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DYNAMIC FACTOR DEMAND MODELS, PRODUCTIVITY MEASUREMENT, AND RATES OF RETURN: THEORY AND AN EMPIRICAL APPLICATION TO THE U.S. BELL SYSTEM

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ABSTRACT

Prucha and Nadiri (1982,1986,1988) introduced a methodology to estimate systems of dynamic factor demand that allows for considerable flexibility in both the choice of the functional form of the technology and the expectation formation process. This paper applies this methodology to estimate the production structure, and the demand for labor, materials, capital and R&D by the U.S. Bell System. The paper provides estimates for short-, intermediate- and long-run price and output elasticities of the inputs, as well as estimates on the rate of return on capital and R&D. The paper also discusses the issue of the measurement of technical change if the firm is in temporary rather than long-run equilibrium and the technology is not assumed to be linear homogeneous. The paper provides estimates for input and output based technical change as well as for returns to scale. Furthermore, the paper gives a decomposition of the traditional measure of total factor productivity growth.

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1. Introduction[1]

In Prucha and Nadiri (1982) we introduced a methodology to estimate systems of dynamic factor demand that allows for considerable flexibility in both the choice of the functional form of the technology and the expectation formation process. This approach was explored further in Prucha and Nadiri (1986). It is based on a firm with a finite but shifting planning horizon. The stocks at the end of the planning horizon are determined endogenously via the assumption that the firm maintains a constant firm size and static expectations beyond the actual planning horizon. Prucha and Nadiri (1982) introduced also a corresponding estimation algorithm that avoids the need for an explicit analytic solution of the firm's control problem and show how at the same time it is possible to evaluate (for reasons of numerical efficiency) the gradient of the statistical objective function from analytic expressions. A generalization of the algorithm is given in Prucha and Nadiri (1988).

In this paper we apply the methodology of Prucha and Nadiri (1982) to estimate the production structure and the demand for labor, materials, capital and R&D in the historic U.S. Bell System. (The merits of the breakup of the U.S. Bell System is still an item of considerable debate. In future research it seems of interest to compare the historic U.S. Bell System with several of the currently operating telephone companies.) We consider alternative specifications of the length of the planning horizon and the expectation formation process; we compare, in particular, results obtained from the finite horizon model with those from an infinite horizon model. The empirical application to the U.S. Bell System not only provides an illustration of the methodology but also contributes several new features to the existing literature on the production structure of AT&T. First, we formulate and estimate a dynamic model in contrast to the static models that were usually applied to AT&T data.[2] Schankerman and Nadiri (1986) find evidence to reject the hypothesis that for AT&T all factors are variable. Second, contrary to conventional studies we include R&D as a factor of production. R&D should be of particular importance in a high technology firm like AT&T.[3]

As a description of the technology we introduce a new restricted cost function that generalizes the restricted cost function introduced by Denny, Fuss and Waverman (1981a) and Morrison and Berndt (1981) from the linear homogeneous to the homothetic case. Furthermore, we discuss measures of technical change if the firm is in temporary rather than long-run equilibrium and if the technology is not a priori assumed to be linear homogeneous.

The paper is organized as follows: In Section 2 we describe the theoretical model under both the assumption of a finite and infinite planning horizon, and derive the factor demand equations used in the empirical analysis. In Section 3 we present the parameter estimates of the model corresponding to different planning horizons and expectation regimes. Adjustment cost characteristics as well as price and output elasticities of the inputs in the short-, intermediate- and long-run are presented in Section 4. Section 5 deals with the formulation of pure measures of technical change and the measurement of returns to scale. In Section 6 we provide a decomposition of the traditional measure of total factor productivity growth into components attributable to technical change, scale and the adjustment costs. We also provide a decomposition of the growth of output and labor productivity. Section 7 deals with the calculation of rates of return on physical and R&D capital. The conclusions are contained in Section 8 followed by a brief technical appendix.

2. Theoretical Model and Empirical Specification

2.1 Theoretical Model

Consider a firm that employs m variable inputs V_i $(i=1,\ldots,m)$ and n quasi-fixed inputs X_j $(j=1,\ldots,n)$ in producing the single output good Y. The firm's production process is described by the following generalized production function:

$$(1) \qquad Y_t = F(V_t, X_{t-1}, \Delta X_t, T_t)$$

where $V_t = \{V_{it}\}_{i=1}^{m}$ is the vector of variable inputs, $X_t = \{X_{jt}\}_{j=1}^{n}$ is the vector of end-of-period stocks of quasi-fixed factors, T_t is a technology index, (and t denotes time). The vector $\Delta X_t = X_t - X_{t-1}$ appears in the production function to model internal adjustment costs in terms of forgone output due to changes in the quasi-fixed factors. It is assumed that $F(.)$ is twice continuously differentiable and that $F_V > 0$, $F_{X_{-1}} > 0$ and $F_{|\Delta X|} < 0$.[4] It is furthermore assumed that the production function is strictly concave in all arguments (except possibly in the index of technology). This implies that the marginal products of the factors of production V and X_{-1} are decreasing and that the marginal adjustment costs are increasing.

The stocks of the quasi-fixed factors accumulate according to $(j=1,\ldots,n)$

$$(2) \qquad X_{jt} = I_{jt} + (1-\delta_j)X_{jt-1} ,$$

where I_{jt} denotes gross investment and δ_j denotes the depreciation rate.

The firm is assumed to face perfectly competitive markets with respect to its factor inputs. We denote the acquisition price for the variable and quasi-fixed factors as $\tilde{w}_{it}$ $(i=1,\ldots,m)$ and $\tilde{q}_{jt}$ $(j=1,\ldots,n)$, respectively. It proves convenient to normalize all prices in terms of the price of the first variable factor. We denote those normalized prices as $w_{it} = \tilde{w}_{it}/\tilde{w}_{1t}$ and $q_{jt} = \tilde{q}_{jt}/\tilde{w}_{1t}$, and define vectors $w_t = \{w_{it}\}_{i=2}^{m}$ and $q_t = \{q_{jt}\}_{j=1}^{n}$.

Instead of describing the production structure in terms of the production function (1) we can describe the productions structure equivalently in terms of the normalized restricted cost function. Let $\{\hat{V}_{it}\}_{i=1}^{m}$ denote the cost minimizing variable factor inputs needed to produce output Y_t conditional on X_{t-1} and ΔX_t; then the normalized restricted cost function is defined as

$$(3) \qquad G(w_t, X_{t-1}, \Delta X_t, Y_t, T_t) = \textstyle\sum_{i=1}^{m} w_{it}\hat{V}_{it} \ .$$

This function has the following properties (compare Lau (1976)): $G_X < 0$, $G_{|\Delta x|} > 0$, $G_Y > 0$, $G_w > 0$. Furthermore $G(.)$ is convex in X_{-1} and ΔX and concave in w.

The firm's cost in period t is given by

$$(4) \qquad C(X_t, X_{t-1}, \pi_t) = G(w_t, X_{t-1}, \Delta X_t, Y_t, T_t) + \textstyle\sum_{j=1}^{n} q_{jt} I_{jt} + A_t \ ,$$

where A_t denotes taxes (which will be specified in detail later on) and π_t is a vector composed of w_t, q_t, Y_t, T_t, as well as tax parameters.

The firm is assumed to minimize the present value of current and future costs. We consider two alternative specifications of the firm's optimization problem regarding the length of the planning horizon. First consider the case of an infinite planning horizon. In this case the firm's objective function in period t is assumed to be given by

$$(5) \qquad \textstyle\sum_{\tau=0}^{\infty} C(X_{t+\tau}, X_{t+\tau-1}, E_t\pi_{t+\tau})(1+r)^{-\tau} \ ,$$

where E_t denotes the expectations operator conditional on information available at the beginning of period t and r denotes the real discount rate. It is assumed that in each period t the firm derives an optimal plan for the quasi-fixed inputs for periods $t,t+1,\ldots$ such that (5) is minimized subject to the initial stocks X_{t-1} and information available at that time; the firm then chooses its quasi-fixed inputs in period t according to this plan. (Note that in each period the firm only implements the initial portion of its optimal input plan.) The firm repeats this process every period. In each period a new optimal plan is formulated as new information on the exogenous variables becomes available and expectations on those variables are modified accordingly.[5]

Next consider the case of a finite but shifting planning horizon. Following Prucha and Nadiri (1982, 1986) we assume that the stocks of the quasi-fixed inputs at the end of the planning horizon are determined endogenously subject to the assumption of static expectations and a constant firm size beyond the planning horizon. This means that under the finite horizon specification the firm minimizes (5) in each period t subject to the constraints $X_{t+\tau} = X_{t+T}$ and $E_t\pi_{t+\tau} = E_t\pi_{t+T}$ for $\tau \geq T$. As in the infinite horizon case the process is repeated every period as new information becomes available. The firm's objective function can now be written as

$$(6)\qquad \sum_{\tau=0}^{T} C(X_{t+\tau},X_{t+\tau-1},E_t\pi_{t+\tau})(1+r)^{-\tau} + \Psi(X_{t+T},E_t\pi_{t+T})$$

with

$$\Psi(X_{t+T},E_t\pi_{t+T}) = \sum_{\tau=T+1}^{\infty} C(X_{t+T},X_{t+T},E_t\pi_{t+T})(1+r)^{-\tau} = C(X_{t+T},X_{t+T},E_t\pi_{t+T})/[r(1+r)^{T}] .$$

Here $\Psi(X_{t+T},E_t\pi_{t+T})$ represents the present value of the cost stream incured by the firm from maintaining its operation beyond the (actual) planning horizon at the same level as at the end of the (actual) planing horizon.

2.2 Empirical Specification

For the empirical analysis we specialize the model to the use of two variable inputs, labor (L) and materials (M), and two quasi-fixed factors, the stock of physical capital (K) and the stock of R&D (R). In the subsequent discussion we use the following notation: $V_t = [V_{1t}, V_{2t}] = [L_t, M_t]$ where L_t and M_t denote, respectively, labor input and material input; $X_t = [X_{1t}, X_{2t}] = [K_t, R_t]$ where K_t and R_t denote, respectively, the end of period stocks of capital and R&D. Further $w_{2t} = v_t$ denotes the price of material goods, and $q_{1t} = q_t^K$ and $q_{2t} = q_t^R$ denote the investment deflators for capital and R&D normalized by the wage rate, respectively.

The technology is (dropping subscripts t) modeled in terms of the following normalized restricted cost functions

$$
\begin{aligned}
(7)\qquad & G(v, K_{-1}, R_{-1}, \Delta K, \Delta R, Y, T) = \\
& h(Y)\{\alpha_0 + \alpha_T T + \alpha_v v + \tfrac{1}{2}\alpha_{vv} v^2\} + \alpha_K K_{-1} + \alpha_R R_{-1} + \alpha_{\dot K}\Delta K + \alpha_{\dot R}\Delta R + \\
& \alpha_{vK} vK_{-1} + \alpha_{vR} vR_{-1} + \alpha_{v\dot K} v\Delta K + \alpha_{v\dot R} v\Delta R + \\
& \{\tfrac{1}{2}\alpha_{KK} K_{-1}^2 + \alpha_{KR} K_{-1} R_{-1} + \tfrac{1}{2}\alpha_{RR} R_{-1}^2 + \alpha_{K\dot K} K_{-1}\Delta K + \alpha_{K\dot R} K_{-1}\Delta R + \\
& \alpha_{R\dot K} R_{-1}\Delta K + \alpha_{R\dot R} R_{-1}\Delta R + \tfrac{1}{2}\alpha_{\dot K\dot K}\Delta K^2 + \alpha_{\dot K\dot R}\Delta K\Delta R + \tfrac{1}{2}\alpha_{\dot R\dot R}\Delta R^2\}/h(Y)
\end{aligned}
$$

where $h(Y) = Y^{\rho_0 + \rho_1 \ell n Y}$.

It is not difficult to see that the normalized restricted cost corresponding to a homothetic production function is in general of the form

$$
g\left(v, \frac{K_{-1}}{H(Y)}, \frac{R_{-1}}{H(Y)}, \frac{\Delta K}{H(Y)}, \frac{\Delta R}{H(Y)}, T\right) H(Y)
$$

where $H(Y)$ is a function in Y. (The scale elasticity is then given by $H(Y)/[Y(dH/dY)]$; compare also Section 5.) We note that $h(Y)$ can (apart from a scaling factor) be viewed as a second order translog approximation of $H(Y)$.

(Suppose we approximate $H(Y)$ in terms of a second order translog expansion, then $\ell nH(Y) \doteq \text{const} + \rho_0 \ell nY + \rho_1 \ell nY^2 = \text{const} + \ell n\{Y^{\rho_0+\rho_1 \ell nY}\}$ and therefore $H(Y) \propto Y^{\rho_0+\rho_1 \ell nY}$.) The restricted cost function (7) can hence be viewed as a second order approximation to that of a general homothetic production function. The functional form (7) is a generalization of the restricted cost function introduced by Denny, Fuss and Waverman (1981a) and Morrison and Berndt (1981) from the constant returns to scale case to the homothetic case. In case of constant returns to scale we have $\rho_0 = 1$ and $\rho_1 = 0$.

Following Denny, Fuss and Waverman (1981a) and Morrison and Berndt (1981) we impose parameter restrictions such that the marginal adjustment costs at $\Delta K = \Delta R = 0$ are zero: $\alpha_{\dot{K}} = \alpha_{\dot{R}} = \alpha_{v\dot{K}} = \alpha_{v\dot{R}} = \alpha_{K\dot{K}} = \alpha_{K\dot{R}} = \alpha_{R\dot{K}} = \alpha_{R\dot{R}} = 0$. We have furthermore tested the hypothesis that $\alpha_{KR} = \alpha_{\dot{K}\dot{R}} = 0$, which implies separability in the quasi-fixed factors. We could not reject this hypothesis; the subsequent analysis hence corresponds to this hypothesis which greatly simplifies the exposition.[6] The convexity of $G(.)$ in K, R, ΔK, ΔR and the concavity in v implies that $\alpha_{\dot{K}\dot{K}} > 0$, $\alpha_{\dot{R}\dot{R}} > 0$, $\alpha_{KK} > 0$, $\alpha_{RR} > 0$, $\alpha_{vv} < 0$.

The firm's cost in period t is now given by:

$$(8) \qquad C(X_t, X_{t-1}, \pi_t) = G_t + q_t^K I_t^K + q_t^R I_t^R + A_t ,$$

with

$$G_t = G(v_t, K_{t-1}, R_{t-1}, \Delta K_t, \Delta R_t, Y_t, T_t)$$
$$A_t = u_t[p_t Y_t - G_t - q_t^R I_t^R - D_t] - s_t q_t^K I_t^K,$$
$$D_t = \textstyle\sum_{i=0}^{M} d_t^i (1 - m_t s_t) q_t^K I_{t-i}^K,$$
$$I_t^K = K_t - (1-\delta_K) K_{t-1}, \qquad I_t^R = R_t - (1-\delta_R) R_{t-1} .$$

Here p denotes the output price deflator normalized by the wage rate, I^K and I^R denote gross investment in capital and R&D, and δ_K and δ_R the depreciation rates of capital and R&D knowledge, respectively. In defining taxes A, R&D expenditures are treated as immediately expensable; u is the corporate tax rate, s is the rate of tax credit for gross investment, m is the portion of the tax credit that must be deducted from the depreciable base, d^i the portion of investment that can be depreciated after i years.

We will explore the model under alternative assumptions on the planning horizon and expectations on output. Expectations on relative prices and tax parameters are taken as static. In case of an infinite planning horizon the firm's objective is defined as to minimize (5) subject to (7) and (8). We restrict the solution space for $\{K_{t+\tau}, R_{t+\tau}\}_{\tau=0}^{\infty}$ to the class of processes that are of mean exponential order less than $(1+r)^{1/2}$. Under static output expectations the control problem is standard; cp., e.g., Hansen and Sargent (1980, 1981), Kollintzas (1985, 1986) and Madan and Prucha (1988). The following conditions (corresponding to the derivatives of the objective function with respect to $K_{t+\tau}$ and $R_{t+\tau}$ for $\tau = 1,2,..$) need to be satisfied by the optimal sequence of the quasi-fixed factors with S=K,R:

$$
\begin{aligned}
(9a)\qquad & -\alpha_{\dot{S}\dot{S}} S_{t+\tau+1} + [\alpha_{SS} + (2+r)\alpha_{\dot{S}\dot{S}}] S_{t+\tau} - (1+r)\alpha_{\dot{S}\dot{S}} S_{t+\tau-1} = \\
& - [\alpha_S + \alpha_{vS} v_t + c_t^S] h(Y_t), \qquad \tau=0,1,\ldots,\infty,
\end{aligned}
$$

where

$$
(9b)\qquad c_t^S = \begin{cases} q_t^K (r+\delta_K)[1 - s_t - u_t(1-m_t s_t)B_t]/(1-u_t) & \text{if } S=K, \\ q_t^R (r+\delta_R) & \text{if } S=R, \end{cases}
$$

with

$$
(9c)\qquad B_t = \textstyle\sum_{i=0}^{M} d_t^i (1+r)^{-i}.
$$

The above described restriction of the solution space rules out the unstable roots of the above sets of second order difference equations. We denote the corresponding optimal input path for capital and R&D as $\{K_{t,\tau}\}_{\tau=0}^{\infty}$ and $\{R_{t,\tau}\}_{\tau=0}^{\infty}$. Solving (9) explicitly for the stable root and assuming $K_t = K_{t,0}$

and $R_t = R_{t,o}$ yields the accelerator equations

$$\Delta K_t = m_{KK}(K_t^* - K_{t-1}) \ , \tag{10a}$$

$$\Delta R_t = m_{RR}(R_t^* - R_{t-1}) \ ,$$

$$K_t^* = - \alpha_{KK}^{-1}[\alpha_K + \alpha_{vK}v_t + c_t^K]h(Y_t), \tag{10b}$$

$$R_t^* = - \alpha_{RR}^{-1}[\alpha_R + \alpha_{vR}v_t + c_t^R]h(Y_t),$$

$$m_{KK} = - (1/2)\{r_t+\alpha_{KK}/\alpha_{\dot{K}\dot{K}} - [(r_t+\alpha_{KK}/\alpha_{\dot{K}\dot{K}})^2+4\alpha_{KK}/\alpha_{\dot{K}\dot{K}}]^{1/2}\} \ , \tag{10c}$$

$$m_{RR} = - (1/2)\{r_t+\alpha_{RR}/\alpha_{\dot{R}\dot{R}} - [(r_t+\alpha_{RR}/\alpha_{\dot{R}\dot{R}})^2+4\alpha_{RR}/\alpha_{\dot{R}\dot{R}}]^{1/2}\} \ .$$

By Shephard's lemma we get the following demand equations for materials and labor:

$$M_t = \partial G(.)/\partial v_t = \{\alpha_v + \alpha_{vv}v_t\}h(Y_t) + \alpha_{vK}K_{t-1} + \alpha_{vR}R_{t-1}, \tag{11}$$

$$\begin{aligned} L_t = G(.) - v_tM_t = h(Y_t)\{[\alpha_o + \alpha_tT_t - \tfrac{1}{2}\alpha_{vv}v_t^2] + \alpha_KK_{t-1}/h(Y_t) \\ + \alpha_RR_{t-1}/h(Y_t) + \tfrac{1}{2}\{\alpha_{KK}[K_{t-1}/h(Y_t)]^2 + \alpha_{RR}[R_{t-1}/h(Y_t)]^2 \\ + \alpha_{\dot{K}\dot{K}}[\Delta K_t/h(Y_t)]^2 + \alpha_{\dot{R}\dot{R}}[\Delta R_t/h(Y_t)]^2\}. \end{aligned} \tag{12}$$

The estimating equations for the infinite horizon model are given by (10), (11) and (12), with random errors added to each of those equations.

In case of a finite planning horizon of, say, T+1 periods the firm's objective is defined as to minimize (6) subject to (7) and (8). Let $Y_{t,\tau} = E_tY_{t+\tau}$. The following conditions (corresponding to the derivatives of the objective function with respect to $K_{t+\tau}$ and $R_{t+\tau}$ for $\tau=0,\ldots,T$) need to be satisfied by the optimal sequence of the quasi-fixed factors with $S=K,R$:

$$\begin{aligned} - \alpha_{\dot{S}\dot{S}}S_{t+\tau+1} + [\alpha_{SS} + (1+\beta_{t,\tau+1})\alpha_{\dot{S}\dot{S}}]S_{t+\tau} - \beta_{t,\tau+1}\alpha_{\dot{S}\dot{S}}S_{t+\tau-1} = \\ - [\alpha_S + \alpha_{vS}v_t + c_t^S]h(Y_{t,\tau+1}), \qquad \tau=0,1,\ldots T-1, \\ [\alpha_{SS} + r\alpha_{\dot{S}\dot{S}}]S_{t+T} - r\alpha_{\dot{S}\dot{S}}S_{t+T-1} = \\ - [\alpha_S + \alpha_{vS}v_t + c_t^S]h(Y_{t,T}), \qquad \tau=T, \end{aligned} \tag{13}$$

with $\beta_{t,\tau} = (1+r)h(Y_{t,\tau+1})/h(Y_{t,\tau})$.[7] We denote the optimal input path for capital and R&D corresponding to the finite horizon model as $\{K^T_{t,\tau}\}^T_{\tau=0}$ and $\{R^T_{t,\tau}\}^T_{\tau=0}$. Assuming $K_t = K^T_{t,0}$ and $R_t = R^T_{t,0}$ we can write the first order conditions for $\tau=0$ as:

$$(14)\qquad \begin{aligned} \Delta K_t &= [\alpha_{KK} + (1+\beta_{t,1})\alpha_{\dot{K}\dot{K}}]^{-1}\{-[\alpha_K + \alpha_{vK}v_t + c^K_t]h(Y_{t,1}) \\ &\quad + \alpha_{\dot{K}\dot{K}}K^T_{t,1} - (\alpha_{KK} + \alpha_{\dot{K}\dot{K}})K_{t-1}\}, \\ \Delta R_t &= [\alpha_{RR} + (1+\beta_{t,1})\alpha_{\dot{R}\dot{R}}]^{-1}\{-[\alpha_R + \alpha_{vR}v_t + c^R_t]h(Y_{t,1}) \\ &\quad + \alpha_{\dot{R}\dot{R}}R^T_{t,1} - (\alpha_{RR} + \alpha_{\dot{R}\dot{R}})R_{t-1}\} . \end{aligned}$$

The demand equations for the variable factors, labor and materials, are the same as in the infinite horizon case. The estimating equations for the finite horizon model are hence given by (11), (12) and (14), with random errors added to each of those equations. The next period plan values $K^T_{t,1}$ and $R^T_{t,1}$ appearing in (14) are unobservable but implicitly defined by (13). In principle we could solve (13) to obtain explicit analytic expressions for $K^T_{t,1}$ and $R^T_{t,1}$, and substitute those expressions into (14). However, because of the complexity of the expressions involved this approach is quite impractical even for short planning horizons. We hence estimate the model using the algorithm developed in Prucha and Nadiri (1982, 1988) for the full information maximum likelihood estimator for systems of equations with implicitly defined variables.[8] This algorithm does not require an explicit analytic solution for $K^T_{t,1}$ and $R^T_{t,1}$ but solves (13) numerically for those values at each iteration step of the estimation algorithm, i.e. for each set of trial parameter values. For numerical efficiency Prucha and Nadiri (1982, 1988) show how the algorithm can be designed such that the gradient of the log-likelihood function can be evaluated from analytic expressions rather than by numerical differentiation.

We note that under static output expectations the "finite horizon" quasi-fixed factor demand equations (14) differ from the "infinite horizon" quasi-fixed factor demand equations (10) only in the expression for the next period plan values. In the infinite horizon case we have $K_{t,1} = m_{KK}(2 - m_{KK})K_t^* + (1 - m_{KK})^2 K_{t-1}$ and $R_{t,1} = m_{RR}(2 - m_{RR})R_t^* + (1 - m_{RR})^2 R_{t-1}$. It is not difficult to see that substituting these expressions for $K_{t,1}^T$ and $R_{t,1}^T$ in (14) yields (10).

3. Estimation and Empirical Results

We have estimated the production structure and factor demand for the Bell System using data from 1951 to 1979. Data on 1967 constant dollar gross output, capital, R&D, labor, and materials, as well as data on the rental prices of capital and R&D, the wage rate, and material prices, were taken from sources provided by AT&T. The sources and construction of the data are described in Nadiri and Schankerman (1981b). We used a simple time trend as our technology index and a real discount rate of 4 percent.[9] Data on output, stocks of capital and R&D, labor and materials were used in mean scaled form; prices were constructed conformably.

For the finite horizon model we considered several different forms of expectations, but because of need for brevity only the results obtained for two expectations processes are reported. First, in order to identify the true effect of changing the planning horizon we consider (as in the infinite horizon case) static expectations: $Y_{t,\tau} = Y_t$ for $\tau = 0,\ldots,T$. To generate the second form of expectations we first estimate an AR model for output[10] and then use the model to generate a sequence of rational expectations.

We have tested several hypotheses (in addition to the hypothesis that the adjustment paths of the two quasi fixed factors are separable, which was, as reported above, accepted). We first considered the hypothesis that the technology is homogeneous, i.e., ρ_1=0, and accepted this hypothesis.[11] The second hypothesis considered the absence of adjustment costs for both of the quasi-fixed factors, i.e. $\alpha_{\dot{K}\dot{K}} = \alpha_{\dot{R}\dot{R}} = 0$. This hypothesis was clearly rejected; similarly the hypotheses of the absence of adjustment costs was rejected individually for K and R. This suggests that a static equilibrium model is inappropriate to describe the technology and the structure of factor demand of the Bell System. A similar conclusion was reached by Schankerman and Nadiri (1986) using a different methodology.

In Table 1 we present the estimation results for the infinite horizon model with static expectations and of the 4-period and 10-period horizon models with static and rational (output) expectations. We allowed for autocorrelation of the disturbances in all equations. The estimation technique used was full information maximum likelihood. The results reported in Table 1 show good R^2's for all four equations and models. The DW-statistics generally do not suggest further autocorrelation. A comparison of the likelihoods corresponding to static and rational expectations on output suggests (somewhat informally) the rejection of the hypothesis of static expectations in favor of rational expectations.

The parameter estimates for the infinite and the 4-period horizon model under static expectations are very similar. The largest change occurs in the estimate for $\alpha_{\dot{R}\dot{R}}$ which is about 90 percent higher for the 4-period horizon model. The estimate of $\alpha_{\dot{K}\dot{K}}$ changes only by 5 percent. We hypothesize from this result that by expanding the planning horizon a bit more we should be able to duplicate (in a numerical sense) the results of the infinite horizon model under static expectations almost exactly with our finite horizon model. This is borne out by the results reported in Table 1 for the 10-period planning horizon. The results for the 10-period and infinite horizon model under static expectations are essentially identical.[12]

By allowing for nonstatic expectations we get further differences in our parameter estimates, especially for α_v, α_R, α_K, α_{KK}, $\alpha_{\dot{K}\dot{K}}$ and $\alpha_{\dot{R}\dot{R}}$. The

Table 1: FIML Estimates of the Demand Equations for Labor, Materials, Capital, and R&D for AT&T, 1951-1979

	Planning Horizon				
	Infinite	4-Period	4-Period	10-Period	10-Period
	Expectations				
	Static	Static	Rational	Static	Rational
α_0	5.639	5.653	5.231	5.642	5.191
	(.74)	(.75)	(.65)	(.74)	(.66)
α_T	-.552	-.572	-.659	-.554	-.631
	(.31)	(.33)	(.30)	(.31)	(.31)
α_v	3.316	3.348	3.529	3.316	3.493
	(.27)	(.27)	(.28)	(.27)	(.28)
α_K	-6.729	-6.749	-6.159	-6.731	-6.118
	(1.08)	(1.08)	(.91)	(1.08)	(.91)
α_R	-.265	-.242	-.203	-.264	-.228
	(.13)	(.13)	(.12)	(.13)	(.12)
α_{vK}	-1.653	-1.649	-1.749	-1.652	-1.760
	(.18)	(.18)	(.18)	(.18)	(.18)
α_{vR}	.265	.237	.192	.264	.227
	(.16)	(.16)	(.15)	(.16)	(.15)
α_{vv}	-2.610	-2.636	-2.750	-2.610	-2.717
	(.41)	(.41)	(.40)	(.41)	(.40)
a_{KK}	5.520	5.535	5.262	5.520	5.229
	(1.00)	(1.01)	(.89)	(1.00)	(.88)
α_{RR}	.130	.118	.108	.130	.127
	(.07)	(.07)	(.07)	(.07)	(.07)
$\alpha_{\dot{K}\dot{K}}$	.375	.389	1.195	.376	1.183
	(.59)	(.59)	(.86)	(.59)	(.85)
$\alpha_{\dot{R}\dot{R}}$	1.454	2.782	3.230	1.463	1.837
	(1.62)	(3.24)	(3.17)	(1.63)	(1.99)

Table 1 (continued)

	Planning Horizon				
	Infinite	4-Period	4-Period	10-Period	10-Period
	Expectations				
	Static	Static	Rational	Static	Rational
ρ_0	.638	.640	.649	.638	.647
	(.03)	(.03)	(.02)	(.03)	(.03)
ρ_1	-.033	-.035	-.038	-.033	-.036
	(.02)	(.02)	(.02)	(.02)	(.02)
ρ_L	.839	.835	.878	.839	.889
	(.09)	(.09)	(.07)	(.09)	(.07)
ρ_M	.608	.598	.658	.608	.671
	(.13)	(.13)	(.13)	(.13)	(.13)
ρ_K	.835	.830	.679	.834	.689
	(.10)	(.10)	(.13)	(.10)	(.13)
ρ_R	.672	.661	.606	.671	.609
	(.19)	(.19)	(.21)	(.19)	(.22)
Log of likelihood	301.845	301.845	306.664	301.846	306.738
L eqn.: R^2	0.953	0.952	0.971	0.953	0.972
DW	1.82	1.81	1.37	1.82	1.43
M eqn.: R^2	0.997	0.996	0.997	0.997	0.997
DW	2.08	2.08	1.77	2.08	1.78
ΔK eqn.: R^2	0.798	0.798	0.754	0.797	0.753
DW	2.16	2.16	2.62	2.15	2.64
ΔR eqn.: R^2	0.869	0.869	0.859	0.869	0.858
DW	1.58	1.58	1.59	1.58	1.62

* Asymptotic standard errors are given in parentheses. With ρ_L, ρ_M, ρ_K, ρ_R we denote the autocorrelation coefficient in the labor, material, capital and R&D equation, respectively.

estimates for $\alpha_{\dot{K}\dot{K}}$ form the 4-period horizon model with rational expectations is 200 percent larger than that from the 4-period horizon model with static expectations. We note, however, that the results for the 4-period and 10-period horizon model under rational expectations are again very similar.

The above results suggest that the optimal plans for the finite horizon model converge rapidly to those of the infinite horizon model as the planning horizon increases. (Similar evidence was reported by Prucha and Nadiri (1986) for a somewhat different model under rational price and output expectations.) We note that this result may be viewed as a justification for why it may be reasonable for a firm to only plan moderately ahead into the future (which is what firms actually do). Additional planning costs will very quickly exceed additional gains from extending the planning horizon. To put it differently, the length of the planning horizon does matter for the investment decision of the firm. However once a reasonable horizon is identified, the finite horizon model approximates the infinite horizon model very well.

In principle, we can estimate all of the technology parameters from the variable factor demand equations, i.e. the labor and material demand equation, alone. Those equations are essentially unaffected by the choice of the planning horizon and the form of the expectations. By estimating the labor and material equations jointly with the demand equations for capital and R&D we hope to increase the precision of our estimates. We can, however, only expect improvements if the demand equations for the quasi-fixed factors and in particular the expectations entering those equations are properly specified. In light of this remark we would not expect that different forms of expectations and different choices for the length of the planning horizon affect all estimates of model parameters equally. Not surprisingly we find the main changes in estimates for parameters that determine the adjustment path of capital and R&D (while the estimates of other parameters that determine characteristics of the technology such as scale are essentially unaffected).

4. Adjustment Costs, Price and Output Elasticities

4.1. Adjustment Process

For the infinite horizon model the optimal paths for the quasi-fixed factors capital and R&D are described by the flexible accelerator equations (10). In each period a fraction of the difference between the initial stocks of capital and R&D and the respective long-run optimal values are closed. (Note that the long-run optimal values, i.e. the targets, are changing over time in response to changes in the variables exogenous to the firm's input decisions.) These fractions correspond to the adjustment coefficients m_{KK} and m_{RR}. For the finite horizon model the optimal input path has no exact accelerator representation. Still, since the expressions for the adjustment coefficients in (10c) only depend on the technology parameters and the discount rate, we can pose the question of what values for m_{KK} and m_{RR} are implied by the estimates of the technology parameters obtained from the finite horizon models.

Table 2 contains estimates of implied adjustment coefficients for capital and R&D, m_{KK} and m_{RR}, for the infinite horizon model under static expectations and the four-period horizon model under static and rational expectations. The estimates of the adjustment coefficients for capital are quite different from those for R&D. The estimates for m_{KK} vary between 0.94 and 0.83 and those for m_{RR} between 0.24 and 0.21. The estimates for m_{KK} and m_{RR} obtained under the assumption of rational expectations are about 10 percent smaller than those obtained under the assumption of static expectations.

Table 2: Adjustment Coefficients for Capital and R&D

	m_{KK}	m_{RR}
Infinite Planning Horizon with Static Expectations	0.94	0.24
Finite 4-Period Planning Horizon with Static Expectations	0.94	0.23
Finite 4-Period Planning Horizon with Rational Expectations	0.83	0.21

Table 3: Percentage Deviations of Actual Values from the Long-Run Optimal Values in Selected Years

Variables	Year			
	1951	1960	1970	1979
Labor	25	13	11	10
Materials	34	2	5	2
Capital	-18	-6	-7	-6
R&D	-12	-30	-19	-20

• Percentage deviations are calculated as $(Z_t - Z_t^*)/Z_t^* \cdot 100$ for, respectively, $Z_t = L_t, M_t, K_{t-1}, R_{t-1}$, and $Z_t^* = L_t^*, M_t^*, K_t^*, R_t^*$.

To give some indication of the disequilibrium (from a long-run perspective) in the factor inputs we have calculated for the infinite horizon model with static expectations the percentage difference of actual values from long-run optimal values for respective inputs. These deviations are given in Table 3. The long-run optimal values for capital and R&D, K_t^* and R_t^*, are defined by (10); the long-run optimal values for labor and material, L_t^* and

M_t^*, are obtained by substituting K_t^* and R_t^* into (11).

At the beginning of the sample period labor and materials exceeded considerably the long-run optimal values; the reverse is true for capital and R&D. Over time there have been changes in the extent to which actual and long-run optimal values differ. For the variable inputs, labor and materials, as well as for capital the (absolute) difference between actual and long-run optimal values declined substantially in the 1950's. The gap between the actual stock of R&D and the long-run optimal value widened in the late 1950's, then declined throughout the 1960's. It widened again slightly in the 1970's; still, the size of the shortfall in the actual stock of R&D from the long-run optimal value in 1979 is about 65% higher than in 1951.

4.2. Elasticities

Tables 4 and 5 contain, respectively, price and output elasticities of the demand for capital, R&D, labor and materials. Elasticities calculated from estimates obtained from the 4-period, 10-period and infinite horizon model under static expectations were found to be quite similar. Likewise elasticities calculated from estimates obtained from the 4-period and 10-period horizon model under rational expectations were found to be similar. In Tables 4 and 5 we hence only report elasticities corresponding to estimates of the 4-period horizon model with static and rational output expectations, respectively.[13] Short-run, intermediate- and long-run elasticities are, respectively, evaluated at $t+\tau$ with $\tau=0$, $\tau=1$ and $\tau=\infty$.

The own-price elasticities for both capital and R&D are small. The long-run own-price elasticity for capital varies between -0.25 and -0.28, that for R&D between -0.12 and -0.14. The cross-price elasticities of R&D are

Table 4: Short-, Intermediate- and Long-Run Price Elasticities of Factor Demand for AT&T, 1967 Values*

Elasticities	4-Period Planning Horizon Static Expectations			4-Period Planning Horizon Rational Expectations		
	SR	IR	LR	SR	IR	LR
ε_{Kw}	0.147	0.158	0.158	0.141	0.164	0.169
ε_{Kv}	0.089	0.094	0.095	0.092	0.108	0.111
ε_{Kc^K}	-0.236	-0.252	-0.253	-0.233	-0.272	-0.279
ε_{Rw}	0.155	0.273	0.663	0.145	0.259	0.673
ε_{Rv}	-0.127	-0.224	-0.545	-0.114	-0.204	-0.529
ε_{Rc^R}	-0.028	-0.049	-0.119	-0.031	-0.055	-0.143
ε_{Mw}	0.861	0.655	0.782	0.856	0.655	0.716
ε_{Mv}	-0.861	-1.048	-1.177	-0.856	-1.030	-1.144
ε_{Mc^K}	0.000	0.401	0.429	0.000	0.381	0.456
ε_{Mc^R}	0.000	-0.008	-0.034	0.000	-0.006	-0.029
ε_{Lw}	-0.314	-0.488	-0.558	-0.311	-0.452	-0.522
ε_{Lv}	0.314	0.239	0.283	0.311	0.239	0.258
ε_{Lc^K}	0.000	0.245	0.260	0.000	0.210	0.250
ε_{Lc^R}	0.000	0.004	0.015	0.000	0.003	0.013

* With ε_{Zs} we denote, respectively, the elasticities of the factor Z = capital (K), R&D (R), materials (M), labor (L) with respect to s = wage rate (w), price of materials (v), rental price of capital (c^K), rental price of R&D (c^R). The symbols SR, IR and LR refer to the short-run, intermediate-run and long-run.

higher reflecting the small share of R&D. Material demand is quite elastic. The long-run own-price elasticity for materials takes on values between -1.14 and -1.18. The long-run own-price elasticity of labor varies between -0.52 and -0.56.

The long-run output elasticities of capital, R&D, materials and labor are estimated to fall between 0.64 and 0.65 reflecting scale economies in AT&T.

Table 5: Short-, Intermediate- and Long-Run Output Elasticities of Factor Demand for AT&T, 1967 Values*

Elasticities	4-Period Planning Horizon Static Expectations			4-Period Planning Horizon Rational Expectations		
	SR	IR	LR	SR	IR	LR
ε_{KY}	0.594	0.632	0.637	0.537	0.626	0.645
ε_{RY}	0.148	0.262	0.637	0.139	0.248	0.645
ε_{MY}	1.531	0.565	0.637	1.564	0.716	0.645
ε_{LY}	1.382	0.739	0.637	1.291	0.786	0.645

* With ε_{ZY} we denote, respectively, the elasticities of the factor Z = capital (K), R&D (R), materials (M), labor (L) with respect to output (Y). The symbols SR, IR and LR refer to the short-run, intermediate-run and long-run.

The respective estimates for the short-run output elasticity of capital are 0.60 and 0.54, those of R&D are 0.14 and 0.15. The estimates of the short-run output elasticity of materials and labor show that both factors overshoot in the short-run. The respective estimates of the short-run output elasticity of materials are 1.53 and 1.56, those of labor are 1.38 and 1.29.

The differences in the short-run output elasticities corresponding to estimates obtained under the assumptions of static and rational output expectations are sizable. Consider a change in output by, say, 7 percent. (The average growth rate of output for AT&T over the sample period was 7.33 percent.) Then the implied differences in the estimated short-run demand for the stock of capital, the stock of R&D, materials and labor would be approximately -0.4, -0.1, -0.2, and 0.6 percent. The average ratios of net capital and R&D investment to the stocks of capital and R&D over the sample period were 5.9 percent and 6.3 percent respectively. Therefore the above reported differences in the demand for stocks of capital and R&D translate themselves into big differences in investment demand. The labor bill and the materials bill of AT&T in 1967 were, respectively, 4329 and 1508 millions of dollars. The above reported differences in the demand for labor and materials hence imply significant dollar differences in how we expect variable costs to react to changes in output.

5. Technical Change and Scale

To avoid ambiguities we explicitly define measures for technical change and scale within the context of our cost of adjustment technology, i.e. within the context of temporary equilibrium. In the following we use again the general notation of Section 2 with $V = [V_1, V_2] = [L, M]$ and $X = [X_1, X_2] = [K, R]$.

Let $a(\vartheta, V, X_{-1}, \Delta X, T)$ be the factor by which output can be increased for given inputs if the technology index shifts by ϑ, i.e. $aF(V, X_{-1}, \Delta X, T) = F(V, X_{-1}, \Delta K, T+\vartheta)$. Similarly let $b(\vartheta, V, X_{-1}, \Delta X, T)$ be the factor by which all inputs can be decreased for a given level of output if the technology index shifts by ϑ, i.e. $F(V, X_{-1}, \Delta X, T) = F(bV, bX_{-1}, b\Delta X, T+\vartheta)$. We then define the following "output and input based" measures of technical change:

$$(15a) \qquad \lambda_Y = \frac{\partial a(.)}{\partial \vartheta}\Big|_{\vartheta=0} = \frac{1}{F}\frac{\partial F}{\partial T}$$

$$(15b) \qquad \lambda_X = -\frac{\partial b(.)}{\partial \vartheta}\Big|_{\vartheta=0} = \frac{\partial F}{\partial T}/[\textstyle\sum_{i=1}^{2}\frac{\partial F}{\partial V_i}V_i + \sum_{j=1}^{2}\frac{\partial F}{\partial X_{j,-1}}X_{j,-1} + \sum_{j=1}^{2}\frac{\partial F}{\partial \Delta X_j}\Delta X_j] .$$

Let $u(\lambda, V, X_{-1}, \Delta X, T)$ be the factor by which output increases if all inputs increase by the factor λ, i.e. $uF(V, X_{-1}, \Delta X, T) = F(\lambda V, \lambda X_{-1}, \lambda\Delta X, T)$. Then the returns to scale, say ε, are defined as

$$(15c) \qquad \varepsilon = \frac{\partial u(.)}{\partial \lambda}\Big|_{\lambda=1} = [\textstyle\sum_{i=1}^{2}\frac{\partial F}{\partial V_i}V_i + \sum_{j=1}^{2}\frac{\partial F}{\partial X_{j,-1}}X_{j,-1} + \sum_{j=1}^{2}\frac{\partial F}{\partial \Delta X_j}\Delta X_j]/F.$$

Of course $\varepsilon = \lambda_Y/\lambda_X$. We note that the definitions adopted here are analogous to those given in Caves, Christensen and Swanson (1981) and Caves, Christensen and Diewert (1982a,b) for technologies without adjustment costs.

The Lemma in the Appendix implies immediately the following relationships between the derivatives of the production function F and the restricted cost function $G = V_1 + w_2 V_2$:[14]

(16) $\partial F/\partial V_1 = 1/[\partial G/\partial Y]$, $\quad \partial F/\partial V_2 = w_2/[\partial G/\partial Y]$,

$\partial F/\partial X_{j,-1} = -[\partial G/\partial X_{j,-1}]/[\partial G/\partial Y]$, $\quad \partial F/\partial \Delta X_j = -[\partial G/\partial \Delta X_j]/[\partial G/\partial Y]$,

$\partial F/\partial T = -[\partial G/\partial T]/[\partial G/\partial Y]$.

Consequently λ_Y and λ_X, and hence ε, can be written as follows in terms of the restricted cost function G(.):

$$\text{(17a)} \qquad \lambda_Y = -\frac{\partial G}{\partial T}/[\frac{\partial G}{\partial Y}Y] \ ,$$

$$\text{(17b)} \qquad \lambda_X = -\frac{\partial G}{\partial T}/[G - \textstyle\sum_{j=1}^{2}\frac{\partial G}{\partial X_{j,-1}}X_{j,-1} - \sum_{j=1}^{2}\frac{\partial G}{\partial \Delta X_j}\Delta X_j],$$

$$\text{(17c)} \qquad \varepsilon = [G - \textstyle\sum_{j=1}^{2}\frac{\partial G}{\partial X_{j,-1}}X_{j,-1} - \sum_{j=1}^{2}\frac{\partial G}{\partial \Delta X_j}\Delta X_j]/[\frac{\partial G}{\partial Y}Y].$$

Given our estimate for the restricted cost function G we can now estimate technical change and scale from the above expressions. Our estimates for technical change and scale are quite stable over models. (As remarked above, this suggests that differences in the specification of expectations and the length of the planning horizon mainly affect the estimates of the dynamic characteristics of the model and not the estimates of basic technological characteristics.) The estimates were also quite stable over time. In the following we report results for the 4-period horizon model with rational expectations. The estimate of scale, ε, for 1967 is 1.60, suggesting that AT&T has experienced substantial economies of scale. This estimate is somewhat lower than that reported in Nadiri and Schankerman (1981b) and within the range of estimates reported in Christensen, Cummings and Schoech (1983). Our 1967 estimates for technical change λ_Y and λ_X are 0.60 and 0.37 percent. Denny, Fuss and Waverman (1981b) report similar results for Bell

Canada.

The expressions in (17) for output based and input based technical change and scale in terms of the restricted cost function were given previously in Nadiri and Prucha (1983, 1984). They generalize analogous expressions given Caves, Christensen and Swanson (1981) for a model without explicit adjustment costs and by Otha (1975) for a model where all factors are variable. (We note that the algebra employed here is completely analogous to that used by Caves, Christensen and Swanson.) All results generalize trivially to the case of m variable and n quasi-fixed factors. Furthermore, the results can be readily generalized along the lines of Caves, Christensen and Swanson to the multiple output case.[15]

The issue of a proper measure of technical change, given the firm is in short-run or temporary equilibrium but not in long-run equilibrium, has also been discussed, in particular, in recent papers by Berndt and Fuss (1981, 1986), Hulten (1986), and Morrison (1983, 1986). Those papers relate the proper measure of technical change to an adjustment of traditional measures in terms of a capacity utilization measure. Berndt, Fuss and Hulten consider technologies with constant returns to scale. Morrison allows for (possibly) non-constant returns to scale and works within an explicit dynamic framework. Given our analysis also allows for (possibly) non-constant returns to scale and is based on an explicit dynamic framework it seems of interest to relate our measures of technical change to those given by Morrison (and hence to that in the papers by Berndt, Fuss and Hulten). Define total cost and shadow cost as

$$\text{(18)} \qquad \begin{aligned} C &= G + \textstyle\sum_{j=1}^{2} c_j X_{j,-1} \\ C^{*} &= G + \textstyle\sum_{j=1}^{2} z_j X_{j,-1} + \textstyle\sum_{j=1}^{2} \dot{z}_j \Delta X_j \end{aligned}$$

where c_j and $z_j = -\partial G/\partial X_{j,-1}$ denotes the long-run rental price and the shadow value for the j-th quasi-fixed factor and $\dot{z}_j = -\partial G/\partial \Delta X_j$ denotes the shadow value of ΔX_j. Consider the following traditional measure of technical change defined in terms of the total cost function: $\varepsilon_{CT} = -(\partial C/\partial T)/C$. Then observing that $\partial C/\partial T = \partial G/\partial T$ it follows immediately from (17a,b) that

(19a) $\lambda_Y = \varepsilon \cdot \varepsilon_{CT} C/C^*$,

(19b) $\lambda_X = \varepsilon_{CT} C/C^*$.

Analogously, let $\varepsilon_{CY} = (\partial C/\partial Y)(Y/C)$ denote the output elasticity of total cost. Then observing that $\partial C/\partial Y = \partial G/\partial Y$ it follows immediately form (17c) that

(19c) $\varepsilon = \varepsilon_{CY}^{-1} C^*/C$.

Morrison's (1983,1986) measures of pure technical change (denoted in her paper by ε'_{FT} and ε'_{CT}) correspond exactly to the expressions on the right hand side of (19a) and (19b), and hence are identical to those considered here. Based on the expressions on the right hand side of (19a) and (19b) and the observation that C^*/C can be viewed as a measure of capacity utilization Morrison emphasizes that the derivation of a pure measures of technical change from ε_{CT} involves an adjustment in terms of capacity utilization to account for temporary equilibrium. The approach taken here, and previously by Nadiri and Prucha (1983, 1984), is to first look for a proper definition of technical change on the production side and then to demonstrate how this measure can be evaluated in terms of the restricted cost function. The two approaches complement each other in terms of interpretation. We emphasize the simplicity in the algebra employed here.

6. Decomposition of Total Factor Productivity and Output Growth

Traditional measures of productivity growth assume, in particular, (1) that all factors are variable, (2) that the technology exhibits constant returns to scale, (3) that output and input markets are perfectly competitive, and (4) that factors are utilized at a constant rate. If any one of those assumptions is not satisfied, traditional measures of total factor productivity growth will not be pure measures of technical change.[16] Given traditional measures of total factor productivity growth are widely used, it seems of interest to analyze the composition of those measures if those assumptions are possbily not satisfied. (The question how to properly estimate technical change under non-constant returns to scale and within a dynamic framework was discussed in Section 5.)

Denny, Fuss and Waverman (1981b) and Nadiri and Schankerman (1981a,b) consider technologies with non-constant returns to scale and provide, within a static framework, a decomposition of the traditional measure of total factor productivity growth into a part attributable to technical change and a part attributable to economies of scale. A similar decomposition exists for our cost of adjustment technology. More specifically, let $\dot{TFP}$ be the rate of growth of total factor productivity as measured by the conventional Divisia index and let ΔTFP be the corresponding Tornquist approximation defined as

$$\Delta TFP_t = \Delta \ell n Y_t - \Delta \ell n N_t, \tag{20a}$$

where $\Delta \ell n Y_t$ denotes the growth rate of output and $\Delta \ell n N_t$ denotes the growth rate of a cost share weighted index of aggregate inputs. The index of aggregate inputs, N, is defined by

(20b) $$\Delta \ell n N_t = \frac{1}{2} \sum_{i=1}^{4} [s_{z_i}(t)+s_{z_i}(t-1)]\Delta \ell n Z_{it}$$

with $Z_1 = V_1 = L$, $Z_2 = V_2 = M$, $Z_3 = X_{1,-1} = K_{-1}$, $Z_4 = X_{2,-1} = R_{-1}$. The cost shares are defined as $s_{z_i}(t) = w_{it}V_{it}/C_t$ for i=1,2 and $s_{z_i}(t) = c_{it}X_{i,t-1}/C_t$ for i=3,4. (Recall that $C_t = G_t + c_{1t}X_{1,t-1} + c_{2t}X_{2,t-1}$ denotes total cost and c_{jt} is the (long-run) rental price for the j-th quasi-fixed factor.) The following decomposition of ΔTFP was first given in Nadiri and Prucha (1983, 1984);[17] the proof is included in the Appendix for completeness:

(21) $$\Delta TFP_t = \Delta TFP_t^1 + \Delta TFP_t^2 + \Delta TFP_t^3 + \Delta TFP_t^4 ,$$

where

$$\Delta TFP_t^1 = \frac{1}{2} [\lambda_x(t) + \lambda_x(t-1)]$$

$$\Delta TFP_t^2 = (1-\varepsilon_t^{-1})\Delta \ell n Y_t$$

$$\Delta TFP_t^3 = -\frac{1}{2} \sum_{\tau=t,t-1} \{\frac{(\partial G_\tau/\partial X_{1,\tau-1}+c_{1\tau})X_{1,\tau-1}}{\varepsilon_\tau(\partial G_\tau/\partial Y_\tau)Y_\tau} [\Delta \ell n X_{1,t-1} - \Delta \ell n N_t^\tau]$$

$$-\frac{1}{2} \sum_{\tau=t,t-1} \{\frac{(\partial G_\tau/\partial X_{2,\tau-1}+c_{2\tau})X_{2,\tau-1}}{\varepsilon_\tau(\partial G_\tau/\partial Y_\tau)Y_\tau} [\Delta \ell n X_{2,t-1} - \Delta \ell n N_t^\tau] ,$$

$$\Delta TFP_t^4 = -\frac{1}{2} \sum_{\tau=t,t-1} \{\frac{(\partial G_\tau/\partial \Delta X_{1\tau})\Delta X_{1\tau}}{\varepsilon_\tau(\partial G_\tau/\partial Y_\tau)Y_\tau} [\Delta \ell n \Delta X_{1t} - \Delta \ell n N_t^\tau] ,$$

$$-\frac{1}{2} \sum_{\tau=t,t-1} \{\frac{(\partial G_\tau/\partial \Delta X_{2\tau})\Delta X_{2\tau}}{\varepsilon_\tau(\partial G_\tau/\partial Y_\tau)Y_\tau} [\Delta \ell n \Delta X_{2t} - \Delta \ell n N_t^\tau] .$$

Observe that $\varepsilon(\partial G/\partial Y)Y = C^*$ as is immediately seen from (17c) and the definition of the shadow cost in (18). The first term in the above decomposition of ΔTFP corresponds to technical change. The second term reflects the scale effect. The third term reflects the difference in the marginal conditions between short and long-run equilibrium, i.e. the difference between the shadow price and the (long-run) rental price, due to

Table 6: Decomposition of the Traditional Measure of Total Factor Productivity Growth for AT&T (in percentages)*

Period	Total Factor Productivity ΔTFP	Technical Change ΔTFP^1	Scale Effect ΔTFP^2	Temporary Equilibrium Effect ΔTFP^3	Direct Adjustment Cost Effect ΔTFP^4	Unexplained Residual
1952-1979	3.12	0.38	2.56	0.02	-0.05	0.22
1952-1965	2.82	0.30	2.13	0.02	-0.09	0.45
1965-1972	2.71	0.39	2.83	0.01	-0.00	-0.52
1972-1979	3.87	0.54	3.03	0.02	-0.02	0.29

*Based on the 4-period horizon model with rational expectations.

the adjustment costs. We refer to this effect as the temporary equilibrium effect. The fourth term reflects the direct effect of the presence of ΔX in the production function. We refer to this term as the direct adjustment cost effect. In long-run equilibrium both of the last two terms are zero since then $\partial G/\partial X_{j,-1} + c_j = \partial G/\Delta X_j = 0$. Furthermore both of the last two terms are zero if all factors (and hence the aggregate input index) grow at the same rate.

Based on (21) we have decomposed ΔTFP for different types of model specifications and different periods. We present in Table 6 the results for the four-period horizon model with rational expectations. The results for the other models were similar to those reported in this table. They indicate that the scale effect is by far the most important contributor to total factor productivity growth. The temporary equilibrium effect and the adjustment cost effect are negligible and technical change contributes about 10 to 12 percent to growth of total factor productivity. The contributions of scale and technical change are comparable to those reported by Denny, Fuss and Waverman

Table 7: Decomposition of Output Growth for AT&T, 1952-1979, Average Annual Rates of Growth (in percentages)*

Period	Output Growth	Labor Effect	Materials Effect	Capital Effect	R&D Effect	Adjustment Costs: Capital	Adjustment Costs: R&D	Technical Change
1952-1979	7.33	1.02	1.13	4.38	0.12	-0.01	-0.07	0.60
1952-1965	6.92	0.67	0.95	4.52	0.09	-0.01	-0.07	0.43
1965-1972	7.82	1.55	1.21	5.18	0.11	__**	-0.04	0.61
1972-1979	7.50	1.10	1.43	3.48	0.19	__**	-0.08	0.90

*Based on the 4-period horizon model with rational expectations.
**Smaller than one percent of a percentage point.

(1981b) for Bell Canada. Average total factor productivity growth for the Bell System was about 2.82 percent in 1952-1965; it declined slightly (2.71 percent) in 1965-1972, but increased substantially in 1972-1979 to 3.87 percent. This pattern is in sharp contrast to the behavior of total factor productivity growth at the level of the total economy and many of the industries (Nadiri (1981)) for the period 1972 to 1979.

The contributions of inputs, technical change and adjustment costs to growth of output are shown in Table 7. This decomposition is based on the approximation

$$\Delta \ell n Y_t = \tfrac{1}{2} \textstyle\sum_{i=1}^{6} [\varepsilon_{FZ_i}(t) + \varepsilon_{FZ_i}(t-1)] \Delta \ell n Z_{it} + \tfrac{1}{2}[\lambda_Y(t) + \lambda_Y(t-1)] \tag{22}$$

with $Z_1 = V_1 = L$, $Z_2 = V_2 = M$, $Z_3 = X_{1,-1} = K_{-1}$, $Z_4 = X_{2,-1} = R_{-1}$, $Z_5 = \Delta X_1 = \Delta K$, $Z_6 = \Delta X_2 = R$ and where the ε_{FZ_i}'s denote respective output elasticities. The output elasticities are computed from the estimates for the

Table 8: Decomposition of Labor Productivity Growth in AT&T Average Annual Rates of Growth (in percentages)*

Years	Labor Productivity Growth	Materials Effect	Capital Effect	R&D Effect	Adjustment Cost Effects Capital	Adjustment Cost Effects R&D	Technical Change	Scale Effect
1952-1979	5.61	0.80	3.04	0.09	-0.01	-0.06	0.60	0.98
1952-1965	5.75	0.77	3.66	0.07	-0.01	-0.07	0.43	0.54
1965-1972	4.99	0.66	2.91	0.06	—**	-0.02	0.61	1.57
1972-1979	5.86	1.04	2.21	0.14	0.01	-0.06	0.90	1.15

*Based on the 4-period horizon model with rational expectations.
**Smaller than one percent of a percentage point.

restricted cost function obtained for the four-period horizon model with rational expectations using the formulae given in (16). Decompositions based on estimates from other models were again very similar to those reported in Table 7. The average growth rate of output of the Bell System has been very high, about 7.33 percent per annum over the entire sample period. The contributions of various inputs to the growth of output differ considerably. The most significant source of the growth of output is the growth of capital which contributes more than 50 percent to the growth of output. Materials and labor inputs contribute about 14 percent while the contribution of technical change is about half as much. Growth of R&D contributes about 2 percent which, given its small share in the production cost, is fairly substantial. The same pattern of contributions are evident over the time periods of 1952-1965, 1965-1972 and 1972-1979. The results suggest that most of the growth of output is accounted for by the growth of the conventional inputs in the Bell system.

In Table 8 we look at the sources of the growth of labor productivity in the Bell System. The results are based on the approximation

$$(23) \qquad \Delta\ell n(Y_t/L_t) = \tfrac{1}{2}\textstyle\sum_{i=1}^{6}[\varepsilon_{FZ_i}(t)+\varepsilon_{FZ_i}(t-1)]\Delta\ell n(Z_{it}/L_t) + \tfrac{1}{2}[\lambda_Y(t) + \lambda_Y(t-1)] + \tfrac{1}{2}[\varepsilon_t + \varepsilon_{t-1} - 1]\Delta\ell nL_t.$$

The major component of labor productivity growth is again due to the growth of capital and to a much lesser extent due to technical change, materials and scale. Growth in R&D also contributed less than 1 percent to the growth of labor productivity; the adjustment costs played a relatively small role in reducing labor productivity.

7. Average Rate of Return on Physical and R&D Capital

In the following we define a measure for the rate of return on the investment expenditures on an individual factor in period t within the present framework of a dynamic factor demand model; cp. also Mohnen, Nadiri and Prucha (1986). In this paper we have assumed that the firm chooses its inputs such that it minimizes, for a given output stream, the discounted value of its costs. For expository reasons, consider for a moment a firm whose objective is to maximize the discounted value of its net revenue stream:

$$\sum_{\tau=0}^{\infty} \Pi(V_{t+\tau}, X_{t+\tau-1}, \Delta X_{t+\tau})/(1+r)^{\tau} \tag{24}$$

where $\Pi(V_t, X_{t-1}, \Delta X_t)$ denotes net revenues in period t. (Since price expectation have been taken to be static, we suppressed, for notational simplicity, prices in the argument list of Π.) Let $\{\hat{X}_{t+\tau}, \hat{V}_{t+\tau}\}_{\tau=0}^{\infty}$ denote the optimizing input sequence.

The firm is assumed to realize the initial portion of its investment plan. The firm's net investment expenditures on (say) the first quasi-fixed factor are then given by $\tilde{q}_{1t}\Delta X_{1t} = \tilde{q}_{1t}(\hat{X}_{1t}-X_{1,t-1})$. To calculate the net returns from this investment we have to compare these returns with the returns from an input sequence where that particular investment is not undertaken. To capture the pure effect of the firm's investment we assume that this alternative input sequence is conditionally optimal, i.e., optimal subject to the condition that the firm's investment in the first quasi-fixed factor in period t is not undertaken and hence zero. More formally, we consider as the alternative input sequence, say $\{\tilde{X}_{t+\tau}, \tilde{V}_{t+\tau}\}_{\tau=0}^{\infty}$, the input sequence that maximizes (24) subject to the constraint $\Delta X_{1t} = 0$. We now define as our rate of return the internal rate ρ that equates the present value of the differences in the two net return streams with the initial investment expenditure, i.e.,:

$$\tilde{q}_{1t}\Delta X_{1t} = \Pi(\hat{V}_t, X_{t-1}, \Delta\hat{X}_{1t}, \Delta\hat{X}_{2t}) - \Pi(\tilde{V}_t, X_{t-1}, 0, \Delta\tilde{X}_{2t}) + \sum_{\tau=1}^{\infty}\{\Pi(\hat{V}_{t+\tau}, X_{t+\tau-1}, \Delta\hat{X}_{t+\tau}) - \Pi(\tilde{V}_{t+\tau}, X_{t+\tau-1}, \Delta\tilde{X}_{t+\tau})\}/(1+\rho)^{-\tau}. \tag{25}$$

The definition generalizes in an obvious way to the case of a finite planning horizon. Further, in case of a cost-minimizing firm we can think of establishing the respective input sequences by optimizing (24) subject to the output constraint. Formally, we can then still use (25) for the calculation of the average rate of return on investment. However, since gross revenues are identical for both input sequences we then effectively compare the difference in cost streams.

In Table 9 we present the estimated internal rates of return on net investment in plant and equipment and R&D for the period 1952-1979 and for three subperiods for both finite and infinite planning horizons and for both static and rational expectations on output. These rates are net of the adjustment costs and depreciation of the two quasi-fixed inputs. They are calculated using equation (25). The gross rate of return will be of course much higher. The gross rate of return on capital will average about 13 percent and that on R&D about 30 percent. The magnitude and pattern of these rates are quite comparable to what has been reported in the literature.[18]

Several interesting points about these results should be noted: First, the net average rates of return for capital and R&D are quite different; the rate of return on R&D is about two to five times larger than that on capital. This result is consistent with the results reported in the literature

Table 9: Internal Rates of Return on Net Investment in Capital and R&D (in percentages)

Time Span	Infinite Planning Horizon Static Expectations		4-Period Planning Horizon Static Expectations		4-Period Planning Horizon Rational Expectations	
	Capital	R&D	Capital	R&D	Capital	R&D
1952-1979	7	21	7	24	7	22
1952-1965	9	17	9	22	9	19
1965-1972	6	19	7	23	7	21
1972-1979	4	29	4	31	5	27

which show that the rate of return on R&D is much higher than that on physical capital. Second, there are variations in the rates of return over time for both capital and R&D. The return on physical capital is fairly stable from 1951 to 1972 at about 7 to 8 percent and then declines to an average rate of return of 4 to 5 percent. The average rate of return on R&D is not only higher than that on physical capital but seems to rise over time and therefore the gap between the two rates widens substantially.

8. Conclusions

In Prucha and Nadiri (1982, 1986, 1988) we developed a methodology that allows for the estimation of systems of dynamic factor demand without strong a priori restrictions on the functional form of the technology and the expectation formation process. In this paper we applied this methodology to estimate the production structure and dynamic factor demand of AT&T. We considered alternative assumptions concerning the planning horizon and the form of expectations. The technology was modeled by a new restricted cost function. This function generalizes the restricted cost function introduced by Denny, Fuss and Waverman (1981a) and Morrison and Berndt (1981) from the linear homogeneous to the homothetic case. The paper computes various short-, intermediate- and long-run price and output elasticities. Furthermore, we present proper measures of technical change for technologies where some of the factors are quasi-fixed and shows how those measures can be evaluated in terms of the restricted cost function. Those measures were first introduced in Nadiri and Prucha (1983,1984) and are related here to measures introduced by Morrison (1983, 1986). The paper also provides a decomposition of (the traditional measure of) total factor productivity growth into technical change and components that are attributable to scale and the adjustment costs.

Our empirical results suggest the following:

(1) The optimal plans for the finite horizon model converge rapidly to those of the infinite horizon model as the planning horizon extends. (The obtained estimation results for the 10-period and the infinite horizon model are found to be nearly identical; Prucha and Nadiri (1986) report similar results.) This observation suggests that additional planning costs will quickly exceed additional gains from extending the planning horizon. This

observation may hence serve as a rational for why many firms only plan moderately into the future.

(2) Not all parameter estimates are equally sensitive to alternative specifications of the expectation formation process. On the one hand estimates of parameters determining the adjustment path of capital and R&D turned out to be sensitive. This, of course, would in turn affect conclusions concerning the effects of tax and monetary policies on investment. On the other hand estimates of other characteristics of the underlying technology such as scale seem to be insensitive to the specification of the expectation formation process.

(3) Using our model we calculate the rates of return on physical and R&D capital. The net rate of return on R&D is about two to five times larger than that on capital. Also the gap between the two rates widens over the sample period. The average net rate of return on R&D investment over the period 1952 to 1979 is approximately 20 percent.

(4) The model generates reasonable estimates of the price and output elasticities for the variable and quasi-fixed inputs in the short-, intermediate- and long-run. We find evidence that the variable inputs overshoot in the short-run their long-run targets and that in particular the estimates of the short-run elasticities are sensitive to the specification of the expectation formation process.

(5) The obtained estimates for output and input based technical change are approximately 0.60 and 0.37, those for the returns to scale are approximately 1.60.

(6) The estimates of the adjustment coefficients suggest a fairly short adjustment period for physical capital and a long adjustment period of about

four to five years for R&D. Our estimates of the adjustment coefficients are sensitive to the form of expectations, but insensitive to the length of the planning horizon (unless it is chosen very short).

(7) Our decomposition of the traditional measure of total factor productivity growth shows that approximately 80 percent of the growth is due to scale effects and only approximately 10 percent is due to pure technical change. That is, the traditional measure of total factor productivity growth would seriously mismeasure technical change in the U.S. Bell System.

Appendix: Decomposition of Total Factor Productivity Growth

Lemma: Let y, m, n, k, and ω be elements of $\mathbb{R}$, $\mathbb{R}$, $\mathbb{R}^p$, $\mathbb{R}^q$ and $\mathbb{R}^p_+$, respectively. Consider the function

(A.1) $$y = f(m,n,k)$$

that maps elements of $\mathbb{R}\times\mathbb{R}^p\times\mathbb{R}^q$ into $\mathbb{R}$. Let $m = m(n,k,y)$ be the unique solution of (A.1) for m for any n, k, y. Let $n = n(\omega,k,y)$ be the unique solution of

(A.2) $$(\partial/\partial n)(m(n,k,y) + \omega' n) = 0$$

for any ω, k, y. Define

(A.3) $$\gamma(\omega,k,y) = m(n(\omega,k,y),k,y) + \omega' n(\omega,k,y),$$

then:

(A.4) $$\partial f/\partial m = 1/[\partial\gamma/\partial y], \quad \partial f/\partial n = \omega/[\partial\gamma/\partial y], \quad \partial f/\partial k = -[\partial\gamma/\partial k]/[\partial\gamma/\partial y],$$

(A.5) $$\partial\gamma/\partial\omega = n, \quad \partial\gamma/\partial k = \partial m/\partial k, \quad \partial\gamma/\partial y = \partial m/\partial y.$$

(Note that we have implicitly assumed that $f(.)$, $m(.)$ and $\gamma(.)$ are differentiable).

<u>Proof</u>: The proof is standard; compare, e.g., the proof of Shephard's or Hotelling's lemma. By definition, $y \equiv f(m(n,k,y),n,k)$. Differentiation yields

(A.6) $$1 = [\partial f/\partial m][\partial m/\partial y], \quad 0 = [\partial f/\partial m][\partial m/\partial n] + \partial f/\partial n,$$
$$0 = [\partial f/\partial m][\partial m/\partial k] + \partial f/\partial k.$$

Furthermore, differentiation of (A.3) and observing (A.2) yields

(A.7) $$\partial\gamma/\partial\omega = n, \quad \partial\gamma/\partial k = \partial m/\partial k, \quad \partial\gamma/\partial y = \partial m/\partial y.$$

Equations (A.4) and (A.5) follow immediately from (A.6) and (A.7). □

In the following we give a proof for the decomposition of total factor productivity growth as stated in equation (21). Recall the definition of the shadow prices z_j and $\dot{z}_j$, the shadow cost C^*, and the total cost C given in Section 5. Recall further that (17c) and (18) imply that $C^* = \varepsilon(\partial G/\partial Y)Y$. Substitution of (16) into the decomposition of output growth (22) then yields:

$$\text{(A.8)} \qquad \Delta\ell nY_t = \tfrac{1}{2}\,[\Delta\ell nY_t^t + \Delta\ell nY_t^{t-1}], \quad \Delta\ell nY_t^\tau = \varepsilon_\tau[\textstyle\sum_{i=1}^2 w_{i\tau}V_{i\tau}\Delta\ell nV_{it} + \sum_{j=1}^2 z_{j\tau}X_{j,\tau-1}\Delta\ell nX_{j,t-1} + \sum_{j=1}^2 \dot{z}_{j\tau}\Delta X_{j\tau}\Delta\ell n\Delta X_{jt}]/C_\tau^* + \lambda_Y(\tau),$$

$\tau=t,t-1$. (We have implicitly assumed that the ΔX_{jt}'s are positive.) Next we rewrite (20b) as

$$\text{(A.9)} \qquad \Delta\ell nN_t = \tfrac{1}{2}\,[\Delta\ell nN_t^t + \Delta\ell nN_t^{t-1}], \quad \Delta\ell nN_t^\tau = [\textstyle\sum_{i=1}^2 w_{i\tau}V_{i\tau}\Delta\ell nV_{it} + \sum_{j=1}^2 c_{j\tau}X_{j,\tau-1}\Delta\ell nX_{j,t-1}]/C_\tau \,,$$

$\tau=t,t-1$. Furthermore observe that the definition of $\Delta\ell nN_t^\tau$ implies

$$\text{(A.10)} \qquad \textstyle\sum_{i=1}^2 w_{i\tau}V_{i\tau}(\Delta\ell nV_{it}-\Delta\ell nN_t^\tau) = -\sum_{j=1}^2 c_{j\tau}X_{j,\tau-1}(\Delta\ell nX_{j,t-1}-\Delta\ell nN_t^\tau).$$

It follows from (A.8) that

$$\begin{aligned}\text{(A.11)} \qquad & \Delta\ell nY_t^\tau - \Delta\ell nN_t^\tau = (1-1/\varepsilon_\tau)\Delta\ell nY_t^\tau + 1/\varepsilon_\tau\Delta\ell nY_t^\tau - \Delta\ell nN_t^\tau = \\ & (1-1/\varepsilon_\tau)\Delta\ell nY_t^\tau + [\textstyle\sum_{i=1}^2 w_{i\tau}V_{i\tau}(\Delta\ell nV_{it} - \Delta\ell nN_t^\tau) + \\ & \textstyle\sum_{j=1}^2 z_{j\tau}X_{j,\tau-1}(\Delta\ell nX_{j,t-1} - \Delta\ell nN_t^\tau) + \sum_{j=1}^2 \dot{z}_{j\tau}\Delta X_{j\tau}(\Delta\ell n\Delta X_{jt} - \\ & \Delta\ell nN_t^\tau)]/C_\tau^* + \lambda_Y(\tau)/\varepsilon_\tau = \\ & (1-1/\varepsilon_\tau)\Delta\ell nY_t^\tau + [\textstyle\sum_{j=1}^2 (z_{j\tau}-c_{j\tau})X_{j,\tau-1}(\Delta\ell nX_{j,t-1} - \Delta\ell nN_t^\tau) + \\ & \textstyle\sum_{j=1}^2 \dot{z}_{j\tau}\Delta X_{j\tau}(\Delta\ell n\Delta X_{jt} - \Delta\ell nN_t^\tau)]/C_\tau^* + \lambda_X(\tau).\end{aligned}$$

The last equality was obtained by utilizing (A.10). The decomposition in (21) now follows upon observing that $\Delta TFP = \frac{1}{2}[\Delta\ell nY_t^t - \Delta\ell nN_t^t] + \frac{1}{2}[\Delta\ell nY_t^{t-1} - \Delta\ell nN_t^{t-1}]$ (The expression for the scale effect is for reasons of notational simplicity given under the assumption that $\varepsilon_t \doteq \varepsilon_{t-1}$.)

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Endnotes

1 An earlier version of this paper (Nadiri and Prucha (1983)) was first presented at the Workshop on Investment and Productivity of the Summer Institute of the National Bureau of Economic Research, Cambridge, July 1983. A first revision was circulated as Nadiri and Prucha (1984). (This revision was submitted as a contribution to a book that remained in the stage of preparation.) The present revision connects the material with recent developments in the theory of dynamic factor demand and productivity measurement. We would like to thank Pierre Mohnen for his assistance. We also gratefully acknowledge the financial support of the National Science Foundation, Grant PRA-8108635, and the Research Board of the Graduate School of the University of Maryland. Furthermore we thank the computer centers of New York University and the University of Maryland for their support with computer time.

2 Christensen, Cummings and Schoech (1983) and Nadiri and Schankerman (1981b) specify a restricted variable cost function and demand equations for the variable factors, but do not estimate dynamic demand equations for the quasi-fixed factors. Similar models have been estimated using Bell Canada data; see Denny, Fuss and Waverman (1981b).

3 Nadiri and Schankerman (1981b) do treat R&D as a factor of production and Christensen, Cummings and Schoech (1983) use R&D as a proxy for an index of technology.

4 We take the production function to be twice differential in all arguments. Let f be some function and let z be some argument of f. Then f_z denotes the partial derivative of f with respect to z.

[5] As an alternative to (5) we could have stated the firm's objective function in period t as

$$(5') \qquad E_t\sum_{\tau=0}^{\infty} C(X_{t+\tau}, X_{t+\tau-1}, \pi_{t+\tau})(1+r)^{-\tau} .$$

It is well known that in case $C(.)$ is linear-quadratic the (certainty equivalence feedback control) solution for X_t corresponding to (5) is identical to that implied by the (closed loop feedback control solution) corresponding to (5'). This result is typically referred to as the certainty equivalence principle. If $C(.)$ is not linear-quadratic certainty equivalence will generally not hold. Malinvaud (1969) derives, however, for this latter case a first-order certainty equivalence result under reasonable conditions. We note that the formulation in (5) may be interpreted as a limited information formulation in that it only depends on knowledge of the first moment of the exogenous variables $\{\pi_{t+\tau}\}_{\tau=0}^{\infty}$, while the formulation in (5') depends (in general) on the knowledge of their entire distribution. For an interesting limited information formulation based on the knowledge of the first and second moments see Bitros and Kelejian (1976).

[6] We have tested the hypothesis that $\alpha_{\ddot{X}\dot{R}} = \alpha_{XR} = 0$ both from the infinite horizon model and from the finite horizon model via the likelihood ratio test. To estimate the model in the infinite horizon case under the alternative we followed the approach developed in Epstein and Yatchew (1985) and Madan and Prucha (1988). In estimating the finite horizon model we followed the approach developed in Prucha and Nadiri (1982, 1988).

[7] Note that $\beta_{t,\tau+1}$ reduces to $(1+r)$ in the case of static output expectations.

[8] We note that the algorithm can be readily modified to apply to alternative objective functions.

[9] We have estimated the model with alternative discount rates and found the results quite insensitive to this specification.

[10] The autoregressive model for output was of the form (t-ratios are given in parentheses)

$$Y_t = -\underset{(0.73)}{0.00373} + \underset{(9.12)}{1.56874}\, Y_{t-1} - \underset{(3.66)}{1.10271}\, Y_{t-2} + \underset{(2.93)}{0.62257}\, Y_{t-3}$$

$$R^2 = .999, \; DW = 1.82.$$

[11] The value of the likelihood ratio test statistic was 1.86 compared to the critical value of 3.84.

[12] To examine the effect of the length of the planning period we estimated the finite horizon model with planning horizons of two, four, five and ten periods. Whatever changes can be observed seem to follow patterns that are smooth with respect to the length of the planning horizon. To conserve space, we report in Table 1 only the estimates for the four and ten period planning horizon.

[13] The elasticities are both a function of the model parameters and expectations. The elasticities reported for the two sets of parameter estimates are in both cases evaluated under static expectations. Therefore any difference in the elasticities are solely due to differences in the parameter estimates.

[14] In applying the Lemma we take y=Y, m=V_1, n=V_2, k=$[X_{-1}, \Delta X, T]$, ω=w_2, $f(.)$=F(.) and $\gamma(.)$=G(.). The results summarized in the Lemma are standard. The Lemma is only given for completeness.

15 Suppose Y, V and X are k, m, and n dimensional vectors. Then

$$\lambda_Y = -\frac{\partial G}{\partial T}/[\textstyle\sum_{s=1}^{k}(\partial G/\partial Y_s)Y_s] ,$$

$$\lambda_X = -\frac{\partial G}{\partial T}/[G - \textstyle\sum_{j=1}^{n}(\partial G/\partial X_{j,-1})X_{j,-1} - \sum_{j=1}^{n}(\partial G/\partial \Delta X_j)\Delta X_j],$$

and $\varepsilon = \lambda_Y/\lambda_X$.

16 For a general discussion of problems in measuring technical change see Griliches (1988).

17 Compare also Nadiri and Prucha (1989).

18 See, e.g., Schankerman and Nadiri (1986) on the Bell System data, and Ravenscraft and Scherer (1982) and Clark and Griliches (1984) on U.S. firm data.