NBER WORKING PAPER SERIES

INCREASING RETURNS AND NEW DEVELOPMENTS IN THE THEORY OF GROWTH

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Working Paper No. 3098

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 1989

This paper was presented at the conference on Equilibrium Theory and Applications held at C.O.R.E. in Brussels in June 1989, and benefitted from comments of participants. This work was supported by the National Science Foundation and the Sloan Foundation, and is part of NBER's research programs in Economic Fluctuations and Growth. Any opinions expressed are those of the author not those of the National Bureau of Economic Research.

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ABSTRACT

From the beginning, growth theory has been faced with technically challenging questions about increasing returns and the way to capture ideas in a model of market exchange. Initially, reliance on perfect competition forced growth theory to narrow its scope. Recently, new tools for studying dynamic equilibria with nonconvexities, externalities, and imperfect competition have allowed growth theory to address broader questions like: Why have growth rates tended to increase over time? Why is it that flows of capital are not sufficient to equalize wages in different countries? How is it that trade policy, or aggregate research and development expenditure, or the extent of patent protection influences the rate of growth?

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With the publication of **The Wealth of Nations**, Adam Smith put two propositions at the center of economic theory. The first was that competition allocates the pre-existing stock of productive inputs in a way that is wealth maximizing. The second was that savings, the accumulation of capital, is the key process whereby the stock of inputs, and therefore wealth, grew over time. In the developments leading up to the acceptance of the neoclassical growth model in the late 1950s and early 1960s, economists steadily moved away from Smith's second proposition, not so much because they were convinced that it was wrong, but rather because it seemed to be incompatible with the theoretical apparatus developed to capture the notion of competition. To a large extent, technical issues about modeling drove the substantive conclusions about growth.

In the last 5 to 7 years, a new round of growth models have been proposed that have rehabilitated the idea that endogenous accumulation causes growth. Economists now have a clearer understanding of how this could arise, and the focus has turned to identifying exactly what it is that is being accumulated: Physical capital? Human capital? Knowledge or technology?

Concurrent with this evolution in the substantive issues considered by growth theory, there has been an expansion in the set of technical models under consideration. Until recently, perfect competition was the only equilibrium concept used in models of growth. Now, there are a several models with price-taking behavior but with external effects or distorting taxes, and hence with suboptimal equilibria. The key technical limitation in modeling growth is no longer an injunction against the use of models where the first welfare theorem fails. In attempts to explain the data, such models are now accepted, sometimes grudgingly, as candidates alongside of models with perfect competition. Rather, the limitation now lies in the insistence that models of growth should preserve price-taking behavior.

In its first section, this paper gives a brief description of the early developments in growth theory from Smith down to Solow. The second section describes the new crop of models that have arisen over the course of the last few years. The question raised in both of these sections is whether purely technical issues have limited and continue to limit the set of models that are used to evaluate the evidence on growth. The conclusion suggested is that the restriction to models with price-taking behavior has been a binding constraint, but it is one that we now know how to relax.

I. Introduction and Historical Background

In the 180 years following the publication of **The Wealth of Nations**, economists steadily improved their understanding of price-taking competition. As they did, the logic of their arguments led them to place less and less emphasis on accumulation as a source of growth. The nadir for growth based on endogenous accumulation occurred with the publication in 1956 of Solow's (1956) and Swan's (1956) demonstration that changes in the savings rate have no lasting effect on the rate of growth of output per worker. According to the thesis argued here, it is no accident that these papers followed immediately on the heels of the rigorous characterizations of perfect competition and rigorous statements of the welfare theorems offered by Arrow (1951), Debreu (1952, 1954), Arrow and Debreu (1954), and McKenzie (1954,1955).

The tension between Smith's two propositions about competition and savings had of course been uncovered long before. Stimulated by increases in food prices and land rents associated with the war with France, Malthus and Ricardo developed the notion of diminishing returns to a particular factor of production, labor, as its quantity increased relative to the quantities of other factors like land. Once the logic of diminishing returns associated with changing factor proportions was clear, it was evident that it would apply equally well to changes in the proportion of capital relative to the stock of labor. This put limits on the potential for capital accumulation to generate increases in per capita income.

By the end of the classical period of economic thought, dated for example by the publication of John Stuart Mill's **Principles of Political Economy**, the notion of a steady state level of income determined by diminishing returns was firmly embedded in conventional economic thought and economics was foreverafter the dismal science. Mill allowed for the possibility that this steady state might move over time in response to changes in our understanding of the physical world, but he argued that the details of this process of technological improvement were beyond the scope of economics. Thus, even by the middle of the 19th century, all of the elements of the neoclassical theory of growth could be discerned: diminishing returns that limited the scope for capital accumulation, and exogenous changes in our understanding of the physical world that could act to raise the returns to capital accumulation. To formulate the neo-classical model, what remained to be done was for economists to gain enough confidence in the concept of perfect competition and in mathematical abstraction to give a stark mathematical demonstration of the rather unappealing and counterintuitive result that the investments made by economic agents have no lasting effects on the rate of growth.

Economists accepted the logic of this argument, and placed exogenous technological change at the center of models of growth. But they did so half-heartedly. Policy pronouncements by economists often seemed closer to those suggested by Smith than by the accepted model. Since the time of Smith, they have never ceased proselytizing for higher levels of savings and investment.

Because they found the conclusions of the theory unattractive, many economists tried to modify it in ways that would retain for savings and accumulation a significant role in the growth process. The most influential of these attempts can be attributed to Alfred Marshall (1890). He squarely faced the tension between the diminishing returns implied by competitive analysis and the steady improvement in per capita income that belied the gloomy forecasts of Malthus and Ricardo. To explain the observed growth, Marshall disputed the presence of diminishing returns to accumulation, and supposed instead that increasing returns might be present. To reconcile increasing returns that were internal to a firm with the existence of decentralized markets with many firms, Marshall relied on his famous but unpersuasive analogy with trees (Chapter XIII, Book IV): firms, like trees, grow big, and then they die. From the point of view of theoretical acceptance, his more successful innovation was the introduction of the notion of an externality. This made possible the notion of external increasing returns that manifest themselves at the level of the industry or economy, even though returns at the level of the firm are nonincreasing.

The lasting success of external increasing returns would not seem to be due to any direct empirical evidence concerning their presence. Marshall's description of internal increasing returns was far more extensive and persuasive than his description of external increasing returns. In one of the better known exchanges in the "cost controversies" of the 1930s, Frank Knight (1924) claimed that Frank Graham's use of a trade model with external increasing returns was inappropriate because no one had been able to cite a convincing example of an industry that was characterized by external increasing returns and price-taking competition, an assertion that Graham was not able to rebut. Except for Marshall's suggestion that trade knowledge that cannot be kept secret is an important source of external effects in production (Chapter XI, Book IV), no convincing example of a productive externality has been put forth. (As noted in Section II.4 below, introducing knowledge explicitly into models of growth poses other problems for price-taking behavior, so even this source of external increasing returns has only recently been systematically modeled.) Questions about their empirical relevance notwithstanding, external increasing returns persisted as an important theoretical device in the static theory of international trade, and have recently reemerged in the theory of growth.

The other main approach to growth theory can be traced to Frank Knight's thesis advisor Allyn Young, but its roots go back to Marshall and even Smith. Young (1929) articulated the view that introduction of new goods was the crucial driving force in economic growth, and argued that the potential for new goods is necessary to understand Smith's emphasis on a role for the extent of the market. Larger markets permit more specialization, not so much in the sense that workers focus their time on a narrower set of tasks, but rather because they support the provision of a larger set of specialized inputs in consumption and production. Like Marshall, Young called the beneficial effects arising from the introduction of a new good a positive external effect. On this basis, he tried to describe a model of growth driven by aggregate increasing returns that were external to individual firms.

From a modern point of view, the analogy with external effects is suggestive but ultimately misleading. It is true that the introduction of new goods and a role for the extent of the market seem to be fundamentally inconsistent with perfect competition, but the channel whereby an increase in the size of the market increases the utility of consumers and the production possibilities of firms is quite different from the channel emphasized in standard discussions of external effects in production or consumption. A natural way to conceive of the introduction of new goods is to suppose that there is some fixed cost associated with the introduction of the good. Once the aggregate demand for the good is large enough, the stream of profit that can be extracted by a firm that introduces the good will be large enough to offset the initial fixed cost.

There is a sense in which this kind of process can lead to something that resembles an externality. Consider the case where the contemplated new good is an intermediate input in production, so its demand curve is simply a marginal productivity schedule. A firm that contemplated supplying the new good but that is incapable of price discrimination will value the new good in terms of the monopoly revenue that it can generate. The social value of the new good is this term plus the area under the derived demand curve. This generates a divergence between the perceived private value of a new good and its social value that is analogous to the divergence that would be present if there were productive externalities. (This point is developed more fully in Romer 1989a.) The difference between this divergence and one introduced by a technological externality in production or consumption is that it arises because of the nature of pricing. It is a function of the market mechanism, not of physical quantities used in production or consumption, and it is precisely the nature of the market and the nature of equilibrium that must be the focus of attention, not the specification of preferences and the technology.

The tools for undertaking the analysis of this kind of market and equilibrium were introduced by another of Allyn Young's students, Edward Chamberlain (1933), and independently by Joan Robinson (1933). These tools have only begun to be used for aggregate analysis of issues in trade and growth. It is sometimes supposed that a predisposition against government intervention explains the preference that economists have for models with perfect competition, but this does not explain why they should prefer models with externalities to ones with imperfect competition. A far more likely determinant of modeling strategy is technical difficulty. Perfect competition is the first pass model for many questions simply because it is the easiest model to describe and understand. Allowing for external effects in the next pass removes the presumption in favor of optimality, but preserves price-taking behavior. The formal tools used in the analysis remain largely the same. Imperfect competition is naturally a last resort because it forces a more drastic change in the formal analysis of equilibrium and is associated with a much higher level of analytical difficulty than competition with external effects.

Until the specification of the additively separable functional form for preferences over many goods (Dixit and Stiglitz, 1977) an aggregate analysis of a model with monopolistic competition was completely infeasible. With the interpretation of this functional as a production functional depending on a large number of intermediate inputs instead of a preference functional (Ethier, 1982), a dynamic aggregate analysis of a model with imperfect competition comparable to the Solow-Swan analysis of the competitive model can now be undertaken.

After Young's article was published in 1929, (back-to-back with Frank Ramsey's famous paper on optimal savings, 1929), the depression of the 1930s focused attention on problems other than long run growth. Monopolistic competition became embroiled in controversies concerning industrial organization, and its aggregate implications were not explored. In contrast, the general equilibrium analysis of competition progressed rapidly. Their qualms about exogenous technological change notwithstanding, economists adopted the neo-classical model with its assumption of perfect competition, and no sustained attempt to put forth an alternative was made. Kaldor and Robinson waged a form of guerrilla warfare against the neoclassical model, and they invoked Young's name among others, in support of their cause. But they offered no tractable alternative model.

In an attempt to capture some of the effects emphasized by Kaldor, Arrow (1962b) described a growth model that allowed for external increasing returns in the Marshallian tradition. But neither this model nor its elaboration by Sheshinski (1967) generated the prediction that higher savings rates would lead to higher growth rates, except perhaps in a borderline case. Nor did these models have any role for explicit research and development, patents, or inventive activity. The main prediction was that an exogenously higher rate of population growth led to a higher rate of per capita income growth. Since accumulation, either in the form of physical capital or in the form of new ideas, played no major role in these models, they may not have offered a very attractive alternative to the Solow model for someone concerned with the role of endogenous accumulation in generating growth. Perhaps for this reason, the direction suggested by these models was not pursued by others.

Because the substantive implications of the model of exogenous technological change are so implausible, its unchallenged supremacy during this period can only be rationalized on methodological grounds. Its fundamental premise implies that private firms do not devote valuable resources to activities like research and development and that the rate of growth in an economy would be unchanged if patents and other forms of intellectual property rights were abolished. Judging from the way the model was used, it seems clear that no one took these implications seriously. The exogeneity of technological change was apparently intended as a provisional assumption that would be relaxed as soon as theoretical developments permitted.

But even early on, it was clear that making technological change endogenous would force major changes in the structure of the model. Arrow (1962a), Shell (1966,1967,1973), Nordhaus (1969), Wilson (1975) and no doubt many others, pointed to the difficulty that competition poses for understanding activities like research and development. The basic point is that expenditures on these activities do not contribute to the marginal cost of producing the new goods that result. They are the fixed costs necessary to introduce a good. Once they have been incurred, they have nothing to do with manufacturing, which most plausibly takes place according to a constant returns to scale technology using conventional inputs like capital and labor. If the equilibrium price of a good must be equal to its marginal cost of production, then firms that spend resources on invention or research and development will not break even. They will have no way to recoup the initial expenditures. One of the advantages of the Solow model is that it made this point so clearly. If output takes the form $Y = F(K,AL)$ for a homogeneous of degree one production function F, then the total value of output is paid as compensation to labor and capital. Nothing is left to pay for increases in A.

Despite the apparent importance of corporate research and development and of private inventive activity, and despite the attention economists paid to them in micro level analyses, few attempts were made to give these variables a realistic role in a model of growth, Shell (1973) being a prominent exception that recieved little attention. Economists seemed to accepted the Solow-Swan finesse of the problem more and more as a final solution, and less and less as a provisional step in the analysis. Because a competitive equilibrium provides no way to compensate resources used in the activities that lead to improvements in the technology, all such resources are assumed not to get compensation. Technology is simply assumed to grow nonetheless.

Section II: Recent Models

When external increasing returns were reintroduced into growth models in the beginning of this decade, attention focused first on how to sustain growth without introducing exogenous technological change. Subsequent work has broadened the horizon to include questions about international comparisons of growth, the spatial location of economic activity, and a more fundamental attack on the role that ideas play in fostering growth.

To organize the discussion of the recent models, it is useful to describe several different reasons that have been suggested for why a model of growth must allow for some departure from the usual convexity assumptions, (convexity in the sense of sets, not of functions). Four main questions can be identified:

1) Do fixed factors of production drive growth rates to zero in a model with a convex technology?
2) Can a convex technology explain the time trend in observed growth rates?
3) Is a convex technology consistent with the patterns of international trade, factor movements, and growth rates?
4) Can research and development and patents be incorporated in a convex model?

These questions, together with models designed to address them, are discussed in the next four sections.

II.1 Fixed Factors

The first paper in the recent round of models of growth theory was Romer (1986), a version of which circulated as a job market paper in 1982. This paper implied that the answer to each of the first three questions listed above was negative: A convex model cannot generate growth if there are fixed factors of production. A convex model cannot explain the time trend of growth rates. A convex model cannot explain international differences in levels and rates of growth of income.

The logic behind the first of these three assertions is simple, and is correct as far as it goes, but it depends on an important qualification to be noted below. Suppose that growth is determined by an accumulation equation of the form

$$\dot{K} = F(K,X)-C, \tag{1}$$

where K denotes a factor like capital that can be accumulated on a per capita basis, C denotes aggregate consumption, and X represents a factor like land or labor that is assumed to be in fixed supply on a per capita basis. The maximum feasible path for K as a function of time is given as the solution to

$$\dot{K}_{max}(t) = F(K_{max}(t),X). \tag{2}$$

Suppose that F is concave, hence that it can be no more than homogeneous of degree 1.

Then dividing by $K_{max}(t)$ gives

$$\frac{\dot{K}_{max}}{K_{max}} \leq F(1, \frac{X}{K_{max}}). \tag{3}$$

Suppose further the fixed factor X is "essential" in the sense that $F(K,X) = 0$ if $X = 0$. Then as K_{max} goes to infinity, X/K_{max} goes to zero, which implies that $\dot{K}_{max}/K_{max}$ must also go to zero. From this it follows that any feasible monotonic path for K (and hence also for output) will have a growth rate that asymptotically approaches 0.

Consideration of feasibility alone suggests that constant exponential growth is not possible in a convex model that has an essential fixed factor in the accumulation equation. Adding the possibility that savings rates may decrease with reductions in the rate of return will only reinforce the arguments for slowing growth. These are the kinds of arguments that concerned the classical economists and led to the focus on the steady state.

To address this concern, Romer (1986) allowed for the possibility that the aggregate production function for economy as a whole, $F(K,X)$, could exhibit nondiminishing marginal productivity of K; for example, it could take the form $K^{\alpha}X^{\beta}$ where $\alpha \geq 1$. From this it follows that F can not be concave. The variable K was assumed to represent a composite of physical capital and knowledge, and the knowledge component was assumed to generate external effects or spillovers. Thus, the model relied on Marshallian external increasing returns to preserve price-taking. For a small firm using inputs k and x when the aggregate stock of knowledge and capital is K, output could take the form

$$y = K^{\varphi}k^{\nu}x^{\beta}, \text{ where } \nu+\beta \leq 1, \text{ but } \varphi+\nu = \alpha \geq 1. \tag{4}$$

(Section II.4 returns to the important questions that are swept under the rug here by tying knowledge to capital and failing to give it a separate existence.)

The next, and very influential contribution to the recent work on growth was the paper by Robert Lucas (1988, a version of which circulated in 1985.) Lucas's model introduced increasing returns and external effects for reasons associated with the third question, the one concerned with international comparisons. (These issues are treated in section II.3 below.) It did not rely on increasing returns to generate persistent growth, a point that was most fully developed by Rebelo (1988). Lucas' paper built on a framework introduced by Uzawa (1965), and as Rebelo showed, a convex version of this model is perfectly capable of generating constant exponential growth even in the presence of essential fixed factors of production. All that is needed is that there be one technology for accumulating a capital good that does not depend on any fixed factors.

Suppose for example, that net output $Y = G(K,H_1,T)$ is a function of capital, K, the portion H_1 of total human capital that is devoted to the production of final output, and land T, which is a fixed factor of production. Suppose that capital is produced from foregone consumption,

$$\dot{K} = Y - C = G(K,H_1,T) - C \,. \tag{5}$$

Suppose that total human capital H grows in proportion to the fraction of human capital that is devoted to education:

$$\dot{H} = \mu(H-H_1). \tag{6}$$

What is crucial here is that although the equation for $\dot{K}$ has fixed factor in it, the equation for $\dot{H}$ does not. If a constant fraction of human capital $(H-H_1)/H$ is devoted to education, H can grow at a constant exponential rate. If, for example, G takes the log–linear form $G(K,H_1,T) = K^{\alpha}H_1^{1-\alpha-\beta}T^{\beta}$, and H grows at the rate g, output can grow at the rate $(1-\alpha-\beta)g$ even if no capital accumulation takes place. In fact, capital can grow at the same exponential rate as output, which implies that growth in output can take place forever at the rate $\frac{1-\alpha-\beta}{1-\alpha}g$. Thus, the argument outlined above, suggesting that essential fixed factors in a convex production technology must cause growth to come to a halt, misses the possibility that the fixed factors might not play a role in the fundamental accumulation equation for the economy.

This argument has been extended by Jones and Manuelli (1988). They point out that fixed factors can arise even in the fundamental accumulation sector for an economy so long as these factors are not essential. For example, suppose that the function F(K,X) used in equations 1–3 above takes the constant elasticity of substitution form

$$F(K,X) = [\alpha K^{\rho}+(1-\alpha)X^{\rho}]^{1/\rho}. \tag{7}$$

If the substitution parameter ρ lies in the range (0,1], so K and X are good substitutes, then neither K nor X is essential for production. Asymptotically, as K goes to ∞ for fixed X, F approaches a function that is linear in K, and constant exponential growth is feasible. It is true, as the classical authors suggested, that an increase in the ratio of K to X is associated with a fall in the marginal product of K. However, a monotonically falling marginal product does not imply that the marginal product converges to 0. The marginal product goes to 0 only in the case where the fixed factor is essential.

The claim that a convex model will not generate persistent growth is true only if one believes that there is an essential fixed factor in all of the accumulation equations in the model. A convex aggregate technology is consistent with the assertions a) that constant exponential growth is feasible, and b) that a fixed factor of production like land is present provided that the fixed factor is not essential or is does not enter in one of the fundamental accumulation equations of the model.

Economists were aware of this result during the 1950s and 60s. Part of the reason that exogenous technological change was adopted instead of a model based on equation 7 is that the shares of capital and labor in total income changed only slowly over time. To the extent that a trend could be observed, the share of capital was falling. This rules out the simple interpretation that K in equation 7 represents physical capital, for this function implies that a growing share of total income is paid to capital. The one result that has been clear ever since is that a convex model based on physical capital accumulation alone does not fit the facts. As Jones and Manuelli suggest, for an explanation based on an equation like 7 to fit, the variable K must be interpreted to stand for an aggregate of both physical capital and human capital, with X representing something like unskilled physical labor. In this case, the measured share of capital in total income is too small because it does not include income from human capital.

As a logical matter, this suggests a way to preserve models of growth based on a convex technology and perfect competition. (It still leaves the problem of explaining why growth accounting leaves an unexplained residual.) Nonconvexities are not logically essential for explaining persistent growth, but the formulation based on human capital accumulation contains the seeds of an idea that is ultimately inimical to convexity and perfect competition. It relies on a notion of human capital that can grow without bound on a per capita basis. It differs in a crucial way from the usual notion of human capital as measured by labor economists in terms of years of schooling or experience in the labor force. This new kind of human capital can be passed down from one individual to the next, or from on generation to the next. This suggests that it contains something like scientific ideas, or the stock of knowledge, or the level of the technology that has an existence outside of any individual. As Arrow, Shell, Nordhaus, Wilson, and others have pointed out and as section II.4 argues once again, this poses a serious problem for convexity and competition.

II.2 Time trends in growth rates

One piece of evidence that distinguishes between convex models like those proposed by Rebelo (1988) and Jones and Manuelli (1988) and models with some form of increasing marginal productivity as in Romer (1986) is the fact that, viewed over a long horizon, growth rates have clearly been increasing over time. Table 1 reproduces a table from Romer (1986) on long run trends. It reports productivity growth rates from Maddison (1982) for the country that is estimated to have the highest level of productivity in each of four different epochs.

A simple calculation shows why earlier growth rates must have been smaller than the recent values. The productivity growth rate of 2.3% per year reported for the US in the most recent period implies that output per hour worked doubles every thirty years. Using the Summers–Heston (1988) cross sectional estimates of per capita income, a rough guess of a borderline subsistence level is something like 1/20 or 1/30 of current per capita GDP in the US. In 1890, output per hour worked stood at something like 1/10 is current value. Because hours worked per capita fell over the last century, the level of per capita income in the US or the UK was slightly higher, roughly 1/8 its current value. Starting from this value for per capital income, it is possible to project backwards a productivity growth rate of 2.3% per year and infer the level of per capita income in earlier eras. Going back in time from 1890, this calculation would imply that income in even the most advanced countries was at or below the borderline of subsistence only 200 years ago. If hours worked stayed at the high levels observed at the end of the 19th century, output per capita in 1830 would have been 1/32 of its current value. Even if one makes the

Leading Country	Time Period	Average Annual Labor Productivity Growth Rate
Netherlands	1700–1785	−0.07%
United Kingdom	1785–1820	0.5%
United Kingdom	1820–1890	1.4%
United States	1890–1970	2.3%

Table 1: Data are taken from Maddison 1982. For each time period, the country cited is the world leader in terms of the level of output per man-hour.

implausible allowance that hours worked per capita were 4 times higher in earlier eras than they were in 1890, this buys only an additional 60 years of survival back into the 18'th century.

A labor productivity growth rate of 2.3% per year has not been sustained in the United States in the years since to 1970, but even higher productivity growth rates have been observed in countries in Western Europe and in Japan, countries that may soon overtake the United States in terms of productivity. Thus, from the point of view of the world as a whole, there is not yet good reason to be convinced that growth is finally slowing down.

Convex models like that of Rebelo and of Jones and Manuelli predict that rates of return to the basic input that is accumulated are nonincreasing as accumulation takes place. In the kind of model considered by Rebelo where there are no fixed factors in the essential accumulation sector, rates of return are constant. In the model of Jones and Manuelli, with inessential fixed factors, rates of return fall. If accumulation responds positively to rates of return, these models can not by themselves explain the time trend of growth rates.

Models with some underlying form of increasing returns suggest the possibility that rates of return could be increasing as accumulation takes place. Romer (1986) shows how this can lead to rates of growth that increase over time in one particular example. A much simpler and more appealing example is given in Xie (1989). More generally, such models can generate the possibility that appreciable growth does not start to occur until some minimal level of output and stocks of productive factors has been achieved.

If all of the explanatory power is forced onto the technology, this kind of evidence weighs in favor of departures from the standard concavity assumptions. An alternative view would be to place some of the weight on institutional factors that are treated as being exogenously given. It is clearly the case that legal and social institutions conducive to the accumulation of private property have evolved dramatically over the last several hundred years. Improvements in these institutions would lead to increased accumulation even if raw rates of return were constant or falling over time.

Changes in institutions can solve the problem of explaining the time trend in growth rates, but they do so by exchanging exogenous technological change for exogenous institutional change. This permits a provisional analysis that lets economists focus on issues that they understand relatively well and set aside ones that they do not; but ultimately, one would like to be able to explain the evolution of social institutions as well. It it seems quite plausible that increasing rates of return and increasing opportunities for private investment and accumulation played a causal role in the evolution of institutions that supported these activities. After all, it is not true that early economies were incapable of accumulating large amounts of physical capital. What is striking about investments like those in pyramids, raised field systems, cathedrals, and possibly even flood control and irrigation systems is both the size of the social investment involved and the relatively low rates of return it generated. If there were large unexploited investment opportunities present in early times, it is hard to understand why they were not pursued.

Even if one agrees in principle with an ambitious program for explaining institutional change and understanding growth in a broad historical context, the concerns raised here are far removed from the issues encountered by someone who looks at post war time series in developed economies or who worries about current policy issues. On this basis, it is unlikely that they will be decisive in the debate over the theoretical foundations of growth theory. If this were the only basis of support for models that depart from the usual convexity assumptions, it seems likely that those convexity assumptions would continue to be maintained.

II.3 International Evidence

The last two subsections argue first, that persistent growth is not a fatal problem for someone who wants to preserve perfect competition as the equilibrium concept for growth theory and secondly, that the long run evidence on trends in growth rates is easy enough to assume away. In contrast, international data on levels of income, growth rates, and factor movements and the problems they pose for convex models are more difficult to dismiss. There is of course a separate and older line of attack on perfect competition as the foundation of static trade theory generated by the perception that convex technologies combined with differences in factor endowments are not sufficient to explain the patterns of trade that are observed. (See for example Helpman and Krugman, 1985.) The discussion here will focus only on the issues most closely related to growth.

The difficulty presented by the neoclassical model of growth in a world where nations have different levels of income is that it seems to imply persistent unexploited profit opportunities. If the exogenous technology is treated as something that is freely available to all firms in a particular economy, then it would seem natural to assume that it is available to all firms in the world as well, leaving only differences in the capital labor ratio to explain differences in income. But this implies differences in the marginal productivity of capital between different countries that are impossibly large. Since the share of capital in total income is judged to be on the order of 0.25, the many countries identified by Summers and Heston that have per capita income that is 1/10 that in the US would have to have a capital labor ratio that is smaller by a factor of 10^4. The marginal product of capital would have to be larger by a factor of 10^3. Productivity differentials of this size are very hard to reconcile with the observed patterns of investment and rates of return.

As a result, most applications of the neoclassical model to a cross section of countries allow the level as well as the rate of growth of the technology to differ across countries. This explains the cross country variation by making it a premise of the theory, ruling out any opportunity for the theory to comment on the sources of the cross country variation. It also seems to leave intact large profit opportunities. If the technology is 10 times more advanced in one country than another, there should be large returns to anyone who can cause the advanced technology to be used in the less advanced country. If technology has the kind of public good character attributed to it in the neoclassical model, this should not be hard to do.

Both Romer (1986) and Lucas (1988) use increasing returns to overturn the usual presumption in favor of convergence and profit opportunities implicit in a convex model. Romer (1986) uses the framework outlined in equation 4 above. If total output in country i takes the form $F(K_i,X_i) = K_i^{\alpha}X_i^{\beta}$ with $\alpha \geq 1$, then the marginal product of additional investment will be highest, not lowest, in the country that is most advanced. Using the specification in equation 4, this will be true of both private and social returns. Even if the spillover benefits of the knowledge implicit in K are assumed to extend to foreign countries, the most advanced country will still have the highest marginal productivity so long as the external effects are stronger in the home country than in the foreign country. With this kind of specification, it is no surprise that accumulation can be larger in the more advanced country and that resources could even flow from the less developed country to the advanced country (a result that is shown explicitly for an example.)

The difficulty with this analysis is that it ties knowledge and capital too tightly together. Lucas (1988) sharpens this analysis by effectively splitting the aggregate variable

K into two parts. One part, call it P, corresponds to physical capital and has no external effects associated with it. The other, H, represents human capital. As in the model in Romer (1986), the model here assumes that output increases more than proportionally with increases in P and H taken together. It also invokes the notion of external effects that are local in extent to decentralize this technology in a price-taking equilibrium. Thus, output for a firm depends on its own stock of human capital, h, and an economy wide value H. If P and H moved together, the results would be similar to those in Romer (1986). The key difference here are the assumptions that the two factors P and H can be varied separately and that of the two, only P is internationally mobile. This implies that rates of return on physical capital will be equated across countries, but returns to human capital need not be.

Suppose that output as a function of P and H can be written as

$$Y = P^{\eta}H^{\theta}h^{\delta}, \text{ with } \eta+\delta=1,\ 0<\theta.$$

The sum $\eta+\theta+\delta>1$ plays the role of the exponent α in the previous analysis; it is the exponent that describes the behavior of the marginal productivity of human and physical capital when it expands along a ray in P-H space. If one of two countries had more of both P and H but these countries had identical ratios of P to H, the fact that this exponent is greater than 1 implies that the marginal product of both P and H will be higher in the more developed country than in the less developed country. For H, this statement is true of both the social and private marginal products. If P is free to move between countries, the ratio of P to H must increase in the developed country and decrease in the less developed country for the private marginal product of P in the two countries to be brought into equality. This reduces the differential in the marginal productivity of physical capital, but raises the differential in the marginal products of human capital H that is assumed not to be mobile. Thus, increasing returns can explain why capital mobility is not sufficient to remove pressures for migration. An individual living in a less developed country could earn more by moving to the more developed country.

If both physical capital and human capital were mobile in this model, they could both migrate to the developed country as suggested in the example in Romer (1986). If, as in that example, there are other factors of production like land that are not mobile, these resources need not shift to the new location all at once, and some positive amount will always stay behind.

By relabeling the developed region a city and the less developed region the countryside, this description seems to give a fairly good description of the process of urbanization that is observed within nations, where both P and H are in fact mobile. Thus, the international evidence on potential migration flows and wage differences, and the national evidence on urbanization are both strongly suggestive of increasing returns of some kind. (For a discussion of urbanization and increasing returns, see Fujita, 1989.).

The kind of model considered by Rebelo or Jones and Manuelli, which relies on constant returns to scale in production, can explain international differences in the level of income but it cannot explain the persistent differential in wages across countries and the resulting pressure for migration. If the increasing returns are dropped from Lucas' model, so that the exponents η, θ, and δ sum to one, free mobility of capital will equalize the ratio of physical capital to human capital in each country. Large differences in per capita income could result from large differences in endowments of both physical capital and human capital. A country with per capita income equal to 1/10 that in the US could have per capita stocks of P and H that are 1/10 as large as the comparable values in the US. Thus, a typical individual transported from the less developed country would still be 10 times less productive than the average individual in the advanced country and will still earn 10 times less even after moving.

This kind of model gives up any attempt to explain the apparently large pressures for migration flows for individuals of all skill levels from less developed countries to developed countries. Moreover, it has no explanation for the extreme concentration of resources in specific locations. It does, however, have the advantage that it avoids the necessity of invoking the ephemeral externalities that the models with increasing returns rely on so heavily.

This criticism, essentially the same as Knight's criticism of Graham, represents a serious challenge for both the Romer (1986) and the Lucas (1988) models. The problems of spillovers seem to be inherently tied to forms of knowledge that can exist outside of any person, the same kinds of knowledge that seem to be crucial to the notion that human capital can be passed down from one generation to the next and can therefore grow without bound. One way to think of both the Romer and Lucas models is that they allow for this special kind of knowledge but they assume that it is only produced as a side effect of other activities, investment in physical capital or investment in schooling respectively. The next section considers the possibility that knowledge is produced intentionally, not as a side effect.

II.4 Abstract Knowledge

The modern definition of a public good identifies two distinct attributes of a good: rivalry and excludability. (Cornes and Sandler, 1986) These attributes are usually applied to consumption goods, but they are equally relevant for inputs in production, especially for an input like knowledge. A good is rivalrous if its use by one person or firm precludes its use by another. Rivalry is a feature of the physical properties of the good and the technology for using it. A good is excludable if the producer of the good can keep others from being able to take advantage of the good without consent. Excludability depends both on the technology and the legal system. The notion of a purely excludable or nonexcludable good, or of a purely rival or nonrival good is an idealization. Most goods fall in between the extremes, but usually close enough to one extreme or the other that the terms can be used without qualification.

Conventional economic goods are said to be both rivalrous and excludable. Public goods are both nonrivalrous and nonexcludable. The interesting intermediate case is a good that is nonrivalrous but at least partially excludable. For example, the music contained on a record album is a nonrivalrous input in the production of entertainment services; it can be copied and used as many times as desired. Under the legal system prevailing in the United States it is also at least partially excludable; it is illegal to copy the music and resell it. In contrast, the notion of a good that is rivalrous but nonexcludable does not correspond to any obvious kind of economic good. Thus, nonrivalry is necessary, but not sufficient, for problems of excludability, or in the language typically used in discussions of knowledge, for spillovers to present.

The feature of abstract knowledge that has attracted the most attention is that it is not excludable, or is at best partially excludable. There are many well known examples of discoveries that seem to have generated large spillover effects, that is, large benefits for individuals who did not have to pay for them. When researchers at IBM developed the technology for magnetic disk drives, or when the biologists at Genentech developed the recombinant DNA techniques needed to produce human insulin, hundreds of other firms learned valuable information that they could exploit.

It is clear that private markets cannot provide goods that are completely nonexcludable. The natural solution to nonexcludability is to use the legal system to strengthen property rights. Naively, one might hope that an extensive patent system that removed problems of excludability would let ideas trade in competitive markets just like other goods. Patents can support private markets for ideas, but they cannot support competitive markets. They do not change the fact that ideas are nonrival. By its very definition, a nonrival input is one that has no opportunity cost. A firm that uses a mechanical drawing for a disk drive or a list of instructions for genetically engineering human insulin does not deprive any other firm from using them at the same time. For the cost of a photocopy, anyone can use them.

A replication argument immediately indicates the problem this poses for competitive equilibrium theory. Let Z be a nonrival input in production, say a mechanical drawing for a disk drive, and let X be a comprehensive list of all the rival inputs (machines, structures, trained production workers, supervisors, managers, ...) used in producing the physical units of the drive. Let $G(X,Z)$ denote the services, measured in megabytes of storage, that are produced each year from inputs X using design Z. The input Z could be measured in terms of the cost of producting the particular design. Neglecting lumpiness, replication implies that G must be homogeneous of degree 1 in the rival inputs X alone. Output could be doubled by merely doubling all of them and replicating all productive activities. If Z has productive value as well, (that is, if devoting additional resources to the design effort could have yielded more megabytes of storage from the same inputs X), then $G(\cdot,\cdot)$ cannot be concave. For $\lambda>1$,

$$G(\lambda Z,\lambda X) > G(Z,\lambda X) = \lambda G(Z,X).$$

Because of this nonconvexity, competitive prices are not feasible. If all of the components of X were paid their marginal value product, then by Euler's theorem, payments to them would exhaust the value of output. There is nothing left to pay for the cost of producing the design. From the point of view of marginal cost pricing, if output is homogeneous of degree 1 in rival inputs, then marginal cost is simply the constant unit cost. If firms charge a price equal to marginal cost, they would not recover the initial expenditure on the nonrival design input.

A common response to this observation (e.g. Nordhaus 1969) is to assume that G(Z,X) is not homogeneous of degree one in the rivalrous inputs X. Then Z can be compensated out of the quasi-rents. This was the approach used by the present author in Romer (1986), but it now seems clearly to be no more than a finesse of the basic issue here. As appealing as this assumption is at a technical level, it flies in the face of everything that we know about the physical world. It has to be true that it is possible to double output by doubling all of the rivalrous inputs used in production. Arguments that try to deny this ultimately fall back to the position that it is not possible to double some of the rivalrous inputs (e.g. the special managerial skill of an owner or supervisor), or that it takes time or is costly to double all of these inputs. These observations have no logical bearing on the question of what would happen to total output if one could double the the rival inputs.

(There is a technical issue here. It is sometimes suggested that adding costs of adjustment can convexify a model that is not convex. This is incorrect, but to make the point rigorously forces one to put the analysis in terms of sets in a function space. Suppose that F(X) is a function that has a portion which is not concave, and suppose the theorist adds a cost of adjustment term that depens on $\dot{X}$, the rate of change of X. To check for convexity of the technology, one must consider the set of feasible paths for inputs X(t) and outputs Y(t), and verify whether this is a convex set. No matter how convex a cost function like $H(\dot{X})$ is, if F(X) has a portion that is not concave, then the set of functions

$$\{Y,X: Y(t) \leq F(X(t))-H(\dot{X}(t))\}$$

will not be a convex set.)

What partial equilibrium theory identifies as quasi-rents are really compensation for nonrival inputs that have not been explicitly specified. McKenzie (1955) is particularly clear about this, and refers to the input that is typically left out as the "entrepreneurial factor." If there are nonrival inputs, then there must be a departure from homogeneity of degree one. As a result, it is not possible to pay each input a return equal to its marginal productivity. If Marshallian quasi-rents are used to compensate research and development, then some other rival factor of production is being undercompensated. The crucial advantage of the kind of general equilibrium formulation used in growth theory is that it forces one to take account of this kind of overall adding up restriction.

The problem then, is how to compensate for the production of ideas in a general equilibrium model. The neo-classical model assumes that they are not compensated at all. Romer (1986) asumes that they are compensated out of quasi-rents. An alternative interpretation of this model, one that removes the reliance on quasi-rents, is that ideas are not compensated, but arise as a side effect of the production of capital as in Arrow (1962b). Lucas (1988) also relies on ideas as the nonrival good that is capable of generating the spillovers he identifies and that cause a nonconvexity. In effect, this model assumes that these ideas are produced as a side effect of schooling.

None of these models gives a very satisfactory description of private sector research and development or any indication of where patents might play a role in growth. They all preserve price-taking for rival inputs, and therefore cannot allow the production of the nonrival inputs to generate any revenue. The growth models in Romer (1987, 1989a, 1989b) abandon price-taking to avoid this limitation. Specifically, firms are assumed to produce knowledge that is used in the production of a new good. The operative analogy is that knowledge is like a design. Each firm is assumed to have an infinitely lived patent on the design, so no other firm can produce this good. As in Dixit and Stiglitz (1977) or Judd (1985), it is assumed that there are a large number of firms, each of which produces a distinct good. The equilibrium is one of monopolistic competition with zero profit. The models in Romer (1987, 1989a, 1989b) follow the static model in Either (1982), and assume that the many different goods are producer inputs. In this setting, growth takes place because of the continual introduction of new goods, as suggested by Allyn Young.

The idea that market power is essential to innovation should not be controversial. The widely acknowledged role of a patent or a copyright is precisely to provide a monopoly on the use of a nonrival input for some period of time. This generates a return to compensate for the cost of generating the input. These models of growth assume that the creation of new nonrival inputs lies at the heart of growth, and that the problem of compensating these kinds of inputs is pervasive. Suggestions of this kind can be found in the later writings of Schumpeter (1942).

Models where new ideas are associated with the design of new goods have the theoretical side effect of suggesting an important role for trade in fostering growth. The basic intuition is that with trade, it is no longer necessary for each country to reinvent the wheel. Two units of research effort can be used to produce two designs for the wheel in two closed economies, or it can be used to design the wheel in one country and to design a harness in the other. Harnesses can then be traded for wheels. This results in a kind of trade that looks superficially like trade based on comparative advantage; countries that are good at producing wheels export them and countries that are good at producing harnesses export them. It is, however, a form of endogenously acquired comparative advantage, not one that is the result of differences in resource endowments.

The key result from Romer (1989b) is not only that trade avoids redundancy in research effort, but also that total world wide research effort will increase when two closed economies open for trade. This dynamic effect is special to the growth model, and would not have been predicted by a simple static trade model. Qualifications for this result have been suggested by Grossman and Helpman (1989a). If consumption goods are country specific and one country is assumed to be better at research than the other, then it is possible that free trade tends to cause too much research to be done in the less capable country. Trade restrictions that realocate research effort from the country that is less capable of doing research to the country that is better at it could actually lead to faster growth. This qualification about the allocation of research effort across countries is a standard kind of observation about second best equilibria; starting from an equilibrium that is not Pareto optimal, policies which in the large would be welfare reducing may in small doses be welfare improving. This observation notwithstanding, the essential result should still hold. Compared to a situation with autarky or severely limited trade, a world with free trade among countries should tend to have a higher total level of resources devoted to the production of new ideas and therefore will have a higher rate of growth.

There are many open questions about the details of models like this. Romer (1987, 1989a,1989b) and Grossman and Helpman (1989b) assume that new ideas are always associated with new goods. Models by Aghion and Howitt (1989) and Grossman and Helpman (1989b) assume instead that new ideas can pertain to production techniques or quality attributes of existing goods. All of these models assume that there are a continuum of firms, each of which has some amount of market power because they sell distinct goods. This avoids the many ambiguities introduced when there is strategic behavior among a small group of firms. At some point, this assumption, itself made purely for technical reasons, will have to be weakened. For now, it seems that there is much to be learned in growth models that do not emphasize strategic behavior.

In any generalization, the basic elements identified so far should persist. If ideas are recognized as nonrival inputs, then firms must recover research and development expenses by charging a price for new goods that is higher than the marginal cost of production; and there will exist a form of increasing returns at the level of the economy as a whole.

III. Conclusion

Classified on methodological grounds, there are now four main types of models of growth:

i) Models that have no technological change and no increasing returns, and that support growth through accumulation of human capital.
ii) Models that allow for technological change but make it exogenous.
iii) Models that treat ideas as nonrival inputs and recognize the resulting nonconvexity but that use Marshallian external increasing returns to support an equilibrium with price-taking for rival inputs.
iv) Models that go one step further, departing from price-taking behavior and assuming instead some form of monopolistic competition.

These models differ in terms of how they handle ideas. The first type of model treats an idea as a good that is intangible, but that is otherwise conventional. Ideas here are assumed to be both rivalrous and excludable. The operative analogy is that they are like an education for an individual.

The second, third, and fourth models treat ideas as nonrival inputs in production. All three therefore imply some form of increasing returns when ideas are included in the set of inputs. The neo-classical model (number ii) and the spillover models (iii) both assume as well that ideas are purely nonexcludable, and hence earn no compensation. In the neo-classical model, ideas continue to arrive for reasons that are outside of the model. In the spillover models, ideas arise as side effects of other activities, in one case from physical capital investment, in another, from acquiring education. These models of side effects or learning by doing rely critically on the assumption that ideas are completely nonexcludable, for, as Dasgupta and Stiglitz (1988) have emphasized, even partial excludability will lead to a failure of price-taking competition. Because of this complete nonexcludability, they cannot contemplate the intentional production of ideas.

Only the fourth model, which drops the assumption of price-taking, can allow for the possibility that ideas are partially or completely excludable. As a result, only this class of models can allow a role for patents and intentional research and development.

As noted earlier, there is empirical evidence that is suggestive of the presence of increasing returns. Trends in growth rates, patterns of spatial location of economic activity and of international trade and factor movements are easier to accommodate in a model that is not convex than in one that is. Ultimately, however, the factor that is most likely to move the profession toward acceptance of models with increasing returns and even departures from price-taking is the incongruity between what economists actually believe and what their models of growth predict. A very large majority of economists believe that private sector research and development expenditure is an important determinant of long run potential for growth and that the presence or absence of intellectual property rights is important as well. Discussions of the economics of the firm or the industry typically reflect this belief. It is only in models of growth for the economy as a whole that these effects have played no role.

The major claim of this paper is that this incongruity reflects the technical constraints economists faced when they first tried to formulate dynamic equilibrium models, and that theory has advanced to the point that these constraints can now be relaxed. In particular, we now have tractable methods for studying dynamic equilibrium models with price-taking and spillovers, and more importantly, we are beginning to develop experience with a range of tractable dynamic models of monopolistic competition.

This is not to say that the insight gained from the other types of models is irrelevant. On the contrary, all of the effects identified in each of the other models should be present in a more complicated model with monopolistic competition. Human capital as acquired in school is of enormous importance for growth, especially if it is a complement with the production and use of new ideas. The neo-classical model made the fundamental contribution of showing that ideas, specifically technological change, had to be taken seriously within models of growth. Models with spillovers that focus on the effects of increasing returns at the level of society as a whole demonstrate effects that will still be present if the equilibrium is one with monopolistic competition instead of price-taking with spillovers.

Nor does this imply that all models of growth need to try to address all possible issues at once. For explaining cross country variation in income and growth rates, it may be sufficient to ignore the mechanisms that generate new ideas in the few developed countries, and concentrate instead on variables like schooling and human capital formulation. (See for example King and Rebelo, 1989.) Or to study the interaction between the rate of growth of income and individual fertility decisions, it may be sufficient to use a very simple model of the technology, for example, one in which output is linear in the stock of human capital. (See Becker, Murphy, and Tamura 1989). Similarly, many of dynamic effects identified in models of increasing returns with spillovers are harder to derive in models with monopolistic competition, but the qualitative results in the two types of models are very much the same. In many cases it will still be appropriate to use models that preserve price-taking behavior for purely technical reasons. The difference now is that this should be recognized as a shortcut. Theorists must at least consider the robustness of any conclusions reached to extensions that allow for departures from price-taking.

The net effect for growth theory is that there is a whole range of new questions that have only just begun to be explored, and a whole set of new modeling techniques that go along with them. Compared to the neo-classical model, the analysis in these extended frameworks will be somewhat harder and the conclusions somewhat less clear cut, but this will be more than offset by the richness of the issues that can finally be acknowledge to exist in the theory of growth.

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