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ABSTRACT

This paper develops a framework for studying the macroeconomic costs of resource misallocation. The framework enables the assessment of the conditions under which the existing estimates in the misallocation literature, which are largely based on a value-added production structure and ignore inter-sectoral linkages, provide an unbiased estimate of misallocation costs in relation to a more general setting, in which production of gross output relies upon input-output linkages across sectors. We show that in the absence of intermediate input distortions, the two approaches are isomorphic and will yield the same estimated aggregate productivity loss. When firm-specific intermediate input distortions are present, however, the value-added model produces biased estimates of TFP losses due to both model misspecification and incorrect inferences of firms' productivity and distortions. Using Chinese and Indian enterprise data, we find quantitatively similar TFP losses from resource misallocation for China, regardless of the model used, while for India, we infer significantly larger TFP losses under the gross output model.

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1 Introduction

A vast literature, pioneered by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), has studied how resource misallocation across heterogeneous firms, induced by idiosyncratic policy distortions, could lower countries' aggregate productivity and provide an explanation for the staggering income differences observed across countries.[1] In particular, Hsieh and Klenow (2009) (HK hereafter) develop an accounting framework to estimate the cost of resource misallocation from different types of market frictions using firm-level data. Subsequent contributions to the literature have provided arguments for why their estimates could be systematically biased for various reasons. Jones (2011, 2013) has posited that ignoring inter-sectoral linkages in production, as HK do, may result in an underestimation of economic losses due to misallocation, as the effects of distortions within sectors will be amplified by inter-sectoral linkages. Other studies show that the dispersion of firms' physical total factor productivity (TFP) is significantly smaller if gross output instead of value-added production functions are used (Gandhi et al., 2017), implying a potential overestimation of losses in the existing misallocation literature that uses value-added data.

This paper develops a framework to study the relevance of these criticisms about the quantitative assessment of aggregate TFP losses resulting from resource misallocation and to study the circumstances under which the existing estimates in the literature, based on value added production functions, may be biased.[2]

We first approach the question theoretically by developing a model of an economy in which output in each sector is produced by heterogeneous firms using primary factors of production and also intermediate inputs from different sectors; i.e., firms are linked through an inter-sectoral input-output (IO) network as in Long and Plosser (1983). With identical primitives (firm-level productivity and distortions), we derive expressions for an economy's aggregate output in the two different settings: one in which it is based on gross output from sectors connected through IO linkages

[1] This literature includes Banerjee and Duflo (2005), Alfaro et al. (2009), Guner et al. (2008), Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Buera et al. (2011), and Bartelsman et al. (2013), among many others. See Restuccia and Rogerson (2013) and Hopenhayn (2014) for reviews.

[2] To compare to HK, we focus on the percentage gain in aggregate TFP from removing distortions. That is, we evaluate efficient TFP using actual TFP as benchmark. It is the opposite of the TFP loss from misallocation, which means evaluating actual TFP using efficient TFP as benchmark. We sometimes use TFP loss in the paper when there is no confusion.

(henceforth the GO model) and another in which it is the sum of sectoral value added (henceforth the VA model) as in HK.[3] As is conventional in the literature, we then examine and compare aggregate TFP gains from the removal of all distortions in the two models. Specifically, we explore the conditions under which the VA model can produce the same aggregate TFP gain as the GO model, i.e, the circumstances under which the VA model is unbiased.

Our main conclusion is that whether the VA model can be used to measure the costs of misallocation accurately depends critically on the presence of intermediate input distortions. If there are no such distortions, we show that one can calibrate the VA model to be consistent with the GO model. Importantly, in the absence of intermediate input distortions, we show that consistency between the two models requires three important "adjustments": (1) rescaling firm productivity; (2) changing the sectoral weights in the final good production function; and (3) transforming the elasticity of substitution between output varieties in the sectoral production function in the GO model to its VA counterpart.

The first adjustment, which requires a rescaling of firm productivity, has been emphasized in earlier studies (e.g., Bruno (1978)). Consider an increase in a firm's output productivity. If the firm does not adjust its intermediate inputs, the percentage increase in value-added will be obviously larger than that in gross output, given that value-added equals gross output minus the costs of intermediate inputs. If the firm increases its demand for intermediate inputs in response to higher productivity, the percentage increase in its value-added will be even higher than that in gross output as its output increases more than the cost of intermediate inputs.[4] Hence, a firm's measured value-added productivity, which ignores the change in intermediate input use, will be larger than its measured output productivity.[5]

The second adjustment involves changing the sectoral weights in an economy's final good pro-

[3]The comparison between the GO model and the VA model is legitimate as the intermediate inputs cancel out in aggregation. For the aggregate economy, both models take the primary factors, capital and labor, as inputs and produce a single final output. In fact, our model offers a more complete description of the economy and reduces to the value-added framework of HK when intersectoral linkages are shut down. This allows us to evaluate the potential bias of the HK framework due to the use of value-added production functions.

[4]Note that the marginal product of additional intermediate inputs equals the market price of intermediate inputs only for the last unit of inputs. Infra-marginal increase intermediate inputs raises output more than the market price.

[5]Incidentally, this finding largely explains why Gandhi et al. (2017) find a larger dispersion of firm productivity when firms' value-added data are used, because differences in firm productivity are magnified when measured in value-added due to the adjustment of intermediate inputs.

duction. In particular, a sector's weight in the GO model, which is the economy's expenditure share, needs to be changed to its value added share in the VA model. Without intermediate input distortions, a sector's value added share is just the product of the sector's sales-to-GDP ratio (the so-called Domar (1961) weight) and its output elasticity with respect to the primary factors of production.[6]

Finally, the elasticity of substitution between varieties in a sector's production function needs to be adjusted when firms' outputs are changed from gross output to value added instead, using the output elasticity with respect to the primary factors of production.

In the absence of intermediate input distortions and with necessary adjustments, a theoretical isomorphism between the two models can be established. This appears to stand in contrast to Jones (2011, 2013), who argues that firm- or sector-level distortions can be amplified through IO networks. Our analysis clarifies that measures of value added implicitly incorporate the amplification effect of inter-sectoral linkages, which is revealed in data as a more dispersed distribution of value added productivity within sectors. Correspondingly, in the GO model, the IO amplification and the smaller dispersion of firm gross output productivity completely offset each other, leaving the TFP gain from removing distortions orthogonal to the economy's IO structure. This invariance is a key theoretical result of this paper.

We also show, in the absence of intermediate input distortions, how to correctly infer firm primitives using the VA model. A firm's value added productivity, after appropriate rescaling, will be identical to its gross output productivity (as mentioned above). Furthermore, a firm's distortions inferred from its value added measure will also be identical to those inferred from its gross output measure. The two models hence lead to identical measured efficiency loss from misallocation when applied to the same data.

The isomorphism between the VA and GO approaches breaks down when there are intermediate input distortions. The VA model will be misspecified and no calibration of the VA model can rationalize the data, given the same underlying distortions. Using data on value added will

[6]In the GO model, the effect of a sectoral productivity shock on aggregate productivity is determined by the propagation from upstream sectors to downstream sectors in the production network, which is summarized in a vector called the "influence vector" (Acemoglu et al., 2012). According to Hulten's (1978) theorem, in the absence of distortions in the production network, the vector of Domar weights equals the "influence vector".

also lead to biased estimation of firm primitives. Firms subject to idiosyncratic distortions face different effective intermediate input prices. As a firm's VA is computed as its revenue minus cost of intermediate inputs, while assuming the same market intermediate input prices, its marginal products and productivity inferred from the VA model will both be biased (Bruno (1978)).[7] In sum, when intermediate input distortions exist, the VA model produces biased estimates of TFP gains from the removal of all distortions due to *both* model misspecification and incorrect inference of firm primitives. However, the VA model does not necessarily underestimate or overestimate the TFP gain. The direction and magnitude of the bias depend on the extent of intermediate input distortions and their correlation with other firm characteristics. The assessment of the potential bias of the VA model is thus an empirical matter.

We hence study the potential biases in the assessment of the cost of misallocation using Chinese and Indian industrial enterprise data, as also used by HK. Interestingly, using the Chinese Annual Survey of Industrial Production (1998-2007), we find for China that the two models produce very similar measured TFP gains from the removal of all distortions. The average difference in the estimated TFP gains between the two models is around 5 percentage points, compared to the average TFP gain of over 100% based on the GO model.[8] The small bias of the VA model indicates that intermediate input distortions should be significantly smaller than those associated with the primary factors, which we confirm in the data. In particular, the variance of the distortions intermediate inputs (in logs) is only about 0.03, compared to 0.53 for the primary factors (a weighted sum of capital and labor). The implied lower TFP gain is amplified by inter-sectoral IO linkages, which renders the overall estimated TFP gain based on the GO model to be close to that according to the VA model in the case of China.

For India, however, based on the Indian Annual Survey of Industries (1999-2009), we find substantially larger TFP gains from removing all distortions based on the GO model than the VA model. The difference averages to around 114 percentage points, with the average TFP gain estimated in the GO model at 180%. According to our model, this must be because of the larger

[7]As the price of intermediate inputs no longer equals its marginal product, the differences in input prices will be embedded in the value-added measures, affecting the meaures of productivity (Basu and Fernald, 1995).

[8]Depending on the year, the aggregate TFP gain measured using the value-added model can be smaller or larger than that using the GO model.

dispersion of the intermediate input distortions in India, as compared to China. Indeed, we find that for India, the variance of the distortions on intermediate inputs equals 0.27, while its dispersion of the primary input distortions is similar to that of China.

It is a common practice in the literature to apply the elasticities of substitution estimated with gross output data to the VA model. In a final exercise, we evaluate the size of the bias in the estimated TFP gains due to such inconsistent use of elasticities.[9] As the output elasticity is larger than 1 in practice, our model suggests that the elasticity chosen by the existing studies using the VA model has been larger than the intended one.[10] In other words, if the correct elasticity is chosen for both models, the TFP gains estimated in the existing VA studies should be reduced. When we impose HK's baseline elasticity (=3) in both models, we find that China's aggregate TFP gains from removing all distortions will increase significantly, implying a mistaken 40 percentage-point additional gain on average as compared to our GO counterpart. For India, the implied TFP gain from removing all distortions also increases significantly based on the VA framework, closing the gap between the estimates from the two models by almost 90 percentage points. That said, our proposed GO model still suggests a larger TFP gain for India, implying that distortions on intermediate inputs are still significantly more severe in India than China. These findings highlight the importance of choosing the right elasticity in calibrating the models with different concepts of output, as has been emphasized in Herrendorf et al. (2013).

This paper contributes to three inter-related strands of literature. First, it contributes to the growing literature on resource misallocation, in particular those studies that complement and extend the analysis of the HK model.[11] Important studies along this line include Asker et al. (2014) and David and Venkateswaran (2019) which allows for dynamic capital adjustment, Bartelsman et al. (2013) which add a fixed cost to production and Song and Wu (2015) and Gollin and Udry (2017) which consider heterogeneity in firm technology. Our results are also related to another line of research that tries to distinguish measurement errors from misallocation (Dong, 2011; Song and

[9]For example, HK choose the elasticity based on estimates from Broda and Weinstein (2006) and Hendel and Nevo (2006) which are more suitable for the GO model.

[10]If the elasticity is less than 1, the output elasticity is smaller than the value-added elasticity. See Proposition 3 for the transformation of the elasticity between the two models.

[11]The HK framework has been used by Kalemli-Ozcan and Sorensen (2012), Ziebarth (2012, 2013) , Brandt et al. (2013), Oberfield (2013), Hsieh and Klenow (2014), Adamopoulos et al. (2015), Chen and Irarrazabal (2015), Gopinath et al. (2017), and Restuccia and Stantaeulàlia-Llopis (2017), among others.

Wu, 2015; Gollin and Udry, 2017; Bils et al., 2018; Haltiwanger and Syverson, 2018).[12]

Second, in exploring the role of intersectoral linkages and the potential isomorphism between the VA and GO approaches, this paper contributes to the literature on production networks, and is thus related to the important studies of Jones (2011, 2013) on the role of production networks in amplifying the effect of given distortions. It also relates to several other recent papers on the network propagation of distortions, which include, among others, Altinoglu (2020), Bartelme and Gorodnichenko (2015), Bigio and La'O (2020), Baqaee and Farhi (2019b, 2020a,b),Caliendo et al. (2017), Liu (2019), Luo (2019), and Osotimehin and Popov (2020).[13] While these latter studies mainly focus on the aggregate effects of idiosyncratic distortions in production networks, we pay close attention to the issue of TFP gains from moving to efficiency between a model with production networks and a model without. Our theoretical analysis extends the seminal work of Hulten (1978) to incorporate both firm heterogeneous productivity and factor distortions.

Third, our finding of the bias of the VA model echoes an earlier literature on the validity of the concept of real value added (Sato, 1976; Bruno, 1978). Bruno (1984) and Basu and Fernald (1995, 1997, 2002) have shown that the use of value added measures can lead to biased results in different scenarios.[14] Most closely related to our paper is Gandhi et al. (2017), which shows that the use of data on value added could lead to overestimated costs of misallocation, given that empirically distortions mostly benefit the less productive firms at the expense of the more productive ones. We show, somewhat to the contrary, that adding inter-sectoral linkages with the required adjustments can completely offset the overestimation, in the absence of intermediate input distortions. Finally, Herrendorf et al. (2013) which examines different sources of structural transformation, discusses how sectors can be viewed either as categories of final expenditure or value added and how these different views require changes in the preference specifications. Our findings confirm the importance of this distinction in the measurement of misallocation. We find a significant upward bias in the VA model's estimates of the cost of misallocation due to the use of an incorrect elasticity of substitution.

[12]Other important contributions to the misallocation literature include Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Guner et al. (2008), Banerjee and Moll (2010), Buera and Shin (2013), Midrigan and Xu (2014), Moll (2014), David et al. (2016), among many others.

[13]Sectoral interlinakges have also been studied by Hirschman (1958), Long and Plosser (1983), Horvath (1998, 2000), Ciccone (2002), Acemoglu et al. (2012), Baqaee (2018), and Baqaee and Farhi (2019a,c), to name a few.

[14]The other studies mainly focus on measuring productivity growth over time, while this paper compares productivity across firms.

The rest of the paper is structured as follows. The next section presents the two models for measuring efficiency loss of misallocation. Section 3 studies the specification of the VA model theoretically, while Section 4 discusses the measurement of firm productivity and wedges using different concepts of output. Section 5 evaluates the bias in data. Section 6 concludes.

2 Two Theoretical Approaches

This section first introduces the GO model, which adds a production network *a la* Long and Plosser (1983) to the HK framework, in preparation for an evaluation of the misallocation costs compared to the VA approach.[15] We view the GO model as a natural representation of production in an economy as firms employ both primary inputs and intermediate inputs in reality. We then briefly present the corresponding HK framework, which we label as the VA model. This framework, which abstracts from inter-sectoral linkages is, in a sense, a simplified version of the GO model. Clearly, both the GO and VA models require capital and labor to produce a final product (GDP). However, the GO model considers endogenous intermediate input uses, while the VA model does not.

2.1 The Gross Output (GO) Model

The are S sectors in the economy. Output from any sector Q_S is used as either intermediate input M_S or for final consumption C_S.[16] A final product, Y, is produced by a representative firm according to,

$$Y = \prod_{s=1}^{S} C_s^{\theta_s}, \text{ with } \sum_{s=1}^{S} \theta_s = 1. \tag{1}$$

Let the final output, Y, be the numeraire. The demand for sectoral output from final good producers is given by $P_s C_s = \theta_s Y$. Market clearing for sectoral output implies

$$Q_s = C_s + M_s, \tag{2}$$

[15] Bils et al. (2018) study a model similar to the GO model.

[16] We use C to denote consumption, but it should be understood as expenditure including both consumption and investment.

where $M_s = \sum_{q=1}^{S} M_{sq}$ with M_{sq} being the demand from sector q for the output of sector s. The sectoral output itself is produced using differentiated intermediate varieties Q_{si},

$$Q_s = \left(\sum_{i=1}^{N_s} Q_{si}^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}}, \tag{3}$$

where N_s is the total number of varieties in sector s and σ_s is the sector-specific elasticity of substitution. We assume that each variety is produced by one single firm, such that firm and variety can be used interchangeably. The inverse demand for a variety is given by $P_{si} = P_s Q_s^{\frac{1}{\sigma_s}} Q_{si}^{-\frac{1}{\sigma_s}}$, with the sectoral price index given by $P_s = \left(\sum_{i=1}^{S} P_{si}^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}}$.

The firms produce goods with two primary inputs, capital and labor, as well as intermediate inputs from all sectors, using a Cobb-Douglas technology:

$$Q_{si} = A_{si} \left(K_{si}^{\alpha_s} L_{si}^{1-\alpha_s} \right)^{\eta_s} \left(\prod_{q=1}^{S} M_{qsi}^{\lambda_{qs}} \right)^{1-\eta_s}, \tag{4}$$

with $\sum_{q=1}^{S} \lambda_{qs} = 1$. Different from HK who assume a monopolistic competition market structure, we assume that firms take prices as given.[17] As we will discuss in detail later, the change in market structure however does not affect our comparison of the two models in the empirical analysis.[18] However, assuming perfect competition makes the GO model easier to implement.[19]

Each firm then solves the following profit maximization problem, facing idiosyncratic distortions

[17] One way to think about this is there are many firms with access to identical technology with negligible entry costs, i.e., the market is contestable. In equilibrium there will be only one firm operating but the firm acts like a competitive producer. Otherwise, we can assume there are many firms producing one variety and they all have the same technology and face the same distortions.

[18] The benchmark results of HK will not be altered by the change in the market structure. In the GO model, the change in market structure distorts between sector allocation of resources for two reasons. First, the presence of markups distorts the relative price of intermediate inputs to primary inputs, which further distorts the sectoral allocation of primary inputs. Second, between-sector allocations can still be distorted if sectors have different markups. However, since we only focus on within-sector misallocation in our empirical analysis, the changes in sectoral allocation does not impact our results other than through the determination of sectoral weights in aggregation. Our calibration procedure in Section 5 makes sure the weights are affected by the same proportion due such distortions in the two models.

[19] As discussed later, under perfect competition we do not actually need information on the input-output matrix because the Domar weights are sufficient statistics for estimating the TFP gain. Under monopolistic competition this is no longer true and we have to solve the Leontief inverse in order to compute the sectoral weights for the GO model.

in the factor markets:

$$\max_{Q_{si},K_{si},L_{si},\{M_{qsi}\ \forall q\}} \pi_{si} = P_{si}Q_{si} - (1+\tau_{Ksi})RK_{si} - (1+\tau_{Lsi})WL_{si} - (1+\tau_{Msi})\sum_{q=1}^{S} P_q M_{qsi}.$$

Following HK, we assume that the wedges will not affect actual output besides distorting firms' input decisions. They correspond to the type of distortions in which payments are not discarded in Bigio and La'O (2020). One way to think of this is to view the wedges as shadow prices for the constraints the firms might face in the factor markets, such as a collateral constraint for renting capital. The wedges could also reflect some genuine differences in prices that firms face.[20] Alternatively, we can think of the wedges as actual taxes that are rebated to the households lump sum.[21]

Finally, factor market clearing for capital and labor requires

$$\sum_{s=1}^{S}\sum_{i=1}^{N_s} K_{si} = K, \text{ and } \sum_{s=1}^{S}\sum_{i=1}^{N_s} L_{si} = L. \tag{5}$$

Sector s's demand for intermediate inputs from sector q is the sum of demand across all variety producers $M_{qs} = \sum_{i=1}^{N_s} M_{qsi}$.

We prove in Appendix A the following lemma that a sector's production function inherits the structure of the firm's production function.

Lemma 1 *Taking the inputs of a sector as given and solving the within-sector allocation problem, the sectoral production function is given by*

$$Q_s = TFP_s \cdot \left(K_s^{\alpha_s} L_s^{1-\alpha_s}\right)^{\eta_s} \left(\prod_{q=1}^{S} M_{qs}^{\lambda_{qs}}\right)^{1-\eta_s}, \text{ and } TFP_s = \left[\sum_{i=1}^{N_s}\left(A_{si}\frac{TFPR_s}{TFPR_{si}}\right)^{\sigma_s-1}\right]^{\frac{1}{\sigma_s-1}}, \tag{6}$$

[20]For example, Luo (2019) argues that trade credits provide by suppliers would show up in the prices of intermediate inputs. Firms differing in their use of trade credits will then face different intermediate input prices.

[21]Bigio and La'O (2020) also discuss the case where payments due to wedges are discarded. They find that wedges with discarded payments are isomorphic to productivity shocks. Since we don't observe the amount of output lost due to distortions in data, we assume firm productivity A_{si} already incorporates this kind of wedges and focus on the case where payments due to wedges are not discarded.

where sectoral and firm revenue productivity (TFPR) are defined as

$$TFPR_s \equiv \frac{P_s Q_s}{(K_s^{\alpha_s} L_s^{1-\alpha_s})^{\eta_s} \left(\prod_{q=1}^{S} M_{qs}^{\lambda_{qs}}\right)^{1-\eta_s}} \tag{7}$$

$$TFPR_{si} \equiv \frac{P_{si} Q_{si}}{\left(K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}\right)^{\eta_s} \left(\prod_{q=1}^{S} M_{qsi}^{\lambda_{qs}}\right)^{1-\eta_s}}, \tag{8}$$

This lemma facilitates further aggregation of production as we now only have to deal with production at the sector level.

As in previous analyses which undertake aggregation of a Cobb-Douglas network economy with distortions (Jones, 2013; Bigio and La'O, 2020), a key to the aggregation turns out to be the Domar weight $v_s = \frac{P_s Q_s}{Y}$, which is simply the sectoral sales-to-GDP ratio (Domar, 1961). We can rewrite the market clearing condition for sectoral output in vector form

$$V = \theta + B \circ \text{diag}(\mathcal{T}_M^{\circ -1})V,$$

where V is the 1 by S vector of Domar weights, θ is the vector of sectoral shares in final consumption, B is the direct requirement matrix with its element given by $b_{sq} = \lambda_{sq}(1 - \eta_q)$, $\mathcal{T}_M$ is the vector of $\mathcal{T}_{Ms} = (1 - \eta_s)\lambda_{qs}\frac{P_s Q_s}{P_q M_{qs}}$ summarizing the distortions in intermediate input use within a sector,[22] diag($\cdot$) transforms a vector into a diagonal matrix, and $\circ$ is the element-wise Hadamard product (with $\circ - 1$ indicating the Hadamard inverse). The Domar weight vector can then be solved as

$$V = \left(I - B \circ \text{diag}(\mathcal{T}_M^{\circ -1})\right)^{-1} \theta$$

The Domar weight hence is determined by both technology and distortions in intermediate input markets. If there is no distortion in the network, namely $\mathcal{T}_{Ms} = 1$ for all sectors, we define $\tilde{V} \equiv (I - B)^{-1}\theta$. $\tilde{V}$ is the influence vector in Acemoglu et al. (2012), which captures how sectoral productivity shocks propagate downstream to other sectors through the production network. The influence vector is a key determinant of the shape of the aggregate production function. We denote

[22] Note that $\mathcal{T}_{Ms} = \frac{1}{\sum_i \frac{P_{si} Q_{si}}{P_s Q_s} \frac{1}{\tau_{Msi}}}$ is a weighted average of firm level distortions.

its element to be $\tilde{v}_s$.[23] With the Domar weights, we can relate the sectoral capital and labor demand to the Domar weights as,

$$\begin{aligned}\frac{K_s}{K} &= \frac{\frac{1}{\mathcal{T}_{Ks}}\alpha_s\eta_s v_s}{\sum_{q=1}^{S}\frac{1}{\mathcal{T}_{Kq}}\alpha_q\eta_q v_q} \equiv \beta_{Ks}, \\ \frac{L_s}{L} &= \frac{\frac{1}{\mathcal{T}_{Ls}}(1-\alpha_s)\eta_s v_s}{\sum_{q=1}^{S}\frac{1}{\mathcal{T}_{Lq}}(1-\alpha_q)\eta_q v_q} \equiv \beta_{Ls},\end{aligned}$$

where $\mathcal{T}_{Ks} = \alpha_s\eta_s\frac{P_sQ_s}{RK_s}$ and $\mathcal{T}_{Ls} = (1-\alpha_s)\eta_s\frac{P_sQ_s}{WL_s}$ summarize the effects of distortions on sectoral capital and labor demand. Without intersectoral distortions, $\mathcal{T}_{Ks} = 1$ and $\mathcal{T}_{Ls} = 1$.

Given these shares, we are ready to present the following aggregation result for this economy.

Proposition 1 *The economy admits an aggregate production function given by*

$$Y = AK^{\alpha}L^{1-\alpha}, \tag{9}$$

with $\alpha = \sum_{s=1}^{S}\tilde{v}_s\alpha_s$ *and total factor productivity given by*

$$A = \underbrace{\gamma}_{\text{network distortions}} \underbrace{\prod_{s=1}^{S}(TFP_s)^{\tilde{v}_s}}_{\text{within-sector distortions}} \underbrace{\prod_{s=1}^{S}\left[(\beta_{Ks})^{\alpha_s\eta_s\tilde{v}_s}(\beta_{Ls})^{(1-\alpha_s)\eta_s\tilde{v}_s}\right]}_{\text{between-sector distortions}}, \tag{10}$$

where $\gamma = \prod_{s=1}^{S}\left(\frac{\theta_s}{v_s}\right)^{\theta_s}\prod_{s=1}^{S}\left[\prod_{q=1}^{S}\left(\frac{1}{\mathcal{T}_{Ms}}b_{qs}\frac{v_s}{v_q}\right)^{(1-\eta_s)\lambda_{qs}}\right]^{\tilde{v}_s}.$

Notice that distortions have no effect on the shape of the production function. The output elasticity of capital in the aggregate production function is an average of the sectoral output elasticities weighted by elements of the influence vector. Distortions, however, result in a lower aggregate TFP which can be decomposed into three terms. The first term, γ, summarizes the distortions in the production network. The second term is the weighted average of sectoral TFP, which reflects the misallocation within sectors. Due to the presence of distortions, Hulten's theorem (Hulten, 1978) breaks down: The output elasticity of sectoral TFP is given by the influence vector instead of the

[23] Distortions affect the allocation of resources across sector. How the technology shocks transmit through the production networks however is not affected. Given the the Cobb-Douglas technology, the impact of a technology shock in an upstream sector on a downstream sector is always given by the input share of the upstream sector in the production of the downstream sector.

Domar weights.[24] On the other hand, an increase in sectoral TFP will not change the allocation of resources given the Cobb-Douglas technology (Baqaee and Farhi, 2019c). Finally, the third term reflects distortions in primary input use.

As the sum of $\tilde{v}_s$ is larger than 1,[25] the propagation of market distortions emphasized by Jones (2011, 2013) comes from the presence of the influence vector in aggregate TFP. In particular, the TFP loss coming from within-sector misallocation as studied by HK will be amplified through the production network.

2.2 The Value-Added (VA) Model

The VA model ignores sectoral linkages and assumes each firm to have simply a value added production. For the whole economy, the single final product is still denoted by Y. It is produced with sectoral value-added,

$$Y = \Phi \prod_{s=1}^{S} Y_s^{\widehat{\theta}_s}, \text{ with } \sum_{s=1}^{S} \widehat{\theta}_s = 1 \tag{11}$$

where Φ is a normalizing constant to ensure that the final products in the two models are identical. We use a hat as notation to indicate a variable that is different between the two models. Profit maximization implies $P_{Ys} Y_s = \widehat{\theta}_s Y$. Sectoral value added is an aggregate over its varieties:

$$Y_s = \left(\sum_{i=1}^{N_s} Y_{si}^{\frac{\widehat{\sigma}_s - 1}{\widehat{\sigma}_s}} \right)^{\frac{\widehat{\sigma}_s}{\widehat{\sigma}_s - 1}}, \tag{12}$$

where the elasticity of substitution between varieties $\widehat{\sigma}_s$ will be different from that in the GO model. The inverse demand for a variety is $P_{Ysi} = P_{Ys} Y_s^{\frac{1}{\widehat{\sigma}_s}} Y_{si}^{-\frac{1}{\widehat{\sigma}_s}}$ with the price index equal $P_{Ys} = \left(\sum_{i=1}^{N_s} P_{Ysi}^{1-\widehat{\sigma}_s} \right)^{\frac{1}{1-\widehat{\sigma}_s}}$.

Each firm i uses only primary factors to produce its varieties:

$$Y_{si} = \widehat{A}_{si} K_{si}^{\widehat{\alpha}_s} L_{si}^{1-\widehat{\alpha}_s}. \tag{13}$$

[24] Without distortions, Hulten (1978) only focus on the impact of sectoral TFPs on aggregate output, but not between-sector allocation. In his study, the influence vector equals the vector of Domar weights. The sales-to-GDP ratio fully measures how sectoral TFP shocks affect aggregate output.

[25] $\sum_{s=1}^{S} \tilde{v}_s > \sum_{s=1}^{S} v_s \eta_s = 1$, where the latter equality simply states the aggregate production function has constant returns to scale.

The firm faces idiosyncratic distortions in the factor markets and maximizes profit as follows[26]

$$\max_{Y_{si},K_{si},L_{si}} \widehat{\pi}_{si} = P_{Ysi}Y_{si} - (1+\tau_{Ksi})RK_{si} - (1+\tau_{Lsi})WL_{si},$$

where we have assumed that the magnitudes of the distortions are identical across the two models. Note that using a value added production function should not affect the size of the distortions.[27] Factor prices of primary inputs too are not distinguished between the two models: as factor prices are observed, they should remain the same if both models can rationalize the data. Factor markets are assumed to clear as usual.

HK show that the sectoral production function is given by

$$Y_s = \widehat{TFP}_s \cdot K_s^{\widehat{\alpha}_s} L_s^{1-\widehat{\alpha}_s}, \text{ with } \widehat{TFP}_s = \left[\sum_{i=1}^{N_s}\left(\widehat{A}_{si}\frac{\widehat{TFPR}_s}{\widehat{TFPR}_{si}}\right)^{\widehat{\sigma}_s-1}\right]^{\frac{1}{\widehat{\sigma}_s-1}}. \tag{14}$$

Thus, the formula for sectoral TFP in the VA model has the same form as that in the GO model, except that now firm revenue productivity (TFPR) and physical TFP are expressed in terms of the value added measures.[28]

Given the sectoral production function, sectoral demand for capital and labor is given by:

$$\begin{aligned} \frac{K_s}{K} &= \frac{\frac{1}{\widehat{\mathcal{T}}_{Ks}}\widehat{\alpha}_s\widehat{\theta}_s}{\sum_{q=1}^{S}\frac{1}{\widehat{\mathcal{T}}_{Kq}}\widehat{\alpha}_q\widehat{\theta}_q} \equiv \widehat{\beta}_{Ks}, \\ \frac{L_s}{L} &= \frac{\frac{1}{\widehat{\mathcal{T}}_{Ls}}(1-\widehat{\alpha}_s)\widehat{\theta}_s}{\sum_{q=1}^{S}\frac{1}{\widehat{\mathcal{T}}_{Lq}}(1-\widehat{\alpha}_q)\widehat{\theta}_q} \equiv \widehat{\beta}_{Ls}, \end{aligned}$$

where $\widehat{\mathcal{T}}_{Ks} = \hat{\alpha}_s\frac{P_{Ys}Y_s}{RK_s}$ and $\widehat{\mathcal{T}}_{Ls} = (1-\hat{\alpha}_s)\frac{P_{Ys}Y_s}{WL_s}$ are defined as above. Plugging these factor demand equations into the production function, we obtain the aggregate production function, as summarized

[26]Another minor difference between our model and the original HK model is that they assume an output and a capital wedge while we assume a capital and a labor wedge. The assumption simplifies the algebra a little but will not affect the results in this paper in any way. The wedges changes the marginal revenue product of the production factors. Because there are three wedges (output, capital, and labor wedge) but only two production factors, we can match the same marginal revenue products with different sets of wedges.

[27]However, inferred distortions using data might be different in these two models, depending on whether there are intermediate input distortions. We will discuss the inference in Section 4.

[28]Sectoral and firm revenue productivity are defined as $\widehat{TFPR}_s \equiv \frac{P_{Ys}Y_s}{K_s^{\widehat{\alpha}_s}L_s^{1-\widehat{\alpha}_s}}$ and $\widehat{TFPR}_{si} \equiv \frac{P_{Ysi}Y_{si}}{K_{si}^{\widehat{\alpha}_s}L_{si}^{1-\widehat{\alpha}_s}}$.

in the following proposition.

Proposition 2 *The VA model also admits an aggregate production function given by*

$$Y = \widehat{A} K^{\widehat{\alpha}} L^{1-\widehat{\alpha}}, \tag{15}$$

where the capital share is $\widehat{\alpha} = \sum_{s=1}^{S} \widehat{\theta}_s \widehat{\alpha}_s$,[29] *and total factor productivity is given by*

$$\widehat{A} = \Phi \underbrace{\prod_{s=1}^{S} \left(\widehat{TFP}_s\right)^{\widehat{\theta}_s}}_{\text{within-sector distortions}} \underbrace{\prod_{s=1}^{S} \left[\left(\widehat{\beta}_{Ks}\right)^{\widehat{\alpha}_s} \left(\widehat{\beta}_{Ls}\right)^{1-\widehat{\alpha}_s}\right]^{\widehat{\theta}_s}}_{\text{between-sector distortions}}, \tag{16}$$

Again, the shape of the production is determined by the technology parameters. The effect of distortions on aggregate TFP can be decomposed into the within-sector and between-sector components. Different from the GO model, there is no network propagation at work in the VA model by definition. The exponents on sectoral TFP and the allocation terms are the sectoral value-added shares in final output, which sum to one.

3 Model Comparison

This section compares measured TFP gains in the two models theoretically. As the VA model ignores intersectoral linkages, it is natural that we view the GO model as the data generating process. Taking the technology parameters, firm-level productivity and wedges in the GO model as given, we construct the measures of TFP gains in both models in terms of these parameters. Importantly, we will also build a mapping of model parameters between the two models.

3.1 Calibration of the VA model

In evaluating the TFP gain, we compare the actual state of the world to the counterfactual one with zero distortion, taking the same observed data as given. For the VA model to be consistent with the GO model, the VA model should be able to rationalize the same data as the GO model

[29] The share of labor is $\sum_{s=1}^{S} \widehat{\theta}_s (1 - \widehat{\alpha}_s)$. It is obvious that they sum to 1.

in both states of the world. In this subsection, we establish the conditions for the VA model to be consistent with the GO model.

As the two models use different concepts of output, we must first define value added in the GO model. We compute firm value added as revenue minus the cost of intermediate inputs:

$$P_{Ysi}Y_{si} = P_{si}Q_{si} - \sum_{q=1}^{S} P_q M_{qsi} \tag{17}$$

A key assumption here is that distortions in intermediate inputs are not recorded in the prices paid by the firms. What we have in mind are shadow prices of financial constraints (Bigio and La'O, 2020), or taxes/subsidies that are not directly recorded as costs of intermediate inputs. This assumption rules out firm-level intermediate input price differences as discussed by Luo (2019). Because we only observe the value of intermediate inputs but not the quantity,[30] allowing for distortions that are recorded as firm-specific prices would make the quantity of intermediate input unobserved.[31] Our simplifying assumption induces a bias in the estimates of the TFP loss from misallocation in the GO model, as the intermediate input distortion will be measured with error if recorded intermediate prices are different across firms. However, it can be shown that the VA model inherits the same bias, so that the differences in measured TFP loss from misallocation between the two models are not affected by this assumption.[32]

With firm value added defined, we are ready to calibrate the VA model. The model parameters that needed to be determined include $\{\widehat{\alpha}_s, \widehat{\sigma}_s, \widehat{\theta}_s\}$ and firm productivity $\widehat{A}_{si}$. We take primary input distortions as given and calibrate the VA model to match the allocation of capital, labor and firm nominal value added as generated by the GO model. Intermediate input distortions are not observed in the VA model. However, whether the VA model is consistent with the GO model depends on the properties of these distortions. Specifically, the following assumption is in order.

Assumption 1 $\tau_{Msi} = 0, \ \forall \ s \in \{1, 2, ...S\}$ *and* $i \in \{1, 2, ...N_s\}$.

[30] An exception is Atalay (2014), who observes firm-specific prices of intermediate inputs in a detailed dataset from the US Census Bureau.

[31] In a sense, HK also make this assumption in measuring capital stock, as they only observe the value of total assets in data. Using sector-specific deflators to deflate the value leads to a biased measure of capital stock, had firms used different prices to value their assets.

[32] This proof is available upon request.

We prove in Appendix A the calibration results that are summarized in the following proposition.

Proposition 3 *Under Assumption 1, the VA model can generate the same observed allocation and nominal output as the GO model if the following conditions hold: (1)* $\widehat{A}_{si} \propto A_{si}^{\frac{1}{\eta_s}}$*, (2)* $\widehat{\alpha}_s = \alpha_s$*, (3)* $\widehat{\sigma}_s = 1 + \eta_s(\sigma_s - 1)$*, (4)* $\widehat{\theta}_s = \eta_s \tilde{v}_s$*, and (5)* Φ *is adjusted such that final production is equated in the two models. If Assumption 1 does not hold, we cannot find a calibration of the VA model that is consistent with the data generated by the GO model.*

The above proposition shows that the value added specification may not generate the data that are consistent with those from the GO model. If firms face intermediate input distortions, the two models are theoretically inconsistent with each other. Since intermediate input distortions are present in actual data (Bigio and La'O, 2020; Liu, 2019), we should not expect the two models to produce the same TFP gain from removing distortions. Before proceeding to construct the TFP gain measures, a discussion about the proposition is in order.

First, note that we cannot determine the absolute level of firm value added TFP. This is because we only observe nominal output in the data, which incorporates both quantity and price. As the production of final goods uses a Cobb-Douglas technology, sectoral nominal value added is not affected by sectoral TFP: an increase in productivity is perfectly offset by a decrease in price.[33] Firm productivity is thus only comparable within a sector but not across sectors. Across sectors, comparison of TFP measures are meaningless as we can always change TFP in a sector by using a different unit of output measurement. This claim is invalidated when output prices and quantities are separately observed. We can also determine the level of firm TFP if the elasticity of substitution between sectoral output is different from one, which means that changes in firm TFP will induce resource reallocation across sectors.

As the firm's GO production function is separable in primary and intermediate inputs, we can simply write the firm's gross output as a function of value-added and intermediate inputs.

$$Q_{si} = Y_{si}^{\eta_s} \left(\prod_{q=1}^{S} M_{qsi}^{\lambda_{qs}} \right)^{1-\eta_s}$$

This gives the intuitive results on factor shares and value added TFP. In particular, value added

[33] The effect of the change in sectoral productivity on final output can be offset by adjusting Φ.

TFP is gross output TFP raised to a power of $\frac{1}{\eta_s} > 1$. This is also the reason why value added TFP is more dispersed than gross output TFP, as documented by Gandhi et al. (2017). There are two economic forces at work. First, in response to an increase in output productivity, if the firm does not adjust its intermediate input, the percentage increase in value-added will be larger than that in gross output.[34] Second, an increase in firm gross output productivity induces an increase in the demand for intermediate inputs, holding the intermediate prices constant. This induces a larger increase in value added because the contribution of additional intermediate inputs is larger than the increased costs.

The scaling up of firm value added productivity has been recognized in the literature. What's new is that the sectoral production function has different elasticity of substitution under different concepts of output. The difference is related to the value added ratio to gross output. As η_s increases, the gap between the two elasticities diminishes. If there is no intermediate input use, the two elasticities converge. The economic reason behind this is as follows. Without loss of generality, let us consider the more reasonable case of $\sigma_s > 1$. If the output productivity increases by one percent, the price of the firm's good will decrease by one percent, holding factor prices constant. In the VA model, the corresponding increase in productivity and the decrease in price would be $\frac{1}{\eta_s}$ percents. The decrease in prices induces the firm to increase the demand for primary inputs by $\sigma_s - 1$ percent in the GO model, and $\frac{1}{\eta_s}(\widehat{\sigma}_s - 1)$ in the VA model. The percentage increase in input demand should be identical in the two models. This leads to the relationship between the two elasticities.[35] The value added elasticity is smaller than the output elasticity if $\sigma_s > 1$. The reverse holds if $\sigma_s < 1$. The two elasticities thus should be on the same side of 1, with the value added elasticity closer to 1.

The calibration of the elasticity of substitution should depend on the output concept of the model. This is a point that has not been appreciated enough in the literature. Since the elasticity is generally estimated for gross instead of value added output, the value added studies have often chosen the wrong parameters. For example, HK cite Broda and Weinstein (2006) and Hendel and

[34]Since value-added is the difference between gross output and the costs of intermediate input, an increase in gross output but not intermediate input raises value-added more in percentage terms.

[35]The two elasticities are the same if the elasticity equals to one, regardless the size of η_s. This is the Cobb-Douglas case, under which a price change will not lead to changes in factor demand.

Nevo (2006) for their choice of elasticity while both studies estimate the parameter for gross output instead of value added. As the estimated TFP gain depends positively on the elasticity, this could be a source of biases of the estimates.[36] Using the adjusted elasticity of substitution will lower the measured TFP gain. Our empirical study below shows that assigning incorrect values of elasticity leads to a significant upward bias in the measured TFP gain in the VA model.

Finally, the value added weights are proportional to the Domar weights up to a factor of η_s. This adjustment corresponds to the scaling up of value added productivity. As both models are supposed to represent the same data, the amplification effect of sectoral linkages are always present regardless of the concept of output used.[37]

The differences in parameters between the two models resemble the differences in utility functions estimated with different data in Herrendorf et al. (2013), except that we study a full general equilibrium model and focus more on the production structure. Like us, Herrendorf et al. (2013) emphasize that we can view sectors as producing either final expenditure or value-added, with the two views connected through the IO structure of the economy. Hulten (1978) also compares the differences in measuring aggregate TFP in a model with production networks and one without. But his model without production networks examines the production of sectoral consumption goods from primary factors instead of value added. This is quite different from the VA model presented here.[38]

3.2 Aggregate TFP Gain

We now proceed to the measurement of TFP gains from removing all distortions. Assumption 1 is assumed to hold such that the VA model is consistent with the GO model observationally. We first

[36]For example, HK use 3 as a conservative measure of elasticity. If we take 3 as the elasticity for output, that for value-added will be 2 if we assume $\eta_s = 0.5$, a reasonable number according to Jones (2011).

[37]This can be more clearly seen from the one sector example in Jones (2013). Final output is produced by labor and intermediate input $Q = AL^{\alpha}X^{1-\alpha}$, which is used for consumption and intermediate inputs in production, $Q = C + X$. GDP in this economy then is simply given by $C = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} X$. If we derive the aggregate production from the output production function, we realize that the effect of the output productivity A is amplified through input-output linkages. The exponent $\frac{1}{\alpha}$ is the Domar weight for this economy. If we are only given data on value-added and primary inputs, we will see the value-added productivity is $A^{\frac{1}{\alpha}}$, which already includes the amplification effect of the production network. The VA model does not simply ignore the production network, it implicitly includes the effect of the network in the value-added measures.

[38]Note that Hulten (1978) requires a producer combining capital and labor from different sectors to produce the final product, which has no counterpart in the data.

construct TFP gain at the sector level. In the GO model, sectoral TFP is rewritten as

$$TFP_s = \left(\sum_{i=1}^{N_s} \left(A_{si} \frac{\mathcal{T}_{Ks}^{\alpha_s \eta_s} \mathcal{T}_{Ls}^{(1-\alpha_s)\eta_s}}{(1+\tau_{Ksi})^{\alpha_s \eta_s}(1+\tau_{Lsi})^{(1-\alpha_s)\eta_s}} \right)^{\sigma_s - 1} \right)^{\frac{1}{\sigma_s}}$$

In the VA model, if we plug in the formula for value added productivity, sectoral TFP is

$$\widehat{TFP}_s = \left(\sum_{i=1}^{N_s} \left(A_{si}^{\frac{1}{\eta_s}} \frac{\mathcal{T}_{Ks}^{\alpha_s} \mathcal{T}_{Ls}^{1-\alpha_s}}{(1+\tau_{Ksi})^{\alpha_s}(1+\tau_{Lsi})^{(1-\alpha_s)}} \right)^{\widehat{\sigma}_s - 1} \right)^{\frac{1}{\widehat{\sigma}_s - 1}},$$

where we have assumed $\widehat{A}_{si} = A_{si}^{\frac{1}{\eta_s}}$. This is without loss of generality as the absolute level of productivity will not affect the measured TFP gain. Comparing the two TFPs, we have $\widehat{TFP}_s = TFP_{si}^{\frac{1}{\eta_s}}$. Sectoral efficient TFP (in the absence of any distortion) for the two models are

$$TFP_s^E = \left(\sum_{i=1}^{N_s} A_{si}^{\sigma_s - 1} \right)^{\frac{1}{\sigma_s}} \text{ and } \widehat{TFP}_s^E = \left(\sum_{i=1}^{N_s} A_{si}^{\frac{\widehat{\sigma}_s - 1}{\eta_s}} \right)^{\frac{1}{\widehat{\sigma}_s - 1}}.$$

Combining all these information, sectoral TFP gain in these two models are related to each other according to

$$\frac{\widehat{TFP}_s^E}{\widehat{TFP}_s} = \left(\frac{TFP_s^E}{TFP_s} \right)^{\frac{1}{\eta_s}}.$$

We thus show that the two models illustrate an identical sectoral TFP gain by eliminating all distortions at the sector level, under Assumption 1. Aggregating over sectors leads to an identical aggregate TFP gain with all within-sector distortions removed.

Without intermediate input distortions, the allocation of primary factors would remain the same in the two models. For example, the allocation of capital is

$$\frac{\alpha_s \eta_s v_s}{\sum_{q=1}^{S} \alpha_q \eta_q v_q} = \frac{K_s}{K} = \frac{\widehat{\alpha}_s \widehat{\theta}_s}{\sum_{q=1}^{S} \widehat{\alpha}_q \widehat{\theta}_q}.$$

Plugging the between-sector allocation terms into the aggregate production function in Proposition 1 and 2, it is clear that the aggregate TFP gain from removing between-sector misallocation is also identical in the two models. We summarize the above results in the following proposition.

Proposition 4 *Under Assumption 1, the two models produce the same theoretical aggregate TFP gain, conditional on observing the same data. If we decompose the TFP gain into a between-sector and a within-sector component. The two components are also identical in the two models.*

The isomorphism between the two models contrasts with the arguments in Jones (2011, 2013) and Gandhi et al. (2017). In particular, Jones (2011, 2013) argues that existing distortions can be amplified through production networks such that the cost of misallocation is larger than HK have estimated. On the other hand, Gandhi et al. (2017) find that the dispersion of firm physical TFP is smaller if it is measured with output data, implying a smaller cost of misallocation. Proposition 4 therefore offers a more complete understanding of the effect of production networks: Value-added measures implicitly incorporate the amplification effect of intersectoral linkages, which is reflected in data as a larger dispersion of firm physical productivity. When there is no intermediate input distortion, the two forces cancel out each other, implying that the TFP gain from removing distortions is orthogonal to the input-output structure of the economy. The points made by Jones (2011, 2013) and Gandhi et al. (2017) are, in fact, just opposite sides of the same coin.

In sum, this section establishes that the a VA model can be consistent with data if there is no distortion in intermediate input use, and the two models are no different in measuring TFP gain from moving to efficiency in this case. This discussion takes firm productivity and market distortions as given. To empirically measure the TFP gain, we also have to infer firm productivity and wedges from the data. Moreover, Assumption 1 may not hold in the data (Bigio and La'O, 2020). We take up these issues in the next section.

4 Empirical Measures of TFP Gain

This section discusses the measurement of TFP gains using firm data. We first discuss the inference of firm productivity and wedges, which are then used to construct the empirical measures of aggregate TFP gains from removing all firm-level distortions.

4.1 Inference of Firm Productivity and Wedges

To recover firm productivity and wedges, imagine we have a firm level dataset with information on output ($P_{si}Q_{si}$), capital (K_{si}), labor (L_{si}), and intermediate inputs ($\sum_{q=1}^{S} P_q M_{qsi}$).[39] Nominal value-added is computed as firm's revenue minus the cost of intermediate inputs. Given this definition, we relate nominal value-added to nominal output as

$$P_{Ysi}Y_{si} = \left(1 - \frac{1-\eta}{1+\tau_{Msi}}\right) P_{si}Q_{si}. \tag{18}$$

This condition is important in understanding the results in this subsection. It says that with distortions, the share of value-added in gross output deviates from the contribution of primary factors.

We also assume that we have correct values for all the technology parameters. Our discussion in the last section shows that the literature might have incorrectly calibrated the elasticity of substitution for the VA model, which can potentially bias the value-added TFP gain upward. We ignore this issue in this section and only discuss its quantitative implications in the next section.

We first discuss the inference of the wedges. For the GO model, the first order conditions for firms imply

$$\begin{aligned}
\frac{\alpha_s \eta_s P_{si} Q_{si}}{R K_{si}} &= 1 + \tau_{Ksi}, \\
\frac{(1-\alpha_s)\eta_s P_{si} Q_{si}}{W L_{si}} &= 1 + \tau_{Lsi}, \\
\frac{(1-\eta_s) P_{si} Q_{si}}{\sum_{q=1}^{S} P_q M_{qsi}} &= 1 + \tau_{Msi},
\end{aligned}$$

where R and W are exogenously set.[40] The left hand side is the ratio of marginal revenue products to factor prices, and the right hand side is inferred firm-level distortions expressed in terms of true

[39] Note that we have assumed that we only observe total value of intermediate inputs, which is what most firm-level database can provide. Given that we assume the intermediate input distortion does not affect the allocation of resources between different intermediate inputs, we can attribute total expenditure on intermediate inputs to different inputs, which can be further used to estimate the real quantity of intermediate inputs from different sectors given appropriate price indexes for sectoral output. This however is unnecessary for the exercise in this paper.

[40] The exact value of factor prices is irrelevant as only the relative size of the wedges matter for measuring misallocation.

distortions. The distortions are thus identified as differences in marginal revenue products. For the VA model, we can infer the distortions as

$$\begin{aligned}\frac{\alpha_s P_{Ysi} Y_{si}}{RK_{si}} &= (1+\tau_{Ksi})\left(1-\frac{1-\eta_s}{1+\tau_{Msi}}\right)/\eta_s,\\ \frac{(1-\alpha_s) P_{Ysi} Y_{si}}{WL_{si}} &= (1+\tau_{Lsi})\left(1-\frac{1-\eta_s}{1+\tau_{Msi}}\right)/\eta_s,\end{aligned}$$

where we have used the relationship between nominal value-added and output in the model. Both of the inferred wedges deviate from true distortions, as the share of value-added in output does not reflect true contribution of the primary factors.

Identifying a firm's physical productivity is more challenging as there is no output price or value-added price at the firm level. Following HK, we use the allocation rule for intermediate inputs and the pricing function for the variety to infer physical productivity in the GO model as

$$\kappa_s \frac{(P_{si} Q_{si})^{\frac{\sigma_s}{\sigma_s-1}}}{\left(K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}\right)^{\eta_s}\left(\sum_{q=1}^{S} P_q M_{qsi}\right)^{1-\eta_s}} = A_{si},$$

where κ_s is a sector level constant. The left hand side is observed up to a constant, and the right hand side is true firm productivity. As discussed above, the absolute level of firm productivity does not affect our results in any sense so we can normalize $\kappa_s = 1$. Firm output productivity thus can be correctly inferred using output measures. Similarly, physical productivity measured in the VA model is given by:

$$\widehat{\kappa}_s \frac{(P_{Ysi} Y_{si})^{\frac{\widehat{\sigma}_s}{\widehat{\sigma}_s-1}}}{K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}} = \left(\frac{1-\frac{1-\eta_s}{1+\tau_{Msi}}}{\eta_s}\right)^{\frac{\widehat{\sigma}_s}{\widehat{\sigma}_s-1}} (1+\tau_{Msi})^{\frac{\eta_s-1}{\eta_s}} A_{si}^{\frac{1}{\eta_s}},$$

where $\widehat{\kappa}_s$ is a sector level normalizing constant. The right hand side is inferred value-added productivity, which deviates from its theoretical counterpart due to the presence of intermediate input distortions. The first two terms on the right hand side reflect the effect of the distortions, which is a combination of two effects with opposite signs. First, the share of intermediate input deviates from its contribution in production (the first term). A positive distortion raises the contribution of primary factors and value-added productivity. Second, a positive distortion also reduces the use of intermediate input and hence lowers value-added productivity (the second term). Taken together, the sign of the total bias in measured value-added productivity is indeterminate. Besides the size of the distortion, the sign depends on the technology parameters. A rise in the elasticity of substitution reduces the first effect while keeping the second effect unchanged. This is because nominal value-added now is raised to a lower power in deriving real value-added. A rise in the importance of intermediate input (a reduction in η_s) increases the second effect. Its effect on the first effect however depends on the sign of the distortion. If $\tau_{Msi} > 0$, the bias due to first effect is reduced. The bias is increased if $\tau_{Msi} < 0$.[41]

Our results on the inference of firm productivity and wedges are summarized as follows.

Proposition 5 *Inference using value-added measures is biased due to the presence of intermediate input distortions.*

One way to understand the economics behind the bias in the inference using value-added measures is to consider our value-added measure as a real value-added, where the prices used to value intermediate inputs deviate from that perceived by the firms due to the presence of the intermediate input distortions. In this case, the value of intermediate inputs recorded in data does not reflect true productive contribution of intermediate inputs, which further leads to a bias in the contribution of primary inputs in our value-added measure. This finding resonates with an earlier literature studying the use of real value-added indexes.[42] As the market prices of intermediate inputs do not equal the effective prices facing firms, we attribute the differences in effective prices to the value-added measure which is essentially a partial Solow residual (Basu and Fernald, 1995). The earlier literature mainly focuses on measuring productivity change over time using real value-added. Changes in the relative price between output and intermediate inputs naturally brings challenges to the valuation of intermediate inputs. Our focus here is on the comparison of productivity across firms where the source of changes in the prices of intermediate inputs is idiosyncratic distortions in

[41] To see this point, note that $\frac{\partial \frac{1-\frac{1-\eta_s}{1+\tau_{Msi}}}{\eta_s}}{\partial \eta_s} = -\frac{1}{\eta_s^2}\frac{1}{1+\tau_{Msi}} < 0$. The last inequality requires $\tau_{Msi} > -1$, which is a natural assumption for the firm's problem to be well-defined. If $\tau_{Msi} > 0$, the bias is positive and an increase in η_s lowers the bias. If $\tau_{Msi} < 0$, the bias is negative and an increase in η_s raises the bias.

[42] For example, Bruno (1978) shows that marginal revenue products and productivity will be incorrectly inferred using the real value-added index if the intermediate input prices used in constructing the real value-added measures are different from the prices the firms face. See also the discussion in Basu and Fernald (2002).

the intermediate input markets.

With correct inference of firm productivity and wedges, the GO model correctly measures the TFP gain. A corollary of Proposition 4 and Proposition 5 regarding the VA model is as follows:

Corollary 1 *If Assumption 1 holds, firm productivity and wedges can be correctly inferred using value-added measures. The VA model and the GO model produce the same measured aggregate TFP gain, conditional on observing the same data.*

This corollary serves as a benchmark for our understanding of the consequences of ignoring the production network for measuring misallocation. Assumption 1, concerning the absence of intermediate input distortions, however is highly unrealistic. We next turn to the measurement of TFP gain in data when the assumption does not hold.

4.2 Measured TFP Gain

We now construct the empirical TFP gain measures using inferred firm productivity and wedges from the previous subsection, allowing for distortions in the intermediate input use. We first make the following simplifying assumption for the rest of our analysis.

Assumption 2 $\mathcal{T}_{Js} = 1$, $\forall J \in \{K, L, M\}$ *and* $s \in \{1, 2, ...S\}$.

$\mathcal{T}_{Js}$, $\forall J \in \{K, L, M\}$ summarizing firm level distortion in a sector are defined above. This assumption says that there is no between market misallocation. It is made mainly with the empirical implementation of the TFP gain measures in mind. Without between sector misallocation, we can use sectoral input shares to calibrate the output elasticity in the production function. Otherwise the distortions and the output elasticity could not be distinguished from each other unless we have direct measures of the distortions or the output elasticity from other sources. $\mathcal{T}_{Ms} = 1$ also implies there is no distortion in the production network such that $\tilde{v}_s = v_s$. In this case, we do not have to actually invert the Leontief matrix but can use the Domar weights to aggregate sectoral TFP in the GO model directly.

Let Y^E be the efficient output, the TFP gains from moving to efficient allocation in the two models are as follows if Assumption 2 holds.

$$\left(\frac{Y^E}{Y}\right)^o = \prod_{s=1}^{S}\left(\frac{TFP_s^E}{TFP_s}\right)^{v_s}, \text{ and } \left(\frac{Y^E}{Y}\right)^{va} = \prod_{s=1}^{S}\left(\frac{\widehat{TFP}_s^E}{\widehat{TFP}_s}\right)^{\widehat{\theta}_s}. \tag{19}$$

The aggregate TFP gain is the average across sectoral TFP gains, with each sector weighted by the corresponding Domar weight in the GO model and the value-added share in the VA model. Given Assumption 2, it can be seen that all errors in measuring the aggregate efficiency loss comes from mismeasured sectoral TFP gains. The VA model can produce the correct empirical aggregate TFP gain as long as the sectoral TFP gain is correctly measured, even if Assumption 1 is violated.

To compare between the two models, we will derive actual TFP and its efficient benchmark as functions of firm productivity and wedges, realizing that inferred measures may deviate from true measures. To the extent that the inferred measures are unbiased based on the GO model, measured sectoral TFP in the GO model is the same as above. We restate the result below

$$TFP_s = \left(\sum_{i=1}^{N_s}\left(A_{si}\frac{1}{(1+\tau_{Ksi})^{\alpha_s\eta_s}(1+\tau_{Lsi})^{(1-\alpha_s)\eta_s}(1+\tau_{Msi})^{1-\eta_s}}\right)^{\sigma_s-1}\right)^{\frac{1}{\sigma_s-1}}$$

where we have applied Assumption 2. Efficient sectoral TFP is $TFP_s^E = \left(\sum_{i=1}^{N_s} A_{si}^{\sigma_s-1}\right)^{\frac{1}{\sigma_s-1}}$.

Measured sectoral TFP based on the VA model however will differ from its theoretical value given the bias in the inference of firm productivity and wedges. To conserve notation, we still use the same notation to denote sectoral TFP. Measured sectoral TFP in the VA model is

$$\widehat{TFP}_s = \left(\sum_{i=1}^{N_s}\left(\frac{A_{si}\left(\eta_s\right)^{\frac{1}{1-\sigma_s}}\left(1-\frac{1-\eta_s}{1+\tau_{Msi}}\right)^{\frac{1}{\sigma_s-1}}}{\left[(1+\tau_{Ksi})^{\alpha_s}(1+\tau_{Lsi})^{1-\alpha_s}\right]^{\eta_s}(1+\tau_{Msi})^{1-\eta_s}}\right)^{\sigma_s-1}\right)^{\frac{1}{\eta_s(\sigma_s-1)}}. \tag{20}$$

Ignoring the power terms, the term in the inner brackets only differs from that in the GO model due to the the deviation of the contribution of primary inputs from the output elasticity. The effect of the distortions on the use of intermediate input is recognized in the value-added sectoral TFP, as shown in the denominator. Efficient sectoral TFP however will be mismeasured due to the presence of both effects,

$$\widehat{TFP}_s^E = \left(\sum_{i=1}^{N_s}\left(A_{si}\left(\eta_s\right)^{\frac{1+\eta_s(\sigma_s-1)}{1-\sigma_s}}\left(1-\frac{1-\eta_s}{1+\tau_{Msi}}\right)^{\frac{1+\eta_s(\sigma_s-1)}{\sigma_s-1}}(1+\tau_{Msi})^{\eta_s-1}\right)^{\sigma_s-1}\right)^{\frac{1}{\eta_s(\sigma_s-1)}} \tag{21}$$

Measurement errors at the firm level thus build up, such that both actual and efficient sectoral TFP will deviate from the correct measures.

Given sectoral actual and efficient TFP, how do the two measured TFP gains differ if intermediate input distortions are present? The comparison is not tractable. Importantly, we cannot claim with certainty that the value-added model overstates or understates measured TFP gain.

To illustrate this point, we approximate the measures by assuming A_{si}, $1+\tau_{Ysi}$,[43] and $(1+\tau_{Msi})$ jointly follow a log-normal distribution, with the log of the latter two having zero mean. We further assume $\eta_s = 0.5$ such that $1-\frac{1-\eta_s}{1+\tau_{Msi}}$ can be approximated by $\frac{1}{2}(1+\tau_{Msi})$. TFP gain for a single sector in the GO model can be approximated by[44]

$$\log\left(\frac{TFP_s^E}{TFP_s}\right)^{v_s} = \frac{N_s v_s(1-\sigma_s)}{2}\left(\frac{\sigma_y^2}{4}+\frac{\sigma_m^2}{4}-\sigma_{ay}-\sigma_{am}+\frac{\sigma_{ym}}{2}\right),$$

where σ_x^2 is the variance of variable x and σ_{xz} is the covariance between variables x and z. TFP gain in the VA model is approximated by

$$\log\left(\frac{\widehat{TFP}_s^E}{\widehat{TFP}_s}\right)^{\widehat{\theta}_s} = \frac{N_s v_s(1-\sigma_s)}{2}\left[\frac{\sigma_y^2}{4}+\left(\frac{1}{4}-\frac{1}{\sigma_s-1}\right)\sigma_m^2-\sigma_{ay}-\sigma_{am}+\left(\frac{1}{2}-\frac{1}{\sigma_s-1}\right)\sigma_{ym}\right]$$

Comparing these two measures, we have the following proposition.

Proposition 6 *Assuming firm productivity and distortions follow a joint log-normal distribution and $\eta_s = 0.5$, the difference between the two TFP gain measures is given by*

$$\log\left(\frac{TFP_s^E}{TFP_s}\right)^{v_s} - \log\left(\frac{\widehat{TFP}_s^E}{\widehat{TFP}_s}\right)^{\widehat{\theta}_s} = -\frac{N_s v_s}{2}(\sigma_m^2+\sigma_{ym}) \tag{22}$$

[43] $1+\tau_{Ysi}$ is defined as $(1+\tau_{Ksi})^{\alpha_s}(1+\tau_{Lsi})^{1-\alpha_s}$, which summarizes the distortion in primary input use.

[44] Notice the measure is raised by the Domar weights. This expression actually gives the contribution of sector s to aggregate efficiency loss. It is the same case for the VA model.

An immediate conclusion from the above proposition is that the measured TFP gain in the VA model can be biased away from that in the GO model in either direction. The sign and the size of the bias depends on how the wedges for intermediate inputs are distributed. A higher variance for the intermediate input distortions would raise the value-added TFP gain relative to the output TFP gain. This is because the dispersion of physical productivity is larger in the VA model in the presence of wedges, which raise the level of efficient TFP. A positive covariance between the primary input and input distortions also increases the value-added TFP gain. In this case, firms facing a higher primary input distortion will also have a larger measured value-added productivity, because firm value-added overstates the contribution of primary factors due to the intermediate input distortions. On the other hand, a negative covariance lowers the value-added TFP gain relative to the output TFP gain. In this case, the most constrained firms in the primary input markets also have lower measured TFP. This reduces the scope for and the gain from reallocation. Obviously, only if the covariance is negative and large enough could the GO model have a larger TFP gain. Finally, if the variance of the intermediate input market distortions and its covariance with the primary market distortions are small, the difference between the two measures will be small.

In sum, we find that intermediate input market distortions induce measurement errors in inferred firm productivity and wedges, when using value-added data, suggesting another source of the bias embedded in the VA model. Proposition 6 shows that the bias can go in either direction, again reflecting the fact that value-added measures already incorporates the amplification effect of production networks. The size of the bias is also indeterminate. However, Proposition 6 only holds under restrictive assumptions. The actual bias of the value-added measure is an empirical matter. This is what we study using micro data in the next section.

5 Applications with Actual Data

In this section we take the models to the data and evaluate the biases in the value-added TFP gain measure quantitatively.

5.1 Data

We use the Chinese Annual Survey of Industrial Production (1998-2007) and the Indian Annual Survey of Industries (1999-2009) for our empirical analysis. The Survey of Industrial Production from China's National Bureau of Statistics covers all non-state firms with more than 5 million yuan (about 0.6 million USD during the sample period) in revenue plus all state-owned firms in the industrial sector, which includes mining, manufacturing, utilities, and construction. We use the information on the firm's industry (at the four-digit level), wage payments,[45] employment, output, value-added, capital stock, and intermediate inputs. Capital stock is defined as the book value of fixed capital net of depreciation. Labor compensation in the data is systematically under-reported such that calculated industrial labor share is much smaller than those in the national accounts.[46] To correct for the under-reporting, HK raise the wage payments of all firms to make aggregate labor share calculated from the survey consistent with national accounts data. This adjustment however will not affect the result so we choose to use the original numbers. Finally, to allow for differences in worker human capital across firms, we use wage bills as our measurement of labor input instead of employment.

For India, the main data set is the annual plant-level data from the Annual Survey of Industries (ASI) conducted by the Indian Ministry of Statistics . We use the annual data from 1999 to 2009. A few caveats are in order. The sample consists of registered plants employing 10 or more workers using power, or 20 or more workers without power. Plants with over 100 workers are surveyed every year, while all other plants with fewer than 100 workers were randomly sampled. We use the information on the firm's industry (at the four-digit ISIC level), labor compensation, sales, intermediate input, and book value of the fixed capital stock. The measure of labor compensation is the sum of wages, bonuses, and benefits, which used as our measure of labor input. Capital is measured as the average of the net book value of fixed capital at the beginning and end of a fiscal year.

[45] Wage payments include wages, retirement and unemployment insurance, health insurance, housing benefits, and employee supplementary benefits.

[46] It can be clearly seen in the data. For example, 59% of all firms report zero unemployment insurance payment, 38% of all firms report zero health insurance payment, while both of which are mandatory by the law.

5.2 Calibration

Production networks introduce complicated sectoral linkages within an economy. However, our analysis above shows that we don't have to consider all the details of the linkages. For our purpose, we only have to find the correct sectoral weights in the aggregate production function. To measure the TFP gain, we thus need to assign values to parameters α_s, η_s, σ_s, and the sectoral weights $\tilde{v}_s$ and $\widehat{\theta}_s$. Following HK, we view the U.S. as an undistorted economy and calibrate the elasticities of output with respect to inputs using U.S. industry shares matched to Chinese and Indian industries. The U.S. shares are for six-digit NAICS industries. The output elasticity of capital α_s is calibrated as 1 minus the share of labor income in industrial value-added.[47] Similarly, the output elasticity of intermediate inputs is set to the share of intermediates of sectoral gross output.[48] To reduce the burden of assigning different elasticities of substitution for different industries, we assume $\widehat{\sigma}_s$ to be 3 for all industries following HK. This implies that $\sigma_s = 1 + (\widehat{\sigma}_s - 1)/\eta_s$ would be different across industries for the VA model if η_s varies across industries. Given the aggregate share of intermediate inputs in gross output is roughly 0.5, the elasticity of substitution for output is around 5 which is substantially larger than that for value-added.[49]

The choice of sectoral weights needs some discussion. If there are distortions across sectors, neither the Domar weights nor the value-added shares of GDP are the correct weights.[50] Given that U.S. has different consumption shares θ_s from China and India across industries, theoretically we should combine the Chinese and India consumption shares and U.S. input-output matrix to derive the weights. However, to make our VA model as comparable to HK as possible, we use Chinese and Indian value-added shares for the VA model and use $\tilde{v}_s = \widehat{\theta}_s/\eta_s$ to be our output weights. We admit the choice of weights might bias our measure of aggregate efficiency. However, it will not have a large effect on the comparison between the two models, because both weights are

[47] In computing the U.S. labor's share, each industry's raw labor share is raised by 3/2 to adjust for the underreporting of non-wage benefits in the data, following HK.

[48] If we calibrate the output elasticities using Chinese and Indian shares, the main conclusion of this paper will not change. We do not report these results in the paper.

[49] In results not reported here, we also experiment with assign 3 for σ_s as in HK and back out the elasticity for output using the formula, this would lower the efficiency loss in both models but the main conclusions does not change.

[50] Remember the Domar weights are given by $V = \left(I - B \circ \mathrm{diag}(\mathcal{T}_M^{\circ -1})\right)^{-1} \theta$. The value-added shares are given by $V \circ (\mathbf{1} - (\mathbf{1} - \bar{\eta}) \oslash \mathcal{T}_M)$. Both of which are affected by distortions in intermediate input use.

Table 1: Model Calibration

Parameters	Targeted Value	Mean	STD
α_s	capital's share of value-added for US industries	0.473	0.143
η_s	intermediate's share of gross output for US industries	0.513	0.100
$\widehat{\sigma}_s$	3, following HK	3	0
σ_s	$1 + (\widehat{\sigma}_s - 1)/\eta_s$, according to the theoretical analysis	5.083(China) 5.091(India)	1.025(China) 0.783(India)
$\widehat{\theta}_s$	sector's value-added share in GDP in China or India	0.0024(China) 0.0082(India)	0.0050(China) 0.0235(India)
$\tilde{v}_s$	$\widehat{\theta}_s/\eta_s$, according to the theoretical analysis	0.0052(China) 0.0200(India)	0.0127(China) 0.0700(India)

Note: Mean and Standard deviation of α_s and η_s are computed for 422 Chinese industries in the data. We do not report the statistics for India but note that they are similar to the Chinese numbers. Though there are substantially fewer industries (122) in the Indian data. Statistics of $\widehat{\theta}_s$, $\tilde{v}_s$, and $\widehat{\theta}_s$ are reported separately for China and India as industries have different value-added shares in manufacturing GDP in the two countries. The statistics are also different across years as we compute sector's value-added share in GDP in each year separately. The reported values are for 2005, which is representative of other years.

biased by the same proportion.[51] This is certainly the case when we compare the TFP gain for each industry instead of at the aggregate level. Our calibration of model parameters is summarized in Table 1.

5.3 Estimation Results

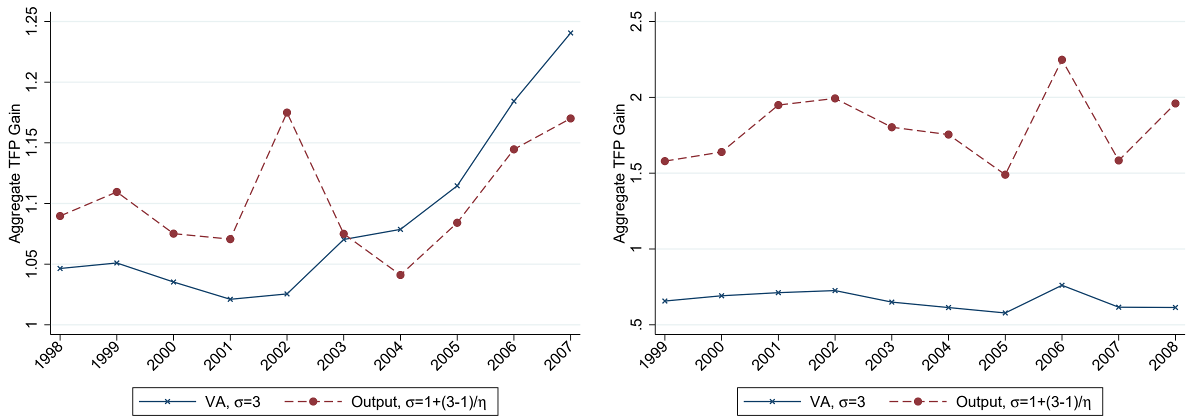
5.3.1 Estimated TFP Gains from Moving to Efficiency

Figure 1 compares measured aggregate TFP gain in the two models. Panel a) shows that the two models give different trends in resource allocation for China. There is a clear upward trend in resource misallocation since 2004 in the VA model, while that in the GO model is more volatile. However, despite the difference in trends, the GO model does not systematically produce larger or smaller TFP gain than the VA model, contrast to the finding of Jones (2011, 2013) and Gandhi et al. (2017).

For India, the estimated TFP gain from removing all distortions is always higher under the out-

[51] Remember that the output weights are given by $\tilde{V} = (I - B)^{-1}\theta$. The value-added weights are given by $\widehat{\theta} = \tilde{V} \circ \bar{\eta}$. As η is correctly calibrated, it is clear both weights are biased by the same proportion.

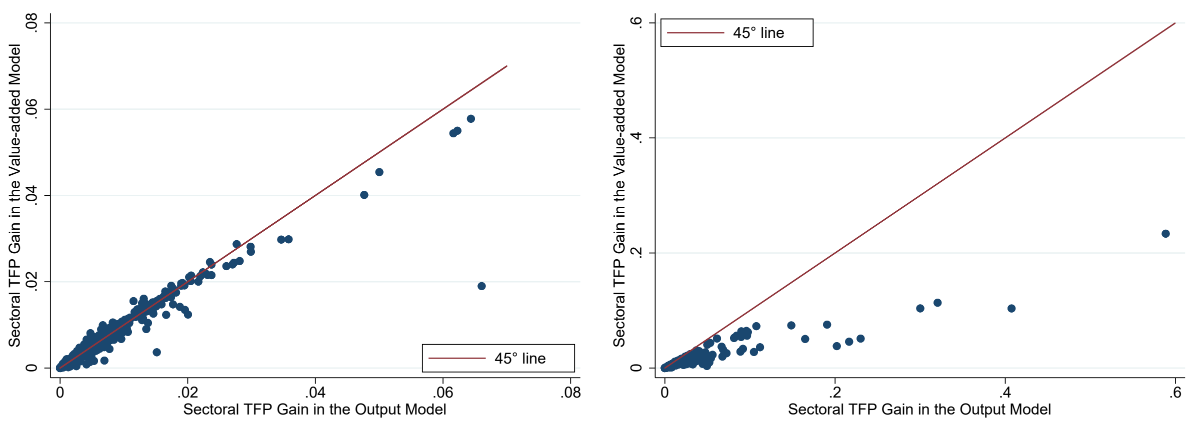
Figure 1: Aggregate TFP Gain in Different Models



(a) China (b) India

Note: Aggregated TFP gain is defined as $\left(\frac{Y^E}{Y}\right)^{o} - 1$ and $\left(\frac{Y^E}{Y}\right)^{va} - 1$.

Figure 2: Sectoral TFP Gain in Different Models



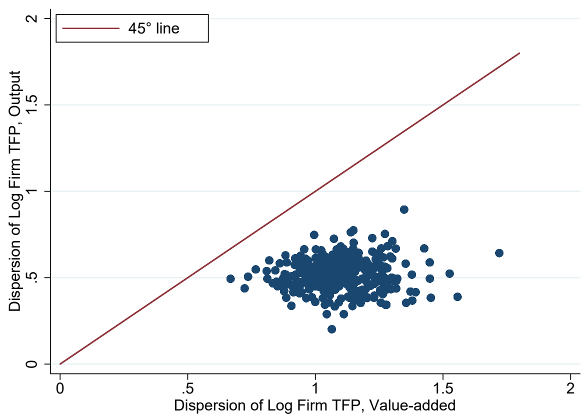
(a) China (b) India

Note: TFP gain at the sector level is measured as $\log\left(\frac{TFP_s^E}{TFP_s}\right)^{\tilde{v}_s}$ and $\log\left(\frac{\widehat{TFP}_s^E}{\widehat{TFP}_s}\right)^{\widehat{\theta}_s}$.

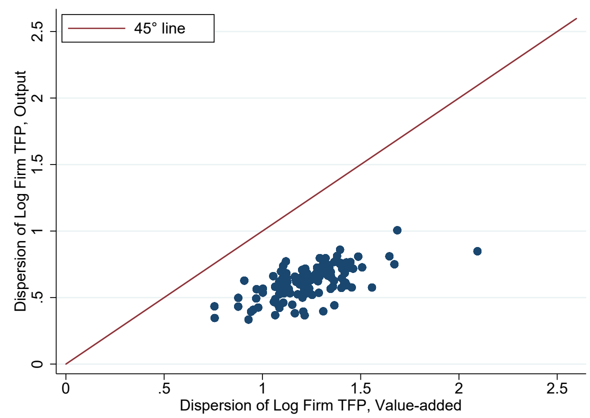
put approach, and the gap averages at 114 percent points in our sample period. If we believe the GO model is correctly specified, this finding suggests that intermediate input distortions are substantially larger in India, and using the VA model might be associated with a significant downward bias in evaluating the actual cost of misallocation.

Next we examine the TFP gain at the sector level. As is shown in Figure 2, the TFP gains measured in the two models are similar to each other for China. The difference between the two is less than 5 percentage points for over 99% of the sector-year observations. For India, we continue to find significantly larger TFP gains using the GO model than the VA model at the sector level, with an exception of a few observations that show higher gain under the value-added framework.

Figure 3: Dispersion of Firm Physical Productivity in Different Models



(a) China



(b) India

Note: The dispersion is measured by the standard deviation of log firm productivity.

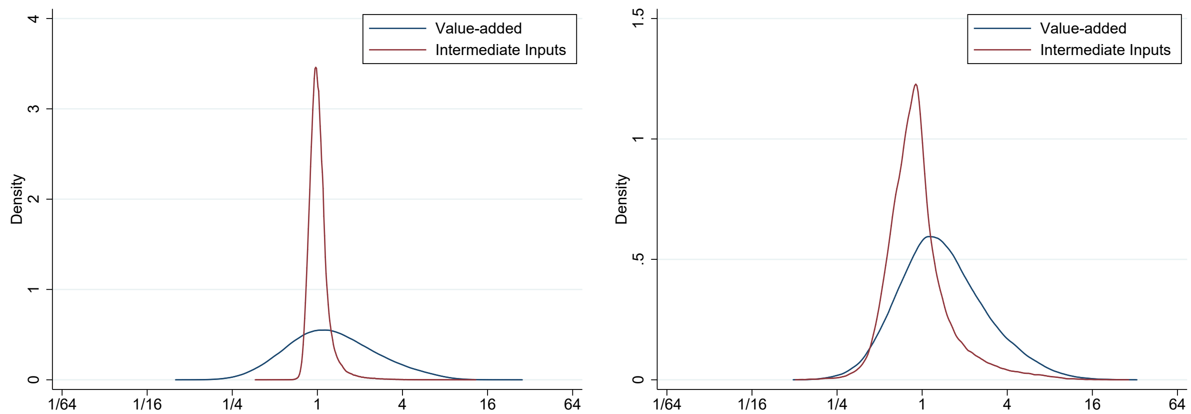
Despite the amplification effect of production networks (Jones, 2011, 2013), a reason why the TFP gain in the GO model is not substantially larger than that in the VA model for China is a lower dispersion of firm TFP measured in the GO model (Gandhi et al., 2017).[52] This can be clearly seen from Panel A of Figure 3, where the standard deviation of log firm TFP in the GO model is plotted against that in the VA model. The pattern also holds for India in Panel B, with the VA model producing substantially lower TFP gains.

5.3.2 Dispersion of Estimated Marginal Revenue Products

Our analysis above shows that the TFP gain measured in the VA model will only deviate from that in the GO model if there are distortions in the use of intermediate input. The lower dispersion in value-added TFP and the multiplier due to the amplification through input-output linkages will offset complete with each other in the absence of intermediate input distortions. The fact that the two measures stay close to each other for China suggests that intermediate input distortions might be mild. Panel A of Figure 4 confirms this assertion in the data, in which we plot the density of the marginal revenue product (in logs) of the primary inputs and intermediate inputs relative to their

[52] Another reason is the boosted elasticity of substitution, as we discuss later.

Figure 4: Dispersion of Marginal Revenue Products



(a) China (b) India

Note: The figures plot the density of the logs of $\left(\frac{\alpha_s \eta_s P_{si} Q_{si}}{K_{si}}\right)^{\alpha_s} \left(\frac{(1-\alpha_s)\eta_s P_{si} Q_{si}}{L_{si}}\right)^{1-\alpha_s}$ and $\prod_{q=1}^{S} \left(\frac{(1-\eta_s) P_{si} Q_{si}}{M_{qsi}}\right)^{\lambda_{qs}}$, both relative to the industry average.

industry average respectively.[53] While the marginal revenue product of primary inputs has a very dispersed distribution with a variance of 0.525, that of intermediate inputs has a variance of merely 0.029. On the other hand, panel B of Figure 4 shows a much larger dispersion of the marginal revenue products of intermediate inputs in India, with the variance reaching 0.267. Though the variance for the marginal revenue product of the primary factors for India is close to that for China. This helps explain why the output measure differs substantially from the value-added measure in India.

Proposition 6 also suggests that a negative covariance between the primary input and intermediate input distortions would lead to a lower value-added TFP gain. [54] We confirm that a negative covariance between the primary input and intermediate input distortions does help explain the much higher output TFP gain in India. The covariance is -0.131 for India and only -0.023 for China. Table 2 reports the variance-covariance matrix of the marginal revenue products for both countries.

[53] They are given by $\left(\frac{\alpha_s \eta_s P_{si} Q_{si}}{K_{si}}\right)^{\alpha_s} \left(\frac{(1-\alpha_s)\eta_s P_{si} Q_{si}}{L_{si}}\right)^{1-\alpha_s}$ and $\prod_{q=1}^{S} \left(\frac{(1-\eta_s) P_{si} Q_{si}}{M_{qsi}}\right)^{\lambda_{qs}}$.

[54] Why does a negative covariance lead to a lower value-added TFP gain? The reason lies in the bias in inferred firm productivity and wedges using value-added measures. First, a negative covariance implies that the inferred TFPR will have a smaller dispersion using value-added measures, given the primary input distortions in the GO model. Second, for firms facing larger distortions in primary markets, we will also infer a lower value-added productivity. This means firms that are highly constrained are not particularly productive in terms of measured value-added productivity. Both biases result in a lower TFP gain in the VA model.

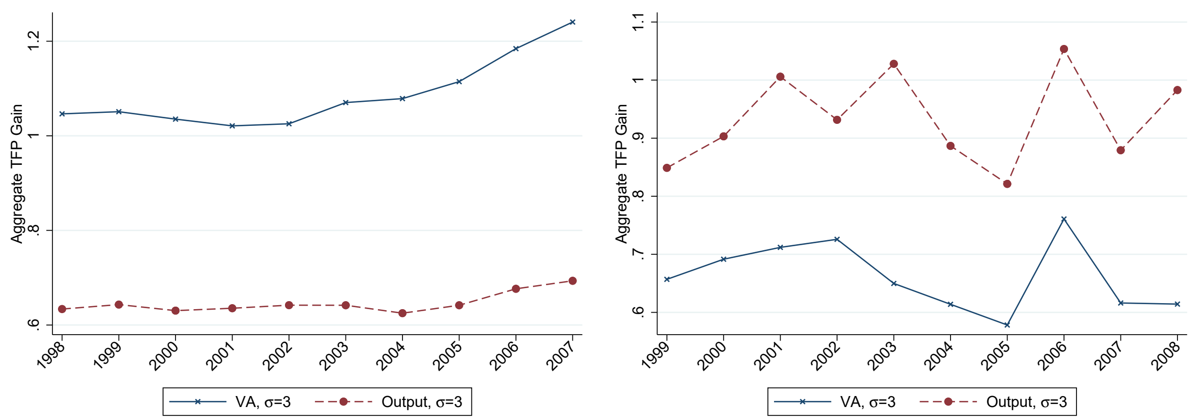
Table 2: Variance-Covariance Matrix of Marginal Revenue Products

	China		India	
	Primary	Intermediate	Primary	Intermediate
Primary	0.525		0.511	
Intermediate	-0.023	0.029	-0.131	0.267

Note: Marginal revenue products are measured as $MRPK_{si}^{\alpha_s} MRPL_{si}^{1-\alpha_s}$ and $\prod_{q=1}^{S} MRPM_{qsi}^{\lambda_{qs}}$ (in logs), both relative to the industry average.

5.3.3 Choice of the Elasticity of Substitution

Figure 5: Aggregate Efficiency Loss in Different Models



(a) China (b) India

Note: Efficiency losses are measured as $\left(\frac{Y^E}{Y}\right)^o - 1$ and $\left(\frac{Y^E}{Y}\right)^{va} - 1$, with $\sigma = 3$ in both models.

HK calibrate the elasticity of substitution with an estimate obtained using gross output data (Broda and Weinstein, 2006; Hendel and Nevo, 2006). The number they choose is thus more suitable for the GO model. If we take 3 as the true elasticity of substitution for the GO model, their calibration procedure suggests an upward bias in the measured cost of misallocation due to the choice of a higher than intended elasticity. This is because the TFP gain is increasing in the elasticity of substitution and the VA model should have a lower elasticity of substitution. Given the average intermediate share of output is roughly 0.5, the value-added elasticity should be around 2 instead of 3. We evaluate this upward bias by assuming that the elasticity is 3 in both models. As is shown in Figure 5, the difference in aggregate TFP gain between the two models is around 40 percentage points over the sample years for China. At the industry level, the measured TFP gain using the VA model is larger for over 95% of industries. For India, the difference between the two models is substantially narrowed by almost 90 percentage points, even though the VA model still underestimates the TFP gain compared to the GO model. This finding suggests that the upward bias in the value-added TFP gain measure due to incorrectly assigned elasticity of substitution is not trivial.[55] The very different results between China and India however shows that it is unclear whether the bias would bring the value-added estimates closer or further away from the output estimates.

6 Conclusion

This paper advances the study of the macroeconomic costs of resource misallocation by incorporating sectoral IO linkages in a gross output model and identifying the conditions under which the existing estimates based on a value added model will be unbiased. We show that in the absence of distortions in the use of intermediate inputs, the two frameworks are isomorphic to each other. Specifically, if there is no distortion in firms' uses of intermediate inputs, a model with a production network can be transformed into the value added model, with the two models suggesting the same TFP gain from the removal of all distortions.

However, when there exist firm-specific intermediate input distortions, the value added model is incorrectly specified and using value added measures may bias inferences of firm productivity and distortions. However, this bias may go in either direction, implying that the widely-used VA framework of HK does not necessarily understate nor overstate the cost of resource misallocation. Furthermore, we show that the literature using the value added model might also calibrate the elasticity of substitution to a higher value than is suitable for the gross output model, offering a potential source of upward bias in the estimated TFP losses from resource misallocation based on the value added model.

The findings in this paper support the idea that sectors can be viewed either as categories of final expenditure or value-added. Both models can be correct representations of the same underlying

[55]Note this conclusion does not rely on the elasticity to be 3. The elasticity is smaller in the VA model for any value larger than 1. Thus as long as we mistake the output elasticity for the value-added elasticity, there is an upward bias in the value-added measure.

data. The findings also suggest that the models can be easily mis-specified or calibrated if we fail to distinguish the two perspectives explicitly, a point also emphasized in Herrendorf et al. (2013). The recent surge in the study of production networks provides many insights regarding how the effect of productivity shocks and market wedges can be amplified by IO linkages. Our study shows that the measured aggregate effects of resource misallocation across firms might be similar between the gross output model with production networks and the value added model without, as the amplification effect via sectoral input-output linkages is already embedded in the measures of value added. The presence of intermediate input distortions, however, may bias the value added estimates in an unpredictable direction and with an unknown magnitude. We suggest the use of the GO model, which can be easily implemented, especially in the studies of evaluating the aggregate efficiency loss from resource misallocation following HK.

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Appendix

A. Proofs

Lemma 1 It is easy to show as firm's demand for production factors is proportional to sectoral factor supply, with the proportionality determined by firm physical productivity and wedges. It then follows that the sectoral production function inherits the Cobb-Douglas form of the firm production function and we only have to find the expression for sectoral TFP. We write the sectoral production function as follows

$$Q_s = TFP_s \cdot \left(K_s^{\alpha_s} L_s^{1-\alpha_s}\right)^{\eta_s} \left(\prod_{q=1}^{S} M_{qs}^{\lambda_{qs}}\right)^{1-\eta_s}.$$

On the other hand, we define sectoral value productivity as

$$TFPR_s \equiv \frac{P_s Q_s}{\left(K_s^{\alpha_s} L_s^{1-\alpha_s}\right)^{\eta_s} \left(\prod_{q=1}^{S} M_{qs}^{\lambda_{qs}}\right)^{1-\eta_s}},$$

Combining these two definitions, we have

$$TFP_s \equiv \frac{Q_s}{\left(K_s^{\alpha_s} L_s^{1-\alpha_s}\right)^{\eta_s} \left(\prod_{q=1}^{S} M_{qs}^{\lambda_{qs}}\right)^{1-\eta_s}} = \frac{TFPR_s}{P_s}.$$

Plugging in CES price aggregator P_s and similarly noticing the prices of varieties is given as $P_{si} = \frac{TFPR_{si}}{A_{si}}$, we have the results presented in Lemma 1.

Proposition 1 With the Domar weights, we can solve for the sectoral intermediate input demand as,

$$M_{qs} = \frac{1}{\mathcal{T}_{Ms}} \lambda_{qs}(1-\eta_s) \frac{P_s Q_s}{P_q} = \frac{1}{\mathcal{T}_{Ms}} b_{qs} \frac{v_s}{v_q} Q_q.$$

Plugging the allocation of production factors described above and in the text into the sectoral production function and taking logs, we have sectoral production function

$$Q_s = TFP_s \cdot \left((\beta_{Ks} K)^{\alpha_s} (\beta_{Ls} L)^{1-\alpha_s}\right)^{\eta_s} \left[\prod_{q=1}^{S} \left(\frac{1}{\mathcal{T}_{Ms}} b_{qs} \frac{v_s}{v_q} Q_q\right)^{\lambda_{qs}}\right]^{1-\eta_s}$$

Taking logs of the sectoral production functions and stacking them in vector form,

$$\bar{q} = \bar{a} + \omega_q + \delta_K \log K + \delta_L \log L + B'\bar{q},$$

where $\bar{q}$ is a vector of $\log Q_s$, δ_K a vector of $\alpha_s \eta_s$, δ_L a vector of $(1-\alpha_s)\eta_s$, and ω_q is a vector of the allocation terms whose s^{th} element is given by $\alpha_s \eta_s \log \beta_{Ks} + (1-\alpha_s)\eta_s \log \beta_{Ls} + (1-\eta_s)\sum_{q=1}^{S} \lambda_{qs} \log\left(\frac{1}{\mathcal{T}_{Ms}} b_{qs} \frac{v_s}{v_q}\right)$. The vector of log output is solved as

$$\bar{q} = (I - B'^{-1}(\bar{a} + \omega_q + \delta_K \log K + \delta_L \log L)$$

Notice that we can write $C_s = \frac{\theta_s Q_s}{v_s}$. Taking logs and stacking it into a vector, we have

$$\bar{c} = \omega_c + \bar{q},$$

where $\bar{c}$ is the vector of $\log C_s$ and ω_c a vector of $\log \frac{\theta_s}{v_s}$. Using the final good production function, we have,

$$\log Y = \theta' \bar{c} = \theta'[\omega_c + (I - B'^{-1}(\bar{a} + \omega_q + \delta_K \log K + \delta_L \log L)]$$

which leads to a Cobb-Douglas aggregate production function with the factor shares for capital and labor are given by $\theta'^{-1}\delta_K$ and $\theta'^{-1}\delta_L$. Since the value-added shares $\delta_K + \delta_L$ are given by $(I - B')\mathbf{1}$ where $\mathbf{1}$ is the vector of ones. It is easy to show that $\theta'(I - B'^{-1}(\delta_K + \delta_L) = 1$, such that the aggregate production function has constant returns to scale.

Proposition 3 Although the model has a complicated structure, it turns out we can calibrate the parameters in a step-by-step procedure. First note that equating the allocation of capital within a sector in the two models requires the following condition,

$$\frac{\frac{P_{Ysi} Y_{si}}{1+\tau_{Ksi}}}{\sum_{q=1}^{S} \frac{P_{Yqi} Y_{qi}}{1+\tau_{Kqi}}} = \frac{K_{si}}{K_s} = \frac{\frac{P_{si} Q_{si}}{1+\tau_{Ksi}}}{\sum_{q=1}^{S} \frac{P_{qi} Q_{qi}}{1+\tau_{Kqi}}}$$

Obviously, this condition holds only if $P_{Ysi}Y_{si} \propto P_{si}Q_{si}$. From the definition of firm value-added and making use of the optimality condition for intermediate input use, we have

$$P_{Ysi}Y_{si} = \left(1 - \frac{1-\eta}{1+\tau_{Msi}}\right) P_{si}Q_{si}$$

Firm value-added will be proportional to firm output only if all firms within a sector face the same distortion τ_{Msi}. For the moment, denote the distortion for all firms within a sector τ_{Ms}.

From the pricing rule of firm products, we can relate firm's nominal output to productivity and inputs in the two models as follows:

$$\begin{aligned} P_{si}Q_{si} &\propto \left(A_{si}\left(K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}\right)^{\eta_s}\left(\prod_{q=1}^{S} M_{qsi}^{\lambda_{qs}}\right)^{1-\eta_s}\right)^{\frac{\sigma_s-1}{\sigma_s}}, \\ P_{Ysi}Y_{si} &\propto \left(\widehat{A}_{si}K_{si}^{\widehat{\alpha}_s}L_{si}^{1-\widehat{\alpha}_s}\right)^{\frac{\widehat{\sigma}_s-1}{\widehat{\sigma}_s}}, \end{aligned}$$

where the term in the brackets on the right hand side is the firm-level production function in the respective model. We have established that firm value-added should be proportional to gross output. Combined with the above conditions we reach a proportional relationship between firm production in the two models. Intermediate inputs show up in the production of gross output but not value-added. We replace intermediate inputs using the optimality condition for intermediate input use. For the VA model to be consistent with data, the following conditions must hold,

$$\left(A_{si}K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}\right)^{\frac{\sigma_s\eta_s}{\sigma_s\eta_s+(1-\eta_s)}} \propto \left(\widehat{A}_{si}K_{si}^{\widehat{\alpha}_s}L_{si}^{1-\widehat{\alpha}_s}\right)^{\frac{\widehat{\sigma}_s-1}{\widehat{\sigma}_s}}$$

From this condition, we establish that $\widehat{\alpha}_s = \alpha_s$ and $\widehat{\sigma}_s = 1 + \eta_s(\sigma_s - 1)$. Firm value-added productivity satisfies the condition, $\widehat{A}_{si} \propto A_{si}^{\frac{1}{\eta_s}}$.

We next look at the allocation at the sector level. Note that a sector's value-added to GDP ratio in the VA model is simply $\widehat{\theta}_s$. In the GO model, it equals to the product of the Domar weight and the value-added to output ratio. The sectoral weight on final good production in the VA model can be calibrated as

$$\widehat{\theta}_s = \left(1 - \frac{1-\eta_s}{1+\tau_{Ms}}\right) v_s,$$

where τ_{Ms} is the constant intermediate input distortion for firms within the sector. The allocation of capital across sectors can be equated across the two models as long as

$$\frac{\frac{1}{\mathcal{T}_{Ks}}\alpha_s \eta_s v_s}{\sum_{q=1}^{S}\frac{1}{\mathcal{T}_{Kq}}\alpha_q \eta_q v_q} = \frac{K_s}{K} = \frac{\frac{1}{\widehat{\mathcal{T}}_{Ks}}\widehat{\alpha}_s \widehat{\theta}_s}{\sum_{q=1}^{S}\frac{1}{\widehat{\mathcal{T}}_{Kq}}\widehat{\alpha}_q \widehat{\theta}_q}$$

Since firm value-added is proportional to firm gross output, the sectoral level wedges are equated across the two models, $\mathcal{T}_{Ks} = \widehat{\mathcal{T}}_{Ks}$. Together with $\alpha_s = \widehat{\alpha}_s$, as we have shown, the above condition holds only if $\widehat{\theta}_s = \eta_s v_s$, which requires $\tau_{Ms} = 0$.

Proposition 6 The proof involves computing sectoral TFP and efficient TFP as the mean of a log-normally distributed random variable. We show how to do this for the more complicated sectoral value-added TFP. Note that if $\eta_s = 0.5$, we can approximate $1 - \frac{1-\eta_s}{1+\tau_{Msi}} = \frac{1}{2}(1+\tau_{Msi})$. Plugging this into sectoral value-added TFP, we have

$$\widehat{TFP}_s = (N_s x_s)^{\frac{1}{\eta_s(\sigma_s - 1)}},$$

where

$$x_s = \frac{1}{N_s}\sum_{i=1}^{N_s}\left(\frac{A_{si}}{(1+\tau_{Ysi})^{\eta_s}(1+\tau_{Msi})^{(1-\eta_s)-\frac{1}{\sigma_s-1}}}\right)^{\sigma_s - 1}, 1+\tau_{Ysi} = (1+\tau_{Ksi})^{\alpha_s}(1+\tau_{Lsi})^{1-\alpha_s}$$

Given the log-normal distribution of firm productivity and wedges and making use of the properties of normal distributions, we have $\log \frac{A_{si}}{(1+\tau_{Ysi})^{\eta_s}(1+\tau_{Msi})^{(1-\eta_s)-\frac{1}{\sigma_s-1}}}$ follows a normal distribution with the variance is given by[56]

$$\sigma_a^2 + \frac{1}{4}\sigma_y^2 + \left(\frac{1}{2} - \frac{1}{\sigma_s - 1}\right)\sigma_m^2 - \sigma_{ay} - \left(1 - \frac{2}{\sigma_s - 1}\right)\sigma_{am} + \left(\frac{1}{2} - \frac{1}{\sigma_s - 1}\right)\sigma_{ym}$$

[56] The mean of this random variable is given by the mean of $\log A_{si}$. The exact value of the mean does not matter as it cancels out in the computation.

This means $\left(\frac{A_{si}}{(1+\tau_{Ysi})^{\eta_s}(1+\tau_{Msi})^{(1-\eta_s)-\frac{1}{\sigma_s-1}}}\right)^{\sigma_s-1}$ also follows a log-normal distribution. We can then approximate x_s as the mean of the log-normal distribution. Repeating this procedure for other TFP measures. We have the sectoral TFP gains given in the text.

B. Data Appendix

B.1 Data Cleaning

In the Chinese industrial enterprise data from the country's National Bureau of Statistics (NBS), we first drop all observations with input or output data missing or having negative values. We also drop firms employing less than 8 workers. After that, we trim the 1% tails of marginal revenue products and TFPQ in each year, relative to the industry mean. Consistently, in the Indian enterprise data from the Annual Survey of Industries (ASI), we first drop all observations with input or output data missing or having negative values. After that, we trim the 1% tails of marginal revenue products and TFPQ in each year, relative to the industry mean.

Table 3 reports the number of plants used to compute the aggregate TFP loss for each country and year.

Table 3: Number of Plants

China		India	
Year	# of plants	Year	# of plants
1998	96,296	1999	10,376
1999	104,520	2000	11,757
2000	102,522	2001	14,995
2001	116,864	2002	16,575
2002	125,264	2003	17,057
2003	142,290	2004	22,936
2004	199,657	2005	19,864
2005	203,003	2006	21,920
2006	228,365	2007	22,421
2007	258,486	2008	20,087

B.2 Additional Results

Table 1: Dispersion of TFPQ and TFPR: China

	TFPQ						TFPR					
	Output			Value-added			Output			Value-added		
Year	S.D.	90-10	75-25	S.D.	90-10	75-25	S.D.	90-10	75-25	S.D.	90-10	75-25
1998	0.476	1.240	0.659	1.718	4.409	2.333	0.349	0.916	0.486	0.742	1.952	1.024
1999	0.485	1.264	0.677	1.789	4.634	2.447	0.348	0.914	0.489	0.742	1.950	1.043
2000	0.484	1.266	0.679	1.790	4.640	2.490	0.341	0.896	0.476	0.733	1.931	1.022
2001	0.478	1.249	0.681	1.772	4.600	2.490	0.341	0.894	0.474	0.727	1.912	1.021
2002	0.482	1.268	0.685	1.800	4.664	2.542	0.339	0.885	0.471	0.723	1.895	1.004
2003	0.484	1.272	0.695	1.819	4.748	2.573	0.336	0.882	0.463	0.725	1.909	1.007
2004	0.461	1.217	0.654	1.748	4.578	2.466	0.321	0.840	0.444	0.709	1.864	0.990
2005	0.474	1.258	0.675	1.815	4.750	2.576	0.325	0.852	0.453	0.723	1.904	1.013
2006	0.488	1.296	0.702	1.833	4.839	2.593	0.331	0.871	0.464	0.730	1.926	1.027
2007	0.495	1.311	0.716	1.857	4.906	2.646	0.334	0.876	0.467	0.740	1.948	1.048

Notes: TFPR is defined as $\frac{P_{si}Q_{si}}{\left(K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}\right)^{\eta_s}\left(\prod_{q=1}^{S}M_{qsi}^{\lambda_{qs}}\right)^{1-\eta_s}}$ and $\frac{P_{Ysi}Y_{si}}{K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}}$ for the output and VA model respectively. TFPQ is defined as $\frac{Q_{si}}{\left(K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}\right)^{\eta_s}\left(\prod_{q=1}^{S}M_{qsi}^{\lambda_{qs}}\right)^{1-\eta_s}}$ and $\frac{Y_{si}}{K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}}$ for the output and VA model respectively. Reported statistics are computed for the log of firm values relative to industry means. S.D. is standard deviation. 90-10 is the difference between 90th and 10th percentile, and 75-25 is that between 75th and 25th percentile. Statistics are computed for each industries and then summed up with industries weighted by their output shares and value-added shares.

Table 2: Dispersion of TFPQ and TFPR: India

	TFPQ						TFPR					
	Output			Value-added			Output			Value-added		
Year	S.D.	90-10	75-25	S.D.	90-10	75-25	S.D.	90-10	75-25	S.D.	90-10	75-25
1999	0.620	1.617	0.934	1.387	3.660	2.171	0.338	0.860	0.471	0.636	1.638	0.898
2000	0.630	1.659	0.948	1.391	3.698	2.151	0.342	0.876	0.479	0.632	1.617	0.861
2001	0.623	1.625	0.936	1.388	3.690	2.080	0.348	0.895	0.461	0.633	1.653	0.855
2002	0.566	1.498	0.820	1.270	3.410	1.815	0.358	0.905	0.496	0.638	1.700	0.862
2003	0.579	1.525	0.794	1.311	3.465	1.921	0.345	0.854	0.469	0.610	1.637	0.819
2004	0.585	1.550	0.802	1.329	3.565	1.729	0.353	0.885	0.463	0.613	1.593	0.855
2005	0.557	1.477	0.802	1.365	3.720	1.987	0.323	0.834	0.442	0.576	1.553	0.770
2006	0.578	1.586	0.820	1.331	3.397	2.012	0.338	0.892	0.460	0.623	1.702	0.714
2007	0.550	1.411	0.810	1.300	3.336	1.998	0.336	0.876	0.465	0.613	1.512	0.824
2008	0.610	1.618	0.921	1.375	3.556	2.132	0.356	0.959	0.472	0.622	1.696	0.780

Notes: TFPR is defined as $\frac{P_{si}Q_{si}}{\left(K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}\right)^{\eta_s}\left(\prod_{q=1}^{S}M_{qsi}^{\lambda_{qs}}\right)^{1-\eta_s}}$ and $\frac{P_{Ysi}Y_{si}}{K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}}$ for the GO and VA model respectively. TFPQ is defined as $\frac{Q_{si}}{\left(K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}\right)^{\eta_s}\left(\prod_{q=1}^{S}M_{qsi}^{\lambda_{qs}}\right)^{1-\eta_s}}$ and $\frac{Y_{si}}{K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}}$ for the GO and VA model respectively. Reported statistics are computed for the log of firm values relative to industry means. S.D. is standard deviation. 90-10 is the difference between 90th and 10th percentile, and 75-25 is that between 75th and 25th percentile. Statistics are computed for each industries and then summed up with industries weighted by their output shares and value-added shares.

Table 3: Aggregate TFP Gain

China					India				
Year	Output	VA	Var(τ_M)	Cov(τ_M,τ_Y)	Year	Output	VA	Var(τ_M)	Cov(τ_M,τ_Y)
1998	109.0	104.6	0.039	-0.031	1999	157.9	65.7	0.260	-0.110
1999	111.0	105.1	0.033	-0.027	2000	164.0	69.2	0.297	-0.152
2000	107.5	103.5	0.031	-0.023	2001	194.9	71.2	0.296	-0.126
2001	107.1	102.1	0.037	-0.025	2002	199.3	72.6	0.336	-0.163
2002	117.5	102.5	0.029	-0.023	2003	180.3	65.0	0.291	-0.115
2003	107.5	107.0	0.036	-0.021	2004	175.4	61.4	0.299	-0.118
2004	104.1	107.9	0.037	-0.026	2005	149.0	57.8	0.255	-0.121
2005	108.4	111.5	0.031	-0.022	2006	224.7	76.1	0.226	-0.076
2006	114.5	118.4	0.031	-0.022	2007	158.3	61.6	0.246	-0.101
2007	117.0	124.1	0.033	-0.023	2008	196.0	61.4	0.328	-0.161

Note: Output and VA indicate aggregate TFP gains computed as $100 \times \left(\frac{Y^E}{Y}\right)^{o} - 100$ and $100 \times \left(\frac{Y^E}{Y}\right)^{va} - 100$. Var($\tau_M$) is the variance of the variance of the marginal revenue products of intermediate inputs (in logs relative to industry means). Cov(τ_M,τ_Y) is the covariance between the variance of the marginal revenue products of intermediate inputs and primary inputs (in logs relative to industry means). The two statistics are averaged over industries with industry output share as weights.