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ABSTRACT

We present evidence that the nature of aggregate consumption dynamics change around the time the bond-stock correlation switches sign. We identify three regimes in a real-time, sequential learning framework: two highly persistent regimes where either permanent or transitory consumption shocks are relatively more dominant, and a disaster regime that is largely transitory. We study implications of this finding for asset prices. The transition from the second to the first regime in the late 1990s makes the correlation between equities and real bonds switch sign from positive to negative as in the data, thus providing an explanation for this fact from the perspective of real consumption dynamics. The findings extend to the international setting.

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1 Introduction

The nature of macroeconomic fluctuations has preoccupied economists since at least Hayek (1933) and, more recently, Lucas (1977). One important aspect of the debate centers on the magnitude of transitory versus permanent shocks (e.g., Campbell and Mankiw, 1987, Cochrane, 1988). In this paper, we document a substantial change in the nature of consumption dynamics along this dimension occurring in the 1990s. In particular, the economy goes from being dominated by shocks that are transitory in nature to shocks that are permanent.

Standard theories imply that this finding has important implications for the dynamics of government bond returns (e.g., Alvarez and Jermann, 2005). Campbell (1986) shows that increasing persistence of consumption growth can make bonds go from being risky to being hedges. Consistent with this idea, the correlation between stock market and inflation-indexed Treasury bond returns changes sign from positive to negative in 1998, coinciding with the change in consumption dynamics that we document (see Campbell, Shiller, and Viceira, 2009 and Figure 1).

We establish the change in consumption dynamics in two steps. First, we show that the autocorrelation of consumption growth is significantly higher post-1998 than pre-1998. We find a similar increase in the persistence of various ingredients of US consumption growth, such as the real growth in capital, labor productivity and hours worked, across the two time periods. The change in consumption dynamics happens not only for the US, but also for the UK and across a set of 19 developed countries. This is an important aspect of the analysis, as the bond-stock correlations also switch internationally. We further observe, in a rolling sample setting, a significant negative relationship between the bond-stock correlation and the autocorrelation of consumption growth.

As a second step, these findings lead us to model the aggregate consumption level as the sum of a mean-reverting and a mean-averting components. We label the former the "transitory component" and the latter the "permanent component." The novelty is that we allow for changes in the relative contribution of these components over time. Using real-time data and inference, we find that the transitory component dominates in the early sample, while the permanent component dominates in the later sample. The switch between these regimes is estimated to occur around the time the bond-stock correlation switches sign.

These dynamics imply that bonds are risky in the early sample and hedges in the later sample, which explains the broad time series pattern in the bond-stock correlation. To see why, consider a negative transitory shock to consumption. Because consumption is expected to revert to its trend, expected consumption growth increases as a result of this shock. Standard intertemporal substitution arguments imply that, in equilibrium, the risk-free rate increases and bond prices decrease. Thus, the correlation between bond returns and consumption growth is positive. In contrast, a negative shock to the permanent component leads to low expected consumption growth in our setting. This reduces the risk-free rate, making the correlation between consumption growth and bond returns negative. We confirm empirically that the correlation of both nominal and real bond returns with consumption growth, i.e., bond return cyclicality, change sign according to this intuition. Stock returns, on the other hand, are positively related to consumption growth regardless of the regime. These features of the model create switching signs of the bond-stock correlation consistent with that in the data.

We use macro data only to estimate our model in order to address the "dark matter" concern (Chen, Dou, and Kogan, 2021). We consider the cases of the US and the UK for the formal model estimation. The results are qualitatively similar across the two countries. The model of consumption dynamics is estimated sequentially to emulate real-time learning of economic agents that do not observe the regime or know the structural parameters of the model. In order to obtain a broad historical perspective on consumption dynamics and, in particular, to capture extreme events, we use the richest data that is available. Specifically, we splice historical annual observations of consumption growth constructed by Barro and Jin (2021) and Barro and Ursua (2008), which start in 1834, with more modern quarterly observations that are available since 1948 and 1960 for the US and the UK, respectively. It is a methodological contribution of this paper to implement sequential learning about the model at mixed frequencies.

The transitory and permanent components of consumption vary in importance as the volatilities of their shocks follow a three-state Markov switching regime process. The changes between two of the three states occur at a very low frequency. The "permanent" state has appeared sporadically in the early sample, primarily at the peak of expansions, and prevails from the mid-1990s and on. The "transitory" state prevails throughout the preceding part of our sample, and re-appears post-Covid. The third state is relatively short-lived, more rare, occurring only infrequently throughout the sample, and features much higher volatility of innovations and a negative mean of consumption growth. We interpret it as a disaster (Barro, 2006, Gabaix, 2012, Wachter, 2013).

Next, we link the model's implications to data on asset prices, namely aggregate equity, real, and nominal bonds. Doing so requires specification and calibration of cash flows, dividends and inflation, respectively. Because our focus is on the role of consumption dynamics for asset pricing, we posit cash flows dynamics that are simple and in line with the literature. In case of dividends we specify a process similar to levered consumption claim of Abel (1999) and calibrate it to match mean and volatility of observed dividends. A realistic process of inflation calls for introduction of inflation-specific regimes within each consumption state that allow for changes in the level of inflation unrelated to the consumption dynamics. These new states require joint estimation of inflation and consumption, which we implement in real time just like with consumption-only estimation. We document that the conclusions about the patterns in consumption growth, such as occurrence of the various states and the timing of shifts between the different states are largely unaffected by this joint estimation.

We proceed by assuming log utility preferences as the simplest model configuration. The bond-stock correlation shifts downwards dramatically in the late 1990s just like in the data. That said, the overall level of the correlation is too low, and the real yield curve is nearly flat and slightly negative. All the shortcomings are manifestations of too little risk in real bonds.

Thus, as a second step, we follow Barro (2006) and Chernov, Schmid, and Schneider (2020) by introducing default risk of high-quality sovereigns. The real curve becomes positively sloped and bond-stock correlation closely matches that in the data. Both the bond curve and the forward equity curve extracted from dividend strips are flat reflecting low risk premiums in this economy.

In our third step we increase risk aversion of the representative agent without changing the elasticity of intertemporal substitution (EIS). That is, we introduce the Epstein and Zin (1989) preferences with EIS of 1 and risk aversion of 7. The increase in risk aversion helps with matching the Sharpe ratio of the equity market, and makes the magnitudes of real bond and forward equity curves more realistic. The bond-stock correlation continues to be in line with the data. We apply an identical configuration of preference parameters to the estimated model of the UK consumption growth with similar conclusions.

Overall, we find that the real channel, that is, the changing nature of consumption dynamics, is driving the sign change in the correlation between real bonds and stocks. This channel is important for nominal bonds as they inherit the dynamics of real bonds. In fact, when estimating a joint model for consumption and inflation, our model also captures the switching sign of the correlation between nominal bonds and stocks.

Prominent extant explanations of the nominal bond-stock correlation focus on a nominal channel, namely the changing relation between inflation and consumption (e.g., David and Veronesi, 2013, Campbell, Pflueger, and Viceira, 2020). In particular, consumption growth and inflation are negatively correlated in the early sample and positively correlated in the late sample, making nominal bonds risky in the former and hedges in the latter.

Our explanation for the switching bond-stock correlation is complementary to this nominal channel. Nominal bond returns are driven both by the consumption dynamics, as the real bond return is an important component of the nominal bond return, and the inflation dynamics. It is difficult to precisely identify their relative importance of two reasons. First, as we show in this paper, the real dynamics and the correlation between inflation and consumption change around the same time. Second, inflation and real consumption dynamics are determined jointly in general equilibrium. We cannot identify the specific nature of the shocks, be they technological or monetary, in the pure exchange economy that we consider.

That said, we find that specifying a realistic inflation process and estimating it jointly with consumption helps match nominal bond risk premiums and the nominal bond term structure. Thus, there is important information in both consumption and inflation data for understanding the stylized facts. But, even if one attributes all innovations to nominal sources, our evidence shows that the effect is present in changing real consumption dynamics. Further, the explanation of nominal bond-stock correlation based on changes in inflation's cyclicality does not speak directly to why the correlation between stocks and real bonds also changes sign at the same time. Such a change arises naturally from changing real consumption dynamics in standard models. This is precisely the channel that is left under-explored by the literature.

Existing empirical work typically addresses a subset of the facts associated with bonds and stocks. Piazzesi and Schneider (2006) is an early paper that explores a mix of transitory and permanent components of the joint inflation-consumption dynamics via adaptive learning (learning about a mean introduces a permanent component in consumption growth as opposed to the consumption level in our model). They explore this mechanism in the context of bond pricing leaning on the nominal channel, i.e., consumption-inflation correlation, as the explanation of the upward sloping nominal yield curves.[1] The model of Bekaert, Engstrom, and Grenadier (2010) that is designed to capture many moments of stock and bonds can generate time-varying nominal bond-stock correlation. Campbell, Pflueger, and Viceira (2020), David and Veronesi (2013), Ermolov (2022), Fang, Liu, and Roussanov (2021), and Pflueger (2023) focus on nominal bond-stock correlations. Song (2017) also examines nominal bond-stock correlations and addresses the challenge of explaining the persistently upward-sloping yield curve despite the shift in bond-stock correlation from positive to negative.

Existing studies often rely on the inflation-consumption correlation, which Duffee (2018a,b) argues is counter-factual due to too high inflation risk relative to the data. Gomez-Cram and Yaron (2021) specifically target that critique. They achieve success in matching the

[1] Bansal and Shaliastovich (2013) and Wachter (2006) use the inflation-consumption relation to explain nominal yield curves as well using the long-run risk and external habit framework, respectively.

nominal and real yield curves via preferences that combine Epstein-Zin utility with shocks to the time preference parameter. They do not discuss joint co-movement between bonds and stocks.

Alvarez, Atkeson, and Kehoe (2002) show that limited participation in financial markets generates transitory consumption responses to monetary shocks, leading to mean-reverting consumption dynamics relevant for bond pricing. Even though the money injection is nominal, the frictions from limited participation and the resulting consumption dynamics affect real pricing. Kozak (2022) uses the production-based framework to capture real bond-stock correlation. We depart from this approach by attributing the switching correlation to shifts in aggregate consumption behavior as opposed to firm-level investment dynamics. Furthermore, we offer more granular information about our model's ability to match the time-series patterns in bond-stock correlations as well as many other asset pricing moments. Gourio and Ngo (2020) show that the correlation between inflation and stock returns turned positive during the ZLB period, and a standard New Keynesian model attributes this to demand shocks becoming more influential when monetary policy is constrained.

Bansal, Kiku, and Yaron (2010) and Hassler and Marfe (2016) advocate permanent, transitory, and disaster components albeit in a functional form that is different from ours.[2] In particular, the composition of permanent and transitory shocks is time-invariant. Research questions are different as well. The former paper focuses on the single-horizon equity premium and interest rate. The latter focuses exclusively on the term structure of dividend strips. Croce, Lettau, and Ludvigson (2015) study the effect of a boundedly rational agent who is filtering unobserved expected consumption and dividend growth rates. They extend the Bansal and Yaron (2004) model by adding additional shocks and allowing for time-invariant conditional covariances between them. The authors demonstrate, in the context of dividend strips, that filtering of the unobserved growth rates may switch the roles of persistent vs transitory shocks to consumption growth. Backus, Boyarchenko, and Chernov (2018) (section 6), motivated by the evidence about real and nominal bond and equity yield curves, advocate a combination of a less persistent expected consumption growth and disasters in an Epstein and Zin (1989) economy. Again, there is no time variation in the relative contribution of these components.

In contemporaneous work, Jones and Pyun (2021) extend the Bansal and Yaron (2004) model with exogenously specified time-varying covariance between realized and expected consumption growth. The time-varying mix of the permanent and transitory components, which we advocate and explicitly model, is implicit in their specification and is referred to

[2] Pohl, Schmedders, and Wilms (2016) consider permanent and transitory components without disasters with application to classic equity moments.

as time-varying consumption growth persistence. The authors primarily focus on the bond-stock covariance and on the leverage effect for equities. Implementation relies on calibration rather than on estimation of the consumption dynamics.

Barro and Jin (2021), Nakamura, Sergeyev, and Steinsson (2017), and Nakamura, Steinsson, Barro, and Ursua (2013) estimate consumption dynamics using a large cross-section of countries and a long sample. They focus on delineating the role of consumption disasters from those of time-variation in its conditional mean and volatility. They do not allow for time-variation in the relative magnitude of permanent and transitory shocks as is our focus.

No-arbitrage models of bond-stock co-movement are represented by Backus, Boyarchenko, and Chernov (2018), Campbell, Sunderam, and Viceira (2017), Koijen, Lustig, and Nieuwerburgh (2017), and Lettau and Wachter (2011). Laarits (2021) uses the latter model to study high-frequency variations in the bond-stock covariance with a focus on the role of precautionary savings. That effect is present in our equilibrium as well with different regimes generating different quantitative impact.

2 Preliminary evidence

The purpose of this section is to demonstrate that there is pervasive evidence of a compositional shift in the mix of permanent and transitory components in the US macro economy, in general, and US consumption, in particular. The same compositional shift takes place in other countries, and it changes the serial correlation of consumption growth making real bonds hedges rather than being risky. Consistent with this intuition, the shift coincides with a change in the sign of the bond-stock correlations both in the US and these other countries. This evidence prompts us to introduce, in the next section, a formal model of consumption dynamics that allows an in-depth exploration of this compositional shift and its asset-pricing implications.

We consider the United States (US) as our main source of data. We complement the US-based evidence with international data from 19 developed economies, referred to as the rest of the world (RoW), when available. A detailed description of all data used in the paper is given in Appendix A. For the preliminary analysis, the consumption data are quarterly country aggregates of real personal consumption expenditures, the stock markets are measured by commonly used market indexes in each country, and the sovereign bonds are nominal with maturity 10 years. The choice of the 19 countries is guided by the availability of financial data. The preliminary evidence of this section excludes the Covid period as summary statistics and regressions are not well-suited to handle the extreme movements in consumption growth that occurred during that period.

2.1 Bond-stock correlation

We first revisit the evidence on the correlation between stocks and government bonds. The correlation is calculated using rolling windows of return data in the respective markets. We complement the US evidence with international data, where we define daily RoW stock and bond returns by taking the cross-sectional average of daily stock and bond return across countries (ex. the US), and then compute the corresponding RoW bond-stock correlation analogously to how we compute it for the US. This evidence also connects to Laarits (2021) who reports individual bond-stock covariances for some of the countries in our sample.

In a thorough empirical analysis, Baele, Bekaert, and Inghelbrecht (2010) argue the US bond-stock correlation changes around 1998 (see also Campbell, Shiller, and Viceira, 2009). Figure 1 shows shows that the sign of the bond-stock correlation indeed switches around this time for both the US and the UK, while Figure 2 demonstrates that the timing of this switch is remarkably consistent also for the RoW bond-stock correlation.

2.2 Autocorrelation of consumption

The changing correlation between real bonds and equity returns suggest that consumption dynamics are different across the two periods. We show that the autocorrelation of consumption growth indeed is significantly and economically different across these periods. We interpret this evidence as arising from a changing mix of permanent versus transitory shocks to the economy.

Specifically, consider the regression:

$$\Delta c_{t+1} = \alpha_0 + \alpha_1 \mathbb{1}_{t<1998} + \beta_0 \Delta c_t + \beta_1 \times (\mathbb{1}_{t<1998} \times \Delta c_t) + \epsilon_{t+1}, \tag{1}$$

where Δc_t is the change in log consumption growth, and $\mathbb{1}_{t<1998}$ equals one if observation t is in the pre-1998 sample and zero otherwise. If β_1 is significantly different from zero, there is different autocorrelation in the period before 1998 versus that after. In this section, we set the cut-off in 1998 on the basis of the extant evidence in Baele, Bekaert, and Inghelbrecht

(2010) and Campbell, Shiller, and Viceira (2009). In later sections we estimate the timing of the regime change from consumption data.

In the case of the US, we also consider regressions where we exclude all NBER recession periods to check that the results are not due to the lower frequency of recessions in the later sample. When we do that, we ensure that the right hand side in the regression (observation t) is always the previous quarter relative to $t+1$ whether a NBER recession or not. Using NBER dates introduces a look-ahead bias, so the nature of these regressions is illustrative.

Table 1 shows the estimated coefficients. The results with and without NBER recessions are qualitatively similar. Our discussion focuses on the latter (middle panel of the table). The column labeled (1) shows that $\beta_0 = 0.25$ and $\beta_1 = -0.39$, significant at the 1% level. Thus, the autocorrelation coefficient in the pre-1998 period is -0.14 ($\beta_0 + \beta_1 = 0.25 - 0.39$), while in the later period it is 0.25 (β_0). The significance of β_1 means this difference is statistically significant, and the economic magnitude is substantial.

The intercept is also statistically different across the two periods. One would expect this because the intercept depends on the mean and persistence of consumption growth, and since the estimated slope, which measures persistence, is significantly different. In order to ensure that this intercept change does not reflect different means of consumption growth across the two time periods, we also run a regression where we impose that the unconditional mean is the same across the two periods. In particular, we demean the consumption growth data across the whole sample and run the regression (1) setting $\alpha_1 = 0$. The results are reported in Table 1 in the column labeled (2), which shows that the effect is slightly stronger in this case.

For robustness checks, we also use revised real per capita nondurable and services consumption, include the post-Covid sample, and use an alternative "unfiltered" measure of consumption (see Kroencke, 2017). The results are similar. See Appendix B.

We construct the RoW consumption growth series by averaging across the individual countries' consumption growth rates each quarter. The correlation of this series with its US counterpart is 0.28. Thus, the RoW consumption is substantively different. Nevertheless, the evidence on its persistence is qualitatively similar. One difference is that the autocorrelation coefficient is basically zero pre-1998 and large afterwards (0.6 or 0.73, depending on how the intercept is treated). Thus, the increase in persistence is even larger internationally.

Another manifestation of the increasing persistence of consumption growth is reflected in changing autocorrelations at multiple lags displayed in Figure 3 for both the US and the RoW consumption data. First, we observe that correlations at different lags are uniformly and significantly higher in the post-1997 sample relative to those in the early sample. Second, such increased persistence is consistent with higher variance in the long-run consumption response relative to the variance of the shock itself. Since the long-run variance of transitory shocks is zero, this evidence suggests this later period has seen an increase in the relative magnitude of permanent versus transitory components in consumption.

2.3 Connection to the macro economy

The evidence prompts a natural question of what drives the changes in consumption persistence, which leads us to dig deeper into the macroeconomic determinants of consumption dynamics. Aggregate goods market clearing in canonical macroeconomic models implies that:

$$Y_t = C_t + I_t, \tag{2}$$

where Y_t is output, C_t is consumption, and I_t is investment. A log-linear approximation of this condition allows us to link consumption growth to various components of macroeconomic growth:

$$\Delta c_t = (1+\omega) \times \Delta y_t - \omega \times \Delta i_t$$

where ω is a log-linearization constant.[3] Data on quarterly aggregate output and investment can be obtained from the Bureau of Economic Analysis.

Adopting a standard Cobb-Douglas production function allows us to decompose output growth as

$$\Delta y_t = (1-\alpha)\Delta z_t + (1-\alpha)\Delta \ell_t + \alpha \Delta k_t,$$

where z_t is log total factor productivity, ℓ_t is log hours worked and k_t is log capital. The Federal Reserve Board at San Francisco provides quarterly data on technology, hours worked, and capital.

Data on capital and productivity presents a number of measurement challenges, and we therefore consider an alternative productivity measure: labor productivity from the Bureau of Labor Statistics. The labor productivity data is defined as output dividend by hours worked, which implies another decomposition of output growth:

$$\Delta y_t = \Delta \ell p_t + \Delta \ell_t,$$

[3] Log-linearize $\frac{C_t}{Y_t} = 1 - \frac{I_t}{Y_t}$ around the mean of I_t/Y_t, denoted IY, to get $c_t = \text{constant} + (1+\omega)\, y_t - \omega i_t$, where $\omega = \frac{IY}{1-IY}$.

where ℓp_t is labor productivity.

Table 2 gives the results from running regressions like the one for consumption growth in Equation (1) for each of these macroeconomic variables. The pattern is qualitatively the same as that documented for consumption growth across all of these variables – the autocorrelation in the pre-98 sample is significantly smaller than that in the later sample. The exceptions are total factor productivity and capital growth, which are arguably the hardest-to-measure variables. In these cases, the coefficients on the interaction term, β_1, are still negative, but not significant. In the other cases, however, the coefficient is lower than -0.33 and significant at the 5% level or stronger, including for labor productivity and investment which can be viewed as alternative measures of productivity and capital growth. In sum, the increase in autocorrelation is a broad macroeconomic phenomenon and not specific to consumption. That said, the consumption dynamics are all we need for evaluating the asset pricing implications of this finding, which is the focus of this paper.

2.4 Connecting the autocorrelation of consumption growth to the bond-stock correlation

To provide preliminary evidence on the connection between consumption growth dynamics and the bond-stock correlation, we regress realized 3-year consumption autocorrelations on contemporaneous realized bond-stock correlations (Table 3). The consumption data is quarterly, which means the realized autocorrelation of consumption is a much noisier estimate than the realized bond-stock correlation, which is estimated based on daily data. We put the consumption-based measure on the left-hand side in the regression to minimize attenuation bias driven by errors-in-variables. The resulting regression relationship is negative and significant, both in the US and the RoW. That is, a decline in the bond-stock correlation is indeed associated with an increase in the persistence of consumption growth.

3 The Model

The preliminary analysis presented in the previous section makes two short-cuts. First, we impose 1998 as the break-point, motivated by the evidence on the bond-stock correlation. Second, we exclude the Covid crisis and, in one specification, business cycle downturns in general. In this section we present a model of consumption dynamics that is motivated by the presented evidence. We estimate it, including the timing of different regimes and accounting for crisis periods. We conclude by presenting and discussing the implications of the estimated dynamics.

3.1 Consumption dynamics

We model aggregate log consumption as the sum of a deterministic trend and two persistent components, $c_{p,t}$ and $c_{\tau,t}$, which we label permanent and transitory, respectively:

$$c_t = c_{p,t} + c_{\tau,t}, \tag{3}$$

We model the permanent and transitory components as follows:

$$\begin{aligned} \Delta c_{p,t+1} &= \mu_p + \rho_p \left(\Delta c_{p,t} - \mu_p \right) \ + \epsilon_{p,t+1}, && (4) \\ c_{\tau,t+1} &= \rho_\tau c_{\tau,t} + \epsilon_{\tau,t+1}, && (5) \end{aligned}$$

where $\epsilon_{j,t+1}$ are mean-zero shocks and $j \in \{p, \tau\}$.

Thus, the permanent component contains a unit root while the transitory component does not. In particular, if $\rho_p = 0$ then the permanent component, $c_{p,t}$, is a random walk. General equilibrium models with production, imply that a combination of permanent and transitory shocks to productivity leads to endogenous consumption dynamics similar to those specified above (e.g., Blanchard, L'Huillier, and Lorenzoni, 2013, Kaltenbrunner and Lochstoer, 2010).

Figure 4 demonstrates the different roles of the two components via impulse responses. A positive shock to the permanent component (the blue line) leads to a persistent increase in consumption since $\rho_p > 0$. Thus, the shock leads to an increase in future expected consumption growth. This is akin to the endogenous consumption response to a permanent technology shock in standard real business cycle models. A positive shock to the transitory component (the red line), however, is associated with a reversal. Thus, in this case a positive shock leads to a negative shock to future expected consumption growth. This is akin to the endogenous consumption dynamics that arises in real business cycle models when the technology shock is transitory.

For simplicity, we set $\rho = \rho_p = \rho_\tau$ and note that

$$\begin{aligned} x_{t+1} &\equiv \rho(\Delta c_{p,t+1} - \mu_p) + (\rho - 1) c_{\tau,t+1} && (6) \\ &= \rho x_t + \rho \epsilon_{p,t+1} + (\rho - 1)\epsilon_{\tau,t+1}. \end{aligned}$$

We can then conveniently express consumption growth as

$$\Delta c_{t+1} = \mu_p + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1}. \tag{7}$$

There are two approaches that would be consistent with the motivating evidence presented in the previous section, ex-ante. One could capture changing autocorrelation of consumption growth by making the coefficient ρ in Equation (6) time-varying. Alternatively, the variance of the shocks ϵ could be changing leading to time-variation in the relative magnitude of the two consumption components. Such variation would manifest itself in changing autocorrelation. We choose the latter approach as it allows us to connect directly to the literature on the nature of macroeconomic fluctuations.

Specifically, we assume $\epsilon_{j,t+1} \sim N(0, \sigma_j^2(S_{t+1}))$, where S_{t+1} is a discrete Markov state variable that takes on N values $S_{t+1} \in \{1, ..., N\}$. Agents observe the current regime S_{t+1} and make forecast of future regime based on the transition matrix below

$$\mathbb{P} = \begin{bmatrix} p_{11} & \cdots & p_{1N} \\ \vdots & \ddots & \vdots \\ p_{N1} & \cdots & p_{NN} \end{bmatrix} \tag{8}$$

where $\sum_{i=1}^{N} p_{ji} = 1$. We also allow the mean of the permanent component to change with the same regimes. We show in Appendix C that this results in the same dynamics of consumption growth (7), but with regime-dependent mean $\mu_p(S_{t+1})$. As a result, conditional expected consumption growth is equal to $E_t(\Delta c_{t+1}) = E_t(\mu_p(S_{t+1})) + x_t$.

The changing volatilities drive time-variation in the relative importance of the long-run and short-run shocks that are depicted in Figure 4 via the relative magnitudes of the permanent and transitory components, as represented by $\sigma_p(S_{t+1})$ and $\sigma_\tau(S_{t+1})$, respectively. Further, Figure 4 implies that a higher proportion of the permanent component in consumption yields higher autocorrelations of consumption *growth*. This occurs as the correlation between shocks to realized versus future expected consumption growth has different signs across the permanent (positive) and transitory (negative) components. The model captures this effect via

$$Cov(\Delta c_{t+1}, E_{t+1}(\Delta c_{t+2})|x_t, S_{t+1}) = \rho\sigma_p^2(S_{t+1}) - (1-\rho)\sigma_\tau^2(S_{t+1}). \tag{9}$$

The time variation in covariance is, again, driven by the relative magnitudes of $\sigma_p(S_{t+1})$ and $\sigma_\tau(S_{t+1})$. It could even switch signs because $\rho < 1$.

These observations prompt us to introduce an explicit measure of the relative contribution of the transitory, c_τ, and permanent, c_p, components to shocks to expected consumption growth via

$$\eta(S_{t+1}) = \frac{(1-\rho)\sigma_\tau^2(S_{t+1})}{\rho\sigma_p^2(S_{t+1})}. \tag{10}$$

When $\eta(S_{t+1}) = 1$ the covariance between realized and expected consumption growth is equal to zero. If $\eta(S_{t+1}) > 1$, the transitory component dominates, and the covariance is negative.

3.2 Estimation

Our preferred model features three different Markov states. We have considered two, three, and four regimes. Two regimes is a natural candidate that captures the preliminary evidence suggesting the compositional shift with respect to permanent – transitory shocks around 1998. When estimating such a specification, we learned that we have to distinguish periods of such compositional shifts in the economy from periods in which there are large changes in volatility in the economy. The former shifts take place at a low frequency whereas the latter can be happening at a business-cycle frequency. That consideration led us to the three-regime specification. We have also considered a version with four regimes in which two regimes capture low-frequency compositional shifts in the economy and then two additional regimes within each low-frequency regime allow the economy to experience either low or high volatility of shocks. We concluded that the more parsimonious specification that allows only three regimes does a good job in capturing the consumption dynamics.

In order to obtain the broadest historical perspective on changing dynamics of US consumption we extend the real PCE data that is used in the previous section until 2024:Q1 and combine it with annual historical consumption data from Barro and Jin (2021) and Barro and Ursua (2008), which start in 1834. The reason we splice the annual data with quarterly data in 1947 is that we want to take advantage of the higher frequency, when available, to develop more granular asset pricing implications. See Appendix A for details. As an out-of-sample check, we also estimate the model using UK consumption data over the same time period.

In order to capture the real-time learning of economic agents, we estimate the model sequentially. Specifically, we follow Kim and Lee (2018) who extend the algorithm in Carvalho and

Lopes (2007) enabling sequential parameter learning and state filtering in real time. Further, we used methods developed by Schorfheide, Song, and Yaron (2018) to handle mixed-frequency data, where we allow for measurement error in consumption in the pre-World War II data (e.g., Romer, 1986 and Romer, 1989). See Appendix D for the regime-switching state-space representation as well as the details of the learning algorithm.

The priors need to be specified such that the regimes can be differentiated, or labeled. In particular, we form prior beliefs in a way that captures extreme volatility variations under the third regime and relatively moderate changes in volatility to be accommodated in the first two regimes. Specifically,

$$\sigma_p(1) > \sigma_p(2), \quad \sigma_\tau(1) < \sigma_\tau(2). \tag{11}$$

By substituting (11) into (10), we can deduce that the first (second) regime allows for a larger (smaller) share of permanent shocks relative to transitory shocks. We set $\mu_p(1) = \mu_p(2)$ so these two regimes are distinguished exclusively by their relative shares of transitory to permanent shocks. To ensure that the third regime is associated with the worst states, we impose in the prior that volatility of shocks in this regime is much larger relative to those in the first two regimes

$$\sigma_j(3) > \max\{\sigma_j(1), \sigma_j(2)\}, \quad j \in \{p, \tau\}. \tag{12}$$

To capture business cycle downturns, we consider the possibility that data exhibit left-skewness captured by $\mu_p(3) < 0$ (in the prior).

To ensure regimes 1 and 2 are capturing phenomena at a lower frequency than the business cycle, we center the prior beliefs about the duration of these regimes at about 48 years versus about 3 years for the third regime. The expected duration of regime j is roughly $1/(1 - p_{jj})$ (e.g., Kim and Nelson, 1999).

3.3 Evidence

Appendix E reports all the estimated parameters. In this section we summarize the key takeaways.

The first regime features higher volatility of the permanent component, 0.47% vs 0.05% for the transitory component. The magnitudes are reversed in the second and the third regimes: 0.15% vs 0.80% (second regime) and 1.43% vs 6.7% (third regime). The overall volatility level is the highest in the third regime. Further, our end-of-sample estimates imply that the relative contribution of the transitory and permanent components, η, is 0.01, 28.68, and 21.62 for the three regimes, respectively. See Equation (10).

We label the first regime as "permanent" as it has the highest relative and absolute contribution of the permanent component in consumption per Equation (4). Both regimes two and three have a relatively larger contribution of the transitory consumption component. We reserve the label "transitory" for the second regime while referring to the third one as "disaster." That is because the third regime features very high volatility and the mean of consumption growth in this regime is negative.

The permanent and transitory regimes are very persistent with expected duration of 36.1 and 36.7 years, respectively (based on the end-of-sample estimates). The disaster regime is less persistent with expected duration of 3.4 years.

Figure 6 displays the filtered probabilities of the regimes along with expected consumption growth, x_t. These beliefs are computed using the then-current parameter beliefs from the real-time learning problem and thus account for the parameter uncertainty agents are faced with. There is a substantial degree of uncertainty about regimes in the first part of the sample. That is not surprising because the frequency of observations is annual and parameter uncertainty is substantial. The delineation is more clear once we switch to quarterly frequency after 1947. The transitory regime was in place from early 1950s until mid 1990s with a resurgence during the post-Covid period. The permanent regime prevails from the mid 1990s to 2020. Qualitatively, the permanent regime has similar consumption dynamics to that of long-run risk models, but due to the relatively low estimated value of ρ, there is actually only limited long-run risk arising from this channel. The main source of consumption risks are the disaster risks arising in the third regime.

The third regime occurs at various points throughout the sample. The pre-World War II sample is characterized by high overall macroeconomic volatility and severe recessions (as compared to the modern era). Thus, the high probability of the third regime at the beginning of some of the recessions reflects these associated extreme risks. Interestingly, the disaster regime exhibits high probability around 1860s, 1880s, 1920s, and 1940s, which is consistent with Barro and Jin (2021) who use a different model. In the post-World War II sample the third regime is activated during the most severe crises, like the Korean war, oil shocks, and the Covid crisis. Yet, we find that our estimated disasters are much more modest than advocated by Barro (2006) and subsequent research on disasters. Specifically, the unconditional probability of transitioning into the disaster state is 4.6%; $\mu_p(3) = -0.28\%$ and $\sqrt{\sigma_p^2(3) + \sigma_\tau^2(3)} = 6.84\%$ (quarterly, based on end-of-sample values).

As a result, this regime is distinct from regular recessions during the modern times, including the Great Recession, which is driven relatively more by the permanent component of consumption. That is reflected in x_t becoming low and remaining so for an extended period. Also, note that periods of high likelihood of disasters coincide with periods of high uncertainty about x_t as shocks overwhelm the trend. Specifically, this is the case during Covid but not the Great Recession period. See the last panel of Figure 6.

To summarize, we demonstrate that agents in a real-time learning problem identify the permanent regime in the US as dominant from the mid 1990s, while the transitory regimes dominate earlier. Interestingly, the transitory regimes again become more dominant from 2020 and on, when the bond-stock correlation switches sign back to positive. The UK results are qualitatively similar and we report them in Appendix F for completeness. Our model contributes to the existing literature by documenting that the mix of transitory and permanent shocks to aggregate consumption is changing over time. That potentially explains disagreement in the earlier literature that reached different conclusions using classical time-series methods: a specific sample could have tilted analyses towards one of the configurations.

4 Asset-pricing implications

In this section we address how the advocated dynamics of consumption growth matter for economists. Cochrane (1988) points out that "... the size or existence of a random walk component in GNP cannot directly distinguish broad classes of economic theories of the business cycle ...". Yet, Alvarez and Jermann (2004, 2005) forcefully demonstrate that asset prices are informative about the marginal rate of substitution and macroeconomic fluctuations. Therefore, it is natural to take an exploration of the consumption dynamics that we have uncovered to asset price data. Specifically, we evaluate if one can make progress on capturing the most recent puzzles that pertain to the interaction between stocks and bonds.

4.1 Intuition using a single-regime model

The regime-switching sign of serial covariance of consumption growth in Equation (9) has dramatic implications for the dynamics of asset prices within the model, such as real bond risk premiums and the bond-stock correlation. To see that we expand on the log utility preferences example considered by Campbell (1986) and Piazzesi (2014).[4] We evaluate a

[4] Campbell (1986) also considers the more general CRRA preferences.

single-regime scenario, that is, where there is constant volatility of shocks. All the derivations are in Appendix G.

In this economy, the n-period bond risk premium is

$$-cov_t(m_{t+1}, r_{t+1}^{(n)}) = q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) = -q_{n-1,x}\rho\sigma_p^2(\eta - 1),$$

where $r_{t+1}^{(n)}$ denotes a log return on a real bond maturing in n periods, and $q_{n-1,x} < 0$ is the exposure of that log bond price to x_t, e.g., $q_{1,x} = -1$. Further, we use a consumption claim with log returns r_{t+1}^c as a metaphor for the stock market. The covariance between real bonds and stocks is

$$cov_t(r_{t+1}^{(n)}, r_{t+1}^c) = q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) = -q_{n-1,x}\rho\sigma_p^2(\eta - 1). \tag{13}$$

Thus, the bond risk premium, bond-stock covariance, and consumption-bond return covariance are literally the same objects under log preferences.

It is evident that the conditional covariance between realized and expected consumption growth highlighted in Equation (9) is the central object. If $\eta < 1$ the permanent component dominates. In this case, consumption growth is positively serially correlated. Thus, states with realized low growth have low growth expectations resulting in a high bond price. Bonds act as hedges to equity, and the short rate is procyclical. If $\eta > 1$ the transitory component dominates and consumption growth is negatively serially correlated. Thus, states with low growth have high growth expectations resulting in a low bond price. Bonds are no longer hedges to equity, and the short rate is countercyclical.

In order to generate switching signs in the bond-stock covariance, $\eta(S_{t+1})$ has to vary over time with values both above and below 1. Existing models have a constant η. Put differently, this quantity does not take values both above and below 1 and thus does not generate switching signs in the bond-stock covariance. Examples include Bansal and Yaron (2004), which always has negative bond-stock covariance, and Blanchard, L'Huillier, and Lorenzoni (2013). In the latter paper the authors explicitly assume this channel away in a single-regime setting by requiring ρ, σ_p and σ_τ to be such that the covariance in Equation (9) is always equal to zero, or, equivalently, $\eta = 1$.

As a motivational example, Figure 7 displays implications of our estimated consumption model combined with log utility, where we set the time discount factor $\delta = 0.9975$ to match the low level of interest rates in the data. In order to relate the estimation to the intuition developed above, we consider only parameters corresponding to one of the three regimes in the sense that all the computations are based on the assumption that the current regime would prevail forever. Because the figure is constructed for illustrative purposes only we use end-of-sample parameter estimates.

Consistent with the intuition, the permanent regime 1 features a correlation between the consumption claim and a 10-year real bond of -0.50, while the transitory regime 2 implies a correlation of 0.68. The disaster regime 3, which is dominated by the transitory component as well, features the consumption claim-bond correlation of 0.64.

The real bond risk premiums in the permanent regime are negative, since bonds are hedges in this state. That effect pushes the yield curve down. The reverse is occurring in the transitory regime. The disaster regime features strongly positive bond risk premiums because of the strong negative correlation between shocks to realized and expected consumption growth.

4.2 Connecting the model to the evidence

Cash flows

In this section we explore how well our model speaks to the key evidence we consider about stock and bond markets. Since we are contemplating an exchange economy, we have to specify the cash flows associated with these assets exogenously. For our purposes, we have to specify two types: aggregate stock dividends and inflation. The latter can be viewed as the cash flow to a nominal bond.

We assume dividend dynamics similar to those in Abel (1999) and let the (log) dividend growth rate be

$$\Delta d_{t+1} = \mu_d + \alpha_d(\Delta c_{t+1} - \mu_c).$$

Here, μ_c is the sample average of consumption growth, the leverage coefficient $\alpha_d = 4$ is calibrated to match the volatility of aggregate dividend growth, and $\mu_d = 0.0025$ is the sample average of aggregate dividend growth rate. The dividend process is intentionally oversimplified to highlight the effects arising from our advocated consumption dynamics.

Our specification of the inflation process is motivated by the observation that models in which monetary policy influences the real economy imply that the cyclicality of real interest rates and the cyclicality of inflation are jointly determined by the monetary policy rule and the stochastic structure of the shocks affecting the economy. Thus, we posit inflation dynamics as a parsimonious function of the real shocks to the economy. Specifically,

$$\pi_{t+1} = \mu_\pi(S_{t+1}, \mathbb{S}_{t+1}) + \alpha_\pi x_{t+1}, \tag{14}$$

where the new state $\mathbb{S}_t$ reflects new inflation-specific regimes.

In the absence of $\mathbb{S}_t$ the variables controlling the conditional mean and volatility of consumption growth control inflation also. The specification is too simple to reflect the full complexities of inflation dynamics. The $\mathbb{S}_t$ states allow μ_π to have both "high" and "low" regimes within each consumption state, enabling the inflation mean to shift accordingly. This specification is in the spirit of previous literature, such as Stock and Watson (2007), which argues for a trend-cycle component in inflation. Specifically, we allow the orthogonal regimes to capture low-frequency changes in inflation means, while also permitting inflation to be correlated with consumption growth. Further, we acknowledge that inflation dynamics during the consumption disaster state differ significantly from those in other states. Therefore, we allow for different high and low values of the inflation mean in this context. As a result, we define four distinct inflation regimes: two are labeled H and L, corresponding to high and low inflation means during the permanent and transitory consumption regimes, respectively. The other two are labeled HD and LD, representing high and low inflation means during the disaster regime. In total, this gives us four possible values for the inflation mean across the consumption-inflation regimes. This approach adds richness in the inflation specification that is necessary to closely match the data, yet remains parsimonious.

The challenge of our specification is that, despite its parsimony, we cannot simply calibrate it because it depends on the new inflation regimes. The posited process has to be estimated. A further challenge is that, because of the real-time learning setting, we cannot estimate inflation dynamics using a two-step procedure where inflation is estimated conditional on a given consumption dynamics. We have to re-estimate the joint dynamics of Δc_t and π_t. We proceed along the same lines as we did with Δc_t alone. We use CPI for the US inflation, which is available from 1913:Q2 onward to 2024:Q1 from the Bureau of Labor Statistics. See Appendix D for details on the estimation procedure. The model provides a good fit to the observed inflation, accounting for small measurement errors, see Figure 5(A). Estimated parameters are reported in Appendix E.

Figure 6 compares the consumption regimes estimated using Δc_t alone vs the joint Δc_t and π_t estimation. The regime probabilities are strikingly similar across the two estimations. There are two noteworthy changes. First, the joint estimation leads to a later transition to the permanent regime in the 1990s. The probability of the permanent regime in 1995 is already 1 for the consumption-only estimate, while it remains below 0.3 for the joint one. Only after

1998 does the permanent regime probability exceed 0.7 for the joint estimation. Second, the disaster regime probability implied by the joint estimation during the years 1980 and 2009 is near 1, in contrast to the consumption-only estimate, which is small. In summary, our main conclusions about the time-variation in consumption dynamics are unaffected by including inflation into estimation. The same conclusion holds for the UK (Appendix F).

Figure 8 displays probabilities of inflation regimes $\mathbb{S}_t$. Although the conventional wisdom is that inflation was low in the post-Volcker sample, the Figure shows that high and low regimes have been present throughout history. In particular inflation has exhibited low mean post the Great depression and post World War II. As regards disasters, the low value of the inflation mean, which is actually deflation, is associated with the Great depression, the Korean War, and Covid. High inflation during disaster is detected only during the Great Financial Crisis, in the post-Volcker era.

We substitute the posited inflation dynamics to obtain

$$\text{cov}(\Delta c_{t+1}, \pi_{t+1} \mid S_{t+1}, x_t) = \alpha_\pi \text{cov}(\Delta c_{t+1}, x_{t+1} | S_{t+1}, x_t).$$

Thus, cyclical properties of inflation are closely aligned with those consumption dynamics if one conditions on consumption states. Given that $\alpha_\pi > 0$ our model connects to the time variation in the cyclicality of inflation (counter-cyclical to pro-cyclical inflation) consistent with existing papers, which advocate the change in inflation's cyclicality as a driver of changing stock-bond correlations. Thus, our model is capable of matching this stylized fact via changing real dynamics. Figure 5(B) demonstrates this mechanism, and compares realized consumption-inflation correlation to the model-based one.

Confronted evidence

Besides the bond-stock correlation and basic equity moments such as the equity premium and return predictability, we focus on the shape of the bond yield curve and the shape of the equity dividend strip curve. As emphasized by Backus, Boyarchenko, and Chernov (2018), yield curve slopes are driven by the persistence of cash flows and the pricing kernel, which is at the heart of our model. For instance, with i.i.d. cash flows and pricing kernel, the yield curve would be flat, while negative autocorrelation yields an upward-sloping curve and positive autocorrelation implies the opposite pattern.

The model produces conditional bond-stock correlations. As mentioned earlier, Figure 1 presents the empirical evidence. In the following, we use the nominal 5-year bond correlations to compare to the model implications as actual observed daily data is available going back to 1961.

As we discussed in the single-regime case, the framework implies that the cyclicality of bond returns should change at the same time as the bond-stock correlation changes sign, per the discussion around Equation (13). To compare the model-implied conditional bond-consumption correlation to the evidence, we compute 10-year rolling correlations between annual consumption growth and excess bond returns, overlapping quarterly, where we proxy for the excess return using the negative of the change in 5-year bond yields minus the 1-year yield. We chose annual growth rates and bond returns as macro data suffer from measurement issues including time-averaging and seasonality adjustments that, at higher frequencies, blur the definition of contemporaneous co-movement relative to the precisely measured asset prices.

Lastly, the previous section has developed intuition about bond risk premiums. We cannot measure their proxy, average bond excess returns, in the case of real bonds. The data on bonds with maturities of less than two years are not reliably available either for the US or the UK. Furthermore, the granularity of available maturities is insufficient for computation of quarterly returns. As a result, we compare the model implications to the evidence on bond slopes. Slopes reflect expectations about the future short-term rates in addition to bond risk premiums.

4.3 Preferences

To evaluate asset-pricing implications we assume a representative agent who learns about consumption dynamics as described in the previous section. Changing parameters create a source of risk in addition to the traditional uncertainty about the state. Solving the full pricing problem with priced parameter and state uncertainty is computationally prohibitive as the dimensionality of the problem is too large. Thus, we follow Cogley and Sargent (2009), Johannes, Lochstoer, and Mou (2016), and Piazzesi and Schneider (2006) by using the Kreps (1998) anticipated utility approach as a realistic and reasonable alternative. In particular, claims are priced at each point in time using current posterior means for the parameters. Thus, the agent does not take into account the fact that she will update parameter beliefs in the future. Further, we assume that the agent sets the current state to the state that is currently most likely according to the state probabilities from the full learning problem. Uncertainty associated with this choice of the state is ignored. The pricing does take into account the fact that states can change in the future.

Logarithmic preferences

We start by extending the motivational example of the previous section to allow for changing regimes. The log utility model is well-known to generate very small risk premiums, so we are evaluating qualitative rather than quantitative effects.

Figures 9(A) and 10(A) display the results. We see that the model is capable of generating sign-switching bond-stock and bond-consumption correlations with patterns similar to those in the data (Figure 9). The real- and nominal-bond correlations with stocks are similar to each other in the model. These correlations are positive in the period when the transitory regime dominates and negative in the period when the permanent regime dominates, both in the data and in the model. In other words, bonds are risky in the early period and hedges in the later period, consistent with the intuition emphasized in the previous section. Finally, Figure 10 shows that the real bond slope is close to zero, but slightly negative, the nominal bond and forward equity yields slopes are positive but smaller than their data counterparts.

That the average real bond slope is negative does not contradict the intuition developed in the previous section, which was developed assuming the current regime prevails forever. Now, yield curves in each regime reflect the possibility of different regimes in the future, and the sample average yield curve reflects regime transitioning probabilities and the slope magnitudes in each regime. All these quantities in the model are such that the sample average is slightly negative. The bonds are risky in regimes 2 and 3, but not risky enough to produce a positive unconditional yield slope in this simple model.

We next allow for default to account for the described challenges with matching properties of real bonds quantitatively. This approach is consistent we several studies. Barro (2006) argues for the importance of introducing a probability of sovereign default conditional on being in a disaster state, while Chernov, Schmid, and Schneider (2020) argue that US sovereign default is reflected in the prices of sovereign credit default swaps. Edwards (2018) and Zivney and Marcus (1989) describe instances of US default. We assume that government bonds lose 50% of their face value in such a case, which is in line with the magnitudes entertained by these authors. The conditional probability of default in the disaster regime is 6%, which is more modest than the one entertained in Barro (2006). Given the end-of-sample estimates of the transition matrix, the unconditional (ergodic) probability of a disaster is 4.3%. Thus, the unconditional probability of a sovereign default is very small at $0.043 \times 0.06 = 0.26\%$.

Figures 9(B) and 10(B) demonstrate the results. The real bond curve is now upward-sloping, and the nominal yield curve is statistically indistinguishable from the observed one, although it is still small. The slope of the equity yields continues to be positive, and the correlation plots continue to be consistent with the patterns in the data. The modest sovereign default risk in fact leads to improvement in the level of the bond-stock correlation by shifting it slightly upwards relative to that in Panel A of Figure 9. The overall magnitude of stock-bond correlation went up as bonds are riskier.

One notable new feature is that the real-bond correlation is now deviating from the data during 1960s while the nominal bond-correlation lines up with the evidence quite well. Indeed, the consumption dynamics indicate the transitory regime during this period correctly implying positive real-bond correlation with stocks. The inflation-specific regimes further finess this prediction to match the specific observed pattern. We revisit this observation more formally later.

We note that our model captures the upward shift in the bond-stock correlation in early 2021. Market observers (e.g., AQR, 2021, Authers, 2021) attribute the spike to declining credibility of central banks and the weak anchoring of inflation expectations in the light of increasing worries about inflation. Our model offers an alternative interpretation of the evidence. We demonstrate that a transition to the disaster regime due to the Covid pandemic injects substantial transitory shocks making the bond-stock correlation behave as if we were in the transitory regime.

To summarize, the changes in relative contribution of permanent and transitory components of consumption growth are responsible for the changing sign in bond-stock correlation. A modest chance of sovereign default, which makes bonds more risky, helps with generating a positive average slope of the real yield curve and a more realistic level of bond-stock correlations. Next, we extend the preference specification to achieve quantitatively realistic magnitudes of risk premiums.

Epstein-Zin preferences

We consider an Epstein and Zin (1989) representative agent. These preferences, which have become a staple in asset pricing, are particularly helpful given our joint focus on stocks and bonds. The separation between risk aversion and EIS offers more flexibility in modeling equities and interest rates.

Specifically, we consider the case that sets the EIS equal to 1, which is the same value of EIS as under logarithmic preferences. The indirect utility of the agent, V_t, takes the form

$$V_t = \max_{C_t} \left[C_t^{1-\delta} \big(E_t(V_{t+1}^{1-\gamma}) \big)^{\frac{\delta}{1-\gamma}} \right],$$

where C_t denotes consumption and γ is the coefficient of relative risk aversion. We keep the value of the time discount factor δ the same as in the previous analysis. Appendix H derives bond and consumption claim prices implied by the consumption dynamics and these preferences.

We set $\gamma = 7$, which is in the range of values commonly used in finance applications to generate quantitatively meaningful asset pricing moments, i.e., the relatively high Sharpe ratio of the equity market. Thus, the only dimension along which we extend the logarithmic-preference analysis is by changing risk aversion. Figures 9(C) and 10(C) display the results for the US. The UK results are qualitatively similar and presented in Appendix F.

The bond-stock correlation and the correlation between excess bond returns and consumption growth have patterns that are similar to the logarithmic case with default in Panel B of Figure 10. As is the case with logarithmic preferences, the real-bond correlation is deviating from the data during 1960s, albeit with correct sign, while the nominal bond-correlation lines up with the evidence quite well. Moving risk aversion from 1 to 7 makes the slopes of both bond and equity curves larger and statistically and visually similar to those in the data. This indicates that consumption and dividend growth, and inflation in the model are reasonable in terms of the dynamics of their persistent components, which is what drives the term structures. However, the level of the quarterly real rate, as measured by the 2-year biod yield, is too high at 3.4% versus 1.50% in the data, and its standard deviation is 0.40% in the model versus 0.93% in the data. This high level is partly due to default risk and setting EIS = 1, which is lower than the value used in, e.g., Bansal and Yaron (2004).

Table 4 shows that the model also does a good job accounting for standard moments of equity returns, such as the equity risk premium, volatility, and, therefore, the Sharpe ratio. As we have seen, transitory shocks are responsible for the bulk of the consumption variance. The sizable equity premium arises due to the disaster regime. Our disasters are akin to the ones discussed in Barro (2006) albeit less likely and severe, which is why our risk aversion is higher than what Barro used.

Equity returns are more volatile than dividend growth, which has an annual standard deviation of 12%, and the price-dividend ratio is volatile and persistent as in the data. We thus address the excess volatility puzzle of Shiller (1981). Nevertheless, the model cannot match the very high volatility and persistence the historical price-dividend ratio, which indicates that the model lacks a sufficiently persistent component in expected returns and/or dividend growth. That said, the final rows of the table shows that the price-dividend ratio in the model is negatively related to future excess market returns, as is widely documented in prior studies (e.g., Cochrane, 1994). This is due to very high conditional risk premiums in the disaster state. Thus, the equity risk premium in the model is counter-cyclical as in the data.

Table 5 provides basic model-implied nominal bond moments. The model matches the mean and volatility of excess bond returns and term spreads relatively well, while the autocorrelation of the term spreads is somewhat lower than in the data. The latter is a manifestation of the hard-wired strong connection to expected consumption growth, which is not very persistent in the estimated model. The moments of term spreads have similar qualities.

The table considers predictability results (failure of the Expectations Hypothesis) as well. The model reconciles both Campbell and Shiller (1991) and Cochrane and Piazzesi (2005) regressions. For the latter regression, we consider only two predictors to avoid multicollinearity within the model. For the two predictors that we consider (1-year rate and 5-year forward) we find similar significance and the same signs as in the data. In contrast to a single predictor, the model R^2 is close to the one in the data.

Counterfactual analysis

In order to delineate the effect of inflation dynamics and those of consumption, we consider a counterfactual where we shut down the effects of variation which resides in inflation regimes $\mathbb{S}_t$. Given a specific value of the consumption regime S_t, we set the value of the inflation mean μ_π to the probability-weighted average of its estimated high and low values from $\mathbb{S}_t$. As a result, the counterfactual model is affected exclusively by consumption dynamics. We report the corresponding evidence in Figurse 9(D) and 10(D), and the last column of Table 5.

The figure highlights two key departures from the baseline model. First, although the model generates an upward sloping nominal yield curve, it falls short of matching its observed magnitude. Second, in contrast to the baseline case, the model implies the same pattern in real- and nominal-bond correlations with stocks during the 1960s. As a result, it cannot capture the observed pattern for nominal bonds highlighting the role of inflation dynamics during this period.

Moving on to other nominal-bond moments in the Table 5, we observe that the counterfactual model cannot match the bond risk premiums and average term spreads (the latter is consistent with the figure). Predictability results are stronger than those in the data, both in terms of the magnitudes of regression coefficients and R^2. This is not surprising as the counterfactual model features a much stronger connection between the endowment and inflation.

Conditional model implications

In this section we explore conditional implications of the model beyond the bond-stock correlation itself. Figure 11 shows the slopes of real and bond as well as forward equity yield curves by regimes and compares them to the data. Because we have real-time parameter and state estimates, we are literally computing the same objects in the data and in the model, that is, averages of asset prices by regime, which are established in real time.

The model is doing a good job in matching real and nominal bond slopes in consumption regimes 1 and 2. There are two differences from the base intuition we have developed under log-preferences for the bond slopes. First, the real slopes are positive in both regimes 1 and 2. Second, despite the same sign of the slopes, the bond-stock correlation does change sign as per Figure 9(C). The log-preference intuition in Figure 7 was developed assuming the current regime prevails forever. In contrast, in the data yield curves reflect the possibility of different regimes in the future. Thus, the model counterpart to the data-based statistics has to allow for the different possible future regimes as well. The positive slopes are a reflection of the possibility of the disaster regime, which generates a strongly upward sloping real curve.

The equity yield slopes, provided in panels on the right, are upward-sloping in regime 1 and flat in regime 2. In the model the curves are upward sloping and quantitatively similar in both regimes. Both regimes are impacted similarly by the chance of entering the disaster regime in the future. The disaster effect dominates quantitatively and leads the model to produce upwards sloping and similar equity term structures. Because of considerable statistical uncertainty, the model-implied slopes are also within confidence bands of the data patterns. Point-wise, the curve is too steep in the second regime. As we noted earlier, we intentionally use a particularly simple specification of dividend dynamics. Its sophistication could be increased to help with matching cash flow persistence implicit in the shape of the dividend curve.

In regime 3 the slope of the real yield curve is upward sloping in the data and downward-sloping in the model; both are insignificantly different from zero (flat). The slight downward-slope of the real yield curve in the model is due to default risk, which is high in the disaster state, but low in the other states. Longer term bond yields reflect this expected decrease in the default risk as the economy is expected to transition out of the disaster state. The nominal yield curve is upward sloping, while it is gently hump-shaped in the model. The two are insignificantly different from each other. The equity yield curve is more strongly downward-sloping in the data than in the model. Thus, there is some tension in matching these slopes quantitatively.

Discussion

To summarize, the proposed model of consumption dynamics is capable of capturing most of the changing bond-stock correlation and the changing correlation between excess bond returns and consumption growth. Capturing other key asset pricing moments requires introduction of a modest government default probability (for the sign of the real yield curve), recursive preferences (for equity moments), and realistic inflation dynamics (for the nominal bond moments and for the bond-stock correlation during the 1960s).

The model cannot match all moments, however, including the level of the risk-free rate and the high volatility and persistence of the price-dividend ratio. We use isoelastic preferences, so that there are no dynamics arising from preference shocks. This provides a particularly transparent framework for evaluating the implications of the change in consumption dynamics that we document in this paper. Preference shocks leading to time-variation in risk aversion and/or the time discount factor, as highlighted in, e.g., Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Campbell, Pflueger, and Viceira (2020), are important elements needed to fully explain asset price dynamics.

5 Conclusion

We establish that four economic objects are changing dramatically at the same time: the correlation of stocks with real bonds, the correlation of stocks with nominal bonds, the mix of permanent and transitory components of consumption, and the cyclicality of inflation. The literature has focused its attention on the shifting cyclicality of inflation (the nominal channel) as an explanation for the changing correlation between stocks and nominal bonds. However, this feature cannot alone explain the switching correlation between stocks and real bonds. We argue theoretically and demonstrate empirically that changes in the dynamics of real consumption growth can explain the changing correlation between real bonds and stocks and goes a long way in capturing the nominal bond-stock correlation.

We emphasize that our exchange economy setting does not preclude inflation from driving real shocks, through, for instance, cost-push shocks or monetary frictions. Whatever the underlying cause, the persistence of consumption growth is changing when bond-stock correlations switch signs, which we attribute to a time-varying mix of transitory and permanent components in consumption. Understanding the fundamental drivers of this shift is a topic for future research.

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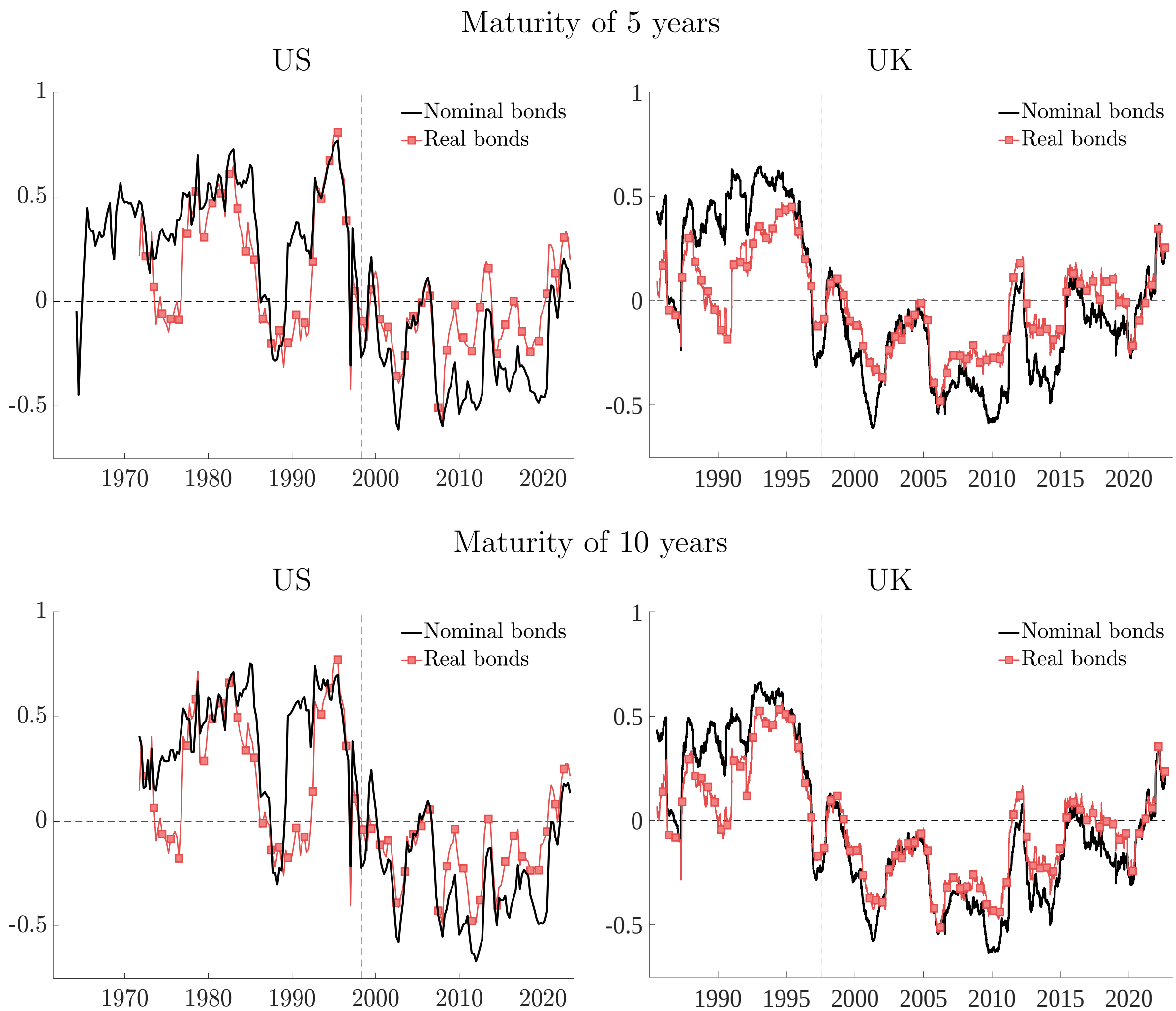
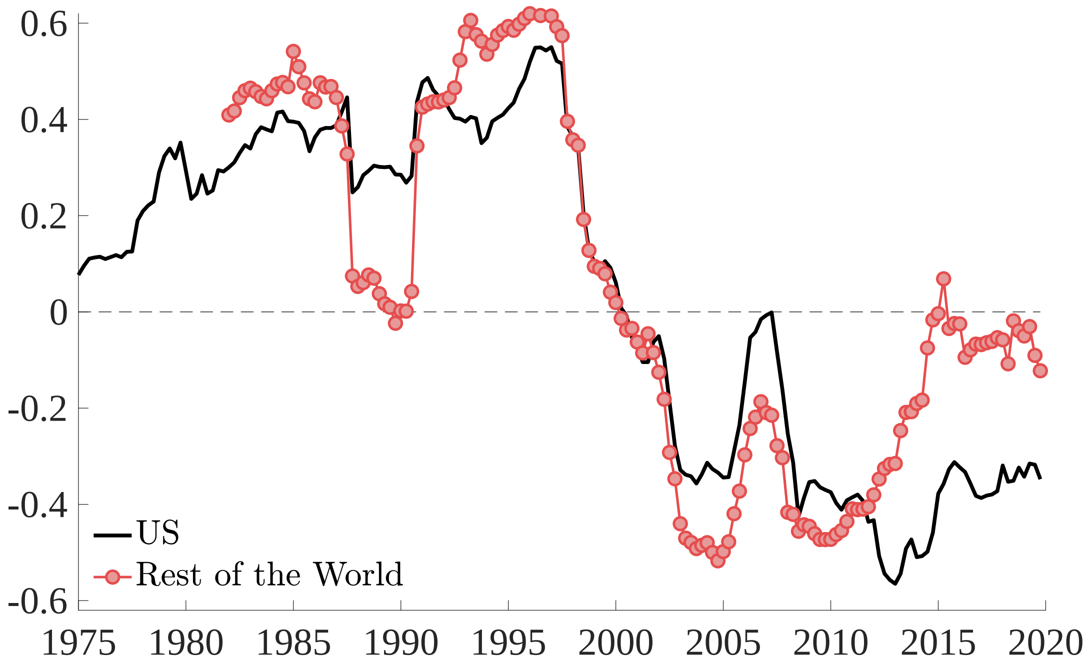
Figure 1: Bond-stock correlation: US and UK



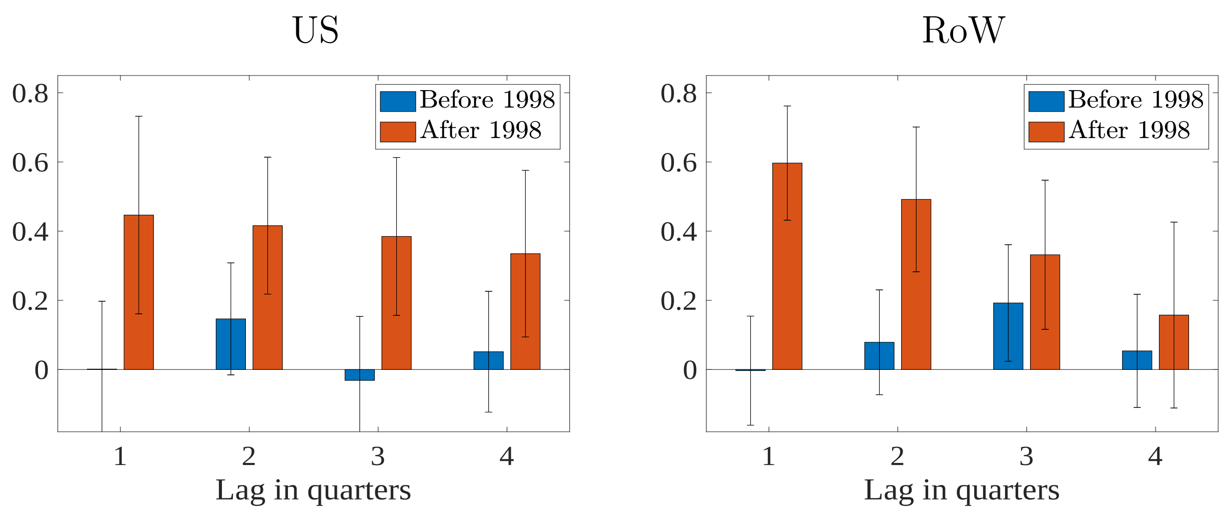
*N*otes: The plot shows bond-stock correlations for US and UK. We compute the realized 1-year correlation based on daily data for the UK. The US real-bond data are available at daily frequency only after 1999. Prior to that we use quarterly data to compute realized 3-year correlation for both real and nominal bonds. Details are provided in the appendix. The vertical dashed lines indicate 1998, the year when nominal bond-stock correlations in the US switch signs. The evidence on nominal bonds updates that from Baele, Bekaert, and Inghelbrecht (2010) and Campbell, Shiller, and Viceira (2009). The evidence on inflation-indexed bonds for the UK updates that from Campbell, Shiller, and Viceira (2009). The corresponding evidence from the US is novel, using real yields extrapolated from nominal yields, survey-based forecasts of inflation, and inflation itself using an affine no-arbitrage model (Chernov and Mueller, 2012) until 1998 and TIPS data thereafter.

Figure 2: Bond-stock correlation: US and Rest of the World



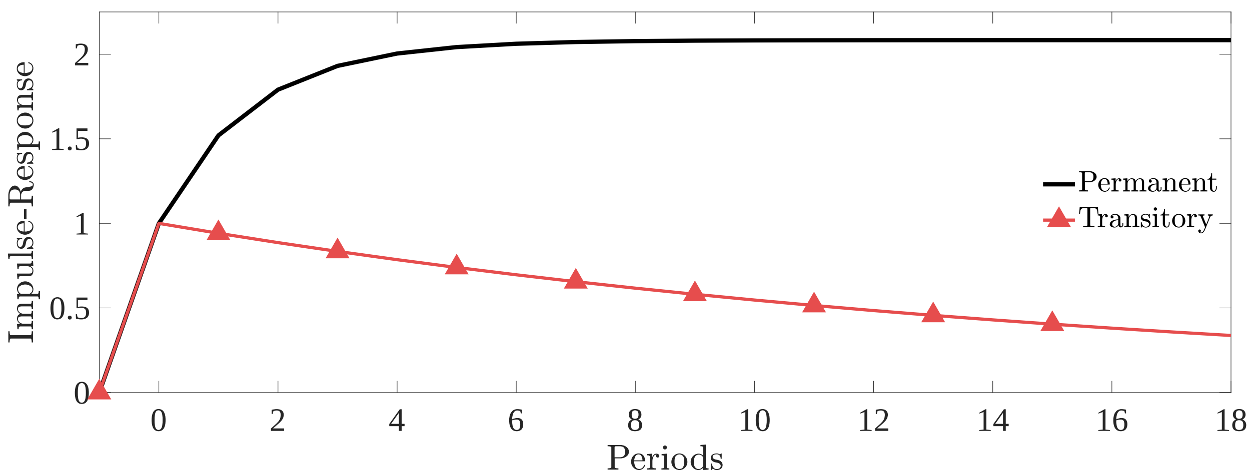
*N*otes: The plot shows bond-stock correlations for the US (1975-2019, black solid line) and the RoW (1982-2019, red dotted line). The US bond-stock correlations are rolling windows of realized 3-year correlations based on daily returns to the CRSP market index and 10-year zero-coupon Treasury bonds, as calculated by the Federal Reserve. The RoW bond-stock correlation is calculated in the same manner using the average daily stock and bond return across countries as the daily RoW data. We use daily stock returns on common market indexes and 10-year nominal sovereign bond return data from 19 developed countries in this exercise. See Appendix A for details on the data construction.

Figure 3: Autocorrelations of consumption growth



*N*otes: The plot shows autocorrelations of quarterly log consumption growth, both for US and internationally (RoW). The US data are quarterly real-time real PCE data. The RoW data is the equal-weighted average real consumption growth across 19 developed countries at each point in time. The blue bars shows the autocorrelations for lags 1 to 4 in the period up until 1997:Q4, while the red bars give the autocorrelations for the same lags in the period post 1997:Q4. The error bars give the +/- two standard error bounds of the estimated autocorrelation coefficients. See Appendix A for details on the data construction.

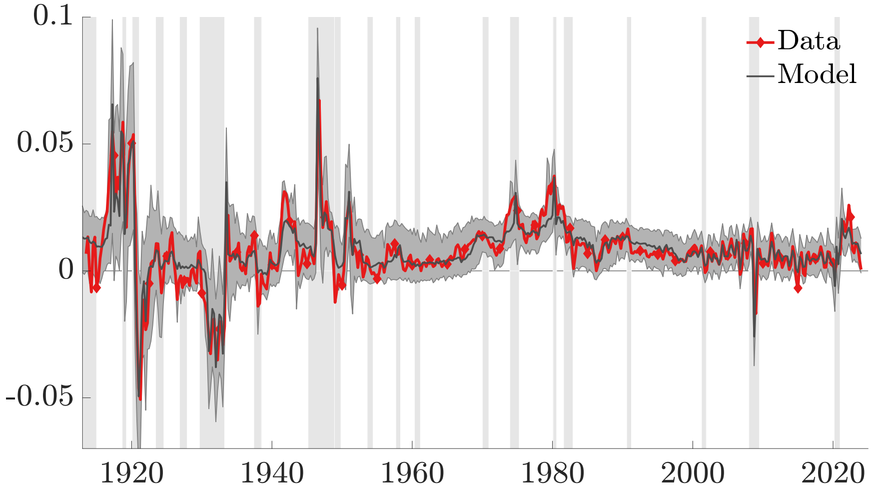
Figure 4: Transitory and permanent effects



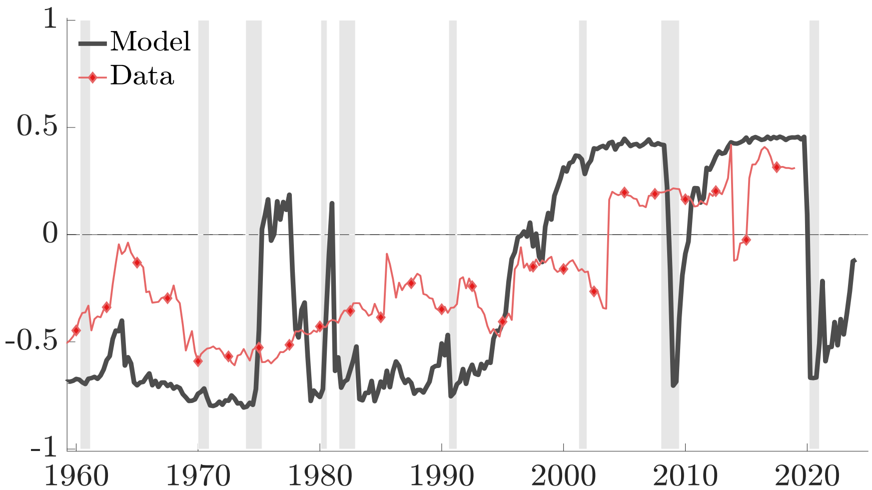
*N*otes: The plot shows the impulse-response of a positive shock to the consumption components we label as permanent (black) and transitory (red). In the former case, a positive shock leads to a positive shock to future expected consumption *growth*, whereas in the latter case a positive shock leads to a negative shock to future expected consumption growth.

Figure 5: Implied inflation and consumption-inflation correlation

(A) Inflation fit

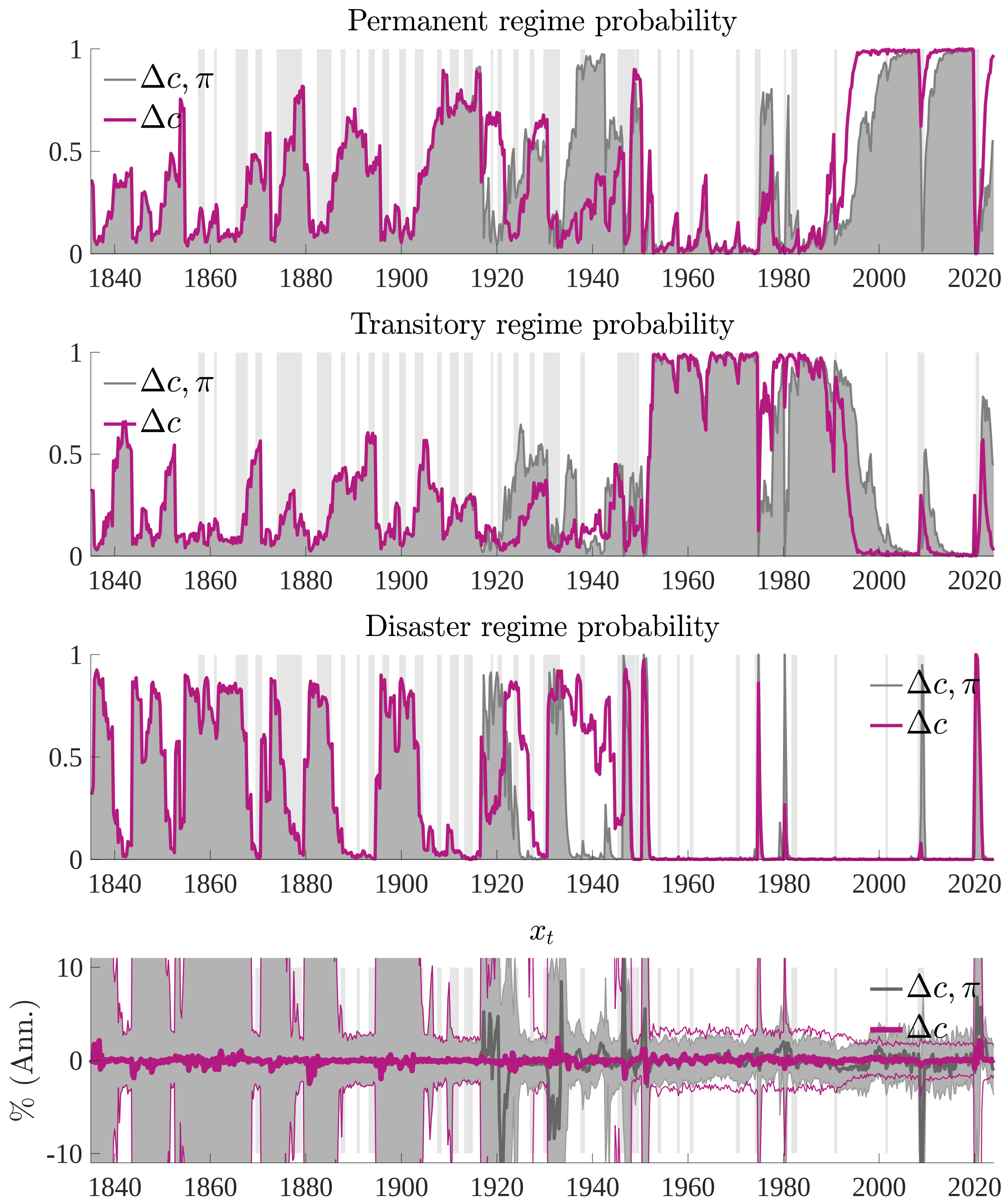


(B) Consumption-inflation correlation



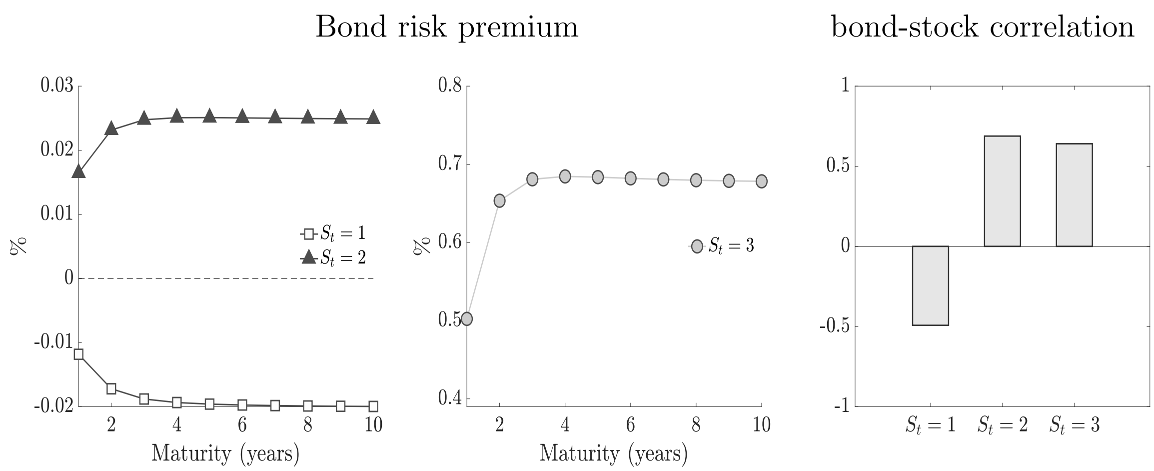
*N*otes: Panel (A): We present the estimated inflation dynamics, with the median and 90% credible interval shown in gray, and the observed data in red. The inflation series is available from 1913 onward. Panel (B): Using the median posterior estimates, we solve the asset pricing model in real time and compute the conditional correlation between consumption growth and inflation from 1960:Q1 to 2024:Q1. We overlay this with rolling correlation estimates based on a 10-year window from quarterly data.

Figure 6: Estimated states for consumption growth



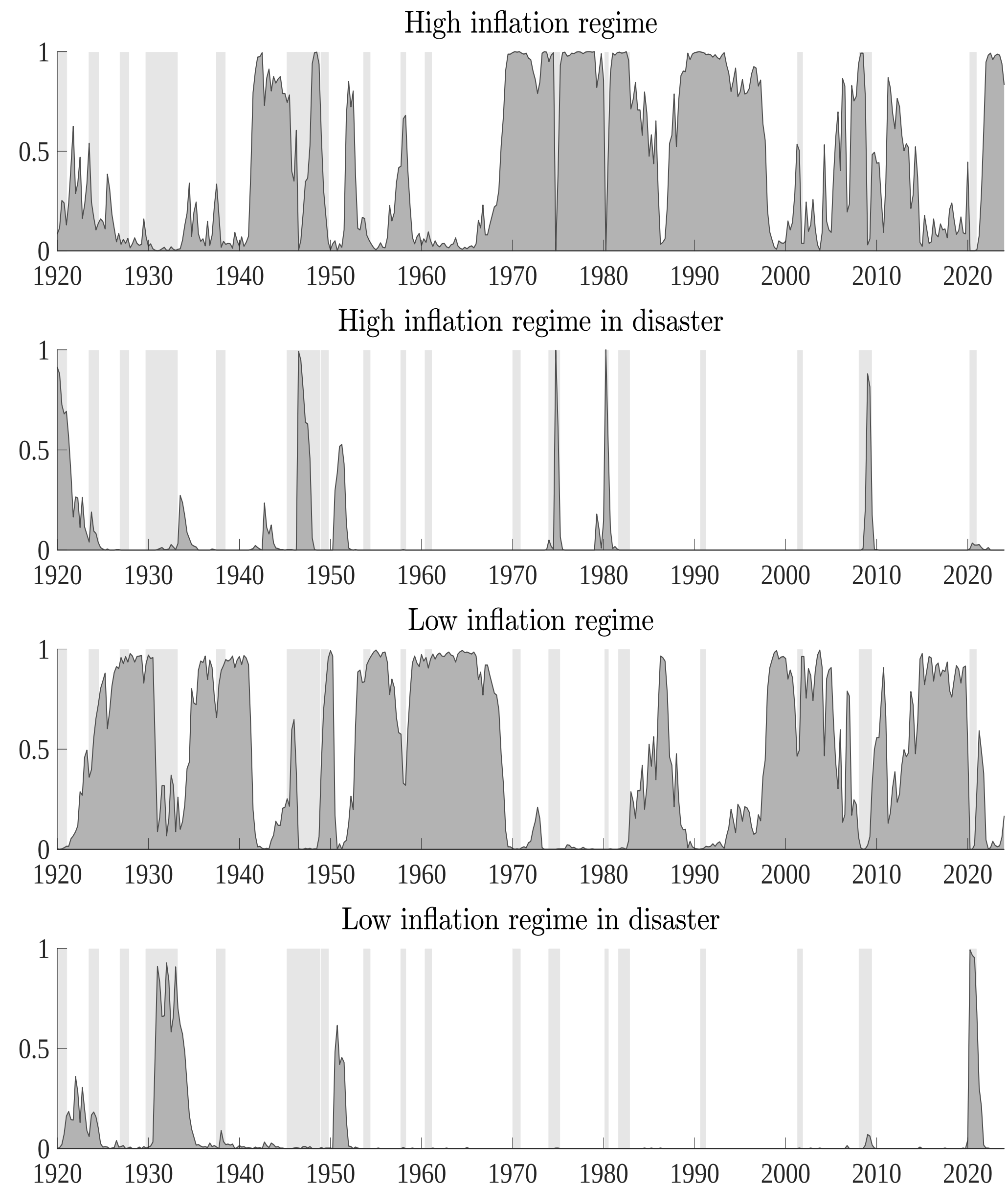
Notes: The plots display the filtering probabilities (average occurrence across all posterior-drawn regimes) for each regime and the expected consumption growth throughout the sample. Grey bars indicate NBER recessions. The first regime corresponds to the "permanent" regime, the second to the "transitory" regime, and the third to the "disaster" regime. The final panel shows the posterior median estimates of expected consumption growth with 90% credible intervals. Estimates from the joint consumption-inflation estimation (gray) are compared with those from the consumption-only estimation (purple).

Figure 7: Model-implied asset pricing conditional moments: Log utility preference



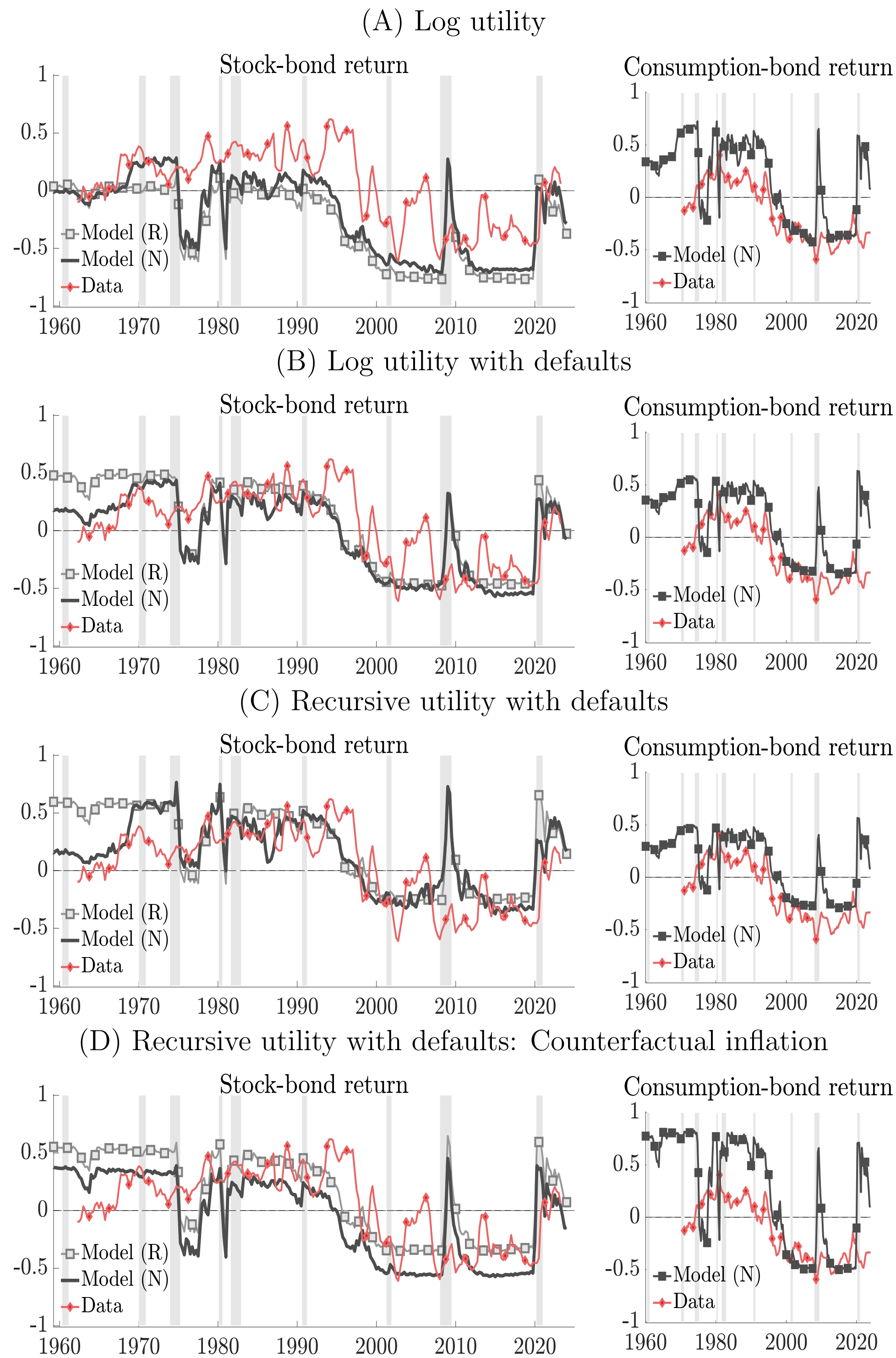
*N*otes: We provide the conditional bond risk premiums and correlation between the returns to real bonds and the consumption claim under log utility preferences. The computations assume that a given regime would prevail forever (a fixed-regime case).

Figure 8: Estimated states for inflation



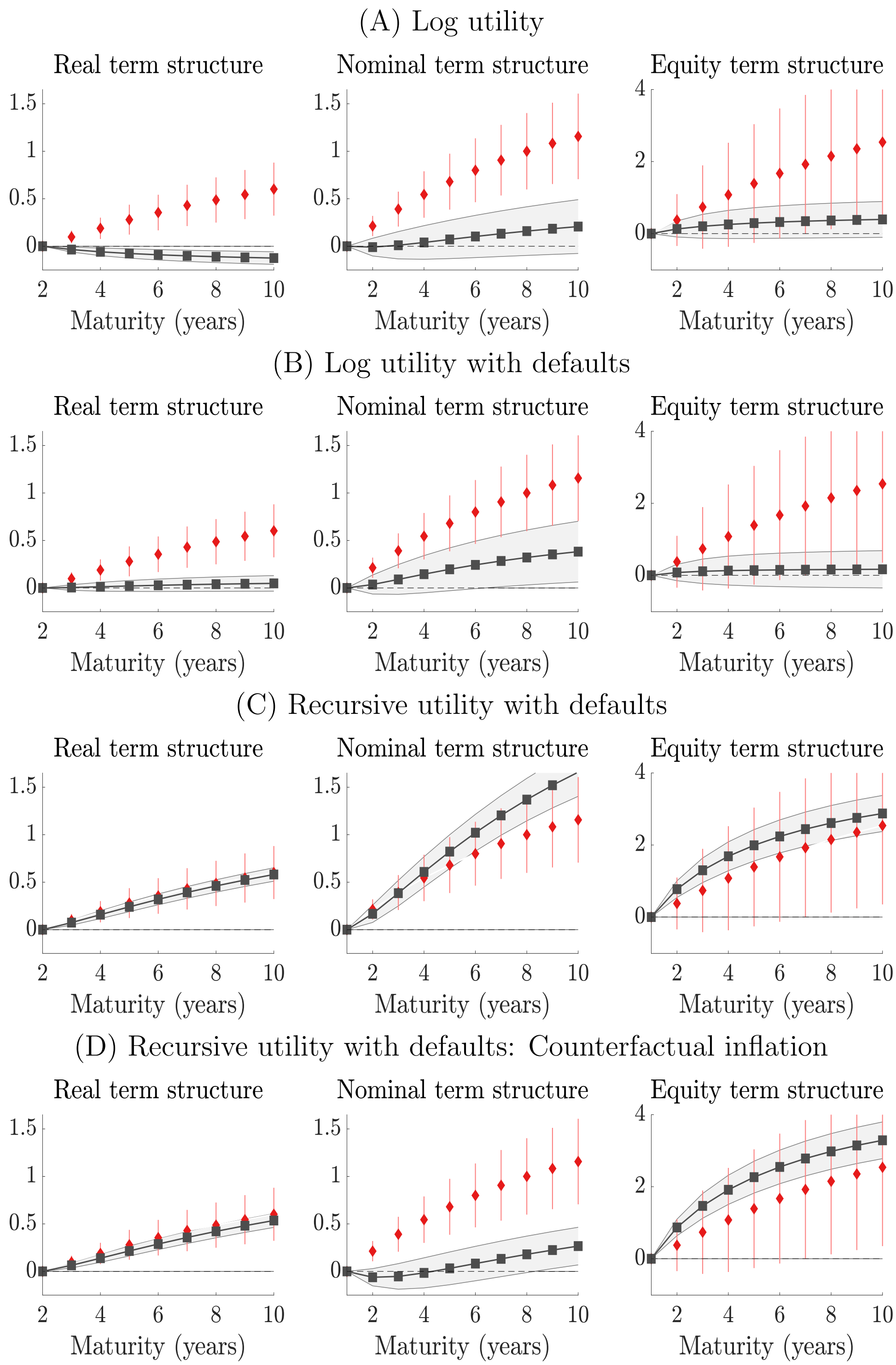
*N*otes: The plots show the filtering probabilities (average occurrence based on the entire posterior-drawn regimes) of each inflation regime through the sample. We consider four inflation regimes: high and low in permanent and transitory consumption regimes, and high disaster and low disaster in the disaster regime. The grey bars represent NBER recessions.

Figure 9: Asset pricing implications: Time-series correlation



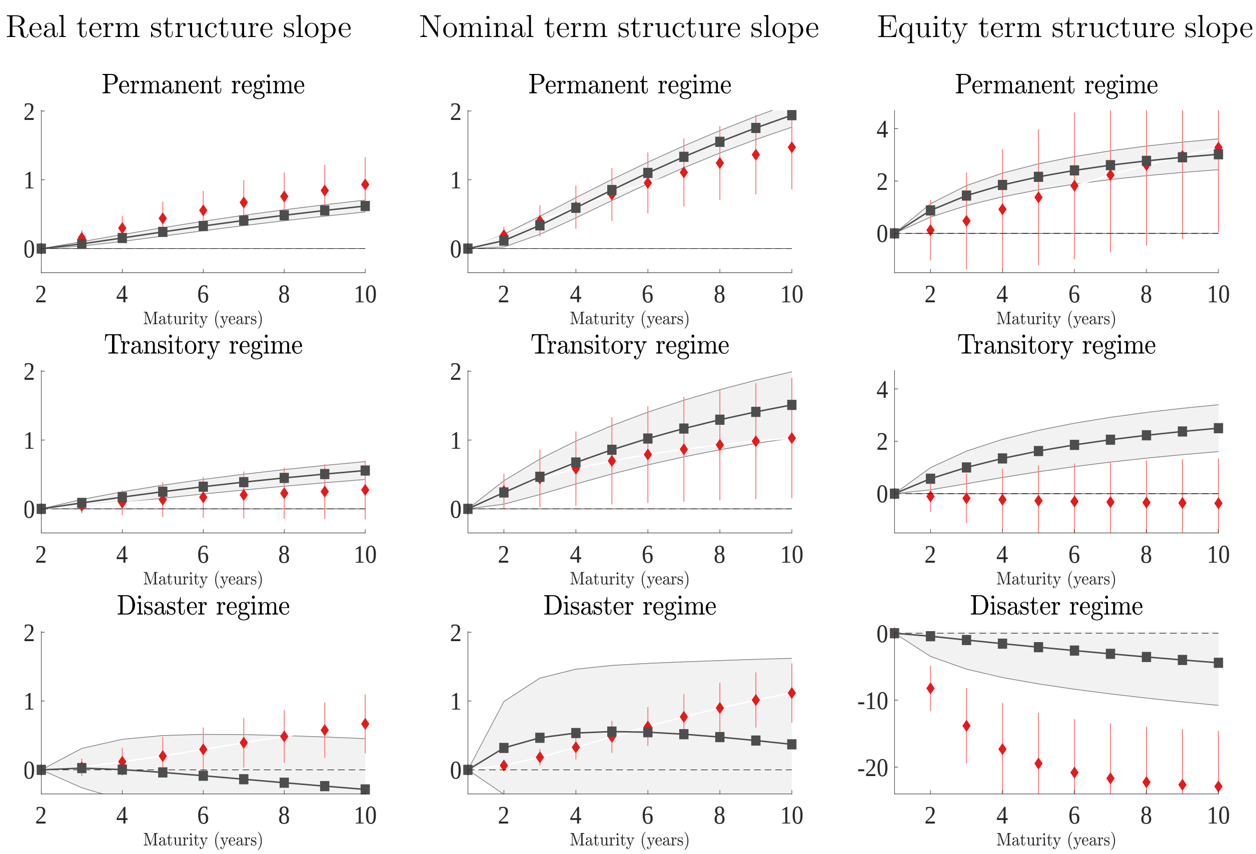
*N*otes: We present the conditional quarterly bond-stock correlation and consumption-bond return correlations. Details on variable construction are provided in the main text. The data is shown in red, with 95% confidence bands for the yield curve slopes, while the model is represented in black. We set $\delta = 0.9975$ for all analyses. For the recursive utility case, we consider $\gamma = 7$ and EIS $= 1$. The probability of default in disaster state $= 6\%$. Grey bars indicate recessions.

Figure 10: Asset pricing implications: Slope



*N*otes: We present the sample average slope of real and nominal bond yields, and real equity yields. Details on variable construction are provided in the main text. The data is shown in red, with 95% confidence bands for the yield curve slopes, while the model is represented in black. We set $\delta = 0.9975$ for all analyses. For the recursive utility case, we consider $\gamma = 7$ and EIS $= 1$. The probability of default in disaster state $=$ 6%.

Figure 11: Model-implied asset pricing conditional moments with data comparison



*N*otes: The plots show the slopes of the term structures at various maturities. The data averages are calculated for each regime assigning an observation to a specific regime when the regime's filtering probability is greater than 0.8, and plotted with 95% confidence bands. The model counterparts are computed as the average yield slopes across the sample for each regime. The data is in red, while the model-based yields are in black.

Table 1: Consumption autocorrelation before and after 1998

	US				RoW	
	Full sample		Sample ex. recessions			
	(1)	(2)	(1)	(2)	(1)	(2)
α_0	0.0039***	0.0001	0.0058***	0.0002	0.0020***	0.0004
	(0.0011)	(0.0005)	(0.0008)	(0.0005)	(0.0007)	(0.0004)
α_1	0.0040***		0.0053***		0.0058***	
	(0.0014)		(0.0014)		(0.0010)	
β_0	0.4424***	0.4560***	0.2463***	0.3460***	0.5968***	0.7332***
	(0.1288)	(0.1317)	(0.1035)	(0.1014)	(0.1141)	(0.1194)
β_1	-0.4411***	-0.4541***	-0.3898***	-0.4848***	-0.6005***	-0.7275***
	(0.1605)	(0.1608)	(0.1585)	(0.1481)	(0.1491)	(0.1477)
R^2	0.0203	0.0191	0.0397	0.0286	0.0747	0.0634

Notes: The table shows the regression estimates from the regression in Equation (1). We present evidence about the US in the left panel. In the middle panel, business cycle downturns are excluded from the US sample, as explained in the main text. In the right panel, we present evidence for the RoW consumption growth based on 19 developed markets. The Covid period is excluded from all regressions. Robust standard errors are reported in parenthesis. The β_0 estimate gives the autocorrelation coefficient in the post-1998 sample, while the β_1 estimate gives the difference between the autocorrelation in the pre- vs. post-1998 samples. The specification in the column labeled (1) estimates the full regression, while the specification in the column labeled (2) is run using demeaned consumption growth and setting $\alpha_1 = 0$, as explained in the main text. One asterisk denotes significance at the 10% level, two at the 5% level, and three at the 1% level. For the US, the data is real-time quarterly real PCE data, from 1947:Q2 to 2019:Q4. The RoW consumption growth series is from 1956:Q1 to 2019:Q4 and computed from the real consumption data for each of the 19 developed countries as available in the FRED database. See Appendix A for details.

Table 2: Production data autocorrelation before and after 1998

	BEA		FRBSF				BLS	
	Δy	Δi	Δy	Δz	$\Delta \ell$	Δk	$\Delta \ell p$	$\Delta \ell$
α_0	0.0159***	0.0225	0.0166**	0.0069*	0.0050	0.0059	0.0135***	0.0027
	(0.0049)	(0.0212)	(0.0088)	(0.0040)	(0.0055)	(0.0037)	(0.0038)	(0.0056)
α_1	0.0217***	0.0345	0.0251**	0.0085	0.0113*	0.0095*	0.0113**	0.0137**
	(0.0071)	(0.0259)	(0.0102)	(0.0053)	(0.0066)	(0.0049)	(0.0049)	(0.0068)
β_0	0.2583	0.1176	0.3059	0.1449	0.3972**	0.7792***	0.3159**	0.4068**
	(0.1630)	(0.2207)	(0.2056)	(0.2180)	(0.1640)	(0.1299)	(0.1328)	(0.1818)
β_1	-0.3399*	-0.4982**	-0.4510**	-0.2930	-0.4217***	-0.1733	-0.4308***	-0.4709**
	(0.1927)	(0.2471)	(0.2264)	(0.2374)	(0.1905)	(0.1545)	(0.1628)	(0.2040)
R^2	0.0610	0.1232	0.0543	0.0256	0.0454	0.5376	0.0252	0.0647

Notes: Each column shows the regression estimates from regressions like that in Equation (1), but where the left- and right-hand side variables are from the production side of the economy and given in the column header. All data is quarterly, from 1947Q1 to 2019Q4, but due to seasonality in some of the data the regressions are all run at the annual frequency overlapping quarterly. BEA refers to the Bureau of Economic Analysis, and Δy and Δi is real per capita growth in log output and private domestic investment. FRBSF refers to the Federal Reserve Board San Francisco, and the data here is the Fernald production data. Here, Δy, Δz, $\Delta \ell$, and Δk refer to real growth in log output, total factor productivity, hours worked, and capital. BLS refers to the Bureau of Labor Statistics, and $\Delta \ell p$ and $\Delta \ell$ is real growth in log labor productivity and hours worked. Robust standard errors are reported in parenthesis. The β_0 estimate gives the autocorrelation coefficient in the post-1998 sample, while the β_1 estimate gives the difference between the autocorrelation in the pre- vs. post-1998 samples. One asterisk denotes significance at the 10% level, two at the 5% level, and three at the 1% level.

Table 3: Consumption autocorrelation and bond-stock correlation

	US	RoW
Constant	0.3673***	0.2202***
	(0.0597)	(0.0748)
Slope	-0.3528**	-0.4042**
	(0.1761)	(0.1996)
R^2	0.1433	0.1509

*N*otes: The table shows the results from regressing 3-year realized consumption autocorrelations contemporaneous bond-stock correlations, overlapping at the quarterly frequency. The consumption autocorrelations are calculated using quarterly data, while the bond-stock correlations are calculated using daily data. The regression is overlapping at the quarterly frequency. The US data is from 1975 to 2019, while the RoW data is from 1982 to 2019 based on data from 19 developed countries. See Appendix A for details on the data construction. Newey-West standard errors are given in parenthesis using 12 lags. One asterisk denotes significance at the 10% level, two at the 5% level, and three at the 1% level.

Table 4: Standard equity pricing moments

		Data	Model
Excess returns	Risk premium	7.54	7.36
	Volatility	16.50	14.45
	Sharpe ratio	0.46	0.51
Log pd ratio	Mean	3.59	3.95
	Volatility	0.41	0.20
	AR(1) coefficient	0.96	0.62
Predictability	1-year excess return on log pd ratio	-0.10	-0.94
		(0.05)	(0.41)
	R^2 value	0.06	0.19

*N*otes: We report basic equity moments from 1960 to 2023. The model-implied moments are calculated using the real-time mean parameter estimates over the sample, while the data moments are calculated using the sample quarterly CRSP stock market returns in excess of the 3-month T-bill rate. The risk premium is the expected return above the risk-free rate, volatility is defined as the standard deviation of excess returns, and the Sharpe ratio measures the ratio of mean excess return to its standard deviation. Model-implied moments are precisely calculated from conditional moments. All return moments are annualized. In the predictability regression, we regress the aggregated one-year excess stock return on the lagged aggregated annual log price-to-dividend (PD) ratio, reporting the slope coefficient (with standard error) and the R^2 value.

Table 5: Nominal bond pricing moments

		Data	Model Baseline	Model Cntrfctl
Excess returns	5-year maturity			
	◦ Risk premium	1.69	1.68	0.33
	◦ Volatility	2.64	3.17	4.22
	◦ Sharpe ratio	0.64	0.53	0.08
	10-year maturity			
	◦ Risk premium	2.59	3.01	0.87
	◦ Volatility	5.05	5.30	9.36
	◦ Sharpe ratio	0.51	0.57	0.09
Term spread	Spread between 5- and 1-year			
	◦ Mean	0.74	0.75	0.03
	◦ Volatility	0.41	0.53	0.59
	◦ AR(1) coefficient	0.89	0.61	0.68
	Spread between 10- and 1-year			
	◦ Mean	1.20	1.52	0.25
	◦ Volatility	0.60	0.65	0.70
	◦ AR(1) coefficient	0.91	0.60	0.68
Predictability	5-year excess return on term spread			
	◦ loading on 5y-1y spread	2.26	1.38	3.05
		(0.44)	(0.70)	(0.82)
	◦ R^2 value	0.12	0.05	0.18
	5-year excess return on yield and forward			
	◦ loading on 1y yield	-1.24	-0.86	-1.68
		(0.56)	(0.47)	(0.52)
	◦ loading on 5y forward	1.48	1.96	2.97
		(0.66)	(0.53)	(0.46)
	◦ R^2 value	0.10	0.12	0.31

Notes: The model-implied moments are calculated using the real-time mean parameter estimates over the sample. In the data, excess bond return is defined as the one-year holding period of the 5- and 10-year maturity nominal bond in excess of the one-year bond yield. The risk premium refers to the expected return over the risk free rate, volatility refers to the standard deviation of excess returns, while the Sharpe ratio is the ratio of the mean excess return to its standard deviation. All return moments are annualized. The term spread is defined as the difference between nominal bond yields of maturities of 5 (or 10) and 1 year. AR(1) refers to the autocorrelation at the one quarter lag. Data moments are computed based on 1970 (1972 for 10 year bonds) to 2023. We consider two types of predictability regressions. In the first predictability regression, we regress one-year excess bond return on the lagged term spread between maturities of 5 and 1 years and report the slope coefficient and the $R2$. In the second predictability regression, we regress one-year excess bond return on 1-year interest rate and forward rates of maturity 5 years in the spirit of Cochrane and Piazzesi (2005). We report the slope coefficient (Newey-West standard errors with 4 lags) and the $R2$. For model implications, we present results for our "Baseline" and "Cntrfctl," which represents a case where the effects of regime-switching inflation mean are removed.

Online Appendix

A Data description

A.1 Consumption data

In the case of US, we use the real-time data series on growth in real personal consumption expenditures (PCE) from 1947:Q2 to 2024:Q1, which are available from the Federal Reserve Bank of Philadelphia. We use real-time consumption data to best align them with investors' information set at the time they set asset prices. PCE offers the longest span of such data. As a robustness check, we also use revised real per capita nondurable and services consumption in some regressions, which is obtained from the NIPA Tables of the Bureau of Economic Analysis. Finally, when estimating the dynamic model of consumption growth, we concatenate Barro and Jin (2021) and Barro and Ursua (2008) annual consumption growth per capita series from 1834 to 1947 with quarterly PCE series from 1948:Q1 to 2024:Q1.

In the case of UK, we construct the extended consumption growth series by first adding the growth rates of the real consumption expenditures in the United Kingdom (1955:Q2 to 2016:Q4) with those of the real final consumption expenditure for Great Britain (2017:Q1 to 2023:Q3) which are obtained from the FRED database of the St. Louis Federal Reserve. We obtain population data from the Bank of England ("A Millennium of Macroeconomic data for the UK") and convert the growth rates to per capita growth rates. Then, we concatenate Barro and Jin (2021) and Barro and Ursua (2008) annual consumption growth per capita series from 1834 to 1955 with quarterly series from 1956:Q1 to 2023:Q3 when estimating the dynamic model of consumption growth.

The rest of the world (RoW) consumption growth is based on consumption data from the following 19 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. This data is obtained from the FRED database of the St. Louis Federal Reserve. We use the longest span of quarterly real personal consumption expenditures available for each country. In each quarter, we take the cross-sectional average consumption growth across countries with available data to arrive at the RoW consumption growth measure. This data is from 1955:Q1 to 2019:Q4.

A.2 Inflation data

We use CPI inflation data for the US (1913:Q2 to 2024:Q1) and the UK (1955:Q2 to 2023:Q4), sourced from the FRED database of the St. Louis Federal Reserve.

A.3 Other macro data

We get quarterly real per capita GDP and gross private domestic investment data from the NIPA Tables of the Bureau of Economic Analysis. We get quarterly data on real output, productivity, capital and hours worked from the San Francisco Federal Reserve (the Fernald data). We get quarterly hours worked and labor productivity from the Bureau of Labor Statistics. All data is for the US from 1947:Q2 to 2019:Q4.

A.4 Asset pricing data

Daily US market returns are obtained from the Center for Research in Security Prices (CRSP) stock market indexes (NYSE/AMEX/NASDAQ/ARCA), while daily nominal zero-coupon Treasury yields at the 5- and 10-year maturities are sourced from Gurkaynak, Sack, and Wright (2007), and daily real zero-coupon Treasury yields at the 5- and 10-year maturities are from Gurkaynak, Sack, and Wright (2010). Nominal yield data spans 6/15/1961 to 12/29/2023 for 5-year bonds and 8/17/1971 to 12/29/2023 for 10-year bonds, while real yield data for both 5- and 10-year maturities is available from 1/5/1999 to 12/29/2023. Stock return data is available from 6/15/1961 to 12/29/2023. When constructing the realized correlation, we compute the correlation between bonds and stocks using a centered one-year rolling window. We approximate daily bond returns as the negative of the change in the yield.

For the RoW data we use daily returns to the main stock market index in each country as well as daily returns to 10-year nominal government bonds for each country. The data is from DataStream. The earliest available data is from 1/3/1979, the latest is 12/31/2019. Each day we take the cross-sectional average of the returns across the countries with available data to arrive at the daily RoW stock and bond returns. We then calculate the n-month rolling correlation between daily RoW bond and stock returns.

We use the following data set to evaluate the pricing implications of our model. Daily market returns from CRSP and zero-coupon nominal and real Treasury (TIPS) yields from Gurkaynak, Sack, and Wright (2007) and Gurkaynak, Sack, and Wright (2010) are aggregated to quarterly frequency. For TIPS, we exclude the initial four years and use post-2003 data to address concerns about the credibility of early TIPS data. To construct long-sample averages of the real bond term structure, we combine the post-2003 TIPS data with the estimated real rates from Chernov and Mueller (2012) (1972:Q1 to 2002:Q4) for selected maturities and interpolate for others. It is important to note that only maturities that are higher than two years are available. The extended TIPS data enables the calculation of the quarterly rolling correlation between stocks and bonds using a centered 3-year rolling window.

We consider two sets of equity strip yield data, one is the synthetic equity strip yield data provided by Giglio, Kelly, and Kozak (2020) available from 1975:Q4 to 2020:Q3 and the other is the traded equity strip yield data from Bansal, Miller, Song, and Yaron (2020) available from 2005:Q1 to 2016:Q4.[5] We take the sample average of the equity strip yield data.

B Robustness of estimation to alternative consumption data and time periods

Since our goal is to estimate the dynamics of consumption growth as perceived by investors, we use real-time consumption data. However, historically, a number of papers use the ex post revised real per capita nondurables + services consumption data from NIPA. In Table A-1 we show that there is a significant change in the autocorrelation of consumption growth in 1998 also using these data when estimating the regression in Equation (1). The results are similar to those obtained using the real-time PCE dataset.

Table A-2 extends the sample to the post-Covid period for both PCE and nondurables + services consumption. We add another consumption interaction term, now with the post-2019 dummy to gauge how this dramatic period affects our conclusions about consumption's serial correlation:

$$\Delta c_{t+1} = \alpha_0 + \alpha_1 \mathbb{1}_{t<1998} + \alpha_2 \mathbb{1}_{t>2019} + \beta_0 \Delta c_t + \beta_1 \times (\mathbb{1}_{t<1998} \times \Delta c_t) + \beta_2 \times (\mathbb{1}_{t>2019} \times \Delta c_t) + \epsilon_{t+1}.$$

[5] We thank Serhiy Kozak for providing data.

The coefficients on both interaction terms are significantly negative. The sign of the coefficients has different meaning. Negative β_1 means, like before, that serial correlation of consumption has increased between 1998 and 2019. Negative β_2 indicates that serial correlation of consumption has declined relative to the baseline 1998-2019 period.

Table A-3 considers an alternative source of data, specifically aggregate consumption series computed by Kroencke (2017). In particular, Kroencke "unfilters" the data to remove the smoothing done by NIPA to arrive at the raw consumption data and shows that this "unfiltered" consumption series explains the equity premium and is priced in the cross-section of returns, unlike the filtered NIPA consumption data. These data are in a different format — the consumption growth is year-on-year change, reported quarterly. The available data end in 2018:Q4. We consider the regression from Equation (1) in a slightly altered form to account for the year-on-year feature of the quarterly data:

$$\Delta c_{t,t+4} = \alpha_0 + \alpha_1 \mathbb{1}_{t<1998} + \beta_0 \Delta c_{t-4,t} + \beta_1 \times (\mathbb{1}_{t<1998} \times \Delta c_{t-4,t}) + \epsilon_{t,t+4}.$$

The standard errors are Newey-West with 8 lags to account for the autocorrelation of the residuals that is induced by the overlapping observations. The coefficient β_1, which captures the change in serial correlation of consumption is strongly significant like in the other regressions.

C Consumption growth and inflation dynamics under a regime-switching mean

We specify aggregate log consumption as

$$c_t = c_{p,t} + c_{\tau,t} \tag{A-1}$$

where the permanent and transitory components are

$$\begin{aligned} \Delta c_{p,t+1} - \mu_p(S^c_{t+1}) &= \rho(\Delta c_{p,t} - \mu_p(S^c_t)) + \epsilon_{p,t+1} \\ c_{\tau,t+1} &= \rho c_{\tau,t} + \epsilon_{\tau,t+1} \end{aligned} \tag{A-2}$$

implying the following growth rate

$$\begin{aligned} \Delta c_{t+1} &= \Delta c_{p,t+1} + \Delta c_{\tau,t+1} \\ &= \mu_p(S^c_{t+1}) + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1} \\ x_{t+1} &\equiv \rho(\Delta c_{p,t+1} - \mu_p(S^c_{t+1})) + (\rho-1)c_{\tau,t+1} \\ &= \rho x_t + \rho\epsilon_{p,t+1} + (\rho-1)\epsilon_{\tau,t+1}. \end{aligned} \tag{A-3}$$

Thus, as long as μ_τ is constant, or simply 0, we obtain the specification considered in the main text.

We specify the inflation dynamics as

$$\pi_{t+1} = \mu_\pi(S^c_{t+1}, S^\pi_{t+1}) + \alpha_\pi x_{t+1}, \tag{A-4}$$

where x_t is the expected consumption growth. S^c_t and S^π_t correspond to S_t and $\mathbb{S}_t$, respectively, in the main text. Here we combine them into a six-state Markov chain $S_t = (S^c_t, S^\pi_t)$ for compactness of notation.

We assume that the joint dynamics evolve according to a six-state Markov chain $S_t = (S^c_t, S^\pi_t)$, which is further decomposed into two independent Markov chains, S_t^c and S_t^π:

$$\mathbb{P}^c = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}, \quad \mathbb{P}^\pi = \begin{bmatrix} p_{11}^\pi & p_{12}^\pi \\ p_{21}^\pi & p_{22}^\pi \end{bmatrix}. \tag{A-5}$$

The transition probability is $\mathbb{P} = \mathbb{P}^c \otimes \mathbb{P}^\pi$. We define

$$S_t = \begin{cases} 1 & \text{if } S_t^c = P \text{ and } S_t^\pi = H, \\ 2 & \text{if } S_t^c = T \text{ and } S_t^\pi = H, \\ 3 & \text{if } S_t^c = D \text{ and } S_t^\pi = H, \\ 4 & \text{if } S_t^c = P \text{ and } S_t^\pi = L, \\ 5 & \text{if } S_t^c = T \text{ and } S_t^\pi = L, \\ 6 & \text{if } S_t^c = D \text{ and } S_t^\pi = L. \end{cases} \tag{A-6}$$

D Sequential learning algorithm with mixed-frequency data

D.1 State-space representation

We first describe the state-space representation with generic notation (y indicating data and z a vector of state) and provide the details.

$$y_{t+1} = \Gamma_{c,t+1} + \Gamma_{z,t+1} z_{t+1} + \Gamma_{\epsilon,t+1}\epsilon_{t+1}, \tag{A-7}$$
$$z_{t+1} = g(z_t, S_{t+1}, \epsilon_{t+1}; \Theta), \quad \epsilon_{t+1} \sim N(0, \Sigma(S_{t+1})). \tag{A-8}$$

State-transition equation. The state dynamics (A-8) are described as

$$
\begin{bmatrix} \Delta c_{p,t+1} \\ \Delta c_{p,t} \\ \Delta c_{p,t-1} \\ c_{\tau,t+1} \\ c_{\tau,t} \\ c_{\tau,t-1} \\ c_{\tau,t-2} \\ x_{t+1} \\ x_t \\ \epsilon_{m,t+1} \\ \epsilon_{m,t} \\ \epsilon_{m,t-1} \end{bmatrix}
=
\begin{bmatrix}
\rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix} \Delta c_{p,t} \\ \Delta c_{p,t-1} \\ \Delta c_{p,t-2} \\ c_{\tau,t} \\ c_{\tau,t-1} \\ c_{\tau,t-2} \\ c_{\tau,t-3} \\ x_t \\ x_{t-1} \\ \epsilon_{m,t} \\ \epsilon_{m,t-1} \\ \epsilon_{m,t-2} \end{bmatrix}
+
\begin{bmatrix} \mu_p(S_{t+1}) - \rho\mu_p(S_t) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
+
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\rho & \rho-1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \epsilon_{p,t+1} \\ \epsilon_{\tau,t+1} \\ \epsilon_{m,t+1} \\ \epsilon_{\pi,t+1} \end{bmatrix}. \tag{A-9}
$$

Measurement equations. Note that the coefficients in the measurement equation (A-7) account for the deterministic changes in observables y_{t+1}.

- From 1835:Q1 to 1913:Q1.

(i) When annual consumption growth series are only available:

$$
\begin{aligned}
y_{t+1} = \Delta c_{a,t+1}, \quad \Gamma_{c,t+1} &= \begin{bmatrix} \frac{1}{2}\mu_p(S_{t+1}) \end{bmatrix}, \quad \Gamma_{\epsilon,t+1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix}, \\
\Gamma_{z,t+1} &= \begin{bmatrix} 0 & 1 & \frac{1}{2} & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 1 & 0 & -1 \end{bmatrix}.
\end{aligned}
$$

(ii) Otherwise:

$$
y_{t+1} = \emptyset, \quad \Gamma_{c,t+1} = \emptyset, \quad \Gamma_{\epsilon,t+1} = \emptyset, \quad \Gamma_{z,t+1} = \emptyset.
$$

- From 1913:Q2 to 1947:Q4.

(i) When annual consumption growth and quarterly inflation series are available:

$$
\begin{aligned}
y_{t+1} = \begin{bmatrix} \Delta c_{a,t+1} \\ \pi_{t+1} \end{bmatrix}, \quad \Gamma_{c,t+1} &= \begin{bmatrix} \frac{1}{2}\mu_p(S_{t+1}) \\ \mu_\pi(S_{t+1}) \end{bmatrix}, \quad \Gamma_{\epsilon,t+1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \alpha_\pi\rho & \alpha_\pi(\rho-1) & 0 & 1 \end{bmatrix}, \\
\Gamma_{z,t+1} &= \begin{bmatrix} 0 & 1 & \frac{1}{2} & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_\pi\rho & 0 & 0 & 0 \end{bmatrix}.
\end{aligned}
$$

(i) When quarterly inflation series are only available:

$$
\begin{aligned}
y_{t+1} = \pi_{t+1}, \quad \Gamma_{c,t+1} &= \begin{bmatrix} \mu_\pi(S_{t+1}) \end{bmatrix}, \quad \Gamma_{\epsilon,t+1} = \begin{bmatrix} \alpha_\pi\rho & \alpha_\pi(\rho-1) & 0 & 1 \end{bmatrix}, \\
\Gamma_{z,t+1} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_\pi\rho & 0 & 0 & 0 \end{bmatrix}.
\end{aligned}
$$

- From 1948:Q1 to 2024:Q1. Both quarterly consumption growth and quarterly inflation are available:

$$y_{t+1} = \begin{bmatrix} \Delta c_{t+1} \\ \pi_{t+1} \end{bmatrix}, \quad \Gamma_{c,t+1} = \begin{bmatrix} \mu_p(S_{t+1}) \\ \mu_\pi(S_{t+1}) \end{bmatrix}, \quad \Gamma_{\epsilon,t+1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ \alpha_\pi \rho & \alpha_\pi(\rho - 1) & 0 & 1 \end{bmatrix},$$

$$\Gamma_{z,t+1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_\pi \rho & 0 & 0 & 0 \end{bmatrix}.$$

Note that due to the singularity issue (meaning that annual consumption series can be constructed from quarterly consumption series) we use quarterly consumption series in the estimation if both annual and quarterly data are present. Also, as explained by Schorfheide, Song, and Yaron (2018), we allow for measurement errors in the level of pre-war annual consumption series.

Temporal aggregation. We concatenate annual consumption growth series with quarterly ones to learn parameters and hidden states. Below we explain how the temporal aggregation works between quarterly and annual consumption growth series.

Note that log annual consumption is

$$c_{a,t+1} = \frac{c_{t+1} + c_t + c_{t-1} + c_{t-2}}{4} \tag{A-10}$$

implying the following growth rate

$$\begin{aligned} \Delta c_{a,t+1} &= \frac{\Delta c_{t+1} + 2\Delta c_t + 3\Delta c_{t-1} + 4\Delta c_{t-2} + 3\Delta c_{t-3} + 2\Delta c_{t-4} + \Delta c_{t-5}}{4} \\ &= \sum_{j=1}^{7} \left(\frac{4 - |j-4|}{4} \right) \Delta c_{t-j+2}. \end{aligned} \tag{A-11}$$

We rewrite the above expression with notation M

$$\Delta c_{a,t+1} = \sum_{j=1}^{2M-1} \left(\frac{M - |j-M|}{M} \right) \Delta c_{t-j+2}. \tag{A-12}$$

We have shown the state-space representation for $M = 2$ to save space.

D.2 Sequential learning algorithm

The algorithm is based on Kim and Lee (2018) who extend the algorithm of Carvalho and Lopes (2007) which enables sequential parameter learning and state filtering in real time.[6] We set $N = 500,000$ for estimation.

Define

$$\Theta = \left\{ \rho, \{\mu_p(i), \sigma_p^2(i), \sigma_\tau^2(i)\}_{i\in\{1,2,3\}}, \{p_{1i}\}_{i\in\{1,2\}}, \{p_{2i}\}_{i\in\{1,2\}}, \{p_{3i}\}_{i\in\{1,2\}}, \{p_i^\pi\}_{i\in\{1,2\}} \right\} \tag{A-13}$$

The state-transition equation (whose specific form is provided in Section D.1) is

$$z_{t+1} = g(z_t, S_{t+1}, \epsilon_{t+1}; \Theta). \tag{A-14}$$

When data are available at t:

[6] We thank Jaeho Kim for his insightful help.

1. Compute for all $i \in \{1, ..., N\}$

   (a) posterior mean and variance statistics

$$m_{t-1}^{(i)} = \alpha \Theta_{t-1}^{(i)} + (1-\alpha)\bar{\Theta}_{t-1} \tag{A-15}$$
$$(1-\alpha^2)V_{t-1} = (1-\alpha^2)\sum_{i=1}^{N} \hat{w}_{t-1}^{(i)}(\Theta_{t-1}^{(i)} - \bar{\Theta}_{t-1})(\Theta_{t-1}^{(i)} - \bar{\Theta}_{t-1})'$$

   where α is a tuning parameter, which we set to $\alpha = 0.9$.

   (b) the latent continuous state for $k \in \{1, ..., 6\}$

$$\tilde{z}_{t|k}^{(i)} = g(z_{t-1}^{(i)}, S_t = k, \epsilon_t = 0; m_{t-1}^{(i)}). \tag{A-16}$$

   (c) the importance weight

$$\hat{w}_{t-1|t,k}^{(i)} = \frac{w_{t-1|t,k}^{(i)}}{\sum_{k=1}^{K}\sum_{i=1}^{N} w_{t-1|t,k}^{(i)}} \tag{A-17}$$
$$w_{t-1|t,k}^{(i)} \propto p(y_t \big| \tilde{z}_{t|k}^{(i)}, S_t = k; m_{t-1}^{(i)}) p(S_t = k \big| S_{t-1}^{(i)}; m_{t-1}^{(i)}) \hat{w}_{t-1}^{(i)}.$$

   To compute the likelihood function $p(y_t \big| \tilde{z}_{t|k}^{(i)}, S_t = k; m_{t-1}^{(i)})$, we use the state-space representation provided in (A-7) and (A-8).

2. Draw for all $i \in \{1, ..., N\}$

   (a) the latent discrete state $S_t^{(i)}$ and resample the particle set based on

$$\left\{\tilde{z}_{t|k}^{(i)}, S_{t-1}^{(i)}, z_{t-1}^{(i)}, \Theta_{t-1}^{(i)}, m_{t-1}^{(i)}\right\}_{i=1}^{N} \quad \sim \quad \left\{\tilde{z}_{t|k}^{(i)}, S_{t-1}^{(i)}, z_{t-1}^{(i)}, \Theta_{t-1}^{(i)}, m_{t-1}^{(i)}\right\}_{i=1}^{N} \tag{A-18}$$

   using the importance weight $\hat{w}_{t-1|t,k}^{(i)}$.

   (b) parameter vector

$$\Theta_t^{(i)} \sim N(m_{t-1}^{(i)}, (1-\alpha^2)V_{t-1}) \tag{A-19}$$

   (c) the latent continuous state

$$z_t^{(i)} \sim p(z_t | S_t^{(i)}, z_{t-1}^{(i)}; \Theta_t^{(i)}) \tag{A-20}$$

3. Compute the importance weight

$$\hat{w}_t^{(i)} = \frac{w_t^{(i)}}{\sum_{j=1}^{N} w_t^{(j)}} \tag{A-21}$$
$$w_t^{(i)} \propto \frac{p(y_t \big| z_t^{(i)}, S_t^{(i)}; \Theta_t^{(i)}) p(S_t^{(i)} \big| S_{t-1}^{(i)}; \Theta_t^{(i)})}{p(y_t \big| \tilde{z}_{t|S_t^{(i)}}^{(i)}, S_t^{(i)}; m_{t-1}^{(i)}) p(S_t^{(i)} \big| S_{t-1}^{(i)}; m_{t-1}^{(i)})}$$

   and

$$\bar{\Theta}_t = \sum_{i=1}^{N} \hat{w}_t^{(i)} \Theta_t^{(i)}. \tag{A-22}$$

When there is missing data at t:

1. Draw for all $i \in \{1, ..., N\}$

   (a) the latent discrete state

$$S_t^{(i)} = S_{t-1}^{(i)} \tag{A-23}$$

   (b) parameter vector

$$\Theta_t^{(i)} = \Theta_{t-1}^{(i)} \tag{A-24}$$

   (c) the latent continuous state

$$z_t^{(i)} \sim p(z_t | S_t^{(i)}, z_{t-1}^{(i)}; \Theta_t^{(i)}) \tag{A-25}$$

E Parameter estimates and fit

Figure A-1 reports the time-series of estimated parameters of US quarterly consumption dynamics. Each point represents the posterior distribution computed using the corresponding history of observations. Confidence sets continue to tighten throughout the sample.

By assumption, the means of consumption growth are identical in the first two regimes, settling at 0.47% (0.45%) by the end of the sample, where the values in parentheses correspond to estimates based solely on consumption. In the third regime, the mean is negative, reaching -0.45% (-0.28%) by the end of the sample. Consequently, the third regime is associated with a negative shift in expected consumption growth.

Figure A-2 presents the time series of the estimated transition probabilities. The permanent and transitory regimes exhibit high persistence, with an expected duration of approximately 36 years (based on end-of-sample estimates for both cases of estimation). In contrast, the disaster regime is less persistent, with an expected duration of 3-4 years.

It is instructive to examine the extent to which sample moments implied by the estimated state-space model mimic the sample moments computed from our actual data set. To do so, we report percentiles of the posterior predictive distribution for various sample moments based on simulations from the posterior distribution of the same length as the data. This is called a posterior predictive check, see Geweke (2005). Typically, the posterior predictive distribution is computed to reflect both parameter and shock uncertainty.

Table A-4 provide these moments and we find that all of the "actual" sample moments, determined from quarterly series from 1947:Q2-2024:Q1, are within the 5th and the 95th percentile of the corresponding posterior predictive distribution. Importantly, the posterior predictive medians closely align with the empirical moments, indicating that the model captures key features of the data.

F UK results

The estimated model for the UK consumption growth is qualitatively similar to the US in terms of parameter estimates and the transitory versus permanent shock allocations across the regimes. To illustrate this we plot filtered probabilities of regimes in Figure A-3. Just like in the US, our model identifies disasters similarly to Barro and Jin (2021) during the pre-War period. Overall, regime allocations are similar to the US during the modern part of the sample. In particular, the model identifies the switch to the permanent regime in the 1990s just like in the US.

When we consider the joint consumption-inflation estimation, consumption regimes remain mostly the same as can be seen in Figure A-3. As is the case with the US, the switch to the permanent regime in the 1990s occurs later in the case of the joint estimation. Lastly, the disaster regime in the late 1970s differs between the two estimations. Figure A-4 reports the estimated inflation regimes. In contrast to the US, the high-inflation regime is dominant throughout the sample. Although the post-1990 period has a large proportion with low inflation mean.

Figure A-5 explores various asset-pricing moments along the lines of US (Figures 9(C) and 10(C)). We use the same recursive preference parameters and assumptions about default. Also, we take advantage of 40 years worth of real bond data in the UK and add the relevant objects computed using these data. Most crucially, the model picks up the change in bond-stock correlation in the late-1990s, consistent with the data on the UK bond-stock correlation, both real and nominal. There is a period around 2008-2013 when the model counter-factually predicts positive bond-stock correlation, as the model is estimated to revert briefly to the transitory regime, but the model-implied bond-stock correlation is negative most of the time after the shift from positive to negative in the late-1990s. The rolling correlation between excess bond returns and consumption growth also goes from positive to negative, both in the data and in the model, though again the period 2008-2013 is problematic for the model. We note that the sample with observable stock-bond correlations is not as long as in the US. Both models produce a realistic real yield curve. They also produce similar nominal curves although in the case of the UK the model overshoots the observed ones. The equity curve is not observed in the UK, but the theoretical one is similar to the one in the US.

G An illustration with log utility preferences

For ease of illustration, we assume homoscedasticity

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1}, \quad \epsilon_p \sim N(0,\sigma_p^2), \\ x_{t+1} &= \rho x_t + \rho\epsilon_{p,t+1} + (\rho-1)\epsilon_{\tau,t+1}, \quad \epsilon_\tau \sim N(0,\sigma_\tau^2), \end{aligned} \tag{A-26}$$

and consider log utility preference implying the following log stochastic discount factor

$$m_{t+1} = \ln\delta - \Delta c_{t+1}. \tag{A-27}$$

In this environment, the unexpected component of the return on consumption claim is

$$r^c_{t+1} - E_t r^c_{t+1} = \Delta c_{t+1} - E_t \Delta c_{t+1} \tag{A-28}$$

because the log price to consumption ratio is constant.

The n-maturity log real bond price is

$$q_t^{(n)} = q_{n,0} + q_{n,x} x_t \tag{A-29}$$

and the return on the n-maturity real bond is

$$r_{t+1}^{(n)} \equiv q_{t+1}^{(n-1)} - q_t^{(n)}. \tag{A-30}$$

We can deduce that its unexpected component is

$$r_{t+1}^{(n)} - E_t r_{t+1}^{(n)} = q_{n-1,x}\big(x_{t+1} - E_t x_{t+1}\big). \tag{A-31}$$

Introduce the relative contribution of permanent and transitory components via

$$\eta = \frac{1-\rho}{\rho} \cdot \frac{\sigma_\tau^2}{\sigma_p^2}. \tag{A-32}$$

We can express the risk premium on a real bond as:

$$-cov_t(m_{t+1}, r_{t+1}^{(n)}) = q_{n-1,x} cov_t(\Delta c_{t+1}, x_{t+1}) = -q_{n-1,x}\rho\sigma_p^2(\eta - 1).$$

Then the sign of the bond risk premium is

$$\text{sign}(-cov_t(m_{t+1}, r_{t+1}^{(n)})) = \text{sign}(\eta - 1)$$

as $q_{n-1,x} < 0$ for $n \geq 2$.

Next, from (A-28) and (A-31), we can express the sign of the conditional covariance as

$$\begin{aligned} \text{sign}\big(cov_t(r_{t+1}^{(n)}, r_{t+1}^c)\big) &= \text{sign}\big(q_{n-1,x}\big) \cdot \text{sign}\big(cov_t(\Delta c_{t+1}, x_{t+1})\big), \\ &= -\text{sign}\big(cov_t(\Delta c_{t+1}, x_{t+1})\big), \\ &= \text{sign}(\eta - 1). \end{aligned} \tag{A-33}$$

H Model solution with recursive preferences

This section provides solutions for the equilibrium asset prices.

H.1 Exogenous dynamics with regime switching

The joint dynamics of consumption and dividend growth are

$$\begin{aligned} \Delta c_{t+1} &= \mu_p(S_{t+1}) + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1}, \\ \pi_{t+1} &= \mu_\pi(S_{t+1}) + \alpha_\pi x_{t+1}, \\ \Delta d_{t+1} &= \mu_d + \alpha_d(\Delta c_{t+1} - \mu_p), \\ x_t &= \rho x_{t-1} + \rho\epsilon_{p,t+1} + (\rho - 1)\epsilon_{\tau,t+1}, \quad \epsilon_{\tau,t+1} \sim N(0, \sigma_\tau(S_{t+1})^2), \quad \epsilon_{p,t+1} \sim N(0, \sigma_p(S_{t+1})^2) \end{aligned} \tag{A-34}$$

where $\bar{\mu}$ is the unconditional mean of consumption growth and the transition matrix is given by $\mathbb{P} = \mathbb{P}^c \otimes \mathbb{P}^\pi$

$$\mathbb{P}^c = \begin{bmatrix} p_{11} & p_{12} & 1 - p_{11} - p_{12} \\ p_{21} & p_{22} & 1 - p_{21} - p_{22} \\ p_{31} & p_{32} & 1 - p_{31} - p_{32} \end{bmatrix}, \quad \mathbb{P}^\pi = \begin{bmatrix} p_1^\pi 1 - p_1^\pi & \\ 1 - p_2^\pi & p_2^\pi \end{bmatrix}. \tag{A-35}$$

H.2 Stochastic discount factor

In this section, we derive the SDF under EZ preference when EIS is 1.

$$V_t = C_t^{1-\delta}\lambda_t^\delta, \quad \lambda_t = \left[E_t(V_{t+1}^{1-\gamma})\right]^{\frac{1}{(1-\gamma)}}. \tag{A-36}$$

We take log and get

$$v_t = (1-\delta)c_t + \frac{\delta}{1-\gamma} \ln E_t\big[\exp((1-\gamma)v_{t+1})\big]. \tag{A-37}$$

Let $vc_t = v_t - c_t$ and re-express as

$$vc_t = \frac{\delta}{1-\gamma} \ln E_t\big[\exp((1-\gamma)vc_{t+1} + (1-\gamma)\Delta c_{t+1})\big]. \tag{A-38}$$

A numerical solution iterates on (A-38) to find vc_t as a function of state variables governing consumption growth dynamics.

The SDF is

$$\begin{aligned} M_{t+1} &= \delta \frac{C_t}{C_{t+1}} \frac{V_{t+1}^{(1-\gamma)}}{E_t(V_{t+1}^{1-\gamma})} = \delta \frac{C_t}{C_{t+1}} C_{t+1}^{(1-\gamma)} \frac{VC_{t+1}^{(1-\gamma)}}{C_t^{(1-\gamma)} E_t(VC_{t+1}^{1-\gamma} C_{t+1}^{1-\gamma} C_t^{-(1-\gamma)})} \\ &= \delta \frac{\exp(-\gamma\Delta c_{t+1} + (1-\gamma)vc_{t+1})}{E_t\big[\exp((1-\gamma)\Delta c_{t+1} + (1-\gamma)vc_{t+1})\big]}. \end{aligned} \tag{A-39}$$

H.3 Consumption claim

The return on consumption claim is

$$R_{t+1}^c = \frac{Q_{t+1}^c + C_{t+1}}{Q_t^c} = \left[\frac{C_{t+1}}{C_t}\right] \cdot \left[\frac{PC_{t+1}+1}{PC_t}\right] \tag{A-40}$$

where $PC_t = Q_t^c / C_t$.

We can deduce from (A-39) and (A-40) that the Euler equation can be expressed as

$$\begin{aligned} 1 &= E_t\Big[M_{t+1} \cdot R_{t+1}^c\Big] \\ PC_t &= E_t\Big[M_{t+1} \cdot \exp\left[\Delta c_{t+1}\right] \cdot \left[PC_{t+1}+1\right]\Big]. \end{aligned} \tag{A-41}$$

H.4 Dividend claim

The return on dividend claim is

$$R_{t+1}^d = \frac{Q_{t+1}^d + D_{t+1}}{Q_t^d} = \left[\frac{D_{t+1}}{D_t}\right] \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right] \tag{A-42}$$

where $PD_t = Q_t^d / D_t$. We can deduce from (A-39) and (A-42) that the Euler equation can be expressed as

$$\begin{aligned} 1 &= E_t\Big[M_{t+1} \cdot R_{t+1}^d\Big] \\ PD_t &= E_t\Big[M_{t+1} \cdot \exp\left[\Delta d_{t+1}\right] \cdot \left[PD_{t+1}+1\right]\Big]. \end{aligned} \tag{A-43}$$

H.5 Zero-coupon bond prices

The n-maturity zero-coupon bond price is determined by

$$Q_t^{(n)} = E_t\left[\widetilde{M}_{t+1} \cdot Q_{t+1}^{(n-1)}\right] \tag{A-44}$$

where

- $\widetilde{M}_{t+1} = M_{t+1}$ for non-defaultable bonds
- $\widetilde{M}_{t+1} = M_{t+1} \cdot \left(1 - \mathbb{I}_{S_{t+1}=3} \cdot \xi \cdot L\right)$ for defaultable bonds; $\mathbb{I}_{S_{t+1}=3}$ indicates disaster realization in $t+1$; If there is a disaster, then with probability ξ bonds default and the loss will be L

with the initial condition $Q_t^{(0)} = 1$. The respective log bond yield is $y_t^{(n)} = -\frac{1}{n} \ln Q_t^{(n)}$.

H.6 Equity premium

Denote the benchmark real rate as

$$R_{f,t} = \frac{1}{E_t[\widetilde{M}_{t+1}]} \tag{A-45}$$

where

- $\widetilde{M}_{t+1} = M_{t+1}$ for non-defaultable bonds
- $\widetilde{M}_{t+1} = M_{t+1} \cdot \left(1 - \mathbb{I}_{S_{t+1}\in\{3,6\}} \cdot \xi \cdot L\right)$ for defaultable bonds; $\mathbb{I}_{S_{t+1}\in\{3,6\}}$ indicates disaster realization in $t+1$; If there is a disaster, then with probability ξ bonds default and the loss will be L;

The expected return on dividend claim in excess of real rate is

$$E_t\left[R_{t+1}^d\right] - R_{f,t} = E_t\left[\exp\left[\Delta d_{t+1}\right] \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right]\right] - \frac{1}{E_t\left[\widetilde{M}_{t+1}\right]}. \tag{A-46}$$

H.7 Zero-coupon dividend equity prices

The n-maturity zero-coupon equity price

$$Q_t^{d,(n)} = E_t\left[M_{t+1} \cdot Q_{t+1}^{d,(n-1)}\right]. \tag{A-47}$$

Denote $Z_t^{(n)} = Q_t^{d,(n)}/D_t$. Instead of (A-47), we solve for

$$Z_t^{(n)} = E_t\left[M_{t+1} \cdot \exp\left[\Delta d_{t+1}\right] \cdot Z_{t+1}^{(n-1)}\right] \tag{A-48}$$

with the initial conditional $Z_t^{(0)} = 1$. The respective log spot equity yield is $e_t^{(n)} = -\frac{1}{n} \ln Z_t^{(n)}$. The log forward equity yield is defined as $e_t^{f,(n)} = e_t^{(n)} - y_t^{(n)}$.

H.8 Zero-coupon nominal bond prices

The n-maturity zero-coupon nominal bond price is determined by

$$Q_t^{\$,(n)} = E_t\left[\left[\frac{\widetilde{M}_{t+1}}{\Pi_{t+1}}\right] \cdot Q_{t+1}^{\$,(n-1)}\right] \tag{A-49}$$

where

- $\widetilde{M}_{t+1} = M_{t+1}$ for non-defaultable bonds
- $\widetilde{M}_{t+1} = M_{t+1} \cdot \left(1 - \mathbb{I}_{S_{t+1}\in\{3,6\}} \cdot \xi \cdot L\right)$ for defaultable bonds; $\mathbb{I}_{S_{t+1}\in\{3,6\}}$ indicates disaster realization in $t+1$; If there is a disaster, then with probability ξ bonds default and the loss will be L;

with the initial condition $Q_t^{\$,(0)} = 1$. The respective log bond yield is $y_t^{\$,(n)} = -\frac{1}{n}\ln Q_t^{\$,(n)}$.

H.9 Equity and bond return correlation

The one-period return on equity is $R_{t+1}^d = \left[\frac{D_{t+1}}{D_t}\right] \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right]$.

H.9.1 With real return on a one-period real bond

The one-period return on zero-coupon bond of maturity n is $R_{t+1}^{(n)} = \frac{Q_{t+1}^{(n-1)}}{Q_t^{(n)}}$. The conditional covariance between the two is

$$\begin{aligned} cov_t\left[R_{t+1}^d, R_{t+1}^{(n)}\right] &= E_t\left[R_{t+1}^d R_{t+1}^{(n)}\right] - E_t\left[R_{t+1}^d\right] E_t\left[R_{t+1}^{(n)}\right] \\ &= E_t\left[\exp(\Delta d_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right] \cdot \left[\frac{Q_{t+1}^{(n-1)}}{Q_t^{(n)}}\right]\right] \\ &\quad - E_t\left[\exp(\Delta d_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right]\right] \cdot E_t\left[\frac{Q_{t+1}^{(n-1)}}{Q_t^{(n)}}\right]. \end{aligned} \tag{A-50}$$

The conditional variances of equity return and bond return of maturity n are

$$\begin{aligned} var_t\left[R_{t+1}^d\right] &= E_t\left[\exp(2\Delta d_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right]^2\right] - \left(E_t\left[\exp(\Delta d_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right]\right]\right)^2, \\ var_t\left[R_{t+1}^{(n)}\right] &= E_t\left[\left[\frac{Q_{t+1}^{(n-1)}}{Q_t^{(n)}}\right]^2\right] - \left(E_t\left[\frac{Q_{t+1}^{(n-1)}}{Q_t^{(n)}}\right]\right)^2. \end{aligned} \tag{A-51}$$

The equity and bond return correlation is

$$corr_t\left[R_{t+1}^d, R_{t+1}^{(n)}\right] = \frac{cov_t\left[R_{t+1}^d, R_{t+1}^{(n)}\right]}{\sqrt{var_t\left[R_{t+1}^d\right] \cdot var_t\left[R_{t+1}^{(n)}\right]}}. \tag{A-52}$$

H.9.2 With real return on a one-period nominal bond

The one-period return on zero-coupon nominal bond of maturity n is $R_{t+1}^{\$,(n)} = \left[\frac{Q_{t+1}^{\$,(n-1)}}{Q_t^{\$,(n)}}\right]$. The real return on a one-period nominal bond of the same maturity is $\frac{R_{t+1}^{\$,(n)}}{\Pi_{t+1}}$. The conditional covariance between the one-period real returns on equity and nominal bond is

$$cov_t\left[R_{t+1}^d, \frac{R_{t+1}^{\$,(n)}}{\Pi_{t+1}}\right] = E_t\left[\exp(\Delta d_{t+1} - \pi_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right] \cdot \left[\frac{Q_{t+1}^{\$,(n-1)}}{Q_t^{\$,(n)}}\right]\right] - E_t\left[\exp(\Delta d_{t+1}) \cdot \left[\frac{PD_{t+1}+1}{PD_t}\right]\right] \cdot E_t\left[\exp(-\pi_{t+1}) \cdot \left[\frac{Q_{t+1}^{\$,(n-1)}}{Q_t^{\$,(n)}}\right]\right]. \tag{A-53}$$

The conditional variances of real return of a one-period nominal bond of maturity n is

$$var_t\left[\frac{R_{t+1}^{\$,(n)}}{\Pi_{t+1}}\right] = E_t\left[\exp(-2\pi_{t+1}) \cdot \left[\frac{Q_{t+1}^{\$,(n-1)}}{Q_t^{\$,(n)}}\right]^2\right] - \left(E_t\left[\exp(-\pi_{t+1}) \cdot \frac{Q_{t+1}^{\$,(n-1)}}{Q_t^{\$,(n)}}\right]\right)^2. \tag{A-54}$$

The equity and bond return correlation is

$$corr_t\left[R_{t+1}^d, \frac{R_{t+1}^{\$,(n)}}{\Pi_{t+1}}\right] = \frac{cov_t\left[R_{t+1}^d, \frac{R_{t+1}^{\$,(n)}}{\Pi_{t+1}}\right]}{\sqrt{var_t\left[R_{t+1}^d\right] \cdot var_t\left[\frac{R_{t+1}^{\$,(n)}}{\Pi_{t+1}}\right]}}. \tag{A-55}$$

H.10 Numerical solution method for endogenous quantities

The log value-consumption ratio, vc_t, is the endogenous state-variable that, along with the exogenously given consumption dynamics

$$\begin{aligned}\Delta c(S_{t+1}, x_t, \epsilon_{t+1}) &= \mu_p(S_{t+1}) + x_t + \epsilon_{p,t+1} + \epsilon_{\tau,t+1}, \quad \epsilon_{j,t+1} \sim N(0, \sigma_j(S_{t+1})^2), \quad j \in \{p, \tau\},\\ x(S_{t+1}, x_t, \epsilon_{t+1}) &= \rho x_t + \rho \epsilon_{p,t+1} + (\rho - 1)\epsilon_{\tau,t+1},\end{aligned}$$

drive the stochastic discount factor and asset prices. Here, the expositions ignore dependence on parameter vector Θ to economize notations. In particular, from (A-38) we have that:

$$\begin{aligned}vc\,(S_t, x_t) &= \frac{\delta}{1-\gamma} \ln E_t\big[\exp((1-\gamma)vc\,(S_{t+1}, x_{t+1}) + (1-\gamma)\Delta c(S_{t+1}, x_t, \epsilon_{t+1}))\big]\\ &= \frac{\delta}{1-\gamma} \ln E_t\Big[\exp\big((1-\gamma)vc\,(S_{t+1}, x(S_{t+1}, x_t, \epsilon_{t+1})) + (1-\gamma)\Delta c(S_{t+1}, x_t, \epsilon_{t+1})\big)\Big]\end{aligned} \tag{A-56}$$

where we have made explicit dependence of the price-consumption ratio on the current state, S_t, and expected consumption growth, x_t. Thus, when we solve for prices and utility at time t, we assume the parameters governing consumption dynamics are known and equal to the posterior means at that time. We assume agent observes the state and set the current state to that most likely according to the time t state probabilities from the full learning problem. This is the anticipated utility assumption we make, which makes the pricing problem computationally feasible.

For simplicity, we assume a conditionally linear representation of $vc\,(S_t, x_t) = a_t(S_t) + b_t(S_t)x_t$ for each regime S_t. Thus, since there are three regimes, we are looking for six parameters $a_t(S_t)$ and $b_t(S_t)$, where the t subscript on these parameters highlights that we re-solve the model at each time t according to the new parameter beliefs. We discretize the state space by considering eleven grid points for x_t that are evenly spaced along the real line between the upper (0.04) and lower (-0.04) bounds. In annual percentage terms, these bounds translate into expected growth rate of ± 16% that can accommodate disasters in the economy. We solve for $vc\,(S_t, x_t)$ for a grid of values for x_t using an initial guess for $vc\,(S_{t+1}, x(S_{t+1}, x_t, \epsilon_{t+1}))$, linear interpolation, and standard numerical integration method (Gauss-Hermite quadrature with 10 quadrature nodes is used over possible values of S_{t+1} and ϵ_{t+1} yielding values for $E_t[\,\cdot\,]$). We iterate on this function until a convergence criteria has been met. Details can be referred to the Online Appendix of Kaltenbrunner and Lochstoer (2010) on policy function iteration to solve models with Epstein-Zin preferences and Albertini and Moyen (2020) for regime-switching models.

The solution to the log price-dividend ratio (A-43) is found using similar methods iterating on:

$$pd\,(S_t, x_t) = \ln E_t \left(M(S_{t+1}, x_t, \epsilon_{t+1}) \exp(\Delta d(S_{t+1}, x_t, \epsilon_{t+1})) \left(e^{pd(S_{t+1}, x(S_{t+1}, x_t, \epsilon_{t+1}))} + 1 \right) \right) \tag{A-57}$$

where

$$\begin{aligned} M(S_{t+1}, x_t, \epsilon_{t+1}) &= \delta \frac{\exp(-\gamma \Delta c(S_{t+1}, x_t, \epsilon_{t+1}) + (1-\gamma) vc\,(S_{t+1}, x(S_{t+1}, x_t, \epsilon_{t+1})))}{E_t\big[\exp((1-\gamma)\Delta c(S_{t+1}, x_t, \epsilon_{t+1}) + (1-\gamma) vc\,(S_{t+1}, x(S_{t+1}, x_t, \epsilon_{t+1})))\big]}, \\ \Delta d(S_{t+1}, x_t, \epsilon_{t+1}) &= \mu_d + \alpha(\Delta c(S_{t+1}, x_t, \epsilon_{t+1}) - \mu_c). \end{aligned}$$

The solution to equity yields are found iterating recursively backwards

$$Z^{(n)}\,(S_t, x_t) = E_t \Big[M(S_{t+1}, x_t, \epsilon_{t+1}) \cdot \exp\left[\Delta d(S_{t+1}, x_t, \epsilon_{t+1})\right] \cdot Z^{(n-1)}\,(S_{t+1}, x(S_{t+1}, x_t, \epsilon_{t+1})) \Big] \tag{A-58}$$

with the initial condition $Z^{(0)}\,(\cdot,\cdot) = 1$; log equity yield is $e^{(n)}\,(S_t, x_t) = -\frac{1}{n} \ln Z^{(n)}\,(S_t, x_t)$.

The solution to real bond yields are found using similar methods iterating recursively backwards

$$Q^{(n)}\,(S_t, x_t) = E_t \Big[\widetilde{M}(S_{t+1}, x_t, \epsilon_{t+1}) \cdot Q^{(n-1)}\,(S_{t+1}, x(S_{t+1}, x_t, \epsilon_{t+1})) \Big] \tag{A-59}$$

where $\widetilde{M}(S_{t+1}, x_t, \epsilon_{t+1}) = M(S_{t+1}, x_t, \epsilon_{t+1}) \cdot \big(1 - \mathbb{I}_{S_{t+1}=3} \cdot \xi \cdot L\big)$ for defaultable bonds; $\mathbb{I}_{S_{t+1}=3}$ indicates disaster realization in $t+1$; If there is a disaster, then with probability ξ bonds default and the loss will be L; the initial condition is $Q^{(0)}\,(\cdot,\cdot) = 1$; log bond yield is $y^{(n)}\,(S_t, x_t) = -\frac{1}{n} \ln Q^{(n)}\,(S_t, x_t)$. Similarly, nominal bond yields are derived

$$Q^{\$,(n)}\,(S_t, x_t) = E_t \left[\left(\frac{\widetilde{M}(S_{t+1}, x_t, \epsilon_{t+1})}{\Pi(S_{t+1}, x_t, \epsilon_{t+1})} \right) \cdot Q^{\$,(n-1)}\,(S_{t+1}, x(S_{t+1}, x_t, \epsilon_{t+1})) \right] \tag{A-60}$$

where $\Pi(S_{t+1}, x_t, \epsilon_{t+1})$ is inflation (specified in the next section); initial condition $Q^{\$,(0)}\,(\cdot,\cdot) = 1$; log bond yield is $y^{\$,(n)}\,(S_t, x_t) = -\frac{1}{n} \ln Q^{\$,(n)}\,(S_t, x_t)$.

Finally, conditional risk premiums and covariances are derived similarly based on the corresponding equations given previously in the Appendix.

I Supplementary Figures and Tables

Table A-1: Robustness: Consumption autocorrelations with revised data

	Full sample		Sample excluding recessions	
	(1)	(2)	(1)	(2)
α_0	0.0012***	0.0002	0.0022***	0.0003
	(0.0004)	(0.0003)	(0.0004)	(0.0003)
α_1	0.0030***		0.0039***	
	(0.0008)		(0.0010)	
β_0	0.6308***	0.6877***	0.4884***	0.6126***
	(0.0821)	(0.0868)	(0.0825)	(0.0918)
β_1	−0.4337***	−0.4855***	−0.4461***	−0.5573***
	(0.1212)	(0.1209)	(0.1415)	(0.1548)
R^2	0.1240	0.1171	0.1004	0.0809

*N*otes: The table shows the regression estimates from the regression in Equation (1). On the right side, business cycle downturns are excluded from the sample as explained in the main text. Covid period is excluded from all regressions. Robust standard errors are reported in parenthesis. The β_0 estimate gives the autocorrelation coefficient in the post-1998 sample, while the β_1 estimate gives the difference between the autocorrelation in the pre- vs. post-1998 samples. The specification in the column labeled (1) estimates the full regression, while the specification in the column labeled (2) is run using demeaned consumption growth and setting $\alpha_1 = 0$, as explained in the main text. One asterisk denotes significance at the 10% level, two at the 5% level, and three at the 1% level. The data is the revised real, per capita, nondurables+services consumption data from 1947:Q2 to 2019:Q4.

Table A-2: Robustness: Consumption autocorrelations with post-Covid data

	Revised PCE		Non-durables and services	
	(1)	(2)	(1)	(2)
α_0	0.0039***	−0.0000	0.0012***	0.0002
	(0.0011)	(0.0006)	(0.0004)	(0.0005)
α_1	0.0040***		0.0030***	
	(0.0014)		(0.0008)	
α_2	0.0027		0.0050	
	(0.0089)		(0.0086)	
β_0	0.4424***	0.4497***	0.6308***	0.6898***
	(0.1288)	(0.1315)	(0.0821)	(0.0945)
β_1	−0.4411***	−0.4465***	−0.4337***	−0.4884***
	(0.1605)	(0.1612)	(0.1212)	(0.1365)
β_2	−0.7386***	−0.7387***	−0.8488***	−0.9077***
	(0.3624)	(0.3626)	(0.3272)	(0.3512)
R^2	0.0551	0.0497	0.0696	0.0676

Notes: The table shows the regression estimates from the regression Equation (1) extended to the post-Covid period:

$$\Delta c_{t+1} = \quad \alpha_0 + \alpha_1 \mathbb{1}_{t<1998} + \alpha_2 \mathbb{1}_{t>2019} + \beta_0 \Delta c_t + \beta_1 \times (\mathbb{1}_{t<1998} \times \Delta c_t) + \beta_2 \times (\mathbb{1}_{t>2019} \times \Delta c_t) + \epsilon_{t+1}.$$

On the left side we implement the regression for real-time PCE and on the left side for the revised real, per capita, nondurables+services consumption. Robust standard errors are reported in parenthesis. The β_0 estimate gives the autocorrelation coefficient between 1998 and 2019, the β_1 estimate gives the difference between the autocorrelation in the pre- vs. 1998-2019 samples, and the β_2 coefficient gives the the difference between the autocorrelation in the post-2019 vs the 1998-2019 samples. The specification in the column labeled (1) estimates the full regression, while the specification in the column labeled (2) is run using demeaned consumption growth and setting $\alpha_1 = \alpha_2 = 0$. One asterisk denotes significance at the 10% level, two at the 5% level, and three at the 1% level. The data are from 1947:Q2 to 2024:Q1.

Table A-3: Robustness: Consumption autocorrelations with filtered consumption data

	(1)	(2)
α_0	0.0098^{***}	0.0013
	(0.0047)	(0.0024)
α_1	0.0179^{***}	
	(0.0060)	
β_0	0.2034	0.3275
	(0.1792)	(0.2151)
β_1	-0.4676^{***}	-0.5825^{***}
	(0.2024)	(0.2362)
R^2	0.0993	0.0715

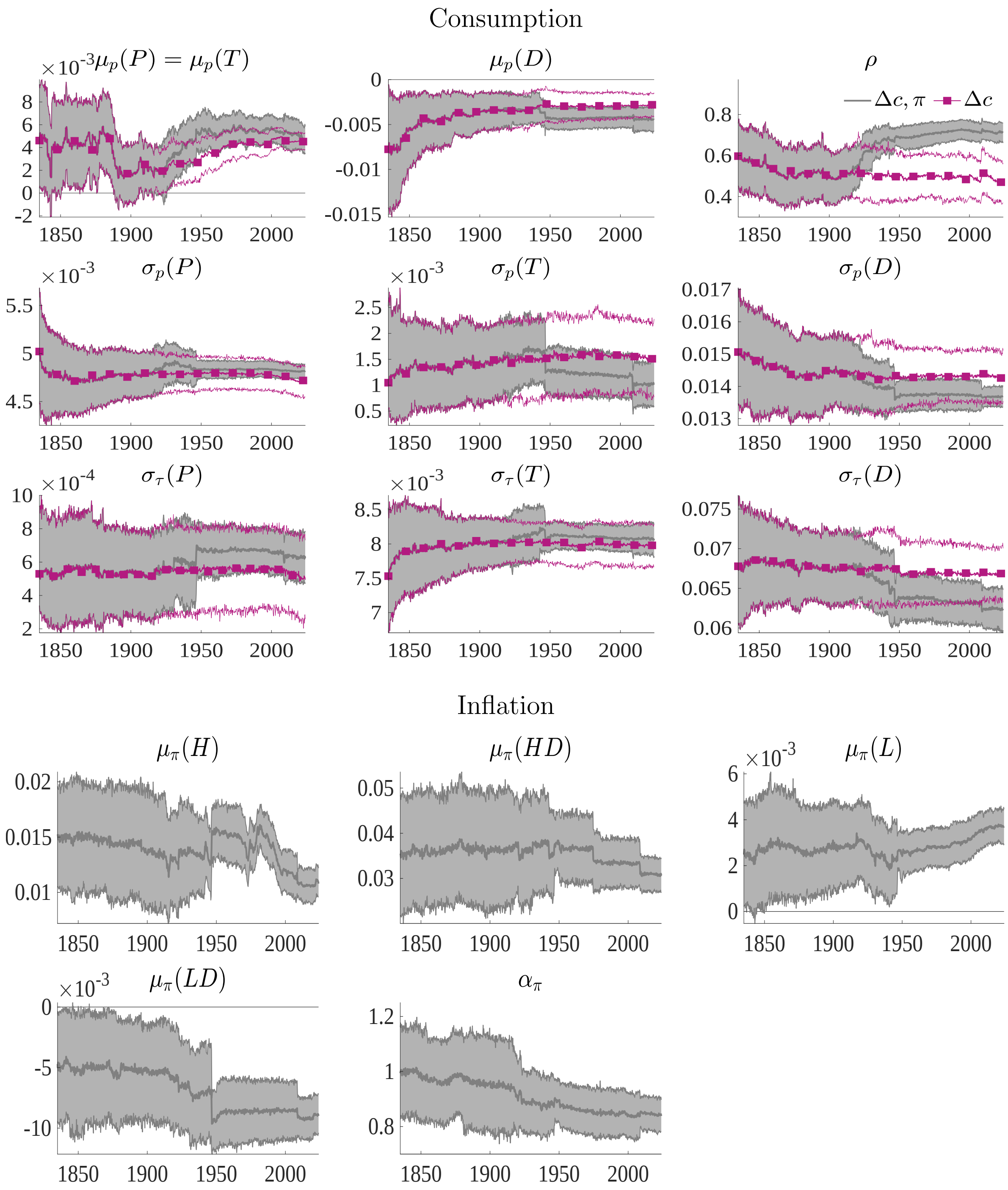
Notes: The table shows the regression estimates from the regression in Equation (1) using filtered consumption data of Kroencke (2017). The data are available from 1949:Q1 to 2018:Q4, which excludes the Covid period. Robust standard errors are reported in parenthesis. The β_0 estimate gives the autocorrelation coefficient in the post-1998 sample, while the β_1 estimate gives the difference between the autocorrelation in the pre- vs. post-1998 samples. The specification in the column labeled (1) estimates the full regression, while the specification in the column labeled (2) is run using demeaned consumption growth and setting $\alpha_1 = 0$, as explained in the main text. One asterisk denotes significance at the 10% level, two at the 5% level, and three at the 1% level.

Table A-4: Posterior predictive checks: Consumption moments

	Data		Model	
		5%	50%	95%
Mean	0.0049	0.0029	0.0040	0.0048
Stdev	0.0114	0.0079	0.0157	0.0256
Skewness	-1.2662	-3.8006	-0.6048	1.6175
Kurtosis	32.8408	3.3256	32.9772	76.3891
Autocorrelation	-0.1267	-0.2776	-0.0512	0.2775

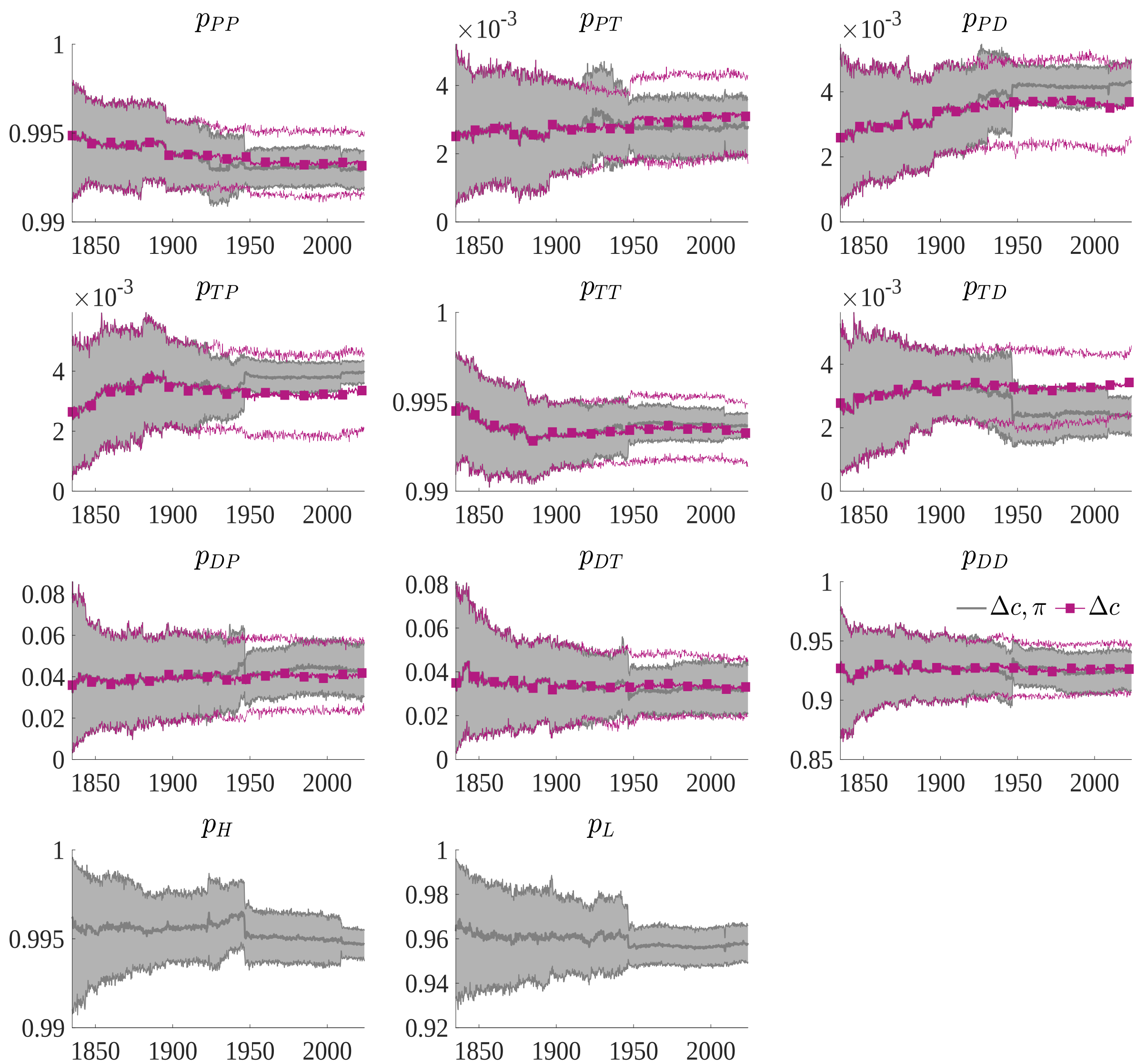
*N*otes: We report percentiles of the posterior predictive distribution for various sample moments based on simulations from the posterior distribution of the same length as the data.

Figure A-1: Estimated parameters



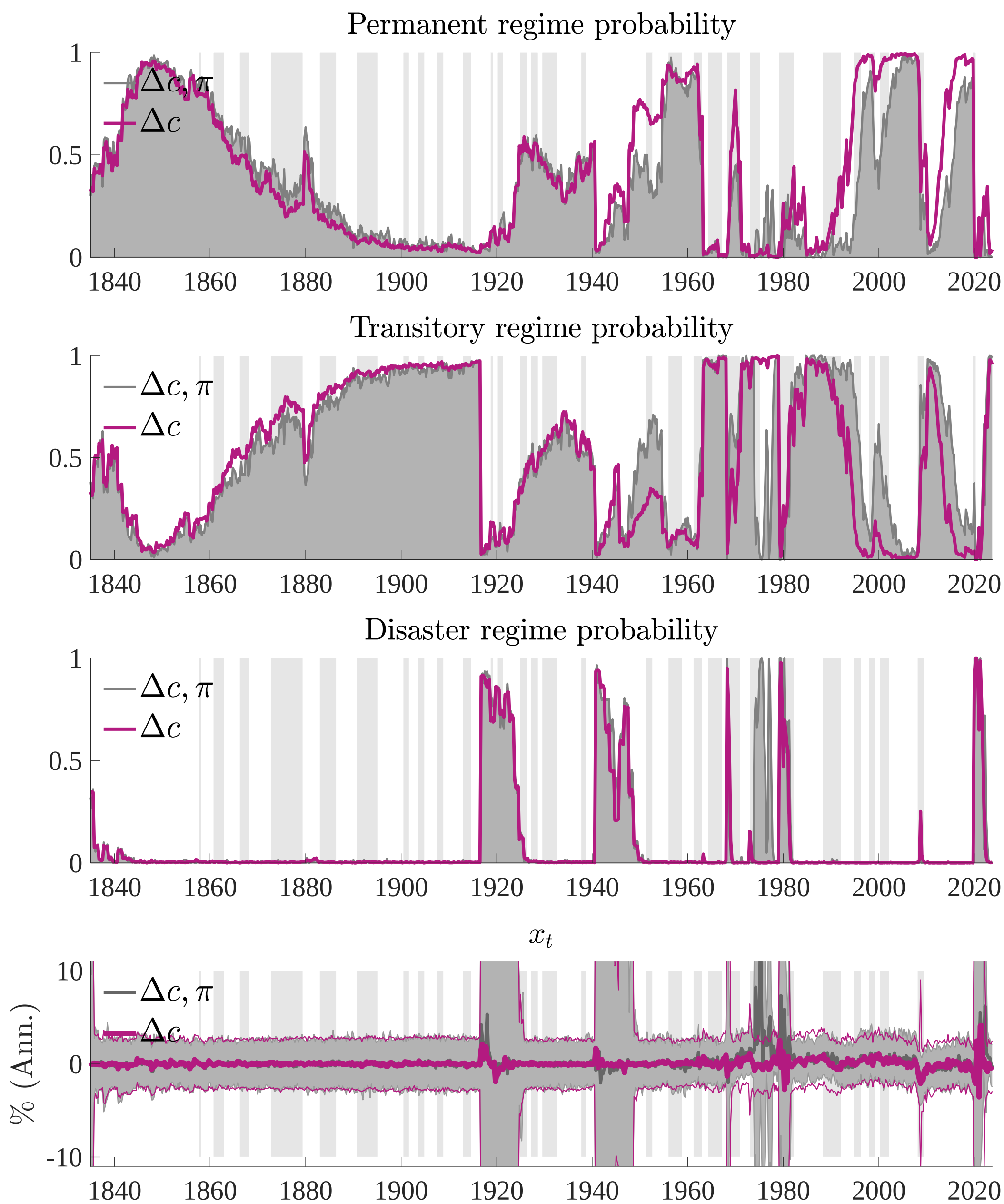
*N*otes: The plots display the median posterior estimates along with 90% confidence bands, derived from the particle learning algorithm. The grey bars represent NBER recessions.

Figure A-2: Estimated transition probabilities



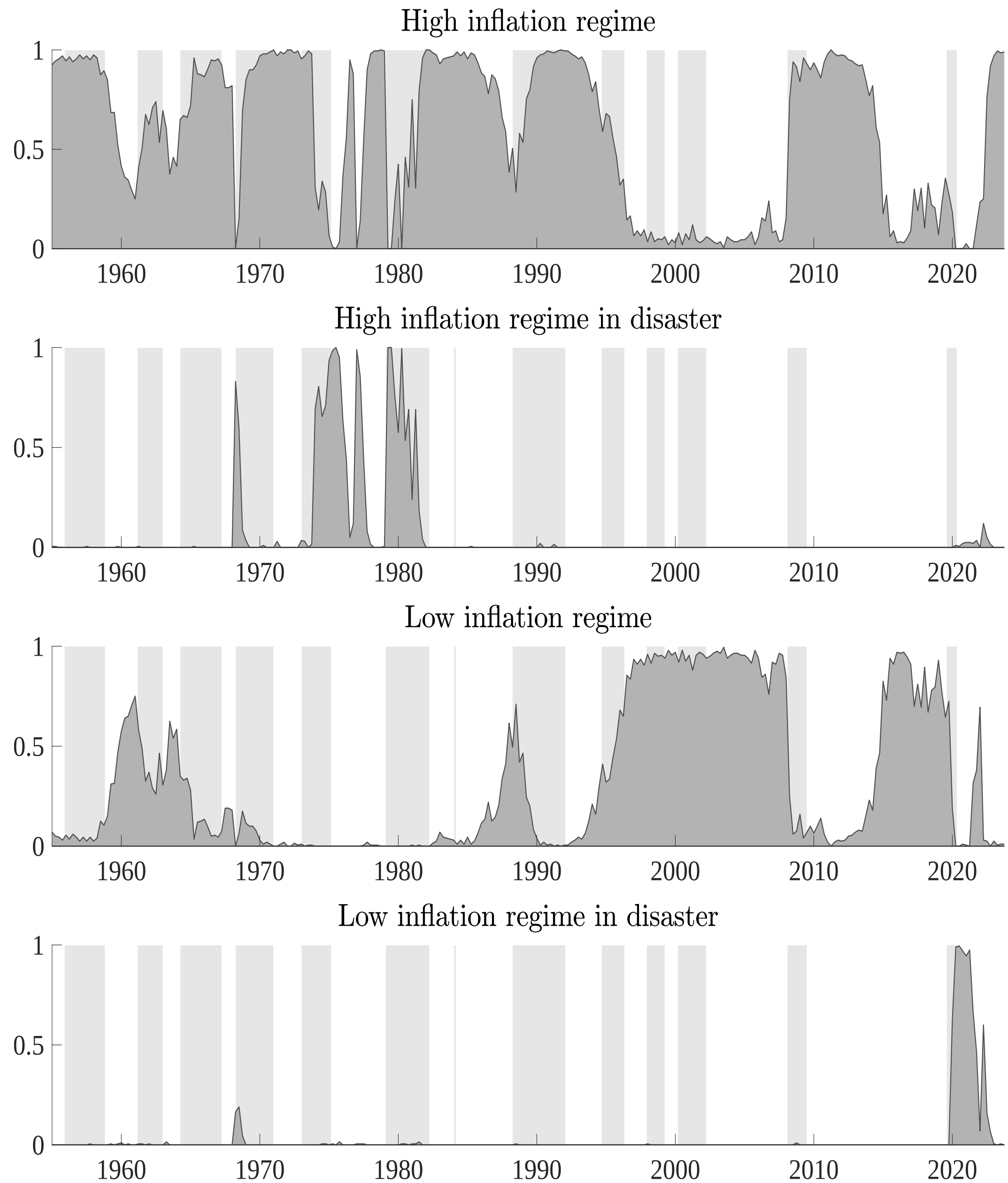
*N*otes: The plots display the median posterior estimates along with 90% confidence bands, derived from the particle learning algorithm. The grey bars represent NBER recessions.

Figure A-3: Estimated states for consumption growth: UK



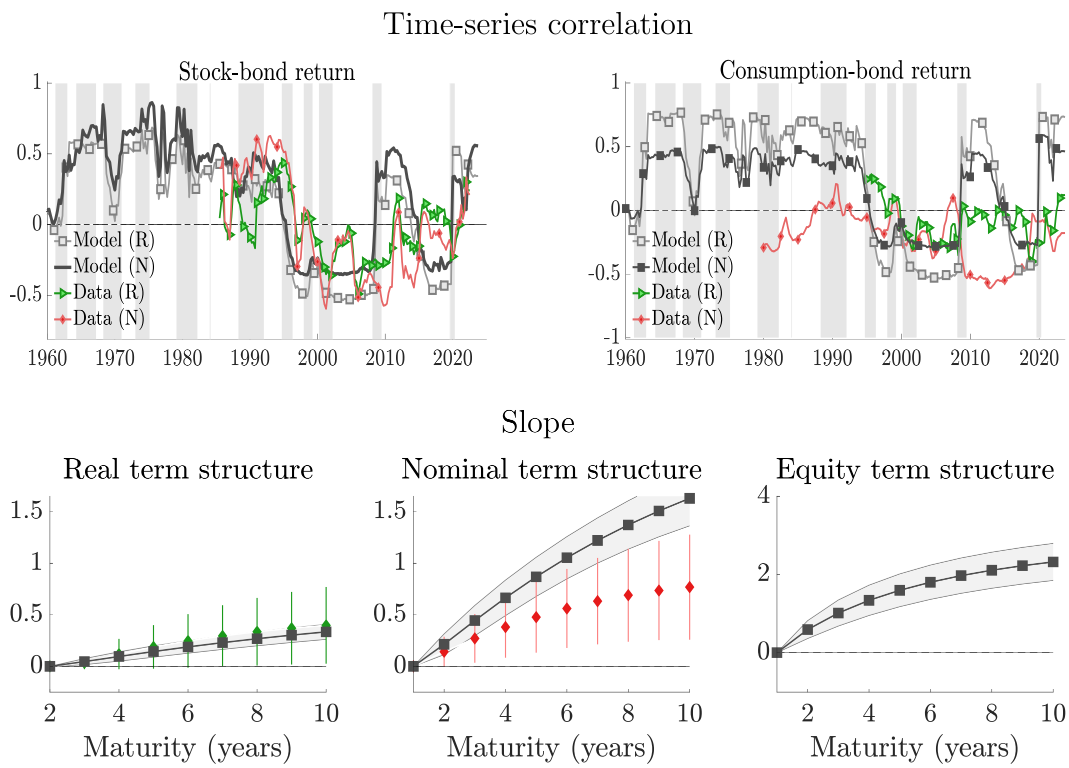
*N*otes: The plots present the filtering probabilities (average occurrence across all posterior-drawn regimes) for each regime, along with the expected consumption growth throughout the sample. Estimates from the joint consumption-inflation estimation (gray) are compared with those from the consumption-only estimation (purple). The grey bars represent recessions.

Figure A-4: Estimated states for inflation: UK



*N*otes: The plots show the filtering probabilities (average occurrence based on the entire posterior-drawn regimes) of each regime through the sample, as derived from the joint consumption-inflation estimation.. The grey bars represent recessions.

Figure A-5: Asset pricing implications for UK



*N*otes: We present the sample average slope of real and nominal bond yields, real equity yields, the conditional quarterly bond-stock correlation, and consumption-bond return correlations. Details on variable construction are provided in the main text. The data is shown in red (nominal bonds) and green (real bonds), with 95% confidence bands for the yield curve slopes, while the model is represented in black. Grey bars mark recessions. The analysis focuses on the recursive utility case only ($\delta = 0.9975$, $\gamma = 7$, EIS $= 1$, and the probability of default in the disaster state $= 6\%$).