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THE FISHERIAN TIME PREFERENCE AND THE EVOLUTION  
OF CAPITAL OWNERSHIP PATTERNS IN A GLOBAL ECONOMY

Kyoji Fukao

Koichi Hamada

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ABSTRACT

Conventionally economic growth theory was based on the assumption of a constant rate of time preference. Uzawa (1968) and Obstfeld (QJE, 1981) introduced the rate of time preference that increases with the utility level. Irving Fisher (The Theory of Interest) has a different opinion, however, that people are more time impatient at the lower level of income.

This paper assumes a non-monotonic time preference schedule such that people are more patient at the middle income levels and are less patient when they are either very poor or rich. Based on a nonlinear savings function out of wealth implied by such a time-preference schedule, this paper develops a single-good, multi-country growth model of a global economy with free capital mobility. The long-run property of this system is characterized by three kinds of long-run equilibrium: the starvation (fatal attractor) equilibrium, the imperialism equilibria dominated by a nation or by a group of nations, and the co-prosperity equilibrium where the wealth and the income of countries in the system grow proportionately. Bifurcation phenomena and the global stability of the system by the Lyapunov function will be discussed.

Our system has a strong resemblance to some models of ecology where species compete for their survival (May, Stability and Complexity in Model Ecosystems). Here we can properly analyze the transition of a debtor to a creditor country from a global perspective, and make a case for the pump-priming foreign aid or debt relief policy.

Please direct correspondence to:

Koichi Hamada  
Economic Growth Center  
Yale University  
27 Hillhouse Avenue  
New Haven, CT 06520  
(203)432-3614

## I. Introduction

If one's income level improves, will one become more time patient or less patient? Many prominent economists discussed this question and the literature of modern economic growth theory explicitly or implicitly assumed some relationship between the level of current economic welfare and the rate of time preference.

Milton Friedman (1957) expressed his view on this matter by drawing a homothetic indifference map for the intertemporal choice of consumption, which implies a constant rate of time preference regardless of the income level. He proceeded even to suggest the absence of "time preference proper," in other words, the zero rate of pure time preference when "the consumer unit correctly assesses the relative value of consumption in the two years" (Friedman, 1957, p. 12).

For a long time economic growth theory had been based on this behavioristic assumption of a constant rate of time preference, until Uzawa (1968), inspired by Koopmans (1960), introduced the rate of time preference that was an increasing function of the instantaneous consumption level.<sup>1</sup> Recent works (e.g. Obstfeld, 1981) utilized this structure to analyze the process of the international asset accumulation. The rationale of this assumption is rarely, however, systematically explicitly stated. Behind this assumption, we believe, is hidden the presumption that the asset holding behavior of economic units or of national economies should exhibit such a pattern that they achieve a certain target level of wealth relative

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<sup>1</sup> Koopmans did not deny the possibility that the rate of time preference might be a decreasing function of the instantaneous utility. See Koopmans, Diamond, and Williamson (1964), p. 98. Iwai (1972) pursues this line of research from a somewhat similar viewpoint as ours.

to income even under a constant return on capital over time. Decreasing rates of time preference with respect to instantaneous consumption would produce unstable asset holding behavior that could hardly be reconciled with the real balance effect.

Irving Fisher, father of modern theory of consumption choice over time, had a different message. It seems worth quoting his argument as a whole even at the risk of slight redundancy.

"Our first step, then, is to show how a person's time-preference depends on the size of his income. In general, it may be said that the smaller the income the higher is the preference for present over future income. It is true that a small income implies a keen appreciation of future wants as well as of immediate wants. Poverty bears down heavily on all parts of a man's life, both that which is immediate and that which is remote. But it enhances the utility of immediate income even more than of future income. This result is partly rational, because of the importance, by supplying present needs, of keeping up the continuity of life and the ability to cope with the future; and partly irrational, because the pressure of present needs blinds one to the needs of the future. As to the rational side, it is clear that present income is absolutely indispensable, not only for the present, but even as a precondition to the attainment of future income. 'A man must live.' Any one who values his life would prefer to rob the future for the benefit of the present, so far, at least, as to keep life going. If one has only one loaf of bread he would not preserve it for next year; for if he did he would starve in the meantime. A single break in the thread of life suffices to cut off all the future. And not only is a certain minimum

of present income necessary to prevent starvation, but the nearer this minimum is approached the more precious does present income appear, relatively to future income.

As to the irrational side, the effect of poverty is often to relax foresight and self-control and tempt one to trust to luck for the future, if only the all-absorbing clamore of present necessities is satisfied

We see, then, that a low income tends to produce a high time-preference, partly from lack of foresight and self-control, and partly from the thought that provision for the present is necessary both for itself and for the future as well."

[Fisher, 1907, Ch. 6, section 5.]

At least, for a low level of income, Fisher's argument seems to be convincing. (For psychological evidence see Maital, 1982). A casual look of the scatter diagram of the savings ratio and GNP per capita (Figure 1) tells us that the saving function is not necessarily a constant or a linear function of the real income level. Even though we exclude the extreme observation of the very low savings ratio of Lesotho,<sup>2</sup> we obtain a mild fit of a nonlinear savings function (see Figure 2).

Thus, it seems reasonable to assume a nonlinear savings schedule as well as a nonlinear time-preference schedule such that people are more time impatient at the lower levels of income, become more patient at the middle levels, and then gradually becomes less patient. The purpose of this paper is not to assess the empirical relationship between savings ratios or rates

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<sup>2</sup> Incidentally, it is interesting to note that another outlier in this diagram is the United States that saves significantly less than the overall average saving ratio.

of time preference and the level of income, but to explore the consequence of the nonlinear savings behavior resulting from the nonlinear time preference schedule. The time preference schedule we consider can be depicted as in Figure 3. The declining portion of the schedule corresponding to lower income levels captures the remarks of Irving Fisher; the increasing portion corresponding to higher income levels captures the behavior analyzed by Uzawa and Obstfeld.

By assuming a nonlinear savings function out of wealth implied by such a nonlinear time-preference schedule, we will develop a nonlinear single-good, multi-country growth model of a global economy.<sup>3</sup> This system can be regarded, in a broader sense, as a multi-country version of the nonlinear growth models developed by Leibenstein (1954) and Nelson (1956) in the earlier years of the development of economic growth theory. By allowing nonlinear savings behavior and studying global rather than local behavior of the system, we will be able to characterize the long-run behavior of this multi-country growth model of the Solow (1956) variety. (See Day (1982, 1983) for a more complex single country model.) In particular, we will discuss three kinds of long-run equilibrium: the starvation (fatal attractor) equilibrium, the imperialism equilibria dominated by a nation or by a group of nations, and the co-prosperity equilibrium where the wealth and the income of countries in the system grow proportionately. Incidentally our system has a strong resemblance to some

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<sup>3</sup> The nonlinearity exploited in this paper concerns the savings behavior. On the other hand, most of the nonlinearity in Leibenstein's work, and part of the nonlinearity in Nelson's, stems from the population behavior. However, the global qualitative property of a multi-country version of their models would be similar to ours.

of the models of ecology where species compete for their survival (see, e.g. May, 1973, also Haken 1983, chapter 10).

Our analysis is global in the two different meanings of the word. It is concerned with the development of the total world economy instead of that of participating countries; it also takes account of the dynamical property of the system in the large instead of the local stability in the neighborhood of an equilibrium.

In Section 2, we will discuss the relationship between the non-monotonic rate of time preference and the savings behavior. In Section 3, we will present a simple growth model of world capitalism with perfect capital mobility and sketch the dynamic property of the model with the monotonic savings behavior. We begin our analysis of the non-monotonic savings behavior in a two country setting in Section 4. The concepts of starvation equilibrium, imperialistic equilibria and co-prosperity equilibrium will be introduced and the possibility of bifurcations as well as the global property of the model will be discussed. In Section 5, we will illustrate the global characteristics of a three-country model by simulating dynamic paths of the asset accumulation, and the Lyapunov stability of a simplified version of the multi-country model. The economic implications of our global analysis will be summarized in the concluding section.

## II. Varying Time Preference and Nonlinear Savings Behavior

In order to discuss the microfoundation of the savings behavior to be introduced below, let us consider a representative family that solves the following optimization problem:

$$\max_{\{c(t)\}} \int_0^{\infty} e^{-\delta t} \ln(c(t)) dt, \quad (1)$$

subject to:

$$\dot{x}(t) = (r(t) - n)x(t) - c(t), \quad (2)$$

$$x(0) = \bar{x}(0), \quad (3)$$

$$x(t) \geq 0, \quad (4)$$

and

$$c(t) \geq 0. \quad (5)$$

Here  $\delta$  is the rate of time preference for the representative family, which we assume to be constant at this outset of our discussion.  $c(t)$  and  $x(t)$  are respectively per capita consumption and per capita wealth at time  $t$ ,  $r(t)$  is the rate of return of wealth at time  $t$ , and  $n$  is the rate of population growth rate. The necessary conditions to these optimization problems are given by the combination of (2),

$$\dot{\lambda}(t) = \lambda(t)(\delta + n - r(t)), \quad (6)$$

and

$$c(t) = 1/\lambda(t)$$

along with the transversality conditions

$$\lim_{t \rightarrow \infty} \lambda(t) e^{-\delta t} \geq 0, \quad (7)$$



and

$$\lim_{t \rightarrow \infty} \lambda(t) x(t) e^{-\delta t} = 0. \quad (8)$$

where  $\lambda(t)$  denotes the current value of the marginal contribution of the state variable  $x(t)$  to the utility.

Under the assumption of logarithmic instantaneous utility, (6) can be written as

$$\dot{c}(t) = (r(t) - n - \delta)c(t) \quad (6')$$

which, in combination with (2), gives the consumption behavior proportional to wealth

$$c(t) = \delta x(t). \quad (9)$$

The resulting dynamical equation can be written as

$$\dot{x}(t) = (r(t) - \delta - n)x(t). \quad (10)$$

This neat form of equation (10) stems from our assumption of logarithmic instantaneous utility function.<sup>4</sup> If we relax this assumption even slightly, we will no longer obtain a simple relationship between

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<sup>4</sup> In fact, this equation gives a microeconomic foundation for a strategic analysis conducted in Hamada (1965) in which game theoretic interactions with varying  $\delta$  are analyzed.

wealth and consumption.<sup>5</sup> Since we are mainly concerned with the global rather than local property of the dynamical system, explicit characterization like (10) would help our analysis a great deal.

It would be ideal if the neat form of equation (10) carries through in the case of varying, non-monotonic time preference. Even with the simplifying assumption of logarithmic utility, the wealth accumulating equation does not take a simple closed form like (10) if we allow  $\delta$  to depend on the value of  $c$ .

In order to analyze the long-term and global consequence of non-monotonous savings behavior of many nations, we postulate in the main part of our paper (Section 4) that savings behavior of  $j$ th nation is governed by the following equations

$$\dot{x}_j(t) = (r(t) - \delta_j(x_j(t)) - n) x_j(t) \quad j=1, \dots, J \quad (11)$$

where  $r(t)$  is the rate of returns to physical capital  $x_j(t)$  that is identical for all the nations due to the reasons to be explained in the next section. This time  $\delta_j(\cdot)$  is a function defined not on the consumption

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<sup>5</sup> For example, assume a Paretian utility

$$U(c(t)) = \frac{c(t)^\gamma}{\gamma} \quad 0 < \gamma < 1,$$

then we get instead of (6')

$$\dot{c}(t) = \frac{1}{1-\gamma} (r(t) - n - \delta) c(t)$$

In this case  $c(t)$  depends not only on  $x(t)$  but also on the whole future time path of the interest rate.

level but on the level of physical wealth  $x_j(t)$ . The shape of  $\delta_j(x_j(t))$  is, however, assumed to be like the rate of time preference schedule drawn in Figure 3. In other words, we will assume in later sections that  $\delta_j(\cdot)$  is a differentiable continuous function such that

$$\delta'_j(x_j(t)) < 0 \quad \text{if } 0 < x_j(t) < x_j^*,$$

and

$$\delta'_j(x_j(t)) > 0 \quad \text{if } x_j^* < x_j(t),$$

where  $x_j^*$  is the turning point of the savings behavior from the Fisherian to the Uzawa region.

The main reason why we consider the accumulation process of physical capital is for the sake of analytical ease. At the same time, this assumption gives our analysis a flavor of the exercise on the long discussed world imperialism in which capitalists compete for concentration of physical capital. Also readers who are keenly concerned with the microfoundation of any macroeconomic models may feel frustrated because our microfoundation appears to fall short of the satisfactory standard except for the case of constant rate of time preference. We hope, however, that intriguing consequences to be presented below will justify at least partly the worthiness of this somewhat bold intellectual experiment.

### III. A Model of World Imperialism with Monotonic Savings Behavior

Let us construct a multi-country growth neoclassical model that sheds light on the long-run property of the asset-liability structure among nations.  $J$  countries produce the same good by identical linear homogeneous well-behaved production functions.

$$Y_j(t) = F(K_j(t), L_j(t)) = L_j(t)f(k_j(t)), \quad j=1, \dots, J, \quad (12)$$

where  $Y_i(t)$ ,  $K_i(t)$ ,  $L_i(t)$ , and  $k_i(t) = K_i(t)/L_i(t)$  are respectively output, capital stock, labor, and per-capita capital stock of the  $j$ th country at time  $t$ .  $f(k_i(t))$  is a per-capita production. Assume that labor force is growing at an identical exponential rate of  $n$ , that is  $L_i(t) = L(0)e^{nt}$ , where we set just for simplicity that the relative scale of each nation is exactly identical.

Assume capital movements take place freely in such a way as to equate the rates of return to capital  $r(t)$ :

$$\frac{\partial F(K_i(t), L_i(t))}{\partial K_i(t)} = \frac{\partial F(K_j(t), L_j(t))}{\partial K_j(t)} \quad \text{for all } i \text{ and } j$$

or

$$f'(k_i(t)) = f'(k_j(t)) = r(t) \quad \text{for all } i \text{ and } j \quad (13)$$

that implies  $k_i(t) = k_j(t) = k(t)$ .

Let  $x_j(t)$  indicate the per-capita capital stock owned by the  $j$ 'th nation. The net per-capita net foreign asset  $z_j(t)$  owned by the  $j$ 'th nation is written as

$$z_j(t) = x_j(t) - k(t). \quad (14)$$

We will conduct our analysis in terms of  $x_j(t)$  rather than  $z_j(t)$ . Needless to say,  $z_j(t)$  is negative if the  $j$ 'th country is a debtor country.

We combine the savings behavior (11) with this simple growth model.<sup>6</sup> The system of differential equations becomes

$$\dot{x}_j(t) = (r(t) - \delta(x_j(t)) - n)x_j(t), \quad j=1, \dots, J \quad (15)$$

where  $r(t) = f'(k(t))$  and  $k(t) = \sum_{j=1}^J x_j(t)/J$ .

Before going into a general analysis of the system with non-monotonic savings behavior, let us start, as a preliminary step, with a two-country model ( $J = 2$ ) accompanied by simpler types of savings behavior corresponding to a constant rate of time preference and that of monotonously increasing consumption out of physical wealth.

(i) Constant  $\delta_j$  in savings function (15)

Suppose  $\delta_j$  is constant in (11). If  $\delta_j$ 's are identical for two countries, say  $\delta_1 = \delta_2$ , then the phase diagram of the pair of differential equations (15) ( $J = 2$ ) looks like Figure 4(a). The initial relative ownership pattern will reproduce itself. Suppose next  $\delta_1 < \delta_2$ , then the phase diagram will look like 4(b). The economy will eventually settle to the imperialism (sink) solution with country 1 as the empire and country 2 as the colony unless the initial condition happens to be at B, an unstable

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<sup>6</sup> This model is an open version of the Solow growth model (Solow 1956) with the Kaldorian savings fraction (Kaldor 1955). For a treatment of this type of model see Hamada (1965, 1966). For open growth models with saving functions proportional to income, see also, for example, Onitsuka (1974) and Ruffin (1979).

solution. This is a special case of the phenomenon that more patient individuals eventually own the whole world (Ramsey 1928, Bewley 1982, Ryder 1985). Equilibrium A and B corresponds to the stable and unstable equilibrium discussed in the strategic analysis with  $\delta_j$  as strategies in Hamada (1965). This abnormality was cleverly avoided in Buiter (1981) because he analyzed the overlapping generation model. There the difference of the rate of time preference has only a limited effect because the chain of planning for the future was blocked by the lack of intergenerational linkage.

(ii) Increasing  $\delta_i$  as a function of the net wealth

If  $\delta_i$  is a monotonic increasing function of wealth  $x_i(t)$ , then the phase diagram will look like 4(c). It can be shown that  $\dot{x}_1(t) = 0$  locus is steeper than  $\dot{x}_2(t) = 0$  locus. The economy will approach to a co-prosperity solution C. The local behavior around C is thoroughly analyzed by Kawai and Okumura (1988).

The age old discussion of the transformation of a country from debtor through creditor to mature creditor should be analyzed from this type of global perspective rather than from a small-country perspective because all the countries cannot end as creditors on the globe which is by itself a closed system. It is amusing to see that competitive species models in ecology have identical analytical structure, and that exactly the same phase diagrams are written for those models (See Haken, 1968, p. 307).

#### IV. Nonmonotonic Savings Behavior and Imperialistic Equilibria

Let us develop a two-country model of the world economy with free capital movements by introducing a non-monotonic savings behavior. Since

the nonlinearity could create rather complicated interaction, we will illustrate the global structure of the model by the following numerical example. Consider a world consisting of symmetric countries. Technology as well as consumption function is identical in the two countries. The only possible difference is in their initial wealth. We may conceive it as two classes of economics, capitalist and labor. The labor class consumes all their income, and the non-human capital is owned by capitalists.

The per-capita production function we adopt is:

$$f(k(t)) = \frac{1}{2\sqrt{2}} k(t)^{\frac{1}{2}},$$

$$\text{where } k(t) = \frac{1}{2} \left\{ \sum_{j=1}^2 x_j(t) \right\}$$

The rate of return  $r(t)$  is thus given by

$$r(t) = \frac{1}{4\sqrt{2}} k(t) - \frac{1}{2}$$

Let  $n = \frac{1}{80}$  or 1.25 percent. The  $\delta_j(x_j(t))$  is symmetric between the two countries, so that we can write it without subscript for the function

$$\delta(x_j(t)) = \frac{1}{8x_j(t)} + \frac{x_j(t)}{800}.$$

Thus the dynamical equation for capital accumulation becomes

$$\dot{x}_j(t) = \left\{ \frac{1}{4} \left[ \sum_{j=1}^2 x_j(t) \right] - \frac{1}{2} - \left[ \frac{x_j(t)}{800} + \frac{1}{80} + \frac{1}{8x_j(t)} \right] \right\} x_j(t)$$

$$j = 1, 2.$$

Then the phase diagram of this system is drawn as Figure 5. There are three kinds of stable equilibria: the co-prosperity equilibrium C, the imperialism equilibria (A with country 1 as the empire and country 2 as the colony, and B with country 2 as the empire and country 1 as the colony), and the starvation (fatal attractor) equilibrium O. There are one unstable equilibrium F and four saddle-point equilibria D, E, G, and H. The phase plane can be divided into four regions by the ridge (water-shed) lines FG, FH, FI and FJ as in Figure 6. From Region I, the path converges eventually to the co-prosperity equilibrium C; from Region II and III, the path converges respectively to A and B the imperialism equilibria; and alas from Region IV to the starvation equilibrium at the Origin!

It is easy to check that, in the closed economy consisting of a single economy Nation 1, point F' (projection of F on  $x_1$  axis) is the unstable equilibrium and C' the stable equilibrium. For the sake of the existence of the foreign nation, Nation 2, the survival region where nation 1 is capable of avoiding starvation is extended to the regions with horizontal shade. Figuratively, in Region FGF' Nation 1 can take off because it can exploit the existence of Nation 2 because of the relative wealth at the initial point; in Region HFF" surrounding B, labor in nation 1 can survive because of the colonization by Nation 2.

Similarly, Region surrounded by F"FJ with vertical shade implies that Nation 1 starting from this region can attain nothing but the colonized



equilibrium B in this world economy even though Nation 1 could attain the state corresponding to C in the absence of Nation 2 after a long while. This path may as well be the Pareto efficient path. Needless to say, the Pareto efficiency does not exclude the situation in which one nation is colonized. In this region, Nation 1 may choose to close the economy against foreign capital, and after a while, may move to Region I. From there Nation 1 can adopt a free capital movement policy to attain the co-prosperity equilibrium C.

Finally, one would be able to imagine the situation when a Nation, say Nation 1 suddenly becomes more frugal. Then  $x_1(t) = 0$  contour GFECDA will shift to the right, point C and D will move to the right and finally collapse into a single point (see Figure 7). At that tangency point a bifurcation phenomenon will take place and the co-prosperity equilibrium will vanish from the globe, leaving only the possibility of the imperialistic equilibria.

Needless to say, this analysis is only an illustration of a typical case. More general analysis can be done, for example, utilizing the topological approach explained in Hirsch and Smale (1977).

#### V. The Global Property of the Multi-Country Model.

We can extend the above exercise into three or more country cases. In order to visualize the global property of a three-country case, let us take the symmetric example with the same production function as in the previous section. Then the basic differential equations become

$$\dot{x}_j(t) = \left( \frac{1}{4\sqrt{2}} \left[ \frac{1}{3} \sum_{j=1}^3 x_j(t) \right]^{-\frac{1}{2}} - \left[ \frac{x_j(t)}{800} + \frac{1}{80} + \frac{1}{8x_j(t)} \right] \right) x_j(t).$$

In this case there are 8 stable long-run equilibria:

- (i) 1 co-prosperity equilibrium
- (ii) 3 oligarchy equilibria, each of which has two empires and one colony.
- (iii) 3 imperialism equilibria, each of which has one empire and two colonies.
- (iv) 1 starvation equilibrium.

Corresponding to these equilibria, the initial endowment in  $x_1, x_2, x_3$  space can be divided into 8 regions. Our simulation exercises give the following pictures.

If we slice the space by simplexes, or more rigorously magnified simplexes, such that

$$\sum_{j=1}^3 x_j = M,$$

and if we reduce the number  $M$ , then the initial condition can be divided as shown in 8(a) to 8(d). Suppose we approach this three dimensional space from the distance towards the origin. First, if  $M > 1.68$ , then we will observe the initial points leading to all the seven equilibria but the starvation equilibrium (Figure 8(a)). If  $1.68 > M > 0.717$ , the initial points leading to starvation equilibrium will replace those leading to the co-prosperity equilibrium. If  $0.717 > M > 0.173$ , then those leading to oligarchy equilibria will disappear. Finally, of course, if  $0.173 > M$ , then the simplex will shrink further leaving no room for survival on this globe.

Then, can we say nothing about the global property of these nonlinear models? The work of ecologists (e.g. May 1973) teaches us that we can at least handle the following simple case with linear increasing function  $\delta(x)$  and the quadratic technology.

Consider a multi-country model with the production technology

$$f(k) = -\frac{k^2}{2} + ak, \quad a > 0$$

where  $k = \sum_{j=1}^J x_j/J$ , and saving function (15) with  $\delta(x) = \rho x$ ,  $\rho > 0$ .

That is,

$$\dot{x}_j = (r(k) - n - \rho x_j) x_j, \quad j=1, \dots, J \quad (16)$$

where  $r(k) = f'(k) = -k + a$ .

Then the system of the nonlinear equation (16) can be written as

$$\dot{x}_j = (a - n - \sum_{m=1}^J \alpha_{jm} x_m) x_j,$$

where  $\alpha_{jj} = \rho + J^{-1}$ , and  $\alpha_{jm} = J^{-1}$  for  $j \neq m$ .

$\|\alpha_{jm}\|$  is a symmetric matrix and positive definite. Assume  $a - n > 0$ , and define the unique co-prosperity equilibrium  $x^* = (x_1^*, x_2^*, \dots, x_J^*)$  where  $x_j^* = (a - n)/(\rho + 1)$ . Then the following magnitude satisfies the conditions of Lyapunov function: (see, e.g. Haken 1983, pp. 124-125).

$$L = \sum_{j=1}^J (x_j - x_j^* \ln x_j) - \bar{L}$$

where  $\bar{L} = \sum_{j=1}^J (x_j^* - x_j \ln x_j^*)$ .

(i)  $L$  is non-negative and vanishes at  $x^*$ ,

(ii)  $L$  is positive,

and

(iii)  $\frac{dL}{dt} = - \sum_{j,m} (x_j - x_j^*) \alpha_{jm} (x_m - x_m^*) \leq 0$  with equality only at  $x^*$ .

Hence, the system is globally stable (May 1973, pp. 54-55, see also Lucas and Stokey 1984 for a discrete analogue of this problem with two economic agents.).<sup>7</sup>

#### VI. Concluding Remarks

By introducing a non-monotonic savings schedule, we have seen that the process of economic developments can be formulated into a dynamical model similar to models of competing species in ecology. Despite its rather simple set of assumptions and dynamical apparatus, this model suggests a wide range of economic implications. We will list some of them as follows.

(1) The old debate on the stages of economic development related to the shift from a debtor status to a lender status (cf Onitsuka 1974). Since every country cannot become net lenders at the same time within the (by itself, closed) world economy, we need a general equilibrium or global-scale analysis like the one analyzed in this paper. Depending on the relative propensity of savings and on the stage of income levels, such

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<sup>7</sup> The reasoning holds if  $\|\alpha_{jm}\|$  is positive definite regardless whether it is symmetric. Also the following function will also serve as a Lyapunov function:

$$L^* = \sum_{j,m} (x_j - x_j^*) \alpha_{jm} (x_m - x_m^*).$$

a development of borrowing or lending phases can actually be explained by the arrows in the phase diagrams that cross the forty-five degree line.

(2) Initial conditions on capital endowments have crucial effect on the long-run destiny of a nation. Depending on which side of the water-shed curve a nation places itself at the initial moment, the long-run path of the nation may take completely different directions. Random shocks into the system may alter the destiny of a nation from a path towards an empire or co-prosperity to a path towards a colonized nation, if we disregard the path towards starvation. In this vein, a case may be made for a pump-priming foreign aid and a debt relief to troubled developing countries. If the Fates push a nation into the path towards a colonized nation or towards starvation, why not let an angelic hand help salvage the nation into being a region of future co-prosperity? Even though the absolute magnitude of aid or relief is small, if it is done in the neighborhood of the critical water shed line, then the nonlinearity of the system may enable the nation to enjoy a long lasting economic benefit.

(3) The savings behavior has a crucial influence on the long-run state of a national economy. If, in Japan, or in Newly Industrialized Economies (NIEs), the saving propensity dramatically rises, then the bifurcation phenomenon explained above may emerge and push these countries into a long-run owners of the world capital. On the other hand, if the savings propensity of, say the United States or the United Kingdom, were to decline for any reason such as fiscal deficits, then the country would fall into the debtor's trap towards possible colonization by high savings countries

(see Hamada and Iwata, 1989, for some possible scenarios regarding the United States, Japan and West Germany).

Needless to say, there is no contradiction between the colonization of a part of the world and Pareto optimality. As suggested in Section 2, the non-linear savings behavior may be a result of optimizing behavior. The global outcome may well be Pareto optimal given certain initial conditions.

The phase diagrams for the two-country case and simulation analysis for the three-country case clarifies the benefit and cost of living in a world community with other nations. A nation that would not survive in absence of other nations may be able to survive because of the presence of other nations that function as possible lenders or borrowers. On the other hand, a nation that would be able to achieve the long-run development corresponding to the co-prosperity equilibrium may end up being colonized if a rich nation exists along with her. The existence of a partner elevates the level of insurance against starvation of a nation, but enhances the risk of being victimized as her colony. The phase diagrams indicate the significance of inequality as well as that of poverty.

(5) Under the above circumstances, there is some case for protectionist policy on the part of the developing country against foreign investment. By closing the country against foreign capital, a country may carry itself to the region from which opening of the country would lead to the co-prosperity equilibrium. In that circumstance, xenophobia or protectionism against foreign capital may prevent the colonization of the country in the long run though at the sacrifice of the present national consumption level.

We do not intend to defend ourselves against all the possible criticisms on our simplicity or arbitrariness of the assumptions employed in our model. In particular, the abstraction from terms of trade by assuming a single commodity growth model may exaggerate the effect of the difference of the savings behavior between low-income and high income countries.

We do hope, however, that this paper will generate more interest in the global analysis of a non-linear economic system. The recent revival of the theory of economic growth (e.g. Lucas 1988, Romer 1986) seems to suggest that the natural direction of research is towards non-linear models of development. In mathematics, in many natural sciences and in some social sciences, new interest is emerging on the global behavior of non-linear systems ... not to mention the highly popularized topics such as chaos (cf. Day 1982, 1983) and fractal geometry. This paper, we believe, presents just one example of many approaches in these growing related areas of study that seem to be potentially effective to improve our understanding of the evolutionary process in our economic universe.

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Saving Ratio (percent)

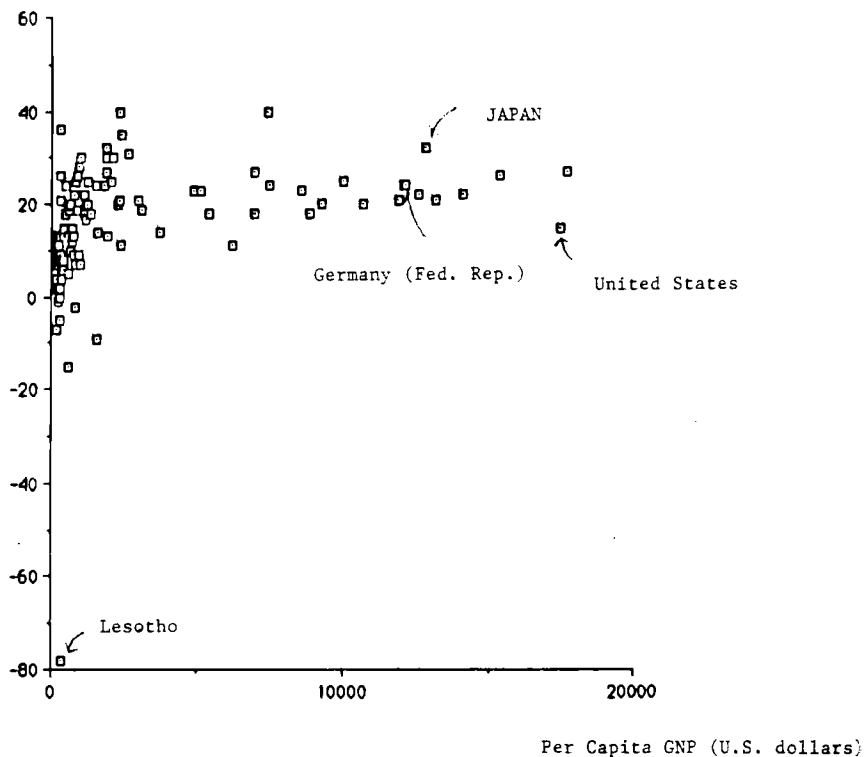
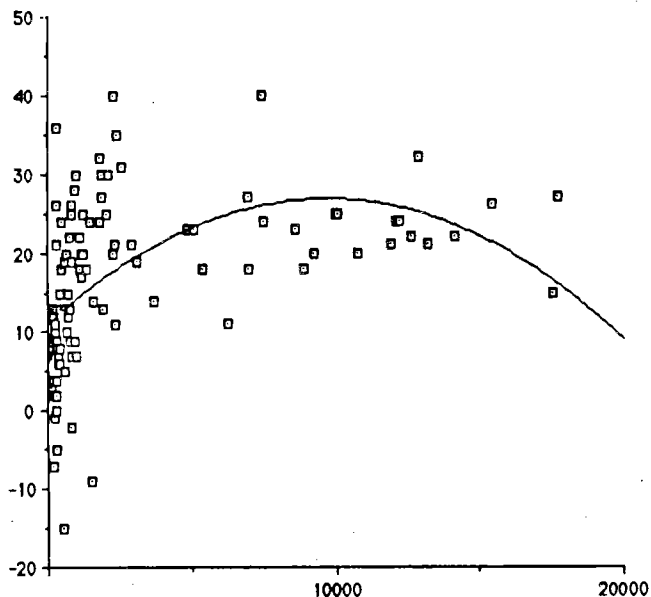


Figure 1

GNP per capita and saving ratio of 104 countries in 1986.

Note: The saving ratio is the ratio of gross domestic saving to gross domestic product. The data taken from World Bank, World Development Report of 1986, Table 1 and Table 5. All the available data are plotted.

Saving Ratio (percent)



Per capita GNP (U.S. dollars)

Figure 2

Fitted nonlinear saving function.

Note: The data are the same as in Figure 1. The extreme observation of Lesotho is excluded. The second order polynomial curve is fitted by ordinary least-squares method. The fitted equation is

$$Y = 11.137 + 3.2X - 0.1675X^2 \quad R = 0.49$$

where Y and X were saving ratio (percent) and GNP per capita (thousand dollars) respectively.

$\delta$  : The Rate of Time Preference

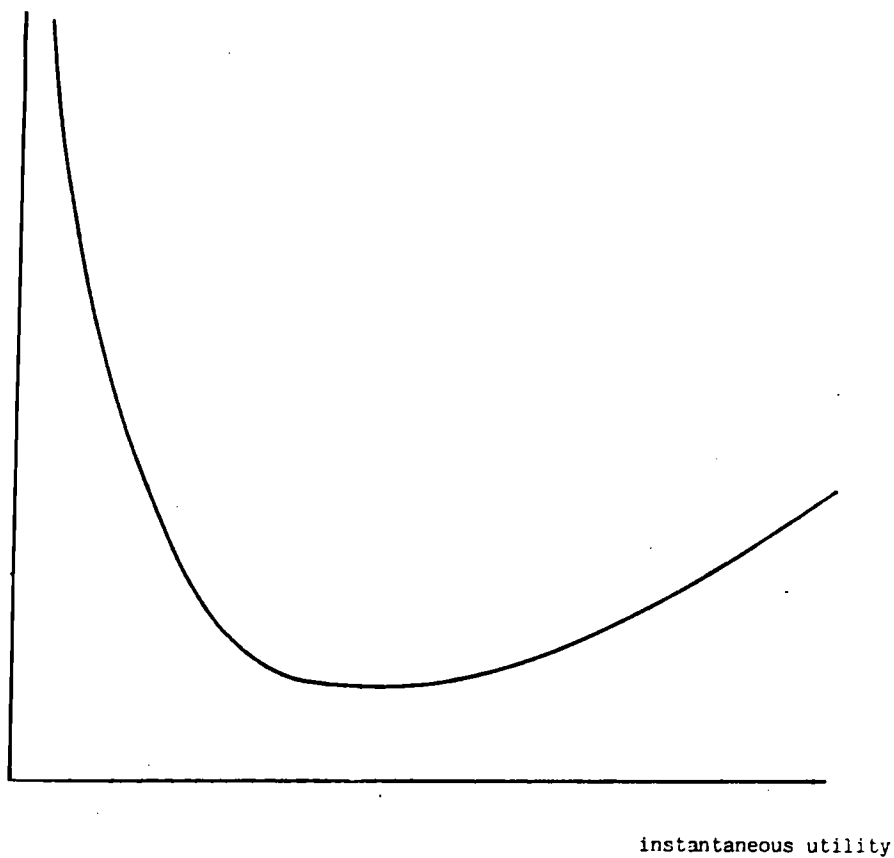


Figure 3  
The Fisherian Time Preference

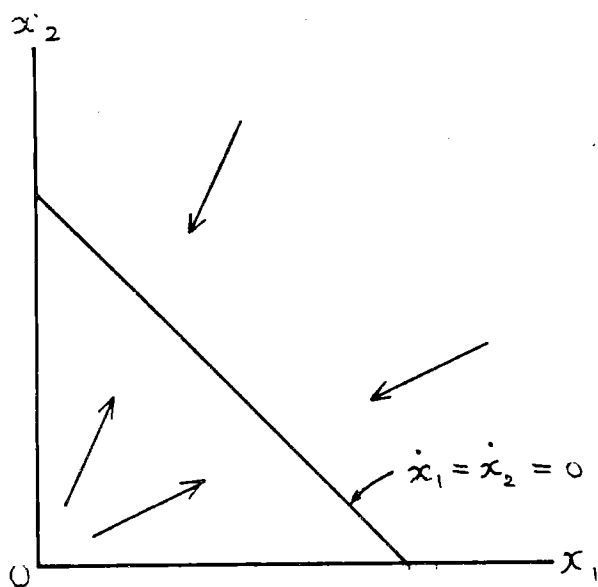


Figure 4(a). Constant and Identical  $\delta_1$

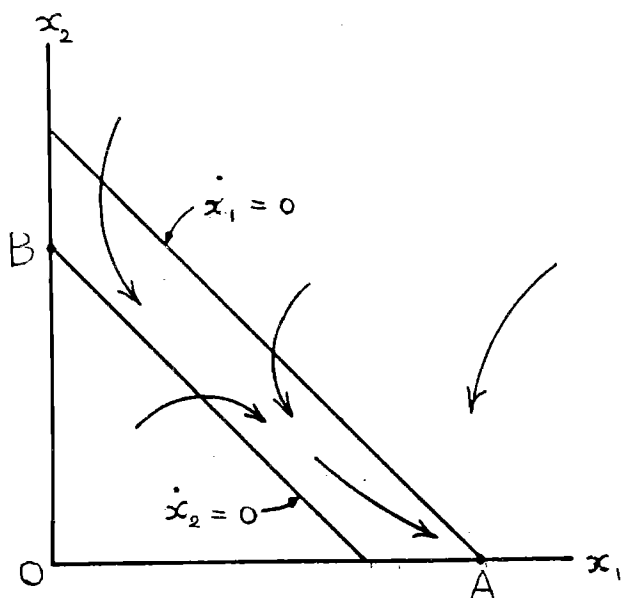


Figure 4(b).  $\delta_1 < \delta_2$

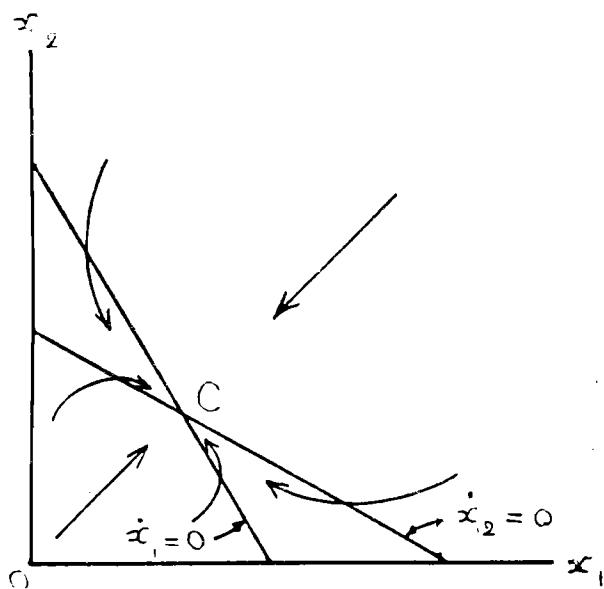


Figure 4(c). Increasing  $\delta_1$  as a Function of the Net Wealth

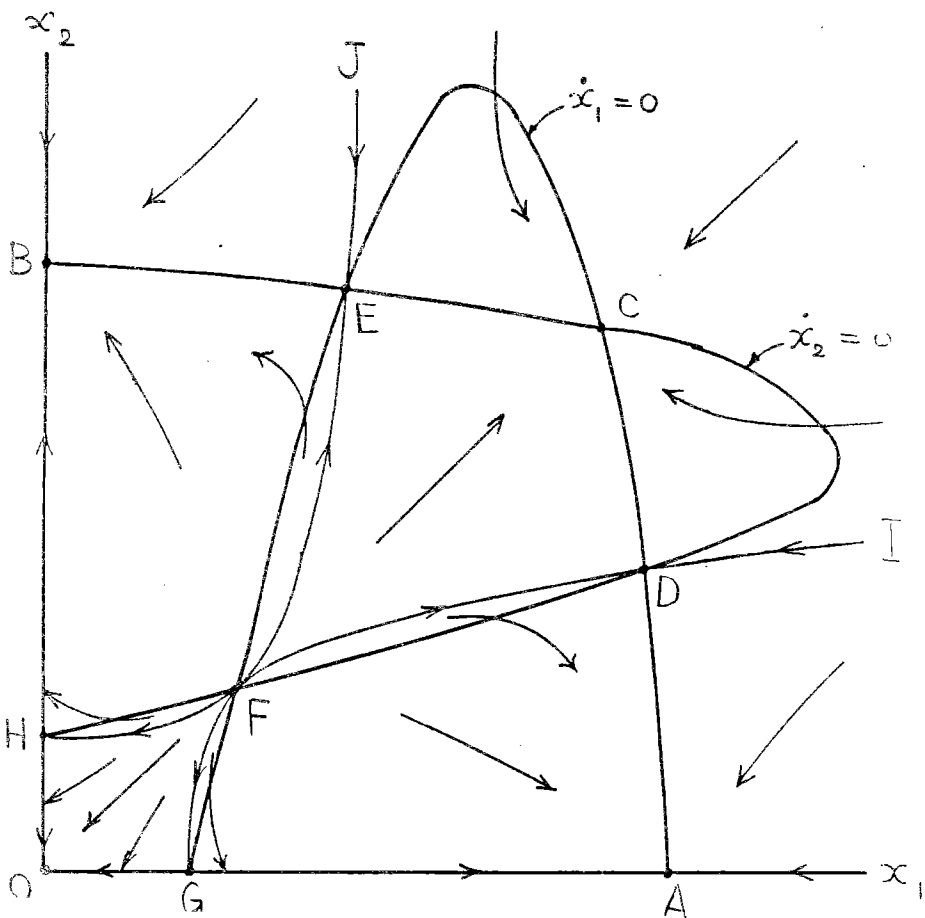


Figure 5.

The Phase Diagram of the System with Nonmonotonic Savings



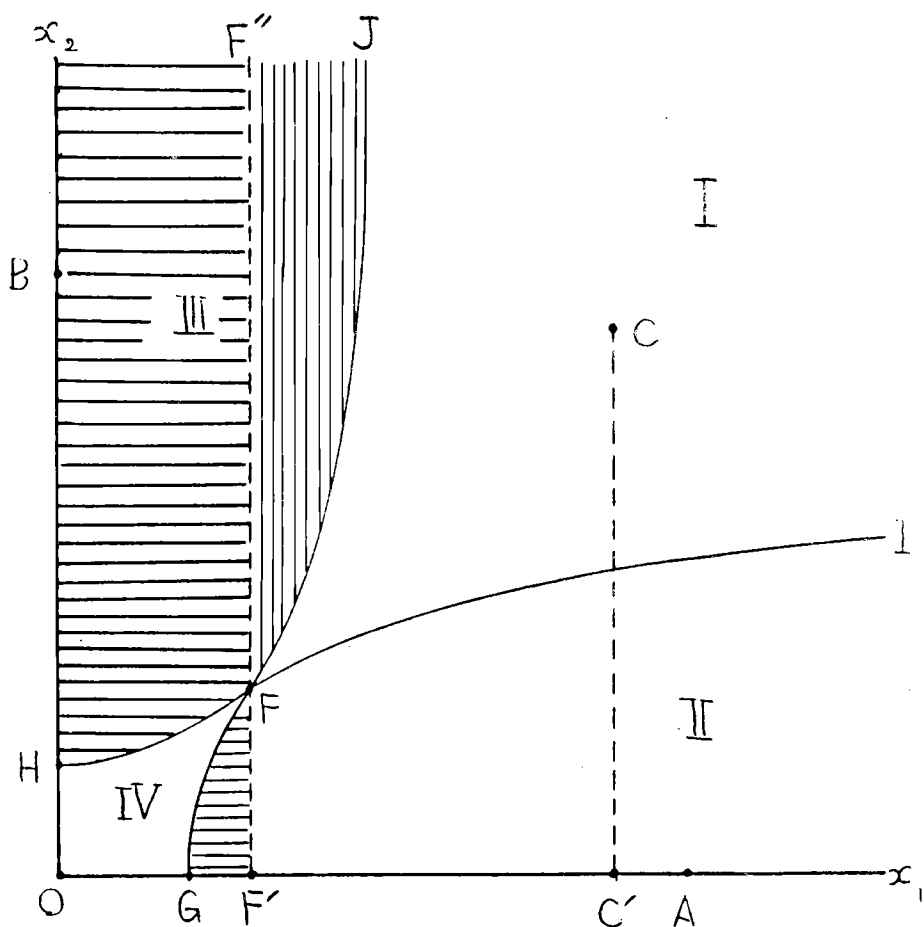


Figure 6

Four Regions

- I: The path converges to the co-prosperity
- II: Country 1 becomes an empire, equilibrium C.
- III: Country 2 becomes an empire
- IV: The path converges to the starvation equilibrium O

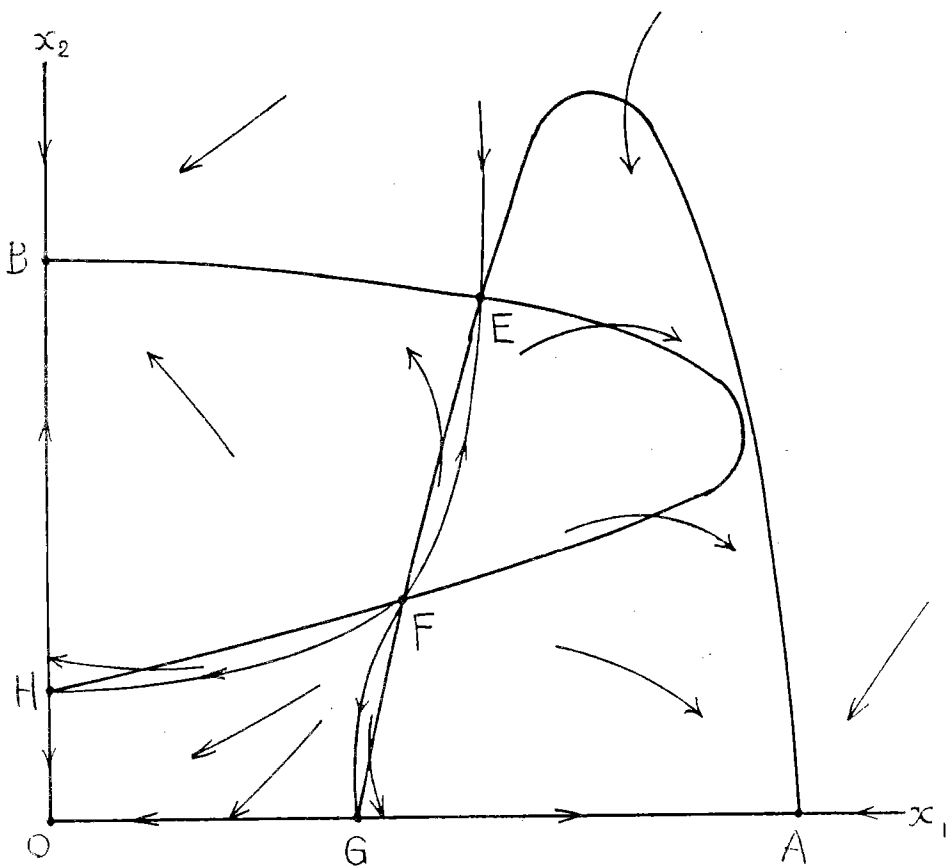


Figure 7

After the Bifurcation

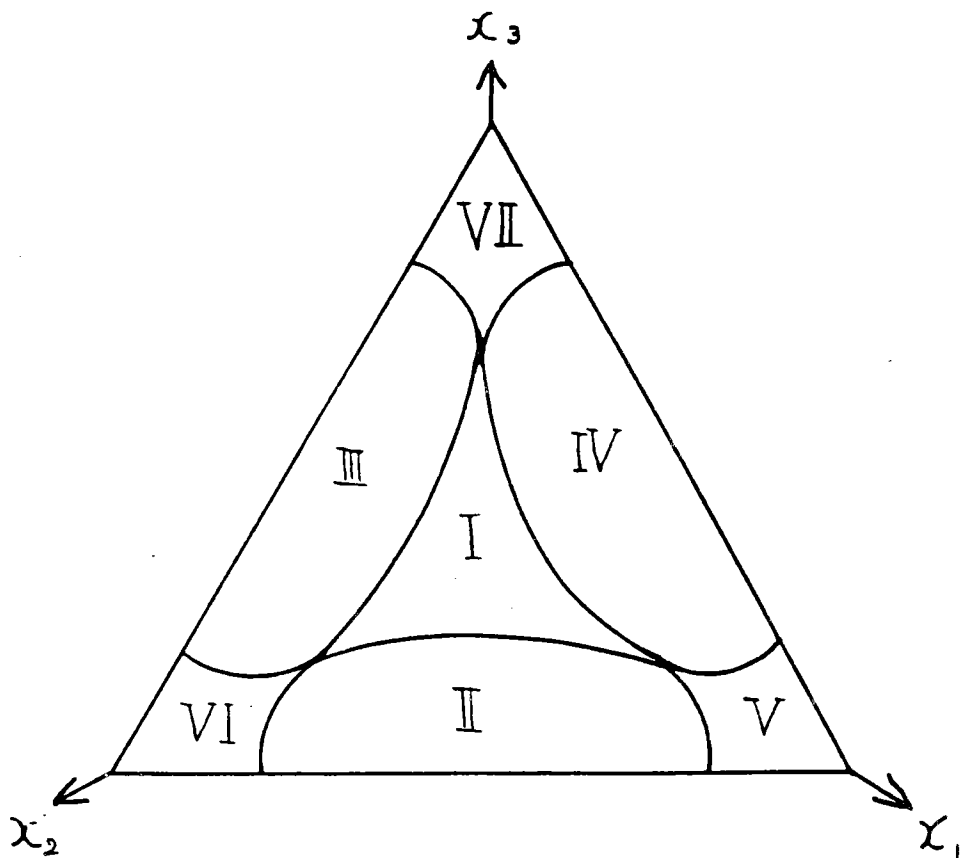


Figure 8(a)  $\sum_{i=1}^3 x_i > 1.68$

- I: The path converges to the co-prosperity equilibrium
- II: Country 1 and 2 become empires.
- III: Country 2 and 3 become empires.
- IV: Country 1 and 3 become empires.
- V: Country 1 becomes an empire.
- VI: Country 2 becomes an empire.
- VII: Country 3 becomes an empire.

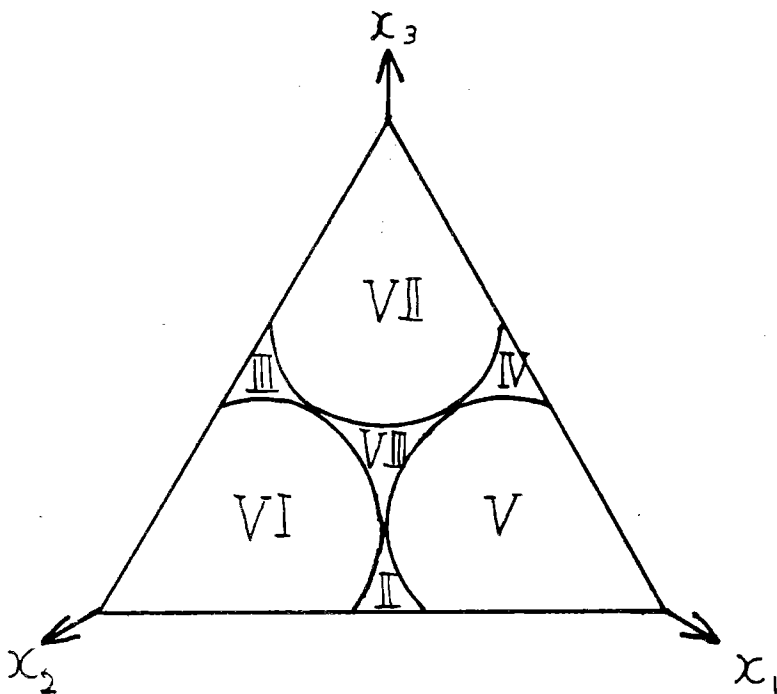


Figure 8(b)  $1.68 > \sum_{i=1}^3 x_i > 0.717$

VIII: The path converges to the starvation equilibrium

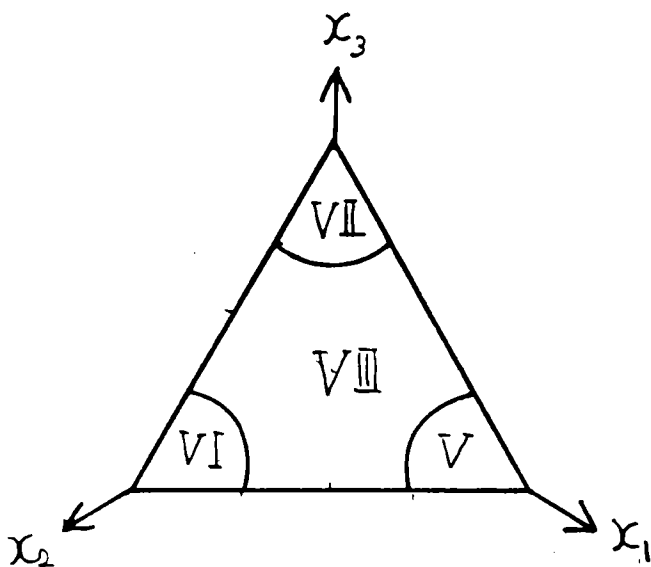


Figure 8(c)  $0.717 > \sum_{i=1}^3 x_i > 0.173$

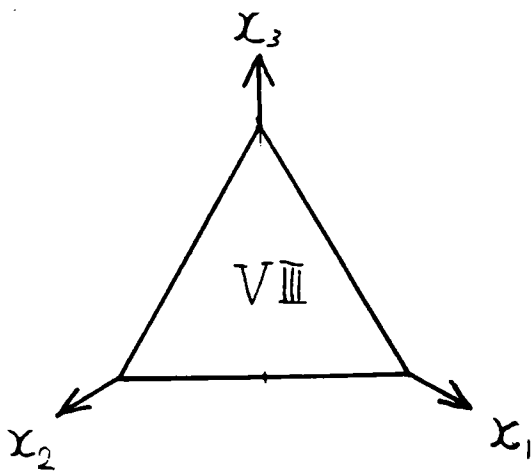


Figure 8(d)  $0.173 > \sum_{i=1}^3 x_i > 0$