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ABSTRACT

Experiences in real-world pollution markets suggest that firms make persistent errors in forecasting allowance and credit prices that inform their investment decisions. The residual uncertainty characterizing allowance and credit trading means that pollution markets may fail to deliver cost-effective abatement. This contrasts with price-based policies under which firms make investments that equate marginal abatement cost to an emission tax. We incorporate the additional cost of forecast errors under quantity-based programs into a standard Weitzman-style prices versus quantities framework. We distinguish between individual firms' uncertainty over competitors' private information and systemic uncertainty over future cost shocks. We show that a welfare-maximizing regulator would favor price instruments in response to the prospect of firm-specific forecast errors under quantity instruments, *ceteris paribus*, and the relative benefit of price instruments increases with forecast error variance. We discuss the role of policy design, such as incorporating price collars, in mitigating cost-inefficiencies from price forecast errors.

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1 Introduction

Public policies – and uncertainty over public policies – play critical roles in firms’ investment and strategic decision-making (Arrow and Fisher 1974; Rodrik 1991; Hassett and Metcalf 1999; Baker, Bloom, and Davis 2016). Even after the regulator selects a policy instrument, thereby resolving uncertainty around policy design, firms may continue to face residual uncertainty over returns to investment due to the characteristics of policy implementation. In the context of carbon pricing, cap-and-trade allowance prices are inherently more uncertain than a carbon tax.

Much of the literature comparing cap-and-trade programs and emission taxes has focused on the regulator’s information deficit (Weitzman 1974; Stavins 2003). Firms typically know their own marginal abatement costs with greater precision than the regulator, and firms lack the incentive to reveal their true marginal abatement costs to the regulator (and to their competitors). A cap-and-trade program enables the regulator to circumvent its information problem by providing incentives for firms to pursue cost-minimizing strategies for reducing pollution. This approach does not eliminate the information problem, however; it simply shifts the information deficit onto firms. To illustrate, suppose the regulator faces a choice between a pollution tax or a cap-and-trade policy. If the regulator chooses a tax, the firm learns the tax rate, identifies its abatement options, and invests in abatement technology that equates marginal cost with the tax. If the regulator chooses a cap-and-trade program, by contrast, the firm identifies its abatement options as in the tax case and then must *form expectations* about the market-clearing price for allowances to guide its abatement technology investment.

Our paper examines this residual uncertainty in allowance prices, which increases the risk that firms may not equate their marginal abatement costs.¹ Firms may err in their allowance price forecasts and make investments that appear optimal *ex ante* but that are too high or

1. This analysis generally applies to various quantity-based pollution markets that enable trading of allowances or credits, including cap-and-trade programs and tradable performance standards, and thus we use allowance and credit prices interchangeably.

too low *ex post*. Such forecast errors may reflect different expectations about (1) other firms’ abatement costs; (2) economic output; and (3) overlapping public policies that may restrict abatement decisions and influence allowance prices. By increasing the aggregate costs of achieving any given emission goal, this residual uncertainty would increase the welfare costs of implementing a quantity-based instrument. Given the potential trillion-dollar scale of carbon markets, such forecast errors under cap-and-trade could be economically significant.

To examine the welfare impacts of firm-level uncertainty, we develop a modified version of Weitzman’s canonical prices versus quantities framework. In our version, quantity orders are not imposed directly on individual firms, as in Weitzman (1974) and much of the subsequent literature. Instead, we model an “aggregate quantity order,” akin to an emissions cap, that determines the total quantity of emission allowances in a given pollution market. With the allocation and subsequent trading of these allowances, firms face uncertainty over the market-clearing allowance price. We first show how firm-level uncertainty over rivals’ cost shocks or overlapping policies gives rise to forecast errors over the market-clearing price for allowances. These firm-level price forecast errors introduce cost inefficiencies for quantity-based regulations. Holding constant other factors, these additional cost inefficiencies would lead a welfare-maximizing regulator to favor price-based instruments over quantity-based instruments, with the relative benefit of price instruments increasing in the variance of the forecast error term. We acknowledge, however, that the cost inefficiencies from these price forecast errors may be augmented or mitigated in cases where individual firms also have residual uncertainty about their own abatement costs when committing to particular compliance strategies.

Since policymakers have revealed their preferences for quantity-based instruments in practice (Santikarn et al. 2021), we also address how to reduce uncertainty-related cost inefficiencies through instrument design, specifically price collars that impose a floor and a ceiling on allowance prices. Price collars may reduce the variance of firm forecast errors and associated welfare losses. We discuss how the information provision role of price collars should

be considered alongside standard arguments about the welfare consequences of introducing price ceilings or floors.

Before turning to the model, let's examine the assumptions behind the cost-minimizing result and how real-world experience deviates from those assumptions. Montgomery (1972) formally showed how, under specific assumptions, firms operating under a cap-and-trade program each have an incentive to equate their marginal abatement costs with the allowance price clearing in the market. In this full-information equilibrium, the allowance price reflects a common, cost-minimizing marginal abatement cost among all firms in the pollution market. Many environmental textbooks illustrate how firms equate marginal abatement costs with allowance prices under cap-and-trade programs through either graphical illustrations or stylized numerical examples.² Stavins (2003) summarizes a point made in these textbooks: "tradable permits can achieve the same cost-minimizing allocation of the control burden as a charge system" (p. 361). This cost-minimization characteristic of cap-and-trade programs has been described in the Economic Report of the President (Council of Economic Advisers 2000, 2010), Energy Information Administration (2009) modeling analyses of carbon dioxide cap-and-trade programs, and the Environmental Protection Agency (2015) fact sheet on the Clean Power Plan.

Complementing the static cost-effectiveness across firms, Rubin (1996) and Kling and Rubin (1997) demonstrate the potential for dynamic cost-effectiveness in cap-and-trade programs with intertemporal trading. The resulting Hotelling-style allowance price path in their theoretical set-up is reflected in virtually every integrated assessment modeling analysis of long-term quantitative climate change goals (Clarke et al. 2014). These analyses report carbon prices that minimize the costs of achieving quantitative goals because they assume full-information, well-functioning, liquid allowance markets; i.e., every emission source equates its marginal abatement cost with the observed carbon price.

2. For example, refer to: Callan and Thomas (2013) pp. 145-148, Field and Field (2016) pp. 258-259, Hanley, Shogren, and White (2019) figure 11.5, Harris and Roach (2017) figure 16.5, Keohane and Olmstead (2016) figure 9-1, Kolstad (2000) p. 163, Tietenberg and Lewis (2018) figure 15-6.

In practice, pollution markets with allowance and credit trading reveal evidence of frequent violations of these assumptions. The Carlson et al. 2000 analysis of SO_2 emission trading in the United States found that more than half of electricity generating units failed to minimize compliance costs at some point during phase I of the program. Bilateral credit transactions under the California Low Carbon Fuel Standard (LCFS) reveal remarkable price dispersion: on days with multiple transactions reported to the regulator, the within-day credit price spread averages about 20 percent (see Figure 1). On only 2 percent of days with multiple transactions is there a single, common price for LCFS credits. In 2020, transportation sector firms paid four times as much, on average, for a CO_2 emission offset as telecommunications sector firms (Ecosystem Marketplace 2021). This variation reflects the dispersion in internal carbon prices used by major corporations, including those operating in the same sector (Aldy and Gianfrate 2019).

Economic and policy shocks have caused unexpected swings in allowance prices. RECLAIM allowance prices increased by two orders of magnitude during the California electricity crisis (Fowlie, Holland, and Mansur 2012). SO_2 allowance prices fell an order of magnitude over two years after the courts overturned related regulations. EU ETS CO_2 allowances have been more volatile than crude oil futures prices since its inception (Aldy and Viscusi 2014).

Overlapping economic and environmental regulations likewise undermine attainment of cost-effective abatement. Fowlie (2010) and Cicala (2015) show that deregulated firms underinvest in capital-intensive compliance strategies for NO_x and SO_2 cap-and-trade programs, combined with evidence of over-investment by regulated firms. In the context of California's CO_2 cap-and-trade program, the uncertainty around non-market overlapping policies, combined with uncertainty around business-as-usual emissions, dwarfs uncertainty over price-responsive abatement quantities (Borenstein et al. 2019).

These illustrations may not be that surprising given that they reflect the challenges a given firm faces in making long-lived capital investments in abatement technology that can

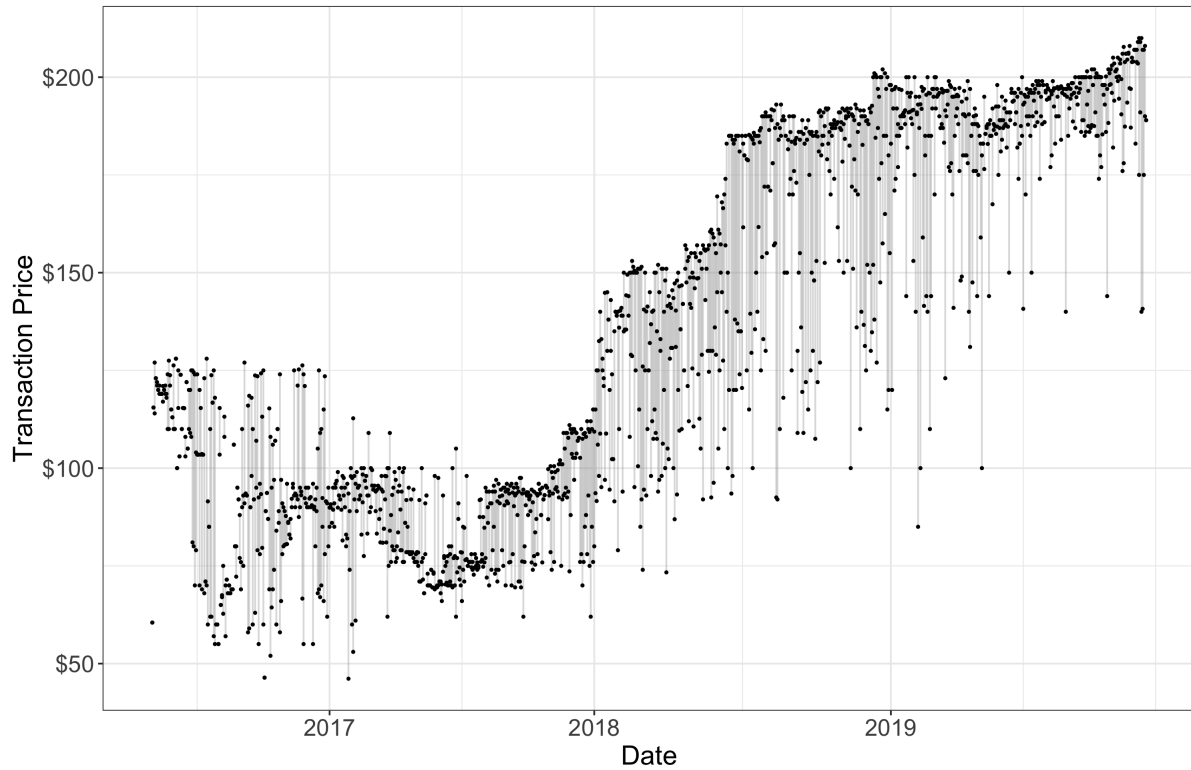


Figure 1: Minimum and Maximum Daily Transaction Prices under California's Low Carbon Fuel Standard

Notes: Notes: Constructed by authors using LCFS transaction prices posted online by the California Air Resources Board. Refer to Appendix Section A.8 for details on construction and data sourcing.

influence the firm’s demand and supply of emission allowances in a market that may not exist, could be illiquid, or lacks risk management financial instruments at the time of the investment decision. Even without such circumstances, there are theoretical and empirical contexts in which markets realize price dispersion – and hence variation in marginal costs of supply – in practice (Salop and Stiglitz 1977; Burdett and Judd 1983; Carlson and McAfee 1983; Dana Jr 1994; Allen 2014).

In the next section, we develop a Weitzman-style prices versus quantities welfare framework to model tradable quantities. Section 3 introduces firm uncertainty over rivals’ abatement costs to illustrate the welfare implications of allowance price forecast errors. Section 4 presents the welfare impacts of uncertainty associated with overlapping policies. Section 5 addresses firms’ uncertainty over their own abatement costs. Section 6 explores the role of policy design in ameliorating the inherent uncertainty of allowance prices. The final section concludes and offers directions for future research.

2 Model of Tradable Quantities

We build on existing theories of cap-and-trade markets to account for cost inefficiencies characterizing firms’ behavior in these markets. Previous studies in the spirit of Weitzman’s canonical work on the relative advantage of price versus quantity instruments have generally focused on uncertainties on the part of the regulator rather than on the part of regulated firms (Hoel and Karp 2002; Pizer 2002; Newell and Pizer 2003; Williams 2002; Yates 2002). Much of this literature assumes that cap-and-trade programs allocate “quantity orders” – Weitzman’s term for assigning a quantity to each firm – directly to individual firms, with the consequence that firms cannot face the types of uncertainties over market-clearing allowance prices that we catalogued above. One exception is Yohe (1978), who considers the possibility that firms may not know their own abatement cost functions with certainty, so quantity orders imposed directly on firms may not be achieved exactly. Williams (2002) and

Yates (2002) have also compared decentralized allowance markets to direct quantity targets, but both assume perfect certainty on the part of firms participating in these markets, even over multiple periods. By contrast, in real-world allowance markets, the regulator issues an “aggregate quantity order” by setting an emission cap. This aggregate quantity order is then implemented through a decentralized process – the allowance market – with firm-level uncertainty over the market clearing price. One consequence of the complex price formation process is that all types of shocks affect the equilibrium price, so regulated firms are affected by more than just direct shocks to their own abatement cost functions.

Given our focus on firm-level uncertainty in allowance trading markets, we distinguish between cross-sectional uncertainty over competitors’ cost shocks and other sources of private information and intertemporal uncertainty over as yet unrealized shocks. In the former case, we would expect firm-specific uncertainty over competitors’ cost shocks to be resolved *simultaneously*, as markets clear and firms’ compliance decisions become known. These idiosyncratic forecast errors are evident, for example, in the range of transaction prices observed even within a single trading day under California’s Low-Carbon Fuel Standard. In the latter case, uncertainty is resolved *sequentially*, as cost shocks are realized over time. Indeed, the former represents epistemological uncertainty over shocks that have been realized but are not perfectly observed, while the latter represents fundamental uncertainty over shocks that have yet to be realized. As we see in the derivation that follows, the intertemporal linkages of equilibrium prices in cap-and-trade markets with banking and borrowing mean that that uncertainty in early periods of a policy may continue to affect the policy’s cost-effectiveness even after information has been fully revealed to participants in the market.

To understand the welfare consequences of firm-level forecast errors, we develop a modified version of the Weitzman (1974, 2020) framework for evaluating the welfare consequences of price-based and quantity-based instruments under uncertainty. Before we introduce firm-level forecast errors, our model also closely resembles the decentralized allowance markets in

Williams (2002) and Yates (2002).³ We first introduce firm-level forecast errors around the cost shocks of competitor firms, but as we demonstrate below, our results are mathematically similar when forecast errors instead stem from uncertainty around the impact of overlapping policies. We then consider the case where firms determine their compliance strategies based on imperfect information about their own abatement costs, to understand interactions with uncertainty over the market-clearing price in a tradable quantities program versus certainty over the regulated price in a tax regime. Finally, while our model captures many key features of multi-period price- and quantity-based regulation in the presence of uncertainty, we adopt the standard assumption that abatement is variable.⁴ Future work will examine in greater detail the impact of price and cost uncertainty on dynamic firm-level investment in abatement.

We begin by defining the benefit function associated with reducing some pollutant and the cost function associated with abatement of that pollutant. Let $B_t(Q_t)$ represent the benefits in period t associated with pollution abatement at level Q_t .⁵ Likewise, let $C_t^i(q_t^i, \theta_t^i)$ be the cost to firm i associated with producing quantity q_t^i of abatement, where θ_t^i represents a firm-specific random shock to the cost function in period t . Therefore, the aggregate costs associated with abatement level Q_t are given by $\sum_{i=1}^N C_t^i(q_t^i, \theta_t^i)$, where $Q_t = \sum_{i=1}^N q_t^i$. We assume uniform mixing of the pollutant in question, such that only the total level of abatement enters into the benefits function, not the identity of each polluting entity. This assumption reflects the characteristics of carbon dioxide and most other greenhouse gas emissions but could be relaxed to model local pollutants. By contrast, the costs of abatement depend on the level achieved by each individual firm.⁶

3. Other closely related papers in this recent literature on prices-versus-quantities with banking and borrowing are Heutel (2020), Pizer and Prest (2020), and Karp (2019). These papers also consider how the relative advantage of prices, quantities, and quantities with banking and borrowing depend on whether the policymaker is able to update policies over time as information is revealed in the market. Our baseline model considers what amounts to an “open loop” policy, or a policy without updating, in which the regulator sets prices or quantities at the start of the two-period regulatory cycle.

4. Weitzman (2020) provides further discussion of the need for this assumption in footnote 8.

5. Here we assume no uncertainty in the benefit function, which is consistent with Weitzman (2020).

6. We also assume that the benefits of abatement can be approximated with a marginal benefit function that is separable over time. This assumption seems reasonable in the case of climate change given evidence

We assume for tractability that there are two periods in the current regulatory cycle.⁷ Let us consider the regulator's problem under two instrument options. Under a tax, the regulator sets an optimal price order in the presence of uncertainty by solving the following maximization problem:

$$\max_{\tilde{p}_1, \tilde{p}_2} E[B_1(\sum_{i=1}^N q_1^i(\tilde{p}_1, \theta_1^i)) - \sum_{i=1}^N C_1^i(q_1^i(\tilde{p}_1, \theta_1^i), \theta_1^i) + B_2(\sum_{i=1}^N q_2^i(\tilde{p}_2, \theta_2^i)) - \sum_{i=1}^N C_2^i(q_2^i(\tilde{p}_2, \theta_2^i), \theta_2^i)] \quad (1)$$

Under a cap-and-trade regime, the regulator sets the optimal (aggregate) quantity order by solving the following maximization problem:

$$\begin{aligned} \max_{\hat{Q}} E[B_1(\sum_{i=1}^N q_1^i(p_1(\hat{Q}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2), \theta_1^i)) - \sum_{i=1}^N C_1^i(q_1^i(p_1(\hat{Q}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2), \theta_1^i), \theta_1^i) \\ + B_2(\sum_{i=1}^N q_2^i(p_2(\hat{Q}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2), \theta_2^i)) - \sum_{i=1}^N C_2^i(q_2^i(p_2(\hat{Q}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2), \theta_2^i), \theta_2^i)] \end{aligned} \quad (2)$$

Here $p_t(\hat{Q}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$ represents the market-clearing price associated with the regulated quantity order $\hat{Q}$ and the vectors of per-period marginal cost shocks $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$.

Following Weitzman and related literature, we expand the cost and benefit functions by taking a second-order Taylor expansion about the quantity $\bar{q}_t^i$. We define each $\bar{q}_t^i$ as the level of abatement that sets expected benefits equal to expected costs for each individual firm.

that the damage function from greenhouse gas emissions may be approximately linear in temperature (Burke, Hsiang, and Miguel 2015). We acknowledge the work of existing papers that have examined how optimal policies may depend on the characteristics of marginal damages (Williams 2002; Yates 2002; Gerlagh and Heijmans 2018), though this discussion is not directly relevant to our modeling of the cost inefficiencies introduced by firm-level uncertainty. We also note that the key insights from our model remain intact when we explicitly model a stock pollutant where avoided damages (benefits) are intertemporally linked; this derivation is available from the authors upon request.

7. As discussed below, we also assume here that there is no discounting between periods and that the intertemporal permit trading ratio is equal to 1. These assumptions greatly simplify our derivation and allow us to highlight the impact of forecast errors. Karp (2019) shows that the choice of discount factor and permit trading ratio may affect the relative advantages of prices, quantities, and quantities with banking and borrowing.

The Taylor expansion of each abatement cost function about $\bar{q}_t^i$ is then given by:

$$C_t^i(q_t^i, \theta_t^i) = a_i(\theta_t^i) + (C' + \theta_t^i)(q_t^i - \bar{q}_t^i) + \frac{C''_i}{2}(q_t^i - \bar{q}_t^i)^2 \quad (3)$$

where C' represents the expected marginal abatement cost at $\bar{q}_t^i$ and C''_i represents the slope of the marginal abatement cost function.⁸ As in Weitzman and much of the subsequent literature, we assume for tractability that the abatement cost function is quadratic or can be well approximated by a second-order Taylor expansion. θ_t^i represents how the random cost shock affects the slope of the abatement cost function for firm i , and $a_i(\theta_t^i)$ represents how the cost shock affects the level of abatement costs. As in Weitzman's derivation, we assume without loss of generality that $E[a_i(\theta_t^i)] = 0$ and $E[\theta_t^i] = 0$. Note that this assumption does not preclude that the *conditional* expectation of a firm's cost shock differs from 0; in general, $E[a_i(\theta_t^i)|a_j(\theta_t^j)] \neq 0$ and $E[\theta_t^i|\theta_t^j] \neq 0$ for some i and j , and $E[a_i(\theta_t^i)|a_i(\theta_{t'}^i)] \neq 0$ and $E[\theta_t^i|\theta_{t'}^i] \neq 0$ for t and t' .

For the benefits function, we also take the Taylor expansion around $\bar{Q}_t = \sum_{i=1}^N \bar{q}_t^i$:

$$B(Q_t) = b + B'(\sum_{i=1}^N q_t^i - \sum_{i=1}^N \bar{q}_t^i) - \frac{B''}{2}(\sum_{i=1}^N q_t^i - \sum_{i=1}^N \bar{q}_t^i)^2 \quad (4)$$

Here B' captures the marginal benefit at $\bar{Q}_t = \sum_{i=1}^N \bar{q}_t^i$, and B'' captures the slope of the marginal benefit function (where $B'' \geq 0$).

For the optimal tax, the derivation here closely follows Weitzman's derivation with multiple production units, except we constrain the regulated price to be the same across all units. We again find that the optimal price order $\tilde{p}_t$ is equal to $B' = C'_i$ for all i (and for all t where these parameters are constant). The full derivation is provided in Appendix Section A.1. Assuming cost minimization, each firm will set its realized marginal cost function equal

8. As we show in the full derivation in Appendix Section A.1 and A.2, the Taylor expansion is defined such that C' is constant for all i . Furthermore, while we could allow for the parameters of the cost and benefit function to differ across periods, we assume for analytic tractability that C''_i and B'' are constant over time.

to this price, yielding the following firm-level response function:

$$\tilde{q}_t^i = q_t^i(\tilde{p}_t, \theta_t^i) = \bar{q}_t^i - \frac{\theta_t^i}{C_i'''} \quad (5)$$

The aggregate quantity produced in each period under the optimal tax will be:

$$\sum_{i=1}^N \tilde{q}_t^i = \sum_{i=1}^N q_t^i(\tilde{p}_t, \theta_t^i) = \sum_{i=1}^N (\bar{q}_t^i - \frac{\theta_t^i}{C_i'''}) = \bar{Q}_t - \sum_{i=1}^N \frac{\theta_t^i}{C_i'''} \quad (6)$$

For the optimal cap-and-trade program, our derivation differs from the original Weitzman derivation in that the regulator sets an overall quantity target but cannot directly determine how that overall quantity is allocated across firms. We find that the optimal quantity order $\hat{Q}$ is equal to $\sum_{i=1}^N \bar{q}_1^i + \sum_{i=1}^N \bar{q}_2^i$; the full derivation is provided in Appendix Section A.2. To determine how this overall quantity is then allocated across firms, we must solve for the market-clearing price such that the aggregate quantity order is achieved after shocks are realized in the market.⁹ Furthermore, because we allow banking and borrowing across the two periods, we must also apply a no-arbitrage condition that relates the first-period price to the expected second-period price. We assume that both the discount factor and the intertemporal trading ratio are equal to 1, so the first-period price and expected second-period price must be equal in equilibrium.

To build intuition for the basic model set-up, we initially follow Weitzman and assume that firms know all shocks in the market before making any compliance decisions and therefore are able to set their marginal abatement costs equal to a known market-clearing price; we then relax this assumption in subsequent sections to show the impact of firm-level uncertainty. Assuming cost minimization, the equilibrium price associated with the overall

9. A cap-and-trade program indirectly implements an aggregate quantity order in the spirit of Weitzman. While an emissions cap and the subsequent trading of allowances allocates the quantity of *allowed emissions*, the quantity order in the Weitzman framework instead determines the quantity of *emissions abatement*. While not explicitly considered in this model, uncertainty over business-as-usual (BAU) emissions – and therefore uncertainty over how the quantity of allowed emissions will translate into a quantity of emissions abatement – is an additional source of firm-level uncertainty. Borenstein et al. (2019) discuss the importance of uncertainty over BAU emissions in driving systematic uncertainty over future allowance prices.

optimal quantity order $\hat{Q}$ is given by:

$$\hat{p}_1(\hat{Q}, \theta_1, \theta_2) = \hat{p}_2(\hat{Q}, \theta_1, \theta_2) = C' + \frac{\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''}}{\sum_{i=1}^N \frac{1}{C_i''}} \quad (7)$$

In the first period, the firm response function dictates that firm i will produce $\hat{q}_1^i = \bar{q}_1^i + \frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\theta_1^i}{C_i''}$; the aggregate first-period quantity produced will be $\hat{Q}_1 = \bar{Q}_1 + \sum_j \frac{\theta_2^j - \theta_1^j}{2C_j''}$.

Likewise, in the second period, firm i will produce $\hat{q}_2^i = \bar{q}_2^i + \frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\theta_2^i}{C_i''}$; the aggregate second-period quantity is then $\hat{Q}_2 = \bar{Q}_2 + \sum_j \frac{\theta_1^j - \theta_2^j}{2C_j''}$. (The full derivation is given in Appendix Section A.2.)

Finally, following Weitzman, we define the relative advantage of prices over quantities as the expected difference between net benefits from the optimal tax and net benefits from the optimal cap-and-trade program with banking and borrowing. That is:

$$\begin{aligned} \Delta = & \mathbb{E}[B_1(\sum_{i=1}^N \tilde{q}_1^i) + B_2(\sum_{i=1}^N \tilde{q}_2^i) - \sum_{i=1}^N C_1^i(\tilde{q}_1^i, \theta_1^i) - \sum_{i=1}^N C_2^i(\tilde{q}_2^i, \theta_2^i)] \\ & - \mathbb{E}[B_1(\sum_{i=1}^N \hat{q}_1^i) + B_2(\sum_{i=1}^N \hat{q}_2^i) - \sum_{i=1}^N C_1^i(\hat{q}_1^i, \theta_1^i) - \sum_{i=1}^N C_2^i(\hat{q}_2^i, \theta_2^i)] \end{aligned} \quad (8)$$

Substituting each firm's response to the price and quantity orders, respectively, we obtain the following expression for the relative advantage of prices over tradable quantities given perfect firm-level certainty:¹⁰

$$\Delta = \mathbb{E} \left[\frac{1}{4} \left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B'' \right) \left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''} \right)^2 \right] \quad (9)$$

10. We can compare this baseline welfare expression directly to similar derivations in existing literature. For example, we can immediately compare this expression to the result in Weitzman (2020) for the comparative advantage of fixed prices over time-flexible quantities with perfect information for a single representative firm (equation 51). Our expression is also consistent with the relative advantage of prices over tradable quantities in Williams (2002) (equation 31), given our assumption that abatement from different firms is perfectly substitutable in the benefits function and if we were to further assume that abatement cost shocks are independent across firms.

The full derivation of this expression is provided in Appendix Section A.3.

While this expression maintains some of the standard logic about comparing the relative slopes of the marginal cost and marginal benefit functions, note that we cannot evaluate the costs associated with the quantity order for each firm separately, since the quantity produced by each firm depends on the shocks to all other firms' marginal abatement cost functions via the market-clearing price. Instead we must compare the slope of marginal benefits to an expression that combines the slopes of all marginal costs. Given this baseline result, we now proceed with relaxing the strong assumption that firms have perfect information about all shocks before making any compliance decisions.

3 Firm Uncertainty Over Rivals' Abatement Costs

In introducing firm-level uncertainty, we distinguish between “idiosyncratic uncertainty” facing specific firms and “systematic uncertainty” facing all firms in the industry. Our goal is to differentiate between the impact of uncertainty on the allocation of abatement across firms and the impact on the allocation of aggregate abatement across periods. We first derive the impact of firm-level uncertainty over rivals' abatement cost shocks, and then show how results are qualitatively similar for firm-level uncertainty over the impact of overlapping policies.

Because the market-clearing price associated with the regulator's aggregate quantity order (emissions cap) depends on shocks to the marginal abatement cost functions of all firms, a given firm may not know this price with certainty when making its abatement decisions. Instead, we assume that before the first compliance period, each firm i observes its current abatement cost shock θ_1^i but none of the other abatement cost shocks in the market.¹¹ Given

11. While it is possible that financial instruments would help regulated firms to mitigate uncertainty associated with volatile allowance and credit prices, evidence on hedging decisions more generally suggests that firms are likely to hedge incompletely, if at all. In studying hedging of input fuel prices by U.S. airlines, Rampini, Sufi, and Viswanathan (2014) find that the airlines in their sample hedge only 20% of expected next-year jet fuel expenses – despite the fact that financial instruments are widely available in this market and jet fuel represents a substantial and highly volatile operating expense for these firms. The authors

its own realized shock and industry-wide common priors over the joint distribution of all cost shocks, each firm i forms a posterior expectation over its rivals' current cost shocks θ_1^{-i} and all future cost shocks θ_2 .¹² Based on its posterior expectations, each firm i then formulates a forecast for the cost-effective price.¹³

$$E[\hat{p}_1(\hat{Q}, \theta_1, \theta_2) | \theta_1^i] = C' + \frac{E[\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''} | \theta_1^i]}{\sum_{j=1}^N \frac{1}{C_j''}} = C' + \frac{\frac{\theta_1^i}{2C_i} + E[\sum_{j \neq i}^{N-1} \frac{\theta_1^j}{2C_j''} + \sum_{j=1}^N \frac{\theta_2^j}{2C_j''} | \theta_1^i]}{\sum_{j=1}^N \frac{1}{C_j''}} \quad (10)$$

Firm i will then set its marginal abatement cost equal to this forecast of the cost-effective price. In real-world emissions trading markets, most compliance options available to firms require time-to-build or other upfront (and often irreversible) investments, motivating our assumption that firms cannot adjust their abatement quantities after the realized market-clearing price is observed.¹⁴

To simplify notation going forward, we suppress the dependence of $\hat{p}_t$ on $\hat{Q}$, θ_1 , and θ_2 and define the difference between firm i 's forecasted price and the cost-effective price as

attribute this imperfect hedging partly to firm financial constraints, which would certainly be relevant in our setting. Moreover, firms may also be unable to hedge their full exposure to uncertain allowance prices since the total quantity of allowances demanded depends on both the uncertain future price and potential additional uncertainties around future abatement cost. Finally, since allowance and credit trading markets are created virtually overnight through regulation, there is considerable uncertainty associated with the start-up of these markets which may reduce the availability of financial instruments in their early phases.

12. Note that this set-up is compatible with a Bayesian Nash equilibrium concept, in which firms choose their optimal actions taking the (expected) actions of their competitors as given, based on their respective information sets and common priors over the distribution of shocks in the market. Because we have not placed any restrictions on potential correlation between firm i 's cost shock and those of specific rivals, this set-up still permits the firm's observation of θ_t^i to be more informative about certain firms' cost shocks than others (or to be more informative about current period shocks than shocks in future periods). For further discussion of firm behavior in emissions trading markets with a Bayesian Nash equilibrium concept, see Malueg and Yates (2009), building on Vives (2002).

13. Throughout this paper, we use the term "cost-effective price" to refer to the market-clearing allowance price that achieves the regulator's aggregate quantity limit at lowest cost. Firms generally do not have perfect foresight about what this cost-effective price will be.

14. In contrast to our modeling approach, Malueg and Yates (2009) assume that firms submit their entire abatement cost schedule to the regulator, who then settles an efficient market-clearing price, which is then communicated to all firms before they choose their actual abatement levels. Consequently, Malueg and Yates find no difference in firm behavior when rivals' cost shocks are known versus held as private information, as the regulator effectively resolves all firm-level uncertainty before (competitive) firms choose their optimal compliance strategies. Real-world compliance behavior more closely matches the model developed here in which firms enter the allowance market with a particular emissions quantity rather than a perfectly flexible emissions schedule.

follows:

$$\mathbb{E}[\hat{p}_1|\theta_1^i] - \hat{p}_1 = \text{forecast error} = \frac{\mathbb{E}[\sum_{j \neq i}^{N-1} \frac{\theta_1^j}{2C_j''} + \sum_{j=1}^N \frac{\theta_2^j}{2C_j''} | \theta_1^i]}{\sum_{j=1}^N \frac{1}{C_j''}} - \frac{\sum_{j \neq i}^{N-1} \frac{\theta_1^j}{2C_j''} + \sum_{j=1}^N \frac{\theta_2^j}{2C_j''}}{\sum_{j=1}^N \frac{1}{C_j''}} \quad (11)$$

The firm's quantity response becomes:

$$q_1^i(\mathbb{E}[\hat{p}_1|\theta_1^i], \theta_1^i) = \frac{\hat{p}_1 + \text{forecast error} - C' - \theta_1^i}{C_i''} + \bar{q}_1^i \quad (12)$$

Next, we decompose each firm's forecast error into two parts: ξ_1 is the component of the forecast error common to all firms (*systemic uncertainty*), and ϵ_1^i is the component of the forecast error specific to firm i (*idiosyncratic uncertainty*). By construction, ξ_1 affects industry-wide abatement levels across periods, while ϵ_1^i affects only the distribution of abatement across firms within the initial compliance period.¹⁵ That is, the aggregate quantity in the first period may be higher or lower than what is intertemporally optimal, depending on the sign of ξ_1 ; likewise, firm i 's share of that industry-wide abatement may be higher or lower than what is cross-sectionally optimal, depending on the sign of ϵ_1^i .

We can rewrite the firm's quantity response as:

$$q_1^i(\hat{p}_1 + \xi_1 + \epsilon_1^i, \theta_1^i) = \frac{\hat{p}_1 + \xi_1 + \epsilon_1^i - C' - \theta_1^i}{C_i''} + \bar{q}_1^i \quad (13)$$

Aggregate first-period abatement is therefore given by:

$$Q_1 = \sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} + \sum_{i=1}^N \frac{\xi_1}{C_i''} + \bar{Q}_1$$

Consequently, after accounting for the impact of firm-level uncertainty, aggregate abatement

15. We construct ϵ_1^i such that $\sum_{i=1}^N \frac{\epsilon_1^i}{C_i''} = 0$, which ensures that firms' idiosyncratic expectation errors collectively cancel with each other in determining the per-period abatement quantity. By contrast, the net impact of firms' systemic expectation errors on industry-wide abatement in the first period is given by $\sum_{i=1}^N \frac{\xi_1}{C_i''} \neq 0$.

differs from the cost-effective level derived in the previous section. To ensure that the regulator's overall quantity limit is still met by the end of the final regulatory period, the aggregate second-period quantity must also shift upwards or downwards from the cost-effective level to compensate, and the market-clearing price in the second period adjusts accordingly. Full details of this derivation are provided in Appendix Section A.4.

By contrast, under a tax, the price is set by regulation and does not depend on private information about other firms' marginal abatement costs or about future marginal abatement costs. We maintain our earlier assumption that the regulated price is known with certainty to all firms under a price-based policy, to maintain the *consistent* assumption across both price and quantity instruments that uncertainty around policy design and stringency has already been resolved. Therefore, firms' responses to a price order do not change from the version derived in Section 2 above.

Based on this set-up, we re-derive the relative advantage of prices over quantities, allowing for the presence of both idiosyncratic and systemic forecast errors under quantity-based regulation but holding constant the net benefits of price-based regulation. Our welfare expression now becomes:¹⁶

$$\begin{aligned}
\Delta = & \mathbb{E} \left[\underbrace{\frac{1}{4} \left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B'' \right) \left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''} \right)^2}_{\text{Original Expression}} + \underbrace{\frac{1}{2} \left(\sum_{i=1}^N \frac{\epsilon_1^{i2}}{C_i''} \right)}_{\text{Cost Inefficiencies from Idiosyncratic Errors} \geq 0} \right. \\
& + \underbrace{\left(\sum_{i=1}^N \frac{\xi_1^2}{C_i''} \right)}_{\text{Cost Inefficiencies from Systemic Errors} \geq 0} + \underbrace{B'' \left(\sum_{i=1}^N \frac{\xi_1}{C_i''} \right)^2 + 2B'' \left(\sum_{i=1}^N \frac{\xi_1}{C_i''} \right) \left(\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} \right)}_{\text{Additional Benefit Term} \geq \text{or} \leq 0} \left. \right] \quad (14)
\end{aligned}$$

We parse Equation 14 in stages. The first term replicates the “relative slopes” comparison from our baseline derivation when we assumed perfect firm-level certainty. The second and

16. This derivation (and those in subsequent sections) assumes that the regulator has no foresight about the extent of firm forecast errors. That is, $\mathbb{E}[\xi_1] = 0$ and $\mathbb{E}[\epsilon_1^i] = 0$ for all i . This follows immediately from the law of iterated expectations.

third terms together indicate that firm-level forecast errors under quantity regulation create an additional advantage of price instruments relative to quantity instruments, with the relative advantage increasing in the variance of both the idiosyncratic error terms and the systemic error terms.¹⁷ To interpret this finding, in light of Weitzman’s original result, we recognize that the regulator is no longer imposing quantity orders directly on individual firms. Instead, as we have noted, the regulator imposes an aggregate quantity order (the emissions cap), which a market mechanism then translates into individual quantity orders. Given idiosyncratic firm-level expectation errors in a single compliance period, the same relative advantages of price and quantity instruments still exist, but we must also consider the possibility that the aggregate quantity order is not distributed in the least-cost manner *across firms*. Likewise, given systemic firm-level expectation errors, we must also consider cost-inefficient distributions of aggregate abatement *across compliance periods*.¹⁸ Both types of abatement re-allocation push the regulator to prefer the price-based instrument over the quantity-based instrument with banking and borrowing.

The final term in Equation 14 captures the idea that systemic forecast errors influence the extent to which the marginal benefits of abatement are equated across time periods. Tradable allowance programs ordinarily deviate from perfect benefit smoothing in response to cost shocks in specific periods. To the extent that the systemic forecast errors dampen firm responses to these period-specific cost shocks, quantity variability may decrease and therefore benefit smoothing may improve; to the extent that these forecast errors amplify

17. The impact of firm-level forecast errors is also decreasing in the slope of the marginal abatement cost function, C''_i . The intuition here is that individual firm quantities are less responsive to (expected) price when the slope of the marginal abatement function is large. (Recall that the firm’s quantity response function is $q_t^i = \bar{q}_t^i + \frac{p_t - \theta_t^i - C'}{C''_i}$.)

18. Both Karp (2019) and Feng and Zhao (2006) also relax the assumption in Weitzman (2020) that the representative firm has perfect information about second-period cost shocks. Karp (2019) assumes instead rational expectations over future shocks which evolve according to an AR-1 process. In the derivation presented here, we instead model heterogeneous firms with different expectations about market-clearing prices. This derivation is still compatible with rational expectations if we require each firm to have rational expectations over the cost shocks of other firms in the market, given the revelation of its own cost shock. However, our derivation depends on heterogeneous expectations about the allowance price. Feng and Zhao (2006) show how the welfare gains from intertemporal trading decrease as firms’ knowledge of current shocks becomes less informative about future shocks, thereby decreasing the extent of information asymmetry between firms and the regulator.

responses to cost shocks, however, benefit smoothing may be further reduced. Consequently, this second set of additional terms has an ambiguous impact on the regulator’s preference for prices over quantities with banking and borrowing, depending on the overall impact on benefit smoothing. The overall impact, in turn, depends on the sign and magnitude of the correlation between $\sum_{i=1}^N \frac{\xi_1}{C_i''}$ and $\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''}$. We note, however, that this additional consideration would disappear given a stock pollutant, such as greenhouse gases, where the timing of production is not a first-order concern over short-time horizons – leaving only the additional cost inefficiencies due to forecast errors under quantity-based policies.¹⁹

Finally, we also note that in this derivation, as in subsequent derivations featuring firm-level uncertainty, we assume that firms incur forecast errors in the first compliance period but not the second compliance period.²⁰ While stylized, this assumption is necessary to ensure that the regulator’s overall quantity order is still met exactly at the end of the specified banking and borrowing horizon, which in turn allows us to apply the consistent assumption across both price and quantity instruments that the regulator’s policy target is met exactly. In real-world emissions trading markets, of course, the firm-level uncertainty that we describe here is likely to persist across all compliance periods, and we rarely observe a terminal date of a credit trading program with banking or borrowing; instead credits are generally allowed to cross over into the next phase of the program, often at an adjusted trading ratio.

19. In the case of greenhouse gases, B'' may be close to zero given linear damages (Burke, Hsiang, and Miguel 2015). This term also drops out for pure stock pollutants, modeled in the spirit of Gerlagh and Heijmans (2018); this derivation is available from the authors upon request.

20. Here we emphasize, however, that although firm-level uncertainty is restricted to the first period, the systemic forecast errors continue to create cost-inefficiencies in later periods due to the intertemporal linkages of cap-and-trade. By contrast, the cost-inefficiencies of idiosyncratic errors are limited to the period in which the uncertainty occurs. This distinction illuminates why we multiply the idiosyncratic forecast error term in Equation 14 by 1/2, but not the systemic error term.

4 Firm Uncertainty Over the Impact of Overlapping Policies

In addition to uncertainty about cost shocks, firms may also make forecast errors around the market-clearing price due to uncertainty about the impact of overlapping policies. As discussed previously, under many market-based environmental policies, a subset of regulated firms are also subject to additional standards from overlapping policies, which are often more stringent or more prescriptive than the price or quantity instrument. For firms where the prescribed technology is not the lowest-cost option for reducing emissions, the existence of this second set of standards raises the total costs for meeting the requirements of the overlapping price- or quantity-based policy.²¹ Under a price instrument, the inefficiencies due to overlapping policies are limited to the subset of firms or business units that are subject to the more prescriptive regulation. Under an allowance or credit trading program, by contrast, these cost-inefficiencies affect both firms covered by the more prescriptive regulation and non-covered firms, by affecting the market-clearing price for allowances or credits. Insofar as firms are uncertain about the impact of these overlapping policies on the allowance price, we again observe expected welfare losses due to forecast errors for a tradable quantity instrument relative to a price instrument.

To illustrate, consider the same two-period market-based policy modeled above, but now assume that a subset R firms are subject to an additional regulatory standard in each compliance period. The remaining M firms are subject only to the market-based policy (where $R+M = N$, the total number of firms in the market). Assume that the supplementary regulation causes firms to abate at the level $(q_R)_t^i$, where $\sum_{j=1}^R (q_R)_t^i - \sum_{j=1}^R \bar{q}_t^i = \kappa \geq 0$. That is, the supplementary regulation results in a total quantity of abatement from this subset of firms that is higher than their ex ante efficient quantity.

21. Insofar as there are fixed costs to participating in allowance or credit trading markets, some firms may elect to treat their allowance allocations as if they represented command-and-control regulation. This behavior may produce similar impacts as overlapping policies, where some subset of firms do not abate at the efficient level. The results described in this section would apply to this scenario as well.

Under a price regime, the M non-covered firms behave no differently, and continue to set $\tilde{q}_1^i = \bar{q}_1^i - \frac{\theta_1^i}{C_i''}$ (and analogously for $\tilde{q}_2^i$). However, under tradable quantities, the aggregate quantity target still determines overall abatement in the market, so the market-clearing price must adjust to account for the excess κ compliance produced by the firms subject to the supplementary regulation. The market-clearing price will therefore adjust to $\hat{p}_1^\kappa = \hat{p}_2^\kappa = C' + \frac{\sum_{i=1}^M \frac{\theta_1^i + \theta_2^i}{2C_i''}}{\sum_{i=1}^M \frac{1}{C_i''}} - \frac{\kappa}{\sum_{i=1}^M \frac{1}{C_i''}}$, and under perfect certainty the non-covered firms will each respond with quantity $q_1^i(\hat{p}_1^\kappa, \theta_1^i) = \frac{\hat{p}_1^\kappa - C' - \theta_1^i}{C_i''} + \bar{q}_1^i$ (and analogously for $q_2^i(\hat{p}_2^\kappa, \theta_2^i)$). When we compare the expected welfare of prices versus tradable quantities under this scenario, we obtain the following relation:

$$\Delta = E \left[\underbrace{\frac{1}{4} \left(\frac{1}{\sum_{i=1}^M \frac{1}{C_i''}} - B'' \right) \left(\sum_{i=1}^M \frac{\theta_1^i + \theta_2^i}{C_i''} \right)^2}_{\text{Standard Comparison}} + \underbrace{\left(\frac{1}{\sum_{i=1}^M \frac{1}{C_i''}} - B'' \right) \kappa^2}_{\text{Impact of Overlapping Policy}} \right]$$

As we see here, the impact of the overlapping policy on the expected welfare of a price instrument relative to a tradable quantity instrument reduces to the standard “relative slopes” comparison. The intuition is straightforward: the overlapping policy causes a subset of firms to over-comply relative to what would be most cost effective; the overcompliance for this subset of firms occurs under either a tax or a cap-and-trade program. Under a tradable quantity regime, the market-clearing price adjusts downward such that the aggregate target is still met, resulting in lower abatement among the remaining firms, relative to their cost-effective level. Under a price instrument, the remaining firms still abate at the cost-effective level, but there is excess compliance across the full set of firms. The tradeoff between these two sources of inefficiency depends on the relative slopes of the marginal benefit and marginal cost functions.

This comparison changes when we introduce the prospect of firm-level uncertainty. Some firms not subject to the supplementary regulation may not know the full extent of overcompliance among doubly regulated firms. Under a price-based policy, this uncertainty about

the impact of overlapping policies does not affect the firm's own compliance decision, so abatement quantities do not change. Under tradable quantities, however, each firm i forms some posterior expectation over the extent of excess compliance, which then informs the firm's forecast of the market-clearing price:²² The firm then chooses its optimal abatement level given its forecasted market-clearing price. Again let $\xi_1 + \epsilon_1^i$ represent the difference between the expected price and the cost-effective price, to capture systemic and idiosyncratic forecast errors, respectively. The expected welfare of a price-based policy relative to tradable allowances or credits is then given by:

$$\begin{aligned} \Delta = E & \left[\underbrace{\left(\frac{1}{\sum_{i=1}^M \frac{1}{C_i''}} - B'' \right) \left(\sum_{i=1}^M \frac{\theta_1^i + \theta_2^i}{2C_i''} \right)^2}_{\text{Standard Comparison}} + \underbrace{\left(\frac{1}{\sum_{i=1}^M \frac{1}{C_i''}} - B'' \right) \kappa^2}_{\text{Impact of Overlapping Policy}} \right. \\ & + \underbrace{\frac{1}{2} \left(\sum_{i=1}^M \frac{\epsilon_1^{i2}}{C_i''} \right) + \left(\sum_{i=1}^M \frac{\xi_1^2}{C_i''} \right)}_{\text{Cost Inefficiencies from Forecast Errors} \geq 0} + \underbrace{B'' \left(\sum_{i=1}^M \frac{\xi_1}{C_i''} \right)^2 + 2B'' \left(\sum_{i=1}^M \frac{\xi_1}{C_i''} \right) \left(\sum_{i=1}^M \frac{\theta_1^i - \theta_2^i}{2C_i''} \right)}_{\text{Additional Benefit Term} \geq \text{or} \leq 0} \Big] \end{aligned} \quad (15)$$

Mathematically, the impact of uncertainty around overlapping policies is almost identical to the results derived above for uncertainty around costs shocks; the full derivation is presented in Appendix Section A.5. Once again, we recover our initial prices versus tradable quantities result, here for the overlapping policies scenario, as well as additional terms that capture the cost inefficiencies associated with both idiosyncratic and systemic firm-level forecast errors. All else equal, these cost inefficiencies again push the regulator to prefer prices over tradable quantities. However, as before, we also have a term with ambiguous sign that depends on whether the forecast errors contribute to more or less benefit smoothing across periods, which in turn depends on the correlation between aggregate cost shocks and systemic forecast errors.

22. Here we return to assuming perfect certainty over cost shocks to isolate the impact of uncertainty over overlapping policies. It is also possible, of course, to use our model set-up to explore interactions between these two sources of uncertainty.

5 Firm Uncertainty Over Own Abatement Costs

Thus far, we have focused on several ways in which firms may face uncertainty over the market-clearing price in allowance trading markets. As we have emphasized, these uncertainties introduce asymmetric cost inefficiencies for quantity-based instruments relative to price-based instruments, since for the latter, regulators set and announce the price of pollution in advance. Yet we have ignored other potential sources of uncertainty that may interact with uncertainty over allowance prices. In practice, these other sources of uncertainty may magnify or mitigate the baseline cost inefficiencies modeled above.

To illustrate, we relax our earlier assumption that firms know their own abatement cost shocks with certainty. Under this version of the model, firms must choose their compliance strategies before they have full information about either their own abatement costs during the compliance period or the market-clearing price for allowances. For example, output prices in the energy market often influence both own abatement costs and the market-clearing price for allowances, and firms may not have full information about energy prices when determining abatement strategies (and often fail to hedge completely their exposure to these prices).

Consequently, under a quantity regime, each firm i chooses its optimal quantity of abatement given its information set $\mathcal{I}_t^i$ and resulting expectations over own costs and the market-clearing price for allowances:

$$q_1^i(\mathbb{E}[\hat{p}_1|\mathcal{I}_1^i], \mathbb{E}[\theta_1^i|\mathcal{I}_1^i]) = \bar{q}_1^i + \frac{\mathbb{E}[\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''} | \mathcal{I}_1^i]}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\mathbb{E}[\theta_1^i | \mathcal{I}_1^i]}{C_i''}$$

Likewise, under a price regime, firm i 's optimal abatement quantity is now a function of the regulated price and its expectation of its own abatement costs, given its current information set:

$$q_1^i(\tilde{p}_1, \mathbb{E}[\theta_1^i|\mathcal{I}_1^i]) = \bar{q}_1^i - \frac{\mathbb{E}[\theta_1^i | \mathcal{I}_1^i]}{C_i''}$$

For this more general model, we define the following firm-level forecast errors over own cost

shocks and the market-clearing price, respectively:

$$\text{own cost forecast errors: } \epsilon_1^i(\theta) + \xi_1(\theta) = E[\theta_1^i | \mathcal{I}_1^i] - \theta_1^i$$

$$\text{price forecast errors: } \epsilon_1^i(\hat{p}) + \xi_1(\hat{p}) = \frac{E[\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''} | \mathcal{I}_1^i]}{\sum_{j=1}^N \frac{1}{C_j''}} - \frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_{j=1}^N \frac{1}{C_j''}}$$

Therefore, we can rewrite the above quantity responses in terms of these two sets of forecast errors. Under a quantity instrument, firm i 's quantity response becomes:

$$q_1^i(E[\hat{p}_1 | \mathcal{I}_1^i], E[\theta_1^i | \mathcal{I}_1^i]) = \bar{q}_1^i + \frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i''} + \frac{\epsilon_1^i(\hat{p}) + \xi_1(\hat{p})}{C_i''} - \frac{\theta_1^i}{C_i''} - \frac{\epsilon_1^i(\theta) + \xi_1(\theta)}{C_i''}$$

Under a price instrument, firm i 's quantity response becomes:

$$q_1^i(\tilde{p}_1, E[\theta_1^i | \mathcal{I}_1^i]) = \bar{q}_1^i - \frac{\theta_1^i}{C_i''} - \frac{\epsilon_1^i(\theta) + \xi_1(\theta)}{C_i''}$$

From these expressions, it is straightforward to see that the overall impact of firm-level uncertainty on each firm's quantity response – which then directly translates into the market's overall cost-effectiveness – may be higher or lower under quantity instruments compared to price instruments. The overall effect depends on the relative magnitude of firms' uncertainty over own cost shocks versus their uncertainty over the market-clearing allowance price and whether these two sources of firm-level uncertainty tend to move in the same or opposite directions. To formalize this intuition, we present the full expression for the relative advantage of prices over quantities when firms face uncertainty over own abatement costs as well

as over allowance prices, with the full derivation in Appendix Section A.6:

$$\begin{aligned}
\Delta = & \underbrace{\frac{1}{4} \left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B'' \right) \left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''} \right)^2}_{\text{Original Expression}} + \underbrace{\frac{1}{2} \left(\sum_{i=1}^N \frac{\epsilon_1^i(\hat{p})^2}{C_i''} \right) + \left(\sum_{i=1}^N \frac{\xi_1(\hat{p})^2}{C_i''} \right) + \frac{1}{2} \left(\sum_{i=1}^N \frac{\xi_1(\theta)^2}{C_i''} \right)}_{\text{Cost Inefficiencies from Idiosyncratic and Systemic Errors} \geq 0} \\
& - \underbrace{\left(\sum_{i=1}^N \frac{\epsilon_1^i(\hat{p}) \epsilon_1^i(\theta)}{C_i''} \right)}_{\text{Correlation in Idiosyncratic Errors} \geq \text{ or } \leq 0} - \underbrace{2 \left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} + B'' \right) \left(\sum_{i=1}^N \frac{1}{C_i''} \right)^2 (\xi_1(\hat{p}) \xi_1(\theta))}_{\text{Correlation in Systemic Errors} \geq \text{ or } \leq 0} \\
& + \underbrace{B'' \left(\sum_{i=1}^N \frac{\xi_1(\hat{p})^2}{C_i''} \right) + 2B'' \left(\sum_{i=1}^N \frac{\xi_1(\hat{p})}{C_i''} \right) \left(\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} \right)}_{\text{Benefit Smoothing for Price Errors} \geq \text{ or } \leq 0} \\
& + \underbrace{\frac{1}{2} B'' \sum_{i=1}^N \left(\frac{\xi_1(\theta)^2}{C_i''} \right) - B'' \left(\sum_{i=1}^N \frac{\theta_2^i}{C_i''} \right) \left(\sum_{i=1}^N \frac{\xi_1(\theta)}{C_i''} \right)}_{\text{Benefit Smoothing for Own Cost Errors} \geq \text{ or } \leq 0}
\end{aligned} \tag{16}$$

We tackle each of these terms in succession. As in all previous derivations, we have a standard comparison of the relative slopes of the benefit function and firms' cost functions. Also as before, we have terms capturing the cost inefficiencies associated with firm uncertainty, whether over prices or own abatement costs; these cost inefficiencies push the regulator to prefer prices over tradable quantities, all else equal. Note that inefficiencies arising from own cost forecast errors drop out in the first period, as they appear across both price and quantity regimes; however, the impact of these errors persists into the second period under quantities but not prices, due to the intertemporal linkages of cap-and-trade with banking and borrowing.

Next, given the multiple sources of firm-level uncertainty affecting tradable quantities markets, we have new terms capturing the correlation between uncertainty over prices and uncertainty over own abatement costs. Whenever these two sources of uncertainty are positively correlated across firms, the regulator is pushed to prefer tradable quantities over prices, as positively correlated errors mitigate the cost inefficiencies associated with each

error individually. That is, forecast errors that cause firms to overestimate the cost-effective market-clearing price induce firms to *overabate* relative to the efficient level, while forecast errors that cause firms to overestimate their own abatement cost shocks induce them to *underabate*. Therefore, these two sources of uncertainty may be, to varying degrees, offsetting under a tradable quantities regime. By contrast, under a price regime, firms only face uncertainty over own abatement costs, so there is no opportunity for mitigating their forecast errors.

Lastly, we must now consider the impact of both types of uncertainty on benefit smoothing. For both price- and quantity-based policies, forecast errors could increase or decrease quantity variability across periods and thereby increase or decrease benefit smoothing, depending on the correlation between firms' errors and their cost shocks. Once again, however, the total effect on benefit smoothing is partially offset for forecast errors around own cost shocks, because they appear under both price and quantity regimes.

Considering all terms together, we see that interactions between firm-level uncertainty over the market-clearing price and over own abatement cost shocks add further nuance to our earlier result that firm-level uncertainty pushes regulators to prefer prices over tradable quantities on cost effectiveness grounds. In the case of greenhouse gas abatement, where the slope of the marginal benefit function is believed to be close to zero and therefore considerations around benefit smoothing are less germane, the final impact of these two sources of firm-level uncertainty depends on the extent of positive or negative correlation across firms' various forecast errors. Where forecast errors over the market-clearing price stem from uncertainty over rivals' cost shocks, it seems reasonable that these errors would be positively correlated with forecast errors around own cost shocks, which would mitigate (but not necessarily eliminate) some of the cost-effectiveness concerns in these markets. However, it is less obvious whether forecast errors around the impact of overlapping policies would be positively or negatively with forecast errors around own costs.

6 Policy Responses to Firm-Level Uncertainty

We have established that firm-level uncertainty may have important implications for the cost-effectiveness of allowance and credit trading programs, by modeling these additional uncertainties in a Weitzman-style framework that has traditionally focused on the regulator’s uncertainty. However, political economy considerations may continue to push policymakers to favor quantity instruments irrespective of any welfare advantage of price instruments (Stavins 2020). For this reason, it is worthwhile to consider what policy design tools may be available to reduce – even if not eliminate – the impacts of this firm-level uncertainty. We discuss here the potential role for price ceilings and price floors (“price collars”), as one example.

Under a tradable quantities program, a price ceiling serves as a maximum price above which the market-clearing price for allowances or credits is not allowed to rise.²³ The regulator commits to selling additional allowances whenever the price ceiling is reached, charging regulated entities this maximum price but no more. The effect is to relax the overall quantity constraint to avoid large increases in firms’ compliance costs. A price floor works analogously, where the regulator commits to buying allowances at a specified minimum price whenever needed, or specifies a reserve price in allowance auctions. Taken together, the price ceiling and price floor represent a hybrid policy between a pure quantity-based instrument and a pure price-based instrument.²⁴

In Appendix Section A.7, we derive a Weitzman-style expression for evaluating the welfare consequences of introducing a price collar in a cap-and-trade market. To establish a baseline, we first evaluate the impact of price collars in the absence of firm-level uncertainty. In general, the impact of the price collar falls into one of three cases, depending on the relationship between the price ceiling or floor, the cost-effective price given the aggregate quantity order

23. In the model that follows, we focus on a “hard” price ceiling where the regulator commits to maintaining a certain maximum price, as opposed to a “soft” price, where the regulator commits only to releasing a finite allowance reserve if the market-clearing price rises above some level.

24. Roberts and Spence (1976) and Weitzman (1978) have demonstrated that this type of hybrid policy may be welfare-enhancing relative to a pure quantity or price instrument.

(emission cap), and the ex post optimal price (first-best price) given the realization of shocks in the market. First, whenever the slope of the marginal cost function is steeper than the slope of the marginal benefit function, the price collar is welfare-improving relative to an unconstrained cap-and-trade program. In this case, a price-based policy is preferred to a quantity-based policy, but may not be possible for political economy or other reasons; a price collar recreates some characteristics of a fixed price policy and therefore increases welfare under the tradable quantities regime. Second, whenever the price ceiling (floor) is higher (lower) than the ex post optimal price, the deadweight loss under the price ceiling (floor) is (weakly) smaller than if the price is permitted to float under the cap-and-trade program. Finally, when the price ceiling (floor) is lower (higher) than the ex post optimal price, the impact on deadweight loss is ambiguous and depends on the relative magnitude of the difference between the ex post optimal price and the price ceiling (floor) versus the difference between the optimal price and the cost-effective price from allowance trading. We illustrate these three scenarios for a market with a representative firm in Appendix Figure 1.

When we account for firm-level uncertainty, the welfare implications of introducing a price collar also depend on how this policy design affects firms' information sets as they make their compliance decisions. A price collar announced in advance means that firms know *ex ante* that the market-clearing price will not rise above the price ceiling or fall below the price floor; this additional information then affects the magnitude of firms' forecast errors. In the Appendix, we extend our Weitzman-style expression for the welfare consequences of introducing price collars by also incorporating firm-level uncertainty over competitors' cost shocks. In this more general welfare expression, we obtain additional terms capturing the difference in the variance of firms' forecast errors with and without the price collar, as well as additional terms capturing changes in benefit smoothing across periods with and without the price collar. Without placing additional structure on the correlation of firms' cost shocks, we cannot determine exactly how the variance of firm forecast errors changes

under a price collar. However, we might expect that the variance would be lower under a price collar, as firms have greater certainty over the market-clearing price when the extremes of the price distribution are truncated. As a consequence of reducing the variance of forecast errors, the cost inefficiencies from firm-level uncertainty would also diminish. These potential reductions in cost inefficiencies should therefore be included in standard evaluations of price collar mechanisms.

7 Conclusion

In this paper, we examine the impact of firm-level uncertainty over allowance prices in cap-and-trade and tradable credit markets – a form of residual uncertainty which is inherent to this type of policy instrument. Motivated by numerous empirical illustrations of cap-and-trade and tradable quantity markets deviating from cost-effectiveness, we develop a theory model that elucidates the welfare implications of instrument choice with allowance and credit price forecast errors. Building on the Weitzman prices-versus-quantities framework, we highlight the additional costs associated with imperfect information about future market-clearing prices when modeling cap-and-trade as an “aggregate quantity order” that allocates quantities in a decentralized manner to firms. All else equal, the cost inefficiencies created by firm forecast errors over the cost-effective price would encourage the welfare-maximizing regulator to favor price instruments over tradable quantity instruments. These cost inefficiencies may be partly offset, however, when firms incur additional forecast errors around their own abatement costs when determining compliance strategies, and those errors around own costs are positively correlated with errors around the allowance price.

We focus our model on a uniformly-mixed pollutant. While a special case, it addresses carbon dioxide and other greenhouse gas emissions, which represent the most pressing environmental policy challenge of the 21st century – and one that drew considerable attention from Marty Weitzman in his scholarship over the last several decades of his career. Given the

scale of investment necessary to combat climate change – on the order of trillions of dollars in the coming decades – and the growing policy interest around the world in market-based instruments in abating greenhouse gas emissions, our work highlights another dimension of the instrument choice problem in promoting cost-effective and economically efficient emission reductions. We also show how variations in instrument design, such as price collars, may mitigate some, but not all, of the welfare costs associated with the residual uncertainty in tradable quantity instruments. We also discuss how intertemporal benefit smoothing for flow pollutants (or non-uniformly mixed pollutants) may also affect the regulator’s choice of instrument.

Examining the empirical interactions between firm-level forecast errors, uncertainty over future abatement costs, and other shocks to pollution markets represents a fruitful direction for research. Many economists responded to the insights of the standard prices-versus-quantities framework by exploring its empirical implications, such as the estimation of the damage and abatement cost functions for a variety of pollutants, including carbon dioxide (e.g., Pizer (2002)). In our previous work, we simulated the impacts of forecast errors on a hypothetical U.S. cap-and-trade market, and estimated that it would increase compliance costs 21 percent relative to an otherwise equivalent tax (Aldy and Armitage 2020). Further examination of the economic fundamentals driving allowance and credit prices, and rigorous evaluations of cost-effectiveness anomalies in these markets could further enhance understanding of instrument choice and design. In future research, we plan to build on the static abatement decisions analyzed here, to understand the impact of these various shocks on long-lived firm investment, including R&D.

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A Appendix to “The Welfare Implications of Carbon Price Certainty”

A.1 Price Instrument

First, we derive a variant of Weitzman’s 1974 and 2018 results with multiple production units and two compliance periods, when the regulator sets a single price order. The problem set-up is given in the main text. The regulator’s optimization problem is given by:

$$\max_{\tilde{p}_1, \tilde{p}_2} E[B_1(\sum_{i=1}^N q_1^i(\tilde{p}_1, \theta_1^i)) - \sum_{i=1}^N C_1^i(q_1^i(\tilde{p}_1, \theta_1^i), \theta_1^i) + B_2(\sum_{i=1}^N q_2^i(\tilde{p}_2, \theta_2^i)) - \sum_{i=1}^N C_2^i(q_2^i(\tilde{p}_2, \theta_2^i), \theta_2^i)]$$

which yields the following first-order conditions:

$$\begin{aligned} E[\sum_{i=1}^N \frac{\partial B_1}{\partial Q_1} \cdot \frac{\partial Q_1}{\partial q_1^i} \cdot \frac{dq_1^i}{d\tilde{p}_1}] &= E[\sum_{i=1}^N \frac{\partial C_1^i}{\partial q_1^i} \cdot \frac{dq_1^i}{d\tilde{p}_1}] \\ E[\sum_{i=1}^N \frac{\partial B_2}{\partial Q_2} \cdot \frac{\partial Q_2}{\partial q_2^i} \cdot \frac{dq_2^i}{d\tilde{p}_2}] &= E[\sum_{i=1}^N \frac{\partial C_2^i}{\partial q_2^i} \cdot \frac{dq_2^i}{d\tilde{p}_2}] \end{aligned}$$

As in Weitzman, we assume that firms set marginal cost equal to price, which yields the following response function for firm i facing price p_t :

$$\frac{\partial C_t^i}{\partial q_t^i} = p_t = C' + \theta_t^i + C_i''(q_t^i - \bar{q}_t^i)$$

Rearranging gives:

$$q_t^i(p_t, \theta_t^i) = q_t^i = \frac{p_t - C' - \theta_t^i}{C_i''} + \bar{q}_t^i$$

Differentiating with respect to p_t gives:

$$\frac{dq_t^i}{dp_t} = \frac{1}{C_i''}$$

Substituting this result into the regulator’s first-order condition for the optimal price order and recognizing that all firms set $\frac{\partial C_t^i}{\partial q_t^i} = \tilde{p}_t$, we have:

$$\begin{aligned} \frac{\partial B_1}{\partial Q_1} \cdot \sum_{i=1}^N \frac{1}{C_i''} &= \tilde{p}_1 \sum_{i=1}^N \frac{1}{C_i''} \\ \frac{\partial B_2}{\partial Q_2} \cdot \sum_{i=1}^N \frac{1}{C_i''} &= \tilde{p}_2 \sum_{i=1}^N \frac{1}{C_i''} \end{aligned}$$

Therefore, the optimal price order is given by $\tilde{p}_1 = B'$ and $\tilde{p}_2 = B'$. Using the same steps as Weitzman (1974), we can then show that $C' = \tilde{p}_1 = \tilde{p}_2$. Therefore, plugging this optimal price order into our price response function, we arrive at the same quantity response as the original Weitzman derivation:

$$q_t^i(\tilde{p}_t, \theta_t^i) = \tilde{q}_t^i = \frac{-\theta_t^i}{C_i''} + \bar{q}_t^i$$

A.2 Quantity Instrument

To derive the outcomes a single quantity order, we retain the set-up given in the main text. Now the optimal aggregate quantity order is given by the solution to the following optimization problem facing the regulator:

$$\begin{aligned} \max_{\hat{Q}} E[B_1(\sum_{i=1}^N q_1^i(\hat{p}_1(\hat{Q}, \boldsymbol{\theta}), \theta_1^i)) - \sum_{i=1}^N C_1^i(q_1^i(\hat{p}_1(\hat{Q}, \boldsymbol{\theta}), \theta_1^i), \theta_1^i) \\ + B_2(\sum_{i=1}^N q_2^i(\hat{p}_2(\hat{Q}, \boldsymbol{\theta}), \theta_2^i)) - \sum_{i=1}^N C_2^i(q_2^i(\hat{p}_2(\hat{Q}, \boldsymbol{\theta}), \theta_2^i), \theta_2^i)] \end{aligned}$$

which yields the first-order condition:

$$\begin{aligned} E[\sum_{i=1}^N \frac{\partial B_1}{\partial Q_1} \cdot \frac{\partial Q_1}{\partial q_1^i} \cdot \frac{\partial q_1^i}{\partial \hat{p}_1} \cdot \frac{d\hat{p}_1}{d\hat{Q}} + \sum_{i=1}^N \frac{\partial B_2}{\partial Q_2} \cdot \frac{\partial Q_2}{\partial q_2^i} \cdot \frac{\partial q_2^i}{\partial \hat{p}_2} \cdot \frac{d\hat{p}_2}{d\hat{Q}}] \\ = E[\sum_{i=1}^N \frac{\partial C_1^i}{\partial q_1^i} \cdot \frac{\partial q_1^i}{\partial \hat{p}_1} \cdot \frac{d\hat{p}_1}{d\hat{Q}} + \sum_{i=1}^N \frac{\partial C_2^i}{\partial q_2^i} \cdot \frac{\partial q_2^i}{\partial \hat{p}_2} \cdot \frac{d\hat{p}_2}{d\hat{Q}}] \end{aligned}$$

To simplify the first-order condition, note first that we still have $\frac{\partial q_t^i}{\partial p_t} = \frac{1}{C_i''}$, a constant. We can also use the price response equation from above to derive $\frac{d\hat{p}_t}{d\hat{Q}}$ as follows:

$$\sum_{i=1}^N q_t^i(p_t, \theta_t^i) = \sum_{i=1}^N \frac{p_t - C' - \theta_t^i}{C_i''} + \bar{q}_t^i \Rightarrow Q_t = \sum_{i=1}^N \frac{p_t - C' - \theta_t^i}{C_i''} + \bar{Q}_t$$

Therefore, aggregate quantity is given by:

$$Q = \sum_{i=1}^N \frac{p_1 - C' - \theta_1^i}{C_i''} + \bar{Q}_1 + \sum_{i=1}^N \frac{p_2 - C' - \theta_2^i}{C_i''} + \bar{Q}_2$$

Differentiating with respect to Q gives:

$$1 = \sum_i \frac{1}{C_i''} \frac{dp_1}{dQ} + \frac{1}{C_i''} \frac{dp_2}{dQ} \Rightarrow \frac{dp_1}{dQ} + \frac{dp_2}{dQ} = 1 / (\sum_i \frac{1}{C_i''})$$

From the regulator's perspective at the time of setting the aggregate quantity limit, the no-arbitrage condition requires $p_1 = p_2$. Therefore, we have $\frac{dp_1}{dQ} = \frac{dp_2}{dQ} = 1 / (2 \sum_{i=1}^N \frac{1}{C_i''})$. Note that this expression is also a constant that can be pulled outside the expectation operator. Therefore, we can substitute these terms into the expression above and invoke the linearity of expectation to rewrite the regulator's first-order condition as follows:

$$\begin{aligned} \sum_{i=1}^N B' \cdot \frac{1}{C_i''} \cdot \frac{1}{2 \sum_j \frac{1}{C_j''}} + \sum_{i=1}^N B' \cdot \frac{1}{C_i''} \cdot \frac{1}{2 \sum_j \frac{1}{C_j''}} \\ = \sum_{i=1}^N E[\frac{\partial C_1^i}{\partial q_1^i}] \cdot \frac{1}{C_i''} \cdot \frac{1}{2 \sum_j \frac{1}{C_j''}} + \sum_{i=1}^N E[\frac{\partial C_2^i}{\partial q_1^i}] \cdot \frac{1}{C_i''} \cdot \frac{1}{2 \sum_j \frac{1}{C_j''}} \end{aligned}$$

Recognizing that each $\frac{\partial C_t^i}{\partial q_t^i}$ is set equal to $\hat{p}_t(\hat{Q}, \theta_t, \theta_{t'})$ by cost minimization and that $E[\hat{p}_1(\hat{Q}, \theta_1, \theta_2)] = E[\hat{p}_2(\hat{Q}, \theta_1, \theta_2)]$ from the perspective of the regulator deciding on optimal policy, we can rewrite this expression as:

$$\sum_{i=1}^N B' \cdot \frac{1}{C_i'''} \cdot \frac{1}{\sum_j \frac{1}{C_j''}} = \sum_{i=1}^N E[\hat{p}(\hat{Q}, \theta_1, \theta_2)] \cdot \frac{1}{C_i'''} \cdot \frac{1}{\sum_j \frac{1}{C_j''}}$$

Which ultimately yields (for the regulator's optimal aggregate quantity order $\hat{Q}$):

$$B' = E[\hat{p}(\hat{Q}, \theta_1, \theta_2)]$$

Note that we have defined $\bar{q}_t^i$ such that $E[\frac{\partial B_t}{\partial q_t^i}] = E[\frac{\partial C_t^i}{\partial q_t^i}]$. Therefore, the regulator's first-order condition for Q is satisfied when $\hat{Q} = \sum_{i=1}^N \bar{q}_1^i + \sum_{i=1}^N \bar{q}_2^i$.

Nonetheless, the realized cost-effective price resulting from the optimal quantity instrument is not necessarily equal to B' or C' , but instead depends of the realizations of cost shocks θ_1 and θ_2 . As in the initial derivation in the main body of the paper, we assume that firms initially have perfect information about first- and second-period cost shocks. Therefore, the first-period market-clearing price is obtained from the following relationship:

$$\hat{Q}_1 + \hat{Q}_2 = \bar{Q}_1 + \sum_{i=1}^N \frac{\hat{p}_1 - C' - \theta_1^i}{C_i'''} + \bar{Q}_2 + \sum_{i=1}^N \frac{\hat{p}_2 - C' - \theta_2^i}{C_i''}$$

We apply the two key conditions governing this market: one, the aggregate quantity limit must be met ($\hat{Q}_1 + \hat{Q}_2 = \hat{Q} = \bar{Q}_1 + \bar{Q}_2$), and two, the no-arbitrage condition requires that the first-period market-clearing price is equal to the (expected) second-period market-clearing price ($\hat{p}_1 = \hat{p}_2$). Imposing these conditions and rearranging terms then yields the efficient market-clearing price under a multi-period quantity instrument with banking and borrowing:

$$\hat{p}_1(\hat{Q}, \theta_1, \theta_2) = \hat{p}_2(\hat{Q}, \theta_1, \theta_2) = C' + \frac{\sum_i \frac{\theta_1^i + \theta_2^i}{2C_i''}}{\sum_i \frac{1}{C_i''}}$$

Plugging this expression for $\hat{p}_t(\hat{Q}, \theta_1, \theta_2)$ into each price response function, we see that the individual quantities $\hat{q}_t^i$ realized as a consequence of the optimal quantity order will generally not be equal to the *ex ante* optimal quantities $\bar{q}_t^i$. In the first period:

$$\hat{q}_1^i = q_1^i(\hat{p}_1, \theta_1^i) = \frac{\hat{p}_1 - C' - \theta_1^i}{C_i'''} + \bar{q}_1^i = \frac{C' + \frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - C' - \theta_1^i}{C_i'''} + \bar{q}_1^i = \frac{\frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_1^i}{C_i'''} + \bar{q}_1^i$$

Likewise, in the second period:

$$\hat{q}_2^i = q_2^i(\hat{p}_2, \theta_2^i) = \frac{\hat{p}_2 - C' - \theta_2^i}{C_i'''} + \bar{q}_2^i = \frac{C' + \frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - C' - \theta_2^i}{C_i'''} + \bar{q}_2^i = \frac{\frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_2^i}{C_i'''} + \bar{q}_2^i$$

By our construction of $\bar{q}_t^i$, we have imposed that C' is constant for all i and all t . Since we generally do not have $\theta_t^i = \theta_t^j$ or $\theta_t^i = \theta_{t'}^i$, this expression does not reduce to $\bar{q}_t^i$ except under very special conditions.

A.3 Relative Advantage of Prices Over Quantities with Banking and Borrowing

Here we derive the relative advantage of prices over quantities with banking and borrowing, relying on the baseline assumption from Weitzman (2020) that firms have perfect information about both first- and second-period cost shocks before making any compliance decisions. We substitute our expressions for $q_t^i(\hat{p}_t, \theta_t^i)$ and $q_t^i(\tilde{p}_t, \theta_t^i)$ into the Taylor expansions for (expected) benefits and costs. For the expected benefits of the quantity order, we obtain:

$$\begin{aligned}
& \mathbb{E}[B_1(\sum_{i=1}^N \hat{q}_1^i) + B_2(\sum_{i=1}^N \hat{q}_2^i)] \\
&= \mathbb{E}[b + B'(\sum_{i=1}^N \hat{q}_1^i - \bar{q}_1^i) + \frac{-B''}{2}(\sum_{i=1}^N \hat{q}_1^i - \bar{q}_1^i)^2] + \mathbb{E}[b + B'(\sum_{i=1}^N \hat{q}_2^i - \bar{q}_2^i) + \frac{-B''}{2}(\sum_{i=1}^N \hat{q}_2^i - \bar{q}_2^i)^2] \\
&= \mathbb{E}[b + B'(\sum_{i=1}^N \frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_1^i) + \frac{-B''}{2}(\sum_{i=1}^N \frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_2^i)^2] \\
&+ \mathbb{E}[b + B'(\sum_{i=1}^N \frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_1^i) + \frac{-B''}{2}(\sum_{i=1}^N \frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_2^i)] \\
&= \mathbb{E}[2b - B''(\sum_{i=1}^N \frac{\theta_1^i - \theta_2^i}{2C_i''})^2]
\end{aligned}$$

Expected costs from the quantity order:

$$\begin{aligned}
& \sum_{i=1}^N \mathbb{E}[C_1^i(\hat{q}_1^i, \theta_1^i)] + \sum_{i=1}^N \mathbb{E}[C_2^i(\hat{q}_2^i, \theta_2^i)] \\
&= \sum_{i=1}^N \mathbb{E}[a_i(\theta_1^i) + (C' + \theta_1^i)(\hat{q}_1^i - \bar{q}_1^i) + \frac{C_i''}{2}(\hat{q}_1^i - \bar{q}_1^i)^2] \\
&+ \sum_{i=1}^N \mathbb{E}[a_i(\theta_2^i) + (C' + \theta_2^i)(\hat{q}_2^i - \bar{q}_2^i) + \frac{C_i''}{2}(\hat{q}_2^i - \bar{q}_2^i)^2] \\
&= \sum_{i=1}^N \mathbb{E}[a_i(\theta_1^i) + \theta_1^i \left(\frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_1^i \right) + \frac{C_i''}{2} \left(\frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_1^i \right)^2] \\
&+ \sum_{i=1}^N \mathbb{E}[a_i(\theta_2^i) + \theta_2^i \left(\frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_2^i \right) + \frac{C_i''}{2} \left(\frac{\sum_j \frac{\theta_1^j + \theta_2^j}{2C_j''}}{\sum_j \frac{1}{C_j''}} - \theta_2^i \right)^2] \\
&= \mathbb{E} \left[-\frac{1}{2} \left(\sum_{i=1}^N \frac{\theta_1^{i2}}{C_i''} \right) - \frac{1}{2} \left(\sum_{i=1}^N \frac{\theta_2^{i2}}{C_i''} \right) + \frac{1}{4} \left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} \right) \left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''} \right)^2 \right]
\end{aligned}$$

Expected benefits from the price order:

$$\begin{aligned}
& \mathbb{E}[B_1(\sum_{i=1}^N \tilde{q}_1^i) + B_2(\sum_{i=1}^N \tilde{q}_2^i)] \\
&= \mathbb{E}[b + B'(\sum_{i=1}^N \tilde{q}_1^i - \bar{q}_1^i) + \frac{-B''}{2}(\sum_{i=1}^N \tilde{q}_1^i - \bar{q}_1^i)^2] + \mathbb{E}[b + B'(\sum_{i=1}^N \tilde{q}_2^i - \bar{q}_2^i) + \frac{-B''}{2}(\sum_{i=1}^N \tilde{q}_2^i - \bar{q}_2^i)^2] \\
&= \mathbb{E}[2b + \frac{-B''}{2}(\sum_{i=1}^N \frac{-\theta_1^i}{C_i''})^2 + \frac{-B''}{2}(\sum_{i=1}^N \frac{-\theta_2^i}{C_i''})^2]
\end{aligned}$$

Expected costs from the price order:

$$\begin{aligned}
& \sum_{i=1}^N \mathbb{E}[C_1^i(\tilde{q}_1^i, \theta_1^i)] + \sum_{i=1}^N \mathbb{E}[C_2^i(\tilde{q}_2^i, \theta_2^i)] \\
&= \mathbb{E}[a_i(\theta_1^i) + (C' + \theta_1^i)(\tilde{q}_1^i - \bar{q}_1^i) + \frac{C_i''}{2}(\tilde{q}_1^i - \bar{q}_1^i)^2] \\
&+ \mathbb{E}[a_i(\theta_2^i) + (C' + \theta_2^i)(\tilde{q}_2^i - \bar{q}_2^i) + \frac{C_i''}{2}(\tilde{q}_2^i - \bar{q}_2^i)^2] \\
&= \sum_{i=1}^N \mathbb{E}[a_i(\theta_1^i) + \theta_1^i \left(\frac{-\theta_1^i}{C_i''} \right) + \frac{C_i''}{2} \left(\frac{-\theta_1^i}{C_i''} \right)^2] + \sum_{i=1}^N \mathbb{E}[a_i(\theta_2^i) + \theta_2^i \left(\frac{-\theta_2^i}{C_i''} \right) + \frac{C_i''}{2} \left(\frac{-\theta_2^i}{C_i''} \right)^2] \\
&= \mathbb{E} \left[\sum_{i=1}^N -\frac{\theta_1^{i2}}{2C_i''} + \sum_{i=1}^N -\frac{\theta_2^{i2}}{2C_i''} \right]
\end{aligned}$$

Combining all terms to form the relative advantage of prices over quantities yields:

$$\Delta = E[\frac{1}{4}(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B'')(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''})^2]$$

This expression matches Equation 9 in the main text.

A.4 Relative Advantage of Prices Over Quantities with Uncertainty Over Rivals' Cost Shocks

We now relax the assumption that firms have perfect certainty over all marginal cost shocks before making any abatement decisions. Suppose that firm i develops a posterior expectation about cost shocks in the market, given its realized cost shock in the current period and industry-wide common priors over the joint distribution of cost shocks in the industry. Given this posterior expectation, firm i then develops a forecast of the market-clearing price and optimizes with respect to this expected price.

Let $E[\hat{p}_1(\hat{Q}, \theta_1, \theta_2)|\theta_1^i] = \hat{p}_1(\hat{Q}, \theta_1, \theta_2) + \xi_1 + \epsilon_1^i$, where $\xi_1 + \epsilon_1^i$ represents firm i 's forecast error over the market-clearing price. Specifically, ξ_1 represents the “systemic” component of this overall forecast error and ϵ_1^i represents the “idiosyncratic” component. We construct these two components of the forecast error such that $\sum_{i=1}^N \frac{\epsilon_1^i}{C_i''} = 0$ and ξ_1 is the average forecast error across all firms.

Given the market-clearing price $\hat{p}_1$ and the associated forecast error, firm i 's quantity response in the first period is given by:

$$q_1^i(\hat{p}_1 + \xi_1 + \epsilon_1^i, \theta_1^i) = \frac{\hat{p}_1 + \xi_1 + \epsilon_1^i - C' - \theta_1^i}{C_i''} + \bar{q}_1^i$$

where $\hat{p}_1$ is the cost-effective price defined above (again suppressing the dependence on $\hat{Q}$, θ_1 , and θ_2 to simplify notation).

The overall quantity produced in the first period is now given by:

$$Q_1 = \frac{\hat{Q}}{2} + \sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} + \sum_{i=1}^N \frac{\xi_1}{C_i''}$$

As discussed in the main text, the second-period price must now adjust to ensure that the aggregate quantity limit is still met, given this adjustment to first-period production. The new market-clearing price in the second period is now given by:

$$\hat{p}_2'(\hat{Q}, \theta_1, \theta_2, \xi_1) = C' + \frac{\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} - \sum_{i=1}^N \frac{\xi_1}{C_i''}}{\sum_{i=1}^N \frac{1}{C_i''}}$$

This market-clearing price then yields the following overall quantity in the second compliance period:

$$Q_2 = \frac{\hat{Q}}{2} + \sum_{i=1}^N \frac{\theta_1^i - \theta_2^i}{2C_i''} - \sum_{i=1}^N \frac{\xi_1}{C_i''}$$

Consequently, expected benefits (over both compliance periods) from the quantity order are now given by:¹

$$E[2b - B''(\sum_{i=1}^N \frac{\theta_1^i - \theta_2^i}{2C_i''} - \sum_{i=1}^N \frac{\xi_1}{C_i''})^2]$$

First-period expected costs from the quantity order are now given by:

$$E[-\frac{1}{2}(\sum_{i=1}^N \frac{\theta_1^{i2}}{C_i''}) + \frac{1}{2}(\frac{1}{4\sum_{i=1}^N \frac{1}{C_i''}})(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''})^2 + \sum_{i=1}^N \frac{\epsilon_1^{i2}}{2C_i''} \\ + \sum_{i=1}^N \frac{\xi_1^2}{2C_i''} + (\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}})(\sum_{i=1}^N \frac{\xi_1}{C_i''})(\sum_{i=1}^N \frac{\theta_1^i}{2C_i''}) + (\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}})(\sum_{i=1}^N \frac{\xi_1}{C_i''})(\sum_{i=1}^N \frac{\theta_2^i}{2C_i''})]$$

Second-period expected costs are given by:

$$E[-\frac{1}{2}(\sum_{i=1}^N \frac{\theta_2^{i2}}{C_i''}) + \frac{1}{2}(\frac{1}{4\sum_{i=1}^N \frac{1}{C_i''}})(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''})^2 \\ + \sum_{i=1}^N \frac{\xi_1^2}{2C_i''} - (\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}})(\sum_{i=1}^N \frac{\xi_1}{C_i''})(\sum_{i=1}^N \frac{\theta_1^i}{2C_i''}) - (\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}})(\sum_{i=1}^N \frac{\xi_1}{C_i''})(\sum_{i=1}^N \frac{\theta_2^i}{2C_i''})]$$

Combining terms and rearranging yields the following modified expression for the relative advantage of prices over quantities with banking and borrowing, where firms are subject to forecast errors and market-level information is revealed over time:

$$\Delta = E[\frac{1}{4}(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B'')(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''})^2 + \frac{1}{2} \sum_i \frac{\epsilon_1^{i2}}{C_i''} \\ + \sum_{i=1}^N \frac{\xi_1^2}{C_i''} + 2B''(\sum_{i=1}^N \frac{\xi_1}{C_i''})(\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''}) + B''(\frac{\xi_1}{C_i''})^2]$$

This expression is Equation 14 in the main text.

A.5 Relative Advantage of Prices Over Quantities with Overlapping Policies

We first derive the welfare comparison between prices and tradable quantities with overlapping policies in a setting without firm-level forecast errors. As described in the main text, some subset of R firms are subject to both the overlapping regulation and the market-based policy, abating at level $(q_R)_t^i$; without loss of generality, we denote the total abatement costs of this subset of firms by C^R . For simplicity, we assume that the impact of the overlapping policies is the same in both periods of our two-period model, though this assumption could be easily relaxed.

1. We could equivalently write this expression as: $E[2b - B''(\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} + \sum_{i=1}^N \frac{\xi_1}{C_i''})^2]$.

For the M firms that are only subject to the market-based policy, firm behavior is unchanged in the presence of overlapping policies under a price regime, with each firm setting $q_1^i = \frac{-\theta_1^i}{C_i''}$. The total expected cost associated with this subset of firms is therefore given by:

$$\begin{aligned} & E\left[\sum_{i=1}^M a_i(\theta_1^i) + \sum_{i=1}^M (C' + \theta_1^i)\left(\frac{-\theta_1^i}{C_i''}\right) + \sum_{i=1}^M \frac{C_i''}{2}\left(\frac{-\theta_1^i}{C_i''}\right)^2\right. \\ & \quad \left. + \sum_{i=1}^M a_i(\theta_2^i) + \sum_{i=1}^M (C' + \theta_2^i)\left(\frac{-\theta_2^i}{C_i''}\right) + \sum_{i=1}^M \frac{C_i''}{2}\left(\frac{-\theta_2^i}{C_i''}\right)^2\right] \end{aligned}$$

By contrast, the total quantity produced by these M firms does change in the presence of the overlapping policy under tradable quantities. In this case, the market-clearing price will fall in response to the over-compliance among the doubly regulated firms, and the quantity produced by the M firms will decrease correspondingly, thereby ensuring that the aggregate quantity target is still met. The market clearing price becomes:

$$\hat{p}_1^\kappa = \hat{p}_2^\kappa = C' + \frac{\sum_{i=1}^M \frac{\theta_1^i + \theta_2^i}{2C_i''}}{\sum_{i=1}^M \frac{1}{C_i''}} - \frac{\kappa}{\sum_{i=1}^M \frac{1}{C_i''}}$$

The corresponding first-period quantity response by firm i is therefore $q_1^i(p_1^\kappa, \theta_1^i) = \bar{q}_1^i + \frac{\hat{p}_1^\kappa - C' - \theta_1^i}{C_i''}$ (and analogously in the second period). The total expected costs for the M firms subject to only the market-based regulation is given by:

$$\begin{aligned} & E\left[\sum_{i=1}^M a_i(\theta_1^i) + \sum_{i=1}^M (C' + \theta_1^i)\left(\frac{\sum_{j=1}^M \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\kappa}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\theta_1^i}{C_i''}\right)\right. \\ & \quad \left. + \sum_{i=1}^M \frac{C_i''}{2}\left(\frac{\sum_{j=1}^M \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\kappa}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\theta_1^i}{C_i''}\right)^2\right] \\ & E\left[\sum_{i=1}^M a_i(\theta_2^i) + \sum_{i=1}^M (C' + \theta_2^i)\left(\frac{\sum_{j=1}^M \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\kappa}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\theta_2^i}{C_i''}\right)\right. \\ & \quad \left. + \sum_{i=1}^M \frac{C_i''}{2}\left(\frac{\sum_{j=1}^M \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\kappa}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\theta_2^i}{C_i''}\right)^2\right] \end{aligned}$$

Total expected benefits depend on the quantity produced in the market as a whole in each period, including those firms subject to both regulations and those subject to only the market-based policy. In the case of a price-based policy, these expected benefits are given

by:

$$\begin{aligned}
& E[b + B'(\sum_{i=1}^M \frac{-\theta_1^i}{C_i'''} + \sum_{j=1}^R ((q_R)_1^i - \bar{q}_1^i)) - \frac{B''}{2}(\sum_{i=1}^M \frac{-\theta_1^i}{C_i'''} + \sum_{j=1}^R ((q_R)_1^i - \bar{q}_1^i)^2)] \\
& E[b + B'(\sum_{i=1}^M \frac{-\theta_2^i}{C_i'''} + \sum_{j=1}^R ((q_R)_2^i - \bar{q}_2^i)) - \frac{B''}{2}(\sum_{i=1}^M \frac{-\theta_2^i}{C_i'''} + \sum_{j=1}^R ((q_R)_2^i - \bar{q}_2^i)^2)] \\
& = E[2b + B'(\sum_{i=1}^M \frac{-\theta_1^i}{C_i'''} + \sum_{i=1}^M \frac{-\theta_2^i}{C_i'''} + 2\kappa) - \frac{B''}{2}(\sum_{i=1}^M \frac{-\theta_1^i}{C_i'''} + \kappa)^2 - \frac{B''}{2}(\sum_{i=1}^M \frac{-\theta_2^i}{C_i'''} + \kappa)^2]
\end{aligned}$$

Under the tradable quantities policy, expected benefits are given by:

$$\begin{aligned}
& E[b + B'(\sum_{i=1}^M \frac{\theta_2^i - \theta_1^i}{2C_i'''} - \kappa + \kappa) - \frac{B''}{2}(\sum_{i=1}^M \frac{\theta_2^i - \theta_1^i}{2C_i'''} - \kappa + \kappa)^2] \\
& b + B'(\sum_{i=1}^M \frac{\theta_1^i - \theta_2^i}{2C_i'''} - \kappa + \kappa) - \frac{B''}{2}(\sum_{i=1}^M \frac{\theta_1^i - \theta_2^i}{2C_i'''} - \kappa + \kappa)^2] \\
& = E[2b - B''(\sum_{i=1}^M \frac{\theta_1^i - \theta_2^i}{2C_i'''})^2]
\end{aligned}$$

Combining all of these terms yields the expression presented in the text for the relative advantage of prices over quantities in the presence of overlapping policies:

$$\begin{aligned}
\Delta & = E[-\frac{B''}{2}(\sum_{i=1}^M \frac{-\theta_1^i}{C_i'''} + \kappa)^2 - \frac{B''}{2}(\sum_{i=1}^M \frac{-\theta_2^i}{C_i'''} + \kappa)^2 + B''(\sum_{i=1}^N \frac{\theta_1^i - \theta_2^i}{2C_i'''})^2] \\
& + \frac{1}{2} \sum_{i=1}^M \frac{\theta_1^{i2}}{C_i'''} + \frac{1}{2} \sum_{i=1}^M \frac{\theta_2^{i2}}{C_i'''} - \frac{1}{2} \sum_{i=1}^M \frac{\theta_1^{i2}}{C_i'''} - \frac{1}{2} \sum_{i=1}^M \frac{\theta_2^{i2}}{C_i'''} + (\frac{1}{\sum_{i=1}^M \frac{1}{C_i'''}}) \kappa^2 + (\frac{1}{\sum_{i=1}^M \frac{1}{C_i'''}}) (\sum_{i=1}^M \frac{\theta_1^i + \theta_2^i}{2C_i'''})^2] \\
& = E[(\frac{1}{4} (\frac{1}{\sum_{i=1}^M \frac{1}{C_i'''}} - B'') (\sum_{i=1}^M \frac{\theta_1^i + \theta_2^i}{C_i'''})^2 + (\frac{1}{\sum_{i=1}^M \frac{1}{C_i'''}} - B'') \kappa^2]
\end{aligned}$$

Next we introduce forecast errors around the impact of the overlapping policy in the market for allowances. The M firms subject to only the market-based regulation expect the impact of over-compliance on the equilibrium price to be $E[\frac{\kappa}{\sum_{i=1}^M \frac{1}{C_i''}} | \mathcal{I}_t^i] = \frac{\kappa}{\sum_{i=1}^M \frac{1}{C_i''}} + \xi_1 + \epsilon_1^i$, where $\mathcal{I}_t^i$ represents the information set of firm i in period t , for each $i \in M$. Consequently, their expectation of the market-price for allowances is given by: $E[\hat{p}_t^\kappa | \mathcal{I}_t^i] = C' + \frac{\sum_{i=1}^M \frac{\theta_1^i + \theta_2^i}{2C_i''}}{\sum_{i=1}^M \frac{1}{C_i''}} - \frac{\kappa}{\sum_{i=1}^M \frac{1}{C_i''}} - \xi_1 - \epsilon_1^i$.

As noted in the main text, we construct the idiosyncratic forecast errors ϵ_1^i such that $\sum_{i=1}^M \frac{\epsilon_1^i}{C_i''} = 0$. Therefore, expected benefits under the tradable quantities regime are now

given by:

$$\begin{aligned} & E[b + B'(\sum_{i=1}^M \frac{\theta_2^i - \theta_1^i}{2C_i''} - \kappa - \sum_{i=1}^M \frac{\xi_1}{C_i''} + \kappa) - \frac{B''}{2}(\sum_{i=1}^M \frac{\theta_2^i - \theta_1^i}{2C_i''} - \kappa - \sum_{i=1}^M \frac{\xi_1}{C_i''} + \kappa)^2 \\ & b + B'(\sum_{i=1}^M \frac{\theta_1^i - \theta_2^i}{2C_i''} - \kappa + \sum_{i=1}^M \frac{\xi_1}{C_i''} + \kappa) - \frac{B''}{2}(\sum_{i=1}^M \frac{\theta_1^i - \theta_2^i}{2C_i''} - \kappa + \sum_{i=1}^M \frac{\xi_1}{C_i''} + \kappa)^2] \end{aligned}$$

Expected costs under the tradable quantities regime are now given by:

$$\begin{aligned} & E[\sum_{i=1}^M a_i(\theta_1^i) + \sum_{i=1}^M (C' + \theta_1^i) (\frac{\sum_{j=1}^M \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\kappa}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\xi_1}{C_i''} - \frac{\epsilon_1^i}{C_i''} - \frac{\theta_1^i}{C_i''}) \\ & + \sum_{i=1}^M \frac{C_i''}{2} (\frac{\sum_{j=1}^M \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\kappa}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\xi_1}{C_i''} - \frac{\epsilon_1^i}{C_i''} - \frac{\theta_1^i}{C_i''})^2 \\ & + \sum_{i=1}^M a_i(\theta_2^i) + \sum_{i=1}^M (C' + \theta_2^i) (\frac{\sum_{j=1}^M \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\kappa}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} + \frac{\xi_1}{C_i''} - \frac{\theta_2^i}{C_i''}) \\ & + \sum_{i=1}^M \frac{C_i''}{2} (\frac{\sum_{j=1}^M \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} - \frac{\kappa}{C_i'' \sum_{j=1}^M \frac{1}{C_j''}} + \frac{\xi_1}{C_i''} - \frac{\theta_2^i}{C_i''})^2] \end{aligned}$$

Expected welfare under prices does not change, as forecast errors relate only to the market-clearing price under tradable quantities. Combining the elements derived above, the expected welfare of prices relative to tradable quantities in the presence of overlapping policies is now given by:

$$\begin{aligned} \Delta = & E[\frac{1}{4}(\frac{1}{\sum_{i=1}^M \frac{1}{C_i''}} - B'')(\sum_{i=1}^M \frac{\theta_1^i + \theta_2^i}{C_i''})^2 + (\frac{1}{\sum_{i=1}^M \frac{1}{C_i''}} - B'')\kappa^2 \\ & + \frac{1}{2} \sum_{i=1}^M \frac{\epsilon_1^i{}^2}{C_i''} + \sum_{i=1}^M \frac{\xi_1^2}{C_i''} + B''(\sum_{i=1}^M \frac{\xi_1}{C_i''})^2 + 2B''(\sum_{i=1}^M \frac{\theta_1^i - \theta_2^i}{2C_i''})(\sum_{i=1}^M \frac{\xi_1}{C_i''})] \end{aligned}$$

which matches the expression given in the main text (Equation 15).

A.6 Relative Advantage of Prices Over Quantities with Uncertainty Over Own and Rivals' Cost Shocks

Now we consider the implications of interactions between uncertainty over rivals' abatement cost shocks (and therefore uncertainty over the market-clearing allowance price) and uncertainty over own abatement cost shocks. The idea is that firms may need to determine their compliance strategies before all information about the allowance price or their own costs is observed.

To derive this result, we generalize the notation used in previous sections. Let firm i 's expectation of the first-period market-clearing price, given its information set, be given by:

$$E[\hat{p}_1|\mathcal{I}_1^i] = \hat{p}_1 + \xi_1(\hat{p}) + \epsilon_1^i(\hat{p})$$

Likewise, let firm i 's expectation of its own first-period cost shocks, given its information set, be given by:

$$E[\theta_1^i|\mathcal{I}_1^i] = \theta_1^i + \xi_1(\theta) + \epsilon_1^i(\theta)$$

Under tradable quantities, firm i 's quantity response is now given by:

$$q_1^i(E[\hat{p}_1|\mathcal{I}_1^i], E[\theta_1^i|\mathcal{I}_1^i]) = \bar{q}_1^i + \frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} + \frac{\xi_1(\hat{p})}{C_i''} + \frac{\epsilon_1^i(\hat{p})}{C_i''} - \frac{\theta_1^i}{C_i''} - \frac{\xi_1(\theta)}{C_i''} - \frac{\epsilon_1^i(\theta)}{C_i''}$$

Aggregate first-period quantity is therefore given by:

$$Q_1 = \bar{Q}_1 + \sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} + \sum_{i=1}^N \frac{\xi_1(\hat{p})}{C_i''} - \sum_{i=1}^N \frac{\xi_1(\theta)}{C_i''}$$

Second-period prices then adjust accordingly, to ensure that the regulator's overall quantity target is still met:

$$\hat{p}_2'(\hat{Q}, \theta_1, \theta_2, \xi_1(\hat{p}), \xi_1(\theta)) = C' + \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} - \xi_1(\hat{p}) + \xi_1(\theta)$$

Firm i 's second-period quantity response is therefore:

$$q_2^i(\hat{p}_2', \theta_2^i) = \bar{q}_2^i + \frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\xi_1(\hat{p})}{C_i''} + \frac{\xi_1(\theta)}{C_i''} - \frac{\theta_2^i}{C_i''}$$

Given these quantity responses, expected benefits under tradable quantities are now:

$$E[b + B'(\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} + \sum_{i=1}^N \frac{\xi_1(\hat{p})}{C_i''} - \sum_{i=1}^N \frac{\xi_1(\theta)}{C_i''}) - \frac{B''}{2}(\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} + \sum_{i=1}^N \frac{\xi_1(\hat{p})}{C_i''} - \sum_{i=1}^N \frac{\xi_1(\theta)}{C_i''})^2 \\ b + B'(\sum_{i=1}^N \frac{\theta_1^i - \theta_2^i}{2C_i''} - \sum_{i=1}^N \frac{\xi_1(\hat{p})}{C_i''} + \sum_{i=1}^N \frac{\xi_1(\theta)}{C_i''}) - \frac{B''}{2}(\sum_{i=1}^N \frac{\theta_1^i - \theta_2^i}{2C_i''} - \sum_{i=1}^N \frac{\xi_1(\hat{p})}{C_i''} + \sum_{i=1}^N \frac{\xi_1(\theta)}{C_i''})^2]$$

Expected costs under tradable quantities are given by:

$$\begin{aligned}
& \mathbb{E}[\sum_{i=1}^N a(\theta_1^i) + \sum_{i=1}^N (C' + \theta_1^i) (\frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} + \frac{\xi_1(\hat{p})}{C_i''} + \frac{\epsilon_1^i(\hat{p})}{C_i''} - \frac{\theta_1^i}{C_i''} - \frac{\xi_1(\theta)}{C_i''} - \frac{\epsilon_1^i(\theta)}{C_i''}) \\
& + \sum_{i=1}^N \frac{C_i''}{2} (\frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} + \frac{\xi_1(\hat{p})}{C_i''} + \frac{\epsilon_1^i(\hat{p})}{C_i''} - \frac{\theta_1^i}{C_i''} - \frac{\xi_1(\theta)}{C_i''} - \frac{\epsilon_1^i(\theta)}{C_i''})^2 \\
& + \sum_{i=1}^N a(\theta_2^i) + \sum_{i=1}^N (C' + \theta_2^i) (\frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\xi_1(\hat{p})}{C_i''} + \frac{\xi_1(\theta)}{C_i''} - \frac{\theta_2^i}{C_i''}) \\
& + \sum_{i=1}^N \frac{C_i''}{2} (\frac{\sum_{j=1}^N \frac{\theta_1^j + \theta_2^j}{2C_j''}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\xi_1(\hat{p})}{C_i''} + \frac{\xi_1(\theta)}{C_i''} - \frac{\theta_2^i}{C_i''})^2]
\end{aligned}$$

By contrast, firm i 's first-period quantity response under a price regime will only change as a consequence of forecast errors over own cost shocks.

$$q_1^i(\tilde{p}_1, \mathbb{E}[\theta_1^i | \mathcal{I}_1^i]) = \bar{q}_1^i - \frac{\theta_1^i}{C_i''} - \frac{\xi_1(\theta)}{C_i''} - \frac{\epsilon_1^i(\theta)}{C_i''}$$

There is no change to the firm's second-period quantity response under a price regime, since we are only considering the impact of uncertainty in the first-period and compliance strategies are not intertemporally linked under a price regime.

Consequently, expected first-period benefits under the price regime are now given by:

$$\mathbb{E}[b + B'(\sum_{i=1}^N \frac{-\theta_1^i}{C_i''} - \frac{\xi_1(\theta)}{C_i''}) - \frac{B''}{2}(\sum_{i=1}^N \frac{-\theta_1^i}{C_i''} - \frac{\xi_1(\theta)}{C_i''})^2]$$

Expected first-period costs under the price regime are now given by:

$$\mathbb{E}[\sum_{i=1}^N a(\theta_1^i) + \sum_{i=1}^N (C' + \theta_1^i) (-\frac{\theta_1^i}{C_i''} - \frac{\xi_1(\theta)}{C_i''} - \frac{\epsilon_1^i(\theta)}{C_i''}) + \sum_{i=1}^N \frac{C_i''}{2} (-\frac{\theta_1^i}{C_i''} - \frac{\xi_1(\theta)}{C_i''} - \frac{\epsilon_1^i(\theta)}{C_i''})^2]$$

Expected second-period benefits and costs under the price regime are unchanged.

Combining these terms and those derived above, the relative advantage of prices over tradable quantities, given firm-level uncertainty over market-clearing prices and own abate-

ment costs, is given by:

$$\begin{aligned}
\Delta = & \frac{1}{4} \left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B'' \right) \left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{C_i''} \right)^2 + \frac{1}{2} \left(\sum_{i=1}^N \frac{\epsilon_1^i(\hat{p})^2}{C_i''} \right) + \left(\sum_{i=1}^N \frac{\xi_1(\hat{p})^2}{C_i''} \right) + \frac{1}{2} \left(\sum_{i=1}^N \frac{\xi_1(\theta)^2}{C_i''} \right) \\
& - \left(\sum_{i=1}^N \frac{\epsilon_1^i(\hat{p}) \epsilon_1^i(\theta)}{C_i''} \right) - 2 \left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} + B'' \right) \left(\sum_{i=1}^N \frac{1}{C_i''} \right)^2 (\xi_1(\hat{p}) \xi_1(\theta)) \\
& + B'' \left(\sum_{i=1}^N \frac{\xi_1(\hat{p})^2}{C_i''} \right) + 2B'' \left(\sum_{i=1}^N \frac{\xi_1(\hat{p})}{C_i''} \right) \left(\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} \right) \\
& + \frac{1}{2} B'' \sum_{i=1}^N \left(\frac{\xi_1(\theta)^2}{C_i''} \right) - B'' \left(\sum_{i=1}^N \frac{\theta_2^i}{C_i''} \right) \left(\sum_{i=1}^N \frac{\xi_1(\theta)}{C_i''} \right)
\end{aligned} \tag{1}$$

which matches the expression given in the text (Equation 16).

A.7 Price Collars

Here we provide more detailed derivations of the expected welfare impacts of introducing a price collar under a tradable quantities policy. We start with the baseline scenario without firm-level uncertainty. First, note that any price ceiling or floor can be expressed as an equivalent ceiling or floor on the weighted average of cost shocks across the market. That is, a price ceiling $\bar{p}$ is equivalent to a ceiling $\bar{t}$ such that $\bar{p} = \frac{\bar{t}}{\sum_{i=1}^N \frac{1}{C_i''}} + C'$. Therefore, the probability that the equilibrium allowance price reaches the price ceiling is equivalent to the probability that the weighted sum of cost shocks rises above this threshold $\bar{t}$. If we define $G(\cdot)$ as the CDF of the weighted sum of cost shocks ($\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''}$ in our two-period set-up), then we can express the probability that the price ceiling is triggered as $1 - G(\bar{t})$; the probability that the price ceiling is not triggered is $G(\bar{t})$.

$$\text{Prob}(\hat{p} > \bar{p}) = \text{Prob}\left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} > \bar{t}\right) = 1 - G(\bar{t})$$

Likewise, the probability of triggering the price floor is equivalent to the probability that the weighted sum of cost shocks falls below some threshold $\underline{t}$:

$$\text{Prob}(\hat{p} < \underline{p}) = \text{Prob}\left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} < \underline{t}\right) = G(\underline{t})$$

When the price ceiling is triggered, expected benefits are given by:

$$\begin{aligned}
& E[B_1(\sum_{i=1}^N q_1^i(\bar{p}, \theta_1^i) + B_2(\sum_{i=1}^N q_2^i(\bar{p}, \theta_2^i))] \\
&= E[b + B'(\sum_{i=1}^N \frac{\bar{t}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\theta_1^i}{C_i''}) - \frac{B''}{2}(\sum_{i=1}^N \frac{\bar{t}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\theta_1^i}{C_i''})^2 \\
&+ b + B'(\sum_{i=1}^N \frac{\bar{t}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\theta_2^i}{C_i''}) - \frac{B''}{2}(\sum_{i=1}^N \frac{\bar{t}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\theta_2^i}{C_i''})^2 | \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} > \bar{t}] \\
&= E[b + B'(\bar{t} - \sum_{i=1}^N \frac{\theta_1^i}{C_i''}) - \frac{B''}{2}(\bar{t} - \sum_{i=1}^N \frac{\theta_1^i}{C_i''})^2 \\
&+ b + B'(\bar{t} - \sum_{i=1}^N \frac{\theta_2^i}{C_i''}) - \frac{B''}{2}(\bar{t} - \sum_{i=1}^N \frac{\theta_2^i}{C_i''})^2 | \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} > \bar{t}]
\end{aligned}$$

Likewise, when the price floor is triggered, expected benefits are given by:

$$\begin{aligned}
& E[B_1(\sum_{i=1}^N q_1^i(\underline{p}, \theta_1^i) + B_2(\sum_{i=1}^N q_2^i(\underline{p}, \theta_2^i))] \\
&= E[b + B'(\underline{t} - \sum_{i=1}^N \frac{\theta_1^i}{C_i''}) - \frac{B''}{2}(\underline{t} - \sum_{i=1}^N \frac{\theta_1^i}{C_i''})^2 \\
&+ b + B'(\underline{t} - \sum_{i=1}^N \frac{\theta_2^i}{C_i''}) - \frac{B''}{2}(\underline{t} - \sum_{i=1}^N \frac{\theta_2^i}{C_i''})^2 | \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} < \underline{t}]
\end{aligned}$$

When the price ceiling is triggered, expected costs are given by:

$$\begin{aligned}
& E[\sum_{i=1}^N C_1^i(q_1^i(\bar{p}, \theta_1^i) + C_2^i(q_2^i(\bar{p}, \theta_2^i))] \\
&= \sum_{i=1}^N E[a_i(\theta_1^i) + (C' + \theta_1^i)(\frac{\bar{t}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\theta_1^i}{C_i''}) + \frac{C_i''}{2}(\frac{\bar{t}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\theta_1^i}{C_i''})^2 \\
&+ a_i(\theta_2^i) + (C' + \theta_2^i)(\frac{\bar{t}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\theta_2^i}{C_i''}) + \frac{C_i''}{2}(\frac{\bar{t}}{C_i'' \sum_{j=1}^N \frac{1}{C_j''}} - \frac{\theta_2^i}{C_i''})^2 | \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} > \bar{t}]
\end{aligned}$$

When the price floor is triggered, expected costs are defined analogously.

To obtain the expected change in welfare from introducing a price collar to a tradable quantities program, we weight these benefit and cost expressions by the probability of triggering the price ceiling or price floor, respectively. We then subtract expected welfare for tradable quantities without a price collar. We finally obtain that the expected welfare of introducing a price ceiling $\bar{p} = \frac{\bar{t}}{\sum_{i=1}^N \frac{1}{C_i''}} + C'$ and a price floor $\underline{p} = \frac{\underline{t}}{\sum_{i=1}^N \frac{1}{C_i''}} + C'$ in a two-period

tradable quantities program is given by:

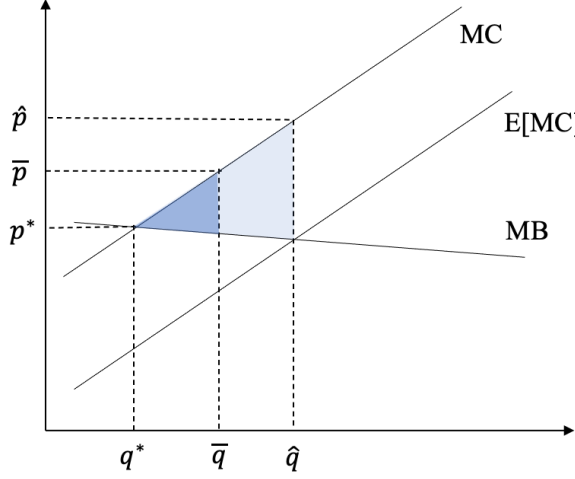
$$\begin{aligned}
\Delta = & (1 - G(\bar{t})) \cdot \mathbb{E} \left[\left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B'' \right) \left(\bar{t} - \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} \right)^2 \right. \\
& \left. + 2\bar{t} \left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} - \bar{t} \right) \left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} \right) \middle| \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} > \bar{t} \right] \\
& + G(\underline{t}) \cdot \mathbb{E} \left[\left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B'' \right) \left(\underline{t} - \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} \right)^2 \right. \\
& \left. + 2\underline{t} \left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} - \underline{t} \right) \left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} \right) \middle| \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} < \underline{t} \right] \tag{2}
\end{aligned}$$

To interpret this expression, we note first that the expected welfare of tradable quantities with and without a price collar is exactly the same with probability $G(\bar{t}) - G(\underline{t})$, assuming no firm-level uncertainty. Therefore, the expected difference in welfare depends only on the scenarios where the price ceiling or price floor is triggered. In this expression, the second term in each set of square brackets is (weakly) positive, since $\bar{t} \geq 0$ for a price ceiling and $\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} > \bar{t}$ whenever the price ceiling is binding, and $\underline{t} \leq 0$ for a price floor and $\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} < \underline{t}$ whenever the price floor is binding.² The first term in each set of square brackets depends on the relative slopes of the marginal benefit and marginal cost functions; if the marginal cost function is steeper than the marginal benefit function, then the introduction of the price collar is unambiguously welfare enhancing. If instead the marginal benefit function is steeper, then whether the price collar is welfare enhancing depends on the relative magnitudes of the first versus second terms, which reflects the expected relationship between the ex post optimal price and the price collar.

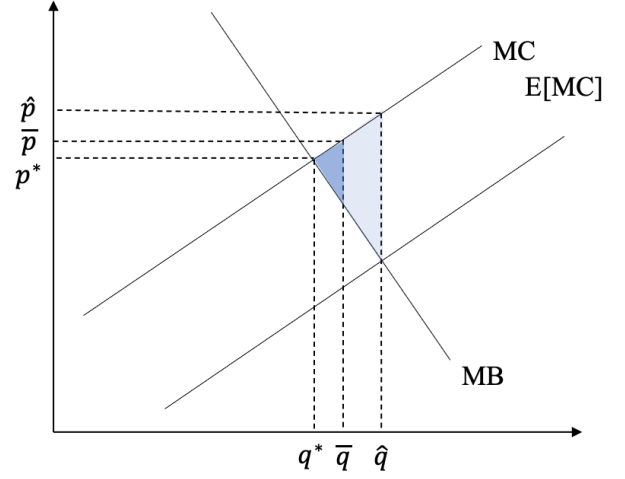
To develop further intuition, we illustrate three potential impacts of price collars for a representative firm, in Appendix Figure 1. In particular, we consider the case in which the realized marginal cost of abatement is higher than the expected cost, such that it is relevant to consider whether the price ceiling is triggered. Results are analogous when the realized marginal cost is lower than the expected cost, in which case it is relevant to consider whether the price floor is triggered.

From this set-up, it is straightforward to introduce firm-level uncertainty. We again assume that firms form posterior expectations over their rivals' cost shocks, given their own realized cost shocks and their common priors over the joint distribution of all cost shocks in the market, which then allows them to forecast the market-clearing price. However, under the policy with price collars, firms also know that the market-clearing price will not rise above the price ceiling or fall below the price floor; this additional information then affects the magnitude of firms' forecast errors. Therefore, once we consider the presence of firm-level uncertainty, we must also consider the information provision role of price collars in

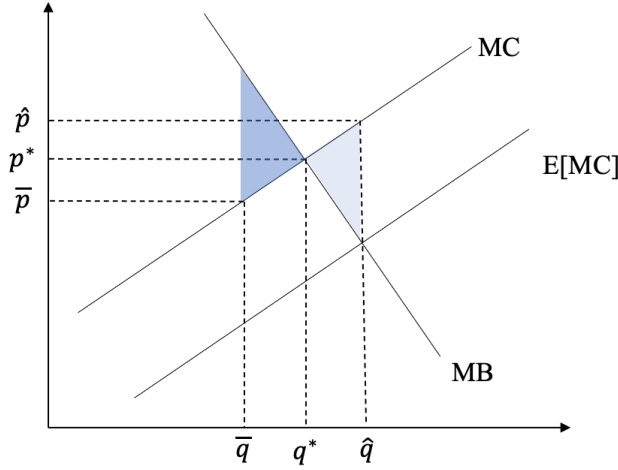
2. We assume that the regulator does not set the price ceiling below the expected optimal price, $\mathbb{E}[p^*] = C'$, since this policy could never be optimal ex ante, so $\bar{t} \geq 0$. Likewise, the regulator does not set the price floor above the expected optimal price, so $\underline{t} \leq 0$.



(a) Price Ceiling: $|MB| < |MC|$



(b) Price Ceiling: $|MB| > |MC|$ and $\bar{p} > p^*$



(c) Price Ceiling: $|MB| > |MC|$ and $\bar{p} < p^*$

Figure 1: Impact of Price Ceiling on Deadweight Loss, Relative to Pure Quantities Policy. In general, the realized welfare impacts of a price collar depend on the relationship between the cost-effective cap-and-trade price ($\hat{p}$), the ex post optimal price (p^*), and the level of the price ceiling or floor ($\bar{p}$ or $\underline{p}$). The figures depict market outcomes when the realized marginal cost shock is greater than 0, so $MC > E[MC]$ and it is relevant to consider whether the price ceiling is triggered. The lighter triangles represent the deadweight loss associated with the cost-effective price from allowance trading ($p^{C\&T}$), while the darker triangles represent the deadweight loss associated with the price ceiling $\bar{p}$. (There is no deadweight loss associated with the ex post optimal price p^* .) In panel (a), MC is steeper than MB and whenever the price ceiling binds (i.e., $\bar{p} > \hat{p}$), it is welfare enhancing. In panels (b) and (c), MB is steeper than MC , and the welfare impact of the price ceiling depends on whether $|p^* - \bar{p}| > |p^* - \hat{p}|$.

evaluating the welfare consequence of this hybrid policy design. Indeed, it is possible that the expected change in welfare from introducing the price collar would be negative in the absence of firm-level uncertainty around prices, but becomes positive given the reduction in variances of price forecast errors.

With probability $1 - G(\bar{t}) + G(\underline{t})$, either the price ceiling or the price floor binds, and the change in expected welfare is given by the expression derived above, plus the difference in forecast errors across the two regimes. This impact of the price collar on firm-level expectations of price comports with the intuition provided in Salant, Shobe, and Uler (2020), that price collars may affect firms' expectations of allowance prices in a dynamic emissions trading market with banking, even when those price collars are not binding. With probability $G(\bar{t}) - G(\underline{t})$, neither the price ceiling nor the price floor is binding, but the information provided to firms about the range of possible prices changes the variance of forecast errors in the regime with the price collar.

Let $\tilde{\xi}_1^i$ and $\tilde{\epsilon}_1^i$ denote firm forecast errors under a price collars regime. The overall welfare impact of introducing price collars in a tradeable quantities regime is now given by:

$$\begin{aligned}
\Delta = & (1 - G(\bar{t})) \cdot E \left[\left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B'' \right) \left(\bar{t} - \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} \right)^2 + 2\bar{t} \left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} - \bar{t} \right) \left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} \right) \right. \\
& \left. + B'' \left(\left(\sum_{i=1}^N \frac{\xi_1}{C_i''} \right)^2 - \left(\sum_{i=1}^N \frac{\tilde{\xi}_1}{2C_i''} \right)^2 \right) + B'' \left(\left(\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} \right) \left(\sum_{i=1}^N \frac{\xi_1}{C_i''} \right) - \left(\bar{t} - \sum_{i=1}^N \frac{\theta_1^i}{2C_i''} \right) \left(\sum_{i=1}^N \frac{\tilde{\xi}_1}{C_i''} \right) \right) \mid \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} > \bar{t} \right] \\
& \text{Difference in Benefit Smoothing, Price Ceiling Binds} \\
& + G(\underline{t}) \cdot E \left[\left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} - B'' \right) \left(\underline{t} - \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} \right)^2 + 2\underline{t} \left(\sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} - \underline{t} \right) \left(\frac{1}{\sum_{i=1}^N \frac{1}{C_i''}} \right) \mid \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} < \underline{t} \right] \\
& \left. + B'' \left(\left(\sum_{i=1}^N \frac{\xi_1}{C_i''} \right)^2 - \left(\sum_{i=1}^N \frac{\tilde{\xi}_1}{2C_i''} \right)^2 \right) + B'' \left(\left(\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} \right) \left(\sum_{i=1}^N \frac{\xi_1}{C_i''} \right) - \left(\underline{t} - \sum_{i=1}^N \frac{\theta_1^i}{2C_i''} \right) \left(\sum_{i=1}^N \frac{\tilde{\xi}_1}{C_i''} \right) \right) \mid \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} < \underline{t} \right] \\
& \text{Difference in Benefit Smoothing, Price Floor Binds} \\
& + (1 - G(\bar{t}) + G(\underline{t})) \cdot E \left[\underbrace{\sum_{i=1}^N \frac{1}{C_i''} \left(\xi_1^2 - \frac{1}{2} \tilde{\xi}_1^2 \right) + \frac{1}{2} \left(\sum_{i=1}^N \frac{(\epsilon_1^i)^2}{C_i''} - \sum_{i=1}^N \frac{(\tilde{\epsilon}_1^i)^2}{C_i''} \right)}_{\text{Difference in Forecast Errors, Either Price Ceiling or Price Floor Binds}} \mid \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} \in [\underline{t}, \bar{t}] \right] \\
& + (G(\bar{t}) - G(\underline{t})) \cdot E \left[\underbrace{B'' \left(\left(\sum_{i=1}^N \frac{\xi_1}{C_i''} \right)^2 - \left(\sum_{i=1}^N \frac{\tilde{\xi}_1}{C_i''} \right)^2 \right) + B'' \left(\left(\sum_{i=1}^N \frac{\xi_1}{C_i''} - \sum_{i=1}^N \frac{\tilde{\xi}_1}{C_i''} \right) \left(\sum_{i=1}^N \frac{\theta_2^i - \theta_1^i}{2C_i''} \right) \right)}_{\text{Difference in Benefit Smoothing, Neither Price Ceiling Nor Price Floor Binds}} \right. \\
& \left. + \underbrace{\sum_{i=1}^N \frac{1}{C_i''} \left(\xi_1^2 - \tilde{\xi}_1^2 \right) + \frac{1}{2} \left(\sum_{i=1}^N \frac{(\epsilon_1^i)^2}{C_i''} - \sum_{i=1}^N \frac{(\tilde{\epsilon}_1^i)^2}{C_i''} \right)}_{\text{Difference in Forecast Errors, Neither Price Ceiling Nor Price Floor Binds}} \mid \sum_{i=1}^N \frac{\theta_1^i + \theta_2^i}{2C_i''} \notin [\underline{t}, \bar{t}] \right]
\end{aligned} \tag{3}$$

Relative to our earlier expression for the welfare implications of introducing a price collar in a tradeable quantities market, we obtain two new sets of terms here. First, we must now consider the difference in the cost inefficiencies due to firm-level uncertainty with and without a price collar. Second, the impact of firm forecast errors on benefit smoothing also

changes.³

A.8 Construction of Figure 1

Since April 2016, the State of California has reported transaction-level data (credit prices and number of credits) on credit trades on 842 days (through December 1, 2019). We accessed these data on December 5, 2019 from: <https://ww3.arb.ca.gov/fuels/lcfs/credit/Weekly%20LCFS%20Credit%20Activity%20%28upto%201%20December,%202019%29.xlsx>. On 83 percent of these days with trades, California reported at least two completed credit trades – what we refer to as a multi-transaction trading days. On only 15 of these 699 multi-transaction trading days did the transactions include the same credit prices. Figure 1 in the main text plots the minimum and maximum daily transaction price, illustrating the significant range in daily LCFS prices.

3. Note that if firms in aggregate underabate when the price ceiling is binding, the market-clearing price cannot then rise in the next period to ensure that the aggregate quantity target is met, due to the presence of the price ceiling. The converse is also true when the price floor is binding.