NBER WORKING PAPER SERIES

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Working Paper No. 3054

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
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August 1989

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NBER Working Paper #3054
August 1989

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ABSTRACT

This paper addresses the issue of how to give optimal advice about monetary policy when it is known that the advice may not be heeded. We examine a simple macroeconomic model in which monetary policy has the ability to stabilize output by offsetting exogenous shocks to aggregate demand. The optimal policy rule for such a model is easily derived. But an advisor who knows that his advice may not be followed should not recommend the optimal policy rule. This is true because, in giving activist advice, such an advisor increases uncertainty about what monetary policy will be followed. We solve for the rule that such an advisor should use in giving advice.

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Macroeconomists giving policy advice know that their advice is followed only intermittently. In contrast, academic analysis of optimal monetary policy usually assumes that the monetary authority chooses a policy rule that is followed thereafter without interruption.[1] This paper takes a step toward bridging this gap. It addresses the issue of how to give optimal advice about monetary policy when the advisor knows that the advice might be ignored.

The possibility that advice might be ignored introduces at least two new considerations into the advisor's problem. First, the advisor may try to increase the likelihood that his advice will be heeded. Second, the possibility that the advice will not be heeded may alter the optimal advice.

The problem of getting policy-makers to do what one wishes is the more obvious consideration. It finds various expressions in common wisdom. For example, an advisor may moderate his views in order to be gain favor. Alternatively, an advisor may exaggerate his views knowing that actual policy will be a water. down version of what he recommends -- and thus equivalent to what he actually thinks should be done. Or an advisor may take into account how the advice he gives today affects his probability of being listened to in the future: he might take an unpopular stand, for example, with the knowledge that he will not be heeded today, but will ultimately be proven right and thus gain credibility.

The second consideration is more subtle: the fact that policy advice may or may not be heeded changes its impact on the economy, both when the advice is taken and when it is ignored. Advice has an effect even when it is ignored because the possibility that it might have been followed was incorporated into the expectations of agents in the economy, and these expectations in turn affect outcomes.

In this paper we choose to focus only on the second of these issues. We assume that there is some fixed probability that the advisor will be listened to, and that this probability is known to all the agents in the economy. We assume further that this probability cannot be affected by the advisor's actions.[2] Finally, we assume that the advisor is disinterested -- that is, he cares about outcomes both when he is listened to and when he is not.[3] It might seem that these simplifications would make the advisor's problem trivial, but this turns out to be far from true. The decision problem facing our advisor is much more subtle than one might anticipate.

The purpose of this paper is thus to solve for the optimal rule for policy advice and compare it to the optimal policy rule. The notion of policy rules is often criticized on the grounds that, in practice, policy is made not by rule but by discretion.[4] Yet economists are often asked for their advice on monetary policy, and there is no reason that they cannot follow a rule in providing advice. Unless such advice is always sought and followed, however, the optimal rule for policy advice is generally not the same as the optimal policy rule.

We examine a simple macroeconomic model in which monetary policy has the ability to stabilize output by offsetting exogenous shocks to aggregate demand. The optimal policy rule, when the the policy-maker's objective is to stabilize output, is a common one: offset the demand shocks to the extent possible. In other words, optimal policy attempts to stabilize nominal GNP. If economists' advice were always followed, this optimal policy rule would also be the optimal rule for policy advice. Yet if economists' advice is followed only with some probability, the optimal rule for policy advice can differ substantially from the optimal policy rule.

The difference between optimal policy and optimal advice arises because the advisor has two conflicting goals. First, he would like to offset exogenous shocks to aggregate demand, which motivates him to advise an activist monetary policy. Second, he would like to make monetary policy more predictable, which motivates him to mimic the alternative policy followed when his advice is ignored. Optimal advice entails trading off these two objectives.

This problem is in some ways analogous to those studied by Friedman (1953) and Brainard (1967). These authors show that uncertainty about the economy makes optimal policy less activist. In our model, the uncertainty is not about the economy but about whether the advice will be followed. We show that this advisor's uncertainty also makes the advisor less activist, in the sense that he recommends a policy that deviates less from what will be done if he is not heeded.

There are, however, important differences between the advisor's uncertainty we examine and the uncertainty of Friedman and Brainard. As uncertainty increases in the Friedman/Brainard framework, policy loses none of its power to affect the state of the economy, while in the model presented here, as his chance of being heeded falls, the policy advisor has reduced power to do good or harm. The two kinds of uncertainty also affect optimal behavior differently. As the policy-maker in the Friedman/Brainard framework is less and less certain about the economy, his optimal policy approaches full passivity; in our model, as the chance of an advisor being heeded goes to zero, his optimal advice remains partially activist.

Our analysis proceeds as follows. In Section One, we present a simple model of the economy and solve for the optimal policy rule where the objective of the monetary authority is to minimize the variance of output. In Section Two, we consider the problem of an advisor who knows that his advice will only be followed with a certain probability, the alternative being a passive monetary policy. We show that the smaller is the probability that the advice will be followed, the less activist the advice should be, that is, the smaller the recommended offset of aggregate demand shocks. Even if the probability of the advice being followed is close to zero, however, some non-negligible offset of demand shocks should be recommended.

The optimal advice problem considered in Section Two is constrained in several ways to simplify the exposition and to provide intuition. Specifically, we restrict the optimal advice rule to have the same functional form as the optimal policy rule derived in Section One. In Section Three we show that the fully optimal advice is qualitatively different from the constrained advice found in Section Two. In particular, optimal advice depends on the lagged "money surprise," the difference between actual and anticipated money, which was caused by the uncertainty as to whether the advice would be followed. In Section Four we allow for a very general specification of the alternative monetary policy and once again solve for the unconstrained optimal advice.

In Section Five, we return to the simple model of Section Two to explore the value of compromise. We consider an economy with two advisors who have different objective functions. After deriving the advice that the advisors would give in a non-cooperative game, we show that there exist a set of compromise policies that both advisors would prefer. Giving optimal advice is thus no substitute for compromising. In many situations, however, we would not expect the compromise solution to be the outcome of a one-shot game.

We conclude in Section Six.

I. The Setup: The Economy and the Optimal Policy Rule

In this section we first present a standard model of the role of monetary policy in stabilizing output, and then solve the model in the traditional fashion for the optimal monetary policy rule. Following Fischer (1977) and Taylor (1980), we examine a model in which aggregate demand has real effects because of nominal stickiness, which takes the form of overlapping wage contracts. All variables are in logarithms, and constants have been omitted.

$$m_t + v_t = p_t + y_t \tag{1}$$

$$y_t = p_t - w_t \tag{2}$$

$$w_t = 1/2\ ({}_{t-1}p_t + {}_{t-2}p_t) \tag{3}$$

$$v_t = v_{t-1} + \epsilon_t \tag{4}$$

Equation (1) is aggregate demand and equation (2) is aggregate supply. The wage is fixed, and firms are able to hire as much labor as they want; an increase in the price level lowers the real wage, increases the quantity of labor demanded, and thus increases output. The money supply in period t, m_t, is set by the monetary policy authority at the end of period t-1.

Equation (3) represents the wage setting process. Contracts are two periods long. Hence, in period t, half of the workforce is governed by contracts written in period t-1, and half by contracts written in period t-2. The expectation of prices in period t formed in period t-i, which is denoted ${}_{t-i}p_t$, is based on the observation of shocks that take place in t-i and on a complete knowledge of the rule governing monetary policy. Equation (4) determines the evolution of velocity, which is assumed to follow an exogenous random walk.

We can solve equations (1) - (4) to get output as a function of monetary policy, expectations of monetary policy, and the realization of the shock to velocity:

$$y_t = 1/6\ (m_t - {}_{t-1}m_t) + 1/3\ (m_t - {}_{t-2}m_t) + 1/2\ \epsilon_t + 1/3\ \epsilon_{t-1}. \tag{5}$$

We assume that the goal of the monetary policy authority is to minimize the variance of output around its natural level of $y=0$. The standard result for this model is that an optimal policy is to set $m_t = -v_{t-1}$ or, equivalently, $m_t - m_{t-1} = -\epsilon_{t-1}$. In doing this, the policy authority is able to undo the effect of the shock in period t-1 on contracts written in period t-2, thus allowing workers covered by those contracts to receive a wage closer to the equilibrium real wage. Workers who write contracts in period t-1 (i.e., contracts that cover periods t and t+1) are not affected by the policy in period t because they observe ϵ_{t-1} before they write their contracts. Since they know the policy rule, they know the value of m_t. By engaging in such a policy, the monetary authority is able to reduce the variance of output to the level that would prevail in a model with one-period contracts. Hence the monetary authority can eliminate at least part of the adverse effects of the nominal wage rigidity.

II. Optimal Advice when the Alternative Policy is Passive

We now consider the problem of an advisor asked to suggest a policy when that advisor knows that there is a chance his advice may not be heeded. As in all of the models below, we assume that the probability that the advisor's advice will be followed is exogenous: with probability λ the monetary policy authority will set m_t at the level suggested by the advisor, and with probability $(1-\lambda)$ the authority will engage in no policy at all, setting $m_t = m_{t-1}$.[5] We consider policy advice rules of the form $m_t = m_{t-1} + \psi\,\epsilon_{t-1}$; this set of rules includes the optimal policy rule derived above as the special case $\psi = -1$ and the completely passive policy as the special case $\psi = 0$. We show that the optimal advice is not the same as the optimal policy. The variance of output under the optimal advice rule that we consider here is lower than that which would obtain if the advisor were to recommend the optimal policy rule derived above. We show in Section Three, however, that the rule for advice derived in this section is not the unconstrained optimum.

To solve equation (5) for the variance of output, we must first solve for expected money. At time t - 1, when workers are signing wage contracts, the shock ϵ_{t-1} has been observed, and the expected value of monetary policy is :

$$ {}_{t-1}m_t = \lambda(m_{t-1} + \psi\epsilon_{t-1}) + (1-\lambda)m_{t-1} = m_{t-1} + \lambda\psi\epsilon_{t-1}. \quad (6) $$

The two period ahead expectation of monetary policy in this case is ${}_{t-2}m_t = {}_{t-2}m_{t-1} = m_{t-2} + \lambda\psi\epsilon_{t-2}$. From equation (5), it is clear that current output y_t depends on velocity shocks and the setting of monetary policy in both the current and immediately preceeding period. To obtain the unconditional variance of output we must consider four possible permutations: advice heeded in both periods, advice heeded only in period t, advice heeded only in period t-1, and advice heeded in neither period. Taking the square of output in each case (since its mean is zero) and multiplying by the corresponding probabilities (e.g., λ^2 for the case where advice is followed in both periods) yields:

$$ \mathrm{var}(y) = \Big(1/4 + \lambda([1/3 + 1/3\,\psi - 1/6(\lambda-1)\psi]^2 + [1/3(\lambda-1)\psi]^2) + (1-\lambda)([1/3 - 1/6\,\lambda\psi]^2 + [1/3\,\lambda\psi]^2) \Big) \times \sigma_\epsilon^2 . \quad (7) $$

We can minimize the variance of output with respect to ψ to find the optimal policy advice rule:

$$ \psi^* = \frac{-4}{13 - 9\lambda} . \quad (8) $$

When $\lambda=1$ the advisor is certain that his advice will be followed and this rule returns the optimal policy derived in Section One. As the probability of being heeded declines, the strength of the advice given falls. That is, the smaller is the probability that the advisor is listened to, the less activist is his advice.

What is perhaps surprising is that as the probability of being heeded goes to zero, the strength of advice given remains non-negligible. The intuition for this result is as follows. In early analyses of optimal policy, such as the work of Friedman and Brainard, monetary policy that becomes unpredictable nonetheless loses none of its power. As the uncertainty of policy in these models increases, activist policy becomes a rogue elephant--powerful, but doing far more harm than good. Under such circumstances, the optimal policy is a completely passive one. But in our model, the harm of activist advice comes through the rational expectations effect of distorting people's expectations about the future. The assumption of rational expectations, however, guarantees that the harm of the advice falls as λ goes to zero, since people know that the chance of the advisor being listened to is small and thus do not modify their expectations significantly. Thus, in our model, the harm caused by activist advice falls as its probability of being heeded declines. Of course the power of the advice to do good falls in the same proportion. In the limit as λ shrinks, the desirability of activist advice must be calculated by weighing two effects that are similar in magnitude but of opposite sign. Thus, there is no general presumption that advice should become completely passive as the probability it will be heeded goes to zero.[6]

III. The General Solution when the Alternative Policy is Passive

This section generalizes the previous results by allowing the optimal advice function to take a more general form. In particular, we allow the advice rule to accept arguments in addition to the one that appears in the optimal policy rule. We show that the constrained advice rule studied in Section Two does not achieve the smallest possible variance of output. For now we retain the assumption that the alternative policy is passive.

The most interesting conclusion that we can draw from allowing for a more general specification is that the optimal advice will depend not only on past velocity shocks, but also on past "money surprises" -- that is, the difference between actual policy and policy expected one period earlier. In the standard optimal policy problem there are, of course, no such money surprises. Their existence in the advice problem is attributable to the uncertainty about whether advice will be followed. Money surprises in period t-1 are random shocks that affect the money supply in period t in ways that could not have been anticipated by workers signing contracts in period t-2. In this sense they are analogous to the velocity shocks, ϵ, and thus a natural determinant for output-stabilizing policy. We thus derive the interesting result that the unconstrained optimal advice in a setting where the advice is not always heeded is different qualitatively as well as quantitatively from the certainty case.

To anticipate the result, Figure One plots the variance of output as a function of the probability of being listened to for four advice rules: (1) passive advice which mimics the alternative policy, (2) advice recommending the optimal policy derived in Section One, (3) the constrained optimal advice derived in Section Two, and (4) the unconstrained optimal advice derived in this section. Perhaps the most surprising implication of Figure One is that, if the probability of being heeded is small (low λ), the "optimal policy rule" is the worst advice; it produces the highest variance of output.

We now proceed with the derivation of the unconstrained optimal advice. For convenience, let $z_t = m_t - m_{t-1}$ be the change in the money stock. The equation for output can be written as:

$$y_t = 1/2\ (z_t - {}_{t-1}z_t) + 1/3\ (z_{t-1} - {}_{t-2}z_{t-1}) + 1/3({}_{t-1}z_t - {}_{t-2}z_t) + 1/2\ \epsilon_t + 1/3\ \epsilon_{t-1}. \qquad (9)$$

Output is thus determined by the current and lagged money surprises, the revision at time t-1 of the expectation of time t money growth that had been formed at t-2, and the current and lagged velocity shocks. If the monetary authority followed any single feedback rule with certainty, the first and second terms would be identically zero. In our problem, however, these terms play a crucial role. Let π_t be an i.i.d. binomial random variable that takes on the value one if the advisor is heeded and zero if the alternative policy is followed. The expected value of π_t is λ. Also define the zero mean random variable $\eta_t = \pi_t - \lambda$ as the surprise in whose advice was followed in period t. We now write the optimum advice rule as:

$$x_{t-1} = \phi\eta_{t-1}x_{t-2} + \psi\epsilon_{t-1}, \qquad (10)$$

where x_{t-1} is the advice regarding the optimal setting of z_t.[7] In Section Two, where we derived a constrained advice rule, we set ϕ equal to zero. In Section Four, we will show that when the alternative policy is passive, the optimal advice takes the form of (10), that is, the arguments given are the only ones that appear in the unconstrained advice rule. Since the advice x_{t-1} is followed if π_t equals one and the alternative policy $z_t=0$ is followed if π_t equals zero, the actual change in the money stock is given by

$$z_t = \pi_t x_{t-1}, \qquad (11)$$

while the anticipated change in money is

$${}_{t-1}z_t = \lambda\ x_{t-1}. \qquad (12)$$

Using this notation, the money surprise is simply

$$z_t - {}_{t-1}z_t = (\pi_t - \lambda)x_{t-1} = \eta_t x_{t-1}. \qquad (13)$$

From equations (9), (12) and (13), we can express output as follows:

$$y_t = 1/2\ \eta_t x_{t-1} + 1/3\ \eta_{t-1} x_{t-2} + 1/3\ [\lambda(x_{t-1} - {}_{t-2}x_{t-1})] + 1/2\ \epsilon_t + 1/3\ \epsilon_{t-1}. \tag{14}$$

Substituting (10) for x_{t-1} where it appears in the third term of (14) and eliminating ${}_{t-2}x_{t-1}$, which is zero, gives

$$y_t = 1/2\ \eta_t x_{t-1} + 1/3(1+\lambda\phi)\eta_{t-1} x_{t-2} + 1/3(1+\lambda\psi)\epsilon_{t-1} + 1/2\ \epsilon_t\ . \tag{15}$$

We have arranged matters so that each of the terms in (15) is orthogonal to all of the others. By assumption, η_t and ϵ_t are independent of each other and are both i.i.d. random variables. Even though x_{t-1} is correlated with both ϵ_{t-1} and x_{t-2}, the first term will not be correlated with either the second or third terms because η_t has a mean of zero conditional on everything known at time t-1.[8]

Again we minimize the variance of output, which is

$$\text{var}(y) = [1/4\ \lambda(1-\lambda)\ E(x_{t-1}^2) + 1/9\ (1+\lambda\phi)^2\ \lambda(1-\lambda)\ E(x_{t-2}^2)\ ] + [1/4 + 1/9(1+\lambda\psi)^2]\sigma_\epsilon^2. \tag{16}$$

The variance of x can be found from equation (10). All of the terms on the right hand side of that equation are mutually orthogonal, allowing us to write:

$$E(x_{t-1}^2) = \phi^2\lambda(1-\lambda)\ E(x_{t-2}^2) + \psi^2\sigma_\epsilon^2. \tag{17}$$

As long as ϕ is not too large, x is stationary and $E(x_{t-1}^2) = E(x_{t-2}^2) = \sigma_x^2$, where

$$\sigma_x^2 = \frac{\psi^2\sigma_\epsilon^2}{1-\lambda(1-\lambda)\phi^2}\ . \tag{18}$$

Substituting this expression[9] into (16),

$$\mathrm{var}(y) = \frac{[1/4 + 1/9\ (1+\lambda\phi)^2\ ]\lambda(1-\lambda)(\ \psi^2\sigma_\epsilon^2\ )}{1-\lambda(1-\lambda)\phi^2}$$

$$+ \quad [1/4 + 1/9(1+\lambda\psi)^2]\sigma_\epsilon^2. \qquad (19)$$

Minimizing this expression for var(y) yields the optimal settings of the two policy parameters:

$$\phi^* = \psi^* = \frac{9\lambda - 13 + \sqrt{169 - 298\lambda + 145\lambda^2}}{8\lambda(1-\lambda)}. \qquad (20)$$

Figure Two plots the optimal value of the policy parameter ψ obtained in this case and in the constrained case treated in Section Two, both as functions of λ, the probability of being heeded. Note that the extent to which velocity shocks are offset is always smaller in the constrained case ($\phi = 0$) than in the case of unconstrained policy advice. The reason for this result provides insight into the general advice problem: a large ψ has a directly beneficial effect in that it reduces the variance of output due to velocity shocks, but it indirectly increases the variance of output to the extent that it increases uncertainty about the level of the money supply. In the case of the unconstrained advice, however, the effects of monetary surprises are somewhat mitigated because the advisor partially offsets lagged money surprises. Thus, in his advice with respect to velocity shocks, the advisor can afford to be more activist. When λ is very low, the probability that a money surprise will be offset is reduced, and so the gap between the unconstrained and constrained offsets to velocity shocks is small.

The fact that the optimal values of ϕ and ψ are the same when the alternative policy is passive can be explained as follows: from the point of view of a person signing contracts in period t-2, a velocity shock in t-1 and a policy surprise have the same effect on nominal GNP (m+v). This result is not robust to different specifications of the alternative policy, however, as will be seen in Section Four.

As λ approaches one, ϕ^* and ψ^* approach negative one. As the likelihood of being listened to approaches certainty, the optimal advice approaches the optimal policy: both policy shocks and velocity shocks are fully offset. Of course, when λ actually equals one, ϕ is irrelevant, since there are no policy surprises. As λ approaches zero, ϕ^* and ψ^* approach -4/13, which is the same limiting value as we obtained in Section Two. The fact that these parameters do not tend toward zero confirms the result obtained in the restricted model above: the deviation from the alternative policy does not vanish even as the likelihood of being listened to declines to zero. Of course, as in Sectin Two, ϕ^* and ψ^* are not defined when λ=0. In such a case, advice has no ability to affect the variance of output and so it does not make sense to talk about any particular value of the advice parameter being optimal.

IV. Optimal Advice for a General Alternative Policy

We can generalize the previous results along yet another dimension by allowing the alternative policy to take a more general form. The general form we consider can readily incorporate an alternative policy that is itself the result of an optimization problem for another advisor with a different objective function. An example of such an alternative advisor is provided in Section Five. The derivation of optimal advice in this case is tedious, and we defer it to an Appendix. In this section we simply set up the problem and state some of our results.

We specify a very general alternative policy as follows:

$$z_t^{alt} = a(z_{t-1} - {}_{t-2}z_{t-1}) + b\epsilon_{t-1} + s_{t-1} + q_{t-2}, \qquad (21)$$

where q_{t-2} can be anything known at time t-2, and s_{t-1} is a random variable known at t-1 but orthogonal to everything known at time t-2 and to the lagged money surprise $z_{t-1} - {}_{t-2}z_{t-1}$. The only restrictions that we impose on the alternative policy are that the parameters a and b be constant and that s be covariance stationary. The previous section considered the special case in which $a = b = s_{t-1} = q_{t-2} = 0$.

The alternative policy, z^{alt}, has been given a general linear form that nests a number of interesting economic phenomena. For example, s_{t-1} may be viewed as a "sunspot," a variable which is not inherently significant. We show in the Appendix that the optimal advice will react to sunspots that affect the alternative policy. The parameter a dictates the response of the alternative policy to money stock surprises. These surprises might appear in the alternative policy rule for the same reason that they appear in the optimal advice rule. The term q_{t-2} can include any deterministic factors in the alternative policy rule; for example, if the alternative rule were a "k percent growth rule," then k would appear as part of q_{t-2}. In the Appendix we show that the optimal advice exactly matches the alternative policy in its responses to q_{t-2}. For example, if the alternative policy is inflationary (high q_{t-2}) in certain periods like election years, the optimal advice tends to be inflationary then as well. Finally, since the general optimal policy has three reaction parameters, we show in the Appendix that it is possible for the advisors to agree completely on some elements of policy advice and still disagree on others.

V. The Virtue of Compromise

In this section we show that optimal advice is not a substitute for compromise. We show that two advisors with different objective functions can be made better off by compromising with each other over what advice to give than by non-cooperatively giving "optimal advice."

For purposes of simplicity, we once again restrict the advisor's reaction functions to have a single parameter. That is, our advisor is the one considered in Section Two, who gives advice of the form $m_t = m_{t-1} + \psi \epsilon_{t-1}$. To explore the set of possible compromises, we also endow the "alternative advisor" of Section Two with a simple objective function that justifies the alternative policy considered in that section. Specifically, we posit that the alternative advisor is a "monetarist" in the sense that he wants to minimize the variance of growth in the money stock. Such an advisor always advises $\Delta M = 0$, regardless of the advice of his competitor. Thus the optimal advice that we derived in Section Two is also the Nash equilibrium of a game between these two advisors.

Our goal here is to find a set of "compromise" values of the advice parameter ψ that, if adhered to all of the time, would make both advisors better off. That is, if both advisors were to advise this compromise value (and so it was applied all of the time), both the variance of output and the variance of money growth would be lower than if the advisors played the Nash equilibrium.

We begin by finding $\underline{\psi}$, defined as the smallest absolute value of the reaction parameter that, if applied all of the time, would produce the same level of the variance of output as the Nash equilibrium. Any value of ψ between $\underline{\psi}$ and negative one, if agreed to by both advisors, would make the output-stabilizing advisor better off than he would be under the Nash equilibrium. Substituting the equation for the optimal advice (8) into the expression for the variance of output (7), yields an expression for the variance of output under the Nash solution, as a function of λ. We define this expression as $K(\lambda)\sigma_{\epsilon}^2$. The variance of output when both advisors agree on policy $\underline{\psi}$ is:

$$\text{var}(y) = (1/4 + [1/3 + 1/3\ \underline{\psi}]^2)\ \sigma_\epsilon^2 \quad . \tag{22}$$

Setting these two expression equal yields an expression for $\underline{\psi}$

$$\underline{\psi} \quad = \quad -1 \pm 3\sqrt{1/4 \quad + \quad K(\lambda)}. \tag{23}$$

Only the larger solution of this equation is between zero (the value that the monetarist advisor would choose) and negative one (the value that the output stabilizer would choose if he were sure of being listened to), and so there is no ambiguity regarding the value of $\underline{\psi}$.

We define $\bar{\psi}$ analogously to $\underline{\psi}$. That is, $\bar{\psi}$ is the highest absolute value of ψ that, if applied all of the time, would produce the same variance of money growth as the Nash equilibrium. The monetarist advisor would be happy to agree to any value of ψ between $\bar{\psi}$ and zero. To find $\bar{\psi}$, note that under the Nash equilibrium ($\psi = \psi^*$), the variance of changes in the money stock is $\lambda\psi^{*2}\sigma_\epsilon^2$. When both advisors agree on a policy $\bar{\psi}$, the variance is $\bar{\psi}\sigma_\epsilon^2$. Setting these two equal yields an expression for $\bar{\psi}$:

$$\bar{\psi} \quad = \quad \psi^*\sqrt{\lambda}$$

Figure Three graphs $\underline{\psi}$ and $\bar{\psi}$ as functions of λ. Also included in the figure is the value of ψ^*, the optimal advice of the output stabilizing advisor under Nash equilibrium. As can be seen, for any value of λ, there is always a set of compromise values of ψ that will make both advisors better off. For any value of λ strictly between zero and one, there is a multiplicity of values of ψ that satisfy both advisors: the exact value of ψ chosen would presumably be the outcome of some bargaining process between the two.

It should also be noted that any of the "compromise" values of ψ derived above could be viewed as the subgame perfect equilibrium of an infinitely repeated game with sufficiently little discounting (see Fudenberg and Maskin, 1986).[10] To the extent that giving advice is better characterized as a one-shot game, however, it may be the case that compromise cannot be achieved, and thus we would expect the advisors to arrive at the Nash solution of the one-shot game.

VI. Conclusion

In this paper we have explored the problems facing an economist asked to give prescriptions for monetary policy who knows that his advice might not be heeded. We have shown that the optimal advice rule generally differs from the conventionally-derived optimal policy rule. Specifically, optimal advice tends to resemble the alternative policy that will be followed if the advice is ignored. This is because giving advice that is very different from the alternative policy raises the variance of unanticipated money and thus of output. The advisor faces a dilemma: his optimal advice reflects a compromise between his desire to make monetary policy predictable by copying the alternative policy and his desire to offset exogenous shocks to aggregate demand.

The rule that we have derived is clearly sensitive to the choice of model. The model that we have chosen is representative of a class of models in which systematic monetary policy can be used to stabilize output because of the existence of nominal stickiness. The informational advantage of the monetary authority in these models comes from its ability to set the money supply after some prices have been set in nominal terms. The tension between recommending the optimal monetary policy and mimicking the alternative policy disappears if the monetary authority has no such informational advantage. In such a model (e.g., Lucas's, 1973, misperceptions model), unanticipated money can only raise the variance of output, and the optimal advice exactly copies the alternative policy.

Various extensions of this work are possible. One step toward realism is to imagine that there are many different advisors. It is easy to show that if there are n advisors, all of whom have the same objective function and each with probability of being heeded λ_i, then they will all give the same advice, and this advice will be the same as the advice given by a single advisor with a probability of being heeded of $\sum_i \lambda_i$. Thus the "advisor" in our model could be reinterpreted as a particular school of thought comprised of many like-minded individuals. This result follows immediately from the fact that all n advisors are maximizing the same function. Another type of extension of our model is to deal explicitly with the issue, raised in the introduction, of how the probability that advice will be followed is determined. In the notation of the model, we would like to make λ endogenous. Future work on this problem might thus combine the framework for optimal advice developed here with a richer model of the political decision-making process.

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Notes

* We are grateful to Kenneth Kuttner, David Romer, Lawrence Summers, and the referees for helpful comments. Mankiw is grateful to the National Science Foundation for financial support.

1. See, for example, the discussions of optimal policy in Fischer (1977), Blinder and Mankiw (1984), and Aizenman and Frenkel (1986).

2. Making these simplifications does indeed rule out many of the circumstances in which advice is given, but by no means all of them. Consider, for example, the case of a presidential election in which each candidate has a trusted economic "guru" to whom he listens. The candidates' attachment to their gurus may not depend on the gurus' view of monetary policy, and the candidates' chances of getting elected are almost certainly not strongly dependent on their positions on monetary policy. The problem facing such a guru exactly conforms to the way in which we have set up the advice problem in this paper.

3. Thus, the policy advisor depicted in our model has objectives different from those found in models of bureaucratic self interest. In such a model, an advisor would be concerned only with outcomes that take place "on his watch," that is, when his advice is listened to.

4. See, for example, the arguments of Sims (1984).

5. In Section Five we motivate this policy as the advice of a "monetarist" whose objective is to minimize the variance of money growth.

6. When $\lambda = 0$, that is, when there is no chance that the advice will be heeded, then the variance of output is not affected by the choice of ψ. This can be seen in equation (7), where setting λ to zero eliminates ψ from the equation. In such a case, it obviously makes no sense to talk about optimal advice, and so equation (8) should only be seen as applying to values of λ greater than zero.

7. Throughout the paper we use the convention that a variable is dated in the period in which it is first known. Thus the advice about how to set money in period t is dated x_{t-1} because it is based on information available at time t-1.

8. If f_{t-1} is anything known at time t-1, $cov(\eta_t, f_{t-1}) = E(\eta_t f_{t-1}) = E(E_{t-1}(\eta_t f_{t-1})) = E(f_{t-1} E_{t-1}(\eta_t)) = 0$.

9. When the denominator is negative or zero, the variance of x becomes infinite. Minimizing the variance of y with respect to ϕ guarantees that the variance of x is finite.

10. Note that in previous sections we have implicitly assumed zero discounting by taking the variance of output, rather than the discounted sum of squares of future deviations of output from its mean, as the argument in the advisor's objective function. Note also that the complete set of equilibria of the long-run game is larger than the set of "compromise" equilibria that we identify, because reversion to the Nash equilibrium is not the worst possible punishment that can be imposed upon a player who deviates.

Appendix

In this appendix we present the derivation of optimal advice when the "alternative policy" is of a very general form.It is convenient to specify the optimal advice as a deviation from the alternative policy. Let x_{t-1} represent this deviation. Above we used x_{t-1} to represent the advice itself rather than its deviation from the alternative policy; since the alternative was passive, however, these came to the same thing.

Using the notation developed above, the change in the money stock is given by:

$$z_t = a(z_{t-1} - {}_{t-2}z_{t-1}) + b\epsilon_{t-1} + s_{t-1} + q_{t-2} + \pi_t x_{t-1}. \qquad (13')$$

Proceeding as in the last section, we can express output as follows:

$$y_t = 1/2\ \eta_t x_{t-1} + 1/3\ \eta_{t-1} x_{t-2} + 1/3\ [a\eta_{t-1} x_{t-2} + b\epsilon_{t-1} + s_{t-1} + \lambda(x_{t-1} - {}_{t-2}x_{t-1})] + 1/2\ \epsilon_t + 1/3\ \epsilon_{t-1}. \qquad (14')$$

We now specify x in a manner that allows us to separate the elements that are essential to reaching an optimum from those that are extraneous:

$$x_{t-1} = \phi\eta_{t-1} x_{t-2} + \psi\epsilon_{t-1} + \omega s_{t-1} + g_{t-1} + h_{t-2}, \qquad (10')$$

where h_{t-2} can be anything known at time t-2 and g_{t-1} is orthogonal to all information at time t-2 and to $\eta_{t-1}x_{t-2}$, ϵ_{t-1}, and s_{t-1} . We will prove below that for the optimal advice g and h are always zero. This will allow us to exclude some variables that do not appear explicitly in (10') that one might expect would appear in the optimal advice rule. For example, we can establish that the value of π_{t-1} , which indicates which advisor was heeded in period t-1, does not enter the advice rule since it is potentially part of g_{t-1}.

Substituting (10') into (14') gives:

$$y_t = 1/2\ \eta_t x_{t-1} + 1/3(1+a+\lambda\phi)\eta_{t-1}x_{t-2} + 1/2\ \epsilon_t + 1/3(1+b+\lambda\psi)\epsilon_{t-1} + 1/3(1+\lambda\omega)s_{t-1} + 1/3\ \lambda g_{t-1}. \qquad (15')$$

Again we have arranged matters so that each of the terms in (15') is orthogonal to all of the others. By assumption, η_t and ϵ_t are independent of each other and with everything known at time t-1; therefore the first and third terms are orthogonal to each other and to all other terms. For the same reason, the second and fourth terms are uncorrelated. They are orthogonal to the last two terms by construction. Finally, g_{t-1} is constructed to be orthogonal to s_{t-1}.

We minimize the variance of output, which is

$$\text{var}(y) = [1/4\ \lambda(1-\lambda)\ E(x_{t-1}^2) + 1/9\ (1+a+\lambda\phi)^2\ \lambda(1-\lambda)\ E(x_{t-2}^2)\ ] + [1/4 + 1/9(1+b+\lambda\psi)^2]\sigma_\epsilon^2 + 1/9(1+\lambda\omega)^2\sigma_s^2 + 1/9\ \lambda^2\sigma_g^2. \qquad (16')$$

The variances of the policy deviations can be found from equation (10'). All of the terms on the right hand side of that equation are mutually orthogonal, allowing us to write:

$$E(x_{t-1}^2) = \phi^2\lambda(1-\lambda)\ E(x_{t-2}^2) + \psi^2\sigma_\epsilon^2 + \omega^2\sigma_s^2 + E(g_{t-1}^2) + E(h_{t-2}^2). \qquad (17')$$

It is clear from (16') and (17') that g and h can only add to the variance of output. Thus the optimal advice will always set them equal to zero. Once g and h are set to zero, x is covariance stationary and

$$\sigma_x^2 = \frac{\psi^2\sigma_\epsilon^2 + \omega^2\sigma_s^2}{1-\lambda(1-\lambda)\phi^2}. \qquad (18')$$

Substituting this expression into (17),

$$\mathrm{var}(y) = \frac{[1/4 + 1/9\ (1+a+\lambda\phi)^2\ ]\lambda(1-\lambda)(\ \psi^2\sigma_\epsilon^2 + \omega^2\sigma_s^2\ )}{1-\lambda(1-\lambda)\phi^2} +$$

$$[1/4 + 1/9(1+b+\lambda\psi)^2]\sigma_\epsilon^2 + 1/9(1+\lambda\omega)^2\sigma_s^2. \qquad (19')$$

The minimization of variance with respect to ϕ is separable from the rest of the problem. By minimizing

$$F(\phi) = \frac{1/4 + 1/9\ (1+a+\lambda\phi)^2}{1-\lambda(1-\lambda)\phi^2} \qquad (24)$$

with respect to ϕ, we minimize the variance of y with respect to ϕ. Taking the derivative of F with respect to ϕ and setting it equal to zero yields the following quadratic equation for the optimal ϕ:

$$4\lambda(1-\lambda)(1+a)\phi^2 + [4(1-\lambda)(1+a)^2 + 9 - 5\lambda]\phi + 4(1+a) = 0. \qquad (25)$$

For the demominator of F to be positive, ϕ must fall in the range $(-1/\sqrt{\lambda(1-\lambda)}\ ,\ 1/\sqrt{\lambda(1-\lambda)}\ )$. This criterion eliminates one of the roots of (25) for the following reason: the quadratic in ϕ is always negative when $\phi = -1/\sqrt{\lambda(1-\lambda)}$ and positive when $\phi = 1/\sqrt{\lambda(1-\lambda)}$; therefore the quadratic must cross the axis an odd number of times in this range. Since the quadratic can cross the axis two times in all, it must cross it exactly once in this range.

Note that if $a = -1$ (that is, the alternative policy involves fully offsetting the money surprise), then the optimal ϕ is zero, which implies agreement with this element of the alternative policy. There could still be disagreement about other elements of monetary policy, such as the response to ϵ_{t-1}, so that there may still be money surprises. Once we have found the minimized value of F, it is straightforward to solve for the optimal ψ

and ω. These are:

$$\psi^{*} = -(1+b) / [9(1-\lambda) F(\phi^{*}) + \lambda] \quad \text{and} \tag{26}$$

$$\omega^{*} = -1 / [9(1-\lambda) F(\phi^{*}) + \lambda]. \tag{27}$$

The equivalence of ϕ^{*} and ψ^{*} which held in Section Three breaks down if either a or b does not equal zero. From (26) it is clear that if the alternative policy completely offsets velocity shocks (if $b = -1$), then the optimal advice is in agreement with the alternative policy's response to these shocks ($\psi^{*} = 0$). The similarity of form in the expressions for ψ^{*} and ω^{*} can be explained as follows. The part of the velocity shock that is left uncancelled by the alternative policy is $(1+b)\epsilon_{t-1}$. In terms of the optimal advice, the part of the velocity shock that is not offset by the alternative policy is just like any other extraneous action of the alternative policy that is a surprise dated t-1 (which is exactly what s_{t-1} represents).

Since $F \geq 1/4$,

$$1 / [9(1-\lambda) F(\phi^{*}) + \lambda] \leq 1.$$

Therefore ψ^{*} and ω^{*} bring the optimal advice only part of the way from the alternative policy toward the first best policy.

As λ approaches one, ϕ^{*} approaches $-(1+a)$, so that that the policy parameter $\phi^{*} + a$ approaches negative one. The reaction to the sunspot s_{t-1}, which is $\omega^{*} + 1$, approaches zero. As λ approaches zero, ϕ^{*} approaches $-4(1+a)/[4(1+a)^{2} + 9]$ and ω^{*} approaches $-1 / [(9/4) + (1+a+\phi^{*})^{2}]$. The fact that neither of these quantities tends toward zero confirms the result obtained in the restricted models above: the deviation from the alternative policy does not vanish even as the likelihood of being listened to declines to zero.

Figure One

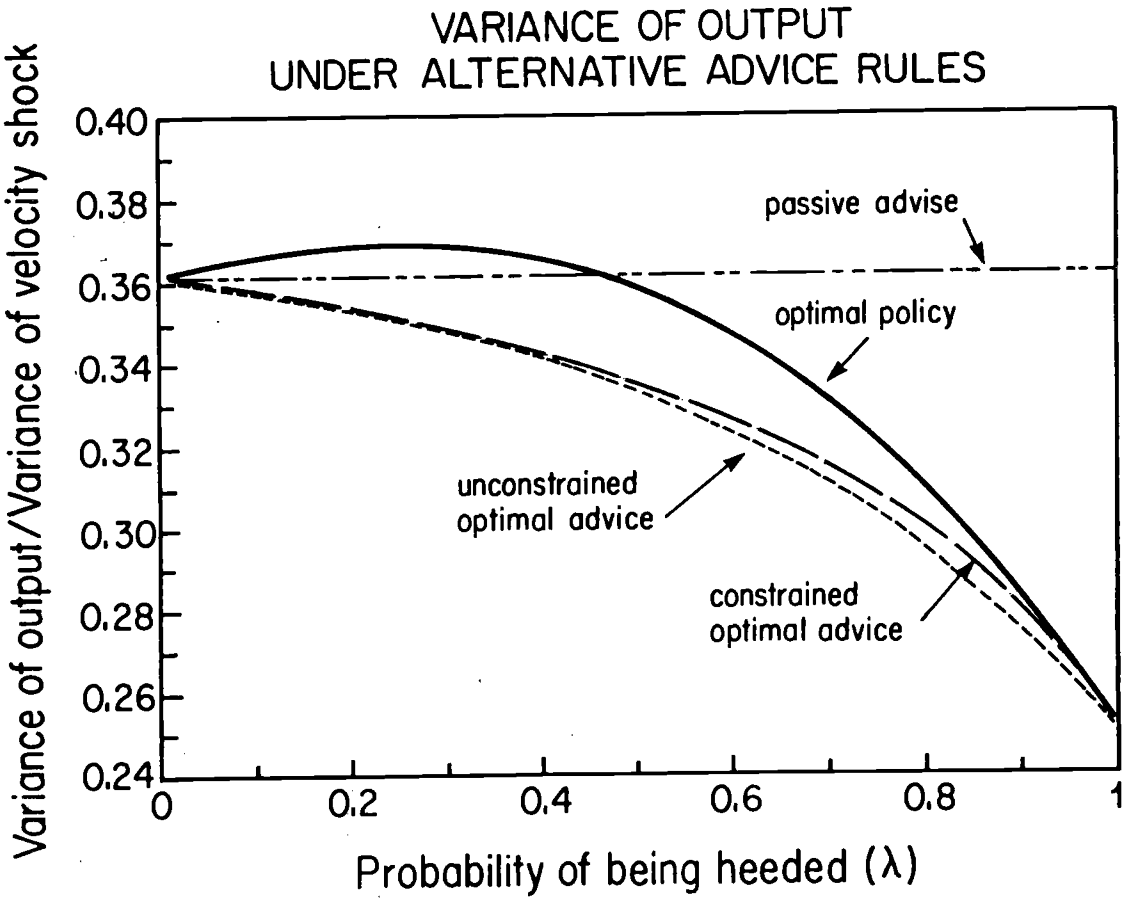
VARIANCE OF OUTPUT
UNDER ALTERNATIVE ADVICE RULES
Variance of output/Variance of velocity shock
0.40
0.38
0.36
0.34
0.32
0.30
0.28
0.26
0.24
0
0.2
0.4
0.6
0.8
1
Probability of being heeded (λ)
passive advise
optimal policy
unconstrained optimal advice
constrained optimal advice

Figure Two

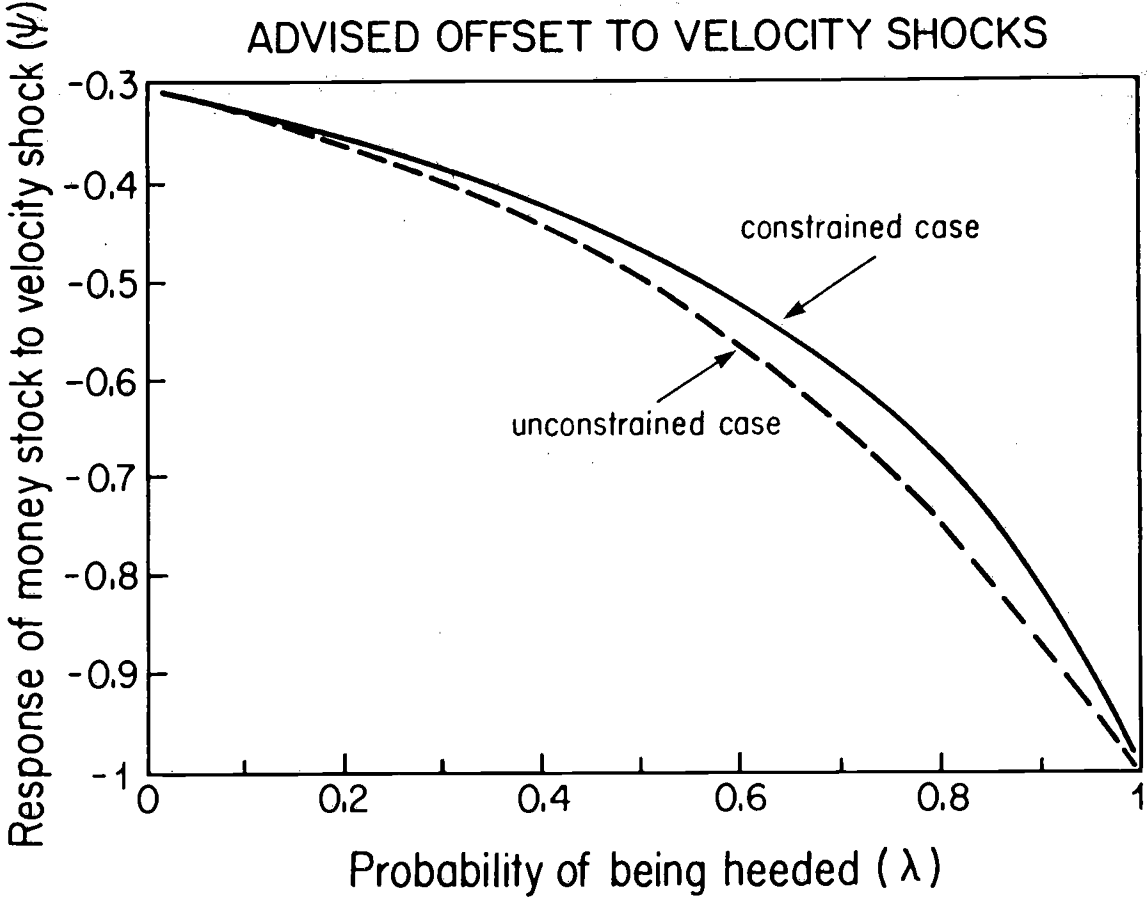
ADVISED OFFSET TO VELOCITY SHOCKS
Response of money stock to velocity shock (ψ)
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
0
0.2
0.4
0.6
0.8
1
constrained case
unconstrained case
Probability of being heeded (λ)

Figure Three