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ABSTRACT

While the mechanism design paradigm emphasizes notions of efficiency based on agent preferences, policymakers often focus on alternative objectives. School districts emphasize educational achievement, and transplantation communities focus on patient survival. It is unclear whether choice-based mechanisms perform well when assessed based on these outcomes. This paper evaluates the assignment mechanism for allocating deceased donor kidneys on the basis of patient life-years from transplantation (LYFT).We examine the role of choice in increasing LYFT and compare realized assignments to benchmarks that remove choice. Our model combines choices and outcomes in order to study how selection affects LYFT. We show how to identify and estimate the model using instruments derived from the mechanism. The estimates suggest that the design in use selects patients with better post-transplant survival prospects and matches them well, resulting in an average LYFT of 9.29, which is 1.75 years more than a random assignment. However, the maximum aggregate LYFT is 14.08. Realizing the majority of the gains requires transplanting relatively healthy patients, who would have longer life expectancies even without a transplant. Therefore, a policymaker faces a dilemma between transplanting patients who are sicker and those for whom life will be extended the longest.

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1 Introduction

Assignment mechanisms are commonly used to allocate scarce resources such as public schools, public housing, and organ allocation. While the design of these mechanisms takes choice-theoretic notions of efficiency as a primary objective (Roth and Sotomayor, 1992; Abdulkadiroglu and Sonmez, 2003), this desideratum often differs from policymakers' situation-specific goals – school districts emphasize student achievement, for example, and organ transplant systems emphasize patient survival. Because canonical choice-based mechanisms are not designed to optimize these outcomes, they may not perform well in these dimensions. Agents' choices may not be well-informed and coordination failures may undercut intended objectives.[1] If so, a planner who can dictate assignments based on estimated benefits may improve performance. However, agents may also have private information about likely outcomes and using a choice-based mechanism may serve policymakers' objectives.

This paper evaluates the mechanism through which deceased donor kidneys are allocated on the basis of survival outcomes. We compare the performance and distributional consequences of the mechanism to those of alternative assignments. Our benchmark assignments investigate whether maximizing survival conflicts with distributional concerns (Atkinson, 1970) or prioritarianism which targets the sickest or neediest (c.f. Persad et al., 2009; Waldinger, 2017). We also assess the role of choice by examining its relationship to survival and considering alternatives that dictate assignments using observables alone.

We make two contributions in service of this objective. First, we present the first quasi-experimental estimates of the Life-Years from Transplantation (LYFT), defined as the difference between median survival with and without a transplant, as a function of patient/donor-specific observed and unobserved characteristics. The current standard in the medical literature relies on observational approaches (Wolfe et al., 2008), in part because conducting randomized controlled trials presents both practical and ethical challenges. Second, we use insights from the literature on generalized Roy selection to analyze a joint model of choices and outcomes in an assignment mechanism in which agents are offered sequentially arriving

[1] Moreover, in the kidney allocation context, surgeons who advise patients may suffer from agency problems that can misalign decisions relative to maximizing survival outcomes.

heterogeneous objects that they may choose to decline. In contrast to the standard framework with multiple treatments (e.g. Lee and Salanie, 2018; Heckman and Pinto, 2018), the number of treatments grows with the size of the market in many assignment contexts, in our case because each organ is unique. Similar considerations arise in other sequential assignment contexts such as the allocation of public housing units and of jobs in certain gig economy contexts. We therefore model potential outcomes as a function of agent (patient), object (organ), and match-specific characteristics, some of which are unobserved. Our results show how to identify and estimate the effects of counterfactual assignments by using variation in offers made to agents and choice shifters that are excluded from outcomes.

Deceased donor organs are a scarce and valuable resource. Only one-sixth of the approximately 100,000 patients waiting for a kidney are transplanted annually, and thousands die while waiting.[2] Increasing LYFT is an important policy goal: transplantation committees use observational estimates of LYFT to evaluate proposed reforms.[3] When a kidney becomes available, patients on the waitlist are offered the organ in a priority order. Patients, or surgeons acting on their behalf, may choose to reject an offer and instead wait for a future organ. This decision may depend on the perceived benefits of transplanting the offered organ.

We jointly model acceptance decisions and survival outcomes to incorporate the potential for selection. The first component of our model considers the choices patients make; the second and third components model patient untransplanted survival and post-transplant survival with the offered organ, respectively. These models use a rich set of patient and organ attributes as well as time to treatment. Given our focus on evaluating alternative assignments, we also include patient- and patient-donor level unobservables.

The model is challenging to identify because transplanted patients can be selected on untransplanted survival, post-transplant survival from an average kidney, or patient-kidney match-specific survival. Selection on these margins can be induced both because choices can depend on survival prospects and because the mechanism prioritizes patient waiting time.

[2] See https://optn.transplant.hrsa.gov/data/view-data-reports/national-data/.

[3] Alternative design reports, generated by the Scientific Registry of Transplant Recipients (SRTR) for the OPTN Kidney Transplantation Committee, use average LYFT as a summary measure of performance. The committee's meeting minutes indicate that this measure is focal. In fact, the U.S. has considered a priority system based on LYFT in the past, and the U.K. uses a "transplant benefit score" to allocate kidneys (Watson et al., 2020).

We identify our model by combining two sources of variation. The first source is randomness in the offers made to a given patient, conditional on the patient's priority-type in the mechanism. The second source is a continuous shifter of choices that is excluded from outcomes.

Variation in the randomness of offers allows us to compare the survival outcomes of patients whose final assignments differed due to the organs they were offered. Using standard arguments (e.g., Imbens and Angrist, 1994), we show that this instrument identifies a treatment effect for the select group of patients whose assignment is affected by an offer. An important limitation of this estimand is that it does not allow us to predict survival from counterfactual assignments. It cannot consider changes in the set of patients who are transplanted or changes in the kidneys to which a patient is matched.

To overcome this limitation and identify the effects of alternative assignments, we use novel arguments and a shifter of choices to identify our choice and survival models. Related approaches have been used in other settings by Geweke et al. (2003); Heckman and Navarro (2007); Lewbel (2007); Hull (2018) to correct for selection and to estimate marginal treatment effects (Heckman and Vytlacil, 2005). Our choice shifter is based on organ scarcity controlling for geography and time. We estimate the model using a Gibbs' sampler similar to Geweke et al. (2003).

Our estimates suggest that choices and assignments are positively correlated with survival outcomes due to both observed and unobserved factors. Patients are more likely to accept kidneys that result in longer survival and those with match-specific benefits. Partly because of this, transplanted patients have a higher LYFT from the average organ as compared to untransplanted patients. Thus, prior approaches that do not account for selection on unobservable factors (e.g., Wolfe et al., 2008) yield biased estimates.

Next, we benchmark the observed assignment from the perspective of a utilitarian planner whose objective is to maximize LYFT. We focus on survival because it is a focal outcome for kidney allocation. We compare the observed assignment to alternatives ranging from a random assignment to one that maximizes LYFT.[4] Because distributional constraints may

[4]The narrower focus on alternative assignments rather than mechanisms avoids solving for choices in counterfactual equilibria, a computationally demanding task for waitlist mechanisms (Agarwal et al., 2021). Accounting for this channel is left for future work.

limit the ability to select which patients receive a transplant, we also consider alternatives that re-assign organs while fixing the set of transplanted patients. Finally, we measure the LYFT increase that can be achieved by a planner who can dictate assignments based only on observed patient and donor characteristics.

Our analysis reveals that observed assignment produces higher LYFT than random allocation – 9.29 years versus 7.54. Most of this gain comes from allowing patient choice. Assignment to patients based on existing priority rules without allowing for choice only achieves an average LYFT of 8.05. The drop from the observed assignment suggests that choice may not be dispensable if the unobserved types are private information.

Nevertheless, there is significant room for improvement – the maximum possible LYFT given the available organs is 14.08. The increase comes from selecting patients who benefit more from the transplant and matching these patients to more suitable donors. A significant portion of these gains can be achieved if a planner can dictate assignments using observables in our dataset.

These potential improvements in LYFT have important distributional consequences that may present real-world challenges. Although it is a priori unclear because the sickest may also have benefited the most from a transplant, increasing LYFT requires transplanting patients who would have lived longer without a transplant because LYFT and survival without a transplant are strongly correlated. Such re-distribution creates distributional concerns because it increases the dispersion in remaining life-years (Atkinson, 1970). While some medical ethicists may still support maximizing total survival benefits, especially in the presence of scarce resources, others suggest worst-off prioritarianism for the sickest (see Persad et al., 2009, and references therein). Our results indicate that the planner faces a dilemma rooted in the tension between these two goals.

Related Literature: For evaluating assignments, we provide an alternative perspective to the literature studying assignment mechanisms, which typically focuses on revealed preference-based measures. (Roth and Sotomayor, 1992).[5] For example, the theory of school choice

[5]Robinson-Cortes (2019) is an early exception that assumes social workers minimize disruptions when placing children into foster care. A recent set of papers, released after our work, consider how changes in assignment systems affect downstream outcomes, mostly in education markets. Kapor et al. (2020); Otero et al. (2021), and Larroucau and Rios (2022) study student achievement effects of, respectively, expanding

typically bases welfare on student preferences (Abdulkadiroglu and Sonmez, 2003), and the empirical literature uses a willingness to travel measure for welfare comparisons (see Agarwal and Somaini, 2020, for a survey).

The economics literature on organ donation focuses either on the number of transplants (e.g., Teltser, 2019; Dickert-Conlin et al., 2019) or on decision-theoretic notions of welfare (Agarwal et al., 2021), with an influential literature focusing on expanding living donor kidney exchange (e.g., Roth et al., 2004; Agarwal et al., 2019). Our paper, by contrast, focuses on survival outcomes and deceased donor organs, which provide the vast majority of transplanted kidneys.

Our paper also relates to approaches that leverage quasi-experimental variation in school choice mechanisms to estimate school quality arising either from tie-breakers (e.g., Cullen et al., 2006, Abdulkadiroglu et al., 2017) or from instruments that shift assignment probabilities (e.g., Abdulkadiroglu et al., 2020). This literature estimates either a local average treatment effect, which is not sufficient for analyzing outcomes from counterfactual assignments because of changes in the set of compliers, or value-added for a school, which abstracts away from match-specific effects. Our approach combines quasi-random variation in assignments with a choice shifter to solve both issues simultaneously. In contemporaneous work, Kapor et al. (2020) use this message of our paper to study outcomes in a college admissions setting.

The techniques we use build on a large literature studying selection models (Roy, 1951). Our model is related to those that combine outcomes with choice models to correct for selection when estimating treatment effects (Geweke et al., 2003; Heckman and Navarro, 2007; Lewbel, 2007; Hull, 2018), causal survival models (Abbring and den Berg, 2003), and models of multi-valued treatments (Lee and Salanie, 2018; Heckman and Pinto, 2018). The main difference between our work and these papers is that patients may have match-specific benefits from an organ, resulting in a large number of unique treatments. This issue is important in assignment contexts whenever there are a large number of heterogeneous objects. We address it by using a model with rich observed heterogeneity across objects and unobserved heterogeneity in outcomes along three dimensions – baseline outcomes, average outcomes

college admission platforms, affirmative action policies, and reapplying to college, while Bates et al. (2022) study teacher assignment.

given observable characteristics of the transplanted organ, and match-specific effects – with each dimension correlated with unobservables in the choice model.

Overview: Section 2 describes the institutions and the data. The model and the instruments are described in Sections 3 and 4. Section 5 presents the identification results and the empirical model. The estimates, LYFT in the observed mechanism, and counterfactuals are in Sections 6, 7, and 8, respectively.

2 Background, Data, and Descriptive Evidence

2.1 Institutional Features

Basics of Kidney Transplantation: Approximately 750,000 patients are afflicted with End-Stage Renal Disease (ESRD) in the United States (USRDS, 2018). Medicare provides near-universal coverage for costs related to ESRD, irrespective of patient age, and this coverage costs taxpayers $35.4 billion in 2016 (7.2% of Medicare claims (USRDS, 2018), approximately 1% of the federal budget). Transplantation is considered the best treatment for ESRD. Each transplant is estimated to extend a patient's life by several years (Wolfe et al., 2008) while also saving $195,000 – $400,000 in dialysis costs (Irwin et al., 2012; Held et al., 2016). These estimates are based on survival models and comparisons of healthcare costs with and without a transplant. We here improve on such estimates by using quasi-experimental variation.

There is significant potential for heterogeneity in survival effects, even amongst compatible patient-donor pairs (Danovitch, 2009). First, survival with or without a transplant can differ across patients. Some patients tolerate dialysis better than others and comorbidities influence post-transplant survival prospects. Second, donor quality—the donor's death circumstances, kidney function, and health prior to death—can significantly influence transplant outcomes. Finally, match-specific factors, such as size and weight match as well as tissue-protein similarity between patient and donor, may also affect post-transplant survival.

The Allocation of Deceased Donor Kidneys: Deceased donor organ allocation is organized using a prioritized waiting list. Patients receive offers when an organ becomes available and may choose to accept or reject it. Each donor's kidneys are allocated to the highest-priority

patients on the waitlist who are willing to accept the organs. During our sample period, priority was based primarily on waiting time and tissue-type similarity between the patient and donor. Each kidney was first offered to patients with a perfect tissue-type match, then to patients from the local area in which the organ was recovered, then regionally, and finally nationally. Within each priority group, patients were ordered according to a points system that emphasized waiting time (see OPTN, 2014, for details). This allocation system evolved over time with incremental changes to enhance efficiency (Smith et al., 2012).[6]

Three features of the kidney allocation system are worth highlighting. First, unlike the assignment systems for some other organs (for example, livers and hearts), the kidney assignment system does not use patient urgency to determine priority. Second, patients who reject an offer remain on the list and may choose to accept the next offer with no penalty in priority for refusing an offer. Third, the design is based on heuristics aided by simulations and compromises in consideration of distributional effects rather than a formal mechanism design approach (see Stegall et al., 2017, for a historical perspective).

2.2 Data and Descriptive Analysis

2.2.1 Data Sources

This study uses data from the Organ Procurement and Transplantation Network (OPTN). The OPTN data system includes data on all donors, wait-listed candidates, and transplant recipients in the US, submitted by the members of the OPTN. The Health Resources and Services Administration (HRSA), U.S. Department of Health and Human Services provides oversight to the activities of the OPTN contractor.

The data include detailed information from the Standard Transplantation Analysis and Research dataset on patient and donor characteristics and survival outcomes as well as, from the Potential Transplant Recipient dataset, all offers made by the system and accept/reject decisions. These data are populated using information gathered during the allocation process,

[6]The system was revised on December 4, 2014, with the aim of improving survival benefits by implementing a priority-based waiting list that emphasizes waiting time, geography, and patient sensitization. This change prioritizes patients in the top quintile of expected post-transplant survival for the top quintile of predicted organ quality. We focus on patients who registered and received offers before this change partly because of the limited period for which we would otherwise observe survival outcomes.

forms transplant centers submitted from patient follow-ups after a transplant is performed, and patient death dates merged from social security records.

We restrict attention to patients who first joined the kidney waiting list between January 1, 2000, and December 31, 2010. From this set, we exclude patients who needed multiple organ transplants and those who received a living donor kidney (see Appendix A for a detailed discussion). Correspondingly, we only use data on donor offers to and acceptance decisions by patients in our sample. We track patient survival until February 29, 2020, to avoid confounding effects due to the COVID-19 pandemic. Thus, we track survival outcomes for up to twenty years and two months from registration for our sample of patients. For patients without death records, we use information from the waitlist for untransplanted patients and from annual post-transplant follow-ups for transplanted patients to construct a censored measure of patient survival.

2.2.2 Descriptive Analysis

Table 1: Patient Characteristics

	All Patients		Received Deceased Donor Transplant	
	Mean	S.D.	Mean	S.D.
New Patients per Year	15967		8648	
Panel A: Outcomes				
Died by Year Five (%)	27.4	44.6	9.1	28.7
Survived Five Years (%)	66.6	47.2	87.5	33.1
Censored by Year Five (%)	6.1	23.9	3.4	18.2
Transplanted by Year Five (%)	47.2	49.9	87.2	33.5
Panel B: Characteristics				
Age at Registration	51.4	14.2	48.8	15.1
On Dialysis at Registration (%)	77.2	41.9	74.9	43.4
Diabetic Patient (%)	42.9	49.5	33.4	47.2
BMI at Registration	28.2	5.9	27.7	5.7

Notes: Sample includes 175640 patients who registered between 2000 and 2010. Transplant and survival data are available through 12/31/2015. Patients for whom we do not observe death are censored. The observed survival duration is computed based on the date and status of the patient when we last observe her. See A.4 for detailed computation of observed survival. Durations presented in Panel A are time since registration.

Patients and Donors: Patients face extreme scarcity, with a significant fraction dying while awaiting a transplant. Panel A of Table 1 shows that an average of 15967 patients registered on the kidney waiting list each year, of which 27.4% die within five years of registering and only 47.2% receive a transplant during this time period. The chances of receiving a transplant decline after the first five years, with only 54.3% of patients ultimately receiving a deceased donor kidney. The remaining patients either still await a kidney or leave the list.

Panel B shows that patients receiving a transplant are younger and appear to have been in better health at the time of registration. Transplanted patients are less likely to be on dialysis at the time of registration, are less likely to be diabetic, and have a lower body mass index. Thus, observed characteristics induce correlation between probability of receiving a transplant and survival without a transplant.

Table 2: Donor Characteristics

	All Donors		Any Kidney Discarded			
			Yes		No	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
Number of Donors per Year	6195		1171		5023	
Median Number of Offers per Donor	51		483		40	
Average Number of Offers per Donor	547.8	1936.0	1892.5	3684.3	234.2	968.0
Donor Age	39.2	18.4	52.0	16.6	36.2	17.5
Cause of Death -- Head Trauma (%)	39.7	48.9	19.5	39.6	44.4	49.7
Hypertensive Donor (%)	28.6	45.2	55.3	49.7	22.4	41.7
Donor Creatinine	1.2	1.0	1.4	1.1	1.1	0.9
Non-Heart Beating Donor (%)	8.0	27.2	10.6	30.7	7.4	26.2
KDPI	0.5	0.3	0.8	0.2	0.4	0.3

Notes: Sample includes deceased donor organs offered between 2000 and 2010 to patients in the sample.

Patients exercise choice despite scarcity, often rejecting undesirable organs. Table 2 shows that the number of offers per donor is 547.8, but the median is much lower, at 51. This skewed distribution arises because undesirable kidneys are rejected by many, while desirable kidneys are accepted quickly. Indeed, 18.9% of donors have at least one viable kidney discarded. Organs from these donors are refused by 1892.5 patients on average.

Organ quality predictors correlate with number of offers and discards in expected ways. Donors whose kidney(s) was/were discarded are older, less likely to have died from head

trauma, more likely to be diabetic or hypertensive, more likely to have donated after cardiac death, and have higher creatinine levels (an indicator of lower kidney function) (Table 2). An aggregate of these and other characteristics is the Kidney Donor Profile Index (KDPI), which indicates the fraction of donors with a lower estimated risk of graft failure.

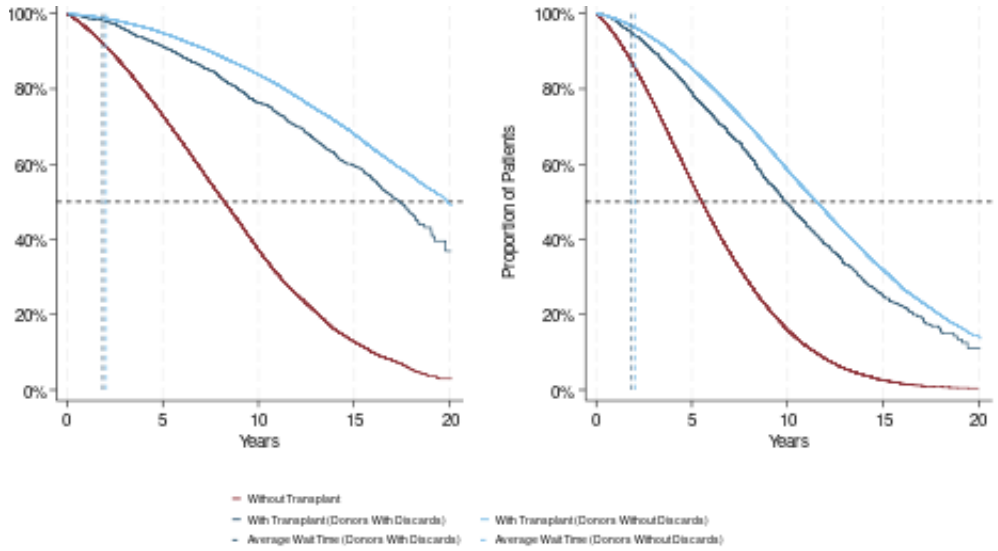
Survival: We focus on survival as the primary outcome of interest for several reasons. First, this outcome is arguably the most important one from the perspectives of both the patient and the policy-makers. The OPTN Kidney Transplantation Committee explicitly used predicted LYFT from observational models to evaluate proposed designs. Second, moving an ESRD patient from dialysis to transplantation saves on expensive dialysis treatment. While we do not directly evaluate this component, future research can use our estimates to revisit cost-benefit analyses. Third, as an outcome, survival can be measured relatively easily. Transplantation's other most commonly discussed effect is quality of life, which is hard to quantify.

Figure 1 shows survival curves for transplanted and untransplanted patients, separated by young and old patients (above/below the median age of 54) and by whether or not the transplanted patient received a kidney from a donor with a discarded kidney. Donors with a discarded kidney are more likely to be undesirable because only one patient accepted the donor's kidneys. As indicated by the waiting times shown via the vertical dashed lines, the average waiting time for a patient who receives a kidney from a donor without a discard is higher than that for a donor with a discard.

These survival curves show that transplanted patients live significantly longer than patients who do not receive a transplant. Moreover, the survival curves are substantially different for young versus old patients and for patients transplanted with desirable versus undesirable organs. Only about half of the young patients who do not receive a transplant survive more than 8.2 years, but more than half of the young patients who receive a transplant from a donor with desirable organs live more than 19.8 years. These statistics are 5.5 and 11.5 years, respectively, for older patients, indicating that older patients have shorter half-lives both with and without a transplant.[7] For both groups of patients, a transplant from an

[7]We focus on median survival instead of expected life-years because we can track survival for up to twenty years. This choice is consistent with prior work measuring the life-year benefits from transplantation (see

Figure 1: Patient Survival



Notes: The figure shows the Kaplan-Meier survival curve for young and old patients (above/below the median age of 54) who registered on the waitlist between 2000 and 2010. Survival with transplant is measured as time since registration.

undesirable organ is associated with half-lives that are shorter by about a year or more. These observations also point to the potential for choices and assignments to be correlated with survival outcomes. Next, we turn to a model that incorporates these features.

3 A Model of Decisions and Outcomes

Our model considers assignment mechanisms in which agents, indexed by i, receive offers for sequentially arriving objects, indexed by j. Agents must decide to accept or reject each offer. These decisions translate into an assignment, and an outcome that may be agent-object specific is realized. Agents may also depart from the mechanism prior to assignment. Given our empirical setting, we will refer to the agents as patients and the objects as organs.[8] However, the model is applicable to other markets where heterogeneous objects are sequentially assigned and decisions potentially induce selection in outcomes. Examples include the allocation of public housing (Waldinger, 2017) or jobs on certain gig economy platforms; in

Wolfe et al., 1999, 2008, for example).

[8] In our empirical context, patient decisions may be delegated or made jointly with a surgeon. We do not distinguish between these alternatives.

the latter, for instance, drivers must accept or reject a ride before considering the next one (Liu et al., 2019).

3.1 Assignment Mechanism and Observed Outcomes

Organs arrive sequentially, and their index j denotes their arrival order. The mechanism orders patients on the waiting list according to an organ-specific priority score that may depend on the time that a patient has waited. Offers are made in this priority order. Acceptance by i of an offer for organ j is denoted with $D_{i,j} = 1$. Organs are assigned to the highest-priority patients who accept an offer. Finally, patients who have been assigned an organ are removed from the list. Other patients may also leave the list.

Consider the set of organs that are feasible for patient i. Holding fixed the decisions of the other patients, let J_i be the ordered set of organs offered to patient i if they refused all offers made to them and they were registered indefinitely. A patient will only receive offers from this set either until the time at which they leave the list without a transplant, denoted A_i, or until they are transplanted. The patient is assigned the first organ they accept in the set of offered organs. Patient i's assignment $T_{i,j}$ therefore depends both on the set of offered organs and their decisions,

$$T_{i,j} = 1\{A_i \geq t_{i,j}\} \prod_{j'<j,\, j' \in J_i} (1 - D_{i,j'})\, D_{i,j},$$

where $D_{i,j} = 1$ if patient i accepts organ j, $t_{i,j}$ is the time between patient i's registration and donor j's arrival. Therefore, each patient i is assigned to the first organ that they accept from the set J_i that arrives before they leave the list. We assume that the analyst observes the decisions $D_{i,j}$ for each observed offer. Observing the choice set and decisions is typical when administrative data from an assignment mechanism are available.

The observed outcome Y_i depends on whether a patient is assigned and to which organ they are assigned. It is given by

$$Y_i = \sum_{j \in J_i} T_{i,j} Y_{i,j} + \left(1 - \sum_{j \in J_i} T_{i,j}\right) Y_{i,0},$$

where $Y_{i,j}$ is the outcome of patient i from being assigned organ j.

In our empirical setting, Y_i denotes survival time since registration on the organ waiting list.[9] For some patients, we will observe a censored outcome with an observed censoring time Y_i^C. Although the formulation above abstracts away from censoring for simplicity of notation, we will account for it based on the standard assumption that the censoring time is independent of the latent duration (see equation 20.22 in Wooldridge, 2010).

3.2 Latent Outcomes and Decisions

There are three key sets of primitives in our model:

Unassigned Outcome: The outcome for patient i if the patient is not assigned any organ is given by

$$Y_{i,0} = g_0\left(x_i, \nu_{i,0}\right), \tag{3.1}$$

where $x_i \in \mathbb{R}^{d_x}$ are patient-specific observables; $\nu_{i,0} \in \mathbb{R}$ denotes a patient-specific unobservable; and $Y_{i,0} \in \mathbb{R}$.

Assignment Outcome: The outcome of patient i from being assigned organ j is given by

$$Y_{i,j} = g_1\left(q_j, x_i, \nu_{i,1}, \varepsilon_{i,j,1}\right), \tag{3.2}$$

where $x_i \in \mathbb{R}^{d_x}$ is a vector of patient-specific observed characteristics; $q_j \in \mathbb{R}^{d_q}$ denotes the observed characteristics of organ j, which we will refer to as organ-types; $\nu_{i,1} \in \mathbb{R}$ denotes a patient-specific unobservable; $\varepsilon_{i,j,1} \in \mathbb{R}$ denotes an unobservable that is patient- and organ-specific; and $Y_{i,j} \in \mathbb{R}$.

Since $Y_{i,j}$ and $Y_{i,0}$ denote survival outcomes in our application, they can be written as arising from survival models with time-varying hazard rates.

Because each organ and patient is potentially unique, this model reduces dimension by parameterizing outcomes in terms of characteristics while allowing for rich heterogeneity

[9] It is not essential that A_i and Y_i are both durations, although this is the case in our empirical setting.

arising from both observables and unobservables.[10] It also includes time between patient registration and donor arrival $t_{i,j}$ since x_i and q_j contain the dates on which patient i and organ j arrive. Moreover, there are multiple levels of unobserved heterogeneity. Outcomes are heterogeneous across i due to $\nu_{i,1}$ and $\nu_{i,0}$ and within treatment types (defined by q_j) for a given i because of $\varepsilon_{i,j,1}$.

Decision Equation: We model the acceptance decision as

$$D_{i,j} = g_D\left(q_j, x_i, z_i, \nu_{i,D}, \varepsilon_{i,j,D}\right) \in \{0,1\} \tag{3.3}$$

where $D_{i,j} = 1$ denotes accept; $\nu_{i,D} \in \mathbb{R}$ denotes unobserved selectivity of patient i; $\varepsilon_{i,j,D} \in \mathbb{R}$ is a shock that is specific to the patient and the organ; and $z_i \in \mathbb{R}^{d_z}$ are observables that influence a patient's decision. Without loss of generality, we assume that g_D is non-increasing in $v_{i,D}$ and non-decreasing in $\varepsilon_{i,j,D}$.

The choice model nests several primitive models of decisions. It is consistent with both myopic decision rules and a dynamic decision process in which patients do not have foresight over future offers but instead base their decisions on their beliefs about the distribution of offers. Although we remain agnostic about the micro-foundations, this formulation and our empirical specification nest the optimal stopping problem in Agarwal et al. (2021).[11]Further assumptions micro-founding the choice model would be necessary for predicting the effects of changes to the mechanism as patients' choices are endogenous (Agarwal et al., 2021).

[10]Angrist et al. (2020) and Bacher-Hicks et al. (2019) also reduce the dimension of treatment effects of schools by parametrizing them in terms of mediating school characteristics. While the underlying models are non-nested, as in these papers, we will use instruments that affect individual assignments to measure the relationship between treatment effects and object characteristics.

[11]In this model, an offer is accepted if the (perceived net present) value from accepting the organ exceeds the option value of waiting. Omitting the dependence on time for simplicity, let $g_D = 1$ if $U_{ij} = U_{i,}\left(q_j, x_i, \nu_{i,D}, \varepsilon_{i,j,D}\right) > V\left(x_i, \nu_{i,D}\right) = V_i$ where $U\left(\cdot\right)$ is the net present value of accepting an offer for j, and $V\left(\cdot\right)$ is the option value of waiting. Agarwal et al. (2021) estimate this model by first estimating conditional choice probabilities using a probit model where $g_D = 1\left\{f\left(q_j, x_i, \varepsilon_{i,j,D}; \theta\right) > 0\right\}$ using a reduced-form function f parametrized in terms of θ. Their empirical specification is more restrictive than ours as it omits $\nu_{i,D}$ and z_i and does not consider survival effects from transplantation. Observe that this formulation allows for $(U_{i,j}, V_i)$ to be correlated with $(Y_{i,j}, Y_{i,0})$, for example, because patients value survival outcomes. Separating $U_{i,j}$ and V_i requires a dynamic discrete choice model with beliefs about future offers as in Agarwal et al. (2021), assumptions that would not be necessary in a static context (see Abdulkadiroglu et al., 2020, for example). We leave this combination of dynamic discrete choice model and survival outcomes to future work because separating these components is not required for the counterfactuals we consider.

However, this micro-foundation comes at the cost of additional assumptions and analytical burden and is not necessary for the alternative benchmarks that we will consider. We leave an approach that extends our work by including a structural model of choice to future work.

The main difference between x_i and z_i is that the latter is excluded from the outcome equations. For example, z_i could include variables that influence this decision, say through the distribution of future offers, but is unrelated to the benefits of accepting a given organ. This exclusion restriction, combined with Assumption 1(i) below, introduces instruments in the model that we will use in the empirical strategy. The specific instruments z_i used in our application are discussed in Section 4.

Our data-generating process samples a set of organs with characteristics q_j and a set of patients with characteristics (x_i, z_i, ν_i, A_i) independently, where $\nu_i = (\nu_{i,0},\, \nu_{i,1},\, \nu_{i,D})$. The process then samples independently and identically distributed (i.i.d.) match-specific unobservables $\varepsilon_{i,j} = (\varepsilon_{i,j,1},\ \varepsilon_{i,j,D})$ for each patient i and organ j. Denote the random vector $(\varepsilon_{i,1}, \varepsilon_{i,2}, \ldots, \varepsilon_{i,j}, \ldots)$ with ε_i. These restrictions allow for dependence between $Y_{i,j}$, $Y_{i,0}$, and $D_{i,j}$ conditional on all observables because they allow for dependence between the components of ν_i and dependence between the components of $\varepsilon_{i,j}$. For simplicity, this model and the identification results in the main text abstract away from unobserved heterogeneity in organ quality. The empirical model we estimate allows for donor-level unobserved heterogeneity in organ quality. Results in appendix C.3 prove identification for this extension.

We assume that z_i is an instrument that shifts acceptance decisions but is excludable from the outcome equations:

Assumption 1. *ε_i, ν_i, and z_i are mutually independent conditional on x_i.*

We normalize the marginal distributions of $\nu_{i,0}, \nu_{i,1}, \nu_{i,D}$, $\varepsilon_{i,j,1}$, and $\varepsilon_{i,j,D}$ to be uniform. These normalizations are without further loss of generality because we have not placed restrictions on the functional forms of $g_0(\cdot), g_1(\cdot)$ and $g_D(\cdot)$.

Our goal is to identify the function $g_D(\cdot)$ and the marginal distributions of $Y_{i,j}$ and $Y_{i,0}$ conditional on the vector $(x_i,\ q_j,\ z_i,\ \varepsilon_{i,j,D},\ \nu_{i,D})$. The residual uncertainty in the distribution of $Y_{i,0}$ is only because of patient-specific unobservables $\nu_{i,0}$, whereas it is due to both match-specific

effects $\varepsilon_{i,j,1}$ and patient-specific effects $\nu_{i,1}$ for $Y_{i,j}$.[12] Incorporating these sources is necessary for capturing unobserved match-specific drivers of outcomes. Since $g_D(\cdot)$ parametrizes the choice model, the conditional distributions of $Y_{i,j}$ and $Y_{i,0}$ given $(x_i,\, q_j,\, z_i,\, \varepsilon_{i,j,D},\, \nu_{i,D})$ yield the distributions of outcomes conditional on observed choices and the resulting selection. In addition, we will also consider counterfactual assignments that condition only on a subset of the variables $(x_i,\, q_j,\, z_i,\, \varepsilon_{i,j,D},\, \nu_{i,D})$. The outcomes in these counterfactuals are also identified since we can integrate over the characteristics that are not conditioned on.

The model and Assumption 1 together impose three main restrictions. First, unobserved patient selectivity, $\nu_{i,D}$, is one-dimensional and fixed across all organs and time. This implies a fixed ordering of patients on selectivity for all organ types. This single-index assumption rules out certain models with random coefficients, for example, on organ characteristics q_j or on scarcity z_i.[13] Second, selectivity and survival outcomes can be correlated through ν_i, but we abstract away from time-varying information about survival that is unobserved by the econometrician and also affects decisions. Relaxing these two restrictions is challenging. Identifying time-varying unobserved heterogeneity in survival is challenging because we only observe a single survival outcome for each patient (see Abbring and den Berg, 2003; Unkel et al., 2014, for related issues). Similarly, identifying general models of time-varying unobserved heterogeneity in selectivity is complicated because patients can accept at most one offer.[14] Third, a patient's decision does not depend directly on the specific decisions of other patients for a given organ since ν_i and $\varepsilon_{i,j}$ are independent of $\nu_{i'}$ and $\varepsilon_{i',j'}$.

In addition, we rule out statistical dependence between the subset of organs offered to a patient and their unobservables:

Assumption 2. *The sequence of offers J_i is conditionally independent of (ν_i, ε_i) given x_i*

[12] For example, the first moments of the marginals we identify are $E[Y_{i,0} | x_i,\, z_i,\, \nu_{i,D}] = \int g_0(x_i, \nu)\, f_{\nu_0|\nu_D=\nu_{i,D}}(\nu)\, d\nu$ and $E[Y_{i,j} | x_i,\, q_j,\, z_i,\, \varepsilon_{i,j,D},\, \nu_{i,D}] = \int\int g_1(q_j, x_i, \nu, \varepsilon)\, f_{\varepsilon_1|\varepsilon_D=\varepsilon_{i,j,D}}(\varepsilon)\, f_{\nu_1|\nu_D=\nu_{i,D}}(\nu)\, d\nu d\varepsilon$, where the distributions of $\nu_{i,1}$ and $\nu_{i,0}$ may depend on $\nu_{i,D}$, and the distribution of $\varepsilon_{i,j,1}$ may depend on $\varepsilon_{i,j,D}$.

[13] Our identification arguments will condition on patient observables x_i. This approach will admit models in which patient-specific random coefficients interact only with x_i in order to preserve a single-index structure, for instance, if $D_{i,j} = g_D(q_j, f(x_i; \nu_{i,D}), z_i, \varepsilon_{i,j,D})$ where x_i and $\nu_{i,D}$ are multi-dimensional and $f(\cdot)$ is real-valued.

[14] While we conjecture that assuming time-invariant unobserved selectivity is testable using information about the timing of rejected offers, we were unable to use this information to prove positive identification results.

and z_i.

Assumption 2 is satisfied if x_i and z_i control for a sufficiently rich set of patient characteristics such that the remaining variation in potential offers is independent of unobserved determinants of a patient's outcomes and decisions. The assumption allows for J_i to depend on the unobservables of other patients i'. But, because J_i is excluded from i's potential outcomes and affects assignment, it is an instrument for which organ is assigned to i. Section 4.1 argues that the assumption is plausible in our empirical setting.

An implication of this assumption is that patients cannot alter their decisions or their outcomes in response to specific future offers, ruling out foresight over the organs that will be offered in the future. This restriction parallels the "no anticipation" assumption in Abbring and den Berg (2003). Nonetheless, recall that our choice model nests the model in Agarwal et al. (2021), where forward-looking patients strategically refuse offers based on the distribution of future offers.

The sequential nature of choices and treatment assignment in our model resembles that of Heckman and Navarro (2007), with two main differences. First, a patient's outcomes and choices from different organs of the same type q_j are heterogeneous in our framework whereas the standard framework uses a finite set of known types. This allows for the realistic possibility that patient choices and survival outcomes can vary across two observationally identical donors. Capturing such match-specific effects can be important in other assignment problems with highly heterogeneous agents. Second, our choice shifter z_i varies at the individual level, not at the individual-treatment level. As we discuss below, we combine this instrument with variation in offers J_i to identify treatment effects.

3.3 Sources of Selection

The model allows for selection on unobservables into transplantation along three dimensions: untransplanted survival $Y_{i,0}$; average survival across transplants $\bar{Y}_i = \frac{1}{|J|}\sum_j Y_{i,j}$; and match-specific survival $Y_{i,j} - \bar{Y}_i$. There are two potential sources of selection: selection due to patient choices and selection due to patient mortality. Selection on these sources creates endogeneity in $T_{i,j}$ that our framework addresses.

Selection due to choice occurs if choices $D_{i,j}$ are correlated with survival outcomes $Y_{i,0}$ or $Y_{i,j}$. Choice can induce selection on $Y_{i,0}$ if, for example, patients with higher expected survival without a transplant due to unobserved health conditions are more selective. That is, if $E\left[Y_{i,0}\left|\nu_{i,D}, x_i\right.\right]$ varies with $\nu_{i,D}$, where expectations are taken over $\nu_{i,0}$. Similarly, choice can induce selection on average transplanted survival, $\bar{Y}_i$, if $E\left[Y_{i,j}\left|\nu_{i,D}, x_i, q_j\right.\right]$ varies with $\nu_{i,D}$, where expectations are taken over $\nu_{i,1}$ and $\varepsilon_{i,j,1}$. Choice can also induce selection on match-specific survival $Y_{i,j} - \bar{Y}_i$ if patients are more likely to accept an organ with high $Y_{i,j} - \bar{Y}_i$.

Selection due to mortality occurs because longer-lived patients (high $Y_{i,0}$) are prioritized and have a higher chance of receiving a transplant. Moreover, such selection can also occur due to either time-to-treatment effects or correlation between $\nu_{i,0}$ and ν_{i1}. Our model also features mortality-induced selection because only organs that arrive prior to A_i are offered, and given our focus on survival outcomes, $A_i \leq Y_{i,0}$.

In addition, there is selection on a number of observable dimensions. For example, the mechanism prioritizes patients who are perfect tissue-type matches for a patient because the resulting transplants may have better survival outcomes. We account for these sources by including all characteristics that influence priority as observables in the model.

4 Instruments

We now describe and probe the two instruments described above. Section 5 will formally prove identification.

4.1 Conditionally Independent Potential Offers

The first instrument exploits randomness in the objects offered to an agent, relying on Assumption 2. We argue that this assumption is theoretically and empirically plausible in our setting. Our theoretical justification is based on the mechanism used to allocate deceased donor kidneys. Recall that J_i is the sequence of offers to agent i if the agent refused all offers made to them and participated in the mechanism indefinitely. Thus, J_i depends only on the kidneys that arrive after a patient registers on the waiting list, the decisions of other

patients, and the determinants of the agent's priority.It does not depend on the decisions made by agent i or their survival outcome. Our knowledge of the mechanism allows us to include determinants of each patient's priority in x_i as controls. The remaining variation in J_i is only due to the stochastic arrival of organs and the decisions of agents other than i. It is plausible to assume that organ arrival is independent of (ν_i, ε_i) because it depends primarily on deaths in the local area. Furthermore, the decisions of other agents are independent of (ν_i, ε_i) in a natural equilibrium model of the the waiting list (Agarwal et al., 2021).

We now empirically investigate these assumptions using a specific function of J_i. To do this, we construct a set of desirable donors that are achievable for patient i in the two years following the patient's registration. Specifically, we calculate whether patient i would be placed above the patient in the 10th position on the list for a given donor. A patient is highly likely to receive an offer for an organ from such a donor because only 22.7% of deceased donors are offered to fewer than ten patients. We then calculate the number of donors that would satisfy these criteria for each patient in the two years following the patient's registration date.

The variation in this variable comes from two sources: variation in the organs that arrived in the two years following patient i's registration and variation in the patients on the waiting list and their decisions when the organ arrived. Our results use fixed effects to control for differences in a patient's priority, geographical area, and time trends. Therefore, Assumption 2 needs to be satisfied conditional on these controls. The first source of variation is independent of i's decisions because specific patients are not considered in organ donation decisions. Indeed, we cannot detect a correlation between patient characteristics and donor characteristics conditional on the controls mentioned above (not reported due to space constraints, available on request). The second source of variation is also plausibly exogenous because, given a particular organ, other patients' decisions should be independent of the selectivity and outcomes of patient i.[15] Consistent with this claim, Appendix Table D.1 shows that this measure varies substantially across patients and is not significantly correlated with the vast majority of patient characteristics conditional on determinants of priority-type, which

[15]The only potential effect is if patient i accepts a kidney that would otherwise have been accepted by another patient who would been pivotal in determining whether i would be in the top ten positions for a different donor.

are exogenous and fixed at the time of registration.

Given this exclusion restriction, we establish relevance by showing that potential offers strongly influence whether or not a patient receives a transplant and also the type of organ transplanted. Columns (1) to (4) in Table 3 present estimates from linear probability models to examine the relationship between whether the transplanted organ is high quality (as measured by KDPI) and the number of potential top 10 offers from donors in the corresponding group. Columns (1) and (2) show that the numbers of offers in both donor categories are positively related to the probability of a transplant, whether or not we control for a rich set of patient characteristics. Columns (3) and (4) show that the type of organ transplanted is positively correlated with the number of potential offers from the corresponding type of donor. The F-statistics point to a strong first-stage relationship as they are much higher than the conventional cutoff of 10 used to assess whether an instrument is strong (Stock and Watson, 2012).

4.2 A Choice Shifter: Scarcity

Our second set of instruments are measures of scarcity z_i that alter an agent's acceptance decisions $D_{i,j}$ but are excluded from latent outcomes $Y_{i,j}$. Patients who expect better transplant opportunities in the future (lower scarcity) should be less willing to accept a given kidney than otherwise identical patients with fewer opportunities (higher scarcity). These instruments must be correlated with decisions but independent of latent outcomes. Formally, Assumption 1(i) requires that, conditional on x_i, (ν_i, ε_i) is distributed independently of z_i.

We construct two measures of scarcity. The first is a predictor of offers a patient can expect in the future. Fix an offer for donor j made to patient i in the calendar quarter t. Consider the set of offers made in the four quarters before t to other patients in a comparison group consisting of other patients with the same blood type as i and that registered in the same DSA as i. We count the subset of offers made to this group of patients when they had the same number of waiting time priority points as patient i when they received the offer for donor j. The second is a predictor of donor supply, which is constructed analogously to the first but counts the number of unique donors in this set of offers.

Table 3: Top 10 Offers: First Stage

	Transplant			
	Any Kidney (1)	Any Kidney (2)	KDPI <= 50% (3)	KDPI > 50% or Missing (4)
log(1 + # Top 10 Offers in 2 Years)				
KDPI <= 50%	0.0479	0.0481	0.0602	-0.0122
	(0.00460)	(0.00466)	(0.00358)	(0.00251)
KDPI > 50% or Missing	0.0234	0.0247	-0.0167	0.0414
	(0.00386)	(0.00387)	(0.00292)	(0.00245)
DSA FE, year FE, and blood type FE	x	x	x	x
Control for Pediatric at Listing	x	x	x	x
CPRA Category Controls	x	x	x	x
Patient Characteristics		x	x	x
F-statistic	142.6	147.9	153.9	162.7
Number of Observations	132507	130923	132507	132507
R-Squared	0.171	0.180	0.140	0.048

Notes: The sample restricts to non-pediatric patients who registered between 2000 and 2008 because the instrument is calculated using offers in the two years post registration. All regressions control for donor service area (DSA) fixed effect, registration year fixed effect, blood type fixed effect, and priority characteristics (indicators for CPRA = 0, 20 <= CPRA < 80, CPRA >= 80, and CPRA missing at registration and time on dialysis). Patient characteristics include an indicator for female; indicators for age 18-35, 35-50, and 50-65; indicators and linear controls for dialysis time 1-3, 3-5, 5-10, and >10 years; and an indicator for diabetes. Standard errors, clustered by DSA, registration year, and blood type are in parentheses. F-test tests against the null hypothesis that the coefficients on the instruments are zero.

Our analysis will include fixed effects for the DSA, blood-type, and calendar year of the assignment. Therefore, both instruments exploit variation in scarcity in a patient's DSA while controlling for secular trends. To assess balance, we investigated whether variations in our measures of scarcity significantly correlate with the characteristics of patients that register in a given year. Reassuringly, Table D.2 in the appendix shows that our scarcity instruments are not significantly correlated with patient characteristics (age, diabetes, female, height, and weight). Our scarcity instruments are also uncorrelated with measures of donor quality (not reported due to space constraints, available on request). The threat to the instrument therefore needs to be a DSA-specific trend in scarcity that is correlated with survival outcomes due to factors beyond patient or donor characteristics.[16]

These instruments are relevant to decisions if they are correlated with beliefs about future

[16]We also estimate a model with DSA-specific time trends. The results from this model are very similar to those from our preferred specification. Compare columns 2 and 9 of Appendix Table D.3.

Table 4: Scarcity Instruments: First Stage

	Acceptance							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Log(1 + No. Donors)	-0.0434		-0.0429		-0.0393		-0.0355	
	(0.00209)		(0.00208)		(0.00204)		(0.00206)	
Log(1 + No. Offers)		-0.039		-0.0387		-0.0336		-0.0316
		(0.00106)		(0.00106)		(0.00106)		(0.00109)
Offer Year FE	x	x	x	x	x	x	x	x
Priority Type FE	x	x	x	x	x	x	x	x
DSA FE and blood type FE	x	x	x	x	x	x	x	x
Years Waited at Offer FE	x	x	x	x	x	x	x	x
Patient Characteristics			x	x			x	x
Donor Characteristics					x	x	x	x
Match Characteristics					x	x	x	x
F-statistic	431.9	1361.8	426.1	1337.5	371.3	998.3	296.8	838.4
Number of Observations	863073	863073	863073	863073	851753	851753	851753	851753
R-Squared	0.025	0.030	0.026	0.031	0.069	0.072	0.097	0.100

Notes: We use the first 100 offers from each donor between 2000 and 2009, and the dependent variable is acceptance of an offer. All regressions control for DSA fixed effect, blood type fixed effect, and a fixed effect for the number of years waited at the offer, as well as priority characteristics (an indicator for pediatric at registration and indicators for CPRA = 0, 20 <= CPRA < 80, CPRA >= 80, and CPRA missing at registration). Patient characteristics include an indicator for female; indicators for age <=18, 18-35, 35-50, and 50-65; indicators and linear controls for dialysis time 1-3, 3-5, 5-10, >10 years; and an indicator for diabetes. Donor characteristics include linear age; indicators and linear controls for donor creatinine > 0.6 and >1.8; and indicators for diabetes, donation after cardiac death, and expanded criteria donor. Match characteristics include the number of Human Leukocyte Antigen (HLA) mismatches via indicators for 0 HLA mismatch, 0 and 1 DR antigen mismatch, identical blood type, local offers; linear controls for (+) and (-) age difference; and interactions between CPRA indicators and # HLA mismatches and between donor and patient age. Standard errors clustered by DSA, offer year, number of years waited at offer, and blood types are in parentheses.

offers. This hypothesis is based on the idea that transplant surgeons, who advise patients on decisions, are likely aware of the recent availability of kidneys. Columns (1) to (8) of Table 4 show the results from a linear probability model that regresses a dummy on whether an offer is accepted on the two measures of scarcity and a variety of controls. Both measures of scarcity are negatively correlated with acceptance. Columns (1) and (2) show that the number of donors or number of past offers made to patients in the comparison group is negatively correlated with acceptance rates, controlling for patient priority type and fixed effects for DSA, allocation year, and years waited. These magnitudes are robust to adding an extensive set of controls for patient characteristics (columns 3 and 4), and are not very sensitive to additional controls for donor and match-specific characteristics (columns 5 through 8). A residualized binscatter plot suggests that these relationships are monotonic (not reported

due to space constraints, available on request).

5 Identification and Estimation

We now show that the instruments J_i and z_i introduced in the previous section can identify the target quantities described in Section 3. Our results condition on the patient type x_i and omit it for simplicity of notation. We observe the organ types q_j, the choices $D_{i,j}$ if i is offered j, the set of organs offered to each patient, and the survival outcome for each patient. We do not require observing either A_i or the potential offer sequence J_i as long as Assumption 2 is satisfied.[17]

The argument proceeds in three parts. First, we use standard arguments to show that variation in the offers received by a patient can be used to recover distributions of the outcomes conditional on certain sequences of choices. Second, we show that the choice model described in equation (3.3) is identified. Third, we combine continuous variation in scarcity with results from the first part to identify the effect of key unobservables on the distribution of outcomes. All proofs are in the online appendix C.

5.1 Identifying Conditional Expected Outcomes

We start by using variation in offers. Given a realization of J_i, let $j(i,n)$ denote the n-th organ offered to i and $q_i = \left(q_{j(i,1)}, q_{j(i,2)}, \ldots, q_{j(i,|q_i|)}\right)$ be the sequence of offer-types offered to i. Our first result shows that variation in the offer-types can identify a conditional average treatment effect for patients who accept the $n-$th offer.[18] Formally, let N_i be one greater than the number of offers that i rejects prior to the first acceptance, that is, $N_i = \min\left\{n : D_{i,j(i,n)} = 1\right\}$.

Lemma 1. *Suppose that Assumption 2 is satisfied. Fix z and q_i. The marginal distributions of $Y_{i,j(i,n)}$ and $Y_{i,0}$ conditional on $N_i = n$, $z_i = z$, q_i, and $A_i \geq t_{j(i,n)}$ are identified for all $n \leq |q_i|$ such that $P\left(N_i = n | q_i, z, A_i \geq t_{j(i,n)}\right) > 0$, and $\left(q_{j(i,1)}, \ldots, q_{j(i,n)}\right)$ and*

[17] Nonetheless, we can simulate J_i in our context using knowledge of the mechanism and data on the offers made for each donor.

[18] Observe that our model and setting do not allow for always takers since a patient cannot be assigned an organ without receiving an offer for one.

$\left(q_{j(i,1)}, \ldots, q_{j(i,n-1)}\right)$ *belong to the support of the distribution of offer-types induced by the distribution of* J_i.

This result uses standard arguments (e.g., Imbens and Angrist, 1994) to identify counterfactual outcomes for patients who would have accepted and been assigned to the $n-$th organ offered. For simplicity, assume that all kidneys are observationally identical and fix z. Start with the case in which patients may either receive one offer or no offers. This yields a standard binary instrument setup in which the compliers are the patients for whom $N_i = 1$. Thus, the results in Imbens and Angrist (1994) imply that the marginal distributions of $Y_{i,1}$ and $Y_{i,0}$ conditional on $N_i = 1$ are identified. Lemma 1 extends this argument to the general case using an experiment that compares otherwise identical patients with offer-type sequences $\left(q_{j(i,1)}, \ldots, q_{j(i,n)}\right)$ and $\left(q_{j(i,1)}, \ldots, q_{j(i,n-1)}\right)$. We directly observe the outcomes $Y_{i,j(i,n)}$ for patients who are offered $\left(q_{j(i,1)}, \ldots, q_{j(i,n)}\right)$ and are assigned to the $n-$th organ. These patients are the compliers in this experiment because patients cannot be assigned an organ without receiving an offer, i.e. our model does not have always-takers. Defiers are ruled out by Assumption 2 because N_i is independent of J_i. To complete the proof, we need to identify the expected outcomes of unassigned outcomes for the compliers.[19] The group of unassigned patients that received $\left(q_{j(i,1)}, \ldots, q_{j(i,n)}\right)$ contains only never-takers since $N_i > n$ whereas the group that received $\left(q_{j(i,1)}, \ldots, q_{j(i,n-1)}\right)$ contains both compliers and never-takers since $N_i > n-1$. The weights on these groups is implied by the distribution of N_i conditional on belonging to the experiment, which is given by $P\left(N_i = k \middle| q_i, z, A_i \geq t_{i,j(i,n)}\right)$ and is directly observed.

This result allows us to evaluate the life-years gained in the observed assignment because the alternative is that all patients are unassigned. Identifying the distributions above, however, is not sufficient for evaluating their values under a counterfactual assignment of kidneys to patients because the distributions condition on $N_i = n$, and are therefore selected on both patient-specific unobserved selectivity $\nu_{i,D}$ and match-specific shocks $\varepsilon_{i,j,D}$. We address this selection problem below using a choice shifter and our model of choice.

[19] Identification of the expected outcomes $E\left[\psi\left(Y_{i,j(i,n)}\right) \middle| N_i = n\right]$ and $E\left[\psi\left(Y_{i,0}\right) \middle| N_i = n\right]$ for any known bounded function $\psi(\cdot)$ implies the identification of the marginal distributions of $Y_{i,j(i,n)}$ and $Y_{i,0}$ conditional on $N_i = n$ because $\psi(\cdot)$ includes functions of the form $1\{Y < \bar{y}\}$ for all $\bar{y} \in \mathbb{R}$. Monotonicity follows because Assumption 2 and our choice model rule our defiers.

5.2 Identifying the Choice Model

The next step identifies the function $g_D(\cdot)$. To simplify exposition, focus on the case when $t_{i,j} = 0$ where $t_{i,j}$ denotes the time difference between donor arrival and patient arrival. In this case, ν_i is unselected due to survival while waiting on the list. Because our empirical setting involves dynamic assignments, we prove results for the case when $t_{i,j} > 0$ and differs across j in appendix C.4.

We need to introduce some notation in order to develop our result. For each value of z and donor type q_j, consider two sets of pairs (ν_D, ε_D) such that one set yields $g_D(q_j, z, \nu_D, \varepsilon_D) = 0$ and the other yields $g_D(q_j, z, \nu_D, \varepsilon_D) = 1$. These two sets are separated by the function $v(\varepsilon_D; q_j, z) = \sup\left\{\nu_D \in \left[0,1\right] : g_D(q_j, z, \nu_D, \varepsilon_D) = 1\right\}$, where we adopt the convention that the supremum of the empty set is 0. Since ε_D and ν_D are uniformly distributed, observe that $v(\varepsilon_D; q_j, z)$ is equal to the fraction of patients that reject an offer of an organ with type q_j with probability at most ε_D when faced with scarcity z. Therefore, identifying the function $v(\varepsilon_D; q_j, z)$ is equivalent to identifying $g_D(\cdot)$.

Our next result makes the following assumption on $v(\cdot; q_j, z)$:

Assumption 3. *For each q_j and z, $v(\cdot; q_j, z)$ is absolutely continuous, $v(0; q_j, z) = 0$, and $v(1; q_j, z) = 1$.*

This assumption requires that there are no (interior) values of ν_D for which the patient either accepts or rejects all organs of type q_j when faced with scarcity z. In other words, there are high (low) enough match-specific shocks ε_D that would result in acceptance (rejection) of an offer, where the pivotal value of ε_D depends on ν_D, q_j, and z. This condition would be violated only if acceptance probabilities were degenerate for some q_j, z, and $\nu_D \in (0,1)$. With this assumption, we show that variation in offers can be used to identify the function $g_D(\cdot)$:

Lemma 2. *Let q_j^n be a sequence composed by n offers of type q_j with $t_{i,j} = 0$, and let $v_{n-1}(\cdot; q_j, z)$ be the $(n-1)$-st order Fourier-Legendre approximation of $v(\cdot; q_j, z)$. If Assumptions 1 - 3 are satisfied, and q_j^n is in the support of the distribution of offer-types induced by J_i, then $v_{n-1}(\cdot; q_j, z)$ is identified for each z and q_j. In particular, if the hypotheses hold*

for all n, then $v(\cdot; q_j, z)$ and therefore $P(D_{i,j} = 1 | \nu_{i,D} = \nu_D)$ is identified.

The main challenge is that there are two latent reasons behind a patient's decisions, namely $\nu_{i,D}$ and $\varepsilon_{i,j,D}$, and we need to disentangle the two. Fix q_j and z, and consider a model that abstracts away from heterogeneity in patient selectivity by omitting $\nu_{i,D}$. In this model, the acceptance probability for the $k-$th offer is equal to the acceptance probability for the $k+1-$st offer. In fact, the distribution of N_i is geometric. However, the data can reject this implication. In a model that includes $\nu_{i,D}$, the patients who reject the $k-$th offer are more likely to be selective (high $\nu_{i,D}$) and the acceptance probability for the $k+1-$st offer will therefore be lower.

Formally, consider the observed probability $P\left(N_i > k | q_j^n, z\right)$ for $k \leq n$. Because $v(\varepsilon_D; q_j, z)$ is the CDF of rejection probability across patients given q_j and z, we can write

$$P\left(N_i > k | q_j^n, z\right) = \int_0^1 \varepsilon_D^k \mathrm{d}v(\varepsilon_D; q_j, z). \tag{5.1}$$

Therefore, the observed quantity $P\left(N_i > k | q_j^n, z\right)$ is the k-th moment of a random variable with cumulative distribution function (cdf) $v(\cdot; q_j, z)$. Learning the function $v(\cdot; q_j, z)$ is related to the Hausdorff moment problem. In general, the cdf of a random variable with bounded support is uniquely determined by its infinitely many moments (Theorem 2.3.11 in Casella and Berger, 2002). Under the absolute continuity assumption we obtain a stronger result: we show that data with finite n are informative even without variation in the number of offers because $v(\cdot)$ can be well-approximated by observing decisions from a *given* sequence of offer-types q_j^n. This follows because the moments described above determine the n-th order Fourier-Legendre approximation of $v(\cdot)$. The partial mean of these approximations converges to the true function $v(\cdot; q_j, z)$ as n becomes large.

5.3 Identifying Selection on Unobservables

Next, we turn our attention to identifying the components that determine selection on unobservables by using an additional regularity assumption:

Assumption 4. *(i) For each z and q_j, the derivative $v'(\cdot; q_j, z) = \frac{\partial}{\partial \varepsilon_D} v(\cdot; q_j, z)$ is a continu-*

ous function for $\varepsilon_D \in (0,1)$. *(ii) For each* q_j, *the functions* $E[Y_{i,0}|\,\nu_D]$ *and* $E[Y_{i,j}|\,\nu_D, \varepsilon_{i,j,D} \geq \varepsilon_D, q_j]$ *are continuous in* ν_D *and* ε_D *for* $(\nu_D, \varepsilon_D) \in (0,1)^2$. *(iii) The unconditional expectations of* $Y_{i,0}$ *and* $Y_{i,j}$ *exist.*

The first part strengthens absolute continuity of $v(\varepsilon_D; q_j, z)$, imposed in Assumption 3, by requiring the existence of a continuous derivative. Given the interpretation of $v(\cdot)$ above, observe that $v'(\cdot; q_j, z)$ is the density function of the distribution of the probability that a patient rejects an offer of an organ with type q_j. The second part imposes weak regularity assumptions on conditional expectations of $Y_{i,0}$ and $Y_{i,j}$, where the expectation is taken over $\nu_{i,0}$ and $(\nu_{i,1}, \varepsilon_{i,j,1})$, respectively. The third part requires that these conditional expectations are integrable over the random variables $\nu_{i,D}$ and $(\nu_{i,D}, \varepsilon_{i,j,D})$, respectively.

Our main result shows identification of the expected values of $Y_{i,0}$ and $Y_{i,j}$ given $\nu_{i,D}$ and $\varepsilon_{i,j,D}$. The result also implies identification of the analogous quantities for any bounded transformation $\psi(\cdot)$ of $Y_{i,0}$ and $Y_{i,j}$, thereby implying identification of their marginal distributions.

Theorem 1. *Suppose that Assumption 4 and the hypotheses for Lemma 2 hold for all* n. *Then, the quantities* $E[Y_{i,0}|\,\nu_{i,D} = \nu_D]$ *and* $E[Y_{i,j}|\,\nu_{i,D} = \nu_D, \varepsilon_{i,j,D} \geq \varepsilon_D, q_j]$ *are identified for all* $\varepsilon_D \in (0,1)$ *and* $\nu_D \in (0,1)$ *such that there exists* z *in the support of its distribution with* $\nu_D = v(\varepsilon_D; q_j, z)$.

Thus, the expected value of outcomes conditional on values of selectivity and idiosyncratic preferences is identified. We sketch the argument for $E[Y_{i,0}|\,\nu_D]$ since the intuition for identifying $E[Y_{i,j}|\,\nu_D, \varepsilon_{i,j,D} \geq \varepsilon_D, q_j]$ is similar in spirit.[20] The proof begins by using results in Lemma 1 to identify the conditional expectations given scarcity z, offer-types, and N_i. Next, we use the identification results for $v(\cdot)$ and arguments in Lemma 2 to recover the objects of interest. For example, Lemma 1 implies that $E\left[Y_{i,0} \times 1\{T_i = 0\}|\, q_j^k, z_i\right]$ is identified from variation in offers. This quantity can be re-written as

$$E\left[Y_{i,0} \times 1\{T_i = 0\}|\, q_j^k, z_i\right] = \int_0^1 E[Y_{i,0}|\,\nu_D = v(\varepsilon_D; z_i, q_j)]\, \varepsilon_D^k \mathrm{d}v(\varepsilon_D; z_i, q_j). \tag{5.2}$$

[20] One qualitative difference is that identifying $E[Y_{i,0}|\,\nu_D]$ allows us to use variation in either z or ε_D to "trace-out" ν_D, i.e., $E[Y_{i,0}|\,\nu_D]$ is overidentified, whereas the result for $E[Y_{i,j}|\,\nu_D, \varepsilon_{i,j,D} \geq \varepsilon_D, q_j]$ must condition on ε_D.

If we observe this quantity for all $k \leq n$, then we can recover the $n-$th order Fourier-Legendre approximation of $E\left[Y_{i,0} \middle| \nu_D = v\left(\varepsilon_D; q_j, z\right)\right] v'\left(\varepsilon_D; q_j, z\right)$ when viewed as a function of ε_D, which converges uniformly to the true function in Cesàro mean (Freud, 1971). Finally, since $v'\left(\varepsilon_D; q_j, z\right) > 0$ and bounded and the function $v\left(\varepsilon_D; q_j, z\right)$ is identified (Lemma 2), we can identify $E\left[Y_{i,0} \middle| \nu_D\right]$ for all $\nu_D \in (0,1)$ if we can find values of z and ε_D such that $v\left(\varepsilon_D; q_j, z\right) = \nu_D$. A similar argument yields identification of $E\left[Y_{i,j} \middle| \nu_{i,D} = \nu_D, \varepsilon_{i,j,D} \geq \varepsilon_D\right]$ provided that we observe values of z such that $\nu_D = v\left(\varepsilon_D; q_j, z\right)$. As is common, identification of $E\left[Y_{i,0}\right]$ and $E\left[Y_{i,j}\right]$ will require full support of $v\left(\varepsilon_D; q_j, z\right)$ for each ε_D and q_j.

This last step resembles strategies in Heckman and Vytlacil (2005); Lewbel (2007); Heckman and Navarro (2007) whereby a continuous instrument is used to "trace-out" the expected values of potential outcomes conditional on an unobservable. The scarcity instrument z does this by changing the set of (ν_D, ε_D) whose treatment status changes in response to the offer instrument. Two differences are worth noting. First, our scarcity instrument is not treatment-specific because the discrete offer instrument generates variation in treatment assignments (c.f. Heckman and Navarro, 2007; Hull, 2018, for example). Our assumption that $\nu_{i,D}$ does not vary across j allows us to use an instrument that varies only across patients i but is fixed across j. Second, we do not use "identification at infinity" arguments as values of z need not push choice probabilities to degenerate values that obviate the selection problem.

The results in Lemma 2 and Theorem 1 use data from the case when organs arrive at the same time as the patient ($t_{i,j} = 0$). Extending our results to the case when $t_{i,j} > 0$ and differs across j introduces two issues. First is the direct effect of time to treatment, which can be captured by including the patient's registration date and organ's arrival date in x_i and q_j. The second issue, which is the main challenge, is that the distribution of $\nu_{i,D}$ conditional on waiting until $t_{i,j}$ is no longer unselected.

Our extension in Appendix C.4 addresses these issues and implies identification of the marginal distributions and survival hazard functions of $Y_{i,0}$ and $Y_{i,j}$ (Theorem 3). As in generalized Roy models more broadly, the joint distribution of outcomes is not identified. Thus, we cannot attribute the effect of waiting time $t_{i,j}$ on $Y_{i,j}$ to either time-to-treatment or to correlation between survival outcomes. We ignore this distinction because it is not relevant for evaluating outcomes under counterfactual assignments.

5.4 Estimation

Although our results above show non-parametric identification, directly estimating these quantities is challenging for several reasons. First, we wish to incorporate rich observed and unobserved heterogeneity governing both choices and outcomes. Such heterogeneity includes patient-specific, donor-specific, match-specific and time-to-treatment effects. Second, that we observe only censored versions of our outcome complicates a non-parametric analysis. Finally, we would like to incorporate correlations between discrete choices and these censored outcomes.

To solve these challenges, we employ a Gibbs' sampling technique to estimate a parametrized version of equations (3.1) – (3.3):[21]

$$y_{i,0} = B\left(Y_{i,0}; \rho_0\right) = x_i \beta_x + \nu_{i,0} \tag{5.3}$$

$$y_{i,j} = B\left(Y_{i,j}; \rho_1\right) = \chi\left(x_i, q_j\right) \alpha_{x,q} + \alpha_\eta \eta_j + \nu_{i,1} + \varepsilon_{i,j,1} \tag{5.4}$$

$$D_{i,j} = 1\left\{\chi\left(x_i, q_j\right) \gamma_{x,q} + z_i \gamma_z + \eta_j - \nu_{i,D} + \varepsilon_{i,j,D} > 0\right\}, \tag{5.5}$$

where $Y_{i,0}$ is survival since registration without a transplant; $Y_{i,j}$ is survival since transplantation if patient i is transplanted organ j; $B\left(\cdot; \rho\right)$ denotes a Box-Cox transformation of the argument with parameter ρ (Box and Cox, 1964);[22] $\chi\left(x_i, q_j\right)$ is a flexible function of patient observables x_i and organ observables q_j; η_j is distributed $\mathcal{N}\left(0, \sigma_\eta^2\right)$ with the parameter σ_η^2 to be estimated; $\varepsilon_{i,j} = \left(\varepsilon_{i,j,D}, \varepsilon_{i,j,1}\right)'$ is distributed $\mathcal{N}\left(0, \Sigma_\varepsilon\right)$ where $\Sigma_{\varepsilon,11}$ is normalized to 1; and ν_i is a mean-zero multi-variate normal with a distribution induced by the following factor structure, which is without loss of generality:

$$\nu_{i,1} = \delta_{1,D} \nu_{i,D} + \nu_{i,f} \tag{5.6}$$

$$\nu_{i,0} = \delta_{0,D} \nu_{i,D} + \delta_{0,f} \nu_{i,f} + \tilde{\nu}_{i,0}, \tag{5.7}$$

[21] It is common to use functional form restrictions that are stronger than those necessary for identification when estimating a model that involves selection due to choices and several types of treatments (see Geweke et al., 2003; Hull, 2018, for example).

[22] Formally, $B\left(Y; \rho\right) = \frac{Y^\rho - 1}{\rho}$. In the special case when $\rho = 0$, $B\left(Y, \rho\right) = \log Y$. We set ρ by comparing an estimated survival curve using the non-parametric Kaplan-Meier estimator to those implied by assuming that $B\left(Y, \rho\right)$ is normally distributed.

where $\nu_{i,D}$, $\nu_{i,f}$ and $\tilde{\nu}_{i,0}$ are independently distributed mean-zero normal random variables with variances to be estimated.

This empirical model maps the patient and kidney types into characteristic space, which reduces the number of parameters. The characteristics include η_j, which represents unobserved heterogeneity in organ quality due to characteristics observed by patients and surgeons but not incorporated into the empirical specifications. We include this term because it may be empirically important. Appendix C.3 shows identification results analogous to theorem 1 in a non-parametric model that allows for organ-level heterogeneity η_j. The argument leverages the feature that the same donor's organs are offered to multiple patients on the waiting list. The correlation between these patients' decisions that cannot be explained by organ-level observables provides information about η_j.

This choice of functional form is motivated by several considerations. First, we wish to allow for correlations between $\nu_{i,0}$, $\nu_{i,1}$, and $\nu_{i,D}$ and between $\varepsilon_{i,j,1}$ and $\varepsilon_{i,j,D}$. For example, the factor $\nu_{i,f}$ captures the component of a patient's unobserved frailty that is not correlated with decisions. Second, decisions are binary, which suggests using probit choice models. These two considerations direct us to employ multivariate normals to model the distributions of ν_i and $\varepsilon_{i,j}$. Third, the parametrization allows us to handle censored data and also fit the shape of the survival curve. Box-Cox transformations yield a tractable likelihood function while generalizing the functional form (see Spitzer 1982, for example). We hold the Box-Cox transformation parameters ρ_0 and ρ_1 fixed and conduct robustness analysis to alternative choices (see Table D.3).

Directly computing and maximizing the likelihood of this model is difficult because each patient's data involve decisions about many donors over time as well as (potentially censored) survival outcomes. Calculating this likelihood requires integrating a nonlinear function over a high-dimensional space. Instead, we estimate the parameters of the model using a Gibbs' sampler (McCulloch and Rossi, 1994; Geweke et al., 2003; Gelman et al., 2014). This method generates a sequence of draws of the model's parameters, collected in θ, and the latent variables ν_i, $\varepsilon_{i,j}$, and η_j given the parameters from their respective posterior distributions. Our chosen parametrization is amenable to this approach because the latent variables can be partitioned so that each group has a posterior distribution given the draws of the other

groups that can be solved in closed form. Details on the method are provided in Appendix B.1. Based on the Bernstein-von-Mises Theorem (see van der Vaart, 2000, Theorem 10.1), we interpret our estimator as equivalent to maximum likelihood. We conducted Monte Carlo simulations to assess the properties of our estimator using one hundred simulated datasets with 10,000 patients and 2,500 donors. These exercises demonstrated that our estimator has good coverage and convergence properties.[23]

6 Survival and Choice Estimates

Table 5 presents estimates for survival without and with a transplant as well as the probability of acceptance in panels A, B, and C, respectively (detailed estimates are available on request). Our specifications contain a rich set of patient and donor covariates to capture medical history and match quality, including characteristics used in the leading models for predicting pre- and post-transplant survival for patients with kidney failure (see Wolfe et al., 2008, for example) as well as determinants of patient priority. Survival estimates show the marginal half-life effects associated with select characteristics. Effects are shown for a one standard deviation increase in a continuous characteristic or a unit change in an indicator.

We present estimates from three different specifications. The first specification (in column 1) replicates the observational approach in Wolfe et al. (2008) for our dataset: this specification only relies on offer randomness; does not employ the scarcity instruments (columns 1); and abstracts away selection on unobservables by assuming that $\nu_{i,D}$, $\nu_{i,0}$ and $\nu_{i,1}$ and $\varepsilon_{i,j,D}$ and $\varepsilon_{i,j,1}$ are mutually independent. The second specification, which is our preferred one, includes the number of past donors as the scarcity instrument (columns 2). To assess robustness, we estimate a third specification with our past offers instrument (columns 3). Table D.3 in the appendix shows our headline findings are robust to numerous variations.

Survival: Proxies for baseline patient health predict survival both with and without a transplant. A patient who is older, diabetic, or on dialysis at registration has a significantly shorter half-life either with or without a transplant, with slightly larger effects for post-transplant survival. For example, a diabetic patient's half-life with or without transplant is

[23] Detailed results and code are available in the replication package associated with the manuscript.

shorter than that of a non-diabetic patient by 3.58 or 1.45 years, respectively.

Measures of donor quality, waiting time, and tissue-type similarity also predict post-transplant survival, but donor characteristics have lower estimated effects than tissue-type matching and patient characteristics. For example, a donor with a history of hypertension yields a shorter half-life by 0.40 years, a difference which is much smaller than those produced by the patient characteristics described above. Receiving a kidney with a perfect tissue-type match has a large effect on half-life, consistent with a lower likelihood of an immune response.

Choice: Measures of donor quality and match-specific benefits are also positively correlated with acceptance. Patients are significantly more likely to accept kidney offers from younger donors; donors who died of head trauma; donors without a history of hypertension; and donors with whom they have a perfect tissue-type match. Kidneys that have higher unobservable quality, η_j, are also more likely to be accepted, suggesting that decisions respond to organ information that is not perfectly captured by the observable characteristics.[24]

The last two rows record the scarcity instruments' effects on acceptance. Consistent with the results in Table 4, each instrument has a significant negative effect on the probability of acceptance. Other parameter estimates are similar across the instrumented specifications, suggesting that the choice between these two instruments is unlikely to be an important driver of our results.

A comparison of estimates across the panels indicates that many organ quality measures positively affect both choice and survival. Tissue-type match and donor death by head trauma are both strongly associated with both choice and survival. That said, the association is not perfect: organs from younger donors are more likely to be accepted even though the survival effects are not significant.

These results qualitatively differ from those in Abdulkadiroglu et al. (2020), who find that preferences for schools are not correlated with value-added after controlling for peer characteristics. An important difference in the institutional context is that choices in our setting are advised by doctors who, given their significant experience and expertise, may have more

[24]In column 10 of Appendix Table D.3, we present results in which the coefficients in the choice and outcome equations are allowed to differ by DSA. The results from this specification are similar to those using the main specification.

Table 5: Survival and Choice Estimates

	Panel A: Survival without Transplant			Panel B: Survival with Transplant			Panel C: Acceptance Model		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Patient Characteristics									
Diabetic	-1.464	-1.452	-1.452	-3.433	-3.580	-3.573	0.000	-0.005	-0.005
	(0.031)	(0.031)	(0.031)	(0.094)	(0.105)	(0.104)	(0.000)	(0.001)	(0.001)
On Dialysis at Registration	-1.067	-1.066	-1.066	-2.492	-2.574	-2.567	0.000	0.002	0.002
	(0.042)	(0.042)	(0.042)	(0.111)	(0.115)	(0.114)	(0.000)	(0.001)	(0.001)
Age at Registration	-1.133	-1.128	-1.128	-3.674	-3.737	-3.724	0.000	0.004	0.004
	(0.026)	(0.026)	(0.026)	(0.115)	(0.121)	(0.120)	(0.000)	(0.001)	(0.001)
Donor Characteristics									
Age < 18				1.988	2.010	2.056	0.035	0.156	0.156
				(0.859)	(0.845)	(0.843)	(0.002)	(0.009)	(0.009)
Age 18-35				0.397	0.341	0.368	0.014	0.091	0.091
				(0.941)	(0.920)	(0.918)	(0.001)	(0.009)	(0.009)
Age 50+				2.882	2.840	2.757	-0.003	-0.073	-0.071
				(2.119)	(2.111)	(2.101)	(0.000)	(0.003)	(0.003)
Cause of Death - Head Trauma				0.669	0.706	0.733	0.009	0.071	0.069
				(0.315)	(0.314)	(0.313)	(0.000)	(0.007)	(0.007)
Expanded Criteria Donor (ECD)				-0.573	-0.624	-0.657	0.002		
				(0.181)	(0.197)	(0.195)	(0.000)	(0.002)	(0.002)
History of Hypertension				-0.371	-0.395	-0.409	0.000	-0.028	-0.027
				(0.120)	(0.121)	(0.121)	(0.000)	(0.001)	(0.001)
Unobservable (η_j)					0.183	0.252		0.220	0.215
					(0.173)	(0.168)		(0.002)	(0.002)
Offer Characteristics									
Perfect Tissue Type Match				2.477	2.338	2.392	0.037	0.124	0.126
				(0.891)	(0.892)	(0.891)	(0.002)	(0.009)	(0.009)
Log Waiting Time (Years)				-0.559	-0.948	-0.934	0.002	0.027	0.017
				(0.059)	(0.145)	(0.142)	(0.000)	(0.001)	(0.001)
Scarcity									
Log(1+#Past Donors)								-0.010	
								(0.001)	
Log(1+#Past Offers)									-0.020
									(0.001)
Instruments	No Instruments	# Past Donors	# Past Offers	No Instruments	# Past Donors	# Past Offers	No Instruments	# Past Donors	# Past Offers

Notes: Select estimates of the marginal effect on the probability of acceptance and half-life. Marginal effects are computed at the median value of observable covariates, integrating over the distribution of all unobservables. All effects are shown for a one standard deviation increase in each continuous covariate and a unit increase in each binary covariate. We generate 250000 draws and burn-in the first 50000 draws. We thin the chain by selecting every 10 draws. We check for convergence visually and also ensure that the potential scale reduction factor for each parameter is at most 1.1. All columns control for DSA fixed effects, blood type fixed effects, and registration year fixed effects. Other patient characteristics include dialysis time at registration, BMI at departure, patient serum albumin, and indicators for female, diabetic, CPRA=0, and prior transplant. Donor characteristics include indicators for other causes of death, expanded criteria donor, donation after cardiac death, male, and bins of creatinine levels. Other offer characteristics include indicators for 2 A, 2 B, 2 DR mismatches, not the same blood type but compatible, regional offer, local offer, and interactions between several patient and donor characteristics. Standard errors are in parentheses.

Table 6: Correlation Table

	Panel A: Selectivity ($\nu_{i,D}$)		Panel B: Match value ($\varepsilon_{ij,D}$)	
	(1)	(2)	(1)	(2)
Probability of Acceptance	-0.040	-0.040	0.066	0.065
	(0.001)	(0.001)	(0.001)	(0.001)
Post-Transplant Survival	-0.054	-0.086	-0.003	0.092
	(0.134)	(0.131)	(0.249)	(0.245)
Survival without a Transplant	0.316	0.311		
	(0.062)	(0.061)		
Instruments	# Past Donors	# Past Offers	# Past Donors	# Past Offers

Notes: Estimates of how a one standard deviation increase in choice unobservables affects acceptance and survival probabilities. Survival durations are calculated using half-lives. Survival effects from changes in $\varepsilon_{ij,D}$ are computed using the expected change in $\varepsilon_{ij,1}$ from a one standard deviation rise in $\varepsilon_{ij,D}$ from zero, given the estimated covariance between $\varepsilon_{ij,D}$ and $\varepsilon_{ij,1}$. Likewise, survival effects from changes in $\nu_{i,D}$ are computed using the expected changes in $\nu_{i,1}$ and $\nu_{i,0}$ from a one standard deviation increase in $\nu_{i,D}$ from zero, given the estimated covariances between $\nu_{i,D}$, $\nu_{i,1}$, and $\nu_{i,0}$. All effects are computed at the median value of observable covariates.

accurate beliefs about survival effects than parents have about value-added.

The point estimates on each characteristic are similar across the three specifications in Table 5, with perhaps more similarities between estimates in columns 2 and 3 than the other pairings. These small differences between column 1 and the others accumulate to more appreciable differences between the estimated effects on life-years from transplantation, which will be presented in Table 7 below. The difference will partly stem from selection on unobservables, which is omitted in the specification in column 1 but included in the other two. We now describe the estimates of the unobservables that drive this selection.

Selection on Unobservables: Our model measures the correlation that unobservable characteristics induce between survival and choice.Table 6 shows how a one-standard-deviation increase in $\nu_{i,D}$ (selectivity) and $\varepsilon_{i,j,D}$ (match value) affects acceptance and survival. The selectivity effects are measured by computing the changes to $\nu_{i,0}$ and $\nu_{i,1}$ induced by their estimated correlation with $\nu_{i,D}$. Likewise, the correlation between $\varepsilon_{i,j,D}$ and $\varepsilon_{i,j,1}$ yields the effects of match value.

Selective patients typically survive longer without a transplant and benefit less from the typical transplant. A one-standard-deviation rise in selectivity lowers the probability of

acceptance by 4.0 percentage points. This effect is of a similar order as that of a kidney from a donor with a history of hypertension. Therefore, there is positive selection into treatment on the patient-specific component of survival benefits. Observational studies that ignore this source of selection may underestimate the benefits of transplantation. In contrast to selectivity, patient-donor specific factors do not induce significant selection via choices. While we estimate the covariance between $\varepsilon_{ij,D}$ and $\varepsilon_{ij,1}$ to be positive, the effect is not statistically significant.

7 Estimated LYFT

7.1 Calculating Life Years from Transplant (LYFT)

For each patient-donor pair, we compute the difference between the median survival time with a transplant and median survival time without a transplant, measured from the date of transplant. Specifically, for each pair (i, j), we define LYFT conditional on a set of covariates $I_{i,j} = \{x_i, q_j, D_{ij}, \eta_j, \nu_{i,D}, \nu_{i,f}\}$ as follows:

$$LYFT\left(I_{i,j}\right) = M\left(Y_{i,j} \middle| I_{i,j}, Y_{i,0} \geq t_{i,j}\right) - M\left(Y_{i,0} \middle| I_{i,j}, Y_{i0} \geq t_{i,j}\right), \tag{7.1}$$

where $M\left(Y \middle| X\right)$ is the median of random variable Y conditional on X and $t_{i,j}$ is the time between patient i's registration and the arrival of kidney j.[25,26] Therefore, this measure accounts for selection on unobservables induced by the mechanism.

7.2 Life Years from Transplant in the Mechanism

Table 7 presents the average estimated LYFT over all realized transplants. The first row accounts for patient- and kidney-specific unobservables and the decision to accept. The second row conditions only on patient and donor observables, integrating $LYFT\left(I_{i,j}\right)$ over

[25]Some estimates of LYFT place a weight of 0.8 on life years without a functioning kidney to account for the lower quality of life (e.g. Wolfe et al., 2008). Our approach omits this arbitrary quality-adjustment.

[26]We use a Gibbs' sampler to compute the expectation of $LYFT\left(I_{ij}\right)$ by drawing η_j, $\nu_{i,D}$, and $\nu_{i,f}$ from their conditional distributions given observables, decisions, and observed survival outcomes. We fix the parameters at the estimate $\hat{\theta}$, generate 200,000 draws, burn-in the first half, and use every 1,000-th draw.

D_{ij}, η_j, $\nu_{i,D}$, $\nu_{i,f}$. The average LYFT from our preferred specification is 8.93 years (column 2). Ignoring selection on unobservables yields a lower estimate of 7.97, suggesting positive selection on LYFT into transplantation based on unobservables. The specification that does not use scarcity instruments yields biased estimates, about two-thirds of a year less than our preferred estimate (column 1). This suggests observational methods used in the medical literature may underestimate gains from transplantation.

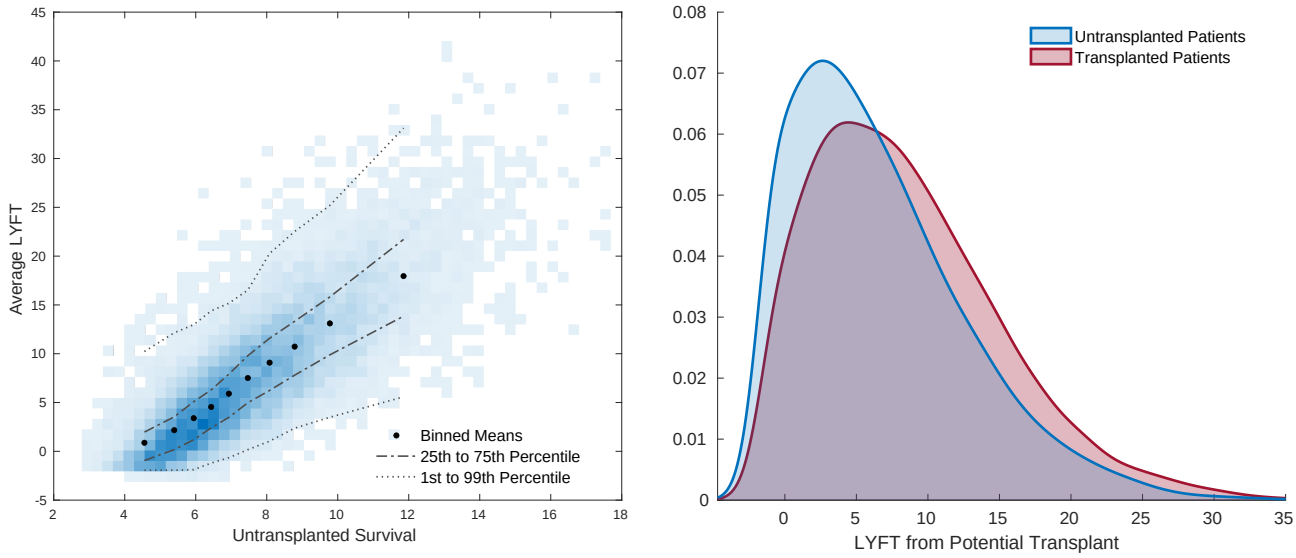
Table 7: Life Years from Transplant

	(1)	(2)	(3)	(4)
Life Years from Transplant				
Accounting for Unobservables	8.25	8.93	8.93	8.91
	(0.07)	(0.12)	(0.12)	(0.10)
Observables Only	8.25	7.97	7.87	8.09
	(0.07)	(0.26)	(0.25)	(0.24)
Untransplanted Survival				
All Patients	7.21	7.21	7.21	7.21
	(0.02)	(0.04)	(0.04)	(0.03)
Transplanted Patients	7.58	7.53	7.53	7.52
	(0.03)	(0.05)	(0.05)	(0.05)
Post-Transplant Survival	15.82	16.46	16.46	16.43
	(0.06)	(0.09)	(0.09)	(0.09)
Instruments	No Instruments	# Past Donors	# Past Offers	# Past Donors
Additional Donor Characteristics				X

Notes: Life years from transplant and survival durations presented in the table are calculated using half-lives. Future donors (offers) is defined as the number of donors (offers) in the next four quarters (see Table 4 for detailed definition). All columns control for patient, donor, and offer characteristics, which are defined analogously as in Table 5 Panel B and Table 5. Standard errors are in parentheses.

The second pair of rows report average survival without a transplant, separately, for all patients and the subset of patients who received a transplant. Across specifications, the untransplanted survival for patients who are transplanted is higher than for patients who are not. Thus, choices and the mechanism result in selection on untransplanted survival into transplantation. Results are robust to using past offers rather than past donors (column 3) and to including further covariates, such as time between organ extraction and transplantation, present in the medical literature (e.g., Rao et al., 2009).[27]

[27]Our preferred specification omits these additional covariates because it is not possible to compute cold ischemic times for the counterfactual allocations in Section 8. In particular, cold ischemic time is determined



(a) LYFT and Untransplanted Survival

(b) LYFT by Transplant Status

Figure 2: Patient Selection

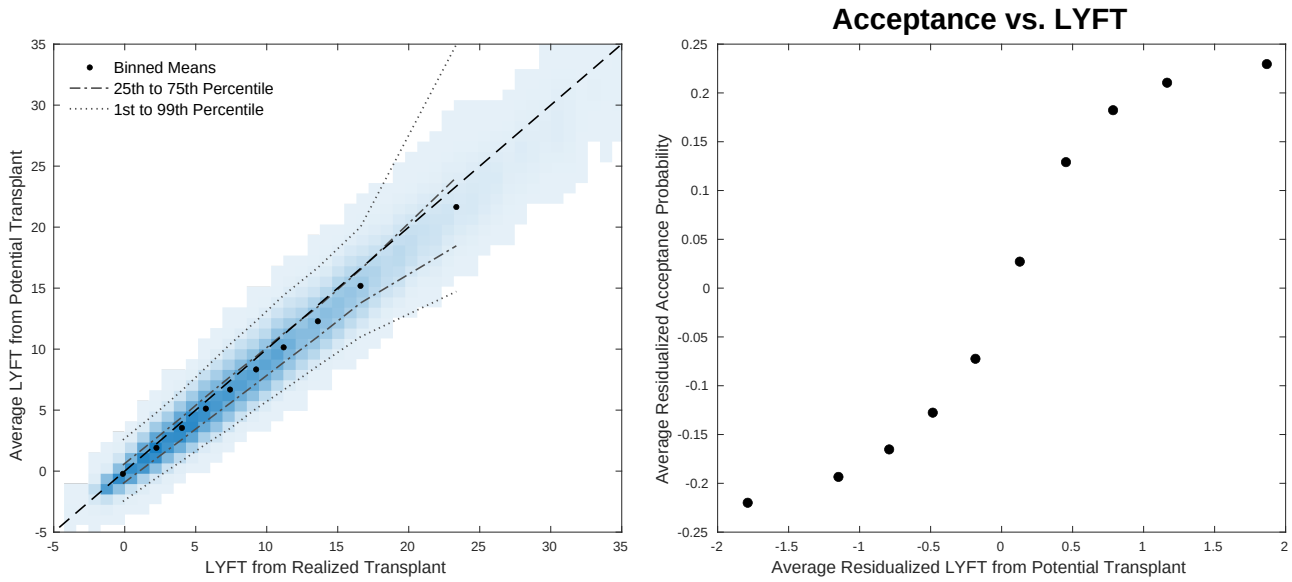
7.3 Selection and LYFT

The selection on LYFT and untransplanted survival reported in Table 7 above can take place along two margins: the patients who are transplanted and the kidneys to which they are matched. We further investigate these sources below.

Patient Selection: There are strong complementarities between baseline health and transplantation. Figure 2(a) presents the joint density of (median) untransplanted survival and the average (median) LYFT from all potential donors for each patient, overlaid with a binscatter plot. LYFT and untransplanted survival are strongly positively correlated. Patients who are expected to live longer without a transplant also have the largest life-year gains.

When combined with the observation in Table 7 that transplanted patients have longer baseline survival, this complementarily suggests that patients who are transplanted likely have more LYFT due to selection on baseline health. In addition, there may be patient selection into transplantation from choice and from the priorities in the mechanism.

by the full set of offers made for a particular kidney, not just the final allocation. Futhermore, including cold ischemic time as a control is potentially problematic since it is an outcome of patients' acceptance decisions.



(a) Transplanted Survival from Potential and Realized Donors

(b) LYFT and Choice

Figure 3: Patient-Kidney Matching

The overall selection into transplantation is presented in Figure 2(b), which shows the distribution of predicted LYFT across all potential transplants. This distribution is shifted to the right for transplanted patients, with an average that is 1.1 years higher. Thus, the mechanism selects patients with larger average LYFT, and some of this selection comes from transplanting patients who are relatively healthy at baseline.

Patient-Kidney Matching: The realized allocation also matches patients to kidneys from which they receive greater survival benefits as compared to the average kidney. Figure 3(a) plots the joint distribution of LYFT from the realized donor for a transplanted patient against LYFT from all potential donors. The binscatter is below the 45-degree line, indicating that the realized transplants generate greater than average LYFT for a patient. This finding that *matches* are selected advantageously complements the finding that the mechanism selects *patients* with higher than average gains from transplantation.

Part of this advantageous matching comes from the correlation between patients' acceptance decisions and LYFT. Figure 3(b) presents a binscatter plot of kidney–patient acceptance probability against LYFT for all potential transplants and shows that the predicted proba-

bility of accepting an offer increases LYFT. As our estimates suggest, patients are more likely to accept kidneys with greater life-year benefits (based on both observable and unobservable characteristics). [28]

In sum, we find that the allocation matches kidneys to patients based on LYFT and that at least some of this selection is induced by choices in the mechanism.

Patient Selection vs. Rematching: Figure 3(a) also provides insight into which of these two assignment margins dominates. The heterogeneity in survival across patients swamps the heterogeneity across donors within a patient. In fact, a decomposition of the total variance in LYFT into patient-specific, donor-specific, and match-specific components (the last being the remainder) shows that the patient-specific component contributes to 3.26 years of the standard deviation in LYFT. The donor-specific and match-specific components are much smaller, accounting for 0.99 years and 0.41 years, respectively.

Thus, the potential for increasing life-years by improving the match between patients and donors without changing which patients are transplanted (rematching) is limited. Distributional constraints may therefore limit the potential gains from improved matching. In particular, maximizing life year gains may mean reallocating organs away from the most urgent cases towards patients with longer expected survival without a transplant, pointing to a potential trade-off between efficiency and worst-off prioritarianism for the sickest.

8 Potential for Further Increasing LYFT

We now evaluate the performance of the mechanism on LYFT and quantify the importance of patient selection versus rematching. We compare the average LYFT achieved by the realized assignment to benchmarks ranging from a random assignment to one that maximizes LYFT. Extending patients' lives is a prima facie objective of the medical profession. However, this objective may raise distributional concerns or conflict with allocation principles discussed in medical ethics. We highlight these trade-offs by comparing the types of patients who are

[28] To verify this point, we regressed the expected value of $LYFT_{ij}$ conditional on $\{x_i, q_j, \eta_j, \nu_{i,D}, \nu_{i,f}\}$ on the probability of acceptance given these same covariates, controlling for patient- and donor-specific fixed effects. A one standard deviation increase in the match-specific component of LYFT raises the probability of acceptance by 0.24 percent.

transplanted under the benchmarks.

We focus on our preferred specification and, to ease computation, we restrict the sample to the set of patients who registered in 2005. Our results are not sensitive to choice of instrument; varying the Box-Cox shape parameters of our specification; omitting donor unobserved heterogeneity η_j; or including time between organ extraction and transplantation (see Table D.3).

8.1 Comparison with Benchmark Assignments

We start with two extremal benchmarks, random assignment, and optimal assignment:

Random assignment is simulated by successively assigning patients to kidneys at random from the set of feasible kidneys. Feasibility requires that the patient must be biologically compatible and the kidney should arrive between the patient's registration date and a simulated death date without a transplant. The latter is drawn from that patient's predicted survival distribution.

Optimal assignment is computed by maximizing total LYFT from all transplants. This benchmark considers an omniscient planner who knows x_i, q_j, $\nu_{i,D}$, $\nu_{i,f}$, η_j, each patient's arrival and untransplanted death dates, and each kidney's arrival date. The planner computes LYFT conditional on these characteristics and can dictate assignments. Only feasible transplants are allowed and each patient can receive at most one transplant.[29]

The comparison to the random assignment measures the increase in LYFT achieved by the mechanism. Both selecting patients and advantageously matching kidneys to patients drives the difference. To decompose these sources, we evaluate an alternative that allocates kidneys randomly among transplanted patients:

The **random amongst transplanted** assignment is simulated by re-assigning *transplanted* patients to a kidney at random from the set of feasible kidneys.

The increase in LYFT due to the mechanism results from both the mechanism's priority

[29] Call the s-th simulated draw for each patient/donor pair $LYFT^s_{ij}$. Let $a_{ij} = 1$ if i is assigned j and $a_{ij} = 0$ otherwise. Let $c_{ij} = 1$ if i is feasible for j and $c_{ij} = 0$ otherwise. We solve the problem $\max_a \sum_{i,j} a_{ij} LYFT^s_{ij}$ subject to $a_{ij}(1 - c_{ij}) = 0$, $\sum_i a_{ij} \leq k_j$, where k_j is the number of kidneys available from donor j, and $\sum_j a_{ij} \leq 1$.

rules for kidney offers and the choices made by patients on the waiting list. To separate the gains achieved due to the mechanism's priority structure from the gains from choice, we evaluate a counterfactual assignment with no patient choice.

The **no choice assignment** is computed by assigning each kidney to the patient with the highest priority among untransplanted patients. Offers cannot be rejected by patients.

Comparing the realized assignment to the optimal assignment bounds the maximum theoretical gain in LYFT that could be achieved by any mechanism. As in the comparison between the realized and random assignments, this gain is driven both by selecting patients and matching patients to kidneys. To decompose these sources, we evaluate an alternative that only reassigns kidneys among transplanted patients.

The **optimal rematching** assignment maximizes the total LYFT using the same information set as in the optimal assignment. In addition to the feasibility constraint, a patient in this assignment can be transplanted only if they were transplanted in the data.

Optimal assignment uses information about factors that induce selection: $\nu_{i,D}$, $\nu_{i,f}$, and η_j. However, the first two factors may not be observed by the planner and may be hard to elicit in a mechanism. Similarly, η_j may be difficult to condition on. These observations motivate a benchmark that uses only observable information:

The **optimal assignment based on observables** is calculated by maximizing the total expected LYFT conditional on x_i and q_j by assigning patients to a feasible kidney. [30]The solution describes the highest possible LYFT that can be achieved by a planner who can dictate assignments based on this information.

Figure 4 presents the results. The average LYFT for the realized assignment amongst patients who registered in 2005 is 9.29 years. This is analogous to the results in Table 7 above.

The realized assignment achieves a 1.75-year increase in average LYFT over random assignment. Both selecting patients and matching patients to kidneys are important: random amongst transplanted yields only 11.7 more months. The remainder of the gain is due to patient-kidney matching.

[30]For tractibility, we assume the planner has foresight about when patients arrive and depart and when kidneys arrive. Relaxing foresight would require solving a dynamic assignment problem with uncertainty about the future.

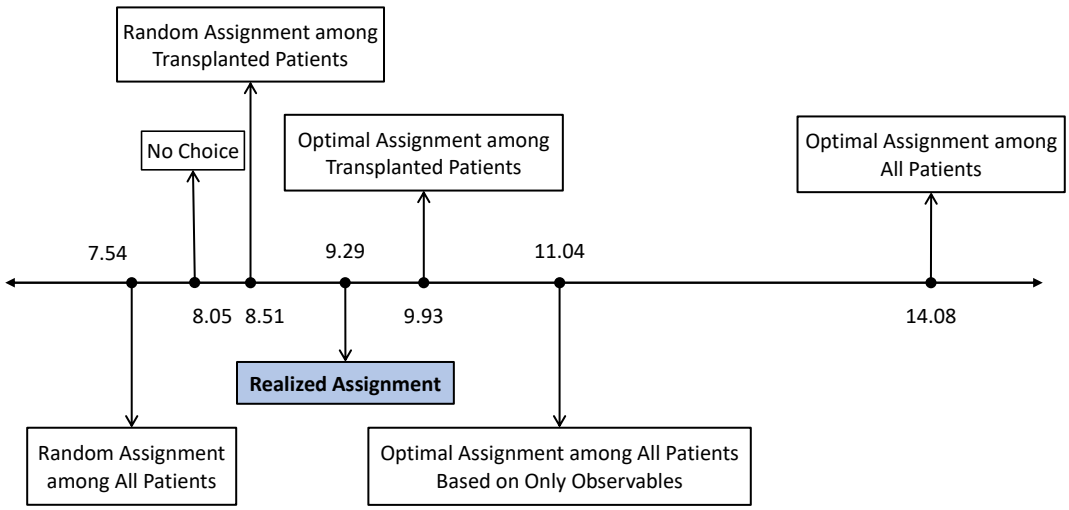


Figure 4: LYFT Under Counterfactual Assignments

Patient choice is a key contributor to the mechanism's gains in LYFT over random assignment. The no choice assignment results a in similar LYFT as the random assignment. Thus, if the priority rules were used to dictate assignments, then only 29.6% of the LYFT increase in the realized assignment would be achieved.[31]

Although the mechanism does better than a random assignment, there is significant scope for further increasing LYFT. The average LYFT under the theoretical upper bound given by the optimal assignment is 4.8 years higher than the LYFT achieved in the realized assignment. Bias in estimates based on observational studies would miss the potential for these gains. A significant fraction, 13.4%, of the increase can be achieved by rematching patients and kidneys while keeping the set of transplanted patients fixed. Nevertheless, consistent with Figure 3(a), most of the improvements in optimal allocation come from changing the set of patients who are transplanted.

A planner who can dictate assignments using the observable characteristics could achieve a significant fraction, but not all, of the potential increase. These observables have been either used to determine priority or considered explicitly in proposed reforms. The average LYFT under the optimal assignment based on observables is 11.04 years. Although less than the theoretical maximum, it is about 1.8 years more than the average LYFT achieved by the

[31] We also simulated the no choice assignment using priorities in place after 2014 and found similar results for LYFT.

mechanism. Therefore, in principle, using observed characteristics rather than choices to target transplants could substantially raise the average LYFT.

Finally, note that a planner who uses the observational model, which is employed in the medical literature and does not account for selection on unobservables (column 1, Table 5), to allocate kidneys would not obtain the same life year gain. The optimal rematching implied by this specification yields an average LYFT of 11.75 years (as opposed to 14.08 years) when this assignment is evaluated using our preferred specification. This allocation obtains only 51% of the maximum possible increase in LYFT over the realized assignment.

8.2 The Planner's Dilemma

Achieving the gains in LYFT described above would require changing the set of patients who are transplanted. We now show that this change shifts the demographics and health conditions of transplanted patients, thereby creating a potential barrier due to distributional considerations or the desire to prioritize patient urgency.

Table 8: Characteristics of Transplanted Patients

		Random Assignment		No Choice		Realized Assignment		Optimal Assignment	
	All Patients	Transplanted Patients	LYFT	Transplanted Patients	LYFT	Transplanted Patients	LYFT	Transplanted Patients	LYFT
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Age < 18	3.1%	3.1%	15.36	6.3%	15.49	5.7%	16.57	5.9%	18.23
Age 18 - 35	11.6%	12.2%	11.91	12.0%	12.41	13.1%	13.81	18.3%	15.89
Age 36 - 59	54.8%	55.5%	7.83	52.6%	8.12	54.3%	9.33	59.1%	13.90
Age >= 60	30.4%	29.2%	4.34	29.1%	4.53	26.9%	5.44	16.8%	11.31
White	42.0%	42.5%	7.02	47.8%	7.79	46.4%	8.66	39.1%	13.90
Black	32.7%	31.8%	7.62	30.0%	7.90	30.7%	9.37	33.0%	14.04
Hispanic	16.7%	16.7%	8.28	15.2%	9.09	14.8%	10.56	18.4%	14.42
Other	8.6%	9.0%	8.33	7.1%	8.29	8.2%	10.24	9.5%	14.35
Diabetic	41.4%	40.0%	5.05	37.0%	5.19	33.3%	6.24	27.7%	11.98
On Dialysis at Registration	83.0%	82.4%	7.28	81.8%	7.73	80.1%	8.99	79.6%	13.91
0 HLA Mismatches	-	0.1%	9.19	16.6%	8.10	12.9%	9.34	7.4%	15.69
0 DR Mismatches	-	4.0%	7.87	36.1%	8.13	22.3%	9.46	12.2%	14.93
HLA Mismatches	-	4.77	-	3.60	-	3.90	-	3.81	-
Untransplanted Survival	6.90	6.95	-	6.97	-	7.11	-	7.61	-

The LYFT increases, from random assignment to the mechanism and finally to the optimal solutions, require transplanting relatively healthy patients. Table 8 presents the distributions of patient age, health, and untransplanted survival for patients transplanted under

the random assignment, the no choice assignment, the actual assignment, and the optimal assignment. Patients transplanted under the realized assignment are healthier than average – younger, less likely to be diabetic, less likely to be on dialysis – and have longer untransplanted survival. Similarly, transplanted patients are also healthier under the optimal assignment than under the realized assignment. The optimal assignment also reallocates kidneys towards racial/ethnic minority patients, who have higher LYFT on average than white patients.

Comparing the realized assignment and the no choice assignment illustrates the role of choice in increasing LYFT. The existing priority rules target transplants between patients and donors with no HLA mismatches. The fraction of zero-mismatch assignments is lower under the realized and optimal assignments as compared to no-choice. Yet, choice also dramatically changes the selection of who is transplanted towards patients with high LYFT by shifting the age distribution towards younger patients and those with longer untransplanted survival. Therefore, while patients benefit from kidneys with a perfect tissue-type match, reassigning kidneys to the right set of patients without perfect tissue-type matches can increase LYFT.[32]

These shifts highlight the distributional effects of optimizing LYFT – the realized outcome increases LYFT by selecting younger, healthier patients to transplant. The optimal assignment exacerbates these distributional changes. These results are driven by the strong correlation between survival with and without a transplant, illustrated in Figure 2(b). Thus, in order to maximize LYFT given the scarcity of kidneys available, the planner must transplant healthier patients and let sicker patients go untransplanted.[33]

This stark trade-off represents a dilemma. Society may have a moral imperative to prioritize sick patients who may soon die, as done in deceased donor liver allocation. But some medical ethicists discard this principle when faced with scarcity, arguing instead for maximizing total survival or treating people equally (random assignment) (see Persad et al., 2009). Our

[32] Appendix table D.4 shows the optimal assignment using this observational model, which does not account for selection on observables, substantially moves kidney reallocation towards younger patients, as compared to the optimal assignment using our preferred specification. This suggests that relying on an observational model may not only result in biased estimates of LYFT, but also lead to misallocation that appears to exacerbate the planner's dilemma.

[33] Indeed, an assignment that transplants the sickest patients first (as measured by $Y_{i,0}$) results in an LYFT of 3.41 years.

results suggest that these two principles are in conflict for kidney allocation, with utilitarian principles also raising concerns about both discrimination based on patient characteristics such as age and increased inequality in patient survival.

9 Conclusion

An important but understudied goal in designing assignment mechanisms is to produce matches that improve associated outcomes such as patient survival or student achievement. With few exceptions (noted in the introduction), the prior empirical literature focuses on revealed preference measures of welfare. We take a first step towards an empirical analysis that incorporates downstream outcomes by studying the LYFT generated through the transplantation of deceased donor kidneys. To do this, we show how to use variation generated in an assignment mechanism to estimate and identify a model that jointly considers choices and outcomes.

We find that the waitlist mechanism used to allocate deceased donor kidneys does better than a random allocation but leaves much scope for improvement. The mechanism transplants patients for whom life would be extended longer, as compared to the average patient, and matches them to more suitable than average kidneys. However, average LYFT could be boosted by several years. The potential economic value of realizing these gains is enormous. Aldy and Viscusi (2007) value a statistical life year at $300,000. At even half this value and ignoring costs savings on dialysis, the potential benefits from one more year of life from the approximately 13,000 deceased donor kidneys transplanted each year accrues to almost $2 billion per year.

Achieving most of these gains will require confronting important distributional considerations because survival without a transplant is a strong predictor of life year gains. Therefore, the planner faces a dilemma between transplanting the sick and transplanting those for whom life will be extended the longest.

This work opens several avenues for further research. First, our approach avoids micro-founding the choice model at the cost of evaluating benchmark assignments rather than the equilibria of alternative mechanisms. This leaves counterfactual selection in an equilibrium

model to future work. Second, we focus on an aggregate measure of LYFT that abstracts away from distributional or non-utilitarian ethical considerations. Formalizing these considerations and incorporating them into the design problem could yield a valuable policymaking tool. The underlying trade-offs are particularly central to designing mechanisms when outcomes are the target and deserve further research in other contexts as well.

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A Data Appendix

A.1 Obtaining Original Data Files

The data reported here have been supplied by UNOS as the contractor for the Organ Procurement and Transplantation Network (OPTN). The interpretation and reporting of these data are the responsibility of the author(s) and in no way should be seen as an official policy of or interpretation by the OPTN or the U.S. Government.

We will retain copies of the data until permitted by our Data Use Agreement with the Organ Procurement and Transplantation Network (OPTN). Further, we plan to send OPTN a copy of our replication archive if and when we are required to destroy our dataset. Researchers interested in using our dataset should directly contact OPTN to obtain permission: https://optn.transplant.hrsa.gov/data/request-data/. We are happy to provide copies of our data to researchers with permission and a data use agreement with the OPTN.

A.2 Data Description

Our data on patients, donors, transplants, and offers are based on information submitted to the Organ Procurement and Transplant Network (OPTN) by its members. The main datasets are the Potential Transplant Recipient (PTR) dataset and the Standard Transplantation Analysis and Research (STAR) dataset.

The PTR dataset contains offers made to patients on the deceased donor kidney waitlist who were not automatically rejected based on pre-specified criteria. Information includes identifiers for the donor, patient, and patient history record that generated the offer; the order in which the offers were made; each patient's acceptance decision; and, if the offer was not accepted, a reason for rejection. Each offer record also contains certain characteristics of the match, including the number of tissue type mismatches.

The STAR dataset contains separate files on deceased donor characteristics, patient histories, patient characteristics and transplant outcomes, and follow-up data, which are collected at six months and then annually, for kidney transplants. The patient and donor characteristics from these datasets are used to estimate our models of acceptance behavior and patient

survival. The patient characteristics and transplant outcomes dataset contains patient death information. For patients who received a transplant through the deceased kidney donor waitlist, the follow-up dataset records whether the patient is still alive at the follow-up point. This information allows us to compute the survival duration for each patient. UNOS also provided supplemental information, including the ordering of distinct match runs conducted for the same deceased donor; the transplant centers of donors and patients in our dataset; and dates of birth for pediatric candidates, who joined the waitlist before turning 18 years of age.

The data contain identifiers that allow us to link the offer and acceptance data to patient and donor characteristics. Each deceased donor has a unique identifier. Similarly, each patient registration generates a unique patient waitlist identifier. Because patients may move to different transplant centers or be registered with multiple centers simultaneously, some individual patients have multiple waitlist identifiers. For this study, we focus on each patient's earliest registration. The follow-up data contain a unique identifier for each transplant, allowing us to connect follow-up information to each transplanted patient. The patient history file contains a unique patient record identifier corresponding to a particular state of the patient on the waitlist, including the patient's CPRA, activity status, and pre-set screening criteria. Each offer in the PTR dataset contains the identifiers for the donor, the patient registration, and the patient history record that were used in the match run. When appropriate, we de-duplicate offers so that each patient can receive at most one offer from each donor.

A.3 Sample Selection

We consider the first waiting period for patients who were actively waiting for a deceased donor kidney between January 1, 2000, and December 31, 2010. This restriction is to avoid selection arising from patients who remain on the list at the beginning of the sample period. We omit patients who received a living donor transplant as their first transplant or were cross-registered for other organs simultaneously. The outcomes for these patients are likely very different from patients who receive only a kidney from a deceased donor. Most patients

who can receive a living donor receive one within the first year of registration and would prefer such a transplant to a deceased donor transplant. The latter restriction is made to focus on a more homogeneous group of patients.

In addition, we made a number of other minor adjustments to work with a more cohesive sample of patients. The number of patients that survive each step of the sample selection process is described in Table A.1.

A small minority of patients are simultaneously registered in multiple donor service areas, indicating that multiple listings and moves are not common. Our analysis keeps only one waitlist record from each patient. If the patient received a kidney transplant through the deceased donor waitlist before December 31, 2015, we keep the waitlist record with the earliest transplant date; if the patient remained untransplanted as of December 31, 2015, we keep the waitlist record with the earliest registration date.[34] Next, we exclude a small number of patients who received a prior kidney transplant to focus on the survival effects of the first transplant. We also exclude patients removed for administrative reasons. These are patients who were listed on the waitlist by error, who departed because a transplant took place but no transplant was recorded in the STAR dataset, and who could no longer be contacted while waiting on the waitlist. These departure reasons are recorded in the STAR patient and the transplant outcome dataset.

Then, we keep the waitlist records with registration dates between Janurary 1, 2000, and December 31, 2010, because we do not have data on offers prior to 2000. For example, an untransplanted patient active between 2000 and 2010 may not be included in the final sample because said patient's first waitlist registration is before 2000. This step amounts to one of the largest cuts.

Finally, we exclude patients who received a transplant through non-standard allocation rules. This can occur, for example, if the donor is an armed service member; if the donor specified a particular recipient (directed donation); if there is a medical emergency or expedited placement attempt; or if the kidney is not offered due to operational issues. We identify these cases by analyzing the PTR data as a large number of offers will be bypassed with a code indicating one of these reasons. In some cases, there is also text specifying specific

[34] In the sample selection process, we use transplant data through December 31, 2015.

circumstances justifying a rejection, and we parse such text to identify invalid offers in cases where the refusal code does not provide a specific reason.

Table A.1: Sample Selection: Patients

	Number of Patients	Number of Wait List Records
Patient's first waiting period that intersects the period 2000-2010	308,370	372,681
Exclude patients who received living donor transplants in their first waiting period	241,209	295,075
Exclude patients were waiting for other organs in their first waiting period	213,685	244,580
Keep one kidney waitlist record for each patient	213,685	213,685
Patients with multiple waitlist records	32,191	32,191
Patients with single waitlist record	181,494	181,494
Exclude patients who had a previous kidney transplant	212,258	-
Exclude patients with administrative waitlist removal reason	207,316	-
Restrict to patients whose remaining waitlist registration is between 2000 and 201(	178,944	-
Exclude patients who received non-standard kidney allocations	175,695	-
Exclude patients with poor death data	175,640	-

Our sample of deceased kidney donors comes from the intersection of the STAR deceased donor dataset and the PTR dataset. These are deceased donors whose kidneys were allocated to patients on the waitlist between January 1, 2000, and December 31, 2010. We further exclude donors allocated using non-standard rules or not offered to patients in the sample. Table A.2 details the number of donors that survive each filter. The largest cuts come from the last step. This is because the priority for waiting time implies that many offers are only given to patients who registered prior to 2000.

Table A.2: Sample Selection: Donors

	Number of Do
Deceased donors offered to any kidney waitlist patients between 2000 and 2(	71738
Exclude deceased donors offered through non-standard kidney allocations	68140
Restrict to deceased donors offered to patients in the sample	58466

We consider a sample of offers made between January 1, 2000, and December 31, 2010, that could have resulted in transplants between our donor and patient samples. The PTR dataset

includes records of all initial patient contacts and patients skipped due to administrative reasons irrespective of whether an offer was made. Such skipping happens mainly for three reasons. First, some patients who were contacted have lower priority than the patients who accepted and were transplanted with the kidneys from a donor. In this case, we determine the cutoff point for each donor and exclude all offers made after the cutoff. Second, some match runs were abandoned due to logistical reasons and then re-run. We only keep the offers from the last match run for a donor. Third, in some cases, the PTR dataset records administrative or logistical reasons for skipping patients in the offer sequence. This can occur, for example, if the kidney has antigens that would result in an immune response; a patient was bypassed due to logistical reasons; or if the kidney does not meet the patient's minimum criteria. We also exclude non-responsive offers, for example, because either the surgeon or the patient is unavailable or because the patient is temporarily inactive/unsuitable for transplantation. Finally, we restrict to offers made to the patients in the sample. This step cuts the offer sample by 41% because many offers are made to patients who were not in our sample, for example, patients who registered prior to 2000. Table A.3 describes how we arrive at the final sample of offers.

Table A.3: Sample Selection: Offers

	Number of Offe
Offers made between 2000 and 2010 from donors in the sample	14,982,656
Exclude non-responsive offers	14,335,386
Restrict to offers made to patients in the sample	8,508,757

A.4 Patient Survival

The patient characteristics and transplant outcomes dataset collects patient death dates from the waitlist record and periodically from the social security master file. In a small minority of cases, death dates are inconsistent across multiple waitlist records for a patient; for these, we assume that earlier death dates take precedence over later ones. Transplant dates and death dates are truncated on February 29, 2020, because death records after this date are

inconsistently populated. For patients who received a transplant or died after February 29, 2020, we treat them as untransplanted or alive, respectively, as of February 29, 2020.

Among 175640 patients in the sample, we observe death dates before February 29, 2020, for 101481 of them. Of these, 63911 are untransplanted patients and 37570 are transplanted. Patients for whom we do not observe death are censored. The censoring rules differ for transplanted and untransplanted patients. For transplanted patients, we censor on the date of the second transplant if a second transplant took place before February 29, 2020; on the day after transplant if there is no follow-up information for the patient corresponding to the transplant; on the date when the patient is lost to follow-up if the patient is lost to follow-up prior to February 29, 2020; and on February 29, 2020, if the patient is known to be alive as of February 29, 2020. For untransplanted patients, we censor on February 29, 2020, if the patient is known to be alive as of February 29, 2020; and on the date when the patient exits the waitlist if no death date is available and the exit day is prior to February 29, 2020.

Table A.4 presents a breakdown of censor reasons and their corresponding censor dates for the patient sample. Nearly one half of the patient sample is uncensored, and among censored patients, the vast majority (64.1%) are censored on February 29, 2020. Since February 29, 2020, is an exogenously determined date, patients censored on that date should be similar to uncensored patients in terms of potential outcomes.

Table A.4: Censor Reason

Censor Reason	Censor Date	# Patient
Transplanted Patients		
Retransplant before Feb. 29, 2020	Retransplant date	5,649
No follow-up information	One day after transplant	463
Lost to follow-up before Feb. 29, 2020	Date lost to follow up	6,827
Known to be alive as of Feb. 29, 2020	February 29, 2020	44,748
Untransplanted Patients		
Known to be alive as of Feb. 29, 2020	February 29, 2020	2,786
No death date and depart the waitlist before Feb. 29, 2020	Date departing waitlist	13,686

B Estimation Appendix

B.1 Gibbs' Sampler

Recall that our model is given by

$$\begin{aligned}
y_{i0} &= B\left(Y_{i0};\rho_0\right) = x_i\beta_x + \nu_{i,0} \\
y_{ij} &= B\left(Y_{ij};\rho_1\right) = \chi\left(x_i,q_j\right)\alpha_{x,q} + \alpha_\eta\eta_j + \nu_{i,1} + \varepsilon_{ij,1} \\
D_{ij} &= 1\left\{y_{ij,D} = \chi\left(x_i,q_j\right)\gamma_{x,q} + z_i\gamma_z + \eta_j + \nu_{i,D} + \varepsilon_{ij,D} > 0\right\},
\end{aligned}$$

where we allow for $\nu_i = (\nu_{i,0},\nu_{i,1},\nu_{i,D}) \sim \mathcal{N}(0,\Sigma_\nu)$ and $\varepsilon_{ij} = (\varepsilon_{ij,1},\varepsilon_{ij,D}) \sim \mathcal{N}(0,\Sigma_\varepsilon)$.

There are several challenges in estimating this model. First, we often observe censored values of y_{i0} and y_{ij}. We perform a data augmentation step given the parameters and the censoring point to solve this issue. For y_{ij}, the data augmentation step is necessary only in cases for which $T_{ij} = 1$.

Second, D_{ij} is a binary variable. As is standard in discrete choice models, we perform a data augmentation step to draw $y_{ij,D}$ given the observed decisions. This step is necessary for the observed values of D_{ij}.

Third, the model incorporates rich correlations between the different observations via η_j, ν_i and ε_{ij}. In particular, due to these terms, the covariance matrix between $\{y_{i0}\}_i$ $\{y_{ij}\}_{ij}$ and $\{y_{ij,D}\}_{ij}$ conditional on the observables and the parameters does not have a simple block-diagonal structure that would allow us to compute simple posterior distributions. To solve this problem, we re-write these variables using a factor structure such that the posterior distribution of the parameters of each equation is conditionally independent of the others given the factors. Specifically, we rewrite ν_i as

$$\begin{aligned}
\nu_{i,D} &= f_{i,1} \\
\nu_{i,1} &= \alpha_{\nu 1} f_{i,1} + f_{i,2} \\
\nu_{i,0} &= \beta_{\nu 1} f_{i,1} + \beta_{\nu 2} f_{i,2} + \tilde{\varepsilon}_{i0}
\end{aligned}$$

where $f_{i,1}$, $f_{i,2}$ and ε_{i0} are each independently distributed mean-zero normal random variables with variances σ_1^2, σ_2^2 and $\sigma_{\tilde{\varepsilon},0}^2$. This structure places no restrictions on the covariance matrix Σ_ν. Similarly, we write ε_{ij} as

$$\varepsilon_{ij,1} = \alpha_\varepsilon f_{ij,3} + \tilde{\varepsilon}_{ij,1}$$
$$\varepsilon_{ij,D} = f_{ij,3} + \tilde{\varepsilon}_{ij,D}$$

where $f_{ij,3}$, $\tilde{\varepsilon}_{ij,1}$ and $\tilde{\varepsilon}_{ij,D}$ are independently distributed mean-zero normal random variables with variances σ_3^2, $\sigma_{\tilde{\varepsilon},1}^2$ and $\sigma_{\tilde{\varepsilon},D}^2$. We normalize the variances σ_3^2, and $\sigma_{\tilde{\varepsilon},D}^2$ to 1. Finally, set

$$\eta_j = f_{j,4}$$

with variance σ_4^2. The main difference between $f_\cdot$and $\tilde{\epsilon}_\cdot$ is that it is sufficient to condition on the former in order to render the models above as conditionally independent.

Therefore, the parameters we are interested in estimating are the coefficients in each equation, $\beta = (\beta_x, \beta_{\nu 1}, \beta_{\nu 2})$, $\alpha = (\alpha_{x,q}, \alpha_\eta, \alpha_{\nu 1}, \alpha_\varepsilon)$, $\gamma = (\gamma_{x,q}, \gamma_z)$, and the variances $\sigma_{\tilde{\varepsilon},0}^2 = V[\tilde{\varepsilon}_{i0}]$, $\sigma_{\tilde{\varepsilon},1}^2 = V[\tilde{\varepsilon}_{ij,1}]$ and $\sigma_l^2 = V[f_l]$ where $l \in \{1, 2, 4\}$ is the l-th factor.

For simplicity of notation, we will collect the coefficients in the vector θ and the standard deviations in the vector σ, with $\sigma_{\tilde{\varepsilon}}$ and σ_f denoting the sub-vectors for $\tilde{\varepsilon}$ and f, respectively. And, with some abuse of notation, we collect y_{i0}, y_{ij} and $y_{ij,D}$ for all i and j in y.

Following standard practice, we assume diffuse conjugate and independent priors for each of these parameters. Specifically, we model the priors α, β and γ using a mean-zero independent normal distribution with variances equal to 1000 and the prior for the variances $\sigma_{\tilde{\varepsilon},0}^2$, $\sigma_{\tilde{\varepsilon},1}^2$ and σ_l^2 using independent inverse-Wishart distributions with parameters $(3, 3)$. These priors are diffuse; thus, they have a negligible impact on our estimates.

The Gibbs' sampler starts with an initial draw y^0, θ^0, σ^0 and f^0 and generates a chain of length K by iterating through the following steps for each $k \in \{0, \ldots, K-1\}$:

1. **Data Augmentation:** Sample y_{i0}^{k+1}, y_{ij}^{k+1} for censored observations and $y_{ij,D}^{k+1}$ for observed decisions given θ^k, σ^k and f^k from truncated normal distributions.

2. **Sample Coefficients:** Sample θ^{k+1} given y^{k+1}, f^k, the standard deviations σ^k and the prior distribution from a multi-variate normal distribution.

3. **Sample Variances:** Sample $\sigma^{2,k+1}_{\tilde{\varepsilon},0}$ and $\sigma^{2,k+1}_{\tilde{\varepsilon},1}$ given y^{k+1}, f^k, the parameters θ^{k+1} and the prior distribution from an inverse-Wishart distribution.

4. **Sample Factors:** For each $l \in \{1,2,3,4\}$, sample $f^{k+1}_{\cdot,l}$ given y^{k+1}, the parameters θ^{k+1}, $\sigma^{k+1}_{\tilde{\varepsilon}}$, σ^k_f, and the remaining factors $f^{k+1}_{\cdot,1},\ldots,f^{k+1}_{l-1}$ and $f^k_{\cdot,l+1},\ldots,f^k_4$.

5. **Sample Factor Variances:** Sample $\sigma^{2,k+1}_l$ for $l \in \{1,2,4\}$ given f^{k+1} and the prior distribution from an inverse-Wishart distribution.

We draw a chain of length K =200,000 and burn 50,000 draws to allow the chain to convergence. We only keep one every 10 draws to save some computation time and reduce the autocorrelation in the resulting chain. To diagnose the potential for non-convergence, we visually inspect the chains and, as recommended in Gelman et al. (2014), we also ensure that the potential scale reduction factor is below 1.1 for each of the parameters. The distributions in each step can be solved for in closed-form as detailed below:

1. Conditional distributions for y_{i0}, y_{ij} and $y_{ij,D}$ given θ, f and σ:

   (a) For each i,j pair with D_{ij} is observed, the distribution of $y_{ij,D}$ conditional on γ, f and D_{ij} is a truncated normal with mean $E\left[g_{ij,D}|\gamma,f_{ij}\right]$ and unit standard deviation. The distribution is truncated below at 0 if $D_{ij}=1$ and above at 0 otherwise.

   (b) For each i such that y_{i0} is censored, the distribution of y_{i0} conditional on β and f is a one-sided truncated normal with mean $E\left[y_{i0}|\beta,f_{i1},f_{i2}\right]$ and standard deviation $\sigma_{\tilde{\varepsilon},0}$. The distribution of y_{i0} is truncated below at the censoring duration.

   (c) For each observed transplant with y_{ij} censored, the distribution of y_{ij} conditional on α^k, f^k is a one-sided truncated normal with mean $E\left[y_{ij}|\alpha,f\right]$ and standard deviation $\sigma_{\tilde{\varepsilon},1}$. The distribution of y_{ij} is truncated below at the censoring duration.

2. Posterior distributions of the coefficients α, β and γ given y, f, σ and the priors. Since y_{i0}, y_{ij} and $y_{ij,D}$ are mutually independent conditional on f, the parameters α,

β and γ are each coefficients in a linear regression model with normally distributed errors. Therefore, the posterior distributions of each of these terms are given by a multivariate normal distribution with closed-form means and variances (Gelman et al., 2014, Chapter 14.2).

3. Posterior distributions of $\sigma^2_{\tilde{\varepsilon},0}$ and $\sigma^2_{\tilde{\varepsilon},1}$ given y, f, σ and the priors. As above, y_{i0}, y_{ij} are mutually independent conditional on f. Therefore, the distributions of $\sigma^2_{\tilde{\varepsilon},0}$ and $\sigma^2_{\tilde{\varepsilon},1}$ are inverse-Wishart with parameters given in Chapter 14.2 of Gelman et al. (2014).

4. Posterior distributions of f given y, θ and σ:

   (a) The distribution of $f_{i,1}$ conditions on the residual

   $$f_{i,1} + \frac{1}{\beta_{\nu 1}}\tilde{\varepsilon}_{i0} = \frac{1}{\beta_{\nu 1}}\left(y_{i0} - \left(x_i\beta_x + \beta_{\nu 2}f_{i,2}\right)\right)$$

   and σ_1 throughout; on the residual

   $$f_{i,1} + \tilde{\varepsilon}_{ij,D} = y_{ij,D} - \left(\chi\left(x_i, q_j\right)\gamma_{x,q} + z_i\gamma_z + \eta_j + f_{ij,3}\right)$$

   for all j such that D_{ij} is observed; and on the residual

   $$f_{i,1} + \frac{1}{\alpha_{\nu 1}}\tilde{\varepsilon}_{ij,1} = \frac{1}{\alpha_{\nu 1}}\left(y_{ij} - \left(\chi\left(x_i, q_j\right)\alpha_{x,q} + \alpha_\eta\eta_j + f_{i,2} + \alpha_\varepsilon f_{ij,3}\right)\right)$$

   (b) if $T_{ij} = 1$. These residuals have prior mean zero and variances $\sigma_1^2 + \frac{\sigma^2_{\tilde{\varepsilon},0}}{\beta^2_{\nu 1}}$, $\sigma_1^2 + \sigma^2_{\tilde{\varepsilon},1}$ and $\sigma_1^2 + \frac{\sigma^2_{\tilde{\varepsilon},1}}{\alpha^2_{\nu 1}}$ repectively. The posterior mean of $f_{i,1}$ is the precision-weighted average of the residuals corresponding to i, and the posterior variance is the inverse of the sum of σ_1^{-2} and the precisions of each residual.

   (c) The distribution of $f_{i,2}$ is analogous, where we condition on σ_2 and the residual

   $$\frac{1}{\beta_{\nu 2}}\left(y_{i0} - \left(x_i\beta_x + \beta_{\nu 1}f_{i,1}\right)\right)$$

   throughout, and on the residual $y_{ij} - \left(\chi\left(x_i, q_j\right)\alpha_{x,q} + \alpha_\eta\eta_j + \alpha_{\nu 1}f_{i,1}\right)$ if $T_{ij} = 1$.

(d) The distribution of $f_{ij,3}$ is analogous, where we condition on α_ε throughout; on $y_{ij,D} - (\chi(x_i, q_j)\gamma_{x,q} + z_i\gamma_z + \eta_j + f_{i,1})$ for all j such that D_{ij} is observed; and on $\frac{1}{\alpha_\varepsilon}(y_{ij} - (\chi(x_i, q_j)\alpha_{x,q} + \alpha_\eta\eta_j + f_{i,2}))$ if $T_{ij} = 1$. Note that σ_3 is normalized to 1.

(e) The distribution of $f_{j,4}$ is analogous, where we condition on σ_4 throughout; on $y_{ij,D} - (\chi(x_i, q_j)\gamma_{x,q} + z_i\gamma_z + f_{i,1} + f_{ij,3})$ for all i such that D_{ij} is observed; and on $\frac{1}{\alpha_\eta}(y_{ij} - \chi(x_i, q_j)\alpha_{x,q} + f_{i,2} + \alpha_\varepsilon f_{ij,3})$ if $T_{ij} = 1$.

5. The variances σ_l^2 for $l \in \{1, 2, 4\}$ follow inverse-Wishart distributions given the prior and respectively, $\{f_{i,1}\}$, $\{f_{i,2}\}$ and $\{f_{j,4}\}$.