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IF LABOR IS INELASTIC, ARE TAXES STILL DISTORTING?

Don Fullerton

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ABSTRACT

Three recent papers measure the marginal excess burden of labor taxes in the United States. They obtain very different results even where they all use a zero uncompensated labor supply elasticity and assume that the additional revenue is spent on a public good that is separable in utility. The impression is that other parameters must explain the differences in results.

Yet each paper uses a different concept of excess burden. Here, I calculate all three measures in one model and show how conceptual differences explain the results. Only one of these measures isolates the distortionary effects of taxes in a way that depends on the compensated labor supply elasticity. The other two measures incorporate income effects and thus depend on the actual change in labor. This result was obscured because those papers report positive marginal excess burden even with a zero uncompensated labor supply elasticity. This paper shows conditions under which their measure is zero, and it interprets the measures in light of recent literature.

Don Fullerton  
National Bureau of Economic Research  
1050 Massachusetts Avenue  
Cambridge, MA 02138

Much of welfare economics has been plagued by differences in the concepts used to measure changes in consumer welfare. One example of this problem is presented in striking clarity by the case of a wage tax when the uncompensated labor supply is vertical, that is, when labor is inelastic. Three recent papers in the American Economic Review consider this case, but they reach very different conclusions.

All three papers calculate the "marginal excess burden" of a dollar of additional tax revenue in the case where labor is inelastic and where the marginal dollar is spent on a public good that is separable in utility. Their results are summarized in Table 1. Charles Stuart (1984) builds a simple two-sector general equilibrium model and finds in this case that "marginal excess burden" is 7 cents. Charles Ballard, John Shoven, and John Whalley (1985) use a more complicated general equilibrium model where their "marginal excess burden" is 12 cents. Edgar Browning (1987) employs a simple partial equilibrium model and obtains a corresponding figure of 21 cents. He concludes that "almost all of the differences in results can be traced to different assumptions about key parameter values" (p.11).

Certainly the results are sensitive to assumptions about the marginal tax rate, the degree of progressivity, the use of the revenue, and the compensated and uncompensated labor supply elasticities. As I show in this paper, however, almost all of the differences in these cited results can be traced to differences in their definitions of marginal excess burden.

These three papers have been discussed in other papers that introduce new models and further definitions.<sup>1</sup> Some confusion remains, so one purpose of this paper is to clarify the different definitions and reconcile the

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<sup>1</sup>See, e.g., Ballard (1987), Ingemar Hansson and Stuart (1988), Robert Triest (1988), Shaghil Ahmed and Dean Croushore (1988), and Joram Mayshar (1988a).

different results. The main contribution of this paper, however, is that I return to the exact model of Stuart, replicate his result, and add calculations for the other two definitions of marginal excess burden (MEB).<sup>2</sup> I also show how Stuart's general equilibrium model reduces to Browning's partial equilibrium model. Thus I am able to show all three measures in Browning's model as well. The first section relates these three papers to some others in the literature. The second section illustrates the different concepts of welfare, and the third section performs the actual calculations.

### I. Discussion

The standard public finance textbook defines excess burden of a wage tax as the difference between consumer welfare under that tax and under a lump sum tax with the same revenue (e.g. Harvey Rosen, 1985, Joseph Stiglitz, 1986). Synonyms include welfare cost, efficiency cost, or dead-weight loss. In the literature, much discussion involves the choice of consumer welfare measure.<sup>3</sup> For a change from the original equilibrium with a pre-existing wage tax, Stuart uses the compensating surplus (CS), Ballard, Shoven, and Whalley use the equivalent variation (EV), and Browning uses the compensating variation (CV). Together with the equivalent surplus (ES), these constitute "The Four Consumer's Surpluses" of Sir John Hicks (1943), as renamed in Hicks (1954).

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<sup>2</sup>Stuart (1984) and Ballard (1987) perform two kinds of policy experiments. Using Ballard's terms, revenue is spent by government in a "balanced-budget" experiment, and it is returned as a lump sum transfer in a "differential" experiment. Here, I consider only one experiment and calculate different measures. Balanced budget spending is the only experiment that appears in all three papers, but that one equilibrium outcome is described very differently by the three burden measures.

<sup>3</sup>See, e.g., Peter Diamond and Daniel McFadden (1974), John Kay (1980), Alan Auerbach and Rosen (1980), and Mayshar (1988b).

The distinction among welfare measures is important for total excess burden, but not for small changes where all such measures are very close to each other.<sup>4</sup> The big difference among the three papers that I review is introduced by the revenue figure subtracted. Browning subtracts the change in revenue along the compensated labor supply curve ( $dR^*$ ), and divides this difference by the actual change in revenue ( $dR$ ). His MEB is  $(CV-dR^*)/dR$ . Stuart subtracts the actual change in revenue and calculates  $(CS-dR)/dR$ . Ballard, Shoven, and Whalley calculate  $(EV-dR)/dR$ .<sup>5</sup> Because the welfare measures CS and EV are numerically close for small changes, the latter two MEB measures are always similar in my results below.

Given the textbook definition of total excess burden, a corresponding concept at the margin would compare the increment in the wage tax to a small lump sum tax with the same revenue (Auerbach, 1985). Only Browning's concept of "marginal excess burden" corresponds to this definition. His measure depends on the marginal tax rate and on the compensated labor supply elasticity. It is positive in the special case where labor is "inelastic," so he would answer that the tax is still distorting. Unfortunately, though, Browning leaves the incorrect impression that the cost-benefit analyst should use one plus his marginal excess burden ( $1+MEB$ ) as the marginal cost of funds (MCF). This MCF can be defined as the change in consumer welfare (CS, EV, or CV) divided by the actual change in revenue ( $dR$ ). In the case where labor does not change, the MCF equals one.<sup>6</sup>

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<sup>4</sup>Mayshar (1988b) provides a formal proof of their equivalence at the margin.

<sup>5</sup>They never state this expression explicitly, however, so others misunderstand what was calculated. Triest (1988) assumes that this MEB is measured in pre-tax prices, when it is actually in prices of the original cum-tax equilibrium. Mayshar (1988b) introduces  $EV-dR$  as a "new" measure, not knowing that this is the measure of Ballard, Shoven, and Whalley.

<sup>6</sup>This result is in Anthony Atkinson and Nicholas Stern (1974), Stuart (1982).

The best intuition for this result derives from Atkinson and Stern (1974). They isolate three reasons to deviate from the conventional rule of Paul Samuelson (1954) that the sum of the marginal rates of substitution ( $\sum MRS$ ) should equal the marginal rate of transformation (MRT). The first modification is that revenue may be affected by any complementarity or substitutability between the public project and the taxed good. All three papers ignore this possibility. Second, the rule is modified if taxes are not lump sum. This "distortionary effect" of Atkinson and Stern is measured by Browning's paper. Third, the rule is modified for income effects that change the amount of the taxed good. This "revenue effect" of Atkinson and Stern exactly offsets the distortionary effect in the special case where actual labor does not change. The project is worthwhile if  $\sum MRS > MRT$ , which means that the  $MCF=1$ .

The measures of Stuart and of Ballard, Shoven, and Whalley effectively combine the distortionary effect and the revenue effect. As can be seen from the above formulas, they define "marginal excess burden" as the marginal cost of funds minus one.<sup>7</sup> With this definition, the cost-benefit analyst can use  $(1+MEB)$  as the MCF. In the special case where labor does not change and the  $MCF=1$ , however, this MEB is zero. Because of the terminology of excess burden, they leave the incorrect impression that the tax is not "distorting." The wage tax is distorting in the conventional sense that it leaves consumers worse off than a lump sum tax.

In addition, with a pre-existing wage tax and MEB defined as  $MCF-1$ ,

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David Wildasin (1984), Ballard (1987), Triest (1988), Mayshar (1988a,b), and others. Also, Wildasin shows that Browning could use  $1+MEB$  for the MCF if he had assumed that the public good has no effect on compensated labor supply. Instead, Browning assumes no effect on actual labor supply.

<sup>7</sup>Papers that use similar definitions include Stuart (1982), Ballard (1987), Ahmed and Croushore (1988), and Mayshar (1988b), among others.

a lump sum tax increment has a negative "marginal excess burden." The lump sum increment has no distortionary effect, but it has a revenue effect because it reduces disposable income and thus causes an increase in labor subject to the wage tax.

For these reasons, some say that MCF-1 should not be called "marginal excess burden" but should instead be called "marginal welfare cost," (e.g. Ballard, 1987) or "marginal efficiency cost" (e.g. David Bizer and Stuart, 1987). These terms are generally regarded as synonyms, however.

I conclude that MCF-1 should not be used to define any concept like marginal excess burden, welfare cost, efficiency cost, or deadweight loss. All such definitions leave the incorrect impression that the wage tax is not distorting. The marginal cost of funds is the relevant concept in any case, so the cost-benefit analyst must simply add back the one subtracted.

Finally, the fundamental difference among the three definitions of "marginal excess burden" is obscured by the fact that Stuart (1984) reports 7 cents and Ballard, Shoven, and Whalley report 12 cents, even though the uncompensated labor supply elasticity is zero. Ballard (1987) shows that this measure of "MEB" is zero in a simple model when the uncompensated labor supply elasticity is zero, and he concludes that complications in the models (discussed below) lead to non-zero results. As shown in Stuart (1982) and Mayshar (1988b), however, the condition for MCF-1 (or that this "MEB"-0) is not that the uncompensated elasticity be zero but that actual labor supply be unchanged. In more general models, labor supply can be affected by income and by prices other than the net wage. In simulations below, using the model of Stuart, I show that a non-zero uncompensated labor supply elasticity can exactly offset these other income and price changes such that the actual labor supply is unchanged and this "MEB" is zero.

## II. Concepts of Excess Burden

Figure 1 depicts the choice between leisure (on the horizontal axis) and labor which earns units of the numeraire good denoted in dollars (on the vertical axis). A proportional tax on labor pivots the budget around its horizontal intercept, to a flatter line marked "new prices." In the special case of inelastic labor, the new choice point B must lie directly below the initial point A. Tax revenue is this vertical distance, AB, in dollars. Government is assumed to spend the revenue in a manner that does not affect the labor-leisure choice, so the public good is separable in utility.

The equivalent variation, distance AC in the diagram, is the number of dollars taken away from the consumer at old prices that would reduce utility by an amount "equivalent" to the tax. Since AC exceeds revenue AB, the excess burden is BC (Kay, 1980). The compensating variation, distance DF, is the number of dollars at new prices that would "compensate" for the tax and raise utility back to the old level. Excess burden in this case is DF minus EF, the revenue that would occur at new prices if the consumer were compensated (Diamond and McFadden, 1974). Using the two variation measures, excess burden is related to a compensated change in labor supply.

The less commonly used compensating surplus is the amount of the numeraire good that must be added to the consumer's new bundle to reach the old utility, the distance from B back to A. The consumer may not re-optimize. Stuart (1982) notes that both the MRT and  $\sum$ MRS are measured in terms of a particular commodity and that the compensating surplus can measure welfare in the same commodity.<sup>8</sup> The compensating surplus is exactly

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<sup>8</sup>It is perfectly consistent, however, to measure costs and benefits of the public project by income equivalents and use the EV. Besides, Slutsky

equal to revenue in this case, so Stuart's measure of excess burden is zero. Using this measure with inelastic labor, the tax is entirely nondistorting. His measure yields zero "marginal excess burden" in this model. The reason he obtains 7 cents is explained below, but in general his marginal excess burden is related to the actual change in labor supply.

In contrast, Browning follows the standard approach. The wage tax is still distorting even though labor supply does not change, because a lump sum tax with the same revenue would leave the consumer on a higher indifference curve. Browning's actual calculations can be described using Figure 2, where the gross wage is fixed at  $w$ . The marginal tax rate  $m$  may exceed the average tax rate  $t$ . The initial net wage is  $w(1-m^0)$ , and labor supply is  $L_1^0$ . Initial excess burden is the area ABC, the area between the tax wedge AC and the compensated labor supply curve  $S^*$ . When the marginal tax rate is increased by  $dm$  ( $=m'-m^0$ ), this measure of excess burden increases by the shaded area, ACDE. Actual labor supply is still  $L_1^0$ , but compensated labor is  $L_1^*$ . The change in excess burden is:

$$ACDE = AGHE - (EDFH-ACFG) = CV - dR^* \quad , \quad (1)$$

where AGHE is the compensating variation, EDFH is the revenue at the new rate  $m'$  when the consumer is compensated, and ACFG is the revenue at the old rate  $m^0$ . In other words, the change in excess burden is the CV minus  $dR^*$ , the change in revenue measured at marginal rates along the compensated curve. This formula corresponds to Auerbach (1985, p.72).

Alternatively, with no initial tax, excess burden can be defined as the

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(undated) shows that results depend on the choice of reference commodity.

equivalent variation minus actual revenue. This measure also compares the wage tax to a lump sum tax. With a pre-existing wage tax, however, the corresponding concept at the margin would compare a small additional wage tax to an equal-yield lump sum increment. Auerbach (1985, p.72) shows that such a measure also requires a calculation of the change in revenue along a compensated curve. This point could be demonstrated using a figure that looks just like Figure 2, except that consumers are held to the new utility level along  $S^*$  when  $m^0$  is raised to  $m'$ . Then AGHE is the equivalent variation, and  $dR^*$  is the change in revenue using marginal rates while holding to the new utility. The change in excess burden is ACDE. Thus the CV and EV measures converge as the tax change gets very small.

For total excess burden, Ballard, Shoven and Whalley use the equivalent variation, and they subtract actual revenue. For "marginal excess burden," however, they subtract the actual change in revenue rather than the change along a compensated curve. Thus their "marginal excess burden" is not the increase in their total excess burden for a small increase in the tax. In fact, for the simple model of Figure 1, they would get large total excess burden and (approximately) zero "marginal excess burden."

To see this point, consider a small wage tax with no other distortions in Figure 1. When the equivalent variation AC and the revenue AB are both arbitrarily small, the excess burden BC is approximately zero. Only as the tax becomes large does the excess burden become noticeable. Now reinterpret Figure 1 so that "old prices" represent a large pre-existing wage tax, and "new prices" indicate a small additional tax. Then the equivalent variation AC and the actual change in revenue AB are still arbitrarily small. The difference, BC, is the "marginal excess burden" of Ballard, Shoven, and Whalley. It is near zero for the same reason that the excess burden of a

small tax is near zero, as if they ignore the pre-existing tax.

Thus Ballard, Shoven, and Whalley use a measure that yields zero "marginal excess burden" in this simple model with inelastic labor. The reason they obtain 12 cents is discussed below.

### III. The Calculation of Marginal Excess Burden

For the change in excess burden, Browning assumes that  $S^*$  is linear and calculates the shaded area of Figure 2. This assumption is unimportant, however, because  $S^*$  only needs to be approximately linear over a small range. Thus the area  $(CV-dR^*)$  is  $0.5(wm^0+wm')(L_1^0-L_1^*)$ . The compensated change in labor is  $[\eta L_1^0/(1-m^0)]dm$ , where  $\eta$  is the compensated labor supply elasticity, and  $dm$  is the change in marginal rate. Actual labor supply  $L_1^0$  does not change. If  $dt$  is the change in average tax rate, then  $dR=wL_1^0dt$  is the actual change in revenue. Thus Browning calculates:<sup>9</sup>

$$\frac{CV - dR^*}{dR} = \left[ \frac{m^0 + 0.5dm}{1 - m^0} \right] \cdot \eta \cdot \frac{dm}{dt} \quad (2)$$

The parameter  $dm/dt$  indicates the progressivity of the tax change. Assuming that the change maintains progressivity,  $dm/dt$  is the same as  $r-m/t$  in Stuart's paper. Browning sets the initial  $m=.43$  and  $t=.31$ , so  $dm/dt$  is 1.39. He also sets  $dm=.01$  and  $\eta=0.2$ , so equation (2) yields 21 cents for "marginal excess burden" as shown in Table 1. The measure in

<sup>9</sup>Browning prefers his other case, where government spending restores the original utility level and thus has an income effect on labor supply. This other case is not the same as Stuart's other case where the revenue is returned lump sum (since the latter is not enough to reach the old utility). I use the case where public spending has no effect on labor, because it is the only case considered by all three papers.

(2) clearly increases with the marginal tax rate  $m$ , the progressivity  $dm/dt$ , and the compensated elasticity  $\eta$ .

To substitute Stuart's parameters, set  $m=.427$ ,  $t=.273$ , and  $r=1.564$ . The "MEB" from equation (2) is then 24 cents as shown in Table 1. Thus the difference in published results is not due to parameters.<sup>10</sup>

To calculate his "marginal excess burden" in a general equilibrium model, Stuart first assumes that a single aggregate consumer has an endowment  $L$  that can be supplied to the market sector as labor  $L_1$  or used in home production as leisure  $L_2$ :

$$L = L_1 + L_2 \quad (3)$$

Also, production in each sector is Cobb-Douglas:

$$Y_1 = AL_1^a \quad \text{and} \quad (4)$$

$$Y_2 = BL_2^b \quad , \quad (5)$$

where capital stocks in each sector are constant and hence subsumed in  $A$  and  $B$ . The consumer uses total income  $I$  to maximize

$$U(\bar{Y}_1, \bar{Y}_2) = \left[ \alpha \bar{Y}_1^{-\rho} + (1-\alpha)(\bar{Y}_2 - \delta)^{-\rho} \right]^{-1/\rho} \quad (6)$$

where  $\bar{Y}_1$  and  $\bar{Y}_2$  are consumption of the two outputs. The  $\delta$  is a "minimum required purchase,"  $\alpha$  is a share parameter, and the elasticity of substitution is  $\sigma = 1/(1-\rho)$ . The nonmarket sector has no tax, so  $\bar{Y}_2 = Y_2$ .

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<sup>10</sup>In the model of Ballard, Shoven, and Whalley, each of 12 households has its own linear tax function with its own  $m$ ,  $t$ , and  $\eta$ .

The market sector pays tax  $twL_1$ , so  $\tilde{Y}_1$  is less than  $Y_1$ . The government spends marginal revenue on the market good.

The Appendix further describes the derivation of parameters and solution of the model. Using equilibrium outcomes, the compensating surplus is derived from equation (6) and  $U(\tilde{Y}_1^0, \tilde{Y}_2^0) = U(\tilde{Y}_1^1 + CS, \tilde{Y}_2^1)$ :

$$CS = \left[ (\tilde{Y}_1^0)^{-\rho} + [(\tilde{Y}_2^0 - \delta)^{-\rho} - (\tilde{Y}_2^1 - \delta)^{-\rho}] (1-\alpha)/\alpha \right]^{-1/\rho} - \tilde{Y}_1^1 \quad (7)$$

The Appendix also describes the additional steps necessary to calculate the other measures. It derives the expenditure function  $E(P, U) = U\bar{P} + \delta P_2$ , where  $P_2$  is the price of the nonmarket good, and  $\bar{P}$  is a composite index of the two output prices. Then:<sup>11</sup>

$$EV = E(P^0, U') - E(P^0, U^0) - \bar{P}^0(U' - U^0) \quad \text{and} \quad (8)$$

$$CV = E(P', U') - E(P', U^0) - \bar{P}'(U' - U^0) \quad (9)$$

but the signs get reversed to measure the loss as a positive amount. Stuart calculates  $(CS-dR)/dR$  and Ballard, Shoven, and Whalley calculate  $(EV-dR)/dR$ . Finally, the appendix derives the compensated quantities and revenue change  $dR^*$  needed to calculate Browning's  $(CV-dR^*)/dR$ .

Using Stuart's model with Stuart's parameters, I simulate a one percent increase in the marginal tax rate  $m$ . Stuart's measure of "marginal excess

<sup>11</sup>Equations (8) and (9) follow Varian (1984, p.264), where the EV and CV are both positive for a utility gain. Note that  $E(P', U')$  is equal to  $I'$ , so the EV in (8) is  $-[E(P', U') - E(P^0, U')] + (I' - I^0)$ . In other words, it includes both the change in consumer surplus and the change in money income. The EV in Auerbach (1985) is only the change in consumer surplus. These definitions differ here because money income changes. To help clarify this distinction, the concepts in (8) and (9) are called the "equivalent gain" and "compensating gain" by King (1983).

burden" is 7 cents, just as in published results. Table 1 shows that the measure of Ballard, Shoven, and Whalley is also 7 cents. Using the same equilibrium outcome, however, Browning's measure is 25 cents. Thus the results differ because the measures differ.

When labor is inelastic in Figure 1, Stuart's measure was shown to provide zero excess burden. So why does he get 7 cents? The reason is that this simulation in his model with a zero uncompensated labor elasticity does not lead to zero change in actual labor supply.

Ballard (1987) gets zero "marginal excess burden" in a similar model with a flat wage tax, and he concludes that Stuart did not get zero because of progressivity. It is true that this change in the progressive tax structure effectively changes the "virtual" income of the consumer, but the point here is that this "MEB" will not be zero whenever any aspect of the reform causes a change in income or in other prices that leads to a general equilibrium response in the quantity of labor.

To set his parameters, Stuart differentiates labor supply with respect to the net wage and imposes three conditions: the uncompensated labor supply elasticity is zero, the compensated  $\eta$  is .2, and the initial  $P_2$  is 1. The implied  $\delta$  is 1968.36,  $\alpha$  is .9429, and  $\rho$  is 1.0625. This procedure is consistent with the definition of an elasticity, since the differentiation varies only the net wage and holds all other prices constant. As an alternative procedure, I search a three-dimensional grid for values of  $\delta$ ,  $\alpha$ , and  $\rho$  that satisfy three conditions for this simulation in general equilibrium:  $\eta = .2$ ,  $P_2^0 = 1$ , and actual labor does not change. The resulting parameters are  $\delta=1720.44$ ,  $\alpha=.9300$ , and  $\rho=2.2180$ . In this case the same one percent increase in  $m$  leaves actual labor unchanged, and the Table shows that Stuart's measure is exactly zero.

These new parameters should not be preferred to Stuart's. Indeed, they imply that the uncompensated labor supply elasticity is not exactly zero. They are used here only to clarify the important conceptual point that Stuart's measure is zero when actual labor supply does not change. This point was obscured in Stuart's paper because readers naturally thought that his 7 cent figure was comparable to Browning's 21 cent figure.

Using these new parameters, the measure of Ballard, Shoven, and Whalley is also zero. So why do they get 12 cents? Their uncompensated elasticity is similarly set to zero in a model with progressive taxes and with varying prices, so their actual labor supply also changes. In addition, as pointed out by Ballard (1987), they have other taxes that introduce second-best effects. The simulation here demonstrates that they would find zero "marginal excess burden" if they reduced their model to one consumer, two sectors, no other taxes, and unchanged supply of labor.<sup>12</sup>

The model of Ballard, Shoven, and Whalley is not used here to calculate the three different measures. The reason is that the calculation of  $dR^*$  would be extremely difficult in a model with many tax instruments. It would require not only the compensated labor supply, but also the compensated demand for each commodity, the sales tax on each compensated quantity, and all factor taxes on producers at those quantities.<sup>13</sup>

As shown in Table 1, the use of these new parameters reduces all three

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<sup>12</sup>The point of this paper does not arise when their measure is used in a revenue neutral reform, as in every previous application of their model. They use (8), the "equivalent gain" of King (1983).

<sup>13</sup>Hansson and Stuart (1988) point out this difficulty with the Browning measure. In this case, one could solve numerically for the lump sum tax that is "equivalent" in terms of utility to the simulated tax change. With more consumers, however, one needs to solve for more "equivalent" tax amounts. Alternatively, the revenue could be returned as lump sum gifts while just using the CV or EV. In Stuart's model, with one consumer, this alternative is extremely close to the Browning measure.

measures of marginal excess burden. While the first two fall from 7 cents to zero, Browning's measure falls from 25 cents to 20 cents.

Finally, I impose partial equilibrium conditions on Stuart's model by setting  $a=A-b-B-1$ . In this case,  $Y_1=L_1$  and  $Y_2=L_2$ , so the consumer's problem reduces to that of Figure 1. An additional grid search is performed to impose the three conditions discussed above ( $\eta=.2$ ,  $P_2^0=1$ , and actual labor unchanged). The necessary utility parameters are  $\delta=2082.31$ ,  $\alpha=.7420$ , and  $\rho=.9810$ . In this case, Stuart's model reduces exactly to Browning's model. Table 1 shows that Browning's "MEB" is 24 cents while the other measures are zero. Browning's partial equilibrium model therefore overstates his own MEB by 20 percent relative to a comparable general equilibrium model.

#### IV. Conclusion

In all of these calculations, the compensating and equivalent variations are very close to the compensating surplus. In Figure 1, the vertical distance between the two indifference curves is always close to the distance between two parallel price lines. The big difference is introduced by the revenue figure subtracted. Browning subtracts  $dR^*$  and thus compares the distorting tax increase to a lump sum increment with the same revenue. The other two measures are always similar to each other because they both subtract the actual change in revenue. These other measures are zero in Stuart's model when actual labor supply is unchanged.

With only the distortionary effect, Browning's "marginal excess burden" is a familiar concept. It is analogous to a concept of total excess burden that compares the whole tax to a lump sum tax. However, it is not enough information to set public spending. With the addition of the revenue effect, the other two measures do provide enough information to decide on a

project that is separable in utility. Because they are zero for a distorting tax, however, they should not be called marginal excess burden, welfare cost, efficiency cost, or deadweight loss. They are defined as the marginal cost of funds minus one, but there is really no need for any concept other than the MCF itself. Given the tax used to fund it, the project is worthwhile if the benefits exceed the MCF.

An important implication is that the decision to fund a public project depends irrevocably on the nature of the project. The assumption that the project has no effect on labor supply may be convenient, but it is unlikely to be valid. The best procedure is to specify the particular project in the utility of consumers, including any complementarity to labor or leisure, and then calculate whether the tax and spending package increases welfare.

Table 1

Different Measures of "Marginal Excess Burden" for U.S. Labor Taxes

	<u>(CS-dR)/dR</u> <u>(Stuart)</u>	<u>(EV-dR)/dR</u> <u>(BSW)</u>	<u>(CV-dR*)/dR</u> <u>(Browning)</u>
From the literature <sup>a</sup>			
Stuart (1984)	.07	--	--
Ballard-Shoven-Whalley (1985)	--	.12	--
Browning (1987)	--	--	.21
Using Browning's model <sup>b</sup>			
with Stuart's parameters	--	--	.24
Using Stuart's model <sup>c</sup>			
calculate other measures	.07	.07	.25
labor unchanged in general equil.	.00	.00	.20
labor unchanged in partial equil.	.00	.00	.24

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CS=compensating surplus, EV=equivalent variation, CV=compensating variation, dR=change in actual revenue using average tax rates, and dR\* =change in revenue on compensated curve, using marginal rates.

a. Estimates from the literature all employ a zero uncompensated labor supply elasticity where additional revenue is spent on government consumption, but they differ on other assumptions.

b. The other assumptions borrowed from Stuart's model are:  $\eta=.2$  for the compensated labor supply elasticity;  $m=.427$  for the marginal tax rate; and  $r-dm/dt=1.564$  for the constant ratio of the marginal tax rate to the average tax rate.

c. It is not possible to use the same assumptions in the Ballard-Shoven-Whalley model, for reasons described in the text.

Figure 1

Different Measures of Excess Burden

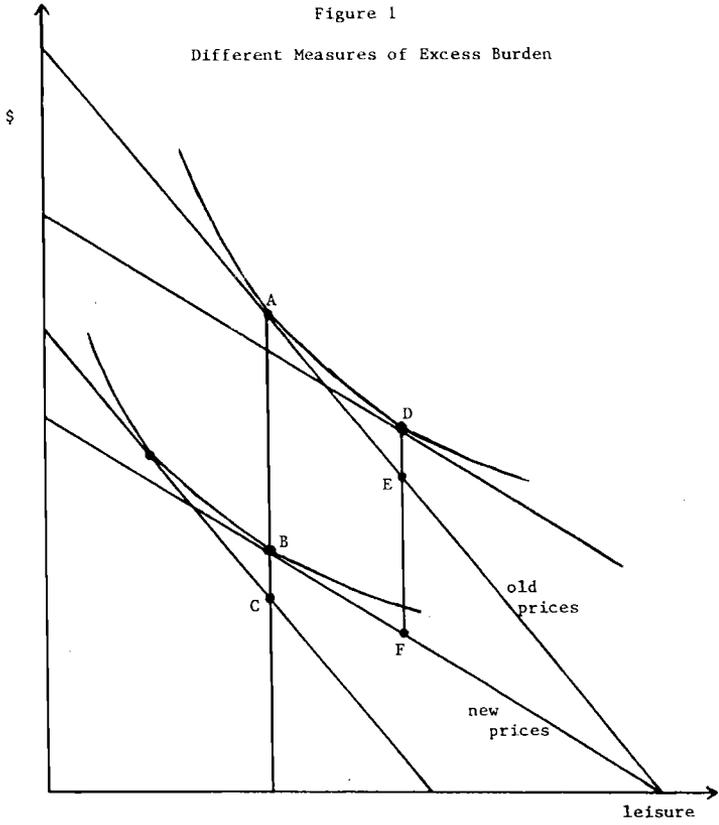
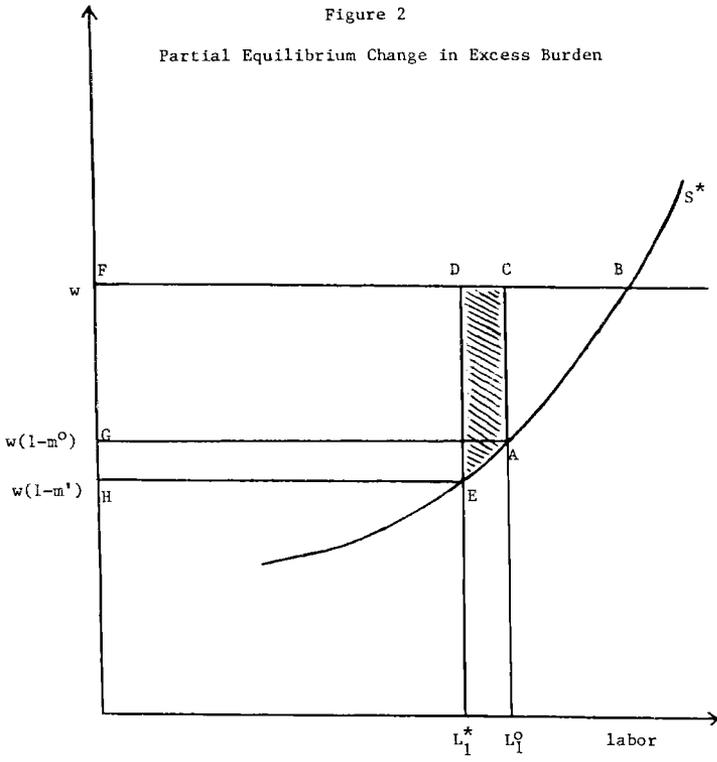


Figure 2

Partial Equilibrium Change in Excess Burden



Appendix: General Equilibrium Welfare Effects

In Stuart's model, the only tax is on labor in the market sector. Total revenue is split between redistributions,  $Tr$ , and government spending,  $G$ . Marginal revenue is spent only on  $G$ , so:

$$G = g_0 + \tau w L_1 \quad (A1)$$

First order conditions provide the relative price:

$$P_2 = \left[ \frac{(1-\alpha)/\alpha [(\bar{Y}_2 - \delta)/\bar{Y}_1]}{\dots} \right]^{-\rho-1} \quad (A2)$$

where the first good is numeraire ( $P_1=1$ ). Stuart uses competitive behavior, equilibrium conditions, and specifications for parameters ( $L, m, r, A, a, B, b, \delta, \alpha, \rho$ , and  $g_0$ ) to obtain ten equations in ten unknowns ( $\tau, L_1, L_2, Y_1, Y_2, \bar{Y}_1, \bar{Y}_2, w, G$ , and  $Tr$ ). A simple iteration solves the model.

From sources in the literature, Stuart sets the initial  $m=.427$  and  $\tau=.273$ , so  $r=1.564$ . From the Economic Report of the President, he sets the initial  $G=227.7$  billion dollars for 1976. Out of a maximum  $L=3660$  hours per year, the average employable person works  $L_1=1008.14$  hours. From the Survey of Current Business, Stuart sets  $\bar{Y}_1 = 1527.4 - 227.7 = 1299.7$ , and  $a=.72$ , so  $A$  must be 10.506. He also uses .72 for  $b$ , and 1 for the initial  $P_2^0$ , so  $B=7.892$ . Revenue is  $\tau(wL_1) = \tau(aY_1) = (.273)(.72)(1527.4) = 300.2$  billion, so from (A1),  $g_0$  must be  $G - \tau w L_1 = -72.5$  billion.

Stuart makes a small error in reporting that  $g_0$  is -63.1, but the correct value only changes his marginal excess burden slightly.

To add calculations for the compensating and equivalent variations, it is convenient to reformulate the model in terms of demands:

$$\bar{Y}_1 = \alpha^\sigma (I - P_2 \delta) / \{ \alpha^\sigma + (1 - \alpha)^\sigma P_2^{1 - \sigma} \} \quad \text{and} \quad (A3)$$

$$\bar{Y}_2 = \delta + (1 - \alpha)^\sigma (I - P_2 \delta) / P_2^\sigma \{ \alpha^\sigma + (1 - \alpha)^\sigma P_2^{1 - \sigma} \} \quad (A4)$$

Substitution back into (6) yields the indirect utility function:

$$V(P, I) = (I - P_2 \delta) / \bar{P} \quad \text{where} \quad \bar{P} = \{ \alpha^\sigma + (1 - \alpha)^\sigma P_2^{1 - \sigma} \}^{1 / (1 - \sigma)} \quad (A5)$$

$\bar{P}$  is the composite price, an index of  $P_1=1$  and  $P_2$ . Solution for  $I$  yields the the expenditure function  $E(P, U) = U \bar{P} + \delta P_2$ .

To calculate Browning's measure using this model, I follow Auerbach (1985, p.75). He defines  $dR^*$  as  $(R' - R^0)$ , where  $R'$  is the difference between the new producer price and the new consumer price all times the compensated quantity, and  $R^0$  is the old price difference times the old quantity. To get the compensated demand  $\bar{Y}_2^*$ , I use  $U^0 \bar{P}'$  in place of  $(I - P_2 \delta)$  in equation (A4). Production  $Y_2^*$  equals that demand. Next I use equations (3), (4), and (5) to solve for the production possibility frontier, the maximum  $Y_1^*$  that can be produced for  $Y_2^*$ . Taxable labor income is then  $aY_1^*$ , but the amount  $m(aY_1^*)$  is expressed in units of the production good  $Y_1$ . The consumption good  $\bar{Y}_1$  is the numeraire. Since the consumer price  $P_1=1$  includes the tax, the producer price of  $Y_1$  is only  $(1 - m)$ . The change in revenue along the compensated curve measured in units of the numeraire at new prices is:

$$dR^* = [m'(aY_1^*) - m^0(aY_1^0)](1 - m') \quad (A6)$$

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