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SOCIAL SECURITY AND THE DETERMINANTS OF FULL AND PARTIAL RETIREMENT: A COMPETING RISKS ANALYSIS

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ABSTRACT

Empirical analyses of retirement typically assume a single form of retirement. In this paper, I consider the determinants of retirement in a competing risks model which allows for full and partial retirement. Simulation results indicate that the large increase in Social Security benefits in the early 1970s has had moderate effects upon retirement, increasing the probability of early full retirement (before age 65) by less than 5 percent and reducing the probability of partial retirement by 1–2 percent.

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1 Introduction

Current demographic trends suggest that the elderly will assume an increasingly important role in the economy. Census Administration projections indicate that by the year 2030, upwards of 32 percent of the U.S. population will be over the age of 55, versus 21 percent in 1985. More significantly, the estimates suggest that the percentage who are very old will increase dramatically, with the over 70 age-group comprising 15 percent of the population and with 6 percent of the population over 80 years of age.[1]

Accompanying the changes in the age-composition of the workforce will be important changes in the institutional environment and incentives faced by the elderly. The elimination of mandatory retirement and alteration of Social Security benefit adjustments for continued work beyond age 65 suggests that future cohorts of aged workers will exhibit patterns of work and retirement that differ from those of the current generation.[2] Accordingly, it will be ever more important for economists to gain an understanding of the factors which influence the work and retirement decisions of older workers.

Central to an analysis of retirement decisions is an examination of the effect that the incentives associated with the Social Security system have upon behavior.

[1]These figures are up from 8 and 2.5 percent respectively. U.S. Bureau of the Census, *Current Population Reports (Series P-25, no. 952)*, Washington D.C.—U.S. Government Printing Office, 1984, Table 6.

[2]The 1983 Social Security Amendments provide for the Normal Age of Retirement, the age at which full benefits can be received, to increase gradually to 67 by the year 2027 beginning in 2003.

It is well known that Social Security affects individual intertemporal budget constraints via a combination of taxes and transfers. Despite the complexity of the task, researchers have been able to trace the impact of a variety of Social Security rules upon the budget constraints of individuals.[3] Unfortunately, the labor supply effects of changes in budget constraints are less well understood. Even with considerable academic interest in the issue, important unanswered questions remain regarding the relationship between Social Security and retirement. Characteristic of the debate is a wide range of results, from Boskin's [1977] claim that the annual probability of retirement has been increased by 40 percent, to Gordon and Blinder [1980] who find that Social Security has negligible effects upon retirement. More recent studies by Fields and Mitchell [1983], Boskin and Hurd [1984], Diamond and Hausman [1984, 1985], Hausman and Wise [1985], and Burtless [1985] conclude that the effects lie somewhere between these two extremes.[4]

This paper addresses the interrelated questions of what factors influence the decision to retire and form of initial retirement. I extend the previous analysis of retirement by considering a hazard model of employment duration in which partial retirement is treated distinctly from full retirement. The majority of studies have relied on a single measure of retirement. This has the effect of confounding the differences in underlying behavior and leads to misleading inferences about the

[3]See Blinder, Gordon and Wise [1980] and Aaron [1984] for comprehensive discussions of the issues.

[4]Aaron [1984] expresses the pessimistic view by noting that "...about all that can be said is that the preponderance of studies, whose evidentiary value is quite low, concludes that Social Security has an indeterminate impact upon retirement."

effects of various factors upon retirement.[5]

Using recently developed techniques for the analysis of competing risks duration models (Han and Hausman [1986], Sueyoshi [1988]), which allow for general hazard shapes and error structures, I find that partial retirement differs substantively from full retirement. These results extend those of Diamond and Hausman [1984, 1985] and Hausman and Wise [1985] to the treatment of partial retirement, and may be contrasted with the structural results of Gustman and Steinmeier [1987].[6]

Simulation results indicate that the early 1970s growth in the Social Security system, which has been targeted by some as the source of dramatic reductions in labor force participation, accounts for a portion of the observed decline, but cannot be viewed as the primary factor. Early full retirement probabilities have increased less than 5 percent, while partial retirement probabilities have fallen by 1 percent. I conclude that rule changes have encouraged full retirement slightly at the expense of some partial retirement, but that the overall responses have been moderate.

The remainder of this paper is organized as follows. In Section 2, I examine the empirical relevance of partial retirement. An econometric specification based upon a competing risks model of retirement is outlined in Section 3 and the Retirement History Survey data are described in Section 4. Section 5 contains the

[5]Notable exceptions include Gustman and Steinmeier [1986], Zabalza, *et al.* [1980] and Boskin [1977]. In addition, Burtless and Moffitt [1984] consider the joint determination of retirement age and hours of "post-retirement" work.

[6]More recent attempts to include partial retirement in dynamic programming models of retirement may be found in Rust [1987] and Berkovec and Stern [1988].

estimates derived from duration models of retirement. In addition to presenting the parameter estimates, I discuss the results of simulations designed to analyze the effects of the changes in Social Security upon retirement. There is a concluding section.

2 Partial Retirement

There is no natural way to define retirement, consequently a variety of definitions have been used in the empirical literature. Among them are complete withdrawal from the labor force (Gordon and Blinder [1980]), transition from job held at the start of the sample (Fields and Mitchell [1983]), self-reported status (Hausman and Wise [1985]), receipt of pension income (Burkhauser [1985]), and work less than a specified number of hours (Boskin [1977]).

Underlying the difficulty in choosing a single measure of retirement is the fact that a dichotomous measurement of retirement is unable to capture the behavior of individuals who are not employed in a "standard" full-time job, but are still in the labor force. Depending upon the classification system chosen, different forms of mismeasurement of the extent of labor force participation are likely to contaminate attempts to analyze retirement decisions.

Research by Gustman and Steinmeier [1984, 1986] has shed some light upon the extent and the importance of partial retirement. Using a sample drawn from the Longitudinal Retirement History Survey (RHS) they find that 28.2 percent of individuals who are observed exiting from full-time employment partially retire.[7] Over one-third of the individuals report partial retirement in at least one the four sample periods. These results are consistent with those of Zabalza, *et al.* [1980] who report that their sample of elderly in Great Britain exhibits a well-defined bimodal distribution of hours worked, and by Parnes and Nestel [1981] who find that about 20 percent of their 1966-1975 National Longitudinal Survey sample reports post-retirement labor market activity.[8]

Drawing upon data derived from the Retirement History Survey (RHS), I find figures on the extent of partial retirement that are similar to those of Gustman and Steinmeier, despite the use of a definition based upon labor market activity rather than self-reported status.[9] Partial retirement is defined as employment at a job in which the individual reports less than 35 hours of work per week, or employment for less than 46 weeks.[10] Table 1 shows the number of individuals exiting from full employment at various ages, aggregated across cohorts, broken down by type

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[7]In these studies, both partial and full retirement are self-reported.

[8]In the latter study, retirement is defined by self-reported status. Among those classified as retired who reported having worked in the 12 month period prior to the 1976 survey, approximately 17 percent reported working more than 2,000 hours. Assuming work over the entire year, this translates into about 38.5 hours per week. 23 percent of the sample reported working more than 29 hours per week, 42 percent more than 19 hours per week, and 71 more than 10 hours per week.

[9]The RHS is a ten-year biennial longitudinal panel study of appoximately 11,000 elderly individuals aged 58–63 in 1969, who were interviewed over the period from 1969 to 1979. The numbers reported here are calculated from a subset of the original data.

[10]46 weeks is the "weeks-equivalent" of a full year, 35 hour, work week. Weeks of unemployment are counted as employment if the individual reports looking for work. It is possible that individuals will report vacation time as weeks of no work so that the weeks definition will overstate partial retirement. This does not appear to be a problem as almost all individuals who are classified as partially retired under the weeks definition are also partially retired by an hours criterion.

of retirement for a sample drawn from the RHS.[11] Out of the 1633 individuals in the sample, over 500 exit from full employment via partial retirement; a little over a third of the 1490 individuals who retire over the sample period partially retire.

The behavior of partially retired individuals differs substantively from that of individuals who retire fully. In table 2 I present data on the mean age of retirement for the various forms of exit. The mean age of full retirement is 64.75 or 66.19 depending upon whether one defines retirement as exit from employment or as complete withdrawal from the labor force. The mean ages of initial retirement are, however, quite close—the mean of 64.68 for partial retirees is only slightly less than the corresponding age of 64.79 for full retirees.

The closeness of the latter values hides considerable variation in work activity. For those individuals who partially retire and are subsequently observed to retire fully, the mean duration of partial retirement is approximately 5.5 years.[12] As one might expect, the durations are negatively correlated (-0.446) with the age of initial retirement so that individuals who retire earlier remain in partial retirement for a longer period of time.

There are also large differences in hours worked. Stratifying the data at 35

[11]The censoring category refers to those individuals who do not retire over the sample interval. The age at censoring is the last age at which they were observed.

[12]The average duration is in excess of the durations studied by Gustman and Steinmeier. They find that most of those who partially retire spend a relatively short period of time, on average three years, in that state. The discrepancy probably results from the use of different definitions of full and partial retirement. I use a definition of full retirement based upon complete withdrawal from the labor force while Gustman and Steinmeier use self-reported retirement. The latter definition would understate work relative to the former if individuals do not differentiate between full and partial retirement in answering the questions on self-reported status and report their status as "retired".

hours per week yields mean hours of work of 20.31 hours per week for those working less than 35 hours and a mean of 43.98 for those working greater than that amount. Conditional on working less than 35 hours per week, the median effort is 20 hours per week and a quarter of the sample work less than 14 hours a week. Modal hours of work for this group is 20 hours per week.[13] These results are consistent with other data sources which find considerable half-time work (Zabalza, *et al.* [1980]).

The combination of partial retirement lasting, on average, for several years and involving a substantial reduction in normal hours of work implies that misclassifying partial retirement as either retirement or full-time work will mask considerable variation in labor force activity. To the extent that this variation is related to the exogenous variables of interest, ignoring partial retirement will lead to incorrect inferences about the relationship between the variables and retirement.

Hazard rates for both forms of retirement are presented in Figure 1. The hazards represent the probability of retiring at a given age, conditional on not having retired prior to that age. The hazards are calculated by dividing the number of retirees of a given type (full or partial) at a given age by the number of unretired and uncensored individuals up to that age.[14] Up to age 64, the hazards are simi-

[13]The data were pooled for these calculations so that there are multiple observations for individuals. The sample sizes were 1,148 for hours less than 35 and 2,719 for work in excess of 35 hours per week.

[14]It is easy to show that this is the Kaplan-Meier [1958] maximum likelihood estimator of the hazard rate for a homogeneous sample and continuously observed failure times. A standard life-table estimator for the same hazard rates would adjust the risk set at each age to account for the fact that durations are observed in discrete intervals. For further details, see Kalbfleisch and Prentice [1984].

lar in shape and magnitude. The major differences between the two risks become most apparent at the ages surrounding 65, with the pronounced spike in the hazard for the full retirement risk at age 65 absent in the partial retirement hazard. The partial retirement risk does have spikes (in particular at ages 64 and 67), but the overall risk appears quite constant. The evidence is consistent with the hypothesis that the underlying risks for the two forms of retirement are different and motivates the econometric specifications outlined below. In particular, more sophisticated statistical techniques and greater structure are required to estimate hazard rates in the presence of individual heterogeneity.

3 Econometric Specification

The econometric techniques employed in this study are derived from the extensive literature on duration models. These models are also referred to as hazard or failure time models. Standard references in the statistical literature include Cox and Oakes [1984] and Kalbfleisch and Prentice [1980]. A recent survey of the use of these models in economics may be found in Kiefer [1988]. Duration models have enjoyed considerable popularity in empirical work since Lancaster [1979] introduced them to the economics profession in his study of unemployment durations. Recent studies by Diamond and Hausman [1984] and Hausman and Wise [1985] have applied these techniques to the study of retirement behavior.

While most of the existing duration studies of retirement assume a single form of retirement, it is not difficult to extend the analysis to treat situations involving competing risks. In addition, this paper uses recently developed techniques (Han and Hausman [1988]) for the estimation of competing risks models with relatively few parametric restrictions on the form of the hazard shapes. These techniques also allow for correlation between risks. The estimator is outlined in the remainder of this section and in Appendices A and B.

The competing risks model can be cast in terms of a familiar latent variables specification. Suppose that there are random variables for K potential failure times $T_j, j = 1, 2, \ldots, K$, where only the first realized failure time is observed so that $T = \min_j\{T_1, T_2, \ldots, T_K\}$. In the current context, one can think of having two failure types: full retirement and partial retirement.

The idea underlying the hazard approach is that the joint cumulative distribution function of the T_j, the density function of the T_j, or the conditional probability function of T_j given that the failure has not occurred previously, provide the same information concerning the probabilities associated with the failure times. Rather than specifying and working directly with the joint probability distribution of the durations, it is convenient to work with the hazard rates.

Following Kalbfleisch and Prentice, define the cause-specific hazard functions associated with the durations as

$$\lambda_j(t) = \lim_{\Delta \downarrow 0} \frac{\Pr(t \le T_j \le t + \Delta | T_k \ge t, k = 1, \ldots, K)}{\Delta} \tag{1}$$

$j = 1, \ldots, K$. I assume that the cause-specific hazard rates at time period t for cause j may be specified as

$$\lambda_j(t, X_j, \beta_j, \nu_j) \;=\; \nu_j \lambda_j(t, X_j, \beta_j) \tag{2}$$

$$= \nu_j \lambda_{jo}(t) \phi_j(X_j, \beta_j) \tag{3}$$

where for each risk j, X_j denotes observed characteristics, ν_j is a non-negative error term representing unobserved characteristics, $\phi_j : \Re^k \rightarrow \Re^+$ is a continuously differentiable, non-negative, aggregator function, β_j is a parameter vector, and where the non-negative function for the baseline hazard λ_{jo}, specifies the form of the duration dependence. It is convenient to define the integrated baseline hazards associated with a given duration t as

$$z_j(t) = \int_0^t \lambda_{jo}(s) ds. \tag{4}$$

Ruling out simultaneous failures, the overall hazard is simply $\sum_j \lambda_j$.

Under the assumption of independence of the T_j, the likelihood associated with this model factors into separate components for each risk where failures of an alternative type are treated as censored observations (Kalbfleisch and Prentice [1984]). This model may also be written as

$$\alpha_t^j = -\log \phi_j(X_j, \beta_j) + \epsilon_j + \tilde{\nu}_j \tag{5}$$

for $j = 1, \dots, K$, where the ϵ_j are independent extreme value errors, $\tilde{\nu}_j = \log \nu_j$ and the ν_j follow an arbitrary non-negative distribution and $\alpha_t^j = \log z_j(t)$.[15] More generally, the specification of the joint-distribution of the $\tilde{\nu}_j$ can be used to create dependence between the underlying T_j. Alternatively, one could specify directly the joint distribution of the T_j. Diamond and Hausman [1985] allow for dependence between durations by specifying a model with log bivariate-normal

[15]See appendix A.

durations. The current method has the advantage of relating the dependence between the risks to the observed characteristics through the parameters, and to the unobserved characteristics through the distribution of the error term. While the aggregator functions ϕ_j may take quite general forms, I specify the $\phi_j = \exp(X_j\beta_j)$ so that the logarithm in (5) simplifies to $X_j\beta_j$.

To estimate this model without parametric restriction upon the shapes of the baseline hazards, I take advantage of the fact that retirement dates are observed on a yearly basis. With discrete data on durations, the model given in (5), is simply a latent variables model with discrete censoring, where the probabilities associated with various durations can be specified in terms of the error distribution. For example, the probability that the latent duration for risk j lies in the interval $(t-1, t]$ is simply the probability that the error term $\epsilon_j + \tilde{\nu}_j$ lies in the interval $(\alpha_{t-1}^j + X_j\beta_j, \alpha_t^j + X_j\beta_j)$. The likelihood is specified so that the integrated hazard terms α_t^j are treated as unknown parameters and estimated jointly with the β_j. The baseline hazards $\lambda_{jo}(t)$ are estimated in piecewise fashion. The contributions to the likelihood are slightly more complicated than the probabilities associated with the latent durations since they are based upon the observed durations. See Appendix B for the derivation of the actual likelihood contributions and further details about the specification.

This specification eliminates the strong, non-testable functional form restrictions placed upon the shapes of the baseline hazards in both the Diamond-Hausman and Hausman-Wise studies. The primary advantages of this hazard approach are that it allows for a flexible functional specification of the probability that individuals retire at a given period while incorporating the information that they did not retire previously, as well as allowing for correlation between the unobserved components.[16]

4 Data

The data are derived from the Social Security Administration's Longitudinal Retirement History Survey (RHS). The RHS is a ten-year biennial longitudinal panel study of approximately 11,000 elderly individuals aged 58-63 in 1969, who were interviewed over the period from 1969 to 1979. The RHS is particularly attractive for the study of retirement and Social Security because of the matching of individual Social Security records to the core panel. Moreover, the period from 1969 to 1979 was one of large exogenous increases in Social Security benefits, providing considerable time-series variation on a key explanatory variable.[17]

The original extract consisted of 4,016 male heads of households from the Retirement History Survey.[18] Excluded were male, non-farm workers in private, but not self-employment for whom there were insufficient data to allow for observa-

[16]The proportionality assumption (the multiplicative form of (2), restricts the effects of explanatory variables on the existing hazards to be multiplicative. This is a strong restriction and can, with considerable effort, be relaxed somewhat (Sueyoshi [1989]). From the regression form of the likelihood, it should be clear that the assumption of proportionality is, in essence, a regression assumption.

[17]Minimum Primary Insurance benefits increased by 19.4 percent from 1970 to 1975, and by 26.8 over the period from 1970–1975. The previous decade's growth was less than half that amount, around 9.2 percent (Social Security Administration, *Annual Statistical Supplement* 1981). Burtless [1984] discusses the unanticipated nature of the increases in greater depth.

[18]The characteristics of this extract are described more fully in Hausman and Paquette [1987].

tion over at least one of the five RHS waves from 1971-1979. The first (1969) wave of the RHS was not merged with the remaining five waves because of problems with comparability of data across succeeding surveys. A subsample of 1,633 observations was used to derive the results reported below.[19]

Since the nature of work effort is central to the analysis, it is important to distinguish carefully between full and partial retirement. I use an hours/weeks based definition of the extent of work. Individuals who are reported as fully or partially retiring in a given sample year are traced back through the previous sample period to see whether they have held an intervening partial retirement job, with the type of retirement and age at retirement adjusted accordingly. Reported hire and termination dates associated with the previous job and job held during the last RHS survey are used to piece together the employment history during the period between survey dates. The age at initial retirement is then adjusted so that a reported age of 65 indicates that the individual retired between age 64 and 65. This adjustment is merely a normalization.

The variables used in the estimation include both heterogeneity controls and economic variables. The control variables include the number of persons in the house, number of years of education, and indicators for poor health, spouse

[19] 697 individuals were deleted from the sample because the data did not support the determination of industrial and occupational classifications. And additional 384 were lost because of insufficient data to allow imputation of asset holdings for any of the five waves. The remaining individuals were deleted because I was unable to follow their employment status for at least one period or determine their date of retirement, or because of poor data, over the entire sample period. For example, it was impossible to use existing earnings records or to impute them for 138 individuals. Further work is required to examine the possible biases resulting from this sample selection procedure and to increase the sample sizes used in the estimation.

present, non-white, and the presence of mandatory retirement provisions on the job. I also define a set of indicators for occupation. Most of these are self-explanatory. The economic variables are related to the income and compensation streams that individuals receive, and include earnings, wealth, pension eligibility, and Social Security benefits.

The indicator for poor health is based upon a self-reported measure.[20] Previous results, in particular those of Diamond-Hausman and Hausman-Wise, have suggested that health is one of the primary determinants of retirement age. Poor health is expected to change preferences for leisure and to create exogenous changes in the wage rate, and should have a positive impact upon retirement probabilities. Most importantly, it is expected that health status will have a different impact upon the probabilities of full and partial retirement. Individuals with health limitations may be more likely to retire fully than partially, given that they should face a poorer distribution of partial retirement wage-leisure offers.[21]

The economic variables follow closely the variables used in existing studies. Earnings are included as a measure of the opportunity cost of retiring. For individuals with adequate information, actual earnings from the basic RHS survey were used, otherwise, earnings were imputed on the basis of the Social Security records, adjusted for truncation using the Fox [1981] imputation method.[22] These

[20]The response is based upon the answer to the question, "Is your health worse than others your age?"

[21]Surprisingly, Gustman and Steinmeier [1986] report that poor health has relatively little impact upon partial retirement wages. In their model of jointly determined retirement age and hours of post-retirement work, Burtless and Moffitt [1984] include health status in their retirement equation, but not in the hours of work equation.

[22]The Fox method uses information on the calendar quarter in which the truncation point

values are net of payroll taxes, but not of income taxes because of difficulties in measuring non-labor income. Theory suggests that earnings will reduce the probability of retiring since they raise the implicit cost of leisure. The assets data are based upon estimates of the non-housing wealth held by individuals.[23]

Private pensions enter via indicators for eligibility. Individuals are defined as eligible for a full or partial pension if they report for at least one wave that they expect to receive a full or reduced pension, respectively, when they retire. Ideally, expected pension income would be considered, but the RHS data do not appear rich enough to support the computation of pension benefits for those individuals where pension income is not observed. Both indicators are expected to increase the probability of retirement.

Social Security benefits entered in two ways. First, a measure of the real benefits that the individual would receive if he retired at age 62 is calculated. In computing these benefits, the rules for the given year and given cohort are applied to the matched individual earnings records for the years 1951-1974. Thus, benefits levels are calculated on the basis of individaul's earnings history. Since the rules applicable to the actual year of potential retirement are used, the substantial time-series variation in Social Security rules is taken into account.[24] If

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was reached to adjust upward the earnings data. Out of sample earnings were imputed using estimated age earnings profiles. There may be sample selection biases associated with this procedure.

[23]A more detailed discussion of the construction of the assets variables may be found in Hausman and Paquette [1987].

[24]Benefits are ultimately related to the individual's work history, so that they may be related to retirement indirectly through preferences for leisure rather than directly through their effects on work incentives. The hope is that the progressive, unanticipated increases in benefits during

appropriate, separate calculations are made on the basis of the wife's earnings record. I then calculate benefit levels using the appropriate rules for one or two-earner households. The incentives created by automatic benefit recomputation (see Blinder, Gordon and Wise [1980]) and the increases in the generosity of the system are approximated by calculating real benefit levels were the individual to retire at age 65, and then looking at the difference between the two levels. The corresponding "delta" variables are then entered into the specification along with the age 62 benefit levels.

Variable means and standard deviations are presented in Table 3 for the subsample used in the estimation.

5 Empirical Results

I consider results from estimation of a general hazard model which accounts for the presence of partial retirement and which does not place restrictions upon the shapes of the hazard functions. The models is estimated for both the independent and the correlated risk cases. It is important to keep in mind that the specification implies that the individual hazard rates depend upon the underlying baseline hazards as well as explanatory variables. Thus, inferences about the effects of variables upon retirement will be affected by the shapes of baseline hazards as well as by differences in the coefficients.

Parameter estimates for the independent risk specification are presented in

the 1970s provide sufficient exogenous variation to identify to Social Security effect.

the first two columns of Table 4. The error term in the ordered likelihood was assumed to follow the extreme value distribution. This corresponds to a model with proportional hazards and no heterogeneity. There is considerable variation in the parameters across risks, with the single risk estimates appearing to be a combination of the competing risks estimates. The parameters for a single form of retirement model (not presented here) lie, almost uniformly, in the interval bounded by the competing risk estimates. A Lagrange Multiplier (LM) test for equality of coefficients across risks in the independent model yields a test statistic value of 655 (df 20), strongly rejecting the null at conventional significance levels. These estimates suggest that treating both full and partial retirement as a single form of retirement convolutes the separate influences of exogenous factors upon the retirement decision.[25]

For example, additional Social Security benefits provide significant, positive incentives for full retirement but not for partial retirement. Conversely, the Social Security delta reduces the conditional probabilities of partial retirement, but has little effect on the full retirement hazard. Individuals who are better able to adjust their behavior marginally, in the form of partial retirement, appear to be more responsive to the incentive effects of Social Security. Thus, changes in the Social Security system appear to affect full retirement probabilities primarily through

[25]For completeness, I also test a parametric Weibull specification (Lancaster [1979]) against the single risk model. The baseline hazard is specified as a two-parameter Weibull hazard $\lambda_0(\alpha,\gamma) = \alpha t^{\gamma-1}$. The test statistic value of 155 (df 14) is well in excess of standard critical values. Moreover, the restricted estimated hazard seriously misses the shape of the baseline hazard, predicting a monotonically increasing hazard in contract to the quadratic shape predicted by the unrestricted model.

benefit levels while altering partial retirement propensities through changes in the delta variable.

The finding that there is variation in the impact of Social Security across risks is quite important. One implication is that policies designed to increase benefit levels and increase marginal incentives through higher deltas will have the effect of leading individuals to fully retire who might otherwise partially retire. Reduced labor force activity will come, not only from those who retire earlier, but also from those who retire fully rather than partially retire.

There are also significant differences across risks in the parameters for economic variables. I find a small, but statistically significant, negative effect of earnings upon partial retirement and little effect upon full retirement. This result is a little surprising, since earnings are expected to have a negative impact on both forms of retirement. Nevertheless, this finding is consistent with the results for Social Security which indicate that partial retirement is more sensitive to benefits. One other interpretation of this result is that individuals with high current earnings are more likely to be subject to the earnings test in a partial retirement job so that the high implicit marginal tax rates discourage that form of retirement. These implicit marginal tax rates are not imposed on individuals who exit the labor force. Wealth has a small and statistically significant positive impact upon partial retirement probabilities. The impact of additional assets upon full retirement is unclear, as is the reason for the observed differences across risks. If anything, the impact of asset holdings upon full retirement probabilities should be greater since, conditional upon a level of Social Security benefits, individuals will be relying more heavily on their asset income if they are out of the labor force.

Poor health status increases significantly the probability of full retirement but not the probability of partial retirement. Similarly, being non-white reduces substantially the partial retirement hazard, but not the full retirement hazard. Both sets of parameters are consistent with a model where individuals receive partial retirement job offers from a distribution which depends upon individual characteristics. Poor health should increase the desire for leisure leading to increased retirement risk. The individual should, however, face poorer job opportunities because of ill health, mitigating the desire for more leisure. This difference in the impact of health across retirement risks is not captured in structural models which assume that an individual has a single partial retirement wage offer independent of health status.

The variation across risks in the parameters for the occupation variables are also consistent with the opportunity interpretation of partial retirement decisions. The larger impact that being a laborer has upon full retirement relative to partial retirement accords with the notion that laborers are likely to have relative few acceptable reduced work opportunities; the opposite is true for craft individuals. Again, the single-risk measure of the occupation effect masks variation across the different forms of retirement.[26]

The parameter estimates for the correlated competing risks model in Table 4 point to the importance of allowing for a general error structure. The unrestricted

[26]The job opportunity interpretation is consistent with the observation that most partial retirement is accompanied by a job change.

error model allows for precise estimates of the effects of Social Security upon retirement. The parameter estimates for benefits and the delta have the expected sign and are statistically significant. While the basic character of the results remains—additional benefits have a greater positive impact upon full retirement probabilities than upon partial retirement and increases in the delta variables reduce partial retirement hazards by more than full retirement hazards, the independent competing risk model seriously overstates the differences between the two forms of retirement relative to the correlated risk model. Allowing for correlation in the error terms significantly reduces the magnitude of the differences. Wald tests for non-zero differences between coefficients for the two risks fail to reject the null that the coefficents are equal at the 5% significance level. The same is true for separate tests of the β. The coefficients for health, Social Security benefits and the the delta variable generate the smallest p-values of .136, .128 and .110 respectively. The corresponding tests for the hazard shapes also fail to reject equality. However, a LM test for equality of the both the coefficents and the hazards across risks in the correlated model yields a test statistic value of 596.8 (df 21), strongly rejecting the null hypothesis at conventional significance levels.

To calibrate the importance of the differences in the estimated hazard rates associated with the various models, I consider a simple simulation of the impact of a large change in the Social Security system. Focusing on the 381 individuals in the youngest RHS cohort, I calculate the predicted survivor functions for each individual given his observed characteristics. In particular, Social Security benefits are calculated under the rules in effect for that cohort in each year. For comparison, I then calculate the average survivor functions under the relevant Social Security law for 1969. The simulation yields an approximate measure of the extent of labor force participation were the system to have remained unchanged over the 1970's. The difference between the estimates of the average survivor function and the corresponding hazards provides a rough approximation of the impact of the changes in Social Security during that decade.

Differences in the survivor functions are calculated for a single risk model as well as the independent and the correlated competing risks models. The simulated survivors are depicted in Figure 2.

The lower portion of the Figure 2 shows that assuming that the error terms are independent, the survivor function for full retirement is substantially lower under current law. For example, the average full retirement survivor function is 5 percent lower at age 64. The difference between the survivor function under the two regimes peaks at nearly 10 percentage points at age 67. The upper portion of Figure 2 shows the corresponding calculations for partial retirement. Individuals have a higher partial retirement survivor function under current law. The difference ranges from 1 percent at age 59 to 9 percent at age 67. While it is difficult to generalize these results without taking into account the latent nature of the failure times, the simulation indicates that the change in Social Security law in the 1970s has increased the probability of early full retirement (before age 65) by around 10 percent, but reduced the corresponding partial retirement probability by a like amount.

For the correlated model, the effects are more modest. The lower portion of the figure shows that the change in Social Security rules has reduced the full retirement survivor function by 5-6 percent at ages 63-64. These reductions are considerably smaller than the 10 percent changes predicted by the independent model. It is interesting to note that the shape of the changes in the survivor functions follows closely the shape for a single risk model. In contrast to the independent error model, the reduction in the overall survivor peaks at 65, and becomes negligible in later years. The corresponding increase in the partial retirement survivor function, on the order of 1 to 2 percentage points, are considerably smaller than those predicted by the independent model.

The larger predicted effects under the assumption of independent risks appear to result directly from the lack of accounting for correlation in the error distribution. The differences in the survivor functions can be attributed to the fact that the independent model treats failures of alternative types as censored observations when calculating the risk for a given failure. In contrast, the correlated model uses the information about the correlation to more precisely calculate the probabilities of survival, in essence using the information that partial retirement is more like full retirement than it is like censoring. One interpretation of this finding is that the correlation between risks is related to unobserved preferences for leisure. Individuals who prefer less work are more likely to reduce work effort partial retirement, and are also more likely to fully retire. Failure to account for the correlation mixes the influences of the unobserved and observed components affecting retirement. This interpretation is consistent with the view that the primary factor influencing the decline in elderly labor force participation has been a secular change in preferences for leisure on the part of the elderly.[27]

The simulation results are summarized in Table 5 where the percentage change in early retirement is presented at various ages for both single and dual-risk models.[28] The predicted responses of full retirement to changes in Social Security falls across specifications, first as the Weibull specification is relaxed, and then as partial retirement and correlated errors are introduced. Similarly, the impact of the reforms upon partial retirement is lower in the correlated error model than in the independent model. It is appararent that relaxing restrictions upon the statistical models reduces the apparent influence of Social Security upon retirement.

6 Conclusion

This paper presents competing risk models of retirement which allow for differences between full and partial retirement, general hazard shapes and error structures. Single risk models models predict larger changes in retirement behavior from Social Security reform than do the competing risk models. In turn, these effects are larger than those found upon adding correlated errors to the competing risks specification.

Results from a simulation of the changes in Social Security over the 1970s suggest that the rapid growth in benefits has had only modest effects upon condi-

[27]This interpretation is not however, testable within the current framework.

[28]The single risk models are based upon both a parametric Weibull specification and an unrestricted baseline hazard model with extreme value errors.

tional retirement probabilities. Early full retirement probabilities have increased by less than 6 percent while partial retirement probabilities have fallen by around 1 percent. Thus, the dramatic decline in labor force participation among the elderly in recent decades is not fully explained by the Social Security increases and remains somewhat of a mystery.

The results presented here can be extended in a number of ways. For one, alternative data sources might allow for better an investigation of the interaction between private and public pension plans.

Perhaps most importantly, better accounting for the time-varying nature of Social Security benefits upon decisions may better detect the responses to changes in retirement incentives. Recent extensions of the Han and Hausman model to allow variables to change over time (Sueyoshi [1989]) should make it possible for a more detailed examination of retirement hazards.

Table 1: Age distribution of retirement or censoring by type of initial retirement, 1633 person RHS subsample.

Retirement Age	Type of Initial Retirement (Number of Persons)			
	Full	Partial	Censored	Total
58	4	1	0	5
59	6	0	0	6
60	13	11	7	24
61	27	14	6	41
62	86	61	7	147
63	124	56	7	180
64	117	130	4	247
65	275	75	8	350
66	173	65	1	238
67	67	52	1	119
68	36	20	29	56
69	19	16	23	35
70	14	5	16	19
71	5	6	14	11
72	7	1	12	8
73	2	2	8	4
Totals	975	515	143	1633

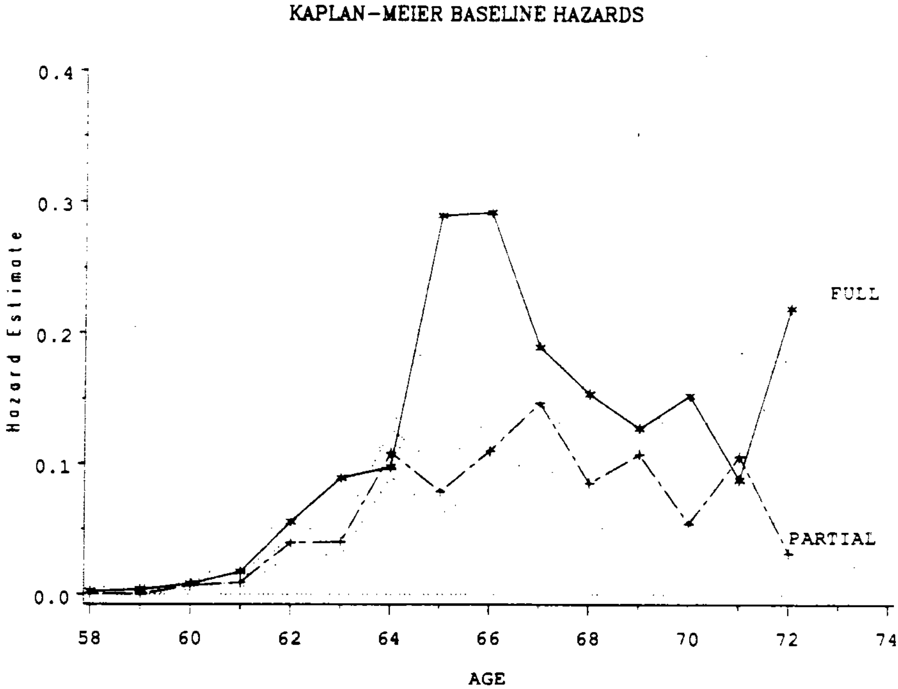
Note: Based upon author's calculations. The various forms of retirement are defined in the text.

Table 2: Mean age of retirement under different retirement paths, RHS subsample.

Type of Retirement	Mean	Std. Error	N
Initial Retirement	64.752	2.201	1490
Full	64.791	2.169	975
Partial	64.678	2.260	515
Final Retirement	66.186	3.151	1347
Initial Full	64.791	2.169	975
Initial Partial	69.841	2.269	372
Duration of Partial Retirement	5.500	2.037	372

Note: Based upon author's calculations. All samples are limited to completed spells for the retir ment in question. The partial retirement category for mean age of final retirement refers to those individuals observed to fully retire who first partially retired.

Figure 1: Kaplan-Meier Hazard Rates for Full and Partial Retirement



Note: The dashed lines represent ±2 standard error bars.

Table 3: Variable definitions and descriptive statistics, 1633 observation RHS subsample.

Variable	Mean	Std. Deviation
Control Variables		
education (years)	10.805	3.447
health (poor health)	0.319	0.466
married	0.890	0.313
non-white	0.097	0.296
household size	2.532	1.240
mandatory retire	0.372	0.484
Occupation Indicators		
clerical (service, sales)	0.219	0.413
craft	0.247	0.431
labor	0.292	0.455
manager	0.129	0.335
professional (technical)	0.067	0.251
Economic Variables		
earnings (x $1,000)	6.926	4.800
wealth (non-housing x $10,000)	1.025	3.194
full pension	0.141	0.348
part pension	0.160	0.367
ss62 (monthly x $100)	2.295	0.644
ss-delta (x $10)	2.004	2.627

Note: Based upon author's calculations. Amounts are expressed in real 1967 dollars. The indicator variables take the value 1 if true, 0 otherwise.

Table 4: Parameter estimates for full and partial retirement models, with correlated and independent unrestricted baseline hazards.

Variable	Independent (1)		Correlated (2)		(3)	
	Full	Partial	Full	Partial	Full	Partial
education	-0.020	-0.028	-0.034	-0.032	-0.019	-0.021
	(0.012)	(0.016)	(0.009)	(0.010)	(0.010)	(0.011)
health	0.474	-0.012	0.368	0.268	0.384	0.220
	(0.073)	(0.100)	(0.064)	(0.087)	(0.065)	(0.098)
married	-0.480	0.001	-0.442	-0.337	-0.443	-0.300
	(0.134)	(0.178)	(0.100)	(0.113)	(0.100)	(0.117)
non-white	-0.012	-0.350	-0.089	-0.169	-0.087	-0.177
	(0.123)	(0.164)	(0.098)	(0.099)	(0.102)	(0.099)
household size	0.017	0.010	0.044	0.037	0.041	0.034
	(0.026)	(0.038)	(0.021)	(0.023)	(0.021)	(0.024)
mandatory retire	0.466	0.081	0.305	0.218	0.293	0.164
	(0.071)	(0.101)	(0.064)	(0.082)	(0.065)	(0.086)
clerical	0.073	-0.029	—	—	-0.022	-0.064
	(0.188)	(0.233)			(0.118)	(0.135)
craft	0.164	0.465	—	—	0.081	0.176
	(0.189)	(0.228)			(0.124)	(0.138)
labor	0.407	0.115	—	—	0.221	0.121
	(0.182)	(0.225)			(0.116)	(0.141)
manager	-0.097	-0.090	—	—	-0.218	-0.196
	(0.205)	(0.266)			(0.138)	(0.155)
professional	-0.208	0.236	—	—	-0.209	-0.038
	(0.234)	(0.277)			(0.159)	(0.174)

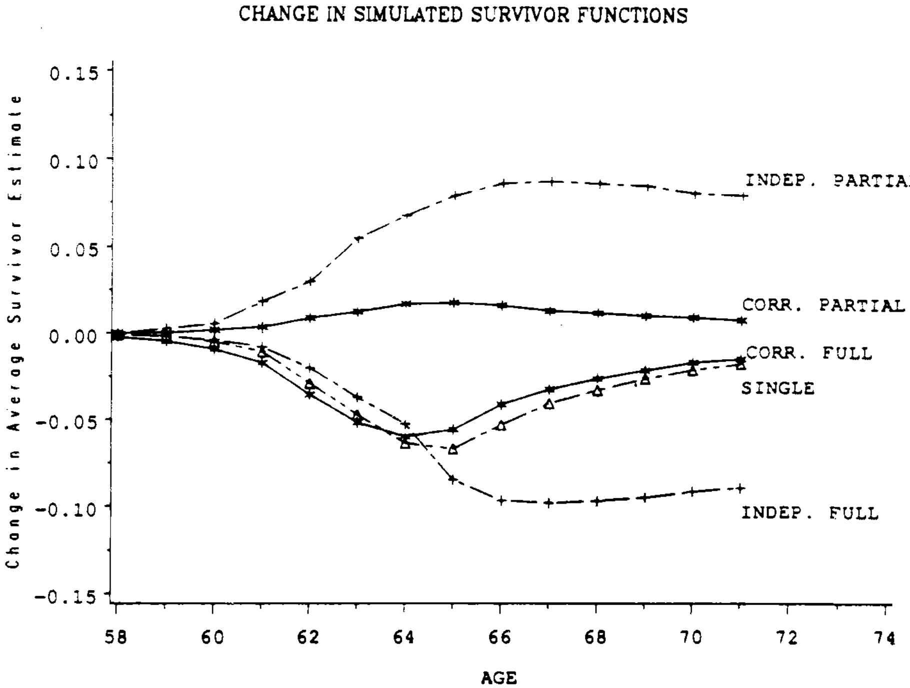
Note: Asymptotic standard errors in parentheses.

Table 4: (continued). Parameter estimates for full and partial retirement models, with correlated and independent unrestricted baseline hazards.

Variable	Independent (1)		Correlated (2)		Correlated (3)	
	Full	Partial	Full	Partial	Full	Partial
earnings	0.001	-0.028	-0.009	-0.014	-0.003	-0.012
	(0.010)	(0.014)	(0.007)	(0.007)	(0.008)	(0.007)
wealth	0.009	0.036	0.011	0.016	0.015	0.022
	(0.015)	(0.016)	(0.012)	(0.011)	(0.013)	(0.012)
full pension	0.111	-0.065	0.077	0.037	0.090	0.032
	(0.097)	(0.131)	(0.078)	(0.082)	(0.080)	(0.083)
part pension	-0.057	0.201	0.022	0.071	0.006	0.079
	(0.096)	(0.117)	(0.073)	(0.076)	(0.075)	(0.080)
ss62	0.587	0.084	0.411	0.310	0.411	0.257
	(0.077)	(0.100)	(0.065)	(0.078)	(0.067)	(0.085)
ss-delta	0.020	-0.100	-0.039	-0.061	-0.035	-0.071
	(0.015)	(0.021)	(0.012)	(0.016)	(0.013)	(0.017)
ρ	—		0.935		0.842	
			(0.119)		(0.197)	
Log Likelihood	-2616.14	-1799.50	-4341.15		-4319.47	

Note: Asymptotic standard errors in parentheses.

Figure 2: Change in Survivor Function Resulting From Change in Social Security Law: Various Models.



Note: The simulations are based upon average survivor functions calculated from the 381 individuals in the youngest RHS cohort.

Table 5: Estimated changes in survivor probabilities resulting from changes from 1969 Social Security law, various models and ages.

	Age 62	Age 65	Age 70
Change in Survivor Probabilities ×100			
Single Risk			
Weibull	-4.056	-7.636	-1.456
Unrestricted	-2.921	-6.708	-2.163
Dual Risk			
Independent			
Full	-2.035	-8.476	-9.153
Partial	2.973	7.842	7.984
Correlated			
Full	-3.603	-5.596	-1.726
Partial	0.863	1.722	0.861

Note: The simulations are based upon average survivor functions calculated from the 381 individuals in the youngest RHS cohort.

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A Hazard Model Transformation

The transformation of the probability density for the T_j to (5) is well-known in the statistics literature.[29] Recall that the cause-specific hazard functions are defined in (1) as

$$\lambda_j(t) = \lim_{\Delta \downarrow 0} \frac{\Pr(t \leq T_j \leq t + \Delta | T_k \geq t, k = 1, \dots, K)}{\Delta} \tag{6}$$

Ruling out ties, the overall hazard is given by $\lambda(t) = \sum_j \lambda_j(t)$ and the overall survivor function by

$$Q(t) = \Pr(T_k \geq t, k = 1, \dots, K). \tag{7}$$

Since the overall density, survivor, and hazard functions are related by $f(t)/Q(t) = \lambda(t)$ and since $-dQ(t) = f(t)\,dt$, solving the differential equation $-d \ln Q(t) = \lambda(t)\,dt$ yields

$$Q(t) = \exp(-\int_0^t \lambda(s)\,ds) \tag{8}$$

Equations (8), and the definition (1) imply that for $j = 1, \dots, K$,

$$f_j(t) = \lambda_j(t)\,Q(t). \tag{9}$$

Suppose that data of the form (t_i, δ_i, j_i), $i = 1, \dots, N$ are observed for the sample, where t_i is the i_{th} individual's observed failure time, δ_i is an indicator for censoring (δ_i takes the value 0 if the observation is censored, and 1 otherwise), j_i is the failure type. If censoring is independent and non-informative, the likelihood

[29] See for example, Kalbfleisch and Prentice [1984] who refer extensively to the "regression form" of the hazard model.

of the sample is given by

$$\prod_{i=1}^{N} \lambda_{j_i}(t_i)^{\delta_i}\, Q(t_i). \tag{10}$$

Using the definition of (8) and rearranging terms, the likelihood factors into separate components for each risk

$$\prod_{j=1}^{K}\prod_{i=1}^{N} \lambda_{j_i}(t_i)^{\delta_i}\, Q_j(t_i) \tag{11}$$

where $Q_j(t) = \exp(-\int_0^t \lambda_j(s)\,ds)$. For a given risk, the likelihood is equivalent to the likelihood generated by treating failures of alternative types as censored observations.[30]

The assumed independence of the hazards implies that we can treat each risk separately by specifying the appropriate $\lambda_j(t)Q_j(t)$ for each risk. The "regression-fo. ." of the model is derived via a change of variables from t to $u = \log(\int_0^t \lambda_j(s)\,ds)$. The Jacobian of the inverse transformation is given by

$$\left|\frac{du^{-1}}{dt}\right| = \frac{\int_0^t \lambda_j(s)\,ds}{\lambda_j(t)} = \frac{\exp u}{\lambda_j(t)}. \tag{12}$$

Then the likelihood terms for each risk are proportional to

$$\lambda_j(t)\exp(-\exp u)\frac{\exp u}{\lambda_j(t)} = \exp(u - \exp u) \tag{13}$$

which is the density function for an extreme value random variable.

Given a parameterization of $\lambda_j(t) = \lambda_{jo}(t)\,\psi(X_j, \beta_j)$, and recalling that $z_j(t) = \int_0^t \lambda_{jo}(s)\,ds$,

$$\log(\int_0^t \lambda_{jo}(s)\phi(X_j,\beta_j)\,ds) \;=\; u_j \tag{14}$$

[30]Note that although the form of $Q_j(t)$ is similar to $Q(t)$, it does not generally have a survivor interpretation.

$$\log z_j(t) \quad = \quad -\log \phi(X_j, \beta_j) + u_j \tag{15}$$

where the u_j are distributed as extreme value random variables.

B Likelihood Contributions

Given the regression form of the model, it is a straightforward task to specify the likelihood contributions associated with various observations. For simplicity, I will consider the case of bivariate competing risks ($k = 2$) although the analysis extends readily.

There are two basic types of observations on durations: censored failures and observed failure (of two types). If an observation is censored, all that is known is that neither failure has occurred. Suppose that the duration is censored at time t. Then the likelihood contribution is given by

$$L_t^3(\theta) = \int_{\alpha_t^1 + X_1\beta_1}^{\infty} \int_{\alpha_t^2 + X_2\beta_2}^{\infty} f(\epsilon_1, \epsilon_2)\, d\epsilon_1\, d\epsilon_2 \tag{16}$$

where $\alpha_t^j = \log z_j(t)$ as defined earlier, and where the density function f is the joint density function of the errors $(\epsilon_1 + \tilde{\nu}_1, \epsilon_2 + \tilde{\nu}_2)$, and the set θ includes all of the α_t^j and β_j parameters for all t and j, and any additional parameters of the density function.[31]

In evaluating the likelihood of an observed failure of a given type one must insure that the latent failure time is greater than the observed failure time for

[31] I will denote the censored failure type as "type 3". Keep in mind, however, that censoring is not analogous to the failures of the first two types.

every point in the discrete interval. If, without loss of generality, a failure of type 1 is observed in the interval $(t-1, t\,]$, the likelihood is given by

$$L_t^1(\theta) = \int_{\alpha_{t-1}^1+X_1\beta_1}^{\alpha_t^1+X_1\beta_1} \int_{g_1(\epsilon_1)}^{\infty} f(\epsilon_1, \epsilon_2)\, d\epsilon_1\, d\epsilon_2. \tag{17}$$

where the $g_1(\cdot)$ function insures that the required relationship between the failure times holds. The derivation of $g_1(\cdot)$ uses the assumption that the evaluation points change at a constant rate over the discrete interval. Then for a failure of type 1, and an arbitrary duration t^* in the interval $(t-1, t\,]$,

$$\begin{aligned} \epsilon_1 &= \alpha_{t-1}^1 + X_1\beta_1 + \kappa(\alpha_t^1 - \alpha_{t-1}^1) \\ \epsilon_2 &= \alpha_{t-1}^2 + X_2\beta_2 + \kappa(\alpha_t^2 - \alpha_{t-1}^2). \end{aligned}$$

where κ represents the fraction of the interval represented by $t^* - (t-1)$. Solving for t_2 in terms of t_1, yields the function g_1 relating the error for risk 1 to the error for risk 2,

$$g_1(\mu) = \alpha_{t-1}^2 + X_2\beta_2 + (\mu - (\alpha_{t-1}^1 + X_1\beta_1))\,\frac{\alpha_t^2 - \alpha_{t-1}^2}{\alpha_t^1 - \alpha_{t-1}^1} \tag{18}$$

Suppose that sample data of the form (t_i, k_i), $i = 1, 2, \ldots, N$, are observed, where t_i represents the observed duration length, and k_i is an indicator of the observed duration type $(k_i = 1, 2, 3)$. Define the indicator variables y_{itk} where $y_{itk} = 1$ if individual i experiences a duration of length t and type k $(t_i = t, k_i = k)$ and $y_{itk} = 0$ otherwise. The probabilities associated with observed durations and censored observations are derived by integration over appropriate regions as described above. Then the log likelihood for the sample is

$$\log L_n(\theta) = \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{k=1}^{3} y_{itk} \log L_{it}^k. \tag{19}$$

This is the likelihood function associated with a general multinomial model with the categories defined by the failure times $t = 1, \ldots, T$ and the failure types $k = 1, 2, 3$.

Completion of the statistical model requires the specification of the bivariate heterogeneity distribution or equivalently, the joint distribution of the composite errors. For computational convenience, I assume that the composite error distribution is jointly normal with correlation coefficient ρ. The likelihood is maximized over the parameters (α, β, ρ) where $\alpha = (\alpha_1^1, \ldots, \alpha_T^1, \alpha_1^2, \ldots, \alpha_T^2)$, and $\beta = (\beta_1, \beta_2)$ where β_j $(j = 1, 2)$ is a M_j-dimensional parameter vector.[32]

[32] All of the likelihood functions in this paper were maximized using standard maximum likelihood techniques. The algorithm used is a modified version of the Berndt, Hall, Hall and Hausman [1974] gradient technique that employs quadratic interpolation to determine stepsizes. Convergence for all of the models was achieved with little difficulty. The actual estimation technique involved estimating the β and α with retirement ages past a given age treated as censored failures. The estimated parameters were then used as starting values for a model with retirement ages unconstrained. The results do not appear to be sensitive to starting values.