NBER WORKING PAPER SERIES

THE MACROECONOMICS OF TRADE CREDIT

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Working Paper 31026
http://www.nber.org/papers/w31026

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 2023, Revised June 2026

First draft: December 2022. We thank Alessandro Dovis and Nobuhiro Kiyotaki for extensive discussions that shaped the direction and scope of our paper. We also thank Adrien Auclert, Javier Bianchi, Saki Bigio, Laura Castillo-Martinez, Chris Clayton, Eduardo Davila, Thomas Drechsel, Joao Gomes, Luigi Iovino, Urban Jermann, Ernest Liu, Guillermo Ordonez, Pablo Ottonello, Margit Reischer, Andreas Schaab, Rob Townsend, Victoria Vanasco, Jaume Ventura, Thomas Winberry, and seminar participants at several universities and conferences for insightful comments. We thank Mohamad Adhami for excellent research assistance. Luigi Bocola thanks the Alfred P. Sloan Foundation and the National Science Foundation for generous funding. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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JEL No. E44, G01, G30

ABSTRACT

In most countries, suppliers of intermediate goods and services are also major providers of short-term financing to firms. This paper studies the macroeconomic implications of these financial links. In our model, trade credit arises from long-term relationships between firms linked in production and is sustained by reputation: customers repay in order to preserve their relationships with suppliers. These financial links generate a credit multiplier. Suppliers can enforce repayment from their customers and discount receivables with banks to obtain liquidity. This process can either dampen or amplify the output effects of financial shocks, depending on suppliers' borrowing capacity. Using Italian data, we find that the credit multiplier is sizable and that trade credit substantially amplified the output costs of the Great Recession.

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1 Introduction

The Great Recession spurred a large body of work on the role of financial frictions in macroeconomic fluctuations. Most of this work, building on the contributions of Kiyotaki and Moore (1997b) and Bernanke, Gertler, and Gilchrist (1999), focuses on capital markets and financial intermediaries, assuming that firms finance their operations exclusively through these channels. In reality, however, most firms around the world address the bulk of their short-term financing needs through trade credit—the financing that suppliers of intermediate inputs provide in the form of extended payment terms. For many non-financial firms, trade credit is larger than short-term loans and bonds combined, underscoring the importance of understanding its drivers and macroeconomic effects.

In this study, we propose a model for understanding the coexistence of bank and trade credit. In our framework, firms can borrow both from banks and from their suppliers of intermediate inputs. Unlike bank debt, which is partially upheld by law, the enforcement of trade credit relies on reputation, with customers having an incentive to repay it in order to maintain the relationship with their suppliers. We show that these financial links between firms work as a *credit multiplier* for the economy. On the one hand, firms that are constrained in their access to bank credit can borrow from their suppliers, as they will have an incentive to repay that extra dollar in order to maintain those trade relationships. On the other hand, suppliers can discount their trade credit bills with banks and obtain liquid funds to pay for their expenses. This process increases the amount of credit flowing from banks to firms and reduces the economic distortions due to financial frictions. However, it can also make the economy more sensitive to financial shocks. Indeed, when calibrating our model to Italian data, we find that trade credit amplified the output costs of the Great Recession.

We consider an economy in which households consume a basket of differentiated goods. Production is organized in two-layer vertical supply chains: upstream firms produce intermediate goods using labor, and downstream firms use these inputs to produce consumption goods. Following Bigio and La'o (2020), we assume that downstream firms receive part of their revenues only after purchasing inputs, so they need credit to operate. Banks can lend to firms, but only up to a fraction of firms' revenues. As in Jermann and Quadrini (2012), this borrowing limit is time-varying and stochastic, and we interpret an unexpected tightening as a negative credit-supply shock. The main innovation of our framework is that downstream firms can also borrow from their input suppliers in the context of a long-term relationship. The resulting contract between a supplier and a customer determines not only the quantity of the input and its price, but also how payments are split between the beginning and the end of the period. We refer to the deferred component as trade credit.

Trade credit emerges in equilibrium as part of the optimal contract between downstream and upstream firms. The enforcement of these credit relationships is sustained by a reputation mechanism, with the supplier excluding the customer from future provisions of the input in case of a default. Thus, upstream firms can supply more trade credit when the value of their relationship with downstream firms is high, as in those cases, there are more incentives to repay. This value is endogenous and depends on the characteristics of the supplier and the customer, as well as the shocks that hit the economy. For example, the value of the relationship is high when it is costly for the downstream firm to substitute a given intermediate input. Equilibrium trade credit is also affected by factors that govern the demand for funds by downstream firms. We allow some of these characteristics that affect the demand and supply of trade credit to differ across sectors in order to obtain a rich set of testable cross-sectional predictions.

We use this framework to study the macroeconomic implications of trade credit. To do so, we compare the behavior of our economy to that of a counterfactual "spot economy", which is identical in all respects to the former, with the exception that all economic entities engage in spot transactions. Focusing on a special case of the model that is analytically tractable, we present two sets of results.

First, we show that trade credit reduces the economic distortions due to financial frictions and brings the economy closer to the "first best", the allocation achieved when credit markets are frictionless.

To understand this first result, let us consider the spot economy. Here, downstream firms need credit at the beginning of the period to purchase intermediate inputs from their suppliers, who then use these resources to pay workers and to collect rents. Financial frictions lower the amount of credit that goes to final-good firms and, ultimately, they reduce what the economy can direct to remunerate the productive input (labor). This has the effect of decreasing the overall output produced.

In the economy with trade credit, suppliers can also receive payment at the end of the period. This has two main effects. First, a larger share of the payments made by downstream firms in the morning can be directed toward the payment of workers, since trade credit allows suppliers to shift the payments of their rents to the end of the period.[1] Second, suppliers can discount their accounts receivable—the trade credit bill issued to their customers—with banks and obtain liquid funds in the morning. This process allows the economy to increase the amount of credit that finances working capital, partly overcoming the distortions induced by financial frictions.

[1]This mechanism is closely linked to the "financial cost advantage of trade credit" explored in previous work by Garcia-Marin, Justel, and Schmidt-Eisenlohr (2019).

The second result is that trade credit can make the economy *more* sensitive to financial shocks. In the spot economy, a tightening of firms' borrowing constraints reduces output. In the trade credit economy, the effect depends on suppliers' borrowing capacity. If suppliers have unused borrowing capacity, they can absorb part of the shock by extending more trade credit and discounting the resulting receivables with banks, which dampens the effect on bank credit and output. If suppliers are themselves constrained, however, they may respond by cutting trade credit. This reduces their own borrowing capacity and further contracts trade credit, amplifying the effects of the original shock on both credit and output.

We quantify the model using Italian firm-level balance sheet data from the historical Orbis dataset. We aggregate this dataset at the sectoral level and collect data on the size of trade credit claims as well as other factors that in our model shape the cross-sectoral heterogeneity in trade credit, namely the share of intermediate-input costs over sales, the accounts receivable of the sector, and the market concentration of input suppliers. In our theory, the first two factors determine the demand for trade credit while the third factor affects the supply, as trade credit is better enforced when it is more costly for downstream firms to switch suppliers. Consistently, we find that sectors that have high working capital needs and that source intermediate inputs from more concentrated sectors obtain more trade credit from their suppliers.

We calibrate the model by fitting these sectoral characteristics, aggregate bank borrowing and its share backed by receivables, and the average decline in trade credit, bank credit, and output during the Great Recession. We show that the model replicates all these moments and that it fits well key cross-sectoral patterns out of sample, such as the cross-sectoral distribution of trade credit usage in normal times, and how different sectors behaved during the Great Recession. Specifically, an important prediction is that sectors characterized by more financially constrained suppliers should be relatively more sensitive to financial shocks, as in those cases suppliers cannot absorb the shock along the supply chain. Using the model to rank sectors along this dimension, we find that those connected to more financially constrained suppliers experienced larger declines in output during the Great Recession. This finding mirrors a large body of empirical work showing that suppliers' balance sheet positions have important implications for the customers via a trade credit channel (Garcia-Appendini and Montoriol-Garriga, 2013; Costello, 2020).[2]

[2]Using U.S. data, Garcia-Appendini and Montoriol-Garriga (2013) shows that suppliers that entered the Great Recession with more liquidity increased their trade credit during the crisis, while Costello (2020) finds that suppliers more exposed to liquidity shocks during the Great Recession reduced their trade credit supply. Other studies have documented substantial spillovers of suppliers' funding ability onto their customers' via a trade credit channel, see for instance Boissay and Gropp (2013), Adelino, Ferreira, Giannetti, and Pires (2023), Giannetti, Serrano-Velarde, and Tarantino (2021) and Bottazzi, Gopalakrishna, and Tebaldi (2023).

We use the calibrated model to quantify the aggregate implications of trade credit for the Italian economy. For that purpose, we perform two exercises. First, to assess the importance of the credit multiplier, we compare the steady state of our economy with that of the spot economy. We find that bank credit in the former is much larger, and it allows the economy with trade credit to support 14% more output. Second, we study the response to financial shocks. Relative to the spot economy, the economy with trade credit is 4.4 times as responsive to financial shocks of a depth and persistence similar to those observed during the Great Recession. We also consider a counterfactual that shuts down supplier trade credit while keeping fixed the timing of payments received by final-good firms. In this counterfactual, trade credit raises the output response during the Great Recession by about 30%.[3]

Literature. There is a large literature on trade credit in corporate finance. Researchers have documented that trade credit is one of the largest liabilities of firms across several countries. These loans have short maturity (typically between 30 and 90 days), they carry large implicit interest rates, and they are a fundamental tool used by firms to manage liquidity.[4] Several papers in this literature try to explain why we observe so much borrowing and lending between firms. Most of them build on the hypothesis that suppliers of intermediate goods have some advantages over financial institutions when lending to their customers (Biais and Gollier, 1997; Burkart and Ellingsen, 2004; Frank and Maksimovic, 1998). The paper closest to ours in this literature is Cuñat (2007), who proposes a theory in which trade credit is supported in equilibrium by suppliers' threat of cutting off customers from all future provisions of the input.[5] The main contribution of our paper is to incorporate this idea in a business cycle model with aggregate shocks and study the macroeconomic implications of trade credit.

Recent work incorporates trade credit into quantitative business cycle models to study the propagation of financial shocks. Altinoglu (2021) and Luo (2020) introduce trade credit relationships in the production network economy of Bigio and La'o (2020), but treat it as exogenous and unresponsive to aggregate shocks. Closer to our approach, Reischer (2024) and Shao (2025) develop quantitative models in which trade credit responds endogenously to aggregate conditions and study how it affects the transmission of financial disturbances. Relative to those papers, our main innovation is to micro-found trade credit as part of a

[3]Specifically, final-good firms continue to receive the same share of sales revenues with delay, but suppliers are not allowed to extend trade credit to them.

[4]See Amberg, Jacobson, Von Schedvin, and Townsend (2021) for evidence on the role of trade credit as a liquidity management tool.

[5]See Wilner (2000) and Brugues (2023) for related theories and empirical evidence. See also Clayton, Maggiori, and Schreger (2023) for a related idea in the context of power relationships across countries.

dynamic contract between firms.[6] This, in turn, leads to different economic trade-offs and mechanisms governing the supply of trade credit.

In Reischer (2024) and Shao (2025), trade credit is perfectly enforced and arises from a static financial decision from the supplier's perspective, equating the financial return on lending with their cost of liquidity. In contrast, in our model, suppliers extend credit because doing so allows their customers to operate closer to efficient scale, increasing the surplus of the supply chain. In equilibrium, this benefit is weighed against two costs: customers must receive sufficient continuation value to be incentivized to repay, and suppliers must bear the cost of providing liquidity. A key advantage of our micro-founded theory is that the forces sustaining trade credit are supported by extensive empirical evidence (Cuñat and Garcia-Appendini, 2012).

These differences in modeling strategy have important implications for how shocks and policies propagate. In our model, for example, changes in expected future fundamentals affect the value of relationships along the supply chain and influence trade credit: shocks that increase the value of relationships lead suppliers to extend more trade credit today and move production closer to efficiency. This channel operates conditional on any persistent shock, such as the credit supply shock we study. These effects are absent in existing quantitative models, where trade credit is determined by static considerations and does not depend on the continuation value of relationships.

The concept of a credit multiplier is related to the seminal work of Holmstrom and Tirole (1997). In that paper, a moral hazard problem limits the amount of funds that investors can give to firms. Banks, who have access to a monitoring technology, can borrow from the first group and lend to the second, a process that increases overall credit available. In our framework, credit is multiplied because suppliers can enforce repayment of trade credit *and* they discount their invoices with financial institutions. This mechanism has deep historical roots. The rise of negotiable bills of exchange in early modern Europe allowed merchants to finance trade by discounting claims with financial intermediaries, and these markets became a central part of commercial finance (Steinsson, 2025).[7] As we discuss in the paper, invoice discounting still represents a large fraction of short-term credit to non-financial firms around the world.

Our paper is also related to the literature on financial networks, which studies how linkages across agents shape the propagation of shocks. Seminal contributions such as

[6] In contemporaneous work, Miranda-Pinto and Zhang (2024) proposes a micro-foundation for trade credit based on input-quality uncertainty and costly monitoring to study how trade credit influences sectoral comovement. While such asymmetric-information models explain why trade credit may arise, they do not capture the dynamic, relational features we focus on.

[7] Importantly, the discounting of bills often occurred through endorsement, so previous holders remained liable if the debtor failed to pay (de Vries and van der Woude, 1997)—something our model can rationalize.

Allen and Gale (2000), Eisenberg and Noe (2001), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) emphasize how financial linkages can generate both risk-sharing and contagion. We contribute to this literature by making the strength of inter-firm financial linkages endogenous through trade credit relationships, allowing credit conditions within supplier-customer relationships to respond to aggregate shocks.

Finally, our paper is related to studies that have introduced optimal contracts in general equilibrium models with aggregate shocks.[8] The paper closest to ours is Cooley, Marimon, and Quadrini (2004), which studies the implications of limited enforcement of financial contracts for the propagation of aggregate technological shocks. We instead focus on trade credit and its role for the amplification of financial shocks.

Layout. The remainder of the paper is organized as follows. Section 2 presents the model environment and defines equilibrium. Section 3 analyzes a simplified one-sector version that is analytically tractable and derives the main theoretical results. Section 4 extends the analysis to the full multi-sector model and describes the numerical solution. Section 5 presents the quantitative analysis. Section 6 concludes.

2 The model

We consider an economy populated by infinitely lived households, firms, and banks. Households supply labor and consume a bundle of imperfectly substitutable goods. These final goods are produced by a continuum of firms that combine capital with intermediate inputs, while the intermediate inputs are produced by monopolists using labor. We will refer to the production process for a specific final good as a supply chain or sector. There is a lag between production and the full receipt of payments, so final-good firms need credit to pay for intermediate inputs. Credit is provided by competitive banks and by the suppliers of intermediate inputs. We describe the environment in detail in Section 2.1, define an equilibrium in Section 2.2, and discuss some of the simplifying assumptions we made in Section 2.3.

2.1 Environment

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. Uncertainty is described by a Markov process that takes finite values in the set $\mathcal{S}$. We denote by s_t the state of the process at time

[8] See, for instance, the work of Kehoe and Perri (2002), Dovis (2019), and Aguiar, Amador, and Gopinath (2009) for applications to international capital flows; Boldrin and Horvath (1995) and Souchier (2022) for the role of long-term wage contracts for labor market fluctuations; and Di Tella (2017), who studies how the optimal state contingency of financial contracts affects the financial amplification mechanism.

t, governed by the transition matrix $\pi(s_{t+1}|s_t)$, and denote by $s^t = (s_0, s_1, \dots, s_t)$ the history of states up to period t. All equilibrium variables are, in general, functions of the history s^t, but whenever no confusion is possible, we leave this dependence implicit.

Households. Households supply labor to firms at the competitive wage W_t and every period receive the profits from firms in the economy, Π_t. They use this income to purchase a continuum of imperfectly substitutable final goods $\{y_{i,t}\}$ at prices $\{p_{i,t}\}$. So, the budget constraint of the representative household is given by

$$\int p_{i,t} y_{i,t} di = W_t L_t + \Pi_t.$$

Households choose labor and final goods to maximize their lifetime utility,

$$U = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[C_t - \chi \frac{L_t^{1+\psi}}{1+\psi} \right] \right\},$$

where β is the rate of time preference, ψ is the inverse Frisch elasticity of labor supply, and C_t is a CES aggregator of final goods:

$$C_t = \left(\int y_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}.$$

This optimization problem yields two familiar optimality conditions, one for labor supply and one for the demand for final good i,

$$\chi L_t^{\psi} = W_t \quad (1)$$

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t} \right)^{-\gamma} C_t, \quad (2)$$

where P_t is the price index for the consumption bundle C_t, and we normalize it to 1.

Production. The production of final goods is carried out by a continuum of firms. These firms produce final goods using capital k and N_i imperfectly substitutable intermediate inputs. We denote by $\{x_{ij,t}\}$ the j-th variety of intermediate good used in the production of a final good of type i at date t. The technology to produce the final good is

$$y_{i,t} = k^{1-\eta_i} \left\{ \left[\sum_{j=1}^{N_i} x_{ij,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}^{\eta_i},$$

where σ is the elasticity of substitution across intermediate inputs and η_i governs the intermediate-input share of production. As in Atkeson and Burstein (2008), we assume that $\sigma > \gamma$; that is, intermediate inputs are more substitutable than final goods.

The intermediate inputs $x_{ij,t}$ are produced by monopolists using a linear technology with labor as the sole production input,

$$x_{ij,t} = l_{ij,t}.$$

There are two sources of technological heterogeneity across different supply chains: (i) the share of intermediate inputs in the production process, governed by η_i, and (ii) the number of intermediate-input suppliers, N_i.[9]

Financial markets. Each period is split into two stages. In the first stage (the morning), final-good firms receive the intermediate inputs, start the production process and obtain a fraction of their sales. In the second stage (the afternoon), final-good firms finish production and receive the remainder of their sales.

The lag between production and the receipt of sales is due to two factors. First, there is time to build: firms produce a fraction δ of output in the morning and the remainder in the afternoon.[10] Second, a fraction π_i of the morning sales is paid by households in the afternoon. Therefore, final-good firms receive only a fraction $\delta(1-\pi_i)$ of their overall sales in the morning. Because of these assumptions, final-good firms may need credit in the morning in order to purchase intermediate inputs. Credit can be obtained from two sources: competitive banks and the suppliers of intermediate inputs.

Competitive banks collect wages from households in the morning and offer loans to downstream and upstream firms. We denote by $b_i(s^t)$ the amount borrowed by final-good firms that produce final good i at time t for history s^t, which limits their morning spot payments to $\sum_{j=1}^{N_i} p_{ij}^s \leq \delta(1-\pi_i)rev_i + b_i$. Similarly, $b_{ij}(s^t)$ denotes the amount borrowed by the intermediate good monopolist that sells variety j to final-good firms of type i to cover their morning wage bill, $Wx_{ij} \leq p_{ij}^s + b_{ij}$. As in Bocola and Lorenzoni (2022), firms cannot commit to repaying the debt in the afternoon, and if they default, they suffer a penalty equal to a fraction $1-\theta(s_t)$ of their afternoon revenues. Given these assumptions, final-good firms have an incentive to repay their debt in the afternoon as long as

$$b_i(s^t) \leq [1-\theta(s_t)]\,[1-\delta(1-\pi_i)]\, rev_i(s^t), \tag{3}$$

[9]Cross-sector heterogeneity is not needed for our qualitative theoretical results, which we derive using a single-sector model in Section 3. It plays a role only in the quantitative analysis.

[10]We follow most of the macro literature and assume that time to build is exogenous. See Antràs (2023) for a recent paper that endogenizes this feature in a general equilibrium model.

where $rev_i(s^t) \equiv p_i(s^t)y_i(s^t)$ is the revenue of firm i in history s^t. As in Jermann and Quadrini (2012), we interpret $\theta(s_t)$ as a "financial shock": when $\theta(s_t)$ increases, final-good firms will face a tighter borrowing constraint with banks, and this will potentially impact their production choices.[11] These shocks are the only source of uncertainty in the model.

In addition to borrowing from banks, final-good firms can borrow from their suppliers of intermediate goods. That is, when selling a good $x_{ij}(s^t)$ to a final good firm, the supplier specifies a spot payment to be made in the morning, $p^{\text{s}}_{ij}(s^t)$, and a payment to be made in the afternoon, $p^{\text{tc}}_{ij}(s^t)$. Effectively, $p^{\text{tc}}_{ij}(s^t)$ is the *trade credit* offered by the supplier. We will discuss momentarily how the terms of the contract are determined.[12]

Depending on the terms of the contract, the upstream producers may also need to borrow from banks. This happens when the payments they receive in the morning are smaller than their costs of production (the wages of workers), that is when $p^{\text{s}}_{ij}(s^t) \leq W(s^t)x_{ij}(s^t)$. In that case, the upstream firm needs to borrow the difference from banks, facing the same incentive problem described earlier. Their borrowing constraint is thus

$$b_{ij}(s^t) \leq [1-\theta(s_t)]p^{\text{tc}}_{ij}(s^t). \tag{4}$$

That is, upstream firms can discount a fraction $[1-\theta(s_t)]$ of their afternoon revenues (their accounts receivable $p^{\text{tc}}_{ij}(s^t)$), and use the proceeds to pay for their workers.

Trade credit. The trade credit contract is the outcome of a long-term relationship between upstream and downstream firms operating in the supply chain i. Upstream firms make a take-it-or-leave-it offer to downstream firms, specifying the terms of the contract, $\{x_{ij}(s^t), p^{\text{s}}_{ij}(s^t), p^{\text{tc}}_{ij}(s^t)\}$ for all $\{s^t\}$. Unlike bank credit, firms that default on their trade credit do not suffer any direct loss of revenues. The enforcement of these contracts is instead guaranteed by a reputation mechanism: a final good firm that defaults at time t on its supplier is permanently excluded from purchasing that intermediate good from time $t+1$ onward.[13]

We denote by $J_i(s^t)$ the expected discounted value of a final good firm operating in

[11] An alternative is to model financial shocks as fluctuations in the interest rate at which firms borrow, as in Neumeyer and Perri (2005). We adopt the debt-limit formulation because, in our application, the quantity of bank credit to Italian firms fell substantially during the Great Recession while interest rate spreads applied to borrowers barely moved (Arellano, Bai, and Bocola, 2017).

[12] Trade credit in our framework is state-contingent, and can respond to the current realization of shocks. Hardy, Saffie, and Simonovska (2022) also model trade credit as a state-contingent contract in the context of a two-period small open economy, and study how the state-contingency allows trade credit to absorb international financial shocks; see also Hardy and Saffie (2024) for related empirical evidence.

[13] The threat made by the supplier is credible given our assumption that there is a continuum of final good producers.

sector i at time t under the contract,

$$J_i(s^t) = \sum_{\tau=0}^{\infty} \sum_{s^{t+\tau}} \beta^\tau \pi\left(s^{t+\tau} \middle| s^t\right) \left[rev_i(s^{t+\tau}) - \sum_{j=1}^{N_i} \left(p_{ij}^{\mathrm{s}}(s^{t+\tau}) + p_{ij}^{\mathrm{tc}}(s^{t+\tau}) \right) \right],$$

where $\pi\left(s^{t+\tau} \middle| s^t\right) = \prod_{l=1}^{\tau} \pi\left(s_{t+l}|s^{t+l-1}\right)$ is the probability of arriving at history $s^{t+\tau}$ from s^t. We denote by $J_i^{(-j)}(s^t)$ the corresponding value when the firm cannot purchase intermediate inputs from firm j starting at period t. The *value of the relationship* with supplier j for a downstream firm is then given by $\tilde{J}_i^j(s^t) \equiv J_i(s^t) - J_i^{(-j)}(s^t)$.

Since a permanent break in the trade relationship is the only cost of defaulting on trade credit, final-good firms will have an incentive to repay their trade credit as long as

$$p_{ij}^{\mathrm{tc}}(s^t) \leq \beta \mathbb{E}\left[\tilde{J}_i^j(s^{t+1})|s^t\right]. \tag{5}$$

The constraint in (5) is a key condition in the model. It says that the greater the value of the relationship between the customer and the supplier of variety j, the more trade credit the supplier can extend.[14]

Each supplier j chooses the optimal contract to maximize the present discounted value of its profits,

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[p_{ij}^{\mathrm{s}}(s^t) + p_{ij}^{\mathrm{tc}}(s^t) - W(s^t) x_{ij}(s^t) \right] \right\} \tag{6}$$

taking as given the wage $W(s^t)$, the consumers' demand for industry i goods in equation (2), the morning budget constraints, the no-default constraints (3) and (4), the trade credit constraint (5), the participation constraint of the final good producer, $\tilde{J}_i^j(s^{t+1}) \geq 0 \; \forall s^{t+1}$, and the feasibility requirement that $rev_i(s^t) - \sum_{j=1}^{N_i} \left[p_{ij}^{\mathrm{s}}(s^t) + p_{ij}^{\mathrm{tc}}(s^t) \right] \geq 0 \; \forall s^t, j$.[15] The suppliers of intermediate goods take as given aggregate quantities and the trade credit contracts of all other suppliers but internalize how their trade credit contract affects $y_i(s^t)$ and $p_i(s^t)$.

[14] In our economy trade credit is always repaid in equilibrium. This is an abstraction: in practice, firms do default on these claims. See Kiyotaki and Moore (1997a) for a theoretical model discussing how default on trade credit along the supply chain affects output, and Mateos-Planas and Seccia (2021) for a recent quantitative model of this phenomenon.

[15] In principle, we should also include the incentive constraints that allow for multiple deviations (e.g., final-good firms defaulting on both banks and firms or defaulting on multiple firms at the same time). If final-good firms were to default on banks and a supplier j, they would lose $[1 - \theta(s_t)] [1 - \delta(1 - \pi_i)] \, rev_i(s^t)$ and the expected discounted surplus of the match with supplier j. So, if (3) and (5) are satisfied, there is no incentive to default on both. It is possible to show that in a deterministic steady state, the trade credit constraints (5) also imply that firms have no incentives to default on the other suppliers as long as $\sigma > \gamma$. This is also true outside the steady state in all our numerical experiments.

2.2 Equilibrium

We restrict attention to a symmetric equilibrium in which all suppliers in a sector offer the same trade credit contract. A symmetric equilibrium is a set of aggregate variables $\{L(S^t),$ $C(s^t)$, $W(s^t)$, $\Pi(s^t)\}$ for all s^t, final good firm quantities and prices $\{y_i(s^t), p_i(s^t)\}$ for all i and s^t, and intermediate-input firm quantities and prices $\{x_i(s^t), p_i^{\mathrm{s}}(s^t), p_i^{\mathrm{tc}}(s^t)\}$ for all i and s^t, such that:[16]

1. Given aggregate prices and profits, aggregate consumption and labor solve the household's problem.
2. Given aggregate prices and quantities as well as the trade credit contracts of all other suppliers, the set $\{y_i(s^t), p_i(s^t), x_i(s^t), p_i^{\mathrm{s}}(s^t), p_i^{\mathrm{tc}}(s^t)\}$ solve the problem of the intermediate goods firm supplying to firm i.
3. Goods markets clear, aggregate profits equal the sum of all profits in the economy, and the labor market clears for every history s^t:

$$L(s^t) = \int_i N_i x_i(s^t) di.$$

The equilibrium concept follows the structure of a dynamic Nash-in-Nash framework in bilateral contracting environments: each supplier chooses a contract taking as given aggregate variables and the continuation values implied by the contracts of all other suppliers, and equilibrium requires a fixed point across these bilateral best responses. Unlike canonical Nash-in-Nash models, however, bilateral surplus is not divided through Nash bargaining. Instead, the supplier makes a take-it-or-leave-it offer, which corresponds to the limiting case of Nash bargaining with the supplier's bargaining weight equal to one.

2.3 Discussion

Before moving on, let us discuss some of the key assumptions of our model.

First, our model features an asymmetry in the enforcement of bank and trade credit: a firm that defaults on a bank loan loses part of its revenues, a legal punishment that is not present when a firm defaults on trade credit. This assumption captures, in a stylized fashion, the fact that bank credit typically carries much better legal protections than trade credit, see Jacobson and Von Schedvin (2015) for a discussion.[17]

[16] We do not include a j subscript for intermediate-input goods because in a symmetric equilibrium, all suppliers to firm i will have the same quantities and prices.

[17] The assumption of a static penalty when defaulting on banks is made for tractability. We could allow for a dynamic punishment by assuming that a firm that defaults on banks is temporarily excluded from financial

The enforcement of trade credit in our model is instead supported by a reputation mechanism. There is ample empirical evidence consistent with this idea (Antras and Foley, 2015; Benguria, Garcia-Marin, and Schmidt-Eisenlohr, 2023). In our model, these financial links are supported by the fact that relationships between a supplier and a customer are valuable and that they will be destroyed in the event a customer defaults on the supplier. This mechanism is supported by a number of studies that have documented large economic costs when a customer loses a link to a supplier (Barrot and Sauvagnat, 2016; Carvalho, Nirei, Saito, and Tahbaz-Salehi, 2021). In addition, Garcia-Marin, Justel, and Schmidt-Eisenlohr (2019) show that suppliers with larger markups provide more trade credit to their customers, consistent with the idea that the higher the market power of the supplier, the more incentives for customers to service trade credit.

Second, credit between firms in our economy flows only from suppliers to customers. In practice, suppliers occasionally receive advances from their customers before supplying the good, an arrangement referred to as a cash-in-advance contract.[18] It is straightforward to modify our framework to generate this pattern, for example by introducing time to build on the side of the suppliers: if suppliers' borrowing needs are sufficiently strong, the optimal contract would feature credit flowing to upstream firms. Trade credit is, however, the dominant arrangement in the available evidence: Garcia-Marin, Justel, and Schmidt-Eisenlohr (2019) document that nearly 90% of inter-firm transactions in Chilean export data are paid via trade credit, compared with about 5% paid cash in advance.

Third, firms in our framework cannot accumulate cash holdings. In making this assumption, we stay close to the literature that studies the misallocation of production due to financial frictions in a multi-sector framework, see Bigio and La'o (2020) and the literature that followed. It is well known that allowing firms to accumulate cash holdings reduces the costs of financial constraints. However, the key qualitative insights of our paper survive in this setting as long as financial constraints are binding in the economy where firms can accumulate cash.

Fourth, suppliers within each sector are assumed to be homogeneous, with identical financial conditions and the same trade credit contract. This abstracts from within-sector heterogeneity in supplier balance sheets, which would allow customers to substitute toward less-constrained suppliers and absorb idiosyncratic supplier shocks. Modeling this margin would expand the state space by replacing the per-sector continuation value with a distribution across heterogeneous suppliers, and we leave it for future research. Con-

markets as in Arellano (2008), which would make the debt limits (3) and (4) depend on future values. The key insights of our analysis would go through in this version of the model.

[18]See Bigio, Méndez, and Van Patten (2026) for a model that features this type of contract and an endogenous network of payments.

ditional on supplier homogeneity, the number of suppliers N_i plays a multi-faceted role. Together with the elasticity of substitution σ, it determines the revenue loss to a customer in sector i from losing access to a single supplier, which shapes both the customer's outside option and the enforcement of trade credit. With $N_i = 1$, the supplier extracts all the rents when the customer is unconstrained; with $N_i > 1$, the customer retains a share of the surplus, and the trade credit a relationship can support shrinks accordingly, since the punishment for defaulting is the loss of one supplier among many.

3 Macroeconomic implications of trade credit

We now analyze the macroeconomic implications of trade credit. For this purpose, we compare the predictions of our economy to those of a "spot" economy—identical in all respects except that suppliers cannot issue trade credit to final-good firms.

We start in Section 3.1 by characterizing the equilibrium of the spot economy, while in Section 3.2 we focus on the economy with trade credit. We derive two main results. First, we show that trade credit relationships allow the economy to support a higher level of output on average due to two forces: increasing the amount of credit flowing from banks to firms *and* allocating it toward more productive uses. Second, we show that trade credit can dampen or amplify the macroeconomic effects of financial shocks depending on the borrowing capacity of suppliers. Section 3.3 discusses some extensions of our framework.

To make the analysis transparent, we consider a special case of our model that is analytically tractable. We focus on an economy with only one sector and one supplier of intermediate inputs, $N_i = 1$. We normalize the level of capital of the final-good firms to 1 so that the production function is $y = x^\eta$.[19] We further assume that $\pi_i = 0$, so final-good firms receive a fraction δ of their revenues in the morning and the remainder in the afternoon. In addition, we set the inverse of the Frisch elasticity to zero, $\psi = 0$. This last assumption implies that the wage is given by $W = \chi$. Due to these assumptions, we can fully characterize the equilibrium by solving the decision problem of the monopolist supplier.

3.1 The spot economy

In the spot economy, final-good firms need to pay their suppliers in the morning. They can do that out of the cash they receive in the morning, $\delta x^\eta(s^t)$, or by borrowing from banks with a debt limit given by $[1 - \theta(s_t)](1 - \delta)x^\eta(s^t)$. This implies that the payment

[19] In a slight abuse of notation, we do not index variables by sector i and the identity of the supplier j as we did in Section 2 because there is only one sector and one supplier in this example.

that final-good firms make in the morning to the supplier, $p^s(s^t)$, cannot be larger than $\{\delta + [1-\theta(s_t)](1-\delta)\}\, x^\eta(s^t)$.

The supplier chooses $\{x(s^t), p^s(s^t)\}$ to maximize the present discounted value of profits subject to the borrowing constraints of the final-good firms and their participation constraints. Since $N_i = 1$, the participation constraints boil down to $J(s^{t+1}) \geq 0$ for all s^{t+1}. This problem is equivalent to solving a static profit maximization problem, which yields the first-order condition

$$[\delta + (1-\theta)(1-\delta)]\eta x^{\eta-1} = W. \tag{7}$$

Suppose first that credit markets are frictionless, i.e., $\theta = 0$. In this case, the supplier chooses labor to equate its marginal product to the wage, $\eta x^{\eta-1} = W$, and it extracts all the rents from the final good producers by setting $p^s = x^\eta$.[20] In this case there are no output distortions even though the supplier has monopoly power, because the latter can set prices non-linearly.[21]

The above solution is not feasible when $\theta > 0$. Due to the borrowing constraints, final-good firms cannot borrow in the morning the entire revenue stream, so the supplier cannot extract all the rents. This induces a "wedge", equal to $[\delta + (1-\theta)(1-\delta)]$, between the marginal product of labor and the wage, distorting down the scale of production.

Using equation (7), we can further characterize the impulse response function (IRF) of output to a financial shock

$$\varepsilon_{y,\theta}^{\text{spot}} \equiv \frac{d \ln y^{\text{spot}}}{d \ln \theta} = -\frac{\eta}{1-\eta}\frac{\theta}{1-\theta}\alpha^{\text{spot}}, \tag{8}$$

where $\alpha^{\text{spot}} \equiv \frac{(1-\theta)(1-\delta)}{\delta+(1-\theta)(1-\delta)}$ is the share of bank credit over the funds available to final-good firms in the morning. This is a notion of *leverage* for the operations of the supply chain.

When θ increases, the supply of bank credit to final-good firms decreases, and so do the funds available in the morning to purchase intermediate inputs. This leads to a reduction in output in the economy, $\varepsilon_{y,\theta}^{\text{spot}} < 0$.

Importantly, the strength of this channel depends on the leverage of the supply chain, as captured by the term α^{spot} in equation (8). This term ranges between 0 and 1, depending on the value of δ. When δ is close to one, α^{spot} is close to zero and financial shocks have limited effects on output. This is because final-good firms in this case receive most of their revenues in the morning, so bank credit is not critical for financing the supply chain.

[20] Because the intermediate good is produced one-for-one with labor, $\eta x^{\eta-1}$ is also the marginal product of labor.

[21] For a discussion of how non-linear pricing breaks the link between markups and misallocation, see Bornstein and Peter (2025).

In contrast, when δ is close to zero, bank credit is essential for financing the purchase of intermediate inputs, so a credit contraction has a greater impact on output.

3.2 The economy with trade credit

In the trade credit economy the supplier has one more instrument to maximize profits: allowing its customers to delay payments to the afternoon, up to the trade credit limit (5). This constraint makes the decision problem dynamic, because future rents promised to customers influence how much trade credit can be supported today.

We can write this decision problem using recursive methods. Let J be the rents that the supplier has promised to a final good firm after history s^t, and let θ be the realization of the financial shock at s_t. The supplier chooses labor x, the morning and afternoon payments $\{p^{\mathrm{s}}, p^{\mathrm{tc}}\}$, as well as the future continuation values for final-good firms conditional on any realization of the financial shock next period, $J'(\theta')$, to maximize the present discounted value of profits,

$$V(J,\theta) = \max_{x, p^{\mathrm{s}}, p^{\mathrm{tc}}, J'(\theta')} \left(p^{\mathrm{s}} + p^{\mathrm{tc}} - Wx\right) + \beta \mathbb{E}\left[V(J'(\theta'),\theta')|J,\theta\right].$$

subject to the debt limits of the final-good firms and the supplier,

$$\begin{aligned} p^{\mathrm{s}} &\leq [\delta + (1-\delta)(1-\theta)]x^{\eta}, && (9)\\ Wx - p^{\mathrm{s}} &\leq (1-\theta)p^{\mathrm{tc}}, && (10) \end{aligned}$$

the trade credit limit,

$$p^{\mathrm{tc}} \leq \beta \mathbb{E}[J'(\theta')|J,\theta], \tag{11}$$

the promise-keeping constraint,

$$J = x^{\eta} - (p^{\mathrm{s}} + p^{\mathrm{tc}}) + \beta \mathbb{E}[J'(\theta')|J,\theta], \tag{12}$$

the participation constraints $J'(\theta') \geq 0$, and the feasibility requirement that $p^{\mathrm{s}} + p^{\mathrm{tc}} \leq x^{\eta}$.

The outcome of this problem is a policy function $\{x(J,\theta), p^{\mathrm{s}}(J,\theta), p^{\mathrm{tc}}(J,\theta)\}$ and a law of motion for future promises for each possible state θ', $J'(\theta'|J,\theta)$. Appendix A.1 derives the first-order conditions of this problem and discusses the key economic trade-off faced by the supplier. In what follows, we first characterize the deterministic steady state, and then move to analyze how financial shocks propagate in this economy.

The steady state. We start by considering the case in which θ_t is deterministic and constant over time, $\theta_t = \theta$ for all t. The next proposition characterizes the steady state of the model: the limit of the optimal contract as $t \to \infty$.

Proposition 1. *Consider the problem of the supplier and let $t \to \infty$. Let $\bar{\theta}$ be the value of θ such that $\delta + (1-\theta)(1-\delta)(1+\beta\theta) = \eta$. Then:*

1. *If $\theta = 0$, the optimal contract converges to*

$$x = \left[\frac{\eta}{W}\right]^{\frac{1}{1-\eta}}, \qquad p^s = x^\eta$$
$$p^{tc} = 0, \qquad J' = 0. \tag{13}$$

2. *If $\theta \in (0, \bar{\theta}]$, the optimal contract converges to*

$$x = \left[\frac{\eta}{W}\right]^{\frac{1}{1-\eta}}, \qquad p^s = [\delta + (1-\theta)(1-\delta)]x^\eta$$
$$p^{tc} = \beta(1-\delta)\theta x^\eta, \qquad J' = (1-\delta)\theta x^\eta. \tag{14}$$

3. *If $\theta \in (\bar{\theta}, 1]$, the optimal contract converges to*

$$x = \left[\frac{\delta + (1-\theta)(1-\delta)(1+\beta\theta)}{W}\right]^{\frac{1}{1-\eta}}, \qquad p^s = [\delta + (1-\theta)(1-\delta)]x^\eta$$
$$p^{tc} = \beta(1-\delta)\theta x^\eta, \qquad J' = (1-\delta)\theta x^\eta. \tag{15}$$

Figure 1 illustrates numerically this result by plotting the steady state level of output, bank credit, and trade credit in the benchmark economy as a function of θ. In the middle panel, the solid line illustrates total bank credit, and the dashed line bank credit to final-good firms. We can distinguish three regions of the parameter space.

In the first region, $\theta = 0$, the optimal contract coincides with that of the spot economy: there are no output distortions in steady state, the supplier takes all the rents from the relationship, and no trade credit is offered, $p^{\text{tc}} = 0$. The supplier does not extend trade credit because this is "expensive" from their point of view, as they need to promise future rents to final-good firms in order to incentivize repayment. So, when there are no financial frictions ($\theta = 0$), the supplier only asks for spot payments and final-good firms pay by using their cash in hand (morning revenues) and by borrowing from banks. This property is consistent with the "pecking order" theory in corporate finance (Petersen and Rajan, 1997), whereby trade credit will be used as a last resort in the financing of the supply chain, only when the customers have exhausted the other means of payment—cash or bank credit.

In the second region, $\theta \in (0, \bar{\theta}]$, output is still at the first-best. However, in order to support this level of production, the supplier extends trade credit, $p^{\text{tc}} > 0$. In this region, θ only affects the financing of the supply chain, but not the output produced. As we can see from Figure 1, an economy with a higher θ in this region has less credit flowing from banks to final-good firms and more credit flowing from the supplier to customers.

Importantly, in this second region, trade credit increases with θ. This raises the question: how can the supplier finance its morning input costs when an increasing share of its revenues is delayed until the afternoon? The answer is that the supplier can obtain liquidity by discounting their accounts receivable with banks. This can be seen by looking at the difference between the dashed and solid lines in the middle panel of Figure 1, which equals bank credit flowing to the supplier.

For sufficiently large values of θ, the borrowing constraint of the supplier binds. This happens when θ reaches $\bar{\theta}$. From that point on, the economy falls in the third region, in which output is distorted down relative to the first-best. In this region, higher levels of θ are associated with lower bank *and* trade credit, and lower output levels. This is because a higher θ tightens the supplier's borrowing constraint, and the supplier responds by cutting trade credit to customers.

The red lines in Figure 1 report the steady state of the spot economy. The trade credit economy always has a higher level of output relative to the spot economy, with the distance between the two economies increasing in θ when $\theta < \bar{\theta}$, and decreasing otherwise. Thus, trade credit relationships alleviate the economic costs of credit market frictions, especially when the supplier is unconstrained.

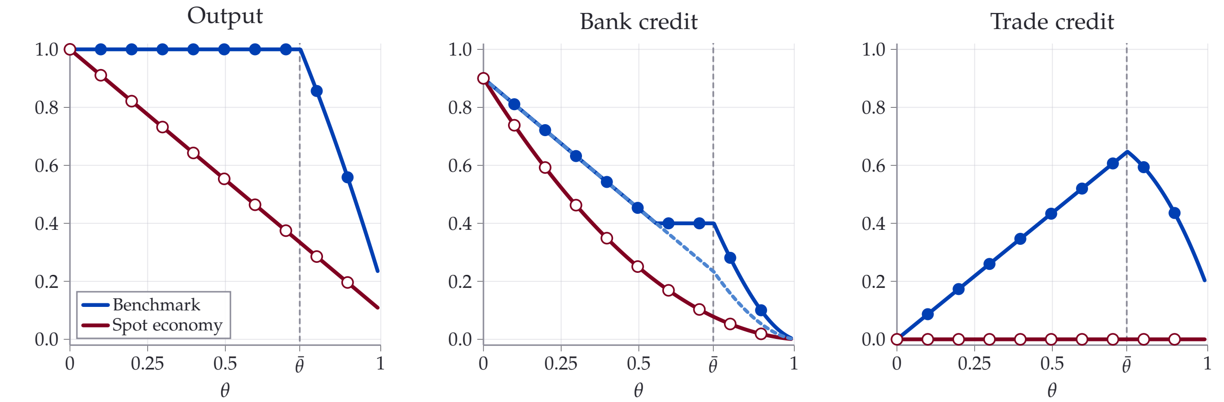
Why is the trade credit economy producing more output in steady state than the spot economy? This difference is driven by two forces.

First, the supplier in the trade credit economy receives a larger share of revenues relative to what they get in the spot economy. In the latter, payments to the supplier only occur in the morning, so they cannot exceed a fraction $\delta + (1 - \theta)(1 - \delta)$ of revenues. In the trade credit economy, instead, payments to the supplier also occur in the afternoon, which allows the supplier to capture a greater share of the revenues of the supply chain. This raises the incentives of operating the supply chain at its efficient level.

Second, in the trade credit economy banks end up financing, directly and indirectly, more working capital relative to what they do in the spot economy. To understand this last point, it is useful to compare the financing of the supply chain in the two economies.

In the spot economy, bank credit flows only from banks to final-good firms. Final-good

Figure 1: The deterministic steady state



Notes: For the numerical illustration, we set $\beta = 0.97$, $\eta = 0.5$, $\chi = 0.5$, and $\delta = 0.1$. The solid blue line reports the steady state level of output, total bank credit, and trade credit to final-good firms in the benchmark economy for different values of θ. The red line reports the same values for the spot economy. The dashed blue line in the middle panel reports bank credit to final-good firms, excluding bank credit given to suppliers.

firms pay for intermediate goods in the morning, and the supplier uses these funds for two purposes: paying for their production costs (the workers' wages) and remunerating their rents. Effectively, the payments of the supplier's rents in the morning crowd out the resources that can be directed to the payments of the productive input, labor.

The economy with trade credit can channel more resources to remunerate labor. This happens both because the economy as a whole can sustain a larger amount of bank credit *and* because a larger share of it can be directed toward labor. On the one hand, in the trade credit economy the supplier borrows up to a fraction $(1-\theta)$ of its accounts receivable; this process of discounting trade credit bills increases the overall amount of bank credit. On the other hand, the supplier can shift part of the payments of its rents to the afternoon, reducing the crowding out effects.

Thus, trade credit relationships act as a *credit multiplier*, increasing the overall amount of liquid funds that supply chains can direct to the payments of inputs. The size of this multiplier is critical to quantify the additional output that can be produced thanks to trade credit relationships. To clarify this point, in the region in which suppliers are borrowing constrained, we can express the difference between the output produced in the economy with trade credit and the one produced in the spot economy as

$$\log\left(y^{\text{tc}}/y^{\text{spot}}\right) = \frac{\eta}{1-\eta}\left\{\log\frac{1}{\eta} + \log\left[1 + \frac{(1-\theta)(1-\delta)\beta\theta}{\delta + (1-\theta)(1-\delta)}\right]\right\}.$$

From the above expression we can see that the macroeconomic effects of trade credit are governed by two forces. The first term, $\log\frac{1}{\eta}$, captures the fact that trade credit allows the supplier to shift part of its rents from the morning to the afternoon, freeing up morning liquidity for wage payments. The second term captures the additional liquidity generated when suppliers discount trade-credit bills with banks.

Trade credit and the response to financial shocks. We now turn to study how trade credit affects the propagation of financial shocks. To this purpose, we assume that θ can take two values: $\{\theta_L, \theta_L + \varepsilon\}$ with $\varepsilon > 0$ and transition matrix $p(\theta'|\theta)$. We then study how output responds to a tightening of credit standards: a switch from the low to the high θ state. By comparing this response to that of the spot economy, derived in equation (8), we can assess whether trade credit dampens or amplifies the economic effects of financial shocks.

In the economy with trade credit, the output effects of financial shocks depend on whether the supplier is financially constrained. The following result characterizes the IRF when the borrowing constraint of the supplier does not bind.

Proposition 2. *Suppose that, for each state $\theta \in \{\theta_L, \theta_H\}$,*

$$\eta \leq 1 - \theta(1-\delta) + (1-\theta)\beta(1-\delta)\mathbb{E}[\theta'|\theta].$$

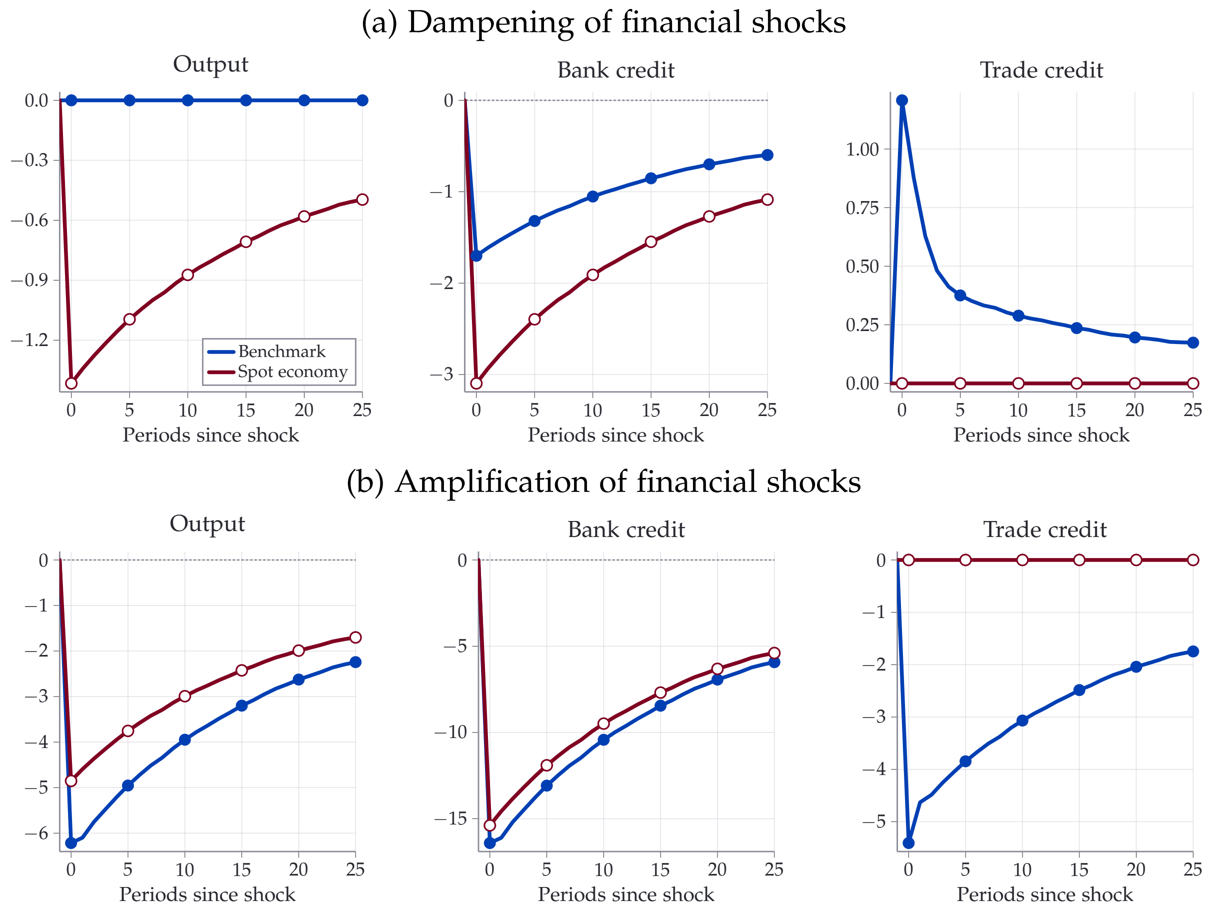
Then the first-best level of output can be supported in both states. In particular, after a switch from θ_L to θ_H, output does not change.

When $\theta_L < \bar{\theta}$ the financial constraint of the supplier does not bind in steady state. In this scenario, a small financial shock has no effects on the operations of the supply chain: after the tightening of bank credit, the supplier extends more trade credit to customers so that the economy can support the same level of production. The solid lines in panel (a) of Figure 2 provide a numerical illustration of this case by plotting the IRFs to the financial shock. As a reference, the line with circles reports the IRFs for the spot economy. As we can see, the presence of trade credit *dampens* the output effect of financial shocks relative to what happens in the economy without trade credit.

When $\theta_L > \bar{\theta}$, the borrowing constraint of the supplier binds in steady state. In this region, the supplier cannot provide more liquidity to its customers, so the decline in bank credit leads to a fall in output. Is the effect smaller or larger relative to what happens in the spot economy? The answer to that question will typically depend on the model parameters. Panel (b) of Figure 2 provides a numerical illustration of a case in which trade credit *amplifies* the negative effects of the financial shock. In that illustration, we can see that bank credit and output fall more in the economy with trade credit than in the spot economy.

To understand why trade credit can amplify the effects of financial shocks on output

Figure 2: Impulse response functions to a financial shock



Notes: For the numerical illustration, we set $\beta = 0.97$, $\eta = 0.5$, $\chi = 0.5$, $\delta = 0.1$, $\varepsilon = 0.01$, $p(\theta_L|\theta_L) = 0.99$, and $p(\theta_L + \varepsilon|\theta_L + \varepsilon) = 0.95$. For the top panel, we set $\theta_L = 0.4$, while for the bottom panel we set $\theta_L = 0.9$. The solid blue line reports the response of output, total bank credit, and trade credit to final-good firms in log-deviations (pts) from their steady state values in the benchmark economy. The red line reports the same information for the spot economy. Impulse response functions are computed by simulations following the methodology in Koop, Pesaran, and Potter (1996).

when $\theta_L > \bar{\theta}$, it is useful to consider a perturbation of the steady state level of output with respect to θ in this region of the parameter space. Using the expression for x in Proposition 1 and rearranging terms, we obtain

$$\varepsilon^{\text{tc}}_{y,\theta} = -\frac{\eta}{1-\eta}\frac{\theta}{1-\theta}\alpha^{tc} + \frac{\eta}{(1-\eta)}\frac{(1-\theta)\beta(1-\delta)\theta}{\delta + (1-\theta)(1-\delta)(1+\beta\theta)}, \tag{16}$$

where $\alpha^{\text{tc}} \equiv \frac{(1-\theta)(1-\delta)(1+\beta\theta)}{\delta+(1-\theta)(1-\delta)(1+\beta\theta)}$ is the ratio between the amount of bank credit going to *all firms* in the economy (both final-good firms and the supplier) over their total funds available in the morning. This is the analogue of the notion of leverage of the supply chain we have introduced for the spot economy in Section 3.1.

The first term on the right hand side of equation (16) captures the same mechanism present in the spot economy and described in equation (8): a fall in credit supply has larger effects on output the more levered the supply chain is. Importantly, the trade credit economy is more levered than the spot economy, $\alpha^{\text{tc}} > \alpha^{\text{spot}}$: in the former, suppliers discount their accounts receivable with banks, a force that increases the quantity of bank credit available in the morning to finance wages. As a result, a negative shock to bank credit supply has larger output effects in the trade credit economy due to this leverage effect.

The second term on the right hand side of equation (16) captures a countervailing force. Holding x constant, an increase in θ increases the surplus that final-good firms obtain in equilibrium, see the expression for J in (15). *Ceteris paribus*, this force allows for a better enforcement of trade credit, loosening the trade credit constraint, and mitigating the effect of the financial shock on output.

Which force dominates depends on model parameters. In the steady state, one can show that $\left|\varepsilon^{\text{tc}}_{y,\theta}\right| > \left|\varepsilon^{\text{spot}}_{y,\theta}\right|$ if and only if $\theta > \frac{1}{1+\sqrt{\delta}}$.

3.3 Extensions

We now dig deeper into some of the key mechanisms of the model and study their robustness with respect to some of our modeling assumptions.

The role of expectations. In our theory, trade credit is sustained through the future value of the bilateral relationship between the customer and the supplier. The forward-looking nature of this constraint implies that news about the future can affect how much trade credit suppliers can offer to their customers, and impact output via this channel. In what follows, we illustrate this point by studying the effect of changes in the discount factor β, as this parameter directly affects the future values of firms.

Consider first comparing the steady state for two different levels of the discount factor, which can be done using the results in Proposition 1. From there, we can see that a lower discount factor has two effects on the allocation. First, it directly reduces the present value of the future relationship surplus, $\beta\mathbb{E}[J'(\theta')]$, which tightens the trade credit limit (A.3). If the supplier is currently constrained, that is if $\theta > \bar{\theta}$, the tightening of the trade credit limit results in lower output in the economy. A reduction in the discount factor does not only reduce output within the constrained region, but also widens the size of the constrained region. This is because $\bar{\theta}$ is increasing in β.

This discussion highlights the importance of expectations about the value of future relationships as a potentially important driver of trade credit and output. A decline in β makes future relationships less valuable today, tightening current trade credit limits even when current credit market conditions, represented by θ, remain unchanged. This mechanism is absent in models where trade credit is governed by a static reduced-form constraint, as in these environments only current conditions affect trade credit availability.[22]

This expectation channel also affects the transmission of shocks other than the financial shock studied here, such as changes in real interest rates or news about future productivity. Unlike the spot economy, where such shocks have no immediate effect, the trade credit economy responds on impact through adjustments in inter-firm credit relationships.

The role of commitment and of the bargaining protocol. Next, we examine the role of commitment and the bargaining protocol. In our baseline model, suppliers make take-it-or-leave-it offers and can commit to future prices and quantities. Commitment allows suppliers to respond to adverse shocks by promising to deliver more surplus to their customers in the future, thereby expanding current trade credit capacity. We now show that the qualitative predictions of the model extend to an environment without commitment.

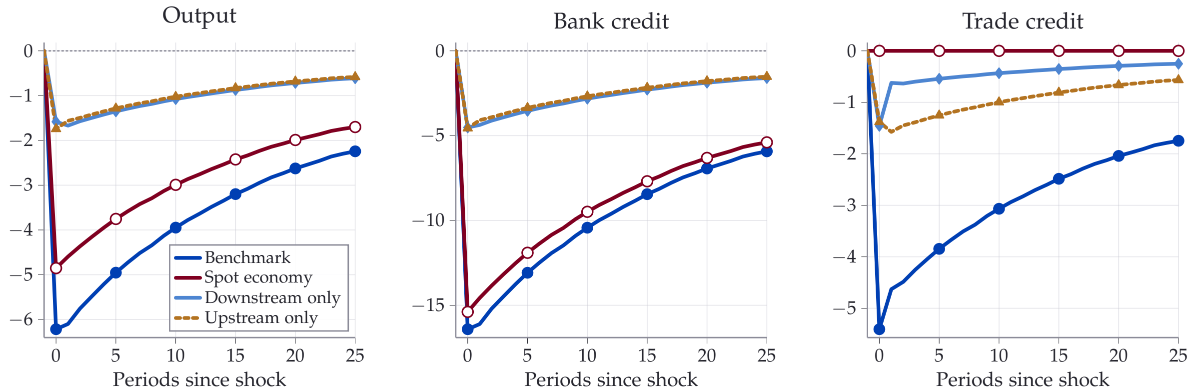
We make two modifications to our framework. First, we assume that suppliers cannot commit to future prices and quantities, and consider the Markov Perfect Equilibrium (MPE). In this environment, suppliers take the future surplus of their customers as given, as it is no longer a choice variable. Second, we consider an environment in which suppliers post per-unit prices for spot and trade credit payments, while the customers choose quantities. Appendix A.5 provides the full characterization.

Like the baseline model, the MPE environment features three regions: (i) a region where trade credit is not used, (ii) a region where the unconstrained level of production is sustained with positive trade credit, and (iii) a region where trade credit is at its limit and output falls below its unconstrained level. Trade credit continues to allow the economy to sustain higher output than the spot economy, and the economy can exhibit either smoothing or amplification of financial shocks via these mechanisms. So, qualitatively, all the predictions of our model survive in this setup. The main difference is quantitative: the region in which the unconstrained level of output is attainable is smaller without commitment. This occurs because committed suppliers can promise larger future surpluses to customers, relaxing current trade credit constraints.

An important remark here is that this version without commitment, while missing some important margins that suppliers can exploit to incentivize repayments, is substantially

[22] Bocola and Fornaro (2026) highlight the importance of this mechanism for global trade and productivity. In their model, changes in global interest rates have negative spillovers through trade credit because they reduce the present value of trading relationships. The reduction in trade credit, in turn, hampers gross trade flows and reduces productivity for the global economy.

Figure 3: Financial shocks to suppliers vs. buyers



Notes: For the numerical illustration, we set $\beta = 0.97$, $\eta = 0.5$, $\chi = 0.5$, $\delta = 0.1$, $\theta_L = 0.9$, $\varepsilon = 0.01$, $p(\theta_L|\theta_L) = 0.99$, and $p(\theta_L + \varepsilon|\theta_L + \varepsilon) = 0.95$. This figure shows how the baseline economy responds to an isolated financial shock to downstream firms (solid light blue) and upstream firms (dashed orange line). The solid blue and red lines report the responses in the benchmark and spot economies, respectively, where the financial shock affects all firms.

more tractable and easier to solve than our benchmark model: in the MPE, one does not need to keep track of promised utility as a state variable, which notably simplifies the computation of the multi-sector economy.

Financial shocks to suppliers vs. buyers. So far, we have studied the response of the economy to aggregate financial shocks that affect all firms symmetrically. We now distinguish between shocks that affect suppliers and shocks that affect their customers. Let θ_u denote the financial friction faced by suppliers and θ_d the friction faced by downstream final-good firms. In the benchmark economy, financial shocks raise both frictions simultaneously. Here, instead, we consider two alternative economies: one in which only θ_d varies while θ_u remains fixed, and another in which only θ_u varies while θ_d remains fixed.

We focus on the parameterization from panel (b) of Figure 2, where trade credit amplifies aggregate financial shocks. Figure 3 reports the impulse responses in these two alternative economies. The solid light blue line corresponds to a shock that affects only downstream firms. A rise in θ_d reduces the ability of final-good firms to pay suppliers upfront, increasing their demand for trade credit. At the same time, tighter financial constraints reduce the surplus generated by customer-supplier relationships, lowering the continuation value J. Because trade credit is constrained by relationship value, the decline in J tightens trade credit limits even though suppliers' own borrowing conditions (θ_u) are unchanged. This mechanism highlights a complementarity between bank credit and trade credit operating through the customer side of the relationship.

The dashed orange line shows the response to a shock that affects only suppliers ($\theta_u \uparrow$), holding θ_d fixed. In this case, suppliers become less able to discount trade credit claims with banks, forcing a contraction in trade credit provision. Overall, the shock to suppliers leads to a contraction in output.

Taken together, these results reveal two distinct complementarities between bank credit and trade credit. The first operates through relationship value: financial distress among customers lowers the surplus generated by the relationship, which in turn tightens endogenous trade credit limits. The second operates through suppliers' financing conditions: tighter bank credit directly limits suppliers' ability to extend trade credit. In the quantitative analysis, we focus on aggregate shocks that simultaneously affect both θ_d and θ_u, but the decomposition here clarifies that both channels contribute to the amplification of financial shocks.

4 Sectoral heterogeneity and computations

The special case studied in the previous section abstracts from two main ingredients that are instead present in our model. First, the economy of Section 2 has many supply chains with heterogeneous characteristics, each with potentially many suppliers. Second, the fully fledged model has general equilibrium forces.

Despite these differences, most of the results we discussed in the previous section extend to the full model. Indeed, the decision problem of one supplier operating in sector i is mathematically very similar to the monopolist case studied in Section 3, and it can be written recursively in terms of promised surplus for the final-good firms $\tilde{J}_i$ and the aggregate financial shock θ. While we leave the analysis of that problem to Appendix A.2, we now discuss some properties of the optimal contract, starting from the deterministic steady state.

4.1 The deterministic steady state

The following proposition characterizes the deterministic steady state of the optimal trade credit contract offered by a supplier in sector i.

Proposition 3. *Fix (W, C) and let*

$$rev_i(x) = C^{\frac{1}{\gamma}} k^{\frac{\gamma-1}{\gamma}(1-\eta_i)} (x)^{\eta_i \frac{\gamma-1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma}}$$

be the revenues of final-good firms operating in sector i as a function of x. Let

$$x_i^{unc} = \left[\frac{k^{(1-\eta_i)\frac{\gamma-1}{\gamma}} \eta_i \frac{\gamma-1}{\gamma} C^{\frac{1}{\gamma}}}{W N_i^{1-\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}} \right]^{\frac{\gamma}{\eta_i+(1-\eta_i)\gamma}}, \tag{17}$$

$$x_i^{con} = \left[\frac{(1-\beta(1-\theta))\left[1-\theta(1-\delta+\delta\pi_i)\right] + (1-\theta)\beta N_i \left[1-\left(\frac{N_i-1}{N_i}\right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}\right]}{\frac{\gamma-1}{\gamma}\eta_i} \right]^{\frac{\gamma}{\eta_i+(1-\eta_i)\gamma}} x_i^{unc}. \tag{18}$$

There exist two thresholds, $\underline{\theta}_i$ and $\overline{\theta}_i$, with $\underline{\theta}_i$ defined as

$$\underline{\theta}_i = \left\{ 1 - N_i \left[1 - \left(\frac{N_i-1}{N_i} \right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} \right] \right\} \frac{1}{(1-\delta)+\delta\pi_i}, \tag{19}$$

and $\overline{\theta}_i$ being the value of θ such that $x_i^{con} = x_i^{unc}$, such that

1. *If $\theta \le \underline{\theta}_i$, the optimal contract offered by each supplier in sector i converges to*

$$x_i = x_i^{unc}, \qquad p_i^s = \left[1 - \left(\frac{N_i-1}{N_i} \right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} \right] rev_i(x_i^{unc}), \qquad p_i^{tc} = 0.$$

2. *If $\theta \in (\underline{\theta}_i, \overline{\theta}_i]$, the optimal contract offered by each supplier in sector i converges to*

$$x_i = x_i^{unc}, \qquad p_i^s = \frac{1-\theta(1-\delta+\delta\pi_i)}{N_i} rev_i(x_i^{unc}),$$

$$p_i^{tc} = \beta \left\{ \left[1 - \left(\frac{N_i-1}{N_i} \right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} \right] - \frac{1}{N_i}\left[1-\theta(1-\delta+\delta\pi_i)\right] \right\} rev_i(x_i^{unc}).$$

3. *If $\theta > \overline{\theta}_i$, the optimal contract offered by each supplier in sector i converges to*

$$x_i = x_i^{con}, \qquad p_i^s = \frac{1-\theta(1-\delta+\delta\pi_i)}{N_i} rev_i(x_i^{con}),$$

$$p_i^{tc} = \beta \left\{ \left[1 - \left(\frac{N_i-1}{N_i} \right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} \right] - \frac{1}{N_i}\left[1-\theta(1-\delta+\delta\pi_i)\right] \right\} rev_i(x_i^{con}).$$

The structure of the contract is identical to that of the special case of Section 3, with trade credit being used by the supplier only when the financial constraint of the customer binds.

Unlike in the special case studied earlier, the monopolists cannot extract all the rents from their customers when $N_i > 1$ because of competitive forces. As a result, there is a range of values of θ, $\theta < \underline{\theta}_i$ in which the borrowing constraints of the final-good firms do not bind and the contract supports the efficient level of output.[23]

Comparative statics. Proposition 3 is also useful to study how production and financing decisions of the supply chain vary with industry characteristics, which in our model are represented by the parameters $\{\eta_i, \pi_i, N_i\}$.

Lemma 1. *Suppose that the economy is in steady state, and consider a supply chain i. If $\theta > \underline{\theta}_i$, we have the following comparative static results:*

$$\frac{\partial\left(N_i p_i^{tc}/rev_i\right)}{\partial \eta_i} > 0, \qquad \frac{\partial\left(N_i p_i^{tc}/rev_i\right)}{\partial \pi_i} > 0, \qquad \frac{\partial\left(N_i p_i^{tc}/rev_i\right)}{\partial N_i} < 0.$$

Lemma 1 tells us that supply chains with high intermediate-input share (high η_i), a high share of accounts receivable over sales (high π_i), and that source intermediate goods from concentrated markets (low N_i) will have more trade credit as a fraction of their revenues. These results are quite intuitive, as we discuss next.

High η_i and π_i imply that final-good firms in sector i have large morning working-capital needs: they must purchase more intermediate inputs, but receive little cash in the morning because they sell much of their output to households on credit. Therefore, their demand for funds in the morning will be high and, ceteris paribus, the supply chain will require more trade credit to operate.

A low N_i means that there are few suppliers providing intermediate goods in sector i. Because of that, losing a relationship to one of them will be quite costly for the final-good firms. As a result, the final-good firms in low N_i sectors will have less incentives to default on their suppliers, which allows the supply chain to sustain more trade credit in equilibrium.

In Section 5 we will use Italian data to test these cross-sectional predictions of the model.

4.2 Dynamic adjustment to financial shocks

The analysis of how the economy adjusts to financial shocks in the full model is somewhat different from the special case considered in Section 3 because financial shocks now trigger general equilibrium effects that were absent in the previous analysis. These forces are also present in the spot economy, and they can be studied analytically in this case. After some

[23] In the special case, this happened only when $\theta = 0$.

manipulations of the optimality condition for labor, we can derive the elasticity of y_i in sector i to a marginal increase in θ in the spot economy:

$$\varepsilon^{\text{spot}}_{y_i,\theta} = -\frac{\eta_i}{1-\eta_i\frac{\gamma-1}{\gamma}}\frac{\theta}{1-\theta}\left\{\frac{(1-\theta)[1-\delta(1-\pi_i)]}{\delta(1-\pi_i)+(1-\theta)[1-\delta(1-\pi_i)]}\right\}+\frac{\eta_i}{1-\eta_i\frac{\gamma-1}{\gamma}}\left(\frac{1}{\gamma}\varepsilon_{C,\theta}-\varepsilon_{W,\theta}\right). \quad (20)$$

The first term on the right-hand side is the full-model analogue of the elasticity derived in the spot economy (8), adjusted for delayed customer payments, π_i, and downward-sloping sectoral demand. The second part of the expression isolates the general equilibrium forces. When θ increases, it reduces the demand for all the other goods in the economy, $\varepsilon_{C,\theta} < 0$: to the extent that varieties are not perfectly substitutable ($\gamma < \infty$), the demand for variety i will fall as a result. This "aggregate demand" channel amplifies the impact of the shock on the output produced by sector i, and it is stronger the smaller γ is. In addition, an increase in θ lowers labor demand and depresses wages, $\varepsilon_{W,\theta} < 0$. This reduces the cost of production and dampens the effect of the shock on output. These two general equilibrium forces are at play in the economy with trade credit, and they will contribute to shaping the response of the economy to financial shocks.

Numerical solution. We now briefly discuss the algorithm for the numerical solution of the model, and leave a detailed description to Appendix A.7. As we show in Appendix A.2, one can set up the supplier's problem recursively in the spirit of Thomas and Worrall (1988), where the state variables in the economy include not only the exogenous financial state θ, but also the surplus each supplier promised to its customers in the previous period. Since sectors are heterogeneous but suppliers in each sector are identical, this amounts to one additional state variable per sector $\{\tilde{J}\}_i$.

When suppliers choose the optimal trade credit contract, they need to take into account not only the current wage W and aggregate demand C, but also their values in the future. We follow the logic of Krusell and Smith (1998) and conjecture that $\{W, C\}$ follow an AR(1) process in logs, whose coefficients depend on the current state of the aggregate financial shock, θ. So that the law of motion for aggregate consumption, for example, is given by

$$\ln C_t = (1-\rho_c(\theta_t))\mu_c(\theta_t) + \rho_c(\theta_t)\ln C_{t-1}. \quad (21)$$

The law of motion for the aggregate wage follows a similar specification. Our calibrated model features two values of θ, so that these laws of motion are characterized by 8 coefficients. Given these laws of motion, we can solve the problem of each sector independently, where there are only four state variables for each sector: (i) the aggregate financial shock θ, (ii) the promised surplus in the sector $\tilde{J}_i$, (iii) the aggregate wage W, and (iv) the aggregate level of demand C. We solve each sector using policy function iteration. Once we find the policy functions, we can simulate the economy for a large number of periods and obtain realized values of $\{\theta_t, W_t, C_t\}$. We then estimate the coefficients of the AR(1) processes via an OLS regression. We repeat this process until convergence of the AR(1) coefficients.[24]

5 Quantitative analysis

We now move on to a quantitative analysis. Section 5.1 describes the data and Section 5.2 presents the calibration and discusses the in-sample and out-of-sample properties of the model. Section 5.3 presents the main counterfactuals: one aimed at assessing the size of the credit multiplier and one studying the role of trade credit during the Great Recession.

5.1 Data

We use Italian firm-level data from the historical Orbis dataset compiled by Bureau van Dijk. We study a balanced panel of non-financial corporations operating between 2007 and 2015.[25] Appendix A.6 lays out the cleaning procedure. For each firm, we observe at an annual frequency operating revenues, sales, short-term bank loans, expenditures on intermediate inputs, the sector in which the firm operates, as well as accounts payable and receivable.[26] Accounts payable is what the firm owes its suppliers for goods already purchased, while accounts receivable is what the firm needs to receive from customers.

We perform our analysis at the sector level. Using the classification from the Italian input-output tables, we partition the firms in our panel into 54 different sectors.[27] We compute the relevant balance-sheet ratios (such as accounts payable to sales) for each firm, and then we take the average of these firm-level ratios within each sector to obtain our sectoral-level data. Table 1 provides descriptive statistics for 2007. The ratio of expenditures

[24]We assess the accuracy of the approximate law of motion following Den Haan (2010), comparing the aggregate paths obtained by iterating on the law of motion against those computed directly from equilibrium conditions at each date, using the same sequence of shocks. The approximation is highly accurate. The mean absolute errors for wages and consumption equal 0.01%, and the maximum errors are 0.42% and 0.44%, respectively. The R^2 of the forecasting regressions exceeds 0.999.

[25]Although the coverage of firms in Orbis varies across countries, Italy is in the upper tier of coverage during our sample period. Kalemli-Özcan, Sørensen, Villegas-Sanchez, Volosovych, and Yeşiltaş (2024) report that Orbis covers 86% of gross output in Italian manufacturing between 2007 and 2012, above the cross-country unweighted average of 80%.

[26]The Italian Orbis dataset does not contain an explicit variable with expenditures on intermediate inputs. To construct this variable, we subtract earnings before interest and taxes (EBIT), the wage bill, and depreciation from operating revenues.

[27]In addition to financial corporations, we also exclude public administration, education, and health sectors, which are heavily subsidized, and the petroleum refining sector, whose reliance on imported inputs distorts the domestic supplier concentration measures used in our calibration.

Table 1: Sector-level descriptive statistics

	Mean	St. deviation	Min	Max
Accounts payable/sales	0.23	0.05	0.10	0.39
Accounts receivable/sales	0.29	0.08	0.07	0.59
Short-term bank loans/sales	0.16	0.09	0.07	0.62
Intermediate inputs/sales	0.67	0.08	0.41	0.84
$\text{HHI}^{\text{supplier}}$	0.05	0.02	0.02	0.14

Notes: This table presents descriptive statistics on sector-level variables of interest. The sample includes the 54 sectors in Italy's input-output table in 2007. Supplier's HHI is computed in 2010 because of data limitations.

on intermediate inputs to sales is large, 67% on average. On average, firms borrow more from their suppliers than they do from banks, with accounts payable averaging 23% of sales while short-term loans average 16%. The typical firm also provides credit to its customers, with accounts receivable averaging 29% of sales.

We also use sectoral data to construct an empirical counterpart to N_i, which in our model controls the degree of market power of suppliers in sector i. To do so, we exploit the fact that in the symmetric equilibrium of our model suppliers have the same sales shares, so $1/N_i$ corresponds to the Herfindahl-Hirschman index (HHI) of the suppliers' market for sector i. We first use the Orbis data to construct the sales HHI index for each of the 54 sectors. We then use Italy's input-output tables to construct a weighted average HHI of a sector's suppliers,

$$\text{HHI}_i^{\text{supplier}} = \sum_j w_{ij} \text{HHI}_j,$$

where HHI_j is the HHI sales' index for sector j and w_{ij} is the share of intermediate inputs provided by sector j to sector i. We construct this index using 2010 data, as it is the only year in our sample where we have Italian data on input-output relationships at a fine sectoral level. This measure averages 0.05 across sectors, corresponding to a market with 20 symmetric suppliers.

In addition to the sectoral data, we use data from *Base Dati Statistica* (BDS) of the Bank of Italy to quantify bank advances on trade credit bills. As we have discussed in section 3, the extra liquidity that suppliers obtain when discounting accounts receivable is a key statistic governing the size of the two counterfactuals we are interested in.

There are a number of financial instruments that firms use to borrow out of their accounts receivable. In the Italian Credit Register, these operations are recorded as *rischi auto-liquidanti* (self-liquidating risks), because they are automatically repaid by the borrower via a pre-determined cash-flow source, the invoice. In the vast majority of these transactions, the supplier retains ownership of the receivable and bears the credit risk if the debtor defaults.[28] From the BDS we obtain an aggregated figure of all the self-liquidating risks of Italian financial institutions starting from 2010:Q2 (Table TRI30101).[29] We also obtain from the *National Financial Accounts* a quarterly measure for total short-term loans issued by Italian Monetary and Financial Institutions (Table TFAT0001). Taking the ratio of these two series, we can then obtain a measure of the fraction of short-term loans backed by accounts receivable. Over the 2011-2019 period, roughly half of all short-term credit provided by financial institutions to Italian firms was backed by an account receivable.[30]

Mapping between model and data. Before turning to the calibration, it is useful to clarify how we map the two-layer supply-chain structure of the model to data, where many firms are both a buyer and a seller of intermediate inputs. The mapping is at the sector level: each sector i in the model corresponds to sector i in the data, and we use sector-level moments computed from Orbis to discipline the sectoral parameters $\{\eta_i, \pi_i, N_i\}$. The model does not require us to identify a separate population of "supplier-only" firms in the data. The buyer-side moments of sector i—the intermediate-input share, accounts receivable, accounts payable, and short-term bank loans—are computed as averages across the firms operating in sector i and mapped to the corresponding objects for final-good firms in the model. The seller-side characteristics of sector i's suppliers enter through the supplier-concentration index N_i constructed above. There is one calibration target for which we use information on both upstream and downstream firms: the share of bank credit backed by accounts receivable. In our model, both upstream and downstream firms can borrow against their invoices, so the model-implied counterpart aggregates over both.

Validating cross-sectoral predictions. Before moving to the calibration, we can use the sectoral data to validate some of the key predictions of the model. In Lemma 1 we have seen that firms borrow more from their suppliers when (i) they are in need for cash, as proxied by a high intermediate goods shares η_i and high share of accounts receivable π_i

[28]The main instruments are advances on invoices (*anticipi su fatture*) and factoring. With advances on invoices, the bank lends against outstanding invoices while the firm retains them and collects payment directly. These operations are *salvo buon fine*, meaning that if the trade credit bill is not repaid, it is the firm, not the financial institution, that bears the loss. In factoring, the firm assigns its accounts receivable to a financial intermediary who manages collection. Factoring can either involve the transfer of the credit risk to the intermediary (*non-recourse factoring*), or not (*recourse factoring*). Out of 173 billion of self-liquidating risks in 2011:Q2, only 18 billion were non-recourse factoring (BDS Tables TRI30101, TFR20281, TDB10288).

[29]This category also contains loans to households that are backed by their wages ("prestiti cessione stipendio"). The Base Dati Statistica provides a time series for the latter types of loans starting in 2011:Q1 (Table TFR20281). We subtract the latter from the former, obtaining a time series for loans to firms backed by account receivable.

[30]Invoice discounting is also quantitatively important in other countries. Caglio, Darst, and Kalemli-Ozcan (2021), for example, find that roughly one-third of all loans to U.S. firms are backed by accounts receivable and inventories.

Table 2: Cross-sectional sector-level regressions

	Dep. variable: Accounts payable/sales						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Accounts receivable/sales	0.305***			0.331***	0.407***	0.328***	0.319***
	(0.027)			(0.025)	(0.026)	(0.026)	(0.029)
Intermediate inputs/sales		0.060**		0.187***	0.210***	0.174***	0.271***
		(0.029)		(0.022)	(0.021)	(0.024)	(0.025)
$\mathrm{HHI}^{\mathrm{supplier}}$			0.638***	0.328***	0.417***	0.364***	0.196***
			(0.109)	(0.081)	(0.086)	(0.083)	(0.066)
Upstreamness					−0.029***		
					(0.003)		
Eigenvector centrality						0.169***	
						(0.045)	
Cash/sales							0.111***
							(0.038)
Short-term debt/sales							0.079
							(0.099)
Bank loans/sales							0.024
							(0.027)
Adj. R^2	0.385	0.050	0.132	0.481	0.538	0.485	0.584
Obs.	486	486	486	486	486	486	486
Year f.e.	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table presents sector-level regression results. The dependent variable is the ratio of accounts payable to sales for each sector at time t. Specifications (1)–(4) test the baseline predictions of the model. Specifications (5)–(6) add measures of network position. Specification (7) controls for the sector's financial characteristics. All regressions include year fixed effects. Robust standard errors are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

and (ii) their suppliers are difficult to substitute, which is captured by a low value for N_i. We can evaluate these predictions by performing simple linear regressions, with results reported in Table 2.

The dependent variable in all specifications is the ratio of accounts payable to sales for each sector at time t. Specification (1) shows that accounts receivable are positively associated with accounts payable. Specification (2) shows that there is also a significant positive relationship between expenditures on intermediate inputs, a proxy for η_i, and accounts payable, as our model would predict. Specification (3) shows a significant and positive relationship between $\mathrm{HHI}^{\mathrm{supplier}}$ and accounts payable. Specification (4) reports the regression with all three control variables. All coefficients are significant, and their signs are consistent with Lemma 1. These three factors jointly account for 48% of the variation in accounts payable across sectors.

The positive association between $\mathrm{HHI}^{\mathrm{supplier}}$ and accounts payable that we document is consistent with a key prediction of our theory: ceteris paribus, firms can borrow more from their suppliers if the latter have more market power.[31] Of course, this empirical association

[31]This result is consistent with the findings in Garcia-Marin, Justel, and Schmidt-Eisenlohr (2019) of a

could also reflect other mechanisms. For example, the $\text{HHI}_i^{\text{supplier}}$ index may proxy for the amount of liquidity that suppliers have, or it may be correlated with the position of sector i in the supply chain, factors that can also affect the accounts payable of sector i.

While it is outside the scope of this paper to fully correct for these and related endogeneity concerns, Columns (5)–(7) of Table 2 provide a robustness analysis. In columns (5)–(6) we include in the regression variables that are used in the literature to capture the location of a given sector in the input-output network: the measure of upstreamness proposed by Antràs, Chor, Fally, and Hillberry (2012), and eigenvector centrality proposed by Liu (2019). Column (7) adds the sector's financial characteristics to the regression. Importantly, we can see that the introduction of these variables marginally increases the R^2 of the regression but does not affect the size and significance of the coefficient associated with $\text{HHI}^{\text{supplier}}$.

5.2 Calibration

We classify the structural parameters into four groups: preference parameters $[\beta, \psi, \chi, \gamma]$, common financial and technology parameters, $[\delta, \sigma]$, sector-specific parameters, $\{\eta_i, \pi_i, N_i\}_i$, and parameters governing the stochastic process of the financial shock. For the quantitative analysis, we assume that θ_t follows a two-state Markov process $\theta_t \in \{\theta_L, \theta_H\}$. We interpret θ_L as the state of the financial sector in "normal times" and a switch to θ_H as a financial crisis. We equate a time period in the model to two years.[32]

We fix a number of parameters to conventional values in the literature. Specifically, we set $\beta = 0.96$, set the inverse Frisch elasticity of labor supply, ψ, to 1, and choose χ so that, given all other parameters, hours worked in the deterministic steady state with $\theta = \theta_L$ is equal to 1. We set the elasticity of substitution across final goods γ so that, given all other parameters, the average markup of final-good firms (computed as the per-unit price divided by per-unit marginal cost) equals 30% in the same steady state. Following the work of Baqaee, Burstein, Duprez, and Farhi (2025), we set the elasticity of substitution across intermediate inputs, σ, to 4.8. Finally, we set $p(\theta_L|\theta_L) = 0.99$ to be consistent with the idea that financial crises are rare events in advanced economies.

We choose the remaining model parameters $[\{\pi_i, \eta_i, N_i\}_i, \theta_H, \theta_L, p(\theta_H|\theta_H), \delta]$ to replicate a set of sector-specific and aggregate moments. Table 3 reports numerical values of the parameters and the empirical targets. Model-implied moments are computed on a long

positive relationship between suppliers' markups and the amount of trade credit they offer to customers using Chilean transaction-level data.

[32]We set the model period to two years to align with the timing of the crisis in our data. Since the financial crisis intensified in the fall of 2008, annual balance sheet data for that year largely reflect pre-crisis conditions, while 2009 data fully capture the tightening of credit markets. A biennial period allows us to map the transition from θ_L to θ_H onto the 2007–2009 comparison in the data.

stochastic simulation of the calibrated model.[33] Even though these parameters are jointly chosen, we can give a heuristic description of how the selected sample moments help inform specific parameters.

We use the sectoral level data to discipline $\{\pi_i, \eta_i, N_i\}_i$. Specifically, we target for each sector the 2007 value for the ratio of accounts receivable over sales, expenditures on intermediate inputs/services over sales, and the $\text{HHI}_i^{\text{supplier}}$ index we computed earlier. In our model, these statistics have a one-to-one mapping with the sector-specific parameters $\{\pi_i, \eta_i, N_i\}_i$, so they can be used to identify them.[34]

In addition to these sector-level moments, we ask the model to match the behavior of bank credit during the Great Recession, as proxied by short-term bank loans relative to revenues. This ratio was on average 17% in 2007 and fell to 13% in 2009, recovering to 15% in 2011. This information is important for disciplining the values of $[\theta_H, \theta_L, p(\theta_H|\theta_H)]$. Specifically, we compute average bank debt over firms' revenues over the simulation conditional on $\theta = \theta_L$ and $\theta = \theta_H$, and equate the former to the 2007 value and the latter to the 2009 one. We also make sure that, 2 years after a switch from θ_L to θ_H, this ratio is close to 15%. In addition, and to guarantee that the model features a reasonable behavior of output conditional on credit shocks of this magnitude, we target an average drop in firms' revenues of 11% when the economy switches from θ_L to θ_H.

The remaining parameter δ controls the liquid funds that downstream firms obtain in the morning and, as such, determines their borrowing needs.[35] We discipline this parameter by targeting the average decline in firms' accounts payable during the Great Recession and the average share of bank credit backed by accounts receivable in normal times.[36] As we have seen in Section 3.2, the first moment is informative on whether suppliers' borrowing constraints are binding or not. The second moment is informative about the degree of financial amplification: in the constrained region, the higher the extra leverage created by the process of discounting trade-credit bills, the more sensitive the economy is to the credit

[33]We simulate the economy for 20,000 periods and discard the first 300 periods. For the moments corresponding to 2007, we use the average in periods with $\theta_t = \theta_L$. For the moments corresponding to 2009 and 2011, we take the average of the periods where the economy transitions from θ_L to θ_H and the subsequent period, respectively.

[34]As discussed earlier, N_i is equal to the inverse of the $\text{HHI}_i^{\text{supplier}}$ index in our symmetric equilibrium. The expenditure share on intermediate goods is informative about η_i. Finally, the accounts receivable of final-good firms directly maps to π_i in our model. If the implied π_i is greater than one, we set it to one.

[35]The lower the value of δ, the more downstream firms will need to borrow from banks and from suppliers to finance their operations. Therefore, keeping θ and the other parameters fixed, lower values of δ increase the demand for trade credit and move the economy closer to the region in which suppliers are financially constrained.

[36]As described earlier, we observe this ratio for the 2011-2019 period. We average the share over the 2016-2019 period, as the Italian economy experienced major financial distress during the 2011-2015 period (Bocola, 2016; Arellano, Bai, and Bocola, 2017).

Table 3: Model parameters and targeted moments

Parameter	Value	Moment	Data	Model
Mean(π_i)	0.89	Distribution of accounts receivable/sales		
Stdev(π_i)	0.19			
Mean(η_i)	0.88	Distribution of intermediate inputs/sales		
Stdev(η_i)	0.09			
Mean(N_i)	24.10	Distribution of HHI$^{\text{supplier}}$		
Stdev(N_i)	11.26			
θ_L	0.83	Mean(Bank loans/sales) in 2007	0.17	0.17
θ_H	0.85	Mean(Bank loans/sales) in 2009	0.13	0.14
		Average log sales change (2007→2009)	−0.11	−0.13
$p(\theta_H \| \theta_H)$	0.80	Mean(Bank loans/sales) in 2011	0.15	0.15
δ	0.30	Share of bank credit backed by accounts receivable	0.51	0.51
		Average log payable change (2007→2009)	−0.13	−0.12

Notes: This table reports the numerical values of model parameters along with the empirical targets used in the calibration. The first six rows report statistics for the distribution of $\{\pi_i, \eta_i, N_i\}$ across the 54 sectors of our analysis. These parameters are chosen to replicate exactly the value of accounts receivable/sales, intermediate input costs/sales, and the HHI$^{\text{supplier}}$ for each of the 54 sectors. The other rows report the exact numerical values for the remaining parameters along with the moments used to discipline them. The sample moments in the model are computed using simulations.

supply shock.

Despite being over-identified, the model captures all these moments quite accurately. The sectoral-level moments are by construction exactly matched. From Table 3 we can see that the model is consistent, conditional on a credit shock, with the behavior of bank credit, trade credit and output observed in Italy during the Great Recession.

Model validation. We now analyze how the model matches cross-sectoral moments that were not targeted in the calibration. We consider two dimensions: cross-sector patterns during normal times and in response to the financial crisis.

Table 4 reports the results. Panel A focuses on normal times. The cross-sector correlation between the model-predicted ratio of accounts payable to sales and its data counterpart is 0.33. The model also reproduces the cross-sectoral dispersion of trade credit, although it generates somewhat more variation than the data: standard deviation of 0.08 in the model versus 0.05 in the data. Panel B of Table 4 turns to the financial crisis. Here the question is whether the model can predict *which* sectors experienced larger declines in trade credit and output during the 2007–2009 episode. The cross-sector correlation between model-predicted and actual changes in trade credit is 0.31. For output, the correlation is 0.33.

Table 4: Model validation

A. Normal times		B. Financial crisis	
Moment	Value	Moment	Value
corr(payable/sales: model, data)	0.33	corr(Δ payable: model, data)	0.31
sd(payable/sales): data	0.05	corr(Δ sales: model, data)	0.33
sd(payable/sales): model	0.08		

Notes: This table reports the correlation between data and model-implied moments in normal times and during a financial crisis. Panel A reports cross-sector moments in normal times. Panel B reports cross-sector correlations between model-predicted and data-observed changes during the financial crisis (2007–2009).

To assess this prediction more directly, we compare the model's cross-sectional implications with external data on real economic activity. Figure 4 uses quarterly chain-linked value added from the Italian national accounts (ISTAT) to construct an event study around the crisis.[37] We sort sectors into terciles based on the model-predicted output decline and plot average log value added for the top and bottom terciles, normalized to zero at 2007:Q4. The two groups display parallel trends before the crisis: they track each other closely from 2005 through late 2007. After the onset of the crisis, the groups diverge. Sectors in the bottom tercile (those the model predicts will decline most) experienced a larger decline in real value added relative to the top tercile. This pattern is consistent with the model's cross-sectional predictions, even though the calibration targeted only aggregate, not cross-sectional, crisis moments. Moreover, the fit is not only qualitative but also quantitative. The dashed lines in Figure 4 plot the model's predicted average decline in output for each tercile two years into the crisis (one period in the model), and they align closely with the empirical counterparts. Between 2007 and 2009, output in the bottom tercile declines by 18% in the model versus 19% in the data, while output in the top tercile declines by 9% in the model versus 8% in the data.

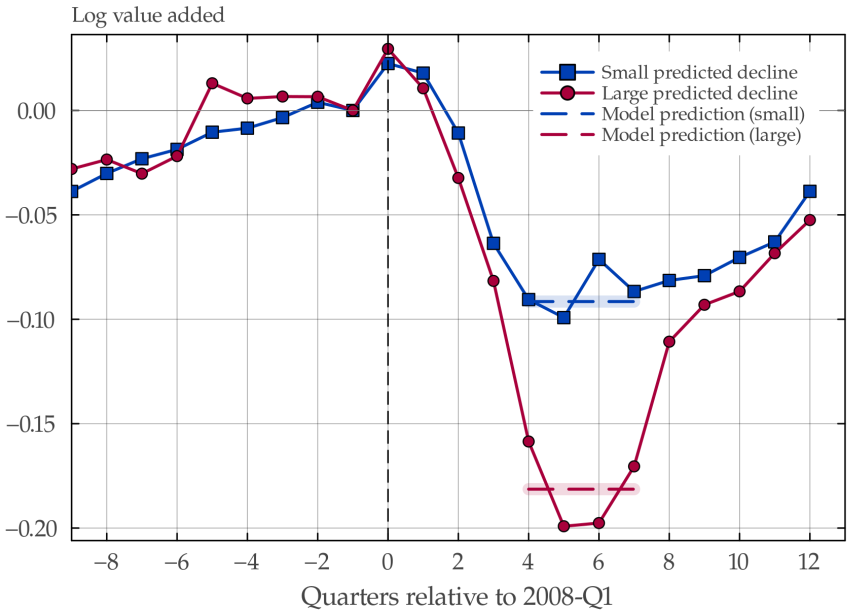
5.3 The macroeconomic implications of trade credit

In Section 3, we focused on two main macroeconomic implications of trade credit. First, we showed that *on average*, trade credit relationships reduce the output costs of financial frictions via a credit multiplier effect. Second, we have seen that trade credit relationships can smooth or amplify the impact of financial shocks depending on suppliers' borrowing capacity. We now use the calibrated model to quantify these two aspects.

We start by measuring the size of the credit multiplier and its implications for output.

[37] We map the 54 sectors in the model to the 16 ISTAT sectors according to the crosswalk table A-3 in Appendix A.6.

Figure 4: Cross-sectional validation: model predictions vs. data



Notes: This figure displays average log real value added, normalized to zero at 2007:Q4, for sectors in the top and bottom terciles of model-predicted output decline between 2007–2009. Time is measured in quarters relative to 2008:Q1. The dashed lines in periods 4–7 correspond to the model's prediction of the average decline in output between 2007–2009 for each group of sectors. The calibration targeted the average fall of output and trade credit during the crisis, but not the cross-sectional output response. Data source: ISTAT quarterly chain-linked value added.

Table 5 compares average credit and output in the benchmark economy with two counterfactuals. In the first, suppliers cannot extend trade credit to final-good firms, so $p_i^{\text{tc}} = 0$ for all i. In the second, suppliers cannot extend trade credit *and* final-good firms are paid on the spot by households, so $p_i^{\text{tc}} = 0$ and $\pi_i = 0$ for all i. We interpret the latter as the spot economy because all transactions among intermediate-good producers, final-good producers, and households are settled immediately rather than through IOUs.

Table 5: Quantifying the credit multiplier

	Benchmark	$p_i^{\text{tc}} = 0$	$p_i^{\text{tc}} = 0$ and $\pi_i = 0$
Bank credit	1.00	0.30	0.42
To final-good firms	0.66	0.30	0.42
To suppliers	0.34	0.00	0.00
Against accounts receivable	0.51	0.08	0.00
Output	1.00	0.46	0.87

Notes: This table reports bank credit and output in the benchmark economy and in the two counterfactual economies. The table reports average values conditional on the economy being in good times ($\theta_t = \theta_L$). We normalize the bank credit and output by their value in the benchmark economy.

Let us start by comparing the benchmark with the $p_i^{\text{tc}} = 0$ economy. Total bank credit in the latter is on average 30% of the size of bank credit in the benchmark. This is due to two effects. First, in the economy with trade credit, final-good firms have higher revenues, so they can mechanically obtain more credit from banks.[38] Second, the suppliers discount their accounts receivable with banks in the economy with trade credit, something that does not happen in the counterfactual economy because suppliers are paid spot and do not need to borrow. Less bank credit and a less efficient allocation of morning liquidity generate a large output loss: as the last row of Table 5 shows, output in the no-trade-credit economy is about half that of the benchmark.

In the counterfactual we just discussed, final-good firms cannot issue IOUs to their suppliers but they do accept IOUs from their customers, the households. In the last column of Table 5, we report what happens to credit and output in an economy in which households also pay on the spot, $\pi_i = 0$ for all i. Final-good firms receive more cash in the morning when $\pi_i = 0$, reducing their need for working capital and mitigating the economic costs of financial frictions. Qualitatively, however, the comparison with the benchmark is similar to that of the previous counterfactual: the benchmark economy features substantially more bank credit relative to the spot economy and 14% higher output ($\frac{1}{0.874} - 1$).

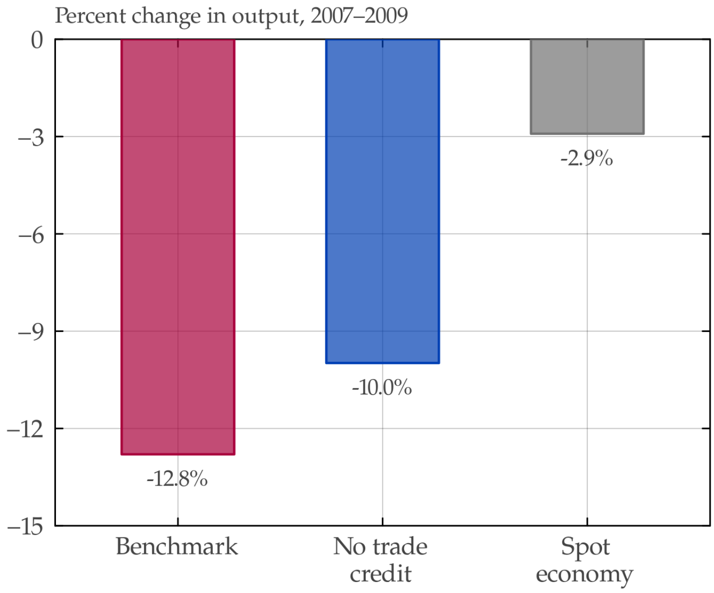
The second quantitative exercise consists of assessing how trade credit shaped the response to a financial shock similar in magnitude to that of the Great Recession. To do so, we study the response of the benchmark economy to a tightening of aggregate financial conditions (a switch from θ_L to θ_H) and compare it to what happens in the two counterfactual economies.

Figure 5 reports the change in output between 2007 and 2009 in the benchmark economy and in the two counterfactuals. In the benchmark economy, output falls 12.8%, consistent with the observed decline in firm revenues over this period. In the no-trade-credit economy, output falls by 10%, while in the spot economy it falls by only 2.9%. Trade credit amplifies the output decline by 28% relative to the no-trade-credit counterfactual, and the economy with trade credit is 4.4 times as responsive as the spot economy. As we explained in Section 4.2, this amplification is due to the fact that supply chains in the economy with trade credit are substantially more levered relative to the counterfactuals, making the economy more sensitive to financial shocks.

The effects of corporate subsidies. We next turn to the effectiveness of corporate subsidies, a standard tool for supporting firms during financial crises. In our environment, such subsidies raise output, but their effectiveness depends on where they are directed within

[38] In both economies, firms' borrowing limit is a fraction $(1 - \theta_t)$ of their afternoon revenues.

Figure 5: The implications of trade credit for the Great Recession



Notes: This figure displays the response of output to a financial shock in the benchmark economy, the counterfactual economy with $p_i^{\mathrm{tc}} = 0$ ("No trade credit") and the counterfactual economy with $p_i^{\mathrm{tc}} = 0$ and $\pi_i = 0$ ("Spot economy").

the supply chain. As we argue below, a dollar of subsidy raises output more when channeled to suppliers than when paid directly to final-good firms. This happens because of the endogenous response of trade credit: a supplier subsidy raises the customer's surplus from the relationship more than a customer subsidy does, and a higher surplus supports more trade credit.

While we leave a detailed analysis of this policy to Appendix A.4, let us briefly explain here the logic of this result. A subsidy to the supply chain relaxes the borrowing constraints by the same direct amount regardless of whether it is targeted at the supplier or customer. Both types of subsidies raise the customer's surplus from the relationship, $\tilde{J}$. However, a subsidy to the supplier raises the customer's surplus from the bilateral relationship more than a direct subsidy to the customer. A supplier subsidy reaches the customer only through the relationship: walking away forfeits the benefit, so the customer's outside option is unchanged. A subsidy paid directly to the final-good firm, by contrast, raises the customer's outside option as well, dampening the rise in $\tilde{J}$. This makes the subsidy to the supplier more effective in relaxing the trade credit constraint and, as a result, at raising output. In the quantitative application, also discussed in Appendix A.4, the subsidy to suppliers is 6% more effective in boosting output than a subsidy to final-good firms.

This analysis identifies a distinct motive for subsidy targeting relative to prior work. Liu (2019) argues that optimal corporate subsidies in a production network should be targeted toward sectors that are more *central* to market imperfections, which he finds typically to be upstream sectors. Glode and Opp (2021) find that corporate subsidies can be more effective when targeted toward downstream producers, as such subsidies can help prevent *default waves*. Relative to these papers, our analysis sheds light on another motive that shapes the design of optimal corporate subsidies: their effects on trade-credit linkages among firms.

6 Conclusion

We have proposed an equilibrium model to explain the prevalence of trade credit as a form of short-term financing for companies around the world and used it to understand the macroeconomic implications of this phenomenon. In our theory, trade credit is enforced in equilibrium by reputational forces, as customers have an incentive to repay their suppliers out of fear of losing that relationship. This process allows the economy to increase credit provided by the financial system because suppliers can enforce these IOUs and at the same time discount these claims with financial institutions to obtain liquidity. We provide evidence consistent with this theory and fit the model to Italian data in order to quantify the macroeconomic implications of the credit multiplier. We show that this process allows the economy to support 14% more output on average, but it also makes the economy more vulnerable to financial shocks, as the presence of trade credit substantially amplified the output costs of the Great Recession.

This framework opens several directions for future research. A first is to extend the analysis to richer supply chains in which firms simultaneously receive and extend trade credit. The mechanism we identify operates at every link of such a chain, so we expect both the credit multiplier and the amplification of financial shocks to be larger in deeper production networks. A second extension is to relax the no-default property of the optimal contract, for instance by introducing mid-period idiosyncratic shocks that the contract cannot condition on. Such environments would generate an additional channel of upstream propagation, in which downstream distress cascades to suppliers through trade-credit losses. Such a framework can also be applied to study how trade credit shapes the propagation of firm-specific shocks through the production network. Finally, the forward-looking nature of trade credit makes these relationships vulnerable to self-fulfilling confidence crises, which could rationalize the sudden disruptions in payment chains observed in some countries during the Great Recession.

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Appendix

A.1 The problem of the supplier in the simplified model

In this section, we discuss the decision problem of the supplier in the simplified model of Section 3. Let J be the rents that the monopolist has promised to a final good firm after history s^t, and let θ be the realization of the financial shock at s_t. The monopolist chooses labor x, the morning and afternoon payments $\{p^{\text{s}}, p^{\text{tc}}\}$, as well as the future continuation values for final-good firms conditional on any realization of the financial shock next period, $J'(\theta')$, to maximize the present discounted value of profits,

$$V(J,\theta) = \max_{x,p^{\text{s}},p^{\text{tc}},J'(\theta')} \left(p^{\text{s}} + p^{\text{tc}} - Wx\right) + \beta \mathbb{E}\left[V(J'(\theta'),\theta')|J,\theta\right].$$

subject to the constraints

$$\begin{aligned} p^{\text{s}} &\leq [\delta + (1-\delta)(1-\theta)]x^{\eta}, && \text{(A.1)}\\ Wx - p^{\text{s}} &\leq (1-\theta)p^{\text{tc}}, && \text{(A.2)}\\ p^{\text{tc}} &\leq \beta\mathbb{E}[J'(\theta')|J,\theta], && \text{(A.3)}\\ J &= x^{\eta} - (p^{\text{s}} + p^{\text{tc}}) + \beta\mathbb{E}[J'(\theta')|J,\theta]. && \text{(A.4)} \end{aligned}$$

In order to understand the trade-off of the supplier, it is useful to derive the first-order conditions of this problem. Let μ, ι, and κ be the Lagrange multipliers associated, respectively, with constraints (A.1), (A.2), and (A.3), and let λ be the multiplier associated with the promise-keeping constraint (A.4). After some rearrangement, we obtain[39]

$$\begin{aligned} \{[\delta + (1-\theta)(1-\delta)]\mu + \lambda\}\,\eta x^{\eta-1} &= W(1+\iota), && \text{(A.5)}\\ 1+\iota &= \mu + \lambda, && \text{(A.6)}\\ 1+(1-\theta)\iota &= \kappa + \lambda, && \text{(A.7)}\\ \lambda'(\theta') &= \lambda + \kappa. && \text{(A.8)} \end{aligned}$$

Equation (A.5) is the optimality condition for x. Relative to the spot economy, increasing x has the additional benefit of relaxing the promise-keeping constraint, an effect represented by the Lagrange multiplier λ on the left hand side of the equation. It also has an additional cost when the supplier debt limit binds, as wages need to be paid in the morning

[39]In what follows, we assume that the participation constraint and the feasibility requirements do not bind in equilibrium. In the steady state, this will be true as long as $\theta > 0$.

and those resources have a shadow cost when the supplier is constrained, an effect that is represented by the Lagrange multiplier ι on the right hand side of the equation.

Equations (A.6) and (A.7) are the optimality conditions with respect to p^{s} and p^{tc}. A marginal increase in p^{s} raises the profits of the supplier by one unit and relaxes its borrowing constraint with the banks (when it binds), which explains why the marginal benefit equals $1+\iota$. The marginal cost of increasing p^{s} is a tightening of the borrowing constraint of final-good firms and of the promise-keeping constraint, $\mu+\lambda$. The trade-off for p^{tc} is very similar. The key difference between p^{s} and p^{tc} is that the latter does not relax the borrowing constraint of the supplier as much because banks advance at most a fraction $(1-\theta)$ of the supplier's accounts receivable. When the borrowing constraint of the monopolist does not bind ($\iota = 0$), we have that $\mu = \kappa$ and the supplier is indifferent between being paid spot or in credit. When the borrowing constraint of the supplier binds ($\iota > 0$), giving trade credit to customers is costly for the supplier relative to being paid spot in the morning. *Ceteris paribus*, a tightening of the supplier's borrowing constraint reduces the incentives to extend trade credit to its customers.

Equation (A.8) combines the first-order condition of the problem with respect to $J'(\theta')$ and the envelope condition. This equation describes a law of motion for the Lagrange multiplier on the promise-keeping constraint, λ. We can see that the multiplier grows over time whenever the trade credit constraint binds, $\kappa > 0$. This is quite intuitive: the supplier has an incentive to increase future rents for his customers when the trade credit constraint binds because this allows them to extend more trade credit today. As the rents of final-good firms grow over time, so does the Lagrange multiplier on the promise-keeping constraint.

We can substitute equations (A.5), (A.6) and (A.8) into (A.7) to better understand the trade-off the supplier faces when extending trade credit to its customers,

$$\{1-\theta(1-\delta)\mu+\iota\}\,\frac{\eta x^{\eta-1}}{W} = \lambda'(\theta') + \theta\iota. \tag{A.9}$$

The right hand side of the expression captures two sources of costs the supplier faces when extending more trade credit. First, in order to enforce it, the supplier needs to promise higher rents to the customer in the future, and their cost is represented by the multiplier of the promise keeping constraint $\lambda'(\theta')$. Second, providing liquidity to the customer is costly for the supplier when they are financially constrained $(\iota > 0)$. The left-hand side captures the benefit: trade credit allows the economy to get closer to the efficient scale $(\eta x^{\eta-1} = W)$, and this has a financial benefit to the supplier as they obtain a share of these surpluses.

A.2 Full model - recursive formulation

In Section 2, we presented the problem of the intermediate-good producer in sequence form. In this section, we lay out the recursive formulation of the problem. We will use the recursive formulation to derive optimality conditions and to prove the different propositions in the paper.

Before writing down the recursive formulation, it is useful to briefly discuss the state variables in the firm's problem. First, the exogenous state variable is θ, the degree of financial frictions in the economy. In addition, each intermediate-good producer carries over a promise made in the previous period to final-good firms. This promise guarantees the final good producer a discounted surplus of the match, $\tilde{J}_i^j$. We study a symmetric equilibrium, under which the promises made by suppliers in production lines in sector i are all identical and we can denote them by $\tilde{J}_i$. The aggregate state of the economy therefore consists of θ as well as the distribution of $\tilde{J}_i$ across all sectors, which we denote by Ω. We will denote the wage that arises in general equilibrium by $w(\theta, \Omega)$ and the revenue of final good firm i by $rev_i(\{x_n\}_{n=1}^{N_i}, \theta, \Omega)$. The latter is given by

$$rev_i(\{x_n\}_{n=1}^{N_i}, \theta, \Omega) = P(\theta, \Omega) C(\theta, \Omega)^{\frac{1}{\gamma}} \left(k^{1-\eta_i} \left\{ \left[\sum_{n=1}^{N_i} x_n^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}^{\eta_i} \right)^{\frac{\gamma-1}{\gamma}} \tag{A.10}$$

with $P(\theta, \Omega)$ and $C(\theta, \Omega)$ denoting the aggregate price index and consumption in the economy.

The recursive problem of the intermediate-good producer supplying goods to final-good firms in sector i is given by

$$\begin{aligned}
V\left(\tilde{J}(\theta), \theta, \Omega\right) = \max_{p_j^s, p_j^{tc}, x_j, \tilde{J}'(\theta')} \; & p_j^s + p_j^{tc} - w(\theta, \Omega) x_j + \beta \mathbb{E}\left[V\left(\tilde{J}'(\theta'), \theta', \Omega'(\theta') \right) \right] \\
s.t. \quad & p_j^s + \sum_{n \neq j} p_n^s \leq [1 - \theta(1 - \delta(1 - \pi_i))] \, rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega), && [\mu] \\
& p_j^{tc} \leq \beta \mathbb{E}\left[\tilde{J}'(\theta') \right], && [\kappa] \\
& w(\theta, \Omega) x_j - p_j^s \leq (1 - \theta) p_j^{tc}, && [\iota] \\
& \tilde{J}(\theta) = rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega) - rev_i(\{0, \vec{x}_{-j}\}, \theta, \Omega) - (p_j^s + p_j^{tc}) + \beta \mathbb{E}_\theta \left[\tilde{J}'(\theta') \right], && [\lambda] \\
& p_j^s + p_j^{tc} + \sum_{n \neq j} (p_n^s + p_n^{tc}) \leq rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega), && [\rho] \\
& \tilde{J}'(\theta') \geq 0, && [\zeta(\theta')]
\end{aligned}$$

where the red letters correspond to the Lagrange multipliers associated with each equation. The first-order conditions are given by

$$[p^s] \qquad 1+\iota = \mu + \lambda + \rho, \tag{A.11}$$

$$[p^{tc}] \qquad 1+(1-\theta)\iota = \kappa + \lambda + \rho, \tag{A.12}$$

$$[x] \qquad [(1-\theta\,(1-\delta(1-\pi_i)))\mu + \rho + \lambda] \frac{\partial rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega)}{\partial x_j} = w(\theta,\Omega)(1+\iota), \tag{A.13}$$

$$[J'(\theta')] \qquad \beta P(\theta'|\theta) V_J \left(J'\left(\theta'\right), \theta'\right) = -\beta P(\theta'|\theta)\left(\kappa + \lambda\right), \tag{A.14}$$

The envelope condition is given by

$$V_{\tilde{J}(\theta)}\left(\tilde{J}\left(\theta\right), \theta\right) = -\lambda(\theta), \tag{A.15}$$

Combining the two sets of conditions above we obtain the following optimality condition

$$\lambda'(\theta') = \lambda(\theta) + \kappa(\theta). \tag{A.16}$$

In deriving these conditions, we have used the fact that the participation constraint $\tilde{J}'(\theta') \geq 0$ does not bind, so that $\zeta(\theta') = 0$ for all θ'. We verify this numerically in our quantitative analysis.

In the symmetric equilibrium, we have that

$$rev_i(x,\theta,\Omega) = P(\theta,\Omega)C(\theta,\Omega)^{\frac{1}{\gamma}} k_i^{(1-\eta_i)\frac{\gamma-1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} x^{\eta_i \frac{\gamma-1}{\gamma}},$$

and[40]

$$\frac{\partial rev_i(x,\theta,\Omega)}{\partial x_j} = \frac{\gamma-1}{\gamma}\eta_i P(\theta,\Omega)C(\theta,\Omega)^{\frac{1}{\gamma}} k_i^{(1-\eta_i)\frac{\gamma-1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1} x^{\eta_i \frac{\gamma-1}{\gamma}-1}. \tag{A.17}$$

A.3 Proofs

Proof of Proposition 1

Proof. In the steady state, $\lambda' = \lambda$ so that equation (A.16) implies $\kappa = 0$. Combining equations (A.11) and (A.12), we obtain $\mu = \theta\iota$. In the steady state, the promise keeping constraint

[40]Note that the second expression is not obtained by differentiating the first with respect to x, but rather by imposing symmetry in the expression of $\frac{\partial rev_i(\{x_j, \vec{x}_{-j}\})}{\partial x_j}$.

boils down to

$$J = \frac{1}{1-\beta}\left(x^{\eta} - p^{s} - p^{tc}\right).$$

Consider first the case where $\mu = 0$. In that case $\iota = 0$ and the first order condition with respect to x implies

$$x = \underbrace{\left(\frac{\eta}{W}\right)^{\frac{1}{1-\eta}}}_{x_{fb}}.$$

From the promise keeping constraint, we obtain that $p^{s} + p^{tc} = x_{fb}^{\eta} - (1-\beta)J$. For a given level of J, the borrowing constraints do not bind if

$$J \geq \theta(1-\delta)x_{fb}^{\eta},$$
$$J \leq \frac{1}{(1-\theta)(1-\beta)}\left[\left((1-\theta)(1+\theta(1-\delta)) + \delta\theta\right)x_{fb}^{\eta} - Wx_{fb}\right],$$

where the first constraint is the trade credit constraint after using the promise keeping constraint together with the borrowing constraint of the final-good firm. The second constraint is the borrowing constraint of the supplier. Therefore, any level of J that satisfies both conditions, if such exists, can be sustained as a steady state. We will restrict attention to the lower bound of such support ($J = \theta(1-\delta)x_{fb}^{\eta}$), which guarantees the supplier the most surplus in the steady state.[41] This level of J is supported by $p^{tc} = \beta\theta(1-\delta)x^{\eta}$ and $p^{s} = (\delta + (1-\delta)(1-\theta))x^{\eta}$.

Depending on parameter values, there could potentially be no level of J such that both borrowing constraints do not bind in the steady state. That is, for some parameters, the borrowing constraints bind in the steady state. Such a case occurs when

$$(1-\theta)(1-\beta)\theta(1-\delta)x_{fb}^{\eta} > \left[\left((1-\theta)(1+\theta(1-\delta)) + \delta\theta\right) - \eta\right]x_{fb}^{\eta},$$

where we have used the definition of x_{fb} to replace the wage bill. Rearranging, we obtain that the borrowing constraints bind in the steady state if and only if

$$\eta > \delta + (1-\theta)(1-\delta)(1+\beta\theta).$$

Note that the RHS is decreasing in θ. Let $\bar{\theta}$ be the level of θ which makes the equation above hold with equality. For any $\theta \geq \bar{\theta}$, the condition above is satisfied and both borrowing constraints bind in the steady state. In this case, we have that $p^{s} = (\delta + (1-\delta)(1-\theta))x^{\eta}$

[41] Note that such equilibrium selection does not affect output in the economy as $x = x^{fb}$ when $\iota = \mu = 0$ regardless of the level of J.

and the promise keeping constraint implies that $(1-\beta)J = \theta(1-\delta)x^\eta - p^{tc}$. The supplier's borrowing constraint is

$$(1-\theta)(1-\beta)J = (\delta + (1-\delta)(1-\theta)(1+\theta))x^\eta - Wx.$$

For the trade credit constraint to be satisfied, we must have $J \geq \theta(1-\delta)x^\eta$. As in the unconstrained case, we restrict attention to the lowest level of J which can be supported in equilibrium. This level of J is supported by the upper limit of trade credit, $p^{tc} = \beta J = \beta\theta(1-\delta)x^\eta$. Plugging into the supplier's borrowing constraint we obtain:

$$Wx = (\delta + (1-\delta)(1-\theta))x^\eta + (1-\theta)\beta\theta(1-\delta)x^\eta,$$

which implies that when $\theta > \bar{\theta}$,

$$x = \left[\frac{\delta + (1-\delta)(1-\theta)(1+\beta\theta)}{W}\right]^{\frac{1}{1-\eta}}$$

□

Proof of Proposition 2

Proof. Let $x^{fb} = (\eta/W)^{1/(1-\eta)}$ and conjecture that the first-best level of production is supported in every state. Set

$$J(\theta) = \theta(1-\delta)(x^{fb})^\eta,$$

$$p^s(\theta) = [\delta + (1-\theta)(1-\delta)]\,(x^{fb})^\eta,$$

and

$$p^{tc}(\theta) = \beta(1-\delta)(x^{fb})^\eta \mathbb{E}[\theta'|\theta].$$

The trade-credit constraint holds with equality because

$$p^{tc}(\theta) = \beta\mathbb{E}[J(\theta')|\theta].$$

The promise-keeping constraint is also satisfied:

$$J(\theta) = (x^{fb})^\eta - p^s(\theta) - p^{tc}(\theta) + \beta\mathbb{E}[J(\theta')|\theta] = \theta(1-\delta)(x^{fb})^\eta.$$

It remains to verify the supplier's borrowing constraint. Using $Wx^{fb} = \eta(x^{fb})^\eta$, this constraint is equivalent to

$$\eta \leq 1-\theta(1-\delta)+(1-\theta)\beta(1-\delta)\mathbb{E}[\theta'|\theta],$$

which holds by assumption. Therefore the first-best allocation is feasible in every state, and output does not change after a switch from θ_L to θ_H. $\square$

Proof of Proposition 3

Proof. We study the deterministic steady state when the wage is W and aggregate consumption is C. In the steady state, the promise-keeping values and Lagrange multipliers do not change over time. Equation (A.16) then implies $\kappa = \zeta = 0$, and combining equations (A.11)–(A.12) gives $\mu = \theta\iota$ and $\rho + \lambda = 1 + (1-\theta)\iota$. In a symmetric equilibrium, all suppliers in a production line supply the same quantity of intermediate inputs. To ease notation, we use $rev_i(x)$ to denote $rev_i(\{x, \ldots, x\})$. In equilibrium,

$$rev_i(x) = C^{\frac{1}{\gamma}} k^{(1-\eta_i)\frac{\gamma-1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} x^{\eta_i \frac{\gamma-1}{\gamma}},$$

and[42]

$$\frac{\partial rev_i(x)}{\partial x_j} = \frac{\gamma-1}{\gamma}\eta_i C^{\frac{1}{\gamma}} k^{(1-\eta_i)\frac{\gamma-1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1} x^{\eta_i \frac{\gamma-1}{\gamma}-1}. \tag{A.18}$$

Finally, in the steady state of a symmetric equilibrium, the promise-keeping constraint for each supplier is

$$\tilde{J}(\theta) = A_i rev_i(x) - p_i^s - p_i^{tc} + \beta\mathbb{E}_\theta[\tilde{J}'(\theta')], \tag{A.19}$$

where

$$A_i \equiv 1 - \left(\frac{N_i - 1}{N_i}\right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}.$$

First, consider an equilibrium in which $\iota = 0$, and study the conditions under which such an equilibrium exists. Using equation (A.13) and equation (A.18), we can solve for x_i and obtain

$$x_i = \underbrace{\left[\frac{k^{(1-\eta_i)\frac{\gamma-1}{\gamma}}\eta_i \frac{\gamma-1}{\gamma} C^{\frac{1}{\gamma}}}{W N_i^{1-\eta_i\frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}}\right]^{\frac{\gamma}{\eta_i+(1-\eta_i)\gamma}}}_{\equiv x_i^{\text{unc}}}. \tag{A.20}$$

[42]Note that the second expression is not obtained by differentiating the first expression with respect to x, but rather by imposing symmetry in $\frac{\partial rev_i(\{x_j, \vec{x}_{-j}\})}{\partial x_j}$.

Let us further conjecture that $p_i^{tc} = 0$ and $\tilde{J} = 0$. From the promise-keeping constraint, it then follows that

$$p_i^s = A_i rev_i(x_i^{\text{unc}}).$$

The final producer's borrowing constraint must hold for this allocation to be sustained in equilibrium. Since there are N_i symmetric suppliers, this requires

$$N_i A_i \leq 1 - \theta\left(1 - \delta(1-\pi_i)\right) \iff \theta \leq \underbrace{\frac{1 - N_i A_i}{1 - \delta(1-\pi_i)}}_{\underline{\theta}_i}. \tag{A.21}$$

Thus, if $\theta \leq \underline{\theta}_i$, the borrowing constraints for both the final-good producer and the supplier do not bind in the steady state, and $x_i = x_i^{\text{unc}}$.

For $\theta > \underline{\theta}_i$, it is still possible that x_i^{unc} can be supported in equilibrium using trade credit. The borrowing limit of the final-good producer binds, so the per-supplier spot payment is

$$p_i^s = \frac{1 - \theta(1 - \delta(1-\pi_i))}{N_i} rev_i(x_i^{\text{unc}}). \tag{A.22}$$

To determine trade credit, note that it must satisfy

$$p_i^{tc} \leq \beta \tilde{J} = \beta \left[A_i - \frac{1 - \theta(1 - \delta(1-\pi_i))}{N_i} \right] rev_i(x_i^{\text{unc}}).$$

To support x_i^{unc} in equilibrium, the supplier's borrowing constraint must hold:

$$W x_i^{\text{unc}} \leq p_i^s + (1-\theta) p_i^{tc}.$$

Plugging in the upper limit for p_i^{tc} and the expression for p_i^s, this condition becomes

$$W N_i x_i^{\text{unc}} \leq \left[1 - \theta(1 - \delta(1-\pi_i))\right] rev_i(x_i^{\text{unc}}) + (1-\theta)\beta N_i \left[A_i - \frac{1 - \theta(1 - \delta(1-\pi_i))}{N_i} \right] rev_i(x_i^{\text{unc}}). \tag{A.23}$$

Let $\overline{\theta}_i$ be the level of θ such that the expression above holds with equality. If $\theta \in (\underline{\theta}_i, \overline{\theta}_i]$, then the supplier's borrowing constraint does not bind and $x_i = x_i^{\text{unc}}$ is the equilibrium allocation with positive trade credit.

Finally, consider the case in which $\theta > \overline{\theta}_i$. In this case, x_i^{unc} cannot be supported in equilibrium, and μ and ι are both strictly positive. The supplier's borrowing constraint binds, so x_i^{con} satisfies

$$W N_i x_i^{\text{con}} = \left[1 - \theta(1 - \delta(1-\pi_i))\right] rev_i(x_i^{\text{con}}) + (1-\theta)\beta N_i \left[A_i - \frac{1 - \theta(1 - \delta(1-\pi_i))}{N_i} \right] rev_i(x_i^{\text{con}}). \tag{A.24}$$

Rearranging using the revenue formula and the expression for x_i^{unc}, we obtain

$$x_i^{\text{con}} = \left[\frac{(1-\beta(1-\theta))\,[1-\theta(1-\delta+\delta\pi_i)] + (1-\theta)\beta N_i A_i}{\frac{\gamma-1}{\gamma}\eta_i} \right]^{\frac{\gamma}{\eta_i+(1-\eta_i)\gamma}} x_i^{\text{unc}}. \qquad \text{(A.25)}$$

The spot and trade-credit payments are then given by

$$p_i^s = \frac{1-\theta(1-\delta(1-\pi_i))}{N_i} rev_i(x_i^{\text{con}}),$$

and

$$p_i^{tc} = \beta \left[A_i - \frac{1-\theta(1-\delta(1-\pi_i))}{N_i} \right] rev_i(x_i^{\text{con}}).$$

$\square$

Proof of Lemma 1

Proof. From Proposition 3, if $\theta > \underline{\theta}_i$, total trade credit relative to revenues is

$$\frac{N_i p_i^{\text{tc}}}{rev_i} = \beta \left\{ N_i A_i - [1-\theta(1-\delta+\delta\pi_i)] \right\},$$

where

$$A_i \equiv 1 - \left(\frac{N_i - 1}{N_i} \right)^{a_i}, \qquad a_i \equiv \eta_i \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma}.$$

First,

$$\frac{\partial \left(N_i p_i^{\text{tc}}/rev_i \right)}{\partial \eta_i} = -\beta N_i \left(\frac{N_i-1}{N_i} \right)^{a_i} \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma} \log \left(\frac{N_i-1}{N_i} \right) > 0,$$

since $(N_i-1)/N_i < 1$. Second,

$$\frac{\partial \left(N_i p_i^{\text{tc}}/rev_i \right)}{\partial \pi_i} = \beta\theta\delta > 0.$$

Finally, let $q_i \equiv \frac{N_i-1}{N_i}$. Then

$$\frac{\partial \left(N_i p_i^{\text{tc}}/rev_i \right)}{\partial N_i} = \beta \left[1 - q_i^{a_i} - a_i \frac{q_i^{a_i-1}}{N_i} \right].$$

Since $1/N_i = 1 - q_i$, the term in brackets equals

$$1 - q_i^{a_i} - a_i(1-q_i) q_i^{a_i-1}.$$

For $a_i \in (0,1)$, the function q^{a_i} is concave, so

$$1 \leq q_i^{a_i} + a_i q_i^{a_i - 1}(1 - q_i),$$

which implies

$$\frac{\partial \left(N_i p_i^{\text{tc}} / rev_i \right)}{\partial N_i} < 0$$

for $N_i > 1$.

$\square$

A.4 Corporate Subsidies

In this section we discuss how to incorporate corporate subsidies into the model. Section A.4.1 writes the recursive problem of the supplier with corporate subsidies in the full model. Section A.4.2 analyzes comparative statics in the special case of the model presented in Section 3. Section A.4.3 presents the effects of corporate subsidies in the quantitative model, showing that corporate subsidies are more effective when targeted toward suppliers rather than final-good firms.

A.4.1 The recursive problem with corporate subsidies

Let us assume that the government uses lump-sum subsidies to support financially constrained firms during financial crises. In particular, $N_i T_i^{\text{final}}$ denotes the total lump-sum subsidy to final-good producers and T_i^{supplier} the lump-sum subsidy to each supplier in production line i. The recursive problem of the intermediate-good producer supplying goods to final-good firms is then given by

$$
\begin{aligned}
V\left(\tilde{J}(\theta), \theta, \Omega\right) = \max_{p_j^s, p_j^{tc}, x_j, \tilde{J}'(\theta')} \; & p_j^s + p_j^{tc} - w(\theta,\Omega)x_j + T_i^{\text{supplier}} \mathbb{1}\left(\theta = \theta^H\right) + \beta \mathbb{E}\left[V\left(\tilde{J}'(\theta'), \theta', \Omega'(\theta')\right)\right] \\
\text{s.t.} \quad & p_j^s + \sum_{n \neq j} p_n^s \leq \left[1 - \theta\left(1 - \delta(1-\pi_i)\right)\right] rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega) + N_i T_i^{\text{final}} \mathbb{1}\left(\theta = \theta^H\right), && [\mu] \\
& p_j^{tc} \leq \beta \mathbb{E}\left[\tilde{J}'(\theta')\right], && [\kappa] \\
& w(\theta,\Omega)x_j - p_j^s \leq (1-\theta) p_j^{tc} + T_i^{\text{supplier}} \mathbb{1}\left(\theta = \theta^H\right), && [\iota] \\
& \tilde{J}(\theta) = rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega) - rev_i(\{0, \vec{x}_{-j}\}, \theta, \Omega) - (p_j^s + p_j^{tc}) + \beta \mathbb{E}_\theta\left[\tilde{J}'(\theta')\right], && [\lambda] \\
& p_j^s + p_j^{tc} + \sum_{n \neq j}(p_n^s + p_n^{tc}) \leq rev_i(\{x_j, \vec{x}_{-j}\}, \theta, \Omega) + N_i T_i^{\text{final}} \mathbb{1}\left(\theta = \theta^H\right), && [\rho] \\
& \tilde{J}'(\theta') \geq 0. && [\zeta(\theta')]
\end{aligned}
$$

A.4.2 Corporate subsidies in the special case of the model

We characterize the effects of lump-sum corporate subsidies in the special case of Section 3. We assume there are no aggregate shocks, that all final goods are received at the end of the period ($\delta = 0$), and that financial frictions bind the supplier in steady state absent subsidies ($\theta > \bar{\theta}$). The supplier's problem with lump-sum subsidies is

$$
\begin{aligned}
V(J) = \max_{\{x, p^{\mathrm{s}}, p^{\mathrm{tc}}, J'\}} \quad & \left(p^{\mathrm{s}} + p^{\mathrm{tc}} - Wx\right) + T_s + \beta V(J'), \\
\text{s.t.} \quad & p^{\mathrm{s}} \leq (1-\theta)x^{\eta} + T_f, \\
& Wx - p^{\mathrm{s}} \leq (1-\theta)p^{\mathrm{tc}} + T_s, \\
& p^{\mathrm{tc}} \leq \beta J', \\
& p^{\mathrm{s}} + p^{\mathrm{tc}} \leq x^{\eta} + T_f, \\
& J = x^{\eta} - (p^{\mathrm{s}} + p^{\mathrm{tc}}) + \beta J',
\end{aligned}
$$

where T_f is the lump-sum subsidy to final-good firms and T_s the one to the supplier.

Proposition 4. *Suppose $\theta > \bar{\theta}$, where $\bar{\theta}$ is defined implicitly by $(1-\bar{\theta})(1+\beta\bar{\theta}) = \eta$. The effects of corporate subsidies on employment in the steady state, evaluated at $T_s = T_f = 0$, are*

$$
\left.\frac{\partial x}{\partial T_s}\right| = \frac{1}{(1-\eta)W}, \qquad \left.\frac{\partial x}{\partial T_f}\right| = \frac{1-\beta(1-\theta)}{(1-\eta)W}.
$$

Proof. This proof is very similar to the proof of Proposition 1. Note that the corporate subsidies do not alter the optimality conditions but only the constraints. In the steady state, $\lambda' = \lambda$ so that equation (A.16) implies $\kappa = 0$. Combining equations (A.11) and (A.12), we obtain $\mu = \theta\iota$. In the steady state, the promise keeping constraint boils down to

$$
J = \frac{1}{1-\beta}\left(x^{\eta} - p^{s} - p^{tc}\right).
$$

Since $\theta > \bar{\theta}$, we know that the monopolist's borrowing constraint binds in the steady state when $T_f = T_s = 0$ (see proof to Proposition 1). We have that $p^{s} = (1-\theta)x^{\eta} + T_f$ and the promise keeping constraint implies $(1-\beta)J = \theta x^{\eta} - p^{tc} - T_f$. The supplier's borrowing constraint is

$$
(1-\theta)(1-\beta)J = \left[(1+\theta)(1-\theta)x^{\eta} - Wx + \theta T_f + T_s\right].
$$

For the trade credit constraint to be satisfied, we must have $J \geq \theta x^{\eta} - T_f$. We restrict attention to the lowest level of J which can be supported in equilibrium. This level of J is supported by the upper limit of trade credit, $p^{tc} = \beta J = \beta\theta x^{\eta} - \beta T_f$. Plugging $J = \theta x^{\eta} - T_f$

into the supplier borrowing constraint we obtain:

$$(1-\theta)(1-\beta)\left(\theta x^{\eta}-T_f\right)=\left[(1+\theta)(1-\theta)x^{\eta}-Wx+\theta T_f+T_s\right].$$

Rearranging we obtain that

$$(1+\beta\theta)(1-\theta)x^{\eta}+(1-\beta(1-\theta))\,T_f+T_s=Wx$$

Totally differentiating with respect to x, T_f, and T_s, we have

$$\frac{1}{x}\eta(1+\beta\theta)(1-\theta)x^{\eta}dx+(1-\beta(1-\theta))\,dT_f+dT_s=Wdx,$$

rearranging we have

$$(1-\beta(1-\theta))\,dT_f+dT_s=\frac{1}{x}\left(Wx-\eta(1+\beta\theta)(1-\theta)x^{\eta}\right)dx.$$

Using the fact that when $T_s=T_f=0$, we have that $(1+\beta\theta)(1-\theta)x^{\eta}=Wx$, we get that when $T_f=T_s=0$, the equation above simplifies to

$$(1-\beta(1-\theta))\,dT_f+dT_s=(1-\eta)\,Wdx, \tag{A.26}$$

so that

$$\left.\frac{\partial x}{\partial T_s}\right|_{T_f=T_s=0}=\frac{1}{(1-\eta)W}, \qquad \left.\frac{\partial x}{\partial T_f}\right|_{T_f=T_s=0}=\frac{1-\beta(1-\theta)}{(1-\eta)W}$$

□

A subsidy to the final-good firm is therefore only a fraction $1-\beta(1-\theta)$ as effective as a subsidy to the supplier. To see why, combine the binding borrowing constraints of the final-good firm and of the supplier:

$$Wx=(1-\theta)x^{\eta}+(1-\theta)p^{\mathrm{tc}}+T_s+T_f.$$

A dollar of either subsidy contributes the same direct cash to the chain. The two differ in how they shift the customer's surplus J, and hence the trade-credit limit $p^{\mathrm{tc}}\leq\beta J$. Substituting the customer's binding borrowing constraint into the expression for J gives, in steady state,

$$p^{\mathrm{tc}}=\beta J=\beta\left[\theta x^{\eta}-T_f\right].$$

Both subsidies raise J because output is higher, but T_s raises it more. The reason is that

T_f accrues to the customer regardless of whether it stays in the relationship: it raises the customer's outside option and partially offsets the rise in J generated by higher output. The slacker trade-credit limit under T_s supports financing beyond the direct injection. That is why T_s is the more effective instrument.

A.4.3 Corporate subsidies in the quantitative model

We now quantify the effects of corporate subsidies in the full model. The government has a budget equal to 1% of steady-state consumption, disbursed as lump-sum transfers during the crisis ($\theta = \theta^H$). We consider six policies that vary along two dimensions, holding the budget fixed. The first dimension is the *recipient*: the transfer goes to all firms (split evenly between final-good firms and suppliers), to suppliers only, or to final-good firms only. The second dimension is the *cross-sector allocation*: the per-firm transfer is either uniform across sectors or scaled by the depth of the financial constraint, $\max(\theta^H - \bar{\theta}_i, 0)$, where $\bar{\theta}_i$ is the threshold level of financial frictions above which the borrowing constraint of suppliers binds in sector i. Sectors deeper into the constrained region receive proportionally larger transfers. Table A-1 reports, for each policy, the impact effect on log consumption of moving from θ_L to θ_H, and the output gain relative to no subsidy.

Every policy cushions the crisis substantially, roughly halving the decline in output from 12.8 to about 6 log points. The composition of the transfer matters, however, even though the budget does not change. Holding the cross-sector allocation fixed, a subsidy to suppliers is more effective than a subsidy to final-good firms: under the uniform allocation it raises output by 6.63 log points versus 6.27, about 6% more. This is the mechanism of Appendix A.4.2 operating in the quantitative model. A transfer to suppliers relaxes the supplier's borrowing constraint directly (allowing them to fund more trade credit), whereas a transfer to final-good firms is partly absorbed into a larger up-front payment to suppliers, which substitutes for trade credit.

Targeting the transfer to the most constrained sectors delivers a further gain. Scaling the per-firm subsidy by the financial constraint of each sector raises effectiveness by about 5% for every recipient group—directing the same budget to the sectors where the constraint binds hardest buys more output. The two margins reinforce each other. The most effective policy, a constraint-scaled subsidy to suppliers, raises output by 6.96 log points: 8% more than an untargeted subsidy spread across all firms, and 11% more than an untargeted subsidy to final-good firms.

Table A-1: The effects of corporate subsidies

Policy	Output response (ln pts)	Output gain vs. benchmark	Effectiveness (rel. to uniform/all)
No subsidy	−12.79	0.00	—
Uniform across sectors			
All firms	−6.34	6.45	1.00
Suppliers only	−6.17	6.63	1.03
Final-good firms only	−6.52	6.27	0.97
Scaled by sectors' financial hardship			
All firms	−6.00	6.79	1.05
Suppliers only	−5.84	6.96	1.08
Final-good firms only	−6.18	6.61	1.02

Notes: Each policy disburses a budget of 1% of steady-state consumption during the crisis. Output response is the impact effect on log consumption (in log points) of a switch from θ_L to θ_H. Effectiveness is the output gain normalized to the uniform subsidy to all firms. Targeted policies scale the per-firm transfer by $\max(\theta^H - \bar{\theta}_i, 0)$, renormalized so the total budget is unchanged.

A.5 Markov Perfect Equilibrium with linear pricing

We characterize the Markov Perfect Equilibrium (MPE) in which the supplier posts per-unit prices and the final-good firm chooses quantities. With a slight abuse of notation, p^s and p^{tc} denote per-unit spot and trade-credit prices throughout this section.

A.5.1 Setup

The final-good firm solves

$$\begin{aligned} \max_x \quad & x^\eta - (p^s + p^{tc})x, \\ \text{s.t.} \quad & p^s x \leq [1 - \theta(1-\delta)]x^\eta. \end{aligned}$$

The resulting demand function is

$$x = \min\left\{\eta^{\frac{1}{1-\eta}}(p^s + p^{tc})^{-\frac{1}{1-\eta}},\ [1-\theta(1-\delta)]^{\frac{1}{1-\eta}}(p^s)^{-\frac{1}{1-\eta}}\right\}, \tag{A.27}$$

where the first branch obtains when the borrowing constraint is slack and the second when it binds. Note that under linear pricing, the supplier's markup drives a wedge between the marginal product of the intermediate input and its cost, distorting production downward even absent financial frictions. This markup distortion is not present in the baseline model, where the take-it-or-leave-it protocol allows the supplier to extract surplus without distorting quantities.

The supplier takes the future customer surplus J' as given and solves

$$
\begin{aligned}
V(\theta) = \max_{x,p^s,p^{tc}} & \; [(p^s + p^{tc})x - Wx] + \beta \mathbb{E}[V(\theta')], \\
\text{s.t.} \quad & W \le p^s + (1-\theta)p^{tc}, \\
& p^{tc} x \le \beta \mathbb{E}[J'(\theta')], \\
& x \text{ satisfies (A.27)},
\end{aligned}
$$

where $J(\theta) = x^\eta - (p^s + p^{tc})x + \beta \mathbb{E} J(\theta')$. Because the supplier's choices do not affect its continuation value, the problem is effectively static.

A.5.2 Steady-state characterization

Proposition 5. *Suppose θ is constant over time, and parameters satisfy $\beta(1-\eta) \ge \eta(1-\delta)(1-\beta)^2$. There exist thresholds $\bar{\theta}_1 \le \bar{\theta}_2$ such that:*

1. *If $\theta \le \bar{\theta}_1$, the supplier's borrowing constraint and trade-credit limit are slack and the allocation is:*

$$
\begin{aligned}
x &= \eta^{\frac{2}{1-\eta}} W^{-\frac{1}{1-\eta}}, \quad p^s = \min\{\eta, [1-\theta(1-\delta)]\} x^{\eta-1}, \\
p^{tc} &= \max\left\{0,\ [\eta - (1-\theta(1-\delta))]x^{\eta-1}\right\}, \quad J = \frac{1-\eta}{1-\beta} x^\eta.
\end{aligned}
$$

2. *If $\theta \in (\bar{\theta}_1, \bar{\theta}_2]$, the supplier's borrowing constraint binds and output falls:*

$$
\begin{aligned}
x &= [(1-\theta)\eta + \theta\,(1-\theta(1-\delta))]^{\frac{1}{1-\eta}} W^{-\frac{1}{1-\eta}}, \quad p^s = [1-\theta(1-\delta)]x^{\eta-1}, \\
p^{tc} &= \frac{\eta - (1-\theta(1-\delta))}{(1-\theta)\eta + \theta\,(1-\theta(1-\delta))} W, \quad J = \frac{1-\eta}{1-\beta} x^\eta.
\end{aligned}
$$

3. *If $\theta > \bar{\theta}_2$, both the supplier's borrowing and trade credit constraints bind:*

$$
\begin{aligned}
x &= [1-\theta(1-\delta) + \beta(1-\theta)\theta(1-\delta)]^{\frac{1}{1-\eta}} W^{-\frac{1}{1-\eta}}, \quad p^s = [1-\theta(1-\delta)]x^{\eta-1}, \\
p^{tc} &= \beta\theta(1-\delta)x^{\eta-1}, \quad J = \theta(1-\delta)x^\eta.
\end{aligned}
$$

where $\bar{\theta}_1$ is the positive root of $\theta^2(1-\delta) - \theta(1-\eta) - \eta(1-\eta) = 0$, and $\bar{\theta}_2$ is defined by $\theta(1-\delta) = (1-\eta)/(1-\beta)$.

Proof. Since the supplier's problem is static, the equilibrium allocation is determined by which constraints bind. We consider each case in turn.

Case 1. When neither the supplier's borrowing constraint nor the trade credit constraint binds, the supplier sets $p^s + p^{tc} = W/\eta$, yielding $x = \eta^{2/(1-\eta)}W^{-1/(1-\eta)}$. The supplier sets p^s at the maximum level consistent with the customer's borrowing constraint, $p^s = \min\{\eta, [1-\theta(1-\delta)]\}\, x^{\eta-1}$, with the residual collected via trade credit. When $1-\theta(1-\delta) \geq \eta$, no trade credit is needed.

In steady state, $J = (1-\eta)x^\eta/(1-\beta)$. Substituting the implied p^{tc} into the supplier's borrowing constraint and rearranging yields the condition $\theta^2(1-\delta) - \theta(1-\eta) - \eta(1-\eta) \leq 0$, which defines $\bar{\theta}_1$.

Case 2. For $\theta > \bar{\theta}_1$, the allocation from Case 1 can no longer satisfy the supplier's borrowing constraint. In this region, the supplier's borrowing constraint binds. Suppose the trade-credit constraint remains slack. The supplier still chooses the total price so that the customer's unconstrained optimality condition holds,

$$p^s + p^{tc} = \eta x^{\eta-1}.$$

However, because spot payments relax the supplier's borrowing constraint one-for-one while trade-credit payments relax it only by $1-\theta$, the supplier sets the spot payment at the customer's borrowing limit:

$$p^s = [1-\theta(1-\delta)]\, x^{\eta-1}.$$

It follows that

$$p^{tc} = [\eta - (1-\theta(1-\delta))]\, x^{\eta-1}.$$

Combining these expressions with the binding supplier borrowing constraint,

$$W = p^s + (1-\theta)p^{tc},$$

yields

$$x = [(1-\theta)\eta + \theta\,(1-\theta(1-\delta))]^{\frac{1}{1-\eta}}\, W^{-\frac{1}{1-\eta}},$$

and the expression for p^{tc} in the proposition follows immediately.

The steady-state customer surplus in this region is

$$J = \frac{1-\eta}{1-\beta}x^\eta.$$

The upper boundary of this region is reached when the trade-credit constraint starts to bind,

$$p^{tc}x = \beta J = \frac{\beta(1-\eta)}{1-\beta}x^{\eta}.$$

Using

$$p^{tc}x = [\eta - (1-\theta(1-\delta))]\, x^{\eta},$$

this condition reduces to

$$\theta(1-\delta) = \frac{1-\eta}{1-\beta}.$$

We denote by $\bar{\theta}_2$ the value of θ satisfying this equation.

Case 3. For $\theta > \bar{\theta}_2$, the trade credit constraint also binds: $p^{tc}x = \beta J$. With both the supplier's borrowing and trade credit constraints binding, the customer's borrowing constraint must bind as well. In steady state, the customer's surplus satisfies

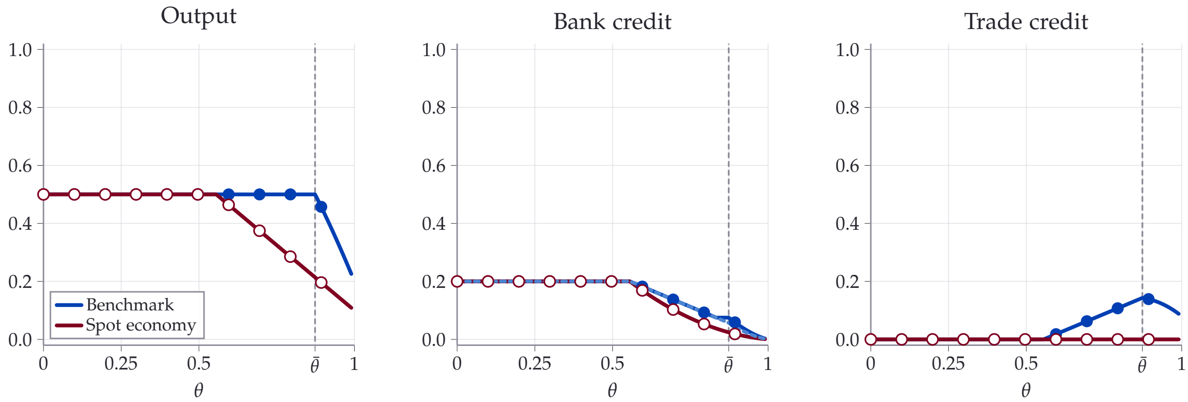
$$(1-\beta)J = \theta(1-\delta)x^{\eta} - \beta J \quad \Longrightarrow \quad J = \theta(1-\delta)x^{\eta}.$$

Substituting $p^{tc} = \beta\theta(1-\delta)x^{\eta-1}$ and $p^{s} = [1-\theta(1-\delta)]x^{\eta-1}$ into the supplier's borrowing constraint gives x as stated.

□

Figure A-1 illustrates the steady-state allocation as a function of θ.

Figure A-1: MPE with linear pricing, deterministic steady state



Notes: For the numerical illustration, we set $\beta = 0.97$, $\eta = 0.5$, $\chi = 0.5$, and $\delta = 0.1$. The solid blue line reports the steady-state level of output, total bank credit, and trade credit for different values of θ. The red line reports the corresponding values for the spot economy. The dashed blue line in the middle panel reports bank credit to final-good firms.

A.5.3 Properties and response to financial shocks

Three qualitative properties of the baseline model carry over to the MPE.

Pecking order. When $1-\theta(1-\delta) \geq \eta$, the unconstrained allocation can be sustained with spot payments alone. In the unconstrained region more broadly, the supplier is indifferent about the split between spot and trade credit payments, since this split does not affect profits or future surplus. This contrasts with the baseline, where trade credit is strictly costly because it requires promising future rents to incentivize repayment. The pecking order nevertheless emerges once financial constraints tighten: a dollar of spot payment contributes more to the supplier's borrowing capacity than a dollar of trade credit (1 vs. $1-\theta$), so the supplier exhausts spot capacity before turning to trade credit.

Credit multiplier. In the spot economy with linear pricing, the constrained allocation is $x^{\text{spot}} = [(1-\theta(1-\delta))\eta]^{\frac{1}{1-\eta}} W^{-\frac{1}{1-\eta}}$. In Cases 2 and 3 of Proposition 5, the economy with trade credit sustains strictly higher output, $x > x^{\text{spot}}$, as trade credit relaxes the financing constraint on the supply chain even when the supplier must itself borrow against trade credit receivables.

State-dependent amplification. Consider a permanent increase in θ. Because the MPE has no endogenous state variable, the economy jumps immediately to the new steady state. In the unconstrained region, output does not respond to θ. In the region where the supplier's borrowing constraint alone binds:

$$\frac{\partial \ln x}{\partial \ln \theta} = -\frac{1}{1-\eta} \frac{2\theta^2(1-\delta) - \theta(1-\eta)}{(1-\theta)\eta + \theta(1-\theta(1-\delta))}.$$

When both constraints bind:

$$\frac{\partial \ln x}{\partial \ln \theta} = -\frac{1}{1-\eta} \frac{(1-\beta)(1-\delta)\theta + 2\beta\theta^2(1-\delta)}{1-\theta(1-\delta) + \beta(1-\theta)\theta(1-\delta)}.$$

For comparison, the spot economy elasticity in the constrained region is

$$\frac{\partial \ln x^{\text{spot}}}{\partial \ln \theta} = -\frac{1}{1-\eta} \frac{\theta(1-\delta)}{1-\theta(1-\delta)}.$$

As in the baseline, the response of output to financial shocks varies across regions, and the trade credit economy can exhibit either dampening or amplification relative to the spot economy depending on the level of θ. The key quantitative difference is that the region of complete smoothing is smaller in the MPE because, absent commitment, suppliers cannot promise larger future surpluses to relax current trade credit constraints.

A.6 Data appendix

In this section, we detail the procedure we use to construct the balanced panel dataset we use in our analysis. We then present some descriptive statistics.

The raw historical dataset contains 18,036,802 firm-year observations. We keep only unconsolidated statements, dropping firms with consolidation codes "NA" (no financial data available), "LF" (limited financial data available), "C1" (consolidated statement with unconsolidated companion), and "C2" (consolidated statement with no unconsolidated companion). This leaves us with 11,304,284 firm-year observations.[43]

We then drop observations where total assets, accounts receivable, accounts payable, operating revenues, or sales are missing. Additionally, we drop observations with negative values for one of the following variables: total assets, employment, sales, wage bill, accounts receivable, accounts payable, short-term bank loans, long-term debt, depreciation, cash holdings, or total inventories. Finally, we drop observations where the year of operation is earlier than the date of incorporation of the firm. These steps leave us with 10,793,282 firm-year observations.

We restrict the sample to 2007–2015 (8,041,927 observations). We drop financial firms (NACE 64–67; 136,677 obs.) and firms in public administration and other heavily subsidized sectors (NACE 84–88, namely public administration, education, health, and social work; 224,416 obs.). We drop firms with less than EUR 10,000 in total assets or annual sales (1,750,006 obs.) and keep only firms with an "active" status (dropping 1,363,336 observations). We then keep firms with observations in all years so that the panel is balanced. The balanced panel contains 2,365,722 observations.

We construct a measure of intermediate inputs by subtracting EBIT, the wage bill, and depreciation from operating revenue. For each firm-year, we compute the ratios of accounts payable, accounts receivable, and short-term bank loans to sales, and of intermediate inputs to operating revenue. We winsorize each ratio at its top and bottom 1% within each year, and drop firms in the top and bottom 1% of log-sales growth (244,998 obs.). We then map firms to the 54 input–output sectors, dropping firms whose sectors cannot be matched to the input–output classification (288 observations), and exclude the refinery sector (1,314 observations), yielding the final sample.[44] The sector-level moments used in the calibration are the equal-weighted averages of these firm-year ratios across firms within each sector.

The final dataset contains a balanced panel of 235,458 firms over 2007–2015, resulting in 2,119,122 observations. Table A-2 displays descriptive statistics using the final balanced

[43] We additionally drop 2,449 duplicate observations.

[44] We drop the refinery sector because most purchases of refinery products are imported.

sample of firms.

Table A-2: Descriptive statistics at the firm-level

Variable	Mean	P10	P25	P50	P75	P90	% of firms with data
Total assets	8,223,811	144,730	340,266	927,730	2,742,296	8,051,413	100%
Sales	6,248,798	85,426	231,170	722,804	2,290,974	7,088,278	100%
Accounts payable	1,144,966	0	7,180	90,826	434,039	1,527,906	100%
Accounts receivable	1,404,430	0	6,540	123,468	581,338	2,038,853	100%
Short-term bank loans	692,610	0	0	7,617	166,490	800,044	100%
Intermediate inputs	6,535,045	115,634	276,327	763,315	2,362,405	7,244,177	78%

Notes: Descriptive statistics from the firm-level balanced panel. All monetary variables are in euros.

For Figure 4, we map the 54 sectors from the input-output table to the 16 ISTAT sectors using the crosswalk in Table A-3.

Table A-3: Crosswalk: ISTAT sectors to IO sectors

ISTAT	ISTAT description	IO codes	IO sector descriptions
A	Agriculture, forestry, fishing	1, 2, 3	Crop & animal production; Forestry; Fishing
B	Mining and quarrying	5	Mining and quarrying
C10-12	Food, beverages, tobacco	10	Food products; beverages and tobacco
C13-18	Textiles, apparel, wood, paper, printing	13, 16, 17, 18	Textiles & apparel; Wood products; Paper products; Printing
C19-21	Chemicals, pharma	20, 21	Chemicals; Pharmaceuticals
C22-25	Rubber, plastics, minerals, metals	22, 23, 24, 25	Rubber & plastics; Non-metallic minerals; Basic metals; Fabricated metal products
C26-28	Electronics, electrical, machinery	26, 27, 28	Computer & electronics; Electrical equipment; Machinery n.e.c.
C29-30	Transport equipment	29, 30	Motor vehicles; Other transport equipment
C31-33	Furniture, other manufacturing, repair	31, 33	Furniture & other manufacturing; Repair & installation of machinery
D-E	Electricity, gas, water, waste	35, 36, 37	Electricity & gas; Water supply; Sewerage & waste management
F	Construction	41	Construction
G-I	Trade, transport, accommodation	45, 46, 47, 49, 50, 51, 52, 53, 55	Wholesale & retail trade; Land transport; Water & air transport; Warehousing; Postal services; Accommodation & food
J	Information and communication	58, 59, 61, 62	Publishing; Film & broadcasting; Telecommunications; IT services
L	Real estate	68	Real estate activities
M-N	Professional, scientific, admin services	69, 71, 72, 73, 74, 77, 78, 79, 80	Legal & accounting; Architecture & engineering; R&D; Advertising; Other professional; Rental; Employment; Travel agencies; Security & office support
R-S-T	Arts, entertainment, other services	90, 93, 94, 95, 96	Arts & entertainment; Sports & recreation; Membership organisations; Repair of household goods; Other personal services

Notes: This table maps the 16 ISTAT value-added sectors (ATECO A21 classification) to the 54 IO sectors used in the model (NACE A64 classification). The ISTAT data are quarterly chain-linked value added from the Italian national accounts.

A.7 Numerical Algorithm

In this section, we lay out the numerical algorithm we use to solve the model. Our approach relies on policy function iteration combined with an approximate law of motion similar to Krusell and Smith (1998). We start by defining some notation. We then show how to solve the partial-equilibrium problem, where aggregate variables are taken as given. Finally, we show how we solve the full general equilibrium.

We describe the algorithm for the general case with corporate subsidies. The benchmark model is the special case in which these subsidies are set to zero. To make clear where the subsidies enter, we color all such terms in blue.

A.7.1 Notation

Before describing the numerical algorithm in detail, it is useful to define some notation. Let

$$\tilde{\theta}_i(\theta) = \theta\left(1 - \delta(1 - \pi_i)\right),$$

and, when no confusion is possible, write $\tilde{\theta}_i$ for $\tilde{\theta}_i(\theta)$ and $\tilde{\theta}_i'$ for $\tilde{\theta}_i(\theta')$.

The revenue of final-good producers in sector i is denoted by

$$rev_i(x) = C(\theta,\Omega)^{\frac{1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}} x^{\eta_i \frac{\gamma-1}{\gamma}}.$$

The first-best labor input in sector i is denoted by

$$x_i^{\text{fb}} = \left[\frac{\gamma-1}{\gamma}\eta_i \frac{C(\theta,\Omega)^{\frac{1}{\gamma}}}{w(\theta,\Omega)} N_i^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1}\right]^{\frac{\gamma}{(1-\eta_i)\gamma+\eta_i}}.$$

The promise-keeping constraint is

$$\tilde{J}(\theta) = A_i rev_i(x) - p^s - p^{tc} + \beta \mathbb{E}_\theta\left[\tilde{J}'(\theta')\right],$$

where

$$A_i \equiv 1 - \left(\frac{N_i - 1}{N_i}\right)^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}.$$

Next, denote by $x_i^{\text{cons}}(\tilde{J}_\theta, \theta)$ the solution to

$$rev_i(x) = \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}}.$$

That is,

$$x_i^{\text{cons}}(\tilde{J}_\theta, \theta) = \left[\frac{\tilde{J}_\theta + T_f(\theta)}{\left(A_i - \frac{1-\tilde{\theta}_i}{N_i}\right) C(\theta,\Omega)^{\frac{1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}}}\right]^{\frac{\gamma}{\eta_i(\gamma-1)}}.$$

A.7.2 Partial equilibrium

We start by solving the behavior of suppliers in a single sector, taking the aggregate values $C(\theta,\Omega)$ and $w(\theta,\Omega)$ as given. For notational simplicity, we denote current aggregate variables by C and w, and next-period aggregate variables by C' and w'. We assume that θ can take one of two values, θ_L and θ_H.

The numerical algorithm uses policy function iteration to find the equilibrium policy function. The policy function maps the current state, which includes the aggregate financial state θ and the promised value $\tilde{J}_\theta$, into promised values next period for each possible realization of θ'. We denote the policy function by

$$\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta).$$

This is the promised value that the supplier must deliver in state θ', given the current state $\{\tilde{J}_\theta, \theta\}$.

Our policy function iteration proceeds in three steps:

1. Given a guess for the policy function and the current state $\{\tilde{J}_\theta, \theta\}$, compute future labor inputs x' for every θ' as a function of the current state and the current Lagrange multipliers on the supplier borrowing constraint, ι, and the feasibility constraint, ρ.
2. Using the implied function $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$, update the policy function.
3. If the implied policy function is close to the guessed one, the policy function has converged. Otherwise, update the guess and repeat from step 1.

The state-space grid is over $\{\tilde{J}_\theta, \theta\}$. The lower bound for $\tilde{J}_\theta$ is close to zero, and the upper bound is given by

$$\left(A_i - \frac{1-\tilde{\theta}_i}{N_i}\right) rev_i(x_i^{\text{fb}}) - T_f(\theta).$$

A.7.2.1 Calculating x' given the policy function and current Lagrange multipliers

The optimality condition for x' can be written as

$$\frac{\gamma-1}{\gamma}\eta_i \left(C'\right)^{\frac{1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1} \left(x'\right)^{\eta_i \frac{\gamma-1}{\gamma}-1} = \frac{w'(1+\iota')}{(1-\tilde{\theta}_i')\mu' + \rho' + \lambda'}. \tag{A.28}$$

In this section, we show how to use the current Lagrange multipliers ι and ρ, together with the state variables $\{\tilde{J}_\theta, \theta\}$ and the guessed policy function $\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta)$, to compute x'. That is, we construct the function

$$x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho).$$

To find x', we need to use promised values two periods ahead. Given the current state $\{\tilde{J}_\theta, \theta\}$ and a realization θ', define

$$\mathbb{E}_{\theta'}\left[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)\right] \equiv \mathbb{E}_{\theta'}\left[\tilde{J}'\left(\theta'' \mid \tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta), \theta'\right)\right]. \tag{A.29}$$

When we apply the policy function a second time, the promised value $\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta)$ need not lie exactly on the grid. We use Chebyshev approximation to evaluate the policy function between grid points.

We start by deriving an equation connecting the current multipliers ι and ρ with future multipliers. From the optimality condition with respect to p^s, we have

$$\mu' + \rho' - \iota' = 1 - \lambda'.$$

Using the law of motion for the multiplier on the promise-keeping constraint, together with the optimality condition with respect to p^{tc} in the current period, we obtain

$$\rho - (1-\theta)\iota = \mu' + \rho' - \iota'. \tag{A.30}$$

Thus, the current multipliers ι and ρ determine the value of $\mu' + \rho' - \iota'$. We split the derivation of x' into two cases, depending on the sign of $\rho - (1-\theta)\iota$.

Case A: $\rho - (1-\theta)\iota \geq 0$**.**

Suppose first that the supplier borrowing constraint and the feasibility constraint do not bind next period, so that $\iota' = \rho' = 0$. Then

$$\mu' = \rho - (1-\theta)\iota.$$

Using the optimality condition with respect to p^s, we have $\lambda' = 1 - \mu'$. The optimality condition for x' becomes

$$\frac{\gamma - 1}{\gamma}\eta_i \left(C'\right)^{\frac{1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1}\frac{\gamma-1}{\gamma}-1} \left(x'\right)^{\eta_i \frac{\gamma-1}{\gamma}-1} = \frac{w'}{1 - \tilde{\theta}_i' \mu'}.$$

This equation pins down x'. We then check whether the supplier borrowing constraint and feasibility constraint are satisfied:

$$w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') \leq (1-\theta')\beta \mathbb{E}_{\theta'}\left[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)\right] + T_s(\theta') + T_f(\theta'), \tag{A.31}$$

$$\tilde{\theta}_i' rev_i(x') \geq N_i \beta \mathbb{E}_{\theta'}\left[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)\right]. \tag{A.32}$$

In the first expression, final-good producers receive T_f per supplier, so the supplier's borrowing constraint is relaxed by $T_f + T_s$. If both inequalities hold, we have found $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$. Otherwise, we proceed as follows.

1. If both constraints (A.31) and (A.32) are violated, then $\mu' > 0$, $\rho' > 0$, and $\iota' > 0$.

Combining the binding supplier borrowing constraint and feasibility constraint gives

$$w'x' = \frac{1-\theta'\tilde{\theta}_i'}{N_i} rev_i(x') + T_f(\theta') + T_s(\theta'). \tag{A.33}$$

This equation pins down x'.

2. If only the supplier borrowing constraint (A.31) is violated, then $\mu' > 0$ and $\iota' > 0$. Conjecture that the feasibility constraint is slack, so that $\rho' = 0$. From equation (A.30),

$$\mu' - \iota' = \rho - (1-\theta)\iota \geq 0.$$

From the optimality conditions with respect to p^s and p^{tc}, we have $\mu' = \theta'\iota' + \kappa'$. Thus, $\kappa' > 0$, so the final-good producer borrowing constraint and the trade-credit constraint bind. Combining the supplier borrowing constraint, final-good producer borrowing constraint, and trade-credit constraint gives

$$w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') = (1-\theta')\beta\mathbb{E}_{\theta'}\left[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)\right] + T_f(\theta') + T_s(\theta').$$

This equation pins down x'. We then verify that the feasibility constraint is satisfied:

$$\tilde{\theta}_i' rev_i(x') \geq N_i \beta \mathbb{E}_{\theta'}\left[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)\right].$$

If this inequality holds, we have found x'. Otherwise, $\rho' > 0$ and x' is obtained from equation (A.33).

3. If only the feasibility constraint (A.32) is violated, then $\mu' > 0$ and $\rho' > 0$. Conjecture that the supplier borrowing constraint is slack, so that $\iota' = 0$. From the optimality conditions with respect to p^s and p^{tc}, we have $\mu' = \theta'\iota' + \kappa'$, and therefore $\kappa' = \mu' > 0$. Thus, the final-good producer borrowing constraint, the trade-credit constraint, and the feasibility constraint bind. Combining these constraints gives

$$\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x') = \tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) + T_f(\theta').$$

This equation pins down x'. We then check whether the supplier borrowing constraint is satisfied:

$$w'x' \leq \frac{1-\theta'\tilde{\theta}_i'}{N_i} rev_i(x') + T_f(\theta') + T_s(\theta').$$

If this inequality holds and the feasibility constraint is satisfied, we have found x'. Otherwise, $\iota' > 0$ and x' is obtained from equation (A.33).

Case B: $\rho - (1-\theta)\iota < 0$.

From equation (A.30), $\iota' > 0$. Since μ' and ρ' are nonnegative, and since $\mu' = \theta'\iota' + \kappa'$ with $\kappa' \geq 0$, we also have $\mu' > 0$.

1. Suppose first that $\rho' = 0$. We consider two subcases.

   (a) Suppose $\kappa' = 0$. Then $\mu' = \theta'\iota'$, and equation (A.30) implies

   $$\iota' = \frac{(1-\theta)\iota - \rho}{1-\theta'}.$$

   Given ι' and $\mu' = \theta'\iota'$, the optimality condition with respect to p^s gives

   $$\lambda' = 1 + \iota' - \mu'.$$

   Plugging these multipliers into equation (A.28) pins down x'. We then check whether the trade-credit constraint is satisfied. Using the supplier borrowing constraint,

   $$p_j^{tc\prime} = \frac{1}{1-\theta'}\left[w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') - T_f(\theta') - T_s(\theta')\right].$$

   Thus, the trade-credit constraint requires

   $$w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') - T_f(\theta') - T_s(\theta') \leq (1-\theta')\beta\mathbb{E}_{\theta'}\left[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)\right].$$

   If this inequality holds, we have a candidate value for x' and proceed to verify that $\rho' = 0$. Otherwise, we move to the next subcase.

   (b) Suppose $\kappa' > 0$. Then $\rho' = 0$, while $\iota' > 0$, $\kappa' > 0$, and $\mu' > 0$. The supplier borrowing constraint, the final-good producer borrowing constraint, and the trade-credit constraint all bind. Combining these three constraints gives

   $$w'x' - \frac{1-\tilde{\theta}_i'}{N_i} rev_i(x') = (1-\theta')\beta\mathbb{E}_{\theta'}\left[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)\right] + T_f(\theta') + T_s(\theta').$$

   This equation pins down x', and in this case

   $$p_j^{tc\prime} = \beta\mathbb{E}_{\theta'}\left[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta)\right].$$

After obtaining x' and $p_j^{tc\prime}$ from one of the two subcases above, we verify that the feasibility constraint is not violated:

$$p_j^{tc\prime} \leq \frac{\tilde{\theta}_i'}{N_i} rev_i(x').$$

If this condition holds, we have found x'. Otherwise, $\rho' > 0$ and we move to the next case.

2. If $\rho' > 0$, then ι', μ', and ρ' are all strictly positive. The supplier borrowing constraint, final-good producer borrowing constraint, and feasibility constraint bind. Combining these constraints gives equation (A.33):

$$w'x' = \frac{1-\theta'\tilde{\theta}_i'}{N_i} rev_i(x') + T_f(\theta') + T_s(\theta').$$

This equation pins down x'.

A.7.2.2 Updating the policy function

The previous subsection derived the function $x'(\theta'; \tilde{J}_\theta, \theta, \iota, \rho)$, which depends on the guessed policy function. We now use this function to find current allocations and update the policy function.

Given x', the updated policy function is

$$\tilde{J}'(\theta') = \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x') - T_f(\theta').$$

This expression reflects the fact that, in the relevant constrained states, the final-good producer borrowing constraint and the trade-credit constraint bind.

We proceed by considering cases corresponding to different combinations of binding constraints. There are six relevant cases. Cases I–III have $\iota = 0$, so the supplier borrowing constraint is slack. Cases IV–VI have $\iota > 0$, so the supplier borrowing constraint binds.

I) $\mu = \kappa = \rho = \iota = 0$.
This case applies when

$$wx_i^{\text{cons}}(\tilde{J}_\theta, \theta) \leq \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + (1-\theta)\beta\mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta')\right] + T_s(\theta) + T_f(\theta)$$

and

$$\tilde{J}_\theta + T_f(\theta) \geq \left(A_i - \frac{1-\tilde{\theta}_i}{N_i}\right) rev_i(x_i^{\text{fb}}).$$

The solution is

$$\begin{aligned}
x &= x_i^{\mathrm{fb}}, \\
p_j^s &= \frac{1-\tilde{\theta}_i}{N_i} rev_i(x_i^{\mathrm{fb}}) + T_f(\theta), \\
p_j^{tc} &= \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}'_i}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta') \right], \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}'_i}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta').
\end{aligned}$$

II) $\mu = \kappa > 0$ **and** $\rho = \iota = 0$.
This case applies when

$$wx_i^{\mathrm{cons}}(\tilde{J}_\theta, \theta) \leq \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + (1-\theta)\beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}'_i}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta') \right] + T_s(\theta) + T_f(\theta),$$

$$\tilde{J}_\theta + T_f(\theta) < \left(A_i - \frac{1-\tilde{\theta}_i}{N_i} \right) rev_i(x_i^{\mathrm{fb}}),$$

and

$$\tilde{J}_\theta + T_f(\theta) \geq \beta N_i \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}'_i}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta') \right] \frac{A_i - \frac{1-\tilde{\theta}_i}{N_i}}{\tilde{\theta}_i}.$$

The solution is

$$\begin{aligned}
x &= x_i^{\mathrm{cons}}(\tilde{J}_\theta, \theta), \\
p_j^s &= \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta), \\
p_j^{tc} &= \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}'_i}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta') \right], \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}'_i}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta').
\end{aligned}$$

III) $\mu = \kappa > 0$**,** $\rho > 0$**, and** $\iota = 0$.

This case applies when

$$wx_i^{\mathrm{cons}}(\tilde{J}_\theta, \theta) \leq \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + (1-\theta) \left[\frac{\tilde{\theta}_i(\tilde{J}_\theta + T_f(\theta))}{N_i A_i - (1-\tilde{\theta}_i)} \right] + T_s(\theta) + T_f(\theta),$$

$$\tilde{J}_\theta + T_f(\theta) < \beta N_i \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 0)) - T_f(\theta') \right] \frac{A_i - \frac{1-\tilde{\theta}_i}{N_i}}{\tilde{\theta}_i},$$

and

$$\tilde{J}_\theta + T_f(\theta) \geq \beta N_i \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, 1)) - T_f(\theta') \right] \frac{A_i - \frac{1-\tilde{\theta}_i}{N_i}}{\tilde{\theta}_i}.$$

The solution is

$$\begin{aligned}
x &= x_i^{\text{cons}}(\tilde{J}_\theta, \theta), \\
p_j^s &= \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta), \\
p_j^{tc} &= \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, \bar{\rho})) - T_f(\theta') \right], \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, \bar{\rho})) - T_f(\theta').
\end{aligned}$$

The value of $\bar{\rho}$ solves

$$\beta N_i \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, \bar{\rho})) - T_f(\theta') \right] = \tilde{\theta}_i \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}}.$$

If the state $\{\tilde{J}_\theta, \theta\}$ does not satisfy the conditions for cases I–III, then the supplier borrowing constraint binds, so $\iota > 0$. This also implies $\mu > 0$. We first consider the case in which the trade-credit constraint and the feasibility constraint are slack. If one of these constraints is violated, we move to cases V and VI.

IV) **$\mu > 0$, $\iota > 0$, and $\kappa = \rho = 0$.**

Since $\kappa = 0$, the optimality conditions imply $\mu = \theta\iota$. The optimality condition for x becomes

$$\left[1 + (1 - \theta\tilde{\theta}_i)\iota\right] \frac{\gamma - 1}{\gamma} \eta_i C^{\frac{1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma} - 1} x^{\eta_i \frac{\gamma-1}{\gamma} - 1} = w(1+\iota).$$

Hence

$$x(\iota) = \left[\frac{1 + (1-\theta\tilde{\theta}_i)\iota}{w(1+\iota)} \frac{\gamma-1}{\gamma} \eta_i C^{\frac{1}{\gamma}} N_i^{\eta_i \frac{\sigma}{\sigma-1} \frac{\gamma-1}{\gamma} - 1} \right]^{\frac{\gamma}{(1-\eta_i)\gamma + \eta_i}}.$$

Since the final-good producer borrowing constraint and the supplier borrowing constraint both bind, trade credit is

$$p_j^{tc} = \frac{wx}{1-\theta} - \frac{1-\tilde{\theta}_i}{N_i(1-\theta)} rev_i(x) - \frac{T_f(\theta)+T_s(\theta)}{1-\theta}.$$

The promise-keeping constraint becomes

$$\tilde{J}_\theta = \left(A_i + \frac{\theta(1-\tilde{\theta}_i)}{N_i(1-\theta)}\right) rev_i(x) - \frac{wx}{1-\theta} + \frac{\theta T_f(\theta)+T_s(\theta)}{1-\theta} + \beta \mathbb{E}_\theta \left[\tilde{J}'(\theta')\right].$$

We find $\bar{\iota}$ such that

$$\tilde{J}_\theta = \left(A_i + \frac{\theta(1-\tilde{\theta}_i)}{N_i(1-\theta)}\right) rev_i(x(\bar{\iota})) - \frac{wx(\bar{\iota})}{1-\theta} + \frac{\theta T_f(\theta)+T_s(\theta)}{1-\theta} + \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \bar{\iota}, 0)) - T_f(\theta')\right].$$

The solution is

$$\begin{aligned}
x &= x(\bar{\iota}), \\
p_j^s &= \frac{1-\tilde{\theta}_i}{N_i} rev_i(x(\bar{\iota})) + T_f(\theta), \\
p_j^{tc} &= \frac{wx(\bar{\iota})}{1-\theta} - \frac{1-\tilde{\theta}_i}{N_i(1-\theta)} rev_i(x(\bar{\iota})) - \frac{T_f(\theta)+T_s(\theta)}{1-\theta}, \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \bar{\iota}, 0)) - T_f(\theta').
\end{aligned}$$

To verify this case, we check

$$\tilde{J}_\theta \geq \left(A_i - \frac{1-\tilde{\theta}_i}{N_i}\right) rev_i(x(\bar{\iota})) - T_f(\theta), \tag{A.34}$$

$$wx(\bar{\iota}) \leq \left[1-\theta+\theta(1-\tilde{\theta}_i)\right] \frac{rev_i(x(\bar{\iota}))}{N_i} + T_f(\theta) + T_s(\theta). \tag{A.35}$$

Equation (A.34) ensures that the trade-credit constraint is satisfied, while equation (A.35) ensures that the feasibility constraint is satisfied. If both conditions hold, the solution above is valid. If (A.35) holds but (A.34) fails, then $\kappa > 0$ and we move to case V. If (A.35) fails, we move to case VI.

V) $\mu > 0$, $\iota > 0$, $\kappa > 0$, **and** $\rho = 0$.

Since the final-good producer borrowing constraint and the trade-credit constraint bind,

$$x = x_i^{\text{cons}}(\tilde{J}_\theta, \theta),$$

and

$$rev_i(x) = \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}}.$$

Since the supplier borrowing constraint also binds,

$$wx = \frac{1-\tilde{\theta}_i}{N_i} rev_i(x) + T_f(\theta) + T_s(\theta) + (1-\theta)\beta \mathbb{E}_\theta \left[\tilde{J}'(\theta') \right].$$

We find ι^* from

$$wx_i^{\text{cons}}(\tilde{J}_\theta, \theta) = \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta) + T_s(\theta) + (1-\theta)\beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta') \right].$$

We then verify that the feasibility constraint is satisfied:

$$\beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta') \right] \leq \frac{\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}}.$$

If this condition is violated, we move to case VI. Otherwise, the solution is

$$\begin{aligned}
x &= x_i^{\text{cons}}(\tilde{J}_\theta, \theta), \\
p_j^s &= \frac{1-\tilde{\theta}_i}{N_i} \frac{\tilde{J}_\theta + T_f(\theta)}{A_i - \frac{1-\tilde{\theta}_i}{N_i}} + T_f(\theta), \\
p_j^{tc} &= \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta') \right], \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i} \right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, \iota^*, 0)) - T_f(\theta').
\end{aligned}$$

VI) **$\mu > 0$, $\iota > 0$, $\rho > 0$, and $\kappa \geq 0$.**

When the final-good producer borrowing constraint, the supplier borrowing constraint, and the feasibility constraint bind, we obtain

$$\begin{aligned}
p_j^s &= \frac{1-\tilde{\theta}_i}{N_i} rev_i(x) + T_f(\theta), \\
p_j^{tc} &= \frac{\tilde{\theta}_i}{N_i} rev_i(x), \\
wx &= \frac{1-\theta\tilde{\theta}_i}{N_i} rev_i(x) + T_f(\theta) + T_s(\theta).
\end{aligned}$$

The final equation pins down the level of x, which we denote by $\bar{x}$. The promise-keeping constraint is

$$\tilde{J}_\theta = \left(A_i - \frac{1}{N_i}\right) rev_i(\bar{x}) - T_f(\theta) + \beta \mathbb{E}_\theta \left[\tilde{J}'(\theta')\right].$$

The function $x'(\cdot)$ depends on the current multipliers ρ and ι only through the value of $\rho - (1-\theta)\iota$. Let

$$z \equiv \rho - (1-\theta)\iota.$$

The value of z solves

$$\tilde{J}_\theta = \left(A_i - \frac{1}{N_i}\right) rev_i(\bar{x}) - T_f(\theta) + \beta \mathbb{E}_\theta \left[\left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, z)) - T_f(\theta')\right].$$

The solution is

$$\begin{aligned}
x &= \bar{x}, \\
p_j^s &= \frac{1-\tilde{\theta}_i}{N_i} rev_i(\bar{x}) + T_f(\theta), \\
p_j^{tc} &= \frac{\tilde{\theta}_i}{N_i} rev_i(\bar{x}), \\
\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta) &= \left(A_i - \frac{1-\tilde{\theta}_i'}{N_i}\right) rev_i(x'(\theta'; \tilde{J}_\theta, \theta, 0, z)) - T_f(\theta').
\end{aligned}$$

A.7.3 General equilibrium

The previous section detailed the algorithm for solving the policy function of suppliers in a sector, taking current and future aggregate variables C_t and w_t as given. We now explain how we solve the full general equilibrium model with aggregate fluctuations.

Firms need to form beliefs about future aggregate consumption and wages. The full aggregate state contains the financial shock θ_t as well as the distribution of promised values across all sectors, $\{\tilde{J}_{it}\}_i$. Since the model contains 54 sectors, solving directly as a function of all promised values would be infeasible. Instead, we conjecture that aggregate consumption and the real wage follow AR(1) processes in logs, with coefficients that depend on the current financial state:

$$\begin{aligned}
\ln C_t &= (1-\rho_c(\theta_t))\, \mu_c(\theta_t) + \rho_c(\theta_t) \ln C_{t-1}, \\
\ln w_t &= (1-\rho_w(\theta_t))\, \mu_w(\theta_t) + \rho_w(\theta_t) \ln w_{t-1}.
\end{aligned}$$

Since θ_t takes two values, these laws of motion are characterized by eight coefficients. We denote the vector of coefficients by $\vec{\zeta}$.

Given these laws of motion, suppliers only need to know the current promised value, the current financial state, lagged aggregate consumption, and the lagged real wage. Thus, the policy function can be written as

$$\tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta, \ln C^L, \ln w^L).$$

We adapt the partial-equilibrium algorithm to solve for this policy function in general equilibrium, given the law-of-motion coefficients $\vec{\zeta}$. The state space is now

$$\{\tilde{J}_\theta, \theta, \ln C^L, \ln w^L\}.$$

The algorithm is the same as in the partial-equilibrium case, with two adjustments. First, future aggregate consumption and wages depend on the future value of θ' through the laws of motion. Second, the expected promised value two periods ahead becomes

$$\begin{aligned}&\mathbb{E}_{\theta'}\left[\tilde{J}''(\theta''; \tilde{J}_\theta, \theta, \ln C^L, \ln w^L)\right]\\ &\quad\equiv \mathbb{E}_{\theta'}\left[\tilde{J}'\left(\theta'' \mid \tilde{J}'(\theta' \mid \tilde{J}_\theta, \theta, \ln C^L, \ln w^L), \theta', (1-\rho_c(\theta))\mu_c(\theta) + \rho_c(\theta)\ln C^L, (1-\rho_w(\theta))\mu_w(\theta) + \rho_w(\theta)\ln w^L\right)\right].\end{aligned}$$

We proceed as follows:

1. Start with a guess for $\vec{\zeta}$.
2. Given $\vec{\zeta}$, solve the policy function for each of the 54 sectors independently.
3. Simulate θ_t for T periods using the transition matrix.[45] Use $\vec{\zeta}$ to obtain C_t and w_t, starting from the deterministic steady state when $\theta = \theta_L$.
4. Using the policy functions together with the sequence $\{\theta_t, C_t, w_t\}$, compute employment by suppliers in each sector and the output produced by each sector. Denote the implied output of sector i by $y_{it}(\vec{\zeta})$ and the labor input of each supplier in sector i by $x_{it}(\vec{\zeta})$.
5. Use the definition of aggregate consumption and the household optimality condition to obtain the implied levels of aggregate consumption and the real wage:

$$C_t(\vec{\zeta}) = \left[\frac{1}{54}\sum_i y_{it}(\vec{\zeta})^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}},$$

$$w_t(\vec{\zeta}) = \chi\left[\frac{1}{54}\sum_i N_i x_{it}(\vec{\zeta})\right]^{\psi}.$$

6. Regress $\ln C_t(\vec{\zeta})$ on $\ln C_{t-1}(\vec{\zeta}) \times \mathbb{1}(\theta_t = \theta_L)$, $\ln C_{t-1}(\vec{\zeta}) \times \mathbb{1}(\theta_t = \theta_H)$, $\mathbb{1}(\theta_t = \theta_L)$, and $\mathbb{1}(\theta_t = \theta_H)$, with no constant. Run the same regression for $\ln w_t(\vec{\zeta})$. Denote the

[45] We set $T = 20{,}000$ when solving the model.

coefficients from these regressions by $\bar{\zeta}$.

7. If $\bar{\zeta}$ is sufficiently close to $\vec{\zeta}$, the algorithm has converged. Otherwise, update the law-of-motion coefficients according to

$$\vec{\zeta}^{new} = 0.5\vec{\zeta} + 0.5\bar{\zeta},$$

set $\vec{\zeta} = \vec{\zeta}^{new}$, and repeat from step 2.