

and $\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R < \frac{\theta R}{4}$.

Proof: Consider the following cooperation strategy. Players choose $G_1 = G_2 = G \in \left[0, \frac{\theta R}{4}\right)$ and $S_1 = S_2 = 1$, provided that no deviation has occurred. If a player deviates from this, starting in the next period, players revert to the equilibrium described in Proposition 1.1 forever. Moreover, a deviation from $G_i = G$ is immediately punished with $S_1 = S_2 = 0$. If both players use this strategy, the payoff of player i is

$$V_i^C = \frac{R}{2} - G.$$

If player i deviates, its optimal deviation is given by

$$\arg \max_{G_i \in \left[0, \frac{\theta R}{4}\right]} \left\{(1-\delta)\left[\pi_i(G_i, G)\theta R - G_i\right] + \delta V^N\right\},$$

where $V^{NN} = \theta R/4$. Solving, we obtain that the most profitable deviation for player i is

$$g^D(G, \theta R) = \sqrt{G\theta R} - G.$$

Thus, the associated deviation payoff is

$$V_i^D = (1-\delta)\left[\frac{g^D(G,\theta R)}{g^D(G,\theta R)+G}\theta R - g^D(G,\theta R)\right] + \delta\frac{\theta R}{4}.$$

Player i does not have an incentive to deviate if and only if $V_i^C \geq V_i^D$, or, equivalently,

$$F(G) = -(2-\delta)G + 2(1-\delta)\sqrt{G\theta R} + \left[\frac{2-(4-3\delta)\theta}{4}\right] R \geq 0.$$

To obtain the most cooperative equilibrium, we solve

$$\max_{G \in \left[0, \frac{\theta R}{4}\right)} \left\{V_i^C = \frac{R}{2} - G\right\}$$
$$\text{s.t. } F(G) = -(2-\delta)G + 2(1-\delta)\sqrt{G\theta R} + \left[\frac{2-(4-3\delta)\theta}{4}\right] R \geq 0.$$

There are two cases to consider:

Case 1: Suppose that $\theta \leq \frac{2}{4-3\delta}$. Then $F(0) \geq 0$ and, hence, the solution to the above optimization problem is $G^{FB} = 0$.

Case 2: Suppose that $\theta > \frac{2}{4-3\delta}$. Then $F(0) = \frac{[2-(4-3\delta)\theta]R}{4} < 0$ and $F\left(\frac{\theta R}{4}\right) = \frac{(1-\theta)R}{2} > 0$. Moreover,

$$\frac{\partial F(G)}{\partial G} = -(2-\delta) + (1-\delta)\sqrt{\frac{\theta R}{G}},$$

which implies that $\frac{\partial F(G)}{\partial G} > 0$ if $G < \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R$, $\frac{\partial F}{\partial G}\left(\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right) = 0$, and $\frac{\partial F(G)}{\partial G} < 0$ if $G > \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R$. Finally, note that $\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R < \frac{\theta R}{4}$. Thus, there is a unique $G^{SB} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ such that $F(0) < 0$ for all $G \in (0, G^{SB})$, $F(G^{SB}) = 0$, and $F(G) > 0$ for all $G \in (G^{SB}, \theta R/4)$. Therefore, the unique solution to the above optimization problem is G^{SB}.

This completes the proof of Proposition 2. ■

A.3 Permanent Elimination

This section presents the proofs of Lemmas 1 and 2, Propositions 3, 4, 5, and 6 and Corollary 1.

Lemma 1. *Under the non-cooperative norm,*[1] *let* $\overline{X} = \frac{[(4-\theta)(1-\delta)+3\phi\delta]\delta R}{(1-\delta)[4(1-\delta)+3\phi\delta]}$ *and* $R^E = \theta R - X + \frac{\delta R}{1-\delta}$.

1. *Suppose that* $X > \overline{X}$. *Then, the worst equilibrium is* $G^{NN} = \frac{\theta R}{4}$ *for* $Z = 0$ *and* $G^{NN} = \frac{\theta R}{4}$ *with no elimination for* $Z = 1$. *The associated punishment expected payoff is* $V^{PU} = V^{NN} = \frac{\theta R}{4}$.

2. *Suppose that* $X \leq \overline{X}$. *Then, the worst equilibrium is* $G^{NN} = \frac{\theta R}{4}$ *for* $Z = 0$ *and* $G^E = \frac{R^E}{4}$ *with elimination of the loser for* $Z = 1$. *The associated punishment expected payoff is*

$$V^{PU} = V^E = \frac{(1-\delta)\big(\phi R^E + (1-\phi)\theta R\big)}{4(1-\delta+\phi\delta)}.$$

Proof: Suppose that both players always select $S_1 = S_2 = 0$. If $S_{-i} = 0$, there will be open conflict regardless of S_i. Thus, in the settlement stage there is no unilateral profitable deviation from $S_1 = S_2 = 0$. Then, for $Z = 0$ we have

$$V_i^{PU}(0) = \max_{G_i^{PU}(0)>0} \left\{(1-\delta)\left[\pi_i(0)\,\theta R - G_i^{PU}(0)\right] + \delta V_i^{PU}\right\},$$

where $\pi_i(0) = \dfrac{G_i^{PU}(0)}{G_1^{PU}(0)+G_2^{PU}(0)}$. For $Z = 1$ we have

$$V_i^{PU}(1) = \max_{G_i^{PU}(1)>0} \left\{\pi_i(1)\left[(1-\delta)\theta R + V_i^{PU}(1,w)\right] + \left[1-\pi_i(1)\right]V_i^{PU}(1,l) - (1-\delta)G_i^{PU}(1)\right\},$$

$$V_i^{PU}(1,w) = \max_{W_i\in\{0,1\}} \left\{W_i\left[-(1-\delta)X + \delta R\right] + (1-W_i)\,\delta V_i^{PU}\right\},$$

$$V_i^{PU}(1,l) = (1-W_{-i})\,\delta V_i^{PU},$$

where $\pi_i(1) = \dfrac{G_i^{PU}(1)}{G_1^{PU}(1)+G_2^{PU}(1)}$ and V_i^{PU} is the expected value of the game, that is,

$$V_i^{PU} = \phi V_i^{PU}(1) + (1-\phi)V_i^{PU}(0).$$

We look for a symmetric equilibrium in which $V_i^{PU} = V^{PU}$ for both players. Thus, in equilibrium, there are only two possible situations to consider: either $-(1-\delta)X + \delta R \geq \delta V^{PU}$ and, hence, $W_i = 1$ for $i = 1, 2$, or $-(1-\delta)X + \delta R < \delta V^{PU}$ and, hence, $W_i = 0$ for $i = 1, 2$.

Equilibrium with elimination: Assume that, in equilibrium, $-(1-\delta)X + \delta R \geq \delta V^{PU}$. Then, we have

$$V_i^{PU}(1) = \max_{G_i^{PU}(1)>0} \left\{(1-\delta)\big(\pi_i(1)R^E - G_i^{PU}(1)\big)\right\},$$

$$V_i^{PU}(0) = \max_{G_i^{PU}(0)>0} \left\{(1-\delta)\left[\pi_i(0)\,\theta R - G_i^{PU}(0)\right] + \delta V^{PU}\right\},$$

$$V^{PU} = \phi V_i^{PU}(1) + (1-\phi)V_i^{PU}(0),$$

[1] That is, assume that in every contestable state, $S_1 S_2 = 0$ for all (G_1, G_2) and Z.

where $R^E = \theta R - X + \dfrac{\delta R}{1-\delta}$.

For $Z = 1$, the unique Nash equilibrium gun profile is $G_1^{PU}(1) = G_2^{PU}(1) = G^E = R^E/4$, which implies $V_i^{PU}(1) = \frac{(1-\delta)R^E}{4}$.

For $Z = 0$, the unique Nash equilibrium gun profile is $G_1^{PU}(0) = G_2^{PU}(0) = G^{NN} = \theta R/4$, which implies $V_i^{PU}(0) = \frac{(1-\delta)\left(\theta R+\delta\phi R^E\right)}{4\left[1-\delta(1-\phi)\right]}$.

Therefore,

$$V^{PU} = V^E = \frac{(1-\delta)\big(\phi R^E + (1-\phi)\theta R\big)}{4(1-\delta+\phi\delta)}.$$

Finally, we must verify that $-(1-\delta)X + \delta R \geq \delta V^{PU}$, which holds if and only if

$$X \leq \frac{\big[(4-\theta)(1-\delta) + 3\phi\delta\big]\delta R}{(1-\delta)\big[4(1-\delta)+3\phi\delta\big]}.$$

Moreover, note that $-(1-\delta)X + \delta R \geq \delta V^{PU}$ implies that $R^E = \theta R - X + \frac{\delta R}{1-\delta} > 0$, and hence, $G^E = R^E/4 > 0$.

Equilibrium without elimination: Assume that, in equilibrium, $-(1-\delta)X + \delta R \leq \delta V^{PU}$. Then, we have

$$V_i^{PU}(Z) = \max_{G_i^{PU}(Z)>0} \left\{(1-\delta)\big[\pi_i(Z)\,\theta R - G_i^{PU}(Z)\big] + \delta V^{PU}\right\}, \quad Z = 0, 1,$$
$$V^{PU} = \phi V_i^{PU}(1) + (1-\phi)V_i^{PU}(0).$$

Solving, we obtain that the unique Nash equilibrium gun profile is $G_1^{PU}(Z) = G_2^{PU}(Z) = G^{NN} = \theta R/4$ for $Z = 0, 1$, which implies

$$V_i^{PU}(0) = V_i^{PU}(1) = V^{PU} = \frac{\theta R}{4}.$$

Finally, we must verify that $-(1-\delta)X + \delta R \leq \delta V^{PU}$, which holds if and only if

$$X \geq \frac{(4-\theta)\delta R}{4(1-\delta)}.$$

Note that

$$\overline{X} = \frac{\big[(4-\theta)(1-\delta) + 3\phi\delta\big]\delta R}{(1-\delta)\big[4(1-\delta)+3\phi\delta\big]} > \frac{(4-\theta)\delta R}{4(1-\delta)}.$$

Thus, we have three possible cases to consider:

Case 1: If $X < \frac{(4-\theta)\delta R}{4(1-\delta)}$, then the only equilibrium is $G_1^{PU}(0) = G_2^{PU}(0) = \frac{\theta R}{4}$ for $Z = 0$ and $G_1^{PU}(1) = G_2^{PU}(1) = \frac{R^E}{4}$ and $W_1 = W_2 = 1$ for $Z = 1$. Thus, the equilibrium discounted expected payoff is $V^{PU} = V^E$.

Case 2: If $X > \overline{X}$, then the only equilibrium is $G_1^{PU}(0) = G_2^{PU}(0) = \frac{\theta R}{4}$ for $Z = 0$ and $G_1^{PU}(1) = G_2^{PU}(1) = \frac{\theta R}{4}$ and $W_1 = W_2 = 0$ for $Z = 1$. Thus, the equilibrium discounted expected payoff is $V^{PU} = V^{NN} = \frac{\theta R}{4}$.

Case 3: If $\frac{(4-\theta)\delta R}{4(1-\delta)} \leq X \leq \overline{X}$, then there are two possible equilibria, described in cases 1 and 2, respectively. We have $V^E < V^{NN}$ if and only if $X > \frac{\delta(1-\theta)R}{1-\delta}$. Moreover,

$$\frac{\delta(4-\theta)R}{4(1-\delta)} > \frac{\delta(1-\theta)R}{1-\delta},$$

which implies that $V^E < V^{NN}$ for all $\dfrac{(4-\theta)\delta R}{4(1-\delta)} \leq X \leq \overline{X}$. Thus, whenever both equilibria exist, the equilibrium described in case 1 is more severe.

This completes the proof of Lemma 1. ■

Lemma 2. *Suppose that countries are following either the permanent cooperation strategy with* $G(0) \in \left[0, \frac{\theta R}{4}\right)$ *and* $G(1) \in \left[0, \frac{\theta R}{4}\right)$ *or the temporary cooperation strategy with* $G(0) \in \left[0, \frac{\theta R}{4}\right)$.

1. *Assume that* $Z = 0$. *Then, the most profitable deviation is*

$$g^D\big(G(0), \theta R\big) = \sqrt{G(0)\,\theta R} - G(0).$$

The associated deviation payoff is

$$V^D\big(G(0)\big) = (1-\delta)\left[\frac{g^D\big(G(0),\theta R\big)}{g^D\big(G(0),\theta R\big) + G(0)}\,\theta R - g^D\big(G(0),\theta R\big)\right] + \delta V^{PU},$$

where V^{PU} *is given by Lemma 1.*

2. *Assume that* $Z = 1$.

(a) *Suppose that* $X > \overline{X}$. *Then, the most profitable deviation is*

$$g^D\big(G(1), \theta R\big) = \sqrt{G(1)\,\theta R} - G(1),$$

inducing countries to fight an open conflict, after which the victor will not eliminate the loser. The associated deviation payoff is

$$V^D\big(G(1)\big) = (1-\delta)\left[\frac{g^D\big(G(1),\theta R\big)}{g^D\big(G(1),\theta R\big) + G(1)}\,\theta R - g^D\big(G(1),\theta R\big)\right] + \frac{\delta\theta R}{4(1-\delta)}.$$

(b) *Suppose that* $X \leq \overline{X}$. *Then, the most profitable deviation is*

$$g^D\big(G(1), R^E\big) = \sqrt{G(1)\,R^E} - G(1),$$

inducing countries to fight an open conflict, after which the victor will eliminate the loser. The associated deviation payoff is

$$V^D\big(G(1)\big) = (1-\delta)\left[\frac{g^D\big(G(1),R^E\big)}{g^D\big(G(1),R^E\big) + G(1)}\,R^E - g^D\big(G(1),R^E\big)\right].$$

Proof: We must consider possible deviations when $Z = 0$ and $Z = 1$ and for each state deviations in the settlement as well as the gun choice stages.

Suppose that $Z = 0$.

Deviation in the settlement stage: Suppose that $\big(G_1(1), G_2(1)\big)$ is given and $S_1 = S_2 = 1$. Consider a unilateral deviation to open conflict, i.e., $S_i = 0$. Then, the expected payoff of player i under this deviation is

$$V_i(0) = (1-\delta)\big(\pi_i(0)\,\theta R - G_i(0)\big) + \delta V^{PU},$$

where $\pi_i(0) = \dfrac{G_1(0)}{G_1(0)+G_2(0)}$. But if player i chooses $S_i = 1$, then it gets

$$V_i(0) = (1-\delta)\big(\pi_i(0) R - G_i(0)\big) + \delta V^{C},$$

where $V^C \geq V^{PU}$ is the expected payoff under cooperation. Therefore, for $Z = 0$, players do not have an incentive to unilaterally deviate to open conflict.

Deviation in the gun choice: Suppose that $\big(G_1(0), G_2(0)\big)$ is given. If i deviates, its expected payoff is

$$\max_{G_i} \left\{ V_i^D = (1-\delta)\left[\frac{G_i}{G_i + G_{-i}(0)}\,\theta R - G_i\right] + \delta V^{PU} \right\},$$

where V^{PU} is the punishment payoff. Note that, regardless of V^{PU}, the most profitable deviation for player i is

$$g^D\big(G_{-i}(0), \theta R\big) = \sqrt{G_{-i}(0)\,\theta R} - G_{-i}(0).$$

Thus, the associated deviation payoff is

$$V_i^D = (1-\delta)\left[\frac{g^D\big(G_{-i}(0), \theta R\big)}{g^D\big(G_{-i}(0), \theta R\big) + G_{-i}(0)}\,\theta R - g^D\big(G_{-i}(0), \theta R\big)\right] + \delta V^{PU}.$$

Suppose that $Z = 1$.

Deviation in the settlement stage: Suppose that $\big(G_1(1), G_2(1)\big)$ is given and $S_1 = S_2 = 1$. Consider a deviation to open conflict, i.e., $S_i = 0$. Then, the expected payoff of player i under this deviation is

$$V_i^D = \pi_i(1)\big[(1-\delta)\theta R + V_i(1,w)\big] + \big[1 - \pi_i(1)\big] V_i(1,l) - (1-\delta) G_i(1),$$
$$V_i(1,w) = \max_{W_i \in \{0,1\}} \big\{ W_i\big[-(1-\delta)X + \delta R\big] + (1 - W_i)\,\delta V^{PU} \big\},$$
$$V_i(1,l) = (1 - W_{-i})\,\delta V^{PU},$$

where $\pi_i(1) = \dfrac{G_i(1)}{G_1(1)+G_2(1)}$ and V^{PU} is the punishment payoff. From Lemma 1, there are two cases to consider.

Case 1: Suppose that $X > \overline{X}$. Then $V^{PU} = V^{NN} = \dfrac{\theta R}{4}$. Therefore, $-(1-\delta)X + \delta R < \delta V^{PU}$ if and only if $-(1-\delta)X + \delta R < \delta\,\frac{\theta R}{4}$, which holds if and only if $X > \frac{(4-\theta)\delta R}{4(1-\delta)}$. Since $\overline{X} > \frac{(4-\theta)\delta R}{4(1-\delta)}$, this inequality always holds. Thus, $W_i = W_{-i} = 0$ and hence

$$V_i^D = (1-\delta)\big[\pi_i(1)\,\theta R - G_i(1)\big] + \delta\frac{\theta R}{4}.$$

Case 2: Suppose that $X \leq \overline{X}$. Then $V^{PU} = V^E$. Therefore, $-(1-\delta)X + \delta R \geq \delta V^{PU}$ if and only if

$$-(1-\delta)X + \delta R \geq \delta\, \frac{(1-\delta)\theta R + \phi\big[-(1-\delta)X + \delta R\big]}{4(1-\delta+\phi\delta)},$$

which always holds for $X \leq \overline{X}$. Thus, $W_i = W_{-i} = 1$ and hence

$$V_i^D = (1-\delta)\left[\pi_i(1)\left(\theta R - X + \frac{\delta R}{1-\delta}\right) - G_i(1)\right].$$

Deviation in the gun choice: Since any deviation in guns will be immediately punished with open conflict, we must consider the two cases above.

Case 1: Suppose that $X > \overline{X}$. Then, the optimal deviation for player i is

$$\max_{G_i}\left\{V_i^D = (1-\delta)\left[\frac{G_i}{G_i + G_{-i}(1)}\,\theta R - G_i\right] + \delta\frac{\theta R}{4}\right\}.$$

Solving, we obtain that the most profitable deviation for player i is

$$g^D\big(G_{-i}(1), \theta R\big) = \sqrt{G_{-i}(1)\,\theta R} - G_{-i}(1).$$

Thus, the associated deviation payoff is

$$V^D\big(G_{-i}(1)\big) = (1-\delta)\left[\frac{g^D\big(G_{-i}(1),\theta R\big)}{g^D\big(G_{-i}(1),\theta R\big) + G_{-i}(1)}\,\theta R - g^D\big(G_{-i}(1),\theta R\big)\right] + \delta\frac{\theta R}{4}.$$

Case 2: Suppose that $X \leq \overline{X}$. Then, the optimal deviation for player i is

$$\max_{G_i}\left\{V_i^D = (1-\delta)\left[\frac{G_i}{G_i + G_{-i}(1)}\,R^E - G_i\right]\right\},$$

where $R^E = \theta R - X + \dfrac{\delta R}{1-\delta}$. Solving, we obtain that the most profitable deviation for player i is

$$g^D\big(G_{-i}(1), R^E\big) = \sqrt{G_{-i}(1)\,R^E} - G_{-i}(1),$$

and the associated deviation payoff is

$$V^D\big(G_{-i}(1)\big) = (1-\delta)\left[\frac{g^D\big(G_{-i}(1),R^E\big)}{g^D\big(G_{-i}(1),R^E\big) + G_{-i}(1)}\,R^E - g^D\big(G_{-i}(1),R^E\big)\right].$$

Selecting $G_1(Z) = G_2(Z)$ for $Z = 0, 1$, we finalize case 2. This completes the proof of Lemma 2. ■

Proposition 3. *Suppose that $\delta \to 1$. Then it is impossible to sustain a permanent cooperative equilibrium, but it is always possible to sustain a temporary cooperative equilibrium. Moreover, a temporary cooperative equilibrium is not Pareto efficient (regardless of whether $\delta < 1$ or $\delta \to 1$).*

Proof: In the proof of Proposition 5 we show that the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ can be sustained as an equilibrium if and only if $F^{PC,0}\big(G(0), G(1)\big) \geq 0$ and $F^{PC,1}\big(G(0), G(1)\big) \geq 0$. In particular, we need

$$F^{PC,1}\big(G(0), G(1)\big) = -\,(2 - 2\delta + \delta\phi)\, G(1) + 2(1-\delta)\sqrt{G(1)R^E} - \delta(1-\phi)\, G(0) + \frac{R}{2} - (1-\delta)R^E \geq 0,$$

where $R^E = \theta R - X + \dfrac{\delta R}{1-\delta}$. Note that

$$\lim_{\delta \to 1} F^{PC,1}\big(G(0), G(1)\big) = -\left[\frac{R}{2} + \phi G(1) + (1-\phi)G(0)\right] < 0.$$

Thus, when $\delta \to 1$, we have $F^{PC,1}\big(G(0), G(1)\big) < 0$ and hence it is not possible to sustain a permanent cooperative equilibrium.

In the proof of Proposition 5 we show that the temporary cooperation strategy with $G(0) \in \left[0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R\right]$ can be sustained if and only if $F^{TC}\big(G(0)\big) \geq 0$, where

$$F^{TC}\big(G(0)\big) = -(2 - \delta + \delta\phi)\, G(0) + 2(1 - \delta + \delta\phi)\sqrt{G(0)\,\theta R} + \left(\frac{2 - \theta(4 - 3\delta + 3\phi\delta)}{4}\right) R \geq 0.$$

Note that

$$\lim_{\delta \to 1} F^{TC}\big(G(0)\big) = -(1+\phi)G(0) + 2\phi\sqrt{G(0)\,\theta R} + \left(\frac{2 - \theta(1+3\phi)}{4}\right) R.$$

Hence

$$\lim_{\delta \to 1} F^{TC}(0) = \left(\frac{2 - \theta(1+3\phi)}{4}\right) R, \;\; \lim_{\delta \to 1} F^{TC}\left(\frac{\theta R}{4}\right) = \frac{(1-\theta)R}{2} > 0.$$

Thus, if $\theta \leq \dfrac{2}{1+3\phi}$, then $\lim_{\delta \to 1} F^{TC}(0) \geq 0$ and hence it is possible to sustain a temporary cooperative equilibrium with $G(0) = 0$. On the contrary, if $\theta > \dfrac{2}{1+3\phi}$, then $\lim_{\delta \to 1} F^{TC}(0) < 0$. Since $\lim_{\delta \to 1} F^{TC}\big(G(0)\big)$ is strictly increasing in $G(0)$ for all $G(0) \in \left[0, \frac{\theta R}{4}\right]$ and $\lim_{\delta \to 1} F^{TC}\left(\frac{\theta R}{4}\right) > 0$, there must exist $G(0) \in \left(0, \frac{\theta R}{4}\right)$ such that $\lim_{\delta \to 1} F^{TC}\big(G(0)\big) > 0$. Thus, a temporary cooperative equilibrium with $G(0) \in \left(0, \frac{\theta R}{4}\right)$ can be sustained.

This completes the proof of Proposition 3. ∎

Proposition 4. *Suppose that $X > \overline{X}$.*

1. *If $\theta \leq \frac{2}{4-3\delta}$, then permanent complete cooperation, i.e., disarmed peace with $G(Z) = 0$ for $Z = 0, 1$, can be sustained.*

2. *If $\theta > \frac{2}{4-3\delta}$, then the best possible cooperative equilibrium that can be sustained is permanent partial cooperation (i.e., armed peace) with $G^{PC} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ for $Z = 0, 1$, where G^{PC} is the unique solution to*

$$F^{PC,0}(G, G) = F^{PC,1}(G, G) = F\,(G) = -\,(2-\delta)\, G + 2\,(1-\delta)\,\sqrt{G\theta R} + \left[\frac{2 - (4-3\delta)\,\theta}{4}\right] R = 0$$

and $\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R < G^{NN} = \frac{\theta R}{4}$.

Proof: Suppose that countries are following the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Then, the expected payoffs for each state $Z = 0, 1$ are given by

$$V^{PC}(0) = (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta\left[\phi V^{PC}(1) + (1-\phi) V^{PC}(0)\right],$$
$$V^{PC}(1) = (1-\delta)\left(\frac{R}{2} - G(1)\right) + \delta\left[\phi V^{PC}(1) + (1-\phi) V^{PC}(0)\right].$$

Solving, we have

$$V^{PC}(0) = \frac{R}{2} - G(0) + \delta\phi\left[G(0) - G(1)\right],$$
$$V^{PC}(1) = \frac{R}{2} - G(1) + \delta(1-\phi)\left[G(1) - G(0)\right].$$

Sustainability for $Z = 0$. Suppose that $Z = 0$ and player i deviates from cooperation. From Lemma 2, the most profitable deviation for player i is $g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0)$ and the associated deviation payoff is given by

$$V^D(G(0)) = (1-\delta)\left[\frac{g^D(G(0), \theta R)}{g^D(G(0), \theta R) + G(0)}\theta R - g^D(G(0), \theta R)\right] + \delta V^{PU}.$$

From Lemma 1, $X > \overline{X}$ implies that $V^{PU} = V^N = \frac{\theta R}{4}$. Thus, player i does not have an incentive to deviate if and only if $V^{PC}(0) \geq V^D(G(0))$ or, equivalently,

$$F^{PC,0}(G(0), G(1)) = -(2-\delta-\delta\phi) G(0) + 2(1-\delta)\sqrt{G(0)\theta R} - \delta\phi G(1) + \frac{\left[2-(4-3\delta)\theta\right]R}{4} \geq 0.$$

Sustainability for $Z = 1$. Suppose that $Z = 1$ and player i deviates from cooperation. From Lemma 2, the most profitable deviation for player i is $g^D(G(1), \theta R) = \sqrt{G(1)\theta R} - G(1)$ and the associated deviation payoff is given by

$$V^D(G(1)) = (1-\delta)\left[\frac{g^D(G(1), \theta R)}{g^D(G(1), \theta R) + G(1)}\theta R - g^D(G(1), \theta R)\right] + \delta V^{PU}.$$

From Lemma 1, $X > \overline{X}$ implies that $V^{PU} = V^N = \frac{\theta R}{4}$. Thus, player i does not have an incentive to deviate if and only if $V^{PC}(1) \geq V^D(G(1))$ or, equivalently,

$$F^{PC,1}(G(0), G(1)) =$$
$$-(2-2\delta+\delta\phi) G(1) + 2(1-\delta)\sqrt{G(1)\theta R} - \delta(1-\phi) G(0) + \frac{\left[2-(4-3\delta)\theta\right]R}{4} \geq 0.$$

Best possible permanent cooperation equilibrium: To determine the best possible cooperative equilibrium, we solve

$$\max_{G(0)\geq 0,\, G(1)\geq 0} \left\{ \phi V^{PC}(1) + (1-\phi) V^{PC}(0) = \frac{R}{2} - \phi G(1) - (1-\phi) G(0) \right\}$$
$$\text{s.t. } F^{PC,0}(G(0),G(1)) \geq 0 \quad \text{and} \quad F^{PC,1}(G(0),G(1)) \geq 0.$$

We start by proving the following two results.

Result 1: If a solution exists, it must satisfy $G(0) \in \left[0, \overline{G}(0)\right]$ and $G(1) \in \left[0, \overline{G}(1)\right]$, where $\overline{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^2 \theta R < \frac{\theta R}{4}$ and $\overline{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 \theta R < \frac{\theta R}{4}$.

Proof: Take the derivative of $F^{PC,0}(G(0),G(1))$ with respect to $G(0)$ and $G(1)$:

$$\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} = -(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}, \qquad \frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(1)} = -\delta\phi.$$

Thus, $\lim_{G(0)\to 0} \frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} = \infty$, $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} > 0$ for all $G(0) \in \left(0, \overline{G}(0)\right)$, $\frac{\partial F^{PC,0}\left(\overline{G}(0),G(1)\right)}{\partial G(0)} = 0$, and $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} < 0$ for all $G(0) > \overline{G}(0)$. Thus, $F^{PC,0}(G(0),G(1))$ is strictly increasing in $G(0)$ for all $G(0) \in \left[0, \overline{G}(0)\right]$, and strictly decreasing in $G(0)$ for all $G(0) \geq \overline{G}(0)$. Since $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(1)} < 0$ for all $G(0)$, $F^{PC,0}(G(0),G(1))$ is strictly decreasing in $G(1)$.

Similarly, take the derivative of $F^{PC,1}(G(0),G(1))$ with respect to $G(0)$ and $G(1)$:

$$\frac{\partial F^{PC,1}(G(0),G(1))}{\partial G(0)} = -\delta(1-\phi), \qquad \frac{\partial F^{PC,1}(G(0),G(1))}{\partial G(1)} = -(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(1)}}.$$

Thus, it is easy to verify that $F^{PC,1}(G(0),G(1))$ is strictly increasing in $G(1)$ for all $G(1) \in \left[0, \overline{G}(1)\right]$, strictly decreasing in $G(1)$ for all $G(1) \geq \overline{G}(1)$, and strictly decreasing in $G(0)$.

Suppose that $(G(0), G(1))$ satisfies $F^{PC,0}(G(0),G(1)) \geq 0$ and $F^{PC,1}(G(0),G(1)) \geq 0$ and $G(0) > \overline{G}(0)$. Then, for any $G^*(0) \in \left[\overline{G}(0), G(0)\right)$ we also have $F^{PC,0}(G^*(0), G(1)) \geq 0$ and $F^{PC,1}(G^*(0), G(1)) \geq 0$. Thus, $(G(0), G(1))$ is not a solution. Similarly, suppose that $(G(0), G(1))$ satisfies $F^{PC,0}(G(0),G(1)) \geq 0$ and $F^{PC,1}(G(0),G(1)) \geq 0$ and $G(1) > \overline{G}(1)$. Then, for any $G^*(1) \in \left[\overline{G}(1), G(1)\right)$ we also have $F^{PC,0}(G(0), G^*(1)) \geq 0$ and $F^{PC,1}(G(0), G^*(1)) \geq 0$. Thus, $(G(0), G(1))$ is not a solution. Therefore, any solution must satisfy $G(0) \in \left[0, \overline{G}(0)\right]$ and $G(1) \in \left[0, \overline{G}(1)\right]$. ■

Result 2: $F^{PC,0}$ and $F^{PC,1}$ are quasiconcave functions on $\mathbb{R}^2_+$.

Proof: Note that $F^{PC,0}(G(0),G(1)) \geq c$ can be written as

$$F^{PC,0}(G(0),G(1)) = \frac{1}{\delta\phi}\left[F^{PC,0}(G(0),0) - c\right] - G(1) \geq 0,$$

where

$$F^{PC,0}(G(0),0) = -(2-\delta-\delta\phi) G(0) + 2(1-\delta)\sqrt{G(0)\theta R} + \frac{[2-(4-3\delta)\theta] R}{4}.$$

Note that $F^{PC,0}(G(0),0)$ is strictly concave in $G(0)$ for all $G(0)$, which implies that $\frac{1}{\delta\phi}\left[F^{PC,0}(G(0),0) - c\right]$ is also strictly concave in $G(0)$ for all $G(0)$ and c. Thus, $\{(G(0), G(1)) : G(1) \leq$

$\frac{1}{\delta\phi}\left[F^{PC,0}\left(G\left(0\right),0\right)-c\right]\}$ is a convex set for every c. Therefore, $\{(G\left(0\right),G\left(1\right)):F^{PC,0}\left(G(0),G(1)\right)\geq c\}$ is a convex set for all c, which implies that $F^{PC,0}\left(G(0),G(1)\right)$ is quasiconcave.

Similarly, $F^{PC,1}\left(G(0),G(1)\right)\geq c$ can be written as

$$F^{PC,1}\left(G(0),G(1)\right)=\frac{1}{\delta\left(1-\phi\right)}\left[F^{PC,1}\left(0,G(1)\right)-c\right]-G\left(0\right)\geq 0,$$

where

$$F^{PC,1}\left(0,G(1)\right)=-\left(2-2\delta+\delta\phi\right)G\left(1\right)+2\left(1-\delta\right)\sqrt{G\left(1\right)\theta R}+\frac{\left[2-\left(4-3\delta\right)\theta\right]R}{4}.$$

Note that $F^{PC,1}\left(0,G(1)\right)$ is strictly concave in $G\left(1\right)$ for all $G\left(1\right)$, which implies that $\frac{1}{\delta(1-\phi)}\left[F^{PC,1}\left(0,G(1)\right)-c\right]$ is also strictly concave in $G\left(1\right)$ for all $G\left(1\right)$ and c. Thus, $\{(G\left(0\right),G\left(1\right)):G\left(0\right)\leq\frac{1}{\delta(1-\phi)}\left[F^{PC,1}\left(0,G(1)\right)-c\right]\}$ is a convex set for every c. Therefore, $\{(G\left(0\right),G\left(1\right)):F^{PC,1}\left(G(0),G(1)\right)\geq c\}$ is a convex set for all c, which implies that $F^{PC,1}\left(G(0),G(1)\right)$ is quasiconcave. ■

Since the objective function is linear and the constraints are quasiconcave, the following Kuhn-Tucker conditions are sufficient for a global maximum:

$$-\left(1-\phi\right)+\lambda^{0}\left[-\left(2-\delta-\delta\phi\right)+\left(1-\delta\right)\sqrt{\frac{\theta R}{G\left(0\right)}}\right]-\lambda^{1}\delta\left(1-\phi\right)+\mu_{L}^{0}-\mu_{H}^{0}=0,$$

$$-\phi-\lambda^{0}\delta\phi+\lambda^{1}\left[-\left(2-2\delta+\delta\phi\right)+\left(1-\delta\right)\sqrt{\frac{\theta R}{G\left(1\right)}}\right]+\mu_{L}^{1}-\mu_{H}^{1}=0,$$

$$\lambda^{0}\geq 0,\ F^{PC,0}\left(G\left(0\right),G\left(1\right)\right)\geq 0,\ \lambda^{0}F^{PC,0}\left(G\left(0\right),G\left(1\right)\right)=0,$$
$$\lambda^{1}\geq 0,\ F^{PC,1}\left(G\left(0\right),G\left(1\right)\right)\geq 0,\ \lambda^{1}F^{PC,1}\left(G\left(0\right),G\left(1\right)\right)=0,$$
$$\mu_{L}^{0}\geq 0,\ G\left(0\right)\geq 0,\ \mu_{L}^{0}G\left(0\right)=0,$$
$$\mu_{H}^{0}\geq 0,\ \overline{G}\left(0\right)-G\left(0\right)\geq 0,\ \mu_{H}^{0}\left[\overline{G}\left(0\right)-G\left(0\right)\right]=0,$$
$$\mu_{L}^{1}\geq 0,\ G\left(1\right)\geq 0,\ \mu_{L}^{1}G\left(1\right)=0,$$
$$\mu_{H}^{1}\geq 0,\ \overline{G}\left(1\right)-G\left(1\right)\geq 0,\ \mu_{H}^{1}\left[\overline{G}\left(1\right)-G\left(1\right)\right]=0.$$

There are several cases to consider:

Case 1: Suppose that $G\left(0\right)=G\left(1\right)=0$. Solving, we obtain $\mu_{H}^{0}=\mu_{H}^{1}=0$, $\lambda^{0}=\lambda^{1}=0$, $\mu_{L}^{0}=\left(1-\phi\right)$, $\mu_{L}^{1}=\phi$, $F^{PC,0}\left(0,0\right)\geq 0$ and $F^{PC,1}\left(0,0\right)\geq 0$. Thus, we must check that $F^{PC,0}\left(0,0\right)=F^{PC,1}\left(0,0\right)=\frac{\left[2-\left(4-3\delta\right)\theta\right]R}{4}\geq 0$, which holds if and only if $\theta\leq\frac{2}{4-3\delta}$.

Case 2: Suppose that $G\left(0\right)\in\left(0,\overline{G}\left(0\right)\right)$ and $G\left(1\right)=0$. Solving, we obtain $\mu_{L}^{0}=\mu_{H}^{0}=\mu_{H}^{1}=0$, $\lambda^{1}=0$,

$$\lambda^{0}=\left(1-\phi\right)\left[-\left(2-\delta-\delta\phi\right)+\left(1-\delta\right)\sqrt{\frac{\theta R}{G\left(0\right)}}\right]^{-1}>0,$$

$\mu_{L}^{1}=\phi\left(1+\lambda^{0}\delta\right)>0$, $F^{PC,0}\left(G\left(0\right),0\right)=0$ and $F^{PC,1}\left(G\left(0\right),0\right)\geq 0$. Since $F^{PC,0}\left(G\left(0\right),0\right)$ is strictly increasing in $G\left(0\right)$ for all $G\left(0\right)\in\left[0,\overline{G}\left(0\right)\right]$, at most there is one solution to $F^{PC,0}\left(G\left(0\right),0\right)=0$. Moreover, there is a solution that satisfies $G\left(0\right)\in\left(0,\overline{G}\left(0\right)\right)$ if and only if $F^{PC,0}\left(0,0\right)<0$ and

$F^{PC,0}\left(\overline{G}(0),0\right)>0$. Note, however, that $F^{PC,0}(0,0)<0$ implies $F^{PC,1}(0,0)<0$, and $F^{PC,1}(0,0)<0$ implies $F^{PC,1}(G(0),0)<0$ because $F^{PC,1}(G(0),0)$ is strictly decreasing in $G(0)$. Thus, $F^{PC,0}(0,0)<0$ is incompatible with $F^{PC,1}(G(0),0)\geq 0$. Summing up, there is no solution such that $G(0)\in\left(0,\overline{G}(0)\right)$ and $G(1)=0$.

Case 3: Suppose that $G(0)=0$ and $G(1)\in\left(0,\overline{G}(1)\right)$. Solving, we obtain $\mu_H^0=\mu_L^1=\mu_H^1=0$, $\lambda^0=0$,

$$\mu_L^0=(1-\phi)+\lambda^1\delta(1-\phi)>0, \qquad \lambda^1=\phi\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(1)}}\right]^{-1}>0,$$

$F^{PC,1}(0,G(1))=0$ and $F^{PC,0}(0,G(1))\geq 0$. Since $F^{PC,1}(0,G(1))$ is strictly increasing in $G(1)$ for all $G(1)\in\left[0,\overline{G}(1)\right]$, at most there is one solution to $F^{PC,1}(0,G(1))=0$. Moreover, there is a solution that satisfies $G(1)\in\left(0,\overline{G}(1)\right)$ if and only if $F^{PC,1}(0,0)<0$ and $F^{PC,1}\left(0,\overline{G}(1)\right)>0$. Note, however, that $F^{PC,1}(0,0)<0$ implies $F^{PC,0}(0,0)<0$, and $F^{PC,0}(0,0)<0$ implies $F^{PC,0}(0,G(1))<0$ because $F^{PC,0}(0,G(1))$ is strictly decreasing in $G(1)$. Thus, $F^{PC,1}(0,0)<0$ is incompatible with $F^{PC,0}(0,G(1))\geq 0$. Summing up, there is no solution such that $G(0)=0$ and $G(1)\in\left(0,\overline{G}(1)\right)$.

Case 4: Suppose that $G(0)\in\left(0,\overline{G}(0)\right)$ and $G(1)\in\left(0,\overline{G}(1)\right)$. Solving, we obtain $\mu_L^0=\mu_L^1=\mu_H^0=\mu_H^1=0$, and

$$\lambda^0\left[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]-\lambda^1\delta(1-\phi)=(1-\phi),$$
$$-\lambda^0\delta\phi+\lambda^1\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(1)}}\right]=\phi.$$

The above system of linear equations has a solution if and only if

$$\left[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(1)}}\right]\neq\delta^2\phi(1-\phi).$$

Given the Kuhn–Tucker conditions $\lambda^0\geq 0$ and $\lambda^1\geq 0$, any interior solution must satisfy $\lambda^0>0$ and $\lambda^1>0$. In that case, the complementary slackness conditions imply $F^{PC,0}(G(0),G(1))=0$ and $F^{PC,1}(G(0),G(1))=0$, which further implies

$$G(1)-G(0)=\sqrt{\theta R}\left[\sqrt{G(1)}-\sqrt{G(0)}\right].$$

Thus, one possibility is that $G(1)=G(0)=G^{PC}$. In that case, we must find G^{PC} such that $F^{PC,0}\left(G^{PC},G^{PC}\right)=F^{PC,1}\left(G^{PC},G^{PC}\right)=F\left(G^{PC}\right)=0$, where

$$F(G)=-(2-\delta)G+2(1-\delta)\sqrt{G\theta R}+\frac{[2-(4-3\delta)\theta]R}{4}.$$

We have $F(0)=\frac{[2-(4-3\delta)\theta]R}{4}<0$ if and only if $\theta>\frac{2}{4-3\delta}$, and

$$\frac{\partial F(G)}{\partial G}=-(2-\delta)+(1-\delta)\sqrt{\frac{\theta R}{G}},$$

which implies that $\frac{\partial F(G)}{\partial G} > 0$ for $G < \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R$, $\frac{\partial F}{\partial G}\left(\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right) = 0$, and $\frac{\partial F(G)}{\partial G} < 0$ for $G > \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R$. Also note that $\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R < \overline{G}(Z) < \frac{\theta R}{4}$ for all Z. Finally, $F\left(\frac{\theta R}{4}\right) = \frac{(1-\theta)R}{2} > 0$, which implies that $F\left(\overline{G}(Z)\right) > 0$ and $F\left(\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right) > 0$. Therefore, there are two possible situations to consider. If $\theta \leq \frac{2}{4-3\delta}$, then $F(G) > 0$ for all $G \in \left(0, \frac{\theta R}{4}\right)$, and thus there is no solution such that $G(0) \in \left(0, \overline{G}(0)\right)$ and $G(1) \in \left(0, \overline{G}(1)\right)$. On the contrary, if $\theta > \frac{2}{4-3\delta}$, then there is a unique $G^{PC} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ such that $F(G) < 0$ for all $G \in \left(0, G^{PC}\right)$, $F\left(G^{PC}\right) = 0$, and $F(G) > 0$ for all $G \in \left(G^{PC}, \overline{G}(Z)\right)$. Finally, we must check that G^{PC} satisfies

$$\left[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G^{PC}}}\right]\left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G^{PC}}}\right] > \delta^2 \phi (1-\phi),$$

or, equivalently,

$$H\left(G^{PC}\right) = 2(2-\delta) G^{PC} - (4-3\delta)\sqrt{G^{PC}\theta R} + (1-\delta)\theta R > 0.$$

Since $H(G) > 0$ for all $G \in \left[0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right]$, $H\left(G^{PC}\right) > 0$ always holds.

Summing up, $G(0) = G(1) = G^{PC}$, where $G^{PC} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ is the unique solution to $F(G) = 0$, is a solution if and only if $\theta > \frac{2}{4-3\delta}$.

Case 5: Suppose that $G(0) = \overline{G}(0)$ or $G(1) = \overline{G}(1)$. If $G(0) = \overline{G}(0)$, we have $\mu_L^0 = 0$ and, hence, $(1-\phi) + \lambda^1 \delta (1-\phi) + \mu_H^0 = 0$, a contradiction because $\lambda^1 \geq 0$ and $\mu_H^0 \geq 0$. If $G(1) = \overline{G}(1)$, we have $\mu_L^1 = 0$ and, hence, $\phi + \lambda^0 \delta\phi + \mu_H^1 = 0$, a contradiction because $\lambda^0 \geq 0$ and $\mu_H^1 \geq 0$.

This completes the proof of Proposition 4. ■

Proposition 5. *Suppose that $X \leq \overline{X}$. Let*

$$\overline{X}_{low} = \theta R + \frac{\left[2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right] R}{2(1-\delta)} \quad \text{and} \quad \overline{X}_{high} = \theta R + \frac{(2\delta-1)}{2(1-\delta)} R.$$

1. *Suppose that $\theta \leq \frac{1-2\delta}{2(1-\delta)}$ or $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ and $X \geq \overline{X}_{high}$. Then, permanent complete cooperation, i.e., disarmed peace with $G(Z) = 0$ for $Z = 0, 1$, can be sustained.*

2. *Suppose that $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ and $\overline{X}_{low} < X < \overline{X}_{high}$. Let $\overline{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^2 \theta R < G^N$, $\overline{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 R^E < G^E$, and $\hat{G}(1) \in \left(0, \overline{G}(1)\right)$ be the unique solution to $F^{PC,1}(0, G(1)) = 0$.*

   (a) *If $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with $G(0) = 0$ and $G(1) = \hat{G}(1)$.*

(b) *If $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with $G(0) \in \left(0, \overline{G}(0)\right)$ and $G(1) \in \left(\hat{G}(1), \overline{G}(1)\right)$ given by the unique solution to*

$$F^{PC,0}(G(0), G(1)) = F^{PC,1}(G(0), G(1)) = 0$$

that satisfies

$$\frac{\partial F^{PC,0}(G(0), 0)}{\partial G(0)} \frac{\partial F^{PC,1}(0, G(1))}{\partial G(1)} > \delta^2 (1-\phi)\phi.$$

3. *If $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$, then the best temporary cooperative equilibrium that can be sustained is $G(0) = 0$.*

4. *If $\theta > \frac{2}{4-3\delta+3\phi\delta}$, then the best temporary cooperative equilibrium that can be sustained is $G(0) \in \left(0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R\right)$ given by the unique solution to $F^{TC}(G(0)) = 0$. Moreover, $\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R < G^{NN} = \frac{\theta R}{4}$.*

Proof Parts 1 and 2: Suppose that countries are following the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Then, the expected discounted payoffs for each state $Z = 0, 1$ are given by

$$V^{PC}(0) = \frac{R}{2} - G(0) + \delta\phi\left[G(0) - G(1)\right],$$
$$V^{PC}(1) = \frac{R}{2} - G(1) + \delta(1-\phi)\left[G(1) - G(0)\right].$$

Sustainability for $Z = 0$: Suppose that $Z = 0$ and player i deviates from cooperation. From Lemma 2, the most profitable deviation for player i is $g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0)$ and the associated deviation payoff is given by

$$V^D(G(0)) = (1-\delta)\left[\frac{g^D(G(0), \theta R)}{g^D(G(0), \theta R) + G(0)}\theta R - g^D(G(0), \theta R)\right] + \delta V^{PU}.$$

From Lemma 1, $X \leq \overline{X}$ implies that $V^{PU} = V^E$. Thus, player i does not have an incentive to deviate if and only if $V^{PC}(0) \geq V^D(G(0))$ or, equivalently,

$$F^{PC,0}(G(0), G(1)) = -(2-\delta-\delta\phi)G(0) + 2(1-\delta)\sqrt{G(0)\theta R} - \delta\phi G(1) + F^{PC,0}(0,0) \geq 0,$$

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\theta R - \frac{(1-\delta)\delta\left[\phi R^E + (1-\phi)\theta R\right]}{4(1-\delta+\phi\delta)}.$$

Sustainability for $Z = 1$: Suppose that $Z = 1$ and player i deviates from cooperation. From Lemma 2, the most profitable deviation for player i is $g^D\left(G(1), R^E\right) = \sqrt{G(1)R^E} - G(1)$, which leads to open conflict and wipe-out. The associated deviation payoff is given by

$$V^D(G(1)) = (1-\delta)\left[\frac{g^D\left(G(1), R^E\right)}{g^D\left(G(1), R^E\right) + G(1)}R^E - g^D\left(G(1), R^E\right)\right].$$

Thus, player i does not have an incentive to deviate if and only if $V^{PC}(1) \geq V^{D}(G(1))$ or, equivalently,

$$F^{PC,1}(G(0), G(1)) =$$
$$-(2-2\delta+\delta\phi) G(1) + 2(1-\delta)\sqrt{G(1) R^{E}} - \delta(1-\phi) G(0) + F^{PC,1}(0,0) \geq 0,$$

where

$$F^{PC,1}(0,0) = \frac{R}{2} - (1-\delta) R^{E}.$$

Best possible permanent cooperation equilibrium: To determine the best possible permanent cooperative equilibrium, we solve

$$\max_{G(0)\geq 0,\, G(1)\geq 0} \left\{ \phi V^{PC}(1) + (1-\phi) V^{PC}(0) = \frac{R}{2} - \phi G(1) - (1-\phi) G(0) \right\}$$
$$\text{s.t. } F^{PC,0}(G(0), G(1)) \geq 0 \quad \text{and} \quad F^{PC,1}(G(0), G(1)) \geq 0.$$

We start by proving the following three results.

Result 1: If a solution exists, it must satisfy $G(0) \in \left[0, \overline{G}(0)\right]$ and $G(1) \in \left[0, \overline{G}(1)\right]$, where $\overline{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^{2} \theta R < \frac{\theta R}{4}$ and $\overline{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^{2} R^{E} < \frac{R^{E}}{4}$.

Proof: Take the derivative of $F^{PC,0}(G(0), G(1))$ with respect to $G(0)$ and $G(1)$:

$$\frac{\partial F^{PC,0}(G(0), G(1))}{\partial G(0)} = -(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}, \qquad \frac{\partial F^{PC,0}(G(0), G(1))}{\partial G(1)} = -\delta\phi.$$

Thus, $\lim_{G(0)\to 0} \frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} = \infty$, $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} > 0$ for all $G(0) \in \left(0, \overline{G}(0)\right)$, $\frac{\partial F^{PC,0}\left(\overline{G}(0),G(1)\right)}{\partial G(0)} = 0$, and $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} < 0$ for all $G(0) > \overline{G}(0)$. Thus, $F^{PC,0}(G(0), G(1))$ is strictly increasing in $G(0)$ for all $G(0) \in \left[0, \overline{G}(0)\right]$, and strictly decreasing in $G(0)$ for all $G(0) \geq \overline{G}(0)$. Since $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(1)} < 0$ for all $G(0)$, $F^{PC,0}(G(0), G(1))$ is strictly decreasing in $G(1)$.

Similarly, take the derivative of $F^{PC,1}(G(0), G(1))$ with respect to $G(0)$ and $G(1)$:

$$\frac{\partial F^{PC,1}(G(0), G(1))}{\partial G(0)} = -\delta(1-\phi), \qquad \frac{\partial F^{PC,1}(G(0), G(1))}{\partial G(1)} = -(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{R^{E}}{G(1)}}.$$

Thus, it is easy to verify that $F^{PC,1}(G(0), G(1))$ is strictly increasing in $G(1)$ for all $G(1) \in \left[0, \overline{G}(1)\right]$, strictly decreasing in $G(1)$ for all $G(1) \geq \overline{G}(1)$, and strictly decreasing in $G(0)$.

Suppose that $(G(0), G(1))$ satisfies $F^{PC,0}(G(0), G(1)) \geq 0$ and $F^{PC,1}(G(0), G(1)) \geq 0$ and $G(0) > \overline{G}(0)$. Then, for any $G^{*}(0) \in \left[\overline{G}(0), G(0)\right)$ we also have $F^{PC,0}(G^{*}(0), G(1)) \geq 0$ and $F^{PC,1}(G^{*}(0), G(1)) \geq 0$. Thus, $(G(0), G(1))$ is not a solution. Similarly, suppose that $(G(0), G(1))$ satisfies $F^{PC,0}(G(0), G(1)) \geq 0$ and $F^{PC,1}(G(0), G(1)) \geq 0$ and $G(1) > \overline{G}(1)$. Then, for any $G^{*}(1) \in \left[\overline{G}(1), G(1)\right)$ we also have $F^{PC,0}(G(0), G^{*}(1)) \geq 0$ and $F^{PC,1}(G(0), G^{*}(1)) \geq 0$. Thus, $(G(0), G(1))$ is not a solution. Therefore, any solution must satisfy $G(0) \in \left[0, \overline{G}(0)\right]$ and $G(1) \in \left[0, \overline{G}(1)\right]$. ■

Result 2: $F^{PC,0}(G(0), G(1))$ and $F^{PC,1}(G(0), G(1))$ are quasiconcave functions for all $(G(0), G(1)) \in \mathbb{R}_{+}^{2}$.

Proof: Note that $F^{PC,0}\left(G\left(0\right),G\left(1\right)\right)\geq c$ can be written as

$$F^{PC,0}\left(G\left(0\right),G\left(1\right)\right)=\frac{1}{\delta\phi}\left[F^{PC,0}\left(G\left(0\right),0\right)-c\right]-G\left(1\right)\geq 0,$$

where

$$F^{PC,0}\left(G\left(0\right),0\right)=-\left(2-\delta-\delta\phi\right)G\left(0\right)+2\left(1-\delta\right)\sqrt{G\left(0\right)\theta R}+F^{PC,0}\left(0,0\right).$$

Note that $F^{PC,0}\left(G\left(0\right),0\right)$ is strictly concave in $G\left(0\right)$ for all $G\left(0\right)$, which implies that $\frac{1}{\delta\phi}\left[F^{PC,0}\left(G\left(0\right),0\right)-c\right]$ is also strictly concave in $G\left(0\right)$ for all $G\left(0\right)$ and c. Thus, $\left\{\left(G\left(0\right),G\left(1\right)\right):G\left(1\right)\leq\frac{1}{\delta\phi}\left[F^{PC,0}\left(G\left(0\right),0\right)-c\right]\right\}$ is a convex set for every c. Therefore, $\left\{\left(G\left(0\right),G\left(1\right)\right):F^{PC,0}\left(G\left(0\right),G\left(1\right)\right)\geq c\right\}$ is a convex set for all c, which implies that $F^{PC,0}\left(G\left(0\right),G\left(1\right)\right)$ is quasiconcave.

Similarly, $F^{PC,1}\left(G\left(0\right),G\left(1\right)\right)\geq c$ can be written as

$$F^{PC,1}\left(G\left(0\right),G\left(1\right)\right)=\frac{1}{\delta\left(1-\phi\right)}\left[F^{PC,1}\left(0,G\left(1\right)\right)-c\right]-G\left(0\right)\geq 0,$$

where

$$F^{PC,1}\left(0,G\left(1\right)\right)=-\left(2-2\delta+\delta\phi\right)G\left(1\right)+2\left(1-\delta\right)\sqrt{G\left(1\right)R^{E}}+F^{PC,1}\left(0,0\right).$$

Note that $F^{PC,1}\left(0,G\left(1\right)\right)$ is strictly concave in $G\left(1\right)$ for all $G\left(1\right)$, which implies that $\frac{1}{\delta(1-\phi)}\left[F^{PC,1}\left(0,G\left(1\right)\right)-c\right]$ is also strictly concave in $G\left(1\right)$ for all $G\left(1\right)$ and c. Thus, $\left\{\left(G\left(0\right),G\left(1\right)\right):G\left(0\right)\leq\frac{1}{\delta(1-\phi)}\left[F^{PC,1}\left(0,G\left(1\right)\right)-c\right]\right\}$ is a convex set for every c. Therefore, $\left\{\left(G\left(0\right),G\left(1\right)\right):F^{PC,1}\left(G\left(0\right),G\left(1\right)\right)\geq c\right\}$ is a convex set for all c, which implies that $F^{PC,1}\left(G\left(0\right),G\left(1\right)\right)$ is quasiconcave. ■

Result 3: If $F^{PC,1}\left(0,0\right)\geq 0$, then $F^{PC,0}\left(0,0\right)\geq 0$.

Proof: $F^{PC,0}\left(0,0\right)\geq 0$ if and only if

$$\frac{R\left[1-2\left(1-\delta\right)\theta\right]}{2\left(1-\delta\right)}\geq\delta V^{E}=\frac{\phi R^{E}+\left(1-\phi\right)\theta R}{4\left(1-\delta+\phi\delta\right)}.$$

$F^{PC,1}\left(0,0\right)\geq 0$ if and only if

$$\frac{R\left[1-2\left(1-\delta\right)\theta\right]}{2\left(1-\delta\right)}\geq -X+\frac{\delta R}{1-\delta}.$$

Since $X\leq\overline{X}$ implies $-X+\frac{\delta R}{1-\delta}\geq\delta V^{E}$, we have that $F^{PC,1}\left(0,0\right)\geq 0$ implies $F^{PC,0}\left(0,0\right)\geq 0$. ■

Since the objective function is linear and the constraints are quasiconcave, the following Kuhn–Tucker conditions are sufficient for a global maximum:

$$-(1-\phi)+\lambda^0\left[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]-\lambda^1\delta(1-\phi)+\mu_L^0-\mu_H^0=0,$$

$$-\phi-\lambda^0\delta\phi+\lambda^1\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{R^E}{G(1)}}\right]+\mu_L^1-\mu_H^1=0,$$

$$\lambda^0\geq 0,\ F^{PC,0}(G(0),G(1))\geq 0,\ \lambda^0 F^{PC,0}(G(0),G(1))=0,$$

$$\lambda^1\geq 0,\ F^{PC,1}(G(0),G(1))\geq 0,\ \lambda^1 F^{PC,1}(G(0),G(1))=0,$$

$$\mu_L^0\geq 0,\ G(0)\geq 0,\ \mu_L^0 G(0)=0,$$

$$\mu_H^0\geq 0,\ \overline{G}(0)-G(0)\geq 0,\ \mu_H^0\left[\overline{G}(0)-G(0)\right]=0,$$

$$\mu_L^1\geq 0,\ G(1)\geq 0,\ \mu_L^1 G(1)=0,$$

$$\mu_H^1\geq 0,\ \overline{G}(1)-G(1)\geq 0,\ \mu_H^1\left[\overline{G}(1)-G(1)\right]=0.$$

There are several cases to consider.

Case 1: Suppose that $G(0)=G(1)=0$. Solving, we obtain $\lambda^0=\mu_H^0=0$, $\mu_L^0=1-\phi$, $\lambda^1=\mu_H^1=0$, $\mu_L^1=\phi$, $F^{PC,0}(0,0)\geq 0$, and $F^{PC,1}(0,0)\geq 0$. Since $F^{PC,1}(0,0)\geq 0$ implies $F^{PC,0}(0,0)\geq 0$, we must check that $F^{PC,1}(0,0)\geq 0$. $F^{PC,1}(0,0)\geq 0$ if and only if

$$\frac{R[1-2(1-\delta)\theta]}{2(1-\delta)}\geq -X+\frac{\delta R}{1-\delta},$$

or, equivalently,

$$X\geq \overline{X}_{high}=\frac{[2\delta-1+2(1-\delta)\theta]R}{2(1-\delta)}.$$

If $\theta\leq\frac{1-2\delta}{2(1-\delta)}$, this constraint always holds because $X>0$. If $\theta>\frac{1-2\delta}{2(1-\delta)}$ we need to check that

$$\frac{[2\delta-1+2(1-\delta)\theta]R}{2(1-\delta)}\leq\overline{X},$$

which holds if and only if

$$\theta\leq\frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}.$$

Summing up, $G(0)=G(1)=0$ is a solution if and only if $[\theta\leq\frac{1-2\delta}{2(1-\delta)}]$ or $[X\geq\overline{X}_{high}$ and $\frac{1-2\delta}{2(1-\delta)}<\theta\leq\frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}]$.

Case 2: Suppose that $G(0)\in\left(0,\overline{G}(0)\right)$ and $G(1)=0$. Solving we obtain $\mu_L^0=\mu_H^0=\mu_H^1=0$, $\lambda^1=0$,

$$\lambda^0=(1-\phi)\left[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]^{-1}>0,$$

$\mu_L^1=\phi\left(1+\lambda^0\delta\right)>0$, $F^{PC,0}(G(0),0)=0$ and $F^{PC,1}(G(0),0)\geq 0$. Since $F^{PC,0}(G(0),0)$ is strictly increasing in $G(0)$ for all $G(0)\in\left[0,\overline{G}(0)\right]$, at most there is one solution to $F^{PC,0}(G(0),0)=0$. Moreover, there is a solution that satisfies $G(0) \in \left(0, \overline{G}(0)\right)$ if and only if $F^{PC,0}(0,0) < 0$ and $F^{PC,0}\left(\overline{G}(0), 0\right) > 0$. Note, however, that $F^{PC,0}(0,0) < 0$ implies $F^{PC,1}(0,0) < 0$ (due to Result 3), and $F^{PC,1}(0,0) < 0$ implies $F^{PC,1}(G(0), 0) < 0$ (because $F^{PC,1}(G(0), 0)$ is strictly decreasing in $G(0)$). Thus, $F^{PC,0}(0,0) < 0$ is incompatible with $F^{PC,1}(G(0), 0) \geq 0$. Summing up, there is no solution such that $G(0) \in \left(0, \overline{G}(0)\right)$ and $G(1) = 0$.

Case 3: Suppose that $G(0) = 0$ and $G(1) \in \left(0, \overline{G}(1)\right)$. Solving we obtain $\mu_H^0 = \mu_L^1 = \mu_H^1 = 0$, $\lambda^0 = 0$,

$$\mu_L^0 = (1-\phi) + \lambda^1 \delta (1-\phi) > 0, \qquad \lambda^1 = \phi \left[-(2 - 2\delta + \delta\phi) + (1-\delta)\sqrt{\frac{R^E}{G(1)}} \right]^{-1} > 0,$$

$F^{PC,1}(0, G(1)) = 0$ and $F^{PC,0}(0, G(1)) \geq 0$. Since $F^{PC,1}(0, G(1))$ is strictly increasing in $G(1)$ for all $G(1) \in \left[0, \overline{G}(1)\right]$, at most there is one solution to $F^{PC,1}(0, G(1)) = 0$. Moreover, there is a solution that satisfies $G(1) \in \left(0, \overline{G}(1)\right)$ if and only if $F^{PC,1}(0,0) < 0$ and $F^{PC,1}\left(0, \overline{G}(1)\right) > 0$. $F^{PC,1}(0,0) < 0$ if and only if

$$X < \theta R + \frac{(2\delta - 1)}{2(1-\delta)} R,$$

while $F^{PC,1}\left(0, \overline{G}(1)\right) > 0$ if and only if

$$X > \theta R + \frac{\left(2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right)}{2(1-\delta)} R.$$

Therefore, we need

$$\theta R + \frac{\left(2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right)}{2(1-\delta)} R < X < \theta R + \frac{(2\delta - 1)}{2(1-\delta)} R.$$

Thus, we must check that

$$\theta R + \frac{(2\delta - 1)}{2(1-\delta)} R > 0,$$

which holds if and only if $\theta > \frac{1-2\delta}{2(1-\delta)}$. We must also check that

$$\theta R + \frac{(2\delta - 1)}{2(1-\delta)} R \leq \overline{X},$$

which holds if and only if

$$\theta \leq \frac{4(1-\delta) + 3\phi\delta}{2(1-\delta)(4 - 3\delta + 3\phi\delta)}.$$

Finally, we must verify that the unique solution to $F^{PC,1}(0, G(1)) = 0$ satisfies $F^{PC,0}(0, G(1)) \geq 0$ or, equivalently, that $G(1) \leq F^{PC,0}(0,0)/\delta\phi$. Summing up, $G(0) = 0$ and $G(1) = \hat{G}(1) \in \left(0, \overline{G}(1)\right)$, where $\hat{G}(1)$ is the unique solution to $F^{PC,1}(0, G(1)) = 0$, is a solution if and only if $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$,

$$\theta R + \frac{\left(2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right)}{2(1-\delta)} R < X < \theta R + \frac{(2\delta - 1)}{2(1-\delta)} R,$$

and $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$.

Case 4: Suppose that $G(0) \in \left(0, \overline{G}(0)\right)$ and $G(1) \in \left(0, \overline{G}(1)\right)$. Solving, we obtain $\mu_L^0 = \mu_L^1 = \mu_H^0 = \mu_H^1 = 0$, and

$$\lambda^0 \left[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right] - \lambda^1 \delta(1-\phi) = (1-\phi),$$

$$-\lambda^0 \delta\phi + \lambda^1 \left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{R^E}{G(1)}}\right] = \phi.$$

The above system of linear equations has a solution if and only if

$$\left[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]\left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{R^E}{G(1)}}\right] \neq \delta^2\phi(1-\phi).$$

A solution must satisfy either $[\lambda^0 > 0$ and $\lambda^1 > 0]$ or $[\lambda^0 < 0$ and $\lambda^1 < 0]$. We have $\lambda^0 > 0$ and $\lambda^1 > 0$ if and only if

$$\left[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]\left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{R^E}{G(1)}}\right] > \delta^2\phi(1-\phi). \tag{1}$$

$\lambda^0 > 0$ and $\lambda^1 > 0$ imply that $F^{PC,0}(G(0), G(1)) = 0$ and $F^{PC,1}(G(0), G(1)) = 0$. Thus, we must solve the following system of equations:

$$F^{PC,0}(G(0), 0) - \delta\phi G(1) = 0,$$
$$F^{PC,1}(0, G(1)) - \delta(1-\phi)G(0) = 0.$$

We need $F^{PC,1}\left(0, \overline{G}(1)\right) > 0$ or, otherwise, $F^{PC,1}(0, G(1)) < 0$ for all $G(1) \in \left[0, \overline{G}(1)\right]$ and, hence, there is no solution to the above system of equations satisfying $G(0) > 0$. Also assume that $F^{PC,1}(0,0) < 0$. Otherwise, the solution is $G(0) = G(1) = 0$ (see Case 1). Since $F^{PC,1}(0, G(1))$ is strictly increasing in $G(1)$ for all $G(1) \in \left[0, \overline{G}(1)\right]$, for a solution of the above system of equations to satisfy $G(0) > 0$ it must be the case that $G(1) > \hat{G}(1)$, where $\hat{G}(1)$ is the unique solution to $F^{PC,1}(0, G(1)) = 0$. Define

$$Q(G(1)) = F^{PC,0}\left(\frac{F^{PC,1}(0, G(1))}{\delta(1-\phi)}, 0\right) - \delta\phi G(1).$$

Since $F^{PC,1}(0, G(1))$ is strictly concave in $G(1)$ and $F^{PC,0}(G(0), 0)$ is strictly concave in $G(0)$, $Q(G(1))$ is strictly concave in $G(1)$. Moreover, note that

$$\begin{aligned}\lim_{G(1)\to\hat{G}(1)} \frac{\partial Q(G(1))}{\partial G(1)} &= \lim_{G(1)\to\hat{G}(1)} \left[\frac{\partial F^{PC,0}\left(\frac{F^{PC,1}(0,G(1))}{\delta(1-\phi)}, 0\right)}{\partial G(0)} \frac{\partial F^{PC,1}(0, G(1))}{\partial G(1)} \frac{1}{\delta(1-\phi)} - \delta\phi\right] \\ &= \left(\lim_{G(0)\to 0} \frac{\partial F^{PC,0}(0,0)}{\partial G(0)}\right) \frac{\partial F^{PC,1}\left(0, \hat{G}(1)\right)}{\partial G(1)} \frac{1}{\delta(1-\phi)} - \delta\phi,\end{aligned}$$

where $\frac{\partial F^{PC,1}\left(0,\hat{G}(1)\right)}{\partial G(1)} > 0$ and $\lim_{G(0)\to 0} \frac{\partial F^{PC,0}(0,0)}{\partial G(0)} = \infty$. Thus, for $G(1) \to \hat{G}(1)$, $Q(G(1))$ is strictly increasing, which implies that $Q(G(1))$ is either strictly increasing for all $G(1) \in \left[\hat{G}(1), \overline{G}(1)\right]$ or it is strictly increasing for all $G(1) \in \left[\hat{G}(1), G^m\right]$, attains a maximum at $G(1) = G^m$, and is strictly decreasing for all $G(1) \in \left[G^m, \overline{G}(1)\right]$. Thus, there are five possible situations to consider:

Case 4.a: Suppose that $Q\left(\hat{G}(1)\right) > 0$ and $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), \overline{G}(1)\right]$. Then, there is no solution to $Q(G(1)) = 0$.

Case 4.b: Suppose that $Q\left(\hat{G}(1)\right) > 0$ and $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), G^m\right]$, attains a maximum at $G(1) = G^m$, and is strictly decreasing for all $G(1) \in \left[G^m, \overline{G}(1)\right]$. Then, there is at most one solution to $Q(G(1)) = 0$, which exists if and only if $Q\left(\overline{G}(1)\right) \leq 0$. Note, however, that for such a solution it must be the case that $\frac{\partial Q(G(1))}{\partial G(1)} \leq 0$, which holds if and only if

$$\frac{\partial F^{PC,0}\left(\frac{F^{PC,1}(0,G(1))}{\delta(1-\phi)}, 0\right)}{\partial G(0)} \frac{\partial F^{PC,1}(0, G(1))}{\partial G(1)} \leq \delta^2 (1-\phi)\phi,$$

a violation of (1).

Case 4.c: Suppose that $Q\left(\hat{G}(1)\right) = 0$. Assume that $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), \overline{G}(1)\right]$. Then, the unique solution to $Q(G(1)) = 0$ is $G(1) = \hat{G}(1)$, which implies $G(0) = 0$, a contradiction. Assume instead that $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), G^m\right]$, attains a maximum at $G(1) = G^m$, and is strictly decreasing for all $G(1) \in \left[G^m, \overline{G}(1)\right]$. If $Q\left(\overline{G}(1)\right) \leq 0$, then there are two solutions to $Q(G(1)) = 0$. One solution is $G(1) = \hat{G}(1)$, which implies $G(0) = 0$, a contradiction. For the other solution, it must be the case that $\frac{\partial Q(G(1))}{\partial G(1)} \leq 0$, which violates (1). If $Q\left(\overline{G}(1)\right) > 0$, then the unique solution to $Q(G(1)) = 0$ is $G(1) = \hat{G}(1)$, which again implies $G(0) = 0$, a contradiction.

Case 4.d: Suppose that $Q\left(\hat{G}(1)\right) < 0$ and $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), \overline{G}(1)\right]$. Then, there is at most one solution to $Q(G(1)) = 0$, which exists if and only if $Q\left(\overline{G}(1)\right) > 0$. Moreover, for this solution it must be the case that $\frac{\partial Q(G(1))}{\partial G(1)} > 0$.

Case 4.e: Suppose that $Q\left(\hat{G}(1)\right) < 0$ and $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), G^m\right]$, attains a maximum at $G(1) = G^m$, and is strictly decreasing for all $G(1) \in \left[G^m, \overline{G}(1)\right]$. If $Q(G^m) < 0$, then there is no solution to $Q(G(1)) = 0$. If $Q(G^m) = 0$, then the unique solution to $Q(G(1)) = 0$ is $G(1) = G^m$. If $Q(G^m) > 0$ and $Q\left(\overline{G}(1)\right) < 0$, then there are two solutions to $Q(G(1)) = 0$, one with $G(1) < G^m$ and another with $G(1) > G^m$. Note, however, that for the solution with $G(1) > G^m$ we have $\frac{\partial Q(G(1))}{\partial G(1)} < 0$, which violates (1). Thus, only the solution with $G(1) < G^m$ satisfies the Kuhn–Tucker conditions. Finally, if $Q(G^m) > 0$ and $Q\left(\overline{G}(1)\right) > 0$, there is a unique solution to $Q(G(1)) = 0$ for which $\frac{\partial Q(G(1))}{\partial G(1)} > 0$.

Summing up, $G(0) \in \left(0, \overline{G}(0)\right)$ and $G(1) \in \left(0, \overline{G}(1)\right)$, given by the unique solution to

$$F^{PC,0}\left(G(0), G(1)\right) = F^{PC,1}\left(G(0), G(1)\right) = 0$$

that satisfies

$$\frac{\partial F^{PC,0}\left(G(0), 0\right)}{\partial G(0)} \frac{\partial F^{PC,1}\left(0, G(1)\right)}{\partial G(1)} > \delta^2 (1-\phi)\phi$$

is a solution if and only if $F^{PC,1}(0,0) < 0$, $F^{PC,1}\left(0, \overline{G}(1)\right) > 0$, and $Q\left(\hat{G}(1)\right) < 0$, where $\hat{G}(1) \in \left(0, \overline{G}(1)\right)$ is the unique solution to $F^{PC,1}\left(0, G(1)\right) = 0$. In Case 3 we have already proved that $F^{PC,1}(0,0) < 0$ and $F^{PC,1}\left(0, \overline{G}(1)\right) > 0$ if and only if

$$\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta) + 3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)},$$

$$\theta R + \frac{\left(2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right)}{2(1-\delta)} R < X < \theta R + \frac{(2\delta-1)}{2(1-\delta)} R.$$

Finally,

$$Q\left(\hat{G}(1)\right) = F^{PC,0}\left(\frac{F^{PC,1}\left(0, \hat{G}(1)\right)}{\delta(1-\phi)}, 0\right) - \delta\phi\hat{G}(1) = F^{PC,0}(0,0) - \delta\phi\hat{G}(1).$$

Thus, $Q\left(\hat{G}(1)\right) < 0$ if and only if $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$.

Case 5: Suppose that $G(0) = \overline{G}(0)$ or $G(1) = \overline{G}(1)$. If $G(0) = \overline{G}(0)$, we have $\mu_L^0 = 0$ and, hence,

$$(1-\phi) + \lambda^1\delta(1-\phi) + \mu_H^0 = 0,$$

a contradiction because $\lambda^1 \geq 0$ and $\mu_H^0 \geq 0$. If $G(1) = \overline{G}(1)$, we have $\mu_L^1 = 0$ and, hence,

$$\phi + \lambda^0\delta\phi + \mu_H^1 = 0,$$

a contradiction because $\lambda^0 \geq 0$ and $\mu_H^1 \geq 0$.

This completes the proof of 5.1 and 5.2.

Proof of Parts 3 and 4: Suppose that countries are following the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Then, the expected discounted payoffs for each state $Z = 0, 1$ are given by

$$V^{TC}(0) = (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta\left[\phi V^{TC}(1) + (1-\phi)V^{TC}(0)\right],$$

$$V^{TC}(1) = (1-\delta)\frac{R^E}{4}.$$

Solving, we obtain

$$V^{TC}(0) = \frac{(1-\delta)\left(\frac{R}{2} - G(0) + \frac{\delta\phi}{4}R^E\right)}{1-\delta(1-\phi)}.$$

Sustainability for $Z=0$: Suppose that $Z=0$ and player i deviates from cooperation. From Lemma 2, the most profitable deviation for player i is $g^D(G(0),\theta R)=\sqrt{G(0)\theta R}-G(0)$ and the associated payoff is given by

$$V^D(G(0))=(1-\delta)\left[\frac{g^D(G(0),\theta R)}{g^D(G(0),\theta R)+G(0)}\theta R-g^D(G(0),\theta R)\right]+\delta V^{PU}.$$

From Lemma 1, $X\leq\overline{X}$ implies that $V^{PU}=V^E$. Thus, player i does not have an incentive to deviate if and only if $V^{TC}(0)\geq V^D(G(0))$ or, equivalently,

$$F^{TC}(G(0))=-(2-\delta+\delta\phi)G(0)+2(1-\delta+\delta\phi)\sqrt{G(0)\theta R}+F^{TC}(0)\geq 0,$$

where

$$F^{TC}(0)=\left[\frac{2-\theta(4-3\delta+3\phi\delta)}{4}\right]R.$$

Best possible temporary cooperation equilibrium: To determine the best possible temporary cooperative equilibrium, we solve

$$\max_{G(0)\geq 0}\left\{\ \phi V^{TC}(1)+(1-\phi)V^{TC}(0)=\left(\tfrac{1-\delta}{1-\delta+\delta\phi}\right)\left[\tfrac{\phi R^E}{4}+(1-\phi)\tfrac{R}{2}-(1-\phi)G(0)\right]\ \right\}$$
$$\text{s.t. } F^{TC}(G(0))\geq 0.$$

There are two cases to consider.

Case 1: Suppose that $\theta\leq\frac{2}{4-3\delta+3\phi\delta}$. Then, $F^{TC}(0)\geq 0$ and, hence, $G(0)=0$ is the solution to the above maximization problem.

Case 2: Suppose that $\theta>\frac{2}{4-3\delta+3\phi\delta}$. Then, $F^{TC}(0)<0$. Moreover,

$$\frac{\partial F^{TC}(G(0))}{\partial G(0)}=-(2-\delta+\delta\phi)+(1-\delta+\delta\phi)\sqrt{\frac{\theta R}{G(0)}}.$$

Thus, $F^{TC}(G(0))$ is strictly increasing in $G(0)$ for all $G(0)\in\left[0,\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2\theta R\right]$ and strictly decreasing in $G(0)$ for all

$$G(0)\in\left[\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2\theta R,\frac{\theta R}{4}\right].$$

Moreover,

$$\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2\theta R\leq\frac{\theta R}{4},$$

with strict inequality for $\delta<1$. Finally, note that

$$F^{TC}\left(\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2\theta R\right)=\frac{(1-\delta+\delta\phi)^2}{2-\delta+\delta\phi}\theta R+\left[\frac{2-\theta(4-3\delta+3\phi\delta)}{4}\right]R>0.$$

Thus, the solution to the above maximization problem is

$$G(0)\in\left(0,\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2\theta R\right)$$

given by the unique solution to $F^{TC}(G(0)) = 0$.

This proves 5.3 and 5.4. ■

Corollary 1. *Open conflict is unavoidable if and only if*

$$X \leq \begin{cases} \overline{X} = \dfrac{[(4-\theta)(1-\delta)+3\phi\delta]\,\delta R}{(1-\delta)[4(1-\delta)+3\phi\delta]} & \text{if } \theta > \dfrac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}, \\ \overline{X}_{low} = \theta R + \dfrac{\left[2\delta - \dfrac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right] R}{2(1-\delta)} & \text{if } \dfrac{1-2\delta}{2(1-\delta)} < \theta \leq \dfrac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}. \end{cases}$$

Moreover:

1. *The lower the cost of the elimination technology (X lower) and the higher the chances that countries can use it (ϕ higher), the less likely open conflict can be avoided.*

2. *The destructiveness of open conflict has a non-monotonic effect on the likelihood of open conflict. Initially, as destructiveness declines (θ higher), open conflict becomes more likely, but eventually further reductions in destructiveness make open conflict easier to avoid. This non-monotonicity holds for any fixed X lying between $\overline{X}_{low}(\theta,\phi)$ and $\overline{X}(\theta,\phi)$.*

Proof: In our terminology, *avoiding open conflict is exactly equivalent to sustaining a permanent cooperative equilibrium.* That is, open conflict is unavoidable precisely when no permanent cooperative equilibrium can be sustained.

From Proposition 5.2, it is not possible to sustain a permanent cooperative equilibrium if and only if

$$\left[\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)} \quad \text{and} \quad X \leq \overline{X}_{low}\right]$$

or

$$\left[\theta > \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)} \quad \text{and} \quad X \leq \overline{X}\right].$$

Clearly, the lower the cost of the elimination technology (the lower X), the more likely that one of these conditions holds.

Rearranging terms, it follows that sustaining a permanent cooperative equilibrium is impossible if and only if

$$X \leq \begin{cases} \overline{X} = \dfrac{[(4-\theta)(1-\delta)+3\phi\delta]\,\delta R}{(1-\delta)[4(1-\delta)+3\phi\delta]} & \text{if } \phi > \dfrac{2(1-\delta)(2-4\theta+3\theta\delta)}{2\theta(1-\delta)-1}, \\ \overline{X}_{low} = \theta R + \dfrac{\left[2\delta - \dfrac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right] R}{2(1-\delta)} & \text{if } \phi \leq \dfrac{2(1-\delta)(2-4\theta+3\theta\delta)}{2\theta(1-\delta)-1}. \end{cases}$$

Both $\overline{X}_{low}$ and $\overline{X}$ are strictly increasing in ϕ. Moreover, $\overline{X}_{low} < \overline{X}$ whenever $\phi \leq \dfrac{2(1-\delta)(2-4\theta+3\theta\delta)}{2\theta(1-\delta)-1}$. Hence, increases in ϕ make the inequality above more likely to hold.

Finally, $\overline{X}_{low}$ is strictly increasing in θ, while $\overline{X}$ is strictly decreasing in θ, and $\overline{X}_{low} < \overline{X}$ for

$$\theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}.$$

Thus, increasing θ makes open conflict more likely when θ lies below this threshold, and less likely when θ is above it.

This completes the proof of Corollary 1. ■

Proposition 6. *Suppose that* $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ *and* $\overline{X}_{low} < X < \overline{X}_{high}$. *Then, partial permanent cooperation induces a higher ex-ante expected payoff than temporary cooperation if and only if*

$$\Delta = \; F^{PC,0}(0,0) - \delta\phi\, G^{PC}(1) - \delta(1-\phi)\, G^{PC}(0) - \frac{(1-\delta)}{1-\delta+\delta\phi}\left[F^{TC}(0) - \delta(1-\phi)\, G^{TC}(0)\right] \; \geq 0$$

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\,\theta R - \frac{(1-\delta)\,\delta\left[\theta R + \phi\left(-X + \frac{\delta R}{1-\delta}\right)\right]}{4(1-\delta+\phi\delta)},$$
$$F^{TC}(0) = \frac{2-\theta(4-3\delta+3\phi\delta)}{4}R.$$

$G^{PC}(0)$ *and* $G^{PC}(1)$ *are given by Proposition 5.2 and* $G^{TC}(0)$ *by Propositions 5.3 and 5.4. Moreover, if* $\theta > \frac{2}{4-3\delta+3\phi\delta}$ *and* $\widetilde{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, *then* $\Delta \geq 0$, *while if* $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$ *and* $\widetilde{G}(1) > F^{PC,0}(0,0)/\delta\phi$, *then* $\Delta < 0$.

Proof: The ex-ante expected payoff under partial permanent cooperation is given by

$$V^{PC} = \frac{R}{2} - \phi\, G^{PC}(1) - (1-\phi)\, G^{PC}(0),$$

where $G^{PC}(0)$ and $G^{PC}(1)$ are given by Proposition 5.2.

The ex-ante expected payoff under temporary cooperation is given by

$$V^{TC} = \phi V^{TC}(1) + (1-\phi)\, V^{TC}(0),$$

where $V^{TC}(0) = \dfrac{(1-\delta)\left(\frac{R}{2} - G^{TC}(0) + \frac{\delta\phi R^E}{4}\right)}{1-\delta(1-\phi)}$ and $V^{TC}(1) = \dfrac{(1-\delta)\,R^E}{4}$. Therefore, we obtain

$$V^{TC} = \frac{(1-\delta)\left[\frac{\phi}{4}R^E + (1-\phi)\left(\frac{R}{2} - G^{TC}(0)\right)\right]}{1-\delta+\delta\phi},$$

where $G^{TC}(0)$ is given by Proposition 5.3 or Proposition 5.4.

$V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi\, G^{PC}(1) - \delta(1-\phi)\, G^{PC}(0) \geq \frac{(1-\delta)\left[F^{TC}(0) - \delta(1-\phi)\, G^{TC}(0)\right]}{1-\delta+\delta\phi},$$

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\,\theta R - \frac{(1-\delta)\,\delta\left[\theta R + \phi\left(-X + \frac{\delta R}{1-\delta}\right)\right]}{4(1-\delta+\phi\delta)},$$

$$F^{TC}(0) = \frac{2-\theta(4-3\delta+3\phi\delta)}{4}R.$$

Note that

$$\frac{1-2\delta}{2(1-\delta)} < \frac{2}{4-3\delta+3\phi\delta} < \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$$

for $\delta > 0$. So, there are several cases to consider:

Case 1: Suppose that $\frac{1-2\delta}{2(1-\delta)} < \theta < \frac{2}{4-3\delta+3\phi\delta}$, $\overline{X}_{low} < X < \overline{X}_{high}$, and $\widetilde{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, where $\widetilde{G}(1) \in \left(0, \overline{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 5.2.a, the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with $G^{PC}(0) = 0$ and $G^{PC}(1) = \widetilde{G}(1)$. From Proposition 5.3, the best temporary cooperative equilibrium that can be sustained is $G^{TC}(0) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi\,\widetilde{G}(1) \geq \frac{(1-\delta)\,F^{TC}(0)}{1-\delta+\delta\phi}.$$

Moreover, $\theta < \frac{2}{4-3\delta+3\phi\delta}$ implies $F^{TC}(0) > 0$ (and $F^{TC}(0) = 0$ at $\theta = \frac{2}{4-3\delta+3\phi\delta}$).

Case 2: Suppose that $\frac{2}{4-3\delta+3\phi\delta} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$, $\overline{X}_{low} < X < \overline{X}_{high}$, and $\widetilde{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, where $\widetilde{G}(1) \in \left(0, \overline{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 5.2.a, the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with $G^{PC}(0) = 0$ and $G^{PC}(1) = \widetilde{G}(1)$. From Proposition 5.4, the best temporary cooperative equilibrium that can be sustained is $G^{TC}(0) \in \left(0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R\right)$ given by the unique solution to $F^{TC}(G(0)) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi\,\widetilde{G}(1) \geq \frac{(1-\delta)\left[F^{TC}(0) - \delta(1-\phi)\,G^{TC}(0)\right]}{1-\delta+\delta\phi}.$$

Since $\widetilde{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, and $\theta > \frac{2}{4-3\delta+3\phi\delta}$ implies $F^{TC}(0) < 0$, the above inequality always holds.

Case 3: Suppose that $\frac{1-2\delta}{2(1-\delta)} < \theta < \frac{2}{4-3\delta+3\phi\delta}$, $\overline{X}_{low} < X < \overline{X}_{high}$, and $\widetilde{G}(1) > F^{PC,0}(0,0)/\delta\phi$, where $\widetilde{G}(1) \in \left(0, \overline{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 5.2.b, the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with $G^{PC}(0) > 0$ and $G^{PC}(1) > \widetilde{G}(1)$ given by the unique solution to $F^{PC,0}(G(0),G(1)) = F^{PC,1}(G(0),G(1)) = 0$ that satisfies

$$\frac{\partial F^{PC,0}(G(0),0)}{\partial G(0)}\frac{\partial F^{PC,1}(0,G(1))}{\partial G(1)} > \delta^2(1-\phi)\,\phi.$$

From Proposition 5.3, the best temporary cooperative equilibrium that can be sustained is $G^{TC}(0) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi\,G^{PC}(1) - \delta(1-\phi)\,G^{PC}(0) \geq \frac{(1-\delta)\,F^{TC}(0)}{1-\delta+\delta\phi}.$$

Since $G^{PC}(1) > \widetilde{G}(1) > F^{PC,0}(0,0)/\delta\phi$, and $\theta < \frac{2}{4-3\delta+3\phi\delta}$ implies $F^{TC}(0) > 0$, the above inequality never holds.

Case 4: Suppose that $\frac{2}{4-3\delta+3\phi\delta} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$, $\overline{X}_{low} < X < \overline{X}_{high}$, and $\widetilde{G}(1) > F^{PC,0}(0,0)/\delta\phi$, where $\widetilde{G}(1) \in \left(0, \overline{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 5.2.b, the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with $G^{PC}(0) > 0$ and $G^{PC}(1) > \widetilde{G}(1)$ given by the unique solution to $F^{PC,0}(G(0), G(1)) = F^{PC,1}(G(0), G(1)) = 0$ that satisfies

$$\frac{\partial F^{PC,0}(G(0),0)}{\partial G(0)} \frac{\partial F^{PC,1}(0,G(1))}{\partial G(1)} > \delta^2 (1-\phi)\phi.$$

From Proposition 5.4, the best temporary cooperative equilibrium that can be sustained is $G^{TC}(0) \in \left(0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R\right)$ given by the unique solution to $F^{TC}(G(0)) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi\, G^{PC}(1) - \delta(1-\phi)\, G^{PC}(0) \geq \frac{(1-\delta)\left[F^{TC}(0) - \delta(1-\phi)\, G^{TC}(0)\right]}{1-\delta+\delta\phi},$$

and $\theta > \frac{2}{4-3\delta+3\phi\delta}$ implies $F^{TC}(0) < 0$.

This completes the proof of Proposition 6. ■

A.4 Temporary Elimination

This section presents the proof of Lemmas 3 and 4, and Propositions 7, 8, and 9, and Corollary 2.

Lemma 3. *Under the non-cooperative norm,*[2] *let*

$$\overline{X}(\gamma) = \frac{[(4-\theta)(1-\delta+\delta\gamma) + 3\delta\phi]\,\delta R}{(1-\delta+\delta\gamma)[4(1-\delta+\delta\gamma)+3\delta\phi]} \quad \text{and} \quad R^E(\gamma) = \theta R - X + \frac{\delta R}{1-\delta+\delta\gamma}.$$

1. *Suppose that $X > \overline{X}(\gamma)$. Then, the worst equilibrium is $G^{NN} = \dfrac{\theta R}{4}$ for $Z = 0$ and $G^{NN} = \dfrac{\theta R}{4}$ and no elimination (i.e. $W_1 = W_2 = 0$) for $Z = 1$. The associated punishment expected payoff is $V^{PU} = V^{NN} = \dfrac{\theta R}{4}$.*

2. *Suppose that $X \leq \overline{X}(\gamma)$. Then, the worst equilibrium is $G^{NN} = \dfrac{\theta R}{4}$ for $Z = 0$ and $G^E(\gamma) = \dfrac{R^E(\gamma)}{4}$ and eliminate the loser (i.e. $W_1 = W_2 = 1$) for $Z = 1$. The associated punishment expected payoff is*

$$V^{PU} = V^E(\gamma) = \frac{(1-\delta+\delta\gamma)\left[\phi R^E(\gamma) + (1-\phi)\,\theta R\right]}{4(1-\delta)[1-\delta(1-\phi-\gamma)]}.$$

Proof: Suppose that when the resource is contestable both players always select $S_1 = S_2 = 0$. If $S_{-i} = 0$, there is open conflict regardless of S_i. Thus, in the settlement stage there is no unilateral profitable deviation from $S_1 = S_2 = 0$.

[2] That is, assume that in every contestable state, $S_1 S_2 = 0$ for all (G_1, G_2) and Z.

When the resource is contestable and elimination is not possible ($Z=0$), the expected discounted payoff of player i is given by

$$V_i^{PU}(C=1,Z=0)=\max_{G_i(0)>0}\left\{(1-\delta)\left[\pi_i(0)\,\theta R-G_i^{PU}(0)\right]+\delta V_i^{PU}(C=1)\right\},$$

where $\pi_i(0)=\dfrac{G_i^{PU}(0)}{G_1^{PU}(0)+G_2^{PU}(0)}$ and $V_i^{PU}(C=1)$ is the expected value of the game conditional on the resource being contestable, i.e.,

$$V_i^{PU}(C=1)=(1-\phi)\,V_i^{PU}(C=1,Z=0)+\phi V_i^{PU}(C=1,Z=1).$$

When the resource is contestable and elimination is possible ($Z=1$), the expected discounted payoff of player i is given by

$$\begin{aligned}
V_i^{PU}(C=1,Z=1) &= \max_{G_i(1)>0}\left\{\pi_i(1)\left[(1-\delta)\,\theta R+V_i^{PU}(w)\right]\right.\\
&\qquad\left.+(1-\pi_i(1))\,V_i^{PU}(l)-(1-\delta)\,G_i^{PU}(1)\right\},\\
V_i^{PU}(w) &= \max_{W_i\in\{0,1\}}\left\{W_i\left[-(1-\delta)\,X+\delta V_i^{PU}(C=0,w)\right]+(1-W_i)\,\delta V_i^{PU}(C=1)\right\},\\
V_i^{PU}(l) &= W_{-i}\,\delta V_i^{PU}(C=0,l)+(1-W_{-i})\,\delta V_i^{PU}(C=1),
\end{aligned}$$

where $\pi_i(1)=\dfrac{G_i^{PU}(1)}{G_1^{PU}(1)+G_2^{PU}(1)}$.

Finally, when the resource is not contestable ($C=0$), the expected discounted payoff of player i is given by

$$\begin{aligned}
V_i^{PU}(C=0,w) &= (1-\delta)\,R+\delta\left[(1-\gamma)\,V_i^{PU}(C=0,w)+\gamma V_i^{PU}(C=1)\right],\\
V_i^{PU}(C=0,l) &= \delta\left[(1-\gamma)\,V_i^{PU}(C=0,l)+\gamma V_i^{PU}(C=1)\right].
\end{aligned}$$

Solving for $V_i^{PU}(C=0,w)$ and $V_i^{PU}(C=0,l)$ as a function of $V_i^{PU}(C=1)$ we have

$$V_i^{PU}(C=0,w)=\frac{(1-\delta)\,R+\gamma\delta V_i^{PU}(C=1)}{1-\delta(1-\gamma)},\quad V_i^{PU}(C=0,l)=\frac{\delta\gamma V_i^{PU}(C=1)}{1-\delta(1-\gamma)}.$$

Also note that $W_i=1$ if and only if

$$-(1-\delta)\,X+\delta V_i^{PU}(C=0,w)\geq\delta V_i^{PU}(C=1).$$

Introducing $V_i^{PU}(C=0,w)$ we have that $W_i=1$ if and only if

$$-X+\frac{\delta R}{1-\delta(1-\gamma)}\geq\frac{\delta V_i^{PU}(C=1)}{1-\delta(1-\gamma)}.$$

We look for a symmetric equilibrium in which $V_i^{PU}(C=1)=V^{PU}$ for both players. Thus, in equilibrium, there are only two possible situations to consider: either

$$-X+\frac{\delta R}{1-\delta(1-\gamma)}\geq\frac{\delta V^{PU}}{1-\delta(1-\gamma)},$$

and hence $W_i = 1$ for $i = 1, 2$, or

$$-X + \frac{\delta R}{1 - \delta (1 - \gamma)} < \frac{\delta V^{PU}}{1 - \delta (1 - \gamma)},$$

and hence $W_i = 0$ for $i = 1, 2$.

Equilibrium with elimination: Assume that, in equilibrium,

$$-X + \frac{\delta R}{1 - \delta (1 - \gamma)} \geq \frac{\delta V^{PU}}{1 - \delta (1 - \gamma)}.$$

Then we have

$$\begin{aligned}
V_i^{PU} (C = 1, Z = 0) &= \max_{G_i(0)>0} \left\{(1 - \delta) \left[\pi_i (0) \theta R - G_i^{PU} (0)\right] + \delta V^{PU}\right\}, \\
V_i^{PU} (C = 1, Z = 1) &= \max_{G_i(1)>0} \left\{(1 - \delta) \left[\pi_i (1) R^E (\gamma) - G_i^{PU} (1)\right] + \frac{\delta^2 \gamma V^{PU}}{1 - \delta + \delta\gamma}\right\}, \\
V^{PU} &= (1 - \phi) V_i^{PU} (C = 1, Z = 0) + \phi V_i^{PU} (C = 1, Z = 1),
\end{aligned}$$

where $R^E (\gamma) = \theta R - X + \dfrac{\delta R}{1 - \delta + \delta\gamma}$.

For $Z = 1$, the unique Nash equilibrium guns profile is $G_1^{PU} (1) = G_2^{PU} (1) = G^E (\gamma) = R^E (\gamma) /4$, which implies

$$V_i^{PU} (C = 1, Z = 1) = \frac{(1 - \delta) R^E (\gamma)}{4} + \frac{\delta^2 \gamma V^{PU}}{1 - \delta + \delta\gamma}.$$

For $Z = 0$, the unique Nash equilibrium guns profile is $G_1^{PU} (0) = G_2^{PU} (0) = G^{NN} = \theta R/4$, which implies that

$$V_i^{PU} (C = 1, Z = 0) = \frac{(1 - \delta) \theta R}{4} + \delta V^{PU}.$$

Therefore,

$$V^{PU} = V^E (\gamma) = \frac{(1 - \delta + \delta\gamma) \left[\phi R^E (\gamma) + (1 - \phi) \theta R\right]}{4 (1 - \delta) [1 - \delta (1 - \phi - \gamma)]}.$$

Finally, we must verify that

$$-X + \frac{\delta R}{1 - \delta (1 - \gamma)} \geq \frac{\delta V^{PU}}{1 - \delta (1 - \gamma)},$$

which holds if and only if

$$X \leq \overline{X} (\gamma) = \frac{[(4 - \theta) (1 - \delta + \delta\gamma) + 3\delta\phi] \delta R}{(1 - \delta + \delta\gamma) [4 (1 - \delta + \delta\gamma) + 3\delta\phi]}.$$

Moreover, note that

$$-X + \frac{\delta R}{1 - \delta + \delta\gamma} \geq \frac{\delta V^{PU}}{1 - \delta (1 - \gamma)}$$

implies that $R^E(\gamma) = \theta R - X + \dfrac{\delta R}{1-\delta+\delta\gamma} > 0$, and hence $G^E(\gamma) = R^E(\gamma)/4 > 0$.

Equilibrium without elimination: Assume that, in equilibrium,

$$-X + \frac{\delta R}{1-\delta(1-\gamma)} < \frac{\delta V^{PU}}{1-\delta(1-\gamma)}.$$

Then we have

$$V_i^{PU}(C=1,Z) = \max_{G_i(Z)>0} \left\{(1-\delta)\left[\pi_i(Z)\theta R - G_i^{PU}(Z)\right] + \delta V^{PU}\right\} \quad \text{for } Z=0,1,$$
$$V^{PU} = (1-\phi)V_i^{PU}(C=1,Z=0) + \phi V_i^{PU}(C=1,Z=1).$$

Solving, we have that the unique Nash equilibrium guns profile is $G_1^{PU}(Z) = G_2^{PU}(Z) = G^{NN} = \theta R/4$ for $Z=0,1$, which implies

$$V_i^{PU}(C=1,Z) = V^{PU} = \frac{\theta R}{4}.$$

Finally, we must verify that

$$-X + \frac{\delta R}{1-\delta(1-\gamma)} < \frac{\delta V^{PU}}{1-\delta(1-\gamma)},$$

which holds if and only if

$$X > \frac{(4-\theta)\delta R}{4(1-\delta+\delta\gamma)}.$$

Thus, we have three possible cases to consider:

Case 1: If

$$X < \frac{(4-\theta)\delta R}{4(1-\delta+\delta\gamma)},$$

then the only equilibrium is $G_1^{PU}(0) = G_2^{PU}(0) = \theta R/4$ for $Z=0$ and $G_1^{PU}(1) = G_2^{PU}(1) = R^E(\gamma)/4$ and $W_1 = W_2 = 1$ for $Z=1$. Thus, the equilibrium expected payoff is $V^{PU} = V^E(\gamma)$.

Case 2: If $X > \overline{X}(\gamma)$, then the only equilibrium is $G_1^{PU}(0) = G_2^{PU}(0) = \theta R/4$ for $Z=0$ and $G_1^{PU}(1) = G_2^{PU}(1) = \theta R/4$ and $W_1 = W_2 = 0$ for $Z=1$. Thus, the equilibrium discounted expected payoff is $V^{PU} = V^{NN} = \dfrac{\theta R}{4}$.

Case 3: If

$$\frac{(4-\theta)\delta R}{4(1-\delta+\delta\gamma)} \leq X \leq \overline{X}(\gamma),$$

then there are two possible equilibria, described in cases 1 and 2, respectively. We have $V^E(\gamma) < V^{NN}$ if and only if

$$X > \frac{\delta(1-\theta)R}{1-\delta+\delta\gamma}.$$

Moreover, note that

$$\frac{(4-\theta)\delta R}{4(1-\delta+\delta\gamma)} > \frac{\delta(1-\theta)R}{1-\delta+\delta\gamma},$$

which implies that $V^E(\gamma) < V^{NN}$ for all

$$\frac{(4-\theta)\,\delta R}{4\,(1-\delta+\delta\gamma)} \leq X \leq \overline{X}(\gamma)\,.$$

Thus, whenever both equilibria exist, the equilibrium described in case 1 (the one with elimination) is more severe.

This completes the proof of Lemma 3. ■

Lemma 4. *Suppose that countries are following either the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ or the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$. Assume that $X \leq \overline{X}(\gamma)$.*

1. *Assume that $Z = 0$. Then, the most profitable deviation in guns is*

$$g^D\big(G(0), \theta R\big) = \sqrt{G(0)\,\theta R} - G(0).$$

The associated deviation payoff is

$$V^D\big(G(0)\big) = (1-\delta)\left[\frac{g^D\big(G(0),\theta R\big)}{g^D\big(G(0),\theta R\big) + G(0)}\,\theta R - g^D\big(G(0),\theta R\big)\right] + \delta V^E(\gamma).$$

2. *Assume that $Z = 1$. Then, the most profitable deviation in guns is*

$$g^D\big(G(1), R^E(\gamma)\big) = \sqrt{G(1)\,R^E(\gamma)} - G(1),$$

which leads to open conflict and wipe-out. The associated deviation payoff is

$$V^D\big(G(1)\big) = (1-\delta)\left[\frac{g^D\big(G(1),R^E(\gamma)\big)}{g^D\big(G(1),R^E(\gamma)\big) + G(1)}\,R^E(\gamma) - g^D\big(G(1),R^E(\gamma)\big)\right] + \frac{\gamma\delta^2 V^E(\gamma)}{1-\delta+\delta\gamma}.$$

Proof: Suppose that countries are following either the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ or the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$.

Suppose that $C = 1$ and $Z = 0$.

Deviation in the settlement stage: Consider a unilateral deviation to open conflict, i.e., $S_i = 0$. Then, the expected payoff of player i under this deviation is

$$V_i^D(C=1, Z=0) = (1-\delta)\left[\frac{\theta R}{2} - G(0)\right] + \delta V^{PU},$$

where $V^{PU} = V^E(\gamma)$ (since $X \leq \overline{X}(\gamma)$ by Lemma 3). If instead player i chooses $S_i = 1$, then it gets

$$V_i(0) = (1-\delta)\left[\frac{R}{2} - G(0)\right] + \delta V^C,$$

where $V^C \geq V^{PU}$ is the expected payoff under cooperation. Therefore, for $Z = 0$, players do not have an incentive to unilaterally deviate to open conflict in the settlement stage.

Deviation in the gun choice: If i deviates in guns, its expected payoff is

$$\max_{G_i \geq 0} \left\{ V_i^D = (1-\delta) \left[\frac{G_i}{G_i + G(0)}\, \theta R - G_i \right] + \delta V^E(\gamma) \right\}.$$

The most profitable deviation for player i is

$$g^D\big(G(0), \theta R\big) = \sqrt{G(0)\,\theta R} - G(0).$$

Thus, the associated deviation payoff is

$$V^D\big(G(0)\big) = (1-\delta) \left[\frac{g^D\big(G(0), \theta R\big)}{g^D\big(G(0), \theta R\big) + G(0)}\, \theta R - g^D\big(G(0), \theta R\big) \right] + \delta V^E(\gamma).$$

Suppose now that $Z = 1$.

Deviation in the settlement stage: Consider a deviation to open conflict, i.e., $S_i = 0$. Then, the expected payoff of player i under this deviation is

$$\begin{gathered}
V_i^D(C=1, Z=1) = \pi_i(1)\,[(1-\delta)\theta R + V_i(w)] + \big[1 - \pi_i(1)\big] V_i(l) - (1-\delta) G_i(1), \\
V_i(w) = \max_{W_i \in \{0,1\}} \left\{ W_i\,[-(1-\delta)X + \delta V_i(C=0, w)] + (1 - W_i)\delta V^{PU} \right\}, \\
V_i(l) = W_{-i}\, \delta V_i(C=0, l) + (1 - W_{-i})\, \delta V^{PU}, \\
V_i(C=0, w) = \frac{(1-\delta)R + \gamma\delta V^{PU}}{1 - \delta + \delta\gamma}, \qquad V_i(C=0, l) = \frac{\gamma\delta V^{PU}}{1-\delta+\delta\gamma},
\end{gathered}$$

where $\pi_i(1) = 1/2$ (since both guns are $G(1)$ before deviating) and V^{PU} is the punishment payoff. Since $X \leq \overline{X}(\gamma)$, we have $V^{PU} = V^E(\gamma)$ by Lemma 3. Therefore,

$$-(1-\delta)X + \delta V_i(C=0, w) \geq \delta V^{PU}$$

if and only if

$$-X + \frac{\delta R}{1-\delta+\delta\gamma} \geq \frac{\delta V^E(\gamma)}{1-\delta+\delta\gamma},$$

which holds whenever $X \leq \overline{X}(\gamma)$. Thus $W_i = W_{-i} = 1$ and, hence,

$$V_i^D(C=1, Z=1) = (1-\delta)\left[\pi_i(1) R^E(\gamma) - G(1)\right] + \frac{\gamma\delta^2 V^E(\gamma)}{1-\delta+\delta\gamma}.$$

Deviation in the gun choice: Since any deviation in gun choices will be immediately punished with open conflict and $X \leq \overline{X}(\gamma)$, the optimal deviation for player i solves

$$\max_{G_i \geq 0} \left\{ V_i^D = (1-\delta) \left[\frac{G_i}{G_i + G(1)}\, R^E(\gamma) - G_i \right] + \frac{\gamma\delta^2 V^E(\gamma)}{1-\delta+\delta\gamma} \right\},$$

which is equivalent (up to an additive constant) to maximizing the one-period contest payoff. Solving, the most profitable deviation for player i is

$$g^D\big(G(1), R^E(\gamma)\big) = \sqrt{G(1)\, R^E(\gamma)} - G(1).$$

Thus, the associated deviation payoff is

$$V^D\big(G(1)\big) = (1-\delta)\left[\frac{g^D\big(G(1), R^E(\gamma)\big)}{g^D\big(G(1), R^E(\gamma)\big) + G(1)}\, R^E(\gamma) - g^D\big(G(1), R^E(\gamma)\big)\right] + \frac{\gamma\delta^2 V^E(\gamma)}{1-\delta+\delta\gamma}.$$

This completes the proof of Lemma 4. ■

Proposition 7. *Suppose that $\gamma > 0$ and $\delta \to 1$. Then, permanent complete cooperation, i.e., disarmed peace with $G(Z) = 0$ for $Z = 0, 1$, can be sustained.*

Proof: In the proof of Proposition 8 we show that the permanent cooperation strategy with $G(0) \in \big[0, \frac{\theta R}{4}\big)$ and $G(1) \in \big[0, \frac{\theta R}{4}\big)$ can be sustained as an equilibrium if and only if

$$F^{PC,0}(G(0), G(1)) = -\,(2-\delta-\delta\phi)\,G(0) + 2(1-\delta)\sqrt{G(0)\theta R} - \delta\phi\, G(1) + F^{PC,0}(0,0) \geq 0,$$

and

$$F^{PC,1}(G(0), G(1)) = -\,(2-2\delta+\delta\phi)\,G(1) + 2(1-\delta)\sqrt{G(1)R^E(\gamma)} - \delta(1-\phi)\,G(0) + F^{PC,1}(0,0) \geq 0,$$

where

$$\begin{aligned}
F^{PC,0}(0,0) &= \frac{R}{2} - (1-\delta)\theta R - \frac{\delta(1-\delta+\delta\gamma)\left[\phi R^E(\gamma) + (1-\phi)\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]},\\
F^{PC,1}(0,0) &= \frac{R}{2} - (1-\delta)R^E(\gamma) - \frac{\gamma\delta^2\left[\phi R^E(\gamma) + (1-\phi)\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]},\\
R^E(\gamma) &= \theta R - X + \frac{\delta R}{1-\delta+\delta\gamma}.
\end{aligned}$$

Taking limits as $\delta \to 1$:

$$\begin{aligned}
\lim_{\delta\to 1} F^{PC,0}(0,0) &= \frac{\left[\phi + (2-\theta)\gamma\right]R + \gamma\phi X}{4(\phi+\gamma)} > 0,\\
\lim_{\delta\to 1} F^{PC,1}(0,0) &= \frac{\left[\phi + (2-\theta)\gamma\right]R + \gamma\phi X}{4(\phi+\gamma)} > 0.
\end{aligned}$$

Thus, when $\delta \to 1$, both $F^{PC,0}(0,0) > 0$ and $F^{PC,1}(0,0) > 0$, which implies the permanent cooperation strategy with $G(0) = G(1) = 0$ can be sustained as an equilibrium. This completes the proof of Proposition 7. ■

Proposition 8. *Suppose that $\gamma \geq 0$ and*

$$X \leq \overline{X}(\gamma) = \frac{\big[(4-\theta)(1-\delta+\delta\gamma) + 3\delta\phi\big]\delta R}{(1-\delta+\delta\gamma)\big[4(1-\delta+\delta\gamma) + 3\delta\phi\big]}.$$

There are thresholds $\overline{\theta}^{PC}_{low}$, $\overline{\theta}^{PC}_{high}$, $\overline{\theta}^{TC}$, $\overline{X}_{low}(\gamma)$, and $\overline{X}_{high}(\gamma)$ such that:

1. *Suppose that* $\theta \leq \overline{\theta}_{low}^{PC}$ *or* $\overline{\theta}_{low}^{PC} < \theta \leq \overline{\theta}_{high}^{PC}$ *and* $X \geq \overline{X}_{high}(\gamma)$*. Then, permanent complete cooperation, i.e., disarmed peace with* $G(Z) = 0$ *for* $Z = 0, 1$*, can be sustained.*

2. *Suppose that* $\overline{\theta}_{low}^{PC} < \theta \leq \overline{\theta}_{high}^{PC}$ *and* $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$*. Let*

$$\overline{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^2 \theta R < G^N, \quad \overline{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 R^E(\gamma) < G^E(\gamma),$$

and let $\hat{G}(1) \in (0, \overline{G}(1))$ *be the unique solution to* $F^{PC,1}(0, G(1)) = 0$*.*

   (a) *If* $\hat{G}(1) \leq F^{PC,0}(0,0)/(\delta\phi)$*, then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with* $G(0) = 0$ *and* $G(1) = \hat{G}(1)$*.*

   (b) *If* $\hat{G}(1) > F^{PC,0}(0,0)/(\delta\phi)$*, then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with* $G(0) \in (0, \overline{G}(0))$ *and* $G(1) \in (\hat{G}(1), \overline{G}(1))$ *given by the unique solution to*

$$F^{PC,0}(G(0), G(1)) = F^{PC,1}(G(0), G(1)) = 0$$

   that satisfies

$$\frac{\partial F^{PC,0}(G(0), 0)}{\partial G(0)} \frac{\partial F^{PC,1}(0, G(1))}{\partial G(1)} > \delta^2(1-\phi)\phi.$$

3. *If* $\theta \leq \overline{\theta}^{TC}$*, then the best temporary cooperative equilibrium that can be sustained is* $G(0) = 0$*.*

4. *If* $\theta > \overline{\theta}^{TC}$*, then the best temporary cooperative equilibrium that can be sustained is*

$$G(0) \in \left(0, \left\{\frac{(1-\delta)\left[1-\delta(1-\gamma-\phi)\right]}{(2-\delta+\delta\phi)(1-\delta+\delta\gamma)-2\delta^2\gamma\phi}\right\}^2 \theta R\right)$$

   given by the unique solution to $F^{TC}(G(0)) = 0$*. Moreover,*

$$\left\{\frac{(1-\delta)\left[1-\delta(1-\gamma-\phi)\right]}{(2-\delta+\delta\phi)(1-\delta+\delta\gamma)-2\delta^2\gamma\phi}\right\}^2 \theta R < G^{NN} = \frac{\theta R}{4}.$$

Proof of parts 1 and 2: Suppose that countries are following the permanent cooperation strategy with $G(0) \in [0, \theta R/4)$ and $G(1) \in [0, \theta R/4)$ and that no country deviates. Then, the expected discounted payoffs for each state $Z = 0, 1$ are given by:

$$V^{PC}(C=1, Z=0) = (1-\delta)\left[\frac{R}{2} - G(0)\right] + \delta\left[(1-\phi)V^{PC}(C=1, Z=0) + \phi V^{PC}(C=1, Z=1)\right],$$
$$V^{PC}(C=1, Z=1) = (1-\delta)\left[\frac{R}{2} - G(1)\right] + \delta\left[(1-\phi)V^{PC}(C=1, Z=0) + \phi V^{PC}(C=1, Z=1)\right].$$

Solving, we obtain:

$$V^{PC}(C=1,Z=0)=\frac{R}{2}-G(0)+\delta\phi\big[G(0)-G(1)\big],$$
$$V^{PC}(C=1,Z=1)=\frac{R}{2}-G(1)+\delta(1-\phi)\big[G(1)-G(0)\big].$$

Sustainability for $Z=0$: Suppose that $Z=0$ and player i deviates from cooperation. Then, from Lemma 4, the most profitable deviation for player i is $g^D(G(0),\theta R)=\sqrt{G(0)\theta R}-G(0)$ and the associated deviation payoff is:

$$V^D(G(0))=(1-\delta)\left[\frac{g^D(G(0),\theta R)}{g^D(G(0),\theta R)+G(0)}\theta R-g^D(G(0),\theta R)\right]+\delta V^E(\gamma).$$

Thus, player i does not have an incentive to deviate if and only if $V^{PC}(C=1,Z=0)\geq V^D(G(0))$ or, equivalently,

$$F^{PC,0}(G(0),G(1))=$$
$$-\big(2-\delta-\delta\phi\big)G(0)+2(1-\delta)\sqrt{G(0)\theta R}-\delta\phi G(1)+F^{PC,0}(0,0)\geq 0,$$

where

$$F^{PC,0}(0,0)=\frac{R}{2}-(1-\delta)\theta R-\frac{\delta(1-\delta+\delta\gamma)\big[\phi R^E(\gamma)+(1-\phi)\theta R\big]}{4\big[1-\delta(1-\phi-\gamma)\big]}.$$

Sustainability for $Z=1$: Suppose that $Z=1$ and player i deviates from cooperation. From Lemma 4, the most profitable deviation for player i is $g^D(G(1),R^E(\gamma))=\sqrt{G(1)R^E(\gamma)}-G(1)$, which induces open conflict, after which the victor eliminates the loser. The associated deviation payoff is:

$$V^D(G(1))=(1-\delta)\left[\frac{g^D(G(1),R^E(\gamma))}{g^D(G(1),R^E(\gamma))+G(1)}R^E(\gamma)-g^D(G(1),R^E(\gamma))\right]+\frac{\gamma\delta^2V^E(\gamma)}{1-\delta+\delta\gamma}.$$

Thus, player i does not have an incentive to deviate if and only if $V^{PC}(C=1,Z=1)\geq V^D(G(1))$ or, equivalently,

$$F^{PC,1}(G(0),G(1))=$$
$$-\big(2-2\delta+\delta\phi\big)G(1)+2(1-\delta)\sqrt{G(1)R^E(\gamma)}-\delta(1-\phi)G(0)+F^{PC,1}(0,0)\geq 0,$$

where

$$F^{PC,1}(0,0)=\frac{R}{2}-(1-\delta)R^E(\gamma)-\frac{\gamma\delta^2\big[\phi R^E(\gamma)+(1-\phi)\theta R\big]}{4\big[1-\delta(1-\phi-\gamma)\big]}.$$

Best possible permanent cooperation equilibrium: To determine the best possible permanent cooperative equilibrium, we solve

$$\max_{G(0)\geq 0,G(1)\geq 0}\left\{\phi V^{PC}(C=1,Z=1)+(1-\phi)V^{PC}(C=1,Z=0)=\frac{R}{2}-\phi G(1)-(1-\phi)G(0)\right\}$$
$$\text{s.t. } F^{PC,0}(G(0),G(1))\geq 0,\quad F^{PC,1}(G(0),G(1))\geq 0.$$

Let

$$\overline{G}(0) = \left(\frac{1-\delta}{2-\delta-\delta\phi}\right)^2 \theta R, \quad \overline{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 R^E(\gamma).$$

Following the same procedure as in the proof of Proposition 5, we obtain that there is no solution such that $G(0) > \overline{G}(0)$ or $G(1) > \overline{G}(1)$, and that $F^{PC,0}$ and $F^{PC,1}$ are quasiconcave functions for all $(G(0), G(1))$.

Since the objective function is linear and the constraints are quasiconcave, the following Kuhn–Tucker conditions are sufficient for a global maximum:

$$\begin{gathered}
-(1-\phi) + \lambda^0\Big[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\tfrac{\theta R}{G(0)}}\Big] - \lambda^1\delta(1-\phi) + \mu_L^0 - \mu_H^0 = 0, \\
-\phi - \lambda^0\delta\phi + \lambda^1\Big[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\tfrac{R^E(\gamma)}{G(1)}}\Big] + \mu_L^1 - \mu_H^1 = 0, \\
\lambda^0 \geq 0,\ F^{PC,0}(G(0), G(1)) \geq 0,\ \lambda^0 F^{PC,0}(G(0), G(1)) = 0, \\
\lambda^1 \geq 0,\ F^{PC,1}(G(0), G(1)) \geq 0,\ \lambda^1 F^{PC,1}(G(0), G(1)) = 0, \\
\mu_L^0 \geq 0,\ G(0) \geq 0,\ \mu_L^0 G(0) = 0, \\
\mu_H^0 \geq 0,\ \overline{G}(0) - G(0) \geq 0,\ \mu_H^0\big[\overline{G}(0) - G(0)\big] = 0, \\
\mu_L^1 \geq 0,\ G(1) \geq 0,\ \mu_L^1 G(1) = 0, \\
\mu_H^1 \geq 0,\ \overline{G}(1) - G(1) \geq 0,\ \mu_H^1\big[\overline{G}(1) - G(1)\big] = 0.
\end{gathered}$$

There are several cases to consider.

Case 1: Suppose that $G(0) = G(1) = 0$. Solving, we obtain $\lambda^0 = \mu_H^0 = 0$, $\mu_L^0 = 1-\phi$, $\lambda^1 = \mu_H^1 = 0$, $\mu_L^1 = \phi$, $F^{PC,0}(0,0) \geq 0$, and $F^{PC,1}(0,0) \geq 0$. Since $F^{PC,1}(0,0) \geq 0$ implies $F^{PC,0}(0,0) \geq 0$, we only need to check $F^{PC,1}(0,0) \geq 0$. This holds if and only if

$$X \geq \overline{X}_{high}(\gamma) = \frac{(1-\delta+\rho)\theta R}{1-\delta+\phi\rho} + \frac{\delta R}{1-\delta+\delta\gamma} - \frac{R}{2(1-\delta+\phi\rho)},$$

where

$$\rho = \frac{\gamma\delta^2}{4\big[1-\delta(1-\phi-\gamma)\big]}.$$

If $\theta \leq \overline{\theta}_{low}^{PC}$, where

$$\overline{\theta}_{low}^{PC} = \frac{(1-\delta+\delta\gamma) - 2\delta(1-\delta+\phi\rho)}{2(1-\delta+\rho)(1-\delta+\delta\gamma)},$$

this constraint always holds because $X > 0$. If $\theta > \overline{\theta}_{low}^{PC}$, we need $\overline{X}_{high}(\gamma) \leq \overline{X}(\gamma)$, which holds if and only if

$$\theta \leq \overline{\theta}_{high}^{PC} = \frac{4(1-\delta+\delta\gamma) + 3\delta\phi}{2(1-\delta+\rho)\big[4(1-\delta+\delta\gamma) + 3\delta\phi\big] + 2\delta(1-\delta+\phi\rho)}.$$

Summing up, $G(0) = G(1) = 0$ is a solution if and only if (i) $\theta \leq \overline{\theta}_{low}^{PC}$ or (ii) $X \geq \overline{X}_{high}(\gamma)$ and $\overline{\theta}_{low}^{PC} < \theta \leq \overline{\theta}_{high}^{PC}$.

Case 2: Suppose that $G(0) \in (0, \overline{G}(0))$ and $G(1) = 0$. Solving, we obtain $\mu_L^0 = \mu_H^0 = \mu_H^1 = 0$, $\lambda^1 = 0$,

$$\lambda^0 = \frac{1-\phi}{-\big(2-\delta-\delta\phi\big) + (1-\delta)\sqrt{\theta R/G(0)}} > 0, \quad \mu_L^1 = \phi(1+\lambda^0\delta) > 0,$$

and $F^{PC,0}(G(0),0) = 0$, $F^{PC,1}(G(0),0) \geq 0$. Since $F^{PC,0}(G(0),0)$ is strictly increasing in $G(0)$ for all $G(0) \in [0, \overline{G}(0)]$, there is at most one solution to $F^{PC,0}(G(0),0) = 0$. Moreover, a solution with $G(0) \in (0, \overline{G}(0))$ exists if and only if $F^{PC,0}(0,0) < 0$ and $F^{PC,0}(\overline{G}(0),0) > 0$. However, $F^{PC,0}(0,0) < 0$ implies $F^{PC,1}(0,0) < 0$ (by Result 3) and $F^{PC,1}(0,0) < 0$ implies $F^{PC,1}(G(0),0) < 0$ (since $F^{PC,1}(G(0),0)$ is strictly decreasing in $G(0)$), so $F^{PC,1}(G(0),0) \geq 0$ cannot hold. Thus, there is no solution with $G(0) \in (0, \overline{G}(0))$ and $G(1) = 0$.

Case 3: Suppose that $G(0) = 0$ and $G(1) \in (0, \overline{G}(1))$. Solving, we obtain $\mu_H^0 = \mu_L^1 = \mu_H^1 = 0$, $\lambda^0 = 0$,

$$\mu_L^0 = (1-\phi) + \lambda^1\delta(1-\phi) > 0, \quad \lambda^1 = \frac{\phi}{-\big(2-2\delta+\delta\phi\big) + (1-\delta)\sqrt{R^E(\gamma)/G(1)}} > 0,$$

and $F^{PC,1}(0,G(1)) = 0$, $F^{PC,0}(0,G(1)) \geq 0$. Since $F^{PC,1}(0,G(1))$ is strictly increasing in $G(1)$ for $G(1) \in [0, \overline{G}(1)]$, there is at most one solution to $F^{PC,1}(0,G(1)) = 0$. A solution with $G(1) \in (0, \overline{G}(1))$ exists if and only if $F^{PC,1}(0,0) < 0$ and $F^{PC,1}(0,\overline{G}(1)) > 0$. The inequality $F^{PC,1}(0,0) < 0$ holds if and only if $X < \overline{X}_{high}(\gamma)$; and $F^{PC,1}(0,\overline{G}(1)) > 0$ holds if and only if $X > \overline{X}_{low}(\gamma)$, where

$$\begin{aligned}\overline{X}_{low}(\gamma) = &\left[\frac{(1-\delta+\delta\phi)(1-\delta+\rho)+\rho(1-\delta)}{(1-\delta+\delta\phi)(1-\delta+\rho\phi)+\rho\phi(1-\delta)}\right]\theta R \\ &+ \frac{\delta R}{1-\delta+\delta\gamma} - \frac{(2-2\delta+\delta\phi)R}{2\big[(1-\delta+\delta\phi)(1-\delta+\rho\phi)+\rho\phi(1-\delta)\big]}.\end{aligned}$$

Hence we require $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$. We must also have $\overline{X}_{high}(\gamma) > 0$, which holds if and only if $\theta > \overline{\theta}_{low}^{PC}$, and $\overline{X}_{high}(\gamma) \leq \overline{X}(\gamma)$, which holds if and only if $\theta \leq \overline{\theta}_{high}^{PC}$. Finally, we must verify that the unique solution to $F^{PC,1}(0,G(1)) = 0$ satisfies $F^{PC,0}(0,G(1)) \geq 0$, which is equivalent to $G(1) \leq F^{PC,0}(0,0)/(\delta\phi)$.

Summing up, $G(0) = 0$ and $G(1) = \hat{G}(1) \in (0, \overline{G}(1))$, where $\hat{G}(1)$ is the unique solution to $F^{PC,1}(0,G(1)) = 0$, is a solution if and only if $\overline{\theta}_{low}^{PC} < \theta \leq \overline{\theta}_{high}^{PC}$, $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$, and $\hat{G}(1) \leq F^{PC,0}(0,0)/(\delta\phi)$.

Case 4: Suppose that $G(0) \in (0, \overline{G}(0))$ and $G(1) \in (0, \overline{G}(1))$. Solving, we obtain $\mu_L^0 = \mu_L^1 = \mu_H^0 = \mu_H^1 = 0$, and

$$\begin{aligned}\lambda^0\Big[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\tfrac{\theta R}{G(0)}}\Big] - \lambda^1\delta(1-\phi) &= 1-\phi, \\ -\lambda^0\delta\phi + \lambda^1\Big[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\tfrac{R^E(\gamma)}{G(1)}}\Big] &= \phi.\end{aligned}$$

This linear system has a solution if and only if

$$\Big[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\tfrac{\theta R}{G(0)}}\Big]\Big[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\tfrac{R^E(\gamma)}{G(1)}}\Big] \neq \delta^2\phi(1-\phi).$$

A relevant solution must satisfy either $\lambda^0 > 0$ and $\lambda^1 > 0$, or $\lambda^0 < 0$ and $\lambda^1 < 0$. The case $\lambda^0 > 0$ and $\lambda^1 > 0$ holds if and only if

$$\Big[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\tfrac{\theta R}{G(0)}}\Big]\Big[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\tfrac{R^E(\gamma)}{G(1)}}\Big] > \delta^2\phi(1-\phi). \qquad (*)$$

In that case we also have $F^{PC,0}(G(0),G(1))=0$ and $F^{PC,1}(G(0),G(1))=0$, so we must solve:

$$F^{PC,0}(G(0),0)-\delta\phi G(1)=0,$$
$$F^{PC,1}(0,G(1))-\delta(1-\phi)G(0)=0.$$

We need $F^{PC,1}(0,\overline{G}(1))>0$; otherwise, $F^{PC,1}(0,G(1))<0$ for all $G(1)\in[0,\overline{G}(1)]$ and there is no solution with $G(0)>0$. Also assume $F^{PC,1}(0,0)<0$; otherwise the solution is $G(0)=G(1)=0$ (case 1). Since $F^{PC,1}(0,G(1))$ is strictly increasing in $G(1)\in[0,\overline{G}(1)]$, any solution with $G(0)>0$ must satisfy $G(1)>\hat{G}(1)$, where $\hat{G}(1)$ is the unique solution to $F^{PC,1}(0,G(1))=0$.

Define

$$Q(G(1)) = F^{PC,0}\Big(\frac{F^{PC,1}(0,G(1))}{\delta(1-\phi)},0\Big)-\delta\phi G(1).$$

Since $F^{PC,1}(0,G(1))$ is strictly concave in $G(1)$ and $F^{PC,0}(G(0),0)$ is strictly concave in $G(0)$, $Q(G(1))$ is strictly concave in $G(1)$. Moreover,

$$\begin{aligned}\lim_{G(1)\to\hat{G}(1)}\frac{\partial Q(G(1))}{\partial G(1)} &= \lim_{G(1)\to\hat{G}(1)}\left[\frac{\partial F^{PC,0}\big(\frac{F^{PC,1}(0,G(1))}{\delta(1-\phi)},0\big)}{\partial G(0)}\frac{\partial F^{PC,1}(0,G(1))}{\partial G(1)}\frac{1}{\delta(1-\phi)}-\delta\phi\right]\\ &= \left(\lim_{G(0)\to 0}\frac{\partial F^{PC,0}(G(0),0)}{\partial G(0)}\right)\frac{\partial F^{PC,1}(0,\hat{G}(1))}{\partial G(1)}\frac{1}{\delta(1-\phi)}-\delta\phi,\end{aligned}$$

where $\frac{\partial F^{PC,1}(0,\hat{G}(1))}{\partial G(1)}>0$ and $\lim_{G(0)\to 0}\frac{\partial F^{PC,0}(G(0),0)}{\partial G(0)}=+\infty$. Hence $Q(G(1))$ is strictly increasing as $G(1)\to\hat{G}(1)$, and therefore either it is strictly increasing for all $G(1)\in[\hat{G}(1),\overline{G}(1)]$, or it is increasing on $[\hat{G}(1),G^m]$, attains a maximum at G^m, and is strictly decreasing on $[G^m,\overline{G}(1)]$. This yields the subcases 4.a–4.e discussed in the text. The only subcases compatible with (*) and the Kuhn–Tucker conditions are those where $Q(\hat{G}(1))<0$ and $Q(\overline{G}(1))>0$, which deliver a unique solution $G(1)\in(\hat{G}(1),\overline{G}(1))$ with $\frac{\partial Q(G(1))}{\partial G(1)}>0$.

Summing up, $G(0)\in(0,\overline{G}(0))$ and $G(1)\in(0,\overline{G}(1))$ given by the unique solution to

$$F^{PC,0}(G(0),G(1))=0, \quad F^{PC,1}(G(0),G(1))=0,$$

and satisfying

$$\frac{\partial F^{PC,0}(G(0),0)}{\partial G(0)}\frac{\partial F^{PC,1}(0,G(1))}{\partial G(1)} > \delta^2(1-\phi)\phi,$$

is a solution if and only if $F^{PC,1}(0,0)<0$, $F^{PC,1}(0,\overline{G}(1))>0$, and $Q(\hat{G}(1))<0$, where $\hat{G}(1)\in(0,\overline{G}(1))$ is the unique solution to $F^{PC,1}(0,G(1))=0$. From case 3 we already know that $F^{PC,1}(0,0)<0$ and

$F^{PC,1}(0, \overline{G}(1)) > 0$ hold if and only if $\overline{\theta}^{PC}_{low} < \theta \leq \overline{\theta}^{PC}_{high}$ and $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$. Finally,

$$Q(\hat{G}(1)) = F^{PC,0}\left(\frac{F^{PC,1}(0, \hat{G}(1))}{\delta(1-\phi)}, 0\right) - \delta\phi\hat{G}(1) = F^{PC,0}(0,0) - \delta\phi\hat{G}(1),$$

so $Q(\hat{G}(1)) < 0$ if and only if $\hat{G}(1) > F^{PC,0}(0,0)/(\delta\phi)$.

Case 5: Suppose that $G(0) = \overline{G}(0)$ or $G(1) = \overline{G}(1)$. If $G(0) = \overline{G}(0)$, then $\mu^0_L = 0$ and hence

$$(1-\phi) + \lambda^1\delta(1-\phi) + \mu^0_H = 0,$$

a contradiction because $\lambda^1 \geq 0$ and $\mu^0_H \geq 0$. If $G(1) = \overline{G}(1)$, then $\mu^1_L = 0$ and hence

$$\phi + \lambda^0\delta\phi + \mu^1_H = 0,$$

a contradiction because $\lambda^0 \geq 0$ and $\mu^1_H \geq 0$.

Proof of parts 3 and 4: Suppose that countries are following the temporary cooperation strategy with $G(0) \in [0, \theta R/4)$ and that no country deviates. Then, the expected discounted payoffs are:

$$V^{TC}(C=1, Z=0) = (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta V^{TC}(C=1),$$
$$V^{TC}(C=1) = (1-\phi)V^{TC}(C=1, Z=0) + \phi V^{TC}(C=1, Z=1),$$
$$V^{TC}(C=1, Z=1) = (1-\delta)\frac{R^E(\gamma)}{4} + \frac{\delta^2\gamma V^{TC}(C=1)}{1-\delta(1-\gamma)}.$$

Solving, we obtain:

$$V^{TC}(C=1, Z=0) = \frac{(1-\delta)\left[1 - \frac{\phi\delta^2\gamma}{1-\delta(1-\gamma)}\right]\left(\frac{R}{2} - G(0)\right)}{1 - \frac{\phi\delta^2\gamma}{1-\delta(1-\gamma)} - \delta(1-\phi)} + \frac{(1-\delta)\delta\phi\,\frac{R^E(\gamma)}{4}}{1 - \frac{\phi\delta^2\gamma}{1-\delta(1-\gamma)} - \delta(1-\phi)},$$
$$V^{TC}(C=1) = \frac{(1-\phi)V^{TC}(C=1, Z=0) + (1-\delta)\phi\,\frac{R^E(\gamma)}{4}}{1 - \frac{\phi\delta^2\gamma}{1-\delta(1-\gamma)}}.$$

Sustainability for $Z = 0$: Suppose that $Z = 0$ and player i deviates. From Lemma 4, the most profitable deviation is $g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0)$, with payoff

$$V^D(G(0)) = (1-\delta)\left[\frac{g^D(G(0), \theta R)}{g^D(G(0), \theta R) + G(0)}\theta R - g^D(G(0), \theta R)\right] + \delta V^E(\gamma).$$

Thus player i does not deviate if and only if $V^{TC}(C=1, Z=0) \geq V^D(G(0))$, which is equivalent to

$$F^{TC}(G(0)) = -\left[\frac{(2-\delta)(1-\delta+\delta\gamma) + (1-\delta-\delta\gamma)\delta\phi}{1-\delta+\delta\gamma}\right]G(0)$$
$$+\frac{2(1-\delta)\left[1-\delta(1-\gamma-\phi)\right]}{1-\delta+\delta\gamma}\sqrt{G(0)\theta R} + F^{TC}(0) \geq 0,$$

where

$$F^{TC}(0) = \left[\frac{1-\delta(1-\gamma)-\phi\delta^2\gamma}{1-\delta+\delta\gamma}\right]\frac{R}{2} - \left[\frac{(1-\delta+\delta\gamma)\delta(1-\phi)+4(1-\delta)\big[1-\delta(1-\phi-\gamma)\big]}{4(1-\delta+\delta\gamma)}\right]\theta R.$$

Best possible temporary cooperation equilibrium: We solve

$$\max_{G(0)\geq 0}\Big\{(1-\phi)V^{TC}(C=1,Z=0)+\phi V^{TC}(C=1,Z=1) = \frac{(1-\delta)(1-\delta+\delta\gamma)}{1-\delta(1-\gamma-\phi)}\left[(1-\phi)\left(\frac{R}{2}-G(0)\right)+\frac{\phi R^E(\gamma)}{4}\right]\Big\}$$
$$\text{s.t. } F^{TC}(G(0))\geq 0.$$

There are two cases:
Case 1: If

$$\theta \leq \overline{\theta}^{TC} = \frac{2\big[1-\delta(1-\gamma)-\phi\delta^2\gamma\big]}{(1-\delta+\delta\gamma)\delta(1-\phi)+4(1-\delta)\big[1-\delta(1-\phi-\gamma)\big]},$$

then $F^{TC}(0)\geq 0$ and $G(0)=0$ solves the problem.
Case 2: If $\theta > \overline{\theta}^{TC}$, then $F^{TC}(0)<0$ and

$$\frac{\partial F^{TC}(G(0))}{\partial G(0)} = -\left[\frac{(2-\delta)(1-\delta+\delta\gamma)+(1-\delta-\delta\gamma)\delta\phi}{1-\delta+\delta\gamma}\right] + \frac{(1-\delta)\big[1-\delta(1-\gamma-\phi)\big]}{1-\delta+\delta\gamma}\sqrt{\frac{\theta R}{G(0)}}.$$

Hence $F^{TC}(G(0))$ is strictly increasing on $[0,\tilde{G}(0)]$ and strictly decreasing on $[\tilde{G}(0),\theta R/4]$, where

$$\tilde{G}(0) = \left\{\frac{(1-\delta)\big[1-\delta(1-\gamma-\phi)\big]}{(2-\delta)(1-\delta+\delta\gamma)+(1-\delta-\delta\gamma)\delta\phi}\right\}^2\theta R,$$

and $\tilde{G}(0)\leq \theta R/4$ with strict inequality for $\delta<1$. Moreover, $F^{TC}(\tilde{G}(0))>0$. Thus the solution is $G(0)\in(0,\tilde{G}(0))$ given by the unique solution to $F^{TC}(G(0))=0$.

This completes the proof of Proposition 8. ■

Corollary 2. *Suppose that $\gamma>0$. Open conflict is unavoidable if and only if*

$$X \leq \begin{cases}\overline{X}(\gamma) & \text{if } \theta > \overline{\theta}^{PC}_{high},\\ \overline{X}_{low}(\gamma) & \text{if } \overline{\theta}^{PC}_{low} < \theta \leq \overline{\theta}^{PC}_{high}.\end{cases}$$

Moreover:

1. *A lower cost of the elimination technology (smaller X) enlarges the parameter region in which open conflict is unavoidable.*
2. *The destructiveness of open conflict has a non-monotonic effect on this region. As destructiveness initially declines (higher θ), the range of X for which open conflict is unavoidable expands, but for sufficiently large θ it shrinks, so further reductions in destructiveness make open conflict avoidable for a larger set of (X,γ).*

3. If $\theta > \overline{\theta}^{PC}_{high}$, a higher γ shrinks the set of X for which open conflict is unavoidable.

Proof: From Proposition 8, open conflict cannot be avoided if and only if

$$\left[\theta > \overline{\theta}^{PC}_{high} \text{ and } X \leq \overline{X}(\gamma)\right] \quad \text{or} \quad \left[\overline{\theta}^{PC}_{low} < \theta \leq \overline{\theta}^{PC}_{high} \text{ and } X \leq \overline{X}_{low}(\gamma)\right].$$

Clearly, the lower the cost of the elimination technology (the lower X), the larger the subset of (θ, γ) for which at least one of these conditions holds; hence the larger the parameter region where open conflict is unavoidable.

Note that $\overline{X}_{low}(\gamma)$ is strictly increasing in θ, while $\overline{X}(\gamma)$ is strictly decreasing in θ. Moreover, $\overline{X}_{low}(\gamma) < \overline{X}(\gamma)$ for $\theta \leq \overline{\theta}^{PC}_{high}$. Thus, as θ increases, the cutoff in X that makes open conflict unavoidable shifts upward when $\theta < \overline{\theta}^{PC}_{high}$ and downward when $\theta \geq \overline{\theta}^{PC}_{high}$. Equivalently, the region in (X, θ)–space where open conflict is unavoidable first expands and then shrinks in θ.

Finally, suppose that $\theta > \overline{\theta}^{PC}_{high}$. Since $\overline{X}(\gamma)$ is strictly decreasing in γ, an increase in γ reduces $\overline{X}(\gamma)$ and therefore shrinks the set of X for which open conflict is unavoidable.

This completes the proof of Corollary 2. ■

Proposition 9. *Suppose that $\overline{\theta}^{PC}_{low} < \theta \leq \overline{\theta}^{PC}_{high}$ and $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$. Then, partial permanent cooperation induces higher ex-ante expected payoff than temporary cooperation if and only if*

$$\Delta = F^{PC,0}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi) G^{PC}(0) - \frac{(1-\delta+\delta\gamma)}{1-\delta(1-\gamma-\phi)}\left[F^{TC}(0) - \delta(1-\phi) G^{TC}(0)\right] \geq 0$$

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\theta R - \frac{\delta(1-\delta+\delta\gamma)\left[\phi R^E(\gamma) + (1-\phi)\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]},$$

$$F^{TC}(0) = \left[\frac{1-\delta(1-\gamma) - \phi\delta^2\gamma}{1-\delta+\delta\gamma}\right]\frac{R}{2} - \left[\frac{(1-\delta+\delta\gamma)\delta(1-\phi) + 4(1-\delta)\left[1-\delta(1-\phi-\gamma)\right]}{4(1-\delta+\delta\gamma)}\right]\theta R.$$

$G^{PC}(0)$ and $G^{PC}(1)$ are given by Proposition 8.2 and $G^{TC}(0)$ by Propositions 8.3 and 8.4. Moreover, if $\theta > \overline{\theta}^{TC}$ and $\hat{G}(1) \leq F^{PC,0}(0,0)/(\delta\phi)$, then $\Delta \geq 0$, while if $\theta \leq \overline{\theta}^{TC}$ and $\hat{G}(1) > F^{PC,0}(0,0)/(\delta\phi)$, then $\Delta < 0$.

Proof: The ex-ante expected payoff under partial permanent cooperation is

$$V^{PC} = \frac{R}{2} - \phi G^{PC}(1) - (1-\phi) G^{PC}(0),$$

where $G^{PC}(0)$ and $G^{PC}(1)$ are from Proposition 8.2.

The ex-ante expected payoff under temporary cooperation is

$$V^{TC} = \frac{(1-\delta)\left[(1-\phi)\left(\frac{R}{2} - G^{TC}(0)\right) + \frac{\phi R^E(\gamma)}{4}\right]}{1 - \frac{\phi\delta^2\gamma}{1-\delta(1-\gamma)} - \delta(1-\phi)} =$$

$$= \frac{1-\delta+\delta\gamma}{1-\delta(1-\gamma-\phi)}\left[(1-\phi)\left(\frac{R}{2} - G^{TC}(0)\right) + \frac{\phi R^E(\gamma)}{4}\right],$$

where $G^{TC}(0)$ is given by Propositions 8.3 or 8.4.

Thus, $V^{PC} \geq V^{TC}$ iff

$$F^{PC,0}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi)G^{PC}(0) \geq \frac{(1-\delta+\delta\gamma)\big[F^{TC}(0) - \delta(1-\phi)G^{TC}(0)\big]}{1-\delta(1-\gamma-\phi)}.$$

Since $\overline{\theta}^{PC}_{low} < \overline{\theta}^{TC} < \overline{\theta}^{PC}_{high}$, four cases arise:

Case 1: $\overline{\theta}^{PC}_{low} < \theta \leq \overline{\theta}^{TC}$, $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$, and $\hat{G}(1) \leq F^{PC,0}(0,0)/(\delta\phi)$. From Proposition 8.2.a, $G^{PC}(0) = 0$ and $G^{PC}(1) = \hat{G}(1)$. From Proposition 8.3, $G^{TC}(0) = 0$. Thus,

$$V^{PC} \geq V^{TC} \iff F^{PC,0}(0,0) - \delta\phi\hat{G}(1) \geq \frac{(1-\delta+\delta\gamma)F^{TC}(0)}{1-\delta(1-\gamma-\phi)}.$$

Case 2: $\overline{\theta}^{TC} < \theta \leq \overline{\theta}^{PC}_{high}$, $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$, and $\hat{G}(1) \leq F^{PC,0}(0,0)/(\delta\phi)$. From Proposition 8.2.a, $G^{PC}(0) = 0$ and $G^{PC}(1) = \hat{G}(1)$. From Proposition 8.4, $G^{TC}(0) > 0$ is the solution to $F^{TC}(G(0)) = 0$. Since $\theta > \overline{\theta}^{TC}$ implies $F^{TC}(0) < 0$, the inequality always holds.

Case 3: $\overline{\theta}^{PC}_{low} < \theta \leq \overline{\theta}^{TC}$, $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$, and $\hat{G}(1) > F^{PC,0}(0,0)/(\delta\phi)$. From Proposition 8.2.b, $G^{PC}(0) > 0$ and $G^{PC}(1) > \hat{G}(1)$. From Proposition 8.3, $G^{TC}(0) = 0$. Since $G^{PC}(1) > F^{PC,0}(0,0)/(\delta\phi)$ and $F^{TC}(0) \geq 0$, the inequality never holds.

Case 4: $\overline{\theta}^{TC} < \theta \leq \overline{\theta}^{PC}_{high}$, $\overline{X}_{low}(\gamma) < X < \overline{X}_{high}(\gamma)$, and $\hat{G}(1) > F^{PC,0}(0,0)/(\delta\phi)$. From Proposition 8.2.b, $G^{PC}(0) > 0$ and $G^{PC}(1) > \hat{G}(1)$. From Proposition 8.4, $G^{TC}(0) > 0$. Since $\theta > \overline{\theta}^{TC}$ implies $F^{TC}(0) < 0$, the inequality holds.

This completes the proof of Proposition 9. ■

B. Stochastic Resource Values and Nonaggressive Escalation

In this appendix section we introduce another extension that analyzes a stochastic resource. Namely, R is randomly determined each period. With probability q it takes a high value H and with probability $1-q$ it takes a low value L. Everything else is the same. Since the proofs of the two results (Propositions 10 and 11) are long, we place them at the end of the document, in Section B.3.

We note that when the contested resource is not stochastic, the model can capture two forms of escalation. First, in the permanent cooperation equilibrium described in Proposition 8.2, case (a), countries increase their arming choices when $Z = 1$ (i.e., when the elimination technology is available) to deter open conflict. This is an instance of non-aggressive escalation. Second, in the temporary cooperative equilibrium described in Propositions 8.2 and 8.3, countries increase their gun choices when $Z = 1$ in preparation for a full-fledged open war that temporarily settles the dispute. This is an instance of aggressive escalation. Note that both forms of escalation cannot simultaneously occur in equilibrium. The reason is simple. Either countries can and are willing to avoid open conflict when $Z = 1$, or they cannot. In the first scenario, a nonaggressive escalation occurs when $Z = 1$; otherwise, there will be an aggressive escalation when $Z = 1$.

This section explores an alternative mechanism for nonaggressive escalation. Specifically, we incorporate random fluctuations in resource levels, alternating between high-stakes and low-stakes realizations, $R = H, L$, with $q \in (0, 1)$ denoting the probability that $R = H$. By itself, this extension cannot induce open conflict, but combined with the elimination technology it adds an interesting feature, namely, nonaggressive and aggressive escalations along the same temporary cooperation equilibrium path.

B.1 Stochastic Resources Without the Elimination Technology

As a benchmark, we start by assuming $\phi = 0$, that is, the opportunity to eliminate never arrives.

It is immediate to extend Proposition 1 to the case with stochastic resources. In particular, under the non-cooperative norm (i.e., when countries coordinate on $S_1 = S_2 = 0$ for all (G_1, G_2)), the unique static Nash equilibrium level of arming is $G^{NN}(R) = \theta R/4$, and the associated payoff is $V^{NN}(R) = \theta R/4$, where $R \in \{L, H\}$. Thus, the expected payoff from the static Nash equilibrium in this stochastic version of the game is $V^{NN} = \theta \mathbf{E}(R)/4$, where $\mathbf{E}(R) = qH + (1-q)L$.

For $\delta > 0$, better equilibria than $G^{NN}(L)$ when $R = L$ and $G^{NN}(H)$ when $R = H$ can be sustained. To do so, assume that countries employ the following permanent cooperation strategy: If no deviation has occurred, the countries play $G(L) \in [0, G^{NN}(L))$ and settle the dispute when $R = L$, and they play $G(H) \in [0, G^{NN}(H))$ and settle the dispute when $R = H$. Any deviation triggers, starting in the next period, the static Nash equilibrium under the non-cooperative norm. In addition, a deviation in gun choices triggers an open conflict immediately.

If countries play the permanent cooperation strategy with $G(L)$ and $G(H)$ and nobody deviates, expected payoffs are given by

$$V^{PC}(R) = (1-\delta)\left(\frac{R}{2} - G(R)\right) + \delta\left[qV^{PC}(H) + (1-q)V^{PC}(L)\right]$$

for $R = L, H$. Solving these equations yields $V^{PC}(L)$ and $V^{PC}(H)$.

A deviation can occur in either state L or H. Optimal deviation payoffs are

$$V^D(G(R)) = (1-\delta)\left(\frac{g(G(R),\theta R)}{G(R)+g(G(R),\theta R)}\theta R - g(G(R),\theta R)\right) + \delta V^N$$

for $R = L, H$.

Thus, to sustain the permanent cooperation strategy with $G(L)$ and $G(H)$, it must be that

$$F^R(G(L),G(H)) = V^{PC}(R) - V^D(G(R)) \geq 0, \tag{2}$$

for $R = L, H$.

As in the previous sections, we focus on the best sustainable cooperative equilibrium, that is, we look for the equilibrium that maximizes the ex-ante expected payoff

$$V^{PC} = qV^{PC}(H) + (1-q)V^{PC}(L) = \frac{\mathbf{E}(R)}{2} - q\,G(H) - (1-q)\,G(L)$$

subject to the sustainability constraints $F^R(G(L),G(H)) \geq 0$ for $R = L, H$, i.e., equation (2). The following proposition summarizes the results.

Proposition 10. *Assume that the disputable resource is stochastic and $\phi = 0$.*

1. *Suppose that $\theta \leq \overline{\theta}(H) = \frac{2(1-\delta)H+2\delta[qH+(1-q)L]}{4(1-\delta)H+\delta[qH+(1-q)L]}$. Then permanent complete cooperation, i.e., disarmed peace with $G(R) = 0$ for $R = H, L$, is an equilibrium of the stochastic game. The associated first-best expected payoff is $V^{FB} = \mathbf{E}(R)/2$.*
2. *Suppose that $\theta > \overline{\theta}(H)$. Let $\hat{G}(H)$ be the unique solution to*

$$F^H(0, G(H)) = 0.$$

   (a) *If $\hat{G}(H) \leq F^L(0,0)/(\delta q)$, then the best possible cooperative equilibrium that can be sustained is partial cooperation with $G(L) = 0$ and $G(H) = \hat{G}(H)$.*

   (b) *If $\hat{G}(H) > F^L(0,0)/(\delta q)$, then the best possible cooperative equilibrium that can be sustained is partial cooperation with $G(L) > 0$ and $G(H) > \hat{G}(H)$, given by the unique solution to*

$$F^L(G(L),G(H)) = F^H(G(L),G(H)) = 0,$$

   that satisfies

$$\frac{\partial F^L(G(L),0)}{\partial G(L)}\,\frac{\partial F^H(0,G(H))}{\partial G(H)} > \delta^2 q(1-q).$$

Proof. See Section B.3. □

Proposition 10 generalizes Proposition 2 to the case of stochastic resource values. In the best cooperative equilibrium, there is either eternal unarmed peace or armed peace with reduced military spending compared to the one-shot Nash equilibrium arming levels. When $\theta \leq \overline{\theta}(H)$ (which always holds when countries are sufficiently patient), there is unarmed peace regardless of the value of the disputable resource. Otherwise, arming is required to sustain a peaceful settlement when the resource is high.

The most interesting result is Proposition 10.2, case (a). Countries understand that when the stakes are high ($R = H$), temptation is also high and therefore they cannot commit to unarmed peace. However, they also understand that "gun flexing" does not mean they must start an open conflict and become eternal enemies. Thus, when the stakes are low ($R = L$), countries trust each other and follow the unarmed peace allocation. Crucially, this arms race when $R = H$ is necessary to avoid a deviation that would trigger open conflict. In other words, stochastic resource values provide a simple mechanism to capture non-aggressive escalations and de-escalations.

In summary, allowing for a stochastic disputable resource does not lead to open conflict, but it explains periods of more and less intense arming and security competition. However, the fundamental result of the folk theorem persists: With enough patience, unarmed peace can be sustained.

B.2 Stochastic Resources With the Elimination Technology

To finalize our analysis, we allow the combination of stochastic resources and stochastic opportunities to eliminate the rival. To characterize the best possible equilibrium, we employ a similar approach to the one used in Sections 4 and 5. We use the worst possible equilibrium under open conflict as a threat to sustain more cooperative equilibria. As in Section 5, we focus on the case with low elimination cost, and hence the most severe punishment uses the elimination technology whenever it is available. A permanent cooperation strategy is a strategy in which countries settle in every contestable state, that is, for all $Z = 0, 1$ and $R = H, L$. A temporary cooperation strategy refers to a strategy in which countries always settle for $Z = 0$ (that is, when $Z = 0$ and $R = H, L$) but engage in open conflict for at least one state in which the elimination technology is available (that is, either for $(Z, R) = (1, H)$ or $(Z, R) = (1, L)$). If, in state $(1, R)$ with $R \in \{H, L\}$, countries do not cooperate, they choose $G^E(\gamma) = R^E(\gamma)/4$ and the victor always eliminates the loser, where

$$R^E(\gamma) = \theta R - X + \frac{\delta \mathbf{E}(R)}{1 - \delta + \delta\gamma}.$$

A full characterization of the best cooperative equilibrium is possible, but not particularly informative. Instead, the following proposition focuses on some of the most interesting results.

Proposition 11. *Assume that the disputable resource is stochastic, $\phi > 0$, and*

$$X \leq \mathbf{E}\left[\overline{X}(\gamma)\right] = \frac{[(4-\theta)(1-\delta+\delta\gamma) + 3\delta\phi]\,\delta\mathbf{E}(R)}{(1-\delta+\delta\gamma)\,[4(1-\delta+\delta\gamma) + 3\delta\phi]}.$$

1. *Suppose that $\gamma = 0$ and $\delta \to 1$. Then, it is impossible to sustain a permanent cooperative equilibrium.*

2. *Suppose that $\gamma > 0$ and $\delta \to 1$. Then, permanent complete cooperation, i.e., disarmed peace with $G(Z, R) = 0$ for all $Z = 0, 1$ and $R = H, L$, can be sustained.*

3. *Suppose that $\gamma \geq 0$ and $\delta < 1$. There are thresholds $\theta^{PC}(H)$ and $X^{PC}(H)$ such that if $\theta > \theta^{PC}(H)$ and $X < X^{PC}(H)$, it is impossible to sustain permanent cooperation.*

4. *Let $F^{TC,0,R}(G(0, L), G(0, H))$ denote the sustainability constraint for state $Z = 0$ and R when countries use a temporary cooperation strategy (without cooperation for $Z = 1$). There is a threshold $\overline{\theta}^{TC}(H)$ such that:*

(a) *Suppose that* $\theta \leq \overline{\theta}^{TC}(H)$. *Then, the best temporary cooperative equilibrium (without cooperation for* $Z = 1$*) that can be sustained is* $G(0, R) = 0$ *for* $R = H, L$.

(b) *Suppose that* $\theta > \overline{\theta}^{TC}(H)$ *and* $\hat{G}(0, H) \leq \frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$. *Then, the best temporary cooperative equilibrium (without cooperation for* $Z = 1$*) that can be sustained is* $G(0, L) = 0$ *and* $G(0, H) = \hat{G}(0, H)$*, where* $\hat{G}(0, H)$ *is the unique solution to*

$$F^{TC,0,H}(0, G(0, H)) = 0.$$

(c) *Suppose that* $\theta > \overline{\theta}^{TC}(H)$ *and* $\hat{G}(0, H) > \frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$. *Then, the best temporary cooperative equilibrium (without cooperation for* $Z = 1$*) that can be sustained is* $G(0, L) > 0$ *and* $G(0, H) > \hat{G}(0, H)$*, given by the unique solution to*

$$F^{TC,0,H}(G(0, L), G(0, H)) = F^{TC,0,L}(G(0, L), G(0, H)) = 0$$

that satisfies

$$\left[\frac{\partial F^{TC,0,H}}{\partial G(0, H)}\right]\left[\frac{\partial F^{TC,0,L}}{\partial G(0, L)}\right] > \frac{(1-\delta+\delta\gamma)^2 \delta^2 (1-\phi)^2 q(1-q)}{(1-\delta+\delta\gamma+\delta\phi)^2}.$$

Proof. See Section B.3. □

Proposition 11.1 considers the case in which the uncontestable state is an absorbing state ($\gamma = 0$). For this case, we can extend the result of Proposition 3. For $\delta \to 1$, it is impossible to sustain cooperation when the elimination option is available. To understand the logic behind this result, consider the problem faced by a country that chooses to deviate from cooperation in order to use the elimination technology:

$$\max_{G_i} u_i(G_i, G_{-i}) = \pi_i(G_i, G_j)\left[(1-\delta)(\theta R - X) + \delta \mathbf{E}(R)\right] - (1-\delta) G_i.$$

The solution to this problem is $G_i = g^D\left(G_{-i}, R^E(0)\right)$ and the associated payoff is

$$u_i^D = (1-\delta)\left(R^E(0) - 2\sqrt{R^E(0) G_{-i}} + G_{-i}\right).$$

Thus,

$$\lim_{\delta \to 1} u_i^D = \lim_{\delta \to 1}(1-\delta) R^E(0) = \mathbf{E}(R).$$

That is, when $\delta \to 1$, we have $u_i^D \to \mathbf{E}(R)$, which is an extremely tempting deviation. Intuitively, as countries become very patient, they have the opportunity to permanently eliminate the rival and fully capture the resource at virtually no cost.

In contrast, if the uncontestable state is not absorbing ($\gamma > 0$), Proposition 11.2 extends the result of Proposition 7. As $\delta \to 1$, it is possible to sustain disarmed peace as an equilibrium. Indeed, the payoff of a country that deviates from cooperation when the elimination opportunity is available is given by

$$u_i^D = (1-\delta)\left(R^E(\gamma) - 2\sqrt{R^E(\gamma) G_{-i}} + G_{-i}\right) + \frac{\delta^2 \gamma V^{PU}}{1-\delta+\delta\gamma},$$

where V^{PU} is the punishment payoff. Thus,

$$\lim_{\delta\to 1} u_i^D = \lim_{\delta\to 1} (1-\delta) R^E(\gamma) + \lim_{\delta\to 1} \frac{\delta^2 \gamma V^{PU}}{1-\delta+\delta\gamma} = 0 + \lim_{\delta\to 1} V^{PU}.$$

That is, when $\delta \to 1$, there is no immediate reward for defecting and the deviation payoff converges to the punishment payoff, which is lower than the expected payoff under cooperation. Intuitively, as countries become very patient, the short-term gains from temporarily eliminating the rival are worthless, and all that matters is that in the long term countries will enter into an eternal war, only interrupted by sporadic episodes of disarmed peace after one of the countries temporarily eliminates the other.

The possibility of sustaining disarmed peace as an equilibrium when $\gamma > 0$ and $\delta \to 1$ should not be misinterpreted as implying that open conflict can always be avoided when $\delta > 0$. In fact, Proposition 11.3 shows that for $\delta > 0$ but sufficiently low, it might be impossible to avoid open conflict when $Z = 1$ and $R = H$. The idea is that for $\delta < 1$, countries might still have incentives to start a war to temporarily eliminate the rival if θ is high enough (that is, the destruction of war is not that severe) and X is low enough (that is, it is cheap to temporarily eliminate the rival country).

Even when permanent cooperation cannot be sustained, this does not fully eliminate the possibilities for cooperation. For example, Proposition 11.4 characterizes the best possible temporary cooperative equilibrium in which countries do not cooperate when the elimination technology is available (that is, when $Z = 1$). In particular, note the equilibrium described in Proposition 11.4, case (b), which is characterized by periods of unarmed peace (when $(Z, R) = (0, L)$), armed peace with settlement (when $(Z, R) = (0, H)$), and open conflict (when $Z = 1$). On this equilibrium path, there are two types of escalations. When $Z = 0$ and $R = H$, countries escalate their arming choices to discourage open conflict (an instance of nonaggressive escalation). In contrast, for $Z = 1$, countries escalate their arming choices in preparation for open conflict (an example of aggressive escalation).

Proposition 11.4, case (b), offers a more realistic perspective on international relations. First, when the elimination technology is not possible, there is room for some international cooperation. Second, when stakes are high, arming must be scaled up, but this does not trigger a spiral of arming or open conflict (nonaggressive escalation). Third, when the elimination technology is available, open conflict cannot be avoided (aggressive escalation), but conflict does not settle disputes forever. Eventually, the defeated country reemerges, and a new cycle of armed peace and open conflict starts all over again.

B.3 Proofs of Propositions 10 and 11

Proof of Proposition 10: Suppose that countries are following the cooperation strategy with $G(L) \in \left[0, \frac{\theta L}{4}\right)$ and $G(H) \in \left[0, \frac{\theta H}{4}\right)$ and that no country deviates. Then, the expected payoffs are given by:

$$V^C(R) = (1-\delta)\left(\frac{R}{2} - G(R)\right) + \delta\left[qV^C(H) + (1-q)V^C(L)\right]$$

for $R = H, L$. Solving for $V^C(H)$ and $V^C(L)$ we have:

$$V^C(H) = (1-\delta)\frac{H}{2} + \frac{\delta \mathbf{E}(R)}{2} - (1-\delta+\delta q)G(H) - \delta(1-q)G(L)$$
$$V^C(L) = (1-\delta)\frac{L}{2} + \frac{\delta \mathbf{E}(R)}{2} - (1+\delta q)G(L) - \delta q G(H)$$

where $\mathbf{E}(R) = qH + (1-q)L$.

Sustainability: Suppose that the resource is $R = H, L$ and player i deviates from cooperation. To determine the most profitable deviation for player i we must solve:

$$\max_{G_i} \left\{ V_i^D = (1-\delta)\left(\pi_i\left(G_i, G(R)\right)\theta R - G_i\right) + \delta V^N \right\}$$

where $\pi_i\left(G_i, G(R)\right) = \frac{G_i}{G_i + G(R)}$ and $V^N = \theta \mathbf{E}(R)/4$. Thus, player i's most profitable deviation is

$$g^D\left(G(R), \theta R\right) = \sqrt{G(R)\theta R} - G(R)$$

and the associated deviation payoff is given by:

$$V^D\left(G(R)\right) = (1-\delta)\left(\theta R - 2\sqrt{G(R)\theta R} + G(R)\right) + \frac{\delta\theta\mathbf{E}(R)}{4}$$

Player i does not have an incentive to deviate if and only if $V^C(R) \geq V^D\left(G(R)\right)$ or, which is equivalent, $F^R\left(G(L), G(H)\right) \geq 0$ for $R = H, L$, where

$$\begin{aligned}
F^H\left(G(L), G(H)\right) &= -\left(2 - 2\delta + \delta q\right)G(H) + 2(1-\delta)\sqrt{G(H)\theta H} - \delta(1-q)G(L) + F^{PC,H}(0,0) \\
F^L\left(G(L), G(H)\right) &= -\left(2 - \delta + \delta q\right)G(L) + 2(1-\delta)\sqrt{G(L)\theta L} - \delta q G(H) + F^{PC,L}(0,0) \\
F^R(0,0) &= \frac{(1-\delta)R(1-2\theta)}{2} + \frac{\delta(2-\theta)\mathbf{E}(R)}{4} \quad \textit{for } R = H, L
\end{aligned}$$

Best possible permanent cooperation equilibrium: To determine the best possible permanent cooperative equilibrium, we solve

$$\max_{G(H), G(L)} \left\{ \begin{array}{l} V^C = qV^C(H) + (1-q)V^C(L) \\ = \frac{\mathbf{E}(R)}{2} - qG(H) - (1-q)G(L) \end{array} \right\}$$
$$\text{s.t.: } F^H\left(G(L), G(H)\right) \geq 0 \text{ and } F^L\left(G(L), G(H)\right) \geq 0$$

Using the same procedure that we used in the proof of Proposition 5, we have the following results:

Result 1: If a solution exists, it must satisfy $G(L) \in \left[0, \overline{G}(L)\right]$ and $G(H) \in \left[0, \overline{G}(H)\right]$, where $\overline{G}(L) = \left(\frac{1-\delta}{2-\delta-\delta q}\right)^2 \theta L < \frac{\theta L}{4}$ and $\overline{G}(H) = \left(\frac{1-\delta}{2-2\delta+\delta q}\right)^2 \theta H < \frac{\theta H}{4}$.

Result 2: $F^H\left(G(L), G(H)\right)$ and $F^L\left(G(L), G(H)\right)$ are quasiconcave functions for all $\left(G(L), G(H)\right) \in \Re_+^2$.

Result 3: If $F^H(0,0) \geq 0$, then $F^L(0,0) \geq 0$. **Proof**: $F^H(0,0) \geq 0$ if and only if $\theta \leq \overline{\theta}(H) = \frac{2(1-\delta)H + 2\delta\mathbf{E}(R)}{4(1-\delta)H + \delta\mathbf{E}(R)}$, while $F^L(0,0) \geq 0$ if and only if $\theta \leq \overline{\theta}(L) = \frac{2(1-\delta)L + 2\delta\mathbf{E}(R)}{4(1-\delta)L + \delta\mathbf{E}(R)}$. Moreover, $\overline{\theta}(H) \leq \overline{\theta}(L)$ with strict inequality if $\delta \in (0,1)$.

Since the objective function is linear and the constraints are quasiconcave, the following Kuhn–Tucker conditions are sufficient for a global maximum.

$$-q+\lambda^{H}\left[-(2-\delta-\delta q)+(1-\delta)\sqrt{\frac{\theta H}{G(H)}}\right]-\lambda^{L}\delta(1-q)+\mu_{m}^{H}-\mu_{M}^{H}=0$$

$$-(1-q)-\lambda^{H}\delta q+\lambda^{L}\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta L}{G(L)}}\right]+\mu_{m}^{L}-\mu_{M}^{L}=0$$

$$\lambda^{H}\geq 0,\ F^{H}(G(L),G(H))\geq 0,\ \lambda^{H}F^{H}(G(L),G(H))=0$$

$$\lambda^{L}\geq 0,\ F^{L}(G(L),G(H))\geq 0,\ \lambda^{L}F^{L}(G(L),G(H))=0$$

$$\mu_{m}^{H}\geq 0,\ G(H)\geq 0,\ \mu_{m}^{H}G(H)=0$$

$$\mu_{M}^{H}\geq 0,\ \overline{G}(H)-G(H)\geq 0,\ \mu_{M}^{H}\left[\overline{G}(H)-G(H)\right]=0$$

$$\mu_{m}^{L}\geq 0,\ G(L)\geq 0,\ \mu_{m}^{L}G(L)=0$$

$$\mu_{M}^{L}\geq 0,\ \overline{G}(L)-G(L)\geq 0,\ \mu_{M}^{L}\left[\overline{G}(L)-G(L)\right]=0$$

There are several cases to consider:

Case 1: Suppose that $G(H)=G(L)=0$. Then, $\lambda^{L}=\lambda^{H}=\mu_{M}^{H}=\mu_{M}^{L}=0$, $\mu_{m}^{H}=q$, $\mu_{m}^{L}=(1-q)$, $F^{H}(G(L),G(H))\geq 0$ and $F^{L}(G(L),G(H))\geq 0$. Since $F^{H}(G(L),G(H))\geq 0$ implies $F^{L}(G(L),G(H))\geq 0$, we must check $F^{H}(G(L),G(H))\geq 0$, which holds if and only if $\theta\leq\overline{\theta}(H)$. Summing up, $G(H)=G(L)=0$ is a solution if and only if $\theta\leq\overline{\theta}(H)$.

Case 2: Suppose that $G(L)\in\left(0,\overline{G}(L)\right)$ and $G(H)=0$. Then, $\mu_{M}^{H}=\mu_{m}^{L}=\mu_{M}^{L}=0$, $\lambda^{H}=0$, $\mu_{m}^{H}=\lambda^{L}\delta(1-q)+q>0$, $\lambda^{L}=(1-q)\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta L}{G(L)}}\right]^{-1}>0$, $F^{L}(G(L),0)=0$, and $F^{H}(G(L),0)\geq 0$. Since $F^{L}(G(L),0)$ is strictly increasing in $G(L)$ for all $G(L)\in\left[0,\overline{G}(L)\right]$, at most, there is one solution to $F^{L}(G(L),0)=0$. Moreover, there is a solution that satisfies $G(L)\in\left(0,\overline{G}(L)\right)$ if and only if $F^{L}(0,0)<0$ and $F^{L}\left(\overline{G}(L),0\right)>0$. Note, however, that $F^{L}(0,0)<0$ implies $F^{H}(0,0)<0$ (due to Result 3) and $F^{H}(0,0)<0$ implies $F^{H}(G(L),0)<0$ (because $F^{H}(G(L),0)$ is strictly decreasing in $G(L)$). Thus, $F^{L}(0,0)<0$ is incompatible with $F^{H}(G(L),0)\geq 0$. Summing up, there is no solution such that $G(L)\in\left(0,\overline{G}(L)\right)$ and $G(H)=0$.

Case 3: Suppose that $G(L)=0$ and $G(H)\in\left(0,\overline{G}(H)\right)$. Then, $\mu_{M}^{L}=\mu_{m}^{H}=\mu_{M}^{H}=0$, $\lambda^{L}=0$, $\lambda^{H}=\left[-(2-\delta-\delta q)+(1-\delta)\sqrt{\frac{\theta H}{G(H)}}\right]^{-1}q>0$, $\mu_{m}^{L}=(1-q)+q\lambda^{H}\delta>0$, $F^{H}(0,G(H))=0$, and $F^{L}(0,G(H))\geq 0$. Since $F^{H}(0,G(H))$ is strictly increasing in $G(H)$ for all $G(H)\in\left[0,\overline{G}(H)\right]$, at most, there is one solution to $F^{H}(0,G(H))=0$. Moreover, there is a solution that satisfies $G(H)\in\left(0,\overline{G}(H)\right)$ if and only if $F^{H}(0,0)<0$ and $F^{H}\left(0,\overline{G}(H)\right)>0$. $F^{H}(0,0)<0$ if and only if $\theta>\overline{\theta}(H)$. $F^{H}\left(0,\overline{G}(H)\right)>0$ if and only if

$$\theta<\frac{2(1-\delta)H+2\delta\mathbf{E}(R)}{\frac{4(1-\delta+\delta q)(1-\delta)H}{(2-2\delta+\delta q)}+\delta\mathbf{E}(R)}$$

which always holds because the right-hand side of the above inequality is always greater than or equal to 1. Finally, we must verify that the unique solution to $F^{H}(0,G(H))=0$ satisfies $F^{L}(0,G(H))\geq 0$ or, which is equivalent, that $G(H)\leq F^{L}(0,0)/\delta q$.

Summing up, $G(L)=0$ and $G(H)=\hat{G}(H)\in\left(0,\overline{G}(H)\right)$, where $\hat{G}(H)$ is the unique solution to $F^{H}\left(0,\overline{G}(H)\right)=0$, is a solution if and only if $\theta>\overline{\theta}(H)$ and $\hat{G}(H)\leq F^{L}(0,0)/\delta q$.

Case 4: Suppose that $G(L)\in\left(0,\overline{G}(L)\right)$ and $G(H)\in\left(0,\overline{G}(H)\right)$. Then, $\mu_m^H=\mu_M^H=\mu_m^L=\mu_M^L=0$ and

$$\lambda^{H}\left[-(2-\delta-\delta q)+(1-\delta)\sqrt{\frac{\theta H}{G(H)}}\right]-\lambda^{L}\delta(1-q)=q$$
$$-\lambda^{H}\delta q+\lambda^{L}\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta L}{G(L)}}\right]=1-q$$

The above system of linear equations has a solution if and only if

$$\left[-(2-\delta-\delta q)+(1-\delta)\sqrt{\frac{\theta H}{G(H)}}\right]\left[-(2-2\delta+\delta q)+(1-\delta)\sqrt{\frac{\theta L}{G(L)}}\right]\neq\delta^{2}q(1-q)$$

A solution must satisfy either [$\lambda^H>0$ and $\lambda^L>0$] or [$\lambda^H<0$ and $\lambda^L<0$]. $\lambda^H>0$ and $\lambda^L>0$ if and only if

$$\left[-(2-\delta-\delta q)+(1-\delta)\sqrt{\frac{\theta H}{G(H)}}\right]\left[-(2-2\delta+\delta q)+(1-\delta)\sqrt{\frac{\theta L}{G(L)}}\right]>\delta^{2}q(1-q)$$

$\lambda^H>0$ and $\lambda^L>0$ implies that $F^H(G(L),G(H))=0$ and $F^L(G(L),G(H))=0$. Thus, we must solve the following system of equations:

$$F^{H}(0,G(H))-\delta(1-q)G(L)=0$$
$$F^{L}(G(L),0)-\delta qG(H)=0$$

Following the same procedure that we used in the proof of Proposition ..., we have: $G(L)\in\left(0,\overline{G}(L)\right)$ and $G(H)\in\left(\hat{G}(H),\overline{G}(H)\right)$ given by the unique solution to $F^H(G(L),G(H))=0$ and $F^L(G(L),G(H))=0$ that satisfies $\frac{\partial F^H(G(L),G(H))}{\partial G(H)}\frac{\partial F^L(G(L),G(H))}{\partial G(L)}>\delta^2(1-q)q$ is a solution if and only if $F^H(0,0)<0$, $F^H\left(0,\overline{G}(H)\right)>0$, and $\hat{G}(H)>F^L(0,0)/\delta q$, where $\hat{G}(H)$ is the unique solution to $F^H(0,G(H))=0$. Moreover, $F^H(0,0)<0$ if and only if $\theta>\overline{\theta}(H)$ and $F^H\left(0,\overline{G}(H)\right)$ always holds.

Case 5: Suppose that $G(L)=\overline{G}(L)$ or $G(H)=\overline{G}(H)$. If $G(L)=\overline{G}(L)$, we have $\mu_m^L=0$ and, hence, $(1-q)+\lambda^H\delta q+\mu_M^L=0$, a contradiction because $\lambda^H\geq0$ and $\mu_M^L\geq0$. If $G(H)=\overline{G}(H)$, we have $\mu_m^H=0$ and, hence $q+(1-q)\lambda^L\delta+\mu_M^H=0$, a contradiction because $\lambda^L\geq0$ and $\mu_M^H\geq0$.

This completes the proof of Proposition 10. ■

Proof of Proposition 11:

***Punishment equilibrium*:** Suppose that in a contestable state countries always play $G^{NN}(R)=\theta R/4$ and $S=0$ when $Z=0$ and $G^E(R)=R^E(\gamma)/4=\left(\theta R-X+\frac{\delta\mathbf{E}(R)}{1-\delta+\delta\gamma}\right)/4$, $S=0$ and $W=1$ when

$Z = 1$. Then, expected payoffs for each state are given by:

$$\begin{aligned}
V^{PU}(1,0,R) &= (1-\delta)\left(\frac{\theta R}{2} - G^{NN}(R)\right) + \delta V^{PU}(1) \;\; \textit{for } R = H, L \\
V^{PU}(1,1,R) &= \frac{(1-\delta)(\theta R - X) + \delta V^{PU}(0,w)}{2} + \frac{\delta V^{PU}(0,l)}{2} - (1-\delta) G^{E}(R) \;\; \textit{for } R = H, L \\
V^{PU}(1) &= \phi \mathbf{E}\left[V^{PU}(1,1,R)\right] + (1-\phi)\mathbf{E}\left[V^{PU}(1,0,R)\right] \\
V^{PU}(0,w) &= (1-\delta)\mathbf{E}(R) + \delta\left[\gamma V^{PU}(1) + (1-\gamma) V^{PU}(0,w)\right] \\
V^{PU}(0,l) &= \delta\left[\gamma V^{PU}(1) + (1-\gamma) V^{PU}(0,l)\right]
\end{aligned}$$

where $\mathbf{E}(R) = qH + (1-q)L$. Solving for the uncontestable states, we obtain:

$$V^{PU}(0,w) = \frac{(1-\delta)\mathbf{E}(R) + \delta\gamma V^{PU}(1)}{1-\delta+\delta\gamma} \;\; \textit{and} \;\; V^{PU}(0,l) = \frac{\delta\gamma V^{PU}(1)}{1-\delta+\delta\gamma}$$

Introducing these expressions into the contestable states value functions, we obtain:

$$\begin{aligned}
V^{PU}(1,0,R) &= (1-\delta)\frac{\theta R}{4} + \delta V^{PU}(1) \;\; \textit{for } R = H, L \\
V^{PU}(1,1,R) &= \frac{(1-\delta) R^{E}(\gamma)}{4} + \frac{\delta^{2}\gamma V^{PU}(1)}{1-\delta+\delta\gamma} \;\; \textit{for } R = H, L
\end{aligned}$$

Taking expectations, we have:

$$\begin{aligned}
\mathbf{E}\left[V^{PU}(1,0,R)\right] &= (1-\delta)\frac{\theta \mathbf{E}(R)}{4} + \delta V^{PU}(1) \;\; \textit{for } R = H, L \\
\mathbf{E}\left[V^{PU}(1,1,R)\right] &= \frac{(1-\delta)\mathbf{E}\left[R^{E}(\gamma)\right]}{4} + \frac{\delta^{2}\gamma V^{PU}(1)}{1-\delta+\delta\gamma} \;\; \textit{for } R = H, L \\
V^{PU}(1) &= \phi \mathbf{E}\left[V^{PU}(1,1,R)\right] + (1-\phi)\mathbf{E}\left[V^{PU}(1,0,R)\right]
\end{aligned}$$

where $\mathbf{E}\left[R^{E}(\gamma)\right] = \theta\mathbf{E}(R) - X + \frac{\delta\mathbf{E}(R)}{1-\delta+\delta\gamma}$. Solving we obtain:

$$V^{PU}(1) = \mathbf{E}\left[V^{E}(\gamma)\right] = \frac{(1-\delta+\delta\gamma)\left[(1-\phi)\theta\mathbf{E}(R) + \phi\mathbf{E}\left[R^{E}(\gamma)\right]\right]}{4(1-\delta+\delta\gamma+\delta\phi)}$$

For this to be an equilibrium we need:

$$-(1-\delta)X + \delta V^{PU}(0,w) \geq \delta V^{PU}(1)$$

which holds if and only if

$$X \leq \mathbf{E}\left[\overline{X}(\gamma)\right] = \frac{\left[(4-\theta)(1-\delta+\delta\gamma) + 3\delta\phi\right]\delta\mathbf{E}(R)}{(1-\delta+\delta\gamma)\left[4(1-\delta+\delta\gamma) + 3\delta\phi\right]}$$

Proof of parts 1, 2 and 3:

Permanent cooperation: Suppose that countries play the permanent cooperation strategy with $G(Z,R) \in \left[0, \frac{\theta R}{4}\right)$ for $Z = 0,1$, and $R = H, L$ and that no country deviates. Then, the expected payoffs for each state are given by:

$$V^{PC}(1,Z,R) = (1-\delta)\left(\frac{R}{2} - G(Z,R)\right) + \delta V^{PC}(1) \text{ for } R = H, L \text{ and } Z = 0,1$$
$$V^{PC}(1) = \phi \mathbf{E}\left[V^{PC}(1,1,R)\right] + (1-\phi)\mathbf{E}\left[V^{PC}(1,0,R)\right]$$

Solving for $V^{PC}(1)$, we have:

$$V^{PC}(1) = \frac{\mathbf{E}(R)}{2} - \phi \mathbf{E}(G(1,R)) - (1-\phi)\mathbf{E}(G(0,R))$$

where $\mathbf{E}(G(Z,R)) = qG(Z,H) + (1-q)G(Z,L)$.

Optimal deviations from permanent cooperation:

Suppose that $Z = 0$. Then, the optimal deviation

$$g^D(G(0,R), \theta R) = \sqrt{G(0,R)\theta R} - G(0,R)$$

and the associated expected payoffs are:

$$V^D(G(0,R)) = (1-\delta)\left[\pi^D(G(0,R))\theta R - g^D(G(0,R), \theta R)\right] + \delta V^{PU}(1)$$

where $\pi^D(G(0,R)) = \frac{g^D(G(0,R),\theta R)}{G(0,R)+g^D(G(0,R),\theta R)}$ and $V^{PU}(1) = \mathbf{E}\left[V^E(\gamma)\right]$. Therefore:

$$V^D(G(0,R)) = (1-\delta)\left[\theta R - 2\sqrt{G(0,R)\theta R} + G(0,R)\right] + \delta \mathbf{E}\left[V^E(\gamma)\right]$$

Suppose that $Z = 1$. Then, in order to compute the optimal deviation we must solve the following problem:

$$V^D(G(1,R)) = \max_{G^D}\left\{\begin{array}{c} \pi\left(G(1,R), G^D\right)\left[(1-\delta)(\theta R - X) + \delta V(0,w)\right] \\ +\left[1 - \pi\left(G(1,R), G^D\right)\right]\delta V(0,l) - (1-\delta)G^D \end{array}\right\}$$
$$V(0,w) = (1-\delta)\mathbf{E}(R) + \delta\left[\gamma V^{PU}(1) + (1-\gamma)V(0,w)\right]$$
$$V(0,l) = \delta\left[\gamma V^{PU}(1) + (1-\gamma)V(0,l)\right]$$

where $V(0,w) = \frac{(1-\delta)\mathbf{E}(R)+\delta\gamma V^{PU}(1)}{1-\delta+\delta\gamma}$, $V(0,l) = \frac{\delta\gamma V^{PU}(1)}{1-\delta+\delta\gamma}$, and $V^{PU}(1) = \mathbf{E}\left[V^E(\gamma)\right]$. Solving for G^D we have that the optimal deviation is given by:

$$g^D\left(G(1,R), R^E(\gamma)\right) = \sqrt{G(1,R)R^E(\gamma)} - G(1,R)$$

and the associated deviation payoffs are:

$$V^D(G(1,R)) = (1-\delta)\left[R^E(\gamma) - 2\sqrt{G(1,R)R^E(\gamma)} + G(1,R)\right] + \frac{\delta^2\gamma \mathbf{E}\left[V^E(\gamma)\right]}{1-\delta+\delta\gamma}$$

Sustainability constraints for permanent cooperation: Suppose that players are playing a permanent cooperation strategy with guns choices $\mathbf{G} = (G(Z,R))$ for $Z = 0,1$ and $R = H, L$, where $G(Z,R) \in \left[0, \frac{\theta R}{4}\right)$. Then, for this strategy to be an equilibrium it must be the case that:

$$F^{PC,Z,R}(\mathbf{G}) = V^{PC}(1,Z,R) - V^{D}(G(Z,R)) \geq 0$$

for $Z = 0,1$ and $R = H, L$ or, which is equivalent,

$$\begin{aligned} F^{PC,0,R}(\mathbf{G}) &= (1-\delta)\left(\frac{R}{2} - G(0,R)\right) + \delta V^{PC}(1) \\ &- \left[\theta R - 2\sqrt{G(0,R)\,\theta R} + G(0,R)\right] - \delta \mathbf{E}\left[V^{E}(\gamma)\right] \geq 0 \end{aligned}$$

and

$$\begin{aligned} F^{PC,1,R}(\mathbf{G}) &= (1-\delta)\left(\frac{R}{2} - G(1,R)\right) + \delta V^{PC}(1) \\ &- (1-\delta)\left[R^{E}(\gamma) - 2\sqrt{G(1,R)\,R^{E}(\gamma)} + G(1,R)\right] - \frac{\delta^{2}\gamma \mathbf{E}\left[V^{E}(\gamma)\right]}{1-\delta+\delta\gamma} \geq 0 \end{aligned}$$

where

$$\begin{aligned} V^{PC}(1) &= \frac{\mathbf{E}(R)}{2} - \phi \mathbf{E}(G(1,R)) - (1-\phi)\mathbf{E}(G(0,R)) \\ \mathbf{E}\left[V^{E}(\gamma)\right] &= \frac{(1-\delta+\delta\gamma)\left[(1-\phi)\,\theta \mathbf{E}(R) + \phi \mathbf{E}\left[R^{E}(\gamma)\right]\right]}{4(1-\delta+\delta\gamma+\delta\phi)} \\ R^{E}(\gamma) &= \theta R - X + \frac{\delta \mathbf{E}(R)}{1-\delta+\delta\gamma} \end{aligned}$$

Sustainability of permanent cooperation when $\delta \to 1$:

Suppose that $\gamma = 0$. Take the limit of $F^{PC,1,R}(\mathbf{G})$ when $\delta \to 1$:

$$\lim_{\delta \to 1} F^{PC,1,R}(\mathbf{G}) = V^{PC}(1) - \mathbf{E}(R) < 0$$

Thus, for $\gamma = 0$, when $\delta \to 1$, it is impossible to sustain a permanent cooperative equilibrium.

Suppose that $\gamma > 0$ and consider $\mathbf{G} = \mathbf{0}$, that is complete permanent cooperation. Take the limit of $F^{PC,0,R}(\mathbf{0})$ when $\delta \to 1$:

$$\begin{aligned} \lim_{\delta \to 1} F^{PC,0,R}(\mathbf{0}) &= \frac{\mathbf{E}(R)}{2} - \lim_{\delta \to 1} \mathbf{E}\left[V^{E}(\gamma)\right] \\ &= \frac{\mathbf{E}(R)}{2} - \frac{(\theta\gamma + \phi)\mathbf{E}(R) - \gamma\phi X}{4(\gamma+\phi)} > 0 \end{aligned}$$

Take the limit of $F^{PC,1,R}(\mathbf{0})$ when $\delta \to 1$:

$$\begin{aligned} \lim_{\delta \to 1} F^{PC,1,R}(\mathbf{0}) &= \frac{\mathbf{E}(R)}{2} - \lim_{\delta \to 1}\left[(1-\delta)R^{E}(\gamma)\right] - \lim_{\delta \to 1} \frac{\delta^{2}\gamma \mathbf{E}\left[V^{E}(\gamma)\right]}{1-\delta+\delta\gamma} \\ &= \frac{\mathbf{E}(R)}{2} - \frac{(\theta\gamma + \phi)\mathbf{E}(R) - \gamma\phi X}{4(\gamma+\phi)} > 0 \end{aligned}$$

Thus, provided that $\gamma > 0$, $G(Z,R) = 0$ for $Z = 0,1$ and $R = H, L$ can be sustained as an equilibrium when $\delta \to 1$.

Impossibility *of avoiding open conflict for* $\gamma > 0$. Let

$$\begin{aligned}
\overline{G}(0,H) &= \left[\frac{1-\delta}{2(1-\delta)+\delta(1-\phi)q}\right]^2 \theta H \\
\overline{G}(0,L) &= \left[\frac{1-\delta}{2(1-\delta)+\delta(1-\phi)(1-q)}\right]^2 \theta L \\
\overline{G}(1,H) &= \left[\frac{1-\delta}{2(1-\delta)+\delta\phi q}\right]^2 H^E(\gamma) \\
\overline{G}(1,L) &= \left[\frac{1-\delta}{2(1-\delta)+\delta\phi(1-q)}\right]^2 L^E(\gamma)
\end{aligned}$$

It is easy to prove that $F^{PC,1,H}(\mathbf{G})$ is strictly increasing in $G(1,H)$ for all $G(1,H) \leq \overline{G}(1,H)$ and strictly decreasing in $G(1,H)$ for all $G(1,H) \geq \overline{G}(1,H)$. It is also easy to prove that $F^{PC,1,H}(\mathbf{G})$ is strictly decreasing in $G(0,H)$, $G(0,L)$, and $G(1,L)$. Consider guns choices $\left(\overline{G}(1,H), \mathbf{0}\right)$, that is, $G(1,H) = \overline{G}(1,H)$ and $G(1,L) = G(0,L) = G(0,H) = 0$. Note that $F^{PC,1,H}\left(\overline{G}(1,H), \mathbf{0}\right) \geq F^{PC,1,H}(\mathbf{G})$ for all $\mathbf{G}$. Therefore, if $F^{PC,1,H}\left(\overline{G}(1,H), \mathbf{0}\right) < 0$, then $F^{PC,1,H}(\mathbf{G}) < 0$ for any $\mathbf{G}$ and, hence, if countries are playing a permanent cooperation strategy, it is impossible to avoid a deviation when $(Z,R) = (1,H)$. Moreover, $F^{PC,1,H}\left(\overline{G}(1,H), \mathbf{0}\right) < 0$ if and only if

$$X < \overline{X}^{PC}(H) = \frac{2[aH + b\mathbf{E}(R)]\theta - (1-\delta)H + \delta\mathbf{E}(R)}{2(a+b)} + \frac{\delta\mathbf{E}(R)}{1-\delta+\delta\gamma}$$

where

$$a = \frac{(1-\delta)(1-\delta+\delta\phi q)}{2(1-\delta)+\delta\phi q},\ b = \frac{\delta^2\gamma}{4(1-\delta+\delta\gamma+\delta\phi)}$$

and $\overline{X}^{PC}(H) > 0$ if and only if

$$\theta > \overline{\theta}^{PC}(H) = \frac{(1-\delta+\delta\gamma)[(1-\delta)H + \delta\mathbf{E}(R)] - 2(a+b)\delta\mathbf{E}(R)}{2(1-\delta+\delta\gamma)[aH + b\mathbf{E}(R)]}$$

Summing up, if $\theta > \overline{\theta}^{PC}(H)$ and $X < \overline{X}^{PC}(H)$, it is impossible to sustain permanent cooperation.

Proof of part 4:

Temporary cooperation with no cooperation for $Z = 1$: Suppose that countries play the temporary cooperation strategy with $G(0,R) \in \left(0, \frac{\theta R}{4}\right)$ for $R = H, L$ and that no country deviates. Then, the expected payoffs for each state are given by:

$$\begin{aligned}
V^{TC}(1,0,R) &= (1-\delta)\left(\frac{R}{2} - G(0,R)\right) + \delta V^{TC}(1) \\
V^{TC}(1,1,R) &= \frac{(1-\delta)(\theta R - X) + \delta V^{TC}(0,w)}{2} + \frac{\delta V^{TC}(0,l)}{2} - (1-\delta) G^E(R) \\
V^{TC}(1) &= \phi\mathbf{E}\left[V^{TC}(1,1,R)\right] + (1-\phi)\mathbf{E}\left[V^{TC}(1,0,R)\right] \\
V^{TC}(0,w) &= (1-\delta)\mathbf{E}(R) + \delta\left[\gamma V^{TC}(1) + (1-\gamma)V^{TC}(0,w)\right] \\
V^{TC}(0,l) &= \delta\left[\gamma V^{TC}(1) + (1-\gamma)V^{TC}(0,l)\right]
\end{aligned}$$

Solving for the uncontestable states, we obtain:

$$V^{TC}(0,w) = \frac{(1-\delta)\,\mathbf{E}(R) + \delta\gamma V^{TC}(1)}{1-\delta+\delta\gamma} \quad and \quad V^{TC}(0,l) = \frac{\delta\gamma V^{TC}(1)}{1-\delta+\delta\gamma}$$

Introducing these expressions into the contestable states value functions, we obtain:

$$\begin{aligned} V^{TC}(1,0,R) &= (1-\delta)\left(\frac{R}{2} - G(0,R)\right) + \delta V^{TC}(1) \\ V^{TC}(1,1,R) &= \frac{(1-\delta)\,R^{E}(\gamma)}{4} + \frac{\delta^{2}\gamma V^{TC}(1)}{1-\delta+\delta\gamma} \\ V^{TC}(1) &= \phi\mathbf{E}\left[V^{TC}(1,1,R)\right] + (1-\phi)\,\mathbf{E}\left[V^{TC}(1,0,R)\right] \end{aligned}$$

Taking expectations, we obtain:

$$\begin{aligned} \mathbf{E}\left[V^{TC}(1,0,R)\right] &= (1-\delta)\left[\frac{\mathbf{E}(R)}{2} - \mathbf{E}\left[G(0,R)\right]\right] + \delta V^{TC}(1) \\ \mathbf{E}\left[V^{TC}(1,1,R)\right] &= \frac{(1-\delta)\,\mathbf{E}\left[R^{E}(\gamma)\right]}{4} + \frac{\delta^{2}\gamma V^{TC}(1)}{1-\delta+\delta\gamma} \end{aligned}$$

Solving for $V^{TC}(1)$ we have:

$$V^{TC}(1) = \frac{(1-\delta+\delta\gamma)}{1-\delta+\delta\gamma+\delta\phi}\left\{\phi\frac{\mathbf{E}\left[R^{E}(\gamma)\right]}{4} + (1-\phi)\left[\frac{\mathbf{E}(R)}{2} - \mathbf{E}\left[G(0,R)\right]\right]\right\}$$

Optimal deviations from temporary cooperation with no cooperation for $Z=1$: Suppose that $Z=0$. Then, the optimal deviation is

$$g^{D}\left(G(0,R),\theta R\right) = \sqrt{G(0,R)\,\theta R} - G(0,R)$$

and the associated expected payoffs are:

$$V^{D}\left(G(0,R)\right) = (1-\delta)\left[\pi^{D}\left(G(0,R)\right)\theta R - g^{D}\left(G(0,R),\theta R\right)\right] + \delta V^{PU}(1)$$

where $\pi^{D}\left(G(0,R)\right) = \frac{g^{D}(G(0,R),\theta R)}{G(0,R)+g^{D}(G(0,R),\theta R)}$ and $V^{PU}(1) = \mathbf{E}\left[V^{E}(\gamma)\right]$. Therefore:

$$V^{D}\left(G(0,R)\right) = (1-\delta)\left[\theta R - 2\sqrt{G(0,R)\,\theta R} + G(0,R)\right] + \delta\mathbf{E}\left[V^{E}(\gamma)\right]$$

Sustainability constraints for temporary cooperation with no cooperation for $Z=1$: Suppose that countries play the temporary cooperation strategy with guns choices $G(0,L), G(0,H)$. For this strategy to be an equilibrium it must be the case that:

$$F^{TC,0,R}\left(G(0,L),G(0,H)\right) = V^{TC}(1,0,R) - V^{D}\left(G(0,R)\right) \geq 0$$

or, which is equivalent,

$$
\begin{aligned}
F^{TC,0,R}\left(G\left(0,L\right),G\left(0,H\right)\right) &= \left(1-\delta\right)\left(\frac{R}{2}-G\left(0,R\right)\right)+\delta V^{TC}\left(1\right)\\
&-\left(1-\delta\right)\left[\theta R-2\sqrt{G\left(0,R\right)\theta R}+G\left(0,R\right)\right]-\delta\mathbf{E}\left[V^{E}\left(\gamma\right)\right]\geq 0
\end{aligned}
$$

where

$$
\begin{aligned}
V^{TC}\left(1\right) &= \frac{\left(1-\delta+\delta\gamma\right)}{1-\delta+\delta\gamma+\delta\phi}\left\{\phi\frac{\mathbf{E}\left[R^{E}\left(\gamma\right)\right]}{4}+\left(1-\phi\right)\left[\frac{\mathbf{E}\left(R\right)}{2}-\mathbf{E}\left[G\left(0,R\right)\right]\right]\right\}\\
\mathbf{E}\left[V^{E}\left(\gamma\right)\right] &= \frac{\left(1-\delta+\delta\gamma\right)\left[\left(1-\phi\right)\theta\mathbf{E}\left(R\right)+\phi\mathbf{E}\left[R^{E}\left(\gamma\right)\right]\right]}{4\left(1-\delta+\delta\gamma+\delta\phi\right)}
\end{aligned}
$$

Best possible temporary cooperation equilibrium with no cooperation for $Z=1$. To determine the best possible temporary cooperation equilibrium with no cooperation for $Z=1$, we solve

$$
\max_{\mathbf{G}}\left\{V^{TC}\left(1\right)\right\}\ \textit{subject to}:\ F^{TC,0,R}\left(G\left(0,L\right),G\left(0,H\right)\right)\geq 0\ \textit{for}\ R=H,L
$$

Using the same procedure that we used in the proof of Proposition 5, we have the following results:

Result 1: If a solution exists, it must satisfy $G\left(0,L\right)\in\left[0,\overline{G}\left(0,L\right)\right]$ and $G\left(0,H\right)\in\left[0,\overline{G}\left(0,H\right)\right]$, where

$$
\begin{aligned}
\overline{G}\left(0,L\right) &= \left[\frac{1-\delta}{2\left(1-\delta\right)+\frac{\delta(1-\delta+\delta\gamma)(1-\phi)(1-q)}{1-\delta+\delta\gamma+\delta\phi}}\right]^{2}\theta L<\frac{\theta L}{4}\\
\overline{G}\left(0,H\right) &= \left[\frac{\left(1-\delta\right)}{2\left(1-\delta\right)+\frac{\delta(1-\delta+\delta\gamma)(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}}\right]^{2}\theta H<\frac{\theta H}{4}.
\end{aligned}
$$

Result 2: $F^{TC,0,H}\left(G\left(0,L\right),G\left(0,H\right)\right)$ and $F^{TC,0,L}\left(G\left(0,L\right),G\left(0,H\right)\right)$ are quasiconcave functions for all $\left(G\left(0,L\right),G\left(0,H\right)\right)\in\Re_{+}^{2}$.

Result 3: If $F^{TC,0,H}\left(0,0\right)\geq 0$, then $F^{TC,0,L}\left(0,0\right)\geq 0$. **Proof**: $F^{TC,0,R}\left(0,0\right)\geq 0$ if and only if $\theta\leq\overline{\theta}^{TC}\left(R\right)$, where

$$
\begin{aligned}
\overline{\theta}^{TC}\left(R\right) &= \frac{\left(1-\delta\right)R+\frac{\delta(1-\delta+\delta\gamma)(1-\phi)\mathbf{E}(R)}{(1-\delta+\delta\gamma+\delta\phi)}}{2\left(1-\delta\right)R+\frac{\delta(1-\delta+\delta\gamma)(1-\phi)\mathbf{E}(R)}{2(1-\delta+\delta\gamma+\delta\phi)}}\\
&= \frac{2\left(1-\delta\right)R\left(1-\delta+\delta\gamma+\delta\phi\right)+2\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\mathbf{E}\left(R\right)}{4\left(1-\delta\right)R\left(1-\delta+\delta\gamma+\delta\phi\right)+\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\mathbf{E}\left(R\right)}
\end{aligned}
$$

Moreover, $\overline{\theta}^{TC}\left(H\right)\leq\overline{\theta}^{TC}\left(L\right)$ with strict inequality if $\delta\in\left(0,1\right)$.

Since the objective function is linear and the constraints are quasiconcave, the following Kuhn-Tucker conditions are sufficient for a global maximum.

$$-q+\lambda^{H}\left[-2\left(1-\delta\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)q}{1-\delta+\delta\gamma+\delta\phi}+\left(1-\delta\right)\sqrt{\frac{\theta H}{G\left(0,H\right)}}\right]$$
$$-\lambda^{L}\frac{\left(1-\delta+\delta\gamma\right)\delta\left(1-\phi\right)\left(1-q\right)}{1-\delta+\delta\gamma+\delta\phi}+\mu_{m}^{H}-\mu_{M}^{H}=0$$
$$-\left(1-q\right)+\lambda^{L}\left[-2\left(1-\delta\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\left(1-q\right)}{1-\delta+\delta\gamma+\delta\phi}+\left(1-\delta\right)\sqrt{\frac{\theta L}{G\left(0,L\right)}}\right]$$
$$-\frac{\left(1-\delta+\delta\gamma\right)\delta\left(1-\phi\right)q}{1-\delta+\delta\gamma+\delta\phi}\lambda^{H}+\mu_{m}^{L}-\mu_{M}^{L}=0$$
$$\lambda^{H}\geq0,\ F^{TC,0,H}\left(G\left(0,L\right),G\left(0,H\right)\right)\geq0,\ \lambda^{H}F^{TC,0,H}\left(G\left(0,L\right),G\left(0,H\right)\right)=0$$
$$\lambda^{L}\geq0,\ F^{TC,0,L}\left(G\left(0,L\right),G\left(0,H\right)\right)\geq0,\ \lambda^{L}F^{TC,0,L}\left(G\left(0,L\right),G\left(0,H\right)\right)=0$$
$$\mu_{m}^{H}\geq0,\ G\left(0,H\right)\geq0,\ \mu_{m}^{H}G\left(0,H\right)=0$$
$$\mu_{M}^{H}\geq0,\ \overline{G}\left(0,H\right)-G\left(0,H\right)\geq0,\ \mu_{M}^{H}\left[\overline{G}\left(0,H\right)-G\left(0,H\right)\right]=0$$
$$\mu_{m}^{L}\geq0,\ G\left(0,L\right)\geq0,\ \mu_{m}^{L}G\left(0,L\right)=0$$
$$\mu_{M}^{L}\geq0,\ \overline{G}\left(0,L\right)-G\left(0,L\right)\geq0,\ \mu_{M}^{L}\left[\overline{G}\left(0,L\right)-G\left(0,L\right)\right]=0$$

There are several cases to consider:

Case 1: Suppose that $G(0,H)=G(0,L)=0$. Then, $\lambda^{L}=\lambda^{H}=\mu_{M}^{H}=\mu_{M}^{L}=0$, $\mu_{m}^{H}=q$, $\mu_{m}^{L}=(1-q)$, $F^{TC,0,H}(G(0,L),G(0,H))\geq0$ and $F^{TC,0,L}(G(0,L),G(0,H))\geq0$. Since $F^{TC,0,H}(G(0,L),G(0,H))\geq0$ implies $F^{TC,0,L}(G(0,L),G(0,H))\geq0$, we must check $F^{TC,0,H}(G(0,L),G(0,H))\geq0$, which holds if and only if $\theta\leq\overline{\theta}^{TC}(H)$. Summing up, $G(0,H)=G(0,L)=0$ is a solution if and only if $\theta\leq\overline{\theta}^{TC}(H)$.

Case 2: Suppose that $G(0,L)\in\left(0,\overline{G}(0,L)\right)$ and $G(0,H)=0$. Then, $\mu_{M}^{H}=\mu_{m}^{L}=\mu_{M}^{L}=0$, $\lambda^{H}=0$, $\mu_{m}^{H}=q+\lambda^{L}\frac{(1-\delta+\delta\gamma)\delta(1-\phi)(1-q)}{1-\delta+\delta\gamma+\delta\phi}>0$,

$$\lambda^{L}=\left(1-q\right)\left[-2\left(1-\delta\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\left(1-q\right)}{1-\delta+\delta\gamma+\delta\phi}+\left(1-\delta\right)\sqrt{\frac{\theta L}{G\left(0,L\right)}}\right]^{-1}>0,$$

$F^{TC,0,L}(G(0,L),0)=0$, and $F^{TC,0,H}(G(0,L),0)\geq0$. Since $F^{TC,0,L}(G(0,L),0)$ is strictly increasing in $G(0,L)$ for all $G(0,L)\in\left[0,\overline{G}(0,L)\right]$, at most, there is one solution to $F^{TC,0,L}(G(0,L),0)=0$. Moreover, there is a solution that satisfies $G(0,L)\in\left(0,\overline{G}(0,L)\right)$ if and only if $F^{TC,0,L}(0,0)<0$ and $F^{TC,0,L}\left(\overline{G}(0,L),0\right)>0$. Note, however, that $F^{TC,0,L}(0,0)<0$ implies $F^{TC,0,H}(0,0)<0$ (due to Result 3) and $F^{TC,0,H}(0,0)<0$ implies $F^{TC,0,H}(G(0,L),0)<0$ (because $F^{TC,0,H}(G(0,L),0)$ is strictly decreasing in $G(0,L)$). Thus, $F^{TC,0,L}(0,0)<0$ is incompatible with $F^{TC,0,H}(G(0,L),0)\geq0$. Summing up, there is no solution such that $G(0,L)\in\left(0,\overline{G}(0,L)\right)$ and $G(0,H)=0$.

Case 3: Suppose that $G(0,L)=0$ and $G(0,H)\in\left(0,\overline{G}(0,H)\right)$. Then, $\mu_{M}^{L}=\mu_{m}^{H}=\mu_{M}^{H}=0$, $\lambda^{L}=0$,

$$\lambda^{H}=\left[-2\left(1-\delta\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)q}{1-\delta+\delta\gamma+\delta\phi}+\left(1-\delta\right)\sqrt{\frac{\theta H}{G\left(0,H\right)}}\right]^{-1}q>0$$

$\mu_m^L = 1 - q + \frac{(1-\delta+\delta\gamma)\delta(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}\lambda^H > 0$, $F^{TC,0,H}(0, G(0,H)) = 0$, and $F^{TC,0,L}(0, G(0,H)) \geq 0$. Since $F^{TC,0,H}(0, G(0,H))$ is strictly increasing in $G(0,H)$ for all $G(0,H) \in \left[0, \overline{G}(0,H)\right]$, at most, there is one solution to $F^{TC,0,H}(0, G(0,H)) = 0$. Moreover, there is a solution that satisfies $G(0,H) \in \left(0, \overline{G}(0,H)\right)$ if and only if $F^{TC,0,H}(0,0) < 0$ and $F^{TC,0,H}\left(0, \overline{G}(0,H)\right) > 0$. $F^{TC,0,H}(0,0) < 0$ if and only if $\theta > \overline{\theta}^{TC}(H)$. $F^{TC,0,H}\left(0, \overline{G}(0,H)\right) > 0$ if and only if

$$\theta < \frac{(1-\delta)\frac{H}{2} + \frac{\delta(1-\delta+\delta\gamma)(1-\phi)2\mathbf{E}(R)}{4(1-\delta+\delta\gamma+\delta\phi)}}{\frac{\left[1-\delta+\frac{\delta(1-\delta+\delta\gamma)(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}\right](1-\delta)H}{2(1-\delta)+\frac{\delta(1-\delta+\delta\gamma)(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}} + \frac{\delta(1-\delta+\delta\gamma)(1-\phi)\mathbf{E}(R)}{4(1-\delta+\delta\gamma+\delta\phi)}}$$

which always holds because the right hand side of the above inequality is always greater than or equal to 1. Finally, we must verify that the unique solution to $F^{TC,0,H}(0, G(0,H)) = 0$ satisfies

$$F^{TC,0,L}(0, G(0,H)) = F^{TC,0,L}(0,0) - \frac{\delta(1-\delta+\delta\gamma)(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}G(0,H) \geq 0$$

or, which is equivalent, that $G(0,H) \leq \frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$.

Summing up, $G(0,L) = 0$ and $G(0,H) = \hat{G}(0,H) \in \left(0, \overline{G}(0,H)\right)$, where $\hat{G}(0,H)$ is the unique solution to $F^{TC,0,H}(0, G(0,H)) = 0$, is a solution if and only if $\theta > \overline{\theta}^{TC}(H)$ and $\hat{G}(0,H) \leq \frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$.

Case 4: Suppose that $G(0,L) \in \left(0, \overline{G}(0,L)\right)$ and $G(0,H) \in \left(0, \overline{G}(0,H)\right)$. Then, $\mu_m^H = \mu_M^H = \mu_m^L = \mu_M^L = 0$,

$$\left[\frac{\partial F^{TC,0,H}(G(0,L), G(0,H))}{\partial G(0,H)}\right]\lambda^H - \frac{(1-\delta+\delta\gamma)\delta(1-\phi)(1-q)}{1-\delta+\delta\gamma+\delta\phi}\lambda^L = q$$
$$-\frac{(1-\delta+\delta\gamma)\delta(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}\lambda^H + \left[\frac{\partial F^{TC,0,L}(G(0,L), G(0,H))}{\partial G(0,L)}\right]\lambda^L = (1-q)$$

The above system of linear equations has a solution if and only if

$$\left[\frac{\partial F^{TC,0,H}(G(0,L), G(0,H))}{\partial G(0,H)}\right]\left[\frac{\partial F^{TC,0,L}(G(0,L), G(0,H))}{\partial G(0,L)}\right] \neq \frac{(1-\delta+\delta\gamma)^2\delta^2(1-\phi)^2 q(1-q)}{(1-\delta+\delta\gamma+\delta\phi)^2}$$

A solution must satisfy [$\lambda^H > 0$ and $\lambda^L > 0$] or [$\lambda^H < 0$ and $\lambda^L < 0$]. $\lambda^H > 0$ and $\lambda^L > 0$ if and only if

$$\left[\frac{\partial F^{TC,0,H}(G(0,L), G(0,H))}{\partial G(0,H)}\right]\left[\frac{\partial F^{TC,0,L}(G(0,L), G(0,H))}{\partial G(0,L)}\right] > \frac{(1-\delta+\delta\gamma)^2\delta^2(1-\phi)^2 q(1-q)}{(1-\delta+\delta\gamma+\delta\phi)^2}$$

$\lambda^H > 0$ and $\lambda^L > 0$ implies that

$$F^{TC,0,H}(G(0,L), G(0,H)) = F^{TC,0,L}(G(0,L), G(0,H)) = 0$$

Thus, we must solve the following system of equations:

$$F^{TC,0,H}\left(0,G\left(0,H\right)\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\left(1-q\right)}{1-\delta+\delta\gamma+\delta\phi}G\left(0,L\right)=0$$
$$F^{TC,0,L}\left(G\left(0,L\right),0\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)q}{1-\delta+\delta\gamma+\delta\phi}G\left(0,H\right)=0$$

Following the same procedure that we used in the proof of Proposition ..., we have: $G\left(0,L\right)\in\left(0,\overline{G}\left(0,L\right)\right)$ and $G\left(0,H\right)\in\left(\hat{G}\left(0,H\right),\overline{G}\left(0,H\right)\right)$ given by the unique solution to $F^{TC,0,H}\left(G\left(0,L\right),G\left(0,H\right)\right)=0$, and $F^{TC,0,L}\left(G\left(0,L\right),G\left(0,H\right)\right)=0$ that satisfies

$$\left[\frac{\partial F^{TC,0,H}\left(G\left(0,L\right),G\left(0,H\right)\right)}{\partial G\left(0,H\right)}\right]\left[\frac{\partial F^{TC,0,L}\left(G\left(0,L\right),G\left(0,H\right)\right)}{\partial G\left(0,L\right)}\right]>\frac{\left(1-\delta+\delta\gamma\right)^{2}\delta^{2}\left(1-\phi\right)^{2}q\left(1-q\right)}{\left(1-\delta+\delta\gamma+\delta\phi\right)^{2}}$$

is a solution if and only if $F^{TC,0,H}\left(0,0\right)<0$, $F^{TC,0,H}\left(0,\overline{G}\left(0,H\right)\right)>0$, and $\hat{G}\left(0,H\right)>\frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$, where $\hat{G}\left(0,H\right)$ is the unique solution to $F^{TC,0,H}\left(0,G\left(0,H\right)\right)=0$. Moreover, $F^{TC,0,H}\left(0,0\right)<0$ if and only if $\theta>\overline{\theta}^{TC}\left(H\right)$ and $F^{TC,0,H}\left(0,\overline{G}\left(0,H\right)\right)>0$ always holds.

Case 5: Suppose that $G\left(0,L\right)=\overline{G}\left(0,L\right)$ or $G\left(0,H\right)=\overline{G}\left(0,H\right)$. If $G\left(0,L\right)=\overline{G}\left(0,L\right)$, we have $\mu_{m}^{L}=0$ and, hence, $(1-q)+\lambda^{H}\frac{(1-\delta+\delta\gamma)\delta(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}+\mu_{M}^{L}=0$, a contradiction because $\lambda^{H}\geq0$ and $\mu_{M}^{L}\geq0$. If $G\left(0,H\right)=\overline{G}\left(0,H\right)$, we have $\mu_{m}^{H}=0$ and, hence $q+\lambda^{L}\frac{(1-\delta+\delta\gamma)\delta(1-\phi)(1-q)}{1-\delta+\delta\gamma+\delta\phi}+\mu_{M}^{H}=0$, a contradiction because $\lambda^{L}\geq0$ and $\mu_{M}^{H}\geq0$.

This completes the proof of Proposition 11. ■