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ABSTRACT

We model sudden stops in a small open economy as rare discrete events precipitated by increases in the world risk-free rate. When external debt is large, the model exhibits multiple equilibria, one where external debt and consumption remain high, and one with a collapse in external debt and consumption. Private agents delever following an increase in the world interest rate, but they fail to internalize the impact of deleveraging on the price of collateral. For high levels of debt, even a small increase in the world interest rate can eliminate the high debt equilibrium and the economy experiences a sudden stop. The central bank can use foreign exchange intervention to prevent the sudden stop. If reserves cannot be borrowed, optimal policy is to "lean against the wind", buying foreign reserves ex-ante when private borrowing is high and selling them after an interest rate shock when private agents are deleveraging.

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1 Introduction

From the Latin American crises of the early 1980's, to the Mexican crisis of 1994, to the Taper Tantrum of 2013, recent history is replete with episodes where increasing interest rates in the U.S. and other developed economies lead to financial crises and sudden stops in emerging markets.[1]

Observed sudden stops have distinct characteristics. Calvo et al. (2008) define a systemic sudden stop as a large current account contraction during a time of systemic financial turmoil. Calvo et al. (2006) develop a two-step procedure involving fluctuations in a country's current account balance and either the Fed Funds rate or the aggregate emerging market bond index spread (EMBI) to identify 22 emerging market systemic sudden stop events during the 1980s and 1990s. Korinek and Mendoza (2014) and later Bianchi and Mendoza (2020) expand this process to identify systemic sudden stop episodes in advanced economies, and they identify a total of 51 distinct systemic sudden stop episodes, 36 in emerging markets and 15 in advanced economies from 1980-2016. These episodes are not evenly distributed across time since by definition they are centered around periods of systemic financial turmoil, particularly around times of Federal Reserve monetary tightening.[2] But while these episodes tend to cluster, they are still rare, and even during these events, only a small subset of countries experience a sudden stop episode. A sudden stop is a discrete rare event that exists alongside regular business cycle fluctuations. For this reason, Mendoza (2010) models a sudden stop as happening when an occasionally binding collateral constraint starts to bind.

In this paper we build a model where a systemic sudden stop is a discrete rare event driven by an exogenous increase in the world cost of borrowing. We then model how central bank foreign exchange intervention (FXI) can be used as a tool to prevent a sudden stop and model how a central bank holds a stock of reserves as insurance against a sudden stop

[1] See e.g. Diaz-Alejandro (1984), Calvo et al. (1993 and 1996), Dooley et al. (1996), Aizenman et al. (2016), Ahmed et al. (2017)

[2] The EMBI spread used to filter the systemic sudden stops is not available in the 1980's, so a sharp increase in the Fed Funds rate is used as a systemic filter before 1992.

event.

In order to describe the characteristics of sudden stops and to rationalize the benefits of foreign exchange intervention, our model incorporates three features. First, the economy must have a borrowing constraint which is occasionally binding. A sudden stop is a rare event; most of the time the constraint is slack. Second, an increase in the cost of foreign borrowing must actually increase the probability of a binding borrowing constraint. Finally, central bank foreign exchange intervention must be effective even during normal times, when the borrowing constraint is slack.

We adopt the model of an underborrowing equilibrium from Schmitt-Grohé and Uribe (2021). Schmitt-Grohe and Uribe show how under certain parameter values, when external debt passes a high enough threshold, there are multiple equilibria in the external position of a small open economy. In one equilibrium, consumption and external debt remains high, sustaining a high value of the real exchange rate, and the value of collateral is high enough so that the borrowing constraint is slack. But in another equilibrium, a reduction in borrowing leads to a real depreciation, a falling value of collateral, and a binding borrowing constraint.

Even if agents beliefs coalesce around the 'best' equilibrium, when external debt reaches a high enough point, the joint effect of pecuniary externalities and non-linearity in the tightness of borrowing constraints can lead to a discontinuous collapse in borrowing capacity following a shock to the cost of foreign borrowing. A key feature of the model in Schmitt-Grohé and Uribe (2021) is that in a non-binding equilibrium, as agents delever and the stock of external debt falls by one unit, the fall in the real exchange rate is large enough that the value of collateral falls by more than one unit. So even as agents delever, the slack in the borrowing constraint shrinks. At a high enough level of initial debt, even a small increase in the cost of foreign borrowing can eliminate this slack, and the non-binding equilibrium is no longer possible. The economy then shifts to the only equilibrium that remains and there is a discrete collapse in foreign borrowing capacity. [3]

[3]We note that the feature that an interest rate-induced delevering tightens the borrowing constraint is a particular feature of the model of Schmitt-Grohé and Uribe (2021) when multiple equilibrium levels of

Although our model focuses on fundamentals, rather than self-fulfilling beliefs, a sudden stop is still inefficient. As in Mendoza (2002), agents do not internalize the impact of their deleveraging on the value of collateral. From the perspective of the individual agent, deleveraging in response to a rising cost of foreign borrowing is perfectly rational, but at the point where the borrowing constraint binds, deleveraging is irrational for the economy as a whole. On the other hand, the policy maker internalizes the effect of deleveraging on the value of collateral. At the point where further deleveraging would cause the borrowing constraint to bind, the policy maker should act to prevent further deleveraging and thus keep the value of collateral high enough to sustain the non-binding equilibrium.

This presents a dilemma however. For a successful foreign exchange intervention policy, it is vital for the central bank to prevent the economy from hitting a binding borrowing constraint, since once that happens, it is too late - the sudden stop will have occurred, with a discrete collapse in consumption, borrowing, and the real exchange rate. But absent any frictions in international capital markets, foreign exchange intervention will have no traction in states of the world where the borrowing constraint is slack. To resolve this problem, we introduce intermediary frictions in private capital markets, following Gabaix and Maggiori (2015). These frictions drive a wedge between domestic and world interest rates, and prevent the private sector from fully offsetting a central bank foreign exchange intervention.[4]

The main body of the paper then explores the optimal foreign exchange rate policy in the presence of exogenous shocks to the borrowing costs for the small economy. The central bank engages in sterilized foreign exchange intervention. By selling foreign bonds, the central bank can reduce the cost of borrowing facing households and increasing domestic consumption and external debt. If the central bank were able to short foreign reserves, then it would always be able to engage in enough intervention to prevent a crisis in any period. However, we make the realistic assumption that the bank cannot have a negative reserve position. If the

borrowing exist. This would not be the case in models with a unique equilibrium.

[4]In effect, government borrowing to finance foreign exchange purchases ‘crowds out’ private borrowing, as described in Chang (2018).

central bank runs out of reserves, then it cannot intervene to avoid a crisis. Then the bank must accumulate reserves in advance of the crisis.

But this is costly, since it is forcing the private sector to engage in excess saving. The same intermediary friction which gives foreign exchange intervention traction in averting a crisis in periods of high borrowing costs makes the accumulation of foreign exchange reserves distortionary in normal times. Just as the sale of foreign exchange reserves would lead the economy to consume and borrow more, the central bank purchase of foreign exchange reserves would force the economy to consume less and save more. This distortion of agents' optimal consumption plans represents the cost to acquiring reserves.

The central results of the paper describe the trade-off that the central bank must make to balance the cost of acquiring reserves ex-ante against the benefits of preventing a sudden stop crisis ex-post. We show this first in a simplified version of the model in which there is effectively only a one-period decision problem for the central bank. We then extend the model to allow for a full quantitative evaluation of the optimal reserve management policy. In the calibrated model, we find that reserve management policy is successful in dramatically reducing the frequency and severity of sudden stops, but due to the trade-off between costs and benefits of raising reserves, the optimal policy does not completely eliminate sudden stops.

The paper is organized as follows. Section 2 provides a brief review of the empirical and theoretical literature on the role of foreign exchange reserves in preventing currency and financial crises. The model is presented in Section 3. Section 4 discusses the mechanics of a sudden stop following a shock to the world interest rate and the optimal policy response. Numerical results from a global solution of the model and a numerical solution for optimal policy is presented in Section 5. Finally Section 6 concludes.

2 Recent empirical and theoretical literature on foreign exchange intervention

The empirical literature has identified fluctuations in the cost of foreign borrowing as key factor for explaining business cycles in emerging market economies. Neumeyer and Perri (2005) and Uribe and Yue (2006) argue that changes in world interest rates and external borrowing costs are one of the main factors driving emerging market business cycles. Miranda-Agrippino and Rey (2020) argue that U.S. monetary policy is a major driver of the global financial cycle and cycles of capital flow surges and stops in emerging market economies.

Fratzscher et al. (2019) use daily data on sterilized foreign exchange intervention and argue that foreign exchange intervention (FXI) is an effective tool for exchange rate stabilization. Forbes and Klein (2015) conclude that FXI is an effective policy tool to prevent currency depreciation in the face of shocks to the foreign interest rate. Ghosh et al. (2016) estimate a policy reaction function for central bank foreign exchange accumulation and find that emerging market central banks engage in FXI to smooth fluctuations in the real exchange rate. Obstfeld et al. (2009) show that countries with a larger stock of reserves in 2007 had less exchange rate depreciation during the crisis of 2008.

Frankel and Rose (1996) and Gourinchas and Obstfeld (2012) find that the stock of reserves is negatively associated with increased probability of a crisis. Ahmed et al. (2017) show that emerging market economies with better fundamentals, including a higher stock of central bank reserves to GDP and a lower ratio of short-term external debt to reserves, outperformed their emerging market peers on a number of financial indicators during the "taper tantrum" episode of 2013.[5]

[5] These findings are related to a number of other papers documenting the early warning signs of an emerging market crisis, including Bussiere and Fratzscher (2006), Rose and Spiegel (2011), Frankel and Saravelos (2012).

Obstfeld et al. (2010) regress reserve stocks on financial openness, and find that countries hold more reserves when they become more financially open and thus more vulnerable to external crises. Aizenman and Hutchison (2012) and Aizenman and Sun (2012) discuss the "fear of losing international reserves" whereby during the crisis of 2008-2009 many emerging market countries chose to allow their exchange rate to depreciate rather than losing reserves.

In Section A of the Appendix, we extend the approach of Frankel and Rose (1996), investigating the influence of world interest rates and foreign exchange reserves on the probability of currency crises for a large group of emerging market countries. We find that increases in the world interest rate significantly increase the probability of currency crises, while lagged reserves to GDP reduce the same probability. More importantly, interacting the two variables, we find that the effect of an increase in world interest rates on the probability of a crisis is significantly reduced for countries with a higher stock of foreign exchange reserves.

In the recent theoretical literature on foreign exchange intervention, Jeanne and Ranciere (2011) model reserves as an insurance contract to prevent sudden stops. Durdu et al. (2009) also model reserve accumulation as insurance against a sudden stop resulting from domestic shocks. Chang et al. (2015) and Cavallino (2019) look at optimal foreign exchange intervention in a linear-quadratic New Keynesian model. Adrian et al. (2022) explore how capital flow management and foreign exchange intervention help strengthen monetary trade-offs. In these models foreign exchange intervention is an additional tool that helps to stabilize the economy in the presence of portfolio shocks. Fanelli and Straub (2021) model optimal foreign exchange intervention in a setting where the central bank tries to manipulate the price of non-traded goods for distributional considerations. Hur and Kondo (2016) and Bianchi et al. (2018) both consider the use of reserves to mitigate rollover risk in sovereign debt.

In a stylized three-period model, Cespedes et al. (2017) and Bocola and Lorenzoni (2020) develop models with multiple equilibria ex-post. The central bank can eliminate bad equilibria by implementing a lender of last resort policy if it has accumulated a sufficient stock of reserves. In Jeanne and Sandri (2020) both private and central bank foreign assets can serve as insurance against a sudden stop in foreign liabilities, but private agents do not internalize the insurance role of their stock of foreign assets. The central bank, which does internalize the insurance role of their stock of foreign assets, will acquire reserves and as a result the economy will have a higher level of liquid external assets than in the laissez-faire equilibrium. In Cespedes and Chang (2020), the central bank acquires reserves ex-ante to lend to banks following a shock to banks' collateral constraint.

Our paper is similar in many ways to Arce et al. (2019) , but also differs in some key features. In Arce et al. (2019) the fact that private agents don't internalize the effect of their own external borrowing on the value of collateral represents a pecuniary externality that leads the private sector to borrow more than is efficient. By acquiring foreign exchange reserves the central bank can force the economy as a whole to save more and thus lead to the efficient level of borrowing. But in Arce et al. the borrowing constraint is always binding. Without a binding collateral constraint Ricardian equivalence would hold and FXI policy would have no aggregate effect. By contrast, in our paper it is essential that foreign exchange policy be effective in states where the borrowing constraint does not bind, since as noted above, once the borrowing constraint binds in our model, the sudden stop has already occurred. In our model, given the formulation of Schmitt-Grohé and Uribe (2021), the binding borrowing constraint itself precipitates a discrete sudden stop. Hence it is necessary to use FXI policy to avoid this event. A secondary difference is that we focus on the particular types of sudden stops driven by spikes to the cost of foreign borrowing which lead to private sector deleveraging. When combined with the collateral constraint of Schmitt-Grohé and Uribe (2021), this allows for an endogenous *fall* in borrowing to cause a discrete sudden stop. In a model where collateral values are less sensitive to movements in external borrowing, a fall in borrowing driven by an interest rate spike would actually relax the borrowing constraint.

3 Model

We construct an infinite horizon model of a small open economy. The economy features a representative household, a financial sector, a central bank. Households derive utility from the consumption of a tradable good y^T and a non-tradable good y^N. Households begin each period with an initial stock of debt. They face a borrowing constraint limiting debt to a fraction of the market value of their endowment in a given period. The only source of exogenous variation in the model is a shock to the world interest rate.

3.1 Households

Households maximize utility, described as follows:

$$U = E_t \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

where $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. c_t is defined as

$$c_t = \left[\alpha \left(c_t^T \right)^{\frac{\xi-1}{\xi}} + (1-\alpha) \left(c_t^N \right)^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}} \tag{2}$$

where c_t^T (c_t^N) denotes the traded good (non-traded good) consumption. The budget constraint for households is written as follows:

$$c_t^T + p_t c_t^N + B_t = y^T + p_t y^N + R_{t-1} B_{t-1} + T_{t-1} + \Pi_{t-1} \tag{3}$$

where B_t represents the household's holdings of domestic bonds (which are held by households, the financial sector, and the central bank) and R_t is the interest rate on domestic bonds. The central bank earns a net return T_{t-1} on its bond portfolio which is rebated lump-sum to savers (more on this later). The financial sector also earns net interest income on its bond portfolio, also rebated to households in lump sum Π_{t-1}.

Combining the first order conditions for traded and non-traded goods gives the price of non-traded goods p_t:

$$p_t = \frac{1-\alpha}{\alpha}\left(\frac{c_t^T}{c_t^N}\right)^{\frac{1}{\xi}} \tag{4}$$

Due to limited enforcement of debt contracts, home country borrowers face a borrowing constraint given by:

$$-B_t \leq \kappa\left(y^T + p_t y^N\right) \tag{5}$$

The multiplier on the borrowing constraint is μ_t. The first order condition with respect to B_t is:

$$\lambda_t - \mu_t = E_t \beta \lambda_{t+1} R_t \tag{6}$$

where λ_t is the marginal utility of traded goods consumption:

$$\lambda_t = c_t^{\frac{1-\sigma\xi}{\xi}} \alpha \left(c_t^T\right)^{\frac{-1}{\xi}} \tag{7}$$

and the complementary slackness condition:

$$\mu_t \left[\kappa\left(y^T + p_t y^N\right) + B_t\right] = 0$$

3.2 Financial Intermediaries

A key feature of the model is the presence of frictions in international financial markets. Private households can't directly hold foreign bonds, but must trade with financial intermediaries who can borrow and lend on international financial markets subject to enforcement costs. As explained in the introduction, this is a critical element of the model because it allows for central bank foreign exchange policy to influence private sector net saving in states where the borrowing constraint is non-binding.

In our modeling of financial intermediaries, we follow closely the structure of intermediary financiers developed by Gabaix and Maggiori (2015). Since the details mirror those in Gabbaix and Maggiori we present the microfoundations for the financial sector in the Appendix and here only present the key elements. Financiers are atomistic entities who operate for one period and start each period with a zero balance sheet position. They purchase bonds B_t^{fs} from private households, financed by selling bonds $-F_t^{fs}$ to foreign lenders. Thus, the aggregate balance sheet of financiers is

$$B_t^{fs} + F_t^{fs} = 0. \tag{8}$$

Each financier decides a bond portfolio to maximize the discounted value of net interest income. The net interest income of the financial sector as a whole that is rebated to households can be written as, where R_t is the interest rate on domestic bonds and R_t^W is the interest rate on foreign bonds:

$$\beta\Pi_t = \beta\left(R_t^W F_t^{fs} + R_t B_t^{fs}\right) = \beta\left(R_t^W - R_t\right) F_t^{fs} \tag{9}$$

Note that all bonds are denominated in traded goods, so there is no currency mismatch in financiers balance sheets. Financiers choose their bond position to maximize expected discounted profits, but are are limited by an incentive constraint arising from their ability to abscond with borrowed funds, as in Gabaix and Maggiori (2015). This drives a wedge between the interest rate on domestic bond and the interest rate on foreign borrowing:

$$R_t^W - R_t = \frac{\Gamma}{\beta} F_t^{fs} \tag{10}$$

where Γ is a non-negative constant capturing the share of financier's bond position that can be diverted, and so measures the friction in private capital markets. If $\Gamma = 0$ then the financial sector is simply a veil and the equilibrium condition for foreign bond holding is $R_t = R_t^W$, exactly as it would be if households could borrow directly from foreigners and faced no frictions. But when $\Gamma > 0$, and $F_t^{fs} < 0$, the domestic interest rate will be higher than the foreign interest rate.[6]

3.3 Central Bank and Market Clearing

The central bank also holds a stock of domestic and international bonds. It can vary the composition of that bond portfolio.

$$B_t^{cb} + F_t^{cb} = 0 \tag{11}$$

By the fact that they participate in both the domestic and international bond markets, the central bank is similar to the financial sector. But we assume that the central bank does not face the intermediation friction Γ. However, we impose the constraint that central banks foreign bond holdings cannot be negative, $F_t^{cb} \geq 0$.

As discussed by Obstfeld, Shambaugh, and Taylor (2009 and 2010), the benefit of central bank reserves is that they are assets of the central bank that can be deployed immediately at a time of crisis. Over the last decade, in a series of policy papers, the IMF lays out their approach to quantitatively assessing a country's reserve adequacy (IMF 2011, 2013, and 2015). The 2013 paper, which is devoted exclusively to the issue of precautionary reserve accumulation, discusses alternative forms of insurance against a balance of payments crisis. The most common are central bank swap lines and contingent credit lines from international financial institutions like the IMF. These programs essentially allow a central bank's stock of reserves to be negative.

However, while both can perform the role of reserves when activated, the number of countries that are covered by these programs is very small, the credit lines are subject to caps which may be lower than the amount of reserves needed, their activation is not automatic and they are not intended to be permanent. Thus they can't be counted on as

[6]Note that this equilibrium condition in the market for foreign bonds can also be derived in a reduced form by adding a quadratic adjustment cost to holding foreign bonds in the household budget constraint, as in Schmitt-Grohe and Uribe (2003). This reduced form approach was the preferred way of adding intermediary frictions in models with central bank foreign exchange intervention in Chang et al. (2015) and Davis et al. (2021).

insurance against a future crisis the way hard currency reserves on the balance sheet of the central bank can be.

In the appendix we further discuss these two programs, central bank swap line and IMF credit lines, that in theory allow a central bank to insure against a balance of payments crisis with negative reserves, and we include some references discussing the limited nature of both.

The central bank earns a net return T_{t-1} on its portfolio which is rebated lump-sum to savers.

$$T_{t-1} = R^W_{t-1} F^{cb}_{t-1} + R_{t-1} B^{cb}_{t-1} \tag{12}$$

Domestic bonds B are held by households, the financial sector, and the central bank. The domestic bond market clearing condition is given by:

$$B_t + B^{fs}_t + B^{cb}_t = 0 \tag{13}$$

Finally, non-traded goods market clearing implies:

$$c^N_t = y^N \tag{14}$$

3.4 Balance of payments identity

Substituting the financial sector and central bank net interest income in equations (9) and (12), the financial sector and central bank balance sheets in equations (8) and (11), and the domestic bond market clearing and non-traded goods market clearing conditions in equations (13) and (14), into the household's budget constraint in (3), we obtain the economy-wide budget constraint:

$$c^T_t = y^T - F^{fs}_t + R^W_{t-1} F^{fs}_{t-1} - F^{cb}_t + R^W_{t-1} F^{cb}_{t-1} \tag{15}$$

This condition can be rearranged into the familiar balance of payments identity where the current account equals the capital account plus the change in central bank foreign exchange reserves:

$$CA_t = \Delta F_t^{fs} + \Delta F_t^{cb}. \tag{16}$$

The current account, CA_t, equals net exports: $y^T - c_t^T$ plus interest income from international bonds purchased in $t-1$: $F_{t-1}^{cb}\left(R_{t-1}^W - 1\right) + F_{t-1}^{fs}\left(R_{t-1}^W - 1\right)$. The capital and financial account, ΔF_t^{fs}, is equal to net international bond purchases by financiers, $F_t^{fs} - F_{t-1}^{fs}$. The change in reserves, ΔF_t^{cb}, is equal to net international bond purchases by the central bank, $F_t^{cb} - F_{t-1}^{cb}$.

A current account deficit $CA_t < 0$, can be financed either by net private capital inflows, a negative capital and financial account $\Delta F_t^{fs} < 0$, or the sale of central bank foreign bonds $\Delta F_t^{cb} < 0$. The two types of financing, public and private, are not equal, since private financiers face an intermediary friction $\Gamma > 0$. As in Gabaix and Maggiori (2015) this friction allows the central bank to use the purchase or sale of foreign bonds as an instrument to adjust the current account and thus the economy's total external debt. A central bank sale of foreign bonds reduces the current account surplus, while a purchase of foreign bonds has the opposite effect. A formal proof is presented in the Appendix, but the intuition is as follows.

Suppose the central bank increases their holdings of foreign bonds, $\Delta F_t^{cb} > 0$. Through the central bank's balance sheet, $\Delta B_t^{cb} < 0$, as the purchase of foreign bonds is financed by issuing domestic bonds. This puts upward pressure on the domestic interest rate and creates an arbitrage opportunity for the financial sector to buy domestic bonds and finance this by selling foreign bonds, $\Delta B_t^{fs} > 0$ and $\Delta F_t^{fs} < 0$.

Absent the intermediary friction, i.e. $\Gamma = 0$, private financiers could fully exploit this arbitrage opportunity and the increase in central bank domestic bond sales would be offset one for one by the increase in domestic bond purchases by the financial sector, $\Delta B_t^{cb} = -\Delta B_t^{fs}$. The total stock of debt in the economy would be unaffected and the equilibrium condition in the market for foreign bonds would imply that $R_t = R_t^W$. In that case, the financial sector is a veil and central bank foreign exchange intervention has no effect on aggregate macroeconomic variables, as in Obstfeld (1981), Backus and Kehoe (1989), Gabaix and Maggiori (2015), and Davis et al. (2021).

But if the intermediary friction $\Gamma > 0$ then when the central bank purchases foreign bonds, $\Delta F_t^{cb} > 0$, creating an arbitrage opportunity between foreign and domestic bonds, the intermediary friction means that this opportunity is not fully exploited by financiers. As financiers sell foreign bonds to take advantage of the arbitrage opportunity, the intermediary friction tends to push up the domestic interest rate above the world interest rate, reducing the private sector's incentive to sell domestic bonds in the same volume as their purchase of domestic bonds from the central bank.

3.5 Determination of net external assets

Here we examine the determination of the economy's net external debt in the model. Define a country's net external assets as $F_t = F_t^{fs} + F_t^{cb}$, and thus $-F_t$ represents net external debt. For now, abstract from central bank foreign reserves, so we set $F_t^{cb} = 0$.

A steady state is defined by constant values of consumption, domestic and foreign bond holdings, and domestic and foreign interest rates. Here, and for the rest of the paper, we assume that in a steady state $\beta R^W < 1$. So domestic agents are more impatient than the rest of the world.

Using (4) -(7) and (10) a steady state is defined by the conditions:

$$1 = \beta R^W - \Gamma(F) + \frac{\mu}{\lambda} \tag{17}$$

$$-F \leq \kappa \left(y^T + \frac{1-\alpha}{\alpha} \left(y^T + \left(R^W - 1 \right) F \right)^{\frac{1}{\xi}} \right) \tag{18}$$

$$c^T = y^T + (R^W - 1)F \tag{19}$$

where $\lambda = c^{\frac{1-\sigma\xi}{\xi}} \alpha \left(c^T\right)^{\frac{-1}{\xi}}$. Equations (17)-(19) describe steady state values of μ, F and c^T. The steady state borrowing constraint can be plotted in a chart with total external debt along both the horizontal and vertical axis in Figure 1. In the figure the right hand side of the inequality in equation (18) is represented by the blue downward sloping line, which is identical in both panels of the figure.

Any equilibrium must lie along the 45 degree line. We pick the parameters of the model such that the borrowing constraint is non-binding in the steady state. Specifically, given the subjective discount factor and world interest rate, the financial intermediation parameter Γ determines the steady state level of external debt, $-F = \frac{1}{\Gamma}\left(\beta R^W - 1\right)$, and Γ is set high enough that (18) holds with strict inequality.[7]

Equation (18) represents the steady state borrowing constraint when $F = F_t = F_{t-1}$. But following Schmitt-Grohé and Uribe (2021) we may also define a short-run borrowing constraint, where unlike the steady-state borrowing constraint in (18), the initial level of external debt F_{t-1} is predetermined:

$$-F_t \leq \kappa \left(y^T + \frac{1-\alpha}{\alpha} \left(y^T + R^W_{t-1} F_{t-1} - F_t \right)^{\frac{1}{\xi}} \right) \tag{20}$$

The right hand side of this short-term borrowing constraint (20) is *increasing* in the choice of debt in period t. This is due to the fact that given F_{t-1}, increased borrowing in period t raises the price of the non-traded good and thus the value of collateral.

This short-term borrowing constraint is given by the red upward sloping convex line in both panels in Figure 1. The difference between the two panels is that the initial debt $-F_{t-1}$ is higher in the second panel. As we can see the short term borrowing constraint in the right hand side of equation (20) is shifted down and to the right as debt carried over from the last period $-F_{t-1}$ increases.

[7]Later when presenting the numerical solution to the model, the global solution incorporates the fact that due to uncertainty and risk aversion the stochastic steady state of the expected value of next period's marginal utility of consumption is greater than the stochastic steady state value of the marginal utility of consumption, and thus the left hand side of equation 17 will be less than one in the global numerical solution.

The steady state in each figure corresponds to the level of debt where the short-term and steady state borrowing constraints intersect. For a low initial stock of debt the short-term borrowing constraint does not intersect with the 45 degree line. But for a higher level of initial debt the short-term borrowing constraint intersects the 45 degree line.

Since the short-term borrowing constraint shifts to the right and down when $-F_{t-1}$ increases, for a high enough initial debt level, the constraint intersects the 45 degree line twice and there are three equilibria. These are labeled A, B, and C in the bottom panel of the figure. Point A represents the steady state equilibrium, while B and C represent departures from the steady state where the short run borrowing constraint is binding. The non-binding equilibrium level of external debt is $-F^A$. The levels of external debt where the constraint is binding are given by $-F^B$ and $-F^C$, where $-F^C < -F^B < -F^A$.

Schmitt-Grohé and Uribe (2021) present a formal proof to derive under what conditions the short-term borrowing constraint crosses the 45 degree line twice and thus under what conditions will the model have three equilibria.

The key to the existence of multiple equilibria is that the slope of the borrowing constraint is greater than one when the economy is in a non-binding steady state equilibrium. When the slope is greater than one, each additional unit of debt will have a direct effect of tightening the borrowing constraint by one unit but an indirect effect of loosening the borrowing constraint by more than one unit since additional debt leads to higher traded goods consumption and thus a higher relative price of non-traded goods. If in addition, at the intersection point C, the slope of the short run borrowing constraint is positive (as in figure 1), then both B and C represent borrowing-constrained equilibria. A large part of Schmitt-Grohé and Uribe (2021) is devoted to showing under what combinations of parameters this will hold.

In figure 1, each equilibrium can be sustained by self-confirming beliefs. Which equilibrium will prevail? For this, we need an equilibrium selection rule. In the equilibrium selection rule we use, if the non-binding equilibrium (A) is possible, agents' beliefs will always coalesce around this equilibrium. But if a non-binding equilibrium is not consistent with the borrowing choice of domestic agents given the cost of foreign borrowing (and central bank intervention), then the level of external borrowing falls to the only remaining equilibrium, at point C.

Note that this selection rule eliminates the possibility of self-fulfilling deleveraging. Movement from a "good" non-binding equilibrium to a "bad" binding equilibrium is driven not by beliefs, but by fundamental shocks, which in this model are represented as shocks to the country's cost of external borrowing, R_t^W. [8]

4 Sudden stops with and without policy intervention

4.1 Competitive equilibrium without intervention

Following on the discussion of the previous section, we can describe how the competitive equilibrium without policy intervention evolves. Begin from the non-binding steady state with external debt $-F^A$, represented graphically by point A in Figure 1.

Following a positive shock to R_t^W, agents' desired level of external borrowing $-F_t$ will fall. This can be represented as a movement left along the 45-degree line in Figure 1. After the initial shock, if R_t^W is stationary then external borrowing $-F_t$ will gradually converge back to the steady state at point A.

For a small shock, the desired deleveraging leads to a debt level to the right of point B, and so there still exists an equilibrium where the borrowing constraint does not bind. But there is a critical value of R_t^W where the equilibrium level of external debt $-F_t = -F^B$ and the borrowing constraint is just on the margin of binding. This is indicated at point B. Given an increase in R_t^W above this threshold, agents will delever further. Once external debt falls below $-F^B$ however, it will fall to the next stable equilibrium at point C. Hence,

[8]The interest rate R_t^W is the exogenous cost of foreign borrowing, and is composed of a world risk free rate and an exogenous spread, like the aggregate EMBI spread that is used by the Calvo et al. (2006), Korinek and Mendoza (2014), or Bianchi and Mendoza (2020) to identify the systemic part of systemic sudden stops. When we model exogenous shocks to R_t^W we are agnostic about whether this is a shock to the world risk free rate or a shock to something like an aggregate emerging market spread

in the region of point B, a very small further increase in R_t^W can precipitate a large and discrete fall in external borrowing.

In Figure 2 we illustrate this process in a simplified version of the model. In this version there is only a transitory shock to R_t^W in period $t = 1$, after which R_t^W will return to its steady state level for period $t = 2, \cdots, \infty$. The figure presents the values of time t external debt, time t traded goods consumption, the time t price of non-traded goods, and overall welfare as a function of the size an unexpected transitory shock to R_t^W in period $t = 1$. [9]

The blue solid line presents the case where the central bank holds no foreign exchange reserves, $F_t^{cb} = 0$. The red and green dashed lines consider the cases where the central bank holds foreign exchange reserves before the shock, and will be discussed below.

The steady state of the model is where $R_t^W = 1.04$. To the left of this steady state level of R_t^W, when there is a negative shock to the cost of foreign borrowing, agents respond to the shock by borrowing more, and this leads to an increase in external debt, traded goods consumption and the price of the non-traded good. But the response may be quite asymmetric following a positive shock to R_t^W. For small positive shocks to R_t^W there is gradual deleveraging and a gradual fall in traded goods consumption and the price of the non-traded good. But at a certain point, which in this model with these parameters occurs when $R_t^W \approx 1.10$, external debt reaches point $-F^B$ in Figure 1 and past this point the non-binding equilibrium no longer exists, and there is a sudden stop. External debt falls to $-F^C$ and traded good consumption falls sharply, along with a large real exchange rate depreciation. The figure also shows that this sudden stop leads to a sharp drop in total welfare.

Following the sudden stop the economy will begin the next period with external debt $-R^W F^C$ and since $\beta R^W < 1$ agents will borrow and slowly return to the original steady state level of external debt.[10]

[9] A stochastic version of the model with shocks in every period, not just the first period, is solved with a global solution method and presented in the next section. The parameters in this simplified model are the same as those used in the full solution, and are described in the next section.

[10] Recall that the Gabbaix and Maggiori friction pins down a deterministic steady state level of external

4.2 Equilibrium with Policy Intervention

We now allow F_t^{cb} to be the instrument of a benevolent policy maker. From the initial non-binding steady state with external debt $-F^A$ the central bank could buy a stock of foreign bonds F_t^{cb}. By buying one unit of foreign bonds, the central bank is increasing the economy's total foreign assets by $\frac{\partial F}{\partial F^{cb}} = 1 + \frac{\partial F^{fs}}{\partial F^{cb}}$ units, where the intermediary friction $\Gamma > 0$ ensures that $\frac{\partial F^{fs}}{\partial F^{cb}} > -1$. We can't express $\frac{\partial F}{\partial F^{cb}}$ in a closed form solution, but for the purposes of building intuition, from the numerical model and our benchmark parameters, we find that $\frac{\partial F}{\partial F^{cb}} \approx 0.1$. Thus by setting $F_t^{cb} < 0$ the central bank can increase total external debt and increase current period consumption. And thus in response to any shock to the world interest rate that causes private sector deleveraging, the central bank could always sell enough foreign bonds to keep the economy's total external debt above the critical threshold $-F^B$.

But this ignores the constraint that $F_t^{cb} \geq 0$. In this case, the policy of simply waiting until after the shock and setting $F_t^{cb} < 0$ as needed is not possible. Instead the central bank would need to build up a stock of foreign reserves $\bar{F}^{cb} > 0$ in advance. This will increase total external assets in the current period by $\frac{\partial F}{\partial F^{cb}}\bar{F}^{cb}$, and thus reduce next period's initial external debt. This is represented by the shift to the left in the short run borrowing constraint in Figure 3, where the red convex curve plots the period t borrowing constraint starting from the steady state when the central bank has no foreign bonds, and the green curve is the period t borrowing constraint starting from the steady state when the central bank does hold a positive stock of foreign bonds. Recall that the steady state is given by the level of external debt where the short run borrowing constraint intersects the steady state borrowing constraint, so the new steady state level of total external debt will shift to $-F^{A^*} = -F^A - \frac{\partial F}{\partial F^{cb}}\bar{F}^{cb}$.

At this new steady state the central bank holds a stock of bonds $\bar{F}^{cb} > 0$ in every period,

debt, so this toy model with no shocks after period 1 can still be calibrated with $\beta R^W < 1$, and following a sudden stop the economy slowly returns to the original steady state.

so if in a given period the central bank were to set $F_t^{cb} = 0$ then there would be an increase in consumption and external debt in that period and the equilibrium level of external debt would shift from $-F^{A^*}$ back to $-F^A$.[11]

So unlike unrestricted FXI, where the central bank can simply wait until after the shock and set $F_t^{cb} < 0$ as needed to increase current consumption, when faced with the non-negativity constraint on foreign bonds, FXI has an ex-ante and an ex-post component.

The ex-ante component is the reduction in the economy's total external debt when the central bank shifts to a new steady state where it holds $\bar{F}^{cb}$ in foreign bonds. Graphically this is represented by the shift to the left in the short run borrowing constraint in the figure. As the borrowing constraint shifts left, the distance between the non-binding steady state equilibrium and the threshold point where the constraint starts to bind gets larger. This means that a shock would have to be bigger to trigger a sudden stop.

On the ex-post side is the fact when the central bank goes from holding $\bar{F}^{cb}$ to holding 0 foreign exchange reserves, the economy's total external debt and traded goods consumption will increase by $\frac{\partial F}{\partial F^{cb}}\bar{F}^{cb}$ in that period.

Equivalently one can think of FXI as allowing the central bank to control the spread between the domestic interest rate and the foreign interest rate, and thus control external borrowing in the face of shocks to the cost of foreign borrowing. From the equilibrium condition $R_t = R_t^W - \frac{\Gamma}{\beta}F_t^{fs}$, given the fact that $\frac{\partial F^{fs}}{\partial F^{cb}} < 0$, by buying bonds the central bank raises the domestic interest rate and by selling bonds the central bank lowers the domestic interest rate. In response to a low cost of foreign borrowing that would lead to higher external debt and raise financial stability concerns, for macroprudential purposes the central bank can accumulate bonds and curtail borrowing by increasing the spread between the domestic interest rate and world cost of borrowing (see e.g. Arce et al. (2019)). Likewise following a

[11] This is what happens when the central bank "sells" some of their stock of foreign exchange reserves, of course it doesn't make sense to sell a stock of maturing one-period bonds, and at the steady state the central bank simply rolls these bonds over every period. A central bank "sale" of foreign exchange reserves is when the central bank does not roll over these bonds causing total external debt to increase by $\frac{\partial F}{\partial F^{cb}}\bar{F}^{cb}$ in that period

positive shock to the world cost of borrowing that may lead to excessive deleveraging and a sudden stop, the central bank can limit deleveraging by selling bonds and thus reducing the spread between the domestic interest rate and the world cost of borrowing.

Recall that the blue solid line in Figure 2 represents the case where prior to the shock the central bank held no foreign bonds, $F_{t-1}^{cb} = 0$. We now plot the responses to the same shock in a model where prior to the shock the central bank holds a stock of bonds equal to 25% of the traded goods endowment (about 8% of GDP, red dashed line) or 50% of the traded goods endowment (about 16% of GDP, green dashed line). By acquiring more foreign bonds in period $t-1$ the central bank pushes out the threshold value of R_t^w where the economy falls into a sudden stop.

So acquiring foreign bonds is insurance against a large shock to the world interest rate in period t, but this insurance comes at a cost. When the central bank buys foreign bonds, that forces the economy as a whole to save more and is a distortion to the optimal consumption path of relatively impatient agents. Equivalently one could think of the central bank buying and holding a stock of foreign bonds as crowding-out private sector borrowing (see e.g. Chang (2018)). The top half of Figure 4 presents the economy's total external debt and the cutoff value of R_t^W beyond which a sudden stop is triggered as a function of the stock of foreign bonds held by the central bank prior to the shock. The economy's pre-shock external debt is decreasing in the stock of bonds held by the central bank, but the cutoff value of R_t^W is increasing, meaning that a sudden stop is less likely.

Finding the optimal stock of central bank foreign bonds in this model requires imposing a shock process for R_t^W. In this simplified model, we assume that the transitory shock to R_t^W follows a normal distribution, and we calibrate the standard deviation of this normal distribution to ensure that the probability of a sudden stop in the case without foreign exchange intervention is 5%.

Using this shock process for the one-off shock to R_t^W, the bottom half of Figure 4 plots the probability of a sudden stop and total ex-ante welfare as a function of the central bank's stock of foreign bonds. As the stock of bonds increases total ex-ante welfare rises as the diminished probability of a sudden stop outweighs the increasing distortions from forcing relatively impatient agents to save, but at a point the marginal welfare cost of excess saving outweighs the marginal insurance benefit. In this simplified model this point occurs when the central bank's stock of bonds is about 30% of the traded goods endowment, or 9.6% of GDP.

At this point the probability that a shock to the world interest rate would be large enough to trigger a sudden stop is much lower, but it is important to note that as long as the marginal cost to acquiring bonds ex-ante is positive and the probability distribution for R_t^W is continuous, the probability of a sudden stop at the optimal level of foreign bonds remains positive. If the central bank were to acquire so many foreign bonds ex-ante that the probability of a sudden stop is zero then the marginal benefit of acquiring one extra unit of bonds would be zero and yet the marginal cost would be positive. Thus as long as there is a marginal cost to acquiring foreign bonds, the optimal stock of central bank foreign bonds will occur where the marginal benefit to acquiring those bonds is positive.

5 Numerical results for the optimal stock of reserves

In this section we move away from the simplified model and present the numerical results from the full stochastic model. We focus on the optimal policy under discretion.

5.1 Parameters and Calibration

The top panel of Table 1 presents the parameter values that we use to calculate the numerical results. Following Schmitt-Grohé and Uribe (2021), $\alpha = 0.31$, $\beta = 0.91$, $R^W = 1.04$, and $\sigma = 2$. One of the key parameters is the elasticity of substitution between traded and non-traded goods. We follow Schmitt-Grohé and Uribe (2021) and set $\xi = 0.5$. Bianchi (2011) uses $\xi = 0.83$, Benigno et al. (2013) follow the empirical estimate from Ostry and

Reinhart (1992) of $\xi = 0.76$. Stockman and Tesar (1995) use $\xi = 0.44$. Akinci (2017) surveys the empirical literature estimating this elasticity and finds that estimates of this elasticity vary between 0.43 and 1.50 depending on the estimation methodology and the countries sampled. Akinci argues that empirical estimates tend to be lower for the emerging markets, and estimates using a few different methodologies put the elasticity around 0.5 in emerging market countries like Argentina and Uruguay.

We make one adjustment to the calibration in Schmitt-Grohé and Uribe (2021) in order to ensure that a sudden stop crisis happens with sufficient frequency. While Schmitt-Grohé and Uribe (2021) focus on endowment shocks, our model is driven by shocks to the cost of foreign borrowing. With the different shock process the probability of a crisis is different, and in order to ensure that the probability of a crisis is around 5% in the competitive equilibrium without policy intervention, we lower the coefficient in the borrowing constraint from $\kappa = 0.32R^W$ in Schmitt-Grohe and Uribe to $\kappa = 0.27R^W$.

We have no prior for the value of the financial intermediation friction Γ, and thus Γ is calibrated to match a certain value for the steady state level of private external borrowing:

$$\frac{\lambda}{\beta E(\lambda)} = R^W - \frac{\Gamma}{\beta} F^{fs}$$

where $\frac{\lambda}{E(\lambda)} < 1$ depends on the amount of uncertainty in the economy and the curvature of the utility function. If the value of Γ is too low then the borrowing constraint binds in the steady state. If it is too high then starting from the steady state equilibrium level of $-F^{fs}$ the short-term borrowing constraint does not cross the 45 degree line, and thus sudden stops are very rare. In order to have a meaningful probability of a sudden stop, we set $\Gamma = 0.05$, which makes the steady state level of external debt high enough to make a sudden stop possible following a sufficiently large shock, but low enough that the constraint is not binding in the steady state.[12]

The model is driven by exogenous shocks to the world cost of borrowing, R_t^W. To ap-

[12] Adrian et al. (2022) set $\Gamma = 0.02$ for advanced economies and $\Gamma = 0.06$ for emerging markets.

proximate this process we consider a process for the world risk-free rate. These shocks follow an AR(1) process with persistence coefficient 0.572 and standard deviation 0.02. To approximate the equilibrium, we use a time iteration procedure over a discretized state space, and the bottom panel of Table 1 provides information for the discretization of the state space. We discretize the shock into 11 grid points, the endogenous state $-F_t$ into 300 grid points and central bank reserves F_t^{cb} into 300 grid points. For ease of exposition, we denote the median R^W as the 'steady state R^W'.

There is only one endogenous state variable in the model, the total external debt $-F_{t-1}$. It is important to note that for a given value of the endogenous state in period $t-1$, $-F_{t-1}$, the policy variable F_t^{cb} is a choice variable with the constraint that $F_t^{cb} \geq 0$. The choice of F_t^{cb} will then affect the endogenous state variable in period t, $-F_t$, but in period $t+1$, it is the state variable $-F_t$ that matters.

As discussed in section 4 above, while the model admits multiple expectational equilibria, we maintain a particular equilibrium selection criterion, and focus on the role of shocks to fundamentals to generate sudden stops. Following a shock to R_t^W, if agents' first-order conditions and the other equilibrium conditions in the model are satisfied at a level of external debt $-\hat{F}$ where the borrowing constraint is not binding (to the right of point B in Figure 1), we pick this as the equilibrium. If on the other hand agents' first-order conditions and the other equilibrium conditions in the model are satisfied at a level of external debt $-\hat{F}$ where the borrowing constraint is binding (to the left of point B), the only remaining equilibrium is the sudden stop equilibrium (point C).

5.2 Numerical results

5.2.1 Policy function

The policy function for the optimal choice of F_t^{cb} as a function of the endogenous state, $-F_{t-1}$, and the exogenous state, R_t^W, is presented in Figure 5. The blue solid line shows the central bank's optimal choice of F_t^{cb}, and the red dashed line plots the multiplier on the borrowing constraint, which changes from 0 to a positive number when the sudden stop occurs. All quantity variables such as total external debt and the stock of central bank reserves are presented as a percent of GDP.

Begin with the middle panel in the figure, this panel plots the optimal choice of F_t^{cb} as a function of $-F_{t-1}$ when R_t^W is equal to its steady state value. The figure shows that as long as the initial stock of external debt is less than 28.5 percent of GDP, a crisis does not occur in period t. Intuitively, this implies that the equilibrium $-F_t$ along the 45 degree line in Figure 1 remains to the right of point B when the initial stock of external debt is less than 28.5% of GDP. Then the policy function for F_t^{cb} for a low stock of initial external debt is zero. When external debt is this low, the probability of a sudden stop in period $t+1$ is zero and thus the central bank sees no need to distort the economy today by buying reserves as insurance against a possible crisis tomorrow. But the figure shows that for an external debt greater than 28% but less than 28.5% the central bank will start acquiring reserves F_t^{cb} as precaution against a large positive shock in period $t+1$. As the initial stock of external debt increases the probability of a sudden stop increases, and thus the marginal benefit of F_t^{cb} increases. The central bank's optimal F_t^{cb} increases right up to the point where the debt is high enough that a sudden stop crisis would have happened in period t. At the highest point, central bank reserves are about 10% of GDP.

The top panel of the figure shows the same policy function when R_t^W is below its steady state value, and the bottom panel shows the same policy functions when R_t^W is above its steady state value. Notice that the central bank buys more reserves when the world interest rate is low than when the world interest rate is rate is high. When the world interest rate is low and private agents are selling more foreign bonds and borrowing more from the rest of the world, the central bank leans against the wind and buys more foreign bonds. When the world interest rate is high and private agents are deleveraging and buying more foreign bonds, the central bank again leans against the wind by reducing its stock of foreign bonds.[13]

[13]The empirical evidence in Appendix A shows that when the world real interest rate decreases, countries would accumulate foreign reserves, particularly for emerging market economies.

5.2.2 Density of External Debt

Figure 6 plots the density of external debt in simulations of the model. In this figure we simulate the model over $T = 10^6$ periods and plot the density of the distribution of total external debt, $-F_t$, over these simulations.[14] The blue solid line plots the density when the central bank does not engage in foreign exchange intervention and $F^{cb} = 0$, the red dashed line plots the density when the central bank engages in optimal foreign exchange intervention, described by the policy functions in Figure 5.

When the central bank does not engage in foreign exchange intervention the density of $-F_t$ has a large mass around 28.5 percent of GDP and a long left tail. With no policy intervention, after a long string of shock realizations of zero, the economy would settle to a 'steady state' level of external debt a little less than 29 percent of GDP.[15] Negative shocks to R_t^W would lead agents to hold more debt and positive shocks to R_t^W would lead agents to hold a little less debt. But as the density figure shows, at a point around an external debt level of 28% the density drops. This is where the sudden stop occurs at point B in Figure 1. If the shock is large enough to trigger a sudden stop then the economy's total external debt falls to less than 18 percent of GDP and then begins a slow process of releveraging as the economy returns to the steady state.

The density of external debt under optimal foreign exchange intervention shows that optimal policy nearly eliminates the probability of a sudden stop. While it is difficult to see the scale on this graph, but there is a small weight in the left tail in the optimal FXI density, but the weight is very close to zero. As discussed earlier, this probability is never zero, but in these simulations the probability is small.

But the density plots in Figure 6 show that the density in the non-binding region, where

[14] We use a smoothing parameter 20. That is, figure 6 presents the average density between grid point $k - 19$ and grid point k of the original distribution of external borrowing as the density of grid point k.

[15] The dynamic models have a stationary distribution of external debts. To facilitate comparison with the deterministic steady state without shocks, we still use the term 'steady state' to describe the situation in which the economy stays when world interest rate is at its middle level for a long time given policy functions obtained in a dynamic model. This situation is sometimes called 'risky steady state' (Coeurdacier et al. (2011)).

the economy is not in a sudden stop, is shifted to the left under optimal FXI. As analyzed before, domestic agents would accumulate less external debt in normal times under the optimal FXI than that without FXI, and therefore the median level of external debt under the optimal FXI is lower, while the mean level of debt under the optimal FXI is higher than that without FXI due to the left tail in the density of external debt in the economy without FXI. Therefore, the density of external debt under the optimal FXI clusters around 28.5 percent of GDP. As discussed earlier, optimal foreign exchange accumulation is insurance against a crisis, but the cost is that it forces the economy to save more than it otherwise would in normal times.

5.2.3 Event Analysis

We now turn to event analysis to examine the dynamics of a typical financial crisis in the model to see how foreign exchange intervention reduces the probability of a sudden stop.

We construct an event analysis as follows. In a simulation path with length $T = 10^6$, a crisis is defined as a binding credit constraint $\mu_t > 0$ in period $t = 0$ in the competitive equilibrium with no foreign exchange intervention, and an event is a window of 11 periods from $t = -5$ to $t = 5$. We average all such events along the simulated path above. Figure 7 presents the path of the world interest rate, total external debt, the central bank's stock of foreign exchange reserves, tradable consumption, the price of non-tradables, the current account, the domestic interest rate, the multiplier on the borrowing constraint, and the spread between the domestic interest rate and the world interest rate during the average of these events. The solid blue line in the figure plots the event in the economy with no foreign exchange intervention, and the dashed red line presents the responses of the same variables to the same path of the exogenous world interest rate and conditional on identical external debt in the first period of the event in the economy with optimal foreign exchange intervention.

The exogenous shock that triggers a crisis event is an increase in the cost of foreign borrowing. The figure shows that the average crisis is triggered by an increase from 3% to 7%. It is interesting to note that in the period before the crisis the cost of foreign borrowing was slightly less than the steady state value of 4%. This indicates that at least some of the crisis episodes occur following a period of low borrowing costs which lead to a build-up external debt. This shock is enough to cause agents to delever to a point to the left of point B in Figure 1, triggering a sudden stop and leading to a rapid deleveraging to a much lower level of external debt. This deleveraging implies a sharp fall in traded goods consumption, a large depreciation of the real exchange rate, and a sharp increase in the current account.

The red dashed line plots the same variables in the same events (the window of ± 5 periods around a crisis in the economy without foreign exchange intervention), but now allowing the central bank to implement the optimal foreign exchange intervention, as show in the policy functions in Figure 5. Under optimal foreign exchange intervention, external debt is lower in the periods leading up to the crisis. The same shock now causes some deleveraging, but not enough to trigger a crisis. Thus there is some fall in traded goods consumption and real exchange rate depreciation, but substantially less depreciation relative to the outcome without policy intervention.

The response of the domestic interest rate and the spread over the world cost of borrowing provides an interesting insight into the optimal FXI response. Prior to the crisis there is a spread of about 4.5% in the economy without FXI and 6% in the economy with FXI. By holding foreign bonds and forcing relatively impatient households to save, the central bank is keeping the domestic interest rate higher.

Finally, the graph of the central bank's optimal stock of bonds in the bottom right hand corner shows that in the periods leading up to the crisis the central bank adds to the stock of bonds as a precautionary measure in the face of low world interest rates. Following the shock the central bank does not roll over these bonds in order to maximize consumption in the period of the shock.

Table 2 calculates the probability of a crisis, the 'steady state' level of external debt, and the mean level of external debt as a percent of GDP in the model with and without FXI calculated from the same simulation over $T = 10^6$periods.[16] The probability of a crisis falls from nearly 5 percent to 0.03 percent when the central bank engages in optimal foreign exchange intervention. The 'steady state' level of external debt is lower under optimal FXI than it is under the competitive equilibrium without policy intervention. However, the mean level of external debt is lower in the model without policy intervention due to the strong left skew on the distribution of external debt in this case.

6 Summary and Conclusion

This paper presents a simple model where sudden stops in a small open economy are rare discrete events triggered by an increase in foreign borrowing costs. Deleveraging is the rational response of private agents to this increase in foreign borrowing costs, but there is a pecuniary externality in that agents do not internalize the effect of their deleveraging on the price of collateral. Because of this, for even a relatively small increase in the cost of foreign borrowing, the economy can fall into a sudden stop equilibrium. A policy maker that does internalize the effect of deleveraging on the price of collateral will set policy to keep the economy out of this sudden stop equilibrium.

When the tools available to the policy maker are the central bank purchases and sales of foreign bonds some interesting questions arise. The central bank's holdings of foreign bonds can't be negative, so in order to reduce the stock of bonds purchased following the shock the central bank must buy a positive stock of bonds before the shock. This central bank purchase of foreign bonds is distortionary since it forces economy wide savings to be higher than it otherwise would be. This paper attempts to provide a framework for assessing the marginal costs and marginal benefits of holding a stock of reserves in order to pinpoint the

[16]The dynamic models have a stationary distribution of external debts. To facilitate comparison with the deterministic steady state without shocks, we still use the term 'steady state' to describe the situation in which the economy stays when world interest rate is at its middle level for a long time given policy functions obtained in a dynamic model. This situation is sometimes called 'risky steady state', see Coeurdacier et al. (2011)

optimal stock of foreign bonds held by the central bank to safeguard against a sudden stop driven by an increase in the world risk-free interest rate.

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Table 1: Parameter values

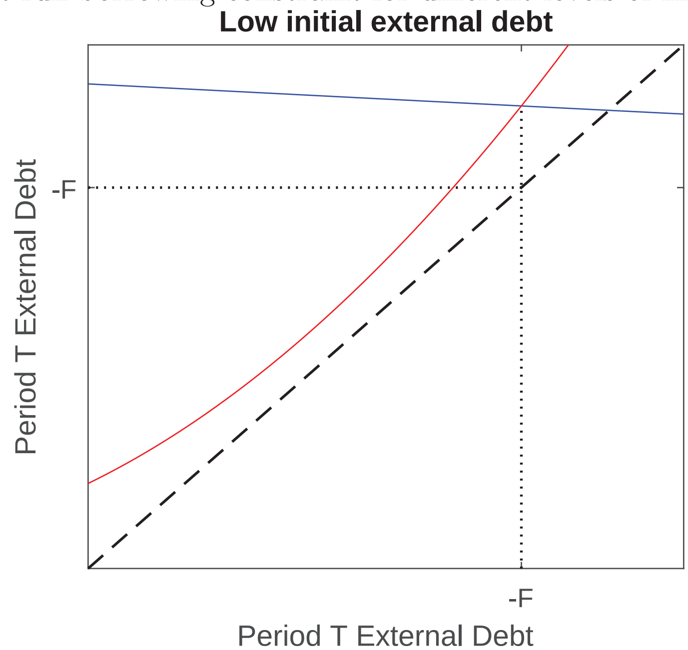
Parameter	Value	Description
		Structural Parameters
R^W	1.04	Annual gross world interest rate
σ	2	Inverse of intertemporal elasticity of consumption
κ	$0.27R^w$	Parameter in borrowing constraint
β	0.91	Subjective discount factor
x	1	endowment of traded goods
y	1	endowment of non-traded goods
α	0.31	Weight on traded goods in CES aggregator
ξ	0.5	Elasticity of substitution between traded/non-traded goods
Γ	0.05	Financial intermediation friction
		Discretization of State Space
$\left[\ln \frac{R^W_{\min}}{R^W} \;\; \ln \frac{R^W_{\max}}{R^W}\right]$	$[-0.071, 0.071]$	Range for the world interest rate
$[-F_{\min} \;\; -F_{\max}]$	$[0.2, 1.0]$	Range for total external debt
$\left[F^{cb}_{\min} \;\; F^{cb}_{\max}\right]$	$[0, 0.5]$	Range for foreign exchange reserves
n_{Rw}	11	number of grid points for $\ln \frac{R^W_t}{R^W}$, equally spaced
n_F	300	number of grid points for $-F_t$, equally spaced
n_{Fcb}	300	number of grid points for F^{cb}_t, equally spaced

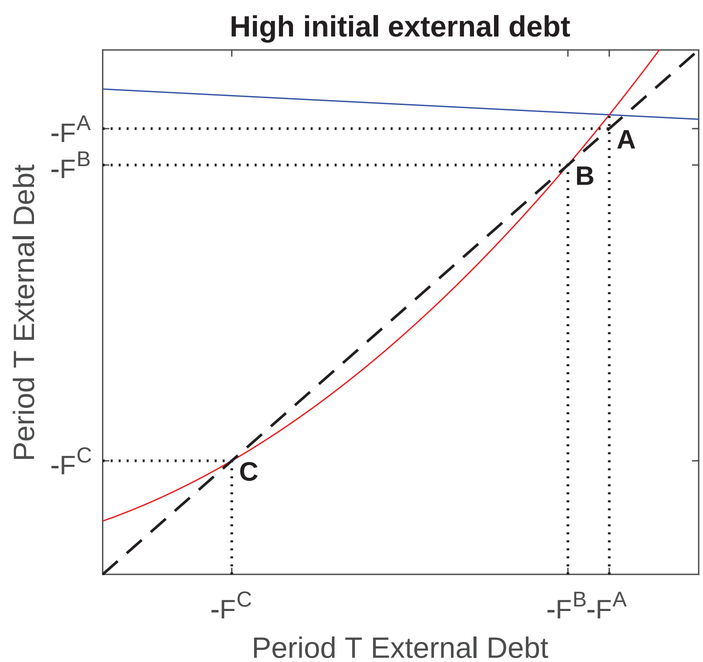
Table 2: Crisis probability, mean, and steady state levels of external debt in the model with and without optimal foreign exchange intervention.

	No FXI	Optimal FXI
Crisis Probability	0.0481	0.0003
‘Steady State’ $-F_t$	28.6	28.4
Mean $-F_t$	26.9	28.4

Notes: This table reports the probability of a crisis and the steady state and mean levels of external debt under the competitive equilibrium with no policy intervention and under optimal foreign exchange intervention. The simulation period is $T = 10^6$.

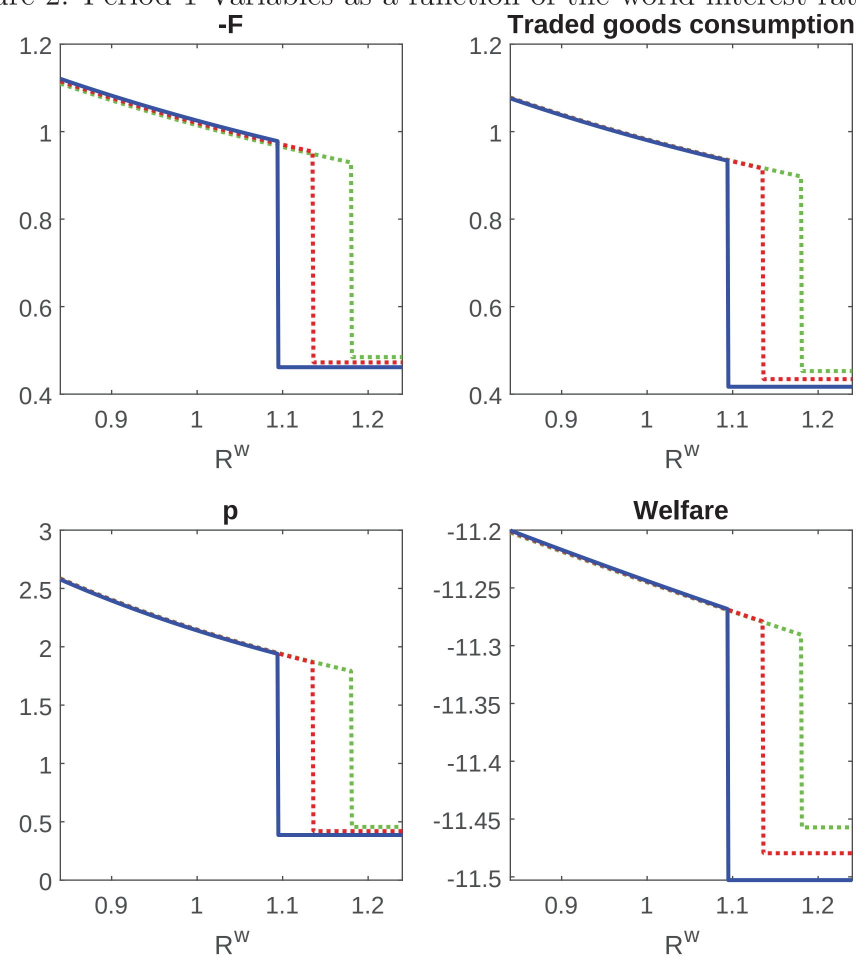
Figure 1: The short run borrowing constraint for different levels of initial external debt.





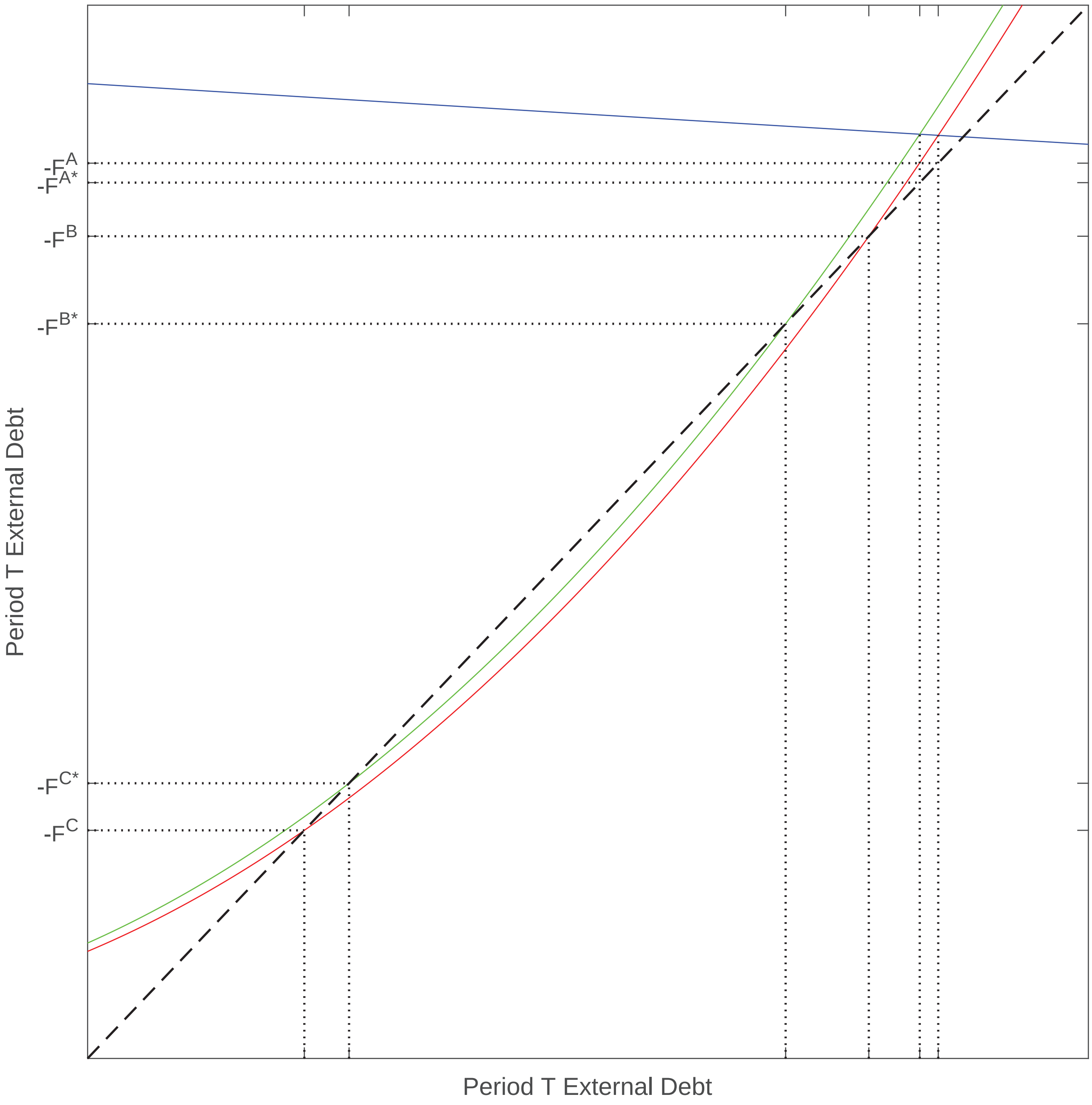
Notes: The blue line represents the steady state borrowing constraint. The red line represents the period t borrowing constraint taking the level of external debt in period $t-1$ as given.

Figure 2: Period 1 Variables as a function of the world interest rate Rw



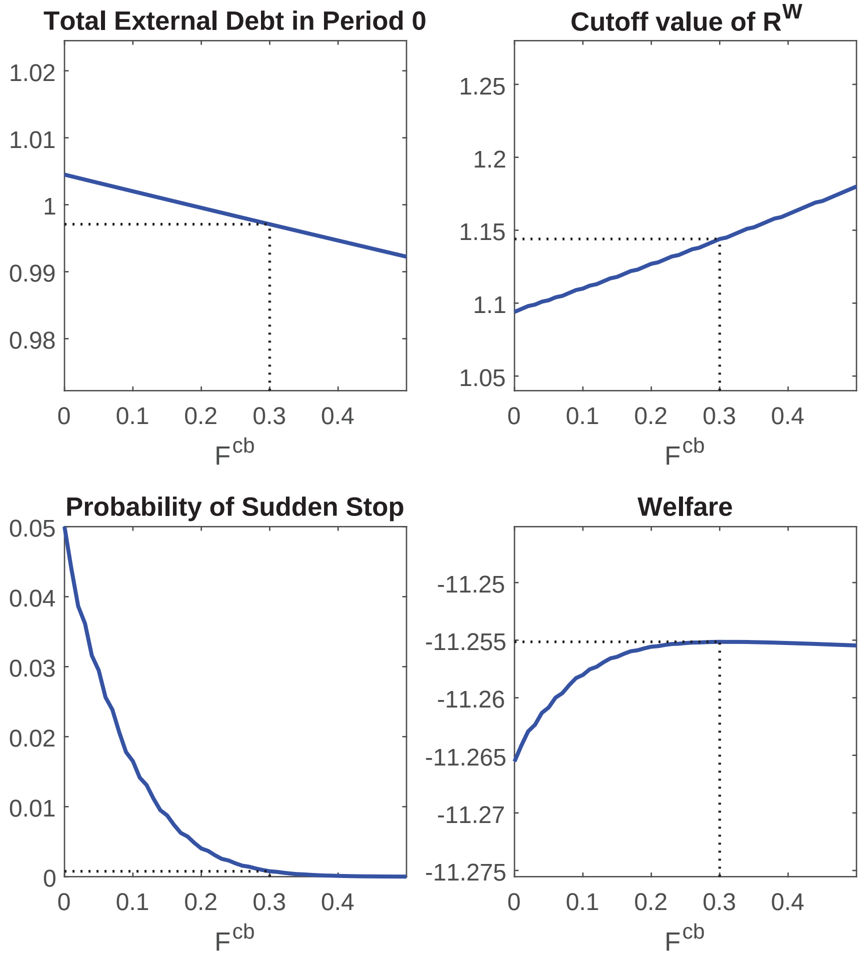
Notes: The blue line is the equilibrium when the central bank acquires no foreign bonds in period 0 and thus there is no foreign exchange intervention in period 1. The red line is when the central bank acquires foreign exchange reserves of 25% of the traded goods endowment. The green line is where the central bank acquires foreign exchange reserves of 50% of the traded good endowment.

Figure 3: Ex-ante and ex-post effect of foreign exchange intervention



Notes: The blue line represents the steady state borrowing constraint. The red line represents the period t borrowing constraint taking the level of external debt in period $t-1$ as given where the parameters in the model ensure that $-F^A$ is the steady state level of external debt and the central bank does not hold a stock of foreign exchange reserves. The green line represents the period t borrowing constraint given the same model parameters as the red line, but the central bank holds a stock of foreign exchange reserves.

Figure 4: Period 0 external debt, the cutoff value of Rw, the probability of a sudden stop, and total welfare as a function of the stock of foreign exchange reserves



Notes: Foreign exchange reserves F^{cb} expressed as a share of the traded goods endowment.

Figure 5: The policy function for reserve accumulation (blue solid line) and the multiplier on the borrowing constraint (red dashed line) as a function of external borrowing and the exogenous state. Low, Mid and High R_t^W denote the lowest level, middle level and highest level of world interest rate, respectively.

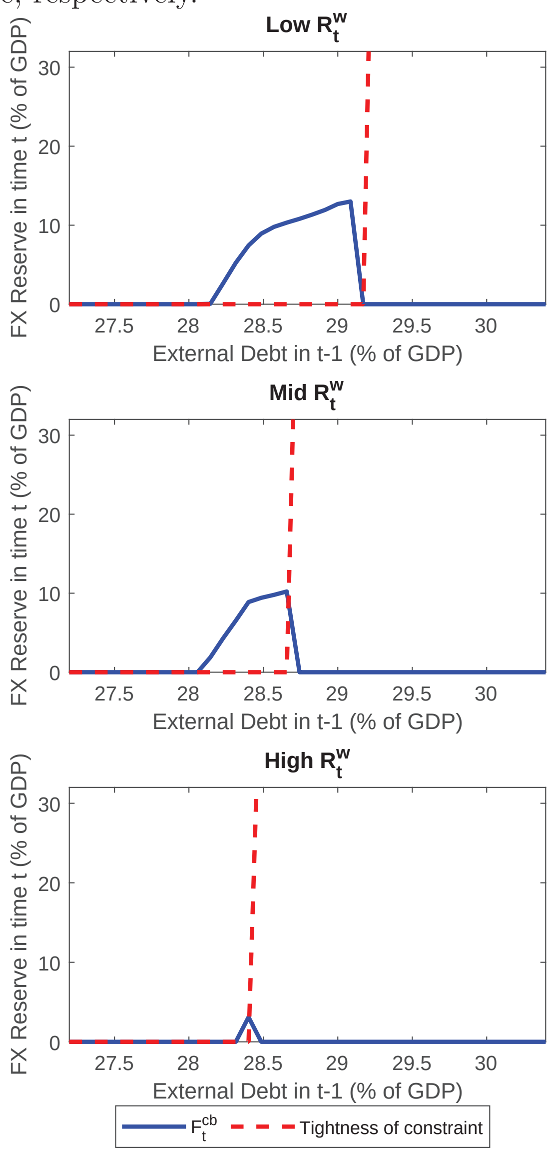


Figure 6: The density of external debt in period t in the model without foreign exchange intervention and in the model with optimal foreign exchange intervention. The smooth parameter is 20 (average of density between grid point $k-19$ and k).

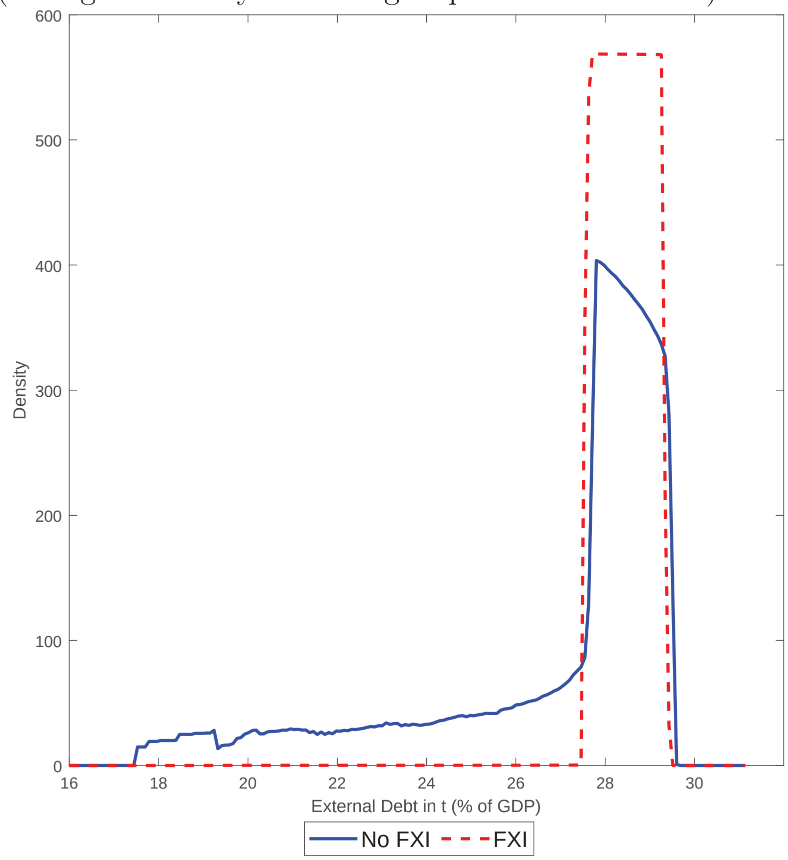
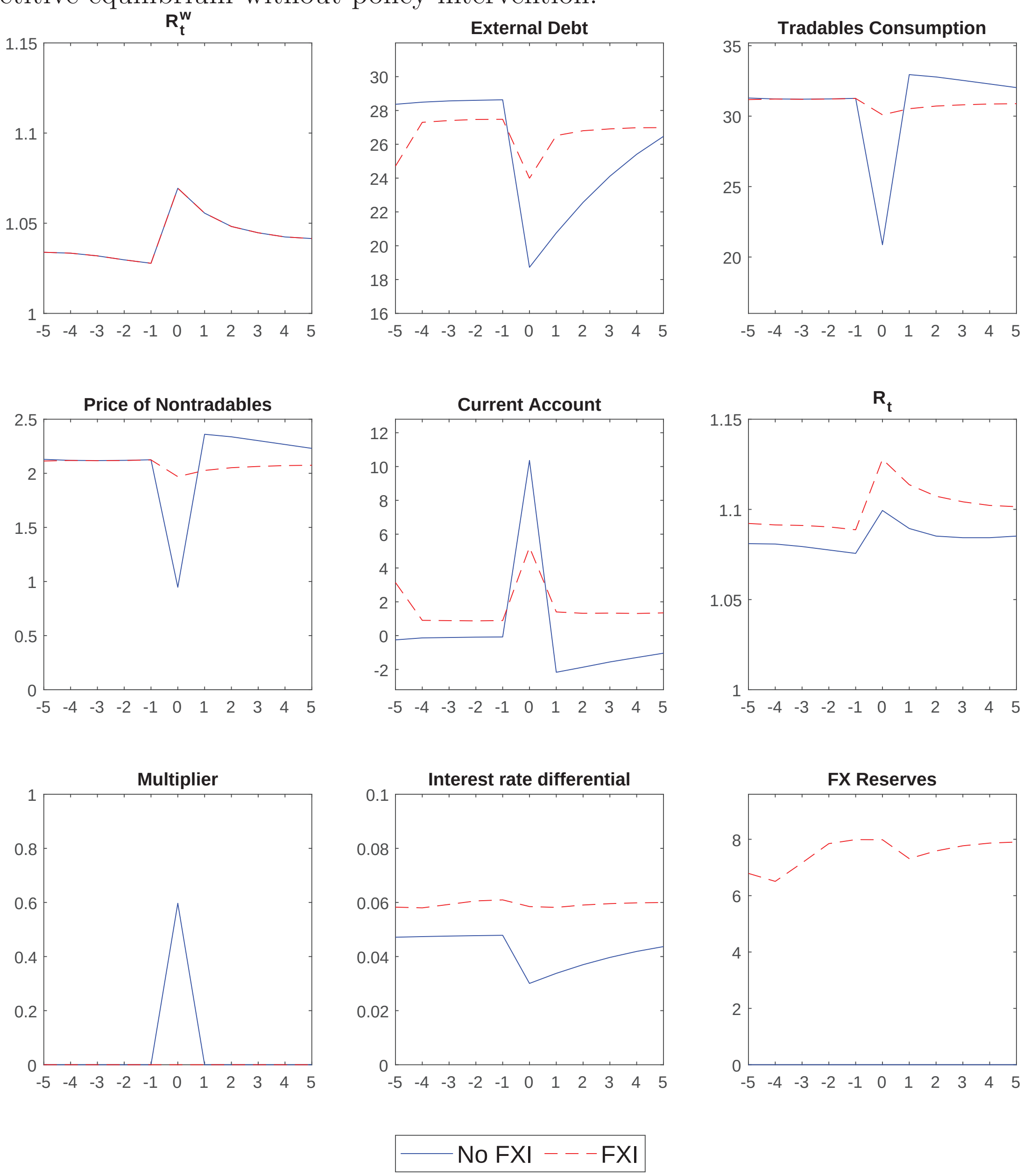


Figure 7: An event in a typical financial crisis when there is no foreign exchange intervention (blue solid lines) and the corresponding event when there's foreign exchange intervention (red dashed lines) along a simulated period of $T = 10^6$. A crisis occurs at period $t = 0$ in the competitive equilibrium without policy intervention.



Notes: Interest rates, Lagrange multiplier and nontraded good price are expressed in level and other variables are expressed as percentage point of GDP.

Online Appendix - Not for Publication

A Empirical motivation

Our paper focuses on how an increase in the cost of foreign borrowing can trigger a sudden stop and consequently lead to a financial crisis, and in normal times when the cost of foreign borrowing stays low, central banks in small economies may accumulate reserves to prevent future financial crisis. Here we provide a supportive evidence to our theoretical model specification and the use of foreign exchange reserves by central banks. We specify an empirical analysis as follows,

$$\frac{\text{FX reserves}_{i,t}}{\text{GDP}_{i,t}} = \gamma_1 R_t^W + \gamma_2 \frac{\text{FX reserves}_{i,t-1}}{\text{GDP}_{i,t-1}} + \gamma_3^{'} X_{it-1} + \gamma_4^{'} Z_t + \eta_{it} \tag{A.1}$$

$$\text{Prob.}(crisis = 1)_{it} = \beta_1 \Delta R_t^W + \beta_2 \frac{\text{FX reserves}_{i,t-1}}{\text{GDP}_{i,t-1}} + \beta_3 \Delta R_t^W \times \frac{\text{FX reserves}_{i,t-1}}{\text{GDP}_{i,t-1}} + \beta_4^{'} X_{it-1} + \beta_5^{'} Z_t + \epsilon_{it} \tag{A.2}$$

The first specification (A.1) shows how foreign reserves in a small open economy change with the world interest rate R_t^W. The second specification (A.2) tries to understand how the world interest rate and foreign reserves affect the incidence of a crisis. We focus on a currency crisis, which is taken from Laeven and Valencia (2020). Independent variables include the world interest rate R_t^W in (A.1) or the change in world interest rate ΔR_t^W in (A.2), lagged foreign exchange reserves over GDP, $\frac{\text{FX reserves}_{i,t-1}}{\text{GDP}_{i,t-1}}$ in country i period $t-1$, other country specific control variables are stacked in vector $X_{i,t-1}$ and other global control variables are stacked in vector Z_t. The last term η_{it} and ϵ_{it} are error terms. The world interest rate is the US 1-Year real treasury constant maturity rate. Definitions of variables and data sources are presented in table 1.

Table 2 shows the empirical results for specification (A.1). When the world real interest rate decreases, countries would accumulate foreign reserves, particularly for emerging markets $\gamma_1 < 0$ (column (1)-(2)). Foreign reserves are highly persistent in the data sample, $\gamma_2 = 0.89$ in column (3)-(4) of table 2; nevertheless, the coefficient for the world interest rate is still significantly negative. Table 3 reports regression results for our empirical specification (A.2) by using a linear probability model and a Probit model. Results show that a rise in US interest rate leads to a significant increase in currency crisis incidence $\beta_1 > 0$, particularly for emerging and developing economies, after controlling for country fixed effects and other domestic and global factors. Movement in the world interest rate is one of key driving forces of a financial crisis. At the same time, a higher stock of foreign exchange reserves before the onset of a financial crisis reduces the incidence of a crisis both for emerging and advanced economies $\beta_2 < 0$. More importantly, the coefficient for the interaction term between the change of world interest rate and the lagged reserves-GDP ratio β_3 is significantly negative for emerging and developing economies.

Figure 1 displays how the lagged reserves-GDP ratios affect the average marginal effects of the change of world interest rate on crisis incidence $\frac{\partial \text{Prob.}(crisis=1)_{it}}{\partial \Delta R_t^W}$. The figures show that higher reserve-GDP ratios before a crisis leads to a lower crisis incidence. When the lagged reserves-GDP ratio is at its medium level (reserves/GDP=0.08), the average marginal effect is 0.006. Note that the standard deviation of percentage change of R_t^W is 2.3, and therefore the currency crisis will increase by $0.006 \times 2.3 = 0.014$ when the world interest rate increases by a standard deviation 2.3. This magnitude is economically significant compared to the unconditional currency crisis incidence 0.04 in the data sample. When reserves-GDP ratios move from 50 percentile (reserves/GDP=0.08) to bottom 5 percentile (reserves/GDP=0.01), currency crisis probability increases by $(0.008-0.006)\times 2.3 = 0.0046$. When foreign reserves increases from the bottom 5 percentile (reserves/GDP = 0.01) to the upper 95 percentile (reserves/GDP=0.37), currency crisis incidence declines by $(0.008 - 0.0005) \times 2.3 = 0.017$ when responding to a standard deviation rise in the world interest rate. This reduction in crisis due to reserve accumulation could account for almost 40% decline in the unconditional probability of currency crisis.

Table 1: Variable definitions and data sources

Variables	Description and data source
Data coverage	1970 – 2015 unbalanced panel data
Currency crisis	from Laeven and Valencia (2020)
R_t^W	US 1-Year Treasury Constant Maturity Rate (Percent) mimus CPI inflation expeaction. Interest rate is taken from Federal Reserve Economic Data and inflation expectation is from the Livingston Survey by Federal Reserve Bank of Philadelphia.
FX reserves/GDP	FX Reserves minus gold/GDP, from Lane and Milesi-Ferretti (2007, 2018)
FI/GDP	Financial integration/GDP. Financial integration is the sum of external assets and liabilities for a given country. External assets and liabilities are taken from Lane and Milesi-Ferretti (2007, 2018)
CA/GDP	Current account/GDP, from Lane and Milesi-Ferretti (2007, 2018)
Real GDP per capita	from IMF International Financial Statistics
Energy	Energy price index, from World Bank Commodity Price Data (The Pink Sheet)
Nonenergy	Non-energy price index, from World Bank Commodity Price Data (The Pink Sheet)
Economies covered by EMBI+	Algeria, Argentina, Azerbaijan, Belarus, Belize, Brazil, Bulgaria, Chile, China, Colombia, Cote d'Ivoire, Croatia, Dominican Republic, Ecuador, Egypt Arab Rep., El Salvador, Gabon, Georgia, Ghana, Greece, Guatemala, Hungary, India, Indonesia, Iraq, Jamaica, Jordan, Kazakhstan, Korea Rep., Lebanon, Lithuania, Malaysia, Mexico, Mongolia, Morocco, Namibia, Nigeria, Pakistan, Panama, Peru, Philippines, Poland, Russian Federation, Senegal, South Africa, Sri Lanka, Thailand, Trinidad and Tobago, Tunisia, Turkey, Ukraine, Uruguay, Venezuela RB, Vietnam, Zambia
Advanced economies	Austria, Australia, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Iceland, Ireland, Israel, Italy, Japan, Latvia, the Netherlands, New Zealand, Norway, Portugal, Slovak Republic, Slovenia, Spain, Sweden

Table 2: US interest rate and foreign exchange reserves

	(1)	(2)	(3)	(4)
	OLS All	OLS EMBI+	OLS All	OLS EMBI+
R_t^W	-0.003***	-0.005***	-0.001**	-0.001*
	(-4.060)	(-4.445)	(-2.015)	(-1.910)
Lagged FX reserves/GDP			0.892***	0.886***
			(19.371)	(15.312)
Lagged FI/GDP	0.009	0.041***	-0.002	-0.002
	(1.186)	(3.198)	(-1.146)	(-0.512)
Lagged CA/GDP	0.140*	0.108	0.025	0.025
	(1.723)	(1.205)	(1.320)	(1.190)
Lagged Real GDP per capita	-0.002**	0.002	0.000	0.000
	(-2.503)	(0.592)	(0.099)	(0.201)
Energy price	0.000***	0.000***	0.000	-0.000
	(3.793)	(3.176)	(0.514)	(-0.069)
Nonenergy price	0.001**	0.000	0.000	0.000
	(2.350)	(1.482)	(0.004)	(0.800)
Observations	2,729	1,917	2,729	1,917
Number of iso_code	78	55	78	55
Adjusted R-squared	0.191	0.286	0.806	0.813

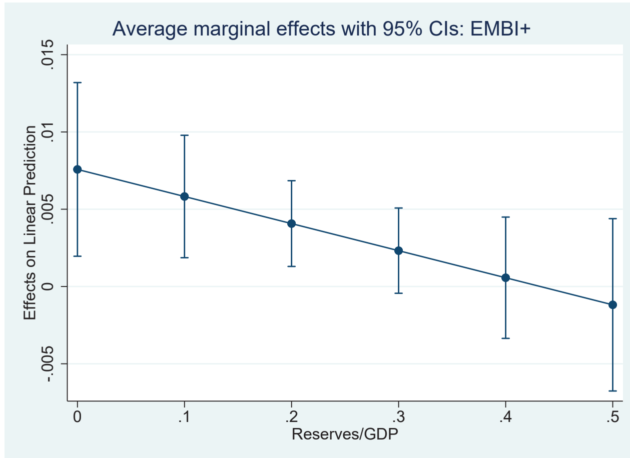
Notes: FX reserves are foreign exchange reserves excluding gold. FI denotes financial integration, which is the sum of external assets and liabilities. CA stands for current account. All is for the whole data sample and $EMBI+$ for economies covered by J.P. Morgan Emerging Markets Bond Indix Global. Significance is denoted as $***\ p < 0.01$, $**\ p < 0.05$, $*\ p < 0.1$. Standard errors are clustered at country code and robust t-statistics is reported in parentheses. OLS denotes a linear probability model using a panel regression with country fixed effects. The data sample covers 78 emerging, developing and advanced economies from $1970 - 2015$. We drop two years immediately after a currency crisis and treat non-crisis years as normal times.

Table 3: US interest rate, foreign exchange reserves and currency crisis incidence

	(1)	(2)	(3)	(4)
	OLS All	OLS EMBI+	Probit All	Probit EMBI+
ΔR_t^W	0.005**	0.008**	0.045*	0.063**
	(2.017)	(2.643)	(1.815)	(2.198)
Lagged FX reserves/GDP	-0.180***	-0.228***	-3.487***	-3.949***
	(-3.316)	(-2.932)	(-2.892)	(-2.962)
$\Delta R_t^W \times$ Lagged FX reserves/GDP	-0.013	-0.018*	-0.143	-0.031
	(-1.552)	(-1.729)	(-0.555)	(-0.162)
Lagged FI/GDP	-0.004	0.008	-0.046	0.082
	(-0.466)	(0.494)	(-0.496)	(0.686)
Lagged CA/GDP	-0.106	-0.073	-1.508**	-1.071
	(-1.522)	(-1.114)	(-2.074)	(-1.423)
Lagged Real GDP per capita	0.001	0.009**	-0.012*	0.040*
	(1.587)	(2.620)	(-1.789)	(1.823)
Energy price	-0.001	-0.001**	-0.008*	-0.011**
	(-1.628)	(-2.408)	(-1.874)	(-2.408)
Nonenergy price	0.001	0.001	0.011	0.012
	(0.960)	(1.296)	(1.500)	(1.531)
Observations	2,729	1,917	2,729	1,917
Number of countries	78	55	78	55
Adjusted R-squared	0.014	0.023		

Notes: Currency crisis is a dummy variable, equals to one when there is a currency crisis in year t and zero otherwise. FX reserves are foreign exchange reserves excluding gold. FI denotes financial integration, which is the sum of external assets and liabilities. CA stands for current account. All is for the whole data sample and $EMBI+$ for economies covered by J.P. Morgan Emerging Markets Bond Indix Global. Significance is denoted as $*** \ p < 0.01$, $** \ p < 0.05$, $* \ p < 0.1$. Standard errors are clustered at country code and robust t-statistics is reported in parentheses. OLS denotes a linear probability model using a panel regression with country fixed effects. Probit denotes a panel Probit model. The data sample covers 78 emerging, developing and advanced economies from $1970 - 2015$. We drop two years immediately after a currency crisis and treat non-crisis years as normal times.

Figure 1: This figure shows the average marginal effects of the change of world interest rate, ΔR_t^W, on currency crisis for countries covered by EMBI+, conditional on different reserves/GDP ratio in the linear probability models.



B Microfoundations for the financial sector

A key feature of the model is the presence of frictions in international financial markets. Private households do not directly hold foreign bonds, but must trade with financial intermediaries who can borrow and lend on international financial markets subject to enforcement costs. The financial sector is made up of a continuum of identical atomistic financiers, indexed $i \in [0, 1]$. Financiers issue bonds on the international market, and use the proceeds to buy home bonds from domestic households. Financiers begin each period with zero net worth. They then issue $F_t^{fs}(i)$ international bonds and purchase $B_t^{fs}(i)$ domestic bonds. By intermediating the borrowing from domestic households, financiers thus take a positive position in B_t^{fs} and a negative position in F_t^{fs}, where $B_t^{fs}(i) + F_t^{fs}(i) = 0$. After aggregating across all atomistic financiers the balance sheet for the financial sector is given by:

$$B_t^{fs} + F_t^{fs} = 0 \tag{B.1}$$

where $F_t^{fs} = \int_0^1 F_t^{fs}(i)\, di$ and $B_t^{fs} = \int_0^1 B_t^{fs}(i)\, di$.

Note that both international bonds and domestic bonds are denominated in units of the traded good. Thus, unanticipated movements in the real exchange rate have no impact on financiers' balance sheets through currency mismatches. But, as we establish below, since financiers act as an intermediary between households and international financial markets, this drives a wedge between the domestic interest rate, R_t, and the exogenous world interest rate R_t^W. Both returns are denominated in traded goods.

The net interest income from the financier's domestic and international bond portfolio is:

$$\Pi_t(i) = R_t^W F_t^{fs}(i) + R_t B_t^{fs}(i) \tag{B.2}$$

The total net interest income from financiers $\Pi_t = \int_0^1 \Pi_t(i)\, di = R_t^W F_t^{fs} + R_t B_t^{fs}$, is remunerated lump-sum to households.

Each atomistic financier is in operation for a single period, and their objective is to maximize the discounted net interest income from bonds purchased in period t:

$$\beta \Pi_t(i) = \beta \left(R_t^W F_t^{fs}(i) + R_t B_t^{fs}(i) \right) = \beta \left(R_t^W - R_t \right) F_t^{fs}(i) \tag{B.3}$$

As in Gabaix and Maggiori (2015) we assume that financiers have an incentive to divert the funds they receive from issuing foreign bonds. After taking the position $F_t^{fs}(i) < 0$, the financier can divert a share $\Gamma \left| F_t^{fs}(i) \right|$ of their credit position $\left| F_t^{fs}(i) \right|$, where Γ is a non-negative constant. If the financier diverts the funds their firm is unwound and the proceeds are returned to the creditor. Since creditors correctly anticipate the ability and motivation of the financier to divert funds, financiers are subject to the following incentive compatibility constraint:

$$\beta \Pi_t(i) \geq \Gamma \left| F_t^{fs}(i) \right| \times \left| F_t^{fs}(i) \right| = \Gamma \left(F_t^{fs}(i) \right)^2 \tag{B.4}$$

The financiers maximization problem is to choose $F_t^{fs}(i)$ to maximize $\Pi_t(i)$ subject to this incentive compatibility constraint. Since the value of the financier's firm, $\Pi_t(i)$, is linear in $F_t^{fs}(i)$ and the right hand side of this constraint is convex in $F_t^{fs}(i)$, the constraint always binds. Thus:

$$R_t^W - R_t = \frac{\Gamma}{\beta} F_t^{fs}(i) \tag{B.5}$$

If $\Gamma = 0$ then the financial sector is simply a veil and the equilibrium condition for foreign bond holding is $R_t = R_t^W$, exactly as it would be if households could borrow directly from foreigners and faced no frictions. But when $\Gamma > 0$, and $F_t^{fs} < 0$, the domestic interest rate will be higher than the foreign interest rate.[1]

C A Discussion of Central Bank Swap Lines and IMF Credit Lines

As discussed by Obstfeld, Shambaugh and Taylor (2009 and 2010), the benefit of central bank reserves is that they are assets of the central bank and can be deployed immediately at a time of crisis. Over the last decade, in a series of policy papers, the IMF lays out their approach to quantitatively assessing a country's reserve adequacy (IMF 2011, 2013, and 2015). In the 2013 paper, which is devoted exclusively to the issue of precautionary reserve accumulation, they discuss alternative forms of insurance against a balance of payments crisis. The most common are central bank swap lines and contingent credit lines from international financial institutions like the IMF. However, while both can perform the role of reserves when activated, their activation is not automatic and they are not intended to be permanent, and thus they can't be counted on as insurance against a future crisis the way hard currency reserves on the balance sheet of the central bank can be.

[1] Note that this equilibrium condition in the market for foreign bonds can also be derived in a reduced form by adding a quadratic adjustment cost to holding foreign bonds in the household budget constraint, as in Schmitt-Grohe and Uribe (2003). This reduced form approach was the preferred way of adding intermediary frictions in models with central bank foreign exchange intervention in Chang et al. (2015) and Davis et al. (2021).

Central bank swap lines established by the Federal Reserve allow a foreign central bank to swap their own currency for U.S. dollars and thus take a short position in U.S. dollar reserve assets. As discussed by Aizenman and Pasricha (2010) and Aizenman et al. (2022), the U.S. Federal Reserve has permanent, unlimited swap lines with only 5 foreign central banks, the Bank of Canada, the Bank of England, the European Central Bank, the Bank of Japan, and the Swiss National Bank. In 2008 the Fed established swap lines with 9 other central banks on a temporary basis, but only four of these were emerging markets (Brazil, Mexico, South Korea, and Singapore, the other 5 were Australia, New Zealand, Sweden, Denmark, Norway). The temporary swap lines with these 9 countries were reactivated at the height of the COVID crisis in March 2020.

The Fed also created the Foreign and International Monetary Authority (FMIA) repurchase facility in March 2020 to allow a foreign central bank to engage in a repo transaction to deposit U.S. treasuries at the Fed in return for U.S. dollar liquidity. This would be useful at a time of extraordinary market stress and prevent foreign central banks from having to sell their U.S. treasury reserves at fire-sale prices, but unlike a swap line, where the central bank swaps its own domestic currency for U.S. dollars, and thus takes a short position in dollars, the FIMA repo facility just allows the foreign central bank to swap one type of dollar asset for another dollar asset.

The second most common form of non-reserves insurance are credit lines from international financial institutions like the IMF. The traditional IMF lending program, like the program negotiated with Argentina in 2018, is usually implemented after the recipient country's commitment to a costly adjustment program. As part of their Global Financial Safety Net program, in the wake of the global financial crisis in 2008 the IMF established credit lines, the Flexible Credit Line (FCL) and the Precautionary and Liquidity Line (PLL), where a country would be pre-screened and then have discretionary access to IMF funds during a crisis or simply for precautionary reasons. (see Essers and Ide (2019)) However, as IMF (2013) highlights, these credit lines do not have the same permanence as reserves held by the central bank; these credit lines last for two years, and are subject to periodic review. Since their establishment in the wake of the global financial crisis, they have had limited uptake (to date only 5 countries have FCL arrangements, Chile, Colombia, Mexico, Peru, Poland, and only 3 have PLL arrangements, North Macedonia, Morocco, Panama).

The IMF has tried on a few occasions to establish a credit line facility since the early 1990's, when the IMF first considered but did not establish the Short-term Financing Facility (STFF). The IMF did establish the Contingent Credit Line (CCL) in the late 1990s in the wake of the East Asian crisis, but no country applied for the CCL and it was terminated in 2003. One problem was that access to the CCL credit line remained at the discretion of the IMF Executive Board even for pre-screened countries, the IMF tried to remedy this with the Reserve Augmentation Line (RAL), but it was never established.

D Variables and Equations

Variables in the model:
c_t^T, c_t^N, R_t, B_t^{cb}, B_t, B_t^{fs}, F_t^{fs}, λ_t, μ_t, p_t, T_t, Π_t, F_t^{cb}, τ_t taking B_0, F_0^{fs}, F_0^{cb} and R_t^W as given
Equations:

c_t^T:

$$\left(\left[\alpha\left(c_t^T\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_t^N\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}\right)^{-\sigma}\times$$

$$\left[\alpha\left(c_t^T\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_t^N\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}-1}\alpha\left(c_t^T\right)^{\frac{-1}{\xi}}=\lambda_t$$

c_t^N:

$$\left(\left[\alpha\left(c_t^T\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_t^N\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}\right)^{-\sigma}\times$$

$$\left[\alpha\left(c_t^T\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_t^N\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}-1}(1-\alpha)\left(c_t^N\right)^{\frac{-1}{\xi}}=p_t\lambda_t$$

$R_t : B_t + B_t^{cb} + B_t^{fs} = 0$
$B_t^{cb} : B_t^{cb} + F_t^{cb} = 0$
$B_t : \frac{\lambda_t - \mu_t}{R_t} = \beta\lambda_{t+1}$
$B_t^{fs} : B_t^{fs} + F_t^{fs} = 0$
$F_t^{fs} : R_t = R_t^W - \frac{\Gamma}{\beta}F_t^{fs}$
$\lambda_t : c_t^T + p_t c_t^N + B_t = y^T + p_t y^N + R_{t-1}B_{t-1} + T_t + \Pi_t$
$\mu_t : \mu_t = 0$ or $-B_t = \kappa\left(y^T + p_t y^N\right)$
$p_t : c_t^N = y^N$
$T_t : T_t = R_{t-1}^W F_{t-1}^{cb} + R_{t-1}B_{t-1}^{cb} - F_t^{cb} - B_t^{cb}$
$\Pi_t : \Pi_t = R_{t-1}^W F_{t-1}^{fs} + R_{t-1}B_{t-1}^{fs} - F_t^{fs} - B_t^{fs}$
F_t^{cb} : *policy*

E Proof of effectiveness of foreign exchange intervention when $\Gamma > 0$

To see how central bank foreign exchange intervention can have a macroeconomic effect when the economy is in a non-binding equilibrium and $\Gamma > 0$, start from the non-binding equilibrium where private sector demand for international bonds is given by F_t^{fs}, the central bank does not change their stock of international bonds $F_t^{cb} = F_{t-1}^{cb}$, and thus traded goods consumption in periods t and $t+1$ is given by:

$$\begin{aligned}
c_t^T &= y^T - F_t^{fs} + R_{t-1}^W F_{t-1}^{fs} - F_t^{cb} + R_{t-1}^W F_{t-1}^{cb}\\
c_{t+1}^T &= y^T - F_{t+1}^{fs} + R_t^W F_t^{fs} - F_{t+1}^{cb} + R_t^W F_t^{cb}
\end{aligned}$$

Now suppose instead the central bank engages in a sale of some of their stock of international bonds, $F_t^{cb\prime} < F_{t-1}^{cb}$. Following this central bank intervention the private sector changes their demand for international bonds from F_t^{fs} to $F_t^{fs\prime}$. The new consumption level following this central bank intervention and private sector response is:

$$
\begin{aligned}
c_t^{T\prime} &= y^T - F_t^{fs\prime} + R_{t-1}^W F_{t-1}^{fs} - F_t^{cb\prime} + R_{t-1}^W F_{t-1}^{cb} \\
c_{t+1}^{T\prime} &= y^T - F_{t+1}^{fs} + R_t^W F_t^{fs\prime} - F_{t+1}^{cb} + R_t^W F_t^{cb\prime}
\end{aligned}
$$

We want to prove that when $\Gamma > 0$, the central bank sale of international bonds led to higher external debt, $-F_t^{fs} - F_t^{cb} < -F_t^{fs\prime} - F_t^{cb\prime}$ and thus by extension, higher traded goods consumption in period t.

If $F_t^{fs\prime} < F_t^{fs}$, the proof is trivial (this would be where the central bank sold foreign bonds, and as a result the private sector also sold foreign bonds). Turning to the case where $F_t^{fs\prime} > F_t^{fs}$: combine the household's first order condition with respect the F in equation (6) with the equilibrium condition for foreign borrowing in equation (10).

$$
\frac{\lambda_t - \mu_t}{\beta \lambda_{t+1}} = R_t^W - \frac{\Gamma}{\beta}\left(F_t^{fs}\right) \tag{E.1}
$$

Thus in a non-binding equilibrium:

$$
\frac{\lambda_t}{\beta \lambda_{t+1}} + \frac{\Gamma}{\beta}\left(F_t^{fs}\right) = \frac{\lambda_t^\prime}{\beta \lambda_{t+1}^\prime} + \frac{\Gamma}{\beta}\left(F_t^{fs\prime}\right) \tag{E.2}
$$

where λ_t and λ_{t+1} are the marginal utilities of consumption when traded goods consumption is c_t^T and c_{t+1}^T, and $\lambda_t^\prime$ and $\lambda_{t+1}^\prime$ are the marginal utilities of consumption when traded goods consumption is $c_t^{T\prime}$ and $c_{t+1}^{T\prime}$. If $F_t^{fs\prime} > F_t^{fs}$ and $\Gamma > 0$ then the above equality becomes:

$$
\frac{\lambda_t}{\beta \lambda_{t+1}} > \frac{\lambda_t^\prime}{\beta \lambda_{t+1}^\prime} \tag{E.3}
$$

which implies that the central bank intervention by selling international bonds lowers the domestic interest rate. This implies:

$$
c_t^T - c_{t+1}^T < c_t^{T\prime} - c_{t+1}^{T\prime}
$$

which implies that $-F_t^{fs} - F_t^{cb} < -F_t^{fs\prime} - F_t^{cb\prime}$. Note that the switch from the equality in equation (E.2) to the inequality in equation (E.3) relied on the fact that $\Gamma > 0$. If there were no intermediary frictions $\Gamma = 0$, then the inequality in equation (E.3) would be an equality and $-F_t^{fs} - F_t^{cb} = -F_t^{fs\prime} - F_t^{cb\prime}$. Any central bank sale of international bonds would be exactly offset by a private sector purchase of international bonds, leaving total external debt unchanged.

F Optimal FXI in an infinite horizon model

This section presents the policy maker's problem under discretion in an infinite horizon model. As usual, the policy maker's problem is defined as follows: a path of reserve holding $\{F_t^{cb}\}_{t=0}^{\infty}$ that maximizes the representative domestic household's objective function, subject to the constraints in the competitive equilibrium defined in the main text. In order to make the expressions of constraints more concisely, we make the following equivalent changes of variables. Let the exogenous variable be $s_t \equiv R_t^w$, total external borrowing at the beginning of period t, $f_{t-1} \equiv -R_{t-1}^W F_{t-1}$ and reserve holding at the beginning of period t, $f_{t-1}^{cb} \equiv F_{t-1}^{cb} R_{t-1}^W$. Therefore the end-of-period portfolio can be written as $F_t = -f_t/R_t^W$ and $F_t^{cb} = f_t^{cb}/R_t^W$. The Bellman equation for the policy maker's problem can be written as,

$$\mathbf{P_1}: v(f_{t-1}, s_t) = \max_{\{c_t^T, f_t, f_t^{cb}\}} \{u(c_t) + \beta \mathbb{E}_t [v(f_t, s_{t+1})]\} \tag{F.1}$$

subject to

$$c_t^T = y^T - f_{t-1} + \frac{f_t}{R_t^w} \tag{F.2}$$

$$u_{c^T}(c_t)(1-\mu_t) = \beta R_t \mathbb{E}_t u_{c^T}(c_{t+1}) \tag{F.3}$$

$$\frac{f_t}{R_t^w} \leq \kappa \left(y^T + p_t y^N\right) \tag{F.4}$$

$$\mu_t \left[\kappa \left(y^T + p_t y^N\right) - \frac{f_t}{R_t^w}\right] = 0, 0 \leq \mu_t < 1$$

$$p_t = \frac{1-\alpha}{\alpha} \left(\frac{c_t^T}{y^N}\right)^{\frac{1}{\xi}} \tag{F.5}$$

$$R_t = R_t^w + \frac{\Gamma}{\beta} \frac{f_t + f_t^{cb}}{R_t^w} \tag{F.6}$$

where aggregate consumption and marginal utility can be written as

$$c_t = \left[\alpha (c_t^T)^{\frac{\xi-1}{\xi}} + (1-\alpha)(y^N)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$$

$$u_{c^T}(c_t) = \alpha c_t^{-\sigma} \left(c_t/c_t^T\right)^{\frac{1}{\xi}}$$

Note that central bank policy f_t^{cb} only appears in equation (F.6) as a control variable and could change domestic bond interest rate R_t, which can facilitate computing an optimal allocation substantially.