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ABSTRACT

I study a version of the search-theoretic model of imperfect competition by Burdett and Judd (1983) in which sellers face a strictly increasing rather than a constant marginal cost of production. The equilibrium exists and is unique, and its structure depends on the extent of search frictions. If search frictions are large enough, the price distribution is non-degenerate and atomless. If search frictions are neither too large nor too small, the price distribution is non-degenerate with an atom at the lowest price. If search frictions are small enough, the price distribution is degenerate. The equilibrium is efficient if and only if the price distribution is degenerate and, hence, if and only if search frictions are small enough. In contrast, in Burdett and Judd (1983), the equilibrium price distribution is always non-degenerate and atomless and the equilibrium is always efficient. As in Burdett and Judd (1983), the equilibrium goes from monopolistic to competitive as search frictions decline.

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1 Introduction

Search frictions provide a natural theory of imperfect competition in product markets. The theory is simple and compelling. Due to search frictions, buyers cannot purchase a product from just any seller, but only from a finite sample of sellers. As the number of non-captive buyers—i.e. buyers who can purchase the product from multiple sellers–relative to the number of captive buyers—i.e. buyers who can purchase the product from only one seller—goes from zero to infinity, the equilibrium spans the spectrum going from monopoly to perfect competition.

The search-theoretic framework of imperfect competition was developed by Butters (1977), Varian (1980), and Burdett and Judd (1983). Traditionally, the framework has been used to study the dispersion of the price of a particular good (Sorensen 2000, Hong and Shum 2006, Galenianos, Pacula and Persico 2012, Kaplan and Menzio 2015, Bethune, Choi and Wright 2020), the dispersion of the relative price of different goods (Kaplan, Menzio, Rudanko and Trachter 2019), and the fluctuations of the price of a particular good at a particular store (Varian 1980, Menzio and Trachter 2018). More recently, the framework has been used to study a broader set of questions, such as price stickiness and monetary policy (Head, Liu, Menzio and Wright 2012, Burdett, Trejos and Wright 2017, Burdett and Menzio 2018), consumption inequality (Pytka 2018, Nord 2022), product design and market concentration (Menzio 2023, Albrecht, Vroman and Menzio 2023), and business cycles (Kaplan and Menzio 2015). The framework, adapted to the labor market, has been used to study residual wage inequality (Burdett and Mortensen 1998, Bontemps, Robin and van den Berg 2000, Mortensen 2003).

In this paper, I contribute to the development of the search-theoretic framework of imperfect competition by characterizing the properties of equilibrium under the assumption that sellers face an increasing marginal cost of production. That is, I characterize the properties of equilibrium when sellers operate a decreasing returns to scale production function. Butters (1977), Varian (1980), and Burdett and Judd (1983) assume that all sellers face the same, constant marginal cost of production. Existing variations of the search-theoretic framework of imperfect competition either maintain the assumption that all sellers face the same, constant marginal cost, or assume that sellers face different but constant marginal costs. That is, the existing literature assumes that sellers operate a constant returns to scale production function. For some applications, the assumption of constant returns to scale may be appropriate (e.g. production with only variable factors that can be purchased or produced at constant prices). For other applications, the assumption of decreasing returns to scale may be more realistic (e.g. production with either fixed factors or with variable factors that are purchased or produced at increasing price). For example, in a version of Lagos and Wright (2005) where the decentralized market operates as in Burdett and Judd (1983), it would be natural to assume that individual sellers face a strictly increasing disutility from producing additional units of output. In a labor market context a la Burdett and Mortensen (1998), it would be natural to assume that an individual firm has a strictly decreasing marginal product of labor because it faces a downward sloping demand for its output.

Specifically, I consider the market for a consumer good. On one side of the market, there is a continuum of identical sellers. Each seller posts a price p for the good, and produces the good according to some cost function $c(q)$ that is strictly increasing and strictly convex in the quantity q. On the other side of the market, there is a double continuum of identical buyers. Each buyer demands one unit of the good and enjoys a utility u from consuming it. Due to informational or physical frictions, a buyer cannot purchase from just any seller, but only from those with which he comes into contact. The process through which buyers contact sellers is such that a buyer contacts one randomly selected seller with probability $1-\alpha$, in which case the buyer is captive, and two randomly selected sellers with probability α, in which case the buyer is non-captive, with $\alpha \in (0,1)$. The parameter alpha is an inverse measure of search frictions, since the ratio of non-captive to captive buyers goes from 0 to infinity, as α goes from 0 to 1. The structure of the market is the same as in Burdett and Judd (1983), except that the sellers' marginal cost of production is strictly increasing rather than constant.

This seemingly modest modification to the environment of Burdett and Judd (1983) affects the structure of equilibrium. In Burdett and Judd (1983), the equilibrium is such that sellers post prices according to a distribution F with support over some interval $[p_\ell, p_h]$, with $p_\ell > c$ and $p_h = u$. The price distribution F is atomless. Indeed, any mass point at a price p_0 creates a downward discontinuity in the demand curve and, in turn, induces an individual seller to deviate from the price p_0 to a price $p_0 - \epsilon$ so as to undercut the mass of competitors at p_0. The price distribution F is such that an individual seller enjoys the same profit by posting any price on the support $[p_\ell, p_h]$. Specifically, a seller at the $(1-x)$-th quantile of the price distribution trades with all the captive buyers and with a fraction x of the non-captive buyers. A seller with the highest price, which always equals the buyers' valuation u, trades only with the captive buyers. The function $p(x)$, which maps the seller's ranking into its price, is such that the seller at the $(1-x)$-th quantile of the price distribution enjoys the same profit as a seller with the highest price. As search frictions vanish, in the sense that α converges to 1, the quantity of output traded by the seller with the highest price converges to zero, and so does its profit. The quantity of output traded by the seller with the lowest price increases and, for its profit to converge to zero, its price must converge to the marginal cost c. If the marginal cost of production is strictly increasing, the seller's price would have to fall below marginal cost to drive its profit to zero. Such a price, however, is not consistent with the seller's profit maximization.

What is then the structure of equilibrium when sellers face a strictly increasing marginal cost of production? The answer, I find, depends on the extent of search frictions. If search frictions are sufficiently large, in the sense that α is smaller than some cutoff α_1, the equilibrium has the same structure as in Burdett and Judd (1983). That is, the equilibrium price distribution F is atomless and its support is some interval $[p_\ell, p_h]$, with $p_h = u$. A seller enjoys the same profit by posting any price on the support of F. If a seller posts a lower price, it sells more units but enjoys a lower profit per unit. because of both the lower price and the higher average cost. If a seller posts a higher price, it sells fewer units but enjoys a higher profit per unit, because of both the higher price and the lower average cost. Intuitively, when search frictions are large enough, the equilibrium has the same structure as in Burdett and Judd (1983) because the difference in the quantity of output traded by sellers at different quantiles of the price distribution is not so large to drive the marginal cost of the best seller above its price.

If search frictions are neither too large, in the sense that α is greater than α_1, neither too small, in the sense that α is smaller than some other cutoff α_2, the equilibrium price distribution F consists of a mass point at p_0 and a density over some interval $[p_\ell, p_h]$, with $p_\ell > p_0$ and $p_h = u$. The structure of equilibrium is different than in Burdett and Judd (1983). There is a mass of sellers that post the lowest price p_0 on the support of F. These sellers do not wish to lower their price to $p_0 - \epsilon$ because the price p_0 is equal to their marginal cost. There is a density of sellers posting prices in the interval $[p_\ell, p_h]$. These sellers trade less output but make it up with their profit margin. Intuitively, as search frictions reach the cutoff point α_1, the quantity of output traded by the best seller drives the marginal cost to the price. Further declines in search frictions would drive the marginal cost above the price and, hence, would be inconsistent with the seller's profit maximization. Therefore, as search frictions decline further, more and more of the top sellers amass at a price p_0 equal to marginal cost. Since any seller posting a price greater than p_0 sells discretely fewer units that a seller posting p_0, profit equalization requires a gap on the support of F between p_0 and p_ℓ.

If search frictions are sufficiently small, in the sense that α is greater than α_2, the equilibrium price distribution F is degenerate at p_0. The equilibrium is the same as in a frictionless and perfectly competitive model, in the sense that every seller trades the same quantity of output at a price that is equal to the marginal cost of production. This finding is intuitive as well. A seller posting any price greater than p_0 must enjoy the same profit as a seller posting the highest price u. A seller posting a price the highest price u must enjoy the same profit as a seller posting the lowest price p_0. As search frictions decline, the profit for a seller posting the price u converges to zero while the profit for a seller posting the price p_0 remains strictly positive. Therefore, at some point, every seller chooses to post the price p_0, and the right tail of the distribution F disappears.

Even though the modification of the Burdett and Judd (1983) environment affects the structure of equilibrium, it does not change the relationship between search frictions and the extent of competition. Just as in Burdett and Judd (1983), the equilibrium limits to the monopoly outcome as the ratio of non-captive to captive buyers converges to zero (i.e., $\alpha \to 0$), and it limits to the competitive outcome as the ratio of non-captive to captive buyers diverges to infinity (i.e., $\alpha \to 1$). Just as in Burdett and

Judd (1983), sellers' profits decline monotonically as the ratio of non-captive to captive buyers increases. The structure of equilibrium is different than in Burdett and Judd (1983) because the competitive outcomes are different. When sellers have a constant marginal cost of production, the competitive outcome is such that every seller posts a price equal to the marginal cost. The competitive outcome does not pin down how much each seller produces. When sellers have an increasing marginal cost of production, the competitive outcome is such that every seller posts a price equal to the marginal cost and every seller produces the same quantity of output. An atomless price distribution generates non-vanishing dispersion in the quantity of output produced by different sellers, even as the extent of price dispersion becomes arbitrarily small. For this reason, an atomless price distribution can approach the competitive outcome when sellers have a constant marginal cost, but cannot approach the competitive outcome when sellers have an increasing marginal cost. When sellers have an increasing marginal cost, the competitive outcome can only be approached if the price distribution becomes degenerate.

For the same reason it changes the structure of equilibrium, the assumption of increasing marginal costs of production affects the welfare properties of equilibrium. In Burdett and Judd (1983), the equilibrium is always efficient. The equilibrium features price dispersion and, in turn, dispersion in the quantity of output produced by different sellers. But, since sellers have the same constant marginal cost of production, dispersion in the quantity of output produced by different seller does not lead to any inefficiency. In contrast, when sellers have a strictly increasing marginal cost of production, the equilibrium is efficient if and only if there is no dispersion in the quantity of output produced by different sellers and, in turn, if and only if there is no dispersion in prices. Hence, when sellers have a strictly increasing marginal cost of production, the equilibrium is efficient if search frictions are sufficiently small, in the sense that α is greater than α_2. If search frictions are not sufficiently small, in the sense that α is smaller than α_2, the equilibrium features price dispersion, quantity dispersion, and it is inefficient.

2 Environment

I consider the market for a homogeneous good. On one side of the market, there is a continuum of ex-ante identical sellers with measure 1. Each seller chooses a price p and produces the good according the cost function $c(q)$, where q is the quantity of output sold by the seller and $c(q)$ is a strictly increasing and strictly convex function such that $c(0) = 0$, $c'(0) = 0$, $c'(\infty) = \infty$. I assume that a seller has the option to refuse trade with some of the buyers who are demanding the good.

On the other side of the market there is a double continuum of ex-ante identical buyers with measure b per seller, with $b > 0$. Each buyer enjoys a utility $u - p$ if he purchases a unit of the good at the price p. Each buyer enjoys a utility of 0 if he does not purchase the good. I assume that the buyer's valuation for the good, u, is greater than the marginal cost, $c'(b(1+\alpha))$, where $b(1+\alpha)$ is the largest quantity of output that a seller may have to produce.

The market is frictional, in the sense that a buyer cannot purchase from any seller, but only from those sellers with which he comes into contact. In the spirit of Burdett and Judd (1983), I assume that a buyer contacts one randomly selected seller with probability $1-\alpha$, in which case the buyer is said to be captive, and two randomly selected sellers with probability α, in which case the buyer is said to be non-captive, with $\alpha \in (0,1)$. The buyer observes the price charged by all of the contacted sellers and, then, he decides whether and where to purchase a unit of the good. Note that the parameter α is an inverse measure of search frictions, as the ratio of non-captive to captive buyers goes from zero to infinity as α goes from 0 to 1.

I am now in the position to define an equilibrium.

Definition 1. (Equilibrium) *An equilibrium is a cumulative distribution $F(p)$ of prices such that:* (i) *for every price p on the support of F, the profit of a seller attain its maximum;* (ii) *for every price p on the support of F, a seller trades with every buyer who demands a unit of the good;* (iii) *a buyer demands the good from the contacted seller with the lowest price, as long as the price is non-greater than u;* (iv) *a buyer demands the good from either of the contacted sellers if both have the same price, as long as the price is non-greater than u.*

The definition of equilibrium is the same as in Burdett and Judd (1983), except for condition (ii). The condition requires that, in equilibrium, sellers find it optimal to meet buyers' demand. The condition is a restriction on the set of equilibria. The condition poses no restrictions on the sellers' strategies, since sellers are allowed to turn down some of the buyers' demand. The condition simplifies the characterization of equilibrium, even though I suspect that it does not have any bite.

3 General properties of equilibrium

In this section, I derive some general properties of equilibrium. These properties imply that the equilibrium has one of the following structures: (i) the distribution F is a mass point at a price p_0 equal to the sellers' marginal cost; (ii) the distribution F is a mass point at a price p_0 equal to the sellers' marginal cost and a density over an interval of prices $[p_\ell, p_h]$, with $p_\ell > p_0$ and $p_h = u$; (iii) the distribution F is a density over an interval of prices $[p_\ell, p_h]$, with $p_h = u$.

In order to derive the general properties of equilibrium, it is useful to start by formulating the problem of an individual seller. The profit $V(p)$ for a seller posting an arbitrary price $p \in [0,u]$ is

$$\begin{aligned} V(p) = \quad & \max_q pq - c(q) \\ & \text{s.t. } q \in [0, q(p)], \end{aligned} \tag{3.1}$$

where

$$q(p) = b\left[(1-\alpha) + 2\alpha(1-F(p)) + \alpha\mu(p)\right], \tag{3.2}$$

where q is the quantity of output traded by the seller, $1-F(p)$ is the measure of sellers posting a price strictly greater than p, and $\mu(p)$ is the measure of sellers posting the price p. The seller's profit is given by revenues, pq, minus costs, $c(q)$. The seller chooses how much output q to produce and sell subject to demand. The quantity of output $q(p)$ demanded by buyers to the seller is easy to understand. The seller meets a measure $b(1-\alpha)$ of captive buyers, i.e. buyers who are not in contact with another seller. A captive buyer demands the good from the seller with probability 1. The seller meets a measure $b2(1-\alpha)$ of non-captive buyers, i.e. buyers who are in contact with a second seller. A non-captive buyer demands the good from the seller with probability $1-F(p)+\mu(p)/2$, where $1-F(p)$ is the probability that the buyer's second contact is a seller posting a price strictly greater than p, $\mu(p)$ is the probability that the buyer's second contact is a seller posting a price equal to p, and $1/2$ is the probability that the buyer chooses to demand the good from the seller. Obviously, if the seller posts a price $p > u$, it enjoys a profit $V(p) = 0$.

As mentioned in the previous section, I restrict attention to equilibria in which sellers find it optimal to meet buyers' demand. In other words, I restrict attention to equilibria in which, for every price p on the support of the distribution F, the solution to the seller's problem (3.1) is to produce and trade a quantity $q(p)$ of output. Since I restrict attention to equilibria without rationing, the expression (3.2) for the seller's demand $q(p)$ does not include any non-captive buyers who are in contact with a seller posting a price lower than p and who are rationed by that seller.

I am now in the position to derive some of the general properties of equilibrium. In the first lemma, I show that the equilibrium profit V^* is strictly positive. The intuition behind this finding is the same as in Butters (1977), Varian (1980), and Burdett and Judd (1983). That is, a seller can guarantee itself a strictly positive profit by posting a price p equal to the buyers' reservation price u, and by trading only with the strictly positive measure of buyers who are not in contact with any other seller.

Lemma 1: *In any equilibrium, the seller's profit V^* is strictly positive.*

Proof: The profit for a seller that posts the price u is

$$\begin{aligned} V(p) = \quad & \max_q pq - c(q), \\ & \text{s.t. } q \in [0, b\left[1-\alpha+2\alpha(1-F(u))+\alpha\mu(u)\right]] \end{aligned}$$

Irrespective of the price distribution F, the seller can choose a quantity q equal to $b(1-\alpha)$. Since $u > c'(b(1+\alpha))$ and $c'(b(1+\alpha)) > c'(b(1-\alpha))$, it follows that $u > c'(q)$. Since c is strictly convex and $c(0) = 0$, $c(q) < qc'(q)$. Combining these observations yields $uq > c'(q)q > c(q)$ and, hence, $V(u)$ is strictly positive. Since $V^* \geq V(u)$, it follows that V^* is strictly positive. ■

In the second lemma, I prove that the equilibrium distribution F cannot have a mass point at a price p if the price is strictly greater than the marginal cost. The intuition behind this finding is essentially the same as in Butters (1977), Varian (1980) and Burdett and Judd (1983). Namely, a mass point at some price p creates a downward discontinuity in the demand curve facing the seller to the left of p, as any price to the left of p gives the seller the option to trade with all, rather than half, of the buyers who are in contact with another seller posting p. If the price p exceeds the marginal cost of production, an individual seller can attain a strictly higher profit by posting the price $p-\epsilon$ rather than p. Unlike in Butters (1977), Varian (1980) or Burdett and Judd (1983), the deviation from p to $p-\epsilon$ may require the seller to ration some of its customers.

Lemma 2: *The equilibrium price distribution F does not have a mass point at p if $p > c'(q(p))$.*

Proof: On the way to a contradiction, let there be a mass point at some price p_0 such that $p_0 > c'(q(p_0))$. The profit for a seller posting the price p_0 is given by

$$V(p_0) = q(p_0)p_0 - c(q(p_0)), \tag{3.3}$$

where

$$q(p_0) = b\left[1 - \alpha + 2\alpha(1 - F(p_0)) + \alpha\mu(p_0)\right]. \tag{3.4}$$

The profit for a seller posting the price $p_0 - \epsilon$, for some $\epsilon > 0$, is given by

$$\begin{aligned} V(p_0 - \epsilon) = \quad & \max_q q(p_0 - \epsilon) - c(q), \\ & \text{s.t. } q \in [0, q(p_0 - \epsilon)] \end{aligned} \tag{3.5}$$

where

$$q(p_0 - \epsilon) \geq b\left[1 - \alpha + 2\alpha(1 - F(p_0)) + 2\alpha\mu(p_0)\right]. \tag{3.6}$$

Let q^* be such that $c'(q^*)$ equals $c'(q(p_0)) + (p_0 - c'(q(p_0)))/2$. First, assume that q^* is smaller than the lower bound on $q(p_0 - \epsilon)$ on the right-hand side of (3.6). Let δ be defined as $q^* - q(p_0)$. Clearly, $\delta > 0$. Therefore, we have

$$\begin{aligned} & V(p_0 - \epsilon) - V(p_0) \\ \geq \quad & -q(p_0)\epsilon + \delta\,(p_0 - \epsilon) - [c(q^*) - c(q(p_0))] \\ > \quad & -q(p_0)\epsilon + \delta\,(p_0 - \epsilon) - c'(q^*)\delta \\ = \quad & -q(p_0)\epsilon + \delta\,(p_0 - \epsilon) - [c'(q(p_0)) + (p_0 - c'(q(p_0)))/2]\,\delta \\ = \quad & -q(p_0)\epsilon - \delta\epsilon - c'(q(p_0))\delta/2, \end{aligned} \tag{3.7}$$

where the second line makes use of the fact that q^* is a feasible quantity for a seller posting the price $p_0 - \epsilon$, the third line makes use of the fact that c is strictly convex, the fourth line makes use of the definition of q^*, and the last line is an algebraic manipulation of the previous one. Since (3.7) holds for any $\epsilon > 0$ and $p_0 > c'(q(p_0))$, there is an ϵ small enough such that $V(p_0 - \epsilon) - V(p_0) > 0$. Therefore, the price p_0 cannot be on the support of F.

Now, assume that q^* is greater than the lower bound on $q(p_0 - \epsilon)$ on the right-hand side of (3.6). In this case, let $\hat{q}$ be equal to $b(1-\alpha)+b2\alpha(1-F(p_0))+b2\alpha\mu(p_0)$. We have

$$\begin{aligned} & V(p_0-\epsilon)-V(p_0) \\ \geq \quad & -q(p_0)\epsilon+\delta(p_0-\epsilon)-[c(\hat{q})-c(q(p_0))] \\ > \quad & -q(p_0)\epsilon+\delta(p_0-\epsilon)-c'(\hat{q})\delta \\ > \quad & -q(p_0)\epsilon+\delta(p_0-\epsilon)-[c'(q(p_0))+(p_0-c'(q(p_0)))/2] \\ = \quad & -q(p_0)\epsilon-\delta\epsilon-c'(q(p_0))\delta/2, \end{aligned} \tag{3.8}$$

where the second line makes use of the fact that $\hat{q}$ is a feasible quantity for a seller posting the price $p_0 - \epsilon$, the third line makes use of the fact that c is strictly convex, the fourth line makes use of the fact that $\hat{q} < q^*$ and, hence, $c'(\hat{q}) < c'(q^*)$ and $c'(q^*)$ equals $c'(q(p_0)) + (p_0 - c'(q(p_0)))/2$. Since (3.8) holds for any $\epsilon > 0$ and $p_0 > c'(q(p_0))$, there exists an ϵ small enough such that $V(p_0 - \epsilon) - V(p_0) > 0$. Therefore, the price p_0 cannot be on the support of F. ■

In the third lemma, I show that the support of the equilibrium price distribution F cannot include the price p if the price is strictly smaller than the marginal cost of production. The intuition for this finding is simple. If, in equilibrium, a seller posted a price p that is strictly smaller than the marginal cost, the seller would be strictly better off by rationing some customers. Rationing, however, is not allowed by the definition of equilibrium.

Lemma 3: *The support of the equilibrium price distribution F does not include any price p such that $p < c'(q(p))$.*

Proof: On the way to a contradiction, suppose that there exists a p_0 on the support of F such that $p_0 < c'(q(p_0))$. The profit of a seller posting the price p_0 is

$$V(p_0) = q(p_0)p_0 - c(q(p_0)), \tag{3.9}$$

where

$$q(p_0) = b\left[1 - \alpha + 2\alpha(1 - F(p_0)) + \alpha\mu(p_0)\right]. \tag{3.10}$$

Since the seller has the option of rationing buyers, we have

$$V(p_0) = \begin{array}{l} \max_q qp_0 - c(q), \\ \text{s.t. } q \in [0, b\left[1 - \alpha + 2\alpha(1 - F(p_0)) + \alpha\mu(p_0)\right]]. \end{array} \tag{3.11}$$

The necessary conditions for the optimality of q are

$$p_0 - c'(q) = \lambda, \tag{3.12}$$

$$\lambda \geq 0,\ q \leq q(p_0), \tag{3.13}$$

where λ is the multiplier on the constraint $q \leq q(p_0)$ and the inequalities in (3.13) hold with complementary slackness. If $q = q(p_0)$, then $p_0 - c'(q(p_0))$ is strictly negative. Hence $q = q(p_0)$ violates the necessary condition for optimality (3.12). In words, if $p_0 > c'(q(p_0))$, the optimal q must be strictly smaller than $q(p_0)$ and the seller must find it optimal to ration its customers. This contradicts the definition of equilibrium. ■

In the next lemma, I show that the equilibrium price distribution F has at most one mass point. If the equilibrium price distribution F does have a mass point at the price p_0, then p_0 is equal to the seller's marginal cost of production $c'(q(p_0))$ and p_0 is the lowest price on the support of F. These findings are easy to understand. The fact that any mass point must be at a price equal to the seller's marginal cost follows immediately from Lemma 2 and Lemma 3. The fact that there is at most one mass point follows immediately from the fact that there is at most one price that is equal to the seller's marginal cost, given that a seller's quantity is strictly decreasing in the price and the seller's marginal cost is strictly increasing in quantity. The fact that the mass point must be the lowest price on the support of F follows immediately from Lemma 3.

Lemma 4: *If the equilibrium price distribution F has mass points, then:* (i) *the mass point p_0 is unique and such that $p_0 = c'(q(p_0))$;* (ii) *the mass point p_0 is the lowest price on the support of F.*

Proof: (i) From Lemma 2 and Lemma 3, it follows that any mass point p_0 of the equilibrium price distribution F must be such that $p_0 = c'(q(p_0))$. On the way to a contradiction, suppose that the equilibrium price distribution F has a second mass point p_1. First, consider the case of $p_1 < p_0$. In this case, the quantity of output $q(p_1)$ produced by a seller posting the price p_1 is strictly greater than the quantity of output $q(p_0)$ produced by a seller posting the price p_0. In turn, this implies that $c'(q(p_1))$ is strictly greater than $c'(q(p_0))$, which is equal to p_0 and strictly greater than p_1. Therefore, $c'(q(p_1)) > p_1$. From Lemma 3, however, we know that the support of equilibrium price distribution F does not include any prices that are strictly smaller than the marginal cost. Next, consider the case of a second mass point p_1 with $p_1 > p_0$. In this case, the quantity of output $q(p_1)$ produced by a seller posting the price p_1 is strictly smaller than the quantity of output $q(p_0)$ produced by a seller posting the price p_0. In turn, this implies that $c'(q(p_1))$ is strictly smaller than $c'(q(p_0))$, which is equal to p_0 and strictly smaller than p_1. Therefore, $c'(q(p_1)) < p_1$. From Lemma 2, however, we know that the equilibrium price distribution F does not admit any mass points at prices that are strictly smaller than the marginal cost.

(ii) On the way to a contradiction, suppose that p_0 is not the lowest price on the support of the equilibrium price distribution F. Then there exists a price $p_1 < p_0$ on the support of F. The seller that posts the price p_1 produces $q(p_1)$ units of output, where $q(p_1) > q(p_0)$. Therefore, $c'(q(p_1))$ is strictly greater than $c'(q(p_0))$, which is equal to p_0 and strictly greater than p_1. From these observations, it follows that p_1 is strictly smaller than $c'(q(p_1))$. Lemma 2, however, states that the equilibrium price distribution F does not admit on its support any price strictly smaller than the marginal cost. ■

The following corollary to Lemma 4 computes the quantity of output sold by a seller at the mass point p_0.

Corollary 1: *If the equilibrium price distribution F has a mass point at p_0, the output $q(p_0)$ produced by a seller posting the price p_0 is $b\,[1+\alpha(1-F(p_0))]$.*

Proof: Let p_0 be a mass point of F. The output $q(p_0)$ produced by a seller posting the price p_0 is given by

$$\begin{aligned} q(p_0) &= b\,[1-\alpha+2\alpha(1-F(p_0))+\alpha\mu(p_0)] \\ &= b\,[1-\alpha+2\alpha(1-F(p_0))+\alpha F(p_0)] \\ &= b\,[1+\alpha(1-F(p_0))], \end{aligned} \tag{3.14}$$

where the first line is the general expression for the quantity of output produced by a seller posting the price p_0, the second line makes use of the fact that p_0 is the lowest price on the support of F, and the third line is an algebraic manipulation of the second one. ■

Lemma 4 states that the equilibrium price distribution F has at most one mass point. Hence, there are three possible types of equilibria: (i) an equilibrium in which F is degenerate at the mass point; (ii) an equilibrium in which F is non-degenerate and has a mass point; (iii) an equilibrium in which F is non-degenerate and does not have a mass point. In Lemmas 5 and 6, I characterize the properties of equilibria of type (iii) and (ii).

Lemma 5 shows that, if the equilibrium price distribution F does not have a mass point, then the support of F is an interval $[p_\ell, p_h]$ such that p_h is equal to the buyers' valuation u. The intuition behind this lemma is the same as in Butters (1977), Varian (1980) and Burdett and Judd (1983). Namely, any gap between p_1 and p_2 in the support of F would create a demand curve that is constant for all p in the interval $[p_1, p_2]$. Hence, an individual seller would strictly prefer posting the price p_2 than the price p_1. Similarly, if the highest price p_h was strictly smaller than u, the demand curve would be constant for any p in the interval $[p_h, u]$. Hence, an individual seller would strictly prefer posting the price u than the price p_h.

Lemma 5: *If the equilibrium price distribution F does not have a mass point, its support is an interval $[p_\ell, p_h]$, with $p_\ell < p_h = u$.*

Proof: If the equilibrium price distribution F does not have a mass point, any p on the support of F is such that $p > c'(q(p))$ and $q(p) = b[1-\alpha+2\alpha(1-F(p))]$.

I first show that there cannot be any gap on the support of F. On the way to a contradiction, suppose that there is a gap between two prices p_1 and p_2, with $p_1 < p_2$. The profit for a seller positing the price p_1 is given by

$$\begin{aligned} V(p_1) &= b\,[1-\alpha+2\alpha(1-F(p_1))]\,p_1 - c\,(b\,[1-\alpha+2\alpha(1-F(p_1))]) \\ &< b\,[1-\alpha+2\alpha(1-F(p_2))]\,p_2 - c\,(b\,[1-\alpha+2\alpha(1-F(p_2))]) \\ &= V(p_2), \end{aligned} \tag{3.15}$$

where the second line makes use of the facts that $F(p_1) = F(p_2)$ and $p_1 < p_2$, and the third line makes use of the definition of $V(p_2)$. Since $V(p_1) < V(p_2)$ and $V(p_2) \leq V^*$, it follows that p_1 cannot be on the support of F. A contradiction.

I next show that the highest price p_h on the support of F is equal to u. Suppose that p_h is strictly greater than u. In this case, the profit $V(p_h)$ for a seller posting the price p_h is equal to 0. Since $V^* > 0$, it follows that p_h cannot be on the support of F. Hence, p_h is non-greater than u. Suppose now that p_h is strictly smaller than u. In this case, the profit $V(p_h)$ for a seller posting the price p_h is equal to $b(1-\alpha)p_h - c(b(1-\alpha))$. The profit $V(u)$ for a seller posting the price u is equal to $b(1-\alpha)u - c(b(1-\alpha))$. Since $p_h < u$, it follows that $V(p_h) < V(u)$. Since $V(u) \leq V^*$, it follows that p_h cannot be on the support of F. Therefore, p_h must be equal to u. ■

Lemma 6 shows that, if the equilibrium price distribution F is non-degenerate but does have a mass point, the support of F is the mass point p_0 and an interval $[p_\ell, p_h]$, with $p_\ell > p_0$ and $p_h = u$. The same logic behind Lemma 5 explains why, away from the mass point, the support of the equilibrium price distribution F is an interval $[p_\ell, p_h]$ with $p_h = u$. The fact that there is a gap between the mass point p_0 and p_ℓ is due to the fact that the quantity of output sold by a seller that posts the price p_0 is discretely greater than the quantity of output sold by a seller that posts the price p_ℓ. For the profit of the two sellers to be identical, p_ℓ needs to be discretely smaller than p_0.

Lemma 6: *If the equilibrium price distribution F has a mass point but it is not degenerate, its support is $p_0 \cup [p_\ell, p_h]$, with $p_\ell > p_0$ and $p_h = u$.*

Proof: Consider an equilibrium such that the price distribution F has a mass point but it is not degenerate. From Lemma 4, it follows that the mass point is at a price p_0 that equals the seller's marginal cost $c'(q(p_0))$. From Corollary 1, it follows that $q(p_0)$ equals $b\,[1 + \alpha(1 - F(p_0))]$. From Lemma 4, it follows that any other price p on the support of F is strictly greater than p_0 and, hence, $q(p)$ is equal to $b\,[1 - \alpha + 2\alpha(1 - F(p))]$.

I first show that there cannot be a gap on the support of F between any two prices p_1 and p_2 that are on the support of F and are different from the mass point p_0. On the way to a contradiction, suppose there is a gap in the support of F between two prices p_1 and p_2. The profit for a seller positing the price p_1 is

$$\begin{aligned} V(p_1) \quad &= b\,[1 - \alpha + 2\alpha(1 - F(p_1))]\,p_1 - c\,(b\,[1 - \alpha + 2\alpha(1 - F(p_1))]) \\ &< b\,[1 - \alpha + 2\alpha(1 - F(p_2))]\,p_2 - c\,(b\,[1 - \alpha + 2\alpha(1 - F(p_2))]) \\ &= V(p_2), \end{aligned} \tag{3.16}$$

where the second line makes use of the facts that $F(p_1) = F(p_2)$ and $p_1 < p_2$, and the third line makes use of the definition of $V(p_2)$. Since $V(p_1) < V(p_2)$ and $V(p_2) \leq V^*$, it follows that p_1 cannot be on the support of F. A contradiction.

I next show that the highest price p_h on the support of F is equal to u. Suppose that p_h is strictly greater than u. In this case, the profit $V(p_h)$ for a seller posting the price p_h

is equal to 0. Since $V^* > 0$, p_h cannot be on the support of F. Hence, p_h is non-greater than u. Suppose now that p_h is strictly smaller than u. In this case, the profit $V(p_h)$ for a seller posting the price p_h is equal to $b(1-\alpha)p_h - c(b(1-\alpha))$. The profit $V(u)$ for a seller posting the price u is equal to $b(1-\alpha)u - c(b(1-\alpha))$. Since $p_h < u$, it follows that $V(p_h) < V(u)$. Since $V(u) \leq V^*$, p_h cannot be on the support of F. Therefore, p_h must be equal to u.

Lastly I show that $p_\ell > p_0$. The profit for a seller positing the price p_0 is

$$\begin{aligned} V(p_0) &= b\left[1+\alpha(1-F(p_0))\right]p_0 - c\left(b\left[1+\alpha(1-F(p_0))\right]\right) \\ &> b\left[1-\alpha+2\alpha(1-F(p_0))\right]p_0 - c\left(b\left[1-\alpha+2\alpha(1-F(p_0))\right]\right). \end{aligned} \tag{3.17}$$

The second line in (3.17) follows from the fact that $1+\alpha(1-F(p_0))$ is strictly greater than $1-\alpha+2\alpha(1-F(p_0))$ for any $F(p_0) > 0$ and from the fact that p_0 is equal to $c'(b\left[1+\alpha(1-F(p_0))\right])$. In fact, $qp - c(q)$ is strictly greater than $\hat{q}p - c(\hat{q})$ for any $\hat{q} < q$ as long as $p \geq c'(q)$. The profit for a seller posting the price p_ℓ is

$$V(p_\ell) = b\left[1-\alpha+2\alpha(1-F(p_0))\right]p_\ell - c\left(b\left[1-\alpha+2\alpha(1-F(p_0))\right]\right). \tag{3.18}$$

Comparing the second line of (3.17) with (3.18) immediately reveals that $V(p_0) > V(p_\ell)$ for any p_ℓ in some interval $(p_0, p_0+\epsilon)$. Since, $V(p_0)$ must be equal to $V(p_\ell)$, it follows that $p_\ell > p_0$. ■

Lemma 7 shows that a seller posting a price p that is greater or equal than the marginal cost $c'(q(p))$ finds it optimal to meet the buyers' demand. Lemma 3 shows that the support of the distribution F contains only prices p such that $p \geq c'(q(p))$. Therefore, Lemma 3 and Lemma 7 imply that, at any equilibrium price, sellers do not ration their buyers and, hence, condition (ii) in the definition of equilibrium is always satisfied.

Lemma 7: *A seller finds it optimal to meet demand for any p such that $p \geq c'(q(p))$.*

Proof: The problem for a seller posting the price p is

$$V(p) = \max_q pq - c(q), \text{ s.t. } q \in [0, q(p)]. \tag{3.19}$$

The objective function in (3.19) is strictly concave with respect to q. When evaluated at $q = q(p)$, the derivative of the objective function with respect to q is $p - c'(q(p))$, which is non-negative. Therefore, the solution to the maximization problem in (3.16) is $q = q(p)$. ■

4 Existence of different types of equilibria

The general properties derived in the previous section mean that there are three possible types of equilibrium: (i) the distribution F is a mass point at a price p_0 equal to the sellers' marginal cost; (ii) the distribution F is a mass point at a price p_0 equal to the sellers' marginal cost and a density over an interval $[p_\ell, p_h]$, with $p_\ell > p_0$ and $p_h = u$; (iii) the distribution F is a density over an interval $[p_\ell, p_h]$, with $p_h = u$. In this section, I solve for the distribution F in each of the three types of equilibrium. In the process, I derive necessary and sufficient conditions on the fundamentals under which each of the three types of equilibrium exists.

4.1 Degenerate distribution

The first and simplest type of equilibrium is such that the distribution F is a mass point at the price p_0. From Lemma 4, it follows that $p_0 = c'(q(p_0))$. From Corollary 1, it follows that $q(p_0) = b$. Therefore, in this equilibrium, the profit for a seller posting the price p_0 is given by

$$V(p_0) = bc'(b) - c(b). \tag{4.1}$$

For the equilibrium to exist, it must be the case that a seller cannot attain a profit strictly greater than $V^* = V(p_0)$ by posting a price different from p_0. If a seller posts any price $p < p_0$, its profit is given by

$$\begin{aligned} V(p) &= \max_q qp - c(q), \text{ s.t. } q \in [0, b(1+\alpha)] \\ &< \max_q qp_0 - c(q), \text{ s.t. } q \in [0, b(1+\alpha)] \\ &= bp_0 - c(b) = V^*. \end{aligned} \tag{4.2}$$

The second line in (4.2) follows from the fact that $p < p_0$. The third line in (4.2) follows from the fact that the solution of the maximization problem in the second line is $q = b$, since $p_0 = c'(b)$. Therefore, a seller never finds it optimal to post a price p strictly smaller than p_0.

If a seller posts a price p equal to the buyers' valuation u, its profit is given by

$$\begin{aligned} V(u) &= \max_q qu - c(q), \text{ s.t. } q \in [0, b(1-\alpha)] \\ &= b(1-\alpha)u - c(b(1-\alpha)). \end{aligned} \tag{4.3}$$

The second line in (4.3) follows from the fact that the solution to the maximization problem in the first line is $q = b(1-\alpha)$, since u is strictly greater than $c'(b(1+\alpha))$ and $c'(b(1+\alpha))$ is strictly greater than $c'(b(1-\alpha))$. Therefore, a seller does not find it optimal to post a price equal to the buyers' valuation u if and only if $V(u) \leq V^*$ or, equivalently, if and only if

$$b(1-\alpha)u - c(b(1-\alpha)) \leq bc'(b) - c(b). \tag{4.4}$$

If a seller posts a price p in the interval (p_0, u), its profit is

$$\begin{aligned} V(p) &= \max_q qp - c(q), \text{ s.t. } q \in [0, b(1-\alpha)] \\ &< \max_q qu - c(q), \text{ s.t. } q \in [0, b(1-\alpha)] \\ &= V(u), \end{aligned} \tag{4.5}$$

where the second line makes use of the fact that $p < u$. Assuming that condition (4.4) holds, a seller never finds it optimal to post a price $p \in (p_0, u)$. Lastly, note that a seller never finds it optimal to post a price $p > u$, since $V(p) = 0$ for all $p > u$ and $V^* > 0$.

The following proposition summarizes the conditions for existence and the properties of an equilibrium in which the price distribution is degenerate.

Proposition 1: (Degenerate distribution)

(i) *An equilibrium with a degenerate price distribution F exists if and only if*

$$b(1-\alpha)u - c(b(1-\alpha)) \leq bc'(b) - c(b). \tag{4.6}$$

(ii) *The equilibrium with a degenerate price distribution F is unique. The equilibrium is such that every seller posts the price $p_0 = c'(b)$ and trades b units of the good.*

4.2 Non-degenerate distribution and a mass point

The second type of equilibrium is such that the distribution F is non-degenerate and has a mass point. From Lemma 3, it follows that the mass point is at a price p_0 equal to the seller's marginal cost $c'(q(p_0))$. From Corollary 1, it follows that $q(p_0)$ is equal to $b(1 + \alpha(1 - F(p_0)))$. From Lemma 6, it follows that the seller that do not post the price p_0 are distributed over some interval $[p_\ell, p_h]$, with $p_\ell > p_0$ and $p_h = u$.

I now proceed to characterize this type of equilibrium and, in the process, identify necessary and sufficient conditions for its existence. The profit for a seller posting p_0 is

$$V(p_0) = b(1 + \alpha x^*)c'(b(1 + \alpha x^*)) - c(b(1 + \alpha x^*)), \tag{4.7}$$

where $x^* = 1 - F(p_0)$ denotes the measure of sellers that post a price strictly greater than p_0, and $1 - x^*$ is the measure of sellers that post a price equal to p_0. Since p_0 is on the support of F, $V(p_0)$ is equal to V^*.

The profit for a seller positing the price p equal to the buyers' valuation u is

$$\begin{aligned} V(u) &= b\left[1 - \alpha + 2\alpha(1 - F(u))\right]u - c\left(b\left[1 - \alpha + 2\alpha(1 - F(u))\right]\right) \\ &= b(1-\alpha)u - c(b(1-\alpha)), \end{aligned} \tag{4.8}$$

where the second line makes use of the fact that $F(u) = 1$. Since u is on the support of F, $V(u)$ is also equal to V^*.

Equating the profit of a seller posting the price p_0 and the profit of a seller posting the price u yields

$$b(1+\alpha x^*)c'(b(1+\alpha x^*)) - c(b(1+\alpha x^*)) = b(1-\alpha)u - c(b(1-\alpha)). \tag{4.9}$$

The equation above pins down x^*, the fraction of sellers that post a price strictly higher than p_0. In equilibrium, x^* must be strictly greater than 0 (i.e., the distribution F has a mass point at p_0) and strictly smaller than 1 (i.e., the distribution F is non-degenerate). The left-hand side of (4.9) is a strictly increasing function of x^*. The left-hand side of (4.9) takes the value $bc'(b) - c(b)$ for $x^* = 0$. The left-hand side of (4.9) takes the value $b(1+\alpha)c'(b(1+\alpha)) - c(b(1+\alpha))$ for $x^* = 1$. Therefore, x^* is strictly greater than 0 if and only if

$$bc'(b) - c(b) < b(1-\alpha)u - c(b(1-\alpha)) \tag{4.10}$$

and it is strictly smaller than 1 if and only if

$$b(1+\alpha)c'(1+\alpha) - c(1+\alpha) > b(1-\alpha)u - c(b(1-\alpha)). \tag{4.11}$$

Assume that conditions (4.10) and (4.11) hold.

The profit for a seller posting a price $p \in [p_\ell, p_h]$ is such that

$$V(p) = b\left[1-\alpha+2\alpha x(p)\right]p - c(b\left[1-\alpha+2\alpha x(p)\right]), \tag{4.12}$$

where $x(p) = 1 - F(p)$ denotes the fraction of sellers posting a price greater than p. Since any $p \in [p_\ell, p_h]$ is on the support of F, $V(p)$ must be equal to V^*.

Since a seller posting the price u and a seller posting any price $p \in [p_\ell, p_h]$ must make the same profit, we have

$$b\left[1-\alpha+2\alpha x(p)\right]p - c(b\left[1-\alpha+2\alpha x(p)\right]) = b(1-\alpha)u - c(b(1-\alpha)). \tag{4.13}$$

The above equation pins down the function $x(p)$. In equilibrium, it must be the case that $F(p_\ell) = 1 - x^*$, $F(p_h) = 1$ and $F'(p) > 0$. Therefore, the function $x(p)$ that solves (4.13) must be such that $x(p_\ell) = 1 - x^*$, $x(p_h) = 0$ and $x'(p) < 0$. Rather than characterizing the function $x(p)$, i.e. the fraction of sellers posting a price greater than p, it is easier and more useful to characterize its inverse $p(x)$, i.e. the price posted by a seller that is at the $(1-x)$-th quantile of the price distribution. The equilibrium restrictions on $x(p)$ are equivalent to $p(x)$ being such that $p(0) = p_h$, $p(x^*) = p_\ell$, and $p'(x) < 0$, with $p_\ell > p_0$ and $p_h = u$.

Totally differentiating (4.13) yields

$$p'(x) = -\frac{2\alpha\left[p(x) - c'(b(1-\alpha+2\alpha x))\right]}{1-\alpha+2\alpha x}. \tag{4.14}$$

The expression above is a differential equation for $p(x)$. The solution to (4.13) is the solution of the differential equation that satisfies the boundary condition $p(0) = u$. Notice that (4.14) is such that $p(x) > c'(b(1-\alpha+2\alpha x))$ implies $p'(x) < 0$, $p(x) = c'(b(1-\alpha+2\alpha x))$ implies $p'(x) = 0$, and $p(x) < c'(b(1-\alpha+2\alpha x))$ implies $p'(x) > 0$. Since $p(x) = c'(b(1-\alpha+2\alpha x))$ implies $p'(x) = 0$ and $c'(b(1-\alpha+2\alpha x))$ is strictly increasing in x, it follows that $p(x)$ can equate $c'(b(1-\alpha+2\alpha x))$ at most once. Since $p(0) > c(b(1-\alpha))$, it follows that $p(x) > c'(b(1-\alpha+2\alpha x))$ for all $x \in [0, x^*]$ if and only if $p(x^*) > c'(b(1-\alpha+2\alpha x^*))$. If that is the case, $p'(x) < 0$ for all $x \in [0, x^*]$.

A seller posting the price $p(x^*)$ makes the same profit as a seller posting the price p_0. Equating these profits yields

$$b(1-\alpha+2\alpha x^*)p(x^*) - c(b(1-\alpha+2\alpha x^*)) = b(1+\alpha x^*)c'(b(1+\alpha x^*)) - c(b(1+\alpha x^*)). \quad (4.15)$$

Notice that, for $p(x^*) \leq c'(b(1-\alpha+2\alpha x^*))$, the left-hand side of (4.15) is strictly smaller than the right-hand side because $qc'(q) - c(q)$ is a strictly increasing function of q and $1-\alpha+2\alpha x^*$ is strictly smaller than $1+\alpha x^*$. Therefore, $p(x^*) > c'(b(1-\alpha+2\alpha x^*))$. From the characterization of the solution to the differential equation (4.14), it follows that $p(x) > c'(b(1-\alpha+2\alpha x))$ for all $x \in [0, x^*]$ and $p'(x) < 0$ for all $x \in [0, x^*]$. Next, notice that, for any $p(x^*) \leq c'(b(1+\alpha x^*))$, the left-hand side of (4.15) is strictly smaller than the right-hand side because $qc'(q^*) - c(q)$ is strictly increasing in q for all $q \in [0, q^*]$. Therefore, $p(x^*) > c'(b(1+\alpha x^*)) = p_0$.

I have thus established that there exists a function $p(x)$ that satisfies the equal profit condition (4.13) for all $x \in [0, x^*]$ and such that $p(0) = p_h = u$, $p'(x) < 0$, and $p(x^*) = p_\ell > p_0$. The inverse of $p(x)$ is thus a function $x(p)$ that satisfies the equal profit condition (4.13) for all $p \in [p_\ell, p_h]$ and such that $x'(p) < 0$, $x(p_h) = 0$ and $x(p_\ell) = x^*$.

Lastly, I need to show that a seller cannot make a profit strictly greater that V^* by posting a price p that is off the support of F. Consider a seller posting a price p strictly smaller than p_0. The seller's profit is given by

$$\begin{aligned} V(p) &= \max_q qp - c(q), \text{ s.t. } q \in [0, b(1+\alpha)] \\ &< \max_q qp_0 - c(q), \text{ s.t. } q \in [0, b(1+\alpha)] \\ &= b(1+\alpha x^*)p_0 - c(b(1+\alpha x^*)) \\ &= V^*, \end{aligned} \quad (4.16)$$

where the second line follows from the fact that $p < p_0$, and the third line follows from the fact that the solution to the maximization problem in the second line is q equal to $b(1+\alpha x^*)$, since p_0 is equal to $c'(b(1+\alpha x^*))$. Therefore, a seller can never increase his profit by posting a price strictly smaller than p_0.

Consider a seller posting a price p in the interval (p_0, p_ℓ). The seller's profit is

$$
\begin{aligned}
V(p) &= \max_q qp - c(q), \text{ s.t. } q \in [0, b(1-\alpha+2\alpha x^*)] \\
&< \max_q qp_\ell - c(q), \text{ s.t. } q \in [0, b(1-\alpha+2\alpha x^*)] \\
&= b(1-\alpha+2\alpha x^*)p_\ell - c(b(1-\alpha+2\alpha x^*)) \\
&= V^*,
\end{aligned}
\tag{4.17}
$$

where the first line makes use of the fact that $1 - F(p) = x^*$ for any $p \in (p_0, p_\ell)$, the second line makes use of the fact that $p < p_\ell$, and the third line makes use of the fact that the solution to the maximization problem in the third line is $q = b(1-\alpha+2\alpha x^*)$. Therefore, a seller never finds it optimal to post a price in $p \in (p_0, p_\ell)$. Lastly, a seller never finds it optimal to post a price p strictly greater than u, since $V(p) = 0$ and $V^* > 0$.

The following proposition summarizes the conditions for the existence and the properties of an equilibrium in which the distribution is non-degenerate and has a mass point.

Proposition 2: (Non-degenerate distribution with a mass point)

(i) *An equilibrium with a non-degenerate price distribution and a mass point exists if and only if*

$$
bc'(b) - c(b) < b(1-\alpha)u - c(b(1-\alpha)) \tag{4.18}
$$

and

$$
b(1+\alpha)c'(1+\alpha) - c(1+\alpha) > b(1-\alpha)u - c(b(1-\alpha)). \tag{4.19}
$$

(ii) *An equilibrium with a non-degenerate price distribution and a mass point is unique. The equilibrium is such that:*

(a) *The price distribution F is a mass point at p_0 and a density over the interval $[p_\ell, p_h]$, with $p_\ell > p_0$ and $p_h = u$.*

(b) *A measure $1 - x^*$ of sellers post the price p_0 and trade $b(1+\alpha x^*)$ units of output, where $p_0 = c'(b(1+\alpha x^*))$ and $x^* \in (0,1)$ is given by (4.9).*

(c) *A measure x^* of sellers post prices in the interval $[p_\ell, p_h]$. For any $x \in [0, x^*]$, a seller at the $(1-x)$-th quantile of the price distribution F posts the price $p(x)$ and trades $b(1-\alpha+2\alpha x)$ units of output, where $p(x)$ is given by (4.13).*

4.3 Atomless distribution

The third type of equilibrium is such that the price distribution F is atomless. From Lemma 5, it follows that the support of F is some interval $[p_\ell, p_h]$, with $p_h = u$.

The profit for a seller posting a price p equal to the buyers' valuation u is

$$
V(u) = b(1-\alpha)u - c(b(1-\alpha)). \tag{4.20}
$$

The profit for a seller posting a price p in the interval $[p_\ell, p_h]$ is

$$V(p) = b(1-\alpha+2\alpha x(p))p - c(b(1-\alpha+2\alpha x(p))), \tag{4.21}$$

where $x(p) = 1 - F(p)$ denotes the fraction of sellers posting a price greater than p. Since u is on the support of F, it follows that $V(u)$ must be equal to V^*. Since any $p \in [p_\ell, p_h]$ is on the support of F, it follows that $V(p)$ must also be equal to V^*.

Equating (4.20) to (4.21) yields

$$b(1-\alpha+2\alpha x(p))p - c(b(1-\alpha+2\alpha x(p))) = b(1-\alpha)u - c(b(1-\alpha)). \tag{4.22}$$

The expression above pins down the function $x(p)$. In equilibrium, the function $x(p)$ that solves (4.22) must have the property that $x(p_\ell) = 1$, $x(p_h) = 0$, and $x'(p) < 0$. Analogously, the inverse of $x(p)$, which I denote as $p(x)$, must have the property that $p(0) = p_h$, $p(1) = p_\ell$, and $p'(x) < 0$, with $p_h = u$.

Totally differentiating (4.22) yields

$$p'(x) = -\frac{2\alpha\left[p(x) - c'(b(1-\alpha+2\alpha x))\right]}{1-\alpha+2\alpha x}. \tag{4.23}$$

The expression above is a differential equation for $p(x)$. The solution to (4.22) is the solution of the differential equation (4.23) that satisfies the boundary condition $p(0) = u$. Recall that $p(x)$ can equate $c'(b(1-\alpha+2\alpha x))$ at most once. Since $p(0) > c(b(1-\alpha))$, it follows that $p(x) > c'(b(1-\alpha+2\alpha x))$ for all $x \in [0,1]$ if and only if $p(1) > c'(b(1+\alpha))$. If that is the case, $p'(x) < 0$ for all $x \in [0,1]$.

From (4.22), it follows that $p(1)$ is given by

$$b(1+\alpha)p(1) - c(b(1+\alpha)) = b(1-\alpha)u - c(b(1-\alpha)). \tag{4.24}$$

Clearly, $p(1) > c'(b(1+\alpha))$ if and only if

$$b(1+\alpha)c'(b(1+\alpha)) - c(b(1+\alpha)) < b(1-\alpha)u - c(b(1-\alpha)). \tag{4.25}$$

If and only if condition (4.25) holds, the solution to the differential equation (4.23) is a function $p(x)$ such that $p(0) = p_h = u$, $p(1) = p_\ell$ and $p'(x) < 0$ for all $x \in [0,1]$. In turn, if and only if (4.25) holds, there exists a solution to (4.22) such that $x(p_h) = 1$, $x(p_\ell) = 0$ and $x'(p) < 0$ for all $p \in [p_\ell, p_h]$.

Since it is clear that a seller cannot attain a profit strictly greater than V^* by posting a price p that is strictly greater than p_h nor by posting a price p that is strictly smaller than p_ℓ, it follows that an equilibrium with an atomless distribution exists if and only if condition (4.25) holds.

Proposition 3: (Atomless distribution)

(i) *An equilibrium with an atomless price distribution exists if and only if*

$$b(1+\alpha)c'(b(1+\alpha)) - c(b(1+\alpha)) < b(1-\alpha)u - c(b(1-\alpha)). \tag{4.26}$$

(ii) *An equilibrium with an atomless price distribution is unique. The equilibrium is such that:*

(a) *The support of the price distribution F is the interval $[p_\ell, p_h]$, with $p_h = u$.*

(b) *For all $x \in [0,1]$, a seller at the $(1-x)$-th quantile of the price distribution F posts the price $p(x)$ and trades $b(1-\alpha+2\alpha x)$ units of output, where $p(x)$ is given by (4.23).*

5 Existence, uniqueness and nature of equilibrium

In the previous section, I derived necessary and sufficient conditions for the existence of the three different types of equilibria. I also proved that, if an equilibrium of a particular type exists, it is always unique. In this section, I prove that an equilibrium always exists and is unique, but that the nature of equilibrium depends on parameter values.

The conditions for the existence of different type of equilibria depend on the value taken by three expressions:

$$bc'(b) - c(b), \tag{5.1}$$

$$b(1-\alpha)u - c(b(1-\alpha)), \tag{5.2}$$

$$b(1+\alpha)c'(b(1+\alpha)) - c(b(1+\alpha)). \tag{5.3}$$

Figure 1 plots the three expressions above as a function of α. Condition (5.1) is the solid line, condition (5.2) is the dotted line, condition (5.3) is the dashed line. The expression in (5.1) is strictly positive and is independent of α. Now consider the expression in (5.2). For $\alpha = 0$, the expression (5.2) takes the value $bu - c(b)$, which is strictly positive because $u > c'(b)$ and $c'(b) > c(b)$, and strictly greater than $bc'(b) - c(b)$ because $u > c'(b)$. For $\alpha = 1$, the expression in (5.2) takes the value 0. The expression in (5.2) is strictly decreasing with respect to α, because its derivative, $-bu + bc'(b(1-\alpha))$, is strictly positive. Lastly, consider the expression in (5.3). For $\alpha = 0$, the expression in (5.3) takes the value $bc'(b) - c(b)$. The expression is strictly increasing in α because $qc'(q) - c(q)$ is strictly increasing in q.

Let α_1 denote the value of alpha for which the expression in (5.2) is equal to the expression in (5.3). Clearly, α_1 exists and belongs to the interval $(0,1)$. Let α_2 denote the value for which the expression in (5.2) is equal to the expression in (5.1). Clearly, α_2 exists and belongs to the interval $(\alpha_1, 1)$. For any $\alpha \in (0, \alpha_1)$, the expression in (5.2) is strictly greater than the expression in (5.3), which is strictly greater than the expression in (5.1).

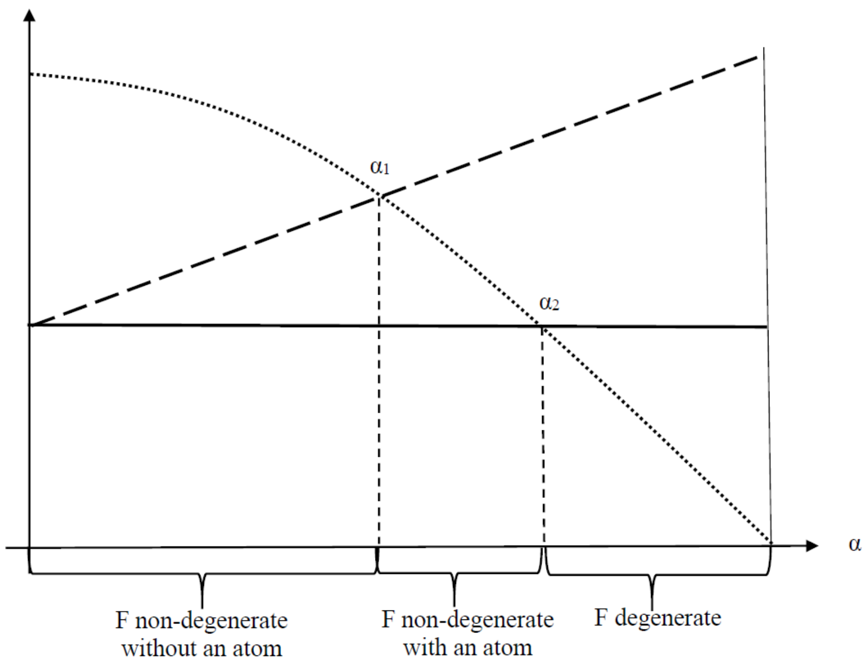

Figure 1: Conditions (5.1), (5.2) and (5.3)

For any $\alpha \in (\alpha_1, \alpha_2)$, the expression in (5.3) is strictly greater than the expression in (5.2), and (5.2) is strictly greater than (5.1). For any $\alpha \in (\alpha_2, 1)$, the expression in (5.3) is strictly greater than the expression in (5.1), which is strictly greater than (5.2).

Proposition 3 states that an equilibrium with an atomless price distribution is unique and exists if and only if (5.2) is strictly greater than (5.3). This is the case if and only if α belongs to the interval $(0, \alpha_1)$. Proposition 2 states that an equilibrium with a non-degenerate price distribution and a mass point is unique and exists if and only if (5.3) is strictly greater than (5.2) and (5.2) is strictly greater than (5.1). This is the case if and only if α belongs to the interval (α_1, α_2). Proposition 1 states that an equilibrium with a degenerate distribution is unique and exists if and only if (5.1) is strictly greater than (5.2). This is the case if and only if α belongs to the interval $(\alpha_2, 1)$. Therefore, for any value of $\alpha \in (0, 1)$, there exists a unique equilibrium, but the nature of equilibrium depends on α.

Theorem 1 below contains a summary of my findings.

Theorem 1: (Existence, uniqueness and nature of equilibrium)

(i) *Equilibrium exists and is unique.*

(ii) *If $\alpha \in (0, \alpha_1)$, the equilibrium features a non-degenerate price distribution without a mass point, and it has the properties described by Proposition 3.*

(iii) *If $\alpha \in (\alpha_1, \alpha_2)$, the equilibrium features a non-degenerate distribution with a mass point, and it has the properties described by Proposition 2.*

(iv) *If* $\alpha \in (\alpha_2, 1)$*, the equilibrium features a degenerate distribution, and it has the properties described by Proposition 1.*

To understand Theorem 1, recall that α is an inverse measure of search frictions, in the sense that the ratio of non-captive to captive buyers ranges from zero to infinity as α goes from 0 to 1. When search frictions are large, in the sense that α is smaller than α_1, the equilibrium is such that sellers post prices from an atomless distribution with support $[p_\ell, p_h]$, with $p_h = u$. The equilibrium price distribution is atomless for the same reason as in Burdett and Judd (1983). That is, any mass point would create a downward discontinuity in the demand curve facing an individual seller and, hence, a seller would always be better off posting a price just below the mass point than at the mass point. In equilibrium, all sellers make the same profit. Sellers that post a lower price trade more units of the good but enjoy a smaller profit per unit, both because their price is lower and because their average cost is higher. Sellers that post a higher price trade fewer units of the good but enjoy a larger profit per unit, both because their price is higher and because their average cost is lower.

When search frictions are intermediate, in the sense that α is between α_1 and α_2, the equilibrium is such that sellers post prices from a non-degenerate distribution with support $p_0 \cup [p_\ell, p_h]$, with $p_\ell > p_0$ and $p_h = u$. At the lowest price p_0, the distribution has a mass point. Over the interval $[p_\ell, p_h]$, the distribution is atomless. The mass point at p_0 is immune from the argument in Burdett and Judd (1983) because the price p_0 is exactly equal to the seller's marginal cost. Therefore, even though the demand curve facing an individual seller has a downward discontinuity on the left side of p_0, a seller has no incentive to undercut the competitors at the mass point because the price is equal to the marginal cost and increasing quantity would not increase profits even if though increasing quantity requires only an infinitesimal decrease in the price. Not every seller, however, can set the price p_0 in equilibrium. Indeed, if every seller set the price p_0, an individual seller would be better off posting a price equal to u and trading only with those buyers who are not in contact with anybody else. Since for any $p < p_0$, the price p exceeds the marginal cost, sellers that do not post p_0 must distribute themselves without a mass point. Since the mass point at p_0 also creates a downward discontinuity in demand to the right of p_0, the highest price posted by a seller not at the mass point must be strictly greater than p_0.

When search frictions are small, in the sense that α is greater than α_2, the equilibrium is such that every seller posts the price p_0. The mass point at p_0 is immune to deviations towards lower prices because it is equal to the marginal cost. The mass point at p_0 is immune to deviations to the buyers' reservation price u because, when search frictions are small, a seller does not meet enough captive buyers to make this deviation profitable.

It is easy to understand why the structure of the equilibrium evolves as search frictions decline. When search frictions are large, the difference in the quantity of output sold by the seller with the lowest price (i.e., the seller that trades with every contacted buyer) and the quantity of output sold by the seller with the highest price (i.e., the seller that trades only with the contacted buyers who have no alternative) is small and so is the difference in their marginal costs. For this reason, the equilibrium has the same structure as in Burdett and Judd (1983), except that the marginal cost is not exactly the same for all sellers. As search frictions decline, the output traded by the seller with the lowest price grows, while the output traded by the seller with the highest price declines. As a result, the marginal cost of the seller with the highest price increases, while the marginal cost of the seller with the lowest price decreases. At some point, the marginal cost of the best seller becomes equal to the price.

As search frictions further decline, the Burdett and Judd (1983) equilibrium breaks down because the best seller is not willing to increase its quantity. Instead, the equilibrium is such that an increasing fraction of the best sellers amasses at the lowest price. The measure of sellers at the lowest price is such that their profits are equated to the profits of the seller with the highest price. Since the profit of the seller with the highest price decreases as search frictions become smaller, the profit of the sellers at the lowest price must decrease as well. This implies that the mass of sellers at the lowest price increases, the quantity sold by each of them decreases, and the lowest price increases. As some point, all of the seller in the market move reach the lowest price and the equilibrium features a single market price. Further reductions in search frictions have no impact on the equilibrium price distribution.

It is also interesting to examine the behavior of equilibrium in the limit for $\alpha \to 0$ and for $\alpha \to 1$. For $\alpha \to 0$, almost every buyer is in contact with a single seller and, for this reason, the equilibrium distribution of prices converges to the monopoly price $p = u$. For $\alpha \to 1$, almost every buyer is in contact with multiple sellers and the equilibrium price distribution is a mass point at the competitive price $p = c'(b)$. As α goes from 0 to 1, the ratio of non-captive to captive buyers goes from zero to infinity, and the sellers' profit declines monotonically. Therefore, as α goes from 0 to 1, the equilibrium outcomes go from monopolistic to competitive.

Let me contrast the findings in Theorem 1 with the behavior of equilibrium in Burdett and Judd (1983), a model in which sellers have a constant marginal cost of production. In Burdett and Judd (1983), the unique equilibrium always features an atomless price distribution. Intuitively, any mass point at a price p_0 generates a downward discontinuity to the left of p_0. Since the sellers' marginal cost is constant and prices are greater than marginal cost (because equilibrium profits are strictly positive), the downward discontinuity can always be exploited by an individual seller to increase profits. Since the equilibrium price distribution is atomless, sellers are strictly ranked by buyers. Hence, the seller with the lowest price trades a quantity of output that keeps increasing as search frictions decline, while the seller with the highest price trades a quantity of output that declines towards zero as search frictions decline. For $\alpha \to 0$, the equilibrium is such that all sellers trade approximately the same quantity of output and they all post prices that are approximately equal to the monopoly price u. For $\alpha \to 1$, the equilibrium is such that the concentration of trade is the highest and almost all sellers post prices that are approximately equal to the constant marginal cost.

Whether sellers have a constant or increasing marginal cost of production, the equilibrium goes from monopolistic to competitive as α goes from 0 to 1, exactly as one would have expected. The difference between the structure of equilibrium when sellers have a constant or increasing marginal cost of production is caused by the difference in the competitive outcome. If sellers have a constant marginal cost, the competitive outcome is such that every seller posts a price equal to the marginal cost. The competitive outcome, however, does not place any restrictions on the output produced and traded by different sellers. If sellers have an increasing marginal cost, the competitive outcome is such that every seller posts a price equal to the marginal cost and trades the same quantity of output. An atomless price distribution generates non-vanishing dispersion in the quantity of output produced by different sellers, even as the extent of price dispersion becomes arbitrarily small. For this reason, an atomless price distribution can approach the competitive outcome when sellers have a constant marginal cost, and, in fact, it does in Burdett and Judd (1983). An atomless price distribution cannot approach the competitive outcome when sellers have an increasing marginal cost. When sellers have an increasing marginal cost, the competitive outcome can only be approached if the price distribution becomes degenerate.

6 Welfare properties of equilibrium

In this section, I examine the welfare properties of equilibrium. For any $\alpha \in (0, \alpha_1)$, the sum of the buyers' and sellers' payoffs is

$$\begin{aligned} W &= \int_0^1 \left[b(1-\alpha+2\alpha x)u - c(b(1-\alpha+2\alpha x)) \right] dx \\ &= bu - \int_0^1 c(b(1-\alpha+2\alpha x))dx \end{aligned} \tag{6.1}$$

For $\alpha \in (0, \alpha_1)$, the equilibrium is such that the price distribution F is atomless. Consider a seller at the $(1-x)$-th quantile of the price distribution F. The payoff to the seller is given by the difference between revenues, $b(1-\alpha+2\alpha x)p(x)$, and production costs, $c(b(1-\alpha+2\alpha x))$. The payoff to the seller's customers is given by $b(1-\alpha+2\alpha x)(u-p(x))$. Therefore, the sum of the payoffs to the seller and its customers is given by the difference between $b(1-\alpha+2\alpha x)u$ and $c(b(1-\alpha+2\alpha x))$. The sum of the buyers' and sellers' payoffs is the integral of the sum of the payoffs enjoyed by each individual seller and their customers.

For any $\alpha \in (\alpha_1, \alpha_2)$, the sum of the buyers' and sellers' payoffs is

$$\begin{aligned} W &= \int_0^{x^*} [b(1-\alpha+2\alpha x)u - c(b(1-\alpha+2\alpha x))]\, dx + \int_{x^*}^1 [b(1+\alpha x)u - c(b(1+\alpha x))]\, dx \\ &= bu - \int_0^{x^*} c(b(1-\alpha+2\alpha x))dx - (1-x^*)c(b(1+\alpha x^*)). \end{aligned} \tag{6.2}$$

For $\alpha \in (\alpha_1, \alpha_2)$, the equilibrium price distribution F is such that there is a measure $1-x^*$ of sellers at the price p_0 and a measure x^* of sellers distributed over some interval $[p_\ell, p_h]$ without atoms. Consider a seller posting the price p_0. The payoff to the seller is given by the difference between revenues, $b(1+\alpha x^*)p_0$, and production costs, $c(b(1+\alpha x^*))$. The payoff to the sellers' customers is given by $b(1+\alpha x^*)(u-p_0)$. Therefore, the sum of the payoffs to the seller and its customers is given by $b(1+\alpha x^*)u - c(b(1+\alpha x^*))$. Consider a seller at the $(1-x)$-th quantile of the price distribution, with $x > x^*$. The sum of the payoffs to the seller and its customers is given by the difference between $b(1-\alpha+2\alpha x)u$ and $c(b(1-\alpha+2\alpha x))$.

For any $\alpha \in (\alpha_2, 1)$, the sum of the buyers' and the sellers' payoffs is

$$W = \int_0^1 (bu - c(b))dx = bu - c(b). \tag{6.3}$$

For $\alpha \in (\alpha_2, 1)$, the equilibrium price distribution F is degenerate at the price p_0. Consider a seller posting the price p_0. The payoff to the seller is given by the difference between revenues, bp_0, and production costs, $c(b)$. The payoff to the sellers' customers is given by $b(u-p_0)$. Therefore, the sum of the payoffs to the seller and its customers is given by $bu - c(b)$.

Now consider the problem of a utilitarian social planner

$$\begin{aligned} W^* = \quad & \max_{g_1, g_2} \int_0^1 q(i)u - c\,(q(i))\, di, \text{ s.t.} \\ & q(i) = b\left[(1-\alpha)g_1(i) + 2\alpha\left(\textstyle\int_0^1 g_2(i,j)dj\right)\right], \\ & g_1(i) \geq 0,\ g_1(i) \leq 1, \\ & g_2(i,j) \geq 0,\ g_2(i,j) + g_2(i,j) \leq 1. \end{aligned} \tag{6.4}$$

Let me explain the optimization problem above. The objective of a utilitarian planner is to maximize the sum of the buyers' and sellers' payoffs. The planner chooses the probability $g_1(i)$ with which a buyer who is in contact with seller i purchases the good, and the probability $g(i,j)$ with which a buyer who is in contact with seller i and seller j purchases the good from seller i. The probability that a buyer who is in contact with seller i purchases the good must be smaller than 1. The probability that a buyer in contact with seller i and seller j purchases the good from either seller must be smaller than 1.

The necessary conditions for the optimality of the planner's solution are

$$b(1-\alpha)\,(u - c'(q(i))) = \lambda_1(i), \tag{6.5}$$

$$2b\alpha\,(u - c'(q(i))) = \lambda_2(i,j), \tag{6.6}$$

$$g_1(i) \leq 1,\ \lambda_1(i) \geq 0, \tag{6.7}$$

and

$$g_2(i,j) + g_2(j,i) \leq 1,\ \lambda_2(i,j) \geq 0, \tag{6.8}$$

where $q(i)$ denotes the quantity produced by seller i, $\lambda_1(i)$ denotes the multiplier on the constraint $g_1(i) \leq 1$, $\lambda_2(i,j)$ denotes the multiplier on the constraint $g(i,j) + g(j,i) \leq 1$, and the inequalities in (6.7) and (6.8) hold with complementary slackness.

From (6.8), it follows that $\lambda_2(i,j)$ equals $2b\alpha\,(u - c'(q(i)))$ for all j. Therefore, $\lambda_2(i,j) = \lambda_2(i)$ for all j. Since $\lambda_2(i,j) = \lambda_2(j,i)$, it follows that $\lambda_2(i) = \lambda_2(j) = \lambda_2$ for all i and j. Therefore, $2b\alpha\,(u - c'(q(i)))$ equals λ_2 for all i and, hence, $q(i) = q$ for all i. In words, the solution to the planner's problem is such that every seller produces the same quantity of output q. The highest feasible q is b and, since $u > c'(b)$, it follows that $g_1(i) - 1$ and $g_2(i,j) + g_2(j,i) = 1$. In other words, the solution to the planner's problem is such that a buyer always purchases the good. These observations imply that

$$W^* = bu - c(b). \tag{6.9}$$

Comparing total welfare in equilibrium and in the solution to the planner's problem leads immediately to the following theorem.

Theorem 2 (Efficiency). *The equilibrium is efficient if α belongs to the interval $(\alpha_2, 1)$. The equilibrium is inefficient if α belongs to the interval $(0, \alpha_2)$.*

Theorem 2 states that the equilibrium is efficient if and only if search frictions are low enough. Irrespective of search frictions, the equilibrium price distribution F is such that every seller posts a price non-greater than the buyer's valuation for the good. Therefore, every buyer purchases a unit of the good and the total value of buyers' consumption is maximized. If search frictions are sufficiently low, the price distribution F is degenerate. Therefore, every seller produces the same quantity of output and the total cost of sellers' production is minimized. Since the total value of buyers' consumption is maximized and the total cost of sellers' production is minimized, the equilibrium is efficient. If search frictions are not low enough, the equilibrium price distribution F is not degenerate. Therefore, not all sellers produce the same quantity of output and the total cost of sellers' production is not minimized. Hence the equilibrium is not efficient.

Theorem 2 identifies another difference between the model in which sellers have an increasing marginal cost of production and the model in which sellers have a constant marginal cost of production (i.e. Burdett and Judd 1983). In Burdett and Judd (1983), the equilibrium is always efficient. Irrespective of search frictions, the equilibrium price distribution F is such that every seller posts a price non-greater than the buyer's valuation for the good. Therefore, every buyer purchases a unit of the good and the total value of buyers' consumption is maximized. Irrespective of search frictions, the equilibrium price distribution is not degenerate. Even though sellers produce different quantities of output, the total cost of sellers' production is minimized because sellers have a constant, common marginal cost. Hence, the equilibrium is always efficient.

7 Conclusions

In this paper, I studied a version of the search-theoretic model of imperfect competition in product markets of Burdett and Judd (1983) where sellers face a strictly increasing rather than a constant marginal cost of production. I showed that the equilibrium exists and is unique, and its structure depends on the extent of search frictions. When search frictions are large enough, the price distribution F is non-degenerate and atomless, and its support is some interval $[p_\ell, p_h]$, with $p_h = u$. When search frictions are neither too large nor too small, the price distribution F is a mass point at the lowest price p_0 and a density over some interval $[p_\ell, p_h]$, where $p_\ell > p_0$, $p_h = u$ and p_0 is equal to the marginal cost for a seller posting p_0. When search frictions are small enough, the price distribution F is degenerate at p_0, where p_0 equals the marginal cost for a seller posting p_0. Hence, unless search frictions are sufficiently large, the structure of equilibrium is different than in Burdett and Judd (1983). I also showed that the equilibrium is efficient if and only if the price distribution F is degenerate. Hence, unless search frictions are sufficiently small, the equilibrium is inefficient, in contrast with Burdett and Judd (1983). However, just as in Burdett and Judd (1983), the equilibrium becomes more competitive as search frictions become smaller and reaches the competitive outcome as search frictions vanish.

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