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HOW ELASTIC IS THE GOVERNMENT'S DEMAND FOR WEAPONS?

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ABSTRACT

We attempt to make inferences about the elasticity of the government's demand for specific weapons by analyzing the statistical relationship between quantity and cost revisions across the population of major weapon systems, using data contained in the Pentagon's Selected Acquisition Reports. The cost revisions are due in part to the arrival of technological information generated in the course of research and development. When we standardize the data by program base year, we find that the elasticity of demand is .55, and is significantly different from both zero and unity. Thus, the government's demand for specific weapons is inelastic, but not perfectly inelastic. The estimates also imply that weapons acquisition is characterized by increasing returns: the mean and median values of the elasticity of total cost with respect to quantity are .78 and .72, respectively.

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The weapons acquisition process has long been recognized to be characterized by considerable uncertainty. A "major thesis" of Peck and Scherer's (1962) seminal monograph on defense procurement was that "there is uniqueness in both the magnitude and the diverse sources of uncertainty in weapons acquisition" (p. 17). They defined two broad classes of uncertainties: internal and external.

> Internal (or technological) uncertainties relate to the possible incidence of unforeseen technical difficulties in the development of a specific weapon system. External uncertainties relate to factors external to an individual project and yet affecting the course and outcome of the project (p. 24).

The extent of internal (and possibly also of external) uncertainty about a weapon system is greatest at the beginning of its "life-cycle." As resources are devoted to the system's research and development (R&D), information about the true cost of acquiring the system is generated, and the degree of technological uncertainty is reduced.

The question analyzed in this paper is, how do defense decision makers -- the people in the Pentagon and Congress who make decisions about the allocation of defense resources -- respond to the arrival of new information concerning the cost of weapons acquisition? Because in most economic settings it is inefficient <u>not</u> to change behavior in response to new information, this question relates to the degree of <u>efficiency</u> of defense procurement, an issue of considerable concern to policy makers and the public.[1]

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[1]In a recent paper, Rogerson (1988b) provides a theoretical model which accounts for the stylized fact that the equilibrium "quality," or

More specifically, we seek to determine the elasticity of the government's demand for individual weapon systems. When the government's estimate of the cost of acquiring a given weapon changes as a consequence of data generated in the course of R&D, how much (if at all) does the desired "buy" (quantity) change? An important determinant of the elasticity of demand for a specific weapon is the degree of substitutability between it and other weapons actually or potentially being acquired. Scherer (1964, pp. 51-53) suggests that even systems that have no obvious technical or operational substitutes are "threatened" by rival systems in the bureaucratic competition for budgetary support. In the early 1960s, for example, (offensive) Polaris missiles and the Nike Zeus ballistic missile defense programs were regarded by top Defense Department (DOD) officials as substitutes, in effect.

Early in a program's life-cycle, there is often intense competition among a few potential suppliers to develop and produce the weapon. But once the design and technical competition is over, the system is likely to be produced on a sole-source basis. The winning contractor then enjoys a monopoly with respect to the supply of the system. The extent of the contractor's market power, and his ability to earn monopoly profits, are inversely related to the elasticity of the government's demand. The higher the demand elasticity, the lower the price the contractor will seek to set and the lower his profit.[2] It may also be

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extent of technical sophistication, of weapon systems tends to be <u>inefficiently high</u>.

[2]Elastic demand is not the only feature of the environment that might reduce monopoly profits. As Rogerson (1988a) has argued, such profits tend to be dissipated through contract- or "rent-seeking" expenditures on R&D during the design competition. Lichtenberg (1988a)

the case that the higher the demand elasticity, the lower the optimal level of government expenditure to monitor and regulate the costs and profits of defense contractors.

Our research strategy for determining the demand elasticity is to examine empirically the relationship between revisions in cost estimates and revisions in quantity estimates across the population of major weapon systems. The revisions are from original or "baseline" estimates (made around the start of full-scale development) to "current" estimates (made at a subsequent date).

A number of previous investigators have presented and analyzed data on weapon system cost "variance factors," defined as the ratio of actual (ex post) system cost to the baseline (ex ante) estimate. Peck and Scherer (1962, p. 22) found that the mean cost variance factor in a sample of 11 programs was 3.2; only one program had a factor less than 2. These findings were consistent with earlier results obtained by Marshall and Meckling (1962).[3] But Rich and Dews (1986, p. 12) found that "acquisition programs of the 1970s and 1980s experienced less percentage cost growth than acquisition programs of the 1960s"; they also found that nondefense programs (except for highway and water projects, generally characterized by only modest technical risk) "experienced greater cost growth than the defense programs, in some cases much greater" (p. 10). Some limited attempts have been made to explain differences across

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provides evidence that firms incur considerable unreimbursed R&D costs in connection with these competitions.

[3]Tirole (1986) develops a theoretical model of contract negotiations which provides a "justification" for equilibrium cost overruns.

systems in cost variance factors in terms of the magnitude of the technical advance sought.

Previous authors have also, in some cases, provided distributions of time and performance variance factors; these are, respectively, the ratio of ex post to ex ante development time (in years) and system performance (e.g., aircraft speed). But we are not aware of any previous analysis of quantity variances, or of attempts to relate these to cost variances in order to make inferences about the elasticity of demand.

In the next section we present a simple model of revisions in weapons system quantity and cost estimates. The following section describes how the data contained in the Defense Department's Selected Acquisition Reports can be used to estimate the parameters of this model. Empirical results are reported and discussed in the next section. The paper closes with a summary and concluding remarks.

I. A Model of Quantity and Cost Revisions

How do defense decision-makers determine the quantities of various weapon systems that the government will buy? One might hypothesize that decision-makers choose those quantities that maximize the amount of an intangible ultimate good called "national security," subject to a budget constraint. There is a technology for producing national security, a "national security production function." The arguments of this function are the quantities of N different weapon systems (X_1, X_2,...,X_N), and the quantity of (a vector of) other inputs Y (such as the number of troops deployed). DOD does not face a perfectly elastic supply of any given weapon system at an exogenously-determined price. Rather, there is a cost function for each system, which indicates the (expected total) cost of producing various quantities of the system. DOD maximizes the national security production function subject to the cost functions (which may be nonseparable) and to an overall resource constraint.[4]

Although we believe that specifying a national security production function might be a useful approach to analyzing procurement behavior, we will adopt the simpler approach of directly specifying demand functions, rather than deriving them from a production function. The particular demand functions we specify implicitly impose (quite strong) restrictions on the form of the production function (e.g., the elasticity of substitution between weapon i and weapon j ($i,j = 1,\ldots,N$) is the same for all i and j).

We postulate that at any given time in the life-cycle of a weapons system, the Pentagon has estimates of (the slopes and intercepts of) both the marginal cost schedule (the supply curve) and the marginal benefit schedule (the demand curve) of the system. In particular, the Pentagon has such estimates at two times: the date at which full-scale development begins (time 0), and at a later date (time t). (Estimates made at time 0 are referred to as "baseline estimates.") We assume that the supply and demand schedules are log-linear. The baseline schedules may be written

$$\ln MC = \delta_0 - \alpha \ln Q \tag{1}$$

$$\ln MB = \theta_0 - \beta^{-1} \ln Q \tag{2}$$

[4]This model obviously abstracts from purely "political" factors that may influence procurement decisions. See Lichtenberg (1989) for a discussion of theory and evidence concerning the possible link between procurement decisions and campaign contributions by government contractors.

where MC denotes marginal cost, MB denotes marginal benefit, Q denotes quantity, and β is the elasticity of demand.[5] For simplicity, we also assume that the baseline quantity chosen by the government is the one satisfying the equality between ℓn MC and ℓn MB[6]:

$$\ell n\ MC = \ell n\ MB \Rightarrow \ell n\ Q_0 = \frac{\theta_0 - \delta_0}{\beta^{-1} - \alpha}. \qquad (3)$$

In order for Q_0 to be an equilibrium quantity, it must be the case that $\beta^{-1} > \alpha$: the demand curve must be more negatively sloped than the supply curve. Because, as we see below, weapons systems typically exhibit decreasing marginal costs, the condition is not a trivial one. Figure 1 illustrates the determination of baseline equilibrium quantity.

As time passes following the start of full-scale development, decision makers will revise their estimates of the supply and/or demand

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[5]The objection might be raised that the MB schedule is unlikely to be log-linear in Q, since there will be a threshold number below which the force is not viable. Below this threshold, MB will be zero (or perhaps negative if adding a unit to an unsurvivable force increases an opponent's incentive for preemptive attack), and it will either jump or rise slowly before starting to fall with Q. However, it is not clear that the sample includes observations where threshold force size effects are important.

[6]By making this assumption, we are abstracting from several problems often thought to severely affect weapons procurement, much as moral hazard, risk aversion and asymmetric information. As Baron and Besanko (1988, p. 342) demonstrate, under these conditions the equality MC=MB will not hold at the second-best optimum. Because we are analyzing <u>changes</u> in equilibrium quantity, however, there are two alternative conditions under which deviations of the difference $\Delta \equiv (\ln MC - \ln MB)$ from zero will not affect the consistency of our estimates. First, if Δ is unchanging over the life of a weapons system, then Δ will cancel out when we analyze revisions in Q over the life cycle. Second, even if Δ does vary over the life cycle, our estimates of α and β will be consistent provided that changes in Δ are uncorrelated across weapons systems with changes in the intercept of the MC schedule.

schedules. Information generated during the course of development about the cost or difficulty of acquiring the system would result in supply-curve revisions.[7] Changes in the actual or perceived nature of the "threat" from enemy forces, and revisions in supply-curve estimates of other (complementary or substitute) systems under development would result in demand-curve revisions. We represent the Pentagon's estimates of the supply and demand curves at time t ($t > 0$) as follows:

$$\ell n\ MC = \delta_t - \alpha\ \ell n\ Q \tag{4}$$

$$\ell n\ MB = \theta_t - \beta^{-1}\ \ell n\ Q \tag{5}$$

We assume that only the intercepts, and not the slopes, of the supply and demand curves are subject to revision; data limitations would not allow us to identify changes in the slopes. Equilibrium quantity at time t therefore satisfies

$$\ell n\ Q_t = \frac{\theta_t - \delta_t}{\beta^{-1} - \alpha} \tag{6}$$

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[7] In addition to changing their estimates of marginal (variable) costs, decisions makers may change their estimate of fixed costs. Eq. (1) implies that the total cost (TC) function of the system may be written

$$TC = \exp(\delta_0)\ \frac{1}{1-\alpha}\ Q^{1-\alpha} + K_0$$

where K_0 is fixed cost. Changes in TC may be due to changes in K as well as to changes in δ. Only changes in δ should affect equilibrium quantity. In our empirical analysis, we use changes in ln TC as our indicator (or measure) of changes in δ. Since TC is also affected by changes in K, our indicator is a "noisy" one, i.e. it is subject to error. If revisions of K are uncorrelated with revisions of δ, then we have classical measurement error and our estimate of the demand elasticity will be biased towards zero. If K and δ revisions are positively correlated (as one might expect), a downward bias also occurs, but it is smaller in magnitude. See Lichtenberg (1988b).

The revision in equilibrium quantity between time 0 and time t can be calculated by subtracting (3) from (6):

$$\ell n(Q_t/Q_0) = \frac{-(\delta_t - \delta_0)}{\beta^{-1} - \alpha} + \frac{\theta_t - \theta_0}{\beta^{-1} - \alpha} \tag{7}$$

The log-change in quantity is due to both supply- and demand-curve revisions, each divided by the difference between the slopes of the two curves.[8] Equation (7), along with the baseline supply curve (1), can under certain assumptions provide a basis for estimating the parameters α and β. We have data, for 84 major weapons systems, on the quantity- and supply-shift variables $\ell n(Q_t/Q_0)$ and $(\delta_t - \delta_0)$. Unfortunately, we do not have data on the demand shift $(\theta_t - \theta_0)$.[9] But suppose, as seems reasonable, that demand shifts are uncorrelated with supply shifts across weapons systems. Moreover, assume that α and β do not vary across weapons systems. Then the regression equation:

$$(\ell n(Q_t/Q_0))_i = -(\beta^{-1} - \alpha)^{-1}(\delta_t - \delta_0)_i + \varepsilon_i \tag{8}$$

where the i subscript denotes weapon system i and ε is a disturbance term, will yield a consistent estimate of the nonlinear function of the

[8] Supply- and demand-curve revisions may result in changes in the performance, or quality, as well as in the quantity, of the system. We will discuss below our admittedly imperfect attempt to adjust the data for such quality change.

[9] We have, however, what may be crude proxies or instruments for demand shifts -- "year effects"; this issue is discussed below.

parameters $-(\beta^{-1} - \alpha)^{-1}$.[10] Of course, neither α nor β can be separately identified from this equation alone, but the available data permit us to estimate another equation which identifies α. By simultaneously estimating the system of two equations, we can identify both parameters.

II. Selected Acquisition Reports Summary Data

The Department of Defense is required by law to submit periodically to Congress Selected Acquisition Reports (SARs) for all major weapons systems it is acquiring. All programs that are estimated to require an eventual expenditure for research, development, test, and evaluation of more than $200 million (in fiscal year 1980 dollars), or an eventual expenditure for procurement of more than $1 billion, are covered by this requirement. Routine acquisitions (such as trucks and common ammunition) and "highly sensitive classified" programs are excluded.[11]

The SAR provides a summary of key cost, schedule and technical information about the program. Current estimates of cost, schedule, and technical data are compared with established and approved baseline estimates, and a "disciplined approach to the calculation of variances [between baseline and current estimates] is applied."[12] Most of the schedule and technical data are classified, and therefore cannot serve as a basis for economic research. But the cost and (in most cases) the

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[10] Even if this function varies across systems, provided that it varies randomly -- i.e., is not correlated with supply or demand shocks -- the equation will yield consistent estimates of its mean.

[11] According to the Center on Budget and Policy Priorities, the share of DOD's budget authority for procurement, R&D, and construction that is classified increased from 13.3 percent in fiscal year 1986 to 18.5 percent in fiscal year 1988.

[12] Department of Defense (1986), p. 3.

quantity data are not classified, and in fact the Assistant Secretary of Defense (Comptroller) regularly publishes a Program Acquisition Cost Summary, which contains such data for all major weapons systems covered by the reporting requirement.[13]

The empirical analysis performed in this paper is based on data reported in the SAR Summary Tables as of December 31, 1987.[14] Among the data items recorded for each weapons system are the following: weapons system name, baseline estimates of quantity (Q_0) and cost (C_0), base year (year in which the baseline estimate was made), and current estimates of quantity (Q_t) and cost (C_t).[15] Thus, we can infer the (absolute or logarithmic) changes in estimates of both quantity and cost between the base year and the end of 1987 for each system. These data alone would not be sufficient to identify the demand and supply curve parameters. But in addition to reporting the total cost change to date, the SAR provides a distribution of the cost change by category (or "reason" for

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[13] Unfortunately, the published Summary doesn't provide data on program attributes such as whether the program is strategic or tactical (strategic programs tend to be given higher budgetary priority) and whether the program is incremental in nature (a follow-on or modification) or represents an entirely new endeavor.

[14] Clearly, this methodology is potentially subject to the problem of censoring of the data. Previously initiated programs that had either been ("successfully") completed or ("unsuccessfully") terminated are absent from our data set. (Relatively few programs are cancelled, however.) This means that the programs we observe are a nonrandom (in a relevant sense) sample of the population of all weapons programs. To address the censoring issue, we would need to collect data on completed and terminated programs as well as programs in progress. This is beyond the scope of this paper.

[15] Both baseline and current estimates of costs are given in both base-year (constant) dollars and in current-year dollars. Revisions in current-year dollar estimates reflect changes in inflation assumptions and projections as well as changes in real program costs. In this paper, we analyze only the estimates expressed in base-year dollars.

the cost change). The six categories and brief descriptions of them are as follows:

> Quantity change. A change in the number of units of an end item of equipment. This does not include changes in support items.
>
> Schedule change. A change in a procurement or delivery schedule, completion date, or intermediate milestone for development or production.
>
> Engineering change. An alteration in the physical or functional characteristics of a system or item, after establishment of such characteristics.
>
> Estimating change. A change in program cost due to correction of an error in preparing the baseline cost estimate, refinement of a prior current estimate, or a change in program or cost estimating assumptions and techniques.
>
> Support change. Cost changes associated with training and training equipment, peculiar support equipment, data, operational site activation, and initial spares and repair parts.
>
> Other. A change in program cost due to natural disasters, work stoppage, and similarly unforeseeable events not covered in other variance categories.

For the moment, we will group the last five categories together, and think of revisions in estimates of total costs as occurring for two distinct reasons: (1) quantity changes, and (2) all other reasons. We will argue that under the accounting framework used for preparation of the SARs, these two categories correspond to movements along the (baseline) supply curve and shifts of the supply curve, respectively.

Figure 2 illustrates how DOD accountants allocate the change in total cost into these two categories. MC_0 and MC_t represent the baseline and current marginal cost schedules, respectively, and Q_0 and Q_t the baseline and current quantities. The area under the MC_0 curve to the left of Q_0 corresponds to the baseline estimate of total cost, C_0. The area under the MC_t curve to the left of Q_t corresponds to the revised estimate of total cost, C_t. The difference $C_t - C_0$ may be represented as the sum of two components, which we denote by ΔC_Q and ΔC_N. ΔC_Q is the change in cost that would have occurred if only the quantity had changed, and the MC curve had not shifted. ΔC_N is the cost change that would have occurred if only the MC curve had shifted, and the baseline quantity had been Q_t. ΔC_Q corresponds to the first of the six categories into which total cost change is allocated, ΔC_N to the sum of the other five.

It is perhaps apparent that information about C_0, ΔC_Q, and ΔC_N enables us to determine both the slope and the shift of the supply function. In fact, our assumptions of log-linear supply curves with constant slopes and (possibly) shifting intercepts imply the following relationships between supply curve parameters and the observable variables:[16]

$$\ell n(1 + \frac{\Delta C_Q}{C_0}) = (1 - \alpha)\ \ell n(Q_t/Q_0) \tag{9}$$

$$\delta_t - \delta_0 = \ell n(1 + \frac{\Delta C_N}{C_0 + \Delta C_Q}) \tag{10}$$

Substituting for $(\delta_t - \delta_0)$ in eq. (8) using eq. (10),

$$(\ell n(Q_t/Q_0))_i = \bar{\epsilon} - [(\beta^{-1} - \alpha)^{-1}(\ell n(1 + \frac{\Delta C_N}{C_0 + \Delta C_Q}))_i] + (\epsilon_i - \bar{\epsilon}) \tag{11}$$

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[16]See the Appendix for derivation of eqs. (9) and (10). It is, of course, possible to solve eq. (9) explicitly for α; we write the equation in this form because we will interpret it as a regression equation for estimating α.

Equations (9) and (11) consitute a system of recursive nonlinear simultaneous equations, which provides a basis for estimating both the demand and supply elasticities.

The intercept $\bar{\varepsilon}$ of eq. (11) may be interpreted as the mean value of the demand shock. Notice that eq. (9) has a zero intercept. This reflects the fact (which applies to our data) that a weapon system with zero quantity change has zero quantity-related cost change. This equation, however, was based on the assumption of a log-linear MC curve. If the true MC curve is not log-linear, then a log-linear curve fitted to it may have a nonzero intercept, even if the true curve passes through the origin. We therefore estimate variants of eq. (9) both excluding and including an intercept.

Although above we tentatively defined ΔC_Q simply as "quantity-related cost change" (the first category) and ΔC_N as "all other cost changes" (the remaining five categories), we need to make two amendments to the latter definition. We will treat both engineering-related and support-related cost changes as neither shifts in nor movements along the supply curve. As defined above, engineering-related cost changes are due to efforts to alter the physical characteristics, or "quality," of the system. (In practice, most such expenditures are incurred to modify "mature" weapons in order to extend their useful lives, and avoid the acquisition of entirely new systems.) Our analysis thus far has been based on the implicit assumption that we were examining the relationship between cost changes and quantity changes of systems of <u>unchanging quality</u>. In the literature on ("hedonic") price measurement and quality change, an accepted technique of "adjusting" for quality change in price indices is to subtract the producer's cost of increasing the product's quality; this technique is used by the Bureau of Labor Statistics, the government agency that produces the official price indices.[17] Thus, eliminating engineering-related cost changes from consideration appears to be appropriate if we want to examine the relationship between "quality-adjusted" changes in cost and quantity.

Support-related cost changes are excluded from both ΔC_Q and ΔC_N because these represent changes in the cost of <u>complementary</u> goods rather than changes in the cost of the weapon system itself.[18] It may be useful to interpret support-related cost changes within the context of a simple two-equation system of demand equations. Let the subscripts 1 and 2 denote end-items and support items, respectively, and P, Q, and C denote price, quantity, and cost. Suppose that the supplies of both products are perfectly elastic at exogenously-determined prices, and that the demand equations are log-linear:

$$\begin{aligned} \ln Q_1 &= \theta_{11} \ln P_1 + \theta_{12} \ln P_2 \\ \ln Q_2 &= \theta_{21} \ln P_1 + \theta_{22} \ln P_2 \end{aligned} \tag{12}$$

Hence

$$\begin{aligned} \ln C_1 &= (\theta_{11} + 1) \ln P_1 + \theta_{12} \ln P_2 \\ \ln C_2 &= \theta_{21} \ln P_1 + (\theta_{22} + 1) \ln P_2 \end{aligned} \tag{13}$$

Since the two goods are assumed to be complements, the cross- as well as own-demand elasticities are assumed to be negative. Changes in C_1 and C_2

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[17] See Early and Sinclair (1983) and Lichtenberg and Griliches (1988) for discussions of the issue of quality change in the context of output and price measurement.

[18] We initially conjectured that support-related cost changes should be included in our definition of ΔC_Q, on the grounds that the baseline cost estimate C_0 includes the cost of support items (e.g., spare parts)

are seen to be jointly endogenous; it would be incorrect to regard support-related cost changes as causing changes in C_1. Although we can observe changes in P_1 (i.e., the supply shift $\delta_t - \delta_0$), C_1, and C_2, unfortunately we cannot observe changes in P_2, so we can't identify the θ_{ij} other than θ_{11}.

To summarize, ΔC_N will be defined as the sum of schedule, estimating, and other changes. We will also estimate models in which ΔC_N is disaggregated into its components.

III. Empirical Results

This section begins by presenting descriptive statistics on quantity and cost revisions for 84 major weapons systems.[19] We then report and analyze estimates of the model developed in the previous section. To facilitate discussion, we adopt the following notation:

$DQ = \log(Q_t/Q_0)$ (log change in system quantity)

$DC.Q = \log(1 + \frac{\Delta C_Q}{C_0})$ (log change in cost due to quantity change)

$DC.NONQ = \log(1 + \frac{\Delta C_N}{C_0 + \Delta C_Q})$ (log change in cost not due to quantity change)

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as well as the cost of the ("end-item") weapons themselves. But this definition led to nonsensical estimates of α.

[19]The SAR Summary Tables contained data for exactly 100 systems. Thirteen systems had classified data on program quantities and therefore had to be excluded from the sample. The value of the ratio Q_t/Q_0 was zero for 2 systems (the Navy HFAJ and the Air Force I-S/A AMPE) and extremely small (3/645) for the Air Force small ICBM. The former two also had to be excluded; the latter one was also because it was a major outlier.

$$DC.SCHED = \log(1 + \frac{\Delta C_{SCHED}}{C_0 + \Delta C_Q})$$ (log change in cost due to schedule change)

$$DC.ESTIM = \log(1 + \frac{\Delta C_{ESTIM}}{C_0 + \Delta C_Q})$$ (log change in cost due to estimating change)

$$DC.OTHER = \log(1 + \frac{\Delta C_{OTHER}}{C_0 + \Delta C_Q})$$ (log change in cost due to other change)

LAGE = log(1988.1 - Base Year) (log of "age" of baseline estimate)

Because $\Delta C_N \equiv \Delta C_{SCHED} + \Delta C_{ESTIM} + \Delta C_{OTHER}$, and because all four of these variables are small, on average, relative to $(C_0 + \Delta C_Q)$, DC.NONQ is approximately equal to the sum of DC.SCHED, DC.ESTIM AND DC.OTHER (since $\ell n(1 + x) \cong x$ when x is "small"). Using this notation eqs. (9) and (11) may be written

$$DC.Q = (1 - \alpha)\ DQ \tag{14}$$

$$\begin{aligned} DQ_i &= \bar{\epsilon} - (\beta^{-1} - \alpha)^{-1}\ DC.NONQ_i + (\varepsilon_i - \bar{\epsilon}) \\ &= \bar{\epsilon} + \pi\ DC.NONQ_i + u_i \end{aligned} \tag{15}$$

where $\pi \equiv -(\beta^{-1} - \alpha)^{-1}$ and $u_i \equiv (\varepsilon_i - \bar{\epsilon})$.

Table 1 presents descriptive statistics and a correlation matrix for these variables. The mean change in log quantity is .23, indicating a 26 percent average increase in the number of units. The huge defense buildup of the early 1980s is no doubt largely responsible for this increase. Mean cost change attributable to quantity change (DC.Q) is lower than quantity change (DQ) itself -- .18 compared to .23 -- evidently reflecting decreasing marginal cost. Mean cost change not due to quantity change (DC.NONQ) is quite small (.03) and not significantly different from zero. Perhaps this is because firms are forced to "eat" most of the costs of schedule, estimating, and other changes.

There are two alternative approaches one can take to estimating the parameters α and β, and we will pursue both of them. One can estimate a separate value of α, α_i, for each weapons system using the equation (based on (14))

$$\alpha_i = 1 - (DC.Q_i/DQ_i) \qquad (16)$$

Obviously, α_i can be calculated only if $DQ_i \neq 0$, which is true in the case of 64 out of 84 observations.[20] Statistics relating to the distribution of the 64 α_i's are as follows:

mean	.222
std. dev.	.630
quantiles	
.95	.779
.75	.513
.50	.284
.25	.099
.05	-.508

The mean and median values of α_i appear to be quite consistent with previous estimates of the elasticity of cost with respect to quantity contained in the empirical learning-curve literature on the defense industry. One can then replace the constant α by the variable α_i in eq. (15) and estimate β by nonlinear OLS (NOLS) estimation of that

[20] Due to the presence of $\bar{\epsilon}$ and u_i in eq. (15), one cannot estimate separate values of β, β_i, from that equation (conditional on α_i) in an analogous fashion. Attempts to do so yield absurd values of the β_i; for example, their mean is -.74, and their standard deviation is 10.5.

equation alone. The alternative approach is to assume (as we did above) that both α and β are invariant across observations, and to interpret eq. (14) as a regression equation for estimating α. Disturbances in this equation arise due to both non-log-linearity of the MC schedule and to deviations of α_i from the assumed common α. Estimates of α and β are obtained via simultaneous estimation of eqs. (14) and (15). The disturbances of the two equations may be correlated, so we may increase the efficiency of the estimates by using nonlinear seemingly unrelated regressions (NSUR), or joint generalized least squares, estimation.

Because eq. (14) has a zero intercept. the OLS formula for the slope of this equation is

$$(1 - \hat{\alpha}) = \frac{\Sigma(DC.Q_i)(DQ_i)}{\Sigma(DQ_i)^2}$$

Hence

$$\hat{\alpha} = 1 - \frac{\Sigma(DC.Q_i)(DQ_i)}{\Sigma(DQ_i)^2}$$

$$= 1 - \frac{\Sigma(DC.Q_i)^2(DC.Q_i/DQ_i)}{\Sigma(DQ_i)^2}$$

$$= 1 - \Sigma w_i(1 - \alpha_i) = \Sigma w_i \alpha_i$$

using (16) and the definition $w_i \equiv DQ_i^2/\Sigma DQ_i^2$ (hence $\Sigma w_i = 1$). $\hat{\alpha}$ is a weighted average of the individual α_i's, with weights proportional to the square of the log-change in quantity. The sign of the difference between $\hat{\alpha}$ and $\bar{\alpha} \equiv \frac{1}{N} \Sigma\alpha_i = .222$ (the <u>unweighted</u> average of the α_i's) depends on the sign of the correlation between α_i and w_i: $\mathrm{sgn}(\hat{\alpha} - \bar{\alpha}) = \mathrm{sgn}(\mathrm{corr}(\alpha_i, w_i))$. The sample value of the correlation coefficient is positive (.176) although not very significant (prob.-value = .16), so $\hat{\alpha}$ will exceed the simple average of the α_i's.[21]

Table 2 present estimates of the demand elasticity for the version of the model with individual α_i's, and estimates of both demand and cost elasticities for the version with common α. The first line shows nonlinear OLS estimates of eq. (15) in which the parameter α is replaced by the computed α_i's as defined in eq. (16). The estimate of β is positive (as hypothesized) but far from being significantly different from zero. The intercept $\bar{\bar{\epsilon}}$ is positive and highly significant. The insignificance of β implies that we cannot reject the hypothesis that quantity demanded does not respond to supply shocks. The correlation matrix in Table 1 reveals, however, that both DQ and DC.NONQ are positively correlated with the age of the baseline estimate (a proxy for the age of the program itself): older programs tend to have experienced both higher quantity growth (perhaps due to larger demand increases) and greater supply shifts. These correlations might tend to bias downward the estimated demand elasticity.

We can, perhaps, eliminate or at least reduce this bias by controlling for system age in the DQ equation. The most general way of doing this is to include (program base) "year effects" as regressors in that equation. When we allow for these effects, we are analyzing the <u>within-year</u> relationship between DQ and DC.NONQ, i.e., we are asking whether programs that experienced larger supply shifts had lower quantity increases than other programs <u>with the same base year</u>. This seems a more

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[21]In view of eq. (16), this positive correlation may be at least partly spurious, induced by errors of measurement of DQ, due for example to "phony" estimates of baseline quantity Q_0. Such errors would also bias upward the estimate $\hat{\alpha}$ from eq. (15).

appropriate reference group than the entire set of programs, which vary considerably in age. To the extent that there are changes over time in overall defense demand (e.g., the 1979-84 defense buildup) that affect all programs then in existence, these year effects may be considered as capturing demand shocks common to systems of given age.

Line (2) of the table displays the estimates when year effects are included in the DQ equation. As expected, the estimate of the demand elasticity increases, to about .54. Its t-ratio is now 1.66, so the elasticity is significantly greater than zero at about the 5 percent level, using a one-tailed test.

The estimates in line (3) and in subsequent lines are from the common-α version of the model, and are therefore based on a larger sample (observations for which DQ = 0 are included). The estimates of β and $\bar{\epsilon}$ in line (3), which correspond to the model without year effects, are similar to, but slightly smaller than, their counterparts in line (1). The estimate of α is larger than the simple average of the α_i, consistent with our earlier discussion of the relationship between $\hat{\alpha}$ and $\bar{\alpha}$. The estimated intercept of the DC.Q equation (α_0) is small but statistically significant.[22]

In line (4) we replace the intercept $\bar{\epsilon}$ of the DQ equation by a complete set of year dummies. This has virtually no effect on the estimates of α and α_0 but almost triples the point estimate of β, to .557. This is virtually identical to the point estimate of β in line 2 (although the standard error is 30 percent lower), which suggests that

[22] The correlation between the disturbances of the DQ and DC.Q eqs. is close to zero (-.01), so the NOLS and NSUR estimates of this model are virtually identical.

the estimate of β is not sensitive to what assumption we make about the heterogeneity of α.[23]

In line (5) of the table we include year effects in the DC.Q equation as well as in the DQ equation. Their inclusion has essentially no effect on the estimates of β and α.

The estimated standard errors reported in Table 2 are correct if and only if the disturbances of the equation(s) are homoskedastic. We tested the null hypothesis of homoskedasticity using the test of first and second moment specification proposed by White (1980), and rejected the null hypothesis in the case of both equations.[24] Because eq. (14) is linear in the parameter α, it is straightforward to compute the heteroskedasticity-consistent standard error of α: it is .064, about 75 percent larger than the uncorrected standard error of .037 on line (5). Because eq. (15) is nonlinear in β, it is unfortunately not straightforward to compute the heteroskedasticity-consistent standard error of β. However we can, perhaps, get an idea of the magnitude of the appropriate correction by computing both the uncorrected and heteroskedasticity-consistent standard errors of the "reduced-form" parameter

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[23] Point estimates of the year dummies corresponding to line (4) are as follows for 1970-87, respectively: .63, .54, .96, .75, NE (not estimated -- no observations with program base year 1974), .35, .22, .81, .48, .12, .03, .22, .09, .70, -.02, .42, .03, -.01. The standard errors on the individual year dummies are large, so that most are not significant, but the coefficients reveal a clear pattern. The average value of the 1970-78 (pre-defense buildup) year dummies is .59, and that of the 1979-87 dummies is .18. Programs initiated before the defense buildup experienced much larger (positive) demand shocks than those started during or after the buildup.

[24] This is somewhat surprising since the logarithmic transformation, which usually attenuates heteroskedasticity, is applied to all of the variables.

$\pi = -(\beta^{-1} - \alpha)^{-1}$: these are .395 and .469, respectively. Since the value of π is predominantly determined by β, this may indicate that the standard errors of β reported in Table 2 are underestimated by no more than 16 percent.

For each equation, we also tested the hypothesis of normality of the residuals using the Kolomogorov D statistic. Despite the fact that plots of both distributions appeared similar to normal curves, the hypothesis was rejected at the .01 level in both cases. Given the moderate size of our sample, however, rejection of normality is unlikely to seriously undermine the validity of our inference procedures.

Because the total shift in the supply curve DC.NONQ is (approximately) the sum of shifts occurring for three different types of reasons -- schedule changes, estimating changes, and other changes -- it seems natural to inquire whether the response of quantity demanded to supply shifts depends on the <u>nature of</u> or <u>reason for</u> the shift. We investigate this issue by removing DC.NONQ from the DQ equation and, instead, including as regressors its three components, DC.SCHED, DC.ESTIM, and DC.OTHER. We allow the β coefficient associated with each of the three components to differ. Estimates (standard errors) of the β coefficients associated with DC.SCHED and DC.ESTIM were as follows:[25]

DC.SCHED	1.215
	(.227)
DC.ESTIM	.333
	(.324)

[25] Estimates of the coefficient on DC.OTHER would not converge after numerous attempts with different starting values. This may be due to the relatively low variability of DC.OTHER: the number of observations with

It is obvious that our finding above of a negative correlation between DC.NONQ and DQ (hence a nonzero demand elasticity) was due primarily to the negative correlation between DC.SCHED and DQ.[26] The response to cost revisions due to schedule changes is large -- indeed, greater than one -- and highly significant. These estimates suggest that procurement officials adjust program quantities mainly in response to cost revisions that are associated with schedule changes.

This interpretation, if it is correct, appears in certain respects consistent with the observations made by Peck, Scherer, and others about tradeoffs between cost, time, and performance in weapon system acquisition. Improving the performance, or quality, of a system entails increases in development cost and/or time. Reducing development time requires increases in cost and/or sacrifices in quality, and reducing cost means either slower development or lower quality. Data presented by Peck and Scherer suggest that decision makers attach the greatest importance to achieving performance objectives, the least importance to achieving cost objectives, and intermediate importance to achieving development time objectives. For a sample of 12 weapon systems, they found that whereas "actual performance more frequently exceeded original promises than fell below them" (p. 23), actual development time was on average 1.36 times as large as the original time estimate, and actual cost was 3.2 times the original cost estimate. This suggests that

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nonzero values of DC.SCHED, DC.ESTIM, and DC.OTHER were 51, 82, and 15, respectively. See also their standard deviations reported in Table 1.

[26]Our previous estimate of β of about .55 is close to a variance-weighted average of the β coefficients associated with DC.SCHED and DC.ESTIM.

decision makers have lower tolerance for time slippage than for cost growth, and are willing to incur substantial additional cost to keep a program close to schedule. A program that experiences significant schedule delays despite the infusion of additional funds might be interpreted as one encountering unusually serious technical difficulties, and therefore one liable to the greatest quantity reduction (or the smallest quantity increase).

Our key empirical finding is that there is an inverse relationship between program quantity changes and (non-quantity-related) changes in program costs, particularly those due to schedule changes. We have interpreted these program cost changes as reflecting exogenous "supply shocks" that occur in the course of weapons acquisition, an activity characterized by considerable technical uncertainty. As Peck and Scherer note, this is "the most common explanation offered for time, cost and quality variances" (p. 435), and it is consistent with data analyzed by Marshall and Meckling, who found that "average production cost and development time variances were an increasing function of the size of the technological advance sought" (p. 435). But Peck and Scherer argue that "technological uncertainty is one likely cause of development cost overruns, but by no means the only cause" (p. 436). They maintain that the most significant cause of schedule delays is the "lack of urgency in connection with lower priority programs" (p. 459).

In this vein, one could argue that the negative correlation across programs between quantity changes and schedule-related cost changes is due to the response of both variables to unobserved demand shocks, rather than the response of the first to the second. In this case, of course, our estimate of β could no longer be interpreted as a demand elasticity.

Suppose that changes in the (actual or perceived) nature of "the threat" change the relative demand for different programs: some programs seem more, others less, attractive than they did under previous evaluations of the military environment. In the case of programs that have become less attractive, decision makers might be hypothesized to do two things: reduce program quantities and "stretch out" the development and acquisition of the remaining quantities. Such "stretch-outs" will generally result in schedule-related cost increases, so that these demand shocks could contribute to the observed negative correlation.

We have both a theoretical and an econometric response to the objection that β may not be identified due to our inability to measure demand shocks. First, Defense Department officials claim that decisions to stretch out programs are generally not related to decisions to change the ultimate size of the program. (In fact, stretching out a program may be the only or easiest way of <u>preserving</u> the original size of the program.) Also, because both positive and negative deviations from the original schedule of program milestones will generally result in schedule-related cost increases, programs that have experienced demand <u>increases</u> would (under the assumptions of the previous paragraph) exhibit both quantity increases and schedule-related cost increases. This would induce a <u>positive</u> correlation between DQ and DC.SCHED, and tend to offset the effect of programs whose demand has declined.

We can attempt to address the potential problem of endogeneity of DC.SCHED by estimating the DQ equation via instrumental variables. The difficulty, of course, is finding appropriate instruments for DC.SCHED, and our choices are severely limited. Because DC.ESTIM was not significant in the DQ equation, and also because based on their definition it seems to be a relatively "pure" supply shock (less likely than DC.SCHED to be contaminated by demand shocks) we dropped it from the DQ equation and instead used it as an instrument for DC.SCHED. We also used DC.OTHER as an instrument. The ("first-stage") linear regression of DC.SCHED on these two instruments is as follows (standard errors in parentheses):

$$\underset{}{\text{DC.SCHED}} = \underset{(.008)}{.024} + \underset{(.039)}{.176}\ \text{DC.ESTIM} + \underset{(.488)}{.167}\ \text{DC.OTHER} + \text{error}$$

When we estimate equation (15), replacing DC.NONQ by the predicted value of DC.SCHED from the first-stage regression above, the estimate (standard error) of β is 1.317 (0.422), about the same as (indeed, slightly larger than) the non-instrumental estimate. The standard error approximately doubles, but the parameter remains highly significantly different from zero. The similarity of the two estimates suggests that (conditional on this set of instruments) we do not commit a specification error in using NSUR, and therefore that the NSUR estimates should be viewed as consistent.

IV. Summary and Conclusions

We have attempted in this paper to make inferences about the elasticity of the government's demand for specific weapons by analyzing the statistical relationship between quantity and cost revisions across the population of major weapon systems. We interpret the cost revisions as due in part to the arrival of technological information generated in the course of research and development. If the government's demand is less than perfectly inelastic, it will react to this information by changing the quantity of the weapon it will buy. The more elastic the demand, the less market power is wielded by sole-source suppliers of particular systems.

When we standardize the data by program base year -- in effect comparing a program only to those other programs entering full-scale development at about the same time -- we find a significant negative correlation between quantity and cost changes. The estimated elasticity of demand is .55, and is significantly different from both zero and unity. This suggests that the government's demand for specific weapons is inelastic, but not perfectly inelastic. The estimates also imply that weapons acquisition is characterized by increasing returns: the mean and median values of the elasticity of total cost with respect to quantity are .78 and .72, respectively.

Further analysis revealed that the negative correlation between quantity and cost revisions -- hence the nonzero demand elasticity -- was entirely attributable to one component of cost revisions: those associated with changes in the acquisition schedule. The elasticity of quantity with respect to schedule-related cost increases is about twice as great as the elasticity with respect to cost increases generally. In principle, it is possible that schedule-related cost increases are due to demand-induced stretch-outs of programs rather than supply-related, or technological, shocks. But it is not clear on theoretical grounds that unobserved demand shocks could account for the correlations we observe, and the demand-shock interpretation is also not supported by one econometric attempt to correct for it.

Figure 1

Determination of Baseline Equilibrium Quantity and Cost

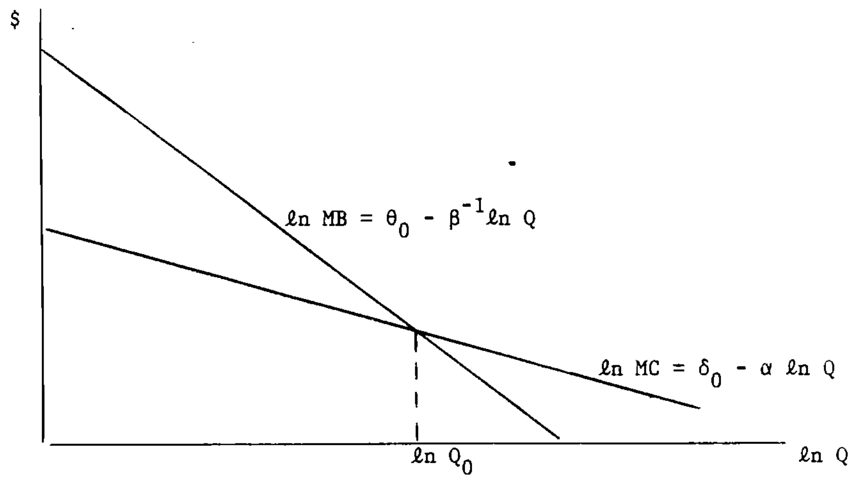

Figure 2

Allocation of ΔC into Components ΔC_Q and ΔC_N

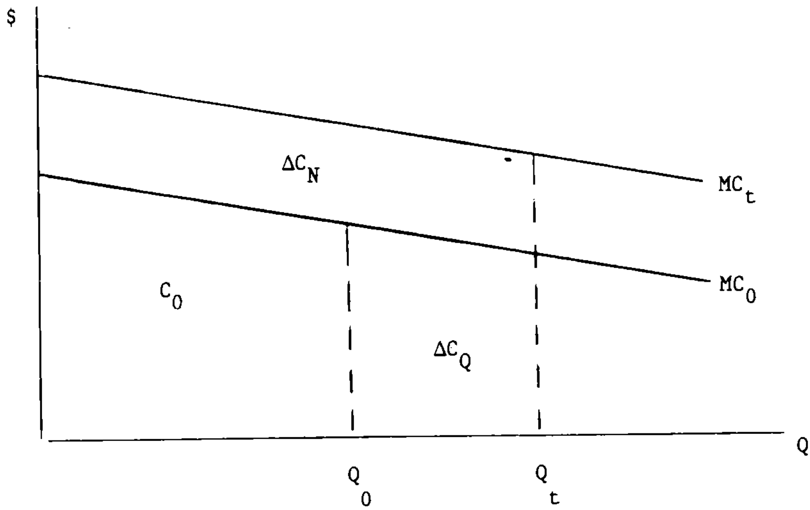

TABLE 1

DESCRIPTIVE STATISTICS:
QUANTITY AND COST REVISIONS,
84 MAJOR WEAPONS SYSTEMS

Variable	Mean	Std. Dev.	Median	Min.	Max.
DQ	.229	.664	.063	-2.52	2.55
DC.Q	.175	.412	.049	-1.17	1.64
DC.NONQ	.052	.237	-.004	-0.28	0.95
DC.SCHED	.029	.080	.000	-0.14	0.45
DC.ESTIM	.027	.208	-.009	-0.43	0.74
DC.OTHER	.003	.017	.000	-0.09	0.08
LAGE	1.80	.915	1.808	-2.30	2.90

CORRELATION MATRIX

	DQ	DC.Q	DC.NONQ	DC.SCHED	DC.ESTIM	DC.OTHER	LAGE
DQ	1.00						
DC.Q	.87	1.00					
DC.NONQ	-.07	-.17	1.00				
DC.SCHED	-.20	-.26	.66	1.00			
DC.ESTIM	-.06	-.14	.97	.47	1.00		
DC.OTHER	.26	.15	.28	.13	.22	1.00	
LAGE	.14	.19	.31	.29	.25	.21	1.00

TABLE 2

ESTIMATES OF DEMAND AND SUPPLY ELASTICITIES

Line	Specification features: Individual or common α	Specification features: Year Effects in DQ, DC.Q eqs.	Estimation technique	N	Parameter estimates (asymptotic standard errors)[a]: β	$\bar{\epsilon}$	α	α_Q
(1)	Indiv.	none	NOLS	64	.255 (.316)	.332 (.098)	--	--
(2)	"	DQ eq.	"	"	.539 (.324)	--	--	--
(3)	Common	none	NSUR	84	.199 (.256)	.240 (.075)	.458 (.034)	.051 (.024)
(4)	"	DQ eq.	"	"	.557 (.227)	--	.443 (.034)	.048 (.024)
(5)	"	DQ, DC.Q eqs.	"	"	.548 (.227)	--	.449 (.037)	--

Note: a. These are not heteroskedasticity-consistent standard errors. See text for discussion of appropriate corrections for heteroskedasticity.

Appendix

Derivation of Equations (9) and (10)

$$\ell n\ MC = \delta_0 - \alpha\ \ell n\ Q$$

$$MC = \delta_0' Q^{-\alpha} \qquad \text{where } \delta_0' \equiv \exp(\delta_0)$$

$$TC(Q) = \int_0^Q \delta_0' q^{-\alpha} dq = \delta_0'(1 - \alpha)^{-1} Q^{1-\alpha}$$

$$\ell n\ TC = \delta_0 - \ell n(1 - \alpha) + (1 - \alpha)\ell n\ Q$$

$$\ell n\ TC\ (Q_t) - \ell n\ TC(Q_0) = (1 - \alpha)[\ell n\ Q_t - \ell n\ Q_0]$$

$$[\ell n\ TC(Q_t)]_{\delta=\delta_t} - [\ell n\ TC(Q_t)]_{\delta=\delta_0} = \delta_t - \delta_0$$

References

Baron, David, and Besanko, David (1988), "Monitoring of Performance in Organizational Contracting: The Case of Defense Procurement," Scandinavian Journal of Economics 90, 329-356.

Department of Defense (1980), "Preparation and Review of Selected Acquisition Reports," DOD 7000.3-G, OASD (Comptroller), May.

_________ (1986), "Instruction 7000.3," April 17.

_________ (1988), "Selected Acquisition Report (SAR) Summary Tables, as of Date: December 31, 1987," OASD (Comptroller), May 13.

Early, John, and Sinclair, James (1983), "Quality Adjustment in the Producer Price Indexes," in Foss, Murrary (ed.), The U.S. National Income and Product Accounts: Selected Topics (Chicago: University of Chicago Press).

Lichtenberg, Frank (1988a), "The Private R&D Investment Response to Federal Design and Technical Competitions," American Economic Review 78, June, 550-59.

_________ (1988b), "Aggregation of Variables in Least-Squares Regression," unpub. paper, Columbia Business School, June.

_________ (1989), "Contributions to Federal Election Campaigns by Government Contractors," Journal of Industrial Economics, forthcoming.

_________ and Griliches, Zvi (1986), "Errors of Measurement in Output Deflators," NBER Working Paper No. 2000; Journal of Business and Economic Statistics.

Marshall, A., and Meckling, W. (1962), "Predictability of the Costs, Time, and Success of Development," in Nelson, Richard (ed.), The Rate and Direction of Inventive Activity: Economic and Social Factors (Princeton: Princeton University Press), 461-75.

Peck, Merton, and Scherer, Frederic (1962), The Weapons Acquisition Process: An Economic Analysis (Boston: Harvard Business School).

Rich, Michael, and Dews, Edmund (1986), "Improving the Military Acquisition Process: Lessons from RAND Research," RAND Corporation Report R-3373-AF/RC, February.

Rogerson, William (1988a), "Profit Regulation of Defense Contractors and Prizes for Innovation: Theory and Evidence," unpublished paper, Northwestern University, January.

_________ (1988b), "Quality vs. Quantity in Military Procurement: An Organizational Theory of Decision Bias," unpublished paper, Northwestern University, February.

Scherer, Frederic (1964), The Weapons Acquisition Process: Economic Incentives (Boston: Harvard Business School).

Tirole, Jean (1986), "Procurement and Renegotiation," Journal of Political Economy 94, April, 235-59.

White, Halbert (1980), "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," Econometrica 48, 817-838.