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ON THE POSSIBILITY OF PRICE DECREASING BUBBLES

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ABSTRACT

It is often argued that a rational bubble, because it is positive, must increase the price of a stock. This argument is not valid in general: as soon as bubbles affect interest rates, the fundamental value of a stock depends on whether or not a bubble is present. The existence of a rational bubble then might, by raising equilibrium interest rates, depress the fundamental to such an extent that the sum of the positive bubble and decreased fundamental falls short of the fundamental, no-bubble price. Under conditions made precise below, there can therefore be price decreasing bubbles, and an asset can be "undervalued."

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A rational speculative bubble on a stock which can be freely disposed of — a form of limited liability — is non-negative by definition:[1] it represents what an investor might be willing to pay to buy a stock *forever* stripped of its dividends. The fundamental, on its part, represents the present discounted value of the dividend stream. In legal terms, the fundamental is the value of the *usufruct*, while the bubble measures the value of the stock minus its usufruct. For instance, a claim to an apple tree could be split into two assets: a claim to the apple crop (whose value is the fundamental), and a claim to the useless[2] wood itself (the bubble). Because of the free disposal assumption, the price of the latter claim cannot be negative; a rational speculative bubble on an asset which can be costlessly destroyed is, therefore, non-negative.

Much has been made of this fact in some recent discussions.[3] It has been in particular argued that the definition given *supra* of a rational bubble poorly captures the essence of bubbles in actual economies. Two reasons are advanced. Firstly, it is proposed, "a bubble cannot start; to exist, it must be present since the origin of time." This is true, but in the somewhat trivial sense that the equilibrium price today of something which has even an infinitesimal probability of being valued tomorrow cannot be zero; by backward recursion, every bubble which may conceivably arise tomorrow must exist today. Secondly, and more importantly, the above definition of a rational bubble is judged to be unsatisfactory because it cannot account for the existence of "undervalued" assets. "Because bubbles are positive," it is claimed, "the presence of a bubble increases the equilibrium price of an asset.".

The latter argument is of course correct if the fundamental is independent of the presence or absence of a bubble: the sum of a positive bubble and a *fixed* fundamental indeed exceeds the fundamental, and the "bubbly" price — in Tirole's terminology — is, therefore, larger than the non-bubbly price. As soon as either dividends or discount rates depend on the presence or absence of a

[1] In the following, I adopt Tirole's [1982, 1985] definition of a rational bubble.

[2] I assume, for the sake of discussion, that apple wood can neither be burned nor serve any other useful purpose, and is costless to destroy.

[3] See, for instance, Diba and Grossman [1987] and West [1988].

bubble, however, the fundamental is affected by the presence of a bubble. For instance — and this is the scenario explored in this paper,[4] the existence of a bubble may lead to an increase in interest rates which so depresses the fundamental that the sum of the positive bubble and of the bubbly fundamental falls short of the non-bubbly fundamental. Hence, *a positive rational bubble may in fact decrease the overall price of a stock* , contrary to what is commonly believed.

The purpose of this paper is to formalize the foregoing argument, and to determine conditions under which price decreasing bubbles on a stock may indeed exist, in general equilibrium, when the discount rate is not exogenous. In section 1, I construct a simple general equilibrium model in which a stock need not be priced at its fundamental. In section 2, I derive existence conditions for price decreasing rational bubbles. The conclusion summarizes the results. research.

1. The model

To make sense of the question asked in this paper — can price decreasing bubbles exist in general equilibrium ? — it is of course necessary to first construct a model in which asset bubbles are at all possible. This is achieved, most simply, by considering a simple two-period overlapping generation model without intergenerational altruism which provides, through the perpetual arrival of new cohorts, the *new* entrants into the market required, as shown by Tirole [1982], for the existence of a rational speculative bubbles. What is therefore crucial, in this overlapping generation setting, for the existence of bubbles is *not* the finiteness of the agents' horizon but, instead, the fact that successive generations are economically distinct.[5]

[4] The other possibility — namely that managers may pay attention, when setting dividends or issuing new shares, to the existence of a bubble on their company's stock — is not analyzed here. A result akin to Wallace's [1980] theorem on the maximum rate of growth of the money supply is likely to hold in that setting: the bubble on a stock must be driven to zero if new bubbly shares are issued "too fast" by the firm.

[5] See my [1987] paper for an example of an economy in which bubbles are possible, despite the infinite horizon of every agent alive, because of the continuous arrival into the economy of *new* infinitely-lived families.

1.1. Consumers

Consumers live for two periods. Population is constant, with reproduction assumed to take place by parthenogenesis.[6] All consumers are identical, both within a generation and across cohorts. There is only one non-storable good, call it a fruit. A (selfish) representative consumer born at time t wishes to maximize the time separable utility

$$U_t \equiv u(c_{1t}) + v(c_{2t}) \tag{1}$$

he derives from his consumption bundle (c_{1t}, c_{2t}) when young and old. The functions $u(.)$ and $v(.)$ are defined on $\mathbf{R}^+$, are continuous, increasing and concave, and satisfy the following assumptions:

Assumption 1 $\lim_{c\to 0} u'(c) = \lim_{c\to 0} v'(c) = +\infty$,

Assumption 2 $\lim_{c\to +\infty} u'(c) = \lim_{c\to +\infty} v'(c) = 0$.

Each agent receives an endowment (e_1, e_2), $e_1 > 0$, $e_2 \geq 0$ of the consumption good. It is in addition assumed that the following inequality holds:

Assumption 3 $u'(e_1)/v'(e_2) < 1$,

which implies the dynamic inefficiency, in this zero population growth economy, of any competitive equilibrium yielding (e_1, e_2) as a consumption allocation. This crucial assumption will be used *infra*.

To smooth out, if desired, his consumption profile, our consumer can buy x_t fruit producing trees, at a price of p_t fruits each. His budget constraints are thus:[7]

$$\begin{aligned} c_{1t} + p_t x_t &= e_1, && (2)\\ c_{2t} &= e_2 + (p_{t+1} + y_{t+1})x_t, && (3)\\ c_{1t}, c_{2t} &\geq 0, && (4) \end{aligned}$$

where $y_{t+1} > 0$ denotes the fruit output of a tree at time $t+1$. The first-order condition for an interior[8] utility maximum is simply

$$p_t\, u'(c_{1t}) = (p_{t+1} + y_{t+1})\, v'(c_{2t}), \tag{5}$$

[6] See Bernheim and Bagwell [1988] for the complications introduced by marriage.

[7] I do not explicitly introduce a consumption-loans market, as it will be inactive in equilibrium.

[8] Corner solutions are not characterized, as equilibrium considerations impose interiority.

Equation (5) can be solved, using (2) and (3), for the optimum tree holding:

$$p_t x_t = S\left(\frac{p_{t+1} + y_{t+1}}{p_t}\right), \tag{6}$$

where S(.) denotes the savings function.

1.2. Technology

Trees have the following features. There is a constant number of identical trees, normalized without loss of generality to be equal to the size of the population; the number of trees per capita is therefore equal to 1. It is known from the origin of time that

$$y_t = \begin{cases} y > 0 & \text{for } 0 < t \leq T-1; \\ 0 & \text{for } t \geq T. \end{cases} \tag{7}$$

where T is a possibly very large but finite integer. In other terms, a tree produces fruits for only a finite number of periods. In the long run ($t \geq T$), trees become sterile. It is further assumed, in the spirit of the discussion above, that the wood of the tree is intrinsically useless and costless to destroy; the only reason a consumer might therefore choose to buy a tree in or after period $T-1$ is for speculative purposes: "buying in order to resell." Prior to period $T-1$, however, trees have a positive fundamental — the present discounted value of future dividends.

The rationale for specifying the dividend process in that fashion is best understood by remembering Tirole's [1985] results: if i) dividends are capitalized *ex ante* (i.e., all the trees ever present in the economy are traded today), and ii) dividends per head do not become zero in the long run, there cannot be a bubble on a asset with a fundamental. Our setup violates the second condition — the dividend per head becomes zero after time T — which leaves open the possibility that a bubble may exist on trees.[9]

[9]See Tirole's [1985] Proposition 1.

1.3. *Competitive equilibria*

In a competitive equilibrium, a representative consumer must hold — because of the choice of units — precisely one tree:

$$x_t = 1 \qquad \forall t \geq 0. \tag{8}$$

Therefore, substituting (8) into (5), one finds that the equilibrium fruit price of trees must satisfy the following first-order difference equation:

$$p_t\, u'(e_1 - p_t) = (p_{t+1} + y_{t+1})\, v'(e_2 + p_{t+1} + y_{t+1}), \tag{9}$$

which is forced by the dividend process given in (7). Equivalently, the equilibrium fruit price of a tree must follow

$$p_t = S\left(\frac{p_{t+1} + y_{t+1}}{p_t}\right). \tag{10}$$

Given the structure of the dividend process, it is simplest to divide the solution procedure into two parts: $t \geq T-1$ and $t \leq T-2$.

1.3.1. Long run $(t \geq T-1)$

Trees become intrinsically useless as of period T. Their fundamental value is thus zero as of period $T-1$ whether or not a bubble is present. Determining the equilibrium price of trees thus amounts to studying whether or not a bubble on an intrinsically useless asset may exist in this economy starting in period $T-1$. Now we know, from Gale [1973], from Wallace's [1980] results on fiat money equilibria and Tirole's [1985] related work on bubbles, that, in many cases, the dynamic inefficiency of the *bubbleless* economy is necessary and sufficient for the existence of bubbly equilibria. As the following shows, this holds true in the present model.

If trees are not valued in and after period $T-1$ ($p_t = 0$ for $t \geq T-1$), the competitive equilibrium consumption allocation is, trivially, the autarkic one (e_1, e_2) — which, by construction [see (3)], is dynamically inefficient. This equilibrium is one in which there is no bubble: the price of a tree is equal to its zero fundamental.

But then, because Assumption 3 holds, it is easy to check that there also exists a stationary *golden rule* bubbly equilibrium characterized by

$$p_t = \hat{p} \qquad \forall t \geq T-1, \tag{11}$$

where $\hat{p}$ is given, from (10) and the fact that the dividend is zero after $T-1$, by

$$\hat{p} = S(1) > 0, \tag{12}$$

with the inequality implied by the dynamic inefficiency of the autarkic equilibrium imposed by assumption 3.[10] *Because the long run fundamental is constructed to be zero independently of equilibrium interest rates,* $\hat{p}$ represents a pure bubble. Bubbles are thus trivially price increasing *in the long run* in this economy — in accordance with the intuition given in the introduction.

Before proceeding further, it is necessary to note that there exists many non-stationary equilibria in addition to the stationary equilibria described above. Following standard[11] arguments, it is easy to show that if the interest elasticity of savings is not too negative, these non-stationary paths converge to the inefficient non-bubbly steady state. If income effects are strong ($S'(.) \ll 0$), however, cyclical or chaotic equilibria may occur. For the sake of simplicity, I henceforth concentrate on equilibria which are stationary as of period $T-1$ —i.e., equilibria which are such that either $p_t = 0$ or $p_t = \hat{p}$ for all $t \geq T-1$.

1.3.2. Short-run ($t < T-1$)

In the short run, trees have a positive fundamental. Let

$$R_{t+1} \equiv \frac{u'(e_1 - p_t)}{v'(e_2 + p_{t+1} + y)} \tag{13}$$

denote the (implicit) interest rate on consumption loans between periods t and $t+1$. The fundamental value f_τ of a tree at some date $\tau < T-1$ is simply the present discounted value of dividends

[10] Equivalently, $\hat{p}$ is the stationary solution to (9).
[11] See, for instance, Grandmont [1985], Tirole [1985], and Wallace [1980].

distributed by trees until $T-1$, i.e.,

$$f_\tau \equiv \sum_{i=\tau+1}^{T-1} \frac{y}{\prod_{s=\tau+1}^{i} R_s}. \tag{14}$$

Note that the discount factors depend on the asset price itself, so that the fundamental will differ in general differ across bubbly and non-bubbly equilibria.

The equilibrium behavior of tree prices in the short run is characterized, from (7) and (10), by *backward*[12] iterations of the difference equation

$$p_t\ u'(e_1 - p_t) = (p_{t+1} + y)\ v'(e_2 + p_{t+1} + y), \tag{15}$$

subject to either of the following "terminal" conditions:

$$p_{T-1} = 0 \qquad \text{or} \qquad p_{T-1} = \hat{p}. \tag{16}$$

Because the function $p\ u'(e_1 - p)$ is increasing and maps, using (1) and (2), the interval $[0, e_1)$ onto $[0, +\infty)$, it has an inverse, so that the backward perfect foresight dynamics are uniquely defined, and can be represented, from (15), as

$$p_t = \varphi(p_{t+1}), \tag{17}$$

where φ, continuous and differentiable, maps $[0, \infty)$ onto $[0, e_1)$.

Using (17) and the terminal conditions (16), two competitive (long-run stationary) equilibrium paths emerge:

- a non-bubbly equilibrium path $\bar{\mathbf{p}} = (\bar{p}_0, \bar{p}_1, \ldots, \bar{p}_{T-2}, 0, 0, \ldots)$;
- a bubbly equilibrium path $\hat{\mathbf{p}} = (\hat{p}_0, \hat{p}_1, \ldots, \hat{p}_{T-2}, \hat{p}, \hat{p}, \ldots)$,

where $^{-}$'s and $\hat{}$'s henceforth denote, respectively, the non-bubbly and bubbly price sequences. Notice that, for each type of equilibrium, the associated fundamental (which need not be explicitly computed) is unique.[13] It of course differs *across* equilibria.

Although in the long run a bubble is necessarily price increasing since $\hat{p} > 0$, it is possible, as the next section will demonstrate, that it be price decreasing in the short run.

[12]Backward perfect foresight dynamics *are* the appropriate solution concept in this model, and not just an analytical device as in Grandmont [1985].

[13]This is true in this model because the asset stops paying dividends in finite time. In more general frameworks, there might be, for instance, several non-bubbly fundamentals (e.g. Obstfeld and Rogoff [1983].)

2. Price decreasing bubbles

I now turn to the comparison of the non-bubbly and bubbly price sequences, $\bar{\mathbf{p}}$ and $\hat{\mathbf{p}}$, and ask whether it is possible to have, for some $\tau < T-1$, $\hat{p}_\tau < \bar{p}_\tau$. From basic economic reasoning, the answer to this question must crucially depend on the preferences of consumers, and more precisely on on the sign and magnitude of their interest elasticity of savings.

To understand why this is so, it is necessary to consider in detail the channels through which the presence of a bubble might lead to a decrease of the overall price of the asset. As mentioned *supra*, the main requirement is that the presence of a bubble decrease the asset's fundamental. In this world in which the dividends distributed by the tree are exogenous, such a decrease can only occur if the interest rates at which future dividends are discounted are higher in the bubbly path then in the non-bubbly path. The existence of a price decreasing bubble therefore must require that *higher* equilibrium interest rates be associated with a *lower* overall real value of the stock of trees — which cannot occur if the interest elasticity of savings is positive. The critical role of income effects is formalized in the following proposition:

Proposition 1 *If the interest elasticity of savings, $S'(.)$, is positive, bubbles are price increasing in the short run as well as in the long run.*

Proof. The long run part of the proposition was proved in 1.3.1. The short run part is proved by backward induction. We know that, as of $T-1$, the bubble is price increasing, since $\hat{p} > 0$. Can it be non-price increasing at $T-2$? Suppose it is, so that $\hat{p}_{T-2} \leq \bar{p}_{T-2}$. Then $(\hat{p}+y)/\hat{p}_{T-2} > y/\bar{p}_{T-2}$. But if $S'(.) > 0$, this implies in equilibrium that $\hat{p}_{T-2} = S[(\hat{p}+y)/\hat{p}_{T-2}] > S[y/\bar{p}_{T-2}] = \bar{p}_{T-2}$ — a contradiction! Hence $\hat{p}_{T-2} > \bar{p}_{T-2}$: the bubble is price increasing at $T-2$. By a similar argument, one shows that if the bubble is price increasing at $\tau \leq T-2$, it is price increasing at $\tau-1$ if $S'(.) > 0$. Therefore the bubble is price increasing at every date, in the short run and in the long run.

The intuition underlying the proof of this proposition is straightforward. For interest rates to be higher in the bubbly equilibrium than in the non-bubbly one (a necessary condition for a bubble to be price decreasing), it must be the case, if the interest elasticity of savings is positive, that trees crowd out consumption loans in the bubbly equilibrium relative to the non-bubbly case.[14] But if the bubble is price decreasing, the real value of the stock of trees is lower in the bubbly than in the non-bubbly equilibrium — i.e., trees crowd consumption loans *in, not out*! Whence the contradiction on which the proof is based, and which can only be resolved by allowing for a negative interest elasticity of savings.[15]

What happens when the interest elasticity of savings is negative critically depends, as in endogenous business cycle theory,[16] on the particular specification of the utility function and of the endowment vector. The striking pattern of a bubble which is price increasing in, say, even periods and price decreasing in odd periods emerges in the following case:

Proposition 2 *Suppose the interest elasticity of savings in negative, and that the second period endowment is zero. Then, in the short run, the bubble on the tree is price decreasing in periods $T-2k$, and price increasing in periods $T-2k+1$, where k is a positive integer.*

Proof. Since $e_2 = 0$ (a specification which satisfies, because of assumption 1, condition 3), it is straightforward to show from (5) that the interest elasticity of savings is negative if the function $g(x) \equiv xv'(x)$ is decreasing over the interval $[0, e_1)$. But then, from (15) and (17), the map φ is decreasing. Therefore, $\hat{p}_{T-2} = \varphi(\hat{p}) < \varphi(0) = \bar{p}_{T-2}$: the bubble is price decreasing at $T-2$. This implies that $\hat{p}_{T-3} = \varphi(\hat{p}_{T-2}) > \varphi(\bar{p}_{T-2}) = \bar{p}_{T-3}$: the bubble is price increasing at $T-3$. Similarly, it is price decreasing at $T-4$, price increasing at $T-5$, etc., until the origin of time is reached.

Proposition 2 applies, for instance, to the case of isoelastic utility: $u(c) = v(c) = c^{1-\rho}/(1-\rho)$, with $\rho > 1$ to yield a negative interest elasticity of savings. Extending this Proposition to allow for a non-zero second period endowment or an interest elasticity of savings non-uniformly negative presents little interest, except that

[14] This crowding out is of course only incipient, since the interest rate adjusts to make the consumption loans market inactive in equilibrium.

[15] The borderline case in which intertemporal substitution and income effects cancel each other out to yield $S'(.) = 0$ is not considered for the sake of brevity. It can be agglomerated into the positive interest elasticity of savings case with a slight change in the proofs.

[16] See Grandmont [1985].

of a theoretical *curiosum*, since Proposition 2 decisively establishes that there is indeed no reason to presume, as is often done, that the presence of a bubble always raises the price of an asset. On the contrary, complex patterns — in general more complex than the one exhibited here and related to those described by Grandmont [1985] — emerge as soon as even a small (in absolute value) but negative interest elasticity of savings is allowed. As for the argument that economies with negative interest elasticities of savings are pathological so that their behavior can be disregarded, it is best countered by the remark that the empirical evidence on the sensitivity of life-cycle savings to changes in interest rates is mixed and can not rule out an $S'(.)$ negative but close to zero.

Conclusion

This paper has established that, contrary to what is often claimed, rational speculative bubbles, which are positive by definition, need not increase the price of the asset to which they are attached — because the fundamental is not in general independent, in equilibrium, of the bubble. It is therefore possible to observe situations in which bubbles are price decreasing: the price of an asset subject to a bubble might be lower than the price of that asset in the absence of a bubble.

Whether that situation should be construed as one in which the asset is "undervalued" is mainly a question of semantics. If by undervaluation it is meant that the price falls short of the fundamental in the particular equilibrium under consideration, an asset is indeed, by this narrow definition, never undervalued: the price of an asset which can be freely disposed of always is never lower than the value of its usufruct. If undervaluation is taken in the broader acceptation of "existence of a bubbly equilibrium with lower asset prices than the non-bubbly equilibrium", then an asset can indeed be undervalued.

Psychological factors, "animal spirits" and extraneous waves of pessimism can, in any case, depress as well as raise an asset price relative to its "fundamental" non-bubbly value.

References

Bernheim D. and K. Bagwell (1988): "Is Everything Neutral?" ? *Journal of Political Economy*, 96, 308–338.

Diba B. and H. Grossman (1987): " On the Inception of Rational Bubbles," *Quarterly Journal of Economics*, 697–700.

Gale D. (1973): "Pure Exchange Equilibrium of Dynamic Economic Models," *Journal of Economic Theory*, 6, 12–36.

Grandmont J.-M. (1985): "On Endogenous Competitive Business Cycles," *Econometrica*, 53, 995–1045.

Obstfeld M. and K. Rogoff (1983): "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" *Journal of Political Economy*, 91, 675–687.

Tirole J. (1982): "On the Possibility of Speculation Under Rational Expectations," *Econometrica*, 50, 1163–1181.

Tirole J. (1985): "Asset Bubbles and Overlapping Generations," *Econometrica*, 53, 1071–1100.

Wallace N. (1980): "The Overlapping Generation Model of Fiat Money," in *Models of Monetary Economies*, ed. by J. Kareken and N. Wallace, Federal Reserve Bank of Minneapolis, Minneapolis.

Weil P. (1987): "Overlapping Families of Infinitely Lived Agents," mimeo, Harvard University.

West K. (1988): "Bubbles, Fads, and Stock price Volatility Tests: A Partial Evaluation," *National Bureau of Economic Research* Working Paper No. 2574.