

NBER WORKING PAPER SERIES

AVERAGE INFLATION TARGETING:
TIME INCONSISTENCY AND AMBIGUOUS COMMUNICATION

Chengcheng Jia
Jing Cynthia Wu

Working Paper 29673
<http://www.nber.org/papers/w29673>

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 2022, Revised May 2023

We thank the Editor, Yuriy Gorodnichenko, and the referee for important feedback on the manuscript. We are also grateful to Todd Clark, Oli Coibion, Drew Creal, Kinda Hachem, Sylvain Leduc, Andy Levin, Chen Lian, Zheng Liu, Kurt Lunsford, Thomas Mertens, Marianne Nessén, Luba Petersen, Ayşegül Şahin, Sebastian Schmidt, Eric Sims, Eric Swanson, and Michael Woodford, as well as seminar and conference participants at the NBER Monetary Economics Program Meeting, Annual Macroeconomics and Monetary Policy Conference at the Federal Reserve Bank of San Francisco, AEA Annual Meeting, the First Annual BEAR conference at the Bank of England, SNB-FRB-BIS High-Level Conference on Global Risk, Uncertainty, and Volatility, Federal Reserve Bank of Cleveland, and Bank of Israel. Further, we thank Dario Cardamone and Christopher Healy for excellent research assistance. Wu acknowledges support from the National Science Foundation under Grant No. SES-1949107. The paper was formerly titled “Average Inflation Targeting: Time Inconsistency and Intentional Ambiguity.” The views stated herein are those of the authors and are not necessarily those of the Federal Reserve Bank of Cleveland or the Board of Governors of the Federal Reserve System, or the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2022 by Chengcheng Jia and Jing Cynthia Wu. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Average Inflation Targeting: Time Inconsistency and Ambiguous Communication
Chengcheng Jia and Jing Cynthia Wu
NBER Working Paper No. 29673
January 2022, Revised May 2023
JEL No. E31,E52

ABSTRACT

We study the implications of average inflation targeting (AIT). AIT improves the inflation-output trade-off when the private sector believes the central bank's announcement. Ex post, the central bank has the incentive to implement inflation targeting instead to maximize social welfare. Next, we examine whether and how the central bank can convince the private sector, and find ambiguous communication helps the central bank gain credibility and improve welfare. These results apply to several key aspects of AIT announcement and do not rely on specific modeling assumptions.

Chengcheng Jia
Federal Reserve Bank of Cleveland
PO Box 6387
Cleveland, OH 44101
Chengcheng.Jia@clev.frb.org

Jing Cynthia Wu
Department of Economics
3060 Jenkins Nanovic Hall
University of Notre Dame
Notre Dame, IN 46556
and NBER
j.cynthia.wu@gmail.com

1 Introduction

At the 2020 Jackson Hole Economic Policy Symposium, Federal Reserve (Fed) Chair Jerome Powell announced a revision to the Fed’s long-run monetary policy framework, replacing inflation targeting (IT) with average inflation targeting (AIT) to achieve its dual mandate; see Powell (2020). Various speeches made by Fed officials and observers highlight the Fed’s ambiguous communication about AIT and associate it with time inconsistency and central bank credibility; for example, see Powell (2020), Clarida (2020), Brainard (2020), Kozicki (2019), and Brunnermeier (2021). Ambiguous communication for this new policy seems to be incompatible with the Fed’s long-term effort of promoting transparency started from Chair Alan Greenspan. Our paper investigates the implications of such ambiguous communication, and discusses how it is a result of time inconsistency and how it affects central bank credibility.

We focus on two key issues of AIT: time inconsistency and ambiguous communication. We show that AIT is not time consistent, similar to other path-dependent policy. By convincing the private sector of its intention to implement AIT, the central bank can improve the trade-off between inflation and real activity, which is captured by the Phillips curve. Ex post, the central bank has the incentive to deviate from its communication and implement IT instead to improve social welfare. Next, we assess whether and how the central bank can convince the private agents with its AIT announcement. We show that although the central bank implements IT de facto, it can still attract followers with its AIT announcement because it is possible for AIT believers to have a smaller nowcast error than IT believers. Moreover, announcing AIT without some specifics helps the central bank further gain credibility and improve welfare compared with the case of clear communication.

Our paper studies AIT under the assumption that expectations in the private sector deviate from the full-information rational expectations (FIRE). We introduce AIT into an otherwise textbook three-equation New Keynesian model by replacing current inflation in the central bank’s objective function with average inflation over L periods. In the FIRE

benchmark, the central bank minimizes its loss subject to a forward-looking Phillips curve. Although the central bank is discretionary and solves its problem period by period, equilibrium is path dependent and inflation oscillates around its steady state after a cost-push shock.

The Phillips curve captures the trade-off between inflation and real activity the central bank faces. Although AIT does not alter the forward-looking Phillips curve, it tilts its reduced form in a favorable way: compared with IT, AIT is associated with a Phillips curve that has a smaller intercept and slope. This works through the expectations channel of AIT: when inflation is above its target today, the expected inflation next period will be below the target. A lower expected inflation lowers inflation today through the expectation term in the forward-looking Phillips curve.

Although AIT presents the central bank with a better inflation-output trade-off by tilting the Phillips curve, committing to an AIT policy does not necessarily yield higher welfare, because the central bank's objective function is different from social welfare. After the central bank convinces the private sector about its intention to implement AIT, it has an incentive to deviate from its communicated objective and optimize social welfare instead. This strategy improves welfare but is time inconsistent. We show this time-inconsistent equilibrium cannot be replicated by a different objective function in a time-consistent manner.

The central bank's strategy to announce AIT but implement IT ex post could be welfare-enhancing, but is it sustainable? Or can agents learn the truth in the long run? We answer these questions through social learning, whereby agents meet and update their beliefs based on their nowcast performances. We allow two layers of heterogeneity. First, agents can have different beliefs about a key aspect in the central bank's objective. Second, agents with the same belief observe different private signals about economic fundamentals. By contrast, the central bank has perfect information.

Our main findings are ambiguous communication helps the central bank gain credibility and improve social welfare in the long run. In the baseline, we focus on the central bank's

communication regarding the horizon over which it averages inflation. Two competing forces are at play: the longest horizon delivers the best Phillips curve but the fewest followers. However, the central bank does not need to face this trade-off if it communicates ambiguously, which delivers a larger following and better welfare than clear communication with any horizon.

These results do not depend on whether the central bank is credible initially, whether inflation expectations are fully forward looking or partially backward looking, whether the central bank cares about volatility of the interest rate, if the cost-push shock is persistent, how agents weigh nowcast errors between inflation and the output gap, or whether the central bank is concerned about its reputation.

Finally, we further investigate our main results in the context of other aspects of AIT communication. We focus on two key elements: the central bank’s relative weight between inflation and output, and a decay rate in the central bank’s calculation of the average inflation in a weighted manner. In all cases we study, our main results hold except when AIT believers have beliefs that are either extreme or close to the IT benchmark.

The rest of the paper proceeds as follows: Section 1.1 draws a connection to the existing literature. Section 2 motivates our research question with some empirical evidence on ambiguous communication. Section 3 sets up our model in the FIRE benchmark, and Section 4 discusses the time-inconsistent issue. Section 5 quantitatively assesses communication on AIT horizon, and Section 6 provides robustness checks. Section 7 extends to two additional aspects of AIT communication. Finally, Section 8 concludes.

1.1 Literature

We begin with the literature on AIT. Coibion et al. (2020) use survey data to empirically assess how AIT changes expectations. Among theoretical work, Amano et al. (2020) and Honkapohja and McClung (2021) study AIT with adaptive learning. Both of these papers focus on a simple policy rule, whereas we introduce AIT to the central bank’s optimization

problem. Nessén and Vestin (2005) and Budianto, Nakata, and Schmidt (2020) also model a discretionary central bank. The difference is we focus on time inconsistency and ambiguous communication.

Several other papers discuss AIT as one of many alternative strategies when advanced economies face a low-interest-rate environment. Mertens and Williams (2019a,b) show AIT can mitigate the effects of the ZLB by raising inflation expectations when inflation is low. Svensson (2020) compares average inflation targeting with annual inflation targeting, price-level targeting, temporary price-level targeting, and nominal GDP targeting, and discusses its advantages. Andrade et al. (2021) show AIT can work as an alternative to a higher inflation target for the euro area, which faces a low r^* . Papell and Prodan (2020) show how the new policy framework is consistent with an alternative policy rule of forward guidance.

Our paper is related to the literature on agent-based modeling. We focus on its applications in economics, which refers to it as social learning or social dynamics. Burnside, Eichenbaum, and Rebelo (2016) use social learning to study booms and busts in housing markets, and Bohren and Hauser (2021) study social learning in a theoretical framework. Our paper is related to Hachem and Wu (2017) in the sense that both papers study inflation. It is also related to Arifovic, Bullard, and Kostyshyna (2013) in the sense that both papers use a New Keynesian model. The difference is that we focus on AIT.

Our paper speaks to the literature on the Phillips curve. Many recent papers, for example, Stock and Watson (2020), find empirical evidence suggesting that the slope of the Phillips curve has flattened since the Great Recession. Our paper is especially related to Coibion and Gorodnichenko (2015) and Hazell et al. (2020) in the sense that we all emphasize the importance of inflation expectation formation to the slope of the Phillips curve. The difference is that the above-referenced papers show empirically how the Phillips curve has changed recently, whereas our paper studies theoretically how AIT changes the reduced-form Phillips curve, where monetary policy plays a role in inflation expectations.

2 Empirical Motivations

Since Chair Greenspan, the Fed has been promoting transparency. However, its communication on AIT seems to be an exception. This section provides some empirical evidence from both narratives and survey data that AIT communication is ambiguous, which motivates our research question.

2.1 Narratives

On August 27, 2020, when Chair Powell (2020) announced AIT at the Jackson Hole Economic Policy Symposium, he explained the Fed’s intention to make the policy rule ambiguous:

In seeking to achieve inflation that averages 2 percent over time, we are not tying ourselves to a particular mathematical formula that defines the average. Thus, our approach could be viewed as a flexible form of average inflation targeting.

Subsequently, on August 31, 2020, Vice Chair Clarida (2020) emphasized his skepticism about the benefit and credibility of clear communication for the AIT policy:

To be clear, “inflation that averages 2 percent over time” represents an ex ante aspiration, not a description of a mechanical reaction function – nor is it a commitment to conduct monetary policy tethered to any particular formula or rule. Indeed, as summarized in the minutes of the September 2019 FOMC meeting, the Committee (and, certainly, I) was skeptical about the benefit, credibility, or practicality of adopting a formal numerical price level or average inflation target rule, just as it has been unwilling to implement its existing flexible inflation-targeting strategy via any sort of mechanical rule.

Later, speeches made by other Fed officials provided further insights into why the Fed does not provide clear communication on the policy rule. For example, Governor Brainard (2020) pointed to the challenges of communication due to time consistency:

While a formal average inflation target (AIT) rule is appealing in theory, there are likely to be communications and implementation challenges in practice related to time-consistency and the mechanical nature of such rules... a mechanical AIT rule is likely to become increasingly difficult to explain and implement as conditions change over time.

In addition, Governor Waller (2022) highlighted the Fed’s ambiguous communication on the AIT horizon:

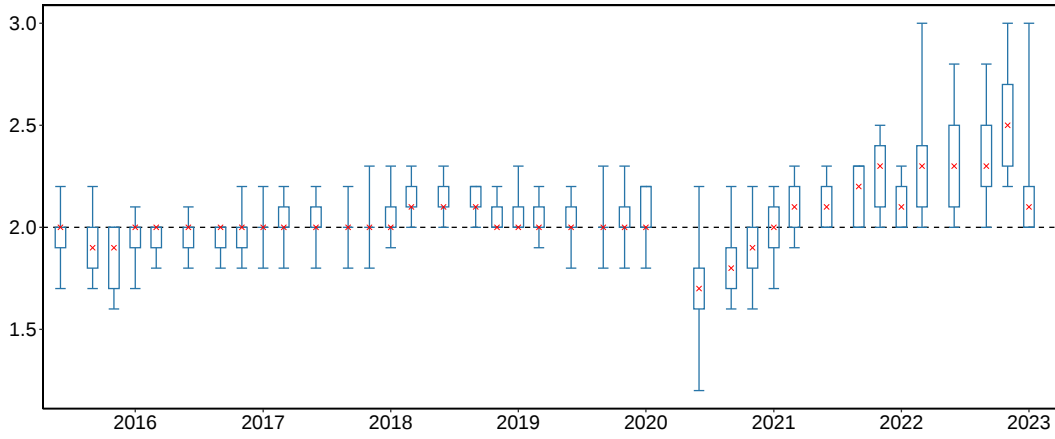
Meanwhile, the FOMC said that it would keep rates near zero until our employment goal had been reached and until inflation had reached 2 percent and was “on track to moderately exceed 2 percent for some time... The phrases [e.g., “for some time”], admittedly, are not concrete in their meaning. Inflation averaging doesn’t define how much above 2 percent is moderate and how long some value of elevated inflation should be tolerated.

The Fed’s ambiguous communication about the AIT policy, especially its specific horizon, has also drawn questions from outsiders. For example, Bank of Canada’s Kozicki (2019) made the following comment at the Fed Listens event in 2019: “... *need to choose average period for inflation...*” Similarly, Brunnermeier’s (2021) panel discussion at the Jackson Hole Economic Policy Symposium in 2021 raised the following questions: “*Average inflation targeting over how many periods?*”

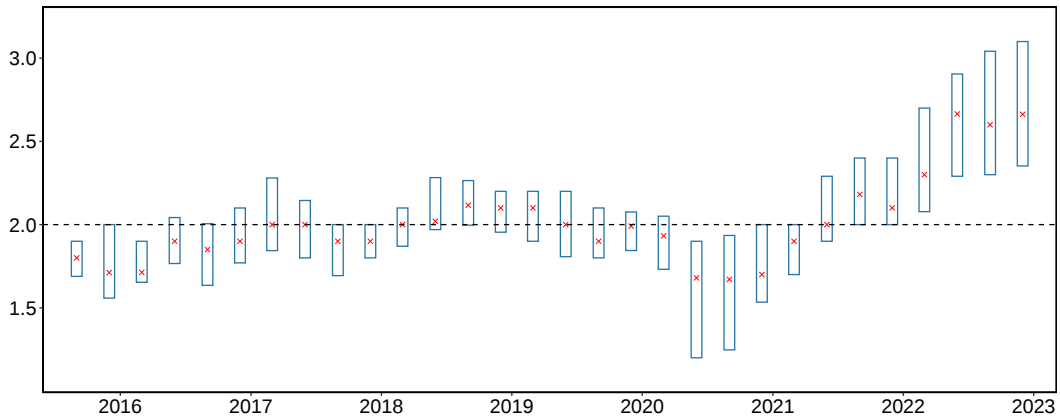
To summarize, the discussion by Fed officials and observers highlight the following issues. First, communication on AIT is ambiguous. Second, a key component of ambiguity is on the horizon of AIT. Third, a potential reason for ambiguous communication is time inconsistency. Finally, ambiguous communication as opposed to clear communication might be beneficial in terms of central bank credibility. Our paper discusses all these aspects of the AIT policy.

Figure 1: Inflation expectations

(a) Summary of Economic Projections (SEP)



(b) Survey of Professional Forecasters (SPF)



Notes: X-axis: time. Units on the y-axis: percentage points. Red “x”: median. Black dashed line: 2% inflation target. Box plots: in panel (a), the central tendency (which excludes the three highest and lowest projections), the minimum, and the maximum; in panel (b), the 25th percentile and the 75th percentile of the projections. Sources: <https://www.federalreserve.gov/monetarypolicy/fomccalendars.htm> for panel (a); <https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/survey-of-professional-forecasters> for panel (b).

2.2 Survey Evidence

In this section, we use the survey data from the Summary of Economic Projections (SEP) and the Survey of Professional Forecasters (SPF) to investigate how the introduction of the AIT policy changes inflation expectations. While the SEP surveys FOMC participants, the SPF measures the private sector’s expectations.

Figure 1 plots the summary statistics of SEP and SPF projections of core PCE inflation from September 2015 to December 2022. Panel (a) shows two-year-ahead projections. The box plots present the median (with red “x”), the central tendency (which excludes the three highest and lowest projections), as well as the minimum and maximum. Panel (b) plots the median and 25th and 75th percentiles of the four-quarter-ahead projections.

Since AIT came into sight in 2020, we observe two changes. First, the median displays a wider movement since 2020, whereas it stays close to 2% prior. This reflects the fact that with AIT, the Fed is expected to tolerate inflation that deviates temporarily from its 2% target. More importantly, the dispersion, whether it is among FOMC members or professional forecasters, has increased drastically since 2020. The dispersion among FOMC members might originate from their disagreements about some elements of the AIT policy, or more broadly, monetary policy strategy, which motivates the Fed’s ambiguous communication. The disagreement among private agents could be the consequence of ambiguous communication. For example, when the horizon of the AIT policy is not communicated clearly, people tend to have diverse expectations regarding the future evolution of the US economy.

3 Model

In this section, we lay out our analytical framework in the FIRE benchmark. We begin with social welfare, whose period loss is defined as

$$\mathcal{L}_t = \frac{1}{2} (\pi_t^2 + \lambda \hat{y}_t^2), \quad (3.1)$$

where $\hat{y}_t$ is the output gap, defined as the log deviation of real output from its natural level, and π_t is the rate of inflation. λ captures the relative weight between the output gap and inflation, and is derived from the second-order approximation of the household’s utility function; see Rotemberg and Woodford (1999).

3.1 Average Inflation Targeting

We model average inflation targeting with the following objective function of the central bank: a weighted sum between squared average inflation over L periods and the squared output gap¹:

$$\mathbb{L}_t^{cb}(L) = \frac{1}{2} \left(\left(\frac{\pi_t + \pi_{t-1} + \dots + \pi_{t-L+1}}{L} \right)^2 + \lambda^{cb}(L) \hat{y}_t^2 \right) + \beta \mathbb{E}_t \mathbb{L}_{t+1}^{cb}(L), \quad (3.2)$$

where we choose the optimal weight $\lambda^{cb}(L)$ for each L such that it minimizes the unconditional welfare loss,

$$\mathbb{E}_0 \mathcal{L}_t = \frac{1}{2} (var[\pi_t] + \lambda var[\hat{y}_t]). \quad (3.3)$$

Intuitively, $\lambda^{cb}(L) \approx \lambda/L^2$ so that the relative weight between current inflation and the output gap is comparable to the social welfare objective function. For a formal analysis of optimal $\lambda^{cb}(L)$, see Appendix C.2. In general, the central bank's objective function is different from the social welfare. They only coincide when $L = 1$.

The central bank minimizes its loss in (3.2) by choosing inflation and the output gap subject to a standard forward-looking Phillips curve (see, e.g., Woodford (2011), Clarida, Galí, and Gertler (1999), and Galí (2015)):

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \hat{y}_t + u_t, \quad (3.4)$$

where β is the discount factor and κ depends on nominal rigidity. For the most part, we assume the cost-push shock u_t is iid and is the only shock in the economy.

Our central bank is discretionary because it solves the optimization problem period by

¹We define AIT as flexible average inflation targeting, which also puts weight on the output gap. This definition is similar to flexible inflation targeting in the literature.

period. However, unlike a standard model, the equilibrium is path dependent:

$$\pi_t = a_{\pi,1}^{(L)} \pi_{t-1} + \dots + a_{\pi,L-1}^{(L)} \pi_{t-L+1} + b_{\pi}^{(L)} u_t \quad (3.5)$$

$$\hat{y}_t = a_{y,1}^{(L)} \pi_{t-1} + \dots + a_{y,L-1}^{(L)} \pi_{t-L+1} + b_y^{(L)} u_t. \quad (3.6)$$

For analytic tractability and to gain some intuition, we start with two-period AIT, and we extend to multi-period in Section 5. For two-period AIT ($L = 2$), the coefficients are given by the following set of fixed-point equations:

$$a_{\pi} \equiv a_{\pi,1}^{(2)} = -\vartheta \frac{\kappa}{4(1 - \beta a_{\pi})} \quad (3.7)$$

$$a_y \equiv a_{y,1}^{(2)} = \frac{1}{\kappa} (1 - \beta a_{\pi}) a_{\pi} \quad (3.8)$$

$$b_{\pi} \equiv b_{\pi}^{(2)} = \vartheta \frac{\lambda^{cb}}{\kappa} \quad (3.9)$$

$$b_y \equiv b_y^{(2)} = \frac{1}{\kappa} (1 - \beta a_{\pi}) b_{\pi} - \frac{1}{\kappa}, \quad (3.10)$$

where $\lambda^{cb} \equiv \lambda^{cb}(2)$. See Appendix A.1 for derivations and the expression for ϑ .

We characterize the equilibrium with the following lemma:

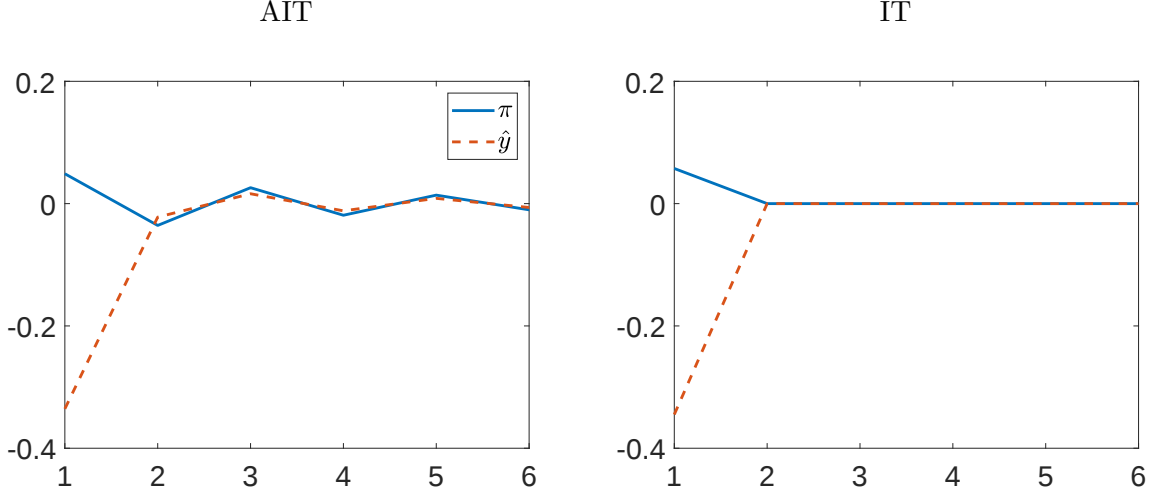
Lemma 1 $a_{\pi} < 0$, $a_y < 0$.

Proof: See Appendix B.1.

Lemma 1 says higher inflation leads to lower inflation and a lower output gap in the next period. This lemma, especially a negative $a_{\pi} \equiv \frac{\partial \pi_t}{\partial \pi_{t-1}}$, leads to an important result – the expectations channel of AIT, which we discuss in Section 3.3. We corroborate the result of a negative a_{π} with the left panel of Figure 2, which plots impulse responses of inflation and the output gap to a 1% cost-push shock.² The cost-push shock introduces a trade-off for the central bank by increasing inflation and decreasing the output gap. When the central bank implements AIT, inflation oscillates around zero. In particular, the central bank tightens

²Parameters are calibrated in line with the New Keynesian literature at an annual frequency. For details, see Appendix E.

Figure 2: Impulse responses to a cost-push shock



Notes: X-axis: horizon. Units on the y-axis: percentage points. Blue solid lines: inflation; red dashed lines: output gap. Left panel: AIT; right panel: IT. $u_t = 1$.

monetary policy in the second period, which leads to a negative inflation rate and a negative output gap.

3.2 Comparison with Inflation Targeting

We define IT as the textbook version of optimal discretionary policy (see, e.g., Woodford (2011) and Galí (2015)). Its objective function is social welfare in (3.1). The equilibrium for IT is given by

$$\pi_t = b_\pi^{(1)} u_t = \left(\kappa + \frac{\lambda}{\kappa} \right)^{-1} \frac{\lambda}{\kappa} u_t \quad (3.11)$$

$$\hat{y}_t = b_y^{(1)} u_t = \left[\frac{1}{\kappa} \left(\kappa + \frac{\lambda}{\kappa} \right)^{-1} \frac{\lambda}{\kappa} - \frac{1}{\kappa} \right] u_t. \quad (3.12)$$

See Appendix A.2 for derivations.

Unlike the AIT equilibrium in (3.5) - (3.6), IT equilibrium in (3.11) - (3.12) does not depend on lagged inflation. On the right side of Figure 2, we plot the impulse responses under IT. After the first period, the economy is back to the steady state, which is in contrast

to the oscillation in the left panel.

3.3 Phillips Curve

This section investigates how AIT affects the Phillips curve, which captures the central bank's available trade-off between inflation and the output gap. We begin with an important property of the model, which is a direct result of Lemma 1:

Proposition 1 *The **Expectations Channel** implies lower (higher) positive (negative) inflation under AIT than that under IT for a given output gap.*

Proof: See Appendix B.2.

For AIT, the expectations channel works as follows: when inflation is positive, the expected inflation next period is negative per Lemma 1, which in turn lowers inflation today via the expectation term in the forward-looking Phillips curve (3.4). By contrast, IT does not have such an expectations channel.

As a result, although IT and AIT share the same structural-form Phillips curve in (3.4), the reduced-form Phillips curves are different. For AIT, it is

$$\pi_t = \frac{\kappa}{1 - \beta a_\pi} \hat{y}_t + \frac{1}{1 - \beta a_\pi} u_t, \quad (3.13)$$

and the reduced-form Phillips curve for IT is

$$\pi_t = \kappa \hat{y}_t + u_t. \quad (3.14)$$

For the derivation of (3.13), see Appendix A.1. We obtain (3.14) by imposing $\mathbb{E}_t \pi_{t+1} = 0$ on (3.4).

Comparing the Phillips curves under AIT in (3.13) and under IT in (3.14) leads to the following proposition:

Proposition 2 *AIT implies a smaller slope of the reduced-form Phillips curve than IT.*

Proof: See Appendix B.3.

Why does AIT flatten the reduced-form Phillips curve? We use the right panel of Figure 3 for illustration (a similar argument can be made using the left panel). Points A and B are both associated with more expansionary policy and have a 0.1% higher output gap than the origin, where the Phillips curves of IT and AIT intersect. The direct effect from the forward-looking Phillips curve (3.4) is that both A and B have inflation that is $0.1\kappa\%$ higher than the origin. But they differ in terms of the indirect effect through the expectations channel described in Proposition 1. IT does not have such an indirect effect. By contrast, with an AIT policy, higher inflation today lowers expected inflation in the next period, which feeds back to lower inflation today. This expectations channel makes point B lower than point A. Therefore, AIT is associated with a smaller slope of the Phillips curve than IT.

With a flatter reduced-form Phillips curve, we next argue that AIT presents a better trade-off between inflation and the output gap than IT:

Proposition 3 *AIT yields a better available trade-off for the central bank between inflation and the output gap.*

Proof: See Appendix B.4.

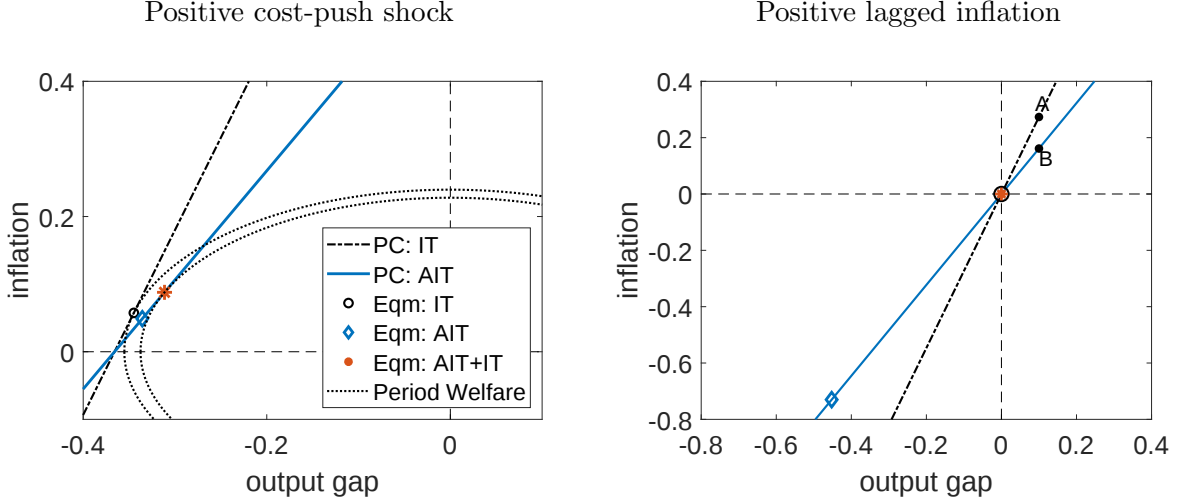
This is one key result of the paper: AIT tilts the Phillips curve in a favorable way – closer to the origin in the relevant space (the second quadrant). We show the intuition of this result in two steps, and for each step, we focus on one state variable. First, we start with a cost-push shock.

Lemma 2 *The reduced-form Phillips curve under AIT has*

- *the same x -intercept as*
- *and a smaller absolute value of the y -intercept than*

that under IT after a cost-push shock.

Figure 3: Reduced-form Phillips curves and equilibria



Notes: Left: $u_t = 1, \pi_{t-1} = 0$; right: $\pi_{t-1} = 1, u_t = 0$. Units: percentage points. Lines are Phillips curves. Black circle: IT equilibrium; blue diamond: AIT equilibrium with commitment; red star: the equilibrium where the private sector forms expectations based on the AIT announcement, but the central bank deviates to implement IT ex post. The ellipses are period social welfare.

Proof: See Appendix B.5.

The results in Lemma 2 imply that AIT offers a better trade-off for the central bank than IT. The intercept (which we refer to as the y -intercept in the proposition) of the Phillips curve captures the equilibrium when the output gap is stabilized after a cost-push shock. The two upward-sloping lines in the left panel of Figure 3 are Phillips curves after a positive cost-push shock, which leads to a positive inflation rate and hence a positive intercept. The expectations channel discussed in Proposition 1 makes the intercept of the Phillips curve smaller under AIT than under IT. The x -intercept of the Phillips curve represents the equilibrium when inflation is completely stabilized after a cost-push shock. Zero inflation this period implies zero expected inflation and a zero expected output gap next period for both policies, eliminating any feedback from the expectations to the current period. Therefore, the two policies share the same x -intercept.

Next, we turn to the case of non-zero lagged inflation, and we show the central bank does not face a trade-off for either IT or AIT:

Lemma 3 *The reduced-form Phillips curves under two-period AIT and IT both cross the*

origin after non-zero lagged inflation.

Proof: See Appendix B.6.

What drives this result? The private sector forms its expectations next period conditional on current inflation only. Therefore, lagged inflation does not introduce an indirect effect via the expectations channel.³

Although AIT allows a better available trade-off between inflation and the output gap, it does not guarantee an improvement in welfare, because the central bank's objective function is different from social welfare. In Section 4, we discuss a welfare-enhancing strategy in which the central bank implements a policy that is different from its announced AIT.

3.4 An Example of the Zero Lower Bound

Although the rest of our paper is structured with positive inflation in light of the 2021-2022 inflation surge, this section discusses the implications of the ZLB, which was a primary reason for introducing AIT.

We model the ZLB with an interest rate peg and a negative shock to the natural rate of interest, for which we need to introduce the IS curve:

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\gamma} (\underline{i} - r_t^n - \mathbb{E}_t \pi_{t+1}), \quad (3.15)$$

where the nominal interest rate is $\underline{i} = 0$ and r_t^n is the natural rate of interest. γ is the inverse of the intertemporal elasticity of substitution. We model a one-period ZLB, which corresponds to one year in our calibration. The forward-looking Phillips curve is the same as equation (3.4).

We first solve the equilibrium during normal times when the ZLB does not bind. In this case, the natural rate shock does not change the equilibrium, because the central bank sets the interest rate optimally to track the natural rate and offset any effect the natural rate

³Note this result is specific to $L = 2$.

shock might have on the equilibrium. Therefore, the central bank can achieve dual stability when only the natural rate shock is present. See details in Appendix A.3. However, this is not the case when the ZLB binds, because the central bank cannot lower the nominal interest rate further to compensate for a negative shock to the natural rate.

With our assumption on the ZLB, $\mathbb{E}_t \pi_{t+1} = \mathbb{E}_t \hat{y}_{t+1} = 0$ under IT. Therefore, the IS curve is

$$\hat{y}_t = \frac{1}{\gamma} r_t^n, \quad (3.16)$$

and the Phillips curve is the same as equation (3.14).

For two-period AIT,

$$\mathbb{E}_t \pi_{t+1} = a_\pi \pi_t \quad (3.17)$$

$$\mathbb{E}_t \hat{y}_{t+1} = a_y \pi_t, \quad (3.18)$$

where a_π and a_y satisfy equations (3.7) and (3.8). For derivation, see Appendix A.3.

Therefore, the IS curve is

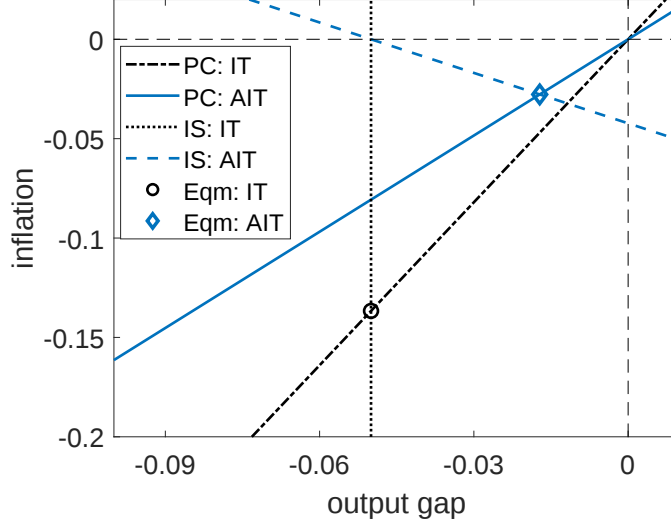
$$\hat{y}_t = \left(a_y + \frac{1}{\gamma} a_\pi \right) \pi_t + \frac{1}{\gamma} r_t^n, \quad (3.19)$$

and the Phillips curve is the same as equation (3.13).

Figure 4 plots the implications of the ZLB. The upward-sloping lines are the Phillips curves. The vertical dotted black and downward-sloping dashed blue lines are the IS curves, and the circle and diamond are the equilibria. Black represents IT and blue is AIT. Although Figure 4 illustrates a case without the cost-push shock, our results go through as long as the cost-push shock is not too positive and the equilibria stay in the same quadrant.

Similar to our discussion of Proposition 2, the AIT Phillips curve is flatter than the IT curve. The IS curve for IT is vertical per equation (3.16), whereas that for AIT is downward

Figure 4: Zero lower bound



Notes: $r_t^n = -1, u_t = 0$. Units: percentage points. The upward-sloping lines are the Phillips curves; the downward-sloping and vertical lines are the IS curves; the circle and diamond are the equilibria. Black represents IT and blue represents AIT.

sloping per equation (3.19) and Lemma 1. Moreover, the two IS curves cross the horizontal axis at the same point because $\pi_t = 0$ implies $\mathbb{E}_t \pi_{t+1} = 0$ regardless of whether the central bank implements IT or AIT. In both cases, the equilibrium entails negative inflation and a negative output gap, which together are the hallmark of the ZLB.

However, AIT achieves a better equilibrium than IT with a less negative inflation and less negative output gap. This result again works through the expectations channel discussed in Proposition 1. Therefore, AIT could be a useful tool for an economy that is stuck in a liquidity trap.

4 Time Inconsistency

4.1 Time Inconsistent Nature of AIT

This section deviates from the FIRE framework we rely on in Section 3 and demonstrates the time inconsistency of AIT. After the central bank convinces the private sector of its

intended policy objective, the Phillips curve is fixed. The central bank can then improve welfare ex post by picking the point on the Philips curve that minimizes the period loss of social welfare in equation (3.1), and we formalize this idea in the following proposition:⁴

Proposition 4 *Time inconsistency of AIT:* *the central bank has the incentive to announce AIT but implement IT ex post. The consequent time-inconsistent equilibrium is given by*

$$\pi_t = \left(\frac{\kappa}{1 - \beta a_\pi} + \lambda \frac{1 - \beta a_\pi}{\kappa} \right)^{-1} \frac{\lambda}{\kappa} u_t \quad (4.1)$$

$$\hat{y}_t = \left[\left(\frac{\kappa}{1 - \beta a_\pi} + \lambda \frac{1 - \beta a_\pi}{\kappa} \right)^{-1} \frac{\lambda}{\kappa} \frac{1 - \beta a_\pi}{\kappa} - \frac{1}{\kappa} \right] u_t. \quad (4.2)$$

Proof: See Appendix A.4.

The equilibrium described by Proposition 4 dominates both the AIT and IT equilibriums. We explain the intuition with Figure 3. We discuss one state variable at a time. First, after non-zero lagged inflation, equations (4.1) - (4.2) suggest the central bank achieves dual stabilization $\pi_t = x_t = 0$ (the red dot in the right panel of Figure 3). This equilibrium has the same zero loss as the IT equilibrium (black circle), and improves welfare from the AIT equilibrium (blue diamond).

Next, in response to a cost-push shock, the time-inconsistent equilibrium (the red star in the left panel) is on the tangent point between the more favorable Phillips curve (the blue solid line) and the period loss of social welfare (the ellipses). Therefore, it dominates both IT, which is on the worse Phillips curve (the black dashed line), and AIT with commitment, where the central bank minimizes the AIT loss in equation (3.2) but not the period welfare.⁵

⁴In this section, we assume the central bank can successfully convince the private sector of its intention to implement AIT, although it deviates from its announcement ex post. In our quantitative exercises in Sections 5 - 7, we assess this assumption and investigate whether the central bank can fool the public consistently.

⁵We use “AIT with/under commitment” to refer to a central bank that announces *and implements* the AIT policy. Notice the central bank is still discretionary and solves its optimization problem period by period.

4.2 Can a Different Objective Function Replicate the Time Inconsistent Equilibrium?

In this section, we discuss if we can solve the time inconsistency issue of AIT by appointing the central bank with a different objective function, in the spirit of Rogoff (1985). We first show this is not possible with a contemporaneous objective function that simply modifies the weight in the social welfare (3.1).

Proposition 5 *It is impossible to achieve the equilibrium in (4.1) - (4.2) with a time consistent policy where the central bank's objective function has the following form:*

$$\mathcal{L}_t = \pi_t^2 + \tilde{\lambda} \hat{y}_t^2. \quad (4.3)$$

Proof: See Appendix B.7.

Why does this happen? With the equilibrium condition in (4.1), the Phillips curve's forward looking component drops out, and hence, it is identical to the one for IT in (3.14). But the IT Phillips curve is not consistent with the equilibrium summarized by (4.1) - (4.2). We can see this from Figure 3. The time-inconsistent equilibrium is represented by the red star, which is on the AIT Phillips curve and generally not on the IT Phillips curve (see the left panel).

Although Proposition 5 only proves a contemporaneous objective function cannot deliver the time-inconsistent equilibrium described in (4.1) - (4.2), the impossibility result extends to objective functions that include lagged terms. That is because such an objective function delivers a path dependent equilibrium, which is not consistent with (4.1) - (4.2).

5 Ambiguous Communication About AIT Horizon

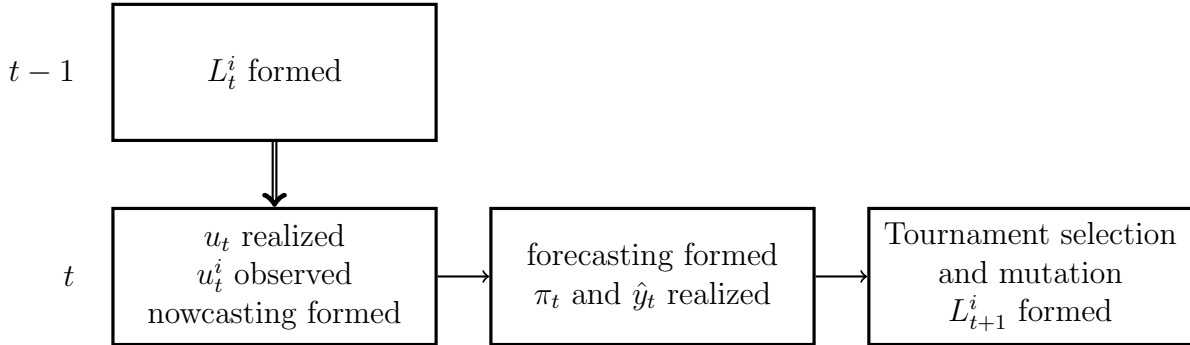
We start our quantitative analysis by focusing on central bank' communication regarding the horizon of AIT. The central bank can announce IT, AIT by specifying a horizon ("clear

communication”), or AIT without specifying a horizon (“ambiguous communication”). The central bank follows the time-inconsistent strategy discussed in Section 4. Different from Section 4, where we assume the central bank can successfully convince the private sector of its intention to implement AIT, we relax this full credibility assumption in this section. Rather, the central bank can endogenously accumulate or lose credibility, which we model as the fraction of private agents who believe the central bank’s announced policy.

5.1 Social Learning

This section sets up the environment of social learning. We model social learning similar to Arifovic, Bullard, and Kostyshyna (2013) and Hachem and Wu (2017). Agents are grouped by their beliefs about $L \in \{1, 2, \dots, L_{max}\}$, and we also allow heterogeneous information within each group. Agents update their beliefs via tournament selection and mutation. We assume each agent believes all other agents in the economy have the same information set that he/she does, whereas the central bank observes everything.

Figure 5: The sequence of events



The sequence of events is drawn in Figure 5. Before entering period t , each agent has a belief about L , and we label it L_t^i . When agents enter period t , a shock u_t is realized, and

agent i observes it with a private signal u_t^i :

$$u_t^i = u_t + v_t^i, \quad v_t^i \sim N(0, \sigma_v^2). \quad (5.1)$$

For the baseline model, we assume a temporary cost-push shock: $u_t \sim N(0, \sigma_u^2)$, and we relax this assumption in Section 6.4. Within-group heterogeneity is present when $\sigma_v \neq 0$. With this information, agent i forms his/her expectations about aggregate variables at time t using equations (3.5) - (3.6):

$$\mathbb{E}(\pi_t | \mathcal{I}_{t-1}, L_t^i, u_t^i) = a_{\pi,1}^{(L_t^i)} \pi_{t-1} + \dots + a_{\pi,L_t^i-1}^{(L_t^i)} \pi_{t-L_t^i+1} + b_{\pi}^{(L_t^i)} u_t^i \quad (5.2)$$

$$\mathbb{E}(\hat{y}_t | \mathcal{I}_{t-1}, L_t^i, u_t^i) = a_{y,1}^{(L_t^i)} \pi_{t-1} + \dots + a_{y,L_t^i-1}^{(L_t^i)} \pi_{t-L_t^i+1} + b_y^{(L_t^i)} u_t^i, \quad (5.3)$$

where $\mathcal{I}_{t-1}$ is the realized equilibrium path up to time $t-1$. The nowcasting is used for social learning in the last subperiod in Figure 5.

Next, we describe the central bank's problem in the second subperiod, including how macro aggregates realize and agents form their forecasts. Agent i forms expectations about inflation in the next period:

$$\mathbb{E}_t^i \pi_{t+1} \equiv \mathbb{E}(\pi_{t+1} | \mathcal{I}_t, L_t^i) = a_{\pi,1}^{(L_t^i)} \pi_t + \dots + a_{\pi,L_t^i-1}^{(L_t^i)} \pi_{t-L_t^i+2}. \quad (5.4)$$

The average expectation is

$$\bar{\mathbb{E}}_t \pi_{t+1} = \frac{1}{N} \sum_i \mathbb{E}_t^i \pi_{t+1}, \quad (5.5)$$

where N is the total number of agents.

Therefore, the Phillips curve in equation (3.4) becomes

$$\pi_t = \beta \bar{\mathbb{E}}_t \pi_{t+1} + \kappa \hat{y}_t + u_t. \quad (5.6)$$

The central bank picks π_t and $\hat{y}_t$ to minimize the period loss $\mathcal{L}_t$ defined in equation (3.1) subject to the Phillips curve in equation (5.6). The realized laws of motion for π_t and $\hat{y}_t$ are different from equations (3.5) - (3.6) and can be found in Appendix D.1.

Tournament selection works as follows: at each time period t , agents are randomly drawn to meet in pairs. When two agents meet, they update their beliefs by comparing nowcast errors, which are defined as

$$\varepsilon_t^i = |\mathbb{E}(\pi_t|\mathcal{I}_{t-1}, L_t^i, u_t^i) - \pi_t| + w|\mathbb{E}(\hat{y}_t|\mathcal{I}_{t-1}, L_t^i, u_t^i) - \hat{y}_t|, \quad (5.7)$$

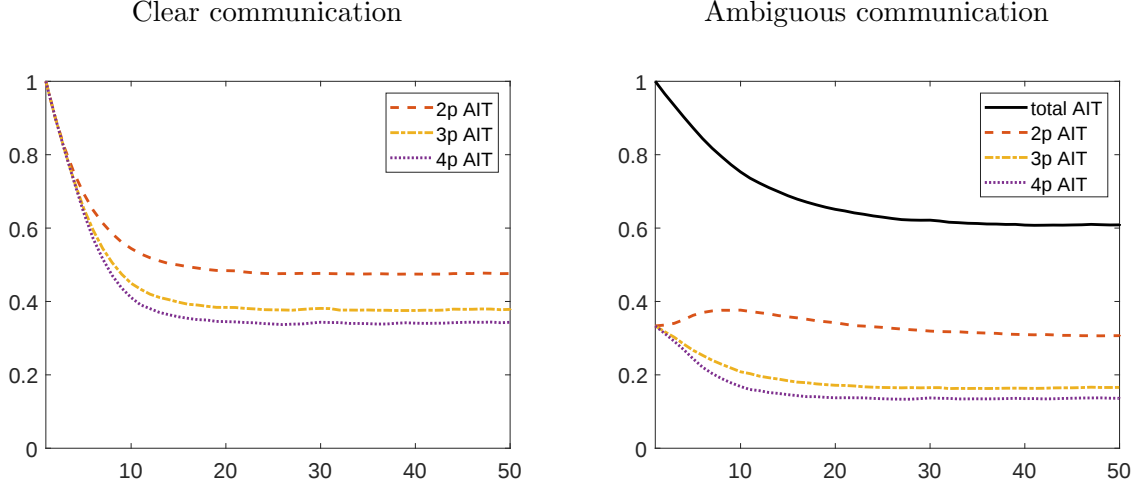
where we use nowcasting expectations in equations (5.2) - (5.3), and w is the weight agents put on the error on the output gap.⁶ When two agents are from the same group, that is, $L_t^i = L_t^j$, they stay in this group. When two agents come from two different groups, suppose agent i has a smaller nowcast error $\varepsilon_t^i < \varepsilon_t^j$. Agent i stays in his/her current group, whereas agent j switches. At the end of period t , agents can mutate with a probability, that is, randomly switch to another rule. Mutation is a standard device in the literature to avoid the algorithm getting stuck prematurely in a corner solution. This step updates the belief to L_{t+1}^i , which will be used at time $t + 1$. Parameter values are in Appendix E.

5.2 Central Bank Credibility and Welfare

This section demonstrates that ambiguous communication helps improve the central bank's credibility and welfare. In this section, we assess the full credibility assumption we made in Section 4 and ask if the central bank eventually loses its following because agents figure out that it deviates from its promise and implements IT ex post. To better connect with Section 4, we initialize the central bank with full credibility in our main specification. As a robustness check, Section 6.1 shows our results are robust when the central bank starts with no or partial credibility.

⁶We use equal weight $w = 1$ in the benchmark case and discuss the implications of different weighting schemes in Section 5.4.

Figure 6: Fraction of AIT believers

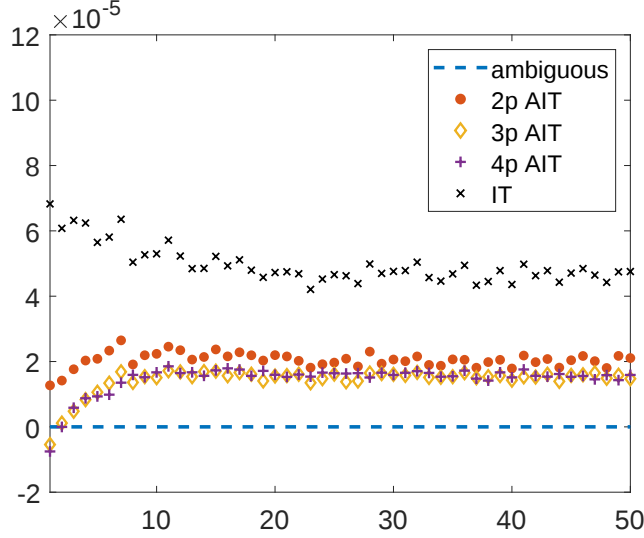


Notes: X-axis: horizon. Left panel: L is clearly communicated; right panel: AIT is announced without explicitly specifying L . Red dashed lines: fraction of two-period AIT believers, yellow dash-dotted lines: fraction of three-period AIT believers; purple dotted lines: fraction of four-period AIT believers. Black solid line in the right panel: fraction of total AIT believers. All the results are averaged over 1,000 simulations.

We define the fraction of agents in each group as $p^{(L)}$. The left panel of Figure 6 plots the fraction of AIT believers when the central bank announces AIT with a clear horizon $L \in \{2, 3, 4\}$. In each case, only two groups of beliefs exist: IT and L -period AIT, or $p^{(1)} + p^{(L)} = 1$. The red dashed, yellow dash-dotted, and purple dotted lines represent a central bank that announces two-period, three-period, or four-period AIT, respectively. All our results are averaged over 1,000 simulations. At each point in time, the fraction of AIT believers is the highest when the central bank announces two-period AIT and lowest when the central bank announces four-period AIT.

The right panel plots the evolution of beliefs under ambiguous communication, where the central bank only announces AIT as its tool to manage inflation but does not specify its length. At the beginning, an equal fraction of agents believe in AIT with different lengths: $p^{(2)} = p^{(3)} = p^{(4)}$. Over time, these fractions evolve through social learning with $p^{(1)} + p^{(2)} + p^{(3)} + p^{(4)} = 1$. The red dashed, yellow dash-dotted, and purple dotted lines capture the fraction of agents in each AIT group. The black solid line is the relevant metric, which is the sum of the three colored lines and captures the total fraction of AIT believers.

Figure 7: Welfare loss



Notes: X-axis: horizon. y-axis: welfare loss. Blue dashed line: ambiguous communication, which is normalized to zero. Red dots: the central bank announces two-period AIT; yellow diamonds: the central bank announces three-period AIT; purple plus signs: the central bank announces four-period AIT; black crosses: IT. All the results are averaged over 1,000 simulations.

Comparing the black solid line in the right panel with each line in the left panel, we find ambiguous communication allows a larger fraction of agents to believe in the AIT strategy that the central bank announces, even though it implements IT ex post. Therefore, ambiguous communication helps the central bank gain more credibility in the long run.

Comparing the three colored lines in the right panel, similar to the left panel, more agents believe in two-period AIT than in three- or four-period AIT. Notice that in the left panel, the three colors represent three different types of announcements. For each announcement, only the corresponding color is relevant, whereas in the right panel, only one type of announcement - ambiguous announcement - exists, but agents can have different beliefs about L in this scenario.

We have established that ambiguous communication can help the central bank gain a larger following for its communicated AIT than clear communication. But does it improve welfare in the long run? In Figure 7, we plot the period loss of social welfare for five different announcements. We take the ambiguous communication in the blue dashed line as

the benchmark and plot loss in other cases relative to it. Therefore, the blue dashed line coincides with the horizontal axis by construction.

Ambiguous communication is associated with the smallest loss for all but the initial few periods; see the blue dashed line. On the contrary, announcing IT and sticking to it (black “x”), which is the operating framework before introducing AIT, is universally the worst in terms of welfare.

5.3 State Dependency of Optimal Horizon

This section discusses the state dependent nature of the optimal AIT horizon and uses it to explain the relative performance between two-, three-, and four-period AIT in Figure 7. Within the confines of clear communication, the time-inconsistency of AIT makes it optimal for the central bank to announce AIT of different horizons depending on the state of the economy. As a result of Proposition 4, the central bank always maximizes the period welfare ex post regardless of its announcement. Therefore, comparing between equilibria where the central bank announces AIT with different horizons is equivalent to comparing their corresponding Phillips curves, which are characterized by

$$\pi_t = \frac{\kappa}{1 - \beta a_{\pi,1}^{(L)}} \hat{y}_t + \sum_{l=1}^{L-2} \frac{\beta a_{\pi,l+1}^{(L)}}{1 - \beta a_{\pi,1}^{(L)}} \pi_{t-l} + \frac{1}{1 - \beta a_{\pi,1}^{(L)}} u_t, \quad (5.8)$$

where the coefficients are solved in Appendix C.1.

Suppose the economy only faces a positive cost-push shock at t . The central bank’s optimal strategy is to announce the largest feasible L at time t and announce $L = 2$ subsequently. To show why, we begin with period t , for which the intercept of the Phillips curve (5.8), $\frac{1}{1 - \beta a_{\pi,1}^{(L)}} u_t$, is a decreasing function of L ; see Figure F.1. In other words, the Phillips curve with the largest L ($L = 4$ years in this example) is associated with the strongest effect of the expectations channel and provides the best trade-off between inflation and the output gap. Hence, the largest L forms the optimal horizon for the AIT announcement at time t .

At time $t + 1$, it is optimal for an AIT central bank to announce $L = 2$. In this case, the Phillips curve does not have an intercept; see equation (5.8). Consequently, we can achieve the equilibrium of zero inflation and a zero output gap. By contrast, if the central bank announces an $L > 2$, non-zero past inflation causes a non-zero intercept in the Phillips curve, which implies a non-zero loss in the period social welfare. At time $t + h$, the central bank can announce any $L \in \{2, \dots, h + 1\}$ and implement IT to achieve dual stability.

This state dependency explains the relative performance between two-, three-, and four-period AIT in Figure 7. Initially, four-period AIT has the smallest loss because the largest L offers the best Phillips curve; see Figure F.1. But the advantage of announcing a large L disappears quickly, because four-period AIT has a smaller following over time; see Figure 6. At some point, we see a smaller welfare loss if the central bank announces $L = 3$ instead of $L = 4$.⁷ Two competing forces are at play: a larger L is associated with a more favorable Phillips curve but fewer followers. However, the central bank does not have to face this trade-off and can take advantage of ambiguous communication.

5.4 Can AIT Believers Perform Better When the Truth Is IT?

This section provides explanation for the result in Section 5.2 that AIT believers' nowcast could potentially outperform IT believers' when the central bank always implements IT ex post.

We use the left panel of Figure 3 to illustrate the intuition. The red star marks the realized equilibrium after a positive cost-push shock in which all agents believe the central bank's announcement about two-period AIT but the central bank implements IT. The black circle and blue diamond mark the expected equilibria of the IT believers and the AIT believers. Neither AIT believers nor IT believers predict perfectly, or the red star does not overlap with either the blue diamond or the black circle. The IT believers have a smaller prediction

⁷Notice we interpret time in a relative sense instead of an absolute sense and focus our discussion on the long run. Although we calibrate structural parameters to an annual frequency, the time to converge depends on the specifics of social learning, which are not the focus of the paper.

error on inflation, whereas AIT believers have a smaller error on the output gap.

Although the central bank implements IT ex post, AIT believers could have a smaller nowcast error if agents put more weight on the output gap. We further elaborate on this point in Panels (a) and (b) of Figure 8, where we extend Figure 6 with two extreme cases: in Panel (a), agents put all the weight on inflation, or $w = 0$; in Panel (b), agents put all the weight on the output gap, or $w \rightarrow \infty$. Comparing the two panels, we find AIT has a larger following when agents put more weight on the output gap (or a larger w). This result holds regardless of whether the central bank communicates the horizon of AIT clearly or ambiguously, which confirms the intuition provided by Figure 3.

6 Robustness

This section provides robustness checks of our main results presented in Section 5.2.

6.1 Initial Credibility

Section 5.2 makes a stark assumption that the central bank starts with full credibility. For example, Coibion et al. (2020) use survey data to show that the private sector does not understand the impact of AIT soon after its introduction. In this section, we assess what happens when the central bank is partially credible or not credible initially. We compute the evolution of the fraction of AIT believers with different levels of initial credibility: 100%, which is our baseline assumption, 50%, and 0. In the long run, the fractions of AIT believers converge to the same point, where over 60% of agents believe AIT; for details, see Figure F.2. In summary, the initial credibility does not matter for the long run.

6.2 Backward-Looking Inflation Expectations

In this section, we consider (partially) backward looking expectations; for example, see Ball and Mazumder (2011) and Coibion and Gorodnichenko (2015). We model the private sector's

inflation expectation as follows:

$$\mathbb{E}_t \pi_{t+1} = \phi \pi_{t-1} + (1 - \phi) \mathbb{E}_t^{\text{FIRE}} \pi_{t+1}, \quad (6.1)$$

where ϕ captures the degree of backward-looking and $\mathbb{E}_t \pi_{t+1}^{\text{FIRE}}$ is the inflation expectation formed under FIRE.

With the modified expectation formation process, the Phillips curves in (3.4) becomes

$$\pi_t = \beta [\phi \pi_{t-1} + (1 - \phi) \mathbb{E}_t \pi_{t+1}^{\text{FIRE}}] + \kappa \hat{y}_t + u_t. \quad (6.2)$$

When expectations are completely backward looking, i.e., $\phi = 1$, the Phillips curve is

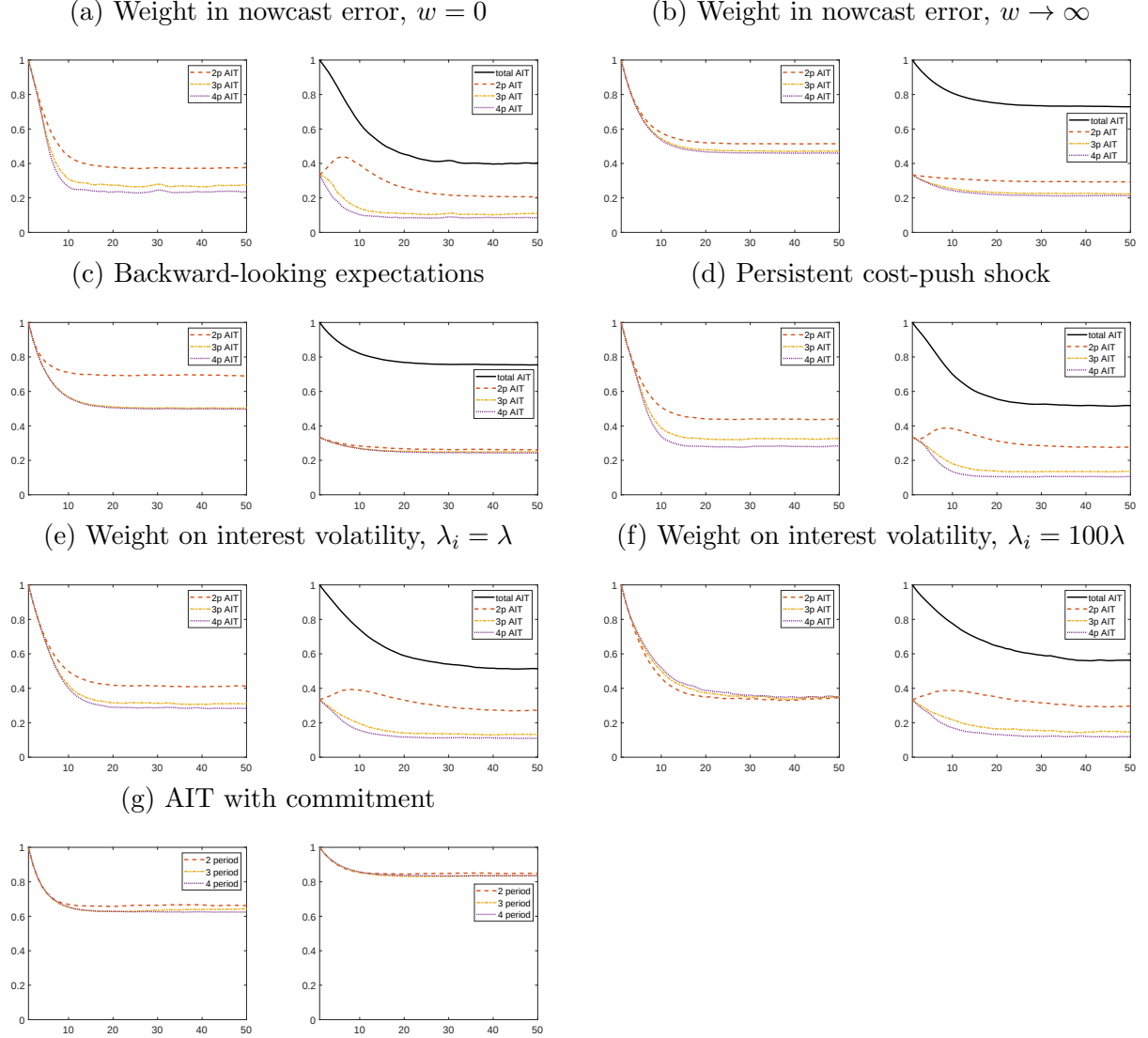
$$\pi_t = \beta \pi_{t-1} + \kappa \hat{y}_t + u_t, \quad (6.3)$$

which is the same for both IT and AIT believers, because the expectations channel of AIT is muted. Consequently, different beliefs of L are irrelevant. Instead, we focus on the more interesting case in which agents are partially backward looking with $\phi < 1$. For details of implementing backward-looking expectations, see Appendix D.2.

We plot the case of $\phi = 0.5$ in Panel (c) of Figure 8. Compared with the baseline model where agents are fully forward-looking in Figure 6, Figure 8(c) shows backward-looking expectations lead to more AIT followers. That is because the backward-looking component reduces the difference in nowcast errors between agents in different groups. Specifically, it reduces the nowcast errors of AIT believers relative to the IT believers, which makes AIT believers less likely to switch.

More importantly, comparing the two panels in Figure 8(c), our conclusion that ambiguous communication on the AIT horizon attracts more followers than clear communication still holds when expectations are backward looking.

Figure 8: Fraction of AIT believers: Robustness



Notes: X-axis: horizon. Each panel is structured similarly to Figure 6, where the left plot captures clear communication, and the right plot corresponds to ambiguous communication. Red dashed lines: fraction of two-period AIT believers, yellow dash-dotted lines: fraction of three-period AIT believers; purple dotted lines: fraction of four-period AIT believers. Black solid line in the right panel: fraction of total AIT believers. All the results are averaged over 1,000 simulations.

6.3 Weight on Interest Rate Volatility

This section examines the case in which the central bank cares not only about volatilities of inflation and the output gap as in the social welfare (3.1), but also about volatility of the

interest rate. Specifically, we modify (3.1) to

$$\mathcal{L}_t^i = \frac{1}{2} (\pi_t^2 + \lambda \hat{y}_t^2 + \lambda_i i_t^2), \quad (6.4)$$

where λ_i is the weight on interest-rate volatility.

As a result, agents form their expectations the same as in the baseline model described in Section 5.1. What differs is the subsequent central bank's problem: it now picks π_t , $\hat{y}_t$, and i_t to minimize the loss $\mathcal{L}_t^i$ defined in equation (6.4) subject to the Phillips curve in equation (5.6) and an IS curve

$$\hat{y}_t = \bar{\mathbb{E}}_t \hat{y}_{t+1} - \frac{1}{\gamma} (i_t - \bar{\mathbb{E}}_t \pi_{t+1}). \quad (6.5)$$

For implementation details, see Appendix D.3.

We plot two cases in Figure 8: in Panel (e), we plot the case in which the central bank cares about the volatility of the interest rate as much as that of the output gap, $\lambda_i = \lambda$; in Panel (f), we show a case in which the central bank puts a heavier weight on the interest rate, $\lambda_i = 100\lambda$. Note the baseline case in Figure 6 corresponds to $\lambda_i = 0$.

Comparing the two cases, the order of lines with different colors changes for the case of clear communication. More importantly, our main result that ambiguous communication leads to higher central bank credibility is robust with any λ_i .

6.4 Persistent Cost-Push Shock

In the baseline model, we assume the cost-push shock is temporary. In this section, we relax this assumption and allow a more general case with potentially a persistent cost-push shock. Specifically,

$$u_t = \rho u_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2). \quad (6.6)$$

The temporary shock is a special case with $\rho = 0$.

In the general case, the Phillips curve under AIT becomes

$$\pi_t = \frac{\kappa}{1 - \beta a_\pi} \hat{y}_t + \frac{\beta b_\pi \rho + 1}{1 - \beta a_\pi} u_t, \quad (6.7)$$

where the intercept depends on b_π , which further depends on the value of λ^{cb} .

We repeat the exercise in Section 5.2 by increasing ρ , and we set $\rho = 0.8$ for an annual calibration. Note we keep the variance of the cost-push shock, $\sigma_u^2 = \frac{\sigma_\epsilon^2}{1-\rho^2}$, as well as $\lambda^{cb}(L)$ the same as in the baseline model. Comparing the colored dashed lines in the left panel of Figure 8(d) with the solid black line in the right panel, we find ambiguous communication keeps a higher fraction of followers compared to clear communication. Moreover, Panel (a) of Figure 9 shows ambiguous communication improves welfare. Both of these results are consistent with our baseline model. Interestingly, comparing clear communication with ambiguous communication, although a larger ρ yields a smaller difference in AIT followers, it gives rise to a larger difference in welfare.

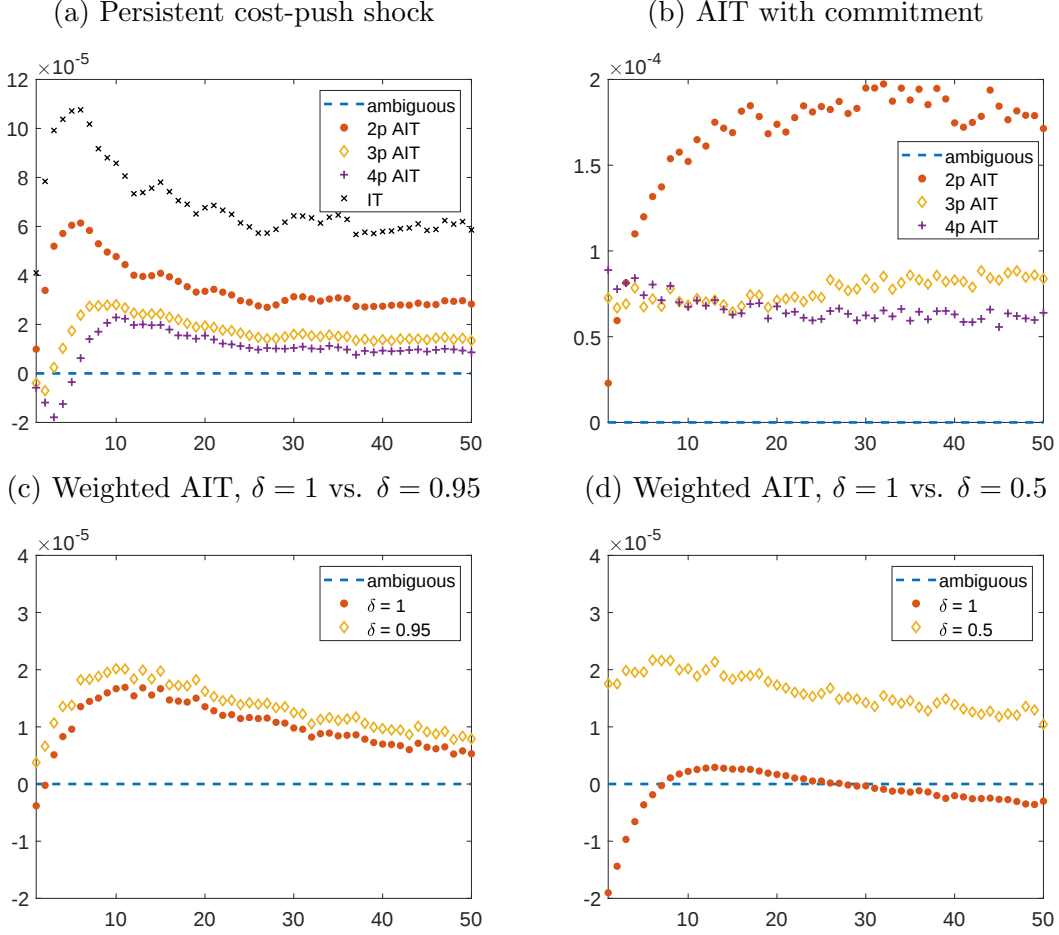
6.5 Weight in Nowcast Error

Panels (a) and (b) of Figure 8 inspect how the weight agents put on their nowcast errors plays a role in our result. In the long run, the central bank always arrives with more credibility when using ambiguous communication than when using clear communication; compare the black lines in the right panels with the colored lines in the left panels. This result further confirms that the qualitative result in Figure 6 does not depend on how agents weigh nowcast errors between inflation and the output gap.

6.6 AIT with Commitment

Up to this point in Section 6, the central bank always announces AIT but implements IT ex post per the argument of Proposition 4. What if the central bank is concerned about its reputation and therefore commits to implementing AIT ex post? This section investigates

Figure 9: Welfare loss: Robustness



Notes: X-axis: horizon. y-axis: welfare loss. All the results are averaged over 1,000 simulations.

our model implications for such a case.

In Panel (g) of Figure 8, We compare the fraction of AIT believers under clear communication (in the left panel) with that under ambiguous communication (in the right panel). The three different line styles capture $L = 2, 3, 4$. Clear communication means the central bank announces and implements L -period AIT. For ambiguous communication, the central bank announces AIT without specifying its horizon and implements L -period AIT. Regardless of whether communication is clear or ambiguous, the central minimizes the period loss of the AIT objective in (3.2). For implementation details, see Appendix D.4.

Figure 8(g) shows that regardless of L , ambiguous communication always results in more

followers when the central bank is under commitment, which serves as a robustness check of our main result. If we compare Figure 8(g) with Figure 6, we find committing to AIT gains credibility over the time inconsistent strategy. However, does higher credibility resulting from commitment necessarily lead to higher welfare? We answer this question with Panel (b) of Figure 9. The blue dashed line is our baseline, where the central bank announces AIT without specifying the horizon but implements IT ex post. Similar to Figure 7, we normalize its welfare loss to zero. The other colored lines capture a central bank that makes a clear announcement of $L = 2, 3, 4$, and keeps its promise. Figure 9(b) shows a central bank under commitment always has a larger welfare loss compared with the baseline, and the differences capture the “reputation cost.” This result highlights the time inconsistent nature of AIT discussed in Section 4.

7 Other Aspects of AIT Communication

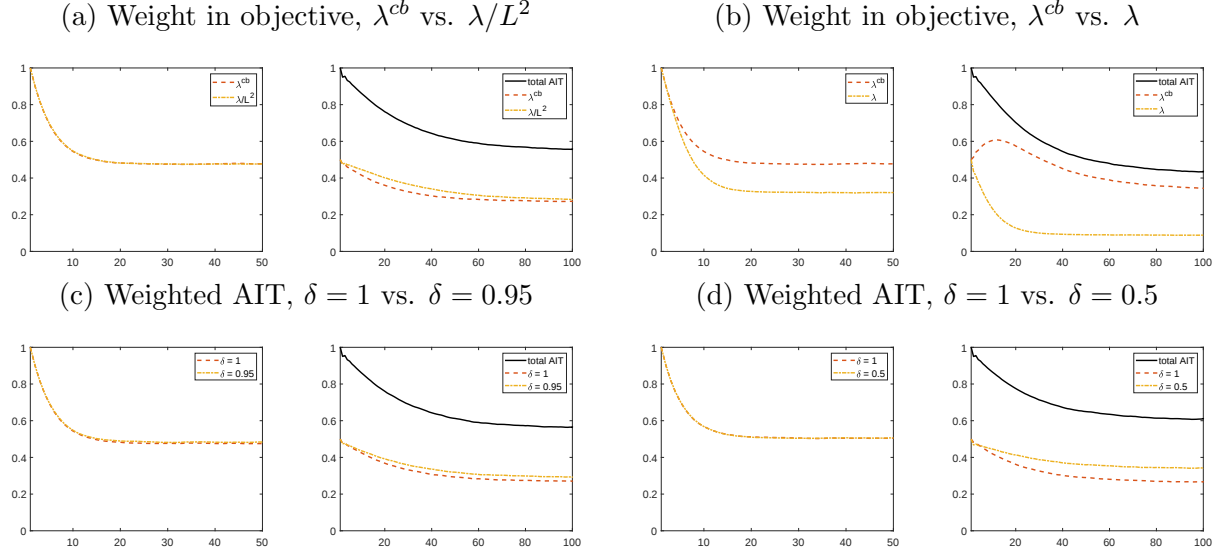
Sections 5 and 6 focus on the central bank’s communication about AIT horizon. This section explores two additional aspects. In this section, we abstract from the horizon by fixing it at $L = 2$.

7.1 Weight between Inflation and Output

First, we discuss how the central bank communicates about its relative weight between inflation and output, or λ^{cb} in (3.2). So far, we have assumed λ^{cb} is chosen optimally by minimizing the unconditional welfare loss in (3.3) and its value is known to the private sector. In this section, we allow the central bank to communicate and agents to believe different values for this weight.

In Panel (a) of Figure 10, AIT followers’ belief set includes $\{\lambda^{cb}, \lambda/L^2\}$. The former is the optimal weight and the latter mimics how $\lambda^{cb}(L)$ depends on L , and they are slightly different; for details, see Appendix C.2. The fractions of believers for both weights are very close; see

Figure 10: Fraction of AIT believers: Other aspects



Notes: X-axis: horizon. Each panel is structured similarly to Figure 6, where the left plot captures clear communication, and the right plot corresponds to ambiguous communication. Black solid line in the right panel: fraction of total AIT believers. All the results are averaged over 1,000 simulations.

the red dashed lines and yellow dash-dotted lines. More importantly, consistent with our main result, ambiguous communication gains more credibility than clear communication in the context of the central bank's weight.

The Panel (b) of Figure 10 illustrates a case in which our main result that ambiguous communication accumulates more followers no longer holds. In this case, agents can either believe the optimal λ^{cb} or the weight in the social welfare function, λ , without adjusting by the horizon. In this case, the yellow dash-dotted line always has the least amount of believers. Why does our result break in this scenario? For agents who believe the weight λ , their nowcast error is much larger than any other group; see Figure F.3. This is because when using the social weight, the relative weight between current inflation and the output gap is different by a factor of 4 between the central bank's objective function in (3.2) and the social welfare. Therefore, in the case of ambiguous communication, AIT believers as a whole have a larger nowcast error. This demonstrates that our result may break down when AIT believers can potentially have extreme beliefs.

7.2 Weighted Average Inflation Targeting

So far, the average inflation is calculated as a simple average. In this section, we allow a weighted average by modifying (3.2) as follows:

$$\mathbb{L}_t^{cb} = \frac{1}{2} \left(\left(\frac{\pi_t + \delta \pi_{t-1}}{2} \right)^2 + \lambda^{cb} \hat{y}_t^2 \right) + \beta \mathbb{E}_t \mathbb{L}_{t+1}^{cb}, \quad (7.1)$$

where δ is the decay parameter on lagged inflation. When $\delta = 1$, the objective coincides with our baseline AIT objective in (3.2). $\delta = 0$ corresponds to the IT case.

In Panel (c) of Figure 10, AIT followers' beliefs are $\delta = 1$ and $\delta = \beta = 0.95$. For Panel (d), $\delta \in \{0.5, 1\}$. For both panels, we compare clear communication in the left plots with ambiguous communication in the right plots. The fraction of agents who believe $\delta = 1$ is captured by the red dashed lines and the fraction of those who believe a smaller δ is plotted in yellow dash-dotted lines. Panels (c) and (d) display a common result. When communication regarding δ is clear, different announcements yield a similar number of followers. More importantly, the total fraction of AIT followers under ambiguous communication (the black solid lines in the right plots) is higher than either case under clear communication (the colored dashed lines in the left plots). This again corroborates our main result that ambiguous communication helps the central bank build credibility even with a different dimension of the central bank communication.

Next, we compare welfare between ambiguous communication and clear communication in Figure 9. Panel (c) shows that ambiguous communication yields the lowest welfare loss when $\delta \in \{0.95, 1\}$, which is consistent with our main result.⁸

Interestingly, Panel (d) reports that when $\delta \in \{0.5, 1\}$, ambiguous communication does not yield the best welfare. This result illustrates that not only the total fraction of AIT believers matters, but also its composition. The Phillips curve is more favorable with a larger δ . The Phillips curves for $\delta = 1$ and $\delta = 0.95$ are close to each other; whereas the

⁸When we extend the plot out to 100 periods, the fractions of AIT believers converge, and the same conclusion holds.

Phillips curve for $\delta = 0.5$ is closer to the IT Phillips curve than that for $\delta = 1$; for details, see Figure F.4. Therefore, compared to clear communication, although ambiguous communication yields more AIT followers overall, the followers with $\delta = 0.5$ do not compensate for the welfare loss due to a smaller fraction of $\delta = 1$.

8 Conclusion

Our paper studies the implications of AIT. We focus on two key issues: time inconsistency and ambiguous communication. AIT can improve the available trade-off between inflation and real activity that the central bank faces as captured by the Phillips curve. To improve social welfare, the central bank has the incentive to deviate from its communicated objective and implement IT ex post. The time-inconsistent strategy is welfare improving, assuming the central bank can convince the private sector of its intention to implement AIT. We assess this assumption using social learning. We show ambiguous communication helps the central bank gain credibility and improve welfare in the long run despite AIT being time inconsistent. We demonstrate this result applies to several key aspects of AIT announcements and does not rely on specific modeling assumptions.

Although AIT was introduced as a policy tool to combat the ZLB and its associated low inflation expectations, our paper shows this policy can also help the central bank manage inflation expectations in the current environment of high inflation. In addition, ambiguous communication can enhance the central bank's credibility and hence improve social welfare in the long run.

References

- Amano, Robert, Stefano Gnocchi, Sylvain Leduc, and Joel Wagner. 2020. “Average Is Good Enough: Average-Inflation Targeting and the ELB.” Federal Reserve Bank of San Francisco Working Paper 2020-21.
- Andrade, Philippe, Jordi Galí, Hervé Le Bihan, and Julien Matheron. 2021. “Should the ECB Adjust its Strategy in the Face of a Lower r^* ?” CREI working paper.
- Arifovic, Jasmina, James Bullard, and Olena Kostyshyna. 2013. “Social Learning and Monetary Policy Rules.” *The Economic Journal* 123 (567):38–76.
- Ball, Laurence and Sandeep Mazumder. 2011. “Inflation dynamics and the great recession.” *Brookings Papers on Economic Activity* :337–406.
- Bohren, J. Aislinn and Daniel N. Hauser. 2021. “Learning with Heterogeneous Misspecified Models: Characterization and Robustness.” *Econometrica* 89:3025–3077.
- Brainard, Lael. 2020. “Bringing the Statement on Longer-Run Goals and Monetary Policy Strategy into Alignment with Longer-Run Changes in the Economy.” Hutchins Center on Fiscal and Monetary Policy at the Brookings Institution. <https://www.federalreserve.gov/newsevents/speech/brainard20200901a.htm>.
- Brunnermeier, Markus. 2021. “Averaging and Anchoring: Is there an Inflation Dilemma?” Jackson Hole Economic Policy Symposium “Macroeconomic Policy in an Uneven Economy”.
- Budianto, Flora, Taisuke Nakata, and Sebastian Schmidt. 2020. “Average inflation targeting and the interest rate lower bound.” .
- Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo. 2016. “Understanding booms and busts in housing markets.” *Journal of Political Economy* 124 (4):1088–1147.

- Clarida, Richard, Jordi Galí, and Mark Gertler. 1999. “The science of monetary policy: a new Keynesian perspective.” *Journal of economic literature* 37 (4):1661–1707.
- Clarida, Richard H. 2020. “The Federal Reserve’s New Monetary Policy Framework: A Robust Evolution.” Peterson Institute for International Economics. <https://www.federalreserve.gov/newsevents/speech/clarida20200831a.htm>.
- Coibion, Olivier and Yuriy Gorodnichenko. 2015. “Is the Phillips curve alive and well after all? Inflation expectations and the missing disinflation.” *American Economic Journal: Macroeconomics* 7 (1):197–232.
- Coibion, Olivier, Yuriy Gorodnichenko, Edward S Knotek II, and Raphael Schoenle. 2020. “Average inflation targeting and household expectations.” National Bureau of Economic Research Working Paper 27836.
- Galí, Jordi. 2015. *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press.
- Hachem, Kinda and Jing Cynthia Wu. 2017. “Inflation announcements and social dynamics.” *Journal of Money, Credit and Banking* 49 (8):1673–1713.
- Hazell, Jonathon, Juan Herreno, Emi Nakamura, and Jón Steinsson. 2020. “The slope of the Phillips Curve: evidence from US states.” Tech. rep., National Bureau of Economic Research.
- Honkapohja, Seppo and Nigel McClung. 2021. “On robustness of average inflation targeting.” *Bank of Finland Research Discussion Paper, No. 6* .
- Kozicki, Sharon. 2019. “Monetary Policy Strategies for the Federal Reserve: Discussion of Practical Considerations.” A Fed Listens Event “Conference on Monetary Policy Strategy, Tools, and Communication Practices”.

- Mertens, Thomas M and John C. Williams. 2019a. “Monetary policy frameworks and the effective lower bound on interest rates.” In *AEA Papers and Proceedings*, vol. 109. 427–32.
- Mertens, Thomas M. and John C. Williams. 2019b. “Tying down the anchor: monetary policy rules and the lower bound on interest rates.” *FRB of New York Staff Report* (887).
- Nessén, Marianne and David Vestin. 2005. “Average inflation targeting.” *Journal of Money, credit and Banking* 37 (5):837–863.
- Oudiz, Gilles and Jeffrey Sachs. 1985. “International policy coordination in dynamic macroeconomic models.” In *International economic policy coordination*. Cambridge University Press, 274–330.
- Papell, David H and Ruxandra Prodan. 2020. “Policy Rule Forward Guidance Following the COVID-19 Recession.”
- Powell, Jerome H. 2020. “New Economic Challenges and the Fed’s Monetary Policy Review.” Jackson Hole Economic Policy Symposium “Navigating the Decade Ahead: Implications for Monetary Policy”. <https://www.federalreserve.gov/newsevents/speech/powell20200827a.htm>.
- Rogoff, Kenneth. 1985. “The optimal degree of commitment to an intermediate monetary target.” *The quarterly journal of economics* 100 (4):1169–1189.
- Rotemberg, Julio J and Michael Woodford. 1999. “Interest rate rules in an estimated sticky price model.” In *Monetary policy rules*. University of Chicago Press, 57–126.
- Stock, James H and Mark W Watson. 2020. “Slack and cyclically sensitive inflation.” *Journal of Money, Credit and Banking* 52 (S2):393–428.
- Svensson, Lars E. O. 2007. “Optimization under commitment and discretion, the recursive saddlepoint method, and targeting rules and instrument rules: Lecture notes.”

- Svensson, Lars E.O. 2020. “Monetary Policy Strategies for the Federal Reserve.” *International Journal of Central Banking* 16 (1):133–193.
- Waller, Christopher. 2022. “Lessons Learned on Normalizing Monetary Policy.” Dallas Society for Computational Economics. <https://www.federalreserve.gov/newsevents/speech/waller20220618a.htm>.
- Woodford, Michael. 2011. *Interest and prices: Foundations of a theory of monetary policy*. princeton university press.

Appendices

A Two-Period AIT

A.1 Two-Period AIT under FIRE

We solve the equilibrium with the method of undetermined coefficients. We guess the following linear functions for π_t and $\hat{y}_t$:

$$\pi_t = a_\pi \pi_{t-1} + b_\pi u_t \quad (\text{A.1})$$

$$\hat{y}_t = a_y \pi_{t-1} + b_y u_t, \quad (\text{A.2})$$

which yield

$$\mathbb{E}_t \pi_{t+j} = a_\pi^j \pi_t \quad (\text{A.3})$$

$$\mathbb{E}_t \hat{y}_{t+j} = a_y a_\pi^{j-1} \pi_t. \quad (\text{A.4})$$

Substitute the expression for $\mathbb{E}_t \pi_{t+1}$ from (A.3) into the Phillips curve in (3.4) and rearrange, and we get (3.13). Consequently, the slope of the Phillips curve is

$$\frac{\partial \pi_t}{\partial \hat{y}_t} = \frac{\kappa}{1 - \beta a_\pi}. \quad (\text{A.5})$$

Rearranging (3.13), we get

$$\hat{y}_t = \frac{1}{\kappa} (1 - \beta a_\pi) \pi_t - \frac{1}{\kappa} u_t. \quad (\text{A.6})$$

Rewrite the central bank's objective function in (3.2) by substituting out the expectation terms using (A.3) - (A.4):

$$\begin{aligned} \mathbb{L}_t^{cb}(2) &= \frac{1}{2} \left(\left(\frac{\pi_t + \pi_{t-1}}{2} \right)^2 + \lambda^{cb} \hat{y}_t^2 \right) + \frac{1}{2} \sum_j \mathbb{E}_t \beta^j \left(\left(\frac{\pi_{t+j} + \pi_{t+j-1}}{2} \right)^2 + \lambda^{cb} \hat{y}_{t+j}^2 \right) \\ &= \frac{1}{2} \left(\left(\frac{\pi_t + \pi_{t-1}}{2} \right)^2 + \lambda^{cb} \hat{y}_t^2 \right) + \frac{1}{2} \pi_t^2 \sum_j \beta^j \left(\left(\frac{a_\pi + 1}{2} \right)^2 a_\pi^{2(j-1)} + \lambda^{cb} a_y^2 a_\pi^{2(j-1)} \right). \end{aligned} \quad (\text{A.7})$$

Minimize the central bank's objective function with respect to $\hat{y}_t$, which yields the following first-order condition (using (A.5) - (A.6)):

$$\begin{aligned} 0 &= \frac{1}{4} (\pi_t + \pi_{t-1}) \frac{\kappa}{1 - \beta a_\pi} + \lambda^{cb} \left(\frac{1}{\kappa} (1 - \beta a_\pi) \pi_t - \frac{1}{\kappa} u_t \right) \\ &\quad + \pi_t \frac{\kappa}{1 - \beta a_\pi} \sum_j \beta^j \left(\left(\frac{a_\pi + 1}{2} \right)^2 a_\pi^{2(j-1)} + \lambda^{cb} a_y^2 a_\pi^{2(j-1)} \right). \end{aligned} \quad (\text{A.8})$$

Rearrange

$$0 = \left[\frac{\kappa}{4(1-\beta a_\pi)} + \lambda^{cb} \frac{1}{\kappa} (1-\beta a_\pi) + \frac{\kappa}{1-\beta a_\pi} \sum_j \beta^j \left(\left(\frac{a_\pi+1}{2} \right)^2 a_\pi^{2(j-1)} + \lambda^{cb} a_y^2 a_\pi^{2(j-1)} \right) \right] \pi_t \quad (\text{A.9})$$

$$+ \frac{\kappa}{4(1-\beta a_\pi)} \pi_{t-1} - \lambda^{cb} \frac{1}{\kappa} u_t.$$

Define

$$\vartheta \equiv \left[\frac{\kappa}{4(1-\beta a_\pi)} + \lambda^{cb} \frac{1}{\kappa} (1-\beta a_\pi) + \frac{\kappa}{1-\beta a_\pi} \sum_j \beta^j \left(\left(\frac{a_\pi+1}{2} \right)^2 a_\pi^{2(j-1)} + \lambda^{cb} a_y^2 a_\pi^{2(j-1)} \right) \right]^{-1} \quad (\text{A.10})$$

then,

$$\pi_t = -\frac{\kappa}{4(1-\beta a_\pi)} \vartheta \pi_{t-1} + \lambda^{cb} \frac{1}{\kappa} \vartheta u_t. \quad (\text{A.11})$$

Comparing it with (A.1), we get (3.7) and (3.9).

Next, substituting (A.1) in (A.6) and rearranging,

$$\hat{y}_t = \frac{1}{\kappa} (1-\beta a_\pi) a_\pi \pi_{t-1} + \left(\frac{1}{\kappa} (1-\beta a_\pi) b_\pi - \frac{1}{\kappa} \right) u_t. \quad (\text{A.12})$$

Comparing it with (A.2), we obtain (3.8) and (3.10).

A.2 IT

The central bank minimizes the period loss of welfare in (3.1) with respect to $\hat{y}_t$, which results in the following first-order condition:

$$\pi_t \frac{\partial \pi_t}{\partial \hat{y}_t} + \lambda \hat{y}_t = 0. \quad (\text{A.13})$$

The constraint is the IT Phillips curve in (3.14). Using the Phillips curve to substitute out $\frac{\partial \pi_t}{\partial \hat{y}_t}$ and $\hat{y}_t$ in (A.13), we obtain

$$\kappa \pi_t + \frac{\lambda}{\kappa} (\pi_t - u_t) = 0, \quad (\text{A.14})$$

which is equivalent to (3.11). Combining it with (3.14) to cancel out π_t , we get (3.12).

A.3 Two-Period AIT under FIRE with Natural Rate Shock

This appendix outlines an economy that faces both natural rate and cost-push shocks. The IS curve and the Phillips curve that characterize the economy are

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\gamma} (i_t - r_t^n - \mathbb{E}_t \pi_{t+1}) \quad (\text{A.15})$$

$$\pi_t = \beta \mathbb{E} \pi_{t+1} + \kappa \hat{y}_t + u_t. \quad (\text{A.16})$$

We conjecture the following equilibrium:

$$\pi_t = a_\pi \pi_{t-1} + b_\pi u_t + b_\pi^r r_t^n \quad (\text{A.17})$$

$$\hat{y}_t = a_y \pi_{t-1} + b_y u_t + b_y^r r_t^n, \quad (\text{A.18})$$

which leads to

$$\mathbb{E}_t \pi_{t+j} = a_\pi^j \pi_t \quad (\text{A.19})$$

$$\mathbb{E}_t \hat{y}_{t+j} = a_y a_\pi^{j-1} \pi_t. \quad (\text{A.20})$$

Because the expectations in (A.19) - (A.20) and the Phillips curve in (A.16) are the same as those in Appendix A.1, the remaining derivations in Appendix A.1 follow through and we find expressions for a_π, a_y, b_π, b_y as in (3.7) - (3.10).

Moreover, we obtain $b_\pi^r = 0$ and $b_y^r = 0$. Suppose the natural rate shock is the only shock in the economy, $\pi_{t+h} = \hat{y}_{t+h} = 0$ for any h . This equilibrium path can be achieved by setting the policy rate to track the natural rate:

$$i_t = r_t^n. \quad (\text{A.21})$$

A.4 Two-Period AIT with Time Inconsistency

The central bank minimizes the period loss of welfare in (3.1) with respect to $\hat{y}_t$, which results in the first-order condition in (A.13). The private sector believes AIT announcement. Therefore, the Phillips curve is given by (3.13). Substituting (A.5) - (A.6) in, we obtain

$$\frac{\kappa}{1 - \beta a_\pi} \pi_t + \frac{\lambda}{\kappa} (1 - \beta a_\pi) \pi_t - \frac{\lambda}{\kappa} u_t = 0, \quad (\text{A.22})$$

which is equivalent to (4.1). Substituting (4.1) into (A.6),

$$\hat{y}_t = \frac{1}{\kappa} (1 - \beta a_\pi) \left(\frac{\kappa}{1 - \beta a_\pi} + \lambda \frac{1 - \beta a_\pi}{\kappa} \right)^{-1} \frac{\lambda}{\kappa} u_t - \frac{1}{\kappa} u_t, \quad (\text{A.23})$$

which is equivalent to (4.2).

B Proofs

B.1 Proof of Lemma 1

We first prove $a_\pi < 0$ by contradiction. Suppose $a_\pi > 0$. For the system to have a stationary solution, $0 < a_\pi < 1$. Together with $0 < \beta < 1$, this implies $1 - \beta a_\pi > 0$. Together with $\kappa > 0$ and $\lambda^{cb} > 0$, this implies $\vartheta > 0$ in (A.10). Consequently, $a_\pi = -\vartheta \frac{\kappa}{4(1-\beta a_\pi)} < 0$, and is thus a contradiction. We have proved $a_\pi < 0$, which implies $1 - \beta a_\pi > 0$, and it follows that $a_y = \frac{1}{\kappa} (1 - \beta a_\pi) a_\pi < 0$. ■

B.2 Proof of Proposition 1

The expectation term in the forward-looking Phillips curve (3.4) is $\mathbb{E}_t \pi_{t+1} = a_\pi \pi_t$ for AIT per (3.5), and $\mathbb{E}_t \pi_{t+1} = 0$ for IT per (3.11). Lemma 1 shows $a_\pi < 0$. Therefore, $a_\pi \pi_t < 0$ when $\pi_t > 0$ and $a_\pi \pi_t > 0$ when $\pi_t < 0$. ■

B.3 Proof of Proposition 2

The slope of the Phillips curve of AIT in (3.13) is $\frac{\kappa}{1-\beta a_\pi}$, and that of the IT Phillips curve in (3.14) is κ . We have shown $a_\pi < 0$ in Lemma 1. Therefore, $1 - \beta a_\pi > 1$, and hence, $\frac{\kappa}{1-\beta a_\pi} < \kappa$. ■

B.4 Proof of Proposition 3

The AIT Phillips curve in (3.13) can be written as

$$(1 - \beta a_\pi) \pi_t^{AIT} = \kappa \hat{y}_t + u_t,$$

where we label inflation under AIT π_t^{AIT} . Compare it with the IT Phillips curve in (3.14):

$$\pi_t^{IT} = \kappa \hat{y}_t + u_t.$$

Given $\hat{y}_t$ and u_t ,

$$(1 - \beta a_\pi) \pi_t^{AIT} = \pi_t^{IT}.$$

We have shown $a_\pi < 0$ in Lemma 1. Therefore, $1 - \beta a_\pi > 1$, and

$$|\pi_t^{IT}| > |\pi_t^{AIT}|.$$

■

B.5 Proof of Lemma 2

The y -intercept of the Phillips curve of AIT in (3.13) is $\frac{1}{1-\beta a_\pi} u_t$, and that of the IT Phillips curve in (3.14) is u_t . We have shown $a_\pi < 0$ in Lemma 1. Therefore, $1 - \beta a_\pi > 1$, and hence, $\frac{1}{1-\beta a_\pi} |u_t| < |u_t|$ for $u_t \neq 0$.

Next, the x -intercept of the Phillips curve of AIT in (3.13) is characterized by $0 = \frac{\kappa}{1-\beta a_\pi} \hat{y}_t + \frac{1}{1-\beta a_\pi} u_t$, or $0 = \kappa \hat{y}_t + u_t$. For the IT Phillips curve in (3.14), it is also $0 = \kappa \hat{y}_t + u_t$. Hence, they have the same x -intercept. ■

B.6 Proof of Lemma 3

The AIT Phillips curve in (3.13) is

$$\pi_t = \frac{\kappa}{1 - \beta a_\pi} \hat{y}_t,$$

when $u_t = 0$. When $\hat{y}_t = 0$, $\pi_t = 0$.

Similarly, the IT Phillips curve in (3.13) is

$$\pi_t = \kappa \hat{y}_t,$$

when $u_t = 0$. When $\hat{y}_t = 0$, $\pi_t = 0$. ■

B.7 Proof of Proposition 5

We prove by contradiction. Assume a time consistent objective function exists that yields the equilibrium in (4.1) - (4.2). We begin with the Phillips curve in (3.4). The equilibrium in (4.1) is time consistent. Therefore, agents form expectations based on (4.1), and the reduced-form Phillips curve is the same as the one for IT in (3.14).

The central bank minimizes (4.3) with respect to $\hat{y}_t$. The first order condition is similar to (A.13) but with a different λ :

$$\pi_t \frac{\partial \pi_t}{\partial \hat{y}_t} + \tilde{\lambda} \hat{y}_t = 0. \quad (\text{B.1})$$

Using the Phillips curve in (3.14) to substitute out $\frac{\partial \pi_t}{\partial \hat{y}_t}$ and $\hat{y}_t$, we obtain

$$\kappa \pi_t + \frac{\tilde{\lambda}}{\kappa} (\pi_t - u_t) = 0, \quad (\text{B.2})$$

Therefore, similar to (3.11) - (3.12), we get

$$\pi_t = \left(\kappa + \frac{\tilde{\lambda}}{\kappa} \right)^{-1} \frac{\tilde{\lambda}}{\kappa} u_t \quad (\text{B.3})$$

$$\hat{y}_t = \left[\frac{1}{\kappa} \left(\kappa + \frac{\tilde{\lambda}}{\kappa} \right)^{-1} \frac{\tilde{\lambda}}{\kappa} - \frac{1}{\kappa} \right] u_t. \quad (\text{B.4})$$

To equate (B.3) with (4.1), we can solve $\tilde{\lambda}$ via

$$\left(\frac{\kappa}{1 - \beta a_\pi} + \lambda \frac{1 - \beta a_\pi}{\kappa} \right)^{-1} \frac{\lambda}{\kappa} = \left(\kappa + \frac{\tilde{\lambda}}{\kappa} \right)^{-1} \frac{\tilde{\lambda}}{\kappa}. \quad (\text{B.5})$$

However,

$$\begin{aligned}
& \frac{1}{\kappa} \left(\kappa + \frac{\tilde{\lambda}}{\kappa} \right)^{-1} \frac{\tilde{\lambda}}{\kappa} - \frac{1}{\kappa} \\
&= \frac{1}{\kappa} \left(\frac{\kappa}{1 - \beta a_\pi} + \lambda \frac{1 - \beta a_\pi}{\kappa} \right)^{-1} \frac{\lambda}{\kappa} - \frac{1}{\kappa} \\
&\neq \left(\frac{\kappa}{1 - \beta a_\pi} + \lambda \frac{1 - \beta a_\pi}{\kappa} \right)^{-1} \frac{\lambda}{\kappa} \frac{1 - \beta a_\pi}{\kappa} - \frac{1}{\kappa}.
\end{aligned} \tag{B.6}$$

Or, the equilibrium in (B.4) is different from (4.2). Hence the contradiction. ■

C Multi-Period AIT

C.1 Solving Equilibrium under FIRE

We solve multi-period AIT following the standard solution method of rational-expectation models with linear-quadratic optimization, which is developed in Oudiz and Sachs (1985) and further discussed in Svensson (2007).

The equilibrium in the private sector, which serves as the constraint of the central bank's optimization problem, is given by

$$\begin{bmatrix} X_{t+1} \\ \mathbb{E}_t x_{t+1} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + B i_t + C \epsilon_{t+1}, \tag{C.1}$$

where X_t are the predetermined state variables, x_t are the forward-looking variables, and i_t are instruments.

Take conditional expectation of equation (C.1) at t :

$$\begin{bmatrix} \mathbb{E}_t X_{t+1} \\ \mathbb{E}_t x_{t+1} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + B i_t. \tag{C.2}$$

Then, we conjecture that

$$i_{t+1} = F_{t+1} X_{t+1} \tag{C.3}$$

$$x_{t+1} = G_{t+1} X_{t+1}, \tag{C.4}$$

where F_{t+1} and G_{t+1} are determined by the problem in period $t + 1$ but are known in period t .

Then, (C.4) and the upper block of (C.2) imply

$$\mathbb{E}_t x_{t+1} = G_{t+1} \mathbb{E}_t X_{t+1} = G_{t+1} (A_{11} X_t + A_{12} x_t + B_1 i_t), \tag{C.5}$$

where

$$A \equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B \equiv \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad C \equiv \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}. \quad (\text{C.6})$$

The lower block is

$$\mathbb{E}_t x_{t+1} = A_{21}X_t + A_{22}x_t + B_2i_t. \quad (\text{C.7})$$

Suppose

$$x_t = \bar{A}_t X_t + \bar{B}_t i_t. \quad (\text{C.8})$$

Equating (C.5) and (C.7) and using (C.8), we get

$$\bar{A}_t = (A_{22} - G_{t+1}A_{12})^{-1}(G_{t+1}A_{11} - A_{21}) \quad (\text{C.9})$$

$$\bar{B}_t = (A_{22} - G_{t+1}A_{12})^{-1}(G_{t+1}B_1 - B_2). \quad (\text{C.10})$$

The upper block of (C.1) and (C.8) imply

$$X_{t+1} = \tilde{A}_t X_t + \tilde{B}_t i_t + C_1 \epsilon_{t+1}, \quad (\text{C.11})$$

where

$$\tilde{A}_t = A_{11} + A_{12}\bar{A}_t \quad (\text{C.12})$$

$$\tilde{B}_t = B_1 + A_{12}\bar{B}_t. \quad (\text{C.13})$$

The period loss in the central bank's objective function is given by

$$L_t = \frac{1}{2} \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}' W \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}. \quad (\text{C.14})$$

Use (C.8), and we obtain

$$\mathcal{L}_t = \frac{1}{2} \begin{bmatrix} X_t \\ i_t \end{bmatrix}' \begin{bmatrix} Q_t & N_t \\ N_t' & R_t \end{bmatrix} \begin{bmatrix} X_t \\ i_t \end{bmatrix}, \quad (\text{C.15})$$

where

$$Q_t = W_{XX} + W_{Xx}\bar{A}_t + \bar{A}_t' W_{xX} + \bar{A}_t' W_{xx} \bar{A}_t \quad (\text{C.16})$$

$$N_t = W_{Xx}\bar{B}_t + \bar{A}_t' W_{xx} \bar{B}_t + W_{Xi} + \bar{A}_t' W_{xi} \quad (\text{C.17})$$

$$R_t = W_{ii} + \bar{B}_t' W_{xx} \bar{B}_t + \bar{B}_t' W_{xi} + W_{ix} \bar{B}_t, \quad (\text{C.18})$$

and

$$W \equiv \begin{bmatrix} W_{XX} & W_{Xx} & W_{Xi} \\ W_{xX} & W_{xx} & W_{xi} \\ W_{iX} & W_{ix} & W_{ii} \end{bmatrix}. \quad (\text{C.19})$$

Since the period loss function is quadratic and the constraint is linear, the value function is quadratic and satisfies the following Bellman equation:

$$\frac{1}{2} [(1 - \beta) X_t' V_t X_t + \beta w_t] = \min_{i_t} \left\{ (1 - \beta) L_t + \beta \mathbb{E}_t \frac{1}{2} [(1 - \beta) X_{t+1}' V_{t+1} X_{t+1} + \beta w_{t+1}] \right\}, \quad (\text{C.20})$$

where V_{t+1} is a positive semidefinite matrix and w_{t+1} is a scalar, and both are known at time t .

Solve the first-order condition subject to (C.11) and (C.14):

$$0 = X_t' N_t + i_t' R_t + \beta \left(X_t' \tilde{A}_t' V_{t+1} \tilde{B}_t + i_t' \tilde{B}_t' V_{t+1} \tilde{B}_t \right). \quad (\text{C.21})$$

Therefore,

$$i_t = F_t X_t, \quad (\text{C.22})$$

which is consistent with the conjecture in (C.3) and

$$F_t = - \left(R_t + \beta \tilde{B}_t' V_{t+1} \tilde{B}_t \right)^{-1} \left(N_t' + \beta \tilde{B}_t' V_{t+1} \tilde{A}_t \right). \quad (\text{C.23})$$

Substitute i_t in (C.8) with (C.22) and get

$$x_t = G_t X_t, \quad (\text{C.24})$$

which is consistent with the conjecture in (C.4) and

$$G_t = \bar{A}_t + \bar{B}_t F_t. \quad (\text{C.25})$$

Substituting (C.11), (C.14), and (C.22) into (C.20), we get

$$V_t = Q_t + N_t F_t + F_t' N_t' + F_t' R_t F_t + \beta \left(\tilde{A}_t + \tilde{B}_t F_t \right)' V_{t+1} \left(\tilde{A}_t + \tilde{B}_t F_t \right). \quad (\text{C.26})$$

(C.9), (C.10), (C.12), (C.13), (C.16) - (C.18), (C.23), (C.25), and (C.26) define a mapping from (G_{t+1}, V_{t+1}) to (G_t, V_t) , which also determine F_t .

Iterate the coefficients backward from (G_{t+1}, V_{t+1}) to (G_t, V_t) until they converge to a fixed point:

$$G = G_{t+1} = G_t \quad (\text{C.27})$$

$$V = V_{t+1} = V_t. \quad (\text{C.28})$$

Finally, the equilibrium conditions are

$$x_t = G X_t. \quad (\text{C.29})$$

C.1.1 Example: $L = 3$

The state variables and forward-looking variables are

$$X_t = \begin{bmatrix} u_t \\ \pi_{t-2} \\ \pi_{t-1} \end{bmatrix}, \quad x_t = \begin{bmatrix} \pi_t \\ \hat{y}_t \end{bmatrix}, \quad (\text{C.30})$$

and there is only one policy instrument in i_t , and $\epsilon_t = u_t$.

Write out linear system

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \frac{1}{\gamma} & 1 \end{bmatrix} \begin{bmatrix} u_{t+1} \\ \pi_{t-1} \\ \pi_t \\ \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t \hat{y}_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & -\kappa \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_t \\ \pi_{t-2} \\ \pi_{t-1} \\ \pi_t \\ \hat{y}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\gamma} \end{bmatrix} i_t + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_{t+1}. \quad (\text{C.31})$$

Map it into (C.1),

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \frac{1}{\gamma} & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & -\kappa \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \frac{1}{\gamma} & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\gamma} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \frac{1}{\gamma} & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Next, the objective function maps into (C.14) with

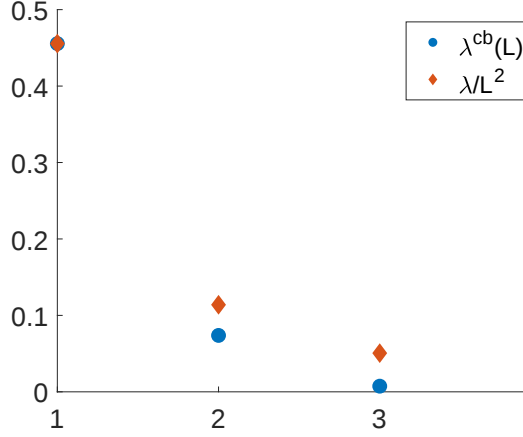
$$W = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 & 0 \\ 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 & 0 \\ 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda^{cb}(L=3) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (\text{C.32})$$

C.2 Optimal λ^{cb}

To solve the unconditional variances in (3.3), we write the equilibrium in (3.5) - (3.6) into the companion form:

$$X_t = \Phi X_{t-1} + \Sigma u_t, \quad (\text{C.33})$$

Figure C.1: $\lambda^{cb}(L)$ vs λ/L^2



Notes: X-axis: L . Blue dots: $\lambda^{cb}(L)$; red diamonds: λ/L^2 .

where

$$X_t = \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ \vdots \\ \pi_{t-L+2} \\ \hat{y}_t \end{bmatrix}, \Phi = \begin{bmatrix} a_{\pi,1}^{(L)} & a_{\pi,2}^{(L)} & \dots & a_{\pi,L-2}^{(L)} & a_{\pi,L-1}^{(L)} & 0 \\ 1 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 \\ a_{y,1}^{(L)} & a_{y,2}^{(L)} & \vdots & a_{y,L-2}^{(L)} & a_{y,L-1}^{(L)} & 0 \end{bmatrix}, \Sigma = \begin{bmatrix} b_{\pi}^{(L)} \\ 0 \\ \vdots \\ 0 \\ b_y^{(L)} \end{bmatrix} \quad (\text{C.34})$$

and the coefficients are solved in Appendix C.1.

Take unconditional variances,

$$\text{var}(X_t) = \Phi \text{var}(X_{t-1}) \Phi' + \Sigma \text{var}(u_t) \Sigma'. \quad (\text{C.35})$$

The unconditional variance $\text{var}(X_t) = \text{var}(X_{t-1})$, and consequently,

$$\text{vec}(\text{var}(X_t)) = (I - \Phi \otimes \Phi)^{-1} \text{vec}(\Sigma \text{var}(u_t) \Sigma'), \quad (\text{C.36})$$

where vec is the vectorization of a matrix, and I is an identity matrix.

We compare $\lambda^{cb}(L)$ and λ/L^2 in Figure C.1. They look very similar despite some small differences. Specifically, both of them decrease in L .

D Social Learning

D.1 Baseline Model

Instead of writing out the general form, we write out how we implement the model where $\max L^i = 4$. Under this assumption, for any L_t^i , we can write out agent i 's inflation forecast

in (5.4) as follows:

$$\mathbb{E}_t^i \pi_{t+1} = a_{\pi,1}^{(L_t^i)} \pi_t + a_{\pi,2}^{(L_t^i)} \pi_{t-1} + a_{\pi,3}^{(L_t^i)} \pi_{t-2}. \quad (\text{D.1})$$

Then, the average expectation in (5.5) is

$$\bar{\mathbb{E}}_t \pi_{t+1} = \bar{a}_{\pi,1} \pi_t + \bar{a}_{\pi,2} \pi_{t-1} + \bar{a}_{\pi,3} \pi_{t-2}. \quad (\text{D.2})$$

Substitute (D.2) into the Phillips curve in (3.4) and get

$$\pi_t = (1 - \beta \bar{a}_{\pi,1})^{-1} \beta \bar{a}_{\pi,2} \pi_{t-1} + (1 - \beta \bar{a}_{\pi,1})^{-1} \beta \bar{a}_{\pi,3} \pi_{t-2} + (1 - \beta \bar{a}_{\pi,1})^{-1} \kappa \hat{y}_t + (1 - \beta \bar{a}_{\pi,1})^{-1} u_t. \quad (\text{D.3})$$

The central bank minimizes the current period welfare loss (3.1) with respect to $\hat{y}_t$. The first-order condition is

$$\pi_t \frac{\kappa}{1 - \beta \bar{a}_{\pi,1}} + \lambda \hat{y}_t = 0. \quad (\text{D.4})$$

Rewrite the Phillips curve in (D.3),

$$\hat{y}_t = \frac{1 - \beta \bar{a}_{\pi,1}}{\kappa} \pi_t - \frac{\beta \bar{a}_{\pi,2}}{\kappa} \pi_{t-1} - \frac{\beta \bar{a}_{\pi,3}}{\kappa} \pi_{t-2} - \frac{1}{\kappa} u_t. \quad (\text{D.5})$$

Substituting the expression of $\hat{y}_t$ into the first-order condition (D.4) solves π_t :

$$\pi_t = \Delta \beta \bar{a}_{\pi,2} \pi_{t-1} + \Delta \beta \bar{a}_{\pi,3} \pi_{t-2} + \Delta u_t, \quad (\text{D.6})$$

where $\Delta = \left(\frac{\kappa}{1 - \beta \bar{a}_{\pi,1}} + \lambda \frac{1 - \beta \bar{a}_{\pi,1}}{\kappa} \right)^{-1} \frac{\lambda}{\kappa}$. Finally, we solve $\hat{y}_t$ by substituting (D.6) to (D.5).

D.2 Backward-Looking Expectations

Substituting (D.2) into the Phillips curve in (6.2), we get

$$\pi_t = (1 - \beta(1 - \phi) \bar{a}_{\pi,1})^{-1} \{ [\beta \phi + \beta(1 - \phi) \bar{a}_{\pi,2}] \pi_{t-1} + \beta(1 - \phi) \bar{a}_{\pi,3} \pi_{t-2} + \kappa \hat{y}_t + u_t \}. \quad (\text{D.7})$$

The central bank minimizes the current period welfare loss (3.1) with respect to $\hat{y}_t$. The first-order condition is

$$\pi_t \frac{\kappa}{1 - \beta(1 - \phi) \bar{a}_{\pi,1}} + \lambda \hat{y}_t = 0. \quad (\text{D.8})$$

Rewrite the Phillips curve in (D.7),

$$\hat{y}_t = \frac{1 - \beta(1 - \phi) \bar{a}_{\pi,1}}{\kappa} \pi_t - \frac{\beta \phi + \beta(1 - \phi) \bar{a}_{\pi,2}}{\kappa} \pi_{t-1} - \frac{\beta(1 - \phi) \bar{a}_{\pi,3}}{\kappa} \pi_{t-2} - \frac{1}{\kappa} u_t. \quad (\text{D.9})$$

Substituting the expression of $\hat{y}_t$ into the first-order condition (D.8) solves π_t :

$$\pi_t = \Delta [\beta \phi + \beta(1 - \phi) \bar{a}_{\pi,2}] \pi_{t-1} + \Delta \beta(1 - \phi) \bar{a}_{\pi,3} \pi_{t-2} + \Delta u_t, \quad (\text{D.10})$$

where $\Delta = \left(\frac{\kappa}{1 - \beta(1 - \phi) \bar{a}_{\pi,1}} + \lambda \frac{1 - \beta(1 - \phi) \bar{a}_{\pi,1}}{\kappa} \right)^{-1} \frac{\lambda}{\kappa}$. We solve $\hat{y}_t$ by substituting (D.10) to (D.9).

The nowcast in (5.2) and (5.3) becomes

$$\mathbb{E}(\pi_t | \mathcal{I}_{t-1}, L_t^i, u_t^i) = \phi \pi_{t-1} + (1 - \phi) \left(a_{\pi,1}^{(L_t^i)} \pi_{t-1} + \dots + a_{\pi,L_t^i-1}^{(L_t^i)} \pi_{t-L_t^i+1} + b_{\pi}^{(L_t^i)} u_t^i \right) \quad (\text{D.11})$$

$$\mathbb{E}(\hat{y}_t | \mathcal{I}_{t-1}, L_t^i, u_t^i) = \phi \pi_{t-1} + (1 - \phi) \left(a_{y,1}^{(L_t^i)} \pi_{t-1} + \dots + a_{y,L_t^i-1}^{(L_t^i)} \pi_{t-L_t^i+1} + b_y^{(L_t^i)} u_t^i \right) \quad (\text{D.12})$$

D.3 Weight on Interest Rate Volatility

The central bank now picks π_t , $\hat{y}_t$, and i_t to minimize the loss $\mathcal{L}_t^i$ (6.4) subject to the Phillips (5.6) and IS curves (6.5).

The average expectation of the output gap is given by

$$\bar{\mathbb{E}}_t \hat{y}_{t+1} = \bar{a}_{y,1} \pi_t + \bar{a}_{y,2} \pi_{t-1} + \bar{a}_{y,3} \pi_{t-2}. \quad (\text{D.13})$$

Substituting expectations in (D.2) and (D.13) into the IS curves in (6.5), we obtain

$$i_t = -\gamma \hat{y}_t + (\gamma \bar{a}_{y,1} + \bar{a}_{\pi,1}) \pi_t + (\gamma \bar{a}_{y,2} + \bar{a}_{\pi,2}) \pi_{t-1} + (\gamma \bar{a}_{y,3} + \bar{a}_{\pi,3}) \pi_{t-2}. \quad (\text{D.14})$$

Substitute (D.14) into (6.4), and then minimize it subject to the Phillips curve (D.3). The first-order condition is

$$\pi_t \frac{\kappa}{1 - \beta \bar{a}_{\pi,1}} + \lambda \hat{y}_t + \lambda_i i_t \left(-\gamma + (\gamma \bar{a}_{y,1} + \bar{a}_{\pi,1}) \frac{\kappa}{1 - \beta \bar{a}_{\pi,1}} \right) = 0. \quad (\text{D.15})$$

Next, substituting (D.5) into (D.14), we get

$$\begin{aligned} i_t = & \left(-\gamma \frac{1 - \beta \bar{a}_{\pi,1}}{\kappa} + \gamma \bar{a}_{y,1} + \bar{a}_{\pi,1} \right) \pi_t + \left(\gamma \frac{\beta \bar{a}_{\pi,2}}{\kappa} + \gamma \bar{a}_{y,2} + \bar{a}_{\pi,2} \right) \pi_{t-1} \\ & + \left(\gamma \frac{\beta \bar{a}_{\pi,3}}{\kappa} + \gamma \bar{a}_{y,3} + \bar{a}_{\pi,3} \right) \pi_{t-2} + \gamma \frac{1}{\kappa} u_t \end{aligned} \quad (\text{D.16})$$

Using (D.5) and (D.16) to substitute out the expressions of $\hat{y}_t$ and i_t in the first-order condition (D.15), we can express π_t as follows:

$$\begin{aligned} \pi_t = & \Delta \left\{ \beta \bar{a}_{\pi,2} - \kappa \frac{\lambda_i}{\lambda} \Theta \left(\gamma \frac{\beta \bar{a}_{\pi,2}}{\kappa} + \gamma \bar{a}_{y,2} + \bar{a}_{\pi,2} \right) \right\} \pi_{t-1} \\ & + \Delta \left\{ \beta \bar{a}_{\pi,3} - \kappa \frac{\lambda_i}{\lambda} \Theta \left(\gamma \frac{\beta \bar{a}_{\pi,3}}{\kappa} + \gamma \bar{a}_{y,3} + \bar{a}_{\pi,3} \right) \right\} \pi_{t-2} + \Delta \left\{ 1 - \frac{\lambda_i}{\lambda} \Theta \gamma \right\} u_t, \end{aligned} \quad (\text{D.17})$$

where

$$\Delta = \left\{ \frac{\kappa}{1 - \beta \bar{a}_{\pi,1}} + \lambda \frac{1 - \beta \bar{a}_{\pi,1}}{\kappa} + \lambda_i \Theta \left(-\gamma \frac{1 - \beta \bar{a}_{\pi,1}}{\kappa} + \gamma \bar{a}_{y,1} + \bar{a}_{\pi,1} \right) \right\}^{-1} \frac{\lambda}{\kappa} \quad (\text{D.18})$$

$$\Theta = -\gamma + (\gamma \bar{a}_{y,1} + \bar{a}_{\pi,1}) \frac{\kappa}{1 - \beta \bar{a}_{\pi,1}}. \quad (\text{D.19})$$

Finally, we solve $\hat{y}_t$ by substituting (D.17) to (D.5).

D.4 AIT with Commitment

The Phillips curve is the same as in the baseline model in (D.3). The central bank's objective function is the period loss of (3.2). For 2-period AIT with commitment, it minimizes

$$\left(\frac{\pi_t + \pi_{t-1}}{L}\right)^2 + \lambda^{cb} \hat{y}_t^2, \quad (\text{D.20})$$

subject to (D.3), where $L = 2$.

The first-order condition with respect to $\hat{y}_t$ is

$$\frac{1}{L^2} (\pi_t + \pi_{t-1}) \frac{\kappa}{1 - \beta \bar{a}_{\pi,1}} + \lambda^{cb} \hat{y}_t = 0 \quad (\text{D.21})$$

Substituting $\hat{y}_t$ from (D.5) into the first-order condition (D.21) solves π_t :

$$\pi_t = \Delta \left(-\frac{\kappa}{1 - \beta \bar{a}_{\pi,1}} + L^2 \lambda^{cb} \frac{\beta \bar{a}_{\pi,2}}{\kappa} \right) \pi_{t-1} + \Delta L^2 \lambda^{cb} \frac{\beta \bar{a}_{\pi,3}}{\kappa} \pi_{t-2} + \Delta L^2 \lambda^{cb} \frac{1}{\kappa} u_t, \quad (\text{D.22})$$

where $\Delta = \left(\frac{\kappa}{1 - \beta \bar{a}_{\pi,1}} + L^2 \lambda^{cb} \frac{1 - \beta \bar{a}_{\pi,1}}{\kappa} \right)^{-1}$. Finally, we solve $\hat{y}_t$ by substituting (D.22) to (D.5).

E Calibration

E.1 Structural Parameters

We calibrate the model at an annual frequency, and our parameter values are in line with the standard New Keynesian literature; see parameters in Table E.1. The top panel displays the structural parameters we calibrate, some of which do not show up in the linear equations in our model, and the bottom panel displays implied parameter values. We set $\beta = 0.95$ to match the annual return on safe assets in the US. We assume a log utility for consumption, $\gamma = 1$, and a unitary Frisch elasticity of labor supply, $\varphi = 1$. We choose η , the price elasticity of demand, to be 6. We set the Calvo parameter, which governs price stickiness, to be $\frac{1}{3}$, implying the average duration between price changes is 1.5 years. We assume the cost-push shock is *i.i.d.* with a standard deviation of $\sigma_\epsilon = 0.1$.

With these underlying parameters, the slope of the Phillips curve, κ , is 2.73, and the weight on output gap stabilization in the welfare function, λ , is 0.46.

E.2 Learning Parameters

We summarize the learning parameters in Table E.2. We set the number of agents in the private sector to be 1,000. We further assume the number of random meetings per period is 1,000. Our results are reported as the average from 1,000 simulations. We follow Arifovic, Bullard, and Kostyshyna (2013) by setting the mutation probability to 10%. For the parameter that governs the level of uncertainty, we use $\sigma_v = 0.1$ for high uncertainty, which is our benchmark case, 0.01 for low uncertainty, and 0 for no uncertainty.

Table E.1: Structural parameters

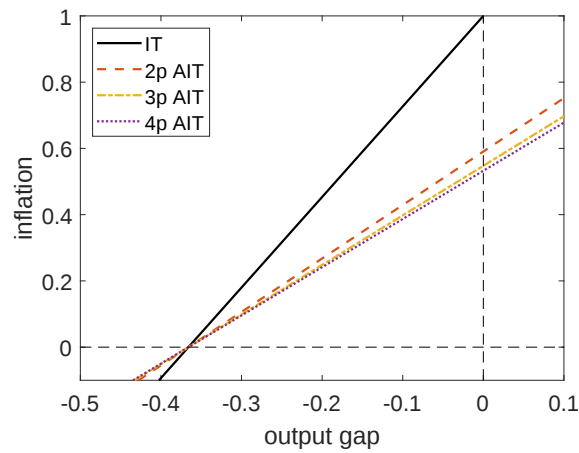
parameter	description	value
β	discount factor	0.95
γ	elasticity of intertemporal substitution	1
φ	elasticity of labor supply	1
η	the price elasticity of demand	6
θ	price rigidity	$\frac{1}{3}$
σ_u	std. dev. of cost-push shock	0.1
κ	slope of the Phillips curve	2.73
λ	weight in welfare	0.46

Table E.2: Learning parameters

parameter	description	value
N	number of agents	1,000
	number of simulations	1,000
	number of meetings every period	1,000
	probability of mutation	0.1
σ_v	std. dev. in private signals	0.1 (benchmark) / 0.01 (low) / 0 (no)

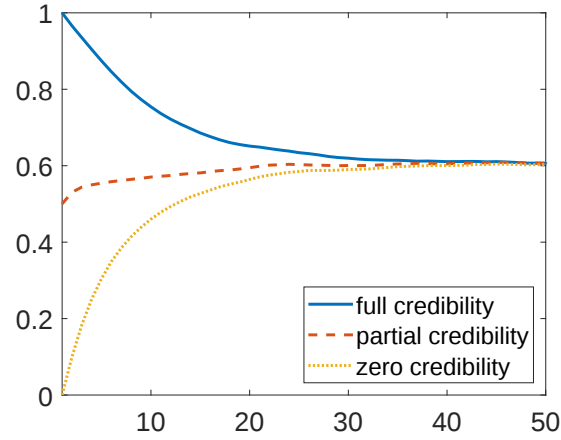
F Additional Figures

Figure F.1: Phillips curves: multi-period AIT



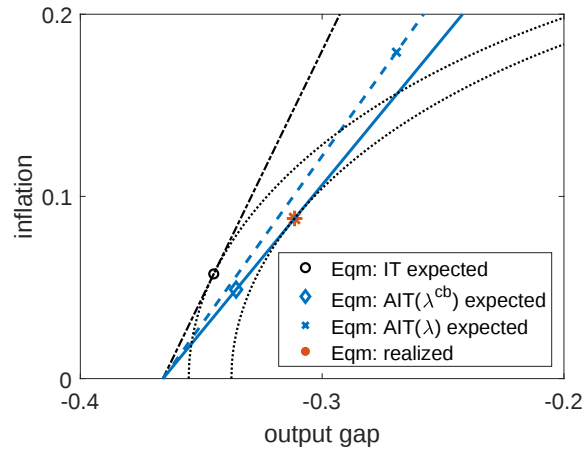
Notes: $u_t = 1, \pi_{t-1} = 0$. Units: percentage points. Black solid line: IT. Colored dashed lines are AIT with $L \in \{2, 3, 4\}$.

Figure F.2: Fraction of AIT believers with different initial credibility



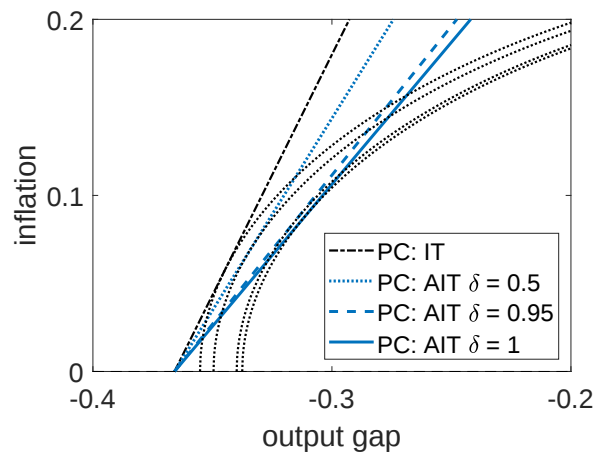
Notes: X-axis: horizon. Blue solid line: the central bank starts with full credibility; red dashed line: the central bank starts with partial credibility; yellow dotted line: the central bank starts with zero credibility. All the results are averaged over 1,000 simulations.

Figure F.3: Expected vs. realized equilibriums



Notes: $u_t = 1, \pi_{t-1} = 0$. Units: percentage points. Black circle: the equilibrium expected by the IT believers; blue diamond: the equilibrium expected by the AIT believers with λ^{cb} ; blue cross: the equilibrium expected by the AIT believers with λ ; red star: the realized equilibrium where the private sector forms expectations based on AIT with λ^{cb} but the central bank deviates to implement IT ex post. Lines are Phillips curves. The ellipses are period social welfare.

Figure F.4: Phillips curves: weighted AIT



Notes: $u_t = 1, \pi_{t-1} = 0$. Units: percentage points. Different lines are Phillips curves with different δ . The ellipses are period social welfare.