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ABSTRACT

Measuring country origins of factor content in bilateral or sector-level exports is important to understand evolution of regional and global value chains and the roles of individual country-sectors in these chains. This paper proposes a method to distinguish between measures based on backward and forward linkages and between net and gross factor content. It is consistent with the System of National Account Standard and has the adding-up property. In comparison, these properties do not hold for the "hypothetical extraction method" proposed by Los et al. (2016). We show a number of examples involving disaggregated trade in which the two methods diverge.

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I. Introduction

A proper measure of national factor content crossing international borders is a necessary component of any empirical analysis of the development of global supply chains or the effects of regional trading agreements on domestic employment or other questions. There are two types of value-added measures associated with international trade. The first type involves estimates of a given country's value added projected forward into final consumption in different marketplaces around the world. For example, how much US factor content is ultimately absorbed in the Chinese market? The answer should include not only US factor content in the US exports to China, but alsoUS factor content in US exports to Japan but ultimately absorbed in China. The second type involves looking backward at the origins of value added in the trade flows for a particular country pair or sector. For example, how much total US factor content is embedded in its exports to China? How much the US value added is created in association with its exports to China? How much is the Chinese GDP induced by its semiconductor exports? Is it the same as total Chinese factor content embodied in its gross semiconductor exports? To answer these questions, one needs to look backwards to find country (or sector) origins of the factor content embedded in the trade flows and distinguish between net and gross measures. The aim of the paper is to derive a set of three measures that can provide answers to these questions.

The earlier answer to the first type of questions was provided by Trefler and Zhu (2010) and Johnson and Noguera (2012) by applying Leontief insight directly, and also computed by Los and Timmer (2018) recently by "hypothetical extraction method(HEM)".The answers to the second types of questions have been tackled by Wang, Wei and Zhu (2013) and Los, Timmer and de Vries (2016) among others, with no consensus yet in the literature. In this paper, we distinguish the concepts of "Domestic Value-added" (DVA, a net measure consistent with GDP accounting) and "Domestic Content" (DC, a gross measure of total factor content consistent with gross output) measures in Koopman, Wang, and Wei (2014, henceforth KWW).

For backward based measures (e.g., how much US factor content is in US exports to China), the ratio of embedded factor content to the bilateral exports is bounded by 100%. In comparison, that in the forward based measures are not bounded in the same way. For example, the ratio of the US factor content ultimately absorbed in China can in principle be more than the value of US gross exports to China (since it can include US factor content absorbed in China via US exports to Japan or Korea). Building on the value-added multiplier (Miller and Blair, 2009), our methodology produces both the backward and forward measures in a consistent framework.

For both the forward and backward linkages, our measures satisfy a simple and intuitive adding-up property. In particular, the sum of a country's DVA or factor content in all of its bilateral exports across all trading partners, or the sum of its DVA in a sector across all sectors, is equal to the total DVA in a country's gross exports, thus can be linked to System of National Accounts (SNA) or GDP accounting in a transparent way. But this property is not satisfied by the HEM. Our method is thus more consistent with GDP accounting in the sense of being able to distribute the total DVA that is exported abroad to the sum of the DVAs in its bilateral exports across all partner countries. Note this is different from applying a constant DVA ratio to all bilateral gross exports. Such adding-up property comes out naturally from our computation method, inherent from the property of a value-added multiplier that we use to define our three measures of factor contents in gross trade flows. We do not need impose separate accounting constraints.

Koopman, Wang, and Wei (2014) provide an accounting framework that decomposes a country's gross exports to into four parts: DVA that is exported and consumed abroad (VAX); DVA that is exported first but re-imported home (RDV); foreign value added used in the production of exports (FVA); and pure double counted items (PDC) arising from multiple border crossing of value added in intermediate inputs. These four terms are mutually exclusive and add up to 100% of the gross exports. It also distinguishes domestic content in exports (which is a gross measure of total factor content but unfortunately is also labeled as "trade in value added" or value-added exports in the literature) and DVA embodied in gross exports. It is important to note that, for a country's aggregate, there is no difference between value added in exports based on forward and backward linkages. That is, asking "how much US value added is exported to the world" and "how much of the US exports reflect its value added" always yield the same answer. In contrast, for bilateral or sectoral exports, the two are different. For example, "how much US value added is absorbed in China" and "how much of the US exports to China reflects US value added" in general require different answers.

Beyond measuring DVA in exports, the spirit of KWW is to build a bridge between two important and commonly reported data series (GDP and trade statistics) and establishes a precise relationship between components of officially reported gross exports (in gross terms) and GDP(in net terms) statistics[1]: VAX is the home country's GDP used to satisfy foreign demand, in which the factor content embodied in gross exports crosses national borders at least once. RDV, while embedded in the gross exports, is eventually consumed at home by being embedded in the home country's imports. Note that the domestic factor content in RDV crosses national borders at least twice. FVA is a part of other countries' GDP, which means that the associated factor content has also crossed national borders at least twice. PDC, the remaining part of the gross exports, has been recorded already by one of the previous three terms in some country's exports, and the associated factor content has crossed national borders at least three times.

By identifying which parts of the gross trade statistics are double counted relative to GDP statistics and what the sources of double counting are, the KWW method provides a way to re-interpret gross trade transactions in value-added terms (relative to GDP) that is fully consistent with the SNA standard, as suggested by Grossman and Rossi-Hansberg (2008)[2]. However, as KWW is not applicable to bilateral or sector level exports, we strive to achieve the same goal as KWW for disaggregated exports.

The rest of the paper is organized as follows: section II derives our measures of domestic factor content in disaggregated trade flows; Section III compares our analytical approach to HEM; Section IV reports the numerical comparison of the two approaches computed from the most recent version of WIOD (Timmer et al, 2016); Section V concludes.

II. Three measures of domestic factor contents in bilateral gross exports

Before defining our three measures of domestic factor content in bilateral trade flows, consider a 3-country example (US, Japan and China) in a production chain with three gross export transactions: First, US exports intermediate goods (e.g., steel) to Japan (E^{uj}= \$300). Second, Japan uses E^{uj} to produce downstream intermediates (e.g, car parts), and then exports back to the US

[1] These relationships are given in equations (15) to (18) of KWW for the two-country case and are extended to the G country, N sector case in Wang, Wei and Zhu (2013).

[2] Grossman and Rossi-Hansberg (2008), page 1996. "The globalization of production processes mandates a new approach to trade data collection, one that records international transactions, much like domestic transactions have been recorded for many years."

(E^{ju}=500), consisting $200 of Japanese value added and $300 factor content originally from the United States. Third, the US uses E^{ju}, combined with its additional $500 value added to produce a gross output (e.g., automobiles) of $1000. Half of the output is sold and consumed at home ($500), and the other half is exported to China and consumed there (E^{uc}=$500).

In this simple example, as summarized in Figure 1A, the total value of US gross exports (to both Japan and China) is $300+$500=$800, the US value added exports that is absorbed abroad (VAX) is ($300+$500)/2=$400. The US value added in its exports (DVA of US exports) is $300+$500/2=$550, and the domestic content (DC) of U.S. exports is $300+$400=$700. Notice these are three different concepts about the relationship between US factor content and US exports. Based on SNA 2008, value added represents the contribution of labor and capital to the production process. "As value added is intended to measure the value created by process of production, it ought to be measured net" (page 95 of 2008 SNA, by Euro Stat, IMF, OECD, UN and World Bank). This means that based on SNA standard, value-added cannot be counted twice in the production process; it should only be recorded at the time it is first generated from the production process as new value that added to a product. For the US exports to Japan, the VAX absorbed in Japan is zero. So the VAX as a share of US exports to Japan is zero, but the VAX absorbed in China as a share of US exports to China is $400/$500=80%. The US DVA in its exports to Japan is 100%, while that in its exports to China is only $250/$500=50%. While the US factor content in its exports to Japan is 100%, that in its exports to China is $400/$500=80%.

The US value added in its exports that is eventually returned to and absorbed in the US (RDV) is $150, half of the $300 US value-added at the first stage that is eventually consumed by US consumers. The domestically sourced double counting (DDC) in US exports is $150, that is, half of the value of the $300 which is counted initially as new value-added in the first stage E^{uj} but is exported again in the third stage (and therefore counted again in E^{uc}).

While the Japanese gross export to the US is $500, its value added in export is $200. As all of its value added is ultimately absorbed abroad, both its VAX and DC are $200. The global gross exports is worth $1300 (=$E^{uj}$ +E^{ju} +E^{uc} =$300+$500+$500), with the total domestic value added ultimately absorbed abroad (VAX^{uj} +VAX^{ju} +VAX^{uc} =0 + $200 + $400 =$600 and DVA in exports DVA^{uj} +DVA^{ju} +DVA^{uc} =$300+$200+$250= $750. The DC in exports at global level is DC^{uj} +DC^{ju} +DC^{uc} =$300+$200+$400=$900, while the total foreign content in exports is worth FC^{uj} +FC^{ju} +FC^{uc} = 0+$300+$100 =$400.

The differences among the three measures are clear: as the VAX is the home-country value-added created that ultimately satisfies foreign demand, it is only a part of the home country's GDP in exports as it excludes DVA that satisfies home final demand. DVA measures home country's GDP induced by its production of exports. While VAX is measured based on forward linkages, DVA is measured based on backward linkages. While both VAX and DVA are "net" measures, DC accounts for all domestic content regardless of whether they have been counted in previous production stages and is therefore in gross term.

We now generalize this example and define the three measures precisely. Assume a three-country (home-u, partner-c and third country-j) world, in which each country produces goods in N differentiated tradable sectors. Goods in each sector can be consumed directly or used as intermediate inputs, and each country exports both intermediate and final goods to the other two countries. The production and trade system can be written as an inter-country input-output (ICIO) accounting system in block matrix notations:

$$\begin{bmatrix} \mathrm{X}^{\mathrm{u}} \\ \mathrm{X}^{\mathrm{c}} \\ \mathrm{X}^{\mathrm{j}} \end{bmatrix} = \begin{bmatrix} \mathrm{a}^{\mathrm{uu}} & \mathrm{a}^{\mathrm{uc}} & \mathrm{a}^{\mathrm{uj}} \\ \mathrm{a}^{\mathrm{cu}} & \mathrm{a}^{\mathrm{cc}} & \mathrm{a}^{\mathrm{cj}} \\ \mathrm{a}^{\mathrm{ju}} & \mathrm{a}^{\mathrm{jc}} & \mathrm{a}^{\mathrm{jj}} \end{bmatrix} \begin{bmatrix} \mathrm{X}^{\mathrm{u}} \\ \mathrm{X}^{\mathrm{c}} \\ \mathrm{X}^{\mathrm{j}} \end{bmatrix} + \begin{bmatrix} \mathrm{y}^{\mathrm{uu}} + \mathrm{y}^{\mathrm{uc}} + \mathrm{y}^{\mathrm{uj}} \\ \mathrm{y}^{\mathrm{cu}} + y^{\mathrm{cc}} + y^{\mathrm{cj}} \\ \mathrm{y}^{\mathrm{ju}} + y^{\mathrm{jc}} + y^{\mathrm{jj}} \end{bmatrix} = \begin{bmatrix} \mathrm{a}^{\mathrm{uu}} & \mathrm{a}^{\mathrm{uc}} & \mathrm{a}^{\mathrm{uj}} \\ \mathrm{a}^{\mathrm{cu}} & \mathrm{a}^{\mathrm{cc}} & \mathrm{a}^{\mathrm{cj}} \\ \mathrm{a}^{\mathrm{ju}} & \mathrm{a}^{\mathrm{jc}} & \mathrm{a}^{\mathrm{jj}} \end{bmatrix} \begin{bmatrix} \mathrm{X}^{\mathrm{u}} \\ \mathrm{X}^{\mathrm{c}} \\ \mathrm{X}^{\mathrm{j}} \end{bmatrix} + \begin{bmatrix} \mathrm{Y}^{\mathrm{u}} \\ \mathrm{Y}^{\mathrm{c}} \\ \mathrm{Y}^{\mathrm{j}} \end{bmatrix}$$

Rearranging as：

$$\begin{bmatrix} \mathrm{X}^{\mathrm{u}} \\ \mathrm{X}^{\mathrm{c}} \\ \mathrm{X}^{\mathrm{j}} \end{bmatrix} = \begin{bmatrix} I - \mathrm{a}^{\mathrm{uu}} & -\mathrm{a}^{\mathrm{uc}} & -\mathrm{a}^{\mathrm{uj}} \\ -\mathrm{a}^{\mathrm{cu}} & I - \mathrm{a}^{\mathrm{cc}} & -\mathrm{a}^{\mathrm{cj}} \\ -\mathrm{a}^{\mathrm{ju}} & -\mathrm{a}^{\mathrm{jc}} & I - \mathrm{a}^{\mathrm{jj}} \end{bmatrix}^{-1} \begin{bmatrix} \mathrm{Y}^{\mathrm{u}} \\ \mathrm{Y}^{\mathrm{c}} \\ \mathrm{Y}^{\mathrm{j}} \end{bmatrix} = \begin{bmatrix} \mathrm{b}^{\mathrm{uu}} & \mathrm{b}^{\mathrm{uc}} & \mathrm{b}^{\mathrm{uj}} \\ \mathrm{b}^{\mathrm{cu}} & \mathrm{b}^{\mathrm{cc}} & \mathrm{b}^{\mathrm{cj}} \\ \mathrm{b}^{\mathrm{ju}} & \mathrm{b}^{\mathrm{jc}} & \mathrm{b}^{\mathrm{jj}} \end{bmatrix} \begin{bmatrix} \mathrm{Y}^{\mathrm{u}} \\ \mathrm{Y}^{\mathrm{c}} \\ \mathrm{Y}^{\mathrm{j}} \end{bmatrix} = \mathrm{BY} \qquad (1)$$

where b^{uc} denotes a N×N block Leontief inverse matrix, which is the total requirement matrix that describes the amount of gross output in the home country u required for a one-unit increase in final demand of the partner country c. Y^c is a N×1 vector that gives the global use of partner's final goods.

Having defined the Leontief inverse, we turn to measuring domestic factor contents. Let v^{u} be the 1×N direct value-added coefficient vector. Each element of v^{u} gives the share of direct domestic value added in total output. This is equal to one minus the intermediate input share from all countries (including domestically produced intermediates):

$$\mathrm{v}^{\mathrm{u}} = \mu(I - \mathrm{a}^{\mathrm{uu}} - \mathrm{a}^{\mathrm{cu}} - \mathrm{a}^{\mathrm{ju}}) \qquad (2)$$

where μ is a 1×N unity vector. Multiply these direct value-added coefficients with the global Leontief inverse B produces a 3N×3N value-added multiplier matrix:

$$\hat{V}B = \begin{bmatrix} \hat{v}^{u} & 0 & 0 \\ 0 & \hat{v}^{c} & 0 \\ 0 & 0 & \hat{v}^{j} \end{bmatrix} \begin{bmatrix} b^{uu} & b^{uc} & b^{uj} \\ b^{cu} & b^{cc} & b^{cj} \\ b^{ju} & b^{jc} & b^{jj} \end{bmatrix} = \begin{bmatrix} \hat{v}^{u}b^{uu} & \hat{v}^{u}b^{uc} & \hat{v}^{u}b^{uj} \\ \hat{v}^{c}b^{cu} & \hat{v}^{c}b^{cc} & \hat{v}^{c}b^{cj} \\ \hat{v}^{j}b^{ju} & \hat{v}^{j}b^{jc} & \hat{v}^{j}b^{jj} \end{bmatrix} \tag{3}$$

Within $\hat{V}B$, the diagonals denote the domestic content share of domestically produced products in a particular sector at home, the off-diagonal elements along the columns denote the share of partner c and the third country j's share of factor contents in these same goods, respectively. The off-diagonal along the row represent how each source country's factor content is used in downstream country-sectors and their sum equals to the source country's total factor content embodied in global production and trade. Each of the first N columns in the $\hat{V}B$ matrix includes all factor content, domestic, partner and third country, needed to produce one additional unit of domestic product at home. The second N columns present factor content shares for production in the partner country c and the third N columns present factor content shares for production in third country j. Because all value added must be either domestic or foreign, the sum along each column is unity. A proof is provided in appendix A.

The $\hat{V}B$ or value-added multiplier matrix contains all necessary information to compute the domestic and imported factor content shares in each country's final goods production or gross output (exports). The diagonal elements of $\hat{V}B$ provide the shares of the domestic content in each country's bilateral gross exports. The off-diagonal elements along the column give the shares of the foreign content embodied in each country's bilateral gross exports. They are the shares of the partner country and third country factor content, respectively. While the off-diagonal elements *along a row* provides information on a country's share of factor content embodied as intermediate inputs in the production of other countries' gross bilateral exports.

We use the diagonals of $\hat{V}B$ to define the DC measure for each country by multiplying the diagonals of $\hat{V}B$ and a 3N by 3 matrix of bilateral gross trade flows. This yields a 3N by 3 matrix of domestic content in bilateral gross trade as follows:

$$DC = \hat{V}B^{D}E = \begin{bmatrix} \hat{v}^{u}b^{uu} & 0 & 0 \\ 0 & \hat{v}^{c}b^{cc} & 0 \\ 0 & 0 & \hat{v}^{j}b^{jj} \end{bmatrix} \begin{bmatrix} 0 & e^{uc} & e^{uj} \\ e^{cu} & 0 & e^{cj} \\ e^{ju} & e^{jc} & 0 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 0 & \hat{v}^{u}l^{uu}e^{uc} & \hat{v}^{u}l^{uu}e^{uj} \\ \hat{v}^{c}l^{cc}e^{cu} & 0 & \hat{v}^{c}l^{cc}e^{cj} \\ \hat{v}^{j}l^{jj}e^{ju} & \hat{v}^{j}l^{jj}e^{jc} & 0 \end{bmatrix}}_{DVA} \tag{4}$$

$$+\underbrace{\begin{bmatrix} 0 & \hat{v}^{\mathrm{u}}(b^{uu}-l^{uu})\mathrm{e}^{\mathrm{uc}} & \hat{v}^{\mathrm{u}}(b^{uu}-l^{uu})e^{\mathrm{uj}} \\ \hat{v}^{\mathrm{c}}(b^{cc}-l^{cc})\mathrm{e}^{\mathrm{cu}} & 0 & \hat{v}^{\mathrm{c}}(b^{cc}-l^{cc})\mathrm{e}^{\mathrm{cj}} \\ \hat{v}^{\mathrm{j}}(b^{jj}-l^{jj})\mathrm{e}^{\mathrm{ju}} & \hat{v}^{\mathrm{j}}(b^{jj}-l^{jj})\mathrm{e}^{\mathrm{jc}} & 0 \end{bmatrix}}_{DDC} = DVA + DDC$$

where $L = \begin{bmatrix} I-\mathrm{a}^{\mathrm{uu}} & 0 & 0 \\ 0 & I-\mathrm{a}^{\mathrm{cc}} & 0 \\ 0 & 0 & I-\mathrm{a}^{\mathrm{jj}} \end{bmatrix}^{-1} = \begin{bmatrix} l^{\mathrm{uu}} & 0 & 0 \\ 0 & l^{\mathrm{cc}} & 0 \\ 0 & 0 & l^{\mathrm{jj}} \end{bmatrix}$ is the domestic Leontief Inverse, a 3N by 3N matrix. It can be further decomposed into two terms: DVA and DDC. A step by step math derivation is given in online appendix B.

The measure of DVA in bilateral exports thus can be written as

$$DVA = \hat{V}LE = \begin{bmatrix} \hat{v}^{\mathrm{u}} & 0 & 0 \\ 0 & \hat{v}^{\mathrm{c}} & 0 \\ 0 & 0 & \hat{v}^{\mathrm{j}} \end{bmatrix} \begin{bmatrix} l^{\mathrm{uu}} & 0 & 0 \\ 0 & l^{\mathrm{cc}} & 0 \\ 0 & 0 & l^{\mathrm{jj}} \end{bmatrix} \begin{bmatrix} 0 & \mathrm{e}^{\mathrm{uc}} & e^{\mathrm{uj}} \\ \mathrm{e}^{\mathrm{cu}} & 0 & \mathrm{e}^{\mathrm{cj}} \\ \mathrm{e}^{\mathrm{jc}} & \mathrm{e}^{\mathrm{ju}} & 0 \end{bmatrix} \qquad (5)$$

$$= \begin{bmatrix} 0 & \hat{v}^{\mathrm{u}} l^{\mathrm{uu}} \mathrm{e}^{\mathrm{uc}} & \hat{v}^{\mathrm{u}} l^{\mathrm{uu}} e^{\mathrm{uj}} \\ \hat{v}^{\mathrm{c}} l^{\mathrm{cc}} \mathrm{e}^{\mathrm{cu}} & 0 & \hat{v}^{\mathrm{c}} l^{\mathrm{cc}} \mathrm{e}^{\mathrm{cj}} \\ \hat{v}^{\mathrm{j}} l^{\mathrm{jj}} \mathrm{e}^{\mathrm{ju}} & \hat{v}^{\mathrm{j}} l^{\mathrm{jj}} \mathrm{e}^{\mathrm{jc}} & 0 \end{bmatrix}$$

It measures the amount of all domestic value added exported via a particular sector (e.g., US electronics), including value added originating from any domestic sector (e.g., including US plastics, chemicals, and automobiles) via backward linkages, and is ultimately absorbed at home or abroad (partner country or third country). There are three key features of this measure. First, it focuses on the identity of the last sector through which DVA from all domestic sectors is exported. Second, it includes the part of DVA that eventually returns home. Third, because it is always physically associated with gross trade in a particular trading route, its value is always bounded by the value of the corresponding gross trade flows. Different from the DC measure in equation (5), this measure is in net term that excludes all double counting, and is consistent to SNA standard at any level of disaggregation.

As with our simple example, while all three measures are factor content employed in the exporting country/sector, DC and DVA are backward looking and do not depend on where that value is ultimately absorbed. For example, some of the factor content in DC or DVA could be returned to the exporting country or sent to third countries. In contrast, VAX is forward looking and describes what is being absorbed in the partner country. Separately, DC is in gross terms, including double counting due to intermediate inputs crossing borders more than once; while DVA and VAX are both in net terms, excluding double counting.

Because VAX depends on where the value-added is consumed and has to be in net term, it is defined by $\hat{V}B$ multiply final products consumed by each country as the following $\hat{V}BY$ matrix after zeroing its diagonal:

$$\hat{V}BY = \begin{bmatrix} \hat{v}^u b^{uu} & \hat{v}^u b^{uc} & \hat{v}^u b^{uj} \\ \hat{v}^c b^{cu} & \hat{v}^c b^{cc} & \hat{v}^c b^{cj} \\ \hat{v}^j b^{ju} & \hat{v}^j b^{jc} & \hat{v}^j b^{jj} \end{bmatrix} \begin{bmatrix} y^{uu} & y^{uc} & y^{uj} \\ y^{cu} & y^{cc} & y^{cj} \\ y^{ju} & y^{jc} & y^{jj} \end{bmatrix}$$

$$= \begin{bmatrix} \hat{v}^u \sum_t b^{ut} y^{tu} & \hat{v}^u \sum_t b^{ut} y^{tc} & \hat{v}^u \sum_t b^{ut} y^{tj} \\ \hat{v}^c \sum_t b^{ct} y^{tu} & \hat{v}^c \sum_t b^{ct} y^{tc} & \hat{v}^c \sum_t b^{ct} y^{tj} \\ \hat{v}^j \sum_t b^{jt} y^{tu} & \hat{v}^j \sum_t b^{jt} y^{tc} & \hat{v}^j \sum_t b^{jt} y^{tj} \end{bmatrix} \quad (6)$$

$$VAX = \begin{bmatrix} 0 & \hat{v} \sum_t b^{ut} y^{tc} & \hat{v} \sum_t b^{ut} y^{tj} \\ \hat{v}^c \sum_t b^{ct} y^{tu} & 0 & \hat{v}^c \sum_t b^{ct} y^{tj} \\ \hat{v}^j \sum_t b^{jt} y^{tu} & \hat{v}^j \sum_t b^{jt} y^{tc} & 0 \end{bmatrix}$$

Where Y is 3N by 3 final demand matrix. The resulting $\hat{V}BY$ is a 3N by 3 value-added production and trade matrix. Its off diagonal elements constitute the 3N by 3 bilateral value-added trade matrix. Because the diagonal of resulted $\hat{V}BY$ matrix is set to zero, value-added produced in home country that satisfies home final demand is excluded. It only measures the part of the source country's GDP induced by its production of exports to satisfy foreign final demand.

This measure accounts for the amount of DVA originating from a specific sector (e.g., from the US electronics sector), via all sectors' gross exports from the source country (i.e., including gross exports from the US automobile and machinery sectors in addition to the US electronics sector), and ultimately consumed in a particular destination country. It has been successfully used by Johnson and Noguera (2012).

It also has three key features. Using the example of VAX from the US electronics sector, we note first that some of the value added from that sector can be exported indirectly via other US sectors such as automobiles, since US electronics products are used as intermediate inputs in the production of automobile exports. Second, as VAX focuses on the value added "ultimately absorbed in a particular destination country," it excludes the part of value added that is eventually returned and consumed at home. Finally, it can deviate from and indeed not bounded by gross trade flows at the sector or bilateral level. That is, the US VAX to China can in principle exceed the value of US gross exports to China.

III. Relations to HEM

The hypothetic extraction method (HEM) compares actual GDP in a country with a hypothetical GDP in which no production activities are related to exporting from that country. The difference between the two describes DVA in exports, which provides good intuition for why the DVA term defined by KWW (2014, equation 37) is the part of GDP generated in the production of gross exports. This measure is labeled as VAX_D in Los and Timmer (2018). The HEM method can also be used to compute value-added exports as done by Johnson and Noguera (2012), which is identical to our VAX measure defined in equation (6).

Similar to our DVA, VAX_D is also always physically associated with bilateral trade flows passing through national borders among countries that sign the trade agreements, while VAX include value-added indirectly traded though third counties outside the treaty members, Los and Timmer (2018) suggest that VAX_D is more appropriate than VAX in quantifying the effects of free trade agreements (FTAs) since a reduction in trade barriers among FTA members may have a smaller effect on VAX than VAX_D if VAX includes a large amount of indirect trade through non-member countries.

However, DVA and VAX_D are different at either bilateral or sector level even though they coincide at the country aggregate level. We will illustrate bellow how VAX_D over-counts DVA but under-counts DC for disaggregated trade.

Let us first illustrate the difference between the two methods using a different example modified from the earlier one. Suppose US exports $150 of intermediate products each to Japan and China, respectively. Each of them then adds $100 worth new value added in producing downstream intermediates and export them ($250 each) back to the US. The final stage production and exports stay the same as shown in Figure 1B.

Applying our method, in the US exports to China, $\mathrm{DVA}^{\mathrm{uc}} = \$150 + \$250 = \$400$, $\mathrm{DC}^{\mathrm{uc}} = \$150 + \$400 = \$550$. Applying HEM, $\mathrm{VAX\_D}^{\mathrm{uc}} = \$150 + \$75 + \$250 = \$475$. $\mathrm{VAX\_D}^{\mathrm{uc}}$ lies in between the DVA and DC measures. While the HEM is able to trace most of the domestic factor content through exports based on backward linkages in each stage of the production, it does not identify how much new value-added (GDP) is created in each production stage, and also excludes some of the factor content. In this example, VAX_D counts half of the US new value-added created at the first stage export twice: first in exports to Japan (E^{uj}) and second in exports to China (E^{uc2}), because the new value added created by US production embodied in E^{uc2} is \$250, not \$325.

It is useful to inspect how double counting is treated differently in the two methods. In our method, US DVA created in the first step is "double counted" because it is recorded twice in gross exports, first in E^{uc1} and E^{uj}, second in E^{uc2}. While the embodied US DVA in its gross exports to China is \$400, US domestic sourced double counting (DDC) is \$150 (\$75 in E^{uc1} and \$75 in E^{uj}). The US GDP associated with producing the total gross exports is still \$550 (including \$150 in exports to Japan), stay the same as the earlier example.

While HEM's VAX_D measure avoids double counting in E^{uc1}, it still counts half of the US value-added in E^{uj} as DVA again when imports from Japan is used in producing E^{uc2}. This example illustrates the point that the HEM's VAX_D should not be interpreted as the part of GDP generated by producing gross exports at the country/sector or bilateral sector level. In addition, the difference between the DVA in aggregate exports and the sum of the DVA from the bilateral (or sectoral) level by the HEM method will under-count DDC, since it only includes part of those domestic sourced double counting terms. In Online Appendix C, we summarize this example by an ICIO table and provide a step-by-step computation.

We now discuss the general 3-country accounting framework[3]. Define $A^{uc} = \begin{bmatrix} 0 & a^{uc} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $A^{D} = \begin{bmatrix} a^{uu} & 0 & 0 \\ 0 & a^{cc} & 0 \\ 0 & 0 & a^{jj} \end{bmatrix}$, $A^{F} = \begin{bmatrix} 0 & a^{uc} & a^{uj} \\ a^{cu} & 0 & a^{cj} \\ a^{ju} & a^{jc} & 0 \end{bmatrix}$, $Y^{uc} = \begin{bmatrix} y^{uc} \\ 0 \\ 0 \end{bmatrix}$, the bilateral exports matrix as $E^{uc} = \begin{bmatrix} e^{uc} \\ 0 \\ 0 \end{bmatrix} = A^{uc}X + Y^{uc} = \begin{bmatrix} a^{uc}x^{c} + y^{uc} \\ 0 \\ 0 \end{bmatrix}$, and the value added coefficient of USA as $V^{u} = [v^{u} \quad 0 \quad 0]$.

[3] Because HEM calculates DVA in bilateral trade flows route by route, the extracted trade flow and the Leontif marix are different each time, it is difficult to summarize them.

After hypothetically extracting e^{uc}, the Leontief inverse for HEM is: $B^{uc\prime} = (I - A + A^{uc})^{-1} = \begin{bmatrix} 1-a^{uu} & 0 & -a^{uj} \\ -a^{cu} & 1-a^{cc} & -a^{cj} \\ -a^{ju} & -a^{jc} & 1-a^{jj} \end{bmatrix}^{-1} = \begin{bmatrix} b^{uu\prime} & b^{uc\prime} & b^{uj\prime} \\ b^{cu\prime} & b^{cc\prime} & b^{cj\prime} \\ b^{ju\prime} & b^{jc\prime} & b^{jj\prime} \end{bmatrix}$, then the HEM's measure of DVA in e^{uc} is given by following equation:

$$\begin{aligned} \mathrm{VAX\_D}^{uc} &= V^u(I-A)^{-1}Y - V^u(I-A+A^{uc})^{-1}(Y-Y^{uc}) \\ &= V^u B^{uc\prime} E^{uc} = v^u b^{uu\prime} e^{uc} \end{aligned} \quad (7)$$

Based on equation (5), our measure of DVA in e^{uc} equals

$$\mathrm{DVA}^{uc} = v^u(I-a^{uu})^{-1}e^{uc} = v^u l^{uu} e^{uc} \quad (8)$$

Based on equation (4), our measure of DC in e^{uc}can be decomposed as follows

$$\mathrm{DC}^{uc} = v^u b^{uu} e^{uc} = \mathrm{DVA}^{uc} + \mathrm{DC}^{uc}$$

$$= \underbrace{\underbrace{\underbrace{v^u l^{uu} e^{uc}}_{\mathrm{DVA}^{uc}} + v^u l^{uu} a^{uj} b^{ju\prime} e^{uc}}_{\mathrm{VAX\_D}^{uc}} + v^u b^{uu\prime} a^{uc} b^{cu} e^{uc}}_{\mathrm{DC}^{uc}} \quad (9)$$

A detailed derivation is in Online Appendix D.

The first term ($v^u l^{uu} e^{uc}$) of equation (9) is US DVA embodied in e^{uc}, which is the first term in equation (4). The second term ($v^u l^{uu} a^{uj} b^{ju\prime} e^{uc}$) is the US value added embodied in the US intermediate exports to Japan and used in Japan's production of its exports to US which in turn is used in US production of its exports to China. This term is treated as a part of the VAX_D because it is sourced from US, but is considered as a part of DDC in our method. The third term ($v^u b^{uu\prime} a^{uc} b^{cu} e^{uc}$) is the value added of US embodied in its intermediate exports to China, used in production by China for its exports to the US, which in turn used it in the production for final exports to China. This trade loop term between US and China is considered as double counted domestic value in both our method and HEM.

In some special cases, when either a^{uj} or $b^{ju\prime}$ in the second term of equation (9) equals to zero, $v^u l^{uu} a^{uj} b^{ju\prime} e^{uc} = 0$, which means without the trade loop of US-Japan-US (back-and-forth intermediate goods trade with a third country), the DVA and VAX_D are consistent ($\mathrm{DVA}^{uc} =$

VAX_D^{uc}). On the other hand, in some special cases when either a^{uc} or b^{cu} in the third term of equation (9) equal to zero, $v^{u}b^{uu\prime}a^{uc}b^{cu}e^{uc} = 0$, which means without the trade loop of US-China-US (back-and-forth intermediate goods trade with partner country only), the VAX_D measured by HEM method are consistent with DC measure ($DC^{uc} = \text{VAX\_D}^{uc}$). When there is no feedback trade loop between home and other countries, which means the second and third terms of equation (9) are both equal to zero, then $DVA^{uc} = \text{VAX\_D}^{uc} = DC^{uc}$. Actually speaking, the HEM measure is neither DVA or DC at disaggregate level. In a sense, the HEM measure is a special case of the more general formulas of equation (4) and (9) in this paper.

At the country aggregate level, because all bilateral flows are summed together, the HEM method will have $b^{uu\prime} = (I - a^{uu})^{-1}$. Therefore, the two DVA measures are exactly the same.

$$\text{VAX\_D}^{u} = v^{u}b^{uu'}e^{u} = v^{u}(I - a^{uu})^{-1}e^{u} = DVA^{u} \tag{10}$$

However, at the bilateral level, VAX_D^{uc} and DVA^{uc} are different：

$$\begin{aligned} &\text{VAX\_D}^{uc} - DVA^{uc} = v^{u}l^{uu}a^{uj}b^{ju'}e^{uc} \\ &= V^{u}(I - A^{D})^{-1}(A^{F} - A^{uc})(I - A + A^{uc})^{-1}E^{uc} \\ &\in V^{u}(I - A^{D})^{-1}A^{F}(I - A)^{-1}E^{uc} = DDC^{uc} \end{aligned} \tag{11}$$

Where $A^{F} - A^{uc} \in A^{F}$, $(I - A + A^{uc})^{-1} \in (I - A)^{-1}$

As demonstrated by equation (11), the VAX_D^{uc} can be decomposed into two parts. The first part is DVA^{uc}. The second part equals the second term of equation (9), which is the difference between DVA^{uc}_{HEM} and DVA^{uc}. This implies that $DC^{uc} \geq \text{VAX\_D}^{uc} \geq DVA^{uc}$ because the last term of (11) is nonnegative. In general, the numerical difference between VAX_D^{uc} and DC^{uc} or DVA^{uc} depends inversely on the relative size of the second and third terms in equation (9). This is because the third term represents the value of a feedback trade loop in a specific sector between two partner countries. When sector classification is sufficiently detailed in an ICIO database, the feedback loop trade would become correspondingly less important within a given sector, and VAX_D^{uc} will become a close approximation of DC^{uc} (a proof is provided in online appendix E).

As equation (11) shows, its right hand term is a part of DDC^{uc}. We leave the derivation to Online Appendix E , only illustrate the economic intuition for the decomposition of DC^{uc} and

VAX_D^{uc} here by Figure 2, with attention to how the double counting has happened, and why tracing the source of the value-added is not sufficient to account for the new value-added at each stage of production to consistent with SNA standard.

As shown in Figure 2, DDC in US gross exports results from intermediate goods trade loop as in our US-Japan-US-China example. In such a feedback loop involve third countries, the correct net measure of DVA (GDP) in exports should only include the new DVA generated from the last stage of production, as illustrated by DVA^{uc}. However, VAX_D^{uc} includes all domestic sourced factor content created from previous stages, causing it to be larger than DVA^{uc}.

Similarly, at the country-sector and bilateral-sector level, we can also derive the difference between VAX_D^{ui} and DVA^{ui}, VAX_D^{uci} and DVA^{uci} in the same way as what we did in equation (11). They can be shown to be nonnegative and is part of DDC defined previously. Detailed proofs are given in Online Appendix F.

In sum, because the HEM computes DVA sector by sector or route by route, it does not in general produce a DVA measure in net term. It coincides with DVA for country's aggregate exports and in some special cases for disaggregated trade flows. In general, it produces a number that is bigger than DVA but smaller than DC. In comparison, the measures of DVA and DC defined by our method are conceptually consistent with the GDP accounting in System of National Accounts and satisfy an intuitive adding-up property. This allows GDP in exports to be distributed in bilateral exports across trading partners or in sectoral exports across all sectors.

In a recent review of the literature on measuring global value chains, Johnson (2018) proposes a gross trade decomposition formula in a similar spirit as the HEM method but also simultaneously define domestic and foreign value-added as well as double counted residuals (similar to what Koopman, Wang, and Wei (2014) did). However, it still has the same limitation as the HEM method discussed in this paper. In particular, its DVA measure in its gross exports decomposition equation (13) is similar to VAX_D, and is not a GDP consistent-measure at the bilateral, sector, or bilateral sector level, and does not possess the adding-up property.[4]

4. Johnson (2018) criticizes KWW (2014) for multiplying gross exports by global Leontief inverse: $(\mathrm{I}-\mathrm{A})^{-1}E$ thus treating input shipments inconsistently. This is a misunderstanding of the KWW method since KWW only multiplies gross exports E by $\mathrm{V}(\mathrm{I}-\mathrm{A})^{-1}=VB$, the value-added multiplier as we discussed in this paper. As we shown, this is

IV. Measurement in Practice: Our Approach versus HEM

We compare the two methods using the most recent version of WIOD (for 2014, released in 2016). As reported in Table 1, the average percentage differences between VAX_D and DVA is larger than that between DC and VAX_D for all non-zero trade routes: 0.8% over 0.2% for country sector level trade flows and 0.98% over 0.04% for bilateral sector level trade flows (the last row). The VAX_D in practice is reasonably close to DC. At the country sector level, in total 2262 non-zero trade flows, only 14 of them have over 0.5% percentage difference and nearly 95% of them with less than 0.1% percentage difference. At bilateral sector level, in total 95,959 non-zero flows, only two of them with a percentage difference larger than 0.5% and more than 99.9% percent of them with a percentage difference less than 0.1%. These numerical results are not surprising and they are consistent with the analytical results of equation (9) in section III. This is due to the VAX_D measure only excludes a portion of the double-counted domestic value when it is exported by the same bilateral sector twice, represented by the third item in equation (9); but includes a portion of the double counted domestic value, represented by the second item in equation (9), and the numerical value of the second term is much larger than the third term for any ICIO data set from the real world.

The numerical results also verify our analytical finding discussed in section III: at the country aggregate level, GDP generated by the production of gross exports computed from our method and the HEM are exactly the same, while at a disaggregated level, VAX_D in general lies between DVA and DC. Because VAX_D includes a part of double counting, it cannot be interpreted as GDP generated by production of exports at disaggregated levels.

Some additional patterns are also interesting. First, at a disaggregated level, the larger the proportion of domestic value-added first exported then returned home via intermediate trade, the greater the difference between VAX_D and DVA. Second, when VAX_D computed at the bilateral-sector level is aggregated into country/sector or bilateral aggregate level, it is always greater than VAX_D computed directly at the aggregated level.

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a technique to decompose the total value of gross exports according to its sources. KWW only uses local Leontief inverse i.e $(\mathrm{I}-\mathrm{A}^{D})^{-1}$, the inverse of diagonal block of the global IO coefficient matrix A to compute the output needed to produce a country's gross exports. This effectively uncouples the part of gross output needed to produce each country's exports as Johnson (2018) attempts to do in his equations (7) and (8).

Finally, in many cases, the difference between VAX_D and DVA is small. This is because, with the relatively coarse definition of a sector in the existing ICIO table, and perhaps at the current stage of GVC development, the component attributed to domestic sourced double counting is a small portion of most countries' gross exports. However, since VAX_D and DVA are conceptually distinct, future ICIO tables with finer sector definitions, or future development of global trade that features more intermediate goods trade back and forth, could enlarge the difference between these measures.

We note that the low percentage of domestically sourced double counting in a country's gross exports could also be related to the homogeneous technology assumption built in to the current Input-Output(IO) accounting framework. Under such a framework the input bundles of a country's exports to all destination countries are assumed to be the same. This assumption does not hold in the data, and future ICIO tables may move away from this assumption. By combining firm level data with the IO table, Alonso de Gortari (2017) has shown that the percentage of US value-added returned via intermediate imports from Mexico is 10% higher than that measured in the current IO accounting framework (17%) if specialized inputs in the bilateral trade are taken into account. This suggests that the domestically sourced double counting is likely higher than what the current input-output accounting framework is able to measure. Once the input bundles in the bilateral exports are allowed to vary by partner country, the difference between VAX_D and DVA will likely become bigger for many country pairs.

Even with the limitations in the current ICIO data, the difference between the two measures is already significant for several country/sector cases. For example, for the refined petroleum products of both Netherlands and Germany, the difference between these two methods is more than 7% (Table 2). For the chemical and air transport industries in the Netherlands, the difference is about 5%. This means if one interprets VAX_D as the part of GDP generated by producing these exports, it would be overestimated by more than 5-7%. This is largely because the Dutch DVA embodied in the products from these upstream industries are important intermediate inputs for many of its downstream manufacturing industries that will be subject to more border crossings and double counting in gross trade statistics before they are absorbed in the final products.

At the bilateral/sector level, except for basic industrial intermediates, trade routes involving large discrepancies between the two methods are typically GVC-dominated sectors and those between partners heavily involved in GVC activities. Examples include the computer and electronic trade between China-Taiwan (12.4%) and China-Korea (6.0%), due to components and parts crossing national borders back and forth several times before they are embodied into final products for sale to final consumers (Table 2). These examples highlight the potential bias on the estimates of GDP in gross exports when one incorrectly interprets HEM's VAX_D as a measure of GDP in gross exports at disaggregate levels.

V. Concluding Remarks

While our analytical method and HEM are fully consistent at the country aggregate level, important differences emerge when computing DVA in exports at the disaggregated level. We trace DVA in "net" terms at each stage of production, while DVA measure computed by HEM is not necessary in "net" terms. Therefore, the DVA in export measure at the bilateral, sector, or bilateral sector level computed by HEM does not have the additivity property as our method and cannot be interpreted as GDP in exports at disaggregated level as that at country aggregate level. Our discussion shows that it is very important to understand what is measured by each measure computed from different methods and what are the correct economic and statistical interpretations for these measures. Which measure is proper to use in a particular case will depend on the nature and characters of the research question at hand. To help readers better understand how to properly use various measures discussed in the paper, we summarize the conceptual differences and the major economic questions these measures may properly addressed in table 3.

The table highlights the different logic under VAX_D and DVA. Since HEM highly depend on its strong assumption of Leontief production function, when the bilateral export of a certain country/industry is prohibited (that is, extracted), the exporting country/sector cannot export to third countries, and the importing country also cannot import from third countries. Therefore, the GDP loss of the exporting country will be equal to VAX_D. However, this is not what happened in the real world. DVA measures new value-added created by the exporting country in the production of exports. If there is "re-export", the added value created by the upstream export producer has already been counted as GDP, so there is no need to count it again according to SNA standard. It is an accounting measure only, not rely on any assumption of production function. This is an advantage over VAX_D.

It is true that IO tables do not record the sequence of the intermediate transactions, when "feedback loop" presence, many different networks can underlie the same IO-table. Therefore, it is a difficult task to retrieve the underlying production chain as argued by Los and Timmer (2018). However, as we discussed earlier, it is feasible to identify factor content that cross-national border once, twice and at least three times in KWW's gross trade flow decomposition, rules thus could be established to decide how 'domestic value-added' or 'double counted' value embodied in a particular bilateral gross trade flows should be identified based on SNA standard (Value-added should only be recorded at the time it is first generated from the production process as new value that added to a product) such as the source based method proposed by Nagengast and Stehrer (2016) and further elaborated by Borin and Mancini (2019).

While the Leontief production function makes a strong assumption about substitutability, one should not confuse that with the usefulness of Input-Output tables as an accounting framework. Indeed, such tables are the core infrastructure underlying GDP statistics in SNA (1993, 2008), The KWW method as a gross trade flow accounting framework, decomposes gross trade flows for a given year on an **ex-post** basis. Therefore, as long as intermediate transaction (Z), gross output (X), direct value-added (VA) and final demand (Y) are known, one can always decompose X in term of Y and express all countries' gross output (gross exports) according to their location of final goods production or destination of final consumption. Such excise is no longer needs to ground on the highly restrictive Leontief type production function as HEM[5], but based on the core framework in SNA. Therefore, the KWW decomposition and its extension in this paper does not require assumptions on the substitutability in the production function and does not present any interpretation difficulties as an accounting framework.

In addition, it will be misleading if one overemphasizes that economic measures must derive from corresponding economic models, because this is not true in the history of economic ideas. For instance, when the modern concept of GDP first present to US congress by Simon Kuznets, an American economist then worked at BEA in 1934, his idea is to capture all economic production by individuals, companies, and the government in a single measure (this could be done in several ways), which was not based on an economic model but on a preliminary

[5] because the economic transactions recorded in the global Supply and use tables (the statistical base of ICIO tables) are statistical reflection of real world economic activities, which already are the results of substitutability among various economic activities.

economic statistical framework. This framework finally improved as the National Income and Product Accounts (NIPA)[6]. Therefore, the gross trade flow decomposition we proposed is not an "arbitrary accounting exercise" as criticized by Johnson (2018) and Los and Timmer (2018). It can serve different types of economic models, especially for general equilibrium models with IO structures at the sector level (such as CGE models and other multi-sector growth models). It sets up a set of accounting rules for these models to follow if GVC analysis needs to be conducted.

[6] It is a set of economic accounts that provide the framework for presenting detailed measures of U.S. output and income. https://www.bea.gov/resources/methodologies/nipa-handbook

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Figure 1 A simple snake type production chain

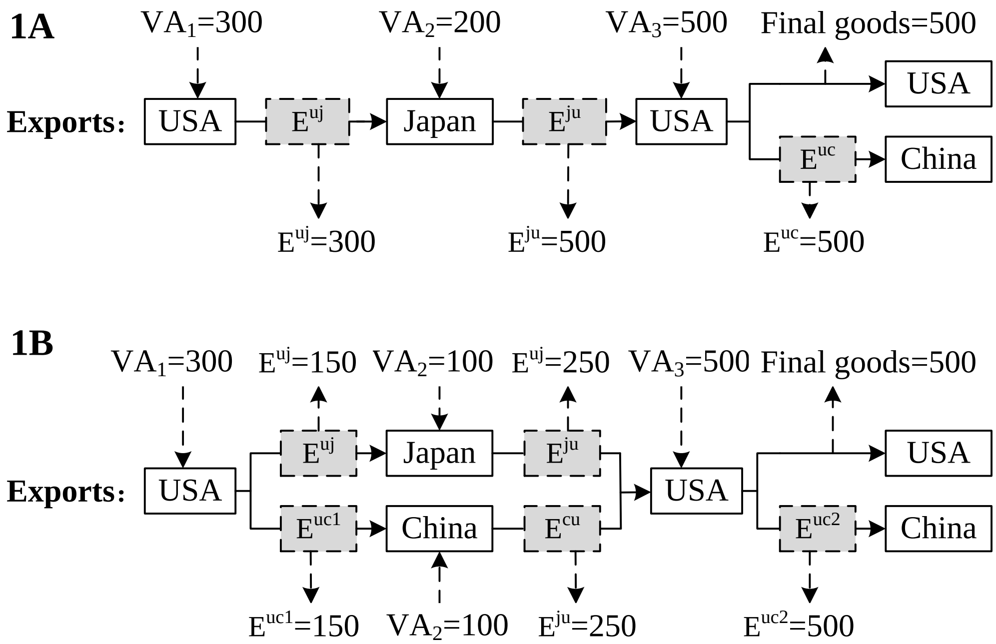


Figure 2 Decomposition of the Value of DC^{uc} and VAX_D^{uc} – An Intuitive Illustration

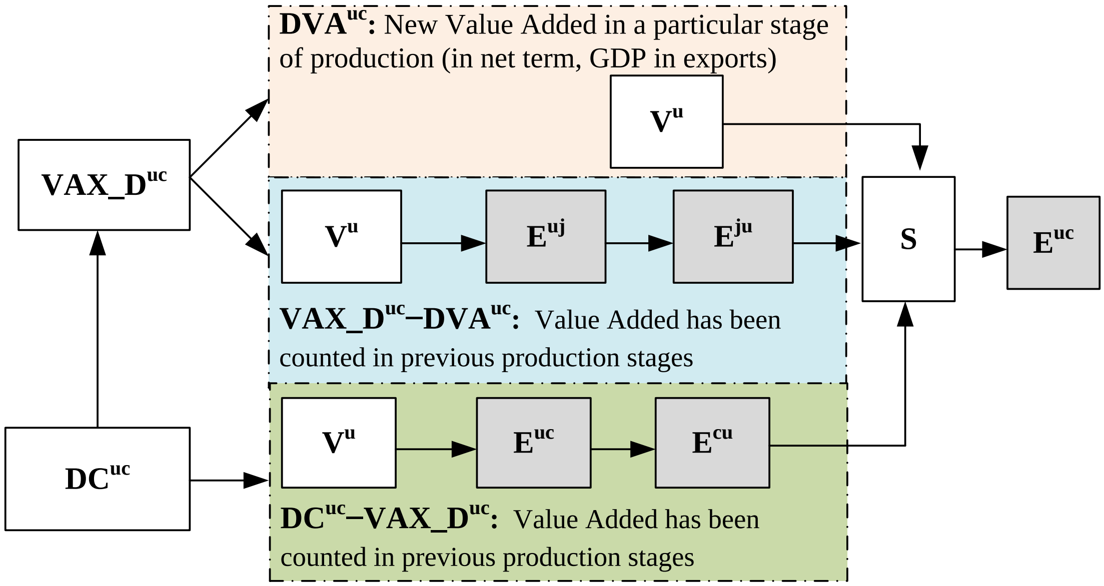


Table 1 Statistical Difference between KWW and HEM DVA measures, 2014

	Country-sector level		Bilateral-sector level	
	$\frac{\mathrm{VAX\_D}}{\mathrm{DVA}} - 1$	$\frac{\mathrm{VAX\_D}}{\mathrm{DC}} - 1$	$\frac{\mathrm{VAX\_D}}{\mathrm{DVA}} - 1$	$\frac{\mathrm{VAX\_D}}{\mathrm{DC}} - 1$
Observations	2464	2464	105952	105952
Exports > 0	2262	2262	95959	95959
Mean	0.28%	0.03%	0.27%	0.00%
Maximum	7.33%	2.06%	12.41%	1.04%
Minimum	0.00%	0.00%	0.00%	0.00%
Standard Error	0.52%	0.09%	0.57%	0.01%
CV	0.97%	0.32%	1.18%	0.12%
0-0.1%	983	2133	5618	95895
0.1%-0.2%	465	74	35849	39
0.2%-0.5%	524	41	40055	23
0.5-1%	173	9	8622	1
1-5%	114	5	5576	1
5-100%	3	0	239	0
Weighted mean	**0.80%**	**0.21%**	**0.98%**	**0.04%**

Data source: Authors computation from **https://www.rug.nl/ggdc/valuechain/wiod/wiod-2016-release**

Table 2 The share of DVA and VAX_D in Domestic Content (DC), Select country-sector and bilateral-sector pairs, 2014

Reporter	Partner	Sector	Gross exports Million USD	DVA (GDP only)	VAX_D (GDP and part of related double counting)	DC (GDP and all related double counting	T^{*}	T^{**}
Netherlands	**World**	Refined petroleum products	35,197	12,962	13,911	13,990	7.33%	0.57%
Germany	**World**	Refined petroleum products	34,013	9,402	10,078	10,125	7.19%	0.47%
Netherlands	**World**	Chemicals and chemical products	59,791	22,177	23,233	23,466	4.76%	1.00%
Netherlands	**World**	Air transport	10,290	5,379	5,627	5,632	4.61%	0.09%
Belgium	**World**	Refined petroleum products	41,998	6,985	7,291	7,339	4.39%	0.66%
China	Taiwan	computer, electronic and optical products	16,442	10,847	12,188	12,232	12.4%	0.36%
Netherlands	Germany	Refined petroleum products	14,581	5,335	5,665	5,673	6.2%	0.14%
China	Korea	computer, electronic and optical products	30,245	21,158	22,439	22,500	6.0%	0.27%
Netherlands	Germany	chemicals and chemical products	21,176	7,732	8,185	8,218	5.9%	0.40%
Belgium	Germany	Refined petroleum products	12,304	1,987	2,073	2,075	4.3%	0.10%
Germany	France	chemicals and chemical products	13,764	8,121	8,450	8,459	4.0%	0.11%
Germany	USA	chemicals and chemical products	11,960	7,148	7,349	7,350	2.8%	0.01%
USA	Mexico	motor vehicles	18,739	13,875	14,265	14,299	2.8%	0.24%
Germany	Italy	motor vehicles	11,062	7,394	7,591	7,593	2.6%	0.03%

Note: T^{*} is the difference between DVA and VAX_D measures computed as 100*VAX_D/DVA-1
T^{**} is the difference between DC and VAX_D measures computed as 100* DC / VAX_D -1.
Data source: Authors computation from **https://www.rug.nl/ggdc/valuechain/wiod/wiod-2016-release**

Table 3: Comparing Four Factor Content Measures: Differences and Implications

	Measures	Examples of Applications
Based on forward linkages	**VAX** Domestic value added in gross exports to satisfy foreign final demand	Impact of changes in foreign demand on the GDP of the exporting country. Example: the change in the final food demand of Chinese consumers on the US value-added embodied in its soybean exports to the world
Based on backward linkages	**VAX_D** Domestic factor content in gross exports that could include double counting.	The loss of an exporting country's domestic value-added when a particular bilateral/sectorial trade flow is eliminated. Examples： the short-term impact of an earthquake in Japan, the interruption of international logistics due to COVID-19, or the US ban on exports of advanced semiconductor chips to China.
	DVA Domestic value-added in gross export flows, exclude any double counting	Domestic value-added (GDP) created by the production of exports (at any level of disaggregation). Fully consistent with the concept of value-added defined by SNA. Can be used to measure the change in a country's GDP due to a change in the export flows at bilateral/sectoral level. Examples: the increase in US GDP due to an expansion of its agricultural exports to China, including US value-added embodied in cotton exports, used by Chinese apparel firms' exports to the US and other countries; as the accounting framework for models with IO structure for general equilibrium trade analysis
	DC Total value of factor contents in gross export flows	A gross measure of factor contents in exports flows, including some double counting. It can be used to decompose gross export flows at any level of disaggregation by value sources, and to measure the extent of vertical specialization of a particular country-sector pair.

Appendices

A. Proof that the sum along each column of the VB matrix is unity.

From equation (2), the direct value-added coefficient vector of country u can be measured as one minus the intermediate input share from all countries (including domestically produced intermediates). Extending this to the three-country world, a 1×3N direct value-added coefficient vector can be obtained:

$$V = (\mathrm{v^u} \quad \mathrm{v^c} \quad \mathrm{v^j}) = \mu \begin{bmatrix} I - \mathrm{a^{uu}} & -\mathrm{a^{uc}} & -\mathrm{a^{uj}} \\ -\mathrm{a^{cu}} & I - \mathrm{a^{cc}} & -\mathrm{a^{cj}} \\ -\mathrm{a^{ju}} & -\mathrm{a^{jc}} & I - \mathrm{a^{jj}} \end{bmatrix} \qquad \text{(A1)}$$

where μ is a 1×3N unity vector. Multiplying the global direct value-added coefficient vector with the global Leontief inverse B results in a unite vector μ.

$$VB = (\mathrm{v^u} \quad \mathrm{v^c} \quad \mathrm{v^j}) \begin{bmatrix} \mathrm{b^{uu}} & \mathrm{b^{uc}} & \mathrm{b^{uj}} \\ \mathrm{b^{cu}} & \mathrm{b^{cc}} & \mathrm{b^{cj}} \\ \mathrm{b^{ju}} & \mathrm{b^{jc}} & \mathrm{b^{jj}} \end{bmatrix}$$

(A2)

$$= \mu \begin{bmatrix} I - \mathrm{a^{uu}} & -\mathrm{a^{uc}} & -\mathrm{a^{uj}} \\ -\mathrm{a^{cu}} & I - \mathrm{a^{cc}} & -\mathrm{a^{cj}} \\ -\mathrm{a^{ju}} & -\mathrm{a^{jc}} & I - \mathrm{a^{jj}} \end{bmatrix} \begin{bmatrix} I - \mathrm{a^{uu}} & -\mathrm{a^{uc}} & -\mathrm{a^{uj}} \\ -\mathrm{a^{cu}} & I - \mathrm{a^{cc}} & -\mathrm{a^{cj}} \\ -\mathrm{a^{ju}} & -\mathrm{a^{jc}} & I - \mathrm{a^{jj}} \end{bmatrix}^{-1} = \mu$$

B. The derivation of equation (4) in the main text

In equation (3), the diagonals of $\hat{V}B$ denote the domestic sourced factor content share of domestically produced products in a particular sector at home. We use the diagonal of $\hat{V}B$ to define the DC measure for each country by multiplying the diagonal of $\hat{V}B$ and a 3N by 3 matrix of bilateral gross trade flows. This yields a 3N by 3 matrix of domestic content in bilateral gross trade as follows:

$$DC = \hat{V}B^D E = \begin{bmatrix} \hat{v}^{\mathrm{u}} b^{\mathrm{uu}} & 0 & 0 \\ 0 & \hat{v}^{\mathrm{c}} b^{\mathrm{cc}} & 0 \\ 0 & 0 & \hat{v}^{\mathrm{j}} b^{\mathrm{jj}} \end{bmatrix} \begin{bmatrix} 0 & \mathrm{e^{uc}} & e^{\mathrm{uj}} \\ \mathrm{e^{cu}} & 0 & \mathrm{e^{cj}} \\ \mathrm{e^{ju}} & \mathrm{e^{jc}} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \hat{v}^{\mathrm{u}} b^{\mathrm{uu}} \mathrm{e^{uc}} & \hat{v}^{\mathrm{u}} b^{\mathrm{uu}} e^{\mathrm{uj}} \\ \hat{v}^{\mathrm{c}} b^{\mathrm{cc}} \mathrm{e^{cu}} & 0 & \hat{v}^{\mathrm{c}} b^{\mathrm{cc}} \mathrm{e^{cj}} \\ \hat{v}^{\mathrm{j}} b^{\mathrm{jj}} \mathrm{e^{ju}} & \hat{v}^{\mathrm{j}} b^{\mathrm{jj}} \mathrm{e^{jc}} & 0 \end{bmatrix} \qquad \text{(B1)}$$

where $B^D = \begin{bmatrix} b^{\mathrm{uu}} & 0 & 0 \\ 0 & b^{\mathrm{cc}} & 0 \\ 0 & 0 & b^{\mathrm{jj}} \end{bmatrix}$ is a diagonal block matrix and part of the global Leontief inverse. Each block in equation (B1) represents domestic factor content embodied in a particular bilateral export flow. It is consistent with equation (41) in Koopman, Wang and Wei (KWW, 2014). Further decomposing (B1), we obtain:

$$DC = \hat{V}B^D E = \hat{V}LE + \hat{V}(B^D - L)E = \begin{bmatrix} 0 & \hat{v}^{\mathrm{u}} l^{\mathrm{uu}} \mathrm{e}^{\mathrm{uc}} & \hat{v}^{\mathrm{u}} l^{\mathrm{uu}} e^{\mathrm{uj}} \\ \hat{v}^{\mathrm{c}} l^{\mathrm{cc}} \mathrm{e}^{\mathrm{cu}} & 0 & \hat{v}^{\mathrm{c}} l^{\mathrm{cc}} \mathrm{e}^{\mathrm{cj}} \\ \hat{v}^{\mathrm{j}} l^{\mathrm{jj}} \mathrm{e}^{\mathrm{ju}} & \hat{v}^{\mathrm{j}} l^{\mathrm{jj}} \mathrm{e}^{\mathrm{jc}} & 0 \end{bmatrix} \tag{B2}$$

$$+ \begin{bmatrix} 0 & \hat{v}^{\mathrm{u}}(b^{uu} - l^{uu})\mathrm{e}^{\mathrm{uc}} & \hat{v}^{\mathrm{u}}(b^{uu} - l^{uu}) e^{\mathrm{uj}} \\ \hat{v}^{\mathrm{c}}(b^{cc} - l^{cc})\mathrm{e}^{\mathrm{cu}} & 0 & \hat{v}^{\mathrm{c}}(b^{cc} - l^{cc})\mathrm{e}^{\mathrm{cj}} \\ \hat{v}^{\mathrm{j}}(b^{jj} - l^{jj})\mathrm{e}^{\mathrm{ju}} & \hat{v}^{\mathrm{j}}(b^{jj} - l^{jj})\mathrm{e}^{\mathrm{jc}} & 0 \end{bmatrix}$$

where $L = \begin{bmatrix} I - \mathrm{a}^{\mathrm{uu}} & 0 & 0 \\ 0 & I - \mathrm{a}^{\mathrm{cc}} & 0 \\ 0 & 0 & I - \mathrm{a}^{\mathrm{jj}} \end{bmatrix}^{-1} = \begin{bmatrix} l^{\mathrm{uu}} & 0 & 0 \\ 0 & l^{\mathrm{cc}} & 0 \\ 0 & 0 & l^{\mathrm{jj}} \end{bmatrix}$ is the domestic Leontief Inverse, a 3N by 3N matrix. Its first term $\hat{V}LE$ measures domestic value-added generated from the production of bilateral gross exports, which is a generalization of equation (37) of KWW(2014) to bilateral sector level.

Based on the definition of Leontief Inverse, we have：

$$B - L = B[I - (I - A^D - A^F)(I - A^D)^{-1}] = BA^F(I - A^D)^{-1} \tag{B3}$$

Where $\mathrm{A}^{\mathrm{D}} = \begin{bmatrix} \mathrm{a}^{\mathrm{uu}} & 0 & 0 \\ 0 & \mathrm{a}^{\mathrm{cc}} & 0 \\ 0 & 0 & \mathrm{a}^{\mathrm{jj}} \end{bmatrix}$, $\mathrm{A}^{\mathrm{F}} = \begin{bmatrix} 0 & \mathrm{a}^{\mathrm{uc}} & \mathrm{a}^{\mathrm{uj}} \\ \mathrm{a}^{\mathrm{cu}} & 0 & \mathrm{a}^{\mathrm{cj}} \\ \mathrm{a}^{\mathrm{ju}} & \mathrm{a}^{\mathrm{jc}} & 0 \end{bmatrix}$. Expand (B3), we have:

$$B - L = \begin{bmatrix} \sum_{r \neq u} b^{ur} a^{ru} l^{uu} & \sum_{r \neq u} b^{ur} a^{rc} l^{cc} & \sum_{r \neq u} b^{ur} a^{rj} l^{jj} \\ \sum_{r \neq c} b^{cr} a^{ru} l^{uu} & \sum_{r \neq c} b^{cr} a^{rc} l^{cc} & \sum_{r \neq c} b^{cr} a^{rj} l^{jj} \\ \sum_{r \neq j} b^{jr} a^{ru} l^{uu} & \sum_{r \neq j} b^{jr} a^{rc} l^{cc} & \sum_{r \neq j} b^{jr} a^{rj} l^{jj} \end{bmatrix} \tag{B4}$$

Because $B^D - L$ is the diagonals of $B - L$ matrix， the second term in (B2) can be written as：

$$\hat{V}(B^D - L)E = \begin{bmatrix} 0 & \hat{v}^{\mathrm{u}}(b^{uu} - l^{uu})\mathrm{e}^{\mathrm{uc}} & \hat{v}^{\mathrm{u}}(b^{uu} - l^{uu}) e^{\mathrm{uj}} \\ \hat{v}^{\mathrm{c}}(b^{cc} - l^{cc})\mathrm{e}^{\mathrm{cu}} & 0 & \hat{v}^{\mathrm{c}}(b^{cc} - l^{cc})\mathrm{e}^{\mathrm{cj}} \\ \hat{v}^{\mathrm{j}}\left(b^{jj} - l^{jj}\right)\mathrm{e}^{\mathrm{ju}} & \hat{v}^{\mathrm{j}}\left(b^{jj} - l^{jj}\right)\mathrm{e}^{\mathrm{jc}} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \hat{v}^{\mathrm{u}} \sum_{r \neq u} b^{ur} a^{ru} l^{uu} \mathrm{e}^{\mathrm{uc}} & \hat{v}^{\mathrm{u}} \sum_{r \neq u} b^{ur} a^{ru} l^{uu} e^{\mathrm{uj}} \\ \hat{v}^{\mathrm{c}} \sum_{r \neq c} b^{cr} a^{rc} l^{cc} \mathrm{e}^{\mathrm{cu}} & 0 & \hat{v}^{\mathrm{c}} \sum_{r \neq c} b^{cr} a^{rc} l^{cc} \mathrm{e}^{\mathrm{cj}} \\ \hat{v}^{\mathrm{j}} \sum_{r \neq j} b^{jr} a^{rj} l^{jj} \mathrm{e}^{\mathrm{ju}} & \hat{v}^{\mathrm{j}} \sum_{r \neq j} b^{jr} a^{rj} l^{jj} \mathrm{e}^{\mathrm{jc}} & 0 \end{bmatrix} \tag{B5}$$

It equals the domestically sourced double counting (DDC) defined in KWW(2014), the sixth terms in their equation (36). Therefore, the domestic factor content measure DC can be further decomposed into DVA and DDC as follows, which is equation (4) in the main text:

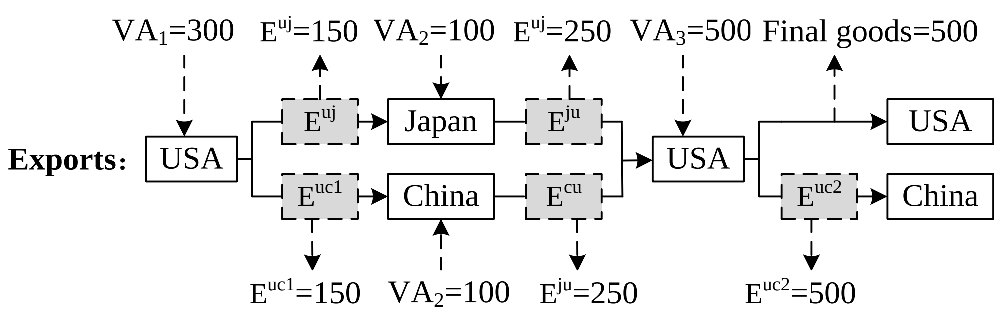
$$DC = \hat{V}B^{D}E = \begin{bmatrix} \hat{v}^{\mathrm{u}}b^{\mathrm{uu}} & 0 & 0 \\ 0 & \hat{v}^{\mathrm{c}}b^{\mathrm{cc}} & 0 \\ 0 & 0 & \hat{v}^{\mathrm{j}}b^{\mathrm{jj}} \end{bmatrix} \begin{bmatrix} 0 & \mathrm{e}^{\mathrm{uc}} & e^{\mathrm{uj}} \\ \mathrm{e}^{\mathrm{cu}} & 0 & \mathrm{e}^{\mathrm{cj}} \\ \mathrm{e}^{\mathrm{ju}} & \mathrm{e}^{\mathrm{jc}} & 0 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 0 & \hat{v}^{\mathrm{u}}l^{\mathrm{uu}}\mathrm{e}^{\mathrm{uc}} & \hat{v}^{\mathrm{u}}l^{\mathrm{uu}}e^{\mathrm{uj}} \\ \hat{v}^{\mathrm{c}}l^{\mathrm{cc}}\mathrm{e}^{\mathrm{cu}} & 0 & \hat{v}^{\mathrm{c}}l^{\mathrm{cc}}\mathrm{e}^{\mathrm{cj}} \\ \hat{v}^{\mathrm{j}}l^{\mathrm{jj}}\mathrm{e}^{\mathrm{ju}} & \hat{v}^{\mathrm{j}}l^{\mathrm{jj}}\mathrm{e}^{\mathrm{jc}} & 0 \end{bmatrix}}_{DVA}$$

$$+ \underbrace{\begin{bmatrix} 0 & \hat{v}^{\mathrm{u}}(b^{uu}-l^{uu})\mathrm{e}^{\mathrm{uc}} & \hat{v}^{\mathrm{u}}(b^{uu}-l^{uu})e^{\mathrm{uj}} \\ \hat{v}^{\mathrm{c}}(b^{cc}-l^{cc})\mathrm{e}^{\mathrm{cu}} & 0 & \hat{v}^{\mathrm{c}}(b^{cc}-l^{cc})\mathrm{e}^{\mathrm{cj}} \\ \hat{v}^{\mathrm{j}}(b^{jj}-l^{jj})\mathrm{e}^{\mathrm{ju}} & \hat{v}^{\mathrm{j}}(b^{jj}-l^{jj})\mathrm{e}^{\mathrm{jc}} & 0 \end{bmatrix}}_{DDC} = DVA + DDC$$

C. The second example and its ICIO table representation

Assume a production chains in the global production network:



It can be expressed as an ICIO table as follows:

Table C. A simple ICIO table

Inputs \ Outputs		Intermediate Uses				Final Uses			Total Outputs
		U1	U2	C	J	U	C	J	
Intermediate Inputs	U1	0	0	150	150	0	0	0	300
	U2	0	0	0	0	500	500	0	1000
	C	0	250	0	0	0	0	0	250
	J	0	250	0	0	0	0	0	250
Value Added		300	500	100	100				
Total Inputs		300	1000	250	250				

Measuring the input coefficient matrix $\mathrm{A} = \begin{bmatrix} 0 & 0 & 0.6 & 0.6 \\ 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \end{bmatrix}$, the value added coefficient vector $\mathrm{V} = (1 \quad 0.5 \quad 0.4 \quad 0.4)$, the exports $\mathrm{E} = \begin{bmatrix} 0 & 150 & 150 \\ 0 & 500 & 0 \\ 250 & 0 & 0 \\ 250 & 0 & 0 \end{bmatrix}$, final goods $\mathrm{Y} = \begin{bmatrix} 0 \\ 1000 \\ 0 \\ 0 \end{bmatrix}$.

Based on the input coefficient matrix, the Leontief Inverse can be computed as $\mathrm{B} = \begin{bmatrix} 1 & 0.3 & 0.6 & 0.6 \\ 0 & 1 & 0 & 0 \\ 0 & 0.25 & 1 & 0 \\ 0 & 0.25 & 0 & 1 \end{bmatrix}$, where the total requirement coefficient matrix for domestic demand is $\mathrm{B}^{D} = \begin{bmatrix} 1 & 0.3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, the Local Leontief Inverse $\mathrm{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Based on the WWZ method, the DVA in exports can be measured as

$$DVA = \widehat{\mathrm{V}}\mathrm{LE} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0.4 \end{bmatrix} \begin{bmatrix} 0 & 150 & 150 \\ 0 & 500 & 0 \\ 250 & 0 & 0 \\ 250 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 150 & 150 \\ 0 & 250 & 0 \\ 100 & 0 & 0 \\ 100 & 0 & 0 \end{bmatrix}$$

And the domestic content in exports can be measured as

$$DC = \widehat{\mathrm{V}\mathrm{B}^{\mathrm{D}}}\mathrm{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0.4 \end{bmatrix}\begin{bmatrix} 0 & 150 & 150 \\ 0 & 500 & 0 \\ 250 & 0 & 0 \\ 250 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 150 & 150 \\ 0 & 400 & 0 \\ 100 & 0 & 0 \\ 100 & 0 & 0 \end{bmatrix}$$

Based on the HEM method, the uc1's exports can be extracted as

$$\mathrm{A} - \mathrm{A}^{\mathrm{uc1}} = \begin{bmatrix} 0 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \end{bmatrix}, \mathrm{B}^{\mathrm{uc1}} = (\mathrm{I} - \mathrm{A} + \mathrm{A}^{\mathrm{uc1}})^{-1} = \begin{bmatrix} 1 & 0.15 & 0 & 0.6 \\ 0 & 1 & 0 & 0 \\ 0 & 0.25 & 1 & 0 \\ 0 & 0.25 & 0 & 1 \end{bmatrix},$$

$$\mathrm{Y}^{\mathrm{uc1}} = \mathrm{Y}$$

Therefore, the domestic value added in uc1's exports can be computed as:

$$\mathrm{V}^{\mathrm{u}}\mathrm{B}\mathrm{Y} - \mathrm{V}^{\mathrm{u}}\mathrm{B}^{\mathrm{uc1}}\mathrm{Y}^{\mathrm{uc1}} = \mathrm{V}^{\mathrm{u}}(\mathrm{B}\mathrm{Y} - \mathrm{B}^{\mathrm{uc1}}\mathrm{Y}^{\mathrm{uc1}})$$

$$= \begin{pmatrix} 1 & 0.5 & 0 & 0 \end{pmatrix}\left\{\begin{bmatrix} 1 & 0.3 & 0.6 & 0.6 \\ 0 & 1 & 0 & 0 \\ 0 & 0.25 & 1 & 0 \\ 0 & 0.25 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1000 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 0.15 & 0 & 0.6 \\ 0 & 1 & 0 & 0 \\ 0 & 0.25 & 1 & 0 \\ 0 & 0.25 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1000 \\ 0 \\ 0 \end{bmatrix}\right\}$$

$$= 150$$

The uc2's exports can be extracted as:

$$\mathrm{A} - \mathrm{A}^{\mathrm{uc2}} = \mathrm{A}\,, \mathrm{B}^{\mathrm{uc2}} = \mathrm{B}, \mathrm{Y}^{\mathrm{uc2}} = \begin{bmatrix} 0 \\ 500 \\ 0 \\ 0 \end{bmatrix}$$

The domestic value added in uc2's exports can be computed as:

$$\mathrm{V}^{\mathrm{u}}\mathrm{B}\mathrm{Y} - \mathrm{V}^{\mathrm{u}}\mathrm{B}^{\mathrm{uc2}}\mathrm{Y}^{\mathrm{uc2}} = \mathrm{V}^{\mathrm{u}}\mathrm{B}(\mathrm{Y} - \mathrm{Y}^{\mathrm{uc1}})$$

$$= \begin{pmatrix} 1 & 0.5 & 0 & 0 \end{pmatrix}\begin{bmatrix} 1 & 0.3 & 0.6 & 0.6 \\ 0 & 1 & 0 & 0 \\ 0 & 0.25 & 1 & 0 \\ 0 & 0.25 & 0 & 1 \end{bmatrix}\left\{\begin{bmatrix} 0 \\ 1000 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 500 \\ 0 \\ 0 \end{bmatrix}\right\}$$

$$= 400$$

The uc's exports can be extracted as:

$$\mathrm{A} - \mathrm{A}^{\mathrm{uc}} = \begin{bmatrix} 0 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \end{bmatrix}, \mathrm{B}^{\mathrm{uc}} = \begin{bmatrix} 1 & 0.15 & 0 & 0.6 \\ 0 & 1 & 0 & 0 \\ 0 & 0.25 & 1 & 0 \\ 0 & 0.25 & 0 & 1 \end{bmatrix}, \mathrm{Y}^{\mathrm{uc}} = \begin{bmatrix} 0 \\ 500 \\ 0 \\ 0 \end{bmatrix}$$

The domestic value added in uc2's exports can be computed as:

$$\mathrm{V^uBY - V^uB^{uc}Y^{uc} = V^u(BY - B^{uc}Y^{uc})}$$

$$= (1 \quad 0.5 \quad 0 \quad 0)\left\{\begin{bmatrix}1 & 0.3 & 0.6 & 0.6\\ 0 & 1 & 0 & 0\\ 0 & 0.25 & 1 & 0\\ 0 & 0.25 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\ 1000\\ 0\\ 0\end{bmatrix} - \begin{bmatrix}1 & 0.15 & 0 & 0.6\\ 0 & 1 & 0 & 0\\ 0 & 0.25 & 1 & 0\\ 0 & 0.25 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\ 500\\ 0\\ 0\end{bmatrix}\right\}$$

$$= 475$$

Therefore, we can get:

$$\mathrm{E^{uc1} = DVA^{uc1} = VAX\_D^{uc1} = DC^{uc1}} = 150$$

$$\mathrm{E^{uc2}} = 500,\ \mathrm{DVA^{uc2}} = 250,\ \mathrm{VAX\_D^{uc2}} = 400,\ \mathrm{DC^{uc2}} = 400$$

$$\mathrm{E^{uc}} = 650,\ \mathrm{DVA^{uc}} = 400,\ \mathrm{VAX\_D^{uc}} = 475,\ \mathrm{DC^{uc}} = 550$$

D. The derivation of equations (7) to (9) in the main text

Suppose there are three countries (USA, China, and Japan) in the world. Define the input coefficient matrix as $\mathrm{A} = \begin{bmatrix}a^{uu} & a^{uc} & a^{uj}\\ a^{cu} & a^{cc} & a^{cj}\\ a^{ju} & a^{jc} & a^{jj}\end{bmatrix}$, $\mathrm{A^{uc}} = \begin{bmatrix}0 & a^{uc} & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$, $\mathrm{A^D} = \begin{bmatrix}a^{uu} & 0 & 0\\ 0 & a^{cc} & 0\\ 0 & 0 & a^{jj}\end{bmatrix}$, $\mathrm{A^F} = \begin{bmatrix}0 & a^{uc} & a^{uj}\\ a^{cu} & 0 & a^{cj}\\ a^{ju} & a^{jc} & 0\end{bmatrix}$ and the final goods matrix as $\mathrm{Y} = \begin{bmatrix}y^{uu} + y^{uc} + y^{uj}\\ y^{cu} + y^{cc} + y^{cj}\\ y^{ju} + y^{jc} + y^{jj}\end{bmatrix}$, $\mathrm{Y^{uc}} = \begin{bmatrix}y^{uc}\\ 0\\ 0\end{bmatrix}$. The bilateral exports matrix as $\mathrm{E^{uc}} = \begin{bmatrix}e^{uc}\\ 0\\ 0\end{bmatrix} = \mathrm{A^{uc}X + Y^{uc}} = \begin{bmatrix}a^{uc}x^{c} + y^{uc}\\ 0\\ 0\end{bmatrix}$. The value added coefficient of USA as $\mathrm{V^u} = [\mathrm{v^u} \quad 0 \quad 0]$.

From the Leontief model, we define the local (or domestic) Leontief inverse, $\mathrm{L} = (\mathrm{I - A + A^F})^{-1} = \begin{bmatrix}(\mathrm{I} - a^{uu})^{-1} & 0 & 0\\ 0 & (\mathrm{I} - a^{cc})^{-1} & 0\\ 0 & 0 & (\mathrm{I} - a^{jj})^{-1}\end{bmatrix} = \begin{bmatrix}l^{uu} & 0 & 0\\ 0 & l^{cc} & 0\\ 0 & 0 & l^{jj}\end{bmatrix}$, the HEM Leontief inverse $\mathrm{B^{uc\prime}} = (\mathrm{I - A + A^{uc}})^{-1} = \begin{bmatrix}1 - a^{uu} & 0 & -a^{uj}\\ -a^{cu} & 1 - a^{cc} & -a^{cj}\\ -a^{ju} & -a^{jc} & 1 - a^{jj}\end{bmatrix}^{-1} = \begin{bmatrix}b^{uu\prime} & b^{uc\prime} & b^{uj\prime}\\ b^{cu\prime} & b^{cc\prime} & b^{cj\prime}\\ b^{ju\prime} & b^{jc\prime} & b^{jj\prime}\end{bmatrix}$, which hypothetical extracted e^{uc}, and global Leontief inverse $B = (I - A)^{-1} = \begin{bmatrix} b^{uu} & b^{uc} & b^{uj} \\ b^{cu} & b^{cc} & b^{cj} \\ b^{ju} & b^{jc} & b^{jj} \end{bmatrix}$. The HEM's measure of DVA in e^{uc} is given by following equation:

$$VAX\_D^{uc} = V^u(I - A)^{-1}Y - V^u(I - A + A^{uc})^{-1}(Y - Y^{uc})$$

$$= V^u(I - A + A^{uc})^{-1}[(I - A + A^{uc})(I - A)^{-1}Y - (Y - Y^{uc})]$$

$$= V^u(I - A + A^{uc})^{-1}[A^{uc}(I - A)^{-1}Y + Y^{uc}] \qquad \text{(D1)}$$

$$= V^u(I - A + A^{uc})^{-1}[A^{uc}X + Y^{uc}]$$

$$= V^u(I - A + A^{uc})^{-1}E^{uc} = V^u B^{uc\prime} E^{uc}$$

$$= [v^u \quad 0 \quad 0] \begin{bmatrix} b^{uu\prime} & b^{uc\prime} & b^{uj\prime} \\ b^{cu\prime} & b^{cc\prime} & b^{cj\prime} \\ b^{ju\prime} & b^{jc\prime} & b^{jj\prime} \end{bmatrix} \begin{bmatrix} e^{uc} \\ 0 \\ 0 \end{bmatrix} = v^u b^{uu\prime} e^{uc}$$

Where $E^{uc} = A^{uc}X + Y^{uc} = A^{uc}(I - A)^{-1}Y + Y^{uc}$.

Based on equation (5) in the main text our measure of DVA in e^{uc} equals

$$DVA^{uc} = V^u(I - A^D)^{-1}E^{uc} = V^u L E^{uc}$$

$$= [v^u \quad 0 \quad 0] \begin{bmatrix} l^{uu} & 0 & 0 \\ 0 & l^{cc} & 0 \\ 0 & 0 & l^{jj} \end{bmatrix} \begin{bmatrix} e^{uc} \\ 0 \\ 0 \end{bmatrix} = v^u l^{uu} e^{uc} \qquad \text{(D2)}$$

The difference of VAX_D^{uc} and DVA^{uc} is：

$$VAX\_D^{uc} - DVA^{uc} = V^u(I - A + A^{uc})^{-1}E^{uc} - V^u(I - A^D)^{-1}E^{uc}$$

$$= V^u(I - A + A^{uc})^{-1}E^{uc} - V^u(I - A + A^F)^{-1}E^{uc}$$

$$= V^u(I - A + A^F)^{-1}[(I - A + A^F) - (I - A + A^{uc})](I - A + A^{uc})^{-1}E^{uc}$$

$$= V^u(I - A + A^F)^{-1}(A^F - A^{uc})(I - A + A^{uc})^{-1}E^{uc} \qquad \text{(D3)}$$

$$= [v^u \quad 0 \quad 0] \begin{bmatrix} l^{uu} & 0 & 0 \\ 0 & l^{cc} & 0 \\ 0 & 0 & l^{jj} \end{bmatrix} \begin{bmatrix} 0 & 0 & a^{uj} \\ a^{cu} & 0 & a^{cj} \\ a^{ju} & a^{jc} & 0 \end{bmatrix} \begin{bmatrix} b^{uu\prime} & b^{uc\prime} & b^{uj\prime} \\ b^{cu\prime} & b^{cc\prime} & b^{cj\prime} \\ b^{ju\prime} & b^{jc\prime} & b^{jj\prime} \end{bmatrix} \begin{bmatrix} e^{uc} \\ 0 \\ 0 \end{bmatrix}$$

$= \mathrm{v}^{\mathrm{u}} l^{\mathrm{uu}} a^{\mathrm{uj}} \mathrm{b}^{\mathrm{ju}\prime} \mathrm{e}^{\mathrm{uc}}$

Therefore， $\mathrm{VAX\_D}^{\mathrm{uc}}$ can be decomposed into two parts:

$$\mathrm{VAX\_D}^{\mathrm{uc}} = \mathrm{DVA}^{\mathrm{uc}} + (\mathrm{VAX\_D}^{\mathrm{uc}} - \mathrm{DVA}^{\mathrm{uc}}) = \mathrm{v}^{\mathrm{u}} l^{\mathrm{uu}} \mathrm{e}^{\mathrm{uc}} + \mathrm{v}^{\mathrm{u}} l^{\mathrm{uu}} a^{\mathrm{uj}} \mathrm{b}^{\mathrm{ju}\prime} \mathrm{e}^{\mathrm{uc}} \quad \text{(D4)}$$

Based on equation (4) in the main text, our measure of DC for e^{uc}can be written as:

$$\mathrm{DC}^{\mathrm{uc}} = \mathrm{V}^{\mathrm{u}} B^{D} \mathrm{E}^{\mathrm{uc}} = [\mathrm{v}^{\mathrm{u}} \quad 0 \quad 0] \begin{bmatrix} \mathrm{b}^{\mathrm{uu}} & 0 & 0 \\ 0 & \mathrm{b}^{\mathrm{cc}} & 0 \\ 0 & 0 & \mathrm{b}^{\mathrm{jj}} \end{bmatrix} \begin{bmatrix} \mathrm{e}^{\mathrm{uc}} \\ 0 \\ 0 \end{bmatrix} = \mathrm{v}^{\mathrm{u}} \mathrm{b}^{\mathrm{uu}} \mathrm{e}^{\mathrm{uc}} \quad \text{(D5)}$$

The difference of $\mathrm{VAX\_D}^{\mathrm{uc}}$ and $\mathrm{DC}^{\mathrm{uc}}$ equals：

$$\mathrm{DC}^{\mathrm{uc}} - \mathrm{VAX_D}^{\mathrm{uc}} = \mathrm{V}^{\mathrm{u}} (\mathrm{I} - \mathrm{A})^{-1} \mathrm{E}^{\mathrm{uc}} - \mathrm{V}^{\mathrm{u}} (\mathrm{I} - \mathrm{A} + \mathrm{A}^{\mathrm{uc}})^{-1} \mathrm{E}^{\mathrm{uc}}$$

$$= \mathrm{V}^{\mathrm{u}} (\mathrm{I} - \mathrm{A} + \mathrm{A}^{\mathrm{sr}})^{-1} [(\mathrm{I} - \mathrm{A} + \mathrm{A}^{\mathrm{uc}}) - (\mathrm{I} - \mathrm{A})] (\mathrm{I} - \mathrm{A})^{-1} \mathrm{E}^{\mathrm{uc}}$$

$$= \mathrm{V}^{\mathrm{u}} (\mathrm{I} - \mathrm{A} + \mathrm{A}^{\mathrm{uc}})^{-1} \mathrm{A}^{\mathrm{uc}} (\mathrm{I} - \mathrm{A})^{-1} \mathrm{E}^{\mathrm{uc}} \quad \text{(D6)}$$

$$= [\mathrm{v}^{\mathrm{u}} \quad 0 \quad 0] \begin{bmatrix} \mathrm{b}^{\mathrm{uu}\prime} & \mathrm{b}^{\mathrm{uc}\prime} & \mathrm{b}^{\mathrm{uj}\prime} \\ \mathrm{b}^{\mathrm{cu}\prime} & \mathrm{b}^{\mathrm{cc}\prime} & \mathrm{b}^{\mathrm{cj}\prime} \\ \mathrm{b}^{\mathrm{ju}\prime} & \mathrm{b}^{\mathrm{jc}\prime} & \mathrm{b}^{\mathrm{jj}\prime} \end{bmatrix} \begin{bmatrix} 0 & \mathrm{a}^{\mathrm{uc}} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{b}^{\mathrm{uu}} & \mathrm{b}^{\mathrm{uc}} & \mathrm{b}^{\mathrm{uj}} \\ \mathrm{b}^{\mathrm{cu}} & \mathrm{b}^{\mathrm{cc}} & \mathrm{b}^{\mathrm{cj}} \\ \mathrm{b}^{\mathrm{ju}} & \mathrm{b}^{\mathrm{jc}} & \mathrm{b}^{\mathrm{jj}} \end{bmatrix} \begin{bmatrix} \mathrm{e}^{\mathrm{uc}} \\ 0 \\ 0 \end{bmatrix}$$

$$= \mathrm{v}^{\mathrm{u}} \mathrm{b}^{\mathrm{uu}\prime} \mathrm{a}^{\mathrm{uc}} \mathrm{b}^{\mathrm{cu}} \mathrm{e}^{\mathrm{uc}}$$

Therefore，we can further decompose $\mathrm{DC}^{\mathrm{uc}}$into the following three terms：

$$\mathrm{DC}^{\mathrm{uc}} = \mathrm{DVA}^{\mathrm{uc}} + (\mathrm{VAX\_D}^{\mathrm{uc}} - \mathrm{DVA}^{\mathrm{uc}}) + (\mathrm{DC}^{\mathrm{uc}} - \mathrm{VAX\_D}^{\mathrm{uc}})$$

$$= \underbrace{\underbrace{\underbrace{\mathrm{v}^{\mathrm{u}} l^{\mathrm{uu}} \mathrm{e}^{\mathrm{uc}}}_{\mathrm{DVA}^{\mathrm{uc}}} + \mathrm{v}^{\mathrm{u}} l^{\mathrm{uu}} \mathrm{a}^{\mathrm{uj}} \mathrm{b}^{\mathrm{ju}\prime} \mathrm{e}^{\mathrm{uc}}}_{\mathrm{VAX\_D}^{\mathrm{uc}}} + \mathrm{v}^{\mathrm{u}} \mathrm{b}^{\mathrm{uu}\prime} \mathrm{a}^{\mathrm{uc}} \mathrm{b}^{\mathrm{cu}} \mathrm{e}^{\mathrm{uc}}}_{\mathrm{DC}^{\mathrm{uc}}} \quad \text{(D7)}$$

E. The derivation of equation (11) in the main text

The difference between HEM and WWZ method in computing DVA in exports is a different treatment of the returned domestic value-added (RDV) via intermediate imports.

$$\mathrm{VAX\_D}^{\mathrm{uc}} - \mathrm{DVA}^{\mathrm{uc}} = \mathrm{V}^{\mathrm{u}} (\mathrm{I} - \mathrm{A} + \mathrm{A}^{\mathrm{uc}})^{-1} \mathrm{E}^{\mathrm{uc}} - \mathrm{V}^{\mathrm{u}} (\mathrm{I} - \mathrm{A}^{\mathrm{D}})^{-1} \mathrm{E}^{\mathrm{uc}}$$

$$= V^{u}(I - A + A^{uc})^{-1}E^{uc} - V^{u}(I - A + A^{F})^{-1}E^{uc}$$

$$= V^{u}(I - A + A^{F})^{-1}[(I - A + A^{F}) - (I - A + A^{uc})](I - A + A^{uc})^{-1}E^{uc} \qquad \text{(E1)}$$

$$= V^{u}(I - A^{D})^{-1}(A^{F} - A^{uc})(I - A + A^{uc})^{-1}E^{uc}$$

According to KWW(2014), DDC is the difference between DC^{uc} and DVA^{uc}.

$$DDC^{uc} = DC^{uc} - DVA^{uc} = V^{u}(I - A)^{-1}E^{uc} - V^{u}(I - A^{D})^{-1}E^{uc}$$

$$= V^{u}(I - A)^{-1}E^{uc} - V^{u}(I - A + A^{F})^{-1}E^{uc}$$

$$= V^{u}(I - A + A^{F})[(I - A + A^{F}) - (I - A)](I - A)^{-1}E^{uc} \qquad \text{(E2)}$$

$$= V^{u}(I - A^{D})A^{F}(I - A)^{-1}E^{uc}$$

From the definition of A^{F}, we know that A^{uc} is a part of A^{F}, and $A^{F} - A^{uc}$ is the remaining terms in A^{F}. Similarly， $A^{uc}$is also a part of the IO coefficient matrix A and $A - A^{uc}$ is the remining terms of A. Therefore,

$$VAX\_D^{uc} - DVA^{uc} = V^{u}(I - A^{D})^{-1}(A^{F} - A^{uc})(I - A + A^{uc})^{-1}E^{uc}$$

$$\in V^{u}(I - A^{D})^{-1}A^{F}(I - A)^{-1}E^{uc} = DDC^{uc} \qquad \text{(E3)}$$

Where $A^{F} - A^{uc} \in A^{F}$, $(I - A + A^{uc})^{-1} \in (I - A)^{-1}$.

Relative to global IO coefficient matrix A， and inter-country IO coefficient matrix A^{F}， The IO coefficient matrix for a particular trade route is only a very small portion of them. In practice， $A - A^{uc}$ or $A^{F} - A^{uc}$ should very be close to A和A^{F} except for some special circumstances. This makes the difference small between VAX_D^{uc} and DVA^{uc}（$V^{u}(I - A^{D})^{-1}(A^{F} - A^{uc})(I - A + A^{uc})^{-1}E^{uc}$）and the difference between $DC^{uc}$and DVA^{uc}（$DDC^{uc} = V^{u}(I - A^{D})^{-1}A^{F}(I - A)^{-1}E^{uc}$）， i.e VAX_D^{uc} is a good approximation of DC^{uc}。

We can further analyze the difference between VAX_D^{uc} and DC^{uc} as follows:

$$DC^{uc} - {VAX_{D}}^{uc} = V^{u}(I - A)^{-1}E^{uc} - V^{u}(I - A + A^{uc})^{-1}E^{uc}$$

$$= V^{u}(I - A + A^{sr})^{-1}[(I - A + A^{uc}) - (I - A)](I - A)^{-1}E^{uc} \qquad \text{(E4)}$$

$$= V^u(I - A + A^{uc})^{-1}A^{uc}(I - A)^{-1}E^{uc}$$

Equation (E4) indicates that the difference between VAX_D^{uc} and DC^{uc} is the DVA of a country returns to the country through a specific bilateral trade route, it is usually a small portion relative to the total domestic factor content (DC) computed through the country's domestic and inter-country industrial linkage.

F. The difference between VAX_D and DVA at the country-sector and bilateral-sector level

Extending the HEM method to county-sector level, the VAX_D can be measured as follow.

$$VAX\_D^{ui} = V^u(I - A)^{-1}Y - V^u(I - A + A^{ui})^{-1}(Y - Y^{ui})$$

$$= V^u(I - A + A^{ui})^{-1}[(I - A + A^{ui})(I - A)^{-1}Y - \left(Y - Y^{ui}\right)]$$

$$= V^u(I - A + A^{ui})^{-1}[A^{ui}(I - A)^{-1}Y + Y^{ui}] \qquad (F1)$$

$$= V^u(I - A + A^{ui})^{-1}[A^{ui}X + Y^{ui}]$$

$$= V^u(I - A + A^{ui})^{-1}E^{ui}$$

And the DVA in exports measured by our method is

$$DVA^{ui} = V^u(I - A^D)^{-1}E^{ui} \qquad (F2)$$

The difference between VAX_D^{ui} and DVA^{ui} can be measured as

$$VAX\_D^{ui} - DVA^{ui} = V^u(I - A + A^{ui})^{-1}E^{ui} - V^u(I - A^D)^{-1}E^{ui}$$

$$= V^u(I - A^D)^{-1}[(I - A^D) - \left(I - A + A^{ui}\right)](I - A + A^{ui})^{-1}E^{ui} \qquad (F3)$$

$$= V^u(I - A^D)^{-1}(A^F - A^{ui})(I - A + A^{ui})^{-1}E^{ui}$$

At the bilateral sector level, the VAX_D can be measured as follow.

$$VAX\_D^{uci} = V^u(I - A)^{-1}Y - V^u(I - A + A^{uci})^{-1}(Y - Y^{uci})$$

$$= V^u(I - A + A^{uci})^{-1}[(I - A + A^{uci})(I - A)^{-1}Y - \left(Y - Y^{uci}\right)]$$

$$= \mathrm{V^u(I - A + A^{uci})^{-1}[A^{uci}(I - A)^{-1}Y + Y^{uci}]} \tag{F4}$$

$$= \mathrm{V^u(I - A + A^{uci})^{-1}[A^{uci}X + Y^{uci}]}$$

$$= \mathrm{V^u(I - A + A^{uci})^{-1}E^{uci}}$$

Finally, the DVA in exports measured by our method is

$$\mathrm{DVA^{uci} = V^u(I - A^D)^{-1}E^{uci}} \tag{F5}$$

The difference between $\mathrm{VAX\_D^{uci}}$ and $\mathrm{DVA^{uci}}$ can be measured as

$$\mathrm{VAX\_D^{uci} - DVA^{uci} = V^u(I - A + A^{uci})^{-1}E^{uci} - V^u(I - A^D)^{-1}E^{uci}}$$

$$= \mathrm{V^u(I - A^D)^{-1}[(I - A^D)(I - A + A^{uci})^{-1} - I]E^{uci}} \tag{F6}$$

$$= \mathrm{V^u(I - A^D)^{-1}(A^F - A^{uci})(I - A + A^{uci})^{-1}E^{uci}}$$