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DEBT AND DEFICITS: FISCAL ANALYSIS WITH STATIONARY RATIOS

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ABSTRACT

We introduce a measure of a government's fiscal position, a loglinear combination of taxes, spending, and the market value of the government debt, that appears stationary in long historical data from the US and the UK, and in postwar data from these and 13 other developed countries. The fiscal position forecasts persistent changes in tax and spending growth, which bring it back to its unconditional mean within a decade, but predicts only transitory variation in debt returns and GDP growth. Mean-reversion substantially reduces the impact of the fiscal position on future growth of the debt-GDP ratio.

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If a government is in a weak fiscal position, then over the long run tax revenues must rise, or spending must fall, or the returns to the holders of government debt must adjust; or some combination of all three possibilities must occur. As we will show, this follows essentially as a matter of accounting. But which channel is most important empirically? And how does the value of the debt evolve while these adjustments take place?

To answer these two questions we introduce a measure of the fiscal position, a loglinear combination of taxes, spending, and the market value of the government debt, that captures deviations from steady-state fiscal balance. We present both theoretical reasoning and empirical evidence that our measure moves in a stationary fashion around its unconditional mean.

The relationship between the average log return on the debt, r, and the average log growth rate of its market value, g, is an important determinant of the fiscal position. If $r = g$, for example, then debt is not burdensome: in this case, the market value of the debt is irrelevant, and our measure of the fiscal position reduces to the log tax–spending ratio. More generally, our measure is well behaved whether the average return on the debt is larger than, smaller than, or equal to its average growth rate, that is, whether $r > g$, $r < g$, or $r = g$.

In long-run historical data from the US over the period 1841–2022 and from the UK over the period 1727–2022, we cannot reject the hypothesis that the average growth rates of debt, tax revenue, government expenditure, and GDP are all equal; equivalently, we find no statistically significant trends in the ratios of debt, tax, or spending to GDP. We impose equal mean growth rates in our empirical work, allowing us to think of g as the growth rate of the economy. In the US the average return on debt is slightly lower than the average growth rate of the economy while in the UK the average return is slightly higher, but in both countries r and g are sufficiently close to one another that our measure of the fiscal position is extremely similar to the log ratio of taxes to spending.

We use our fiscal position measure to explore the dynamics of tax revenue, government expenditure, and debt in the long-run US and UK data. We reframe our two questions as follows. First, what combination of tax growth, spending growth, and returns on debt brings the fiscal position back to its unconditional mean from an arbitrary initial position? (We refer to these three channels collectively as *fiscal adjustment*. In the special case where $r = g$, returns on debt play no role, and this is also true to a close approximation in the data.) Second, how does the debt-GDP

ratio evolve while the fiscal position is returning to its mean from an arbitrary initial position? (Even when $r \approx g$, there can be large movements in debt-GDP along a transition path and we show how to characterize these.)

To analyze fiscal dynamics, we use two alternative econometric methods which deliver similar results: local projections regressions and a vector autoregression (VAR) that includes the return on debt, the growth rate of tax revenue, the growth rate of output, and our measure of the fiscal position. The VAR system includes one extra lag of the fiscal position to ensure that the information set and hence our empirical results are identical (up to approximation error) whether we include tax revenue or spending growth in the VAR.

Our main empirical findings are as follows. First, the fiscal position mean-reverts quite rapidly so that adjustment is essentially complete within a decade. Second, changing expected returns on the government debt play little role in fiscal dynamics. This is true not only for the adjustment of the fiscal position itself (where the fact that $r \approx g$ minimizes the role of returns), but also for the dynamics of debt-GDP in response to the fiscal position. Third, the contributions of tax and spending growth are roughly equal in the US, but spending growth plays a considerably more important role than tax growth in the UK. Fourth, fiscal adjustment proceeds rapidly enough to greatly reduce the impact of the fiscal position on the ten-year growth of the debt-GDP ratio. In both the US and the UK we estimate that a positive 1% fiscal position reduces the debt-GDP ratio by 1-3 percentage points over ten years, a number that would be about 10 percentage points if there were no mean-reversion in the fiscal position, and further effects on the debt-GDP ratio are close to zero after ten years.

We repeat the analysis for post-World War II data from the US, the UK, and 13 other developed countries: Canada, Switzerland, and 11 countries in the eurozone (Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, the Netherlands, and Portugal). While short samples reduce the power of unit root tests, our measure of the fiscal position appears stationary, as in the long-run US and UK data. In some of these countries, the mean return on debt and the mean growth rate of the economy differ substantially. Despite this, our main findings hold up well across countries. Returns make only a minor contribution to fiscal adjustment—which in most countries is driven primarily by spending growth rather than by the growth rate of tax revenue—and play a supporting rather than a leading role in the response of the debt-GDP ratio to the fiscal position.

In principle, other variables may affect the required return on the debt, and our approach can flexibly accommodate any stationary variables that predict debt returns (or GDP growth, or other elements of our VAR). To illustrate this point, we exploit the fact that the log tax-GDP ratio appears to be stationary in postwar US data. High levels of tax-GDP forecast higher returns on the debt and lower tax growth; and (though with only marginal significance) lower GDP growth. Our major conclusion is unaffected—we find, once again, that the fiscal position forecasts future tax and particularly future spending growth rather than bond returns—but the precision of the decomposition increases (see Online Appendix Table OA.48).

While most of our analysis is backward-looking, analyzing the long-run history of fiscal adjustment in developed countries, we can also use our framework to study specific historical episodes and to make forward-looking projections from any initial position under the assumption that future fiscal adjustments will follow similar patterns to those observed historically. In this spirit we discuss US fiscal adjustment after World War II and at the end of the 20th Century, and we forecast future fiscal adjustment from the end of our sample period in 2022.

Our paper is relevant for several literatures in finance and macroeconomics. One literature studies the dynamics of the debt-GDP ratio, assuming it is stationary (see, for example, Bohn (1991, 1998, 2008), Cochrane (2001, 2022, 2023), Blanchard (2019), Jiang et al. (2024*b*), and Eichengreen, Menuet and Donnat (2026)). Contrary to this, we find that the debt-GDP ratio does not appear to behave like a stationary time series in US and UK data. As we have discussed the debt-GDP ratio does not have a statistically significant time trend, but it has drifted persistently up and down for long periods of time in each country, showing no strong tendency to return to a constant mean. Unit root tests do not reject, either in logs or levels, the null hypothesis that the debt-GDP ratio has a unit root. Even if one believes that economic forces act to make the debt-GDP ratio truly stationary in the very long run, the persistence of the time series implies that it is inadvisable to model debt-GDP using the standard techniques of stationary time-series analysis (Campbell and Perron, 1991). Nonstationarity helps to explain the (at first sight puzzling) finding of the literature that the debt-GDP ratio fails to predict fiscal outcomes. We address this problem by using an alternative ratio that does appear to be stationary and that can be used to characterize the growth rate rather than the level of debt-GDP during periods of fiscal adjustment.

Giannitsarou and Scott (2006) and Berndt, Lustig and Yeltekin (2012) develop

approximations that are similar to ours in spirit but rely on the assumption that the tax revenue-debt and government expenditure-debt ratios are stationary. The evidence regarding this assumption is mixed: we can reject the presence of a unit root for both series in long-run US data but not in postwar US data, nor in either long-run or postwar UK data. We find, however, that their logs are cointegrated. Our approach exploits this fact.

A literature on the fiscal theory of the price level emphasizes the possibility that the government's intertemporal budget constraint is satisfied through adjustments in the value of the government debt to reflect the government's plans for taxes and spending (Leeper, 1991; Sims, 1994; Woodford, 1995; Cochrane, 2001, 2023). This force might operate either through movements in conditionally expected real returns or through unexpected contemporaneous return shocks that occur simultaneously with tax or spending shocks, or both.[1] We consider both possibilities, and show that in the countries and time periods we study—in which $r \approx g$, so that debt is not particularly burdensome—returns on the debt have played only a secondary role in fiscal adjustment. However, because we conduct a reduced-form time-series analysis, we cannot make causal statements about fiscal dynamics. Although variations in the fiscal position are largely resolved by adjustments to taxes and spending in the developed countries we study, we do not provide a theory of the mechanisms underlying that fact. It is also possible that shocks to expected returns play a more substantial role in some emerging economies where r is much higher than g and where political and economic conditions are more turbulent.

A literature on tax smoothing, following Barro (1979), has argued that optimal fiscal policy under convex tax distortions sets the tax rate to follow a random walk. According to this theory, any changes in tax rates to achieve fiscal balance should take place immediately rather than gradually. While we do find that the growth rate of tax revenue tends to respond more rapidly than the growth rate of spending to fiscal imbalance, both tax growth and spending growth are predictable from the fiscal position several years in advance. This pattern also holds if we measure the

[1] Movements in expected returns might reflect changes in the equilibrium real interest rate, perhaps associated with anticipated inflation, or changes in the extent of financial repression or in convenience yields. Unexpected return shocks could either reflect shocks to yields (in turn, perhaps due to, say, sudden increases in expected inflation that reduce the value of long-dated nominal debt), or an unexpected inflation that reduces the value of all nominal assets; or even an explicit default. Our approach does not require us to take a view on the sources of shocks to expected or unexpected returns.

difference between tax growth and GDP growth, that is, the growth of the average tax rate. A caveat is that our data do not allow us to measure the evolution of the marginal tax rate.

Finally, a recent literature has asked why the average return on the government debt is so low. Some papers have conjectured that government debt has a "convenience yield" that investors value separately from its return (Greenwood, Hanson and Stein, 2015; Jiang et al., 2024*a*; Krishnamurthy and Vissing-Jorgensen, 2012; Reis, 2022; Mian, Straub and Sufi, 2025). If such a convenience yield is present, it will drive down the average financial return on the debt—possibly to a level below the mean growth rate. We take the returns on government debt as given, measuring them in the data without requiring them to satisfy the restrictions of any particular asset pricing model and without requiring that $r > g$. Thus our framework accommodates any convenience yield that may exist.

The organization of the paper is as follows. In Section 1 we lay out our framework for fiscal analysis. We apply the framework empirically to long-run historical US and UK data in Section 2, which decomposes the variability of the fiscal position and its implications for the accumulation of debt. We examine post-World War II data in the US, the UK, and 13 other developed countries in Section 3. Section 4 concludes. An online appendix (Campbell, Gao and Martin, 2026) presents supplementary details.

1 A framework for fiscal analysis

The primary surplus of a government is an essential ingredient in any measure of the government's fiscal position. The primary surplus—the excess of tax revenue over government expenditure—is the flow of resources that the government devotes to servicing its debt. When it is positive, the debt grows at a slower rate than the return on the debt. When it is negative—that is, when the government runs a primary deficit—the debt grows at a faster rate than the return on debt.

In this paper we assume that both the growth rate of debt and the return on debt are stationary time series that have well defined unconditional means. It is traditional to assume that the unconditional mean return on the debt exceeds the unconditional mean growth rate of the debt (i.e., that "$r > g$"). Our framework allows for this possibility, but as it has been challenged by economists such as Ball, Elmendorf and Mankiw (1998), Blanchard (2019), Mehrotra and Sergeyev (2021),

Mian, Straub and Sufi (2025) and Abel and Panageas (2025), and as it is inconsistent with sample averages in US data, we do not *require* that the assumption holds—and our main results are not sensitive to whether it holds or not.

To be useful in fiscal analysis, the primary surplus must be scaled in some way so that the resulting ratio is stationary. A common approach is to divide both the primary surplus and the value of debt by GDP to create the surplus-GDP and debt-GDP ratios. Given stationary debt growth and returns, stationarity of either of these two ratios implies the stationarity of the other because of the identity that links the primary surplus with the evolving value of the debt. Many papers treat both ratios as stationary and ask what forces return the debt-GDP ratio to its unconditional mean.

Contrary to this approach, the debt-GDP ratio does not appear to behave like a stationary time series in US and UK data. (See Figure 2 and the discussion in Section 2.2.) This is not particularly surprising from a theoretical perspective: for example, Barro (1979) writes, "There is no force that causes the ratio of debt to income to approach some target value".

Is there some other way to scale fiscal variables to make them stationary? One possibility is to scale the primary surplus by the value of debt, and work with the primary surplus-debt ratio. Under our maintained assumptions that the growth rate of debt and the return on the debt are stationary, the primary surplus-debt ratio should also be stationary.[2] The primary surplus-debt ratio is analogous in the fiscal context to the dividend-price ratio on a stock. Just as a corporation pays dividends to the owners of its stock, so the government pays primary surpluses to the owners of its debt. This suggests the possibility of analyzing the primary surplus-debt ratio using a Campbell and Shiller (1988*a*) loglinearization to relate it to future log returns on debt and log growth rates of primary surpluses.

Two problems arise in doing so, and both result from the fact that the primary surplus can be negative. First, the log growth rate of the primary surplus is ill-defined when the surplus is negative. Second, an exogenous increase in the debt can either raise or lower the primary surplus-debt ratio depending on whether the primary surplus is currently positive or negative. Thus, the effect of a given shock to the primary surplus-debt ratio depends on the sign of the ratio at each point in time. Both problems also afflict the standard analysis of the primary surplus-GDP ratio.

[2]We discuss the results of unit root tests for the variables of interest in Section 2.2.

In this paper we develop an alternative loglinear analysis that solves these problems. We write $\tau_t = \log T_t$ for log tax revenue, $x_t = \log X_t$ for log expenditure, and $v_t = \log V_t$ for the log market value of the debt in period t, with all variables measured in real terms. We then define the *fiscal position* fp_t as

$$fp_t = k + \rho\left[a\tau_t - bx_t\right] - (1-\rho)v_t \tag{1}$$

where k is a constant, a and b are linearization parameters that capture the typical scale of the government's tax revenue and expenditure relative to its debt, and $\rho = e^{-(r-g)}$ captures how burdensome debt is: ρ is less than one when $r > g$ and greater than one when $r < g$. The parameters ρ, a, and b are linked by a simple relationship, $a - b = (1-\rho)/\rho$, so that equation (1) can be rewritten in terms of any two of these three parameters. The constant k is a function of a and b. We derive this measure in Section 1.2 and show that it can be loglinearly related to subsequent debt returns and the growth rates of tax revenue and government expenditure. As both revenue and expenditure are always positive, their log growth rates are well defined.[3]

The effect of debt on our fiscal position measure does not depend on whether the primary surplus is currently positive or negative, but only on the long-run average value of the primary surplus, through ρ—equivalently, on the relationship between the unconditional means of the return on government debt and the growth rate of the debt, r and g.

An important special case occurs when $r = g$. In this case $\rho = 1$, so our measure of the fiscal position is just a multiple of the log ratio of taxes to spending: $fp_t = a(\tau_t - x_t)$. Debt drops out because debt is not burdensome when its growth rate and its return are equal: any extra debt that the government may have issued can simply be rolled over forever, without requiring any future fiscal adjustments. Empirically, we find that in the long histories of US and UK data, the average return and growth rate are close enough to one another that this special case of the fiscal position measure is extremely close to the general measure we construct using historical averages.

[3] Cochrane (1998) presents a different approach that linearizes in the level of surplus.

1.1 Taxes, spending, and debt in the US and the UK

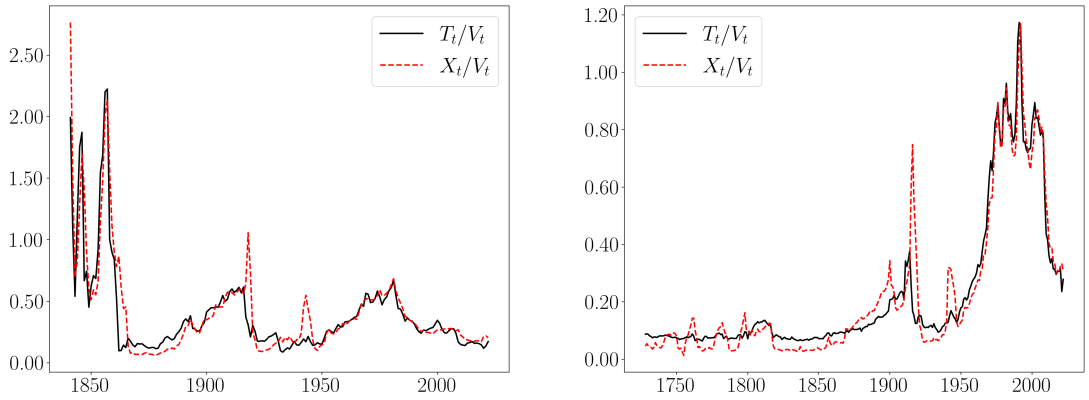
Our goal is to relate a measure of the government's fiscal position to adjustments in tax growth, spending growth, and debt returns within a linear model. Writing r_t for the log return on the government's debt, and recalling that $\tau_t = \log(T_t)$ and $x_t = \log(X_t)$, we want to relate the fiscal position to $\Delta\tau_t$, Δx_t, and r_t. Our framework does not restrict the time-series behavior of conditional expectations, but as noted above we assume that the *unconditional* means of the growth rates of tax revenue, spending, and debt are all equal to each other and to the unconditional mean of GDP growth, so that tax-debt, spending-debt, and debt-GDP ratios do not trend upwards or downwards over time. We write g for this shared growth rate, $\mathbb{E}\,\Delta\tau_{t+1} = \mathbb{E}\,\Delta x_{t+1} = \mathbb{E}\,\Delta v_{t+1} = \mathbb{E}\,\Delta y_{t+1} = g$, and r for the unconditional expected log return on the debt, $\mathbb{E}\,r_{t+1} = r$.

For our model to be linear, we need our fiscal position measure fp_t to be linear in τ_t, x_t, and v_t. Since the fiscal position is homogeneous of degree zero in these inputs (reflecting the arbitrary units used to measure tax revenue, spending, and debt), we can without loss of generality write it as linear in the two ratios $\tau v_t = \log T_t/V_t$ and $xv_t = \log X_t/V_t$.

In postwar US data, and in long-term and postwar UK data, the ratios τv_t and xv_t do not appear stationary, reflecting persistent changes in the size of the government's currently funded activities relative to the size of its debt. Figure 1 plots the history of T_t/V_t and X_t/V_t, with US data in panel (a) and UK data in panel (b). (Section 2.1 and online appendix section OA.4 describe our data sources.) The figure shows persistent variation in the size of government relative to the size of the debt: in the US, the federal government had little debt before the US Civil War and was therefore large relative to its debt, while in the UK the government's tax revenues and expenditures grew relative to the debt with the creation of the modern welfare state after World War II. In postwar US data and the UK data, unit root tests fail to reject the hypothesis that T_t/V_t and X_t/V_t, or their logs τv_t and xv_t, are nonstationary.

But Figure 1 also shows that the tax-debt and spending-debt ratios tend to move together, with transitory deviations between the two reflecting events such as major wars that temporarily drive up spending. Both visual inspection and standard tests for cointegration indicate that the two ratios are cointegrated, in other words that

Figure 1: The spending-debt and tax-debt ratios in the US and UK.



(a) US (b) UK

a stationary linear combination of the two ratios exists.[4] We exploit this important fact in our definition of the fiscal position.

1.2 A loglinear measure of the fiscal position

By definition, the gross return on government debt is

$$R_{t+1} = \frac{V_{t+1} + T_{t+1} - X_{t+1}}{V_t}. \tag{2}$$

This formula takes the perspective of a representative investor who holds all government debt and receives all resources devoted to servicing it, while buying all new issues so as to continue to hold all the debt.

We define the primary surplus as $S_t = T_t - X_t$ and assume throughout that the gross return R_{t+1} is strictly positive: this rules out the possibility of a total default on all government debt obligations with zero recovery. Note that the debt return R_{t+1} should only be interpreted as a riskless interest rate in the special case in which all government debt is short-term real debt. We allow debt to be risky: the realized return on debt is low if, for example, real yields rise, or if there is an explicit default. Inflation can also resolve fiscal difficulties by driving down real returns. It

[4] Johansen tests (in both the trace and eigenvalue form) reject the null hypothesis that there is no cointegrating relationship between τv_t and xv_t at the 1% level.

can do so in two ways: either via an unexpected inflation, which reduces the value of all nominal debt, or via increases in expected inflation that reduce the value of long-dated nominal debt by driving up nominal yields.

We can rewrite equation (2) as

$$R_{t+1} = \frac{V_{t+1}}{V_t}\left(1 + \frac{S_{t+1}}{V_{t+1}}\right). \tag{3}$$

Taking logs of (3), and using lower-case letters to denote logarithms of variables written with upper-case letters, we have

$$\log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right) = r_{t+1} - \Delta v_{t+1}. \tag{4}$$

We choose our fiscal position measure to approximate this logarithmic version of the primary surplus-debt ratio, $\log(1+S_{t+1}/V_{t+1})$. This quantity is analogous to the dividend-price ratio measure, $\log(1+D_{t+1}/P_{t+1})$, studied by Gao and Martin (2021). It allows the primary surplus to go negative; moreover, the measure is in natural units, in the sense that $\log(1 + S_{t+1}/V_{t+1})$ is approximately equal to S_{t+1}/V_{t+1} if surplus-debt is small.

As returns and debt growth are stationary, the logarithmic surplus-debt ratio should also be stationary; and a fiscal position measure that approximates the left-hand side of this equation will be linearly related to future returns and debt growth. Unconditionally,

$$\mathbb{E}\log\left(1 + S_{t+1}/V_{t+1}\right) = r - g. \tag{5}$$

To construct a tractable measure of the fiscal position, we linearize the logarithmic primary surplus-debt ratio in τv_t and $x v_t$. Specifically, we write

$$\log\left(1 + S_t/V_t\right) = \log\left(1 + e^{\tau v_t} - e^{x v_t}\right) \tag{6}$$

and linearize the right hand side of this equation around $(\tau v_t, x v_t) = (\log a, \log b)$, where the linearization points a and b are both positive. If the government's expenditure and tax revenues are large relative to the size of the debt then a and b will be large. The significance of this "big government" case, with large a and b, is that a small shift in tax or spending has a large impact on the fiscal position, as is already apparent in equation (1). Online appendix OA.1 presents a simple

steady-state model to illustrate this point.

We want to linearize around the unconditional mean of $\log(1 + S_t/V_t)$ so that our chosen measure of the fiscal position behaves like $\log(1 + S/V)$ in deterministic or steady-state models. It is notationally convenient to define the parameter $\rho = e^{-(r-g)}$, so that we can rewrite (5) as

$$\mathbb{E} \log(1 + S_t/V_t) = -\log \rho. \tag{7}$$

The parameter ρ lies between zero and one if $\mathbb{E} \log(1 + S_t/V_t) > 0$ and is greater than one if $\mathbb{E} \log(1 + S_t/V_t) < 0$. From equation (5), these two possibilities correspond to the cases where $r > g$ and $r < g$, respectively. Linearization around the unconditional mean then implies that $\rho = 1/(1 + a - b)$, or

$$b = a - \frac{1 - \rho}{\rho}. \tag{8}$$

When $r > g$ and $\rho < 1$, the linearization parameter b must be less than the linearization parameter a because spending must be less than tax revenue in steady state so that the government runs a steady-state primary surplus; hence, proportional changes in spending have a smaller effect on the primary surplus-debt ratio than the same proportional changes in tax revenue. Conversely, when $r < g$ and $\rho > 1$, b exceeds a because spending exceeds tax revenue in steady state.

Our loglinear measure of the fiscal position can be written in terms of the parameters a, b, and ρ as

$$fp_t = k + \rho a\, \tau v_t - \rho b\, x v_t = k + \rho\, [a\tau_t - b x_t] - (1 - \rho) v_t, \tag{9}$$

where we use the letters fp to denote fiscal position, and k is a constant. The online appendix derives equation (9) and a formula for k as a function of a and b.

Equation (8) links the parameters a, b, and ρ and allows us to substitute out any one of them. For example, we can write $fp_t = k + \rho a\, (\tau_t - x_t) + (1 - \rho) x v_t$. This expresses the fiscal position as a weighted average of the current proportional deviation between taxes and spending, multiplied by the parameter a which captures the size of government relative to the size of the debt at the point of linearization, and the current log ratio of spending to debt. When $r > g$, $1 - \rho$ is positive implying that a high spending-debt ratio, or equivalently low debt relative to the

size of government, improves the fiscal position. The converse is true when $r < g$.

In the long historical data from both the US and the UK the sample estimates of r and g are very close together, making the case $r = g$ an important benchmark. The fiscal position (9) simplifies in this case because $\rho = 1$ and $k = 0$, so

$$fp_t = a\left(\tau_t - x_t\right). \tag{10}$$

The fiscal position now depends only on the log ratio of taxes to spending, multiplied by the scale parameter a which captures the size of government relative to debt. When the government is large relative to its debt, a is large and small proportional adjustments in taxes and spending have large effects on the primary surplus-debt ratio. The level of debt does not appear, because debt can be rolled over forever when $r = g$ without requiring any fiscal adjustment: any level of debt is equally fiscally sustainable.

1.3 The dynamics of the fiscal position

To characterize the dynamics of the fiscal position, we use the one-period relationship between the fiscal position, the return on debt, and the growth rate of debt:

$$fp_{t+1} = r_{t+1} - \Delta v_{t+1}. \tag{11}$$

Equation (11) follows from equation (4) by replacing $\log(1 + S_{t+1}/V_{t+1})$ with our loglinear approximation to it, the fiscal position fp_{t+1}. It shows that the fiscal position measure fp has the same units as debt growth: if, for example, the fiscal position is -1%, this implies that debt will grow by 1% more than its return.

We can use equation (11) to substitute out debt growth, Δv_{t+1}, from the first difference of equation (9). Rearranging to put fp_t on the left hand side of the equation, we find that

$$fp_t = (1-\rho)r_{t+1} - \rho\left(a\Delta\tau_{t+1} - b\Delta x_{t+1}\right) + \rho\, fp_{t+1}. \tag{12}$$

This expression includes all three parameters a, b, and ρ; if desired, we can use equation (8) to substitute out any one of these. Taking unconditional expectations of both sides of equation (12), we see that $\overline{fp} = r - g$, consistent with (5).

Equation (12) shows the various ways in which the fiscal position can adjust over

a single period. In the familiar case where $r > g$ so $\rho < 1$, a weak current fiscal position implies some combination of low returns on the government debt, rapid growth in tax revenue, slow growth in spending, and a weak future fiscal position. In the case where $r = g$ so $\rho = 1$, the level of debt drops out of the formula for the fiscal position so debt returns become irrelevant; in this case a weak current fiscal position implies some combination of tax growth that exceeds spending growth and a weak future fiscal position. In the case where $r < g$ so $\rho > 1$, debt enters the fiscal position formula with the opposite sign. In this case debt returns are relevant once again, but a weak fiscal position can be resolved by unusually high returns on the debt rather than unusually low returns.

We can solve (12) forward T periods to find that

$$fp_t = \sum_{j=0}^{T-1} \rho^j \left[(1-\rho) r_{t+1+j} - \rho \left(a \Delta \tau_{t+1+j} - b \Delta x_{t+1+j}\right)\right] + \rho^T fp_{t+T}. \tag{13}$$

At this point it is tempting to take a limit as $T \to \infty$, and indeed if $\rho < 1$ we can. But as we want to allow for the possibility that $\rho \geq 1$, we do not—yet—do so. Instead we take conditional expectations and subtract unconditional expectations, giving

$$\widehat{fp}_t = \sum_{j=0}^{T-1} \rho^j \, \mathbb{E}_t \left[(1-\rho)\widehat{r}_{t+1+j} - \rho(a\widehat{\Delta\tau}_{t+1+j} - b\widehat{\Delta x}_{t+1+j})\right] + \rho^T \, \mathbb{E}_t \, \widehat{fp}_{t+T} \,, \tag{14}$$

where a hat on a variable indicates a deviation from its unconditional mean: for example, $\widehat{r}_t = r_t - \mathbb{E}\, r_t$.

For the right-hand side of equation (14) to converge appropriately as T tends to infinity, we require that the conditional expectations of the various hatted quantities converge to zero faster than ρ^T tends to infinity.[5] This always holds in the $r > g$ case, with $\rho < 1$; in the $r \leq g$ case, with $\rho \geq 1$, we impose this as an assumption.

[5] In our empirical work, we will use a VAR to calculate conditional expectations: in this case the assumption holds if the maximal eigenvalue of the VAR matrix has magnitude less than $1/\rho$, so that all variables mean-revert sufficiently quickly. We find that this is the case in our empirical applications. We discuss convergence issues in more detail in online appendix Section OA.3.

When it holds, we can send $T \to \infty$, giving

$$\widehat{fp}_t = \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \left[(1-\rho)\widehat{r}_{t+1+j} - \rho(a\widehat{\Delta\tau}_{t+1+j} - b\widehat{\Delta x}_{t+1+j}) \right]. \tag{15}$$

The stationarity of the fiscal position implies that the future fiscal position drops out of this formula. Over the infinite future, the current fiscal position implies a combination of future debt returns, spending growth, and tax growth that satisfies equation (15).

When $r > g$, ρ is less than one.[6] There is a sense of urgency in this case: a weak fiscal position can be resolved by changes in tax and spending, but if these changes are delayed then they must be *larger*, because $\rho^j \ll 1$ for large j; or, if fiscal adjustment is accomplished via unusually low returns to bondholders, then the returns must be lower the more delayed they are.

When $r = g$, so that $\rho = 1$, the level of debt drops out of the definition of the fiscal position, as already noted, and as a result returns drop out of equation (15).

The equation continues to hold even when $\rho \geq 1$—that is, when $r \leq g$—under our maintained assumption that the various conditional expectations converge to their unconditional expectations faster than ρ^T tends to infinity. In this case, expectations about deviations of returns and fiscal variables from their unconditional means have considerable weight even—in fact especially—in the distant future.

1.4 The dynamics of the debt-GDP ratio

The formulas (12), (13), and (15) do not track the evolution of the debt-GDP ratio. To do so, we work with the ratios $\tau y_t = \tau_t - y_t$, $xy_t = x_t - y_t$, and $vy_t = v_t - y_t$. The definition of the fiscal position is unchanged whether it is written in terms of the levels of tax revenue, spending and debt or in terms of their ratios to GDP: as $\rho(a-b) = 1-\rho$ by the restriction (8), the definition (9) can be rewritten as

$$fp_t = k + \rho\,(a\,\tau y_t - b\,xy_t) - (1-\rho)\,vy_t\,. \tag{16}$$

Similarly, the identities in Section 1.3 hold with tax replaced by the tax-GDP ratio τy_t, spending by the spending-GDP ratio xy_t, debt by the debt-GDP ratio vy_t,

[6] In this case, the identity (15) also holds without hats on the variables, as can be seen by adding $\overline{fp}$ to the left-hand side and $r-g$ to the right-hand side. (These are equal: $\overline{fp} = r - g$.)

and returns by returns relative to GDP growth $r_t - \Delta y_t$. For example, the identity (13) can be rewritten as

$$fp_t = \sum_{j=0}^{T-1} \rho^j \left[(1-\rho) \left(r_{t+1+j} - \Delta y_{t+1+j} \right) - \rho \left(a \Delta \tau y_{t+1+j} - b \Delta x y_{t+1+j} \right) \right] + \rho^T fp_{t+T} \,. \tag{17}$$

Using (16) to compare the fiscal position at time t with the fiscal position at time $t+T$, we see that

$$fp_{t+T} = fp_t + \rho \sum_{j=0}^{T-1} \left(a \Delta \tau y_{t+1+j} - b \Delta x y_{t+1+j} \right) - (1-\rho) \left(v y_{t+T} - v y_t \right) \,. \tag{18}$$

Using this equation to substitute the terminal fiscal position out of the identity (17), then rearranging and taking conditional expectations,[7] we have the identity

$$\begin{aligned} fp_t = & \sum_{j=0}^{T-1} \frac{(1-\rho)\rho^j}{1-\rho^T} \mathbb{E}_t \left(r_{t+1+j} - \Delta y_{t+1+j} \right) \\ & - \sum_{j=0}^{T-1} \frac{\rho^{j+1} - \rho^{T+1}}{1-\rho^T} \mathbb{E}_t \left(a \Delta \tau y_{t+1+j} - b \Delta x y_{t+1+j} \right) - \frac{(1-\rho)\rho^T}{1-\rho^T} \mathbb{E}_t \left(v y_{t+T} - v y_t \right) . \end{aligned} \tag{19}$$

This identity applies for any $\rho \neq 1$, and it does not require any assumptions regarding the stationarity of the debt-GDP ratio vy_t. The weights on the returns on debt in excess of GDP growth (excess returns, for short) are positive and they sum to one, so the first term of (19) is a weighted average of excess returns. By contrast the coefficients on returns in the corresponding identity (13) sum to $1 - \rho^T$, which is close to zero when ρ is close to one. Thus returns can play an important role in the dynamics of the debt despite playing a much more limited role in the dynamics of the fiscal position.

Using l'Hôpital's rule to take the limit as $\rho \to 1$, we obtain the corresponding

[7] The identity (19) also holds without the conditional expectation (that is, ex post), but when $\rho \geq 1$ it only has a well-behaved limit as $T \to \infty$ when the conditional expectation is included.

identity for the case $r = g$:

$$fp_t = \frac{1}{T}\sum_{j=0}^{T-1}\mathbb{E}_t\left(r_{t+1+j} - \Delta y_{t+1+j}\right) - a\sum_{j=0}^{T-1}\left(1 - \frac{j}{T}\right)\mathbb{E}_t\left(\Delta\tau y_{t+1+j} - \Delta x y_{t+1+j}\right) - \frac{1}{T}\mathbb{E}_t\left(v y_{t+T} - v y_t\right). \quad (20)$$

In both (19) and (20), a weak fiscal position (a negative value on the left-hand side) implies some combination of low excess returns on debt, increases in the tax-GDP ratio relative to the spending-GDP ratio, and an increasing debt-GDP ratio. As ρ is so close to one in our long samples, (20) is an accurate guide to the results we report below.

These identities could be rewritten to express the change in the debt-GDP ratio, over the period from time t to time $t + T$, in terms of the initial fiscal position and subsequent debt returns. We have chosen to put the fiscal position on the left-hand sides of (19) and (20) to reflect how we use the identities in our empirical work: we will ask how a given fiscal position, fp_t, is typically resolved by subsequent movements in bond markets, in fiscal policy, and in the debt-GDP ratio.

1.4.1 The infinite-horizon limit

The relative importance of these channels is determined by $\rho^{-1} = e^{r-g}$, and hence by the relationship between the mean return r and the mean growth rate g. The distinction between the cases $r > g$, $r = g$ and $r < g$ becomes particularly important if we want to derive infinite horizon identities by taking limits of (19) or (20) as T tends to infinity. We now consider how to take these limits in each case, with proofs in the online appendix.

When $r > g$, ρ is less than one. In this case, the government runs a steady-state primary surplus, and a is greater than b so a proportional rise in the tax-GDP ratio has more impact than an equal proportional fall in the spending-GDP ratio. Events that take place in the near future are particularly important. A positive return shock, or negative fiscal shock, that occurs soon after time t raises the level of the debt, which then (on average) grows faster than output. As the horizon T increases in (19), the final term (representing changes in the debt-GDP ratio in future) tends to zero, so a weak fiscal position must ultimately be resolved via bad news for

bondholders and/or rises in the tax-GDP ratio relative to the spending-GDP ratio; and in the limit as T tends to infinity we have

$$fp_t = (1-\rho)\sum_{j=0}^{\infty}\rho^j\,\mathbb{E}_t\left(r_{t+1+j} - \Delta y_{t+1+j}\right) - \rho\sum_{j=0}^{\infty}\rho^j\,\mathbb{E}_t\left(a\Delta\tau y_{t+1+j} - b\Delta x y_{t+1+j}\right). \tag{21}$$

This identity is equivalent to (15), differing only in that debt returns, tax growth and spending growth are each expressed relative to GDP growth.

When $r = g$, excess returns and debt-GDP growth appear as simple averages in (20), while the weights on tax-GDP and spending-GDP growth reflect the fact that tax and spending changes that occur earlier are more effective than later changes in limiting T-period debt growth. The coefficient $\frac{1}{T}$ on the growth of debt-GDP indicates that in the absence of tax-GDP or spending-GDP changes or abnormal returns on the debt, a fiscal position of -1% at time t results in a 1pp rise in the log debt-GDP ratio each year. As the horizon T increases in identity (20), average returns, debt growth and GDP growth converge to their unconditional means, which are equal, so the first and third terms on the right-hand side of (20) vanish in the limit as $T \to \infty$, and we recover (the $\rho = 1$ case of) the identity (21).

The situation is qualitatively different when $r < g$ so that ρ exceeds one. In this case the weights on excess returns increase in j, so at long horizons the weighted average places almost all weight on the distant future, when expected excess returns are close to their unconditional mean $r - g$. Meanwhile, the coefficient on expected debt-GDP growth converges to $\rho - 1$ as $T \to \infty$, rather than to zero.[8] The upshot is that we obtain a new limiting identity for the $r < g$ case when we let $T \to \infty$ in (19):

$$fp_t = (r-g) - \rho\sum_{j=0}^{\infty}\mathbb{E}_t\left(a\Delta\tau y_{t+1+j} - b\Delta x y_{t+1+j}\right) - (\rho-1)\left[\lim_{T\to\infty}\mathbb{E}_t\left(v y_{t+T}\right) - v y_t\right]. \tag{22}$$

Each of the terms in this identity is finite.[9] We might informally label equation (22)

[8] In identities that relate prices to discounted sums of future dividends plus a discounted future price, it is common practice to impose a transversality condition that the discounted future price tends to zero as horizon tends to infinity. If this condition fails, the discounted future price represents a "bubble term". The non-vanishing expected debt-GDP growth term in (22) is analogous to a bubble term.

[9] It therefore avoids a familiar problem that would occur, in the case $r < g$, if we took expectations and then the limit as $T \to \infty$ of (13): as $\rho > 1$, we would find that $\rho^T\,\mathbb{E}_t\,fp_{t+T}$ and

the "politician's dream". As the infinite sum in (22) does not discount the future, there is no pressing need to act quickly to resolve a weak fiscal position: changes to tax-GDP and spending-GDP growth rates that occur in 100 years' time enter the right-hand side of (22) with the same weight as changes that occur tomorrow. In this respect there is a sharp qualitative contrast with the situation (21) when $r > g$.

Moreover, as ρ crosses one the bubble term $(\rho - 1)\left[\lim_{T\to\infty} \mathbb{E}_t\left(vy_{t+T}\right) - vy_t\right]$ springs into being in (22). Even in the absence of changes in tax-GDP or spending-GDP, when $r < g$ a weak fiscal position can be resolved, in expectation, by allowing the debt-GDP ratio to rise over the long run. Higher debt-GDP makes a larger primary deficit sustainable and brings the fiscal position back to its mean.

It is tempting to conclude from equation (22) that politicians can borrow without limit to exploit a so-called "magic money tree". However this conclusion is incorrect because we have assumed that the unconditional mean of log debt growth equals the log growth rate of the economy, or equivalently that the unconditional mean change in the log debt-GDP ratio is zero. Thus a policy that allows the debt-GDP ratio to rise to resolve a weak fiscal position must also allow the debt-GDP ratio to fall to resolve a strong one. Note also that the presence of the factor $(\rho - 1)$ multiplying the change in the debt-GDP ratio means that there is no discontinuity as the $\rho = 1$ threshold is crossed. Near $\rho = 1$, very large changes in the debt-GDP ratio would be needed to contribute meaningfully to resolution of the fiscal position. Thus—despite what might seem the through-the-looking-glass character of the $r < g$ regime—the *quantitative* implications of equation (22) are modest in the countries we study because we estimate $\rho \approx 1$.

2 Debt and deficits in long-term US and UK data

To implement our approach to fiscal analysis, we begin by using these results to study debt and deficits in long-term data from the US and the UK, the series illustrated in Figures 1 and 2.

$\mathbb{E}_t \sum_{j=0}^{T-1} \rho^j \left[(1-\rho)r_{t+1+j} - \rho\left(a\Delta\tau_{t+1+j} - b\Delta x_{t+1+j}\right)\right]$ diverge to infinity in opposite directions, with their sum remaining constant at fp_t. Essentially this issue is noted as an analytical difficulty in the $r < g$ case by Reis (2021), Cochrane (2021), and Brunnermeier, Merkel and Sannikov (2024).

2.1 Data

To measure tax revenue and spending of the US federal government, we use annual data on total receipts, outlays, and interest payments reported by the Office of Management and Budget (OMB). These data are available on the FRED website for recent years (since 1901 for receipts and outlays, and 1940 for interest payments), and we augment them with hand-collected data for earlier years. We use total receipts to measure T_t, and the difference between total outlays and interest payments to measure X_t.[10]

We treat social security taxes as taxes and social security benefit payments as outlays, so we are consolidating the Social Security trust fund with the federal government. For this reason we do *not* include the nonmarketable debt held by the Social Security trust fund in our measure of government debt. Conversely, as our measures of tax receipts and expenditures do include transfers between the Federal Reserve and the US Treasury, we are treating the Federal Reserve as being outside the federal government. We therefore include the marketable debt held by the Federal Reserve in our measure of the debt.

Our framework requires that we measure the market value of the government debt, not the more readily available face value of the debt. We use data on the market value of marketable federal debt provided by the Federal Reserve Bank of Dallas from 1942, and splice this series together with data from Hall and Sargent (2021) for years before 1942.[11] The earliest data available are from the 1790s, but there is a break in the available data between 1838 and 1840, so we begin our series in 1841. To calculate real returns on the debt we apply the accounting identity (2) to the time series of debt, tax revenue, and spending and adjust for inflation.[12]

[10] Receipts include taxes and other collections from the public. See table 17.1 in https://www.whitehouse.gov/wp-content/uploads/2023/03/ap_17_receipts_fy2024.pdf for details. Outlays are payments that liquidate obligations. Details are given in the chapter on outlays in https://www.whitehouse.gov/wp-content/uploads/2023/03/ap_15_concepts_fy2024.pdf. The US federal government also collects income from the public through market-oriented activities. Collections from these activities are subtracted from gross outlays, rather than being added to taxes and other governmental receipts. See table 18.1 in https://www.whitehouse.gov/wp-content/uploads/2023/03/ap_18_offsetting_fy2024.pdf for details.

[11] The Hall and Sargent data align closely with the Federal Reserve Bank of Dallas data in the period where both series are available.

[12] In the postwar period we have confirmed the plausibility of the implied return series by regressing it on contemporaneous variables that explain the returns on short-term and long-term government debt: the short-term realized real interest rate and the change in the long-term bond yield. These regressions have high explanatory power and coefficients with the right sign and

We measure inflation using the GDP deflator from the Bureau of Economic Analysis since 1929, spliced to the historical deflator calculated by Louis Johnston and Samuel H. Williamson and available on the Measuring Worth website for earlier years.[13] We use the same two data sources to measure US GDP.

We proceed in a similar fashion to obtain historical data from the UK. To measure tax revenue and spending of the UK central government, we obtain data from the historical public finances database of the UK Office for Budget Responsibility. To measure the market value of central government debt, we splice together Bank of England data for the period 1727-2016 with BIS data from 2017-2022. We obtain data on UK nominal GDP and the GDP deflator from the Measuring Worth website.

Summary statistics for US and UK data are provided in the online appendix, Section OA.8.

2.2 Unit root tests and linearization parameters

The fiscal position, as we have defined it in equation (9), is a loglinear combination of tax revenue, government expenditure, and the market value of government debt with linearization parameters ρ, a, and b linked by the relationship (8). Before we can begin our analysis we need to choose these parameters.

The value of ρ can be estimated by using the relationship $-\log \rho = \mathbb{E}\, r_{t+1} - \mathbb{E}\, \Delta y_{t+1}$, where r_{t+1} is the log real return on government debt and Δy_{t+1} is the growth rate of real GDP. In the full US sample, $\mathbb{E}\, r_{t+1} = 2.9\%$ and $\mathbb{E}\, \Delta y_{t+1} = 3.5\%$, so $\rho = 1.006$. In the full UK sample, $\mathbb{E}\, r_{t+1} = 2.1\%$ and $\mathbb{E}\, \Delta y_{t+1} = 1.8\%$, so $\rho = 0.997$.[14] We set ρ equal to its sample estimate and compute all results in the general framework. Because both estimates are so close to one the simple equations that hold in the $r = g$ case, (10) and (20), provide an accurate and transparent guide to what we find. In particular they make clear why returns play so small a role in the mean-reversion of the fiscal position. Our results are almost identical if instead we impose $\rho = 1$ exactly.

For consistency with our analysis in Section 1.2 we de-mean the growth rates of real tax revenue and spending using the restriction that their means are equal to the mean real GDP growth rate (3.5% for the US and 1.8% for the UK), and we

strong statistical significance. Results are reported in Section OA.6 of the online appendix.

[13] See https://www.measuringworth.com/ for details.

[14] We also estimate ρ greater than but very close to one in shorter subsamples of US data, and ρ less than but very close to one in shorter subsamples of UK data.

impose the theoretical restriction that the mean of the fiscal position equals $-\log\rho$, which is approximately zero given our estimates of ρ.

We use equation (8) to substitute out the parameter b and choose the remaining linearization parameter a to ensure that our measure of the fiscal position, fp_t, optimally approximates $\log(1+S_t/V_t)$ in a least-squares sense. That is, a is chosen to solve the problem

$$\min_a \sum_t \left(\log\left(1+S_t/V_t\right) - \underbrace{\left[k+(1-\rho)xv_t+\rho a\left(\tau_t-x_t\right)\right]}_{fp_t} \right)^2, \tag{23}$$

where k is given in the online appendix.

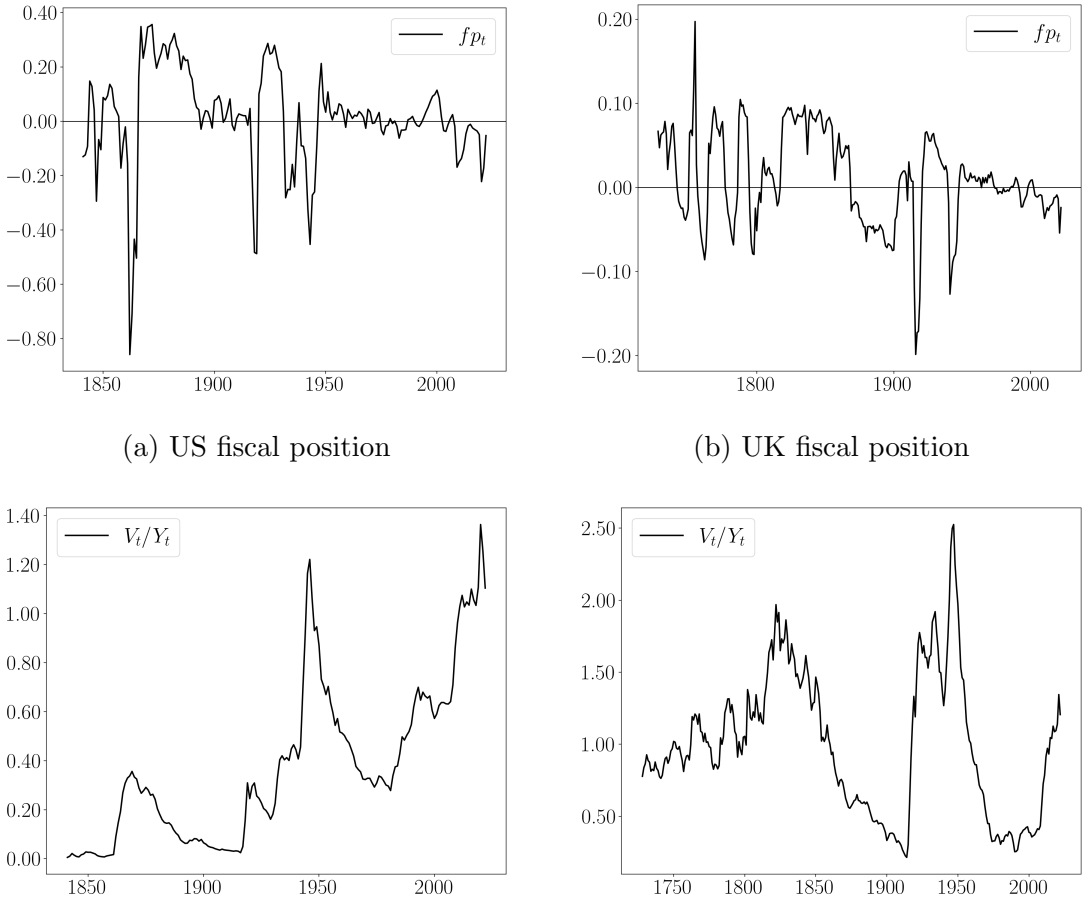
This procedure sets $a=0.394$ in the US and $a=0.128$ in the UK. In the US long historical sample, the sample averages of $(T_t/V_t, X_t/V_t)$ are $(0.387, 0.405)$ and the medians are $(0.276, 0.277)$; in the UK, the averages are $(0.227, 0.222)$ and medians are $(0.110, 0.106)$. Thus a is close to the sample mean in the US, and close to the sample median in the UK.

The top panels of Figure 2 show the history of fp_t over the sample period 1841–2022 in the US and 1727–2022 in the UK. In the US, sharp downturns in the fiscal position are visible around the time of the Civil War, the Great Depression, and the two World Wars. The UK's fiscal position spikes downward most dramatically during the two World Wars.

These major war episodes were, broadly, spending surges in which—despite rising tax revenues—government expenditure rose disproportionately. (The Korean and Vietnam Wars have little effect on the US fiscal position, and the Napoleonic Wars have little effect on the UK position, because they were largely financed by rises in tax revenue.) The shock to the fiscal position at the time of the Great Depression was, by contrast, a revenue collapse associated with a contracting economy.

The bottom panels of Figure 2 show the two countries' debt-GDP ratios, which have drifted persistently up and down for long periods of time, showing no strong tendency to return to a constant mean. We expect the average drift in debt-GDP to be zero, since an unconditionally positive or negative drift would imply arbitrarily large or small values for debt-GDP in the distant future. While the US data plotted in panel (c) of Figure 2 do convey a visual impression of an unconditionally positive (upward) drift, this drift is not statistically significant—equivalently, the mean difference between the growth rate of debt and the growth rate of GDP is

Figure 2: The fiscal position and debt-GDP ratio in the US and UK.



(a) US fiscal position

(b) UK fiscal position

(c) US debt-GDP ratio

(d) UK debt-GDP ratio

statistically insignificant, as shown in Table OA.20—and as already discussed our analysis assumes that debt and GDP have the same unconditional mean growth rates so that the unconditional drift of the debt-GDP ratio is zero. The UK data plotted in panel (d) of Figure 2 are also consistent with this assumption.

In Section OA.9 of the online appendix we report unit root test statistics and sample autocorrelations for the major time series: government debt returns, the growth rates of tax revenue, spending, and output, the ratio of debt to GDP, the ratios of taxes and spending to output and debt, the primary surplus-debt ratio and the logarithmic primary surplus-debt ratio, and finally our loglinear measure of the fiscal position. We report these statistics for the US long sample, UK long sample, and US and UK post-World War II samples beginning in 1947. In all four

specifications, ADF tests reject at the 5% level or better the null of a unit root in log debt returns; in the log growth rates of spending, tax revenue, debt, and GDP; and in the primary surplus-debt ratio and the log primary surplus-debt ratio. We therefore treat these as stationary variables suitable for inclusion in our empirical analysis. By contrast, ADF tests do not reject the null of a unit root in the debt-GDP ratio and spending-GDP ratio (either in logs or levels) in any of the four specifications.[15] Finally, we reject the null of a unit root for our measure of the fiscal position in the US and UK long samples and in postwar US data, but not in postwar UK data. Given the lower power of unit root tests in the shorter postwar samples, we proceed to treat the fiscal position as stationary.

2.3 Local projections

As a first way to understand the drivers of the variation in fp_t shown in Figure 2, we implement a local projections approach following Jordà (2005), Plagborg-Møller and Wolf (2021), and Li, Plagborg-Møller and Wolf (2024). This approach imposes minimal structure on the multivariate dynamic system for the fiscal position and its determinants, but it does require us to truncate the horizons T we can consider.

Based on the identity (13) we expect the fiscal position to positively forecast $b\Delta x_{t+1+j} - a\Delta\tau_{t+1+j}$, thereby moving the fiscal position back towards its unconditional mean. As b and a are close together in the US and UK, in panel (a) of Figure 3 we simply regress US spending growth minus tax growth, $\Delta x_{t+1+j} - \Delta\tau_{t+1+j}$, onto the US fiscal position for each future year from 1 to 10 years ahead. We plot the coefficients along with two-standard-error bands. The coefficients are positive out to 8 years ahead, and statistically significant for the first 5 years. In panel (b) we report the two components—spending growth and tax growth—separately. The figure shows a delayed positive response of spending growth to the fiscal position, with wide standard error bands so that no individual year is statistically significant, and a strong but transitory negative response of tax growth which is statistically significant for the first 2 years. Thus the positive response of spending minus tax growth is driven initially by tax growth, and later by spending growth.

Since a positive fiscal position forecasts increases in spending growth relative to tax growth, we expect to find that the fiscal position is mean-reverting. We

[15] The evidence is somewhat more ambiguous for the tax-debt, spending-debt, and tax-GDP ratios (either in logs or levels), for which the ADF test rejects the presence of a unit root in one of the four specifications.

verify this empirically in panel (c), which regresses the future US fiscal position on the current fiscal position. The coefficients are positive for 6 years but statistically significant for only 3. Mean-reversion is almost complete after only 5 years.

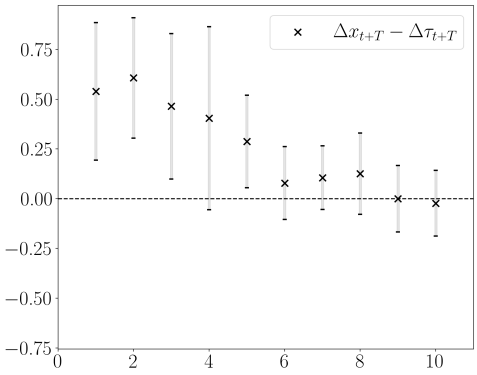
While returns contribute little to the dynamics of the fiscal position—exactly nothing in the $r = g$ benchmark, and very little at our sample value of ρ—they do contribute to the dynamics of debt in equation (19). In panel (d) we regress future debt returns on the fiscal position. There is a small but statistically significant positive coefficient one year ahead, and coefficients close to zero for all future years. This implies that debt returns slightly moderate the changes in debt that would otherwise be implied by the fiscal position. Panel (d) also shows the local projection coefficients of GDP growth on the fiscal position. These are close to zero and statistically insignificant at all horizons.

The predictive coefficients for tax growth, spending growth, returns, and GDP growth imply predictive coefficients for future one-period growth in the debt-GDP ratio. We directly estimate these coefficients and plot them in panel (e). A strong fiscal position implies slower debt-GDP growth, with a coefficient of about -0.5 one year ahead that decays to zero within five years. In the absence of fiscal adjustment (predictability of tax growth, spending growth, returns, or GDP growth) these coefficients would be -1.0 at all horizons. The implications for cumulative debt growth are shown in panel (f). A strong fiscal position today has a cumulative effect on debt-GDP that converges after about 5 years to a value of about -1.2. In other words, a 1% positive fiscal position today implies a long-run reduction in debt-GDP of about 1.2 percentage points. This can be compared with the 5 percentage point decrease that would occur over 5 years in the absence of fiscal adjustment, and the ongoing linear increase beyond 5 years that would occur in this scenario.

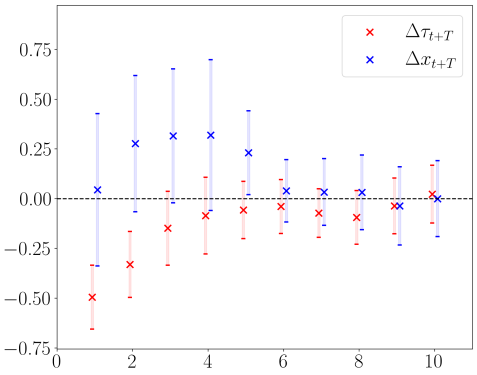
In Figure 4 we repeat this analysis for the UK and find qualitatively similar results. The UK fiscal position is a statistically significant predictor of spending growth minus tax growth up to 8 years ahead. In the UK this is primarily due to predictability of spending growth, rather than tax growth; this may reflect the series of wars that are associated with a weak fiscal position in the UK. Mean-reversion is slower in the UK, taking about 8 years rather than 5 years to complete. The fiscal position positively predicts GDP growth and more strongly predicts returns in the UK, with statistical significance in the first three years. Putting these effects together the predictive coefficients for future UK debt-GDP growth are slightly larger and more persistent than in the US. A 1% fiscal position implies a reduction

Figure 3: What does the US fiscal position predict?

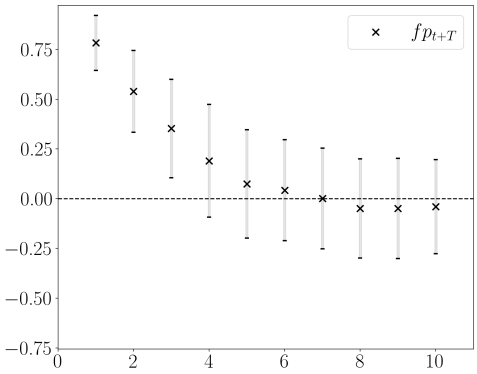
This figure plots estimated slope coefficients (with ±2 Newey–West standard error bands) from regressions $\theta_{t+j} = \alpha + \beta_{\theta,j}\, fp_t + \varepsilon_{\theta,t+j}$, for $j = 1, \ldots, 10$, where the variables θ_{t+j} are indicated in the legend of each subfigure. US data 1841–2022, $\rho = 1.006$, $a = 0.394$, $b = 0.399$.



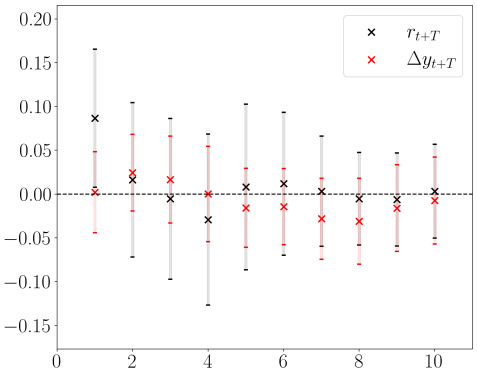
(a) Spending less tax growth



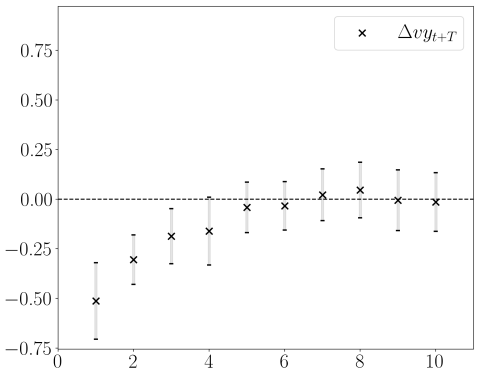
(b) Spending and tax growth



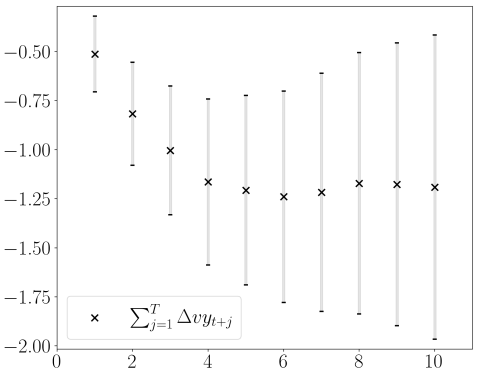
(c) Future fp



(d) Return and GDP growth



(e) Debt-GDP growth



(f) Cumulative debt-GDP growth

in debt-GDP of about 1.9 percentage points after six years, although the effect weakens thereafter with wide confidence intervals.

Figures 5 and 6 illustrate the predictive power of the US and UK fiscal position measures in a different way. These figures aggregate the first ten years, plotting the history of the fiscal position fp_t against the discounted sum of subsequent realized ten-year spending growth minus tax growth in panel (a) and ten-year returns in panel (b). All series in Figures 5 and 6 are de-meaned and divided by their standard deviation to eliminate scale effects.

In each figure, panel (a) shows visually how the fiscal position is associated with subsequent ten-year changes in tax and spending growth rates that bring the fiscal position back to its mean. In both countries wars—the Civil War for the US, wars with France in the 18th century for the UK, and World Wars I and II—are associated with a weak fiscal position and subsequent corrective ten-year tax and spending changes. Panel (b) shows that the relationship between the fiscal position and subsequent returns on the government debt is much less consistent, particularly in the US.

2.3.1 Using local projections to decompose the fiscal position

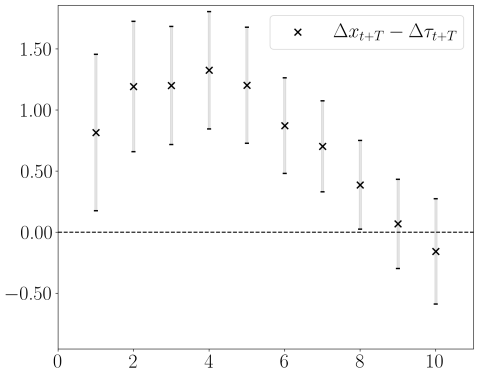
We explore the contributions of returns, tax, and spending to the identity (13) by running regressions

$$\begin{aligned}
(1-\rho)\sum_{j=0}^{T-1}\rho^j r_{t+1+j} &= \alpha_{r,T} + \beta_{r,T} fp_t + \varepsilon_{r,t+T},\\
-\rho a\sum_{j=0}^{T-1}\rho^j \Delta\tau_{t+1+j} &= \alpha_{\tau,T} + \beta_{\tau,T} fp_t + \varepsilon_{\tau,t+T},\\
\rho b\sum_{j=0}^{T-1}\rho^j \Delta x_{t+1+j} &= \alpha_{x,T} + \beta_{x,T} fp_t + \varepsilon_{x,t+T},\\
\rho^T fp_{t+T} &= \alpha_{fp,T} + \beta_{fp,T} fp_t + \varepsilon_{fp,t+T}.
\end{aligned} \tag{24}$$

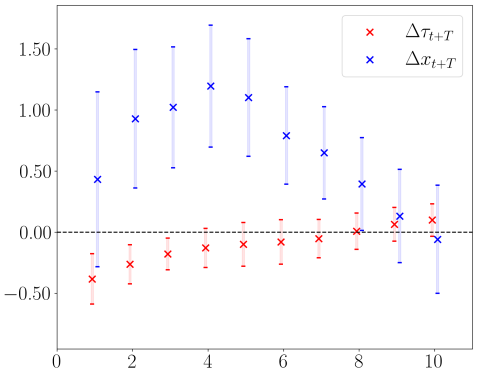
at horizons $T = 1, 3$, and 10 years. The estimated coefficients $\beta_{r,T}$, $\beta_{\tau,T}$, $\beta_{x,T}$, and $\beta_{fp,T}$ are shown (measured in percent) in the second, third, fourth, and fifth columns of Table 1. The quantities reported in square brackets are Newey–West standard errors calculated using 2, 4, and 11 lags, respectively. The identity (13) implies that

Figure 4: What does the UK fiscal position predict?

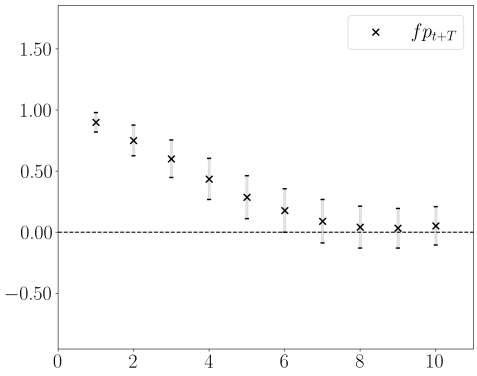
This figure plots estimated slope coefficients (with ±2 Newey–West standard error bands) from regressions $\theta_{t+j} = \alpha + \beta_{\theta,j}\, fp_t + \varepsilon_{\theta,t+j}$, for $j = 1, \ldots, 10$, where the variables θ_{t+j} are indicated in the legend of each subfigure. UK data 1727–2022, $\rho = 0.997$, $a = 0.128$, $b = 0.125$.



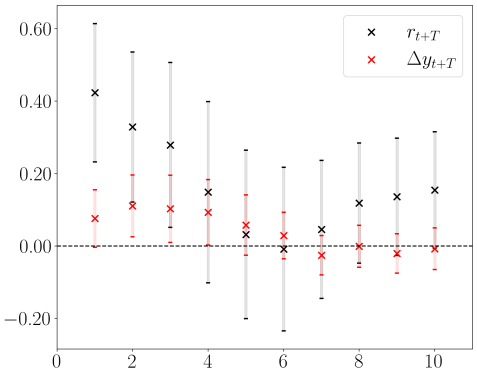
(a) Spending less tax growth



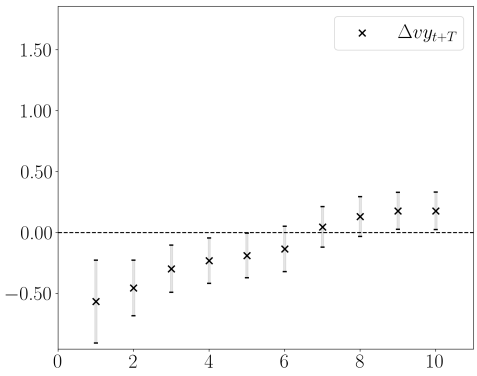
(b) Spending and tax growth



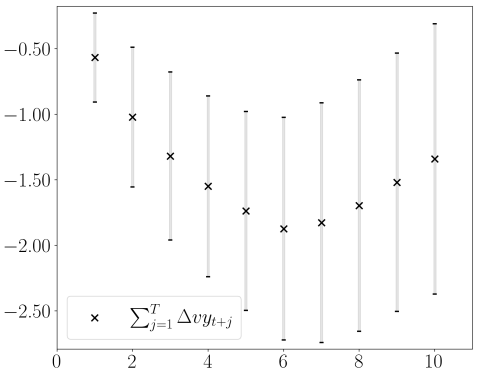
(c) Future fp



(d) Return and GDP growth

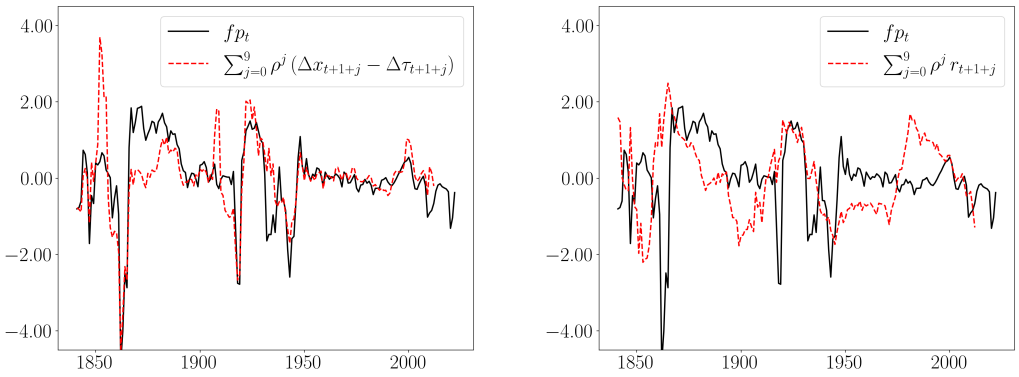


(e) Debt-GDP growth



(f) Cumulative debt-GDP growth

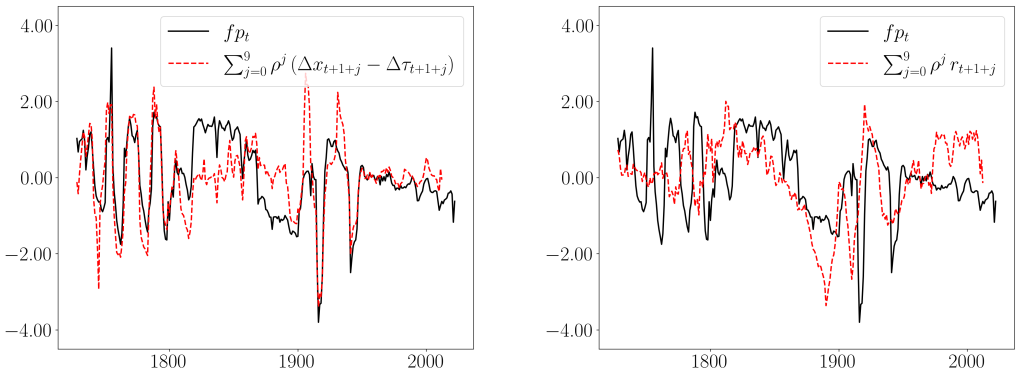
Figure 5: The fiscal position and subsequent spending less tax growth and returns. US data 1841–2022, $\rho = 1.006$, $a = 0.394$, $b = 0.399$.



(a) Spending less tax growth

(b) Return

Figure 6: The fiscal position and subsequent spending less tax growth and returns. UK data 1727–2022, $\rho = 0.997$, $a = 0.128$, $b = 0.125$.



(a) Spending less tax growth

(b) Return

Table 1: Local projections for the fiscal position, US and UK.
All coefficients are multiplied by 100, i.e., measured in percent. The table reports Newey–West standard errors with lags of 2, 4, and 11, respectively, at horizons $T = 1, 3$ and 10.

US: 1841 to 2022, $\rho = 1.006$, $a = 0.394$, $b = 0.399$

horizon	return	tax	spending	future fp
1	-0.1	19.3	2.0	78.8
	[0.0]	[3.2]	[6.8]	[6.6]
3	-0.1	38.4	25.6	36.4
	[0.0]	[8.1]	[13.7]	[13.1]
10	-0.0	53.3	51.3	-4.2
	[0.2]	[13.1]	[24.3]	[18.4]

UK: 1727 to 2022, $\rho = 0.997$, $a = 0.128$, $b = 0.125$

horizon	return	tax	spending	future fp
1	0.1	4.9	5.5	89.5
	[0.0]	[1.3]	[4.3]	[3.8]
3	0.3	10.4	29.5	59.7
	[0.1]	[2.3]	[7.9]	[7.8]
10	0.5	13.0	81.3	5.0
	[0.3]	[4.3]	[16.4]	[14.5]

the coefficients satisfy the restriction

$$\beta_{r,T} + \beta_{\tau,T} + \beta_{x,T} + \beta_{fp,T} = 1 \tag{25}$$

at each horizon, up to an approximation error which vanishes in the case where $r = g$ and is very small when r is close to g as in our data. Measured in percent, the coefficients in Table 1 therefore add up to a number close to 100.

Table 1 shows several important results. First, returns contribute negligibly to the fiscal position at any horizon. The trivial contribution of returns is due in part to the fact that ρ is close to one in both the US and the UK. We are close to the $r = g$ case in which debt can be rolled over forever without imposing a burden on the government. But, in addition, US returns are not highly correlated with the US

fiscal position as we saw in Figure 5, so even values of ρ that are far from one would not deliver a substantial role for returns in the US decomposition.

Second, the contributions of tax and spending growth rise steadily with the horizon while the contribution of the future fiscal position declines. At a one-year horizon, around 20% of the variance of the US fiscal position and 10% of the variance of the UK fiscal position is accounted for by one-year adjustments to tax and to spending, leaving the rest to the future fiscal position. After three years, tax and spending account for 64% of the variance of the US fiscal position and 40% of the variance of the UK fiscal position, and after ten years they account for almost all the variance. This pattern corresponds to the fact we have previously noted, that the fiscal position follows a stationary process that gradually reverts to its mean.

Third, the importance of spending relative to tax increases over time, consistent with the response patterns illustrated in Figures 3 and 4: at the ten-year horizon, spending accounts for around 50% of the variation in the fiscal position in the US and around 80% in the UK.

Table 2 runs regressions to explore the relationship between the fiscal position and the dynamics of the debt, using the identity (19). Since r is very close to g in the data, this identity is very close to the $r = g$ special case (20). The table reports the coefficients from the regressions

$$\begin{aligned}
\sum_{j=0}^{T-1} \frac{(1-\rho)\rho^j}{1-\rho^T} r_{t+1+j} &= \alpha_{r,T} + \beta_{r,T} fp_t + \varepsilon_{r,t+T}, \\
-\sum_{j=0}^{T-1} \frac{(1-\rho)\rho^j}{1-\rho^T} \Delta y_{t+1+j} &= \alpha_{y,T} + \beta_{y,T} fp_t + \varepsilon_{y,t+T}, \\
-a\sum_{j=0}^{T-1} \frac{\rho^{j+1}-\rho^{T+1}}{1-\rho^T} \Delta \tau y_{t+1+j} &= \alpha_{\tau y,T} + \beta_{\tau y,T} fp_t + \varepsilon_{\tau y,t+T}, \\
b\sum_{j=0}^{T-1} \frac{\rho^{j+1}-\rho^{T+1}}{1-\rho^T} \Delta x y_{t+1+j} &= \alpha_{xy,T} + \beta_{xy,T} fp_t + \varepsilon_{xy,t+T}, \\
-\frac{(1-\rho)\rho^T}{1-\rho^T} (v y_{t+T} - v y_t) &= \alpha_{vy,T} + \beta_{vy,T} fp_t + \varepsilon_{vy,t+T}.
\end{aligned} \tag{26}$$

at horizons $T = 1, 3$, and 10 years.

Table 2: Local projections for debt growth, US and UK.

All coefficients are multiplied by 100, i.e., measured in percent. The table reports Newey–West standard errors with lags of 2, 4, and 11, respectively, at horizons $T = 1, 3$ and 10.

US: 1841 to 2022, $\rho = 1.006$, $a = 0.394$, $b = 0.399$

horizon	r	$-\Delta y$	$-\Delta\tau y$	Δxy	$-\Delta vy$	$-(vy_{t+T} - vy_t)$
1	9.6	-0.3	19.5	1.9	49.7	49.5
	[3.9]	[2.3]	[3.0]	[6.3]	[8.8]	[8.8]
3	3.4	-1.5	31.1	12.3	33.7	100.0
	[2.8]	[2.0]	[5.1]	[8.6]	[6.0]	[17.7]
10	0.8	0.7	43.0	38.4	12.3	119.1
	[2.7]	[1.9]	[8.0]	[13.9]	[5.5]	[53.3]

UK: 1727 to 2022, $\rho = 0.997$, $a = 0.128$, $b = 0.125$

horizon	r	$-\Delta y$	$-\Delta\tau y$	Δxy	$-\Delta vy$	$-(vy_{t+T} - vy_t)$
1	42.5	-7.3	5.8	4.5	55.9	56.1
	[10.0]	[4.2]	[1.4]	[4.2]	[18.1]	[18.1]
3	34.1	-9.7	10.2	15.0	44.1	133.0
	[9.5]	[4.1]	[2.4]	[5.5]	[12.2]	[36.7]
10	16.7	-4.2	16.7	49.9	13.2	134.1
	[10.3]	[1.8]	[3.7]	[11.5]	[7.5]	[76.3]

The identity (19) implies that the coefficients satisfy the restriction

$$\beta_{r,T} + \beta_{y,T} + \beta_{\tau y,T} + \beta_{xy,T} + \beta_{vy,T} = 1 \tag{27}$$

at each horizon, up to approximation error. Unlike the identity explored in Table 1, however, in this case approximation error does not vanish when $r = g$. The coefficients in Table 2 are measured in percent, so the coefficients in the first five columns of Table 2 would add up to 100 in the absence of approximation error. The coefficient sums are close to 100 in the UK and for the 10-year horizon in the US, but fall short by about 20 for shorter US horizons.

The last column of Table 2 reports the cumulative effect of the fiscal position on the debt-GDP ratio at each horizon. This column is obtained from the fifth column by dividing by $(1-\rho)\rho^T/(1-\rho^T)$. Given ρ close to one this is similar to multiplying by T.

In the US, the fiscal position has little predictive power for GDP growth at any horizon. One-year changes in returns, tax-GDP growth, and spending-GDP growth reduce the one-year impact of the fiscal position on the debt-GDP ratio by about one half: a 1% positive fiscal position implies a 0.50 percentage point reduction in debt-GDP over one year. As the horizon increases, the contributions of returns decline and the contributions of tax-GDP growth and particularly of spending-GDP growth increase, so that at the ten-year horizon a 1% positive fiscal position implies an average ten-year debt-GDP growth rate of only -0.12%. This cumulates to a 1.19 percentage point reduction in the debt-GDP ratio over ten years, with a standard error of 0.53 percentage points.

In the UK, the results for debt accumulation are similar to those in the US but as before, spending plays a greater role than tax revenue in these adjustments. A strong fiscal position predicts high returns which slow the reduction in the debt-GDP ratio but rapid GDP growth which speeds that reduction. At a ten-year horizon, a 1% fiscal position is associated with a reduction of 1.34 percentage points in the UK debt-GDP ratio. This estimate is less precise than in the US, with a standard error of 0.76 percentage points. In both countries, these results imply that fiscal adjustment reduces the impact of the fiscal position on the ten-year growth of the debt-GDP ratio by almost 90% (from about 10 percentage points in the absence of fiscal adjustment to just over 1 percentage point).

2.4 VAR estimation

The local projections approach is appealingly direct, requiring no assumptions about the dynamics of fiscal variables, but it restricts the longest horizons we can consider. Even in our long historical data sets, there are only 18 decades of US data and not quite 30 decades of UK data, so local projections are likely to be unreliable at any horizons beyond a decade. To estimate longer-run fiscal dynamics, we now assume that fiscal variables can be described by a low-order vector autoregression (VAR) that allows us to calculate expected fiscal adjustment and its components over the infinite future.

We estimate a first-order VAR in the variables r_t, $\Delta\tau_t$, Δy_t, and fp_t. Our inclusion of r_t, $\Delta\tau_t$, and fp_t is motivated by the identities (13) and (15). We do not include Δx_t as it is mechanically related to fp_t, fp_{t-1}, r_t, and $\Delta\tau_t$ via the identity (12). Note however that this inference is possible only if we include an additional lag of the fiscal position, fp_{t-1} as well as fp_t, in the system. We include this additional lag so that (except for approximation error) our results are invariant to the decision to include $\Delta\tau$ in the VAR rather than Δx.[16]

We include GDP growth, Δy_t, both because of its importance for forecasting the other variables in the VAR—for example, we expect a larger economy to be able to raise more tax revenue—and because of our interest in the identity (19).

The estimated VAR for US data is shown in the first four columns of Table 3. The US fiscal position fp_{t+1} is relatively predictable, with an R^2 of about 64%, and is strongly predicted by its lag.

A strong fiscal position (high fp_t) forecasts high returns for debt holders; more precisely, as the coefficient on the lagged fiscal position is of similar magnitude with the opposite sign, an *improving* fiscal position forecasts high returns for debt holders. The fiscal position also strongly forecasts tax growth, and with a negative sign, so that a strong fiscal position predicts slower growth in tax revenue. It has no significant ability to forecast GDP growth, by contrast, so that the growth rate of the tax-GDP ratio is forecastable. This is broadly consistent with findings in the public finance literature that reject the random-walk prediction for tax rates in Barro (1979).[17] Meanwhile, GDP growth (Δy_t) is a highly significant forecaster of

[16] See Engsted, Pedersen and Tanggaard (2012) for a thoughtful discussion of this issue.

[17] See, for example, Sahasakul (1986). The coefficient on fp_t in a regression of $\Delta\tau_{t+1} - \Delta y_{t+1}$ onto the state variables is -0.553, with a standard error of 0.083. The coefficient on fp_{t-1} in the same regression is 0.159, with a standard error of 0.096.

Table 3: VAR coefficient estimates. US data, 1841–2022, $\rho = 1.006$, $a = 0.394$, $b = 0.399$.

OLS standard errors are reported in square brackets. The last two columns show the imputed coefficients for spending growth and spending growth minus tax growth.

	r_{t+1}	$\Delta\tau_{t+1}$	Δy_{t+1}	fp_{t+1}	Δx_{t+1}	$\Delta x_{t+1} - \Delta\tau_{t+1}$
r_t	0.054	0.191	0.040	0.048	0.069	−0.122
	[0.075]	[0.094]	[0.033]	[0.084]	[0.215]	[0.212]
$\Delta\tau_t$	−0.047	0.143	−0.019	−0.018	0.186	0.043
	[0.058]	[0.073]	[0.026]	[0.065]	[0.167]	[0.165]
Δy_t	0.123	1.120	0.304	0.127	0.788	−0.332
	[0.176]	[0.222]	[0.078]	[0.197]	[0.505]	[0.500]
fp_t	0.222	−0.580	−0.028	0.943	−0.441	0.139
	[0.067]	[0.085]	[0.030]	[0.075]	[0.194]	[0.192]
fp_{t-1}	−0.179	0.193	0.034	−0.209	0.710	0.517
	[0.078]	[0.098]	[0.034]	[0.087]	[0.223]	[0.221]
R^2	6.04%	45.20%	10.55%	63.52%	9.10%	13.94%

Table 4: VAR coefficient estimates. UK data, 1727–2022, $\rho = 0.997$, $a = 0.128$, $b = 0.125$.

OLS standard errors are reported in square brackets. The last two columns show the imputed coefficients for spending growth and spending growth minus tax growth.

	r_{t+1}	$\Delta\tau_{t+1}$	Δy_{t+1}	fp_{t+1}	Δx_{t+1}	$\Delta x_{t+1} - \Delta\tau_{t+1}$
r_t	0.164	−0.172	0.065	0.031	−0.429	−0.256
	[0.059]	[0.057]	[0.026]	[0.020]	[0.164]	[0.160]
$\Delta\tau_t$	0.096	0.118	−0.013	−0.046	0.482	0.365
	[0.060]	[0.058]	[0.027]	[0.020]	[0.168]	[0.164]
Δy_t	−0.421	0.173	−0.058	−0.077	0.807	0.634
	[0.130]	[0.124]	[0.058]	[0.044]	[0.359]	[0.351]
fp_t	0.499	−0.659	−0.162	1.136	−1.746	−1.087
	[0.170]	[0.163]	[0.076]	[0.057]	[0.471]	[0.460]
fp_{t-1}	−0.132	0.450	0.229	−0.291	2.793	2.343
	[0.170]	[0.162]	[0.076]	[0.057]	[0.470]	[0.459]
R^2	16.87%	15.41%	6.68%	83.32%	17.43%	16.17%

tax growth with a coefficient above one, consistent with the presence of increasing marginal tax rates.

The last two columns of Table 3 show imputed coefficients for spending growth Δx_{t+1} and spending growth less tax growth $\Delta x_{t+1} - \Delta \tau_{t+1}$. A strong fiscal position predicts high spending growth, but the effect operates with a lag. Rapid GDP growth also predicts high spending growth, although this effect is imprecisely estimated. Combining the effects on spending and tax growth, a strong fiscal position (high fp_t and fp_{t-1}) forecasts increases in government spending relative to tax revenue which gradually bring the fiscal position back to its mean.

The estimated VAR for UK data is shown in the first four columns of Table 4. As in the US, a strong fiscal position (high fp_t) forecasts high returns for debt holders and low tax growth. By contrast with the US, GDP growth is only a weak and statistically insignificant predictor of tax revenue growth. As in the US, a strong fiscal position predicts high spending growth with a lag. When the effects on spending and tax growth are combined, a strong fiscal position predicts a reduction in spending relative to tax revenue one year ahead, but an increase two years ahead. The latter effect is stronger, so again tax and spending responses create mean reversion in the fiscal position.

2.4.1 VAR-implied decompositions of the fiscal position

We can use these VARs to understand what fluctuations in the fiscal position, fp_t, imply about the subsequent evolution of debt returns, tax growth, and spending growth. We repeat the analysis of Section 2.3, but instead of using historical data to estimate the regressions in (24) and (26), we calculate the VAR-implied regression coefficients.[18] This allows us to consider arbitrarily long horizons.

Table 5 reports the VAR-implied decomposition (24) over various different horizons T, for the US in the top panel and the UK in the bottom panel. At each horizon we report the contribution of returns, tax, spending, and future fiscal posi-

[18] This approach can be understood in the variance-decomposition framework of Cochrane (2008). Working on the topic of equity market predictability, Cochrane estimates a model whose only predictor variable is a valuation ratio analogous to our fp_t. He emphasizes the linkage between predictions of returns, cash flow growth, and future valuation ratios in that model. Our approach differs from his in two ways: we allow each variable in the VAR to depend on lagged values of all other variables in the system, not just on the valuation ratio; and we include an additional lag of the valuation ratio itself (fp_{t-1}) so that tax growth and spending growth are treated symmetrically, as discussed in the previous subsection.

Table 5: VAR-implied decomposition (24) of the fiscal position, fp_t, in US and UK data.

All quantities are measured in percent. Bootstrapped standard errors are reported in square brackets; — marks a component that is zero by construction.

US: 1841 to 2022, $\rho = 1.006$, $a = 0.394$, $b = 0.399$

horizon	return	tax	spending	future fp
1	-0.1	18.8	2.1	79.1
	[0.0]	[2.0]	[4.5]	[4.5]
3	-0.1	38.2	25.0	36.8
	[0.1]	[6.9]	[13.4]	[10.4]
10	-0.1	48.2	50.3	1.6
	[0.1]	[15.9]	[18.3]	[4.1]
∞	-0.1	48.6	51.5	—
	[0.1]	[18.0]	[18.0]	

UK: 1727 to 2022, $\rho = 0.997$, $a = 0.128$, $b = 0.125$

horizon	return	tax	spending	future fp
1	0.1	4.8	5.3	89.9
	[0.0]	[0.9]	[2.4]	[2.4]
3	0.3	9.9	29.5	60.3
	[0.1]	[2.6]	[8.2]	[7.4]
10	0.5	13.8	76.8	8.9
	[0.2]	[6.3]	[12.3]	[7.6]
∞	0.6	14.3	85.2	—
	[0.3]	[7.7]	[7.8]	

Table 6: VAR-implied decomposition (26) of the fiscal position, fp_t, in US and UK data.

All quantities are measured in percent. Bootstrapped standard errors are reported in square brackets; — marks a component that is zero by construction.

US: 1841 to 2022, $\rho = 1.006$, $a = 0.394$, $b = 0.399$

horizon	r	$-\Delta y$	$-\Delta\tau y$	Δxy	$-\Delta vy$	$-(vy_{t+T} - vy_t)$
1	9.8	-0.3	18.9	2.0	69.5	69.2
	[4.1]	[1.8]	[2.0]	[4.4]	[6.4]	[6.4]
3	4.0	-1.8	30.5	12.4	54.9	162.9
	[3.8]	[2.0]	[3.8]	[8.6]	[8.0]	[23.7]
10	0.9	-1.1	44.2	32.1	23.9	232.1
	[1.9]	[1.1]	[9.0]	[14.2]	[7.2]	[70.2]
∞	—	—	52.6	46.1	1.3	234.9
			[16.3]	[16.6]	[0.5]	[86.0]

UK: 1727 to 2022, $\rho = 0.997$, $a = 0.128$, $b = 0.125$

horizon	r	$-\Delta y$	$-\Delta\tau y$	Δxy	$-\Delta vy$	$-(vy_{t+T} - vy_t)$
1	41.8	-7.8	5.7	4.3	56.0	56.1
	[7.3]	[3.2]	[1.0]	[2.4]	[7.6]	[7.6]
3	32.6	-9.6	9.9	14.9	52.2	157.7
	[8.0]	[2.9]	[1.8]	[5.2]	[7.8]	[23.5]
10	16.5	-6.2	16.6	44.0	29.1	296.4
	[6.3]	[1.7]	[4.0]	[10.0]	[6.1]	[62.2]
∞	0.6	-0.2	22.8	76.8	—	319.0
	[0.3]	[0.1]	[8.3]	[8.4]		[92.0]

tion, measured in percent, so that the four columns add up to 100% at each horizon.

Bootstrapped standard errors are shown in square brackets under the point estimates. Each bootstrap sample is computed by first drawing a new VAR coefficient matrix using the point estimates and the covariance matrix of the estimated coefficients. Using this VAR coefficient matrix, we generate the news series and do the variance decomposition. We repeat this procedure 2,000 times to generate standard errors.

The first three rows of Table 5 confirm our earlier findings using local projections, suggesting that the VAR accurately captures fiscal dynamics at horizons up to a decade. The fiscal position decays rapidly in the US and a little more slowly in the UK, but after 10 years it has returned almost to its unconditional mean in both countries. In the US, taxes adjust faster than spending, but after 10 years taxes and spending are contributing equally to fiscal adjustment. In the UK, by contrast, taxes and spending begin adjusting equally fast, but soon spending contributes much more than taxes to fiscal adjustment. As before, returns contribute very little in either case. The last row of Table 5 calculates the implications at an infinite horizon. It shows taxes and spending contributing equally to US fiscal adjustment, while spending accounts for 85% of UK fiscal adjustment leaving only 14% to be covered by taxation.

Table 6 reports the VAR-implied decomposition (26). The table is formatted in the same way as the local projections equivalent, Table 2. The first three rows show results that are similar to those in Table 2, except that the cumulative impact of the fiscal position on the ten-year growth of the debt-GDP ratio is greater with coefficients above 2. (There is no approximation error because we impute Δvy from the identity $\Delta vy_{t+1} = r_{t+1} - \Delta y_{t+1} - fp_{t+1}$ so that the columns sum to 100 by construction.) These VAR results imply that fiscal adjustment reduces the ten-year impact of the fiscal position on the growth of the debt-GDP ratio by about three-quarters, as compared with the almost 90% reduction implied by the local projections results.

The last row of Table 6 considers an infinite horizon. In the US, where $r < g$ and $\rho > 1$, the limiting future debt-GDP ratio can contribute to the decomposition of the fiscal position even at an infinite horizon, as shown in equation (22). However, because ρ is close to one this contribution is small. In the UK, where $r > g$ and $\rho < 1$, the limiting future debt-GDP ratio can contribute nothing. Returns and GDP growth also play a very modest role and, as before, tax-GDP and spending-

GDP contribute roughly equally in the US whereas spending-GDP dominates in the UK.[19]

The last entry in the last row of the table shows that the infinite-horizon impact of the fiscal position on the debt-GDP ratio is comparable to the ten-year impact. This reflects the fact that we estimate almost complete fiscal adjustment within ten years. In the US, we estimate that a 1% positive fiscal position today implies a long-run reduction of 2.35 percentage points in the debt-GDP ratio, with a standard error of 0.86 percentage points, while in the UK we estimate a long-run reduction of 3.19 percentage points in this ratio with a standard error of 0.92 percentage points.

2.4.2 Responses to tax, spending, and return shocks

So far we have presented variance decompositions for the fiscal position in relation to its own future value or the accumulation of debt. It is also possible to use our linear framework to decompose the responses to shocks to taxes or spending. Such shocks must be accommodated by current or future returns on the debt, current changes in spending to accommodate tax shocks or taxes to accommodate spending shocks, future changes in taxes or spending, or the future fiscal position—and at an infinite horizon, the fiscal position makes no contribution.

This analysis allows us to consider two issues that we have not yet been able to address. First, is there evidence that contemporaneous changes in the value of the government debt are important in accommodating fiscal shocks? Such changes do not appear in the forward-looking formula for the fiscal position at a point in time, which only includes expected future debt returns and not contemporaneous shocks to returns. Second, are there differences in the dynamic responses when deficits are driven by shocks to taxes or by shocks to spending?

To address these questions we return to the identity (13). We apply the "news

[19]The contributions of tax and spending are not numerically identical in the last rows of Tables 5 and 6 because the former uses identity (15) while the latter uses identity (21). These differ because (21) calculates a separate contribution of output and measures tax and spending relative to output.

operator", $\Delta\,\mathbb{E}_{t+1} = \mathbb{E}_{t+1} - \mathbb{E}_t$ to both sides of (13) and rearrange to obtain

$$\underbrace{\Delta\,\mathbb{E}_{t+1}\,\tau_{t+1}}_{\substack{\text{short-run tax news:}\\ N_{\text{SR tax},t+1}}} = \left(\frac{a-b}{a}\right)\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j r_{t+1+j}}_{\text{return news: } N_{\text{return},t+1}} - \underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=1}^{T-1}\rho^j \Delta\tau_{t+1+j}}_{\substack{\text{long-run tax news:}\\ N_{\text{LR tax},t+1}}} +$$

$$+\left(\frac{b}{a}\right)\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j \Delta x_{t+1+j}}_{\text{spending news: } N_{\text{spending},t+1}} + \frac{1}{a}\underbrace{\Delta\,\mathbb{E}_{t+1}\,\rho^{T-1} fp_{t+T}}_{\substack{\text{future fiscal position news:}\\ N_{\text{future fp},t+1}}} . \quad (28)$$

This identity allows us to trace out the consequences of an unexpected shock to taxes. We refer to such a shock as short-run tax news, $N_{\text{SR tax},t+1} = \Delta\,\mathbb{E}_{t+1}\,\tau_{t+1}$. A positive short-run tax shock must be reflected in some combination of (i) news about returns, $N_{\text{return},t+1} = \Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j r_{t+1+j}$; (ii) news about declines in long-run tax growth, $N_{\text{LR tax},t+1} = \Delta\,\mathbb{E}_{t+1}\sum_{j=1}^{T-1}\rho^j \Delta\tau_{t+1+j}$; (iii) news about spending growth, $N_{\text{spending},t+1} = \Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j \Delta x_{t+1+j}$; and (iv) news about the future fiscal position, $N_{\text{future fp},t+1} = \Delta\,\mathbb{E}_{t+1}\,\rho^{T-1} fp_{t+T}$. This last term becomes negligible once the horizon, T, is sufficiently long.

Section OA.10.7 of the online appendix presents tables that show the contribution of each of the terms on the right hand side to the variance of the left hand side of equation (28), using the same VAR-based methodology as in Tables 5 and 6. In both the US and the UK, returns play almost no role in the response to tax shocks—an unsurprising finding given that r is very close to g. At the shortest horizon—that is, in the year the shock occurs—a contemporaneous adjustment in spending accounts for 56% of the response in the US and 67% in the UK, with the one-year-ahead fiscal position accounting for the remainder of the response. Over subsequent years, the stationarity of the fiscal position implies that its contribution declines (reaching zero at an infinite horizon) and the contribution of spending increases, with little contribution from tax adjustments subsequent to the initial shock.

We can carry out a similar exercise for unexpected shocks to spending, rewriting

the identity (28) as

$$\underbrace{\Delta\,\mathbb{E}_{t+1}\,x_{t+1}}_{\substack{\text{short-run spending news:}\\ N_{\text{SR spending},t+1}}} = -\left(\frac{a-b}{b}\right)\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^{j}r_{t+1+j}}_{\text{return news: } N_{\text{return},t+1}} + \frac{a}{b}\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^{j}\Delta\tau_{t+1+j}}_{\text{tax news: } N_{\text{tax},t+1}} + \\ -\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=1}^{T-1}\rho^{j}\Delta x_{t+1+j}}_{\substack{\text{long-run spending news:}\\ N_{\text{LR spending},t+1}}} - \frac{1}{b}\underbrace{\Delta\,\mathbb{E}_{t+1}\,\rho^{T-1}fp_{t+T}}_{\substack{\text{future fiscal position news:}\\ N_{\text{future fp},t+1}}}. \qquad (29)$$

We write $N_{\text{tax},t+1}$ for the tax news term that appears on the right-hand side of identity (29). This is the sum of short-run tax news and long-run tax news, as defined in (28): $N_{\text{tax},t+1} = N_{\text{SR tax},t+1} + N_{\text{LR tax},t+1}$. Similarly, we write $N_{\text{SR spending},t+1}$ for short-run spending news and $N_{\text{LR spending},t+1}$ for long-run spending news, so that $N_{\text{spending},t+1}$ as defined after identity (28) is equal to the sum $N_{\text{SR spending},t+1} + N_{\text{LR spending},t+1}$. Once again we present tables in the online appendix that use our VAR methodology to decompose the variance of the left hand side of (29) into the contributions of each of the terms on the right hand side.

In both the US and the UK, returns play almost no role in the response to spending shocks, again an unsurprising result given that r is very close to g. Other results, however, are different. In the year a spending shock occurs, a contemporaneous adjustment in tax revenue accounts for only 11% of the response in the US and 9% in the UK, with the one-year-ahead fiscal position accounting for the remainder of the response. Over subsequent years, stationarity implies that the contribution of the fiscal position declines towards zero. By contrast with the results for tax shocks, the adjustment is no longer entirely from spending. At an infinite horizon in the US, less than a quarter of the adjustment results from mean-reversion in spending and the remaining three-quarters comes from tax revenue. In the UK, 63% comes from mean-reversion in spending and the remaining 37% comes from tax revenue. As before, the greater role for spending adjustments in the UK may reflect the large wars that play an important role in the long UK history.

We can also use our VAR system to decompose the variance of return shocks into components attributable to news about future returns and to news about future growth rates of tax revenue and spending. This is analogous to the Campbell (1991) decomposition of the variance of aggregate US stock returns, although returns on

the government debt are considerably less volatile, with an annualized standard deviation around 10% in the US and 7% in the UK, than returns on the aggregate stock market.

We find that in both the US and the UK, the infinite-horizon variance decomposition for government debt returns has a contribution exceeding 100% from future tax and spending growth and a small (in the US) or moderate (in the UK) negative contribution from future returns (see Tables OA.58 and OA.64 of the online appendix). The latter component reflects positive autocorrelation of government debt returns: since an upward revision in expected future returns is associated with a downward shock in current returns, positive autocorrelation dampens the variance of current returns. The result is that more than 100% of the variance in unexpected returns can be attributed to news about future tax and spending growth. (While we cannot clearly separate the contributions of tax and spending, each of which has a very large standard error, the joint contribution of tax and spending is fairly precisely estimated at the infinite horizon in both countries.) These findings contrast with those reported by Campbell (1991) and others for the aggregate US stock market, where long-run mean reversion in returns contributes positively to an explanation of return variance.

2.5 Sensitivity to linearization parameters

Our baseline sets ρ equal to its sample estimate, which is very close to one; at $\rho = 1$ the measure of the fiscal position takes the particularly simple form (10), in which the level of the government debt does not appear and returns play no role in the dynamics of the fiscal position. Here we ask how sensitive our findings are to the choice of ρ. We find that our major conclusions are not sensitive to the choice of ρ within a reasonable range.

We demonstrate this in the online appendix by reproducing the VAR-based infinite-horizon US and UK variance decompositions shown in Tables 5 and 6 for a range of values between $\rho = 1.01$ and $\rho = 0.96$. These different values of ρ represent different assumptions about the true unconditional population expectation of the logarithmic surplus-debt ratio, $\mathbb{E}\log(1 + S_t/V_t)$, ranging from -1.0% when $\rho = 1.01$ to 4.1% when $\rho = 0.96$. Lower values of ρ are associated with higher values of $\mathbb{E}\log(1 + S_t/V_t)$; loosely speaking, lower ρ represents higher "$r - g$", so that issuing debt is more burdensome. The unconditional mean of the log return on US govern-

ment debt is 2.5% when $\rho = 1.01$ and 7.6% when $\rho = 0.96$. We do not consider the lowest values of ρ in the table to be reasonable: we include them merely to show how our results would change in a world in which r is much higher than g.

We reestimate the VAR for each value of ρ (and the associated a, which is calculated as in equation (23)). As in our baseline VAR, we impose consistency on our model by de-meaning with theoretical means. The online appendix also shows the effect of varying a away from the estimated values, while fixing ρ: we consider two boundary values of a corresponding to the lowest and highest values of T_t/V_t observed in the sample.

In each table in online appendix Section OA.10.5 the first four columns report the various values of ρ together with the associated implied unconditional mean return on government debt, the value of a (estimated or set to a boundary value), and the approximation error in (23). The approximation error is generally small except when a high value of a is imposed, corresponding to the highest value of T_t/V_t observed in our data. The fifth column of each table shows the maximum eigenvalue of the coefficient matrix, which must be smaller than $1/\rho$ in magnitude if equation (14) is to converge to the well-defined limit (15) as $T \to \infty$. As we move ρ away from its estimated value the theoretically imposed mean of fp moves further from the observed sample mean, so our VAR system appears increasingly persistent. As a result the maximal eigenvalue increases, but it always remains below $1/\rho$.

The remaining columns of the tables report the infinite-horizon variance contributions of returns, taxes, and spending as in the last row of Table 5, and the contributions of these variables as in the last row of Table 6 along with the implied cumulative impact on the debt-GDP ratio. The US results are generally insensitive to the values of ρ and a, except when a is set to its lowest boundary value and ρ is set at 0.98 or below. The UK results show a similar pattern but additionally become unstable when a is set to a boundary value and ρ is set above 1, implying $r < g$. Provided that the estimated value of a is used, the value of ρ has little impact on any of our results.

3 The postwar experience

We now focus on the decades since World War II. Over this period, data becomes available for a wider range of countries; and a richer set of potentially informative state variables is available for the US.

3.1 International debt and deficits

We first repeat the analysis using a larger cross-section of countries for which we have been able to obtain appropriate data in the decades since World War II. For comparison, we include our previous data from the US and the UK, using the postwar sample period starting in 1947. We add Canada, Switzerland, and a collection of European countries that share the euro as currency.

The key challenge in international data is obtaining a time series for the market value of the government debt. Standard sources often report the face value of the debt instead. We have market value data from 1989 in Canada and 1999 in Switzerland, as well as data for 11 countries in the eurozone (Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, the Netherlands, and Portugal) since the creation of the euro in 1999. See online appendix OA.4 for details. A secondary challenge is confirming the plausibility of imputed debt returns; we conduct this exercise in the online appendix OA.6.

As in our long US and UK samples, we estimate the parameters ρ and a in the general framework that allows r to differ from g. We estimate $\rho = 1.010$ for the US, 0.991 for the UK, 0.955 for Canada, and 0.994 for Switzerland; ρ estimates for the eurozone countries are reported in online appendix section OA.7. In several of these countries, the mean return and the mean growth rate differ more substantially, so the $r = g$ benchmark is no longer as close an approximation as it is for the US and the UK. Given ρ, we choose a to solve the problem (23), as above.

Section OA.5 of the online appendix plots the data for all these countries. The top left panels of each figure show the history of debt-GDP ratios. The nonstationarity of these ratios is visually apparent, and consistent with the results of unit root tests reported in online appendix section OA.9. The top right panels of each figure show the surplus-debt ratio S_t/V_t, the bottom left panels show the tax-GDP and spending-GDP ratios T_t/Y_t and X_t/Y_t, and the bottom right panels show the tax-debt and spending-debt ratios T_t/V_t and X_t/V_t. Section OA.10 reports VAR estimates for the US and the UK in the shorter post-World War II period, and section OA.11 reports VAR estimates for the other countries.

Table 7 reports the implications of these estimates for the variance decomposition of the fiscal position at a 10-year horizon for each country, previously studied in Tables 1 and 5. Given the short sample periods, we use the VAR methodology rather than relying on local projections. Most of the patterns we saw in the long-

Table 7: VAR-implied decomposition (24) of the fiscal position fp_t for 15 countries at horizon $T = 10$.

All quantities are measured in percent. Bootstrapped standard errors are reported in square brackets.

country	return	tax	spending	future fp
US	−1.1	−1.3	89.7	12.7
	[0.9]	[24.4]	[19.9]	[12.2]
UK	−1.8	33.2	57.2	11.4
	[0.7]	[21.2]	[16.6]	[16.3]
Canada	4.0	20.3	74.0	1.7
	[1.9]	[18.3]	[13.5]	[10.9]
Switzerland	0.4	32.7	66.3	0.6
	[0.2]	[30.5]	[31.8]	[10.0]
Austria	1.8	75.1	26.2	−3.1
	[1.9]	[38.7]	[43.9]	[18.5]
Belgium	−0.3	30.5	66.2	3.6
	[0.2]	[30.8]	[25.7]	[17.2]
Germany	0.7	25.3	79.3	−5.4
	[1.5]	[31.9]	[28.2]	[16.4]
Spain	−0.1	18.6	82.7	−1.1
	[0.7]	[44.1]	[37.5]	[23.2]
Finland	14.6	1.7	80.7	3.0
	[5.1]	[33.4]	[20.7]	[13.8]
France	1.1	72.7	26.0	0.2
	[0.8]	[25.6]	[22.0]	[22.5]
Greece	2.6	−7.6	94.7	10.3
	[2.8]	[46.9]	[59.1]	[30.8]
Ireland	−0.3	−6.8	107.1	0.1
	[1.9]	[43.5]	[38.1]	[15.1]
Italy	1.5	56.3	41.3	0.8
	[1.9]	[31.6]	[36.6]	[18.4]
Netherlands	−0.2	20.0	79.7	0.6
	[0.4]	[34.7]	[33.1]	[9.1]
Portugal	−0.6	24.7	58.3	17.6
	[1.7]	[26.6]	[30.3]	[20.3]

Table 8: VAR-implied decomposition (26) of the fiscal position fp_t for 15 countries at horizon $T = 10$.

All quantities are measured in percent. Bootstrapped standard errors are reported in square brackets.

country	r	$-\Delta y$	$-\Delta\tau y$	Δxy	$-\Delta vy$	$-(vy_{t+T} - vy_t)$
US	10.8	−5.0	11.6	54.3	28.2	268.0
	[8.7]	[3.5]	[11.4]	[12.9]	[16.5]	[156.5]
UK	−21.4	−3.6	34.2	30.2	60.6	636.2
	[8.4]	[4.5]	[14.0]	[19.1]	[22.3]	[234.4]
Canada	10.9	−0.1	15.5	62.6	11.1	144.3
	[5.0]	[2.8]	[13.0]	[19.0]	[14.6]	[189.0]
Switzerland	7.1	4.5	−7.2	99.0	−3.4	−35.0
	[3.9]	[0.9]	[25.8]	[27.9]	[11.6]	[120.5]
Austria	40.1	12.3	−0.5	99.1	−51.0	−522.4
	[46.3]	[7.1]	[22.5]	[26.8]	[44.6]	[456.8]
Belgium	36.9	7.1	2.0	73.7	−19.6	−195.3
	[22.1]	[6.6]	[21.0]	[28.8]	[34.8]	[346.3]
Germany	6.7	5.6	−11.6	111.3	−12.0	−128.2
	[14.3]	[3.1]	[22.4]	[18.6]	[15.9]	[169.8]
Spain	−3.2	−3.5	31.0	45.5	30.1	307.1
	[19.4]	[7.7]	[22.9]	[29.7]	[37.0]	[377.6]
Finland	36.3	−0.2	4.4	61.7	−2.3	−30.6
	[13.2]	[4.7]	[11.1]	[22.8]	[13.2]	[176.7]
France	35.1	8.4	29.1	53.9	−26.5	−270.1
	[28.0]	[6.0]	[17.0]	[30.4]	[37.1]	[377.4]
Greece	20.6	−23.2	51.5	5.9	45.2	488.1
	[21.9]	[17.7]	[14.9]	[28.9]	[30.2]	[326.3]
Ireland	1.6	−1.7	7.8	81.3	10.9	98.9
	[9.6]	[5.8]	[24.8]	[32.4]	[28.1]	[253.8]
Italy	12.1	0.5	44.2	40.8	2.5	26.6
	[15.3]	[7.4]	[24.0]	[36.1]	[23.2]	[249.8]
Netherlands	5.8	1.2	9.1	86.9	−3.0	−29.0
	[8.9]	[3.4]	[15.1]	[14.7]	[17.5]	[170.8]
Portugal	−4.5	−3.9	31.5	28.1	48.8	527.3
	[13.6]	[7.7]	[17.3]	[28.4]	[30.7]	[331.5]

sample US and UK data appear again in this table. In all countries except Finland the returns on government debt have a minimal influence on the dynamics of the fiscal position, and in all countries the fiscal position mean-reverts quickly enough that at least 80% of its variability is accounted for by ten-year fiscal adjustment. Spending adjusts more strongly than tax revenue in all countries except Austria, France, and Italy, although the standard errors are large given the relatively short sample periods in this table.

Table 8 reports the implications of these estimates for debt-GDP growth, previously studied in Tables 2 and 6. Again we use the VAR methodology and consider a ten-year horizon. The impact of the fiscal position on the cumulative ten-year growth of debt-GDP depends not only on cumulative fiscal adjustment over ten years, but also on the timing of fiscal adjustment within those years. There are larger effects on debt growth in countries such as Portugal, Greece, and the UK that adjust slowly, and smaller (sometimes even negative) effects in countries such as Austria, Germany, and Switzerland that adjust rapidly. In some countries, such as Austria, Belgium, Finland, and France, returns reduce the impact of the fiscal position on debt growth, while in the UK returns increase that impact. GDP growth rarely plays an important role, with the notable exception of Greece. The standard errors are again large due to the short sample periods.

Section OA.10 in the online appendix reports post-World War II results for the US and the UK based on local projections at 1-, 3-, 10-year horizons. (Given the 10-year horizon, we require a long sample period for the local projections approach to be feasible. The US and the UK are the two countries for which we observe post-World War II data over a sufficiently long period.) In the US, the fiscal position mean-reverts substantially over 10 years, so that the future fiscal position contributes 21% of the variance of the fiscal position at this horizon. Mean-reversion is slower in the UK, where the future fiscal position contributes 49% of the variance at a 10-year horizon. As before returns contribute very little to fiscal adjustment, which is dominated by spending, although the standard errors are once again large.

3.2 Other state variables

In principle, the VAR analysis can be augmented by including any state variables relevant for forecasting fiscal outcomes or debt returns.

3.2.1 Implications of a stationary US tax-GDP ratio

In the postwar US, for example, the log tax-GDP ratio appears to be stationary, as is visually apparent in Figure OA.2. Table OA.13 shows that we can reject a unit root in postwar data, with a p-value less than 0.001 (though not in the longer sample, and not for the log spending-GDP ratio over either sample period). What conclusions would be drawn by an economist prepared to accept that we are in a new, stable, postwar political equilibrium with stationary log tax-GDP ratio?

Table OA.47 in online appendix OA.10.6 reports results for a VAR, estimated over the 1947–2022 sample period, that includes τy_t, and so takes into account the stationary postwar relationship between tax and output. The tax-GDP ratio is quite predictable, notably by its own lag and by tax growth, and in turn it predicts high returns on debt, low future tax growth, and with less precision low GDP growth.

We can use this VAR to conduct a variance decomposition analogous to the one reported in Table 5. The results are shown in Table OA.48; the point estimates are similar to the postwar variance decomposition in our baseline specification (Table OA.35), but the standard error on the spending contribution is markedly reduced.

3.2.2 Yield curve forecasting variables

In the postwar period, yield curve data are available that might potentially help to forecast returns on the debt. We have therefore also considered VAR systems that include bond return forecasting variables based on the yield curve. Unlike the tax-GDP ratio, however, these variables have very little effect on our conclusions. In the interest of space, we only report results for the US.

Table OA.50, in the online appendix OA.10.6, adds the real short rate (calculated as the nominal 1-year yield minus lagged inflation), $yr_{1,t}$, and the 1-10 year yield spread, $spr_{1\to 10,t}$, to the variables considered in our baseline VAR. These variables help to forecast returns, but the variance decomposition using this expanded system (shown in Table OA.51) is almost identical to the postwar US baseline variance decomposition reported in online appendix Section OA.10. This is consistent with the visual impression given by Figure 5: as realized returns exhibit little comovement with the fiscal position, an improved ability to forecast returns has very little effect on our conclusions.

Table OA.53 reports results when both the tax-GDP ratio, τy_t, and the yield curve variables $yr_{1,t}$ and $spr_{1\to 10,t}$ are included in the VAR. The resulting variance

decomposition (Table OA.54) is very similar to that shown in Table OA.48, so once again our conclusions are unaltered by inclusion of the bond return forecasting variables.

3.3 Specific episodes

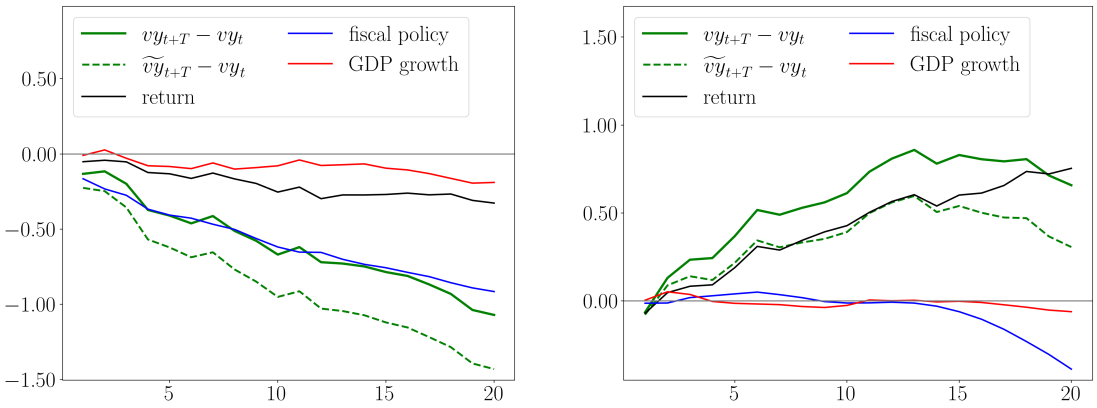
We have presented variance decompositions which tell us about the average importance of different fiscal adjustment mechanisms. But in any specific period returns, taxes, spending, the fiscal position, and the value of the government debt may evolve differently from the average. Some recent papers on US fiscal history have emphasized this. Acalin and Ball (2026), for example, stress that low returns on the government debt contributed to the reduction in the US debt-GDP ratio between 1947 and 1974.

We can use an ex post version of equation (19) to track the contributors to the evolution of debt-GDP in any specific episode. Fixing an initial date t, we plot the cumulative change in the debt-GDP ratio for different horizons T along with the various contributors on the right hand side of (19). To simplify the presentation, we combine the initial fiscal position with subsequent tax and spending growth rates to create a single fiscal policy variable.

The left panel of Figure 7 sets $t = 1947$ and follows the evolution of the debt-GDP ratio over the next 20 years. The solid green line shows the evolution of the log debt-GDP ratio in this period. The solid red line shows the contribution of GDP growth, the solid black line the contribution of returns, and the solid blue line the contribution of fiscal policy (the initial fiscal position combined with subsequent tax and spending growth rates). GDP growth and returns are demeaned using the 1947-2022 sample and the fiscal variables are demeaned using their implied theoretical means, so the lines illustrate the contribution of abnormal GDP growth and returns, and fiscal policy relative to the steady state. The sum of the red, black, and blue lines is the dashed green line. The difference between the solid and dashed green lines is approximation error, which we saw in Table 2 is non-negligible for the US when tracking the evolution of debt.

Consistent with Acalin and Ball (2026), the figure shows that abnormally low returns on the government debt were a more important contributor than abnormally high GDP growth to the reduction in the US debt-GDP ratio that occurred after World War II. However, fiscal policy was considerably more important than either

Figure 7: Cumulative change of debt-to-GDP in the US post WWII. These plots show the contributions of return, fiscal policy and GDP growth to the change in debt-to-GDP ratio. The exact formula of the three components are according to (19) with all series demeaned. Fiscal policy combines the current fiscal position and future tax and spending growth. The dashed line is the imputed change of debt-to-GDP according to (19).



(a) Start year $t = 1947$

(b) Start year $t = 1980$

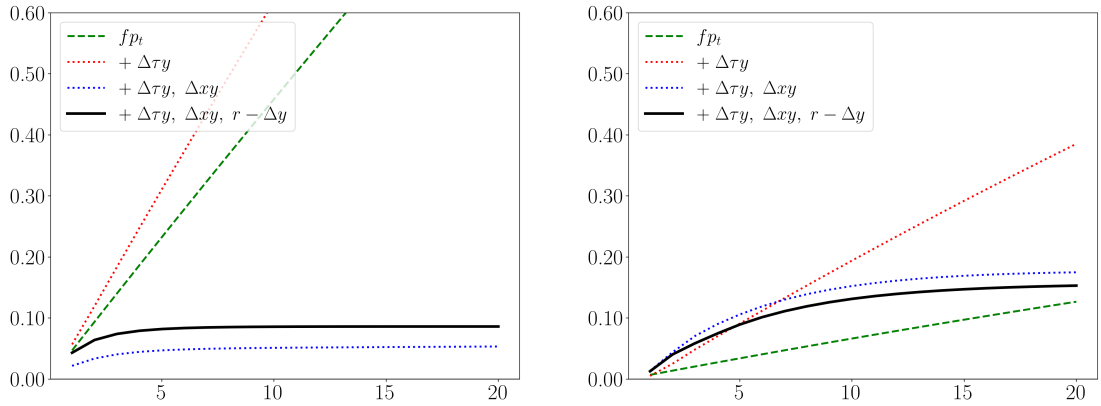
returns or GDP growth. The US reduced its debt-GDP ratio not primarily by growing out of debt or by financial repression, but by running a sequence of primary surpluses.

The right hand side of Figure 7 sets $t = 1980$ and repeats the analysis. In the last two decades of the 20th Century, high returns on the government debt were the most important contributor to the increase in the debt-GDP ratio. GDP growth was close to average throughout this period, and the fiscal position was only slightly weak in the 1980s and strengthened in the second half of the 1990s; but high returns on the debt, driven both by high real interest rates at the start of the period and declining bond yields later on, accounted for the increase in the debt-GDP ratio during this period.

3.3.1 Conditional forecasts

Figure 7 decomposes the realized change in the debt-GDP ratio between two dates. It is possible to use a similar methodology to make conditional forecasts by using our VAR system. We estimate a VAR for the state vector $(r_t,\ \Delta\tau_t,\ \Delta y_t,\ fp_t,\ fp_{t-1})$,

Figure 8: Forecasts of $vy_{t+T} - vy_t$ for the US, starting in 2022, using VARs estimated on the full sample and on the postwar sample. The terms of the identity (19) are added one at a time. The black solid line is the projected total.



(a) Based on sample 1841-2022

(b) Based on sample 1947-2022

stand at the last observation (2022), and generate forecasts by iterating the estimated transition matrix forward. Spending growth is recovered by inverting the one-period identity relating spending to the elements of the state vector. The conditional forecasts enter the decomposition with the weights from equation (19).

This approach requires that we choose a sample period over which to estimate the VAR system. Figure 8 shows conditional forecasts over 2023 to 2042, based on the full-sample VAR in panel (a) and the postwar VAR in panel (b). The model's overall forecast of the change in the debt-GDP ratio is the solid black line, which asymptotes at about 9 percentage points in the left panel and 16 percentage points in the right panel. The green dashed line shows the model's forecast based only on the initial fiscal position; the red line adds the effect of tax changes; and the blue line adds the effect of spending changes. The difference between the blue and the black lines is the contribution of forecast returns and GDP growth.

In both panels of Figure 8 the green dashed line is upward sloping because the initial fiscal position is negative. This is more strongly the case when we use the full-sample system, as the values of the linearization parameters ρ and a used in that system imply a more negative 2022 fiscal position. In both panels the red dashed line is steeper than the green line, showing that the tax-GDP ratio is forecast to amplify the accumulation of debt after 2022 rather than moderate it as would normally be

the case. However the blue line is considerably shallower than the red dashed line, reflecting the model's forecast that reductions in spending-GDP will moderate the accumulation of debt. The blue line and the black line are close to one another because conditionally, as unconditionally, return and output forecasts play only a secondary role in fiscal adjustment.

4 Conclusion

Conventional tests do not reject the presence of a unit root in the government debt-GDP ratio in long-run data for the US and UK, nor (albeit in shorter time series) for 13 other countries. This uncomfortable fact makes it hard to interpret the results of studies that treat the debt-GDP ratio as a stationary variable. We have presented a framework for fiscal analysis that instead makes a loglinear approximation to the surplus-debt ratio—which does appear to be stationary—the central object of interest. We call this approximation the fiscal position of the government and study its dynamics and its implications for changes in the debt-GDP ratio.

Our framework highlights the importance of two separate factors: first, the relation between the average return on the debt r and the average growth rate of the economy g, which determines the average surplus-debt ratio required to service the debt, or what one might call the burden of the debt; and second, the overall size of the government relative to the debt. If the currently funded activities of the government are large relative to the size of its debt, then small proportional changes in tax revenue or government spending have a large impact on the surplus-debt ratio.

We analyze the contributions of taxes and spending separately, and so, for example, we can distinguish between deficits associated with declines in tax revenue and those associated with increases in government expenditure. There are good economic reasons to analyze these two variables separately: in a recession, tax revenue declines at a faster rate than GDP in the presence of increasing marginal tax rates, whereas spending increases, but there is no particular reason to expect tax and spending to adjust symmetrically.

We derive an identity that relates the government's fiscal position to future returns on government debt and to future tax and spending growth rates, just as the identity derived by Campbell and Shiller (1988*a*) relates the dividend yield on a security to that security's future returns and dividend growth rates. Over the long run, a weak fiscal position must be followed by some combination of altered returns

on government debt, high long-run tax growth, and low long-run spending growth. The contribution of returns shrinks when r is close to g, as we estimate empirically, and becomes identically zero when $r = g$.

We use this identity to interpret variation in the fiscal position over time in long-run data from the US and the UK and from these and 13 other developed countries in the post-World War II period. In all these countries the fiscal position has limited forecasting power for future debt returns over the long run; instead, it forecasts long-run changes in the growth rates of tax revenue and government spending. In the long-run US data these changes occur roughly equally in taxes and in spending; in the UK, the postwar US, and most other countries we study they occur more through spending than through taxes. Mean-reversion in the fiscal position is driven by tax and spending adjustments; the contribution of returns is small both because we estimate r close to g and because the fiscal position forecasts only transitory changes in returns.

We derive a second identity that allows us to calculate the implications of fiscal adjustment in returns, taxes, and spending for the growth of the debt-GDP ratio that follows any initial fiscal position. In this identity returns can play a role even when $r = g$, but the contribution of returns remains modest because of their limited forecastability. Changes in taxes and spending occur rapidly enough to substantially reduce the impact of the fiscal position on subsequent debt-GDP growth. In long-run US and UK data we estimate that a positive 1% fiscal position is typically followed by a 1-3 percentage point decrease in the debt-GDP ratio after ten years, whereas this decrease would be about 10 percentage points in the absence of fiscal adjustment. Since mean-reversion in the fiscal position is close to complete after ten years, the 10-year estimate is close to the estimate we obtain for an infinite horizon.

Our two identities can be used not only to characterize the changes in returns, taxes, and spending that are typically associated with variation in the fiscal position, but also the realized changes in these variables that drove changes in the fiscal position and the debt-GDP ratio during specific episodes, and conditional forecasts of such changes at a particular point in time. In this spirit we decompose the reduction in US debt-GDP that occurred after World War II and the increase that occurred towards the end of the 20th Century; and we forecast long-run tax and spending growth from the end of our sample period in 2022, finding that spending forecasts account for more than 100% of forecast fiscal adjustment given the 2022 configuration of state variables.

Our finding that tax and spending growth are the primary drivers of the mean-reversion of the fiscal position and the related evolution of the debt-GDP ratio, with debt returns playing only a secondary role, contrasts with the findings of Campbell and Shiller (1988*a*) and Campbell and Shiller (1988*b*) that the dividend yield and cyclically adjusted price-earnings (CAPE) ratio on the aggregate US stock market primarily predict future stock returns rather than future growth rates of dividends or earnings. Other studies have found a greater role for predictable cash flow growth when noisier measures of corporate cash flows, such as aggregate net payouts, are used to scale the market value of corporate assets (Larrain and Yogo, 2008). One way to understand our results is that primary surpluses, like net corporate payouts but unlike aggregate dividends or smoothed earnings, have important transitory variation which our framework relates to changing growth rates of tax revenue and government spending.[20]

We also analyze long-run responses to tax and spending shocks. Again we find that debt returns, both unexpected returns at the time the shocks occur and subsequent predictable returns, play almost no role in these responses. Instead, mean-reverting tax and spending growth follow stochastic processes that allow debt value to remain stable. While our framework does not allow us to say which variables are exogenous and which are endogenous, this pattern does tell us that governments in the countries we study have chosen fiscal policies that avoid large predictable or unpredictable returns to debtholders.

One reason for these policy choices could be that large swings in the value of the debt are politically risky for incumbent policymakers. James Carville, a political adviser to US President Bill Clinton, is reported to have said, "I used to think that if there was reincarnation, I wanted to come back as the president or the pope or as a .400 baseball hitter. But now I would like to come back as the bond market. You can intimidate everybody." An illustration of this principle was recently provided by the market reaction to unexpectedly large tax cuts in the September 2022 "mini-budget" in the United Kingdom, which led to the rapid departure of both Chancellor of the Exchequer Kwasi Kwarteng and Prime Minister Liz Truss.

[20]The market value of government debt is also far more stable than the value of aggregate stock indexes. Thus, the dividend-price ratio can be thought of as a way to remove a persistent component from volatile stock valuations, while the primary surplus-debt ratio and our loglinear approximation to it can be thought of as a way to remove a persistent component from volatile primary surpluses. We are grateful to John Cochrane for suggesting this interpretation of our findings.

It is possible, perhaps even probable, that our framework would attribute a more significant role to debt returns in countries that have experienced turbulent macroeconomic crises. A priority for future research should be to apply our analysis to other countries, including emerging markets, where data are available on the market value (as opposed to the face value) of the public debt.

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