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Rafael Dix-Carneiro
Sharon Traiberman

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ABSTRACT

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Rafael Dix-Carneiro
Department of Economics
Duke University
210A Social Sciences Building
Durham, NC 27708
and NBER
rafael.dix.carneiro@duke.edu

Sharon Traiberman
Department of Economics
New York University
19 West 4th Street, 6th Floor
New York, NY 10012
and NBER
sharon.traiberman@gmail.com

Globalization, Trade Imbalances and Inequality*

Rafael Dix-Carneiro

Sharon Traiberman

Duke University and NBER

New York University and NBER

June 20, 2022

Abstract

We investigate the role of trade imbalances for the distributional consequences of globalization. We do so through the lens of a quantitative, general equilibrium, multi-country, multi-sector model of trade with four key ingredients: (a) workers with different levels of skills are organized into separate representative households; (b) endogenous trade imbalances arise from households' consumption and saving decisions; (c) production exhibits capital-skill complementarity; (d) labor market frictions across sectors and non-employment. We conduct a series of counterfactual experiments that illustrate the quantitative importance of both trade imbalances and capital-skill complementarity for the dynamics of the skill premium. We show that modelling trade imbalances can lead to stark differences between short- and long-run consequences of globalization shocks for the skill premium.

1 Introduction

Is a trade deficit evidence of some workers losing for the benefit of others? Does it reflect opportunities shifting away from manufacturing workers and towards professionals? Policy makers routinely voice such concerns. Curiously, despite a growing interest in understanding the links between globalization, trade imbalances, and workers' outcomes, the workhorse models of International Trade tend to assume away trade imbalances in studies of trade and inequality. We address this gap in the literature by offering a framework to analyze the distributional consequences of trade when trade imbalances—trade over time—are allowed to respond to the same shocks that determine trade patterns—trade over space.

To accomplish this task, we build a general equilibrium, multi-country, multi-sector model of trade with four crucial ingredients. First, workers have different education levels—allowing us to discuss earnings inequality between these groups. Second, workers of each education level are

*Dix-Carneiro: rafael.dix.carneiro@duke.edu; Traiberman: st1012@nyu.edu.

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organized into separate representative families within each country. Each representative family makes collective consumption and savings decisions, allowing us to parsimoniously and tractably discuss trade imbalances and income differences across groups. Third, production features capital-skill complementarity—an empirically relevant amplification mechanism when studying inequality (Krusell et al., 2000; Parro, 2013; Burstein et al., 2013). Fourth and finally, workers face intersectoral mobility frictions à la Artuç et al. (2010), and the possibility of entering non-employment.

We introduce trade imbalances by allowing our representative families to trade one-period bonds with each other—within and across countries. This approach is the standard starting point in the International Macroeconomics literature to modeling trade imbalances (Obstfeld and Rogoff, 1995). In the International Trade literature, this framework has been used by Reyes-Heroles (2016) to quantify the role of trade costs in explaining rising global imbalances. It was also used by Eaton et al. (2016) to study the determinants of the dynamics of trade in the wake of the 2008 Financial Crisis. A similar framework was also used by Kehoe et al. (2018) to study the relevance of the so-called “global savings glut” for changes in the United States industrial structure—the transition from manufacturing to services—over the 1990s and 2000s. In line with these papers, our framework can incorporate a broad set of shocks to the global economy: changes in bilateral trade costs; in productivity across sectors and countries; and in the demand for savings. The model we build shares many similarities to these papers, but incorporate several additional ingredients, including multiple types of workers, capital-skill complementarity, non-employment, and labor market adjustment frictions. These ingredients are crucial to modeling the distributional consequences of trade in the presence of trade imbalances.

Our modeling strategy is not without its tradeoffs, but has several appealing properties for calibration and simulation of counterfactuals. First, despite featuring many countries, many sectors, and a rich input-output structure, calibration can be performed country by country, facilitating estimation of parameters that are not easily observed in the data, nor estimated elsewhere. Second, despite modeling consumption dynamics as an endogenous choice, our model is amenable to the “hat algebra” procedure of Dekle et al. (2007). This makes solving counterfactual equilibria straightforward, and allows us to evade estimating initial trade costs and productivities. The costs of our approach are two-fold. First, our modeling strategy does not allow for within-group consumption inequality, and so we focus on the wages of workers with a college education versus those without, i.e., the skill premium.¹ Second, we need to assume perfect foresight over all aggregate variables in any counterfactual, foreclosing on possibly important questions related to uncertainty and imperfect information. Nevertheless, we believe that our model provides an important starting point to address the questions posed at the paper’s onset.

¹Although our model has implications for wage inequality across sectors, within groups, we do not discuss this dimension of wage inequality in this paper.

After calibrating the model, we explore the connections between globalization and the skill premium through the lens of our model. Following Parro (2013), we frame our discussion of the skill premium by focusing on two channels. First, there is a *Stolper-Samuelson channel*: shocks to the global environment induce workers to reallocate across industries of different skill intensities, shifting relative labor demands by skill group. Second, there is *capital-skill complementarity*: trade shocks that lower (raise) the price of capital will raise (lower) the relative demand for skilled workers in all industries. Trade imbalances enter this story by changing how workers reallocate across sectors. For example, countries running trade deficits are more likely to shift production toward less tradable industries (such as services), while those running trade surpluses are more likely to shift production to more tradable industries (such as agriculture and manufacturing). In turn, if these industries differ in their factor intensities, then these reallocations will reverberate to the skill premium. In order to quantify the importance of these various modeling ingredients, throughout our counterfactual exercises, we compare responses from our baseline (full) model to three more restricted alternatives: (i) one in which there is no capital-skill complementarity, and trade is balanced in each period; (ii) one in which trade remains balanced in each period, but we introduce capital-skill complementarity; and (iii) one in which there is no capital-skill complementarity, but trade imbalances are endogenously determined.

In our analysis, we focus on three sets of shocks: (a) a trade liberalization episode in China; (b) a productivity boom in China; and (c) a global trade liberalization episode. The lessons that we draw from these sets of exercises are three-fold. First, reallocation patterns, especially in the short run, are quite different across models with and without endogenous imbalances. These differences tend to magnify the Stolper-Samuelson effect in the presence of endogenous imbalances. Second, it is important to model the full set of shocks accruing to the *global economy*, even if one is interested in the skill premium in only one country. For example, in the counterfactuals that liberalize Chinese trade, and then global trade, we reduce Chinese trade costs by the same amount. However, the cumulative spillover/general equilibrium effects of reducing the trade costs in China's trade partners lead to different responses of the skill-premium in China. In particular, the short-run growth in China's skill premium is 2.5% following trade liberalization in China, but less than 1% in a global trade liberalization—even if the changes in China's trade costs are held constant across scenarios. Finally, we show that in the long run, capital-skill complementarity plays a dominant role in understanding the effects of globalization on inequality. More novel to our setting, we uncover an interaction between the skill premium and imbalances that leads to rich and counter-intuitive dynamics of the skill premium. Capital-skill complementarity can amplify the income growth of skilled workers relative to others, leading to higher demand for savings for this group. We find that for certain types and configurations of shocks, this demand for savings can amplify imbalances to such a degree that, in the short run, the Stolper-Samuelson channel can dominate the effect of

lower capital prices. Surprisingly, this can lead to a sizable short-run *decline* in the skill premium in some countries, as workers reshuffle into agriculture or manufacturing (which tend not to be as skill intensive as less tradable sectors, such as services).

Our paper contributes to a discussion of globalization and inequality going back to at least [Goldberg and Pavcnik \(2007\)](#), who point out that the behavior of the skill premium in developing countries following trade liberalization is puzzling, often at odds with the intuition from the simple $2 \times 2 \times 2$ version of the Heckscher-Ohlin model. [Lawrence et al. \(1993\)](#) and [Attanasio et al. \(2004\)](#) also find that standard factor proportions theory is a poor guide to understanding changes in the skill premium following changes in trade costs. This spurred researchers to look at other forces, namely interactions between technology and trade, to understand the skill premium: [Costinot and Vogel \(2010\)](#), [Parro \(2013\)](#) (building on the seminal work of [Krusell et al. \(2000\)](#)), [Burststein and Vogel \(2017\)](#). Our paper adds labor market dynamics and trade imbalances to this literature. We find that the skill premium can behave differently at different horizons, and that there are non-trivial interactions between trade imbalances and the forces shaping wage inequality.

We also build on the rapidly growing literature that seeks to understand the adjustment process to trade shocks. While there has been substantial progress in modeling worker adjustment to trade ([Artuç et al., 2010](#); [Dix-Carneiro, 2014](#); [Coşar et al., 2016](#); [Caliendo et al., 2019](#); [Traiberman, 2019](#)), work on trade imbalances has been more limited. This is in spite of strong evidence indicating that changes in trade costs can have quantitatively meaningful impacts on trade deficits and surpluses ([Reyes-Heroles, 2016](#); [Alessandria and Choi, 2021](#); [Barattieri, 2022](#)). Our work extends the model developed in [Dix-Carneiro et al. \(2022\)](#) in order to incorporate different types of workers and a richer production structure, at the expense of ignoring search frictions in the labor market. Other papers that have sought to model imbalances and labor reallocation are [Kehoe et al. \(2018\)](#) and [Caliendo et al. \(2019\)](#). [Kehoe et al. \(2018\)](#) build a model of structural change that studies the movement of trade, imbalances, and the allocation of labor across manufacturing and services industries. This work abstracts from the skill premium, which is the focus of our study. In addition, our methods allow us to integrate our model into state-of-the-art quantitative, general-equilibrium, trade models. [Caliendo et al. \(2019\)](#) is an important paper in the literature that allows for both worker reallocation and endogenous trade deficits. The key points of contrast are two-fold: (1) we focus on the skill premium and allow for capital-skill complementarity; (2) our model of trade imbalances is more closely linked to the International Macroeconomics literature, wherein demand for savings is the result of agents making optimal economic decisions. Finally, our work is closely related to [Reyes-Heroles et al. \(2020\)](#) who study a model with workers of different skills, capital-skill complementarity, and imbalances. In the current paper, we allow for labor market frictions, and we also develop methods to calibrate the model country by country, and compute transition dynamics for any set of shocks experienced by the global economy.

The remainder of this paper fleshes out the above discussion. In section 2 we lay out our model—an extension of [Dix-Carneiro et al. \(2022\)](#) with the additional ingredients that let us speak to wage inequality across skill groups. We then briefly outline how to bring our model to data in section 3. In section 4, we look in detail at which ingredients are quantitatively important for the evolution of the skill premium in response to different kinds of shocks to the global economy. We also discuss how our findings relate more broadly to empirical findings on the behavior of the skill premium following trade liberalization episodes. Finally, in section 5, we offer concluding remarks, including where we believe research in this topic should go next.

2 Model

Our model combines existing workhorse models of globalization, trade imbalances, capital-skill complementarity and labor market adjustment. Following [Dix-Carneiro et al. \(2022\)](#), we embed a trade block à la [Caliendo and Parro \(2015\)](#), and the labor supply model of [Artuç et al. \(2010\)](#) into a model of endogenous trade imbalances. The latter are a product of household consumption-savings decisions, which serve as the basis for the inter-temporal approach of [Obstfeld and Rogoff \(1995\)](#). We augment the production structure with capital-skill complementarity following [Krusell et al. \(2000\)](#) and [Parro \(2013\)](#).

The economy consists of $i = 1, \dots, N$ countries. Each country i has a constant supply of skilled (S) workers, $\bar{L}_i^S$, and of unskilled (U) workers, $\bar{L}_i^U$. There are four types of goods in the economy: (a) a non-tradable final good; (b) a non-tradable capital good; (c) K non-tradable sectoral composite intermediate goods; and (d) continua of tradable intermediate varieties. Time unfolds discretely, with periods indexed by t . Finally, agents have perfect foresight over all aggregate variables, and we do not consider aggregate uncertainty.

2.1 Technology

In this subsection we describe the technologies that are available to produce each type of good. The final non-tradable good is produced by perfectly competitive firms according to a Cobb-Douglas aggregator. In each country i , these firms combine the K sector-specific composite intermediate goods with expenditure weight on sector k denoted by $\mu_{k,i}$. Note that the weights are allowed to differ across countries. The price of one unit of the final good in country i is denoted by $P_{i,t}^F$.

Capital goods are also produced by perfectly competitive firms operating a Cobb-Douglas technology. In country i , the expenditure weight on sector k for the capital good is denoted by $\alpha_{k,i}$. This model of capital embeds several special cases from the literature. For example, by letting $\alpha_{k,i} = 1$ and $\mu_{k,i} = 0$ for some specific sector k , we capture the idea of a capital sector as in [Parro \(2013\)](#). Alternatively, by letting $\alpha_{k,i}$ vary across sectors we can capture the idea that investment

goods and consumption goods have different compositions by aggregating similar goods, as in [Eaton et al. \(2016\)](#). The price of one unit of the capital good in country i is denoted by $P_{i,t}^K$.

The K sector-specific composite goods are produced by identical, perfectly competitive firms operating in each sector $k \in \{1, \dots, K\}$ of country i . Total output in sector k is given by a constant elasticity of substitution (CES) aggregate over the output of a sector-specific continuum of tradable varieties indexed by $j \in [0, 1]$. The price of one unit of sector k 's composite in country i is denoted by $P_{k,i,t}^I$, and is given by a CES price index over the prices of individual varieties. These sector-specific composite goods are non-tradable and can be used in the production of the final good, the capital good, or as intermediate inputs in the production of varieties, which we describe next.²

Perfectly competitive firms in country i at time t can produce variety j according to a nested CES production function as in [Parro \(2013\)](#). The lowest tier nest aggregates capital and skilled labor into a composite, $h_{k,i,t}(j)$, according to:

$$h_{k,i,t}(j) = \left[\chi_{k,i} [x_{k,i,t}^K(j)]^{\frac{\rho-1}{\rho}} + [S_{k,i,t}(j)]^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \quad (1)$$

where $x_{k,i,t}^K(j)$ is the quantity of capital used in production, $S_{k,i,t}(j)$ is the quantity of skilled labor, $\chi_{k,i}$ is a time and variety invariant weight, and ρ is the elasticity of substitution between capital and skilled labor. This first composite good is then combined with unskilled labor in the second tier according to:

$$v_{k,i,t}(j) = \left[\xi_{k,i} [h_{k,i,t}(j)]^{\frac{\sigma-1}{\sigma}} + [U_{k,i,t}(j)]^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (2)$$

where $U_{k,i,t}(j)$ is the quantity of unskilled labor used in production, $\xi_{k,i}$ is a time and variety invariant weight, and σ is the elasticity of substitution between the capital-skill composite and unskilled labor. Finally, this composite is combined with sector-specific intermediate inputs in a Cobb-Douglas fashion to make the final output according to:

$$Y_{k,i,t}(j) = z_{k,i,t}(j) v_{k,i,t}(j)^{\gamma_{k,i}} \prod_{l=1}^K M_{l,i,t}(j)^{(1-\gamma_{k,i})\nu_{kl,i}}, \quad (3)$$

where $z_{k,i,t}(j)$ is a common productivity across all producers of variety j of sector k in country i at time t , $M_{l,i,t}(j)$ is the quantity of composite intermediate inputs from sector l , $\gamma_{k,i}$ is the value-added share in production in sector k and country i , and the parameters $\nu_{kl,i}$ summarize the input-output structure of the economy in country i .³

The above production structure makes clear the role of capital in our setup. Although capital is treated as a static input, it does not enter production in the same way as sector-specific intermediate

²Given this structure, the price of the final good in country i is given by $P_{i,t}^F = \prod_{k=1}^K (P_{k,i,t}^I / \mu_{k,i})^{\mu_{k,i}}$; and the price of the capital good is given by $P_{i,t}^K = \prod_{k=1}^K (P_{k,i,t}^I / \alpha_{k,i})^{\alpha_{k,i}}$.

³It is easy to allow for time varying parameters χ , ξ , μ , γ , ν , as we do in Appendices C and D.6.

inputs as it differently substitutes for skilled and unskilled labor. In particular, so long as $\sigma > \rho$, the production function exhibits capital-skill complementarity: a reduction in the price of capital raises demand for skilled labor relative to unskilled labor.⁴

2.2 Households and Labor Supply

Within each country, workers of skill type $s \in \{U, S\}$ are aggregated into two separate representative families. The household head of skill group s , taking prices and wages as given, determines consumption, savings, and labor supply decisions for each member of the household, maximizing aggregate family utility. We first describe the utility of individual workers, then we show how the household planner aggregates individual member utilities. Finally, we explain how the household's problem can be decentralized to the worker level and written recursively. For ease of notation, we temporarily omit country and skill group subscripts, i and s , and index individuals by ℓ .

At the end of each period t , workers are allocated to a sector k_{t+1} (including possibly non-employment, which we denote as $k_{t+1} = 0$) for the next period. In order to move from sector k to k' , the worker incurs a common cost of mobility, $C_{kk'}$ (with $C_{kk} = 0 \forall k$), and an additive stochastic idiosyncratic component, $\omega_{k',\ell,t}$. The $\omega_{k,\ell,t}$ shocks are assumed to be *iid* across individuals and time, and are distributed according to a Gumbel distribution with parameters $(-\nu^{EM}\zeta, \zeta)$, where ν^{EM} is the Euler-Mascheroni constant and ζ is a shape parameter. This setup closely follows [Artuq et al. \(2010\)](#). Given this structure, the *flow* utility for worker ℓ at time t , $\mathcal{U}_{\ell,t}$, can be written as:

$$\mathcal{U}_{\ell,t} \equiv \mathcal{U}(c_{\ell,t}, k_t, \tilde{\mathbf{d}}_{\ell,t}, \boldsymbol{\omega}_{\ell,t}) = \log(c_{\ell,t}) + \eta_{k_t} + \sum_{k'=0}^K \tilde{d}_{k',\ell,t} [-C_{k_t,k'} + \omega_{k',\ell,t}], \quad (4)$$

where $c_{\ell,t}$ is worker ℓ 's consumption of the final good at time t , η_{k_t} is a time-invariant, sector-specific utility term, and $\tilde{\mathbf{d}}_{\ell,t} = (\tilde{d}_{1,\ell,t}, \dots, \tilde{d}_{K,\ell,t})$ is a vector with time $t+1$ sectoral choice indicators. That is, $\tilde{d}_{k,\ell,t} = 1$ if worker ℓ chooses to work in sector k at $t+1$ and $\tilde{d}_{k,\ell,t} = 0$ if the worker chooses to work in a sector different from k . Note that $\tilde{\mathbf{d}}_{\ell,t}$ is indexed by t to highlight that this decision is made at time t . We impose that workers can only work in one sector per period, so that $\sum_{k'=0}^K \tilde{d}_{k',\ell,t} = 1$. Thus, the total supply of workers in the family to industry k at time $t+1$ is given by:

$$L_{k,t+1} = \int_0^{\bar{L}} \tilde{d}_{k,\ell,t} d\ell. \quad (5)$$

⁴One can derive this property following Shephard's Lemma. In particular, and temporarily omitting time, country, and sector subscripts, if P^K is the price of capital, and p^h is the price of the capital-skill composite, then the demand of skilled relative to unskilled labor as a function of capital prices is given by,

$$\frac{d \log(S/U)}{d \log P^K} = (\rho - \sigma) \times \left(\frac{P^K}{p^h} \right)^{1-\rho}.$$

This is a decreasing function if and only if $\rho < \sigma$.

The household head's objective is to maximize the net present value of (4) integrated across family members subject to her budget constraint. In addition to consumption and employment decisions, the planner has access to financial markets by means of buying and selling one-period riskless bonds. Bonds can be traded by families within the same country (across skill groups) as well as internationally, and are available in zero net supply globally. International bond markets are frictionless, with a nominal return that is equalized across space and equal to R_t .

The household head of a given skill group chooses the path of consumption $c_{\ell,t}$, labor supply $\tilde{d}_{k,\ell,t}$, and bonds, B_t , to solve:

$$\max_{\{c_{\ell,t}\}, \{\tilde{d}_{k,\ell,t}\}, \{B_t\}} E_0 \left\{ \delta^t \phi_t \int_0^{\bar{L}} \mathcal{U}_{\ell,t} d\ell \right\} \quad (6)$$

$$\text{subj. to } P_t^F \int_0^{\bar{L}} c_{\ell,t} d\ell + B_{t+1} = \sum_{k=1}^K w_{k,t} \int_0^{\bar{L}} \tilde{d}_{k,\ell,t-1} d\ell + R_t B_t, \quad (7)$$

where E_0 refers to the expectation over future idiosyncratic shocks $\{\omega_{\ell,t}\}$. There is no aggregate uncertainty, and households have perfect foresight over the evolution of aggregate variables. The discount factor, δ , is common across all workers, but is subject to family-level shifters, denoted by ϕ_t . These shifters can differ across countries, skill levels, and over time.⁵ The right hand side of the budget constraint (7) reflects total income available to the household at time t . The first term is the wage income aggregated across all individuals ($w_{k,t}$ is the sector-specific wage), and the second term is the revenue accruing from interest payments on bonds purchased in the previous year. The first term in the left hand side of the budget constraint is total expenditure of the household on final goods. The gap between total expenditures and income must be equal to total bonds purchases.

Let $\tilde{\lambda}_t$ be the Lagrange multiplier on the family's budget constraint. The first order condition on consumption implies that $c_{\ell,t}^{-1} = P_t^F \tilde{\lambda}_t$ for all ℓ .⁶ That is, consumption is equalized across a skill group's family members within a period. We denote this *per capita* consumption by c_t .

Before discussing savings and the worker's individual labor supply decision, we will bring back country and skill subscripts, i and s , but drop individual subscripts, ℓ . Turning to the savings

⁵The use of these shifters is common in the International Macroeconomics literature (Stockman and Tesar, 1995; Bai and Ríos-Rull, 2015). As we illustrate in equation (8), these shifters lead to wedges in the Euler Equation commanding how households trade off current with future consumption. The fact that these shifters lead to wedges in Euler Equations implies that they can also be viewed as generated by asset markets frictions. While in our quantitative exercises we do not make use of these shocks, allowing for these wedges can be important for the model to match the dynamics of aggregate expenditures with final goods (Kehoe et al., 2018; Dix-Carneiro et al., 2022). We discuss them here to show how to incorporate these wedges into our framework. While important for matching data, a drawback is that these parameters do not respond to shocks in the global economy. They are assumed to be exogenous.

⁶In an abuse of terminology we will continue to refer to $\tilde{\lambda}_t$ as the Lagrange multiplier. However, the correct shadow price associated with period's t budget constraint is given by $\delta^t \phi_t \tilde{\lambda}_t$.

behavior of the household, the first order condition on bonds implies the following Euler Equation:

$$\frac{P_{i,t+1}^F c_{i,t+1}^s}{P_{i,t}^F c_{i,t}^s} = \delta R_{t+1} \hat{\phi}_{i,t+1}^s, \quad (8)$$

where $\hat{\phi}_{i,t+1}^s \equiv \frac{\phi_{i,t+1}^s}{\phi_{i,t}^s}$ and will be referred to as (skill group s specific) inter-temporal shocks. It is now apparent that these inter-temporal shocks are wedges to the Euler Equation, giving flexibility for our model to match the path of final goods expenditures in the data. Given a path of income and initial conditions on bond holdings $\{B_{i,0}^s\}$, equations (7) and (8) determine the path of bonds $\{B_{i,t}^s\}$ for each type of household $s \in \{U, S\}$.

Similarly to [Dix-Carneiro et al. \(2022\)](#), the labor supply decision of workers can be decentralized and written recursively from the perspective of an individual worker. In the remainder of this subsection, we outline this recursive problem and the implied transition dynamics for labor supply.

The optimal labor supply decision, $\tilde{d}_{k,i,\ell,t}^s$, of worker ℓ of skill level s in sector k in country i at time t facing idiosyncratic shocks ω_t solves:

$$\tilde{V}_{k,i,t}^s(\omega_t) = \tilde{\lambda}_{i,t}^s w_{k,i,t}^s + \eta_{k,i}^s + \max_{k'} \left\{ -C_{kk',i}^s + \omega_{k',i,t}^s + \delta \hat{\phi}_{i,t+1}^s E_{\omega} \left[\tilde{V}_{k',i,t+1}^s(\omega_{t+1}) \right] \right\}, \quad (9)$$

where $\tilde{V}_{k,i,t}^s(\omega_t)$ is the value function of a worker of skill level s in sector k in country i at time t facing idiosyncratic vector of shocks ω_t . Observe that in equation (9), wages are multiplied by the household head's Lagrange multiplier on the budget constraint $\tilde{\lambda}_{i,t}^s$. To understand the role of the Lagrange multiplier, note that $\tilde{\lambda}_{i,t}^s w_{k,i,t}^s$ is the marginal utility accrued to the whole household from the additional consumption brought in by a worker employed in sector k and earning income $w_{k,i,t}^s$. Therefore, for the household problem to be decentralized, individual workers must internalize the effect of their labor supply decisions on the whole family's utility. This is a key difference between our setup and the hand-to-mouth setup in [Artuç et al. \(2010\)](#), and similar papers.

It is convenient to work directly with $E_{\omega} \left[\tilde{V}_{k,i,t}^s(\omega_t) \right]$, and we denote this integrated value function by $V_{k,i,t}^s$. The Gumbel structure on idiosyncratic shocks implies that there is a simple recursive formula for $V_{k,i,t}^s$ given by:

$$V_{k,i,t}^s = \tilde{\lambda}_{i,t}^s w_{k,i,t}^s + \eta_{k,i}^s + \zeta_i \log \left(\sum_{k'=0}^K \exp \left(\frac{-C_{kk',i}^s + \delta \hat{\phi}_{i,t+1}^s V_{k',i,t+1}^s}{\zeta_i} \right) \right). \quad (10)$$

If we aggregate individual policy rules solving (9) across the distribution of idiosyncratic shocks ω , we obtain inter-sectoral transition rates between sectors k and k' , for skill group s following the

familiar multinomial logit form:

$$s_{kk',i,t,t+1}^s = \frac{\exp\left(\frac{-C_{kk',i}^s + \delta \hat{\phi}_{i,t+1}^s V_{k',i,t+1}^s}{\zeta_i}\right)}{\sum_{k''=0}^K \exp\left(\frac{-C_{kk'',i}^s + \delta \hat{\phi}_{i,t+1}^s V_{k'',i,t+1}^s}{\zeta_i}\right)}. \quad (11)$$

Armed with these inter-sectoral transition rates, labor allocations across sectors is governed by:

$$L_{k,i,t+1}^s = \sum_{k'=0}^K L_{k',i,t}^s s_{k'k,i,t,t+1}^s. \quad (12)$$

Before we describe markets and equilibrium, a discussion is in order regarding the family structure we have assumed. The assumption of organizing skill types into households is equivalent to one in which there is complete risk sharing within groups, but not across groups.⁷ The assumption of perfect risk sharing within certain groups (countries or skills) is, of course, unrealistic, as it shuts down within-group consumption inequality. Nevertheless, the family structure has become a common modeling choice to address aggregate savings behavior in the presence of agent heterogeneity, as it leads to substantially more tractable models.⁸ Within our framework, it allows us to separate the labor supply from consumption decisions within groups, allowing for a manageable quantitative model making predictions about both the skill premium and trade imbalances.

We also stress that the framework we have presented here is quite general. One could instead allow for worker heterogeneity by regions, or along any other dimension.⁹ One could also allow for some subset of agents to share risk, and force other agents to consume their income directly, with no access to savings. Thus, while based on strong assumptions, the family structure makes our framework versatile. The plausibility of risk sharing within and across certain groups should be adjudicated on a case-by-case basis.

⁷See [Altug and Miller \(1998\)](#) for a formal treatment of this idea.

⁸Examples of papers that exploit this assumption to study across group inequality include [Bilbiie and Ragot \(2021\)](#) and [Challe and Ragot \(2015\)](#).

⁹If one were to allow for some households to organize into families, and force some households to consume their income in each period (“hand to mouth”), one would have a hybrid between our model and that of [Caliendo et al. \(2019\)](#). The solution method extends very naturally to solving for policy rules for each type of agent. The key difference is that “hand to mouth” households would not use the Lagrange multiplier in solving their labor supply problem. Crucially, the distribution of assets would remain low-dimensional (zero for “hand to mouth” households, and then a finite set of bonds for the families). Since we ultimately shoot on the distribution of assets (bonds) to solve for the final steady state and transition dynamics following a shock, keeping this object low dimensional is the key requirement for tractability.

2.3 International Trade

Intermediate varieties j can be traded across countries, but they are subject to trade costs.¹⁰ Trade costs are sector, but not variety, specific. We denote the cost of shipping a variety *from* origin o to destination i in sector k at time t by $d_{k,oi,t}$.¹¹

Given wages, $w_{k,i,t}^s$, and prices of intermediates and capital, $P_{k,i,t}^I$ and $P_{i,t}^K$ respectively, we can write the unit cost of an input bundle in country i in sector k at t as:

$$c_{k,i,t} = (p_{k,i,t}^v / \gamma_{k,i})^{\gamma_{k,i}} \times \left(\prod_{l=1}^K (P_{l,i,t}^I / \nu_{kl,i})^{\nu_{kl,i}} \right)^{1-\gamma_{k,i}}, \quad (13)$$

where

$$p_{k,i,t}^v = \left(\xi_{k,i}^\sigma [p_{k,i,t}^h]^{1-\sigma} + [w_{k,i,t}^U]^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \quad (14)$$

and

$$p_{k,i,t}^h = \left(\chi_{k,i}^\rho [P_{i,t}^K]^{1-\rho} + [w_{k,i,t}^S]^{1-\rho} \right)^{\frac{1}{1-\rho}}. \quad (15)$$

Since there is perfect competition, the price of variety j in sector k at time t shipped from origin o to destination i is given by the marginal cost:

$$p_{k,oi,t}(j) = \frac{c_{k,o,t}}{z_{k,o,t}(j)} d_{k,oi,t}.$$

Consumers and firms in country i purchase varieties from the lowest cost producer country. Hence, the price of variety j in sector k in country i at time t is given by:

$$p_{k,i,t}(j) = \min_o \{p_{k,oi,t}(j)\}.$$

Following Eaton and Kortum (2002), we assume that $z_{k,i,t}(j) \sim \text{Frechet}(A_{k,i,t}, \lambda)$, where $A_{k,i,t}$ is a scale parameter akin to sector- and country-specific total factor productivity (TFP) and λ is the shape parameter that determines comparative advantage within sectors. The scale parameters are allowed to be country, sector, and time specific. However, the shape parameter is assumed to be common across sectors and countries and to be constant over time. Given these assumptions, the price of sector k goods in country i at time t is written as:

$$P_{k,i,t}^I = B_{k,i} \times \left(\sum_{o=1}^N A_{k,o,t} [c_{k,o,t} d_{k,oi,t}]^{-\lambda} \right)^{-1/\lambda}, \quad (16)$$

¹⁰This is without loss of generality, since one could include non-tradables by setting trade costs to infinity, and could include freely traded goods by setting (proportional) trade costs to 1.

¹¹We abstract from tariffs and tariff revenues, but these can be easily incorporated as in Caliendo and Parro (2015) and Parro (2013).

where $B_{k,i}$ is a constant. Given wages, equations (13) to (16), together with the price of capital $P_{i,t}^K = \prod_{l=1}^K \left(P_{l,i,t}^I / \alpha_{l,i} \right)^{\alpha_{l,i}}$, define a system of equations that can be used to solve for all goods prices in the economy. Moreover, it can be shown that under the Frechet assumption, country i 's share of expenditure on sector k goods from country o at time t takes on the familiar gravity form:

$$\pi_{k,oi,t} = \frac{A_{k,o,t} [c_{k,o,t} d_{k,oi,t}]^{-\lambda}}{\sum_{o'=1}^N A_{k,o',t} [c_{k,o',t} d_{k,o'i,t}]^{-\lambda}}. \quad (17)$$

2.4 Market Clearing

Given an allocation of labor of type s across sectors, $L_{k,i,t}^s$, labor market clearing requires:

$$w_{k,i,t}^s L_{k,i,t}^s = e_{k,i,t}^s \gamma_{k,i} Y_{k,i,t}, \quad (18)$$

where $Y_{k,i,t}$ is gross output in sector k in i at t , and $e_{k,i,t}^s$ is the share of value-added paid to labor of type s . Because we do not assume that $\sigma = \rho = 1$, the expenditure shares $e_{k,i,t}^s$ are endogenous to prices. Goods market clearing requires that gross output in sector k in each country and period is equal to the sum of demand across trade partners. Denote total expenditure on sector k by country i by $X_{k,i,t}$. Using equation (17), goods market clearing implies that output from country o in sector k at time t is given by:

$$Y_{k,o,t} = \sum_{i=1}^N \pi_{k,oi,t} X_{k,i,t}. \quad (19)$$

Expenditure $X_{k,i,t}$ is given by the sum of final consumption by households, capital expenditure, and expenditure on inputs. To obtain the equation determining $X_{k,i,t}$, first define $NX_{i,t}$ as the value of total net exports in country i at time t , $I_{i,t}$ to be total disposable income across all households in country i at time t , and $X_{i,t}^K$ to be total expenditure on capital goods in country i at time t . With this notation in hand, $X_{k,i,t}$ can be written as:

$$X_{k,i,t} = \mu_{k,i} [I_{i,t} - NX_{i,t}] + \alpha_{k,i} X_{i,t}^K + \sum_{l=1}^K (1 - \gamma_{l,i}) \nu_{lk,i} Y_{l,i,t}, \quad (20)$$

where

$$X_{i,t}^K = \sum_{l=1}^K e_{l,i,t}^K \gamma_{l,i} Y_{l,i,t}, \quad (21)$$

and $e_{l,i,t}^K$ is the endogenous share of payments to capital in sector l .

Aggregate disposable income $I_{i,t}$ is given by the sum of disposable income across skill groups

$I_{i,t}^s$, which is given by:

$$I_{i,t}^s = \sum_{k=1}^K w_{k,i,t}^s L_{k,i,t}^s = \sum_{k=1}^K e_{k,i,t}^s \gamma_{k,t} Y_{k,i,t}. \quad (22)$$

Finally, bonds market clearing requires that net aggregate exports are equal to changes in bonds net of interest:

$$NX_{i,t} = I_{i,t}^S + I_{i,t}^U - (E_{i,t}^{C,S} + E_{i,t}^{C,U}) = B_{i,t+1}^S + B_{i,t+1}^U - R_t(B_{i,t}^S + B_{i,t}^U), \quad (23)$$

and that bonds are in global zero net supply:

$$\sum_{i=1}^N \sum_{s \in \{U,S\}} B_{i,t}^s = 0. \quad (24)$$

In equation (23), $E_{i,t}^{C,s} \equiv \bar{L}_i^s P_{i,t}^F c_{i,t}^s$ is the total expenditure of skill group s in final goods.

2.5 Equilibrium

An equilibrium in this model is a set of initial steady-state allocations $\{L_{k,i,0}^s, B_{i,0}^s\}$, a set of final steady-state allocations $\{L_{k,i,\infty}^s, B_{i,\infty}^s\}$ and sequences of policy functions for workers/firms $\{s_{kk',i,t,t+1}^s\}$, value functions for workers $\{V_{k,i,t}^s\}$, bond decisions by the households $\{B_{i,t}^s\}$, bond returns $\{R_t\}$, allocations $\{L_{k,i,t}^s\}$, household consumption per capita $\{c_{i,t}^s\}$, trade shares $\{\pi_{k,io,t}\}$, and price indices $\{P_{k,i,t}^I, P_{k,i,t}^K, P_{k,i,t}^F\}$ such that: (a) Workers' value functions solve (10); (b) Consumption and bonds decisions solve (6) subject to (7), given an initial distribution of bonds $\{B_{i,0}^s\}$; (c) Labor allocations evolve according to equation (12); (d) Trade shares are given by (17); (e) Prices are set competitively and goods markets clear: equations (19) to (22) hold; (f) Labor markets clear: $\sum_{k=1}^K L_{k,i,t}^s = \bar{L}_i^s$ for $s \in \{U, S\}$; (g) Bonds market clears: $\sum_{i=1}^N \sum_{s \in \{U, S\}} B_{i,t}^s = 0$. Appendix D displays the algorithms we designed to compute the steady-state equilibrium and to compute transitional dynamics in response to changes in the global environment.

2.6 Discussion

To understand how trade imbalances arise in our model, assume that there are no inter-temporal preference shocks, and so $\hat{\phi}_{i,t}^s = 1$ for all i, s and t . In this case, equation (8) implies that $E_{i,t+1}^{C,s} = \delta R_{t+1} E_{i,t}^{C,s}$ for each country i and skill group s over the transition path. Normalizing $\sum_{i=1}^N \sum_{s \in \{U, S\}} E_{i,t}^{C,s} = 1$ —so that all nominal variables are expressed as a fraction of world expenditure on final goods—we obtain that $R_t = 1/\delta$ for all t .¹² In turn, this implies that individual

¹²Given that net exports must sum to zero at each point in time, normalizing $\sum_{i=1}^N \sum_{s \in \{U, S\}} E_{i,t}^{C,s} = 1$ implies that world GDP is also normalized to one, i.e., $\sum_{i=1}^N \sum_{s \in \{U, S\}} I_{i,t}^s = 1$.

countries' expenditures on final goods are constant as a share of world expenditure following a shock. Therefore, for any path of shocks, countries immediately smooth final expenditures as a share of global final goods expenditures (global GDP).

To fix ideas, suppose that China suddenly realizes that it will gradually become more productive and richer. In this case, our model predicts that China's final goods expenditure, as a share of world GDP, will immediately jump to its new steady-state value. In turn, this implies that China will consume above production in the short run and then below in the long run, leading to short-run trade deficits and long-run trade surpluses. Nonetheless, in the data, we rarely observe this stark version of expenditure smoothing we have just discussed. As we previously discussed, the inter-temporal preference shocks $\hat{\phi}_{i,t}^s = 1$ are wedges that reconcile our model with the observed data.

3 Calibration and Data

3.1 Preliminaries

We calibrate our model to a global economy with six sectors and six countries. We consider a world comprised of the United States, China, and four country aggregates: Europe, Asia/Oceania, the Americas, and the Rest of the World. Each country's economic activity consists of six sectors: Agriculture; Low-, Mid- and High-Tech Manufacturing; Low- and High-Tech Services. Tables A.1 and A.2 in Appendix A detail these divisions.

Table 1 summarizes the parameters we need to numerically solve the model. We split them into three categories: (i) parameters that are fixed at values previously reported in the literature, as they are difficult to identify given available data (Panel A); (ii) parameters that can be determined without having to solve the model (Panel B); and (iii) parameters that are estimated by the method of simulated moments. We calibrate our model using a variety of datasets for year 2000 or closest year available.

We start by discussing parameters fixed according to values reported in the literature, which are listed in Panel A. First, we calibrate the model at the annual frequency. In this case, annual steady-state international bonds' returns are given by $1/\delta$, so we set $\delta = 0.95$ implying annual returns of 5%.¹³ The estimation of the dispersion of the idiosyncratic ω shocks typically requires panel data and instrumental variable strategies. As a result, we impose this parameter to be common across countries and set $\zeta_i = 1.61 \forall i$ based on the estimate Artuç and McLaren (2015) obtained using US data. The Frechet scale parameter $\lambda = 4$ comes from Simonovska and Waugh (2014). We follow Parro (2013) and set the elasticity of substitution between skilled labor and capital goods

¹³This choice is based on the fact that both the Federal Funds and T-Bill rates in 1999-2000 were between 5% and 6%: <https://fred.stlouisfed.org/series/FEDFUNDS> and <https://fred.stlouisfed.org/series/DTB1YR>.

$\rho = 0.67$, and the elasticity of substitution between unskilled labor and capital goods $\sigma = 1.67$. These elasticities have been originally estimated by [Krusell et al. \(2000\)](#).

Table 1: Summary of Parameters

Panel A. Fixed According to the Literature			
Parameter	Value	Description	Source
δ	0.95	Discount factor	5% annual interest rate
ζ_i	1.61	Dispersion of ω shocks	Artuç and McLaren (2015)
λ	4.00	Frechet Scale Parameter	Simonovska and Waugh (2014)
ρ	0.67	Capital-skilled labor EoS	Krusell et al. (2000)
σ	1.67	Composite-unskilled labor EoS	Krusell et al. (2000)
Panel B. Calibrated Outside of the Model			
Parameter		Description	Source
$\mu_{k,i}$		Final Expenditure Shares	WIOD
$\alpha_{k,i}$		Capital Expenditure Shares	WIOD
$\gamma_{k,i}$		Value-Added Expenditure Shares	WIOD
$\nu_{k\ell,i}$		Input-Output Matrix	WIOD
Panel C. Calibrated Using the Model Structure			
Parameter		Description	
$C_{kk'},^s i$		Mobility Costs	
$\eta_{k,i}^s$		Sector-Specific Utility	

Turning to Panel B, we can directly calibrate final expenditure shares $\mu_{k,i}$, capital expenditure shares $\alpha_{k,i}$, labor expenditure shares $\gamma_{k,i}$, and input-output shares $\nu_{k\ell,i}$, without having to solve the model. To that aim, we employ the World Input Output Database (WIOD), which compiles data from national accounts combined with bilateral international trade data for a large collection of countries. These data cover 56 sectors and 44 countries, including a Rest of the World aggregate, between 2000 and 2014. In the next section, we describe how we obtain the labor supply parameters (mobility costs and sector-specific utilities).

3.2 Labor Supply Parameters

The calibration procedure for mobility costs $C_{kk',i}^s$ and sector-specific utilities $\eta_{k,i}^s$ proceeds in two steps. First, we show that given values for ζ_i and δ , and data on skill-group-specific transition rates across sectors, we can exactly invert the model to obtain these parameters. This is the procedure we

follow for US-specific parameters. For the remaining countries, we do not have data on transition rates by skill groups, and, sometimes, these data are only available at a more aggregate level than what we have in Table A.2 of Appendix A—see Dix-Carneiro et al. (2022) for details. Therefore, we impose $C_{kk',i}^s = \psi_i \times C_{kk',US}^s$ for the remaining countries, where ψ_i is calibrated to target the average sectoral persistence rate of workers in country i .

3.2.1 Inverting the Model for Parameters in the US

To calibrate the labor supply parameters for the US, we assume that the economy is in steady state in the year 2000. Given that we treat ζ_{US} and δ as known, we are able to exactly map our data to mobility costs and sector-specific utilities. In the remainder of this subsection, we omit the country and time subscripts, since we focus on the US in steady state. We also omit skill superscripts for ease of exposition, but note that we calibrate different parameters for each skill group.

Inverting the model proceeds similarly to the estimation strategy of Artuç et al. (2010). In particular, manipulating a steady-state version of equation (11) leads to the following equation:

$$\log\left(\frac{s_{kk'}}{s_{kk}}\right) - \delta \log\left(\frac{s_{kk'}}{s_{k'k'}}\right) = -\frac{(1-\delta)C_{kk'}}{\zeta} + \frac{\delta}{\zeta}\tilde{\lambda}(w_{k'} - w_k) + \frac{\delta}{\zeta}(\eta_{k'} - \eta_k), \quad (25)$$

with the convention that wages in the non-employment sector are zero, $w_0 = 0$, and with the normalization that $C_{kk} = 0 \forall k$.

Appendix D.1 shows that wages w_k and the Lagrange multiplier $\tilde{\lambda}$ can be obtained as a function of trade shares ($\pi_{k,oi}^{Data}$), producers' expenditure shares ($e_{k,i}^{U,Data}$, $e_{k,i}^{S,Data}$), and parameters in Panel B of Table 1. In addition, skill-specific inter-sectoral transition rates are observed in the US. Therefore, equation (25) allows us to perfectly invert the data to obtain mobility costs C and sector-specific utilities η . However, before doing so, we need an additional normalization. This is especially clear for the term $\eta_{k'} - \eta_k$, since shifting the vector of η 's by a constant would leave this difference unchanged. As a consequence, in addition to setting $C_{kk} = 0$ and $w_0 = 0$, we also set the utility of non-employment to zero, i.e., $\eta_0 = 0$. With these normalizations in hand, equation (25) describes a system of linear equations that can be solved for both $C_{kk'}$ and η_k . Appendix D.1 provides the details behind this procedure. Central to this procedure is access to full data on inter-sectoral transition by skill group. This is the case for the United States, but data from other countries is often more limited. In the next subsection, we discuss how we calibrate the labor supply parameters for other countries.

3.2.2 Calibration Routine for Non-US Countries

For countries besides the US, we only observed inter-sectoral transition matrices that are common across skill levels. In addition, for some countries (for example, China), we only have data on

transitions between more aggregated sectors, which don't include Agriculture. Therefore, we cannot apply the procedure we outlined in the previous section to exactly invert mobility costs and sector-specific utilities. Instead, we proceed by jointly calibrating a scaling factor ψ_i (common across skill levels) for mobility costs, so that $C_{kk',i}^s = \psi_i \times C_{kk',US}^s$, and a vector of sector-specific utilities η_i^s for each country.

To recover these parameters we similarly assume that the global economy is initially in steady state and employ a nested procedure. First, we compute output across sectors and countries, $\{Y_{k,i}\}$ using equations (19) to (22) and data on trade shares $\{\pi_{k,oi}^{Data}\}$ —see Step 2 in Appendix D.2 for details. Next, we compute wages implied by $\{Y_{k,i}\}$, expenditure shares on skilled and unskilled workers $\{e_{k,i}^{s,Data}\}$, and the observed labor allocations in the data $\{L_{k,i}^{s,Data}\}$: $w_{k,i}^{s,0} = \frac{\gamma_{k,i} e_{k,i}^{s,Data} Y_{k,i}}{L_{k,i}^{s,Data}}$. In the inner nest, given a value of ψ_i , we recover the η_i^s vector that exactly replicates $w_{k,i}^{s,0}$. Appendix D.2 details an iterative procedure solving for this problem. Here, we outline the basic idea. Given a guess for η_i^s , and conditional on ψ_i and $w_{k,i}^{s,0}$, we solve the Bellman equation in (10) and obtain transition rates (11). These steady-state transition rates imply labor allocations $L_{k,i}^{s,Model}$. We then compute a model-implied wage that rationalizes these allocations: $w_{k,i}^{s,Model} = \frac{\gamma_{k,i} e_{k,i}^{s,Data} Y_{k,i}}{L_{k,i}^{s,Model}}$. At this point, we update η_i^s based on the deviations between the model-implied wage $w_{k,i}^{s,Model}$ and $w_{k,i}^{s,0}$. Intuitively, if the guessed value for $\eta_{k,i}^s$ is too low, then labor supply, evaluated at $w_{k,i}^{s,0}$, will be too low in that sector—and the iterative algorithm will raise $\eta_{k,i}^s$.

The outer loop of our calibration routine finds a value of ψ_i that minimizes the distance between some function of the observed subset of transitions, and their data counterpart. In principle, one could use any function of the observed subset of transition rates, and it may depend on the data one has at hand. In practice, we match the average persistence of workers in a sector, $\sum_k \sum_s s_{kk,i}^{s,Data} \frac{L_{k,i}^{s,Data}}{L_i^s}$.¹⁴ We use golden section search to minimize the squared distance between model and data persistence.

3.3 Data

We use data from the Current Population Survey (CPS) in the United States to obtain employment allocations, including non-employment. For the remaining countries, we obtain unemployment rates from ILOSTAT and employment allocations from WIOD. Average wages across countries and sectors are similarly drawn from the WIOD.¹⁵

¹⁴In some cases, we only observe a subset of sectors or aggregated sectors. For example, in China we only observe total manufacturing. In these cases, we replicate the observed transition matrix in the model, and apply the function to these replicated transition matrix. The details of what we observe for each country can be found in the Data Appendix.

¹⁵Given that workers are homogeneous in our model, we adjust the wage data from WIOD to control for differences in skill composition across sectors. We also adjust wages for differences in industrial composition across countries in each of our four country aggregates. Our Data Appendix provides the details behind this procedure.

To be able to identify mobility costs, we make use of micro-data from several countries. Except for the US and China, all the remaining countries are country aggregates. In these cases, we select one country or set of countries as “representative” for which we measure yearly worker transition rates across sectors. Table 2 lists the representative countries and the datasets we have used to obtain inter-sectoral transition rates.¹⁶ As previously noted, we only use skill-specific transition rates for the US. Transition rates for the remaining countries aggregate across skill groups.

Table 2: Micro Data Used to Compute Inter-Sectoral Transition Rates

Country Aggregate (Representative Country)	Source	Year
United States	Current Population Survey (CPS)	1999-2000
China	Urban Household Survey	2004
Europe (United Kingdom)	Labour Force Survey	1999-2001
Asia/Oceania (Korea, Australia)	Korean Labor and Income Panel Study	1999-2000
	Household, Income and Labour Dynamics in Australia	2001-2002
Americas (Brazil)	Relação Anual de Informações Sociais	1999-2000
Rest of World (Turkey)	Entrepreneur Information Survey	2014

Notes: For Asia/Oceania, we target the population-weighted average of transition rates and coefficient of variation of wages for South Korea and Australia. We were not able to gather information for the year 2000 for all the datasets we employ. In these cases, we selected the closest possible year for which the relevant data are available.

3.4 Calibration Results

We map college workers to the skilled labor group and non-college workers to the unskilled labor group. Tables 3 to 5 contain estimates of the labor supply parameters, while Tables B.1 to B.4 in Appendix B contain estimates of the various preference and production function parameters. We begin with Table 3, which contains estimated inter-sectoral mobility costs in the US. We report the values of mobility costs as a fraction of ζ , the dispersion of idiosyncratic preference shocks for sectors ω . In addition, to make our estimates more directly comparable to those in Artuç et al. (2010) and Artuç and McLaren (2015), we express C_{US}^s/ζ relative to $\tilde{\lambda}_{US}^s \times \bar{w}_{US}^s$, where $\bar{w}_{US}^s$ is the average wage of skill group s in the US.¹⁷ From now on, we refer to the numbers in Table 3 as normalized inter-sectoral mobility costs. Our estimates of normalized inter-sectoral mobility costs are remarkably similar in magnitude across skill groups. If anything, they are slightly larger for college-educated workers. There are potentially many reasons for this finding, ranging from the possibility of skill-group-specific values of ζ to the possibility of un-modeled specific human capital.

¹⁶The Brazilian *Relação Anual de Informações Sociais* and the Turkish *Entrepreneur Information System* (EIS) are administrative datasets. See Dix-Carneiro (2014) and Demir et al. (2021) for descriptions of these data. We are extremely grateful to Wei Huang and Banu Demir for their very generous help with China’s Urban Household Survey and with Turkey’s EIS data, respectively.

¹⁷Artuç et al. (2010) and Artuç and McLaren (2015) normalize the average wage in the US, $\bar{w}_{US} = 1$, and have $\tilde{\lambda}_{US} = 1$.

Nevertheless, the numbers are similar in magnitude to other estimates in the literature ([Artuç et al., 2010](#); [Artuç and McLaren, 2015](#)).

Table 3: Normalized Inter-Sectoral Mobility Costs in the US – $C_{US}^s/(\tilde{\lambda}_{US}^s \times \bar{w}_{US}^s \times \zeta)$ for $s \in \{Non - College, College\}$

From ↓ To →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
<i>Panel A: Non-College</i>						
Agriculture	0	6.05	5.91	6.20	3.54	5.19
LT Manufacturing	4.91	0	3.86	3.70	2.98	4.19
MT Manufacturing	5.91	4.12	0	4.15	2.83	4.42
HT Manufacturing	4.91	3.88	4.01	0	2.84	3.80
LT Services	5.60	5.66	5.36	5.38	0	3.96
HT Services	5.32	5.34	5.41	4.80	2.39	0
Non-Employment	4.70	5.90	5.45	5.72	2.96	3.97
<i>Panel B: College</i>						
Agriculture	0	5.18	6.42	4.42	4.46	3.47
LT Manufacturing	7.56	0	5.64	4.42	3.65	4.20
MT Manufacturing	6.36	4.02	0	5.15	3.73	4.38
HT Manufacturing	6.86	4.75	4.49	0	3.14	2.88
LT Services	6.81	5.27	5.73	5.54	0	3.31
HT Services	6.67	5.75	5.42	5.27	3.53	0
Non-Employment	6.19	7.56	6.76	6.64	3.99	3.51

Notes: Estimates of mobility costs in the literature, such as [Artuç et al. \(2010\)](#) and [Artuç and McLaren \(2015\)](#) (a) normalize the average wage in the US $\bar{w}_{US} = 1$; and (b) have $\tilde{\lambda}_{US} = 1$. To be able to compare our estimates to those, we express C_{US}^s as a fraction of $\tilde{\lambda}_{US}^s \times \bar{w}_{US}^s \times \zeta$ for each skill group.

While the magnitudes are similar across skill groups, there are some notable differences in the patterns of mobility costs. For example, mobility costs for college-educated workers to enter Agriculture are much higher than almost any other cost, with only costs of exiting non-employment at the same level. This is not the case for non-college workers. These steep costs are needed to rationalize the low rates at which college-educated workers switch into the Agriculture sector. While the average off-diagonal element of the transition matrix is on the order of 2%, the average transition rate into Agriculture is less than 1%. These moving costs can matter for the transition dynamics of the skill premium, as differences in the willingness of workers to reallocate across certain sectors can shape the dynamics of wage adjustments.

Table 4 compares mobility costs around the world, appropriately normalized, $C_i/(\tilde{\lambda}_i \bar{w}_i)$, to those in the US. The appropriately normalized values of ψ_i are relatively close to 1, with the Rest of the World (RoW) being a notable exception. Chinese mobility costs are estimated to be smaller than those in the US, but the implied magnitudes are all still within 2 to 4 times marginal-utility-adjusted average wages. However, RoW's costs are much lower.

Sector-specific utilities are shown in Table 5. Given that workers of skill group s choose sectors

Table 4: Mobility Costs Around the World Relative to the US's

$$\frac{C_i/(\tilde{\lambda}_i \bar{w}_i)}{C_{US}/(\tilde{\lambda}_{US} \bar{w}_{US})} = \psi_i \times \frac{\tilde{\lambda}_{US} \bar{w}_{US}}{\tilde{\lambda}_i \bar{w}_i}$$

	Country					
	US	China	Europe	Asia/Oc.	Americas	RoW
$\psi_i \times \frac{\tilde{\lambda}_{US} \bar{w}_{US}}{\tilde{\lambda}_i \bar{w}_i}$	1	0.71	1.05	0.91	1.18	0.54

Notes: C_i stands for the skill-group-size weighted mean of mobility costs in country i , $\tilde{\lambda}_i \bar{w}_i$ stands for the skill-group-size weighted mean of marginal utility adjusted wages in country i . This table reports $\frac{C_i/(\tilde{\lambda}_i \bar{w}_i)}{C_{US}/(\tilde{\lambda}_{US} \bar{w}_{US})} = \psi_i \times \frac{\tilde{\lambda}_{US} \bar{w}_{US}}{\tilde{\lambda}_i \bar{w}_i}$ so that we are better able to compare estimated mobility costs relative to the US.

Table 5: Normalized Sector-Specific Utilities $\eta_{k,i}^s/(\tilde{\lambda}_i^s \times \bar{w}_i^s)$ for $s \in \{Non - College, College\}$

	Country					
Sector	US	China	Europe	Asia/Oc.	Americas	RoW
<i>Panel A: Non-College Workers</i>						
Agriculture	1.85	1.30	0.45	1.80	0.70	2.12
LT Manufacturing	1.14	-0.36	0.03	0.95	-0.32	1.11
MT Manufacturing	1.67	0.40	0.24	1.20	0.29	1.55
HT Manufacturing	0.94	-1.14	-0.54	0.70	-2.71	-1.32
LT Services	1.63	0.09	-0.21	0.98	-0.43	-0.02
HT Services	1.72	0.74	0.20	1.07	-0.03	0.97
Non-Employment	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
<i>Panel B: College Workers</i>						
Agriculture	3.01	2.01	0.67	1.78	0.51	0.82
LT Manufacturing	1.72	1.03	0.24	0.75	-0.06	0.37
MT Manufacturing	2.09	1.35	0.51	0.87	0.30	0.12
HT Manufacturing	1.30	0.09	-0.38	0.43	-1.99	-2.27
LT Services	2.56	1.41	0.09	0.77	-0.22	-1.06
HT Services	2.78	2.07	0.57	0.90	0.30	0.32
Non-Employment	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00

Notes: Workers decide in what sector to search partly based on wages scaled by $\tilde{\lambda}_i^s$. To aid the interpretation of the magnitude of the estimates of $\eta_{k,i}^s$, we express them as a fraction of $\tilde{\lambda}_i^s \times \bar{w}_i^s$, where $\bar{w}_i^s$ is the average wage of skill group s in country i .

based on wages scaled by the Lagrange multiplier $\tilde{\lambda}_i^s$ (see equation (9)), we compare our estimates of $\eta_{k,i}^s$ to the model-implied values of $\tilde{\lambda}_i^s \times \bar{w}_i^s$ across countries. Generally, the appropriately normalized value of η is positive—suggesting that employment tends to be more attractive than non-employment, above and beyond wages. There are some exceptions to this conclusion—mostly in the Americas, where η ’s are often negative. This result likely reflects very high persistence in non-employment in Brazil (our representative country for the Americas), which is around 97%. This region also has high non-employment rates relative to other countries. For example, non-employment among non-college Workers is 9%, second only to Europe. These terms can pick up additional, un-modeled labor market frictions, non-employment benefits, or non-compensating differentials. We are agnostic on what exactly is absorbed into η , but we do assume that these are held constant across different counterfactuals.

Another pattern that stands out is that η ’s in Agriculture are generally high—especially in the US, China, and RoW. This is how our model can rationalize the size of this sector in these countries despite their relatively low wages. For example, in the US, the wage in Agriculture for college workers is nearly half of the average wage in the services sectors, and less than one third of the average wage in the manufacturing sectors. Finally, comparing across skill groups, the sector-specific utilities are larger for college-educated workers in the US, China, Europe and the Americas, and lower in Asia/Oceania and the Rest of the World. This suggests that differences in non-employment between groups are unlikely to be explained by wage differences alone. With these parameters in hand, we are in a position to perform counterfactual experiments. In the next section we use our framework as a laboratory to study the quantitative importance of our mechanisms for the skill premium.

4 Mechanisms and Quantification

In this section, we study our model’s implications for the evolution of the skill premium in response to various shocks in the global environment. In our setting, as in Parro (2013), there are two channels through which shocks can affect the skill premium. The first we call the *Stolper-Samuelson channel*: since skill intensities differ across sectors, globalization shocks change the relative demand for different types of workers depending on which sectors expand and which contract. The second is *capital-skill complementarity*: if trade shocks lower the price of capital goods, relative demand for skilled labor will increase *in all sectors*. Unlike in standard factor proportions theory, a decline in capital prices will favor skilled labor, regardless of factor intensities. In order to gauge the importance of these two channels, we contrast the evolution of the skill premium in our full model with a model that eliminates capital-skill complementarity by imposing Cobb-Douglas production

functions in capital, college-educated workers, and non-college workers.¹⁸

To illustrate the Stolper-Samuelson channel, we analyze which sectors expand and contract following a trade shock, with a focus on their respective factor intensities. To understand the capital-skill complementarity channel, we study the evolution of capital prices. Throughout our exercises, we contrast the results of our model of endogenous trade imbalances to one where trade needs to balance in every period. The long-run steady state of this latter model corresponds most closely to the typical exercises considered in the International Trade literature—a key difference being the presence of labor market frictions in our framework.

General equilibrium forces interact in complicated ways, more so with multiple factors of production (Jones and Scheinkman, 1977), motivating much of the “quantitative” approach in the International Trade literature (Costinot and Rodriguez-Clare, 2014). We follow this quantitative approach, focusing on the implications of changes in trade costs and productivities, which constitute common exercises in this literature. We stress at the outset that while our mechanisms—the Stolper-Samuelson channel and capital-skill complementarity—are general, the numerical results are particular: which forces dominate will depend on the the exact configuration of shocks we feed into the model.

We focus on three sets of shocks to the global environment. First, motivated by its accession to the World Trade Organization in 2001, we focus on a trade liberalization episode in China, where both import and export costs are gradually reduced. Next, we study the implications of our model for slow productivity growth in China. Finally, we contrast our first exercise to the implications of a global reduction in trade costs. This comparison is informative about the mechanisms and implications of our model because a trade shock affecting a single country typically has different effects on trade imbalances relative to shocks that are common across countries.

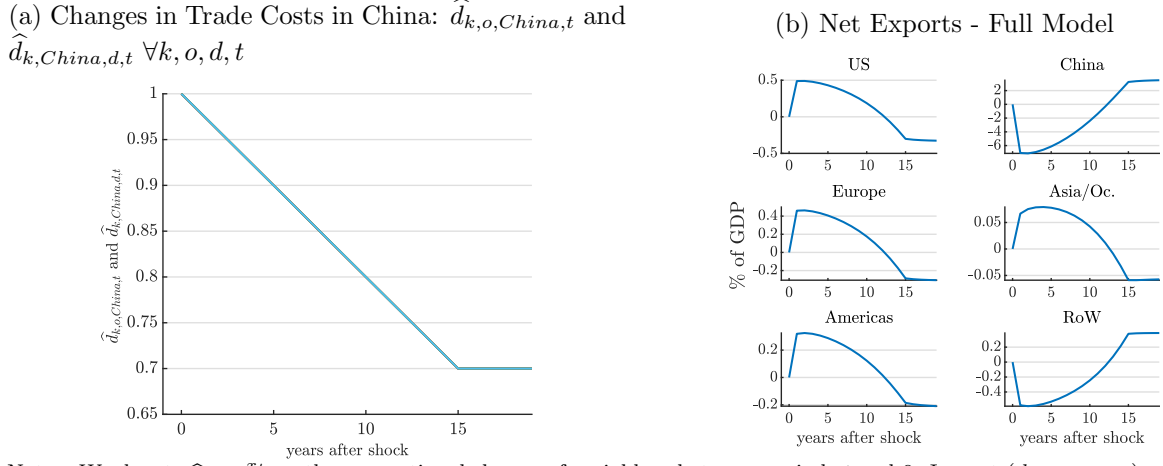
4.1 Trade Liberalization in China

We first outline how a decline in both import and export costs in China—holding all other trade costs constant—impacts the global economy over time: we study its impact on trade imbalances, output, labor allocations, and capital prices across countries. We subsequently turn to the evolution of the skill premium.

From now on, we denote $\hat{x}_t \equiv \frac{x_t}{x_0}$ as the proportional change of variable x between periods t and 0. Figure 1a plots the path of changes in import and export costs in China ($\hat{d}_{k,o,China,t}$ across origin countries o , and $\hat{d}_{k,China,d,t}$ across destination countries d). All import and export costs into and from China slowly decline for 15 years until they reach a total reduction of 30%. The full path of the shock is revealed at the end of period 0, and we impose that all countries start from a balanced-trade steady state. Figure 1b plots the resulting changes in net exports across countries.

¹⁸In practice, we set $\sigma = .99$ and $\rho = 1.01$ in equations (1) and (2).

Figure 1: Trade Liberalization in China and Resulting Trade Imbalances



Notes: We denote $\hat{x}_t \equiv \frac{x_t}{x_0}$ as the proportional change of variable x between periods t and 0. Import ($d_{k,o,China,t}$) and export ($d_{k,China,d,t}$) costs in China gradually decline until they reach a 30% reduction—for all origins o , destinations d and sectors k . Note that $\hat{d}_{k,i,i,t} = 1$ for all sectors k , countries i and periods t . Panel (b) displays resulting outcomes of our full model with trade imbalances and capital-skill complementarity.

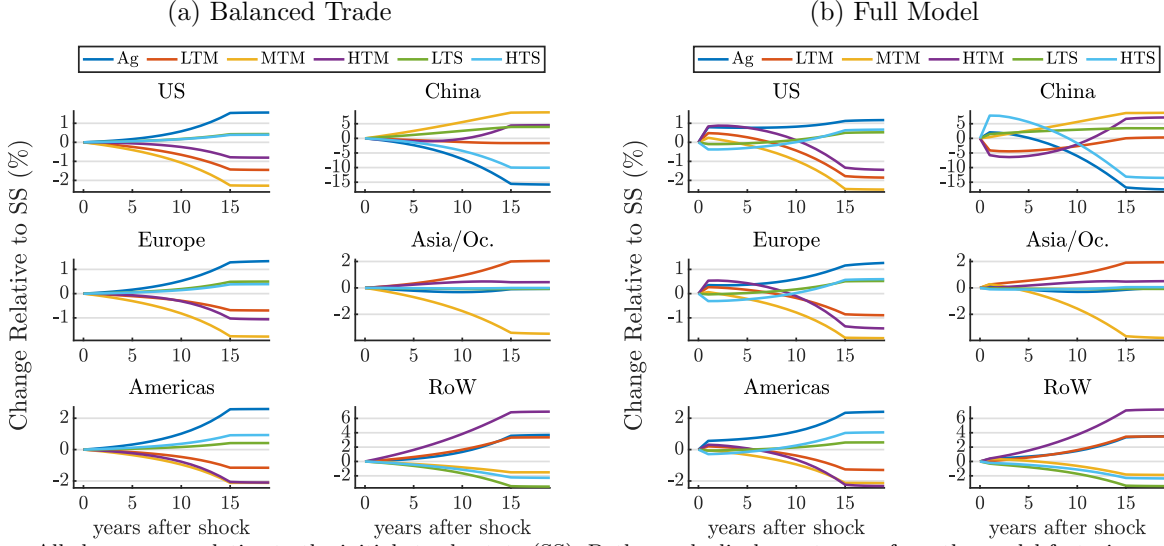
As soon as the reduction in Chinese trade costs is revealed, Chinese household heads anticipate they will be able to enjoy higher consumption in the long run. To smooth consumption, China borrows in the short run—running a trade deficit. In the long run, this debt is paid down by running a trade surplus in perpetuity. The behavior of trade imbalances in the remaining countries mirrors China's: they mostly run initial trade surpluses, followed by perpetual trade deficits (the Rest of the World being an exception). As we show below, these inter-temporal shifts in consumption have rich dynamic impacts on workers of different skill levels.

To investigate the Stolper-Samuelson channel, we start by inspecting what sectors expand or contract in response to the shock in Figure 1a. To that aim, Figures 2a and 2b plot the resulting evolution of gross output shares. Panel (a) plots allocations in the balanced-trade version of our model, where trade is imposed to be balanced period by period.¹⁹ As China grows steadily richer, countries tend to increase their exports to China, tilting global production patterns towards Chinese demand—in terms of final goods consumption, demand for intermediate inputs, and demand for capital production.²⁰ For example, the United States expands output in Agriculture, in which it has a comparative advantage and Chinese final goods demand is high. Simultaneously, China expands its manufacturing sectors, in which it enjoys a comparative advantage. Finally, convergence towards the new steady state is mostly monotonic.

¹⁹More precisely, we impose that international bond holdings $B_{i,t}^s = 0$ for all skill groups s , all countries i and all periods t .

²⁰Figure B.1 in Appendix B plots the $\alpha_{k,i}$ and $\mu_{k,i}$ parameters across countries and sectors. One can see the stark difference in China's expenditure on Agriculture relative to other countries, as well as the importance of Low-Tech Services and High-Tech Manufacturing for Chinese capital goods.

Figure 2: Evolution of Output Shares in Response to Trade Liberalization in China (Figure 1a)

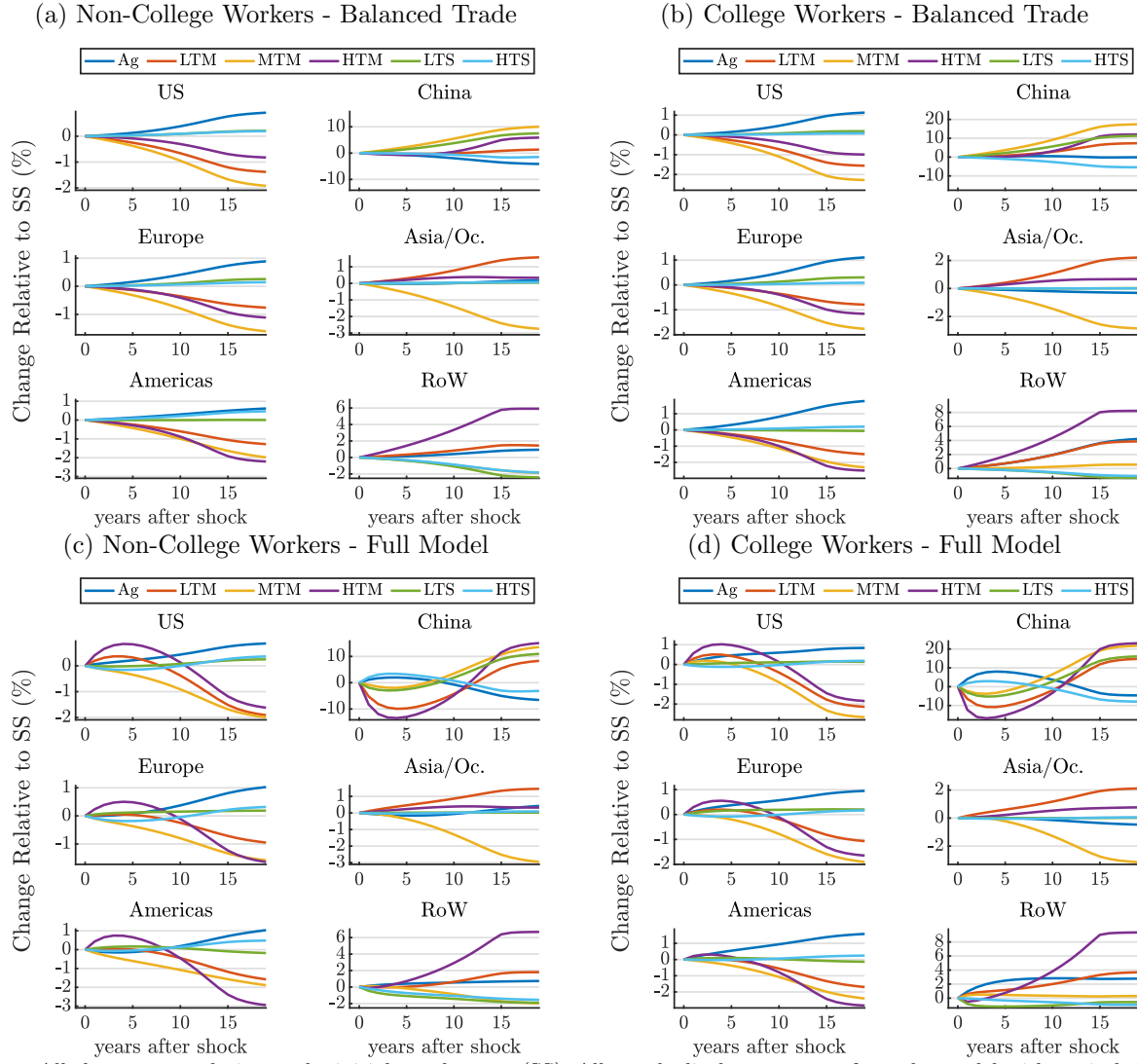


Notes: All changes are relative to the initial steady state (SS). Both panels display outcomes from the model featuring capital-skill complementarity. Panel (a) shows outcomes from a model imposing balanced trade in all periods. Panel (b) shows resulting outcomes of the full model with trade imbalances.

Panel (b) demonstrates that endogenizing imbalances leads to starkly different evolutions of output shares in the short run. Unlike in the case of balanced trade, there is both substantially more reallocation in the short run, and reallocation patterns in most countries display stark non-monotonicities. This increase in short-run reallocation is a consequence of allowing for trade imbalances in a model with trade costs that differ across sectors. With balanced trade, the small and gradual changes in trade costs lead to small and gradual changes in production patterns—which are then mechanically tied to the evolution of final goods expenditures. However, with imbalances, consumption across countries adjusts much faster than production. China, anticipating it will be richer in the long run, immediately increases consumption by borrowing, leading most other countries to sharply increase their exports. This surge in exports induces inter-sectoral reallocation because services are significantly more costly to trade across borders than physical goods (agriculture and manufacturing). Hence, when countries lend to China, they do so by disproportionately exporting physical goods, moving production away from services.

In the long run, China pays the debt it accumulated in the short run by expanding its manufacturing sectors and exporting varieties from these sectors. In turn, countries such as the US import these manufacturing varieties from China, running trade deficits, and reallocating production back to services—beyond the initial steady state levels. Long-run allocations in China reflect both lower trade costs and the long-run expansion of manufacturing to pay off its short-run debt. Since the changes in imbalances are relatively modest—on the order of a few percentage points of GDP in most countries—long-run allocations are relatively similar whether or not trade is balanced.

Figure 3: Labor Reallocation in Response to Trade Liberalization in China (Figure 1a)



We now translate these changes in output allocations to changes in labor allocations. To this end, Figure 3 plots changes in labor allocations for each skill group for both the balanced trade and full models. We highlight two patterns that are important to understand the subsequent discussion of the skill premium. First, comparing across columns, reallocation patterns are generally similar across skill groups. Second, endogenizing trade imbalances leads to the same outsized short-run response and non-monotonic patterns that were observed in the response of output allocations. These patterns of labor reallocation shape the Stolper-Samuelson channel.

To understand the directions of Stolper-Samuelson effects, we map out skill intensities across sectors in order to later connect changes in sector-specific labor demand to the skill premium. We compute the skill intensity of a sector as the expenditure on college workers relative to non-college workers in the initial steady state. To better compare sector-specific skill intensities within and across countries, we use Agriculture as a base sector and define the relative skill intensity, $RSI_{k,i}$, of sector k in country i as:

$$RSI_{k,i} = \frac{e_{k,i,0}^S / e_{Agriculture,i,0}^S}{e_{k,i,0}^U / e_{Agriculture,i,0}^U},$$

where $e_{k,i,0}^s$ is the expenditure share to skill group s in sector k of country i at the initial steady state. When $RSI_{k,i}$ is large, college workers are used intensively in sector k relative to Agriculture in country i . Therefore, changes leading to a shift from Agriculture to sector k puts upward pressure on the demand for college workers in country i , which tends to increase the skill premium. While we have used Agriculture as a base sector, we can equally make comparisons across any pair of sectors k and k' to have a sense of whether output reallocation from k to k' will tend to increase or decrease the relative demand for skills.

Table 6: Relative Skill Intensities, $RSI_{k,i}$

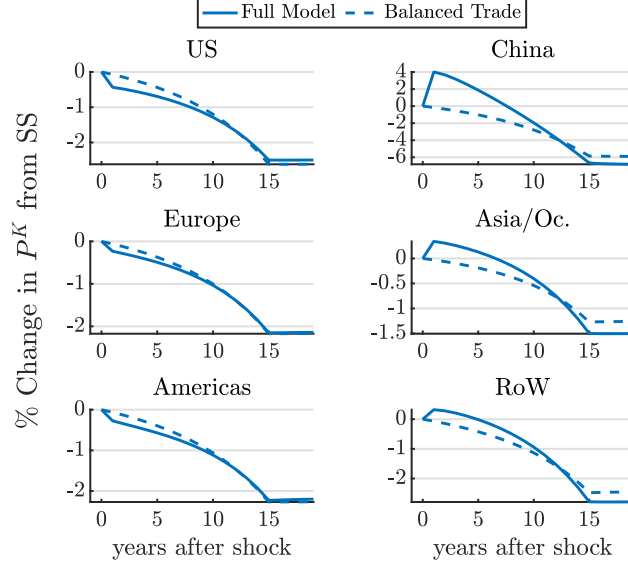
Sector	Country					
	US	China	Europe	Asia/Oc.	Americas	RoW
Agr.	1.00	1.00	1.00	1.00	1.00	1.00
LT Manuf.	1.65	18.35	3.66	0.92	4.78	6.20
MT Manuf.	1.34	12.65	3.44	0.84	2.78	4.69
HT Manuf.	2.68	32.50	3.87	1.17	3.60	12.10
LT Serv.	1.40	58.50	3.26	1.22	2.81	6.57
HT Serv.	4.54	174.57	12.97	3.08	14.38	39.78

Notes: Relative Skill Intensities are computed as $RSI_{k,i} = \frac{e_{k,i,0}^S / e_{Agriculture,i,0}^S}{e_{k,i,0}^U / e_{Agriculture,i,0}^U}$, where $e_{k,i,0}^s$ is the expenditure share to skill group s in sector k of country i at the initial steady state.

Table 6 contains the relative skill intensity measure for all countries and sectors. Three strong patterns emerge. First, High-Tech Services is the sector with highest relative skill intensity within all countries. Second, Agriculture typically has the lowest one (with the exception of Asia/Oceania).

Comparing across countries, it is clear that the skill intensity differences in China, and to a lesser extent the Rest of the World aggregate, are substantially more heterogeneous across sectors than anywhere else. This suggests that the Stolper-Samuelson channel may be more important in these two countries, an observation that will be confirmed in our analysis of the skill premium.

Figure 4: Evolution of the Price of Capital Following Trade Liberalization in China (Figure 1a)



Notes: All changes are relative to the initial steady state (SS). Balanced Trade: outcomes from a model imposing balanced trade in all periods. Full Model: outcomes of the full model with trade imbalances. All outcomes refer to the model with capital-skill complementarity.

To understand the directions and magnitudes of the capital-skill complementarity effects, we now turn to the evolution of the price of capital across countries. Figure 4 plots the evolution of capital prices in response to trade liberalization in China, for the model with and without endogenous trade imbalances. We observe a decline in capital prices in all countries, regardless of assumptions on imbalances. The only exception are short-lived increases in a few countries when trade imbalances are endogenous. Given the reduction in trade costs, the long-run decline in capital prices is expected: approximately 1/3 of capital is composed of physical goods such as agriculture and manufacturing (see Table B.2 in Appendix B). These results show that capital-skill complementarity will work in the direction of increasing the skill premium in all countries, at least in the long run.

We are now in a position to discuss the impacts of a gradual trade liberalization episode in China on the skill premium, and the channels through which they are materialized. Table 7 contains changes in the skill premium in the short run (measured 4 periods after the shock is unveiled), and in the long run (at the final steady state). The first column displays the change in the skill premium with Cobb-Douglas technology and balanced trade in every period, shutting down the capital-skill complementarity channel. This is our baseline of comparison, which focuses on Stolper-Samuelson

effects, while abstracting from capital-skill complementarity and trade imbalances. The next two columns add our model’s ingredients separately. Column (2) maintains the assumption of balanced trade, but allows for capital-skill complementarity—as in [Parro \(2013\)](#)—while column (3) features Cobb-Douglas technologies but relaxes the assumption of balanced trade. The fourth and final column shows outcomes from our full model, featuring both capital-skill complementarity and trade imbalances.

Table 7: Changes (in %) in the Skill Premium in Response to the Decline in Trade Costs in China Depicted in Figure [1a](#)

	<u>Balanced Trade</u>		<u>Full Model</u>	
	CD	K-S	CD	K-S
	(1)	(2)	(3)	(4)
<i>Panel A: Short Run ($t = 4$)</i>				
US	0.01	0.04	-0.07	-0.09
China	0.12	4.93	2.34	10.47
Europe	0.00	0.05	-0.07	-0.05
Asia/Oc.	-0.00	0.09	-0.05	0.06
Americas	0.03	0.10	-0.03	0.00
RoW	-0.08	0.38	-0.09	0.67
<i>Panel B: Long Run</i>				
US	0.04	0.34	0.10	0.43
China	0.70	36.98	-0.22	33.53
Europe	0.02	0.44	0.06	0.50
Asia/Oc.	-0.01	0.75	0.02	0.79
Americas	0.27	0.93	0.31	1.02
RoW	-0.87	3.05	-0.92	2.84

Notes: K-S stands for model with capital-skill complementarity. CD stands for model with Cobb-Douglas production, eliminating the capital-skill complementarity channel. The skill premium is measured as the wage of college workers relative to the wage of non-college workers.

In column (1), when trade is balanced and there is no capital-skill complementarity, changes in Chinese trade costs have a near-zero effect on the skill premium. In this setting, the only source of changes in the skill premium can occur through the Stolper-Samuelson channel. In this case, the incremental short-run changes in trade costs have only small impacts on output and labor reallocation—as shown in Figures [2a](#), [3a](#) and [3b](#). Hence, these small short-run changes in the skill premium are unsurprising. In the long run, we also observe small changes in the skill premium, outside of perhaps the Rest of the World and China itself. This echoes findings in [Parro \(2013\)](#), who found that the Stolper-Samuelson channel plays a minor role in determining responses of the skill premium to changes in trade costs.

Column (2) shows results allowing for capital-skill complementarity, while maintaining balanced

trade. In the short run, the change in the skill premium is still relatively small outside of China. Nevertheless, it is positive in all countries. In the long run, as the price of capital continues to fall, the skill premium now rises in all countries. In China the increase is particularly dramatic—relative wages grow by over 35%. The reduction in capital prices is such that capital-skill complementarity completely dwarfs the Stolper-Samuelson channel—even flipping the sign of the skill-premium change in the Rest of the World. Before reintroducing trade imbalances, we note that similar to the evolution of labor allocations, the changes in the skill premium are largely monotone: short-run impacts are amplified in the long run.

Turning to column (3), we now shut down capital-skill complementarity but allow for endogenous trade imbalances. Here, we find small but non-zero changes in the skill premium both in the short and long runs. In the short run, there is more reallocation of labor across sectors than with balanced trade in each period. This amplification in turn magnifies the Stolper-Samuelson channel. For example, looking back at Figure 2b, in China there is a large jump into High-Tech Services and an immediate decline in manufacturing. From Table 6 one can see that relative skill intensity in High-Tech Services in China is five to ten times larger than in manufacturing sectors, helping to explain the short-run increase in the Chinese skill premium. It is rather startling that such large differences in skill intensities still lead to relatively small changes in the skill premium, especially compared to the effects of capital-skill complementarity.

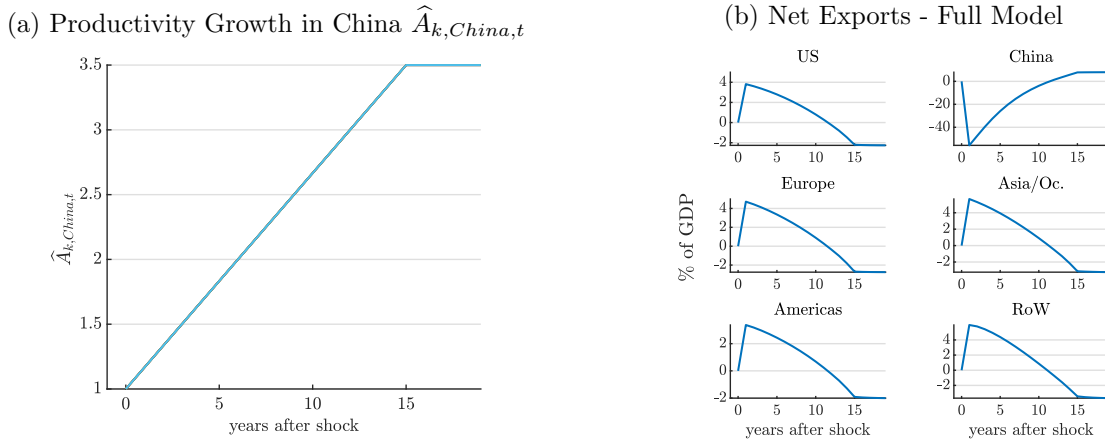
Another interesting pattern is that for many countries the short- and long-run changes in the skill premium are of a different sign. For example, an initial increase of nearly 2.5% in China becomes an approximately quarter percent decline in the long run. This reversal is implied by the aforementioned non-monotonic patterns of adjustment. Figure 3, which showed the initial burst into High-Tech Services, also displays a long-run shift into the more non-college intensive manufacturing industries.

Finally, we turn to column (4), which contains the results of our full model, featuring both capital-skill complementarity and endogenous trade imbalances. Broadly speaking, the patterns are consistent with simply combining the discussion of columns (2) and (3). In particular, introducing capital-skill complementarity pushes the skill-premium up in most countries at every horizon, while allowing for endogenous trade imbalances leads to non-monotonic patterns of adjustment and larger effects in the short run, at least in some countries. There are two caveats. First, in some countries—for example, the Americas—capital-skill complementarity and the Stolper-Samuelson channel offset in the short-run, leading to a very limited response of the skill premium. Second, there is the curious fact that in the US, the skill premium declines by *more* in column (4) than in column (3). The magnitudes are small, but we return to finding this in the next subsection, when we study the consequences of productivity growth in China.

4.2 Productivity Growth in China

Motivated by the strong productivity growth China has experienced since the early 2000's, we study the global impacts of slow productivity growth in the country through the lens of our model. Specifically, we simulate a linear increase in Chinese productivity, $A_{k,China,t}$, uniform across sectors k , reaching a plateau of a 3.5 times increase after 15 years. The magnitude of this shock is in line with the size of actual changes in Chinese productivity that we recover using a shocks-extraction procedure described in Appendix D.6—see Figure C.1a.²¹ We impose that all countries start from a balanced-trade steady state.

Figure 5: Slow Productivity Growth in China and Resulting Trade Imbalances



Notes: We denote $\hat{x}_t \equiv \frac{x_t}{x_0}$ as the proportional change of variable x between periods t and 0. Panel (a) illustrates slow productivity growth in China $\hat{A}_{k,China,t}$, uniform across sectors k . Panel (b) displays resulting outcomes of our full model with trade imbalances and capital-skill complementarity.

Figure 5 plots the path of productivity growth in China that we feed into the model (Panel (a)) and the implied trade imbalances around the world (Panel (b)). As was the case following a gradual trade liberalization in China, as soon as the path of productivity is unveiled at the end of $t = 0$, Chinese households anticipate they will be much richer in the long run. Consumption smoothing dictates that their expenditures will jump above production in the short run, generating trade deficits.²² In the long run, they pay off their accumulated debt by running trade surpluses in perpetuity. One important difference is that the response of net exports is much larger in this case compared to the shock we studied in section 4.1, especially so in China. However, patterns of

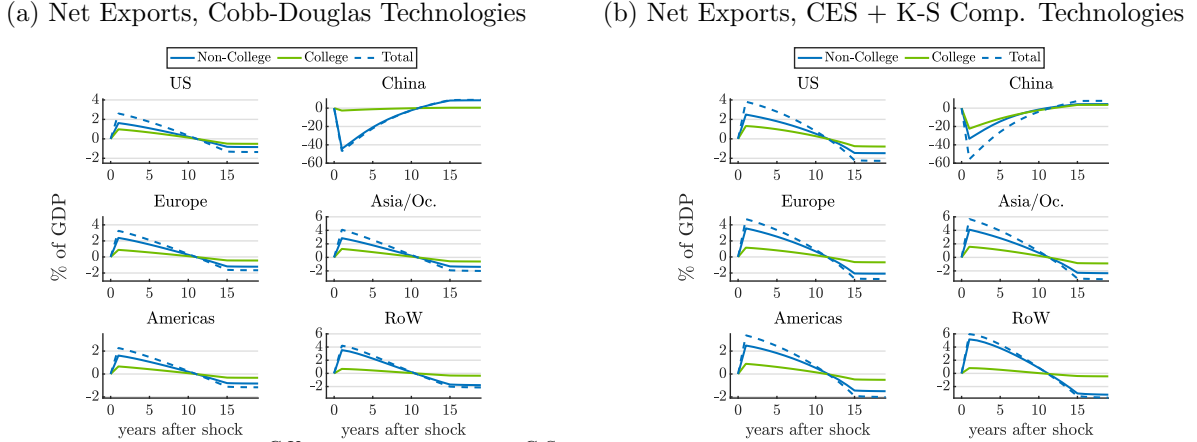
²¹Note that the productivity location parameters $A_{k,i}^t$ are not comparable to productivity in the classic sense of a Solow Residual. In order to make sense of the magnitudes, note that TFP growth, defined as $\hat{c}_{k,i}^t / \hat{P}_{k,i}^{I,t}$, can be expressed as $(\hat{A}_{k,i}^t / \hat{\pi}_{k,ii}^t)^{1/\lambda}$. Therefore, data on changes in trade shares, and imposing $\lambda = 4$, $\hat{A}_{k,China} = 3.5$ is equivalent to an annualized average TFP growth in China of approximately 2% per year.

²²This pattern of adjustment is at odds with the large trade surpluses China has maintained since 2000. These persistent trade surpluses are a challenge for models of trade imbalances generated by consumption-smoothing motives. The inter-temporal preference shocks $\hat{\phi}_{i,t}^s$ play the role of reconciling the path of Chinese trade surpluses in the data to predictions of this class of models (Kehoe et al., 2018).

reallocation are qualitatively similar to those previously obtained.

To better understand the evolution of the skill premium under the different scenarios considered in Table 7, it will help to illustrate the evolution of trade imbalances under different assumptions on technology: full model with capital-skill complementarity, and under Cobb-Douglas technologies, which shuts down this channel. As we explain below, there are important differences on how the skill premium evolves under these two types of technologies, and these are partially driven by how trade imbalances evolve in each case.

Figure 6: Decomposing Trade Imbalances Following the Shock Depicted in Figure 5a



Notes: Define $NX_{i,t}^U = I_{i,t}^U - E_{i,t}^{C,U}$ and $NX_{i,t}^S = I_{i,t}^S - E_{i,t}^{C,S}$. From equation (23), $NX_{i,t} = NX_{i,t}^U + NX_{i,t}^S$. Panels (a) and (b) decompose aggregate net exports $NX_{i,t}$ (Total) into $NX_{i,t}^U$ (Non-College) and $NX_{i,t}^S$ (College). Panel (a) imposes Cobb-Douglas production functions, shutting down the capital-skill complementarity channel. Panel (b) displays outcomes of the full model with capital-skill complementarity.

According to equation (23), aggregate net exports $NX_{i,t} = NX_{i,t}^U + NX_{i,t}^S$, where $NX_{i,t}^U = I_{i,t}^U - E_{i,t}^{C,U}$ and $NX_{i,t}^S = I_{i,t}^S - E_{i,t}^{C,S}$. Figure 6 plots the evolution of trade imbalances for each country, and decompose aggregate net exports $NX_{i,t}$ (Total) into $NX_{i,t}^U$ (Non-College) and $NX_{i,t}^S$ (College). For example, looking at the US in Panel (a), there are three lines. The dashed blue line shows that the total trade surplus in the US rises on impact to approximately 2.5% of GDP; the solid blue line shows that 1.5% (in percentage points) of this rise comes from non-college workers; the solid green lines shows that an additional 1% comes from college workers. There are two important takeaways from these plots. First, the magnitudes involved are larger with capital-skill complementarity. In China, for instance, the trade deficit is under 50% with Cobb-Douglas technologies, but jumps to nearly 60% in Panel (b); in the United States, matching this larger Chinese deficit causes its surplus to nearly double. The second important takeaway is that in China, with capital-skill complementarity, the more pronounced deficit is driven by substantially more borrowing by college workers, while non-college workers lower their savings. Armed with these findings, we now turn to dissecting the evolution of the skill premium. As before, we conduct

our analysis starting from a balanced-trade world where only the Stolper-Samuelson force is active (i.e., imposing Cobb-Douglas technologies), and add different model ingredients one by one.

Table 8: Changes (in %) in the Skill Premium in Response to Slow Productivity Growth in China Depicted in Figure 5a

	<u>Balanced Trade</u>		<u>Full Model</u>	
	CD (1)	K-S (2)	CD (3)	K-S (4)
<i>Panel A: Short Run ($t = 4$)</i>				
US	-0.00	0.07	-0.78	-1.35
China	-0.61	108.85	3.21	155.29
Europe	-0.00	0.10	-2.23	-2.59
Asia/Oc.	0.01	0.14	-0.87	-1.74
Americas	0.04	0.22	-0.28	-0.37
RoW	-0.52	0.36	-3.39	-3.12
<i>Panel B: Long Run</i>				
US	-0.05	0.36	-0.19	0.46
China	-1.16	422.86	-9.11	424.41
Europe	-0.06	0.63	-1.51	-0.45
Asia/Oc.	0.05	0.70	0.68	1.97
Americas	0.12	1.68	0.48	2.53
RoW	-3.35	1.72	-1.87	4.51

Notes: K-S stands for model with capital-skill complementarity. CD stands for model with Cobb-Douglas production, eliminating the capital-skill complementarity channel. The skill premium is measured as the wage of college workers relative to the wage of non-college workers.

Table 8 shows changes in the skill premium at both short- and long-run horizons under different scenarios of our model. Its structure mirrors the layout of Table 7. Column (1) displays outcomes for the model imposing balanced trade and shutting down the capital-skill complementarity channel. The Stolper-Samuelson forces are slightly more salient compared to those in the previous section—especially in China and the Rest of the World; in China, the skill premium moves in the opposite direction. However, the overall changes in the skill premium are still relatively small. For example, in China, which grows precipitously, the skill premium decreases by less than 1.2% in the long run. Even with a large shock in China, with balanced trade, the seemingly stark inter-sectoral differences in skill intensity documented in Table 6 do not seem to translate to large changes in the skill premium.

In column (2), we add capital-skill complementarity but keep imposing balanced trade. Here, the change in the skill premium is substantially more pronounced in China—qualitatively similar to our findings from the previous section. Interestingly, changes in other countries are more muted, but also comparable to our findings in Table 7. In fact, the effects are often larger in other countries

when trade costs decline than when productivity rises in China. This suggests that starting from our steady state, if trade is balanced in each period, then Chinese productivity is quantitatively more important for the skill premium in China, while Chinese trade costs are quantitatively more important for the skill premium in China’s trade partners.

How does incorporating endogenous trade imbalances change this picture? As soon as China realizes that it will grow rapidly in the future, it finances a consumption boom by running a very large short-run trade deficit—over 50% of GDP. As we discussed in section 4.1, this trade deficit induces substantially more short-run reallocation, although the magnitudes involved are larger with the current shock. In particular, there is a substantial amount of reallocation towards High-Tech Services in the short run, pushing the demand for college workers up and leading to an increase in the skill premium (column (3)). However, in the long run, as China starts to pay off its debt, it expands the output of its manufacturing sectors (above the initial steady state), and contracts the output of its service sectors (below the initial steady state), leading to a decline in the skill premium. Therefore, not only trade imbalances can magnify the effects on the skill premium, but they can also affect the direction of the change, especially in the short run.

In column (4) we finally turn to our full model with both capital-skill complementarity and endogenous imbalances. Two findings in this column capture well the role that imbalances play in our understanding of the skill premium. First, comparing columns (2) and (4), endogenizing imbalances amplifies the importance of capital-skill complementarity, especially outside of China. This is a consequence of imbalances leading to more reallocation. Second, there is a short-run decline in the skill premium in most countries that is *larger* than the decline when capital-skill complementarity is absent (compare column (4) to (3)). We initially highlighted this possibility in our discussion of the trade liberalization in China. However, the magnified decline in the skill premium is now larger and occurs in most countries. In the US, for example, the initial decline is doubled between the model with and without capital-skill complementarity.

This is puzzling at first glance: even if imbalances lead to more reallocation, as capital prices fall, demand for skilled workers should rise. In order for the importance of the Stolper-Samuelson channel to *grow* and indeed *dominate* in the model with capital-skill complementarity, it must be the case that either reallocation patterns are significantly different, or that the evolution of trade imbalances substantially differ across assumptions on technology. As we showed in Figure 6, the latter is important: the same technology shock leads to larger imbalances under capital-skill complementarity. Hence, reallocation patterns are further amplified, amplifying the decline in the skill premium through Stolper-Samuelson effects.

One may now wonder *why* trade imbalances are larger with capital-skill complementarity. College workers in China become much richer in response to productivity growth in the model with capital-skill complementarity. Chinese college workers’ income as a share of global GDP grows four-

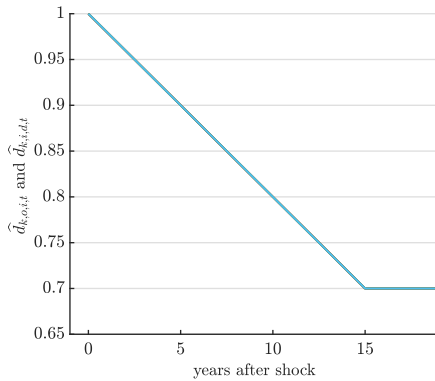
fold when we impose Cobb-Douglas technologies, but *twenty-five* fold when we allow for capital-skill complementarity. This steep income growth is the source of the increased borrowing by Chinese college workers illustrated in Figure 6. Therefore, it is the *interaction* between the long-run growth in income for college workers and the ability to run trade imbalances that generates the sizable short-run decline in the skill premium in China’s trade partners. This interaction demonstrates that carefully modeling both technology and trade imbalances is key for understanding inequality dynamics—decomposing the model ingredient-by-ingredient is far from linear.

4.3 Global Trade Liberalization

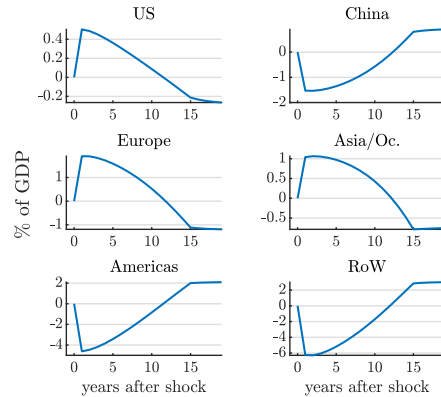
We now analyze the response of our model to global reductions in trade costs. Specifically, we simulate a gradual 30% uniform decline in trade costs in all sectors and countries over a 15 year period. As in the previous simulations, the full path of the shock is unveiled to all countries at the end of $t = 0$, and we impose that all countries start from a balanced-trade steady state. Figure 7a plots the evolution of trade costs across any pair of distinct countries that we feed into the model, and Figure 7b plots the resulting changes in trade imbalances.

Figure 7: Global Trade Cost Decline and Resulting Imbalances

(a) Changes in Trade Costs for All Countries and Sectors



(b) Net Exports



Notes: We denote $\hat{x}_t \equiv \frac{x_t}{x_0}$ as the proportional change of variable x between periods t and 0. Import and export costs $d_{k,oi,t}$ gradually decline until they reach a 30% reduction—for all pairs of origins o and destinations i and for all sectors k . Note that $\hat{d}_{k,i,i,t} = 1$ for all sectors k , countries i and periods t . Panel (b) displays resulting outcomes of our full model with trade imbalances and capital-skill complementarity.

We highlight two findings in this figure. First, despite the fact that the shock experienced *by China*— $\hat{d}_{k,o,China,t}$ and $\hat{d}_{k,China,d,t}$ —is exactly the same, its short-run trade deficit is only 1/4 as large compared to the case where only Chinese trade costs decline (see Figure 1b). To understand this outcome, note that in section 2.6, we emphasized that normalizing world GDP to 1 and shutting down inter-temporal preference shocks in every period ($\hat{\phi}_{i,t}^s = 1$), the interest rate R_t equals $1/\delta$ and skill-group and country-specific final good expenditures are fixed as a share of world GDP.

This implies that when the initial equilibrium features balanced trade, we can sum each household budget constraint over time to obtain the following formula for final good expenditures:

$$E_i^{C,s} = (1 - \delta) \sum_{r=0}^{\infty} \delta^r I_{i,r}^s, \quad (26)$$

where $E_i^{C,s}$ is the constant expenditure on final goods of skill group s in country i denominated in shares of global GDP and $I_{i,t}^s$ is their disposable income, denominated in the same units.²³ That is, as soon as a shock is unveiled, expenditures immediately jump to the present value of future disposable income streams. Plugging this into equation (23) yields the formula:

$$NX_{i,t} = (I_{i,t}^S + I_{i,t}^U) - (1 - \delta) \sum_{r=0}^{\infty} \delta^r (I_{i,r}^S + I_{i,r}^U). \quad (27)$$

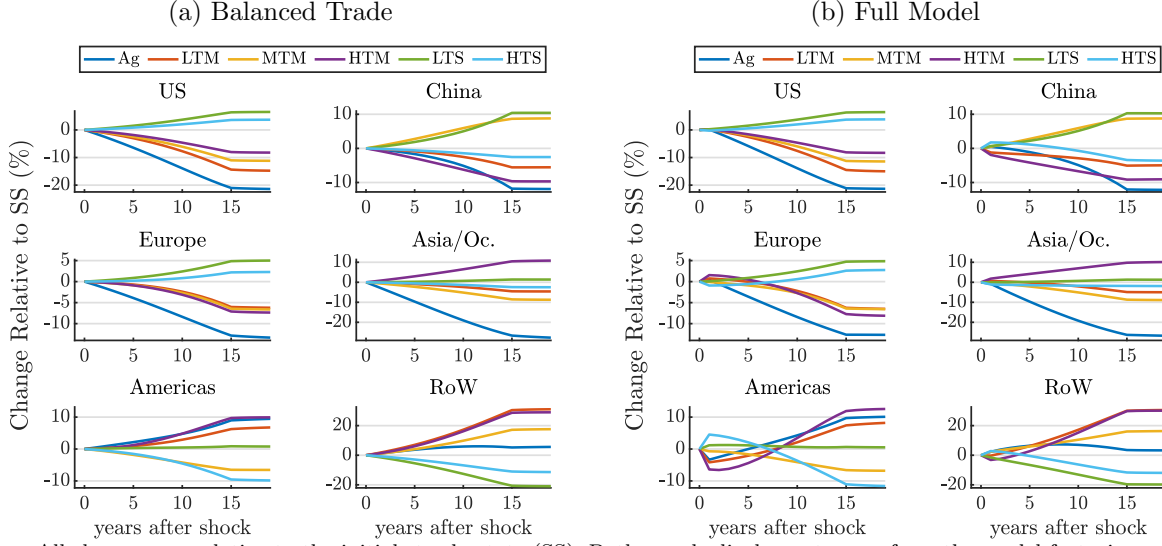
When trade costs are reduced in China only, Chinese income, as a share of world GDP, experiences a very steep growth path. In turn, equation (27) implies that China experiences large trade deficits in the short run, and trade surpluses in the long run. However, in the event where trade costs are similarly reduced in all countries, real incomes all over the world grow in parallel, so that individual country incomes are relatively flat when expressed as shares of world GDP. In that case, equations (26) and (27) imply that the impact of a global trade cost reduction will tend to have more muted impacts on trade imbalances relative to the case we studied in section 4.1, especially in China.

The second finding we highlight is that despite the symmetry of the shock, trade imbalances still emerge. This occurs for two reasons. First, symmetric declines in trade costs can have uneven effects across countries. Even absent labor market frictions, the effect of trade costs on national income depends on myriad factors, among which: initial differences in openness levels, differences in comparative advantage patterns, and third-country effects. Second, even if these factors were otherwise symmetric, countries still differ in the magnitudes of labor market frictions (see Table 4)—leading to different paths of convergence to the final steady state. These two sets of forces generate asymmetries in the path of incomes across countries, which in turn generate imbalances.

Figure 8 plots the evolution of gross output shares following the global shock, both for the model with balanced trade and for our full model. As in the case of the trade liberalization in China, different assumptions on trade imbalances can lead to quite different patterns in output shares, primarily in the short run. With trade imbalances, there are substantially more pronounced non-monotonic patterns—especially in the Americas and the Rest of the World. In section 4.1, reallocation patterns were heavily dependent on Chinese comparative advantage and consumption. This is why, for example, there was a large reallocation towards Agriculture in many countries.

²³Note that this invokes the transversality condition $\lim_{t \rightarrow \infty} \delta^t B_{i,t}^s = 0$.

Figure 8: Evolution of Output Shares in Response to Global Trade Cost Decline



Notes: All changes are relative to the initial steady state (SS). Both panels display outcomes from the model featuring capital-skill complementarity. Panel (a) shows outcomes from a model imposing balanced trade in all periods. Panel (b) shows resulting outcomes of the full model with trade imbalances.

Under the global trade liberalization episode, China is a less important driver of global output patterns. This does lead to some differences: for example, the United States reallocates output shares towards services almost immediately, leading to similar patterns between the model with balanced trade and the full model.

As in our previous two exercises, patterns of labor reallocation generally follow patterns of output. Also as before, capital prices generally decline over time. One observation of note is that the magnitude of the decline in the price of capital is much larger than in the case of the shock affecting only China. For most countries this is unsurprising, but even in China the long-run decline in capital prices is nearly 3 times larger with the global shock. The direct change in trade costs in China— $\hat{d}_{k,o,China,t}$ and $\hat{d}_{k,China,d,t}$ —is the same as in section 4.1, so that the larger magnitude is driven by accumulated indirect effects of the shock in other countries reverberating back to China. With these facts in mind we move to our discussion of the skill premium.

Table 9 displays changes in the short- and long-run skill premium across countries and follows the same structure as Tables 7 and 8. Focusing on column (1), which imposes balanced trade and shuts down the capital-skill complementarity channel, the changes in the skill premium are once again small in the short run. They are larger in the long run, but only in China and the Americas. The most interesting comparison may be to China, where the direct shock (i.e., changes in Chinese trade costs) is the same as in section 4.1. Here, the long-run effects are quite a bit larger. This suggests that the indirect impacts of trade shocks can have sizable impacts on inter-sectoral shifts in labor demand, in turn magnifying Stolper-Samuelson effects.

Table 9: Changes (in %) in the Skill Premium in Response to the Global Decline in Trade Costs Depicted in Figure 7a

	<u>Balanced Trade</u>		<u>Full Model</u>	
	CD	K-S	CD	K-S
	(1)	(2)	(3)	(4)
<u>Panel A: Short Run ($t = 4$)</u>				
US	0.03	1.32	-0.10	1.16
China	0.21	5.18	0.88	7.09
Europe	0.03	1.12	-0.12	1.09
Asia/Oc.	-0.11	1.02	-0.43	0.69
Americas	-0.57	4.86	0.19	8.20
RoW	-0.61	5.67	1.12	10.10
<u>Panel B: Long Run</u>				
US	-0.03	9.31	0.03	9.44
China	2.60	42.86	2.18	41.37
Europe	0.26	8.75	0.27	8.69
Asia/Oc.	-0.84	6.93	-0.66	7.21
Americas	-4.61	27.44	-4.99	25.35
RoW	0.09	41.43	-0.40	38.87

Notes: K-S stands for model with capital-skill complementarity. CD stands for model with Cobb-Douglas production, eliminating the capital-skill complementarity channel. The skill premium is measured as the wage of college workers relative to the wage of non-college workers.

Including capital-skill complementarity but maintaining balanced trade, seen in column (2), has two effects on the findings in column (1). First, magnitudes are much larger at both horizons, so that trade cost reductions matter much more for the skill premium. Second, the skill-premium increases everywhere—showing that capital-skill complementarity dwarfs the Stolper-Samuelson channel in both the short and long runs. Comparing across types of shocks, the change in the skill premium is larger in China’s trade partners when all countries experience trade liberalization than when only China liberalizes. This is perhaps unsurprising, reflecting the aforementioned large decline in the price of capital.

In column (3), we display results for the case of allowing for trade imbalances, but maintaining Cobb-Douglas technologies. The changes in the skill premium are no longer monotone over time for each country. Instead, in most countries, the short- and long- run changes in the skill premium are of different signs. This is most pronounced in the Americas and the Rest of the World, where the non-monotonocities shown in Figure 8b are most salient. In other countries, effects on the skill premium are much smaller.

An interesting example is in China, where the sign of the change in the skill premium does not change over time, while it did when trade costs only fell in China. The indirect impacts of the shocks to other countries raise the Chinese skill premium at both horizons, through shifts in intersectoral labor demand. This suggests that general equilibrium forces are far from second order—the same shock *in* China and similar qualitative behavior of trade imbalances are not sufficient in of themselves to understand how labor allocations will respond to trade shocks.

Finally, in column (4) we turn to our full model with endogenous trade imbalances and capital-skill complementarity. Here the skill premium rises in all countries, in both the short and long runs. Capital prices decline rapidly enough, and in a large magnitude, that the Stolper-Samuelson force is swamped everywhere. This suggests that for large global shocks, capital-skill complementarity is the dominant force driving the skill premium, regardless of imbalances and their consequent effect on the allocation of labor across industries and countries.

4.4 Taking Stock

We have three main takeaways from the different analyses conducted in this section. First, we emphasize that trade imbalances can significantly affect the strength and importance of Stoper-Samuelson effects. This effect is materialized through changes in reallocation patterns. For example, column (3) of Tables 7 and 8 show that once we allow for trade imbalances, we can have non-monotonic responses of the skill premium. In the short run, Stolper-Samuelson effects can be amplified by trade imbalances, as illustrated by comparing columns (2) and (4) of Tables 7, 8, and 9. However, the quantitative importance of trade imbalances is more mixed in the long run. At this horizon, the strength of capital-skill complementarity is the driving force behind the responses

of the skill premium.

Second, our model predicts very nuanced behavior of the skill premium depending on the nature of the shock, the full constellation of shocks across countries, and on specific model ingredients. Our results corroborate the following quote of [Goldberg \(2015\)](#) who concludes that recent research in International Trade “suggests that unqualified statements about the effects of globalization on inequality are unwarranted. Each case is different, and an informed perspective on this topic requires a careful study of the nature of the globalization institutional setting, the production structure, the functioning of the markets in each country and the degree and nature of liberalization.”

Finally, we have shown that capital-skill complementarity can go a long way toward explaining worldwide globalization-fueled increases in inequality. More novel to this paper, we have shown that capital-skill complementarity can significantly interact with trade imbalances. Column (4) of Table 8 is very interesting in this regard, showing that capital-skill complementarity is the dominating force in the long run, but Stolper-Samuelson effects are dominant in the short run. This short run behavior is driven in part by the response of trade imbalances to changes in income growth across groups brought about by capital-skill complementarity.

5 Concluding Remarks

This paper studied a quantitative dynamic model of International Trade with workers of different skills, capital-skill complementarity, inter-sectoral labor market frictions, and trade imbalances. Although capital-skill complementarity tends to dominate the response of the skill premium in the long run, we showed that the behavior of trade imbalances can have quantitatively important impacts in the short run. Overall, we have documented nuanced behavior of the skill premium that ultimately depends on the nature of the shock, the full set of shocks across countries, and on specific model ingredients.

The framework we develop in this paper can be used to answer more questions than we have explored. For example, although our model allows for labor market frictions across sectors and inequality across sectors within skill groups, we left this dimension unexplored. Moreover, there is nothing special about skill groups in our framework. One could allow for different attributes to define a representative household—for example, regions, or capitalists and workers. Of course, one would need to change the production structure commensurately. Given the strong assumptions on within-household risk sharing, the reasonability of our model as an approximation to the real world will depend on the question at hand.

This final caveat points to a natural starting place for future work: moving away from the representative household assumption. We plan to extend our current framework, which relies on different types of workers aggregating into a single representative household, into a more typical model of

individual savings and borrowing. In particular, we plan to allow for two empirically relevant sets of frictions to co-exist: borrowing constraints and labor market frictions. The macroeconomics literature has documented the importance of asset market frictions for understanding inequality in the US (Castañeda et al., 2003; Heathcote et al., 2009). Hence, given our interest in modeling the interaction between savings and labor market reallocation, incomplete markets are the logical next step.

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Online Appendix, Not for Publication

A Country and Sector Definitions	2
B Additional Parameter Estimates	3
C Shocks Extraction Results	5
D Solution Methods	8
D.1 Algorithm to Invert Model Parameters in the US	8
D.2 Algorithm to Calibrate Sector-Specific Preferences, Non-US Countries	10
D.3 Solving for the Initial Steady State	12
D.4 Algorithm: Out-of-Steady-State Transition, Full Model with Bonds	15
D.5 Algorithm: Out-of-Steady-State Transition, Exogenous Deficits (No Bonds)	24
D.6 Algorithm for Shocks Extraction	28

A Country and Sector Definitions

Table A.1 displays how we divide the world according to the country divisions in the World Input Output Database. Table A.2 details how we define the six sectors we consider in our quantitative exercises.

Table A.1: Country Definitions

1	USA
2	China
3	Europe
4	Asia/Oceania
5	Americas
6	Rest of the World (ROW)

Notes: Asia/Oceania = {Australia, Japan, South Korea, Taiwan}, Americas = {Brazil, Canada, Mexico}, Rest of the World = {Indonesia, India, Russia, Turkey, Rest of the World}. This partition of the world was dictated by data availability from the World Input Output Database.

Table A.2: Sector Definitions

1	Agriculture/Mining	Agriculture, Forestry and Fishing; Mining and quarrying
2	Low-Tech Manufacturing	Wood products; Paper, printing and publishing; Coke and refined petroleum; Basic and fabricated metals; Other manufacturing
3	Mid-Tech Manufacturing	Food, beverage and tobacco; Textiles; Leather and footwear; Rubber and plastics; Non-metallic mineral products
4	High-Tech Manufacturing	Chemical products; Machinery; Electrical and optical equipment; Transport equipment
5	Low Tech Services	Utilities; Construction; Wholesale and retail trade; Transportation; Accommodation and food service activities; Activities of households as employers
6	Hi Tech Services	Publishing; Media; Telecommunications; Financial, real estate and business services; Government, education, health

B Additional Parameter Estimates

In this section, we display the various parameters displayed in Panel B of Table 1. Figure B.1 provides a visualization of the parameters shown in Tables B.1 and B.2.

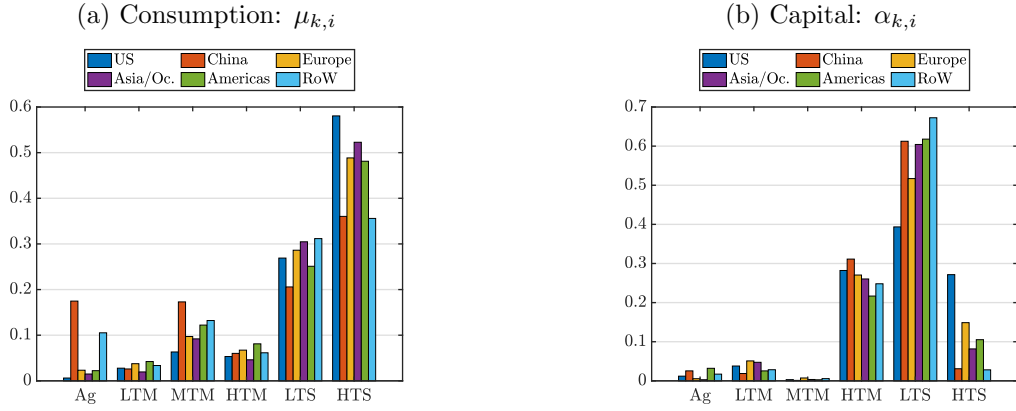
Table B.1: Final Expenditure Shares $\mu_{k,i}$

Sector	Country					
	US	China	Europe	Asia/Oc.	Americas	RoW
Agr.	0.01	0.17	0.02	0.01	0.02	0.11
LT Manuf.	0.03	0.03	0.04	0.02	0.04	0.03
MT Manuf.	0.06	0.17	0.10	0.09	0.12	0.13
HT Manuf.	0.05	0.06	0.07	0.05	0.08	0.06
LT Serv.	0.27	0.21	0.29	0.30	0.25	0.31
HT Serv.	0.58	0.36	0.49	0.52	0.48	0.36

Table B.2: Capital Good Shares $\alpha_{k,i}$

Sector	Country					
	US	China	Europe	Asia/Oc.	Americas	RoW
Agr.	0.01	0.03	0.01	0.00	0.03	0.02
LT Manuf.	0.04	0.02	0.05	0.05	0.03	0.03
MT Manuf.	0.00	0.00	0.01	0.00	0.00	0.01
HT Manuf.	0.28	0.31	0.27	0.26	0.22	0.25
LT Serv.	0.39	0.61	0.52	0.60	0.62	0.67
HT Serv.	0.27	0.03	0.15	0.08	0.11	0.03

Figure B.1: Aggregation Parameters in 2000

Table B.3: Labor Shares in Production $\gamma_{k,i}$

Sector	Country					
	US	China	Europe	Asia/Oc.	Americas	RoW
Agr.	0.45	0.58	0.56	0.54	0.62	0.67
LT Manuf.	0.37	0.25	0.32	0.35	0.28	0.27
MT Manuf.	0.33	0.28	0.31	0.37	0.32	0.28
HT Manuf.	0.39	0.24	0.33	0.32	0.31	0.25
LT Serv.	0.61	0.37	0.49	0.54	0.56	0.48
HT Serv.	0.62	0.55	0.63	0.67	0.67	0.68

Table B.4: Input-Output Table – Averages Across Countries $\frac{1}{N} \sum_{i=1}^N \nu_{k\ell,i}$, Standard Dev. across Countries in Parentheses.

User ↓ Supplier →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Agr.	0.27 (0.05)	0.08 (0.02)	0.12 (0.03)	0.14 (0.03)	0.26 (0.05)	0.14 (0.06)
LT Manuf.	0.19 (0.04)	0.38 (0.06)	0.04 (0.01)	0.08 (0.01)	0.22 (0.04)	0.08 (0.04)
MT Manuf.	0.22 (0.03)	0.07 (0.02)	0.29 (0.04)	0.11 (0.02)	0.22 (0.05)	0.09 (0.04)
HT Manuf.	0.02 (0.02)	0.16 (0.02)	0.07 (0.01)	0.46 (0.05)	0.18 (0.04)	0.11 (0.05)
LT Serv.	0.06 (0.04)	0.14 (0.03)	0.10 (0.02)	0.10 (0.03)	0.34 (0.07)	0.26 (0.10)
HT Serv.	0.01 (0.00)	0.08 (0.02)	0.03 (0.02)	0.11 (0.06)	0.27 (0.06)	0.51 (0.16)

C Shocks Extraction Results

Relying on the model's structure and data from the WIOD, we extract four main sets of shocks affecting the global economy between 2000 and 2014: (a) changes in trade costs $\hat{d}_{k,oi,t}$; (b) productivity shocks $\hat{A}_{k,i,t}$; (c) shocks to the production function weights $\hat{\chi}_{k,i,t}$ and $\hat{\xi}_{k,i,t}$ (d) and inter-temporal preference shocks $\hat{\phi}_{i,t}^s$. We measure changes in trade costs, productivity and production function weights relative to 2000 (which we label $t = 0$): $\hat{d}_{k,oi,t} = \frac{d_{k,oi,t}}{d_{k,oi,0}}$, $\hat{A}_{k,i,t} = \frac{A_{k,i,t}}{A_{k,i,0}}$, $\hat{\chi}_{k,i,t} = \frac{\chi_{k,i,t}}{\chi_{k,i,0}}$, and $\hat{\xi}_{k,i,t} = \frac{\xi_{k,i,t}}{\xi_{k,i,0}}$. On the other hand, shocks to inter-temporal preferences are relative to the previous period: $\hat{\phi}_{i,t+1}^s \equiv \frac{\phi_{i,t+1}^s}{\phi_{i,t}^s}$. In addition to these shocks, we consider changes over time in parameters driving final goods preferences ($\mu_{k,i,t}$), capital production ($\alpha_{k,i,t}$), and technologies ($\gamma_{k,i}^t$ and $\nu_{k\ell,i}^t$).

In essence, we make use of the gravity structure of the model to obtain shocks to productivity and trade costs—the procedure we employ is similar to [Head and Ries \(2001\)](#) and [Eaton et al. \(2016\)](#).²⁴ For inter-temporal preference shocks, we follow [Reyes-Heroles \(2016\)](#) and back out $\hat{\phi}_{i,t}^s$ using the Euler Equation and time-series data on aggregate expenditures. We leave the details of the implementation to Appendix [D.6](#).

²⁴We impose $\hat{A}_{k,i,t} = \hat{A}_{k,i,T_{Data}}$, $\hat{\chi}_{k,i,t} = \hat{\chi}_{k,i,T_{Data}}$, $\hat{\xi}_{k,i,t} = \hat{\xi}_{k,i,T_{Data}}$, and $\hat{d}_{k,oi,t} = \hat{d}_{k,oi,T_{Data}}$ for all $t > T_{Data}$, where T_{Data} is the last period for which we have data ($T_{Data} = 14$ refers to 2014).

Figure C.1: Extracted Productivity and Technology Shocks

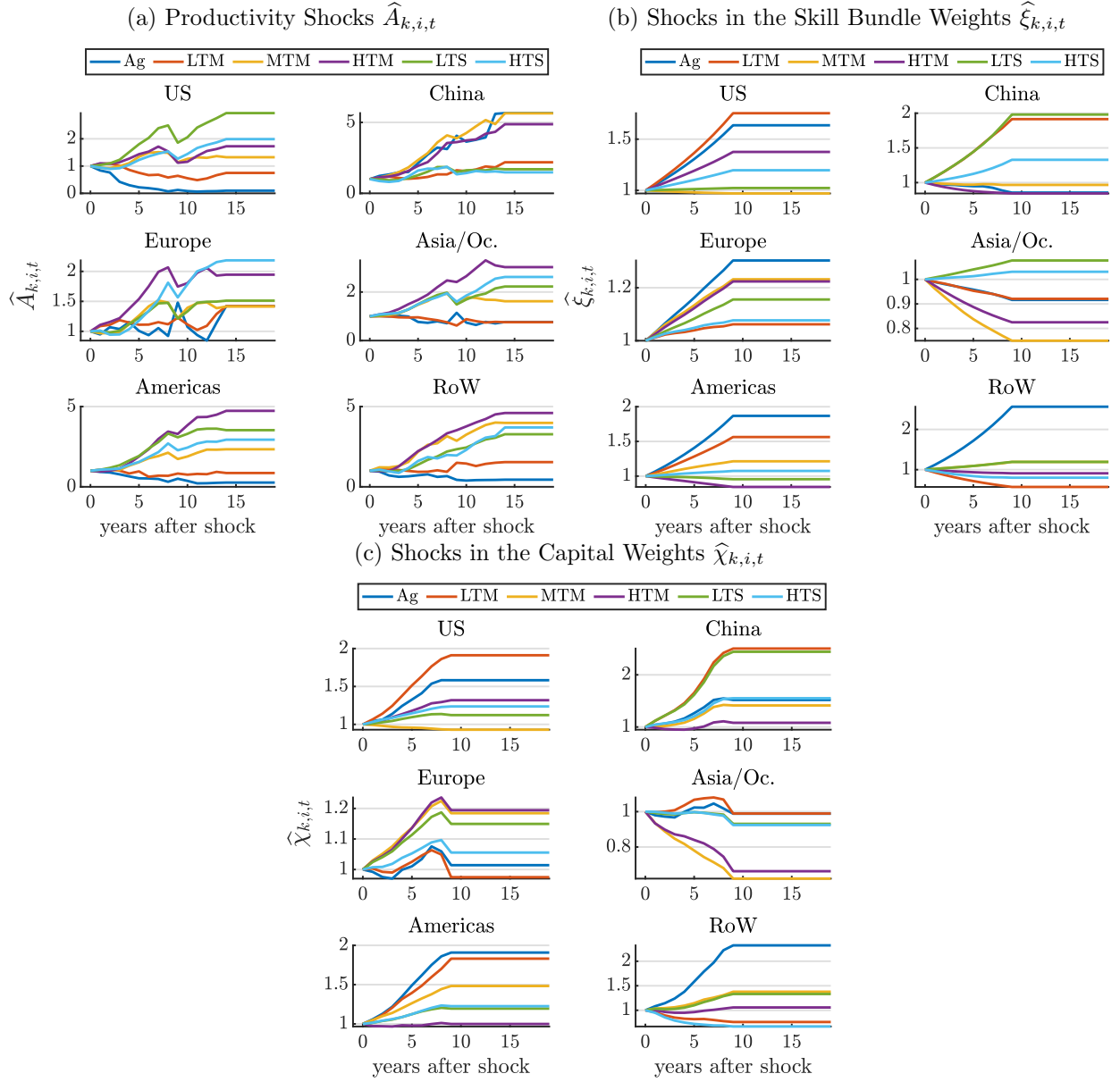


Figure C.2: Extracted Trade Shocks

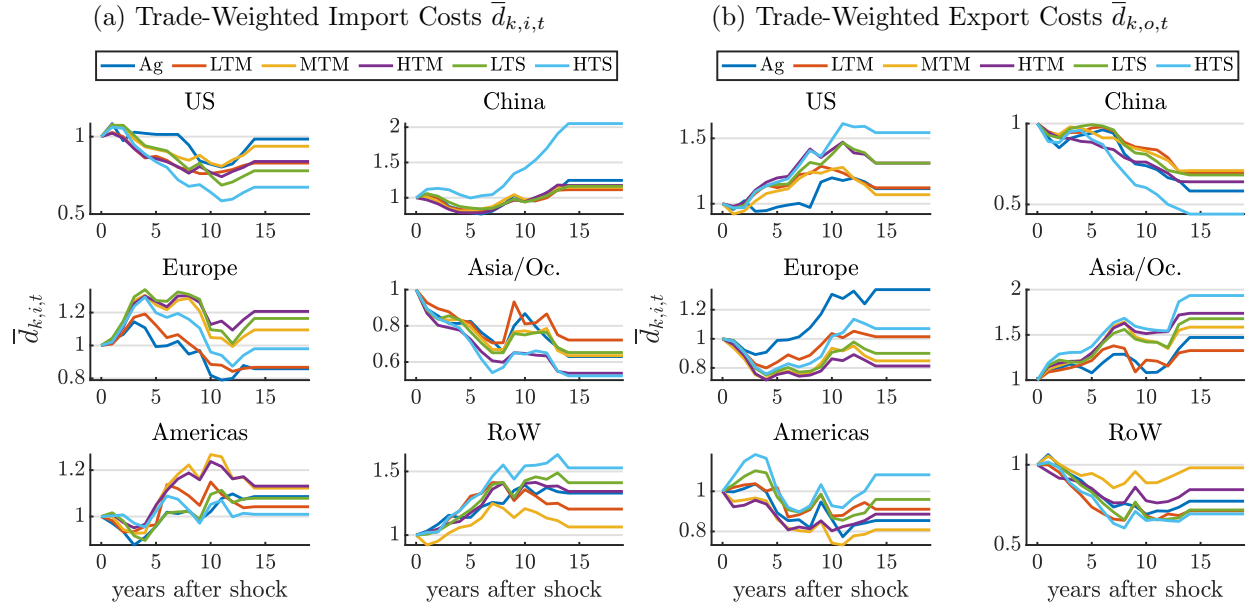
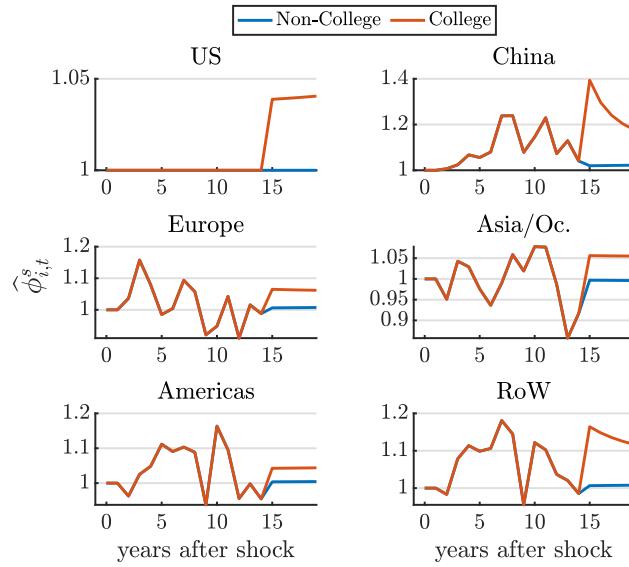


Figure C.3: Extracted Inter-Temporal Preference Shocks $\hat{\phi}_{i,t}^s$



D Solution Methods

D.1 Algorithm to Invert Model Parameters in the US

Step 1: Given data on trade shares $\pi_{k,oi}^{Data}$, net exports NX_i^{Data} , capital expenditure shares $e_{k,i}^{K,Data}$, and parameters $\mu_{k,i}$, $\alpha_{k,i}$, $\gamma_{k,i}$, $\nu_{lk,i}$ solve for the model-implied allocation of gross output $Y_{k,i}$:

$$Y_{k,o} = \sum_{i=1}^N \sum_{l=1}^K \pi_{k,oi}^{Data} \times \left[\mu_{k,i} \gamma_{l,i} (1 - e_{l,i}^{K,Data}) + \alpha_{k,i} \gamma_{l,i} e_{l,i}^{K,Data} + (1 - \gamma_{li}) \nu_{lk,i} \right] Y_{l,i} \\ - \sum_{i=1}^N \pi_{k,oi}^{Data} \mu_{k,i} NX_i^{Data}.$$

In all of the steps below we focus on the US, so that $i = US$ from now onwards.

Step 2: For each $s \in \{U, S\}$, compute $L_{k,i}^{s,0}$, the ergodic allocation of workers across sectors implied by the transition matrix $s_{kk',i}^{s,Data}$, observed in the data.

Step 3: Given data on $e_{k,i}^{s,Data}$ and parameters $\gamma_{k,i}$, compute model consistent wages $w_{k',i}^{s,0}$ for $s \in \{U, S\}$:

$$w_{k,i}^{s,0} = \frac{e_{k,i}^{s,Data} \gamma_{k,i} Y_{k,i}}{L_{k,i}^{s,0}}.$$

Step 4: Compute aggregate consumption

$$E_i^C = \sum_{k=1}^K \left(1 - e_{k,i}^{K,Data} \right) \gamma_{k,i} Y_{k,i} - NX_i^{Data}.$$

Step 5: Split aggregate consumption across skill groups $s \in \{U, S\}$ using:

$$E_i^{C,s} = \frac{I_i^{s,0}}{\sum_{s \in \{U, S\}} I_i^{s,0}} E_i^C,$$

where $I_i^{s,0} = \sum_{s \in \{U, S\}} w_{k,i}^{s,0} L_{k,i}^{s,0}$.

Step 6: Compute $\tilde{\lambda}_i^s = \frac{\bar{L}_i^s}{E_i^{C,s}}$ for $s \in \{U, S\}$.

Step 7: Solve the system in equation (25) for mobility costs $C_{kk',i}^s$ and sector-specific preferences

$\eta_{k,i}^s$, separately for each skill level $s \in \{U, S\}$:

$$\log \left(\frac{s_{kk',i}^{s,Data}}{s_{kk,i}^{s,Data}} \right) - \delta \log \left(\frac{s_{kk',i}^{s,Data}}{s_{k'k',i}^{s,Data}} \right) = -\frac{(1-\delta)C_{kk',i}^s}{\zeta_i} + \frac{\delta}{\zeta_i} \tilde{\lambda}_i^s (w_{k',i}^{s,0} - w_{k,i}^{s,0}) + \frac{\delta}{\zeta} (\eta_{k',i}^s - \eta_{k,i}^s) .$$

D.2 Algorithm to Calibrate Sector-Specific Preferences, Non-US Countries

This a note outlining how we solve for η 's conditional on a guess for ψ_i across countries.

Note that this procedure can be performed separately by country and skill group.

Step 1: Given data on labor supply and gross output, parameters estimated outside the procedure, a current guess of ψ_i , and a current guess of $\eta_{k,i}^{s,(g)}$, construct the mobility costs:

$$C_i^s = \psi_i \times C_{US}^s.$$

Step 2: Given data on trade shares $\pi_{k,oi}^{Data}$, net exports NX_i^{Data} , and capital expenditure shares $e_{k,i}^{K,Data}$, solve for the model-implied allocation of gross output $Y_{k,i}$:

$$Y_{k,o} = \sum_{i=1}^N \sum_{l=1}^K \pi_{k,oi}^{Data} \times \left[\mu_{k,i} \gamma_{l,i} (1 - e_{l,i}^{K,Data}) + \alpha_{k,i} \gamma_{l,i} e_{l,i}^{K,Data} + (1 - \gamma_{li}) \nu_{lk,i} \right] Y_{l,i} \\ - \sum_{i=1}^N \pi_{k,oi}^{Data} \mu_{k,i} NX_i^{Data},$$

and the normalization

$$\sum_{k,o} Y_{k,o} = 1.$$

Step 3: Solve for the Bellman equation across sectors using wages computed from data on worker expenditure shares, labor allocations, and the model-implied allocation of gross output $Y_{k,i}$, $w_{k,i}^{s,0} = \frac{\gamma_{k,i} e_{k,i}^{s,Data} Y_{k,i}}{L_{k,i}^{s,Data}}$:

$$V_{k,i}^s = \tilde{\lambda}_i^s w_{k,i}^{s,0} + \eta_{k,i}^{s,(g)} + \zeta_i \log \left(\sum_{k''=0}^K \exp \left(\frac{-C_{kk'',i}^s + \delta V_{k'',i}^s}{\zeta_i} \right) \right).$$

Step 4: Solve for workers' inter-sectoral transition rates:

$$s_{kk',i}^s = \frac{\exp \left(\frac{-C_{kk',i}^s + \delta V_{k',i}^s}{\zeta_i} \right)}{\sum_{k''=0}^K \exp \left(\frac{-C_{kk'',i}^s + \delta V_{k'',i}^s}{\zeta_i} \right)}.$$

Step 5: Solve for the implied supply of labor, $\tilde{L}_{k,i}^s$, by solving for the unit eigenvector of s and scaling appropriately. I.e., solve

$$[s_{kk',i}^s]' \tilde{L}^s = \tilde{L}^s$$

subject to $\langle \mathbf{1}_{K+1}, \tilde{L}^s \rangle = \bar{L}^s$, where $\tilde{L}^s$ is a $K + 1$ -vector, $\mathbf{1}_{K+1}$ is a $K + 1$ -vector of ones, and $\langle, \rangle$ is the dot product.

Step 6: Solve for the *model* implied wage:

$$w_{k,i}^{s,MODEL} = \frac{\gamma_{k,i} \ell_{k,i}^{s,Data} Y_{k,i}}{L_{k,i}^s}$$

Step 7: Update η according to the gap in wages:

$$\eta^{(g+1)} = \eta^{(g)} + \mathcal{D} \times (\log(\mathbf{w}^{MODEL} - \mathbf{w}^0),$$

where $\mathcal{D}$ is a tuning parameter that prevents overly large jumps in η .

Step 8: If $\|\eta^{(g+1)} - \eta^{(g)}\| < tol$, terminate. Else return to Step 3.

D.3 Solving for the Initial Steady State

In order to solve for the initial steady state equilibrium we first need data on the following:

Symbol	Description
$\mu_{k,i}$	Consumption Expenditure Shares
$\alpha_{k,i}$	Capital Expenditure Shares
$\gamma_{k,i}$	Value-Added Shares
$\nu_{kl,i}$	Input-Output Matrix
$e_{k,i}^{K,Data}$	Capital Expenditure in VA
$e_{k,i}^{S,Data}$	Skilled Labor Expenditure in VA
$e_{k,i}^{U,Data}$	Unskilled Labor Expenditure in VA
$\pi_{k,oi}^{Data}$	Trade Shares
NX_i^{Data}	Net Exports

From here we can solve for the initial steady state *country-by-country*. We do so according to the following algorithm:

Step 1: Given initial data solve for gross output according to:

$$Y_{k,o} = \sum_{i=1}^N \sum_{l=1}^K \pi_{k,oi}^{Data} \times \left[\mu_{k,i} \gamma_{l,i} (1 - e_{l,i}^{K,Data}) + \alpha_{k,i} \gamma_{l,i} e_{l,i}^{K,Data} + (1 - \gamma_{li}) \nu_{lk,i} \right] Y_{l,i} \\ - \sum_{i=1}^N \pi_{k,oi}^{Data} \mu_{k,i} NX_i^{Data},$$

and the normalization

$$\sum_{k,o} Y_{k,o} = 1.$$

Step 2: Solve for consumption expenditure in each country by skill group. To do so, first calculate total consumption expenditure in each country:

$$E_i^C = \sum_{k=1}^K (1 - e_{k,i}^{K,Data}) \gamma_{k,i} Y_{k,i} - NX_i^{Data}.$$

Then allocate this to each skill group according to,

$$E_i^{C,S} = \varrho_i E_i^C \\ E_i^{C,U} = (1 - \varrho_i) E_i^C,$$

where $\varrho_i \equiv \frac{\sum_{k=1}^K e_{k,i}^{S,Data} \gamma_{k,i} Y_{k,i}}{\sum_{k=1}^K (1 - e_{k,i}^{K,Data}) \gamma_{k,i} Y_{k,i}}$. To calculate ϱ_i , we assign consumption shares of each skill group in i in proportion to income shares of each skill group in i .

Step 3: Calculate the Lagrange multiplier of each household:

$$\tilde{\lambda}_i^s = \bar{L}_i^s / E_i^{C,s}$$

Steps 1 to 3 are performed independently of the steps below. The loop below can be done country-by-country.

Step 4: Guess $\{L_{k,i}^s\}$.

Step 5: Given initial data and gross output solve for wages:

$$w_{k,i}^s = \frac{\gamma_{k,i} e_{k,i}^s Y_{k,i}}{L_{k,i}^s}$$

Step 6: Solve for the Bellman equation for each worker's labor supply decision:

$$V_{k,i}^s = \tilde{\lambda}_i^s w_{k,i}^s + \eta_{k,i}^s + \zeta_i \log \left(\sum_{k''=0}^K \exp \left(\frac{-C_{kk'',i}^s + \delta V_{k'',i}^s}{\zeta_i} \right) \right)$$

Step 7: Solve for workers' inter-sectoral transition rates:

$$s_{kk',i}^s = \frac{\exp \left(\frac{-C_{kk',i}^s + \delta V_{k',i}^s}{\zeta_i} \right)}{\sum_{k''=0}^K \exp \left(\frac{-C_{kk'',i}^s + \delta V_{k'',i}^s}{\zeta_i} \right)}.$$

Step 8: Solve for the implied supply of labor, $\tilde{L}_{k,i}^s$, by solving for the unit eigenvector of s and scaling appropriately. I.e., solve

$$[s_{kk',i}^s]' \tilde{L}^s = \tilde{L}^s$$

subject to $\langle \mathbf{1}_{K+1}, \tilde{L}^s \rangle = \bar{L}^s$, where $\tilde{L}^s$ is a $K+1$ -vector, $\mathbf{1}_{K+1}$ is a $K+1$ -vector of ones, and $\langle, \rangle$ is the dot product.

Step 9: Update the guess of labor using a penalty p :

$$L_{k,i}^{s,New} = p L_{k,i}^s + (1-p) \tilde{L}_{k,i}^s$$

Step 10: Go back to Step 5 with the new guess of $L_{k,i}^s$ until $\|\tilde{L}_{k,i}^s - L_{k,i}^s\| < tol$, for some tolerance.

D.4 Algorithm: Out-of-Steady-State Transition, Full Model with Bonds

Algorithm for New Steady State

We will refer to $t = 0$ as the initial steady state and $t = 1$ as the final steady state.

This algorithm normalizes world output to 1: $\sum_{k=1}^K \sum_{i=1}^N Y_{k,i,1} = 1$.

Step 0: We start with an initial steady state characterized by $\{\pi_{k,oi,0}\}$, $\{e_{k,i,0}^U\}$, $\{e_{k,i,0}^S\}$, $\{e_{k,i,0}^K\}$, $\{NX_{i,0}^U\}$, $\{NX_{i,0}^S\}$, $\{P_{k,i,0}^I\}$.

Consider changes in preference and production function parameters $\{\mu_{k,i,0}\} \rightarrow \{\mu_{k,i,1}\}$, $\{\alpha_{k,i,0}\} \rightarrow \{\alpha_{k,i,1}\}$, $\{\gamma_{k,i,0}\} \rightarrow \{\gamma_{k,i,1}\}$ and $\{\nu_{kl,i,0}\} \rightarrow \{\nu_{kl,i,1}\}$, and on net exports $\{NX_{i,0}^U\}, \{NX_{i,0}^S\} \rightarrow \{NX_{i,1}^U\}, \{NX_{i,1}^S\}$

In addition, consider changes in (long-run) productivity $\hat{A}_{k,i}$, trade costs $\hat{d}_{k,oi}$, and shares $\hat{\chi}_{k,i}$, $\hat{\xi}_{k,i}$.

Step 1: Guess new steady-state allocations $\{L_{k,i,1}^U\}$ and $\{L_{k,i,1}^S\}$.

Step 2: Guess new steady-state wages $\{w_{k,i,1}^U\}$ and $\{w_{k,i,1}^S\}$.

Step 3: Compute wage changes $\hat{w}_{k,i}^U = \frac{w_{k,i,1}^U}{w_{k,i,0}^U}$, $\hat{w}_{k,i}^S = \frac{w_{k,i,1}^S}{w_{k,i,0}^S}$.

Step 4: Obtain trade shares $\{\pi_{k,oi,1}\}$.

First, iteratively solve the system below, conditional on $\hat{w}_{k,i}^U$ and $\hat{w}_{k,i}^S$:

$$\hat{P}_i^K = \hat{\mathcal{G}}_i^K \left(\hat{P}_{k,i}^I \right)^{\alpha_{k,i,1}},$$

where

$$\begin{aligned} \hat{\mathcal{G}}_i^K &= \frac{\prod_{k=1}^K \alpha_{k,i,0}^{\alpha_{k,i,0}}}{\prod_{k=1}^K \alpha_{k,i,1}^{\alpha_{k,i,1}}} \left(\prod_{k=1}^K [P_{k,i,0}^I]^{\alpha_{k,i,1} - \alpha_{k,i,0}} \right). \\ \hat{p}_{k,i}^h &= \left[\frac{e_{k,i,0}^K}{1 - e_{k,i,0}^U} \hat{\chi}_{k,i}^\rho \left(\hat{P}_i^K \right)^{1-\rho} + \frac{e_{k,i,0}^S}{1 - e_{k,i,0}^U} \left(\hat{w}_{k,i}^S \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}, \\ \hat{p}_{k,i}^v &= \left[e_{k,i,0}^U \left(\hat{w}_{k,i}^U \right)^{1-\sigma} + (1 - e_{k,i,0}^U) \hat{\xi}_{k,i}^\sigma \left(\hat{p}_{k,i}^h \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \\ \hat{c}_{k,i} &= \hat{\mathcal{G}}_{k,i}^c \left(\hat{p}_{k,i}^v \right)^{\gamma_{k,i,1}} \prod_{l=1}^K \left(\hat{P}_{l,i}^I \right)^{(1-\gamma_{k,i,1})\nu_{kl,i,1}}, \end{aligned}$$

where

$$\begin{aligned} \widehat{\mathcal{G}}_{k,i}^c &= \frac{\gamma_{k,i,0}^{(1-\gamma_{k,i,0})} \prod_{l=1}^K \nu_{kl,i,0}^{(1-\gamma_{k,i,0})\nu_{kl,i,0}}}{\gamma_{k,i,1}^{(1-\gamma_{k,i,1})} \prod_{l=1}^K \nu_{kl,i,1}^{(1-\gamma_{k,i,1})\nu_{kl,i,1}}} \times \\ &\quad (p_{k,i,0}^v)^{\gamma_{k,i,1}-\gamma_{k,i,0}} \prod_{l=1}^K (P_{l,i,0}^I)^{(1-\gamma_{k,i,1})\nu_{kl,i,1}-(1-\gamma_{k,i,0})\nu_{kl,i,0}}, \end{aligned}$$

$$\widehat{P}_{k,i}^I = \left[\sum_{o=1}^N \pi_{k,oi,0} \widehat{A}_{k,o} [\widehat{c}_{k,o} \widehat{d}_{k,oi}]^{-\lambda} \right]^{-1/\lambda}.$$

Compute $\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o} \widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I} \right)^{-\lambda}$.

And finally, $\pi_{k,oi,1} = \pi_{k,oi,0} \times \widehat{\pi}_{k,oi}$.

Step 5: Compute $\{e_{k,i,1}^K\}, \{e_{k,i,1}^U\}, \{e_{k,i,1}^S\}$.

First, obtain changes:

$$\begin{aligned} \widehat{e}_{k,i}^K &= \frac{(\widehat{p}_{k,i}^h)^{\rho-\sigma} (\widehat{P}_i^K)^{1-\rho}}{(\widehat{p}_{k,i}^v)^{1-\sigma}} \widehat{\chi}_{k,i}^\rho \widehat{\xi}_{k,i}^\sigma, \\ \widehat{e}_{k,i}^S &= \frac{(\widehat{p}_{k,i}^h)^{\rho-\sigma} (\widehat{w}_{k,i}^S)^{1-\rho}}{(\widehat{p}_{k,i}^v)^{1-\sigma}} \widehat{\xi}_{k,i}^\sigma, \\ \widehat{e}_{k,i}^U &= \left(\frac{\widehat{w}_{k,i}^U}{\widehat{p}_{k,i}^v} \right)^{1-\sigma}. \end{aligned}$$

Then obtain final levels.

$$\begin{aligned} e_{k,i,1}^K &= e_{k,i,0}^K \times \widehat{e}_{k,i}^K, \\ e_{k,i,1}^S &= e_{k,i,0}^S \times \widehat{e}_{k,i}^S, \\ e_{k,i,1}^U &= e_{k,i,0}^U \times \widehat{e}_{k,i}^U. \end{aligned}$$

Normalize these shares to ensure they sum to 1.

Step 6: Solve for $\{Y_{k,i,1}\}$.

$$Y_{k,o,1} = \sum_{i=1}^N \sum_{l=1}^K \pi_{k,oi,1} \left(\mu_{k,i,1} \gamma_{l,i,1} (1 - e_{l,i,1}^K) + \alpha_{k,i,1} \gamma_{l,i,1} e_{l,i,1}^K + (1 - \gamma_{l,i,1}) \nu_{lk,i,1} \right) Y_{l,i,1} \\ - \sum_{i=1}^N \pi_{k,oi,1} \mu_{k,i,1} N X_{i,1},$$

$$\sum_{k=1}^K \sum_{i=1}^N Y_{k,i,1} = 1.$$

Step 7: Update wages:

$$(w_{k,i,1}^U)^{new} = \frac{e_{k,i,1}^U \gamma_{k,i,1} Y_{k,i,1}}{L_{k,i,1}^U}, \\ (w_{k,i,1}^S)^{new} = \frac{e_{k,i,1}^S \gamma_{k,i,1} Y_{k,i,1}}{L_{k,i,1}^S}.$$

Go back to Step 3 until convergence of wages.

Step 8: Compute aggregate expenditures.

$$E_{i,1}^{C,U} = \sum_{k=1}^K w_{k,i,1}^U L_{k,i,1}^U - N X_{i,1}^U, \\ E_{i,1}^{C,S} = \sum_{k=1}^K w_{k,i,1}^S L_{k,i,1}^S - N X_{i,1}^S.$$

Step 9: Compute Lagrange multipliers $\tilde{\lambda}_{i,1}^U = \frac{\bar{L}_i^U}{E_{i,1}^{C,U}}$, $\tilde{\lambda}_{i,1}^S = \frac{\bar{L}_i^S}{E_{i,1}^{C,S}}$.

Step 10: Solve the Bellman Equations $\{V_{k,i,1}^s\}$:

$$(V_{k,i,1}^s)^{g+1} = \tilde{\lambda}_{i,1}^s w_{k,i,1}^s + \eta_{k,i}^s + \zeta_i \log \left(\sum_{l=0}^K \exp \left(\frac{-C_{kl,i}^s + \delta (V_{l,i,1}^s)^g - \delta (V_{k,i,1}^s)^g}{\zeta_i} \right) \right) + \delta (V_{k,i,1}^s)^g.$$

Step 11: Update allocations $L_{k,i,1}$.

Compute transition rates for $s = U, S$:

$$s_{kl,i,1}^s = \frac{\exp\left(\frac{-C_{kl,i}^s + \delta V_{l,i,1}^s}{\zeta_i}\right)}{\sum_{k'=0}^K \exp\left(\frac{-C_{kk',i}^s + \delta V_{k',i,1}^s}{\zeta_i}\right)}.$$

Obtain their implied steady-state allocations by computing $\left(s_{kl,i,1}^s\right)^\infty$. Multiply these shares by $\bar{L}_i^s$ to obtain $\left(L_{k,i,1}^s\right)'$, and update:

$$\left(L_{k,i,1}^s\right)^{new} = (1 - \lambda_L) L_{k,i,1}^s + \lambda_L \left(L_{k,i,1}^s\right)'.$$

Step 12: Armed with $\left(L_{k,i,1}^s\right)^{new}$ go to Step 2 until $dist\left(L_{k,i,1}^s, \left(L_{k,i,1}^s\right)'\right)$ is very small.

Outer Loop: Iteration on $\{NX_{i,t}^U\}$ and $\{NX_{i,t}^S\}$

Step 0: Consider a given path of shocks $\{\hat{A}_{k,i,t}\}_{t=0}^{T_{SS}}$, $\{\hat{d}_{k,oi,t}\}_{t=0}^{T_{SS}}$, $\{\hat{\phi}_{i,t}^s\}_{t=0}^{T_{SS}}$. Consider as well paths of production function shocks $\{\hat{\chi}_{k,i,t}\}_{t=0}^{T_{SS}}$, $\{\hat{\xi}_{k,i,t}\}_{t=0}^{T_{SS}}$. In addition, consider paths for preference and production function parameters $\{\mu_{k,i,t}\}_{t=0}^{T_{SS}}$, $\{\alpha_{k,i,t}\}_{t=0}^{T_{SS}}$, $\{\gamma_{k,i,t}\}_{t=0}^{T_{SS}}$ and $\{\nu_{kl,i,t}\}_{t=0}^{T_{SS}}$. World expenditure on final goods is normalized to 1 in every period: $\sum_{i=1}^N \sum_{s=U,S} E_{i,t}^{C,s} = 1$ for all t .

Step 1: Start with the estimated equilibrium at $t = 0$. Change the normalization from $\sum_i \sum_k Y_{k,i} = 1$ to $\sum_{i=1}^N \sum_{s=U,S} E_i^{C,s} = 1$ for all nominal variables. Variables to be re-normalized: $\{E_{i,0}^{C,U}\}$, $\{E_{i,0}^{C,S}\}$, $\{Y_{k,i,0}\}$, $\{w_{k,i,0}^U\}$, $\{w_{k,i,0}^S\}$, $\{NX_{i,0}^U\}$, $\{NX_{i,0}^S\}$.

Step 2: Obtain $\{B_{i,0}^U\}$ and $\{B_{i,0}^S\}$ under normalization $\sum_{i=1}^N \sum_{s=U,S} E_{i,t}^{C,s} = 1$.

$$B_{i,0}^s = \frac{NX_{i,0}^s}{(1 - \frac{1}{\delta})}.$$

Step 3: Make initial guesses for $\{NX_{i,T_{SS}}^U\}_{i=1}^N$, $\{NX_{i,T_{SS}}^S\}_{i=1}^N$ (under normalization $\sum_{i=1}^N \sum_{s=U,S} E_i^{C,s} = 1$)

Step 4: Solve for the new steady state at T_{SS} conditional on $\{NX_{i,T_{SS}}^U\}_{i=1}^N$, $\{NX_{i,T_{SS}}^S\}_{i=1}^N$ and the change in parameter values $\{\hat{A}_{k,i,T_{SS}}\}$, $\{\hat{d}_{k,oi,T_{SS}}\}$, $\{\hat{\chi}_{k,i,T_{SS}}\}$, $\{\hat{\xi}_{k,i,T_{SS}}\}$, α 's, γ 's, ν 's, and μ 's.

Notice that the steady-state algorithm uses the normalization $\sum_i \sum_k Y_{k,i} = 1$. Within this algorithm, re-normalize $\{NX_{i,T_{SS}}^U\}_{i=1}^N$, $\{NX_{i,T_{SS}}^S\}_{i=1}^N$ with respect to this normalization. To perform such normalization, use revenues $\{Y_{k,i,0}\}$ in the initial steady state if this is the first outer loop iteration. Otherwise, use revenue $\{Y_{k,i,T_{SS}}\}$ obtained in Step 6 below.

After computing the final steady state, change the normalization from $\sum_i \sum_k Y_{k,i} = 1$ back to $\sum_{i=1}^N \sum_{s=U,S} E_i^{C,s} = 1$. Variables to be re-normalized: $\{E_{i,T_{SS}}^{C,U}\}$, $\{E_{i,T_{SS}}^{C,S}\}$, $\{Y_{k,i,T_{SS}}\}$, $\{w_{k,i,T_{SS}}^U\}$, $\{w_{k,i,T_{SS}}^S\}$, $\{NX_{i,T_{SS}}^U\}$, $\{NX_{i,T_{SS}}^S\}$.

Step 5: Normalize $\sum_s \sum_i E_{i,t}^{C,s} = 1$ for all t . Start at $t = T_{SS} - 1$ and go backward until $t = 1$ and sequentially compute:

$$R_{t+1} = \frac{\sum_{s=U,S} \sum_{i=1}^N \frac{E_{i,t+1}^{C,s}}{\hat{\phi}_{i,t+1}^s}}{\delta \sum_{s=U,S} \sum_i E_{i,t}^{C,s}} = \frac{1}{\delta} \sum_{s=U,S} \sum_{i=1}^N \frac{E_{i,t+1}^{C,s}}{\hat{\phi}_{i,t+1}^s},$$

$$E_{i,t}^{C,s} = \frac{E_{i,t+1}^{C,s}}{\delta \widehat{\phi}_{i,t+1}^s R_{t+1}} \text{ for } s = U, S,$$

to obtain paths for $\{E_{i,t}^{C,U}\}$ and $\{E_{i,t}^{C,S}\}$. Note that because $B_{i,1}^s$ are decided before the shock, $R^1 = R^0 = \frac{1}{\delta}$.

Step 6: Solve for out-of-steady-state dynamics conditional on paths for $\{E_{i,t}^{C,U}\}$ obtained in Step 5 and $\{E_{i,t}^{C,S}\}$. See Inner Loop algorithm below.

Step 7: Using the path for disposable income $I_{i,t}^s$ obtained in Step 6 compute:

$$(NX_{i,t}^s)' = I_{i,t}^s - E_{i,t}^{C,s} \text{ for } t = 0, \dots, T_{SS} - 1,$$

$$(NX_{i,T_{SS}}^s)' = -\frac{1-\delta}{\delta} \frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1} (R_{\tau})^{-1}\right)} \left(B_{i,0}^s + \sum_{t=1}^{T_{SS}-1} \left(\prod_{\tau=1}^t (R_{\tau})^{-1} \right) (NX_{i,t}^s)' \right).$$

Step 8: Compute $dist \left(\{NX_{i,t}^s\}, \left\{ (NX_{i,t}^s)' \right\} \right)$.

Step 9: Update $(NX_{i,T_{SS}}^s)^{new} = (1 - \lambda_o) NX_{i,T_{SS}}^s + \lambda_o (NX_{i,T_{SS}}^s)'$. Go back to Step 4 until convergence of $\{NX_{i,t}^s\}$ for both skill groups.

Inner Loop Algorithm: Conditional on paths $\{E_{i,t}^{C,U}\}$ and $\{E_{i,t}^{C,S}\}$ obtained in Step 5 of Outer Loop Algorithm.

Define $\hat{x}_t \equiv \frac{x_t}{x_0}$.

Step 0: Compute multipliers $\tilde{\lambda}_{i,t}^s = \frac{\bar{L}^s}{E_{i,t}^{C,s}}$ for $s = U, S$ and $t = 1, \dots, T_{SS}$.

Step 1: Guess wage paths $\{w_{k,i,t}^s\}_{t=1}^{T_{SS}}$ for $s = U, S$.

Step 2: Obtain series $\{\pi_{k,oi,t}\}_{t=1}^{T_{SS}}$.

For each $t = 1, \dots, T_{SS}$, compute $\hat{w}_{k,i,t}^U = \frac{w_{k,i,t}^U}{w_{k,i,0}^U}$, $\hat{w}_{k,i,t}^S = \frac{w_{k,i,t}^S}{w_{k,i,0}^S}$ and iteratively solve the system:

$$\hat{P}_{i,t}^K = \hat{\mathcal{G}}_{i,t}^K \prod_{k=1}^K \left(\hat{P}_{k,i,t}^I \right)^{\alpha_{k,i,t}},$$

where

$$\begin{aligned} \hat{\mathcal{G}}_{i,t}^K &= \frac{\prod_{k=1}^K \alpha_{k,i,0}^{\alpha_{k,i,0}}}{\prod_{k=1}^K \alpha_{k,i,t}^{\alpha_{k,i,t}}} \left(\prod_{k=1}^K [P_{k,i,0}^I]^{\alpha_{k,i,t} - \alpha_{k,i,0}} \right), \\ \hat{p}_{k,i,t}^h &= \left[\frac{e_{k,i,0}^K}{1 - e_{k,i,0}^U} \hat{\chi}_{k,i,t}^\rho \left(\hat{P}_{i,t}^K \right)^{1-\rho} + \frac{e_{k,i,0}^S}{1 - e_{k,i,0}^U} \left(\hat{w}_{k,i,t}^S \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}, \\ \hat{p}_{k,i,t}^v &= \left[e_{k,i,0}^U \left(\hat{w}_{k,i,t}^U \right)^{1-\sigma} + (1 - e_{k,i,0}^U) \hat{\xi}_{k,i,t}^\sigma \left(\hat{p}_{k,i,t}^h \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \\ \hat{c}_{k,i,t} &= \hat{\mathcal{G}}_{k,i,t}^c \left(\hat{p}_{k,i,t}^v \right)^{\gamma_{k,i,t}} \prod_{l=1}^K \left(\hat{P}_{l,i,t}^I \right)^{(1-\gamma_{k,i,t})\nu_{kl,i,t}}, \end{aligned}$$

where

$$\begin{aligned} \hat{\mathcal{G}}_{k,i,t}^c &= \frac{\gamma_{k,i,0}^{\gamma_{k,i,0}} (1 - \gamma_{k,i,0})^{(1-\gamma_{k,i,0})} \prod_{l=1}^K \nu_{kl,i,0}^{(1-\gamma_{k,i,0})\nu_{kl,i,0}}}{\gamma_{k,i,t}^{\gamma_{k,i,t}} (1 - \gamma_{k,i,t})^{(1-\gamma_{k,i,t})} \prod_{l=1}^K \nu_{kl,i,t}^{(1-\gamma_{k,i,t})\nu_{kl,i,t}}} \times \\ &\quad \left(p_{k,i,0}^v \right)^{\gamma_{k,i,t} - \gamma_{k,i,0}} \prod_{l=1}^K \left(P_{l,i,0}^I \right)^{(1-\gamma_{k,i,t})\nu_{kl,i,t} - (1-\gamma_{k,i,0})\nu_{kl,i,0}}, \end{aligned}$$

$$\hat{P}_{k,i,t}^I = \left[\sum_{o=1}^N \pi_{k,oi,0} \hat{A}_{k,o,t} \left[\hat{c}_{k,o,t} \hat{d}_{k,oi,t} \right]^{-\lambda} \right]^{-1/\lambda}.$$

Compute $\hat{\pi}_{k,oi,t} = \hat{A}_{k,o,t} \left(\frac{\hat{c}_{k,o,t} \hat{d}_{k,oi,t}}{\hat{P}_{k,i,t}^I} \right)^{-\lambda}$.

And finally $\pi_{k,oi,t} = \pi_{k,oi,0} \times \hat{\pi}_{k,oi,t}$.

Step 3: Compute $\left\{e_{k,i,t}^K\right\}_{t=1}^{T_{SS}}$, $\left\{e_{k,i,t}^U\right\}_{t=1}^{T_{SS}}$, $\left\{e_{k,i,t}^S\right\}_{t=1}^{T_{SS}}$. For each $t = 1, \dots, T_{SS} - 1$ compute:

$$\hat{e}_{k,i,t}^K = \frac{\left(\hat{p}_{k,i,t}^h\right)^{\rho-\sigma} \left(\hat{P}_{i,t}^K\right)^{1-\rho}}{\left(\hat{p}_{k,i,t}^v\right)^{1-\sigma}} \hat{\chi}_{k,i,t}^\rho \hat{\xi}_{k,i,t}^\sigma,$$

$$\hat{e}_{k,i,t}^S = \frac{\left(\hat{p}_{k,i,t}^h\right)^{\rho-\sigma} \left(\hat{w}_{k,i,t}^S\right)^{1-\rho}}{\left(\hat{p}_{k,i,t}^v\right)^{1-\sigma}} \hat{\xi}_{k,i,t}^\sigma,$$

$$\hat{e}_{k,i,t}^U = \left(\frac{\hat{w}_{k,i,t}^U}{\hat{p}_{k,i,t}^v}\right)^{1-\sigma},$$

$$e_{k,i,t}^K = e_{k,i,0}^K \times \hat{e}_{k,i,t}^K,$$

$$e_{k,i,t}^S = e_{k,i,0}^S \times \hat{e}_{k,i,t}^S,$$

$$e_{k,i,t}^U = e_{k,i,0}^U \times \hat{e}_{k,i,t}^U.$$

Normalize these shares to make sure they sum to 1.

Step 4: Solve for $\left\{V_{k,i,t}^s\right\}_{t=1}^{T_{SS}}$ for $s = U, S$.

Step 4a: Solve for Steady-State Bellman for $s = U, S$:

$$\left(V_{k,i,T_{SS}}^s\right)^{g+1} = \tilde{\lambda}_{i,T_{SS}}^s w_{k,i,T_{SS}}^s + \eta_{k,i}^s + \zeta_i \log \left(\sum_{l=0}^K \exp \left(\frac{-C_{kl,i}^s + \delta \left(V_{l,i,T_{SS}}^s\right)^g - \delta \left(V_{k,i,T_{SS}}^s\right)^g}{\zeta_i} \right) \right) + \delta \left(V_{k,i,T_{SS}}^s\right)^g$$

Step 4b: Solve Bellman Equations backwards for $t = T_{SS} - 1, \dots, 1 \Rightarrow \left\{V_{k,i,t}^s\right\}_{t=1}^{T_{SS}-1}$:

$$V_{k,i,t}^s = \tilde{\lambda}_{i,t}^s w_{k,i,t}^s + \eta_{k,i}^s + \zeta_i \log \left(\sum_{l=0}^K \exp \left(\frac{-C_{kl,i}^s + \delta \hat{\phi}_{i,t+1}^s V_{l,i,t+1}^s}{\zeta_i} \right) \right)$$

Step 4c: Obtain transition rates for $t = 1, \dots, T_{SS} - 1$:

$$s_{kl,i,t,t+1}^s = \frac{\exp \left(\frac{-C_{kl,i}^s + \delta \hat{\phi}_{i,t+1}^s V_{l,i,t+1}^s}{\zeta_i} \right)}{\sum_{k'=0}^K \exp \left(\frac{-C_{kk',i}^s + \delta \hat{\phi}_{i,t+1}^s V_{k',i,t+1}^s}{\zeta_i} \right)} \text{ for } s = U, S.$$

Step 5: Compute allocation paths going forward ($t = 0$ to $t = T_{SS} - 1$):

$$L_{k,i,t+1}^s = \sum_{l=1}^K L_{l,i,t}^s s_{lk,i,t,t+1}^s.$$

Step 6: Solve for $\{Y_{k,i,t}\}_{t=1}^{T_{SS}}$:

$$\begin{aligned} Y_{k,o,t} &= \sum_{i=1}^N \pi_{k,oi,t} \mu_{k,i} \left(\sum_{s=U,S} E_{i,t}^{C,s} \right) \\ &+ \sum_{i=1}^N \sum_{l=1}^K \pi_{k,oi,t} (\alpha_{k,i} \gamma_{l,i} e_{l,i,t}^K + (1 - \gamma_{l,i}) \nu_{lk,i}) Y_{l,i,t}. \end{aligned}$$

Step 7: Compute $\left(w_{k,i,t+1}^s\right)' = \frac{e_{k,i,t+1}^s \gamma_{k,i} Y_{k,i,t+1}}{L_{k,i,t+1}^s}$.

Step 8: Compute $dist\left(\left\{w_{k,i,t+1}^s\right\}, \left\{\left(w_{k,i,t+1}^s\right)'\right\}\right)$ and update $w_{k,i,t+1}^s = (1 - \lambda_w) w_{k,i,t+1}^s + \lambda_w \left(w_{k,i,t+1}^s\right)'$ until convergence.

Step 9: Compute disposable income (to be used in Step 7 of Outer Loop):

$$I_{i,t}^s = \sum_{k=1}^K e_{k,i,t}^s \gamma_{k,i} Y_{k,i,t} \text{ for } s = U, S.$$

D.5 Algorithm: Out-of-Steady-State Transition, Exogenous Deficits (No Bonds)

Consider paths $\{A_{k,i,t}\}$, $\{d_{k,oi,t}\}$, $\{\chi_{k,i,t}\}$, $\{\xi_{k,i,t}\}$. Also, consider paths $\{\widehat{\phi}_{i,t}\}$ with $\widehat{\phi}_{i,t} = 1$ for all $t \geq T$ for some $T \ll T_{SS}$. Condition on an exogenous paths $\{NX_{i,t}^s\}$ across countries and skill levels.

Define $\widehat{x}_t \equiv \frac{x_t}{x_0}$.

Step 1: Guess paths for multipliers $\{\widetilde{\lambda}_{i,t}^s\}_{t=1}^{T_{SS}}$ for each country i and $s = U, S$.

Step 2: Compute $E_{i,t}^{C,s} = \frac{\overline{E}_i^s}{\widetilde{\lambda}_{i,t}^s}$

Step 3: Guess wage paths $\{w_{k,i,t}^s\}_{t=1}^{T_{SS}}$ for $k = 1, \dots, K$; $i = 1, \dots, N$; and $s = U, S$.

Step 4: Obtain series $\{\pi_{k,oi,t}\}_{t=1}^{T_{SS}}$.

For each $t = 1, \dots, T_{SS}$, compute $\widehat{w}_{k,i,t}^U = \frac{w_{k,i,t}^U}{w_{k,i,0}^U}$, $\widehat{w}_{k,i,t}^S = \frac{w_{k,i,t}^S}{w_{k,i,0}^S}$ and iteratively solve the system:

$$\widehat{P}_{i,t}^K = \widehat{\mathcal{G}}_{i,t}^K \prod_{k=1}^K \left(\widehat{P}_{k,i,t}^I \right)^{\alpha_{k,i}},$$

where

$$\begin{aligned} \widehat{\mathcal{G}}_{i,t}^K &= \frac{\prod_{k=1}^K \alpha_{k,i,0}^{\alpha_{k,i,0}}}{\prod_{k=1}^K \alpha_{k,i,t}^{\alpha_{k,i,t}}} \prod_{k=1}^K [P_{k,i,0}^I]^{\alpha_{k,i,t} - \alpha_{k,i,0}}. \\ \widehat{p}_{k,i,t}^h &= \left(\frac{e_{k,i,0}^K}{1 - e_{k,i,0}^U} \widehat{\chi}_{k,i,t}^\rho \left(\widehat{P}_{i,t}^K \right)^{1-\rho} + \frac{e_{k,i,0}^S}{1 - e_{k,i,0}^U} \left(\widehat{w}_{k,i,t}^S \right)^{1-\rho} \right)^{\frac{1}{1-\rho}}, \\ \widehat{p}_{k,i,t}^v &= \left[e_{k,i,0}^U \left(\widehat{w}_{k,i,t}^U \right)^{1-\sigma} + (1 - e_{k,i,0}^U) \widehat{\xi}_{k,i,t}^\sigma \left(\widehat{p}_{k,i,t}^h \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \\ \widehat{\mathcal{G}}_{k,i,t}^c &= \frac{\gamma_{k,i,0}^{\gamma_{k,i,0}} (1 - \gamma_{k,i,0})^{(1-\gamma_{k,i,0})} \prod_{l=1}^K \nu_{kl,i,0}^{(1-\gamma_{k,i,0})\nu_{kl,i,0}}}{\gamma_{k,i,t}^{\gamma_{k,i,t}} (1 - \gamma_{k,i,t})^{(1-\gamma_{k,i,t})} \prod_{l=1}^K \nu_{kl,i,t}^{(1-\gamma_{k,i,t})\nu_{kl,i,t}}} \times \\ &\quad (p_{k,i,0}^v)^{\gamma_{k,i,t} - \gamma_{k,i,0}} \prod_{l=1}^K (P_{l,i,0}^I)^{(1-\gamma_{k,i,t})\nu_{kl,i,t} - (1-\gamma_{k,i,0})\nu_{kl,i,0}}. \\ \widehat{c}_{k,i,t} &= \widehat{\mathcal{G}}_{k,i,t}^c \left(\widehat{p}_{k,i,t}^v \right)^{\gamma_{k,i}} \prod_{l=1}^K \left(\widehat{P}_{l,i,t}^I \right)^{(1-\gamma_{k,i})\nu_{kl,i}}, \end{aligned}$$

$$\hat{P}_{k,i,t}^I = \left[\sum_{o=1}^N \pi_{k,oi,0} \hat{A}_{k,o,t} \left[\hat{c}_{k,o,t} \hat{d}_{k,oi,t} \right]^{-\lambda} \right]^{-1/\lambda}.$$

Compute $\hat{\pi}_{k,oi,t} = \hat{A}_{k,o,t} \left(\frac{\hat{c}_{k,o,t} \hat{d}_{k,oi,t}}{\hat{P}_{k,i,t}^I} \right)^{-\lambda}$.

And finally $\pi_{k,oi,t} = \pi_{k,oi,0} \times \hat{\pi}_{k,oi,t}$.

Step 5: Compute $\left\{ e_{k,i,t}^K \right\}_{t=1}^{T_{SS}}, \left\{ e_{k,i,t}^U \right\}_{t=1}^{T_{SS}}, \left\{ e_{k,i,t}^S \right\}_{t=1}^{T_{SS}}$. For each $t = 1, \dots, T_{SS} - 1$ compute:

$$\hat{e}_{k,i,t}^K = \frac{\left(\hat{p}_{k,i,t}^h \right)^{\rho-\sigma} \left(\hat{P}_{i,t}^K \right)^{1-\rho}}{\left(\hat{p}_{k,i,t}^v \right)^{1-\sigma}} \hat{\chi}_{k,i,t}^\rho \hat{\xi}_{k,i,t}^\sigma,$$

$$\hat{e}_{k,i,t}^S = \frac{\left(\hat{p}_{k,i,t}^h \right)^{\rho-\sigma} \left(\hat{w}_{k,i,t}^S \right)^{1-\rho}}{\left(\hat{p}_{k,i,t}^v \right)^{1-\sigma}} \hat{\xi}_{k,i,t}^\sigma,$$

$$\hat{e}_{k,i,t}^U = \left(\frac{\hat{w}_{k,i,t}^U}{\hat{p}_{k,i,t}^v} \right)^{1-\sigma},$$

$$e_{k,i,t}^K = e_{k,i,0}^K \times \hat{e}_{k,i,t}^K,$$

$$e_{k,i,t}^S = e_{k,i,0}^S \times \hat{e}_{k,i,t}^S,$$

$$e_{k,i,t}^U = e_{k,i,0}^U \times \hat{e}_{k,i,t}^U.$$

Normalize $e_{k,i,t}^K, e_{k,i,t}^S, e_{k,i,t}^U$ to make sure they sum to 1.

Step 6: Solve for $\left\{ V_{k,i,t}^s \right\}_{t=1}^{T_{SS}}$ for $s = U, S$.

Step 6a: Solve for Steady-State Bellman for $s = U, S$:

$$\left(V_{k,i,T_{SS}}^s \right)^{g+1} = \tilde{\lambda}_{i,T_{SS}}^s w_{k,i,T_{SS}}^s + \eta_{k,i}^s + \zeta_i \log \left(\sum_{l=0}^K \exp \left(\frac{-C_{kl,i}^s + \delta \left(V_{l,i,T_{SS}}^s \right)^g - \delta \left(V_{k,i,T_{SS}}^s \right)^g}{\zeta_i} \right) \right) + \delta \left(V_{k,i,T_{SS}}^s \right)^g$$

Step 6b: Solve Bellman Equations backwards for $t = T_{SS} - 1, \dots, 1 \Rightarrow \left\{ V_{k,i,t}^s \right\}_{t=1}^{T_{SS}-1}$:

$$V_{k,i,t}^s = \tilde{\lambda}_{i,t}^s w_{k,i,t}^s + \eta_{k,i}^s + \zeta_i \log \left(\sum_{l=0}^K \exp \left(\frac{-C_{kl,i}^s + \delta \hat{\phi}_{i,t+1}^s V_{l,i,t+1}^s}{\zeta_i} \right) \right)$$

Step 6c: Obtain transition rates for $t = 1, \dots, T_{SS} - 1$:

$$s_{kl,i,t,t+1}^s = \frac{\exp\left(\frac{-C_{kl,i}^s + \delta\widehat{\phi}_{i,t+1}^s V_{l,i,t+1}^s}{\zeta_i}\right)}{\sum_{k'=0}^K \exp\left(\frac{-C_{kk',i}^s + \delta\widehat{\phi}_{i,t+1}^s V_{k',i,t+1}^s}{\zeta_i}\right)} \text{ for } s = U, S.$$

Step 7: Compute allocation paths going forward ($t = 0$ to $t = T_{SS} - 1$):

$$L_{k,i,t+1}^s = \sum_{l=1}^K L_{l,i,t}^s s_{lk,i,t,t+1}^s.$$

Step 8: Solve for $\{Y_{k,i,t}\}_{t=1}^{T_{SS}}$:

$$\begin{aligned} Y_{k,o,t} &= \sum_{i=1}^N \pi_{k,oi,t} \mu_{k,i} \left(\sum_{s=U,S} E_{i,t}^{C,s} \right) \\ &+ \sum_{i=1}^N \sum_{l=1}^K \pi_{k,oi,t} (\alpha_{k,i} \gamma_{l,i} e_{l,i,t}^K + (1 - \gamma_{l,i}) \nu_{lk,i}) Y_{l,i,t}. \end{aligned}$$

$$\sum_{o=1}^N \sum_{k=1}^K Y_{k,o,t} = 1$$

Step 9: Compute $(w_{k,i,t}^s)' = \frac{e_{k,i,t}^s \gamma_{k,i} Y_{k,i,t}}{L_{k,i,t}^s}$.

Step 10: Compute $\text{dist}\left(\left\{w_{k,i,t+1}^s\right\}, \left\{(w_{k,i,t+1}^s)'\right\}\right)$ and update $w_{k,i,t+1}^s = (1 - \lambda_w) w_{k,i,t+1}^s + \lambda_w (w_{k,i,t+1}^s)'$ until convergence.

Step 11: Compute disposable income:

$$I_{i,t}^s = \sum_{k=1}^K e_{k,i,t}^s \gamma_{k,i} Y_{k,i,t} \text{ for } s = U, S.$$

Step 12: Update $\{E_{i,t}^{C,s}\}$:

$$E_{i,t}^{C,s} = I_{i,t}^s - N X_{i,t}^s.$$

Step 13a: Compute $(\tilde{\lambda}_{i,t}^s)' = \frac{\bar{L}_i^s}{E_{i,t}^{C,s}}$ for all t .

- Step 13b: Compute $dist\left(\left\{\tilde{\lambda}_{i,t}^s\right\},\left\{\left(\tilde{\lambda}_{i,t}^s\right)'\right\}\right)$
- Step 13c: Update $\tilde{\lambda}_{i,t}^s \leftarrow (1 - \alpha_\lambda) \left(\tilde{\lambda}_{i,t}^s\right)' + \alpha_\lambda \left(\tilde{\lambda}_{i,t}^s\right)'$ for a small step size α_λ and go back to Step 2 until convergence.

D.6 Algorithm for Shocks Extraction

This algorithm uses WIOD time-series data on prices, trade shares, expenditures on final goods, expenditure shares on college workers, non-college workers, and capital, to extract shocks to the global economy: $\hat{A}_{k,i,t}$, $\hat{d}_{k,oi,t}$, and $\hat{\phi}_{i,t}^s$

Important 1: Let T_{Data} denote the last period for which we have data. Let $\tilde{T} > T_{Data}$ be the period after which there are no more shocks and $E_{i,t}^{C,s}$ is assumed to be constant across countries (according to the $\sum_i \sum_s E_{i,t}^{C,s} = 1$ normalization). We will denote $E_{i,t}^C \equiv \sum_s E_{i,t}^{C,s}$, aggregate expenditure across skill groups. The algorithm conditions on data on $\{P_{k,i,0}^I\}$, $\{E_{i,t}^C\}_{t=0}^{T_{Data}}$, $\{\hat{\pi}_{k,oi,t}\}_{t=0}^{T_{Data}}$ and $\{\hat{P}_{k,i,t}^I\}_{t=0}^{T_{Data}}$.

Important 2: We only have $\{(\hat{e}_{k,i,t}^K)_{Data}\}$, $\{(\hat{e}_{k,i,t}^S)_{Data}\}$, $\{(\hat{e}_{k,i,t}^U)_{Data}\}$ between 2000 and 2010. The algorithm also conditions on these data. Let $T_0 = 2010 < T_{Data} = 2014$.

Outer Loop: iteration on $\{NX_{i,t}^s\}$

Step 0: Compute changes in trade costs $\{\hat{d}_{k,oi,t}\}_{t=0}^{T_{Data}}$:

$$\hat{d}_{k,oi,t} = \left(\frac{\hat{\pi}_{k,oi,t}}{\hat{\pi}_{k,oi,0}} \right)^{1/\lambda} \frac{\hat{P}_{k,i,t}^I}{\hat{P}_{k,oi,0}^I}.$$

Set $\hat{d}_{k,oi,t} = \hat{d}_{k,oi,T_{Data}}$ for $t > T_{Data}$.

Step 1: Start with the estimated state equilibrium at $t = 0$. Remember that we used the normalization $\sum_{i=1}^I \sum_{k=1}^K Y_{k,i} = 1$ during the estimation procedure. **Change the normalization** from

$\sum_{i=1}^I \sum_{k=1}^K Y_{k,i} = 1$ to $\sum_{i=1}^I E_i^C = 1$. Nominal variables to be renormalized: $\{Y_{k,i,0}\}$, $\{w_{k,i,0}^U\}$, $\{w_{k,i,0}^S\}$, $\{E_{i,0}^{C,U}\}$, $\{E_{i,0}^{C,S}\}$, $\{NX_{i,0}^U\}$, $\{NX_{i,0}^S\}$.

Step 2: Compute $E_{i,t}^{C,s} = E_{i,0}^{C,s} \frac{(E_{i,t}^C)_{Data}}{(E_{i,0}^C)_{Data}}$ for $t = 1, \dots, T_{Data}$ and $s = U, S$, and where $E_{i,0}^C$ is aggregate consumption expenditure in the estimated steady state, and $(E_{i,t}^{C,s})_{Data}$ comes directly from the data. Normalize $E_{i,t}^C$ to ensure that $\sum_{i=1}^I E_{i,t}^C = 1$ in every period.

Step 3: Normalize $\hat{\phi}_{US,t}^U = 1$ for all $t = 1, \dots, T_{SS}$. This yields:

$$R_{t+1} = \frac{E_{US,t+1}^{C,U}}{\delta E_{US,t}^{C,U}} \text{ for } t = 1, \dots, T_{Data} - 1.$$

Obtain remaining shocks using:

$$\begin{aligned}\widehat{\phi}_{US,t+1}^S &= \frac{E_{i,t+1}^{C,S}}{\delta E_{i,t}^{C,S} R_{t+1}} \text{ for } t = 1, \dots, T_{Data} - 1 \\ \widehat{\phi}_{i,t+1}^s &= \frac{E_{i,t+1}^{C,s}}{\delta E_{i,t}^{C,s} R_{t+1}} \text{ for } t = 1, \dots, T_{Data} - 1 \text{ and } s = U, S.\end{aligned}$$

Step 4: Obtain $B_{i,0}^s$ with respect to the normalization $\sum_{i=1}^I E_i^C = 1$:

$$B_{i,0}^s = \frac{NX_{i,0}^s}{\left(1 - \frac{1}{\delta}\right)} \text{ for } s = U, S.$$

Step 5: Make initial guess for $NX_{i,T_{SS}}^s$ for $s = U, S$ (with respect to the normalization $\sum_{i=1}^I E_i^C = 1$).

Step 6: Compute steady state equilibrium at T_{SS} , conditional on $NX_{i,T_{SS}}^s$, $\widehat{A}_{k,i,T_{SS}}$ and $\widehat{d}_{k,oi,T_{SS}}$. If this is the first iteration of the outer loop, impose $\widehat{A}_{k,i,T_{SS}} = 1$. Otherwise, feed $\widehat{A}_{k,i,T_{SS}}$ resulting from Step 9 below.

- Step 6a: Notice that the steady-state algorithm uses the normalization $\sum_{i=1}^I \sum_{k=1}^K Y_{k,i} = 1$. Normalize $NX_{i,T_{SS}}$ with respect to normalization $\sum_{i=1}^I \sum_{k=1}^K Y_{k,i} = 1$. To perform such normalization, use revenue $\{Y_{k,i,T_{SS}}\}$ obtained in the initial steady state if this is the first outer loop iteration, otherwise use revenue $\{Y_{k,i,T_{SS}}\}$ obtained in Step 9 below.

- Step 6b: After computing the final steady state, **change the normalization** from $\sum_{i=1}^I \sum_{k=1}^K Y_{k,i} = 1$ to $\sum_{i=1}^I E_i^C = 1$ using $\{E_i^C\}$ obtained in Step 3a. Nominal variables to be renormalized: $\{Y_{k,i,T_{SS}}\}$, $\{w_{k,i,T_{SS}}^U\}$, $\{w_{k,i,T_{SS}}^S\}$, $\{E_{i,T_{SS}}^{C,U}\}$, $\{E_{i,T_{SS}}^{C,S}\}$, $\{NX_{i,T_{SS}}^U\}$, $\{NX_{i,T_{SS}}^S\}$.

Step 7: Impose $E_{i,t}^{C,s} = \begin{cases} E_{i,T_{Data}}^{C,s} + \frac{E_{i,T_{SS}}^{C,s} - E_{i,T_{Data}}^{C,s}}{\widetilde{T} - T_{Data}} (t - T_{Data}) & \text{for } t = T_{Data} + 1, \dots, \widetilde{T} \\ E_{i,T_{SS}}^{C,s} & \text{for } t > \widetilde{T} \end{cases}$.

That is, $E_{i,t}^{C,s}$ evolves linearly between T_{Data} and $\widetilde{T}$ when it reaches its steady state value determined in Step 6.

Step 8: Compute

$$R_{t+1} = \frac{E_{US,t+1}^{C,U}}{\delta E_{US,t}^{C,U}} \text{ for } t \geq T_{Data}.$$

And obtain remaining shocks $\left\{\widehat{\phi}_{i,t}^s\right\}_{t=T_{Data}+1}^{T_{SS}}$ for $s = U, S$ using

$$\begin{aligned}\widehat{\phi}_{US,t+1}^U &= 1 \text{ for } t \geq T_{Data} \\ \widehat{\phi}_{i,t+1}^s &= \frac{E_{i,t+1}^{C,s}}{\delta E_{i,t}^{C,s} R_{t+1}} \text{ for } t \geq T_{Data}.\end{aligned}$$

Step 9: Solve for the out-of-steady-state dynamics conditional on aggregate expenditures $\left\{E_{i,t}^{C,U}\right\}_{t=0}^{T_{SS}}$, $\left\{E_{i,t}^{C,S}\right\}_{t=0}^{T_{SS}}$, on preference shifters $\left\{\widehat{\phi}_{i,t}^U\right\}_{t=2}^{T_{SS}}$, $\left\{\widehat{\phi}_{i,t}^S\right\}_{t=2}^{T_{SS}}$ and trade cost shocks $\left\{\widehat{d}_{k,oi,t}\right\}_{t=1}^{T_{SS}}$.

Step 10: Using the path for disposable income $\left\{I_{i,t}^U\right\}_{t=1}^{T_{SS}}$, $\left\{I_{i,t}^S\right\}_{t=1}^{T_{SS}}$ obtained in Step 9 compute:

$$\begin{aligned}(NX_{i,t}^s)' &= I_{i,t}^s - E_{i,t}^{C,s} \text{ for } t = 0, \dots, T_{SS} - 1, \\ (NX_{i,T_{SS}}^s)' &= -\frac{1-\delta}{\delta} \frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1} (R_{\tau})^{-1}\right)} \left(B_{i,0}^s + \sum_{t=1}^{T_{SS}-1} \left(\prod_{\tau=1}^t (R_{\tau})^{-1} \right) (NX_{i,t}^s)' \right).\end{aligned}$$

Step 11: Compute $dist\left(\left\{NX_{i,t}^s\right\}, \left\{(NX_{i,t}^s)'\right\}\right)$.

Step 12: Update $\left(NX_{i,T_{SS}}^s\right)^{new} = (1 - \lambda_o) NX_{i,T_{SS}}^s + \lambda_o \left(NX_{i,T_{SS}}^s\right)'$. Go back to Step 6 until convergence of $\left\{NX_{i,t}^s\right\}$ for both skill groups.

Inner Loop Algorithm: Conditional on paths $\{E_{i,t}^{C,U}\}_{t=0}^{T_{SS}}$ and $\{E_{i,t}^{C,S}\}_{t=0}^{T_{SS}}$, net exports $\{NX_{i,t}^{C,U}\}_{t=0}^{T_{SS}}$, $\{NX_{i,t}^{C,S}\}_{t=0}^{T_{SS}}$ and shocks $\{\widehat{\phi}_{i,t}^U\}_{t=0}^{T_{SS}}$, $\{\widehat{\phi}_{i,t}^S\}_{t=0}^{T_{SS}}$, and $\{\widehat{d}_{k,oi,t}\}_{t=0}^{T_{SS}}$ obtained in the Outer Loop.

As before, we denote changes relative to $t = 0$ by $\widehat{x}_t \equiv \frac{x_t}{x_0}$. This loop conditions on the same data as the outer loop.

Step 1: Given paths $\{E_{i,t}^{C,U}\}_{t=0}^{T_{SS}}$ and $\{E_{i,t}^{C,S}\}_{t=0}^{T_{SS}}$, compute multipliers $\widetilde{\lambda}_{i,t}^s = \frac{\overline{L}_{i,t}^s}{E_{i,t}^{C,s}}$ for $s = U, S$, $t = 0, \dots, T_{SS}$.

Step 2: Guess wage paths $\{w_{k,i,t}^s\}_{t=0}^{T_{SS}}$ for $s = U, S$.

Step 3a: For each $t = 1, \dots, T_0$ compute $\widehat{w}_{k,i,t}^U = \frac{w_{k,i,t}^U}{w_{k,i,0}^U}$, $\widehat{w}_{k,i,t}^S = \frac{w_{k,i,t}^S}{w_{k,i,0}^S}$ and compute:

$$\widehat{P}_{i,t}^K = \widehat{\mathcal{G}}_{i,t}^K \left(\widehat{P}_{k,i,t}^I \right)^{\alpha_{k,i,t}},$$

where

$$\widehat{\mathcal{G}}_{i,t}^K = \frac{\prod_{k=1}^K \alpha_{k,i,0}^{\alpha_{k,i,t}}}{\prod_{k=1}^K \alpha_{k,i,t}^{\alpha_{k,i,t}}} \prod_{k=1}^K [P_{k,i,0}^I]^{\alpha_{k,i,t} - \alpha_{k,i,0}}.$$

Step 3b: Compute $\widehat{p}_{k,i,t}^h$ and $\widehat{p}_{k,i,t}^v$

$$\widehat{p}_{k,i,t}^h = \left(\frac{1 - \left(e_{k,i,t}^U \right)_{Data}}{1 - e_{k,i,0}^U} \frac{e_{k,i,0}^S}{\left(e_{k,i,t}^S \right)_{Data}} \right)^{\frac{1}{1-\rho}} \widehat{w}_{k,i,t}^S,$$

$$\widehat{p}_{k,i,t}^v = \left(\frac{e_{k,i,0}^U}{\left(e_{k,i,t}^U \right)_{Data}} \right)^{\frac{1}{1-\sigma}} \widehat{w}_{k,i,t}^U.$$

Step 3c: Recover $\widehat{\chi}_{k,i,t}$ and $\widehat{\xi}_{k,i,t}$ computing

$$\widehat{\chi}_{k,i,t} = \left(\frac{\left(\widehat{e}_{k,i,t}^K \right)_{Data}}{\left(\widehat{e}_{k,i,t}^S \right)_{Data}} \right)^{\frac{1}{\rho}} \left(\frac{\widehat{w}_{k,i,t}^S}{\widehat{P}_{i,t}^K} \right)^{\frac{1-\rho}{\rho}},$$

$$\widehat{\xi}_{k,i,t} = \left(\frac{\widehat{e}_{k,i,t}^S}{\left(\widehat{w}_{k,i,t}^S \right)^{1-\rho}} \frac{\left(\widehat{p}_{k,i,t}^v \right)^{1-\sigma}}{\left(\widehat{p}_{k,i,t}^h \right)^{\rho-\sigma}} \right)^{\frac{1}{\sigma}}.$$

Step 3d: Compute

$$\hat{c}_{k,i,t} = \hat{\mathcal{G}}_{k,i,t}^c (\hat{p}_{k,i,t}^v)^{\gamma_{k,i,t}} \prod_{l=1}^K (\hat{P}_{l,i,t}^I)^{(1-\gamma_{k,i,t})\nu_{kl,i,t}},$$

where

$$\begin{aligned} \hat{\mathcal{G}}_{k,i,t}^c &= \frac{\gamma_{k,i,0}^{(1-\gamma_{k,i,0})} \prod_{l=1}^K \nu_{kl,i,0}^{(1-\gamma_{k,i,0})\nu_{kl,i,0}}}{\gamma_{k,i,t}^{(1-\gamma_{k,i,t})} \prod_{l=1}^K \nu_{kl,i,t}^{(1-\gamma_{k,i,t})\nu_{kl,i,t}}} \times \\ & (p_{k,i,0}^v)^{\gamma_{k,i,t}-\gamma_{k,i,0}} \prod_{l=1}^K (P_{l,i,0}^I)^{(1-\gamma_{k,i,t})\nu_{kl,i,t}-(1-\gamma_{k,i,0})\nu_{kl,i,0}}. \end{aligned}$$

Compute $\hat{A}_{k,i,t} = \frac{\hat{\pi}_{k,i,i,t}}{(\hat{P}_{k,i,t}^I)^\lambda} (\hat{c}_{k,i,t})^\lambda$.

Note: We can only compute $\hat{\mathcal{G}}_{k,i,t}^c$ up to $(p_{k,i,0}^v)^{\gamma_{k,i,t}-\gamma_{k,i,0}}$. This implies that we can only recover $\hat{A}_{k,i,t}$ up to $(p_{k,i,0}^v)^{\gamma_{k,i,t}-\gamma_{k,i,0}}$. However, this is inconsequential as $\hat{A}_{k,i,t} (\hat{c}_{k,i,t})^{-\lambda}$ does not depend on $(p_{k,i,0}^v)^{\gamma_{k,i,t}-\gamma_{k,i,0}}$.

Step 4: For each $t = T_0 + 1, \dots, T_{Data}$ compute $\hat{w}_{k,i,t}^U = \frac{w_{k,i,t}^U}{w_{k,i,0}^U}$, $\hat{w}_{k,i,t}^S = \frac{w_{k,i,t}^S}{w_{k,i,0}^S}$. Impose $\hat{\chi}_{k,i,t} = \hat{\chi}_{k,i,T_0}$, $\hat{\xi}_{k,i,t} = \hat{\xi}_{k,i,T_0}$ and compute:

$$\begin{aligned} \hat{\mathcal{G}}_{i,t}^K &= \frac{\prod_{k=1}^K \alpha_{k,i,0}^{\alpha_{k,i,0}}}{\prod_{k=1}^K \alpha_{k,i,t}^{\alpha_{k,i,t}}} \prod_{k=1}^K [P_{k,i,0}^I]^{\alpha_{k,i,t}-\alpha_{k,i,0}} \\ \hat{P}_{i,t}^K &= \hat{\mathcal{G}}_{i,t}^K \prod_{k=1}^K (\hat{P}_{k,i,t}^I)^{\alpha_{k,i,t}} \\ \hat{p}_{k,i,t}^h &= \left(\frac{e_{k,i,0}^K}{1 - e_{k,i,0}^U} \hat{\chi}_{k,i,t}^\rho (\hat{P}_{i,t}^K)^{1-\rho} + \frac{e_{k,i,0}^S}{1 - e_{k,i,0}^U} (\hat{w}_{k,i,t}^S)^{1-\rho} \right)^{\frac{1}{1-\rho}} \\ \hat{p}_{k,i,t}^v &= \left[e_{k,i,0}^U (\hat{w}_{k,i,t}^U)^{1-\sigma} + (1 - e_{k,i,0}^U) \hat{\xi}_{k,i,t}^\sigma (\hat{p}_{k,i,t}^h)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\ \hat{\mathcal{G}}_{k,i,t}^c &= \frac{\gamma_{k,i,0}^{(1-\gamma_{k,i,0})} \prod_{l=1}^K \nu_{kl,i,0}^{(1-\gamma_{k,i,0})\nu_{kl,i,0}}}{\gamma_{k,i,t}^{(1-\gamma_{k,i,t})} \prod_{l=1}^K \nu_{kl,i,t}^{(1-\gamma_{k,i,t})\nu_{kl,i,t}}} \times \\ & (p_{k,i,0}^v)^{\gamma_{k,i,t}-\gamma_{k,i,0}} \prod_{l=1}^K (P_{l,i,0}^I)^{(1-\gamma_{k,i,t})\nu_{kl,i,t}-(1-\gamma_{k,i,0})\nu_{kl,i,0}}. \end{aligned}$$

$$\hat{c}_{k,i,t} = \hat{\mathcal{G}}_{k,i,t}^c (\hat{p}_{k,i,t}^v)^{\gamma_{k,i,t}} \prod_{l=1}^K (\hat{P}_{l,i,t}^I)^{(1-\gamma_{k,i,t})\nu_{kl,i,t}}$$

Compute $\hat{A}_{k,i,t} = \frac{\hat{\pi}_{k,i,t}}{(\hat{P}_{k,i,t}^I)^\lambda} (\hat{c}_{k,i,t})^\lambda$.

Note: We can only compute $\hat{\mathcal{G}}_{k,i,t}^c$ up to $(p_{k,i,0}^v)^{\gamma_{k,i,t}-\gamma_{k,i,0}}$. This implies that we can only recover $\hat{A}_{k,i,t}$ up to $(p_{k,i,0}^v)^{\gamma_{k,i,t}-\gamma_{k,i,0}}$. However, this is inconsequential as $\hat{A}_{k,i,t} (\hat{c}_{k,i,t})^{-\lambda}$ does not depend on $(p_{k,i,0}^v)^{\gamma_{k,i,t}-\gamma_{k,i,0}}$.

Step 5: For $t \geq T_{Data} + 1$ impose:

$$\hat{A}_{k,i,t} = \hat{A}_{k,i,T_{Data}},$$

$$\hat{\chi}_{k,i,t} = \hat{\chi}_{k,i,T_0}, \hat{\xi}_{k,i,t} = \hat{\xi}_{k,i,T_0}.$$

Step 6: Obtain $\{\pi_{k,oi,t}\}_{t=T_{Data}+1}^{T_{SS}}$. For each $t = T_{Data} + 1, \dots, T_{SS}$, compute $\hat{w}_{k,i,t}^U = \frac{w_{k,i,t}^U}{w_{k,i,0}^U}$, $\hat{w}_{k,i,t}^S = \frac{w_{k,i,t}^S}{w_{k,i,0}^S}$ and iteratively solve the system:

$$\hat{\mathcal{G}}_{i,t}^K = \frac{\prod_{k=1}^K \alpha_{k,i,0}^{\alpha_{k,i,0}}}{\prod_{k=1}^K \alpha_{k,i,t}^{\alpha_{k,i,T_{Data}}}} \prod_{k=1}^K [P_{k,i,0}^I]^{\alpha_{k,i,T_{Data}} - \alpha_{k,i,0}}$$

$$\hat{P}_{i,t}^K = \hat{\mathcal{G}}_{i,t}^K \prod_{k=1}^K (\hat{P}_{k,i,t}^I)^{\alpha_{k,i,t}},$$

$$\hat{p}_{k,i,t}^h = \left(\frac{e_{k,i,0}^K}{1 - e_{k,i,0}^U} \hat{\chi}_{k,i,t}^\rho (\hat{P}_{i,t}^K)^{1-\rho} + \frac{e_{k,i,0}^S}{1 - e_{k,i,0}^U} (\hat{w}_{k,i,t}^S)^{1-\rho} \right)^{\frac{1}{1-\rho}},$$

$$\hat{p}_{k,i,t}^v = \left[e_{k,i,0}^U (\hat{w}_{k,i,t}^U)^{1-\sigma} + (1 - e_{k,i,0}^U) \hat{\xi}_{k,i,t}^\sigma (\hat{p}_{k,i,t}^h)^{1-\sigma} \right]^{\frac{1}{1-\sigma}},$$

$$\begin{aligned} \hat{\mathcal{G}}_{k,i,t}^c &= \frac{\gamma_{k,i,0}^{\gamma_{k,i,0}} (1 - \gamma_{k,i,0})^{(1-\gamma_{k,i,0})} \prod_{l=1}^K \nu_{kl,i,0}^{(1-\gamma_{k,i,0})\nu_{kl,i,0}}}{\gamma_{k,i,T_{Data}}^{\gamma_{k,i,T_{Data}}} (1 - \gamma_{k,i,T_{Data}})^{(1-\gamma_{k,i,T_{Data}})} \prod_{l=1}^K \nu_{kl,i,T_{Data}}^{(1-\gamma_{k,i,T_{Data}})\nu_{kl,i,T_{Data}}}} \times \\ & (p_{k,i,0}^v)^{\gamma_{k,i,t}-\gamma_{k,i,0}} \prod_{l=1}^K (P_{l,i,0}^I)^{(1-\gamma_{k,i,T_{Data}})\nu_{kl,i,T_{Data}} - (1-\gamma_{k,i,0})\nu_{kl,i,0}}. \end{aligned}$$

$$\hat{c}_{k,i,t} = \hat{\mathcal{G}}_{k,i,t}^c (\hat{p}_{k,i,t}^v)^{\gamma_{k,i,t}} \prod_{l=1}^K (\hat{P}_{l,i,t}^I)^{(1-\gamma_{k,i,t})\nu_{kl,i,t}},$$

$$\hat{P}_{k,i,t}^I = \left[\sum_{o=1}^N \pi_{k,oi,0} \hat{A}_{k,o,t} \left[\hat{c}_{k,o,t} \hat{d}_{k,oi,t} \right]^{-\lambda} \right]^{-1/\lambda}.$$

Compute

$$\hat{\pi}_{k,oi,t} = \hat{A}_{k,o,t} \left(\hat{c}_{k,o,t} \frac{\hat{d}_{k,oi,t}}{\hat{P}_{k,i,t}^I} \right)^{-\lambda}.$$

Finally, $\pi_{k,oi,t} = \pi_{k,oi,0} \times \hat{\pi}_{k,oi,t}$.

Step 7: Compute $\left\{ e_{k,i,t}^K \right\}_{t=1}^{T_{SS}}$, $\left\{ e_{k,i,t}^U \right\}_{t=1}^{T_{SS}}$, $\left\{ e_{k,i,t}^S \right\}_{t=1}^{T_{SS}}$. For each $t = 1, \dots, T_{SS}$ compute:

$$\hat{e}_{k,i,t}^K = \frac{\left(\hat{p}_{k,i,t}^h \right)^{\rho-\sigma} \left(\hat{P}_{i,t}^K \right)^{1-\rho}}{\left(\hat{p}_{k,i,t}^v \right)^{1-\sigma}} \hat{\chi}_{k,i,t}^\rho \hat{\xi}_{k,i,t}^\sigma,$$

$$\hat{e}_{k,i,t}^S = \frac{\left(\hat{p}_{k,i,t}^h \right)^{\rho-\sigma} \left(\hat{w}_{k,i,t}^S \right)^{1-\rho}}{\left(\hat{p}_{k,i,t}^v \right)^{1-\sigma}} \hat{\xi}_{k,i,t}^\sigma,$$

$$\hat{e}_{k,i,t}^U = \left(\frac{\hat{w}_{k,i,t}^U}{\hat{p}_{k,i,t}^v} \right)^{1-\sigma},$$

$$e_{k,i,t}^K = e_{k,i,0}^K \times \hat{e}_{k,i,t}^K,$$

$$e_{k,i,t}^S = e_{k,i,0}^S \times \hat{e}_{k,i,t}^S,$$

$$e_{k,i,t}^U = e_{k,i,0}^U \times \hat{e}_{k,i,t}^U.$$

Normalize $e_{k,i,t}^K$, $e_{k,i,t}^S$, $e_{k,i,t}^U$ to make sure they sum to 1.

Step 8: Compute

$$\hat{P}_{i,t}^F = \hat{\mathcal{G}}_{i,t}^F \prod_{k=1}^K \left(\hat{P}_{k,i,t}^I \right)^{\mu_{k,i,t}},$$

where

$$\hat{\mathcal{G}}_{i,t}^F = \frac{\prod_{k=1}^K \mu_{k,i,0}^{\mu_{k,i,0}}}{\prod_{k=1}^K \mu_{k,i,t}^{\mu_{k,i,t}}} \left(\prod_{k=1}^K [P_{k,i,0}^I]^{\mu_{k,i,t} - \mu_{k,i,0}} \right).$$

Step 9: Solve for $\left\{ V_{k,i,t}^s \right\}_{t=1}^{T_{SS}}$ for $s = U, S$.

Step 9a: Solve for Steady-State Bellman for $s = U, S$:

$$\left(V_{k,i,T_{SS}}^s \right)^{g+1} = \tilde{\lambda}_{i,T_{SS}}^s w_{k,i,T_{SS}}^s + \eta_{k,i}^s + \zeta_i \log \left(\sum_{l=0}^K \exp \left(\frac{-C_{kl,i}^s + \delta \left(V_{l,i,T_{SS}}^s \right)^g - \delta \left(V_{k,i,T_{SS}}^s \right)^g}{\zeta_i} \right) \right) + \delta \left(V_{k,i,T_{SS}}^s \right)^g$$

Step 9b: Solve Bellman Equations backwards for $t = T_{SS} - 1, \dots, 1 \Rightarrow \{V_{k,i,t}^s\}_{t=1}^{T_{SS}-1}$:

$$V_{k,i,t}^s = \tilde{\lambda}_{i,t}^s w_{k,i,t}^s + \eta_{k,i}^s + \zeta_i \log \left(\sum_{l=0}^K \exp \left(\frac{-C_{kl,i}^s + \delta \hat{\phi}_{i,t+1}^s V_{l,i,t+1}^s}{\zeta_i} \right) \right)$$

Step 9c: Obtain transition rates for $t = 1, \dots, T_{SS} - 1$:

$$s_{kl,i,t,t+1}^s = \frac{\exp \left(\frac{-C_{kl,i}^s + \delta \hat{\phi}_{i,t+1}^s V_{l,i,t+1}^s}{\zeta_i} \right)}{\sum_{k'=0}^K \exp \left(\frac{-C_{kk',i}^s + \delta \hat{\phi}_{i,t+1}^s V_{k',i,t+1}^s}{\zeta_i} \right)} \text{ for } s = U, S.$$

Step 10: Compute allocation paths going forward ($t = 0$ to $t = T_{SS} - 1$):

$$L_{k,i,t+1}^s = \sum_{l=1}^K L_{l,i,t}^s s_{lk,i,t,t+1}^s.$$

Step 11: Solve for $\{Y_{k,i,t}\}_{t=1}^{T_{SS}}$:

$$\begin{aligned} Y_{k,o,t} &= \sum_{i=1}^N \pi_{k,oi,t} \mu_{k,i,t} \left(\sum_{s=U,S} E_{i,t}^{C,s} \right) \\ &+ \sum_{i=1}^N \sum_{l=1}^K \pi_{k,oi,t} (\alpha_{k,i,t} \gamma_{l,i,t} e_{l,i,t}^K + (1 - \gamma_{l,i,t}) \nu_{lk,i,t}) Y_{l,i,t}. \end{aligned}$$

Step 12: Compute $\left(w_{k,i,t+1}^s\right)' = \frac{e_{k,i,t+1}^s \gamma_{k,i,t} Y_{k,i,t+1}}{L_{k,i,t+1}^s}$.

Step 13: Compute $\text{dist} \left(\left\{ w_{k,i,t+1}^s \right\}, \left\{ \left(w_{k,i,t+1}^s \right)' \right\} \right)$ and update $w_{k,i,t+1}^s = (1 - \lambda_w) w_{k,i,t+1}^s + \lambda_w \left(w_{k,i,t+1}^s \right)'$ until convergence.

Step 14: Compute disposable income (to be used in Step 10 of Outer Loop):

$$I_{i,t}^s = \sum_{k=1}^K e_{k,i,t}^s \gamma_{k,i,t} Y_{k,i,t} \text{ for } s = U, S.$$