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THE DISTRIBUTION OF INVESTOR BELIEFS, STOCK OWNERSHIP AND STOCK
RETURNS

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ABSTRACT

We study theoretically and empirically the relationship between investor beliefs, ownership dispersion and stock returns. We find that high dispersion, measured by high breadth or low Herfindahl index, forecasts returns positively for large stocks, as in Chen, Hong and Stein (2002), but negatively for small stocks. We explain that relationship in a difference-of-opinion model in which stocks differ in the size of investor disagreements and the extent of belief polarization. These differences are characterized by range and kurtosis, respectively. Proxying investor beliefs by analyst forecasts, we find that range and kurtosis affect ownership dispersion in the way that our model predicts.

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1 Introduction

Does the distribution of an asset’s ownership across investors contain information about the asset’s future returns? Why do some assets have a narrow and some a broad investor base, and do assets with a different investor base perform differently? Two influential theories link the distribution of asset ownership to prices and returns. The first theory, formulated by [Chen, Hong, and Stein \(2002, CHS\)](#), emphasizes differences of opinion across investors, combined with short-sale constraints. According to that theory, which builds on [Miller \(1977\)](#) and [Harrison and Kreps \(1978\)](#), large disagreements across investors about an asset’s payoff result in the asset being held only by the most optimistic investors. Since optimists push the asset price up, a narrow investor base indicates an overvalued asset with low future returns. The second theory, formulated by [Merton \(1987\)](#) as the investor-recognition hypothesis, emphasizes costs of entering asset markets, due to learning or constraints. According to that theory, an asset for which entry costs are high attracts few investors and trades at a deep discount because of imperfect risk-sharing. Hence, a narrow investor base indicates an undervalued asset with high future returns.

Empirical work can help distinguish between the opposite predictions of the two theories. CHS find that stocks with a narrow investor base earn low future returns. [Diether, Malloy, and Scherbina \(2002\)](#) find that stocks for which analyst forecasts reveal large disagreements earn low returns. These findings are consistent with the difference-of-opinion theory. Other findings are more consistent with the entry-cost theory. For example, [Hong and Kacperczyk \(2009\)](#) find that social norms prevent some institutional investors from holding stocks in “sin industries” (alcohol, gaming and tobacco), and this raises the stocks’ future returns. [Lou \(2014\)](#) finds that increased advertising by firms brings in more investors, raises their stock price and lowers their future returns.

In this paper we study theoretically and empirically the relationship between investor beliefs, ownership dispersion and future returns. We confirm the findings of CHS for large stocks, but document the opposite relationship for small stocks: a narrow investor base for a small stock predicts *high* future returns. Hence, the informational content of investor base differs sharply across size groups. Our finding seems puzzling in light of the empirical success of the difference-of-opinion theory for large stocks, especially because the effects implied by that theory are stronger for small stocks. Indeed, since optimists are better able to absorb a smaller supply, the overvaluation that they generate should be more severe for small stocks ([Hong, Scheinkman, and Xiong \(2006\)](#)).

We solve the puzzle by developing a model with a richer description of differences of opinion than in previous literature. Stocks in our model differ not only in the average size of investor disagreements, as in the literature, but also in the extent to which beliefs are polarized. Describing the distribution of investor beliefs in our model requires both the distribution’s range and its *kurtosis*.

Our richer description generates the relationship between ownership dispersion and asset returns that we observe in the data. It also connects the difference-of-opinion theory to the seemingly opposite theory based on entry costs because it emphasizes the imperfect risk-sharing mechanism.

Guided by our model, we examine empirically not only the relationship between ownership dispersion and asset returns, but also that between the distribution of investor beliefs and ownership dispersion. Proxying investor beliefs by analyst forecasts, we find that both the range and the kurtosis of the forecast distribution help explain ownership dispersion, and do so in the way predicted by the model. Overall, our results suggest that a richer description of beliefs than in previous literature is important for understanding the relationship between beliefs, ownership and returns.

In our model, presented in Section 2, there are multiple stocks and multiple investors with different beliefs and short-sale constraints. The distribution of investor beliefs for each stock takes one of three forms. For *baseline* stocks, there are equal numbers of optimists and pessimists, and some rational investors in the middle. For *high-disagreement* stocks, the numbers within each category are the same, but optimists and pessimists hold more extreme beliefs. For *polarizing* stocks, beliefs within each category are the same, but there are more optimists and pessimists (and fewer rationals). Moving from baseline to high-disagreement stocks amounts to raising the range of the distribution of beliefs, holding the kurtosis constant. Moving from baseline to polarizing stocks amounts to lowering the kurtosis, holding the range constant. Stocks differ not only in the distribution of investor beliefs, but also in their size, measured by the number of shares outstanding. We document polarization in the empirical distribution of analyst forecasts, which is bell-shaped with two large peaks at the extremes (Figure 1, Section 4).

We use two measures of dispersion of ownership. The first is Breadth B , defined as the fraction of investors who own a stock. The second is Herfindahl Index H , derived by squaring the fraction of total supply of a stock owned by each investor and summing across investors. More dispersed ownership is associated with high values of B and low values of H .

Within the set of small stocks, B for baseline stocks is the same as for high-disagreement stocks, and is smaller than for polarizing stocks. This is because optimists can fully absorb the supply of small stocks, and polarizing stocks have the most optimists. Within the set of large stocks instead, B for baseline stocks is larger than for the other two stock types. This is because optimists for baseline stocks are the least able to absorb a large supply: they hold less extreme beliefs than their counterparts for high-disagreement stocks, and are less numerous than their counterparts for polarizing stocks. Hence, large baseline stocks are held by optimists and rationals, while large stocks of the other two types are held only by optimists. Most of the comparisons on B carry over to H (in the opposite direction).

Our theoretical results yield three empirical hypotheses. The first concerns the relationship between ownership dispersion and stock size. When moving from small to large stocks, B increases and H decreases. Moreover, these effects are larger for stocks with low range and high kurtosis of the distribution of investor beliefs (i.e., the baseline stocks in the model).

The second hypothesis concerns the relationship between ownership dispersion and the distribution of investor beliefs. Holding the range of the belief distribution constant, B is negatively related and H is positively related to the distribution’s kurtosis for small stocks. Moreover, these relationships (driven by the comparison between baseline and polarizing stocks) reverse for large stocks in the case of B and weaken or reverse in the case of H . Likewise, holding the kurtosis of the belief distribution constant, B is negatively related and H is positively related to the range for large stocks. Moreover, these relationships (driven by the comparison between baseline and high-disagreement stocks) disappear for small stocks.

The third hypothesis concerns the relationship between ownership dispersion and future returns. Since optimists for baseline stocks hold less extreme beliefs or are less numerous than their counterparts for the other two stock types, baseline stocks are cheaper and offer higher expected returns, holding size constant. Hence, B is negatively related and H is positively related to expected returns for small stocks, and these relationships weaken or reverse for large stocks.

We test the three hypotheses using CRSP data on US stock prices and returns, Thompson Reuters data on holdings by 13-F institutional investors, and I/B/E/S data on analyst forecasts. We describe the construction of our dataset in Section 3.

Section 4 presents our empirical findings on the first and second hypotheses. We find that B is positively related to stock size, as CHS also document, and H is negatively related. To test how the effects of size on B and H depend on the range and kurtosis of the distribution of investor beliefs, we regress B and H on: stock size, the range and kurtosis of the distribution of analyst forecasts, the interactions of range and kurtosis with size, and control variables. The results from that regression are also informative about how the effects of range and kurtosis on B and H change when moving from small to large stocks.

The regression results are broadly consistent with our first and second hypotheses. In the B regression, the interaction between range and size is negative, and that between kurtosis and size is positive. Thus, the positive relationship between B and size becomes stronger for stocks with low range and high kurtosis of the distribution of analyst forecasts, consistent with our first hypothesis. The negative relationship between H and size also becomes stronger for high kurtosis stocks.

Coming to our second hypothesis, we find that within the set of small stocks, B is negatively related and H is positively related to kurtosis. Both effects reverse for large stocks because of the

interaction terms. Within the set of large stocks, B is negatively related and H is positively related to range. All of the above effects are consistent with our model.

Section 5 presents our empirical findings on the third hypothesis. We perform three main tests, in which we use the first differences ΔB and ΔH of B and H rather than the levels to account for the high autocorrelation of B and H and their correlation with stock size. One test is to regress future returns on: ΔB or ΔH , stock size, the interactions of ΔB or ΔH with size, and control variables. Another test is to form portfolios of high minus low ΔB or ΔH , and to examine the portfolio returns across size groups. A third test is to regress returns within size groups. All three approaches paint a consistent picture: B predicts negatively the returns of small stocks, and H predicts them positively; the effects weaken when moving from small to large stocks, and reverse in the case of B ; and the effects for large stocks are smaller in absolute value than the effects for small stocks. The results are economically significant. Within the smallest size decile, a stock in the smallest ΔB decile earns 8% more annually than a stock in the largest ΔB decile. Within the largest size decile, the return difference is 5.6% annually in the opposite direction.

We subject our findings to a series of robustness tests. Our findings, derived on quarterly returns, become uniformly stronger when using returns over longer horizons from one to five years. Using levels of B and H rather than first differences weakens the results in the case of B , but only for quarterly returns. Our results also become weaker when we include stocks with very low institutional ownership, which is to be expected since the calculations of B and H are performed only within the set of 13-F institutional investors. Our results remain strong when we compute B and H at the level of aggregate investment styles rather than investors. The style-level results, presented in Section 6, are useful for an additional reason: they help rule out alternative explanations of the relationship between ownership dispersion and returns that can apply to the level of individual investors but not to that of styles. Such explanations may be based on monitoring or rent extraction by large shareholders, or on asymmetric information by corporate insiders.

Our paper is most closely related to CHS. CHS describe ownership dispersion by B , and find that B is positively related to stock size and future returns. The relationship between B and returns in CHS becomes insignificant for small stocks. Unlike CHS, we find a robust negative relationship for small stocks. CHS explain theoretically the relationship between B and future returns based on a distribution of investor beliefs that is parametrized only by its range (or standard deviation). We show instead that a richer description that gives distinct roles to range and kurtosis is important for explaining the data. This applies not only to the relationship between ownership dispersion and future returns, but also to that between dispersion and the distribution of investor beliefs which is not tested in CHS.

While our model is based on differences of opinion and short-sale constraints, it is connected

to the alternative theory based on entry costs (Merton (1987)). Small stocks in our model are held only by the optimists, who correspond to the participating investors in the entry-cost theory. Ownership dispersion within small stocks is driven by variation in the optimists' number, and thus by the imperfect risk-sharing mechanism. For large stocks instead, ownership dispersion is driven by whether or not the less optimistic investors (rationals) participate—the mechanism in CHS. Variation in stock size changes the relative importance of the two mechanisms. Including small stocks in our sample allows us to separate the mechanisms.

Diether, Malloy, and Scherbina (2002) examine the empirical relationship between the distribution of investor beliefs and future returns. Proxying investor beliefs by analyst forecasts, they find that stocks for which disagreements are large earn low returns. They do not explore the relationship between these variables and the distribution of ownership, as we do, nor do they examine the role of kurtosis.

Cen, Lu, and Yang (2013) show theoretically and empirically that the breadth-return relationship can turn negative when investor sentiment is volatile. Sentiment in their model is driven by irrational investors, who trade with rational arbitrageurs. When irrational investors become optimistic, breadth increases and expected return decreases, while the opposite happens when irrational investors become pessimistic. The variation in the number of optimists is central to our mechanism as well.

Barberis and Shleifer (2003) show that style investing affects asset prices and returns through the flows of funds across styles. Flows in their model generate return predictability in the form of momentum, reversal and lead-lag effects. Similar mechanisms are at play with rational investors in Vayanos and Woolley (2013). Our style-level findings indicate that variables associated to styles predict stock returns over horizons longer than those of momentum and lead-lag effects. Moreover, the direction of the predictability switches sign as stock size increases.

2 Theory

We derive our empirical hypotheses from a model in which different investors value stocks differently and there are short-sale constraints. Stocks differ in size: some are in large supply and some are in small supply. They also differ in the distribution of valuations across investors. We describe that distribution by its range and its kurtosis. We do not use the distribution's standard deviation because it mixes the effects of range and kurtosis. We identify the effects of range and kurtosis by allowing sufficient heterogeneity across stocks so that the two characteristics can vary separately.

2.1 Model

There are two periods 0 and 1. There are $I + 1$ assets, indexed by $i = 0, 1, \dots, I$, which pay off in period 1. Asset 0 is riskless. We set its price in period 0 and its payoff in period 1 equal to one. Assets 1, ..., I are risky and we refer to them as stocks. Stock i trades at price P_i per share in period 0 and pays dividend D_i per share in period 1. For simplicity, we assume that D_i is independent and identically distributed across stocks, and its distribution is normal. We denote the mean of the normal distribution by $\bar{D} > 0$ and the variance by σ^2 . We allow stocks to differ in their supply. Large stocks are in supply of $\theta_i = \theta_L$ shares and small stocks are in supply of $\theta_i = \theta_S < \theta_L$ shares.

There are N competitive investors indexed by $n = 1, 2, \dots, N$. All investors have CARA utility with risk-aversion coefficient a . Different investors value stocks differently. We model the differences in valuations as an additional component of dividends that is private to each investor: investor n perceives the dividend from each stock i to be $D_i + \epsilon_{in}$ instead of D_i . The valuation ϵ_{in} could reflect a difference of opinion or a hedging benefit. Adopting the difference-of-opinion interpretation, we refer to investors with $\epsilon_{in} > 0$ as optimists for stock i , to investors with $\epsilon_{in} < 0$ as pessimists, and to investors with $\epsilon_{in} = 0$ as rationals. Since stocks have independent payoffs, the equilibrium price of each stock depends only on the distribution of investors' private valuations for that stock only. Hence, the correlation of an investor's private valuations across stocks does not matter: prices are the same when, e.g., some investors are optimists for all stocks, or when investors' optimism for one stock is independent of their optimism for another stock. We denote by x_{in} the number of shares that investor n holds in stock i . Investors are subject to short-sale constraints: x_{in} must be non-negative.

The distribution of investors' private valuations ϵ_{in} for each stock i is symmetric around zero and takes the general form

$$\epsilon_{in} = \begin{cases} \epsilon & \text{for } K \leq \frac{N}{2} \text{ investors,} \\ 0 & \text{for } N - 2K \text{ investors,} \\ -\epsilon & \text{for } K \text{ investors.} \end{cases} \quad (2.1)$$

There are K optimists with valuation ϵ , K pessimists with valuation $-\epsilon$, and $N - 2K$ rationals. The range of the distribution is 2ϵ , the standard deviation is $\sqrt{\frac{2K}{N}}\epsilon$, and the kurtosis is $\frac{N}{2K}$. Distribution 2.1 captures in a stylized manner the empirical distribution of analyst forecasts shown in the histogram in Figure 1: a bell shape with two large peaks at the extremes. Because all optimists have the same valuation under the distribution (2.1), and the same is true for all pessimists, all investors within each group hold the same number of shares. We denote by x_{iO} , x_{iP} and x_{iR} the number of shares that an optimist, pessimist and rational, respectively, holds in stock n .

Distribution (2.1) is characterized by the parameters K and ϵ . An increase in ϵ raises the range

and the standard deviation, but leaves the kurtosis unchanged. An increase in K leaves the range unchanged. It raises the standard deviation because there are more optimists and more pessimists. It lowers the kurtosis because the extreme values ϵ and $-\epsilon$ become smaller when measuring distance in units of the higher standard deviation.

To examine the separate effects of range and kurtosis, we perform two comparisons. We compare a stock with parameters $K > 0$ and $\epsilon > 0$ to a stock with parameters K and $\epsilon' > \epsilon$. This amounts to increasing the range holding the kurtosis constant. We also compare the same stock with parameters (K, ϵ) to a stock with parameters $\frac{N}{2}$ (i.e., no rationals) and ϵ . This amounts to reducing the kurtosis holding the range constant.

We refer to a stock with parameters (K, ϵ) as a *baseline* stock. We refer to a stock with parameters (K, ϵ') as a *high-disagreement* stock because beliefs of optimists and pessimists are more extreme than for the other stocks. We refer to a stock with parameters $(\frac{N}{2}, \epsilon)$ as a *polarizing* stock because all agents are either optimists or pessimists, and there are no rationals. We assume that stocks of the three types can be large or small. Hence, there are six types of stocks: large and baseline, denoted by LB ; large and high-disagreement, denoted by LD ; large and polarizing, denoted by LP ; small and baseline, denoted by SB ; small and high-disagreement, denoted by SD ; small and polarizing, denoted by SP .

In equilibrium, the investors who are optimistic on a stock hold long positions in the stock. Optimists are better able to absorb the entire supply of the stock when the stock is high-disagreement since in that case their beliefs are more extreme, or when the stock is polarizing since in that case they are more numerous, or when the stock is small. We restrict model parameters so that optimists absorb the entire supply of the stock in all cases except for stocks that are large and baseline. The required restriction is

$$\max \left\{ \frac{\theta_S}{K}, \frac{\theta_L}{N} \right\} < \frac{\epsilon}{a\sigma^2} < \frac{\theta_L}{K} < \frac{\epsilon'}{a\sigma^2}. \quad (2.2)$$

Intuitively, the private valuation ϵ' of optimists for high-disagreement stocks must be large relative to the supply per optimist ($\frac{\theta_L}{K}$ for LD stocks and $\frac{\theta_S}{K}$ for SD stocks). Moreover, the private valuation ϵ of optimists for baseline and polarizing stocks must be high relative to the supply per optimist except in the case of large baseline stocks ($\frac{\theta_L}{K}$ for LB stocks, $\frac{\theta_S}{K}$ for SB stocks, $\frac{\theta_L}{N}$ for LP stocks and $\frac{\theta_S}{N}$ for SP stocks).

2.2 Equilibrium

In equilibrium, the aggregate demand of the different types of investors for each stock i is equal to stock i 's supply. Using CARA utility and *i.i.d.* normal payoffs, we can write the maximization

problem of investor n as

$$\max_{x_{in} \geq 0} (\bar{D} + \epsilon_{in} - P_i)x_{in} - \frac{1}{2}a\sigma^2 x_{in}^2.$$

The investor's demand is

$$x_{in} = \max \left\{ \frac{\bar{D} + \epsilon_{in} - P_i}{a\sigma^2}, 0 \right\}. \quad (2.3)$$

Aggregating across investors and using market clearing

$$\sum_{n=1}^N x_{in} = \theta_i,$$

we find that the equilibrium price P_i of stock i solves

$$\sum_{n=1}^N \max \{ \bar{D} + \epsilon_{in} - P_i, 0 \} = a\sigma^2 \theta_i. \quad (2.4)$$

We next derive the solution to (2.4) for each type of stock.

2.2.1 High-Disagreement Stocks

We conjecture an equilibrium where high-disagreement stocks (large or small) are held only by optimists. Since there are K optimists, (2.4) implies that the stock price is

$$K(\bar{D} + \epsilon' - P_i) = a\sigma^2 \theta_i \Rightarrow P_i = \bar{D} + \epsilon' - \frac{a\sigma^2 \theta_i}{K}. \quad (2.5)$$

Each optimist holds $x_{iO} = \frac{\theta_i}{K}$ shares. Substituting P_i from (2.5) into (2.3), we find that rationals hold a zero position (and hence, pessimists do too) if $\frac{a\sigma^2 \theta_i}{K} < \epsilon'$. This inequality is implied by (2.2).

2.2.2 Polarizing Stocks

We conjecture an equilibrium where polarizing stocks (large or small) are held only by optimists. Since there are $\frac{N}{2}$ optimists, (2.4) implies that the stock price is

$$\frac{N}{2}(\bar{D} + \epsilon - P_i) = a\sigma^2 \theta_i \Rightarrow P_i = \bar{D} + \epsilon - 2\frac{a\sigma^2 \theta_i}{N}. \quad (2.6)$$

Each optimist holds $x_{iO} = \frac{2\theta_i}{N}$ shares. Substituting P_i from (2.6) into (2.3), we find that pessimists hold a zero position if $\frac{a\sigma^2 \theta_i}{N} < \epsilon$. This inequality is implied by (2.2).

2.2.3 Baseline Stocks

We first conjecture an equilibrium where baseline stocks are held only by optimists, and show that such an equilibrium exists for small stocks. We next conjecture an equilibrium where baseline stocks are held by optimists and rationals, and show that such an equilibrium exists for large stocks.

Since there are K optimists, (2.4) implies that the stock price in an equilibrium of the first type is

$$K(\bar{D} + \epsilon - P_i) = a\sigma^2\theta_i \Rightarrow P_i = \bar{D} + \epsilon - \frac{a\sigma^2\theta_i}{K}. \quad (2.7)$$

Each optimist holds $x_{iO} = \frac{\theta_i}{K}$ shares. Substituting P_i from (2.7) into (2.3), we find that rationals hold a zero position (and hence, pessimists do too) if $\frac{a\sigma^2\theta_i}{K} < \epsilon$. This inequality is implied by (2.2) in the case of small stocks ($\theta_i = \theta_S$).

Since there are K optimists and $N - 2K$ rationals, (2.4) implies that the stock price in an equilibrium of the second type is

$$K(\bar{D} + \epsilon - P_i) + (N - 2K)(\bar{D} - P_i) = a\sigma^2\theta_i \Rightarrow P_i = \bar{D} + \frac{K\epsilon}{N - K} - \frac{a\sigma^2\theta_i}{N - K}. \quad (2.8)$$

Substituting P_i from (2.8) into (2.3), we find that the positions of optimists and rationals are

$$x_{iO} = \frac{\theta_i}{N - K} + \frac{(N - 2K)\epsilon}{(N - K)a\sigma^2} \quad (2.9)$$

and

$$x_{iR} = \frac{\theta_i}{N - K} - \frac{K\epsilon}{(N - K)a\sigma^2}, \quad (2.10)$$

respectively. Rationals hold a long position if $\frac{a\sigma^2\theta_i}{K} > \epsilon$, which is implied by (2.2) in the case of large stocks ($\theta_i = \theta_L$). Pessimists hold a zero position if $\frac{a\sigma^2\theta_i}{N} < \epsilon$, which is implied by (2.2).

2.3 Prices and Expected Returns

Prices of baseline stocks are smaller than prices of high-disagreement stocks because the number of optimists is the same in both cases but optimistic beliefs are more extreme for high-disagreement stocks. Prices of baseline stocks are smaller than prices of polarizing stocks because the number of optimists (and pessimists) is larger for polarizing stocks.

In the case of small stocks, (2.5)-(2.7), $\epsilon' > \epsilon$ and $K < \frac{N}{2}$ imply

$$P_{SB} = \bar{D} + \epsilon - \frac{a\sigma^2\theta_S}{K} < \min\{\bar{D} + \epsilon' - \frac{a\sigma^2\theta_S}{K}, \bar{D} + \epsilon - 2\frac{a\sigma^2\theta_S}{N}\} = \min\{P_{SD}, P_{SP}\}. \quad (2.11)$$

In the case of large stocks, (2.5), (2.6) (2.8), $\epsilon < \frac{a\sigma^2\theta_L}{K} < \epsilon'$ and $\frac{a\sigma^2\theta_L}{N} < \epsilon$ imply

$$P_{LB} = \bar{D} + \frac{K\epsilon}{N-K} - \frac{a\sigma^2\theta_L}{N-K} < \min\{\bar{D} + \epsilon' - \frac{a\sigma^2\theta_L}{K}, \bar{D} + \epsilon - 2\frac{a\sigma^2\theta_L}{N}\} = \min\{P_{LD}, P_{LP}\}. \quad (2.12)$$

The price comparison reverses for expected returns. The expected return of stock i is $\frac{\bar{D}}{P_i}$, and is inversely related to the price. (We assume that \bar{D} is large enough so that P_i is positive for all values of the other parameters.) Expected returns of baseline stocks are larger than for high-disagreement and polarizing stocks.

The comparison of prices and expected returns across baseline, disagreement and polarizing stocks holds stock size constant. We can also compare prices and expected returns across large and small stocks holding constant the distribution of investors' private valuations. Prices are smaller and expected returns are higher for large stocks. Indeed, since dividends are independent across stocks and investors hold more shares of large stocks in equilibrium ($\theta_L > \theta_S$), the dividends of large stocks covary more highly with investor wealth. Hence, investors require higher expected returns to hold large stocks. This prediction is at odds with the size effect in the data, whereby small stocks earn higher returns than large stocks (e.g., [Banz \(1981\)](#) and [Fama and French \(1992\)](#)). Our model can be made consistent with the size effect, at the cost of more complexity, by introducing a common component in dividends and assuming that small stocks load more on that component. [Hong and Sraer \(2016\)](#) present a model with differences of opinion on a common risk component.

2.4 Ownership

We use two measures to characterize the dispersion of ownership of each stock i across investors. The first is Breadth B_i , defined as the number of investors who own stock i as a fraction of all investors. Larger values of B_i corresponds to more disperse ownership.

Since high-disagreement stocks (large or small) are owned by K optimists, their breadth is

$$B_{LD} = B_{SD} = \frac{K}{N}. \quad (2.13)$$

Since polarizing stocks (large or small) are owned by $\frac{N}{2}$ optimists, their breadth is

$$B_{LP} = B_{SP} = \frac{\frac{N}{2}}{N} = \frac{1}{2}. \quad (2.14)$$

Since small baseline stocks are owned by K optimists, their breadth is

$$B_{SB} = \frac{K}{N} < \frac{1}{2}. \quad (2.15)$$

Since large baseline stocks are owned by K optimists and $N - 2K$ rationals, their breadth is

$$B_{LB} = \frac{K + (N - 2K)}{N} = \frac{N - K}{N} > \frac{1}{2}. \quad (2.16)$$

Within the set of small stocks, B_i for baseline and high-disagreement stocks is equal, and smaller than for polarizing stocks. This is because small stocks are held only by optimists, whose number for baseline and high-disagreement stocks is equal, and smaller than for polarizing stocks. Within the set of large stocks instead, B_i is larger for baseline stocks than for high-disagreement and polarizing stocks. This is because unlike all other stock types, large baseline stocks are held by both optimists and rationals.

The second measure of dispersion of ownership is the Herfindahl Index H_i , derived by squaring the fraction of total supply of stock i owned by each investor and summing across investors:

$$H_i \equiv \sum_{n=1}^N \left(\frac{x_{in}}{\theta_i} \right)^2.$$

Smaller values of H_i correspond to more dispersed ownership of stock i , and thus tend to be associated with larger values of B_i . Hence, we can conjecture that within the set of small stocks, H_i for baseline and high-disagreement stocks is equal, and larger than for polarizing stocks, while within the set of large stocks, H_i is smallest for baseline stocks. The conjectured comparisons are valid for small stocks. For large stocks, the comparison between baseline and polarizing stocks is valid under the additional parameter restriction

$$\frac{\epsilon}{a\sigma^2} < \frac{\theta_L}{\sqrt{KN}}, \quad (2.17)$$

which can hold jointly with (2.2). This is because while large baseline stocks are held more widely and thus have larger B_i , their ownership is skewed towards the optimists. Skewed ownership has no effect on B_i (holding number of owners constant) but raises H_i because that measure gives disproportionately larger weight to owners holding more shares.

Since a high-disagreement stock i (large or small) is owned by K optimists, with each holding $x_{iO} = \frac{\theta_i}{K}$ shares, its Herfindahl Index is

$$H_{LD} = H_{SD} = \sum_{n=1}^K \left(\frac{1}{K} \right)^2 = \frac{1}{K}. \quad (2.18)$$

Since a polarizing stock i (large or small) is owned by $\frac{N}{2}$ optimists, with each holding $x_{iO} = \frac{2\theta_i}{N}$ shares, its Herfindahl Index is

$$H_{LP} = H_{SP} = \sum_{n=1}^{\frac{N}{2}} \left(\frac{2}{N} \right)^2 = \frac{2}{N}. \quad (2.19)$$

Since a small baseline stock i is owned by K optimists, with each holding $x_{iO} = \frac{\theta_i}{K}$ shares, its Herfindahl Index is

$$H_{SB} = \sum_{n=1}^K \left(\frac{1}{K} \right)^2 = \frac{1}{K}. \quad (2.20)$$

Since a large baseline stock i is owned by K optimists and $N - 2K$ rationals, whose share holdings are given by (2.9) and (2.10), respectively, its Herfindahl Index is

$$\begin{aligned} H_{LB} &= \sum_{n=1}^K \left(\frac{1}{N-K} + \frac{(N-2K)\epsilon}{(N-K)a\sigma^2\theta_L} \right)^2 + \sum_{n=1}^{N-2K} \left(\frac{1}{N-K} - \frac{K\epsilon}{(N-K)a\sigma^2\theta_L} \right)^2 \\ &= \frac{1}{N-K} + \frac{K(N-2K)\epsilon^2}{(N-K)a^2\sigma^4\theta_L^2} \end{aligned} \quad (2.21)$$

2.5 Empirical Hypotheses

We summarize our theoretical results in three propositions. Proposition 2.1 examines the relationship between dispersion of ownership and stock size. In the case of high-disagreement and polarizing stocks, dispersion is independent of size because both large and small stocks are held only by optimists. In the case of baseline stocks, dispersion is strictly larger for large stocks because they are held by both optimists and rationals, while small stocks are held only by optimists. Overall, ownership is more disperse for large stocks than for small stocks. These comparisons hold regardless of whether dispersion is measured by Breadth or Herfindahl Index.

Proposition 2.1. (Ownership dispersion and size)

- *Breadth and Herfindahl Index are independent of size for high-disagreement and polarizing stocks ($B_{LD} = B_{SD}$, $B_{LP} = B_{SP}$, $H_{LD} = H_{SD}$, $H_{LP} = H_{SP}$).*
- *Breadth increases with size and Herfindahl Index decreases with size for baseline stocks ($B_{LB} > B_{SB}$ and $H_{LB} < H_{SB}$).*

Proposition 2.2 examines the relationship between dispersion of ownership and distribution of investor valuations, for given stock size. Within the set of small stocks, ownership for baseline and

high-disagreement stocks is equally disperse, and less disperse than for polarizing stocks. The reason is that small stocks are held only by optimists, whose number for baseline and high-disagreement stocks is equal, and smaller than for polarizing stocks. This comparison holds regardless of whether dispersion is measured by Breadth or Herfindahl Index. Within the set of large stocks, ownership is more disperse for baseline stocks than for high-disagreement and polarizing stocks. The reason is that large baseline stocks are held by both optimists and rationals, while the other two stock types are held only by optimists. This comparison holds when dispersion is measured by Breadth, and extends to Herfindahl Index if (2.17) holds.

Proposition 2.2. (Ownership dispersion and distribution of investor valuations)

- *Within the set of small stocks, Breadth for baseline and high-disagreement stocks is equal, and smaller than for polarizing stocks ($B_{SB} = B_{SD} < B_{SP}$). Herfindahl Index for baseline and high-disagreement stocks is equal, and larger than for polarizing stocks ($H_{SB} = H_{SD} > H_{SP}$).*
- *Within the set of large stocks, Breadth is larger for baseline stocks than for high-disagreement and polarizing stocks ($B_{LB} > \max\{B_{LD}, B_{LP}\}$). Herfindahl Index is smaller for baseline stocks than for high disagreement stocks ($H_{LB} < H_{LD}$). It is also smaller for baseline stocks than for polarizing stocks ($H_{LB} < H_{LP}$) if (2.17) holds. Even if (2.17) does not hold, the difference between Herfindahl Index of baseline and polarizing stocks decreases when moving from small to large stocks.*

Proposition 2.3 examines the relationship between dispersion of ownership and expected returns, for given stock size. The relationship is derived by comparing baseline to high-disagreement stocks (variation in range), and by comparing baseline to polarizing stocks (variation in kurtosis). We do not compare high-disagreement to polarizing stocks, to keep the analysis simple and avoid imposing additional parameter restrictions. The comparison between baseline stocks and the other two stock types suffices to determine the cross-sectional relationship between ownership dispersion and expected returns if baseline stocks constitute a large fraction of all stocks.

The cross-sectional relationship follows by combining Proposition 2.2 with the result in Section 2.3 that expected returns for baseline stocks are larger than for high-disagreement and polarizing stocks. Consider small stocks first. Since Breadth for baseline and high-disagreement stocks is equal, and smaller than for polarizing stocks, it is negatively related to expected returns. Likewise, Herfindahl Index is positively related to expected returns. Consider large stocks next. Since Breadth for baseline stocks is larger than for high-disagreement and polarizing stocks, it is positively related to expected returns. Hence, the relationship between Breadth and expected returns reverses when moving from small to large stocks. Likewise, if (2.17) holds, then the relationship between

Herfindahl Index and expected returns also reverses, becoming negative for large stocks.

Proposition 2.3. (Ownership dispersion and expected returns)

- *Within the set of small stocks, Breadth is negatively related and Herfindahl Index is positively related to expected returns.*
- *Within the set of large stocks, Breadth is positively related to expected returns. Herfindahl Index is negatively related to expected returns if (2.17) holds.*

We next convert Propositions 2.1, 2.2 and 2.3 into respective empirical hypotheses. We maintain the interpretation of investors' private valuations as reflecting differences of opinion.

Hypothesis 1. *Breadth is positively related and Herfindahl Index is negatively related to stock size. These relationships are stronger for stocks for which:*

- *The range of the distribution of investor beliefs is low, holding the distribution's kurtosis constant.*
- *The kurtosis of the distribution of investor beliefs is high, holding the distribution's range constant.*

Hypothesis 2. *The following comparisons hold:*

- *Holding constant the kurtosis of the distribution of investor beliefs, Breadth and Herfindahl Index are unrelated to the distribution's range for small stocks. The relationship becomes negative for large stocks in the case of Breadth and positive in the case of Herfindahl Index.*
- *Holding constant the range of the distribution of investor beliefs, Breadth is negatively related and Herfindahl Index is positively related to the distribution's kurtosis for small stocks. The relationship reverses for large stocks in the case of Breadth, and weakens and can reverse in the case of Herfindahl Index.*

Hypothesis 3. *Breadth is negatively related and Herfindahl Index is positively related to expected returns for small stocks. The relationship reverses for large stocks in the case of Breadth, and weakens and can reverse in the case of Herfindahl Index.*

3 Data Sources and Variables

Our sample consists of common stocks (codes 10 and 11 of CRSP) trading on NYSE, NASDAQ and AMEX between the first quarter of 1997 and the fourth quarter of 2015. The frequency of our

analysis is quarterly, driven by the quarterly availability of the ownership data.

3.1 Stock Returns

We source data on stock prices, stock returns including dividends, trading volume, and number of outstanding shares from CRSP. We calculate a stock’s return over any given quarter by compounding the stock’s monthly returns during the quarter. We measure a stock’s size in any given quarter by market capitalization, which we calculate by multiplying the stock’s share price at the end of the quarter times the number of outstanding shares on the same day. We define small stocks as those with size below the 20th NYSE percentile; mid-cap stocks as those with size between the 20th and the 50th NYSE percentile; and large stocks as those with size above the 50th NYSE percentile. Small stocks cover approximately size deciles one to four in our sample because AMEX and NASDAQ stocks are smaller on average than NYSE stocks. Mid-cap stocks cover approximately size deciles five and six. Including small stocks in our sample allows for sufficient variation in stock size so that each of the two mechanisms in our theory—imperfect risk-sharing as in [Merton \(1987\)](#) or participation by the less optimistic investors as in CHS—can dominate.

We construct a number of stock-level variables that we use as controls. These include idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover. We calculate idiosyncratic volatility over any given quarter using a within-quarter time-series regression of daily excess stock returns over the riskless rate on the daily [Fama and French \(2015\)](#) five factors. The five factors are the excess market return over the riskless rate ($R_M - R_f$), small minus big (*SMB*, size), high minus low (*HML*, book to market), robust minus weak (*RMW*, profitability), and conservative minus aggressive (*CMA*, investment). Idiosyncratic volatility is the standard deviation of the regression residuals. If more than 30 observations are missing within the quarter, we treat the volatility observation as missing. We calculate market beta from rolling time-series regressions of monthly excess stock returns on the monthly [Fama and French \(2015\)](#) five factors. We use a 24- or 36-month rolling window, depending on data availability. We source the riskless rate and the five factors (daily and monthly) from Kenneth French’s website.

We source the ratio of book value of equity to market value from the Financial Ratios Suite of WRDS. WRDS calculates the book-to-market ratio on a quarterly basis and lags all observations by two months to ensure no look-ahead biases. We construct our momentum variable in any given quarter by compounding quarterly returns during the nine-month period ending at the end of the quarter.

We construct turnover over any given quarter by dividing the number of shares traded on each day of the quarter by the total outstanding number of shares on that day, and averaging over the

quarter. Because reported share volume is estimated differently by NYSE/AMEX and NASDAQ (Atkins and Dyl (1997)), with the latter roughly double-counting, we divide NASDAQ share volume by two (Nagel (2005)).

3.2 Institutional Ownership

We source data on institutional ownership from Thomson Reuters (TR). That data are derived from institutional investors' 13-F filings. Institutional investors with more than \$100 million in assets are required to report their stock-level holdings to the SEC on a quarterly basis, within 45 days from the end of the quarter.

We use two different databases of TR. From the first database, TR Stock Ownership, available in WRDS, we source the number of 13-F institutional investors who hold any given stock, the total number of 13-F investors, and the stock's Herfindahl Index (H). TR calculates H by dividing the number of shares held by a given 13-F investor by the number of shares held by all 13-F investors, squaring that fraction, and summing across investors.

The second database, Thomson Eikon, groups 13-F institutional investors into investment styles based on their portfolio characteristics and/or their business type. From that database, we source the number of investment styles that hold any given stock, the fraction of the stock held by each style, and the fraction of the stock held by all 13-F investors. The fraction of the stock held by any given style is calculated by dividing the number of shares held by that style by the number of shares held by all styles. The fraction of the stock held by all 13-F investors is calculated by dividing the number of shares held by all styles by the total number of outstanding shares of the stock. We term the latter variable institutional ownership (IO).¹ In our sample, stocks are held by 29 different styles. The 29 styles include seventeen general styles (e.g. aggressive growth, core growth, core value, deep value, index, etc) and twelve hedge fund styles. Appendix B provides more details on the styles and the TR classification procedure.

We construct Breadth and Herfindahl Index, our measures of ownership dispersion, at the investor and at the style level. The investor-level variables are computed as follows. Breadth for stock i and quarter t is the number of 13-F investors who hold the stock in that quarter, divided by the total number of 13-F investors in the same quarter. Herfindahl Index for stock i in quarter t is computed by TR by dividing the number of shares held by a given 13-F investor by the number of shares held by all 13-F investors, squaring that fraction, and summing across investors. The

¹The TR Stock Ownership dataset also reports an IO variable, calculated by dividing the number of shares held by all 13-F investors by the total number of outstanding shares of the stock. While the IO variables are very close across the two datasets, some discrepancies exist. We use the IO variable from Thomson Eikon since WRDS reports that there are some missing data in its hosted TR Stock Ownership dataset.

style-level variables are computed as follows. Breadth for stock i and quarter t is the number of different styles that hold the stock in that quarter. (We do not divide by the total number of styles as it is constant over time in our sample.) Herfindahl Index for stock i and quarter t is calculated by squaring the fraction of the stock held by any given style, and summing across styles.

3.3 Analyst Forecasts

We source data on analyst forecasts from the Detailed History file of the I/B/E/S database, which is provided by TR. We use analyst forecasts for earnings per share (EPS) one fiscal year ahead (FY1). We examine the EPS FY1 forecasts that appear in each month for each stock. When an analyst reports more than one forecast for the same stock in the same month, we use the most recent forecast.

For any given stock and month, we standardize analyst forecasts by dividing them by the absolute value of the mean forecast (Diether, Malloy, and Scherbina (2002)). This allows us to express the dispersion in forecasts in relative terms: a given dispersion in dollar terms is more significant economically when EPS is low. We calculate the range and kurtosis of the stock’s standardized forecasts. Range is the difference between the maximum and the minimum standardized forecast. Kurtosis is the fourth central moment divided by the square of the variance. We include in our analysis only stock/month observations with at least three analysts. We also consider the stricter criterion that the number of analysts must be at least six.

Dividing forecasts by the absolute value of the mean forecast can generate inflated standardized forecasts, and thus an inflated value of the range, when the mean forecast is close to zero. Such values, however, do not distort our analysis. This is because we transform range and kurtosis into deciles across the population of stocks in any given month, and normalize the units so that the smallest decile corresponds to zero and the largest to one (Nagel (2005)). We perform the same transformation for most other independent variables in our regressions. We compute the quarterly values of the transformed range and kurtosis by averaging over the non-missing months of the quarter. We treat stock-months with zero mean forecast as missing.

3.4 Sample Size

The total number of common stocks with CRSP data during our sample period is 14099. We exclude stocks for which we have less than three quarters of data (4781), or with negative book-to-market ratio (86). This leaves us with 9248 stocks (16 stocks are excluded according to both criteria). We apply a number of additional exclusion criteria related to the institutional ownership data, which we describe next.

For some stocks, the quarterly coverage by TR Stock Ownership and Thomson Eikon does not coincide: some are in only one database and some are in neither. We exclude stocks for which the common coverage by the two databases is less than two quarters (304 stocks). We also exclude stocks for which the common coverage is less than half of the number of quarters that each database provides (58 stocks). We exclude the latter stocks because a large discrepancy between their common coverage and their coverage in each separate database might result from reporting errors. The two exclusion criteria leave us with 8911 stocks (25 stocks are excluded according to both criteria).

For any given stock, we exclude quarter t when IO exceeds 100% in any quarter between $t-1$ and $t+2$. Because of this exclusion criterion, 305 additional stocks do not meet the minimum number of observations, leaving us with 8606 stocks. We also exclude stocks for which IO does not exceed 10% during every quarter for which the stock is observed. We impose the 10% threshold because the calculations of Breadth and Herfindahl Index involve aggregate holdings by 13-F investors in the denominator, so any noise in the data can be amplified for low values of IO . Our results weaken when we do not impose the 10% threshold, but remain significant in some specifications (Table VII). The 10% threshold removes 6022 stocks from our sample, leaving us with 2584 stocks. The excluded stock/quarters constitute 73.3% of the total stock/quarters corresponding to small stocks, 49% of those corresponding to mid-cap stocks, and 35% of those corresponding to large stocks.

The average number of stocks per quarter in the cross-section is 1259 and the number of stock/quarter observations is 91874. In the empirical exercises where we use a full set of controls, the average number of stocks per quarter drops to 1133, as some of the independent variables are missing, and the number of stock/quarter observations drops to 82743.

In Section 4, where we use data on analyst forecasts, the number of observations drops further to 53246 stock/quarters. This is because we exclude stock/month observations with fewer than three analysts. The lost stock/quarters mainly concern small stocks (68.3% of the total stock/quarters that are excluded), and less mid-cap stocks (23.1%) and large stocks (8.6%). When we exclude stock/quarters with fewer than six analysts, the sample shrinks further to 34638 observations.

4 Ownership Dispersion and Analyst Forecasts

In this section we test Hypotheses 1 and 2. Hypothesis 1 concerns the relationship between Breadth (B) or Herfindahl Index (H), our measures of ownership dispersion, and stock size. Hypothesis 2 concerns the relationship between B or H , and the distribution of investor beliefs.

Table I: **Descriptive statistics of Breadth and Herfindahl Index**

	Panel A: Full Sample				Panel B: Small Stocks			
	B	ΔB	H	ΔH	B	ΔB	H	ΔH
Mean	7.43%	0.00%	0.09	-0.00	1.88%	0.01%	0.15	-0.00
Standard Deviation	8.67%	0.63%	0.09	0.03	1.13%	0.28%	0.11	0.04
Minimum	0.03%	-8.79%	0.01	-0.94	0.04%	-2.98%	0.02	-0.90
Median	4.48%	-0.01%	0.06	-0.00	1.69%	-0.00%	0.11	-0.00
Maximum	68.45%	13.27%	1.00	0.82	7.76%	4.85%	1.00	0.52
Autocorrelation	0.997	0.059	0.952	-0.145	0.968	0.102	0.941	-0.136
	Panel C: Mid-cap Stocks				Panel D: Big Stocks			
	B	ΔB	H	ΔH	B	ΔB	H	ΔH
Mean	4.57%	0.03%	0.07	-0.00	14.89%	-0.02%	0.05	-0.00
Standard Deviation	1.55%	0.46%	0.07	0.02	10.32%	0.91%	0.04	0.02
Minimum	0.47%	-3.08%	0.02	-0.63	0.03%	-8.79%	0.01	-0.94
Median	4.47%	0.00%	0.06	-0.00	11.61%	-0.02%	0.04	0.00
Maximum	13.59%	4.53%	0.78	0.38	68.45%	13.27%	1.00	0.82
Autocorrelation	0.956	0.068	0.948	-0.082	0.996	0.053	0.904	-0.242

Note: Mean, standard deviation, minimum, median, maximum, and first-order autocorrelation of the levels and first differences of Breadth (B) and Herfindahl Index (H). Breadth for stock i and quarter t , is the number of 13-F investors who own the stock in quarter t , divided by the total number of 13-F investors in the same quarter. Herfindahl Index for stock i and quarter t is calculated by dividing the number of shares held by a given 13-F investor in quarter t by the number of shares held by all 13-F investors in the same quarter, squaring that fraction, and summing across investors. Panel A reports statistics for the full sample, Panel B for small stocks (size below the 20th NYSE percentile), Panel C for mid-cap stocks (size between the 20th and the 50th NYSE percentiles) and Panel D for large stocks (size above the 50th NYSE percentile). The statistics are based on pooled cross-sectional and time-series samples. The number of stock/quarter observations is 91874 for the full sample, 33960 for the small stock subsample, 23595 for the mid-cap stock subsample, and 34319 for the large stock subsample. The average number of stocks per quarter is 1259 for the full sample, 465 for the small stock subsample, 323 for the mid-cap stock subsample and 470 for the large stock subsample.

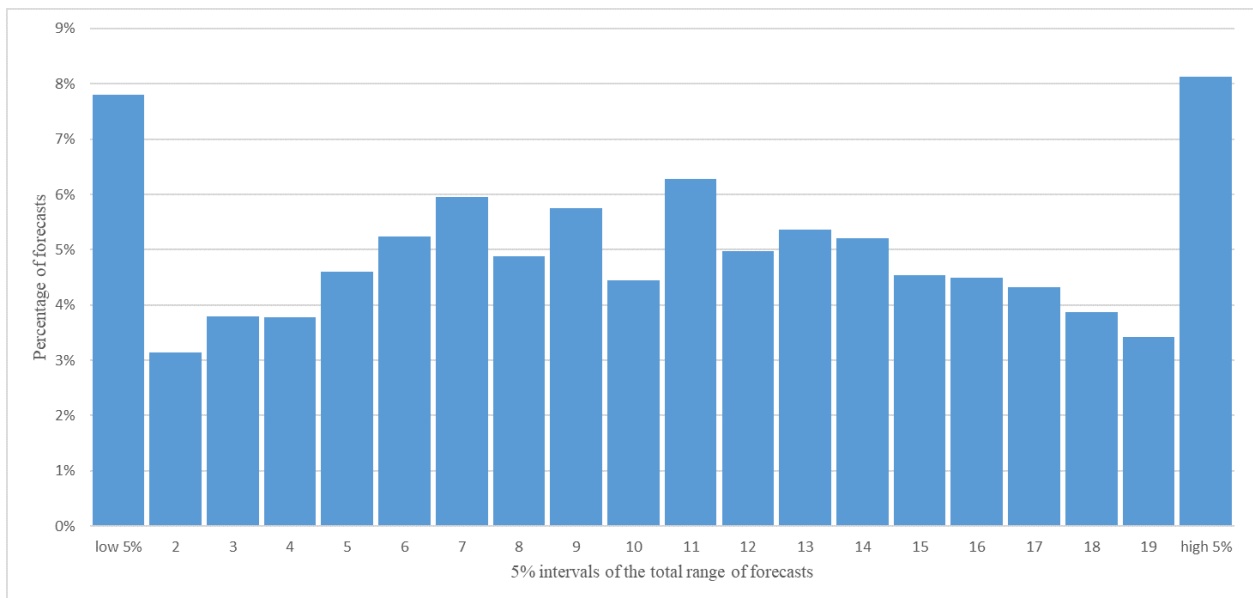
Table I presents descriptive statistics of B and H , and of their first differences ΔB and ΔH , which we compute from the end of quarter $t - 1$ to the end of quarter t . Statistics are computed over the full sample of stocks, and over the subsamples of small, mid-cap and large stocks. The mean of B across the full sample is 7.43%, meaning that the average stock is held by only 7.43% of 13-F investors. The mean of H is 0.09. Consistent with Hypothesis 1, there is a significant positive relationship between B and size. The mean of B for small stocks is 1.88%, for mid-cap stocks is 4.57% and for large stocks is 14.89%. Likewise, there is a significant negative relationship between H and size. The mean of H for small stocks is 0.15, for mid-cap stocks is 0.07 and for large stocks is 0.05.

In addition to indicating what the sign of the relationship between B or H and size should be, Hypothesis 1 identifies variables that should affect the strength of that relationship. According to

Hypothesis 1, the positive relationship between B and size should be more positive for stocks for which the distribution of investor beliefs has low range or high kurtosis. Likewise, the negative relationship between H and size should be more negative for low-range or high-kurtosis stocks. We examine these effects in Table II. The findings in that table are also informative about Hypothesis 2.

Unfortunately, we do not have direct data on investor beliefs to measure their distribution and its moments. We instead use data on the forecasts made by financial analysts. To gain an understanding of the distribution of analyst forecasts and its main properties, we plot in Figure 1 the distribution's empirical histogram for stocks with a large number of analysts. For each stock/month with at least twenty forecasts from different analysts, we calculate the range of forecasts. We then split the range into twenty equal intervals (each covering five percent of the range) and measure how many forecasts fall into each interval as a percent of the total number of forecasts for this stock/month. The average distribution over the pooled sample (10654 stock/months) is not uniform. It exhibits a bell shape, with the important difference that there are two large peaks at the extremes. The distribution of analyst forecasts has a similar flavor as distribution (2.1) in our model: a mass of optimists and pessimists at the extremes, with moderate investors spread out in-between.

Figure 1: **Empirical histogram of analyst forecasts**



Note: Empirical histogram of the distribution of analyst forecasts for earnings per share one fiscal year ahead. For each stock/month with at least twenty forecasts from different analysts, we calculate the range of forecasts. We then split the range into twenty equal intervals (each covering five percent of the range) and measure how many forecasts fall into each interval as a percent of the total number of forecasts for this stock/month. The empirical histogram is calculated by averaging over the pooled sample (10654 stock/months).

Table II: **Breadth and Herfindahl Index on range and kurtosis of analyst forecasts**

	Panel A: At least 3 analysts		Panel B: At least 6 analysts	
	<i>B</i>	<i>H</i>	<i>B</i>	<i>H</i>
range	0.023*** (3.25)	0.084*** (4.12)	0.028** (2.52)	0.111*** (3.09)
range \times size	-0.092*** (-8.64)	0.023 (0.66)	-0.092*** (-6.30)	-0.001 (-0.02)
kurtosis	-0.027*** (-4.64)	-0.012 (-0.74)	-0.016** (-2.34)	0.038** (2.04)
kurtosis \times size	0.055*** (6.62)	-0.014 (-0.57)	0.037*** (4.09)	-0.078*** (-2.95)
number of analysts	0.028*** (9.43)	-0.042*** (-4.39)	0.032*** (8.64)	-0.045*** (-3.54)

Note: Contemporaneous pooled OLS regressions of Breadth (*B*) and Herfindahl Index (*H*) on the range of the distribution of analyst forecasts, the kurtosis of that distribution, and the interactions of range and kurtosis with stock size. The regressions include as additional independent variables: stock size, *IO*, number of analysts who follow the stock, share price, idiosyncratic volatility, market beta, book-to-market ratio, momentum, turnover and quarterly dummies. Stock size, idiosyncratic volatility, market beta, book-to-market ratio and turnover are measured at the end of the last June. Range, kurtosis, share price, *IO*, number of analysts, and momentum are measured at the end of quarter *t*. Both sets of variables are transformed into deciles across the population of stocks at the time when each variable is measured, and the units are normalized so that the smallest decile corresponds to zero and the largest to one. Panel A reports regression results for stock/quarters for which there are at least three analysts in at least one month of the quarter. Panel B reports results when the minimum required number of analysts is raised to six. The number of stock/quarter observations is 53246 for Panel A and 34638 for Panel B. The average number of stocks per quarter is 729 for Panel A and 474 for Panel B. *t*-statistics, in parentheses, are computed using robust standard errors clustered by stock. In this and subsequent tables, three asterisks (***) denote statistical significance at the 1% level, two asterisks (**) at the 5% level, and one asterisk (*) at 10% level.

Table II presents results from contemporaneous pooled OLS regressions of *B* and *H* on the range of the distribution of analyst forecasts, the kurtosis of that distribution, and the interactions of range and kurtosis with stock size. We include as additional independent variables: stock size, *IO*, number of analysts who follow the stock, share price, the five control variables introduced in Section 3.1 (idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover), and quarterly dummies to control for time fixed effects. We compute *t*-statistics using robust standard errors clustered by stock to address firm fixed effects (Petersen (2009)).

Following Nagel (2005), we transform range, kurtosis, stock size, *IO*, number of analysts, share price, idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover into deciles across the population of stocks at the time when each variable is measured, and normalize the units so that the smallest decile corresponds to zero and the largest to one. Thus, the coefficient on range or kurtosis measures the relationship between the respective variable and *B* or *H* for stocks in the bottom size decile. Moreover the sum of that coefficient and of the coefficient on the respective

interaction term measures the relationship between the respective variable and B or H for stocks in the top size decile.

Panel A reports regression results for stock/quarters for which there are at least three analysts in at least one month of the quarter. Panel B reports results when the minimum required number of analysts is raised to six. With more analysts, measures of the distribution of forecasts and its moments become more precise. At the same time, the number of stocks declines, especially of small stocks.

The findings in Table II are consistent with Hypothesis 2, with the exception of the effects of range for small stocks. According to Hypothesis 2, B and H for small stocks should be unrelated to range. The coefficient of range in Table II, however, is positive and significant (at the five or one percent level) for both B and H , which means that B and H are positively related to range for small stocks. The effect of H is what Hypothesis 2 predicts, but for large stocks. The effect of B is the opposite to the predicted effect for large stocks.

Consider next the effects of range for large stocks. The sum of the coefficient of range and of the interaction between range and size is negative in the case of B and positive in the case of H . Hence, B is negatively related and H is positively related to range for large stocks, consistent with Hypothesis 2. These effects are larger and more statistically significant than the effects of range for small stocks.²

Consider next the effects of kurtosis. According to Hypothesis 2, B is negatively related and H is positively related to kurtosis for small stocks. Moreover, the relationship reverses for large stocks in the case of B , and weakens and can reverse in the case of H . The coefficient of kurtosis in Table II is negative and significant in the case of B , and is positive and significant in the case of H for stocks with at least six analysts. This yields the predictions of Hypothesis 2 for small stocks. The sum of the coefficient of kurtosis and of the interaction between kurtosis and size is positive and significant in the case of B and negative and significant in the case of H . This yields the predictions of Hypothesis 2 for large stocks. The effects of kurtosis for large stocks are more statistically significant than its effects for small stocks.

The sign of the interaction term in the Breadth regressions is consistent not only with Hypothesis 2 but also with Hypothesis 1. The negative (positive) coefficient on the interaction term between range (kurtosis) and size means that the relationship between B and size strengthens for low range (high kurtosis) stocks. The interaction term in the H regressions is significant only for kurtosis and for stocks with at least six analysts, and in that case its sign is consistent with Hypotheses 1 and 2.

²Using the same ordering of columns as in Table II, t -statistics for the effects of range for large stocks (top size decile) are -12.35, 5.87, -10.69 and 5.37, and for the effects of kurtosis for large stocks are 8.77, -2.68, 6.82 and -3.89.

Table II reports additionally the regression coefficient on number of analysts. Stocks with more analysts have larger B and lower H , and both relationships are highly statistically significant. Intuitively, the more analysts follow a stock, the more investors invest in the stock, and the more dispersed the stock’s ownership is. Coefficients of other control variables, not shown in Table II, are also intuitive. For example, stocks with higher turnover and higher momentum have higher B and lower H , consistent with these stocks’ attracting more investors.

5 Ownership Dispersion and Stock Returns

In this section we test Hypothesis 3. This hypothesis concerns the relationship between Breadth (B) and Herfindahl Index (H), our measures of ownership dispersion, and future returns.

5.1 Main Results

Tables III-V present our main tests of Hypothesis 3. In these tables we use the first differences ΔB and ΔH of B and H , rather than the levels. This is because B and H are highly autocorrelated (first-order autocorrelation is 0.997 for B and 0.952 for H , as reported in Table I) and highly correlated with stock size (correlation is 0.824 for B and -0.546 for H). Hence, using levels may confound the effects of B and H on returns with the effects of size. CHS use the first difference of B in their regression of returns on B for similar reasons. In Table VII we show that most of our main results carry through when using the levels of B and H rather than the first differences.

Table III presents results from pooled OLS regressions of stock returns in quarter $t+1$ on ΔB or ΔH in quarter t and the interaction of ΔB or ΔH with stock size in the same quarter. In Panel A, we include as additional independent variables: stock size, IO , and quarterly dummies to control for time fixed effects.³ In Panel B, we additionally include: share price and the five control variables introduced in Section 3.1 (idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover). The regression setting and the interpretation of the coefficients are as in Fama and MacBeth (1973). We transform ΔB , ΔH , stock size, IO , share price, idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover into deciles across the population of stocks, and normalize the units so that the smallest decile corresponds to zero and the largest to one. We compute t -statistics using robust standard errors clustered by stock.

³Controlling for IO is consistent with our theoretical model. Indeed, the effects of B and H in our model are generated by a change in the distribution of investor beliefs holding the total number of shares owned by investors constant. We capture the change in the distribution of investor beliefs using B and H , which we can construct based only on 13-F institutional holdings. Holding the total number of shares owned by 13-F institutional investors constant amounts to controlling for IO , which we construct as the ratio of 13-F institutional holdings to total holdings. Our empirical results remain almost identical when we drop IO as a control.

Table III: Returns on first difference of Breadth or Herfindahl Index

	Panel A: Size & IO as controls		Panel B: Full Set of controls	
	ΔB regression	ΔH regression	ΔB regression	ΔH regression
Ownership Variable (OV)	-0.026***	0.021***	-0.020**	0.022***
(ΔB or ΔH)	(-3.32)	(3.59)	(-2.32)	(3.52)
OV \times size	0.035***	-0.025***	0.034***	-0.030***
	(3.36)	(-2.59)	(3.07)	(-2.96)
Size	-0.051***	-0.021***	-0.027***	0.004
	(-8.46)	(-4.06)	(-3.65)	(0.68)
IO	-0.013***	-0.012***	-0.023***	-0.022***
	(-5.24)	(-5.19)	(-7.62)	(-7.58)

Note: Pooled OLS regressions of stock returns in quarter $t + 1$ on the first difference of Breadth (ΔB) or Herfindahl Index (ΔH) in quarter t (computed from end of quarter $t - 1$ to end of quarter t) and on the interaction of ΔB or ΔH with stock size in the same quarter. The regressions in Panel A include as additional independent variables: stock size, IO and quarterly dummies. The regressions in Panel B additionally include: share price, idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover. Stock size, idiosyncratic volatility, market beta, book-to-market ratio and turnover are measured at the end of the last June. ΔB , ΔH , IO , share price and momentum are measured at the end of quarter t . Both sets of variables are transformed into deciles across the population of stocks at the time when each variable is measured, and the units are normalized so that the smallest decile corresponds to zero and the largest to one. The number of stock/quarter observations is 91874 for the regressions in Panel A and 82743 for those in Panel B. The average number of stocks per quarter is 1259 and 1133, respectively. t -statistics, in parentheses, are computed using robust standard errors clustered by stock.

The findings in Table III are consistent with Hypothesis 3. Consider first the effects for small stocks. In both Panels A and B, the coefficient of ΔB is negative and that of ΔH is positive. Thus, B predicts future returns negatively and H predicts them positively. The effects of ΔB and ΔH on small-stock returns are statistically significant in all four regressions (Panels A and B, ΔB and ΔH). They are also economically significant. The coefficient -0.026 on ΔB in Panel A means that within the smallest size decile, a stock in the top ΔB decile ($\Delta B = 1$) earns 2.6% lower expected quarterly return than a stock in the bottom ΔB decile ($\Delta B = 0$). This difference is 10.4% in annualized terms. When including a full set of controls, the effect drops from 2.6% to 2%. Economic significance remains strong when ΔH is used as a predictor: a stock in the top ΔH decile ($\Delta H = 1$) earns 2.1% higher expected quarterly return (2.2% with a full set of controls) than a stock in the bottom ΔH decile ($\Delta H = 0$).

Consider next the effects for large stocks. In both Panels A and B, the coefficient of the interaction of ΔB with size is positive and larger in absolute value than the coefficient of ΔB . Likewise, the coefficient of the interaction of ΔH with size is negative and larger in absolute value than the coefficient of ΔH . Thus, the negative (positive) predictive relationship between ΔB (ΔH) and future returns for small stocks weakens as stock size increases and turns positive (negative) for

large stocks. The interaction terms with size are statistically significant in all four specifications (Panels A and B, ΔB and ΔH). The effects on large-stock returns are statistically significant in the two specifications involving ΔB . In terms of economic significance, within the largest size decile, a stock in the top ΔB decile earns 0.9% higher expected quarterly return (1.4% with a full set of controls) than a stock in the bottom ΔB decile.

The coefficient of ΔB or ΔH and that of the interaction between ΔB or ΔH with stock size capture the two key mechanisms in our theory. For small stocks, ΔB or ΔH are related to future returns only through the imperfect risk-sharing mechanism. That mechanism's strength is captured by the coefficient of ΔB or ΔH . As stock size increases, the relationship between ΔB or ΔH and future returns is also influenced by participation by less optimistic investors. That mechanism's strength is captured by the coefficient of the interaction between ΔB or ΔH with size.

The coefficient of IO is negative and statistically significant in all four specifications of Table III. A number of papers find a positive relationship between IO and future stock returns (e.g., Gompers and Metrick (2001) and Bennett, Sias, and Starks (2003)), while more recent papers find that the coefficient of IO turns negative for return horizons longer than one year (e.g., Dasgupta, Prat, and Verardo (2011) and Edelen, Ince, and Kadlec (2016)). We find a negative relationship between IO and future stock returns even at an one-quarter horizon. This finding could be due to the time-series behavior of the market share held by institutions. In earlier sample periods, the market share of institutional investors was growing rapidly, and high IO could have been a predictor of faster such growth. In our sample period, IO grows rapidly during the first half (from 22.8% in 1997Q1 to 44.2% in 2005Q4) but more slowly during the second half (from 44.2% in 2005Q4 to 59.4% in 2015Q4).

Table IV complements Table III by presenting results from portfolio sorts. We construct nine portfolios based on independent double sorting on size and ΔB or ΔH . We sort stocks on size based on NYSE size percentiles, measuring size at the end of the last June. Small stocks are those below the 20th NYSE size percentile, and cover approximately size deciles one to four in our sample. Mid-cap stocks are those between the 20th and 50th NYSE size percentile, and cover approximately our size deciles five and six. Large stocks are those above the 50th NYSE size percentile. We independently sort stocks on ΔB or ΔH based on sample percentiles in quarter t . Low ΔB (or low ΔH) stocks are those below the 20th percentile, mid ΔB (or mid ΔH) stocks are those between the 20th and 80th percentile, and high ΔB (or high ΔH) stocks are those above the 80th percentile.

Panel A reports the average monthly returns of the nine portfolios created from the size/ ΔB double sort. Portfolio returns are the equally weighted averages of the returns of the stocks in the portfolio over the three months after the portfolio is formed at the end of quarter t .⁴ We compute

⁴Results for value-weighted returns (unreported) are weaker statistically and economically in the case of small

Table IV: **Portfolio double-sorts on size and first difference of Breadth or Herfindahl Index**

Panel A: Average returns of the nine size/ ΔB portfolios				Panel B: Average returns of the nine size/ ΔH portfolios			
	Small	Mid-cap	Large		Small	Mid-cap	Large
Low ΔB	2.69% (3.30)	1.47% (2.77)	0.89% (2.03)	Low ΔH	1.60% (3.45)	1.11% (2.56)	0.96% (2.24)
Mid ΔB	1.62% (3.68)	1.07% (2.73)	0.97% (2.79)	Mid ΔH	1.45% (3.31)	1.24% (3.17)	0.94% (2.68)
High ΔB	1.44% (2.48)	1.41% (3.37)	1.12% (3.16)	High ΔH	2.09% (3.82)	1.16% (2.34)	1.14% (3.16)

Panel C: High ΔB minus Low ΔB					Panel D: High ΔH minus Low ΔH			
	Small	Mid-cap	Large	Large minus Small	Small	Mid-cap	Large	Large minus Small
Average returns	-1.25% (-2.02)	-0.06% (-0.20)	0.23% (0.90)	1.48% (2.80)	0.48% (2.16)	0.05% (0.23)	0.19% (0.76)	-0.29% (-1.08)
CAPM alpha	-1.13% (-2.03)	0.01% (0.05)	0.30% (1.29)	1.43% (2.91)	0.45% (2.14)	0.01% (0.04)	0.11% (0.49)	-0.34% (-1.20)
Carhart 4-factor alpha	-1.62% (-3.15)	-0.33% (-1.53)	0.01% (0.05)	1.63% (3.10)	0.66% (3.32)	0.22% (1.30)	0.32% (1.46)	-0.34% (-1.17)
FF 5-factor alpha	-1.24% (-2.19)	0.03% (0.10)	0.40% (1.56)	1.46% (2.82)	0.58% (2.34)	0.04% (0.18)	0.18% (0.83)	-0.40% (-1.44)

Note: Average returns of nine stock portfolios formed by independent double sorting on size and on the first difference of Breadth (ΔB) or Herfindahl Index (ΔH). We sort stocks on size based on NYSE size percentiles, measuring size at the end of the last June. Small stocks are those below the 20th NYSE size percentile, mid-cap stocks are those between the 20th and 50th NYSE size percentile, and large stocks are those above the 50th NYSE size percentile. We independently sort stocks on ΔB or ΔH based on sample percentiles in quarter t . Low ΔB (or Low ΔH) stocks are those below the 20th percentile, mid ΔB (or mid ΔH) stocks are those between the 20th and 80th percentile, and high ΔB (or high ΔH) stocks are those above the 80th percentile. Panel A reports the average monthly returns of the nine portfolios created from the size/ ΔB double sort. Panel B reports the average monthly returns of the nine portfolios created from the size/ ΔH double sort. Panel C reports average returns and alphas for long-short strategies that go long in the high ΔB portfolio and short in the low ΔB portfolio. Panel D reports average returns and alphas for long-short strategies that go long in the high ΔH portfolio and short in the low ΔH portfolio. Alphas are computed using the CAPM, the [Carhart \(1997\)](#) four-factor model, and the [Fama and French \(2015\)](#) five-factor model. Portfolio returns are the equally weighted averages of the returns of the stocks in the portfolios over the three months after the portfolio is formed at the end of quarter t . The returns are monthly (219 months, starting from 07/1997 to 09/2015, corresponding to the 73 Quarters used in the regressions). t -statistics, in parentheses, are computed using Newey-West standard errors with one lag.

stocks. This is because value-weighting gives larger weight to small stocks in the higher size deciles. The coefficients for “small” in Panels C and D retain their sign, but the average t -statistic across the four rows drops from -2.35 to

t -statistics for portfolio returns using Newey-West standard errors with one lag. Panel C reports average monthly returns and alphas for long-short strategies that go long in the high ΔB portfolio and short in the low ΔB portfolio. Alphas are computed using the CAPM, the Carhart (1997) four-factor model, and the Fama and French (2015) five-factor model. Panels B and D are the counterparts of Panels A and C for ΔH instead of ΔB .

Panel A shows that for small stocks there is a clear decreasing pattern in average return when moving from the low ΔB to the high ΔB portfolio. The average monthly return drops from 2.69% for low ΔB to 1.44% for high ΔB . The average monthly return of a strategy that goes long in the high ΔB portfolio and short in the low ΔB portfolio is -1.25%. That return is statistically significant, with t -statistic -2.02, and economically significant, as it is -15% annualized. Panel B shows a similar decreasing pattern when moving from the high ΔH to the low ΔH portfolio. The long-short average return is 5.76% annualized (0.48% monthly), smaller than in the case of ΔB but statistically significant, with t -statistic 2.16. The returns on the long-short strategies are not driven by exposure to standard risk factors: the strategies' CAPM, four-factor and five-factor alphas are similar to the strategies' average returns.

For mid-cap stocks, there is no clear pattern in average return across ΔB or ΔH portfolios. A slight pattern appears for large stocks. In the case of ΔB , it is opposite to that for small stocks, while in the case of ΔH , it goes in the same direction. The returns of long-short strategies for large stocks are not statistically significant. The change in the long-short strategies' returns when moving from small to large stocks is statistically significant in the case of ΔB , but not significant in the case of ΔH . Going long in the large-stock long-short ΔB portfolio and short in the small-stock long-short ΔB portfolio yields average return 17.76% annualized (1.48% monthly), with t -statistic 2.80.

The findings in Table IV are broadly consistent with Hypothesis 3. For small stocks, B predicts future returns negatively and H predicts them positively. Moreover, the effects weaken when moving from small to large stocks, and the effect of B changes sign. That the effect of H does not change sign (but the effect of B does) is consistent with Hypothesis 3.

Table V complements Tables III and IV by presenting results from pooled OLS regressions of stock returns in quarter $t + 1$ on ΔB or ΔH in quarter t , performed separately on the three size groups constructed as in Table IV (small, mid-cap and large). In Panels A and B we include the same additional independent variables as in Panels A and B, respectively, of Table III. Unlike in Table III, we do not transform the variables into their corresponding deciles, but use logarithms of size, idiosyncratic volatility, book-to-market ratio, turnover, and share price, to address extreme skewness. We compute t -statistics using robust standard errors clustered by stock. For brevity, -1.74 in Panel C, and from 2.49 to 1.90 in Panel D.

Table V: **Returns on first difference of Breadth or Herfindahl Index, by size**

	Panel A: Size & <i>IO</i> as Controls			Panel B: Full Set of Controls		
	Small	Mid-cap	Large	Small	Mid-cap	Large
ΔB	-1.388** (-2.06)	-0.121 (-0.29)	0.355** (2.53)	-1.061 (-1.41)	1.143*** (2.62)	0.371*** (2.72)
ΔH	0.148*** (2.73)	0.039 (0.51)	0.081** (2.19)	0.222*** (2.80)	-0.046 (-0.46)	0.066 (1.46)

Note: Pooled OLS regressions of stock returns in quarter $t + 1$ on the first difference of Breadth (ΔB) or Herfindahl Index (ΔH) in quarter t (computed from end of quarter $t - 1$ to end of quarter t), performed separately on small, mid-cap and large stocks. Size groups are defined as in Table IV. The regressions in Panel A include as additional independent variables: stock size, *IO* and quarterly dummies. The regressions in Panel B additionally include: share price, idiosyncratic volatility, market beta, book-to-market ratio, momentum and turnover. Stock size, idiosyncratic volatility, market beta, book-to-market ratio and turnover are measured at the end of the last June. ΔB , ΔH , *IO*, share price and momentum are measured at the end of quarter t . Stock size, share price, idiosyncratic volatility, book-to-market ratio and turnover are transformed into logarithms. No transformation is done on the remaining variables. The number of stock/quarter observations in Panel A (Panel B) is 33960 (29101) for the small stock subsample, 23595 (20956) for the mid-cap stock subsample and 34319 (32686) for the large stock subsample. The average number of stocks per quarter in Panel A (Panel B) is 465 (399) for the small stock subsample, 323 (287) for the mid-cap stock subsample and 470 (448) for the large stock subsample. Each cell reports the results of a different regression: the line corresponds to the ownership variable (ΔB or ΔH) and the column corresponds to the size group. t -statistics, in parentheses, are computed using robust standard errors clustered by stock.

we report only the coefficients and t -statistics of ΔB and ΔH . Each cell reports the results of a different regression: the line corresponds to the ownership variable (ΔB or ΔH) and the column corresponds to the size group.

The findings in Table V for small stocks are broadly consistent with Hypothesis 3. Consider first the effects for small stocks. In both Panels A and B, the coefficient of ΔH is positive and significant. Moreover, the coefficient of ΔB is negative, although it is significant only in Panel A. Thus, B predicts future returns negatively and H predicts them positively. Consider next the effects for large stocks. In both Panels A and B, the coefficient of ΔB is positive and significant. Hence, the effect of B weakens and changes sign when moving from small to large stocks. The effect of H also weakens but does not change sign. The coefficient of H is positive in both Panels A and B, and significant in Panel A. That the effect of H does not change sign (but the effect of B does) is consistent with Hypothesis 3.

Tables III-V paint a consistent picture. B predicts negatively the returns of small stocks, and H predicts them positively. The effects weaken when moving from small to large stocks, and reverse in the case of B . The effects for large stocks are smaller in absolute value than the effects for small stocks. Within this broad picture, there are differences across the tables, reflecting the differences in empirical methodology. Statistical significance is weaker in Table IV than in Tables III and V because aggregation into portfolios makes the data coarser. Statistical significance for small stocks

is weaker in Tables IV and V than in Table III because the group of small stocks in the former tables includes deciles one to four in our sample, while the coefficient of ΔB or ΔH in Table III concerns decile one.

The effect of ΔB for large stocks is somewhat smaller than in CHS. Using all stocks above the 20th NYSE size percentile, CHS find a coefficient of ΔB equal to 1.187 and a standard deviation of ΔB equal to 0.46%. Thus, a one standard deviation increase in ΔB predicts 0.54% ($=1.187 \times 0.46\%$) higher quarterly return. The standard deviation of ΔB in our combined mid-cap and large stocks (unreported) is 0.76%, and the coefficient of ΔB if we were to pool mid-cap and large stocks in the regressions of Table V (unreported) is 0.553. Hence, a one standard deviation increase in ΔB predicts 0.42% ($=0.553 \times 0.76\%$) higher quarterly return. A discrepancy between CHS and our results may arise because of the different sample periods. Our sample starts in the first quarter of 1997. CHS's sample ends in the fourth quarter of 1998, and Nagel (2005) finds that adding five extra years (1999-2003) renders the coefficient of ΔB insignificant. We return to the effects of sample periods in our robustness analysis in Section 5.3.

5.2 Long-Horizon Results

Table VI presents results from the same regressions as in Panel B of Table III (stock returns in quarter $t + 1$ on ΔB or ΔH in quarter t , its interaction with size, and full controls), except that stock returns are evaluated k quarters ahead (from end of quarter t to end of quarter $t + k$) with $k = 4, 8, 12, 16, 20$. Panel A reports results for the one-year horizon ($k = 4$), Panel B for two years, Panel C for three years, Panel D for four years, and Panel E for five years. We calculate a stock's return over each horizon by compounding the stock's quarterly returns, and we do not express the return in annualized terms (i.e., we leave it as a cumulative return). As in Table III, we transform the independent variables into deciles, and normalize the units so that the smallest decile corresponds to zero and the largest to one. We compute t -statistics using robust standard errors clustered by stock. This addresses the autocorrelation that arises because the periods over which returns are evaluated overlap. Using Newey-West standard errors with a lag corresponding to the overlap yields similar standard errors.

Table VI shows that the relationship between ΔB or ΔH and future returns over long horizons is the same qualitatively as over the one-quarter horizon, but stronger quantitatively. As in Table III, B predicts future returns of small stocks negatively and H predicts them positively. Moreover, the effects weaken when moving from small to large stocks, and change sign in all cases except for the regression of two-year returns on ΔH . The coefficients on ΔB or ΔH and on the interaction term increase with horizon from quarter one (Table III) to quarter twenty (Panel E of Table VI). The

Table VI: Long-horizon return regressions

	Panel A: 4Q		Panel B: 8Q		Panel C: 12Q		Panel D: 16Q		Panel E: 20Q	
	ΔB regression	ΔH regression	ΔB regression	ΔH regression	ΔB regression	ΔH regression	ΔB regression	ΔH regression	ΔB regression	ΔH regression
Ownership Variable (OV)	-0.127*** (-5.77)	0.053*** (3.27)	-0.182*** (-4.38)	0.115*** (3.83)	-0.227*** (-4.09)	0.130*** (3.10)	-0.297*** (-4.12)	0.229*** (3.70)	-0.419*** (-4.52)	0.362*** (3.64)
(ΔB or ΔH)	0.162*** (5.44)	-0.056** (-2.12)	0.235*** (4.21)	-0.087* (-1.82)	0.276*** (3.77)	-0.139** (-2.15)	0.369*** (4.01)	-0.241** (-2.55)	0.560*** (4.58)	-0.437*** (-2.85)
OV \times size	-0.138*** (-5.64)	-0.030 (-1.22)	-0.274*** (-5.09)	-0.115** (-2.15)	-0.347*** (-4.41)	-0.142* (-1.84)	-0.445*** (-4.36)	-0.143 (-1.40)	-0.637*** (-4.76)	-0.145 (-1.01)
size	-0.088*** (-6.38)	-0.087*** (-6.33)	-0.162*** (-5.25)	-0.161*** (-5.22)	-0.140*** (-3.10)	-0.139*** (-3.07)	-0.129* (-1.94)	-0.128* (-1.92)	-0.105 (-1.20)	-0.104 (-1.18)
IO										

Note: Pooled OLS regressions of stock returns from end of quarter t to end of quarter $t + k$ with $k = 4, 8, 12, 16, 20$, on the first difference of Breadth (ΔB) or Herfindahl Index (ΔH) in quarter t and on the interaction of ΔB or ΔH with stock size in the same quarter. Panel A reports results for the one-year horizon ($k = 4$), Panel B for two years, Panel C for three years, Panel D for four years, and Panel E for five years. The regressions include the full set of controls described in Table III. Variables are transformed into deciles and normalized, as described in Table III. The number of stock/quarter observations ranges from 78607 ($k = 4$) to 53195 ($k = 20$). The average number of stocks per quarter ranges from 1107 stocks ($k = 4$) to 967 ($k = 20$). t -statistics, in parentheses, are computed using robust standard errors clustered by stock.

increase is approximately linear, which implies that expressing the coefficients in annualized terms yields a pattern that is approximately constant over time. The coefficients on ΔB or ΔH and on the interaction term remain statistically significant over all horizons. Statistical significance weakens for the interaction term between ΔH and size, but strengthens for ΔB and for the interaction term between ΔB and size. Thus, the predictive effects of B and H are long-lasting, suggesting that the price-pressure effects identified in our model are also long-lasting.

The coefficient of IO is negative over all horizons (as in Table III) and becomes insignificant past the four-year horizon. Since the effects of B and H remain significant past that horizon, Table VI provides additional evidence that the price-pressure effects captured by B and H are conceptually different from those of IO .

5.3 Robustness

Table VII presents results from a series of robustness tests. The table is organized in two panels: Panel A shows results for returns evaluated one quarter ahead and for a full set of controls (analogous to Panel B of Table III), and Panel B presents results for returns evaluated four quarters ahead and for a full set of controls (analogous to Panel A of Table VI).

In Column (1) we use the levels of B and H instead of the first differences ΔB and ΔH . The results are similar to those in Tables III and VI. The only exception is that the coefficient of B in Panel A (one-quarter horizon) becomes insignificant. This may be due to the high correlation of B with size.

In Column (2) we extend our sample to all stocks, dropping the requirement that IO must exceed 10% during every quarter for which the stock is observed. This adds back into our sample a large number of small stocks. The total number of observations rises from 82743 (Panel B of Table III) to 193332 (Panel A, Column (2)). In Panel A the results are insignificant, possibly because of the noise in the calculations of B and H for low values of IO . In Panel B the results become significant in the case of ΔB .

In Columns (3) and (4) we split the sample into two, with the first half covering the period 1997Q1-2005Q4, and the second half the period 2006Q1-2015Q4. The results for the first sub-period (Column (3)) are insignificant in Panel A, but become significant in Panel B in the case of ΔB . The results for the second sub-period (Column (4)) are significant in both panels. The weak results that we find for the first sub-period and Panel A are consistent with the finding of Nagel (2005) that adding five extra years (1999-2003) to the sample of CHS (1980-1998) renders the coefficient of ΔB insignificant. Our results indicate that 1999-2003 was a special period after which the coefficient of ΔB (and of ΔH) recovered its significance.

Table VII: Robustness tests

(1): Levels of B and H		(2): No minimum IO		(3): 1997Q1-2005Q4		(4): 2006Q1-2015Q4		(5): Control for ΔIO	
B	H	ΔB	ΔH	ΔB	ΔH	ΔB	ΔH	ΔB	ΔH
regression	regression	regression	regression	regression	regression	regression	regression	regression	regression
Panel A: 1Q									
Ownership Variable (OV)	0.012	0.029***	-0.002	0.012*	-0.008	0.011	-0.028**	-0.022***	0.022***
$(B, H, \Delta B, \Delta H)$	(0.96)	(3.78)	(-0.44)	(1.80)	(-0.67)	(1.26)	(-2.43)	(-2.57)	(3.39)
OV \times size	0.032***	-0.035***	0.004	-0.007	0.021	-0.016	0.042***	0.039***	-0.031***
	(3.07)	(-3.46)	(0.50)	(-0.82)	(1.35)	(-1.07)	(2.79)	(3.48)	(-2.97)
size	-0.055***	0.010	0.006	0.011**	-0.022**	-0.004	-0.029***	-0.031***	0.003
	(-4.64)	(1.57)	(1.01)	(1.97)	(-2.42)	(-0.47)	(-2.58)	(-4.16)	(0.44)
IO	-0.022***	-0.019***	-0.008***	-0.008***	-0.022***	-0.022***	-0.022***	-0.011***	-0.011***
	(-7.23)	(-6.38)	(-2.89)	(-2.90)	(-5.41)	(-5.36)	(-5.34)	(-3.86)	(-3.83)
Panel B: 4Q									
Ownership Variable (OV)	-0.103**	0.178***	0.021*	-0.103***	-0.096***	0.044*	-0.145***	-0.150***	0.060***
$(B, H, \Delta B, \Delta H)$	(-1.98)	(5.41)	(1.78)	(-5.23)	(-3.11)	(1.75)	(-4.62)	(-6.71)	(3.65)
OV \times size	0.218***	-0.215***	0.009	0.103***	0.149***	-0.057	0.162***	0.208***	-0.066**
	(4.49)	(-4.77)	(0.46)	(4.08)	(3.52)	(-1.37)	(3.92)	(6.78)	(-2.46)
size	-0.201***	0.069**	0.003	-0.044**	-0.159***	-0.059	-0.100***	-0.166***	-0.032
	(-4.01)	(2.41)	(0.16)	(-2.04)	(-4.62)	(-1.55)	(-2.85)	(-6.61)	(-1.27)
IO	-0.070***	-0.063***	-0.005	-0.006	-0.098***	-0.097***	-0.075***	-0.050***	-0.049***
	(-5.02)	(-4.66)	(-0.46)	(-0.53)	(-4.59)	(-4.56)	(-4.48)	(-3.70)	(-3.64)

Note: Panel A shows results for returns evaluated one quarter ahead and for a full set of controls described in Table III. Panel B presents results for returns evaluated four quarters ahead and for a full set of controls. The robustness exercises are: using levels of B and H rather than first differences ΔB and ΔH (Column (1)); dropping the requirement that a stock's IO must exceed 10% during every quarter for which the stock is observed (Column (2)); split the sample into two, with the first half covering the period 1997Q1-2005Q4 (Column (3)) and the second half the period 2006Q1-2015Q4 (Column (4)); and including as additional controls the lagged and contemporaneous first difference ΔIO of IO (Column (5)). We compute the lagged first difference ΔIO from end of quarter $t-1$ to end of quarter t . We compute the contemporaneous first difference ΔIO in Panel A from end of quarter t to end of quarter $t+1$. In Panel B, we use four first differences ΔIO computed from end of quarter $t+k$ to end of quarter $t+1$ with $k=0, 1, 2, 3$. Variables are transformed into deciles and normalized, as described in Table III. This transformation is also performed on B , H and the lagged and contemporaneous first difference ΔIO . The number of stock/quarter observations in Panel A (Panel B) is 82743 (78607) for Column (1), 193332 (177173) for Column (2), 39333 (38536) for Column (3), 43410 (40071) for Column (4) and 81469 (76110) for Column (5). t -statistics, in parentheses, are computed using robust standard errors clustered by stock.

In Column (5) we include as additional controls the lagged and contemporaneous first difference of IO (ΔIO). Doing so may be useful since we also use first differences of B and H . The results are similar to those in Tables III and VI. This provides additional evidence that the price-pressure effects captured by B and H are conceptually different from those of IO .

6 Ownership at the Style Level

In this section we extend our analysis of ownership dispersion and expected returns to the level of investment styles. Investors can adopt different styles, such as value, growth and momentum, because of different preferences or beliefs. Assuming that different styles are adopted by disjoint sets of identical investors in equal numbers, we can apply our theoretical model to the level of styles rather than investors, and test the empirical hypotheses with style-level measures of ownership. The style-level analysis can be viewed as an additional robustness test. It can also help rule out alternative explanations of our findings that can apply to the level of individual investors but not to that of aggregate styles. Examples are explanations that may be based on monitoring or rent extraction by large shareholders (e.g., [Admati, Pfleiderer, and Zechner \(1994\)](#), [Burkart, Gromb, and Panunzi \(1997\)](#), [Bolton and Von Thadden \(1998\)](#)) or on asymmetric information by corporate insiders (e.g., [Kyle \(1985\)](#)).

Breadth and Herfindahl Index are correlated at the investor and at the style level. The correlation between B at the investor and at the style level is 0.50. The counterpart correlation for H is 0.75. These correlations are driven partly by size, but they remain important even within size groups. The correlations within the groups of small, mid-cap and large stocks are 0.64, 0.34 and 0.23, respectively, in the case of B , and 0.73, 0.71 and 0.62, respectively, in the case of H . Given the positive correlation between our measures of ownership dispersion at the investor and at the style level, we can expect our findings to extend to styles.

Table VIII presents descriptive statistics of B and H at the style level. We denote these measures by B_{style} and H_{style} , respectively. Consistent with Hypothesis 1 and the findings in Table I, there is a positive relationship between B_{style} and size, and a negative relationship between H_{style} and size. The mean of B_{style} for small stocks is 9.13, meaning that the average small stock is held by 9.13 out of the 29 styles in our data. The mean of B_{style} rises to 11.19 for mid-cap stocks and to 12.92 for large stocks. The mean of H_{style} for small stocks is 0.28, and drops to 0.22 for mid-cap stocks and to 0.21 for large stocks. The variation of B and H with size may be less pronounced at the style than at the investor level because the total number of styles is small relative to the total number of investors.

Table IX presents results from pooled OLS regressions of stock returns evaluated k quarters

Table VIII: **Descriptive statistics of Breadth and Herfindahl Index at the style level**

	Panel A: Full Sample				Panel B: Small Stocks			
	B_{style}	ΔB_{style}	H_{style}	ΔH_{style}	B_{style}	ΔB_{style}	H_{style}	ΔH_{style}
Mean	11.08	0.07	0.24	-0.00	9.13	0.09	0.28	-0.00
Standard Deviation	2.64	0.93	0.10	0.03	2.53	0.93	0.13	0.04
Minimum	1.00	-11.00	0.10	-0.60	1.00	-5.00	0.10	-0.60
Median	11.00	0.00	0.21	-0.00	9.00	0.00	0.24	-0.00
Maximum	20.00	7.00	1.00	0.55	17.00	6.00	1.00	0.55
Autocorrelation	0.939	-0.212	0.953	0.021	0.933	-0.140	0.950	0.005
	Panel C: Mid-cap Stocks				Panel D: Big Stocks			
	B_{style}	ΔB_{style}	H_{style}	ΔH_{style}	B_{style}	ΔB_{style}	H_{style}	ΔH_{style}
Mean	11.19	0.08	0.22	-0.00	12.92	0.04	0.21	-0.00
Standard Deviation	1.90	0.97	0.08	0.03	1.64	0.90	0.05	0.02
Minimum	3.00	-5.00	0.10	-0.54	1.00	-11.00	0.12	-0.49
Median	11.00	0.00	0.20	-0.00	13.00	0.00	0.20	-0.00
Maximum	18.00	7.00	0.87	0.55	20.00	5.00	1.00	0.42
Autocorrelation	0.872	-0.235	0.942	0.058	0.852	-0.274	0.943	0.031

Note: Mean, standard deviation, minimum, median, maximum, and first-order autocorrelation of the levels and first differences of style-level Breadth (B_{style}) and style-level Herfindahl Index (H_{style}). Style-level Breadth for stock i and quarter t is the number of different styles that hold the stock in that quarter. Style-level Herfindahl Index for stock i and quarter t is calculated by squaring the fraction of the stock held by any given style, and summing across styles. Panel A reports statistics for the full sample, Panel B for small stocks (size below the 20th NYSE percentile), Panel C for mid-cap stocks (size between the 20th and the 50th NYSE percentiles) and Panel D for large stocks (size above the 50th NYSE percentile). The statistics are based on pooled cross-sectional and time-series samples. The number of stock/quarter observations and the average number of stocks per quarter are as in Table I.

ahead, with $k = 1, 4, 8, 20$, on ΔB_{style} or ΔH_{style} , its interaction with size, and full controls. The results are similar to their counterparts at the investor level (Panel B of Table III for $k = 1$ and Panels A, B and E of Table VI for $k = 4, 8$ and 20 , respectively). For small stocks, ΔB_{style} predicts future returns negatively and ΔH_{style} predicts them positively. These relationships weaken when stock size increases and reverse for large stocks. The effects are not significant for the one-quarter horizon, but become significant from four quarters onward in the case of ΔH_{style} and eight quarters onward in the case of ΔB_{style} .

7 Conclusion

We study theoretically and empirically the relationship between investor beliefs, ownership dispersion and stock returns. We find that high dispersion, measured by high breadth or low Herfindahl index, forecasts returns positively for large stocks, as in CHS, but negatively for small stocks. The

Table IX: Returns on first difference of Breadth or Herfindahl Index at the style level

	Panel A: 1Q		Panel B: 4Q		Panel C: 8Q		Panel D: 20Q	
	ΔB_{style}	ΔH_{style}	ΔB_{style}	ΔH_{style}	ΔB_{style}	ΔH_{style}	ΔB_{style}	ΔH_{style}
Ownership Variable (OV)	-0.008	0.010	-0.027*	0.055***	-0.079***	0.096***	-0.299***	0.231**
(ΔB_{style} or ΔH_{style})	(-1.26)	(1.43)	(-1.74)	(2.92)	(-2.66)	(2.91)	(-4.26)	(2.50)
OV \times size	0.018*	-0.019*	0.033	-0.083***	0.119***	-0.104**	0.422***	-0.328**
	(1.86)	(-1.73)	(1.48)	(-2.90)	(2.84)	(-2.15)	(4.20)	(-2.49)
size	-0.016***	-0.001	-0.069***	-0.017	-0.195***	-0.107*	-0.490***	-0.200
	(-3.07)	(-0.21)	(-3.25)	(-0.71)	(-3.69)	(-1.95)	(-3.89)	(-1.40)
IO	-0.022***	-0.022***	-0.087***	-0.087***	-0.161***	-0.161***	-0.102	-0.102
	(-7.57)	(-7.59)	(-6.33)	(-6.34)	(-5.21)	(-5.21)	(-1.16)	(-1.16)

Note: Pooled OLS regressions of stock returns from end of quarter t to end of quarter $t+k$ with $k = 1, 4, 8, 20$, on the first difference of style-level Breadth (ΔB_{style}) or style-level Herfindahl Index (ΔH_{style}) in quarter t and on the interaction of ΔB_{style} or ΔH_{style} with stock size in the same quarter. Panel A reports results for the one-quarter horizon ($k = 1$), Panel B for one year, Panel C for two years, and Panel D for five years. The regressions include the full set of controls described in Table III. Variables are transformed into deciles and normalized, as described in Table III. The number of stock/quarter observations and the average number of stocks per quarter are as in the corresponding panels of Tables III (Panel B for $k = 1$) and VI (Panels A, B and E for $k = 4, 8$ and 20, respectively). t -statistics, in parentheses, are computed using robust standard errors clustered by stock.

effects are economically significant. Within the smallest size decile, a stock in the smallest breadth decile earns 8% more annually than a stock in the largest breadth decile. Within the largest size decile, the return difference is 5.6% annually in the opposite direction.

We explain the non-linear relationship between ownership dispersion and stock returns in a difference-of-opinion model in which stocks differ in the size of investor disagreements and the extent of belief polarization. These differences are characterized by range and kurtosis, respectively. Proxying investor beliefs by analyst forecasts, we find that the effects of range and kurtosis on ownership dispersion depend on stock size as our model predicts.

Our model connects two influential and seemingly opposite theories linking the distribution of asset ownership to asset returns: the difference-of-opinion theory of CHS and the entry-cost theory of [Merton \(1987\)](#). We show that mechanisms from both theories can be at play simultaneously, and that stock size is a key driver of the mechanisms' relative importance. The imperfect risk-sharing mechanism from the entry-cost theory is at play in our model because stocks can differ in the kurtosis of the distribution of investor beliefs. Our results indicate that a rich description of beliefs, in which kurtosis can vary in addition to range, is important for understanding the empirical relationship between beliefs, ownership and returns. Such a description could be useful for future work on differences of opinion in asset markets.

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Appendix

A Proofs

Proof of Proposition 2.1. The comparisons between LD and SD stocks follow from (2.13) and (2.19). The comparisons between LP and SP stocks follow from (2.14) and (2.19). The comparison between LB and SB stocks follows from (2.15) and (2.16) in the case of B , and from (2.20), (2.21) and $\epsilon < \frac{a\sigma^2\theta_L}{K}$ in the case of H . \square

Proof of Proposition 2.2. The comparisons between SB and SD stocks follow from (2.13), (2.15), (2.18) and (2.20). The comparisons between SB and SP stocks follow from (2.14), (2.15), (2.19) and (2.20). The comparisons between LB and LD stocks follow from (2.13) and (2.16) in the case of B , and from (2.18), (2.21) and $\epsilon < \frac{a\sigma^2\theta_L}{K}$ in the case of H . The comparisons between LB and LP stocks follow from (2.14) and (2.16) in the case of B , and from (2.19) and (2.21) in the case of H . Equation (2.19) implies

$$\begin{aligned} H_{SB} - H_{SP} &> H_{LB} - H_{LP} \\ \Leftrightarrow H_{SB} &> H_{LB}. \end{aligned}$$

Since the latter inequality holds from Proposition 2.1, the difference between H of baseline and polarizing stocks decreases when moving from small to large stocks. \square

Proof of Proposition 2.3. Consider first the case of small stocks. Since $B_{SB} = B_{SD}$ and $H_{SB} = H_{SD}$, the comparison between SB and SD stocks has no implications for how B and H are related to expected returns. On the other hand, since $B_{SB} < B_{SP}$ and $H_{SB} > H_{SP}$, and since expected returns of SB stocks are larger than of SP stocks, B is negatively related and H is positively related to expected returns.

Consider next the case of large stocks. Since $B_{LB} > \max\{B_{LD}, B_{LP}\}$ and expected returns of LB stocks are larger than of LD and LP stocks, B is positively related to expected returns. Since $H_{LB} < H_{LD}$, $H_{LB} < H_{LP}$ if (2.17) holds, and expected returns of LB stocks are larger than of LD and LP stocks, H is negatively related to expected returns if (2.17) holds. \square

B Investment Styles of 13-F Investors by Thomson Reuters

Table B.I presents the 29 investment styles in which Thompson Reuters (TR) classifies 13-F investors.

Table B.I: **The 29 investment styles in which Thomson Reuters classifies 13-F investors.**

General Styles	Hedge Fund Styles
Aggressive Growth, Broker Dealer, Core Growth, Core Value, Deep Value, Emerging Markets, GARP (Growth at Reasonable Price), Growth, Hedge Fund, Income Value, Index, Mixed Style, Momentum, Sector Specific, Specialty, VC (Venture Capital) / Private Equity, Yield	Capital Structure Arbitrage, Convertible Arbitrage, CTA (Commodity Trading Advisors) Managed Futures, Distressed Securities, Emerging Markets (Hedge), Event Driven (Merger / Risk Arbitrage), Fixed Income Arbitrage, Funds of Funds, Global Macro, Long Short, Market Neutral, Multi-Strategy (Hedge)

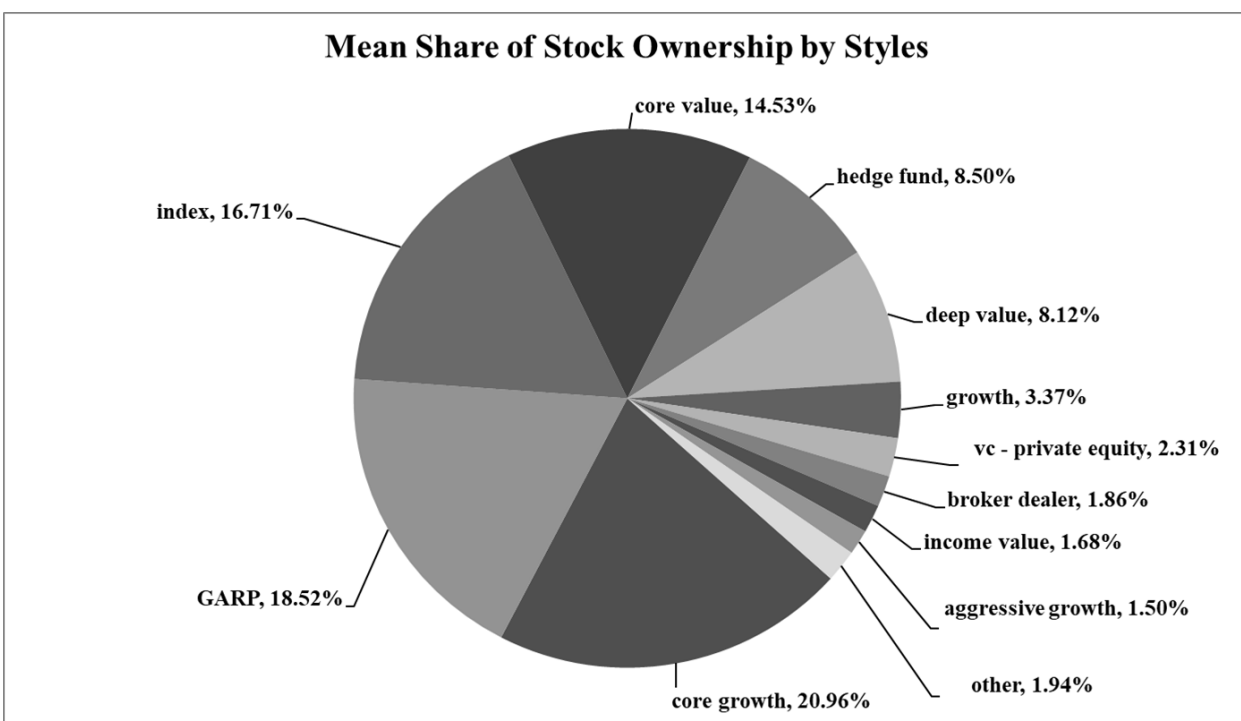
Note: The 29 investment styles in which Thompson Reuters (TR) classifies 13-F investors. The left column reports the seventeen general styles and the right column reports the twelve hedge fund styles. The styles are reported alphabetically on each column. The information is available on http://banker.thomsonib.com/ta/help/webhelp/Ownership_Glossary.htm

TR’s style classification procedure for 13-F investors combines an analysis of the characteristics of the stocks that they hold, their historical investment behavior, their current transactions and their general business type. It first classifies each stock into a certain group or style based on its price-earnings ratio, dividend yield, and the three- to five-year projected earnings-per-share growth relative to the corresponding S&P 500 or sector averages. For each 13-F investor, it then calculates the weights of the different groups or styles of stocks. The group with the biggest weight generally characterizes the investor’s style. The classification also takes into account the historical composition of the investors’ portfolios, the characteristics of the stocks that they buy/sell when they rebalance their portfolios, and their turnover.

Some classifications are more mechanical. 13-F investors whose portfolios follow the composition of certain indices (e.g. S&P 500, Russell 1000/2000/3000, etc) are classified into the Index style. Styles such as “Broker Dealer,” “Hedge Funds” and “VC/Private Equity” are assigned mainly based on the business type of the corresponding investors. Finally, some 13-F investors are classified into hedge-fund styles depending on their exact investment strategy (e.g. “Convertible Arbitrage,” “Quantitative-Statistical Arbitrage,” “Emerging Markets,” “Fund of funds” etc.) The relative importance of hedge-fund styles is small.

The pie chart in Figure B.1 shows the size of each of the 29 styles in our sample, defined as the asset value attributed to the style over the total asset value of all styles. There are eleven styles with size above 1%. The combined size of the remaining eighteen styles is 1.96%.

Figure B.1: Mean share of stock ownership by style



Note: Mean percentage shares in our sample of the 29 investment styles in which Thompson Reuters (TR) classifies 13-F investors. Our sample covers 73 quarters, from 1997Q3 to 2015Q3, and contains 91874 observations of stocks (an average of 1259 stocks per quarter). The average shares above 1% are reported separately (11 styles) and the average shares below 1% are reported all together as “other” (18 styles and undefined style).