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OFFSHORING AND INFLATION

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ABSTRACT

Did trade integration suppress inflation in the United States? Conventional wisdom says “yes,” based on the disinflationary supply-side impacts of trade. We argue that these supply-side arguments are incomplete, because trade integration also influences aggregate demand. Our analysis leverages two facts: trade integration was a long-lasting, phased-in shock, and offshoring accounts for a large share of it. Given these facts, we show trade integration is inflationary in conventional New Keynesian models. This result continues to hold when we account for US trade deficits, the pro-competitive effects of trade on domestic markups, and cross-sector heterogeneity in trade integration.

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Recent decades have seen large increases in trade integration, due to liberalization of trade policies and technological advances. At the same time, consumer and producer price inflation has been low in most advanced countries, with many countries persistently undershooting their inflation targets. The coincidence of these developments has led many to conclude that trade integration has suppressed inflation. This view has been embraced by central bankers; for example, [Carney \(2019\)](#) succinctly states that “the integration of low-cost producers into the global economy has imparted a steady disinflationary bias.”¹ This conventional wisdom has been buttressed by evidence that higher import penetration leads to lower price inflation at the industry level [[Auer and Fischer \(2010\)](#); [Bai and Stumpner \(2019\)](#); [Jaravel and Sager \(2019\)](#)]. It also draws on research and policy discussions about globalization-induced changes in the Phillips Curve, whether manifesting as deflationary cost-push shocks [[Carney \(2017\)](#)], or decreased sensitivity of inflation to domestic slack [[Forbes \(2019\)](#); [Stock and Watson \(2021\)](#)]. More recently, attention has shifted toward parsing how unwinding international integration, triggered by recent trade wars and other de-globalizing forces, may let the inflation genie back out of the bottle [[Moyo \(2017\)](#); [Wolf \(2020\)](#)].

In this paper, we revisit this conventional narrative about globalization and inflation. While parts of it ring true, its exclusive emphasis on supply-side channels omits important demand-side effects of trade integration. In particular, we argue that trade integration is best thought of as a long-lived (arguably permanent) shock, with long phase-in dynamics. With forward-looking consumers, anticipated increases in trade raise aggregate demand prior to their realization, which drives equilibrium inflation up in standard New Keynesian models. Further, this mechanism is particularly strong when trade integration takes the form of increased “offshoring” – i.e., increased use of foreign inputs in production, as occurred in recent decades. This is surprising on its face; because offshoring reduces unit production costs for domestic producers, it is commonly thought to restrain inflation. In another surprising twist, pro-competitive effects of trade, whereby import competition lowers markups set by domestic producers, also raise the inflationary impact of trade integration. Finally, while a multisector version of the model can rationalize facts about prices commonly cited to argue that trade restrains inflation, it too implies that trade integration is inflationary overall. All together, these findings argue for rewriting the conventional narrative.

To orient the discussion, we start by developing a model-based accounting framework that

¹Carney is far from alone in this opinion. [Greenspan \(2005\)](#) remarks: “cross-border trade in recent decades has been expanding at a far faster pace than GDP...The consequent significant additions to world production and trade have clearly put downward pressure on prices in the United States...Over the past two decades, inflation has fallen notably, virtually worldwide...globalization...would appear to be [an] essential [element] of any paradigm capable of explaining the events of the past ten years.” [Yellen \(2006\)](#) states: “the IMF calculates that non-oil import price reductions lowered US inflation by an average of 1/2 percentage point a year over 1997 to 2005. These results are in line with those from...the Federal Reserve Board that estimates that lower (core) import prices have reduced core US inflation by an annual average of 1/2 to 1 percentage point over the last 10 years.” See also [IMF \(2006\)](#), [Bean \(2007\)](#), and [Bernanke \(2007\)](#).

captures two standard, supply-side channels via which trade may impact consumer prices, which reflect prominent arguments in the literature.² The first channel operates via trade in consumption goods. Falling prices for imported consumption goods, and substitution of imports for domestic goods, lower the consumer price level. The second channel reflects the use of imported inputs in production of domestic goods: falling prices for imported inputs reduce domestic production costs, which then bring down prices for domestically produced goods and services. Using data for the United States from the Bureau of Economic Analysis, we show that both these channels appear to be quantitatively important. Industries that are more exposed to offshoring have experienced lower long-run output price inflation. Further, increases in trade appear to lower consumer price inflation by 10-40 basis points per year, relative to growth in the domestic GDP deflator. Increases in offshoring are as important as increases in foreign sourcing of consumption goods in this result.

This initial exercise, which largely confirms prior views about the impact of trade on inflation, serves to set up two broader points. First, while the accounting framework focuses on supply-side impacts of trade on prices, trade integration may change aggregate demand as well. Any demand-side effects would alter factor prices, and thus sectoral value-added deflators, on which the accounting framework is silent. Second, inflation is a monetary phenomenon; endogenous model responses to trade shocks, combined with the conduct of monetary policy, determine inflation. Together, these observations imply that we need a model to provide a full analysis of inflation dynamics.

To that end, we develop an open economy new Keynesian (NK) framework with trade in both intermediate inputs and final goods, and we apply the framework to analyze the inflationary impacts of rising trade in the United States. The framework extends the canonical small open economy NK model [[Galí and Monacelli \(2005\)](#); [Galí \(2015\)](#)] to incorporate “offshoring” – the use of foreign intermediate inputs in production – in addition to trade in final goods.³ This extension is more than window dressing: increases in offshoring are large in the data, so any quantitative account of the impacts of trade on inflation must emphasize offshoring. In addition, we also apply the model in new ways to analyze the rise of trade. First, we use domestic sourcing shares as “sufficient statistics” to assess the impacts of trade in the model, borrowing from the international trade literature [[Costinot and Rodríguez-Clare \(2014\)](#)].⁴ This approach facilitates a concise treatment of the impacts of trade in a “three equation model,” in which dynamics of the domestic sourcing

²A third channel – pro-competitive effects of trade on markups of domestically produced goods – is implicitly captured in the framework, embedded in the price of domestic value added (i.e., the GDP deflator). We do not explicitly address this third channel in the accounting exercise, because doing so would require auxiliary model assumptions needed to back out markups from trade data. We do discuss the role of markups later in our model analysis, however – see Section 3.2.

³Recent related models include [Gopinath et al. \(2020\)](#) and [Auray, Devereux and Eyquem \(2020\)](#). [Amiti, Itskhoki and Konings \(2014\)](#) study the impact of imported inputs on pricing in a real model.

⁴[Baqae and Fahri \(2019\)](#) apply the sufficient statistics argument in a flexible price macro-model.

shares influence inflation via both the Phillips Curve and IS curve. Second, we analyze the impact of permanent, phased-in shocks to trade openness. This shock captures an essential feature of the data – globalization involved a shift in steady states, from a less open to more open world, which took place slowly over time.

Given the historical path of changes in US trade, we show the NK model predicts that the rise of trade during the late 1990's and early 2000's was inflationary, rather than deflationary. This result reflects the impacts of trade on the demand-side of the model. Specifically, anticipated increases in both offshoring and final goods trade shocks appear in the IS curve, where the time path for these shocks is embedded in the real natural rate of interest. A phased increase in offshoring – where domestic sourcing shares are falling over time – generates an increase in the real natural rate, which is itself inflationary. This dynamic response to the rise in offshoring over time drives current inflation up at the outset. On the supply side, only shocks to domestic sourcing of final goods shift the Phillips Curve; because changes in domestic sourcing of final goods were relatively small over this period, this channel is quantitatively modest. Together, these results suggest the dynamics of trade integration were inflationary on net, in contradiction of conventional wisdom. Further, we emphasize that this conclusion holds for virtually all Taylor-type monetary policy rules used in the literature. The only policy rule that would fully negate the inflationary impact of offshoring is one that exclusively targets the (time-varying) real natural rate, which is (in our view) an implausible description of real world policy.

We extend the model in three main ways to probe the robustness of our findings. First, we incorporate financial inflow shocks into the model to match changes in the trade balance over time, which allow the model to match the dynamics of the global savings glut in the early 2000's. We find that the widening of the US trade deficit in the early 2000's actually pushes inflation up, though the magnitude of this effect is modest; this effect works through general equilibrium effects of increasing trade deficits on the real natural rate, similar to anticipated increases in trade.

Second, we introduce variable markups in the model, by assuming that preferences and technologies take the [Kimball \(1995\)](#) form, as in [Gopinath et al. \(2020\)](#). This set up allows trade to have pro-competitive effects, whereby increases in foreign sourcing of final goods and inputs restrain markups set by domestic producers [[Feentra and Weinstein \(2017\)](#); [Jaravel and Sager \(2019\)](#)]. Despite these pro-competitive effects, we find again that trade integration increases inflation, even more than in the baseline model due to general equilibrium effects on the supply side of the economy by which lower markups raise domestic output. At the same time, our simulated data suggests that failure to control for changes in markups induced by trade may help explain the reduced sensitivity of inflation to the domestic output gap – i.e., the flattening of the empirical Phillips curve – over time.

Third, we build out the model to include multiple sectors, with heterogeneous sourcing dy-

namics across sectors and end uses. In the multisector model, we again find trade integration is inflationary. Further, we show the multisector model yields a decline in the relative price of manufacturing output, as observed in the data. Rising trade also appears – in an accounting sense – to restrain inflation in the model, as in our initial exploration of the data. These results emphasize that one cannot draw conclusions about the impact of trade on inflation from studies that link cross-sectional changes in output prices to changes in trade, or studies that decompose prices into components attributable to domestic and import prices. Put differently, data alone is surprisingly uninformative about the role of rising trade in explaining inflation.

As noted at the outset, our work is related to a large prior literature on globalization and inflation. Early work by [Romer \(1993\)](#) and [Rogoff \(2003, 2007\)](#) emphasized the role of trade in modifying the inflation-output tradeoff faced by central banks, making surprise monetary easing less attractive and lowering trend inflation. In contrast, our model features a credible, rule-based policy environment, in which long run inflation rates are anchored at zero; we then study medium run inflation outcomes as the economy transitions from a relatively closed to more open trade equilibrium.⁵ Following on from this early work, many subsequent contributions study supply-side mechanisms through which trade influences inflation dynamics, including how trade modifies the slope of the Phillips Curve, the role of “global slack” in inflation dynamics, and inflation synchronization across countries.⁶ We contribute to this discussion by emphasizing the (lack of any) role for offshoring as a supply-side driver of inflation in baseline NK models, and we re-direct attention to the (largely overlooked) role of trade dynamics on the demand side. We also provide an analysis of how the pro-competitive effects of trade influence the Phillips Curve in a version of the model with variable markups.⁷

In some important respects, our results are also related to the literature on “news shocks” in New Keynesian models. In particular, it is well understood that productivity news shocks – anticipated increases in future productivity – are inflationary in the conventional New Keynesian model [[Barsky and Sims \(2011\)](#); [Sims \(2012\)](#); [Barsky, Basu and Lee \(2015\)](#)]. Our observation is that an anticipated increase in trade has an effect similar to productivity news – it implies higher future consumption for the representative agent, which drives up the real neutral interest rate and inflation.⁸ Thus, while our finding that trade integration is inflationary runs counter to conventional

⁵We take the policy rule as given, though we discuss alternative policy rules in Section 2.4.3. See [Monacelli \(2013\)](#) for a discussion of optimal monetary policy with trade in final goods and inputs, and [Wei and Xie \(2020\)](#) for related analysis in a model with multistage production.

⁶Among others, see [Bianchi and Civelli \(2015\)](#), [Benigno and Faia \(2016\)](#), [Auer, Borio and Filardo \(2017\)](#), [Auer, Levchenko and Saurè \(2019\)](#), [ECB \(2017\)](#), [Forbes \(2019\)](#), and references therein.

⁷[Sbordone \(2007\)](#) studies the impact an increase in varieties available to consumers (akin to an expansion in market size) on the slope of the Phillips Curve in a model with Kimball demand. We instead analyze an increase in import penetration, which shifts the Phillips Curve. Further, we study inflation along the transition path from a relatively closed to more open economy, whereas Sbordone compares Philips Curves in alternative steady states.

⁸In contemporaneous work, [Alessandria and Mix \(2019, 2021\)](#) also investigate the role of anticipated changes in

wisdom, it is closely related to prior results in the monetary literature. However, it is also important to note that trade integration has more nuanced effects on inflation than do productivity news shocks. Whereas a change in future productivity only impacts inflation through aggregate demand channels, changes in trade also have direct effects that work through the Phillips Curve on the supply side. In our baseline model, the dynamics of final goods sourcing matter for consumer price inflation directly; in our augmented model with variable markups, changes in both final goods and input sourcing also influence the path of markups over time, which directly shift the Phillips Curve. Thus, our analysis of trade shocks goes beyond prior analysis of productivity news shocks.

Closely related to this discussion, increases in trade are anticipated by agents in our model analysis. This reflects the basic idea that trade integration entails the transition from a relatively closed initial equilibrium to one with permanently higher openness, where that transition takes time. For example, [Baier and Bergstrand \(2007\)](#) show that increases in trade are phased-in slowly after adoption of regional trade agreements. Further, estimates of long-run elasticities of trade exceed short-run elasticities [see Fact #7 in [Alessandria, Arkolakis and Ruhl \(forthcoming\)](#) or [Boehm, Levchenko and Pandalai-Nayar \(2021\)](#)]. This body of work strongly suggests that, following reductions in trade barriers, one could easily forecast a phased increase in trade. In this vein, [Bombardini, Li and Trebbi \(2020\)](#) use Congressional voting records to show that US politicians were able to correctly predict the impacts of the China shock prior to its realization. Given the major changes in policy and the international economic environment more generally in the 1990's – e.g., the rise of China and other emerging market economies, the phase-in of Uruguay Round tariff cuts, the expansion in US regional trade agreements, and the ICT revolution – we think it quite reasonable to believe that people generally understood the world would become more globalized over time. We build this idea into the model in a simple, tractable way, via a perfect foresight assumption, in order to focus on studying the implications that follow from it.

Lastly, our work is also broadly informed by recent work on trade dynamics. We adopt a perfect foresight approach in analyzing trade shocks, similar to [Eaton et al. \(2011\)](#), [Reyes-Heroles \(2016\)](#), [Kehoe, Ruhl and Steinberg \(2018\)](#), and [Ravikumar, Santacreu and Sposi \(2019\)](#). Whereas these papers focus on real outcomes following trade shocks, we study a monetary economy. The framework in our paper can also be applied to analysis of the inflationary impacts of trade policy, which connects to recent papers on trade policy and macro outcomes [[Erceg, Prestipino and Raffo \(2018\)](#); [Barbiero et al. \(2018\)](#); [Barattieri, Cacciatore and Ghironi \(2019\)](#)]. Finally, our paper is also related to [Rodríguez-Clare, Ulate and Vasquez \(2020\)](#), which examines the impacts of the China shock on employment and unemployment across local labor markets in a model with downward

trade barriers as news shocks, which is closely related to our approach. An important distinction relative to our paper is that [Alessandria and Mix](#) focus on the impacts of trade shocks in a real model, while we work with a New Keynesian model to study inflation dynamics. See [Beaudry and Portier \(2014\)](#) for further discussion of differences in how news shocks affect the economy in real versus monetary models.

nominal wage rigidity.

The remainder of the paper proceeds as follows. In Section 1, we develop an accounting framework to document supply-side channels linking trade and prices. We build these supply-side mechanisms into a baseline New Keynesian model in Section 2, in which we study inflation outcomes during trade liberalization episodes. We then consider extensions to the baseline model in Sections 3 and 4. In Section 3, we study the role of financial shocks that lead to trade imbalances, and we present results for a version of the model with variable markups. In Section 4, we develop a multisector model, in which we link our results back to the accounting framework in Section 1. Section 5 concludes.

1 An Account of Offshoring, Trade, and Inflation

In this section, we develop an accounting framework that ties the rise of trade to industry-level and aggregate prices. To begin, we characterize how industry-level prices for domestic output depend on offshoring and domestic factor costs. We then discuss how the aggregate consumer price level is linked to prices for domestic output versus imported final goods. Using data for the United States from 1997-2018, we decompose changes in the aggregate consumer price level into components attributable to changes in offshoring, domestic factor costs, and imported consumption goods.

Setting this exercise in context, this section emphasizes the supply-side links between trade and prices; we defer attention to demand-side general equilibrium effects until Section 2. The framework in this section shares its basic structure with the multisector model that we analyze in Section 4, where we implement a model-based accounting exercise that mimics this section.

1.1 Linking Offshoring to Domestic Output Prices

Consider a two country environment, with Home (H) and Foreign (F) countries, and many industries $s \in \{1, \dots, S\}$. Within each Home industry, there is a unit continuum of varieties, produced under monopolistic competition. These varieties are aggregated into composite goods, which are then used at Home as final or intermediate goods and exported. Each producer has a nested, constant elasticity of substitution (CES) production function. At the top level, they substitute between labor and a composite intermediate input, in a Cobb-Douglas production function. At the middle level, they substitute across inputs originating from different upstream sectors in forming the composite input, again with Cobb-Douglas aggregation. At the bottom level, they substitute between sector-level inputs coming from Home versus Foreign sources, which are aggregated via a CES function. Because this basic CES monopolistic competition structure is standard, we jump right to results.

Prices for each Home variety are a time-varying markup over marginal costs, and all producers are symmetric. Thus, Home sector-level output prices in sector s at date t are given by:

$$P_{Ht}(s) = \mu_t(s)MC_t(s) \quad (1)$$

$$\text{with } MC_t(s) = Z_t(s)^{-1}W_t^{1-\alpha(s)}P_{Mt}(s)^{\alpha(s)}, \quad (2)$$

where $\mu_t(s)$ and $MC_t(s)$ are time-varying markups and marginal costs in sector s .⁹ Marginal costs depend on productivity ($Z_t(s)$), the price of a composite primary factor (W_t), and the price of a sector-specific composite input ($P_{Mt}(s)$). The parameter $\alpha(s)$ is the Cobb-Douglas share for the composite input in total costs. The composite input price in turn is given by:

$$P_{Mt}(s) = \prod_{s' \in S} P_t(s', s)^{\alpha(s', s)/\alpha(s)} \quad (3)$$

$$\text{with } P_t(s', s) = \left[\xi(s', s)P_{Ht}(s')^{1-\eta(s')} + (1 - \xi(s', s))(\tau_{Mt}(s')P_{Ft}(s'))^{1-\eta(s')} \right]^{1/(1-\eta(s'))}, \quad (4)$$

where $P_t(s', s)$ is the composite price of inputs purchased from sector s' by sector s , $\alpha(s', s)/\alpha(s)$ is the share of inputs from s' in total input expenditure by sector s , $P_{Ft}(s')$ is the price of a composite bundle of foreign varieties, and $\tau_{Mt}(s')$ is an iceberg-type trade cost paid on imports of intermediate inputs.

Consider now the change in prices over time interval $[0, t]$. Further, for a given variable X_t , let $\hat{X}_t = X_t/X_0$ denote the ratio of its value in period t to its value in the base period. Then, the price of output in period t relative to baseline is:

$$\hat{P}_{Ht}(s) = \hat{\mu}_t(s)\widehat{MC}_t(s) = \hat{P}_{Vt}(s)^{1-\alpha(s)}\hat{P}_{Mt}(s)^{\alpha(s)}, \quad (5)$$

where $\hat{P}_{Vt}(s) = \hat{W}_t \left(\hat{\mu}_t(s)/\hat{Z}_t(s) \right)^{1/(1-\alpha(s))}$ is the price of real value added in sector s (equivalently, the sector-specific GDP price deflator). Changes in composite input prices are:

$$\hat{P}_{Mt}(s) = \prod_{s' \in S} \hat{P}_t(s', s)^{\alpha(s', s)/\alpha(s)} \quad (6)$$

$$\hat{P}_t(s', s) = \left[\Lambda_{H0}^M(s', s)\hat{P}_{Ht}(s')^{1-\eta(s')} + \Lambda_{F0}^M(s', s)\hat{P}_{Ft}(s')^{1-\eta(s')} \right]^{1/(1-\eta(s'))}, \quad (7)$$

where $\Lambda_{it}^M(s', s) = \frac{P_{it}(s')M_{it}(s', s)}{P_t(s', s)M_t(s', s)}$ is the share of input spending on inputs from country $i \in \{H, F\}$ and sector s' by Home sector s in total spending in inputs from s' by s by Home, and $\Lambda_{i0}^M(s', s)$ is the

⁹With flexible prices, the markup would be constant given the CES monopolistic competition structure as described. We allow for a time-varying markup $\mu_t(s)$ in Equation 1 to anticipate the full model with price adjustment frictions below, as well as an extension with variable demand elasticities.

base period value of these shares.

This exposition suggests that we could quantify the impact of offshoring on gross output prices using import price data. That is, we could measure price changes for foreign goods $\hat{P}_{Ft}(s')$, and feed them through Equations 5-7 to arrive at predicted changes in output prices. While this approach is straightforward in the model as written, it is not a practical route forward due to shortcomings in standard data sources; We discuss these shortcomings further in Appendix A.

Instead, we proceed here by invoking a sufficient-statistics argument. Using the usual first order conditions for the purchases of domestic inputs, we can write the expenditure share on domestic goods as: $\Lambda_{Ht}^M(s', s) = \frac{P_{Ht}(s')M_{Ht}(s', s)}{P_t(s', s)M_t(s', s)} = \xi(s', s) \left(\frac{P_{Ht}(s')}{P_t(s', s)} \right)^{1-\eta(s')}$. Taking ratios across time, we write $\hat{P}_t(s', s)$ as follows:

$$\hat{P}_t(s', s) = \hat{P}_{Ht}(s') \hat{\Lambda}_{Ht}^M(s', s)^{1/(\eta(s')-1)}. \quad (8)$$

This expression is an analog to the sufficient-statistics approach to counterfactual analysis of the gains from trade, advocated by [Arkolakis, Costinot and Rodríguez-Clare \(2012\)](#) and [Costinot and Rodríguez-Clare \(2014\)](#). Like [Blaum, Lelarge and Peters \(2018\)](#), we apply it on the production side of the economy. When the share of inputs sourced domestically falls ($\hat{\Lambda}_{Ht}^M(s', s) < 1$) – i.e., when offshoring rises – then the cost of the input bundle falls, as long as foreign inputs are gross substitutes for domestic inputs ($\eta(s') > 1$).

Combining Equations 5, 6, and 8, we can write:

$$\hat{P}_{Ht}(s) = \hat{P}_{Vt}(s)^{1-\alpha(s)} \left[\prod_{s' \in S} \hat{P}_{Ht}(s')^{\alpha(s', s)} \right] \left[\prod_{s' \in S} \hat{\Lambda}_{Ht}^M(s', s)^{\alpha(s', s)/(\eta(s')-1)} \right]. \quad (9)$$

The first term captures changes in total factor productivity, markups, or primary factor costs. The second term captures changes in the prices of domestically produced inputs purchased by s from all upstream sectors, including sector s itself. The third term captures the impact of offshoring on unit costs.

Taking logs of Equation 9, and using lower case to denote the log of an upper case variable (i.e., $\hat{x}_t = \ln \hat{X}_t$), yields:

$$\hat{p}_{Ht}(s) = (1 - \alpha(s))\hat{p}_{Vt}(s) + \sum_{s' \in S} \alpha(s', s)\hat{p}_{Ht}(s') + \sum_{s' \in S} \left(\frac{\alpha(s', s)}{\eta(s') - 1} \right) \hat{\lambda}_{Ht}^M(s', s), \quad (10)$$

This expression has input-output logic embedded in it, because the price of Home output in sector s depends on prices of output in all sectors at Home, including itself. The direct effect of a rise in foreign sourcing of inputs is to lower prices in sector s , and then this price reduction spills over across sectors, as sector s is used downstream as an input.

Stacking Equation 10 across sectors, we manipulate it to isolate domestic output prices:

$$\hat{\mathbf{p}}_{Ht} = [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{I} - \alpha] \hat{\mathbf{p}}_{Vt} + [\mathbf{I} - \mathbf{A}']^{-1} \left[\mathbf{A}' \circ \left(\hat{\lambda}_{Ht}^M \right)' \right] [\mathbf{H} - \mathbf{I}]^{-1} \iota, \quad (11)$$

where α is a matrix with $\alpha(s)$ along the diagonal and zeros elsewhere, \mathbf{A} is an input-output matrix with elements $\alpha(s, s')$, $\hat{\lambda}_{Ht}^M$ is a matrix with elements $\hat{\lambda}_{Ht}^M(s, s')$, and \mathbf{H} is a matrix with $\eta(s)$ along the diagonal. Further, ι is a conformable column vector of ones, and \circ denotes the Hadamard (entrywise) product of matrices. The first term is the downstream propagation of cost-push shocks to the price of real value added in all sectors. The second term is the downstream propagation of cost-push shocks attributable to offshoring, where an increase in domestic sourcing of inputs raises the price of domestic gross output. Intuitively, if inputs are increasingly sourced from home, we infer that the price of imported inputs is rising relative to the price of domestic inputs, which implies that gross output prices will grow faster than implied by domestic value-added costs alone.

1.2 Data on Offshoring and Output Prices

We now turn to studying the impact of offshoring on domestic producer prices through the lens to Equation 11. We draw on two complementary data sets from the Industry Economic Accounts of the US Bureau of Economic Analysis (BEA). The first is a data set on the price of gross output by industry, from the GDP-by-industry statistics.¹⁰ The second data set is the Input-Output Accounts, from which we construct annual input-output tables and domestic sourcing shares.¹¹ Both data sets include annual data for 1997-2018 for 71 summary-level industries, of which 26 are goods producing industries.

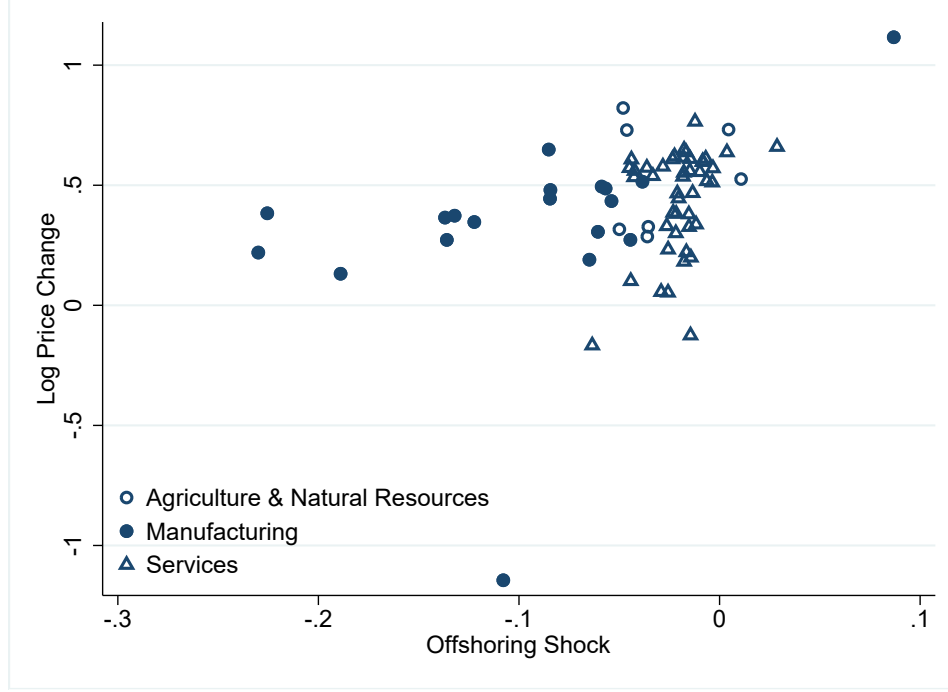
From these data, we construct data analogs to $\hat{\mathbf{p}}_{Ht}$, \mathbf{A} and $\hat{\lambda}_{Ht}^M$ in Equation 11. We measure $\hat{\mathbf{p}}_{Ht}$ using the log of the ratio of gross output price deflators in year t relative to 1997 in each industry. We take averages of the annual input-output matrices to form the time-invariant industry-to-industry input-output matrix: $\mathbf{A} \equiv (1/20) \sum_{t=1997}^{2018} \mathbf{A}_t$, where $\mathbf{A}_t = \mathbf{A}_{Ht} + \mathbf{A}_{Ft}$ is the total direct requirements matrix for year t , \mathbf{A}_{Ht} is the domestic requirements matrix, and \mathbf{A}_{Ft} is the import requirements matrix.¹² We then define domestic sourcing shares $\Lambda_{Ht}^M \equiv \mathbf{A}_{Ht} \oslash \mathbf{A}_t$, and we construct ratios $\hat{\Lambda}_{Ht}^M = \Lambda_{Ht}^M \oslash \Lambda_{H,1997}^M$, where \oslash represents Hadamard (elementwise) division.

¹⁰See [BEA Gross-Domestic-Product-\(GDP\)-by-Industry Data](#). These data are constructed primarily from PPI and CPI data collected by the BLS, and they are used by the BEA to deflate gross output and compute real GDP by industry. Neither CPI data, nor PPI data, have appropriate product coverage to match up to industry-level data sources, so the BEA combines various sources to match the industry definitions in the national accounts.

¹¹See [BEA Input-Output Accounts Data](#). We use make/use tables (after redefinitions, at producer value) and import use matrices (after redefinitions) to form industry-by-industry input-output tables split by domestic and import use.

¹²The (i, j) elements in these matrices are the value of inputs purchased by industry j from industry i as a share of total gross output in industry j . \mathbf{A}_{Ht} records purchases from domestic sources, while \mathbf{A}_{Ft} records purchases from international sources. Note that we suppress changes in this domestic requirements matrix over time in the model, and thus in our empirical analysis as well.

Figure 1: Industry-Level Price Changes vs. the Offshoring Shock from 1997-2018



Note: Agriculture and Natural Resources includes BEA summary-level industries 1-7, Manufacturing includes 8-26, and Services includes 27-71. Each observation is the log difference between sector-level prices in 2018 and 1997: $\hat{p}_{Ht}(s) = \ln p_{H,2018}(s) - \ln p_{H,1997}(s)$.

As a first pass, we assume that elasticities of substitution between home and foreign goods are equal in all sectors: $\eta(s) = \eta$. This implies that we can rewrite Equation 11 as:

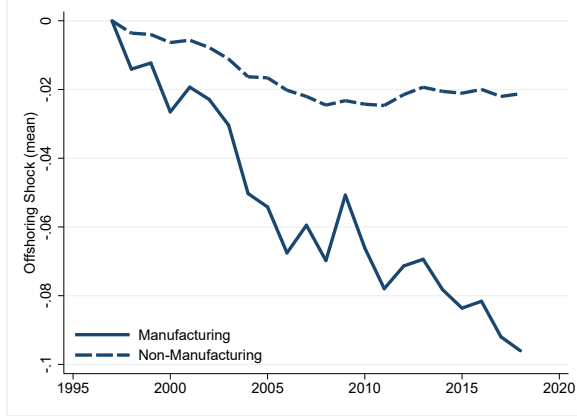
$$\hat{\mathbf{p}}_{Ht} = [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{I} - \alpha] \hat{\mathbf{p}}_{Vt} + \underbrace{\left(\frac{1}{\eta - 1} \right) [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{A}' \circ (\hat{\lambda}_{Ht}^M)']}_{\text{Offshoring Shock}} \boldsymbol{\iota}. \quad (12)$$

Each element of the term labeled Offshoring Shock is the cumulative impact of upstream offshoring on unit costs, accounting for both direct and indirect effects via the input-output structure. We can then relate price changes over a given horizon $\hat{\mathbf{p}}_{Ht}$ to the change in unit costs attributable to offshoring.

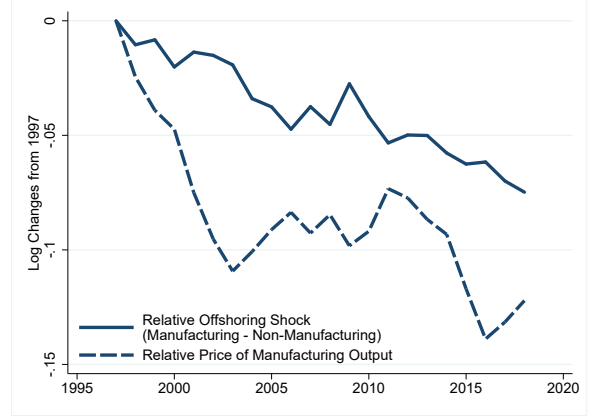
We plot long-run changes in $\hat{\mathbf{p}}_{Ht}$ versus the Offshoring Shock at the industry level in Figure 1. There is a positive correlation between the offshoring shock and price changes in the figure – industries with high exposure to offshoring, and thus the largest declines in direct and indirect home sourcing shares, experienced smaller output price changes. One point that is clear in the figure is that this correlation is driven by two aspects of the data: (a) there is a tight correlation within manufacturing between offshoring exposure and price changes, and (b) there are differences in the evolution of offshoring and prices across composite industry groups (e.g., manufacturing vs. agriculture, natural resources, and services). In contrast, there is only a weak relationship between the

Figure 2: Price Changes and the Offshoring Shock from 1997-2018

(a) Offshoring Shock in Manufacturing vs. Non-Manufacturing



(b) Relative Price and Offshoring Shock for Manufacturing vs. Non-Manufacturing



Note: The relative price of manufacturing in each year is $\frac{1}{|M|} \sum_{s \in M} \hat{p}_{Ht}(s) - \frac{1}{|N|} \sum_{s \in N} \hat{p}_{Ht}(s)$, where M and N denote the set of manufacturing and non-manufacturing industries, respectively. The relative offshoring shock is a similar difference in unweighted averages for manufacturing and non-manufacturing industries.

offshoring shock and price changes within non-manufacturing industries, in part because variation in offshoring changes within non-manufacturing industries are small.

To illustrate the time path of offshoring and price changes, we take simple averages of \hat{p}_{Ht} and the Offshoring Shock for manufacturing and non-manufacturing industries in each year. We plot the time series for these average offshoring shocks in manufacturing and non-manufacturing in Figure 3a. Manufacturing industries are impacted more than are Non-Manufacturing industries by offshoring, nearly five times as intensively. Turning to Figure 3b, we plot average log changes in the prices for manufactured manufacturing industries minus the same for non-manufacturing; More simply, this is an average relative price of manufacturing output. We also include the gap between the offshoring shock that hits the manufacturing and non-manufacturing sectors, which captures the relative offshoring shock across the two sectors. The relative price of manufacturing output declines over the period, coincident with the large relative offshoring shock that hit manufacturing industries.

Together, these data point to a role for offshoring in explaining output price changes over time. To put these in macro-context, we now turn to the evolution of consumer prices.

1.3 Accounting for Consumer Prices

The representative Home consumer has nested, constant elasticity of substitution (CES) preferences. We assume that she has Cobb-Douglas preferences across composite industry-level final goods, and that industry-level final goods are themselves CES composites of Home and Foreign

final goods. The aggregate consumer price level is then given by:

$$P_{Ct} = \prod_{s \in S} P_{Ct}(s)^{\gamma(s)} \quad (13)$$

$$\text{with } P_{Ct}(s) = \left(\nu(s) P_{Ht}(s)^{1-\eta(s)} + (1-\nu(s)) (\tau_{Ct}(s) P_{Ft}(s)) \right)^{1/(1-\eta(s))}, \quad (14)$$

where $P_{Ct}(s)$ is the price of a composite consumption good for industry s and $\tau_{Ct}(s)$ is an iceberg-type trade cost paid on imports of final goods.

Similar to the sufficient statistics argument for input use, we use first order conditions for the purchases of domestic consumption goods to write the expenditure share on domestic goods as: $\Lambda_{Ht}^C(s) = \frac{P_{Ht}(s) C_{Ht}(s)}{P_{Ct}(s) C_t(s)} = \nu(s) \left(\frac{P_{Ht}(s)}{P_{Ct}(s)} \right)^{1-\eta(s)}$. Taking ratios across time yields:

$$\hat{P}_{Ct}(s) = \hat{P}_{Ht}(s) \hat{\Lambda}_{Ht}^C(s)^{1/(\eta(s)-1)}. \quad (15)$$

If $\eta(s) > 1$, this says that aggregate consumer prices decline relative to the price of domestically produced goods when the share of spending on domestic goods falls over time. That is, $\hat{\Lambda}_{Ht}^C(s) < 1$ implies $\hat{P}_{Ct}(s) < \hat{P}_{Ht}(s)$.

The ratio of aggregate consumer prices in period t relative to the base period is: $\hat{P}_{Ct} = \prod_{s \in S} \hat{P}_{Ct}(s)^{\gamma(s)}$. We combine this with Equation 15 and take logs to obtain:

$$\begin{aligned} \hat{p}_{Ct} &= \sum_{s \in S} \gamma(s) \left[\hat{p}_{Ht}(s) + \left(\frac{1}{\eta(s)-1} \right) \hat{\lambda}_{Ht}^C(s) \right] \\ &= \gamma \hat{p}_{Ht} + \gamma [\mathbf{H} - \mathbf{I}]^{-1} \hat{\lambda}_{Ht}^C, \end{aligned} \quad (16)$$

where γ is a row vector with elements $\gamma(s)$, η is a diagonal matrix with elements $\eta(s)$, and $\hat{\lambda}_{Ht}^C$ is a column vector with elements $\hat{\lambda}_{Ht}^C(s)$. The second term in this expression captures the idea that falling prices for imported final goods— e.g., finished manufactured goods from China, like iPhones, clothing, shoes, or laptops – has restrained consumer price growth.

To this, our model adds an additional link between foreign sourcing and consumer prices. Specifically, we can insert Equation 12 into Equation 16 to decompose domestic prices:

$$\hat{p}_{Ct} = \gamma [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{I} - \alpha] \hat{\mathbf{p}}_{Vt} + \underbrace{\left(\frac{1}{\eta-1} \right) \gamma [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{A}' \circ (\hat{\lambda}_{Ht}^M)']}_{\text{Offshoring}} \iota + \underbrace{\left(\frac{1}{\eta-1} \right) \gamma \hat{\lambda}_{Ht}^C}_{\text{C Imports}}, \quad (17)$$

where we have imposed $\eta(s) = \eta$ to simplify the expression.¹³ The first term is a measure of the

¹³Note that taking differences in this expression across adjacent periods (t and $t-1$) yields a decomposition of inflation ($\pi_{Ct} = \hat{p}_{Ct} - \hat{p}_{Ct-1}$) into components attributable to domestic GDP deflators ($\pi_{Vt} = \hat{\mathbf{p}}_{Vt} - \hat{\mathbf{p}}_{Vt-1}$) and changes

dependence of consumer prices on domestic cost growth – changes in the productivity-adjusted prices of domestic factors, i.e., domestic real value added. The second term is an adjustment for the impact of offshoring on consumer prices, which works through the impact that offshoring has in lowering prices for domestically produced goods. The third term captures the role of changes in trade in final goods on consumer prices. Note in both these terms, the formula tells us to aggregate sector-level changes in sourcing using sector expenditure shares in final demand, collected in γ .

We now turn to data to quantify the aggregate roles for offshoring and consumption imports in accounting for consumer prices. Using input-output data, we construct consumption of domestic goods as total personal consumption expenditures (from the use table) less personal consumption expenditures reported in the import use table. From this, we compute Λ_{Ht}^C as the ratio of consumption of domestic goods to total consumption expenditure (as elsewhere, $\hat{\lambda}_{Ht}^C = \ln(\Lambda_{Ht}^C / \Lambda_{H,1997}^C)$), and we compute the time-average share of consumption expenditure allocated to each sector, encoded in γ . To aggregate changes in offshoring, we use γ together with the input output matrix \mathbf{A} , defined previously. We plot the aggregate “shocks” – aggregated changes in domestic sourcing, given by $\gamma \hat{\lambda}_{Ht}^C$ and $\gamma [\mathbf{I} - \mathbf{A}']^{-1} \left[\mathbf{A}' \circ \left(\hat{\lambda}_{Ht}^M \right)' \right]_{\iota}$ – in Figure 3a. As is evident, both “shocks” are negative, consistent with falling domestic sourcing in the aggregate over time. Further, most of the decline is concentrated in the first half of the sample period (before 2010), and it is phased in slowly over time.¹⁴

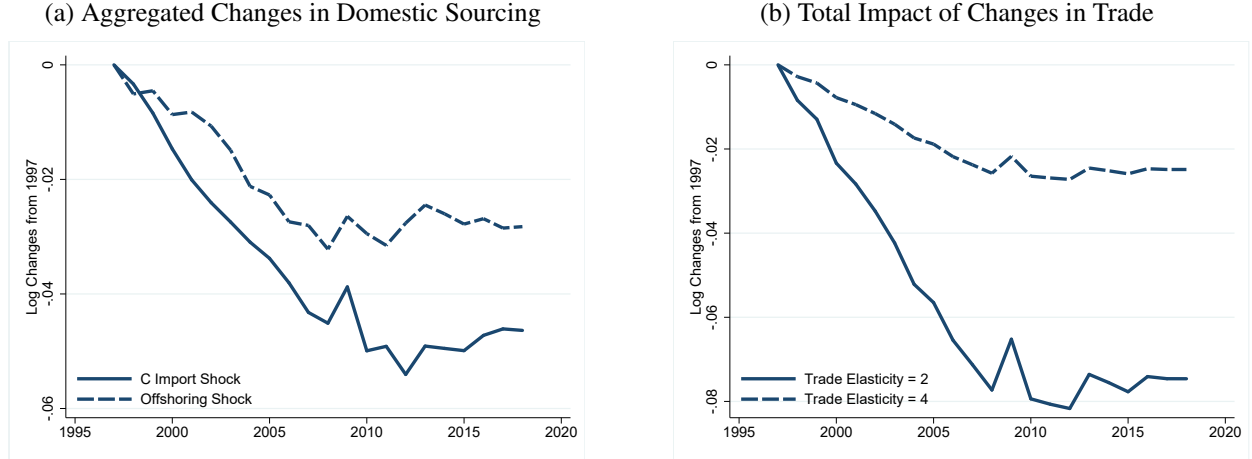
To compute the impact of these changes on consumer prices requires taking a stand on the (matrix) value of η , the industry-level elasticities between home and foreign output. Estimating these separately for the 71 industries is beyond the scope of the exercise we want to perform here, so we impose a homogeneous elasticity ($\eta = \eta \mathbf{I}$). We then consider two alternative values for illustration: $\eta = \{2, 4\}$.¹⁵ We plot the results – i.e., the composite Offshoring and C Imports terms in Equation 17 – in Figure 3b. The cumulative impact of declines in domestic sourcing is to lower consumer prices relative to domestic value-added prices by between 2 and 8 percent over the sample period, depending on the elasticity. Translated into annual effects, rising trade lowers consumer price growth by between 10 and 40 basis points per year relative to growth in value-

in domestic sourcing of final goods and inputs. In Section 4.2, we use this alternative representation to examine inflation in the multisector model.

¹⁴To head off possible confusion later, we note that this aggregation focuses entirely on consumption goods. In it, declines in domestic sourcing of consumption goods are more important than changes in offshoring in determining aggregate consumer prices. In the model below, we will combine sourcing of consumption and investment goods in calibrating shocks, which conforms to standard practice for models without physical capital in the literature.

¹⁵The value $\eta = 2$ is near standard values of the Armington trade elasticity in the international macroeconomics literature. It is also close to a naive estimate of η obtained from the slope of a regression line through the scatter plot in Figure 1, as well as recent estimates of long run macro-elasticities based on tariff changes in [Boehm, Levchenko and Pandalai-Nayar \(2020\)](#). In the trade literature, $\eta = 4$ is in the vicinity of standard values for estimated gravity trade elasticities (see [Simonovska and Waugh \(2014\)](#) for example). In calibrated models below, we set the elasticity between home and foreign goods equal to 3, which is the midpoint between these bounds.

Figure 3: The Role of Offshoring and Consumption Imports in Accounting for Consumer Price Changes from 1997-2018



Note: In Panel (a), “C Import Shock” is $\gamma \hat{\lambda}_{Ht}^C$ and “Offshoring Shock” is $\gamma [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{A}' \circ (\hat{\lambda}_{Ht}^M)'] \iota$. In Panel (b), the total impact of changes in trade is $\left(\frac{1}{\eta-1}\right) \gamma [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{A}' \circ (\hat{\lambda}_{Ht}^M)'] \iota + \left(\frac{1}{\eta-1}\right) \gamma \hat{\lambda}_{Ht}^C$.

added prices. Increases in offshoring account for about 40% of this gap, while foreign sourcing of consumer goods accounts for the remainder.

1.4 Beyond Accounting

These results suggest that offshoring plays an important quantitative role in explaining the evolution of domestic prices across industries, and further that rising trade has lowered the aggregate consumer price level (depressed inflation). We now advance a word of caution about this second conclusion, which motivates the model-based exercises that follow.

All the results presented above about the aggregate consumer price level are based on accounting decompositions. While it is tempting to interpret these decompositions as evidence that offshoring lowered consumer price inflation, this conclusion is not justified based the data alone. For one, the accounting decompositions highlight supply-side channels linking trade to prices, but are silent on general equilibrium responses of factor prices (embedded in the GDP deflator). Moreover, inflation is determined jointly by real factors (i.e., changes in trade) and the reaction of monetary policy to them. Thus, we need a full-fledged model to properly evaluate how changes in trade shape inflation.

Notwithstanding this concern, we also note that evidence presented about price changes across industries speaks to an important potential mechanism linking offshoring and prices. Specifically, differences in price changes across sectors are related to differences in offshoring intensity, such that sectors with higher offshoring intensity have lower output price growth. This points to a

mechanism that may give rise to aggregate price level effects. Thus, we turn to a full general equilibrium model in which we can evaluate the impact of trade in both inputs and final goods on inflation.

2 Baseline New Keynesian Model

This section presents a New Keynesian model with trade in both inputs and final goods. We focus on a one sector model in this section, which serves to highlight the importance of trade dynamics for inflation outcomes, and isolate the distinct roles for trade in final goods versus inputs.

To outline this section, we first develop the model and describe how we use the sufficient statistics argument to link changes in domestic sourcing to inflation. We then simulate the model to establish a “puzzling” result: given the path of observed changes in trade, trade integration drove inflation up. To interpret this result, we turn to a three equation representation of the model – consisting of a Phillips curve, IS equation, and monetary policy rule – that yields sharp analytic results. We present a series of propositions that explain the role of trade dynamics in driving inflation outcomes, and the relationship between our results and “conventional wisdom.” We conclude with a discussion of the robustness of our results with respect to alternative monetary policy rules.

2.1 The Model

The model draws on the standard small open economy New Keynesian structure, as expositied by [Galí \(2015\)](#). We deviate from the textbook model by replacing Calvo-style pricing with Rotemberg pricing, which has no first order consequences for the questions we address. More importantly, we alter the production structure to allow for trade in both final goods and inputs. Further, we develop a new sufficient statistics approach to model analysis.

2.1.1 Consumers

Consumer preferences over labor supply L_t and consumption C_t are represented by:

$$U(\{C_t, L_t\}_{t=0}^{\infty}) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\rho}}{1-\rho} - \mu \frac{L_t^{1+\psi}}{1+\psi} \right] \quad (18)$$

$$C_t = \left(\nu^{1/\eta} C_{Ht}^{(\eta-1)/\eta} + (1-\nu)^{1/\eta} C_{Ft}^{(\eta-1)/\eta} \right)^{\eta/(\eta-1)} \quad (19)$$

$$C_{Ht} = \left(\int_0^1 C_{Ht}(i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}, \quad (20)$$

where C_{Ft} is a composite foreign consumption good. The elasticity $\epsilon > 1$ controls substitution among domestic varieties, while η controls substitution between domestic and foreign goods. The parameter $\nu \in (0,1)$ controls relative demand for home goods, conditional on prices. The parameters $\rho \geq 0$ and $\psi > 0$ govern intertemporal substitution and labor supply.

Financial markets are complete, such that the consumer has access to Arrow-Debreu securities that are traded internationally. The representative consumer's budget constraint is then:

$$\int_0^1 P_{Ht}(i)C_{Ht}(i)di + P_{Ft}\tau_{Ct}C_{Ft} + E_t [Q_{t,t+1}B_{t+1}] \leq B_t + W_tL_t, \quad (21)$$

where B_t is the nominal, domestic currency payoff in period t of the consumer's asset portfolio and $Q_{t,t+1}$ is the stochastic discount factor for nominal payments. The price of the foreign consumption good in domestic currency is P_{Ft} , and $\tau_{Ct} > 1$ is an iceberg trade cost paid on consumption imports. The prices of individual domestic goods are $\{P_{Ht}(i)\}$ and the nominal wage is W_t .

Given prices $\{\{P_{Ht}(i)\}, P_{Ft}, Q_{t,t+1}, W_t\}$ and initial asset holdings B_0 , the consumer chooses consumption $\{C_t, \{C_{Ht}(i)\}, C_{Ft}\}$, labor supply $\{L_t\}$, and asset holdings $\{B_{t+1}\}$ to maximize Equations 18-20 subject to Equation 21 and the standard transversality condition.

2.1.2 Production

The production function for individual domestic varieties is:

$$Y_t(i) = Z_t (L_t(i))^{1-\alpha} (M_t(i))^\alpha \quad (22)$$

$$M_t(i) = \left[\xi^{1/\eta} M_{Ht}(i)^{(\eta-1)/\eta} + (1-\xi)^{1/\eta} M_{Ft}(i)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \quad (23)$$

$$M_{Ht}(i) = \left(\int_0^1 M_{Ht}(j,i)^{(\epsilon-1)/\epsilon} dj \right)^{\epsilon/(\epsilon-1)}, \quad (24)$$

where $L_t(i)$ and $M_t(i)$ are quantities of labor and a composite input used by firm i . The composite input is a nested CES composite of inputs sourced from home and abroad: $M_{Ht}(j,i)$ is the quantity of inputs from Home firm j purchased by firm i , $M_{Ht}(i)$ is the composite home input used by firm i , and $M_{Ft}(i)$ is the quantity of a foreign composite input purchased by firm i . Similar to consumption, $\epsilon > 1$ controls substitution among domestic varieties, while η controls substitution across country sources for inputs. The parameter $\xi \in (0,1)$ controls relative demand for home inputs, conditional on prices.

Producers set the prices of their goods under monopolistic competition, and they select the input mix to satisfy the implied demand. These two problems can be analyzed separately.

Pricing Each Home firm sets its price in domestic currency, which applies to both output sold domestically and exports. It chooses a sequence for $P_{Ht}(i)$ to maximize the present discounted value of profits, inclusive of quadratic adjustment costs incurred when it changes prices, as in [Rotemberg \(1982a,b\)](#). Letting $MC_t(i)$ be the firm's marginal cost of production (defined below), the present value of profits is:

$$E \sum_{t=0}^{\infty} \beta^t \frac{C_t^{-\rho}}{C_0^{-\rho}} \frac{1}{P_{Ct}} \left[P_{Ht}(i) Y_t(i) - MC_t(i) Y_t(i) - \frac{\phi}{2} \left(\frac{P_{Ht}(i)}{P_{H,t-1}(i)} - 1 \right)^2 P_{Ht} Y_t \right],$$

where the last term records the quadratic price adjustment costs. In this adjustment cost term, ϕ is a parameter that controls the degree of price rigidity, $Y_t = \int_0^1 Y_t(i) di$ is total home output, and $P_{Ht} = \left(\int_0^1 P_{Ht}(i)^{(1-\epsilon)/\epsilon} di \right)^{1/(1-\epsilon)}$ is the price of the CES bundle of home output.

Input Demand Firm i in sector s chooses $\{L_t(i), M_t(i), M_{Ht}(i), M_{Ft}(i), M_{Ht}(j, i)\}$ to minimize the cost of producing output $Y_t(i)$. Firms pay iceberg trade costs τ_{Mt} on inputs they import from abroad, so variable production costs are $W_t L_t(i) + P_{Mt} M_t(i)$, with $P_{Mt} M_t(i) = P_{Ht} M_{Ht}(i) + \tau_{Mt} P_{Ft} M_{Ft}(i)$ and $P_{Ht} M_{Ht}(i) = \int_0^1 P_{Ht}(j) M_{Ht}(j, i) dj$. Here $P_{Mt} = \left[\xi P_{Ht}^{1-\eta} + (1-\xi)(\tau_{Mt} P_{Ft})^{1-\eta} \right]^{1/(1-\eta)}$ is the price of the composite input.

2.1.3 Closing the Model

Demand for exports of individual domestic varieties $X_t(i)$ has a CES structure:

$$X_t(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\epsilon} X_t, \quad (25)$$

$$\text{with } X_t = \left(\frac{P_{Ht}}{S_t P_{Ct}^*} \right)^{-\eta} C_t^*, \quad (26)$$

where S_t is the nominal exchange rate (units of domestic currency to buy 1 unit of foreign currency), P_{Ct}^* is the foreign price index in foreign currency, and C_t^* is foreign consumption.

The market clearing conditions for output of each variety and the labor market are:

$$Y_t(i) = C_{Ht}(i) + \int_0^1 M_{Ht}(i, j) dj + X_t(i) + \frac{\phi}{2} \left(\frac{P_{Ht}(i)}{P_{H,t-1}(i)} - 1 \right)^2 Y_t \quad (27)$$

$$\int_0^1 L_t(i) di = L_t. \quad (28)$$

Due to trade in the stage-contingent asset, the usual international risk sharing condition applies:

$$\left(\frac{C_t}{C_t^*}\right)^{-\rho} \left(\frac{S_t P_{Ct}^*}{P_{Ct}}\right) = \Xi, \quad (29)$$

where Ξ is a constant, which depends on initial conditions.¹⁶

Finally, we specify an inflation-targeting monetary policy rule to close the model:

$$1 + i_t = (1 + i_0) \left(\frac{P_{Ct}}{P_{C,t-1}}\right)^\omega, \quad (30)$$

where i_0 is the steady state (date 0) interest rate.

2.1.4 Equilibrium

We define an equilibrium for the small open economy taking foreign variables as given, including the price of foreign goods in domestic currency.¹⁷ Focusing on a symmetric equilibrium in which all domestic producers are identical, we drop the firm/variety index. Given exogenous variables $\{P_{Ft}, C_t^*, P_t^*, Z_t, \tau_{Ct}, \tau_{Mt}\}$, an equilibrium (up to a normalization) is a collection of prices $\{W_t, P_{Ht}, P_{Ct}, P_{Mt}, MC_t, S_t, i_t\}$ and quantities $\{C_t, C_{Ht}, C_{Ft}, L_t, X_t, Y_t, M_t, M_{Ht}, M_{Ft}\}$ that solve the consumer's utility maximization problem, the producer's pricing and input demand problems (maximize profits), and clear the markets for goods, labor, and assets. Further, interest rates are set based on the monetary policy rule. See Appendix B for the full set of equilibrium conditions.

2.1.5 Equilibrium with Domestic Sourcing Shares as Sufficient Statistics

We redefine variables to highlight the role of trade openness in driving the results. Let $\Lambda_{Ht}^C \equiv \frac{P_{Ht} C_{Ht}}{P_{Ct} C_t}$ and $\Lambda_{Ht}^M = \frac{P_{Ht} M_{Ht}}{P_{Mt} M_t}$ be the shares of final and input expenditure that falls on home produced goods, which we refer to as “domestic sourcing shares.” Using first order conditions describing demand

¹⁶Introducing additional notation, $\Xi = \frac{E_0 \theta_0}{\theta_0^*}$, where E_0 is the date zero exchange rate, and θ_0 and θ_0^* are date zero Lagrange multipliers on lifetime budget constraints of home and foreign agents.

¹⁷Ordinarily, the domestic price of foreign goods (P_{Ft}) would be an equilibrium object in a small open economy model, and its behavior would depend on pricing assumptions. We treat it as exogenous here for brevity, since it will not be needed to define the equilibrium in terms of sourcing shares below. We discuss the redundancy of dollar currency import prices again when we model foreign price setting in Section 3.2 and Appendix F.

for final goods and inputs, the relative price of home goods is then given by:

$$\frac{P_{Ht}}{P_{Ct}} = \left(\frac{\Lambda_{Ht}^C}{\nu} \right)^{1/(1-\eta)} \quad (31)$$

$$\frac{P_{Ht}}{P_{Mt}} = \left(\frac{\Lambda_{Ht}^M}{\xi} \right)^{1/(1-\eta)} . \quad (32)$$

These equations imply that the domestic sourcing shares, together with the trade elasticity (η), are sufficient to infer the price of home goods relative to the final goods and input bundles, as in [Arkolakis, Costinot and Rodríguez-Clare \(2012\)](#).

Using this result, we replace these relative prices throughout the equilibrium system, and we redefine the equilibrium for given values of the domestic sourcing shares. Using this method to reduce down the model, the log-linearized equilibrium conditions are presented in Table 1, where all variables are expressed as log deviations from steady state (i.e., $\hat{x}_t = \ln(X_t) - \ln(X_0)$ for variable X and the subscript 0 indexes an initial steady state). In the table, $\widehat{r\bar{m}c}_t \equiv \widehat{m\bar{c}}_t - \widehat{p}_{Ht}$ is real marginal costs and $\widehat{r\bar{w}}_t \equiv \widehat{w}_t - \widehat{p}_{Ht}$ is the real wage. Further, $\hat{r}_t \equiv \ln(1 + i_t) - \ln(1 + i_0) \approx i_t - i_0$, and $\hat{q}_t \equiv \hat{s}_t + \hat{p}_{Ct}^* - \hat{p}_{Ct}$ is the consumption real exchange rate. Finally, inflation rates are given by: $\pi_{Ht} = \hat{p}_{Ht} - \hat{p}_{H,t-1}$ and $\pi_{Ct} = \hat{p}_{Ct} - \hat{p}_{C,t-1}$.

We will analyze the dynamic equilibrium taking the path of domestic sourcing shares as given. Given domestic sourcing shares $\{\hat{\lambda}_{Ht}^C, \hat{\lambda}_{Ht}^M\}$ and exogenous shocks $\{\hat{z}_t, \hat{c}_t^*\}$, an equilibrium is a path for prices $\{\hat{q}_t, \hat{e}_t, \pi_{Ct}, \pi_{Ht}, \widehat{r\bar{w}}_t, \widehat{r\bar{m}c}_t, \hat{r}_t\}$ and quantities $\{\hat{c}_t, \hat{y}_t, \hat{l}_t, \hat{x}_t, \hat{c}_{Ht}, \hat{m}_{Ht}\}$ that satisfies the equilibrium conditions in Table 1.

This equilibrium definition highlights the value of the sufficient statistics approach in the model. In this equilibrium, we need not directly track trade costs, or the price of foreign goods, over time. As a result, this method sidesteps a host of difficult data and theoretical issues. On the data side, we avoid needing to directly measure trade costs or foreign prices. Further, we need not make theoretical assumptions about currency invoicing or pass-through of foreign cost shocks into import prices. Instead, we lean on the model result that the domestic sourcing share – agents' responses to implicit price changes – tells us everything we need to know about relative international prices to study domestic equilibrium outcomes.¹⁸

¹⁸To elaborate further, for historical simulations that take the path of past domestic sources shares as given by data, our approach is identical to the following alternative approach. One could specify an explicit model of import prices (e.g., producer currency pricing for imports, or dollar invoicing of imports). This would link the domestic currency price of imports to foreign marginal costs (in foreign currency) and trade costs. Conditional on foreign marginal costs and the elasticity of substitution between imports and domestic goods, one could then pick the value of trade costs to exactly match the observed share of imports in expenditure over time. Simulating the model with this imputed trade cost series exactly replicates simulation of the model taking the domestic source share as given, as in Table 1. Further, one would obtain identical outcomes for inflation and other macro variables, regardless of the currency invoicing of imports. The currency invoicing of imports would only change the imputed trade cost series needed to match observed

Table 1: Log-Linearization of the Baseline Model

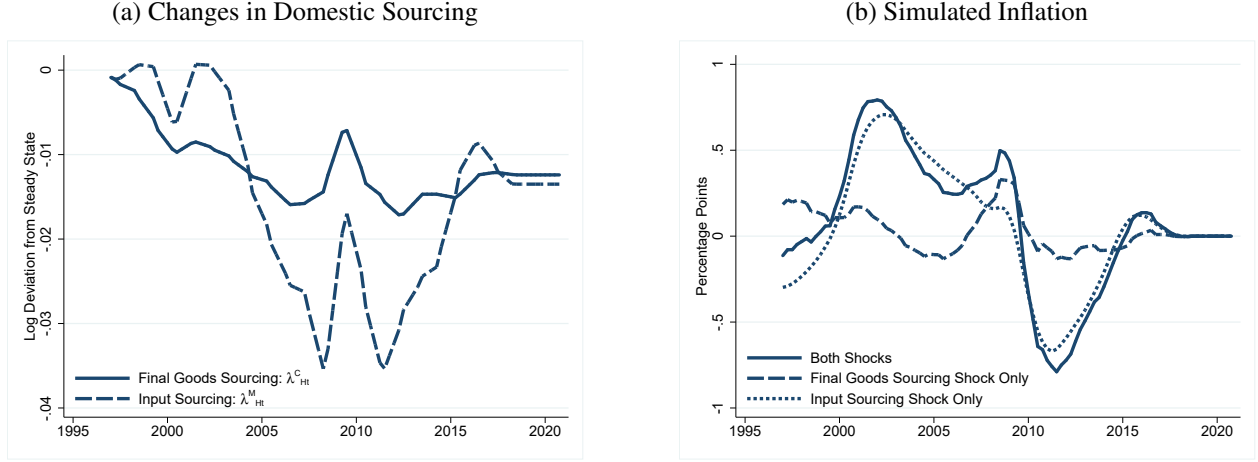
Consumption-Leisure	$\hat{l}_t = -\frac{\rho}{\psi} \hat{c}_t + \frac{1}{\psi} \widehat{r\bar{w}}_t - \frac{1}{\psi(\eta-1)} \hat{\lambda}_{Ht}^C$
Consumption Allocation	$\hat{c}_{Ht} = \frac{\eta}{\eta-1} \hat{\lambda}_{Ht}^C + \hat{c}_t$
Euler Equation	$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\rho} (\hat{r}_t - E_t \pi_{Ct+1})$ $\hat{l}_t = \widehat{r\bar{m}c}_t + \hat{y}_t - \widehat{r\bar{w}}_t$
Input Choices	$\hat{m}_t = \widehat{r\bar{m}c}_t + \hat{y}_t - \frac{1}{\eta-1} \hat{\lambda}_{Ht}^M$ $\hat{m}_{Ht} = \frac{\eta}{\eta-1} \hat{\lambda}_{Ht}^M + \hat{m}_t$
Real Marginal Cost	$\widehat{r\bar{m}c}_t = (1 - \alpha) \widehat{r\bar{w}}_t + \frac{\alpha}{\eta-1} \hat{\lambda}_{Ht}^M - \hat{z}_t$
Domestic Price Inflation	$\pi_{Ht} = \left(\frac{\epsilon - 1}{\phi} \right) \widehat{r\bar{m}c}_t + \beta E_t (\pi_{Ht+1})$
Consumer Price Index	$\pi_{Ct} = \pi_{Ht} + \frac{1}{(\eta-1)} \left(\hat{\lambda}_{Ht}^C - \hat{\lambda}_{Ht-1}^C \right)$
Market Clearing	$\hat{y}_t = \left(\frac{C_{H0}}{Y_0} \right) \hat{c}_{Ht} + \left(\frac{M_{H0}}{Y_0} \right) \hat{m}_{Ht} + \left(\frac{X_0}{Y_0} \right) \hat{x}_t$ $\hat{x}_t = \frac{\eta}{\eta-1} \hat{\lambda}_{Ht}^C + \eta \hat{q}_t + \hat{c}_t^*$ $\hat{c}_t = \hat{c}_t^* + \frac{1}{\rho} \hat{q}_t$
Monetary Policy Rule	$\hat{r}_t = \omega \pi_{Ct}$

Before pushing on to implement this approach, we pause to note one potential shortcoming of it. Specifically, it is best suited to studying the inflationary impacts of “external shocks” on inflation, such as changes in foreign prices (P_{Ft}) or trade costs (τ_{Ct} , τ_{Mt}), because these shocks influence inflation only through domestic sourcing shares. It is less well suited to analyze the inflationary impact of “domestic shocks” – e.g., a change in domestic productivity – that impact inflation directly (i.e., conditional on domestic sourcing shares) as well as indirectly, through their impact on domestic sourcing shares. To simulate the effects of this kind of shock, one would need to directly model domestic sourcing shares, which in turn requires assumptions about import pricing.

Given our focus on understanding the effects of rising trade integration on inflation, the sufficient statistics approach has benefits that outweigh this cost. First, it simplifies algebraic analysis of the model, which we exploit below. Second, in our empirical context, changes in foreign prices (e.g., Chinese productivity growth) and/or trade frictions (e.g., falling tariffs, increases in logistics efficiency, etc.) are likely the first order determinants of declines in domestic sourcing over time. Further, to the extent that domestic shocks have indirect effects via trade, these are included in our

domestic sourcing shares. For prospective analysis of the future impacts of changes in current trade costs, currency invoicing matters in that it determines the mapping from policy changes to domestic sourcing shares.

Figure 4: Shocks and Simulation of Inflation in the Baseline Model



overall tally of the impact of changing trade on inflation under the sufficient statistics approach. Thus, we proceed to implementation.

2.2 Simulated Impact of Trade on Inflation

We apply the model to simulate consumer inflation given the observed evolution of domestic sourcing shares ($\hat{\lambda}_{Ht}^C$ and $\hat{\lambda}_{Ht}^M$) from 1997-2018. Assuming that the economy is in steady state prior to 1997, and that each time period corresponds to one quarter, we calibrate the model using standard external parameter values and the BEA data introduced in Section 1 (see Appendix B.2 for details).

We introduce globalization as an MIT-style shock: starting from steady state, agents learn the sequence of shocks, and we solve for the dynamic equilibrium under perfect foresight.¹⁹ Figure 4a plots the evolution of the domestic sourcing shares. Domestic sourcing falls for both final goods and inputs, with changes that are phased in over time. Moreover, medium term dynamics feature prominently: the input sourcing share follows a u-shaped pattern, and there are sharp adjustments in both sourcing shares around the Great Recession in 2008-2011.

Given these shocks, we plot simulated consumer price inflation (π_{Ct}) in Figure 4b. The solid line records inflation when domestic sourcing for both final goods and inputs changes in the model, while the dashed lines record simulated inflation for each shock separately (i.e., fed into the model one at a time). With both shocks active, inflation doesn't change much at the outset (1997-2000), but then rises by about 75 basis points after the year 2000. It remains positive for the remainder of the 2000-2010 interval, and only falls below zero for a sustained period after 2010. Adding

¹⁹As a terminal condition for the shocks, we assume that agents expect the level of domestic sourcing that prevails in 2018 to persist into perpetuity thereafter. Altering this assumption impacts the last few years of the simulation (changes are modest in size), but has deminimus effects on outcomes in early periods.

up these changes over time, the price level rises by about 17% between 1997 and 2010. It then falls thereafter, but the post-2010 deflation doesn't make up for the pre-2010 inflation – the price level stabilizes at a level about 8% higher than its 1997 level as a result of the changes in domestic sourcing. Thus, changes in trade led to net inflation over this time period, averaging 40 basis points per year.

The path of inflation reflects the impact of changes in final goods and input sourcing. Both contribute to high inflation in the early 2000's, though the role of inputs is far larger. Further, the post-2010 deflation is driven primarily by the dynamics of input sourcing. Referring to the shocks, domestic sourcing is declining during the 2000-2010 period, coincident with positive inflation in the model. In contrast, as domestic sourcing for inputs rises after 2010, inflation is negative. Thus, this simulation suggests that inflation comoves negatively with domestic sourcing: periods of increased globalization are associated with inflation, while retrenchment leads to deflation.

These results may be puzzling on the surface, in that they contradict the conventional wisdom about the impact of trade integration on inflation. Having established this trade and inflation puzzle, we turn back to the model to present theoretical analysis that illuminates the role of trade dynamics in driving inflation.

2.3 The Three Equation Model

To provide intuition regarding the impact of trade on inflation, we distill the equilibrium system in Table 1 into a three equation model, with a Phillips curve, IS curve, and monetary policy rule.

2.3.1 Phillips Curve

We start by deriving a Phillips curve for consumer prices, linking aggregate inflation to a suitably defined output gap. As a first step, we derive the Phillips curve for domestic output prices.

In Appendix B.3, we show that real marginal costs depend on the gap between real wages in the actual and flexible price equilibria: $\widehat{rmc}_t = (1 - \alpha) [\widehat{rw}_t - \widehat{rw}_t^n]$, where superscript n defines the value of a variable under flexible prices. The real wage gap can then be linked to the output, through goods and labor market clearing: $\widehat{rw}_t - \widehat{rw}_t^n = \chi [\hat{y}_t - \hat{y}_t^n]$, where \hat{y}_t^n is the log deviation of gross output in a flexible price equilibrium from its initial steady state value, and $\chi > 0$ is function of fundamental parameters and steady state expenditure shares.²⁰

Inserting these results into the equation for domestic price inflation gives us the relationship between domestic price inflation and the output gap, which we refer to as the domestic price

²⁰While we write the output gap in terms of gross output here, we show the model can be rewritten in terms of the output gap for real value added in Appendix B.3.

Phillips curve:

$$\pi_{Ht} = \left(\frac{(\epsilon - 1)(1 - \alpha)\chi}{\phi} \right) [\hat{y}_t - \hat{y}_t^n] + \beta E_t(\pi_{Ht+1}), \quad (33)$$

with $\frac{(\epsilon - 1)(1 - \alpha)\chi}{\phi} > 0$, so domestic price inflation is increasing in the domestic output gap.

An important point to note here that changes in domestic sourcing have no direct impact on domestic price inflation, conditional on the output gap. This result may seem counterintuitive, because increased offshoring lowers domestic production costs. To elaborate, note that domestic price inflation is given by $\pi_{Ht} = \left(\frac{\epsilon - 1}{\phi} \right) \widehat{r\overline{mc}}_t + \beta E_t(\pi_{Ht+1})$, with $\widehat{r\overline{mc}}_t = (1 - \alpha)\widehat{r\overline{w}}_t + \frac{\alpha}{\eta - 1}\hat{\lambda}_{Ht}^M - \hat{z}_t$. Since a decline in domestic sourcing of inputs ($\hat{\lambda}_{Ht}^M < 0$) lowers real marginal costs, it appears (at first glance) that it would lower domestic price inflation. This logic is misleading, however, because the real wage ($\widehat{r\overline{w}}_t$) is endogenously linked to $\hat{\lambda}_{Ht}^M$. Equation 33 accounts for changes in the real wage through the output gap. Further, by focusing attention on the gap between the actual and flexible price equilibria, this analytical approach nets out changes in domestic sourcing that influence both the actual and flexible price equilibria symmetrically.

Combining Equation 33 with the consumer price index, the consumer price Phillips curve is:

$$\pi_{Ct} = \left(\frac{(\epsilon - 1)(1 - \alpha)\chi}{\phi} \right) [\hat{y}_t - \hat{y}_t^n] + \beta E_t\pi_{Ct+1} + \frac{1}{(\eta - 1)} [\Delta\hat{\lambda}_{Ht}^C - \beta E_t\Delta\hat{\lambda}_{Ht+1}^C], \quad (34)$$

where $\Delta\hat{\lambda}_{Ht}^C \equiv \hat{\lambda}_{Ht}^C - \hat{\lambda}_{Ht-1}^C = \Delta \ln \lambda_{Ht}^C$ is the log change in domestic sourcing across adjacent periods.

In contrast to domestic price inflation, changes in domestic sourcing for final goods directly impact consumer price inflation. Given expected future changes in domestic final goods sourcing ($E_t\Delta\hat{\lambda}_{Ht+1}^C$) and inflation (π_{Ct+1}), and the current output gap ($\hat{y}_t - \hat{y}_t^n$), a reduction in domestic sourcing today ($\Delta\hat{\lambda}_{Ht}^C < 0$) lowers consumer price inflation. This is intuitive, as a reduction in domestic sourcing is associated with a terms of trade improvement, which directly lowers overall consumer price inflation. In contrast, an anticipated reduction in domestic sourcing tomorrow ($E_t\Delta\hat{\lambda}_{Ht+1}^C < 0$) raises consumer price inflation today, all else equal. 2Given π_{Ct+1} , lower domestic sourcing at date $t + 1$ implies that future domestic price inflation (π_{Ht+1}) must be higher, which is associated with higher domestic price inflation today, via Equation 33. These results emphasize that the dynamics of domestic sourcing are important for understanding inflation, and we will return to this point below.

2.3.2 IS Curve

The output gap reflects the structure of the Euler equation, as is standard. Referring again to Appendix B.3 for detailed derivation, the IS curve can be written as:

$$[\hat{y}_t - \hat{y}_t^n] = -\frac{1}{\theta\rho} [\hat{r}_t - \hat{r}_t^n] + E_t [\hat{y}_{t+1} - \hat{y}_{t+1}^n], \quad (35)$$

where $\hat{r}_t \equiv \hat{r}_t - E_t \pi_{Ct+1}$ is the real interest rate, $\hat{r}_t^n \equiv \hat{r}_t^n - E_t \pi_{Ct+1}^n = \hat{r}_t^n - \frac{1}{(\eta-1)} E_t \Delta \hat{\lambda}_{Ht+1}^C$ is the real interest rate in the flexible price equilibrium (the natural real interest rate), and $\theta > 0$ depends on primitive parameters and steady state expenditure shares. To complete the characterization of the IS curve, we solve for the real interest rate in the flexible price equilibrium as a function of exogenous variables:

$$\hat{r}_t^n = \Omega_{C^*} E_t \Delta \hat{c}_{t+1}^* + \Omega_Z E_t \Delta \hat{z}_{t+1} + \Omega_M E_t \Delta \hat{\lambda}_{Ht+1}^M + \Omega_C E_t \Delta \hat{\lambda}_{Ht+1}^C, \quad (36)$$

where $\Omega_{C^*} > 0$, $\Omega_Z > 0$, $\Omega_M < 0$, and $\Omega_C < 0$ are functions of parameters and steady state values, defined in Appendix B.3.

The natural real interest rate depends on expected future changes in domestic sourcing for both final goods and inputs. It is higher when domestic sourcing is expected to decline in the future, equivalently when the terms of trade are expected to improve. There are two distinct channels at work. First, domestic sourcing of final goods directly matters for the path of consumption. An expected terms of trade improvement for consumer goods means that consumers have higher real income in the future, holding their nominal income constant. This leads them to attempt to pull consumption forward, which drives up the flexible price equilibrium interest rate (\hat{r}_t^n) today. Second, domestic sourcing of inputs matters for consumer income via its impacts on the production side of the economy: an improvement in the terms of trade for sourcing inputs has similar effects to an increase in productivity. By lowering future production costs, an expected decline in domestic sourcing of inputs leads to higher future output and income. This increase in future income further leads consumers to attempt to pull consumption forward in time, leading the natural rate to rise. As we shall see below, both these mechanisms are crucial for interpreting the impact of domestic sourcing dynamics on inflation.

2.3.3 Aggregate Demand/Supply Interpretation

Collecting the results above, we can define a three equation model that determines the output gap ($\hat{y}_t - \hat{y}_t^n$), consumer price inflation (π_{Ct}), and the interest rate (\hat{r}_t). The equilibrium system is given by Equation 34, Equation 35 with the solution for the real natural interest rate (Equation 36) and the definition of the real interest rate ($\hat{r}_t \equiv \hat{r}_t - E_t \pi_{Ct+1}$), and the monetary policy rule $\hat{r}_t = \omega \pi_{Ct}$.

Combining the monetary policy rule with the IS curve, one can define a downward sloping “aggregate demand” (AD) schedule in $\{\pi_{Ct}, \hat{y}_t - \hat{y}_t^n\}$, with π_{Ct} on the y-axis and $\hat{y}_t - \hat{y}_t^n$ on the x-axis, where higher inflation today is associated with lower values of the output gap, since the central bank raises interest rates in response. The Phillips curve is then upward sloping, where a higher value of the output gap today raises current inflation (all else equal). Echoing standard textbooks, this Phillips curve can be thought of as an “aggregate supply” (AS) relation. We emphasize the intuition based on this AD/AS version of the model in describing analytical results below.

2.4 Shocks and Inflation in the Three Equation Model

In this section, we use the three equation representation of the model to interpret the inflation results in Section 2.2. As a tool, we present a sequence of propositions that formally characterize the response of inflation to alternative stylized shocks. These propositions highlight that trade dynamics drive inflation: trade integration raises inflation when it is anticipated and permanent. Alternative (unanticipated and/or temporary) shock dynamics may lower inflation, though they are not a realistic description of the past several decades. We expound on these statements below.

2.4.1 Interpreting Simulated Inflation

The following proposition describes the response of current inflation to anticipated, permanent changes in domestic sourcing of final and intermediate goods.

Proposition 1. *Consider (i) an anticipated, permanent decline in the domestic sourcing share for final goods, such that $\Delta\hat{\lambda}_{Ht}^C = 0$, $\Delta\hat{\lambda}_{Ht+1}^C < 0$, and $\Delta\hat{\lambda}_{Ht+k}^C = 0$ for $k > 1$, or (ii) an anticipated, permanent decline in the domestic sourcing share for inputs, such that $\Delta\hat{\lambda}_{Ht}^M = 0$, $\Delta\hat{\lambda}_{Ht+1}^M < 0$, and $\Delta\hat{\lambda}_{Ht+k}^M = 0$ for $k > 1$. For shock sequences (i) or (ii), inflation is positive at date t : $\pi_{Ct} > 0$.*

Proof. See Appendix C.1. □

The intuition behind Proposition 1 is quite simple. An anticipated shock to final goods sourcing – case (i) in the proposition – has two effects. First, the AD schedule shifts up, due to the rise in the real natural rate in response to the anticipated decline in domestic sourcing. Second, the AS schedule also shifts up, due to the expected decline in domestic sourcing of final goods. Both of these shocks drive inflation up. For case (ii), an anticipated decline in domestic sourcing of inputs also shifts the AD schedule up, but it leaves the AS schedule unchanged, because changes in domestic sourcing of inputs only affect inflation through the output gap.²¹ Thus, anticipated declines in domestic input sourcing also raise inflation.

²¹The direct effect of $\hat{\lambda}_{Ht}^M$ on real marginal cost (\widehat{rmc}_t) is subsumed in the output gap, as the direct effect of input sourcing on marginal costs is the same in the flexible and rigid price equilibria. See the discussion above following Equation 33 and corresponding derivations in Appendix B.3.

The shocks described in Proposition 1 are a stylized depiction of the shocks plotted in Figure 4a, abstracting from medium term dynamics. As of the mid-1990's, it was widely understood that the world was on a path toward increased globalization, and this reality materialized over the ensuing long decade. Consistent with the proposition, simulated inflation in Figure 4b is persistently above zero as trade increases between 1997 and 2009. This is the basic story of inflationary globalization in the model.

In addition to this broad perspective, inflation also responds to the medium term dynamics of the sourcing shares.²² These dynamics are most pronounced for the input sourcing share, where domestic input sourcing falls rapidly in the first half of the period and then reverts back toward its initial level, though it remains lower in the long run than it started. This initial rapid decline accentuates the rise in overall inflation in early years. Further, as the shock reverts, the logic inverts: the phased rise in domestic input sourcing (“reshoring”) after 2010 actually drives inflation down (below zero) in the model.

These results testify to the importance of trade dynamics in explaining inflation outcomes. Staying with this theme, we now discuss the conventional wisdom regarding the impact of rising trade on inflation, and how our argument differs from it.

2.4.2 Parsing Conventional Wisdom

Some changes in domestic sourcing do have deflationary impacts. We now consider both transitory and permanent declines in domestic sourcing that are unanticipated.

Proposition 2. *Consider (i) an unanticipated, permanent decline in the domestic sourcing share for final goods, such that $\Delta \hat{\lambda}_{Ht}^C < 0$, and $\Delta \hat{\lambda}_{Ht+k}^C = 0$ for $k \geq 1$, or (ii) an unanticipated, transitory decline in the domestic sourcing share for final goods, such that $\Delta \hat{\lambda}_{Ht}^C < 0$, $\Delta \hat{\lambda}_{Ht+1}^C = -\Delta \hat{\lambda}_{Ht}^C$ and $\Delta \hat{\lambda}_{Ht+k}^C = 0$ for $k \geq 2$. For shock sequences (i) or (ii), inflation is negative at date t : $\pi_{Ct} < 0$.*

Proof. See Appendix C.2. □

This proposition indicates that immediate declines in domestic final goods sourcing – whether temporary or permanent – lower inflation today. Our view is that the conventional wisdom that increasing trade lowers inflation is largely based on thinking through the impacts of these types of shocks. Both in the permanent and transitory cases, the AS curve – equivalently, the Phillips curve – shifts down on impact, directly lowering inflation. Moreover, the shift is more pronounced

²²There are significant short term effects of changes in domestic sourcing surrounding the Great Trade Collapse in 2008-2010. Anticipation of the collapse (though somewhat implausible), accounts for the decline in inflation in the mid-2000s. More plausibly, anticipated recovery from the trade collapse drives inflation up during Great Recession.

for a temporary shock than a permanent shock.²³ This shift in the Phillips curve captures standard terms-of-trade intuition: a decline in the relative price of imports – i.e., an improvement in Home’s terms of trade – reduces consumer price inflation, holding the domestic output gap fixed. This intuition features prominently in central bank policy discussions of the impact of globalization on inflation, where policymakers refer to increases in effective aggregate supply and/or shifting Phillips Curves to explain falling inflation [e.g., Yellen (2006); Bean (2007); Carney (2017)].

While this AS argument is sensible, we note that it omits consideration of the demand side. In the case of an unanticipated, permanent decline in $\hat{\lambda}_{Ht}^C$ – case (i) – there is no shift in the AD schedule, since the natural rate of interest depends on anticipated future changes in domestic sourcing. In case (ii), a transitory decline in $\hat{\lambda}_{Ht}^C$ actually shifts the AD schedule down, as this time path for domestic sourcing lowers the neutral rate of interest. This reinforces the deflationary impact of the downward shift in the AS schedule.

Nonetheless, we recall that the shock dynamics covered in Proposition 2 are not particularly plausible. Case (ii) presumes that agents perceive globalization to be temporary ex ante, which is inconsistent with contemporary analyses and policy discussions in the mid-1990’s and early 2000’s, and it is evidently counterfactual ex post. Case (i) misses the role of phase-in dynamics in trade adjustment, which are an important feature of the historical experience – the economy did not just jump from the closed to the open equilibrium.

Conventional models and analyses also omit any role for changes in input sourcing, which are quantitatively important in US data. We next consider unanticipated changes in input sourcing.

Proposition 3. (i) Consider an unanticipated, permanent decline in the domestic sourcing share for inputs, such that $\Delta\hat{\lambda}_{Ht}^M < 0$, and $\Delta\hat{\lambda}_{Ht+k}^M = 0$ for $k \geq 1$. For this shock sequence, inflation is zero at date t : $\pi_{Ct} = 0$. for $k \geq 0$. (ii) Consider an unanticipated, transitory decline in the domestic sourcing share for inputs, such that $\Delta\hat{\lambda}_{Ht}^M < 0$, $\Delta\hat{\lambda}_{Ht+1}^M = -\Delta\hat{\lambda}_{Ht}^M$, and $\Delta\hat{\lambda}_{Ht+k}^M = 0$ for $k \geq 2$. For this shock sequence, inflation is negative at date t : $\pi_{Ct} < 0$.

Proof. See Appendix C.3. □

In case (i) of the proposition, there is no shift in either the AS curve or the AD curve. Thus, the shock has no impact on inflation. This result strikes us as likely to surprise readers steeped in the literature, who might expect an increase in offshoring to necessarily lower inflation. In case (ii), the temporary decline in domestic sourcing for inputs has no direct impact on the Phillips Curve (AS curve). Instead, it works through aggregate demand, where the AD curve shifts down, because

²³Examining Equation 34, the shocks enter via the term $\frac{1}{(\eta-1)} [\Delta\hat{\lambda}_{Ht}^C - \beta E_t \Delta\hat{\lambda}_{Ht+1}^C]$. For a temporary shock, both the decline in domestic sourcing today ($\Delta\hat{\lambda}_{Ht}^C < 0$) and its subsequent rebound ($\Delta\hat{\lambda}_{Ht+1}^C > 0$) reduce inflation today. A temporary shock also leads the AD curve to shift down, since $\Delta\hat{\lambda}_{Ht+1}^C > 0$ lowers the real natural rate and reduces aggregate demand, thus further lowering inflation in period t .

expected mean reversion in the shock ($\Delta\hat{\lambda}_{Ht+1}^M > 0$) lowers the real natural rate. While rising trade induces a decline in inflation in this scenario – consistent with conventional wisdom – the mechanism is implausible: inflation falls because the transitory shock induces a recession (negative output gap) in period t . While much policy discussion emphasizes the possible deflationary impact of globalization, we know of no prominent discussion that emphasizes its recessionary impacts to explain the decline in inflation.

To conclude, this discussion re-emphasizes two of our main points. Trade dynamics drive inflation, largely through their impact on aggregate demand, and this point is overlooked in prior work on globalization and inflation. Furthermore, how trade dynamics matter depends on whether trade in final goods or inputs is changing.

2.4.3 The Monetary Policy Rule

So far, we have studied the effects of trade integration under pure (consumer price) inflation targeting. In this section, we extend the analysis to consider alternative policy rules. First, we present two propositions that extend Proposition 1 to allow for Taylor-type monetary policy rules, in which the central bank targets either consumer price inflation or domestic price inflation. Second, we identify a monetary policy rule that would eliminate the inflationary impact of an anticipated increase in trade integration and assess its plausibility.

Drawing on the domestic monetary policy literature, the first policy rule posits that the central bank responds to both consumer price inflation and the output gap, as in:

$$\hat{r}_t = \omega\pi_{Ct} + \gamma(\hat{y}_t - \hat{y}_t^n), \quad (37)$$

where $\gamma \geq 0$ is the sensitivity of the interest rate to the output gap.²⁴ The following proposition extends Proposition 1 to this case.

Proposition 4. *Consider (i) an anticipated, permanent decline in the domestic sourcing share for final goods, such that $\Delta\hat{\lambda}_{Ht}^C = 0$, $\Delta\hat{\lambda}_{Ht+1}^C < 0$, and $\Delta\hat{\lambda}_{Ht+k}^C = 0$ for $k > 1$, or (ii) an anticipated, permanent decline in the domestic sourcing share for inputs such that $\Delta\hat{\lambda}_{Ht}^M = 0$, $\Delta\hat{\lambda}_{Ht+1}^M < 0$, and $\Delta\hat{\lambda}_{Ht+k}^M = 0$ for $k > 1$. For shock sequences (i) or (ii), and the monetary policy rule described in Equation 37, inflation is positive at date t : $\pi_{Ct} > 0$.*

Proof. See Appendix C.4. □

Intuitively, adopting a Taylor rule just changes the slope of the AD schedule relative to our baseline case; when the interest rate is more sensitive to the output gap, the aggregate demand

²⁴See Clarida, Gali and Gertler (1999) or Galí (2015) for discussion of simple policy rules of this type.

schedule is steeper. Therefore, an anticipated reduction in home sourcing of final or intermediate goods will have qualitatively the same effects as in the baseline case.

The second policy rule replaces consumer price inflation (π_{Ct}) with domestic producer price inflation (π_{Ht}):

$$\hat{r}_t = \omega\pi_{Ht} + \gamma(\hat{y}_t - \hat{y}_t^n). \quad (38)$$

This second rule follows the international monetary policy literature, in which targeting domestic inflation is consistent with optimal fluctuations in the terms of trade in some benchmark models.²⁵ We provide a final proposition that covers this rule.

Proposition 5. *Consider (i) an anticipated, permanent decline in the domestic sourcing share for final goods, such that $\Delta\hat{\lambda}_{Ht}^C = 0$, $\Delta\hat{\lambda}_{Ht+1}^C < 0$, and $\Delta\hat{\lambda}_{Ht+k}^C = 0$ for $k > 1$, or (ii) an anticipated, permanent decline in the domestic sourcing share for inputs, such that $\Delta\hat{\lambda}_{Ht}^M = 0$, $\Delta\hat{\lambda}_{Ht+1}^M < 0$, and $\Delta\hat{\lambda}_{Ht+k}^M = 0$ for $k > 1$. For shock sequences (i) or (ii), and the monetary policy rule described in Equation 38, inflation is positive at date t : $\pi_{Ct} > 0$.*

Proof. See Appendix C.5. □

To understand this result, note that $\pi_{Ct} = \pi_{Ht} + \frac{1}{(\eta-1)}\Delta\hat{\lambda}_{Ht}^C$. For anticipated shocks, $\Delta\hat{\lambda}_{Ht}^C = 0$, so $\pi_{Ht} = \pi_{Ct}$ in the initial period, prior to realization of the change in domestic sourcing. Therefore, the two alternative policy rules in Equations 37 and 38 coincide at date t . In turn, the same AD/AS intuition from Proposition 1 applies.²⁶

The final question we consider is: what type of monetary policy rule might sterilize the effect of anticipated domestic sourcing shocks on inflation? It is easy to show that the central bank could mute the inflationary effect of trade integration by following a rule that allows for a time-varying natural rate of interest, as in:

$$\hat{r}_t = \hat{r}_t^n + \omega\pi_{Ct}. \quad (39)$$

This type of policy rule is sometimes referred to as a “stochastic intercept rule.” In contrast to the prior rules we have discussed, this rule has a forward looking dimension, because the natural rate responds to expected future variables. The following proposition describes the impact of anticipated changes in domestic sourcing on inflation at date t with this policy rule.

Proposition 6. *(i) Consider an anticipated, permanent decline in the domestic sourcing share for final goods, such that $\Delta\hat{\lambda}_{Ht}^C = 0$, $\Delta\hat{\lambda}_{Ht+1}^C < 0$, and $\Delta\hat{\lambda}_{Ht+k}^C = 0$ for $k > 1$. For this shock sequence, and the monetary policy rule described in Equation 39, inflation at date t is positive: $\pi_{Ct} > 0$. (ii)*

²⁵See discussion in Corsetti, Dedola and Leduc (2011) and Galí (2015).

²⁶For an anticipated change in the domestic sourcing of inputs, then $\Delta\hat{\lambda}_{Ht+1}^C = 0$ as well, so $\pi_{Ht+1} = \pi_{Ct+1}$. Thus, the equilibrium response is identical in all periods. In contrast, for an anticipated change in domestic sourcing of final goods, then $\Delta\hat{\lambda}_{Ht+1}^C \neq 0$. We solve for the full dynamics in this case in order to prove the proposition.

Consider an anticipated, permanent decline in the domestic sourcing share for inputs, such that $\Delta\hat{\lambda}_{Ht}^M = 0$, $\Delta\hat{\lambda}_{Ht+1}^M < 0$, and $\Delta\hat{\lambda}_{Ht+k}^M = 0$ for $k > 1$. For this shock sequence, and the monetary policy rule described in Equation 39, inflation date t is zero: $\pi_{Ct} = 0$.

Proof. See Appendix C.6 □

By targeting the natural rate, the policy rule in Equation 39 fully offsets the aggregate demand effects of anticipated changes in sourcing. In case (ii), this implies that there is no inflation as a result of an anticipated change input sourcing, because anticipated changes in input sourcing work only through aggregate demand. For case (i), the anticipated decline in final goods sourcing still raises inflation, because the AS schedule shifts up, as it did in Proposition 1 as well. Because policy neutralizes the AD effects of the shock, the magnitude of the increase in inflation would be diminished.

While analysis of this final policy rule serves to emphasize and clarify the role of trade dynamics in driving inflation, we do not view it as particularly realistic. First, practical rules for policy typically feature weaker dependence of policy on neutral rates than implied by Equation 39.²⁷ In addition, rules that target the time-varying neutral interest rate are often thought to be impractical, due to informational barriers that prevent the central bank from observing the neutral rate in real time. Lastly, as a historical matter, we know of no policymaker who argued that interest rates needed to be raised more aggressively than called for by current inflation due to globalization, which suggests that Equation 39 is not a good description of the policy rule used in practice.

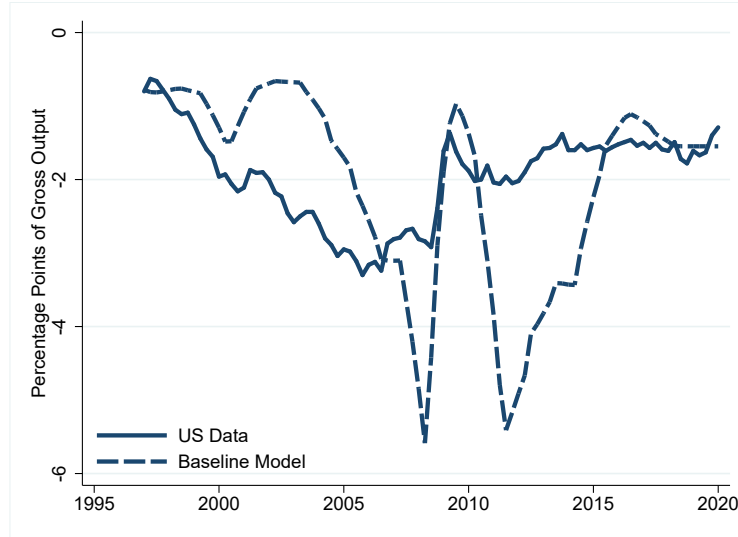
3 Extensions to the Baseline Model: Financial Inflow Shocks and Import Competition

We now consider two substantive extensions to the baseline model.²⁸ The first extension is to introduce financial inflow shocks that allow the model to match the path of the US trade deficit over time. The second extension is to introduce variable markups and dollar currency pricing in the model, allowing for import competition to influence prices. For clarity sake, we examine these modifications one at a time.

²⁷For example, the Federal Reserve itself refers to monetary policy rules that include “long run” neutral interest rates, which are time varying only at low frequencies. Further, it notes that practical considerations often argue for inertial or first difference rules, in which current rates are set based (in part) on lagged values of the policy rate. Both these considerations would imply policy rules that do not fully mute the impacts of anticipated shocks of medium duration, as observed in the data. See <https://www.federalreserve.gov/monetarypolicy/policy-rules-and-how-policy-makers-use-them.htm>.

²⁸We consider a third extension to the baseline model in Appendix D, where we introduce physical capital into the model, in line with medium-scale New Keynesian models commonly used for monetary policy analysis. The punchline is that the evolution of inflation in the baseline model is robust to introducing physical capital.

Figure 5: Trade Deficit (% GDP)



Note: The line labeled US Data is the ratio of the quarterly US trade balance (from the BEA) to gross output. We measure quarterly gross output by dividing quarterly GDP data from the BEA by the ratio of value added to gross output (measured at an annual frequency) in BEA input-output data.

3.1 Financial Inflow Shocks and Trade Imbalances

In addition to trade integration, globalization was also associated with increased financial integration and a US current account (trade) deficit. In the baseline model, we assumed that international financial markets were complete, so current account imbalances were determined by consumption risk sharing. Further, we omitted shocks – to exogenous foreign variables or financial integration itself – that target the actual path of trade and current account deficits over time. As a result, the trade deficit in the baseline model differs from data.

We plot the trade deficit in the baseline model (with shocks to domestic sourcing) and official US quarterly data in Figure 5, both expressed as a share of gross output. While the baseline model generates a widening of the US trade deficit through the mid-2000s, it under predicts the deficit during this “global savings glut” period. Further, the model yields large swings in the deficit around the Great Recession, and fails to pick up on the sustained improvement in the trade deficit during the 2010-2015 interval.

Given these discrepancies, one naturally wonders whether deviations from perfect risk sharing, or foreign shocks that influence international financial flows, which are needed to match the actual evolution of US trade imbalances, are important for our inflation results? Specifically, we emphasized that trade integration drives up the real natural rate of interest in the three equation model, which stokes inflation. A common view is that the global savings glut drove down real interest rates during this period, so one might think this would lower inflation as a result. To investigate this mechanism, we develop an extension to baseline model that allows us to study the role of the trade balance in driving inflation.

To incorporate financial inflow shocks in the model, we drop the complete markets assumption, and replace it with a simple alternative framework that takes the path of the trade deficit as given. Using notation from previous sections, the trade deficit in the model is $TD_t = P_{Ft}\tau_{Ct}C_{Ft} + P_{Ft}\tau_{Mt}M_{Ft} - P_{Ht}X_t$. Now define $TDY_t \equiv \frac{TD_t}{P_{Ht}Y_t}$ to be the ratio of the trade deficit to nominal gross output. Taking the ratio of the trade balance to gross output (TDY_t) as given, we can then replace the risk-sharing condition in the original model with this statement of the (exogenous) trade balance. With slight abuse of language, we refer to TDY_t as a “financial inflow shock” in discussion that follows.

This approach gives us an additional degree of freedom to match the evolution of the US trade deficit. Further, the presence of this shock in the model may also have an independent impact on inflation. To analyze these effects, we turn to the three equation model, augmented to include the financial inflow shock.

3.1.1 Financial Inflow Shocks in the Three Equation Model

In Appendix E, we provide a full treatment of the three equation model with financial inflow shocks. We show first that these shocks do not enter the Phillips curve directly. We then re-derive the IS curve, where financial inflow shocks play an important role. Superficially, the IS curve is still given by Equation 35. However, under the hood, financial inflow shocks influence the real natural rate of interest, which is given by:

$$\hat{r}_t^n = -\tilde{\Upsilon}_M E_t \left(\Delta \hat{\lambda}_{Ht+1}^M \right) - \tilde{\Upsilon}_C E_t \left(\Delta \hat{\lambda}_{Ht+1}^C \right) + \tilde{\Upsilon}_{TD} E_t \left(\Delta \widehat{tdy}_{t+1} \right) \quad (40)$$

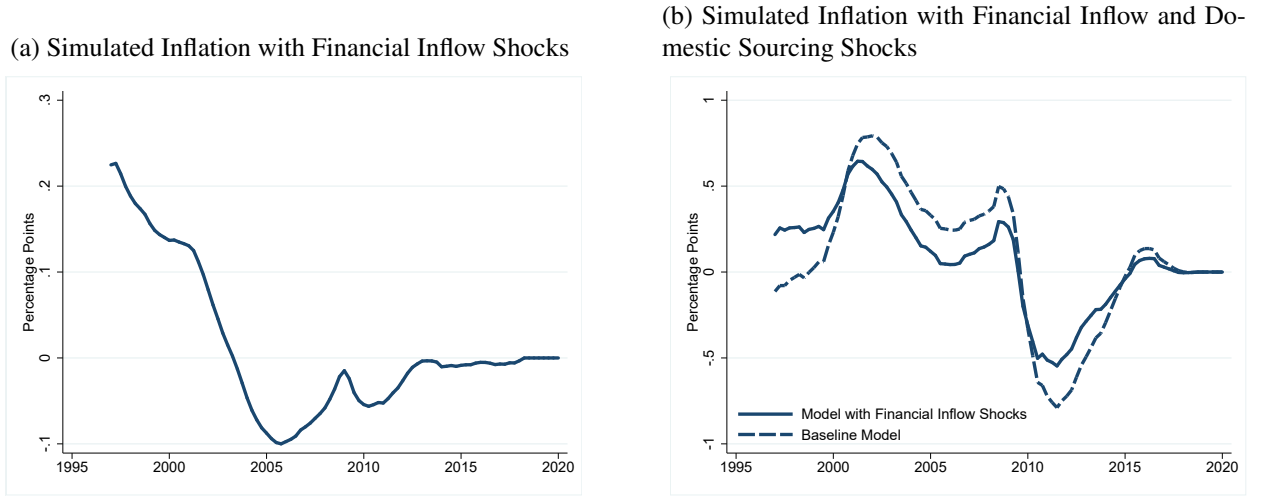
where $\tilde{\Upsilon}_M > 0$, $\tilde{\Upsilon}_C > 0$ if and only if $\rho > 1$, and $\tilde{\Upsilon}_{TD} > 0$.²⁹

In Equation 40, note that the dynamics of the trade deficit across periods determine the natural interest rate, not the level of the trade deficit itself. If the trade deficit is expected to widen from t to $t+1$, such that $\Delta \widehat{tdy}_{t+1} > 0$, then the natural rate of interest rises. The reason is that an expected increase in the trade deficit is associated with higher future consumption relative to present consumption, and this demands a higher interest rate via the Euler equation.

The dependence of the natural rate on changes in capital inflows, rather than their level, runs counter to the intuition we discussed above about how one might think that the global savings glut and resulting US trade deficit should lower the real interest rate. It is the case that a one-time positive shock to capital inflows lowers the natural interest rate. In this case, $\widehat{tdy}_t > 0$ and $E_t \widehat{tdy}_{t+1} = 0$, so that $E_t \Delta \widehat{tdy}_{t+1} < 0$. This temporary capital inflow shock would lead domestic agents to consume more in period t , which lowers the natural rate of interest (via the Euler equation) and ultimately inflation. However, this is not the kind of shock that is relevant in the early 2000’s.

²⁹To simplify this expression, we set changes in foreign consumption and productivity to zero.

Figure 6: Simulation of Inflation in the Model with Financial Inflow Shocks



Referring back to Figure 5, the early 2000's saw sustained increases in the trade deficit year over year. Thus, their effects do not conform to this one-off shock intuition. The phased widening of the deficit led to a sequence of years in which $\Delta tdy_{t+1} > 0$. Thus, anticipation of widening deficits over this period exerted upward pressure on the natural interest rate, rather than downward pressure. Again, this stokes inflation. In contrast, closure of the deficit from the mid-2000s onward exerts downward pressure on the natural rate of interest and thus inflation. We will see these dynamics borne out in simulations that incorporate this shock.

3.1.2 Inflation Dynamics with Financial Inflow Shocks

Following the same procedure for simulating the model as in previous sections, we plot the evolution of inflation given the path of observed trade deficits (\widehat{tdy}_t) in Figure 6a, with domestic sourcing shocks set to zero. Consistent with the narrative above, the widening of the US trade deficit actually serves to push up inflation in early years, and then the anticipated closure of it yields disinflation in the middle years. Note also the magnitudes in this figure: the capital inflow shocks alone yield modest inflation/deflation, less pronounced than the impact of the domestic sourcing shocks.

In Figure 6b, we plot simulated inflation with all three shocks active (the capital inflow shock plus the two domestic sourcing shocks) in this model. For comparison, we also plot inflation from simulation of the baseline model. Consistent with the muted impacts of the capital inflow shocks, the qualitative and quantitative results we obtained in the baseline continue to hold in this extended model. Therefore, we conclude that inflation dynamics do not seem to have been impacted much by current account dynamics during this time frame, above and beyond the impact of changes in domestic sourcing. Further, since this model drops the complete markets assumption, we also

conclude that financial market structure does not play a significant role in our results.

3.2 Import Competition with Variable Markups

Thus far, we have considered models with CES preferences and production functions. While these models allow for variable markups due to price adjustment frictions, they do not allow optimal (flexible price) markups to vary with import competition. This omits any potential role for import competition in lowering markups, and thus influencing inflation, as emphasized by [Bernanke \(2007\)](#) for example. To incorporate pro-competitive effects of trade, we extend the model to incorporate [Kimball \(1995\)](#) aggregators for final goods and inputs and dominant (dollar) currency pricing of imports, as in [Gopinath et al. \(2020\)](#). One distinct contribution of this section will be to demonstrate that our sufficient statistics approach to analyzing the model can be applied in this more sophisticated setting.

3.2.1 Main New Assumptions and Results

There are three important changes in this version of the model relative to the baseline. First, we assume now that aggregators for final and intermediate goods are given by:

$$\nu \int_0^1 \Upsilon \left(\frac{C_{Ht}(i)}{\nu C_t} \right) di + (1 - \nu) \int_0^1 \Upsilon \left(\frac{C_{Ft}(i)}{(1 - \nu) C_t} \right) di = 1 \quad (41)$$

$$\xi \int_0^1 \Upsilon \left(\frac{M_{Ht}(i)}{\xi M_t} \right) di + (1 - \xi) \int_0^1 \Upsilon \left(\frac{M_{Ft}(i)}{(1 - \xi) M_t} \right) di = 1, \quad (42)$$

where $C_{Ft}(i)$ and $M_{Ft}(i)$ are consumption of individual foreign varieties, the parameters ν and ξ govern home bias, and the function $\Upsilon(\cdot)$ satisfies $\Upsilon(1) = 1$, $\Upsilon'(\cdot) > 0$, and $\Upsilon''(\cdot) < 0$. As in [Klenow and Willis \(2016\)](#) and [Gopinath et al. \(2020\)](#), we parameterize $\Upsilon(\cdot)$ using a flexible functional form:

$$\Upsilon(x) = 1 + (\sigma - 1) \exp \left(\frac{1}{\varepsilon} \right) \varepsilon^{\sigma/\varepsilon - 1} \left(\Gamma \left(\frac{\sigma}{\varepsilon}, \frac{1}{\varepsilon} \right) - \Gamma \left(\frac{\sigma}{\varepsilon}, \frac{x^{\varepsilon/\sigma}}{\varepsilon} \right) \right), \quad (43)$$

where $\Gamma(u, z) = \int_z^\infty s^{u-1} e^{-s} ds$ is the incomplete gamma function, with $\sigma > 1$ and $\varepsilon > 0$. Given these aggregators, import demand for final goods and inputs now features variable elasticities, so optimal markups for Home firms vary with aggregate market conditions – in particular, they are lower when import competition is tough.

Second, we assume that import prices are set in dollars, subject to Rotemberg adjustment costs. The solution to the pricing problems for foreign firms yields a dynamic pricing equation for imports, similar to the price setting equations for domestic firms.

Third, we assume that Home and Foreign markets are segmented, so Home producers set prices

independently for domestic and export sales. This simplifies the dynamics of the domestic price level, but plays an otherwise minor role in the results.

We provide a full characterization of the model in Appendix F. Here we emphasize three key results for understanding how we use this model and interpret the results.

The first result is that the sufficient statistics approach to model analysis continues to apply in this model. To sketch the argument, the log linear approximation to the first order condition for demand for domestic final goods in the symmetric firm equilibrium is:

$$\hat{c}_{Ht} = -\sigma \left(\frac{C_{H0}}{\nu C_0} \right)^{-\varepsilon/\sigma} \left(\hat{d}_{Ct} + \hat{p}_{Ht} - \hat{p}_{Ct} \right) + \hat{c}_t \quad (44)$$

where \hat{d}_{Ct} is an endogenous term that indexes the level of demand under Kimball aggregation, and the subscript 0 denotes steady state values. This result implies that the log-linear approximation of demand has a constant elasticity, governed by parameters and steady state values. In the calibrated steady state, $C_{H0} = \nu C_0$. Further, we show in the appendix that $\hat{d}_{Ct} = 0$ in the solution to the log-linearized model. Combining these two observations, we can then solve for the relative price of home goods, just like in the baseline model: $\hat{p}_{Ht} - \hat{p}_{Ct} = -\frac{1}{\sigma-1} \hat{\lambda}_{Ht}^C$. Analogously, $\hat{p}_{Ht} - \hat{p}_{Mt} = -\frac{1}{\sigma-1} \hat{\lambda}_{Ht}^M$ for imported inputs. Thus, we can substitute for these relative prices throughout the model to write the equilibrium in terms of deviations in domestic sourcing shares from the steady state. Taking values for $\hat{\lambda}_{Ht}^C$ and $\hat{\lambda}_{Ht}^M$ as given, these results imply that we need not solve for the relative price of domestic final goods or inputs.³⁰

The second result concerns the domestic price Phillips curve, which reflects optimal pricing for Home firms. In the log-linear symmetric firm equilibrium, it is given by:

$$\pi_{Ht} = -\frac{1}{\phi} \hat{e}_{Ht} + \left(\frac{\epsilon_{H0} - 1}{\phi} \right) \widehat{r\bar{m}c}_t + \beta E_t(\pi_{Ht+1}), \quad (45)$$

where $\widehat{r\bar{m}c}_t = \hat{m}c_t - \hat{p}_{Ht}$. Here ϵ_{H0} is the elasticity of demand faced by Home firms for sales to domestic buyers in steady state, and \hat{e}_{Ht} is the log deviation in this elasticity of demand at date t from its steady state value. When the elasticity of demand is larger than its steady state value ($\hat{e}_{Ht} > 0$), then Home firms reduce their markups and thus domestic price inflation is lower (all else equal). For further insight, we can characterize this demand elasticity as function of the domestic

³⁰This implies that we need not actually solve the import pricing problem to characterize equilibrium variables that determine domestic inflation. To make this argument transparent, we first write down the full equilibrium that includes the solution for the import pricing problem in Appendix F, and then we reduce the model down using the sufficient statistics argument.

sourcing shares in the model (see Appendix F for derivation):

$$\hat{\epsilon}_{Ht} = \frac{C_{H0}}{Y_{H0}} \hat{\epsilon}_{Ht}^C + \frac{M_{H0}}{Y_{H0}} \hat{\epsilon}_{Ht}^M \quad (46)$$

$$\text{with } \hat{\epsilon}_{Ht}^C = -\left(\frac{\varepsilon}{\sigma-1}\right) \hat{\lambda}_{Ht}^C \quad \text{and} \quad \hat{\epsilon}_{Ht}^M = -\left(\frac{\varepsilon}{\sigma-1}\right) \hat{\lambda}_{Ht}^M. \quad (47)$$

Thus, the elasticities by end use $\hat{\epsilon}_{Ht}^C$ and $\hat{\epsilon}_{Ht}^M$ are decreasing functions of domestic sourcing, where the parameter ε controls the elasticity of markups to relative prices. This is the import competition channel: demand elasticities are lower (markups are higher) when domestic sourcing is high. For readers familiar with markup shocks in the New Keynesian literature, import competition restrains markups, operating like a markup shock in this Phillips curve.

While this second result is one channel through which variable markups influence inflation, there is a distinct role for markup dynamics in shaping aggregate demand. The third key result from the model is that markup dynamics influence the natural interest rate, through their impact on consumption dynamics in the flexible price equilibrium. As in standard monopolistic competition models, markups depress equilibrium output – they deter input use and distort labor supply down by lowering real wages. An expected decline in domestic sourcing reduces expected future markups for Home firms. In turn, it raises expected output in the future relative to current output, which generates an expected real exchange rate depreciation. Via risk sharing, the expected depreciation raises the expected growth rate of consumption, and thus the natural interest rate today. We will discuss how this third channel is important for interpreting our simulation results below.

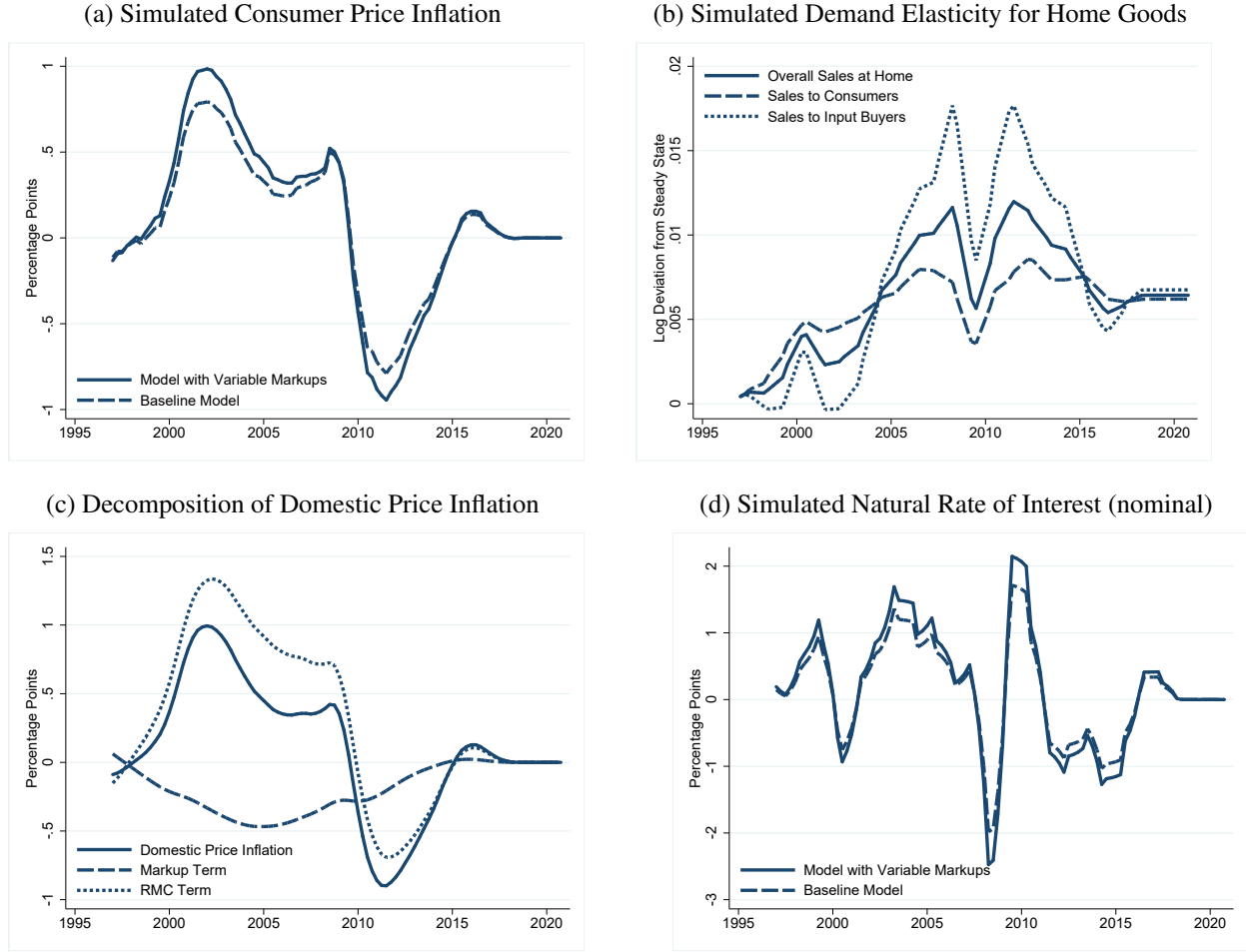
3.2.2 Inflation Dynamics with Variable Markups

With these key results in hand, we proceed directly to simulation.³¹ In Figure 7a, we plot inflation in this model, along with results from the baseline model. The model with variable markups features a larger increase in inflation during the 2000-2010 period than the baseline model, and then a stronger disinflation during the post-2010 period. Far from dampening the impact of rising trade on inflation, variable markups actually make inflation more responsive to changes in trade. Cumulative inflation from 1997 to 2018 is also higher in the model with variable markups, leaving the price level two percentage points higher than in the baseline simulation ($\approx 10\%$ versus $\approx 8\%$ total increase in consumer prices).

To inspect the mechanism, we plot deviations of the elasticity of demand for Home firms from steady state ($\hat{\epsilon}_{Ht}$) in Figure 7b, along with the underlying elasticities of demand for Home final

³¹We calibrate the model with variable markups to match the same initial steady state as the baseline model. Further, we set the parameter $\sigma = 3$ in the Klenow-Willis Υ -function, which controls elasticity of substitution between home and foreign goods in the steady state, to match the baseline model. Following Gopinath et al. (2020), we set $\varepsilon = 1$, which governs the elasticity of the elasticity of substitution with respect to relative prices.

Figure 7: Simulation of Inflation in the Model with Variable Markups



goods ($\hat{\epsilon}_{Ht}^C$) and inputs ($\hat{\epsilon}_{Ht}^M$) separately. Per the discussion above, note these track the inverted path of changes in domestic sourcing over time, so that the elasticity of demand peaks (markups are lowest) in the mid-2000s. These elasticities determine the size of the effective “markup shock” in the domestic price Phillips curve (Equation 45).

To illustrate the influence of these changes in markups on inflation at each point in time, we solve the domestic price Phillips curve forward. However, because markups change in perpetuity due to the permanent nature of the shocks, we adjust the standard inflation accounting equation to account for this feature of our exercise. Denoting the terminal steady state by T , the following relationship between markups and real marginal costs holds: $\widehat{r\bar{m}c}_T = -\left(\frac{1}{\epsilon_{H0}-1}\right)\hat{\epsilon}_{HT}$, and domestic price inflation in the terminal steady state is consequently zero. If the economy jumped immediately to this long run equilibrium, then inflation would be given by $\tilde{\pi}_H = -\frac{1}{\phi}\sum_{s=0}^{\infty}\beta^s\hat{\epsilon}_{HT} +$

$\left(\frac{\epsilon_{H0}-1}{\phi}\right) \sum_{s=0}^{\infty} \beta^s \widehat{rmc}_T = 0$. We look at deviations in actual date- t inflation from this benchmark:

$$\pi_{Ht} = \underbrace{-\frac{1}{\phi} \sum_{s=0}^{\infty} \beta^s E_t [\hat{\epsilon}_{Ht+s} - \hat{\epsilon}_{HT}]}_{\text{Markup Term}} + \underbrace{\left(\frac{\epsilon_{H0}-1}{\phi}\right) \sum_{s=0}^{\infty} \beta^s E_t [\widehat{rmc}_{t+s} - \widehat{rmc}_T]}_{\text{RMC Term}}. \quad (48)$$

We plot the results of this decomposition in Figure 7c. The markup term evidently depresses inflation, by almost 50 basis points when domestic sourcing is lowest in the mid-2000s. Nonetheless, this pro-competitive effect is more than offset by endogenous changes in real marginal costs, which drive domestic price inflation up. Further, pro-competitive effects play no role in explaining the pivot from inflation to deflation after 2010.

To understand why real marginal costs more than offset the direct impacts of markup restraint on inflation, we turn a discussion of the natural interest rate. As noted above, markup shocks not only shift the Phillips Curve, they also influence the level of aggregate demand. In Figure 7d, we see that the (nominal) natural interest rate is more pro-cyclical in this model than in the baseline model with constant markups. As we asserted, the natural rate rises in anticipation of declines in domestic sourcing, and more forcefully in the variable markups model than in the baseline. An anticipated decline in future markups raises the neutral rate and thus aggregate demand today, which reinforces the inflationary impact of anticipated declines in domestic sourcing analyzed in previous sections.

To sum up, like the capital inflow shocks above, allowing for variable markups in the model does not substantially impact our overall conclusions regarding the impact of trade dynamics on inflation. This is surprising, in that pro-competitive effects of trade on domestic prices are commonly cited as an important factor that might restrain inflation. We have shown that this conventional wisdom is incomplete as well, as it fails to account for general equilibrium effects that dominate the partial equilibrium impact of trade on markups and prices. Put differently, aggregate demand effects overwhelm competing aggregate supply (Phillips curve) channels.

3.2.3 The Changing Slope of the Empirical Phillips Curve

To motivate a second application of this model with variable markups, we note that recent empirical work finds that the slope of the empirical Phillips curve – the responsiveness of inflation to proxies for domestic economic slack (e.g., the output or unemployment gap) – appears to have declined since the early 2000's [Ball and Mazumder (2011); Del Negro et al. (2020); Stock and Watson (2021)]. It has been conjectured that the decoupling of inflation from domestic slack may (in part) be a result of globalization [Forbes (2019)], so we use our model to contribute to this debate here.

We start by recalling that the slope of the Phillips Curve (to a first order) is constant in the base-

line model, presented in Section 2. In that model, the sensitivity of domestic price inflation to real marginal costs ($\frac{\epsilon-1}{\phi}$) and the output gap ($\frac{(\epsilon-1)(1-\alpha)\chi}{\phi}$) are pinned down by the value of parameters in the pre-globalization steady state. In the model with variable markups, the sensitivity of inflation to real marginal costs is also constant ($\frac{\epsilon_{H0}-1}{\phi}$). Therefore, trade integration (to a first order) does not affect the sensitivity of inflation to domestic slack in either model.

Nonetheless, globalization may influence *empirical estimates* of the slope of the Phillips curve, if those estimates fail to control for the impact of trade on markups. Referring back to the domestic price Phillips curve in Equation 45, note that one needs to control both for expected inflation ($E_t\pi_{Ht+1}$) and trade-induced changes in markups ($\hat{\epsilon}_{Ht}$) to recover the true relationship between inflation (π_{Ht}) and real marginal costs ($\widehat{r\bar{m}c}_t$). In practice, however, most empirical research examines regressions of π_{Ht} on proxies for $\widehat{r\bar{m}c}_t$ without controlling for markup shocks. This results in omitted variable bias in the estimated slope of the *empirical* Phillips curve – i.e., the partial correlation of π_{Ht} with $\widehat{r\bar{m}c}_t$ (or proxies) conditional on $E_t\pi_{Ht+1}$ – which does not control for markup shocks. The direction of this bias depends on the correlation of $\hat{\epsilon}_{Ht}$ with $\widehat{r\bar{m}c}_t$. In our simulated data, this correlation is negative – markups are low due to import competition when the output gap is positive and real marginal costs are high. As a result, failing to control for trade-induced changes in markups would impart downward bias on the slope of the *empirical* Phillips Curve.

To quantify this bias, we turn to our simulated data. The true theoretical relationship between domestic price inflation and real marginal costs is given by Equation 45, where $\frac{\epsilon_{H0}-1}{\phi} = 0.084$. We then consider the following empirical Phillips Curve regression in our simulated data: $\pi_{Ht} = \varrho_1\widehat{r\bar{m}c}_t + \varrho_2E_t(\pi_{Ht+1}) + u_t$, where $u_t = -\frac{1}{\phi}\hat{\epsilon}_{Ht}$ is treated as a regression residual. The resulting OLS estimates are $\hat{\varrho}_1 = 0.05$ and $\hat{\varrho}_2 = .994$ (note $\hat{\varrho}_2 \approx \beta$). The punchline is that failing to control for markup changes biases the estimated slope of the Phillips Curve downward (toward zero) by about 40%.

To summarize, trade-induced changes in markups restrain inflation relative to what it would have been given changes in real marginal costs (or domestic slack more generally) in our model simulations. In turn, estimates of the Phillips curve during this globalization era that do not control for changing markups suffer from an omitted variable bias, which makes inflation appear less sensitive to domestic slack than it truly is. This bias may help explain why the slope of the empirical Phillips curve has declined in the last couple decades, when pro-competitive effects of trade have plausibly been strongest.

4 Multisector Model

We now turn to a multisector version of the baseline model, in which we allow for heterogeneous changes in domestic sourcing across industries. In doing so, we come full circle to tie our results

for simulated inflation back to the multisector accounting framework we introduced in Section 1.

4.1 Model Overview

The multisector model, with sectors indexed by $s \in \{1, \dots, S\}$, is an extension of the baseline model.³² On the consumption side, we adopt a standard nested CES framework:

$$C_t = \left(\sum_s \zeta(s)^{1/\vartheta} C_t(s)^{(\vartheta-1)/\vartheta} \right)^{\vartheta/(\vartheta-1)} \quad (49)$$

$$C_t(s) = \left(\nu(s)^{1/\eta(s)} C_{Ht}(s)^{(\eta(s)-1)/\eta(s)} + (1 - \nu(s))^{1/\eta(s)} C_{Ft}(s)^{(\eta(s)-1)/\eta(s)} \right)^{\eta(s)/(\eta(s)-1)} \quad (50)$$

$$C_{Ht}(s) = \left(\int_0^1 C_{Ht}(s, i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}, \quad (51)$$

where $C_t(s)$ is aggregate consumption of the sector- s composite goods, which is a CES composite of domestic ($C_{Ht}(s)$) and foreign ($C_{Ft}(s)$) final goods. In the system, $\vartheta \in [0, \infty)$ is the elasticity of substitution across sector composites, and $\eta(s) \in [0, \infty)$ is the sector-specific elasticity of substitution between home and foreign composites. The CES weights satisfy $\sum_s \zeta(s) = 1$ and $\nu(s) \in [0, 1]$.

Individual varieties (indexed by i) are produced using nested CES production functions:

$$Y_t(s, i) = Z_t(s) (L_t(s, i))^{1-\alpha(s)} (M_t(s, i))^{\alpha(s)} \quad (52)$$

$$M_t(s, i) = \left(\sum_{s'} \left(\alpha(s', s) / \alpha(s) \right)^{1/\kappa} M_t(s', s, i)^{(\kappa-1)/\kappa} \right)^{\kappa/(\kappa-1)} \quad (53)$$

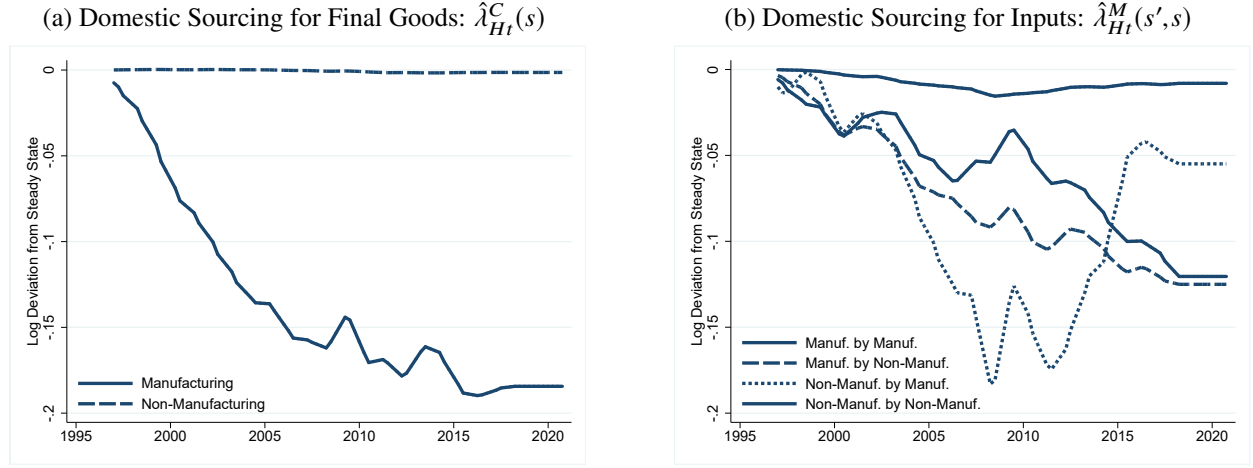
$$M_t(s', s, i) = \left[\xi(s', s)^{\frac{1}{\eta(s')}} M_{Ht}(s', s, i)^{\frac{\eta(s')-1}{\eta(s')}} + (1 - \xi(s', s))^{\frac{1}{\eta(s')}} M_{Ft}(s', s, i)^{\frac{\eta(s')-1}{\eta(s')}} \right]^{\frac{\eta(s')}{\eta(s')-1}} \quad (54)$$

where $M_{Ht}(s', s, i)$ is the quantity of a composite home good from sector s' used by firm i in sector s , $M_t(s', s, i)$ is the composite input from sector s' used by firm i in sector s , which aggregates $M_{Ht}(s', s, i)$ and a composite foreign input $M_{Ft}(s', s, i)$, and $M_t(s, i)$ is the overall composite input used by firm i in sector s . The parameter $\kappa \in [0, \infty)$ is the elasticity of substitution across sectors in input use, and parameter restrictions $\alpha(s) \in [0, 1]$, $\sum_{s'} \alpha(s', s) = \alpha(s)$, and $\xi(s', s) \in [0, 1]$ hold.

As in the baseline model, we again use domestic sourcing shares as sufficient statistics. Due to the multisector structure, there are now $S + S^2$ domestic sourcing shares $\{\lambda_{Ct}(s), \lambda_{Mt}(s', s)\}$ that are proxies for relative prices $\left\{ \frac{P_{Ht}(s)}{P_{Ct}(s)}, \frac{P_{Ht}(s')}{P_{Mt}(s', s)} \right\}$, where $P_{Ht}(s)$ is the price of a representative producer

³²We describe rudiments of the model here, and present the full model in the Appendix G. The model could be extended to include the refinements in Section 3, without altering the main results.

Figure 8: Changes in Domestic Sourcing by Sector



in sector s , $P_{Ct}(s)$ is the price of the composite final good for sector s , and $P_{Mt}(s', s)$ is the price of the composite input purchased by sector s from sector s' .

4.2 Inflation Dynamics with Two Sectors

We apply the same procedure to simulate the multisector model as used in previous sections. Here we present results for a two-sector version of the model, distinguishing manufacturing from a composite non-manufacturing sector.³³

Changes in domestic sourcing are plotted in Figure 8. In Figure 8a, domestic sourcing of final goods falls by almost 20 percent in manufacturing, while it is nearly unchanged in non-manufacturing. In Figure 8b, there is a pronounced decline in domestic sourcing of manufacturing inputs by both the manufacturing and non-manufacturing sectors, of about 12 percent each. There are very heterogeneous developments for sourcing of non-manufacturing inputs across sectors. While domestic sourcing of non-manufacturing inputs by the non-manufacturing sector itself is little changed, there is initially a large decline in domestic sourcing of non-manufacturing inputs by the manufacturing sector, and then a rebound after 2010. We explore the role of this particular shock in several different ways below.

Feeding these changes in domestic sourcing through the model, we plot consumer price inflation in Figure 9a. Similar to the baseline model, inflation is positive throughout most of the period, with the large positive inflation rates in the 2000-2010 period. Thus, dis-aggregating the model to allow for multiple sectors does not qualitatively change the main conclusions; the quantitative

³³New parameters that govern sector expenditure shares, value-added to output ratios, and the input-output structure across sectors are set to match US input-output data in the initial period (see Appendix G). Throughout the simulations that follow, we impose Cobb-Douglas preferences across sectors in final consumption and input use, equivalent to setting $\kappa = 1$ and $\vartheta = 1$. With one exception discussed below, we set $\eta(s) = \eta = 3$.

magnitudes are quite similar as well. In Figure 9b, we examine simulated inflation for final goods and input shocks separately. As in the baseline model, input sourcing shocks drive the medium term dynamics for inflation. Changes in domestic sourcing for final goods raise inflation in the first few years of the simulation, and they never lower inflation much below zero.

In Figure 9c, we disaggregate the shocks by sector and plot simulated data for manufacturing and non-manufacturing shocks separately. One point that stands out here is that the dynamics of sourcing in the non-manufacturing sector are important in explaining the dip in inflation in the post-2010 period, while both shocks drive inflation up in the pre-2010 period.

Another way to emphasize this result is to examine an alternative simulation with unequal elasticities across sectors. We consider a scenario in which the elasticity between home and foreign goods is higher for non-manufacturing than manufacturing, with an elasticity of 2 in manufacturing and an elasticity of 6 in non-manufacturing. This is a plausible case, since non-manufacturing includes agriculture and natural resources. In Figure 9d, we plot the results from this heterogeneous elasticity simulation, along with the equal elasticity simulation (repeated from Figure 9a). Inflation is substantially higher during the 2010-2020 period in this simulation. The reason is that the larger elasticity for the non-manufacturing sectors shrinks the importance of the u-shaped evolution of domestic sourcing of non-manufacturing inputs by the manufacturing sector, because it reduces the implied change in the relative price of home versus foreign non-manufactured inputs. In general, however, allowing heterogeneous elasticities does not change the overall behavior of the inflation.

4.3 Accounting for Price Changes in the Multisector Model

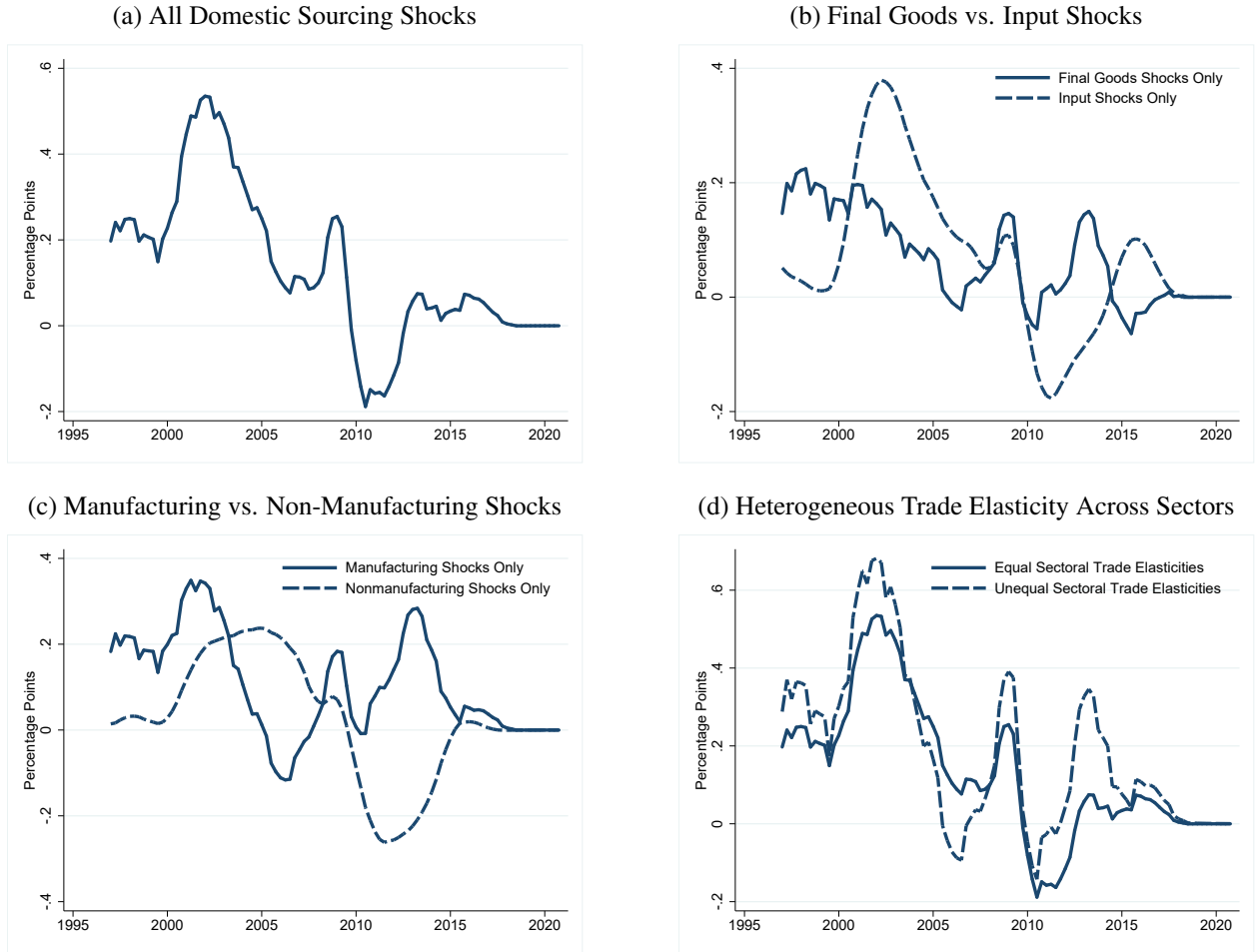
In Section 1, we presented accounting results that suggested rising trade played a large role in explaining price changes over time. In contrast, throughout the model simulations, we have emphasized the exact opposite result: inflation rises due to increasing trade. As a final exercise, we bring these results together, by demonstrating that the basic accounting results hold in this multi-sector model.

In this multisector model, sector-level inflation for domestic goods can be written as:

$$\pi_{Ht} = [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{I} - \alpha] \pi_{Vt} + \underbrace{\left(\frac{1}{\eta - 1} \right) [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{A}' \circ \Delta \hat{\lambda}'_{Ht}]}_{\text{Offshoring Shock}} \iota, \quad (55)$$

where π_{Ht} is a $S \times 1$ vector with elements $\pi_{Ht}(s)$, π_{Vt} is a vector of sector-specific inflation rates for the price of real value added (i.e., sectoral GDP deflators), \mathbf{A} is the steady state input-output matrix, α is a diagonal matrix of steady state shares of inputs in gross output, and $\hat{\lambda}_{Ht}$ is a matrix

Figure 9: Simulated Inflation in the Multisector Model



with elements $\hat{\lambda}(s', s)$ where s' indexes row and s indexes column.³⁴

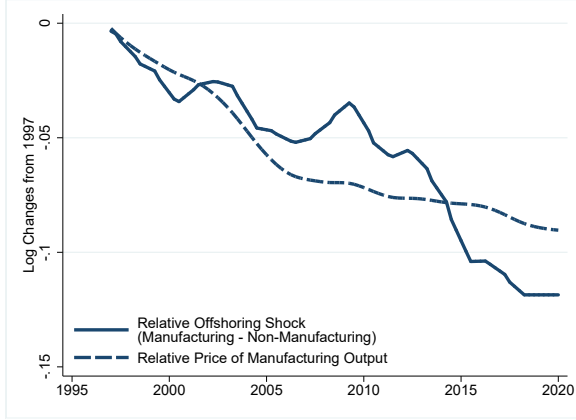
Accumulating these sector-level inflation rates and the offshoring shock over time, we plot the evolution of the simulated relative price of manufacturing goods ($P_{Ht}(m)/P_{Ht}(n)$) over time, along with the cumulative impact of the Offshoring Shock term, in Figure 10a. This figure is analogous to Figure 3b.³⁵ The large change in offshoring for manufacturing industries drives down the relative price of manufacturing in the model, by about twelve percent in the long run. This is a causal statement in the model, where all else is held constant, unlike in the accounting exercise. Note too that the relative price diverges from relative offshoring – both at any given time and in the long run, due to the internal dynamics of π_{Vt} in the model in response to shocks.

³⁴We impose $\eta(s) = \eta$, which facilitates comparison to Equation 12 and matches our parameterization of this model.

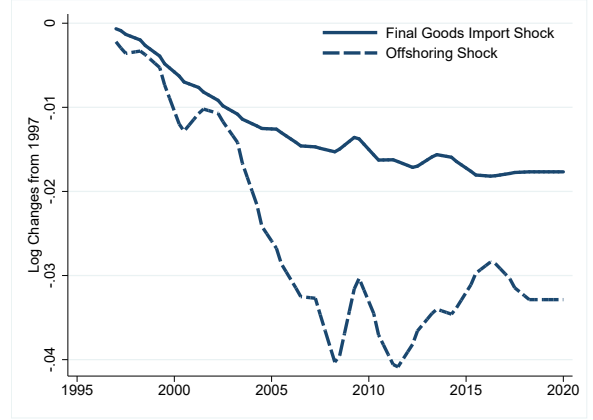
³⁵To be clear, the data series plotted in Figure 3b are not strictly comparable to the simulated data and aggregated shocks in the model. In Figure 3b, we aggregated by taking simple means of highly disaggregated manufacturing and non-manufacturing sectors, while here we are plotting simulation output from the two-sector model.

Figure 10: Accounting for Price Changes in the Multisector Model

(a) Relative Price and Offshoring Shock for Manufacturing vs. Non-Manufacturing



(b) Accounting for Changes in the Price Level



Note: In Panel (a), the relative price of manufacturing output is equal to $\sum_{s=1997}^t D \pi_{Ht}$ and the relative offshoring shock is $\sum_{s=1997}^t D [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{A}' \circ \Delta \hat{\lambda}'_{Hs}] \iota$, where D is a 1×2 matrix with elements 1 and -1 that takes differences across sectors. In Panel (b), the final goods import shock term is $\sum_{s=1997}^t \left(\frac{1}{\eta-1}\right) \gamma \Delta \hat{\lambda}_{Hs}^C$ and the offshoring shock term is $\sum_{s=1997}^t \left(\frac{1}{\eta-1}\right) \gamma [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{A}' \circ \Delta \hat{\lambda}'_{Hs}] \iota$.

Turning to price level accounting, aggregate inflation in the model is:

$$\pi_t = \gamma [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{I} - \alpha] \pi_{Vt} + \underbrace{\left(\frac{1}{\eta-1}\right) \gamma [\mathbf{I} - \mathbf{A}']^{-1} [\mathbf{A}' \circ \Delta \hat{\lambda}'_{Ht}] \iota}_{\text{Offshoring}} + \underbrace{\left(\frac{1}{\eta-1}\right) \gamma \Delta \hat{\lambda}_{Ht}^C}_{\text{Final Goods Imports}}, \quad (56)$$

where γ is a row vector of sector-level consumption shares. This parallels Equation 17 in Section 1.³⁶ In Figure 10b, we plot the impact of changes in offshoring and final goods sourcing on the consumer price level, adding up quarter-on-quarter changes in the model. Like in the accounting exercise, increased offshoring and foreign sourcing of final goods appear to restrain the aggregate price level, accounting for reductions in the price level of 5-6% (25-30 basis points per year).

Having developed the model counterfactuals in full, we are now in a position to emphasize again that these accounting results are not very informative about the actual role of trade in explaining inflation dynamics. As we have shown, rising trade pushes inflation up. The accounting exercises mislead precisely because a rise in offshoring and final goods imports triggers increases in the price of domestic real value added that more than offset the improvement in the terms of trade. This speaks to the importance of well-defined counterfactuals in tying trade to inflation.

³⁶We are careful here to call the final term “Final Goods Imports” rather than “C Imports,” because final consumption in the model includes consumption, investment, and government spending, as in the baseline model.

5 Conclusion

The impact of trade integration on relative prices across sectors is a venerable topic in international economics. So too, much as been written about the impacts of globalization on inflation. In this paper, we have brought these two strands together. There is a plausible case that changes in trade have influenced relative inflation across industries. And, in an accounting sense, rising trade has restrained growth in consumer prices relative to the domestic GDP deflator. Nonetheless, we've developed a suite of models to argue that rising trade actually generates inflation.

One important element of our argument is that much of the increase in trade (at least in the US) has been due to rising imports of intermediate inputs. While conventional wisdom suggests rising input trade restrains producer price inflation, we have demonstrated that it has no such effect in standard New Keynesian models. The second element is that the increase in trade has been spread over time. Anticipated increases in trade – consistent with widespread understanding that globalization was an ongoing process of integration – lead to increased aggregate demand, which generates inflation. Further, we have also shown that neither changes in capital inflows, nor pro-competitive effects of trade on markups overturn these basic forces, and they may in fact strengthen them. Overall, we are left with the conclusion that trade integration is inflationary.

To conclude, we highlight three topics that merit further work. First, while we have developed our argument for a small open economy, it would be natural to revisit the questions we ask in large open economy models. While we believe our conclusions are robust to this extension, multi-country models would provide fertile ground to study how changes in trade influence inflation synchronization across countries.

Second, our framework adopts a sufficient statistics approach to analyze historical developments in trade. To study prospective shocks – whether to trade policy or other exogenous variables – one needs to solve for the impact of those policies on domestic sourcing shares themselves. This requires taking a stand on features of the model that influence trade dynamics in response to those shocks (e.g., the currency invoicing of trade). This is a fertile area for work as well.

Lastly, we have adopted a rule-based approach to characterizing monetary policy. While this is consistent with how many central banks (most importantly, the Federal Reserve) have behaved over this period, additional analysis of monetary policy would be worthwhile. For one, we have not attempted to characterize optimal policy in response to trade shocks, which may have both transitory and permanent components. An analysis of how optimal policy differs with varying assumptions regarding the response of markups and the multisector structure of the economy could lead to important policy insights. Farther afield, the central bank's desired level of inflation may be endogenous, as in [Romer \(1993\)](#) and [Rogoff \(2003\)](#). This deserves more consideration outside the US context, where central bank commitment to low inflation may be less firm.

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A Import Prices in Theory vs. Data

In Section 1, we suggested that one could (in principle) quantify the role of offshoring in driving costs by looking directly at import price data. We pause here to discuss shortcomings in conventional data sources that preclude this approach, and thus motivate the sufficient-statistic approach that we adopt. While this discussion focuses on US data sources, similar issues arise in virtually all standard national accounts sources.

In Equations 5-7, what we need to quantify the role of inputs in driving costs is the true quality-adjusted cost of imported inputs relative to domestic output. A helpful way to think about this is to re-write the import price as $P_{Ft}(s) = B_{Ft}(s)P_{Ft}^{data}(s)$. Think of the last term $P_{Ft}^{data}(s)$ as the observed import price index (IPI), as measured by the US Bureau of Labor Statistics (BLS) international price program, so then $B_{Ft}(s)$ represents sources of bias that lead measured price indexes to deviate from the (production-cost relevant) measure of import prices required by the model.³⁷ There are two key sources for this bias.³⁸

The first is that existing import price deflators do not capture the full impact of offshoring on the cost of inputs. In a multi-country environment, producers have the ability to substitute among foreign input suppliers (e.g., from Japanese to Chinese suppliers). These cost-saving substitutions are not captured in the multilateral import price indexes produced by the BLS, which implies that measured import prices are likely to be biased upward relative to reality, akin to outlet substitution bias in consumer price measurement [Reinsdorf and Yuskavage (2018)]. More broadly, producers also benefit from substituting cheaper foreign for domestic inputs. Because this substitution is not accounted for in measurement of either import prices or domestic purchaser prices by the BLS, it is difficult to estimate the change in the unit cost of the firm's input bundle from existing data. This can be interpreted as a second source of bias in the relative price of imports in our model, which leads the ratio of measured import prices to domestic output prices to understate the decline in costs associated with offshoring.³⁹

A second source of bias is that non-price factors may drive substitution between domestic and foreign inputs. A leading concern is that unmeasured improvements in foreign product quality or variety have driven firms to source inputs from abroad. In our application, quality bias is a

³⁷Throughout the discussion that follows, we assume that data sources record the quality-adjusted price of domestic output accurately. This assumption is a reasonable way to proceed for two reasons. First, in practice, the BLS performs more extensive quality adjustment in the Producer Price Index (PPI) price program than it does under its Import Price Index program. For example, hedonic price adjustment occurs for producer prices, but not for import prices. Second, rapid changes in the international economy and trading environment point to problems of price measurement as likely being most severe for imports [Houseman and Mandel (2015); Moulton (2018)].

³⁸In addition to specific papers cited below, see also Houseman and Mandel (2015) and Moulton (2018).

³⁹Put differently, using PPI and IPI data to construct changes in input costs would overstate cost growth of the composite input. Since this bias would lead to the understatement of the quantity of inputs used in production, it also would tend to overstate productivity growth [Houseman et al. (2011)].

particularly important concern – the BLS is not able to account for quality improvements in imports using the same methods applied to domestic producer prices. For example, while the BLS applies hedonic adjustments in producer price data, it does not do so for import prices.

A final, somewhat different, challenge in using price data is that firms/industries are linked to one another. Even if import prices were measured perfectly, producers may experience cost reductions either because they directly substitute foreign for domestic ones, or because they buy inputs from upstream suppliers who themselves engage in offshoring. Thus, one also needs input use data to track the propagation of offshoring-induced reductions in costs through the network structure of the economy. This industry-level input use data, collected by the BEA, is not directly comparable with BLS-measured prices in terms of coverage and measurement conventions. Thus, it is challenging to combine import and domestic price data with input-output data, or to aggregate the results to examine the ultimate effect of imported input prices on consumer or investment goods price levels.

Our approach to circumventing these problems combines data on changes in imported input expenditure shares with a structural model to impute changes in unit costs attributable to offshoring. While enhanced efforts to improve price collection and measurement are needed, the distinct advantage of our approach is that we are able to use “off the self” data from the national accounts to address the macroeconomic impact of offshoring.

B Baseline Model

B.1 Model Equilibrium Conditions

The equilibrium conditions for the baseline model in Section 2 are collected in Table 2.

B.2 Calibration Details

Parameter values for model simulations are recorded in Table 3.

In this parameterization, we set the elasticity of substitution among domestic varieties (ϵ) equal to the elasticity of substitution between home and foreign goods (η). Put differently, we set the macro-Armington elasticity between home and foreign goods equal to the microeconomic elasticity between domestic varieties. This is consistent with estimates in [Feenstra et al. \(2018\)](#), which fails to reject equality of these elasticities for most goods. This parameterization also facilitates comparison of this baseline model to the model with variable markups to be presented later. Further, relaxing it does not qualitatively change our results. As discussed above, the elasticity of substitution between home and foreign goods is set in the middle of the range of available estimates

Table 2: Baseline Model Summary

Consumption-Leisure	$C_t^{-\rho} \frac{W_t}{P_{Ct}} = \mu L_t^\psi$
Consumption Allocation	$C_{Ht} = \nu \left(\frac{P_{Ht}}{P_{Ct}} \right)^{-\eta} C_t$ $C_{Ft} = (1 - \nu) \left(\frac{\tau_{Ct} P_{Ft}}{P_{Ct}} \right)^{-\eta} C_t$
Euler Equation	$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{P_{Ct}}{P_{C,t+1}} (1 + i_t) \right]$ $W_t L_t = (1 - \alpha) M C_t Y_t$
Input Choices	$P_{Mt} M_t = \alpha M C_t Y_t$ $M_{Ht} = \xi \left(\frac{P_{Ht}}{P_{Mt}} \right)^{-\eta} M_t$ $M_{Ft} = (1 - \xi) \left(\frac{\tau_{Mt} P_{Ft}}{P_{Mt}} \right)^{-\eta} M_t$
Marginal Cost	$M C_t = \frac{W_t^{1-\alpha} P_{Mt}^\alpha}{\alpha^\alpha (1-\alpha)^{(1-\alpha)} Z_t}$ $(1 - \epsilon) + \epsilon \frac{M C_t}{P_{Ht}} - \phi \left(\frac{P_{Ht}}{P_{H,t-1}} - 1 \right) \frac{P_{Ht}}{P_{H,t-1}}$
Price Setting	$+ E_t \left[\beta \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}} \frac{P_{Ct}}{P_{C,t+1}} \phi \left(\frac{P_{H,t+1}}{P_{H,t}} - 1 \right) \frac{P_{H,t+1} Y_{t+1}}{P_{Ht} Y_t} \frac{P_{H,t+1}}{P_{Ht}} \right] = 0$
Price Indexes	$P_{Ct} = \left[\nu P_{Ht}^{1-\eta} + (1 - \nu) (\tau_{Ct} P_{Ft})^{1-\eta} \right]^{1/(1-\eta)}$ $P_{Mt} = \left[\xi P_{Ht}^{1-\eta} + (1 - \xi) (\tau_{Mt} P_{Ft})^{1-\eta} \right]^{1/(1-\eta)}$
Market Clearing	$Y_t = C_{Ht} + M_{Ht} + X_t + \frac{\phi}{2} \left(\frac{P_{Ht}}{P_{H,t-1}} - 1 \right)^2 Y_t$ $X_t = \left(\frac{P_{Ht}}{S_t P_{Ct}^*} \right)^{-\eta} C_t^*$ $\left(\frac{C_t}{C_t^*} \right)^{-\rho} \left(\frac{S_t P_{Ct}^*}{P_{Ct}} \right) = \gamma$
Monetary Policy Rule	$1 + i_t = (1 + i_0) \left(\frac{P_{Ct}}{P_{C,t-1}} \right)^\omega$

in the macroeconomics and trade literatures. Regarding price rigidity, [Sims and Wolff \(2017\)](#) provides the equivalence formula between the parameter governing price rigidity in Rotemberg versus Calvo-style models: $\phi = \frac{\kappa(\epsilon-1)}{(1-\kappa)(1-\beta\kappa)}$, where $1 - \kappa$ is the share of firms that adjust their prices each period in a Calvo-style model. We set $\kappa = .75$, to match the average duration of prices, which leads to the value for ϕ in the table, given other parameters.

In the data, we assign all final goods imports to consumption, consistent with the absence of a government or investment sector in this baseline model. We construct a quarterly series for domestic sourcing shares from annualized data via interpolation. From annual BEA data, we have annual values for λ_{Ht}^C and λ_{Ht}^M , where $t \in \{1997, 2018\}$. We define domestic sourcing shares for the pre-1997 steady state as follows: $\lambda_{H0}^U = \max\{\lambda_{H1997}^U + (\lambda_{H1997}^U - \lambda_{H1998}^U), \lambda_{H1998}^U\}$, for $U = \{C, M\}$. This has the following interpretation. If λ_H^U declines from 1997 to 1998, we assume that the reduction from 1996 to 1997 is the same as from 1997 to 1998. If instead, λ_H^U does not decline from 1997 to 1998, we assume that the value λ_H^U in 1996 is the same as in 1998. This particular formulation captures the notion that home shares tend to decline, but in some cases, they oscillate a bit during the late 90s before they start declining. In the light of these patterns, we set the pre-1997 state as slightly more closed than 1997 in the first scenario, and equal to the state we observe in 1998 in the second scenario. Our results do not hinge on this particular set up, as we have experimented with various approaches. After defining the steady state levels, we compute log deviations of the annual home shares from steady state ($\hat{\lambda}_{Ht}^C, \hat{\lambda}_{Ht}^M$). We then conduct quarterly interpolation as follows: $\hat{\lambda}_{Ht.Q1}^U = 0.4\hat{\lambda}_{Ht-1}^U + 0.6\hat{\lambda}_{Ht}^U$, $\hat{\lambda}_{Xt.Q2}^U = 0.2\hat{\lambda}_{Ht-1}^U + 0.8\hat{\lambda}_{Ht}^U$, $\hat{\lambda}_{Xt.Q3}^U = 0.8\hat{\lambda}_{Ht}^U + 0.2\hat{\lambda}_{Ht+1}^U$, $\hat{\lambda}_{Xt.Q4}^U = 0.6\hat{\lambda}_{Ht}^U + 0.4\hat{\lambda}_{Ht+1}^U$, where subscript $t.Qx$ denotes the value for year t and quarter x . In the paper, t indexes quarters in the model and simulations.

B.3 The Three Equation Model

In this appendix, we convert the baseline model in Table 1 into the three equation model. We first derive the Phillips curve, and then we derive the IS Curve. One additional contribution is that we also discuss the relationship between the gross output gap (used in the text) and the output gap for real value added (i.e., actual versus potential GDP).

B.3.1 Phillips Curve

Domestic price inflation depends on real marginal costs (\widehat{rmc}_t): $\pi_{Ht} = \left(\frac{\epsilon-1}{\phi}\right)\widehat{rmc}_t + \beta E_t(\pi_{Ht+1})$. We seek to replace real marginal costs with the output gap to obtain a domestic price Phillips curve. We do this first for the output gap defined in terms of gross output, and then discuss replacement with the value-added output gap. Conversion of the domestic price Phillips curve into the consumer price Phillips curve is immediate, recognizing that $\pi_{Ct} = \pi_{Ht} + \frac{1}{(\eta-1)}\left(\hat{\lambda}_{Ht}^C - \hat{\lambda}_{Ht-1}^C\right)$. As in the main

Table 3: Calibration

Parameter	Value	Reference/Target
ψ	2	Labor supply elasticity of 0.5
β	.995	Annual risk-free real rate of 2%
ϵ	3	Elasticity of substitution between home varieties
η	3	Elasticity of substitution between home and foreign goods
ρ	2	Intertemporal elasticity of substitution of 0.5
α	.434	To match 1996 input share
ν	.96	To match 1996 home share in consumption
ξ	.925	To match 1996 home share in intermediates
ϕ	23.6453	To yield first order equivalence to Calvo pricing, with average price duration of 4 quarters [Sims and Wolff (2017)].
ω	1.5	Clarida, Gali and Gertler (1999)
Υ	0.008	To match the trade deficit to gross output ratio in 1996, given by $\frac{P_{F0}(\tau_{C0}C_{F0} + \tau_{M0}M_{F0}) - P_{H0}X_0}{P_{H0}Y_0} = 0.0073$.

text, equilibrium objects in the flexible price model have an n in the superscript.

Step One: link $\widehat{r\bar{m}c}_t$ to the real wage gap. Real marginal costs are: $\widehat{r\bar{m}c}_t = (1 - \alpha)\widehat{r\bar{w}}_t + \frac{\alpha}{\eta-1}\hat{\lambda}_{Ht}^M - \hat{z}_t$. In the flexible price equilibrium, markups are constant, so real marginal costs are equal to zero: $\widehat{r\bar{m}c}_t^n = (1 - \alpha)\widehat{r\bar{w}}_t^n + \frac{\alpha}{\eta-1}\hat{\lambda}_{Ht}^M - \hat{z}_t = 0$. Thus, $\widehat{r\bar{m}c}_t - \widehat{r\bar{m}c}_t^n = \widehat{r\bar{m}c}_t$, so we can write:

$$\widehat{r\bar{m}c}_t = (1 - \alpha) [\widehat{r\bar{w}}_t - \widehat{r\bar{w}}_t^n]. \quad (57)$$

Note that the gap between real marginal costs in the actual and flexible price equilibrium only depends on the real wage gap. The direct effects of domestic sourcing of inputs ($\hat{\lambda}_{Ht}^M$) and productivity (\hat{z}_t) are differenced away, as they influence real marginal costs the same way in both equilibria.

Step Two: solve for real wage gap. Combining the first order condition for labor supply with labor demand and the definition of real marginal costs, real wages are given by:

$$\widehat{r\bar{w}}_t = \left(\frac{\alpha\psi}{(1 + \alpha\psi)(\eta - 1)} \right) \hat{\lambda}_{Ht}^M + \left(\frac{1}{(1 + \alpha\psi)(\eta - 1)} \right) \hat{\lambda}_{Ht}^C + \left(\frac{\rho}{1 + \alpha\psi} \right) \hat{c}_t + \left(\frac{\psi}{1 + \alpha\psi} \right) (\hat{y}_t - \hat{z}_t). \quad (58)$$

Evaluating the expression for real wages in the two equilibria, the real wage gap is then:

$$\widehat{rw}_t - \widehat{rw}_t^n = \left(\frac{\rho}{1 + \alpha\psi} \right) [\hat{c}_t - \hat{c}_t^n] + \left(\frac{\psi}{1 + \alpha\psi} \right) [\hat{y}_t - \hat{y}_t^n] \quad (59)$$

$$= \left(\frac{1}{1 + \alpha\psi} \right) [\hat{q}_t - \hat{q}_t^n] + \left(\frac{\psi}{1 + \alpha\psi} \right) [\hat{y}_t - \hat{y}_t^n], \quad (60)$$

where the second line uses the risk sharing condition to eliminate the consumption gap.⁴⁰ This says the gap in real wages depends on differences in the real exchange rate and output across equilibria. Note again that that the direct effect of changes in domestic sourcing and productivity drop away, since they move real wages in identical ways in the two equilibria.

Step Three: solve for the real exchange rate gap. The next step is to swap out the real exchange rate gap, using the goods market clearing condition to link the real exchange rate gap to the output gap and real marginal costs. Collecting pieces, the goods market clearing condition is:

$$\hat{y}_t = s_C \left[\frac{\eta}{\eta - 1} \hat{\lambda}_{Ht}^C + \hat{c}_t^* + \frac{1}{\rho} \hat{q}_t \right] + s_M \left[\frac{\eta}{\eta - 1} \hat{\lambda}_{Ht}^M + \widehat{rmc}_t + \hat{y}_t - \frac{1}{\eta - 1} \hat{\lambda}_{Ht}^M \right] + s_X \left[\frac{\eta}{\eta - 1} \hat{\lambda}_{Ht}^C + \eta \hat{q}_t + \hat{c}_t^* \right]. \quad (61)$$

We rearrange to isolate output on the left-hand side:

$$\hat{y}_t = \frac{\eta}{\eta - 1} \hat{\lambda}_{Ht}^C + \hat{c}_t^* + \left(\frac{s_C / \rho + s_X \eta}{1 - s_M} \right) \hat{q}_t + \left(\frac{s_M}{1 - s_M} \right) \hat{\lambda}_{Ht}^M + \left(\frac{s_M}{1 - s_M} \right) \widehat{rmc}_t, \quad (62)$$

where $s_C \equiv \frac{C_0}{Y_0}$, $s_M \equiv \frac{M_0}{Y_0}$, $s_X \equiv \frac{X_0}{Y_0}$, so $s_C + s_M + s_X = 1$.

Evaluating output at the flexible price equilibrium, and taking differences, the gross output gap is:

$$\hat{y}_t - \hat{y}_t^n = \left(\frac{s_C / \rho + s_X \eta}{1 - s_M} \right) [\hat{q}_t - \hat{q}_t^n] + \left(\frac{s_M}{1 - s_M} \right) \widehat{rmc}_t. \quad (63)$$

Solving for the real exchange rate gap gives us:

$$\hat{q}_t - \hat{q}_t^n = \left(\frac{1 - s_M}{s_C / \rho + s_X \eta} \right) [\hat{y}_t - \hat{y}_t^n] - \left(\frac{s_M}{s_C / \rho + s_X \eta} \right) \widehat{rmc}_t. \quad (64)$$

⁴⁰Note that changes in foreign consumption \hat{c}_t^* do not appear, because they are differenced away when looking at the consumption gap.

Step Four: Link real wages and output gap. Plugging the expression for the real exchange rate gap into the real wage gap equation yields:

$$\begin{aligned}\widehat{rw}_t - \widehat{rw}_t^n &= \left(\frac{1}{1 + \alpha\psi} \right) \left(\frac{1 - s_M}{s_C/\rho + s_X\eta} \right) [\hat{y}_t - \hat{y}_t^n] - \left(\frac{1}{1 + \alpha\psi} \right) \left(\frac{s_M}{s_C/\rho + s_X\eta} \right) \widehat{r\overline{mc}}_t + \left(\frac{\psi}{1 + \alpha\psi} \right) [\hat{y}_t - \hat{y}_t^n] \\ &= \left[\left(\frac{1}{1 + \alpha\psi} \right) \left(\frac{1 - s_M}{s_C/\rho + s_X\eta} \right) + \left(\frac{\psi}{1 + \alpha\psi} \right) \right] [\hat{y}_t - \hat{y}_t^n] - \left(\frac{1 - \alpha}{1 + \alpha\psi} \right) \left(\frac{s_M}{s_C/\rho + s_X\eta} \right) [\widehat{rw}_t - \widehat{rw}_t^n],\end{aligned}\tag{65}$$

where the second line eliminates real marginal costs using Equation 57. Then, we solve for the real wage gap:

$$\widehat{rw}_t - \widehat{rw}_t^n = \chi [\hat{y}_t - \hat{y}_t^n],\tag{66}$$

where $\chi \equiv \left[1 + \left(\frac{1 - \alpha}{1 + \alpha\psi} \right) \left(\frac{s_M}{s_C/\rho + s_X\eta} \right) \right]^{-1} \left[\left(\frac{1}{1 + \alpha\psi} \right) \left(\frac{1 - s_M}{s_C/\rho + s_X\eta} \right) + \left(\frac{\psi}{1 + \alpha\psi} \right) \right]$. It is straightforward to verify that $\chi > 0$ under the parameter restrictions imposed in the main text.

Step Five: Link real marginal costs and output gap. Plugging back into Equation 57, real marginal costs are linked to the gross output gap:

$$\widehat{r\overline{mc}}_t = (1 - \alpha)\chi [\hat{y}_t - \hat{y}_t^n].\tag{67}$$

A positive output gap yields an increase in real marginal costs, since $\alpha \in (0, 1)$ and $\chi > 0$.

Step Six: Write down the domestic price Phillips Curve. Substituting for real marginal costs in the domestic price inflation equation gives us the domestic price Phillips curve:

$$\pi_{Ht} = \left(\frac{(\epsilon - 1)(1 - \alpha)\chi}{\phi} \right) [\hat{y}_t - \hat{y}_t^n] + \beta E_t(\pi_{Ht+1}).\tag{68}$$

Since $\epsilon > 1$ and $\phi > 0$, a positive gross output gap pushes up domestic price inflation, conditional on expected future inflation.

Step Seven (Optional): replace gross output gap with value-added output gap. Equation 67 links real marginal costs to the gross output gap, which results in a domestic price Phillips curve that depends on the gross output gap. Often, the output gap is defined in terms of real value added (GDP), rather than gross output. So we pause here to demonstrate how to write the Phillips curve in terms of real value added.

In our model, real GDP can be constructed via double deflation, as in the national accounts:

$$\widehat{rva}_t = \left(\frac{P_{H0}Y_0}{GDP_0} \right) \hat{y}_t - \left(\frac{P_{M0}M_0}{GDP_0} \right) \hat{m}_t, \quad (69)$$

where \widehat{rva}_t is the log deviation in real value added from steady state and $GDP_0 = P_{H0}Y_0 - P_{M0}M_0$ is value added in the steady state. Using this result, the value-added output gap is:

$$\widehat{rva}_t - \widehat{rva}_t^n = \left(\frac{1}{s_{VA}} \right) [\hat{y}_t - \hat{y}_t^n] - \left(\frac{1}{s_{VA}} - 1 \right) [\hat{m}_t - \hat{m}_t^n], \quad (70)$$

where $s_{VA} \equiv \frac{GDP_0}{P_{H0}Y_0}$. Input use in the flexible price equilibrium is given by $\hat{m}_t^n = \hat{y}_t^n - \frac{1}{\eta-1} \hat{\lambda}_{Ht}^M$, thus:

$$\hat{m}_t - \hat{m}_t^n = \widehat{rmc}_t + [\hat{y}_t - \hat{y}_t^n]. \quad (71)$$

Plugging back gives us:

$$\begin{aligned} \widehat{rva}_t - \widehat{rva}_t^n &= \left(\frac{1}{s_{VA}} \right) [\hat{y}_t - \hat{y}_t^n] - \left(\frac{1}{s_{VA}} - 1 \right) [\widehat{rmc}_t + [\hat{y}_t - \hat{y}_t^n]] \\ &= [\hat{y}_t - \hat{y}_t^n] - \left(\frac{1-s_{VA}}{s_{VA}} \right) \widehat{rmc}_t. \end{aligned} \quad (72)$$

Rewriting yields $\hat{y}_t - \hat{y}_t^n = [\widehat{rva}_t - \widehat{rva}_t^n] + \left(\frac{1-s_{VA}}{s_{VA}} \right) \widehat{rmc}_t$, and we insert this into real marginal costs to get:

$$\widehat{rmc}_t = \left[1 - (1-\alpha)\chi \left(\frac{1-s_{VA}}{s_{VA}} \right) \right]^{-1} (1-\alpha)\chi [\widehat{rva}_t - \widehat{rva}_t^n] \quad (73)$$

The domestic price Phillips Curve is then given by:

$$\pi_{Ht} = \left(\frac{(\epsilon-1)(1-\alpha)\chi}{\phi} \right) \left[1 - (1-\alpha)\chi \left(\frac{1-s_{VA}}{s_{VA}} \right) \right]^{-1} [\widehat{rva}_t - \widehat{rva}_t^n] + \beta E_t(\pi_{Ht+1}). \quad (74)$$

B.3.2 IS Curve

As usual, derivation of the IS curve starts with the Euler Equation. We first convert the Euler Equation into an IS curve that relates the gross output gap to a real interest rate gap. For completeness, we also rewrite the IS curve in terms of real value added. To fully characterize the IS curve, we then solve for the natural real interest rate.

Step One: write Euler Equation with consumption and interest rate gaps. Start with the Euler equation: $\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\rho} (\hat{r}_t - E_t \pi_{Ct+1})$, and take differences between the actual and flexible

price equilibria:

$$\hat{c}_t - \hat{c}_t^n = E_t [\hat{c}_{t+1} - \hat{c}_{t+1}^n] - \frac{1}{\rho} [\hat{r}_t - \hat{r}_t^n], \quad (75)$$

where $\hat{\hat{r}}_t \equiv \hat{r}_t - E_t \pi_{Ct+1}$ and $\hat{\hat{r}}_t^n \equiv \hat{r}_t^n - E_t \pi_{Ct+1}^n = \hat{r}_t^n - \frac{1}{(\eta-1)} E_t \Delta \ln \lambda_{Ht+1}^C$.

Step Two: Link consumption and output gaps. We use the goods market clearing condition to link the consumption and output gaps. The market clearing condition can be written as:

$$[1 - s_M] \hat{y}_t = s_C \left[\frac{\eta}{\eta-1} \hat{\lambda}_{Ht}^C + \hat{c}_t \right] + s_M [\hat{\lambda}_{Ht}^M + \widehat{r\overline{mc}}_t] + s_X \left[\frac{\eta}{\eta-1} \hat{\lambda}_{Ht}^C + \eta \hat{q}_t + \hat{c}_t^* \right]. \quad (76)$$

Then we take differences between actual and flexible price equilibria:

$$[1 - s_M] [\hat{y}_t - \hat{y}_t^n] = s_C [\hat{c}_t - \hat{c}_t^n] + s_M \widehat{r\overline{mc}}_t + s_X \eta [\hat{q}_t - \hat{q}_t^n], \quad (77)$$

where again the direct effects of changes in domestic sourcing and foreign consumption drop away. We proceed to eliminate real marginal costs using Equation 67 and rearrange:

$$\hat{c}_t - \hat{c}_t^n = \left[\frac{(1 - s_M) - s_M(1 - \alpha)\chi}{s_C} \right] [\hat{y}_t - \hat{y}_t^n] - \frac{s_X \eta}{s_C} [\hat{q}_t - \hat{q}_t^n]. \quad (78)$$

And then we can combine Equation 64 and 67 to eliminate the real exchange rate gap:

$$\begin{aligned} \hat{c}_t - \hat{c}_t^n &= \left[\frac{(1 - s_M) - s_M(1 - \alpha)\chi}{s_C} \right] [\hat{y}_t - \hat{y}_t^n] - \frac{s_X \eta}{s_C} \left(\frac{1 - s_M}{s_C/\rho + s_X \eta} - \left(\frac{s_M(1 - \alpha)\chi}{s_C/\rho + s_X \eta} \right) \right) [\hat{y}_t - \hat{y}_t^n] \\ &= \theta [\hat{y}_t - \hat{y}_t^n], \end{aligned} \quad (79)$$

where $\theta \equiv \frac{(1 - s_M) - s_M(1 - \alpha)\chi}{s_C} - \frac{s_X \eta}{s_C} \left(\frac{1 - s_M}{s_C/\rho + s_X \eta} - \frac{s_M(1 - \alpha)\chi}{s_C/\rho + s_X \eta} \right) > 0$.

Step Three: Define IS Curve We replace the consumption gap with the output gap in the Euler Equation and rearrange to get the IS curve for gross output:

$$\hat{y}_t - \hat{y}_t^n = E_t [\hat{y}_{t+1} - \hat{y}_{t+1}^n] - \frac{1}{\theta \rho} (\hat{\hat{r}}_t - \hat{\hat{r}}_t^n). \quad (80)$$

Step Four (Optional): Replace gross output gap with value-added output gap. Reusing results in Equations 72-73 above, we can link the gross output gap and the value-added output gap:

$$\hat{y}_t - \hat{y}_t^n = \Phi [\widehat{rva}_t - \widehat{rva}_t^n], \quad (81)$$

where $\Phi \equiv \left[1 + \left[\frac{s_{VA}}{1-s_{VA}} - (1-\alpha)\chi \right]^{-1} (1-\alpha)\chi \right] > 0$. Then substituting into the IS curve, we get:

$$\widehat{rva}_t - \widehat{rva}_t^n = E_t [\widehat{rva}_{t+1} - \widehat{rva}_{t+1}^n] - \frac{1}{\Phi\theta\rho} (\hat{r}_t - \hat{r}_t^n). \quad (82)$$

Thus, as in the Phillips curve, the translation from gross output to real value added gaps only influences the slope of the IS curve.

B.3.3 The Natural Real Interest Rate

The natural real interest rate is pinned down in the flexible price equilibrium by the Euler Equation:

$$\hat{r}_t^n = \rho E_t [\hat{c}_{t+1}^n - \hat{c}_t^n]. \quad (83)$$

We proceed here to solve for the natural real rate by pinning down consumption growth in the flexible price equilibrium.

Step One: Link consumption to real exchange rate. Under the complete markets assumption, $\hat{c}_t^n = \hat{c}_t^* + \frac{1}{\rho} \hat{q}_t^n$, so we can write:

$$\hat{r}_t^n = \rho E_t [\hat{c}_{t+1}^* - \hat{c}_t^*] + E_t (\hat{q}_{t+1}^n - \hat{q}_t^n) \quad (84)$$

Step Two: Link real exchange rate to output dynamics. The market clearing condition in the flexible price equilibrium is:

$$\hat{y}_t^n = \frac{\eta}{\eta-1} \hat{\lambda}_{Ht}^C + \hat{c}_t^* + \left(\frac{s_C/\rho + s_X\eta}{1-s_M} \right) \hat{q}_t^n + \left(\frac{s_M}{1-s_M} \right) \hat{\lambda}_{Ht}^M. \quad (85)$$

Take differences of this equation and rearrange to get:

$$\Delta \hat{q}_t^n = \left(\frac{1-s_M}{s_C/\rho + s_X\eta} \right) \left[\Delta \hat{y}_t^n - \frac{\eta}{\eta-1} \Delta \hat{\lambda}_{Ht}^C - \left(\frac{s_M}{1-s_M} \right) \Delta \hat{\lambda}_{Ht}^M - \Delta \hat{c}_t^* \right] \quad (86)$$

Note that the phase in of the shocks directly raises $\Delta \hat{q}_t^n$, and if they lead to a boom in output growth (yielding $\Delta \hat{y}_{t+1}^n > 0$) then push it up further.

Step Three: Pin down output growth. From the supply side of the flexible price equilibrium, we know that $\hat{y}_t^n = \hat{z}_t + (1-\alpha)\hat{l}_t^n + \alpha\hat{m}_t^n$ and $\hat{m}_t^n = \hat{y}_t^n - \frac{1}{\eta-1} \hat{\lambda}_{Ht}^M$, so output can be expressed as:

$$\hat{y}_t^n = \frac{1}{1-\alpha} \hat{z}_t + \hat{l}_t^n - \frac{\alpha}{(\eta-1)(1-\alpha)} \hat{\lambda}_{Ht}^M. \quad (87)$$

Then we can pin down \hat{l}_t^n using labor supply and the real wage (obtained from $\widehat{rmc}_t^n = 0$):

$$\hat{l}_t^n = -\frac{\rho}{\psi}\hat{c}_t^* - \frac{1}{\psi}\hat{q}_t^n - \frac{\alpha}{\psi(1-\alpha)(\eta-1)}\hat{\lambda}_{Ht}^M - \frac{1}{\psi(\eta-1)}\hat{\lambda}_{Ht}^C + \left(\frac{1}{\psi(1-\alpha)}\right)\hat{z}_t \quad (88)$$

Combining these two equations and taking differences gives us:

$$\Delta\hat{y}_t^n = \left(1 + \frac{1}{\psi}\right)\left(\frac{1}{1-\alpha}\right)\Delta\hat{z}_t - \frac{\alpha}{(\eta-1)(1-\alpha)}\left[1 + \frac{1}{\psi}\right]\Delta\hat{\lambda}_{Ht}^M - \frac{\rho}{\psi}\Delta\hat{c}_t^* - \frac{1}{\psi}\Delta\hat{q}_t^n - \frac{1}{\psi(\eta-1)}\Delta\hat{\lambda}_{Ht}^C. \quad (89)$$

Step Four: Combine steps to solve for natural real interest rate. We can combine results from steps two, three, and four to write the natural real interest rate as a function of exogenous shocks:

$$\hat{r}_t^n = \Omega_{C^*}E_t\Delta\hat{c}_{t+1}^* + \Omega_Z E_t\Delta\hat{z}_{t+1} + \Omega_M E_t\Delta\hat{\lambda}_{Ht+1}^M + \Omega_C E_t\Delta\hat{\lambda}_{Ht+1}^C, \quad (90)$$

where $\Omega_{C^*} = \frac{\psi(\rho-1)}{1+\psi} > 0$, $\Omega_Z = \frac{1}{1-\alpha} > 0$, $\Omega_M = -\left[\frac{\alpha}{(\eta-1)(1-\alpha)} + \frac{\psi s_M}{(1+\psi)(1-s_M)}\right] < 0$, and $\Omega_C = -\frac{1+\psi\eta}{(1+\psi)(\eta-1)} < 0$.

C Proofs for the Propositions

This section contains the proofs of Propositions 1-4 introduced in Sections 2.4 and 2.4.3. As a prelude, the model equilibrium is defined by Equation 34-36 plus a monetary policy rule. For Propositions 1-3, the monetary policy rule is: $\hat{r}_t = \omega\pi_{Ct}$. Given this setup, the system can be written as:

$$\mathbf{A}_0\mathbf{J}_t = \mathbf{A}_1E_t\mathbf{J}_{t+1} + \mathbf{B}_0\mathbf{\Gamma}_t + \mathbf{B}_1E_t\mathbf{\Gamma}_{t+1}, \quad (91)$$

$$\text{with } \mathbf{J}_t = \begin{bmatrix} \pi_{Ct} \\ \hat{y}_t - \hat{y}_t^n \end{bmatrix} \quad \text{and} \quad \mathbf{\Gamma}_t = \begin{bmatrix} \Delta\hat{\lambda}_{Ht}^C \\ \Delta\hat{\lambda}_{Ht}^M \\ \Delta\hat{z}_t \\ \Delta\hat{c}_t^* \end{bmatrix},$$

where endogenous variables are collected in \mathbf{J}_t and shocks are in $\mathbf{\Gamma}_t$. The coefficient matrices \mathbf{A}_0 , \mathbf{A}_1 , \mathbf{B}_0 , and \mathbf{B}_1 are given by:

$$\mathbf{A}_0 = \begin{bmatrix} 1 & \frac{-(\epsilon-1)(1-\alpha)\chi}{\phi} \\ \frac{\omega}{\theta\rho} & 1 \end{bmatrix}, \quad (92)$$

$$\mathbf{A}_1 = \begin{bmatrix} \beta & 0 \\ \frac{1}{\theta\rho} & 1 \end{bmatrix}, \quad (93)$$

$$\mathbf{B}_0 = \begin{bmatrix} \frac{1}{\eta-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (94)$$

$$\mathbf{B}_1 = \begin{bmatrix} \frac{-\beta}{\eta-1} & 0 & 0 & 0 \\ \frac{\Omega_C}{\theta\rho} & \frac{\Omega_M}{\theta\rho} & \frac{\Omega_Z}{\theta\rho} & \frac{\Omega_{C^*}}{\theta\rho} \end{bmatrix}. \quad (95)$$

Since \mathbf{A}_0 is non-singular, we can rewrite Equation 91 as:

$$\mathbf{J}_t = \mathbf{A}_0^{-1} \mathbf{A}_1 E_t \mathbf{J}_{t+1} + \mathbf{A}_0^{-1} \mathbf{B}_0 \mathbf{\Gamma}_t + \mathbf{A}_0^{-1} \mathbf{B}_1 E_t \mathbf{\Gamma}_{t+1}. \quad (96)$$

We use this common structure to prove Propositions 1-3. We then modify it slightly to prove Proposition 4.

C.1 Proof of Proposition 1

In both case (i) and case (ii) of the proposition, the economy converges to a long-run steady state with $\mathbf{J} = \mathbf{0}$. Since $\mathbf{\Gamma}_{t+k} = \mathbf{0}$ for $k \geq 2$, then $\mathbf{J}_{t+k} = \mathbf{J}$ for $k \geq 2$. Thus, we can solve for \mathbf{J}_{t+1} as follows:

$$\mathbf{J}_{t+1} = \mathbf{A}_0^{-1} \mathbf{B}_0 \mathbf{\Gamma}_{t+1}, \quad (97)$$

$$\text{with } \mathbf{A}_0^{-1} \mathbf{B}_0 = \left[1 + \frac{\omega \bar{\epsilon}}{\theta\rho} \right]^{-1} \begin{bmatrix} \frac{1}{\eta-1} & 0 & 0 & 0 \\ -\frac{\omega}{\theta\rho(\eta-1)} & 0 & 0 & 0 \end{bmatrix}.$$

where the expectation operator is dropped due to perfect foresight. Then, \mathbf{J}_t is given by:

$$\begin{aligned} \mathbf{J}_t &= \mathbf{A}_0^{-1} \mathbf{A}_1 \mathbf{J}_{t+1} + \mathbf{A}_0^{-1} \mathbf{B}_1 \mathbf{\Gamma}_{t+1} \\ &= \mathbf{A}_0^{-1} [\mathbf{A}_1 \mathbf{A}_0^{-1} \mathbf{B}_0 + \mathbf{B}_1] \mathbf{\Gamma}_{t+1} \end{aligned} \quad (98)$$

where we have used $\mathbf{\Gamma}_t = \mathbf{0}$, which reflects the anticipated nature of the shock, and the solution for \mathbf{J}_{t+1} from above.

For case (i), $\mathbf{\Gamma}_{t+1} = \begin{bmatrix} \Delta\hat{\lambda}_{Ht+1}^C \\ 0 \\ 0 \\ 0 \end{bmatrix}$, so $\mathbf{J}_{t+1} = \begin{bmatrix} \frac{\Delta\hat{\lambda}_{Ht+1}^C}{(1+\frac{\omega\bar{\epsilon}}{\theta\rho})(\eta-1)} \\ -\omega\Delta\hat{\lambda}_{Ht+1}^C \\ \frac{\Delta\hat{\lambda}_{Ht+1}^C}{\theta\rho(1+\frac{\omega\bar{\epsilon}}{\theta\rho})(\eta-1)} \end{bmatrix}$, where $\bar{\epsilon} = \frac{(\epsilon-1)(1-\alpha)\chi}{\phi}$. Referring

to Equation 98, then we compute:

$$\begin{aligned} \mathbf{A}_0^{-1}\mathbf{A}_1\mathbf{J}_{t+1} &= \left[1 + \frac{\omega\bar{\epsilon}}{\theta\rho}\right]^{-1} \begin{bmatrix} \beta + \frac{\bar{\epsilon}}{\theta\rho} & \bar{\epsilon} \\ -\frac{\beta\omega}{\theta\rho} + \frac{1}{\theta\rho} & 1 \end{bmatrix} \begin{bmatrix} \frac{\Delta\hat{\lambda}_{Ht+1}^C}{(1+\frac{\omega\bar{\epsilon}}{\theta\rho})(\eta-1)} \\ -\omega\Delta\hat{\lambda}_{Ht+1}^C \\ \frac{\Delta\hat{\lambda}_{Ht+1}^C}{\theta\rho(1+\frac{\omega\bar{\epsilon}}{\theta\rho})(\eta-1)} \end{bmatrix} \\ &= \begin{bmatrix} \beta - \frac{\bar{\epsilon}(\omega-1)}{\theta\rho} \\ -\frac{\beta\omega + (\omega-1)}{\theta\rho} \end{bmatrix} \frac{\Delta\hat{\lambda}_{Ht+1}^C}{\left(1 + \frac{\omega\bar{\epsilon}}{\theta\rho}\right)^2 (\eta-1)} \end{aligned} \quad (99)$$

$$\begin{aligned} \mathbf{A}_0^{-1}\mathbf{B}_1\mathbf{\Gamma}_{t+1} &= \left[1 + \frac{\omega\bar{\epsilon}}{\theta\rho}\right]^{-1} \begin{bmatrix} \frac{-\beta}{\eta-1} + \frac{\Omega_C\bar{\epsilon}}{\theta\rho} & \frac{\Omega_M\bar{\epsilon}}{\theta\rho} & \frac{\Omega_Z\bar{\epsilon}}{\theta\rho} & \frac{\Omega_{C^*}\bar{\epsilon}}{\theta\rho} \\ \frac{\omega\beta}{(\eta-1)\theta\rho} + \frac{\Omega_C}{\theta\rho} & \frac{\Omega_M}{\theta\rho} & \frac{\Omega_Z}{\theta\rho} & \frac{\Omega_{C^*}}{\theta\rho} \end{bmatrix} \begin{bmatrix} \Delta\hat{\lambda}_{Ht+1}^C \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} -\beta + \frac{\Omega_C\bar{\epsilon}(\eta-1)}{\theta\rho} \\ \frac{\omega\beta}{\theta\rho} + \frac{\Omega_C(\eta-1)}{\theta\rho} \end{bmatrix} \frac{\Delta\hat{\lambda}_{Ht+1}^C}{\left(1 + \frac{\omega\bar{\epsilon}}{\theta\rho}\right)(\eta-1)} \end{aligned} \quad (100)$$

Focusing on the first entry of \mathbf{J}_t , consumer price inflation is given by:

$$\pi_{Ct} = \frac{(-\Delta\hat{\lambda}_{Ht+1}^C)}{\left(1 + \frac{\omega\bar{\epsilon}}{\theta\rho}\right)(\eta-1)} * \left[\beta \left(1 - \left(\frac{\frac{\omega\bar{\epsilon}}{\theta\rho}}{1 + \frac{\omega\bar{\epsilon}}{\theta\rho}} \right) \right) + \frac{\bar{\epsilon}(\omega-1)}{\theta\rho(1 + \frac{\omega\bar{\epsilon}}{\theta\rho})} - \frac{\Omega_C\bar{\epsilon}(\eta-1)}{\theta\rho} \right] > 0, \quad (101)$$

since $\Delta\hat{\lambda}_{Ht+1}^C < 0$, $\eta > 1$, $\omega > 1$, $\bar{\epsilon} > 0$, $\theta > 0$, $\rho > 0$, $\beta > 0$, and $\Omega_C < 0$ [see the definition of Ω_C in Step 4 of Section B.3.3].

To prove case (ii), note that $\mathbf{\Gamma}_{t+1} = \begin{bmatrix} 0 \\ \Delta\hat{\lambda}_{Ht}^M \\ 0 \\ 0 \end{bmatrix}$, so $\mathbf{J}_{t+1} = \mathbf{0}$, since the second column of $\mathbf{A}_0^{-1}\mathbf{B}_0$ has

only zeros in it. Referring to Equation 98, then we compute:

$$\begin{aligned} \mathbf{A}_0^{-1} \mathbf{B}_1 \boldsymbol{\Gamma}_{t+1} &= \left[1 + \frac{\omega \bar{\epsilon}}{\theta \rho} \right]^{-1} \begin{bmatrix} \frac{-\beta}{\eta-1} + \frac{\Omega_C \bar{\epsilon}}{\theta \rho} & \frac{\Omega_M \bar{\epsilon}}{\theta \rho} & \frac{\Omega_Z \bar{\epsilon}}{\theta \rho} & \frac{\Omega_{C^*} \bar{\epsilon}}{\theta \rho} \\ \frac{\omega \beta}{(\eta-1)\theta \rho} + \frac{\Omega_C}{\theta \rho} & \frac{\Omega_M}{\theta \rho} & \frac{\Omega_Z}{\theta \rho} & \frac{\Omega_{C^*}}{\theta \rho} \end{bmatrix} \begin{bmatrix} 0 \\ \Delta \hat{\lambda}_{Ht}^M \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \frac{\Omega_M \bar{\epsilon}}{\theta \rho} \\ \frac{\Omega_M}{\theta \rho} \end{bmatrix} \frac{\Delta \hat{\lambda}_{Ht+1}^M}{(1 + \frac{\omega \bar{\epsilon}}{\theta \rho})(\eta - 1)}. \end{aligned} \quad (102)$$

Focusing on the first entry of \mathbf{J}_t , consumer price inflation is given by:

$$\pi_{Ct} = \frac{\Omega_M \bar{\epsilon}}{(\theta \rho + \omega \bar{\epsilon})(\eta - 1)} \Delta \hat{\lambda}_{Ht+1}^M > 0, \quad (103)$$

because $\Delta \hat{\lambda}_{Ht+1}^M < 0$ and $\Omega_M < 0$ [see the definition of Ω_M in Step 4 of Section B.3.3.], with $\frac{\bar{\epsilon}}{(\theta \rho + \omega \bar{\epsilon})(\eta - 1)} > 0$ due to the same parameter restrictions specified above.

C.2 Proof of Proposition 2

For case (i), the unanticipated, permanent shock sequence implies that $\boldsymbol{\Gamma}_t = \begin{bmatrix} \Delta \hat{\lambda}_{Ht}^C \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\boldsymbol{\Gamma}_{t+k} = \mathbf{0}$

for all $k \geq 1$. It then follows that $\mathbf{J}_{t+k} = \mathbf{J} = \mathbf{0}$ for $k \geq 1$. Therefore, Equation 96 reduces to:

$$\begin{aligned} \mathbf{J}_t &= \mathbf{A}_0^{-1} \mathbf{B}_0 \boldsymbol{\Gamma}_t \\ &= \left[1 + \frac{\omega \bar{\epsilon}}{\theta \rho} \right]^{-1} \begin{bmatrix} \frac{1}{\eta-1} & 0 & 0 & 0 \\ -\frac{\omega}{\theta \rho(\eta-1)} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \hat{\lambda}_{Ht}^C \\ 0 \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \quad (104)$$

Current inflation is then given by:

$$\pi_{Ct} = \left[1 + \frac{\omega \bar{\epsilon}}{\theta \rho} \right]^{-1} \frac{\Delta \hat{\lambda}_{Ht}^C}{(\eta - 1)} < 0, \quad (105)$$

since $\Delta \hat{\lambda}_{Ht}^C < 0$ (again imposing the same parameter restrictions as above).

In case (ii), the shock sequence is $\Gamma_t = \begin{bmatrix} \Delta\hat{\lambda}_{Ht}^C \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\Gamma_{t+1} = \begin{bmatrix} -\Delta\hat{\lambda}_{Ht}^C \\ 0 \\ 0 \\ 0 \end{bmatrix}$. Now, $\mathbf{J}_{t+k} = \mathbf{J} = \mathbf{0}$ for $k \geq 2$. Working backwards, Equation 96 then implies:

$$\begin{aligned} \mathbf{J}_{t+1} &= \mathbf{A}_0^{-1} \mathbf{B}_0 \Gamma_{t+1} \\ &= -\mathbf{A}_0^{-1} \mathbf{B}_0 \Gamma_t \\ &= \begin{bmatrix} -1 \\ \frac{\omega}{\theta\rho} \end{bmatrix} \frac{\Delta\hat{\lambda}_{Ht}^C}{(\eta-1)(1+\frac{\omega\bar{\epsilon}}{\theta\rho})}, \end{aligned} \quad (106)$$

where we have used $\Gamma_{t+1} = -\Gamma_t$ in the second line. Then the solution for \mathbf{J}_t is given by:

$$\mathbf{J}_t = \mathbf{A}_0^{-1} \mathbf{A}_1 \mathbf{J}_{t+1} + \mathbf{A}_0^{-1} \mathbf{B}_0 \Gamma_t - \mathbf{A}_0^{-1} \mathbf{B}_1 \Gamma_t, \quad (107)$$

where again we use $\Gamma_{t+1} = -\Gamma_t$. The following provide the solutions for the individual components of this expression:

$$\begin{aligned} \mathbf{A}_0^{-1} \mathbf{A}_1 \mathbf{J}_{t+1} &= \begin{bmatrix} \beta + \frac{\bar{\epsilon}}{\theta\rho} & \bar{\epsilon} \\ -\frac{\beta\omega}{\theta\rho} + \frac{1}{\theta\rho} & 1 \end{bmatrix} \begin{bmatrix} -1 \\ \frac{\omega}{\theta\rho} \end{bmatrix} \frac{\Delta\hat{\lambda}_{Ht}^C}{(\eta-1)(1+\frac{\omega\bar{\epsilon}}{\theta\rho})^2} \\ &= \begin{bmatrix} -\beta + \frac{\bar{\epsilon}}{\theta\rho}(\omega-1) \\ \frac{(\beta+1)\omega-1}{\theta\rho} \end{bmatrix} \frac{\Delta\hat{\lambda}_{Ht}^C}{(\eta-1)(1+\frac{\omega\bar{\epsilon}}{\theta\rho})^2} \end{aligned} \quad (108)$$

$$\begin{aligned} \mathbf{A}_0^{-1} \mathbf{B}_0 \Gamma_t &= \left[1 + \frac{\omega\bar{\epsilon}}{\theta\rho} \right]^{-1} \begin{bmatrix} \frac{1}{\eta-1} & 0 & 0 & 0 \\ -\frac{\omega}{\theta\rho(\eta-1)} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\hat{\lambda}_{Ht}^C \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ \frac{-\omega}{\theta\rho} \end{bmatrix} \frac{\Delta\hat{\lambda}_{Ht}^C}{(\eta-1)(1+\frac{\omega\bar{\epsilon}}{\theta\rho})} \end{aligned} \quad (109)$$

$$\begin{aligned}
-\mathbf{A}_0^{-1}\mathbf{B}_1\Gamma_t &= \left[1 + \frac{\omega\bar{\epsilon}}{\theta\rho}\right]^{-2} \begin{bmatrix} \frac{-\beta}{\eta-1} + \frac{\Omega_C\bar{\epsilon}}{\theta\rho} & \frac{\Omega_M\bar{\epsilon}}{\theta\rho} & \frac{\Omega_Z\bar{\epsilon}}{\theta\rho} & \frac{\Omega_{C^*}\bar{\epsilon}}{\theta\rho} \\ \frac{\omega\beta}{(\eta-1)\theta\rho} + \frac{\Omega_C}{\theta\rho} & \frac{\Omega_M}{\theta\rho} & \frac{\Omega_Z}{\theta\rho} & \frac{\Omega_{C^*}}{\theta\rho} \end{bmatrix} \begin{bmatrix} -\Delta\hat{\lambda}_{Ht}^C \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} \beta - \frac{\Omega_C(\eta-1)\bar{\epsilon}}{\theta\rho} \\ -(\frac{\omega\beta}{\theta\rho} + \frac{\Omega_C(\eta-1)}{\theta\rho}) \end{bmatrix} \frac{\Delta\hat{\lambda}_{Ht}^C}{(\eta-1)(1 + \frac{\omega\bar{\epsilon}}{\theta\rho})^2}
\end{aligned} \tag{110}$$

Combining these to compute \mathbf{J}_t , current inflation is given by:

$$\pi_{Ct} = \frac{\Delta\hat{\lambda}_{Ht}^C}{\left(1 + \frac{\omega\bar{\epsilon}}{\theta\rho}\right)^2 (\eta-1)} \left[1 + \frac{\bar{\epsilon}}{\theta\rho} [2\omega - 1 - \Omega_C(\eta-1)]\right] < 0, \tag{111}$$

where the inequality follows from $\Delta\hat{\lambda}_{Ht}^C < 0$, $\Omega_C < 0$ [as above], and remaining parameter restrictions above.

C.3 Proof of Proposition 3

For case (i), the unanticipated, permanent shock sequence implies that $\Gamma_t = \begin{bmatrix} 0 \\ \Delta\hat{\lambda}_{Ht}^M \\ 0 \\ 0 \end{bmatrix}$ and $\Gamma_{t+k} = \mathbf{0}$

for all $k \geq 1$. It then follows that $\mathbf{J}_{t+k} = \mathbf{J} = \mathbf{0}$ for $k \geq 1$. Then the solution for \mathbf{J}_t reduces to: $\mathbf{J}_t = \mathbf{A}_0^{-1}\mathbf{B}_0\Gamma_t$. Noting that entries in the second column of $\mathbf{A}_0^{-1}\mathbf{B}_0$ are zero (see Equation 104 for example), then it follows that $\pi_{Ct} = 0$.

For case (ii), the shock sequence is $\Gamma_t = \begin{bmatrix} 0 \\ \Delta\hat{\lambda}_{Ht}^M \\ 0 \\ 0 \end{bmatrix}$, $\Gamma_{t+1} = \begin{bmatrix} 0 \\ -\Delta\hat{\lambda}_{Ht}^M \\ 0 \\ 0 \end{bmatrix}$, and $\Gamma_{t+k} = \mathbf{0}$ for all $k \geq 2$.

As a result, $\mathbf{J}_{t+k} = \mathbf{J}$ for $k \geq 2$. It then follows that $\mathbf{J}_{t+1} = \mathbf{A}_0^{-1}\mathbf{B}_0\Gamma_{t+1} = \mathbf{0}$, based again on the observation that the entries in the second column of $\mathbf{A}_0^{-1}\mathbf{B}_0$ are zero. For the same reason, note also that $\mathbf{A}_0^{-1}\mathbf{B}_0\Gamma_t = \mathbf{0}$. Together, these imply that the solution for \mathbf{J}_t is:

$$\mathbf{J}_t = -\mathbf{A}_0^{-1}\mathbf{B}_1\Gamma_t, \tag{112}$$

$$\text{with } -\mathbf{A}_0^{-1}\mathbf{B}_1\Gamma_t = \left[1 + \frac{\omega\bar{\epsilon}}{\theta\rho}\right]^{-2} \begin{bmatrix} \frac{-\beta}{\eta-1} + \frac{\Omega_C\bar{\epsilon}}{\theta\rho} & \frac{\Omega_M\bar{\epsilon}}{\theta\rho} & \frac{\Omega_Z\bar{\epsilon}}{\theta\rho} & \frac{\Omega_{C^*}\bar{\epsilon}}{\theta\rho} \\ \frac{\omega\beta}{(\eta-1)\theta\rho} + \frac{\Omega_C}{\theta\rho} & \frac{\Omega_M}{\theta\rho} & \frac{\Omega_Z}{\theta\rho} & \frac{\Omega_{C^*}}{\theta\rho} \end{bmatrix} \begin{bmatrix} 0 \\ -\Delta\hat{\lambda}_{Ht}^M \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \Omega_M\bar{\epsilon} \\ \Omega_M \end{bmatrix} \frac{(-\Delta\hat{\lambda}_{Ht}^M)}{\theta\rho \left(1 + \frac{\omega\bar{\epsilon}}{\theta\rho}\right)^2}.$$

Extracting current inflation, we arrive at:

$$\pi_{Ct} = -\frac{\Omega_M \bar{\epsilon} \Delta \hat{\lambda}_{Ht}^M}{\theta \rho \left(1 + \frac{\omega \bar{\epsilon}}{\theta \rho}\right)^2} < 0, \quad (113)$$

since $\Omega_M < 0$ (see above; remaining parameter restrictions continue to hold).

C.4 Proof of Proposition 4

With appropriate redefinition of coefficient matrices, the dynamic system is the same as in Equation 91, as in:

$$\begin{aligned} \mathbf{J}_t &= \bar{\mathbf{A}}_0^{-1} \bar{\mathbf{A}}_1 E_t \mathbf{J}_{t+1} + \bar{\mathbf{A}}_0^{-1} \bar{\mathbf{B}}_0 \Gamma_t + \bar{\mathbf{A}}_0^{-1} \bar{\mathbf{B}}_1 E_t \Gamma_{t+1}, \\ \text{with } \bar{\mathbf{A}}_0^{-1} \bar{\mathbf{A}}_1 &= \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho}\right]^{-1} \begin{bmatrix} \beta + \frac{\bar{\epsilon}}{\theta \rho} & \bar{\epsilon} \\ -\frac{\beta \omega}{\theta \rho} + \frac{1}{\theta \rho} & 1 \end{bmatrix}, \\ \bar{\mathbf{A}}_0^{-1} \bar{\mathbf{B}}_0 &= \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho}\right]^{-1} \begin{bmatrix} \frac{1}{\eta-1} & 0 & 0 & 0 \\ -\frac{\omega}{\theta \rho(\eta-1)} & 0 & 0 & 0 \end{bmatrix}, \\ \bar{\mathbf{A}}_0^{-1} \bar{\mathbf{B}}_1 &= \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho}\right]^{-1} \begin{bmatrix} \frac{-\beta}{\eta-1} \left(1 + \frac{\gamma}{\theta \rho}\right) + \frac{\Omega_C \bar{\epsilon}}{\theta \rho} & \frac{\Omega_M \bar{\epsilon}}{\theta \rho} & \frac{\Omega_Z \bar{\epsilon}}{\theta \rho} & \frac{\Omega_{C^*} \bar{\epsilon}}{\theta \rho} \\ \frac{\omega \beta}{(\eta-1)\theta \rho} + \frac{\Omega_C}{\theta \rho} & \frac{\Omega_M}{\theta \rho} & \frac{\Omega_Z}{\theta \rho} & \frac{\Omega_{C^*}}{\theta \rho} \end{bmatrix}. \end{aligned} \quad (114)$$

In case (i), $\mathbf{J}_{t+k} = \mathbf{J} = \mathbf{0}$ for $k \geq 2$, as in Proposition 1. Solving for \mathbf{J}_{t+1} then yields:

$$\mathbf{J}_{t+1} = \bar{\mathbf{A}}_0^{-1} \bar{\mathbf{B}}_0 \Gamma_{t+1} = \begin{bmatrix} \frac{\Delta \hat{\lambda}_{Ht+1}^C}{(1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho})(\eta-1)} \\ \frac{-\omega \Delta \hat{\lambda}_{Ht+1}^C}{\theta \rho(1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho})(\eta-1)} \end{bmatrix}. \quad (115)$$

Since $\Gamma_t = \mathbf{0}$, the solution for \mathbf{J}_t is again $\mathbf{J}_t = \bar{\mathbf{A}}_0^{-1} \bar{\mathbf{A}}_1 \mathbf{J}_{t+1} + \bar{\mathbf{A}}_0^{-1} \bar{\mathbf{B}}_1 \Gamma_{t+1}$, with:

$$\begin{aligned} \bar{\mathbf{A}}_0^{-1} \bar{\mathbf{A}}_1 \mathbf{J}_{t+1} &= \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho}\right]^{-1} \begin{bmatrix} \beta + \frac{\bar{\epsilon}}{\theta \rho} & \bar{\epsilon} \\ -\frac{\beta \omega}{\theta \rho} + \frac{1}{\theta \rho} & 1 \end{bmatrix} \begin{bmatrix} \frac{\Delta \hat{\lambda}_{Ht+1}^C}{(1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho})(\eta-1)} \\ \frac{-\omega \Delta \hat{\lambda}_{Ht+1}^C}{\theta \rho(1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho})(\eta-1)} \end{bmatrix} \\ &= \begin{bmatrix} \beta - \frac{\bar{\epsilon}(\omega-1)}{\theta \rho} \\ -\frac{\beta \omega + (\omega-1)}{\theta \rho} \end{bmatrix} \frac{\Delta \hat{\lambda}_{Ht+1}^C}{(1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho})^2 (\eta-1)} \end{aligned} \quad (116)$$

$$\begin{aligned} \mathbf{A}_0^{-1} \mathbf{B}_1 \mathbf{\Gamma}_{t+1} &= \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right]^{-1} \begin{bmatrix} \frac{-\beta}{\eta-1} \left(1 + \frac{\gamma}{\theta \rho} \right) + \frac{\Omega_C \bar{\epsilon}}{\theta \rho} & \frac{\Omega_M \bar{\epsilon}}{\theta \rho} & \frac{\Omega_Z \bar{\epsilon}}{\theta \rho} & \frac{\Omega_{C^*} \bar{\epsilon}}{\theta \rho} \\ \frac{\omega \beta}{(\eta-1)\theta \rho} + \frac{\Omega_C}{\theta \rho} & \frac{\Omega_M}{\theta \rho} & \frac{\Omega_Z}{\theta \rho} & \frac{\Omega_{C^*}}{\theta \rho} \end{bmatrix} \begin{bmatrix} \Delta \hat{\lambda}_{Ht}^C \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} -\beta \left(1 + \frac{\gamma}{\theta \rho} \right) + \frac{\Omega_C \bar{\epsilon}(\eta-1)}{\theta \rho} \\ \frac{\omega \beta}{\theta \rho} + \frac{\Omega_C(\eta-1)}{\theta \rho} \end{bmatrix} \frac{\Delta \hat{\lambda}_{Ht+1}^C}{(1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho})(\eta-1)} \end{aligned} \quad (117)$$

Current inflation is then given by:

$$\pi_{Ct} = \frac{(-\Delta \hat{\lambda}_{Ht+1}^C)}{\left(1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right)(\eta-1)} * \left[\beta \left(1 + \frac{\gamma}{\theta \rho} - \frac{1}{1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho}} \right) + \frac{\bar{\epsilon}(\omega-1)}{\theta \rho (1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho})} - \frac{\Omega_C \bar{\epsilon}(\eta-1)}{\theta \rho} \right] > 0, \quad (118)$$

using $\Delta \hat{\lambda}_{Ht+1}^C < 0$ and the same parameter restrictions as above (with $\gamma > 0$ as in the text).

For case (ii), $\mathbf{J}_{t+k} = \mathbf{J} = \mathbf{0}$ for $k \geq 1$, again as in Proposition 1. Therefore, with $\mathbf{\Gamma}_t = \mathbf{0}$,

$$\begin{aligned} \mathbf{J}_t &= \bar{\mathbf{A}}_0^{-1} \bar{\mathbf{B}}_1 E_t \mathbf{\Gamma}_{t+1} \\ &= \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right]^{-1} \begin{bmatrix} \frac{-\beta}{\eta-1} \left(1 + \frac{\gamma}{\theta \rho} \right) + \frac{\Omega_C \bar{\epsilon}}{\theta \rho} & \frac{\Omega_M \bar{\epsilon}}{\theta \rho} & \frac{\Omega_Z \bar{\epsilon}}{\theta \rho} & \frac{\Omega_{C^*} \bar{\epsilon}}{\theta \rho} \\ \frac{\omega \beta}{(\eta-1)\theta \rho} + \frac{\Omega_C}{\theta \rho} & \frac{\Omega_M}{\theta \rho} & \frac{\Omega_Z}{\theta \rho} & \frac{\Omega_{C^*}}{\theta \rho} \end{bmatrix} \begin{bmatrix} 0 \\ \Delta \hat{\lambda}_{Ht+1}^M \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \quad (119)$$

Current inflation is then:

$$\pi_{Ct} = \frac{\bar{\epsilon} \Omega_M}{\theta \rho + \omega \bar{\epsilon} + \gamma} \Delta \hat{\lambda}_{Ht+1}^M > 0, \quad (120)$$

since $\Delta \hat{\lambda}_{Ht+1}^M < 0$ and $\frac{\bar{\epsilon} \Omega_M}{\theta \rho + \omega \bar{\epsilon} + \gamma} < 0$ under the prior parameter restrictions.

C.5 Proof of Proposition 5

Relative to the baseline dynamic system (91), the monetary policy rule defined by (39) implies new matrices \mathbf{A}_0 and \mathbf{B}_0 :

$$\tilde{\mathbf{A}}_0 = \begin{bmatrix} 1 & \frac{-(\epsilon-1)(1-\alpha)\chi}{\phi} \\ \frac{\omega}{\theta \rho} & 1 + \frac{\gamma}{\theta \rho} \end{bmatrix}, \quad (121)$$

$$\tilde{\mathbf{B}}_0 = \begin{bmatrix} \frac{1}{\eta-1} & 0 & 0 & 0 \\ \frac{1}{\theta \rho(\eta-1)} & 0 & 0 & 0 \end{bmatrix}, \quad (122)$$

Then,

$$\begin{aligned}
\mathbf{J}_t &= \tilde{\mathbf{A}}_0^{-1} \mathbf{A}_1 E_t \mathbf{J}_{t+1} + \tilde{\mathbf{A}}_0^{-1} \tilde{\mathbf{B}}_0 \Gamma_t + \tilde{\mathbf{A}}_0^{-1} \mathbf{B}_1 E_t \Gamma_{t+1}, \\
\text{with } \tilde{\mathbf{A}}_0^{-1} \mathbf{A}_1 &= \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right]^{-1} \begin{bmatrix} \beta \left(1 + \frac{\gamma}{\theta \rho} \right) + \frac{\bar{\epsilon}}{\theta \rho} & \bar{\epsilon} \\ -\frac{\beta \omega}{\theta \rho} + \frac{1}{\theta \rho} & 1 \end{bmatrix}, \\
\tilde{\mathbf{A}}_0^{-1} \tilde{\mathbf{B}}_0 &= \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right]^{-1} \begin{bmatrix} \frac{1 + \frac{\bar{\epsilon} + \gamma}{\theta \rho}}{\eta - 1} & 0 & 0 & 0 \\ -\frac{(\omega - 1)}{\theta \rho (\eta - 1)} & 0 & 0 & 0 \end{bmatrix}, \\
\tilde{\mathbf{A}}_0^{-1} \mathbf{B}_1 &= \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right]^{-1} \begin{bmatrix} \frac{-\beta}{\eta - 1} \left(1 + \frac{\gamma}{\theta \rho} \right) + \frac{\Omega_C \bar{\epsilon}}{\theta \rho} & \frac{\Omega_M \bar{\epsilon}}{\theta \rho} & \frac{\Omega_Z \bar{\epsilon}}{\theta \rho} & \frac{\Omega_{C^*} \bar{\epsilon}}{\theta \rho} \\ \frac{\omega \beta}{(\eta - 1) \theta \rho} + \frac{\Omega_C}{\theta \rho} & \frac{\Omega_M}{\theta \rho} & \frac{\Omega_Z}{\theta \rho} & \frac{\Omega_{C^*}}{\theta \rho} \end{bmatrix}.
\end{aligned} \tag{123}$$

In case (i), $\mathbf{J}_{t+k} = \mathbf{J} = \mathbf{0}$ for $k \geq 2$, as in Proposition 1. Solving for \mathbf{J}_{t+1} then yields:

$$\mathbf{J}_{t+1} = \tilde{\mathbf{A}}_0^{-1} \tilde{\mathbf{B}}_0 E_t \Gamma_{t+1} = \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right]^{-1} \begin{bmatrix} \frac{1 + \frac{\bar{\epsilon} + \gamma}{\theta \rho}}{\eta - 1} \Delta \hat{\lambda}_{Ht+1}^C \\ -\frac{(\omega - 1)}{\theta \rho (\eta - 1)} \Delta \hat{\lambda}_{Ht+1}^C \end{bmatrix} \tag{124}$$

Since $\Gamma_t = \mathbf{0}$, the solution for \mathbf{J}_t is again $\mathbf{J}_t = \tilde{\mathbf{A}}_0^{-1} \mathbf{A}_1 \mathbf{J}_{t+1} + \tilde{\mathbf{A}}_0^{-1} \mathbf{B}_1 \Gamma_{t+1}$, with:

$$\begin{aligned}
\tilde{\mathbf{A}}_0^{-1} \mathbf{A}_1 \mathbf{J}_{t+1} &= \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right]^{-2} \begin{bmatrix} \beta \left(1 + \frac{\gamma}{\theta \rho} \right) + \frac{\bar{\epsilon}}{\theta \rho} & \bar{\epsilon} \\ -\frac{\beta \omega}{\theta \rho} + \frac{1}{\theta \rho} & 1 \end{bmatrix} \begin{bmatrix} \frac{1 + \frac{\bar{\epsilon} + \gamma}{\theta \rho}}{\eta - 1} \Delta \hat{\lambda}_{Ht+1}^C \\ -\frac{(\omega - 1)}{\theta \rho (\eta - 1)} \Delta \hat{\lambda}_{Ht+1}^C \end{bmatrix} \\
&= \begin{bmatrix} \left(\beta \left(1 + \frac{\gamma}{\theta \rho} \right) + \frac{\bar{\epsilon}}{\theta \rho} \right) (\theta \rho + \bar{\epsilon} + \gamma) - \bar{\epsilon} (\omega - 1) \\ - \left((\beta \omega - 1) \left(1 + \frac{\bar{\epsilon} + \gamma}{\theta \rho} \right) + (\omega - 1) \right) \end{bmatrix} \frac{\Delta \hat{\lambda}_{Ht+1}^C}{\left(1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right)^2 (\eta - 1) \theta \rho}
\end{aligned} \tag{125}$$

$$\begin{aligned}
\tilde{\mathbf{A}}_0^{-1} \mathbf{B}_1 \Gamma_{t+1} &= \left[1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right]^{-1} \begin{bmatrix} \frac{-\beta}{\eta - 1} \left(1 + \frac{\gamma}{\theta \rho} \right) + \frac{\Omega_C \bar{\epsilon}}{\theta \rho} & \frac{\Omega_M \bar{\epsilon}}{\theta \rho} & \frac{\Omega_Z \bar{\epsilon}}{\theta \rho} & \frac{\Omega_{C^*} \bar{\epsilon}}{\theta \rho} \\ \frac{\omega \beta}{(\eta - 1) \theta \rho} + \frac{\Omega_C}{\theta \rho} & \frac{\Omega_M}{\theta \rho} & \frac{\Omega_Z}{\theta \rho} & \frac{\Omega_{C^*}}{\theta \rho} \end{bmatrix} \begin{bmatrix} \Delta \hat{\lambda}_{Ht}^C \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} -\beta \left(1 + \frac{\gamma}{\theta \rho} \right) + \frac{\Omega_C \bar{\epsilon} (\eta - 1)}{\theta \rho} \\ \frac{\omega \beta}{\theta \rho} + \frac{\Omega_C (\eta - 1)}{\theta \rho} \end{bmatrix} \frac{\Delta \hat{\lambda}_{Ht+1}^C}{\left(1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right) (\eta - 1)}
\end{aligned} \tag{126}$$

Current inflation is then given by:

$$\pi_{Ct} = \frac{(-\Delta \hat{\lambda}_{Ht+1}^C)}{\left(1 + \frac{\omega \bar{\epsilon} + \gamma}{\theta \rho} \right) (\eta - 1)} * \left[\left(1 + \beta \left(1 + \frac{\gamma}{\theta \rho} \right) \right) \left(\frac{\bar{\epsilon} (\omega - 1)}{\theta \rho + \omega \bar{\epsilon} + \gamma} \right) + \frac{\bar{\epsilon}}{\theta \rho} \left(\frac{1 + \psi \eta}{1 + \psi} - \frac{\theta \rho + \bar{\epsilon} + \gamma}{\theta \rho + \omega \bar{\epsilon} + \gamma} \right) \right] > 0, \tag{127}$$

using $\Delta\hat{\lambda}_{Ht+1}^C < 0$, the definition of Ω_C and the same parameter restrictions as above (with $\gamma > 0$ as in the text).

To prove case (ii), $\mathbf{J}_{t+k} = \mathbf{J} = \mathbf{0}$ for $k \geq 1$, again as in Proposition 1. Therefore, with $\mathbf{\Gamma}_t = \mathbf{0}$,

$$\begin{aligned} \mathbf{J}_t &= \tilde{\mathbf{A}}_0^{-1} \mathbf{B}_1 \mathbf{\Gamma}_{t+1} \\ &= \left[1 + \frac{\omega\bar{\epsilon} + \gamma}{\theta\rho} \right]^{-1} \begin{bmatrix} \frac{-\beta}{\eta-1} \left(1 + \frac{\gamma}{\theta\rho} \right) + \frac{\Omega_C \bar{\epsilon}}{\theta\rho} & \frac{\Omega_M \bar{\epsilon}}{\theta\rho} & \frac{\Omega_Z \bar{\epsilon}}{\theta\rho} & \frac{\Omega_{C^*} \bar{\epsilon}}{\theta\rho} \\ \frac{\omega\beta}{(\eta-1)\theta\rho} + \frac{\Omega_C}{\theta\rho} & \frac{\Omega_M}{\theta\rho} & \frac{\Omega_Z}{\theta\rho} & \frac{\Omega_{C^*}}{\theta\rho} \end{bmatrix} \begin{bmatrix} 0 \\ \Delta\hat{\lambda}_{Ht+1}^M \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \quad (128)$$

Current inflation is then:

$$\pi_{Ct} = \frac{\bar{\epsilon}\Omega_M}{\theta\rho + \omega\bar{\epsilon} + \gamma} \Delta\hat{\lambda}_{Ht+1}^M > 0, \quad (129)$$

since $\Delta\hat{\lambda}_{Ht+1}^M < 0$ and $\frac{\bar{\epsilon}\Omega_M}{\theta\rho + \omega\bar{\epsilon} + \gamma} < 0$ under the prior parameter restrictions.

C.6 Proof of Proposition 6

Relative to the baseline dynamic system (91), the monetary policy rule defined by (39) implies a new matrix \mathbf{B}_1

$$\hat{\mathbf{B}}_1 = \begin{bmatrix} \frac{-\beta}{\eta-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (130)$$

Note that in both cases, this does not affect the value of which is still given by (97). Then,

$$\mathbf{J}_t = \mathbf{A}_0^{-1} \mathbf{A}_1 \mathbf{J}_{t+1} + \mathbf{A}_0^{-1} \hat{\mathbf{B}}_1 \mathbf{\Gamma}_{t+1} \quad (131)$$

In case (i), as in proposition 1,

$$\begin{aligned} \mathbf{A}_0^{-1} \mathbf{A}_1 \mathbf{J}_{t+1} &= \left[1 + \frac{\omega\bar{\epsilon}}{\theta\rho} \right]^{-1} \begin{bmatrix} \beta + \frac{\bar{\epsilon}}{\theta\rho} & \bar{\epsilon} \\ -\frac{\beta\omega}{\theta\rho} + \frac{1}{\theta\rho} & 1 \end{bmatrix} \begin{bmatrix} \frac{\Delta\hat{\lambda}_{Ht+1}^C}{\left(1 + \frac{\omega\bar{\epsilon}}{\theta\rho} \right)(\eta-1)} \\ \frac{-\omega\Delta\hat{\lambda}_{Ht+1}^C}{\theta\rho \left(1 + \frac{\omega\bar{\epsilon}}{\theta\rho} \right)(\eta-1)} \end{bmatrix} \\ &= \begin{bmatrix} \beta - \frac{\bar{\epsilon}(\omega-1)}{\theta\rho} \\ -\frac{\beta\omega + (\omega-1)}{\theta\rho} \end{bmatrix} \frac{\Delta\hat{\lambda}_{Ht+1}^C}{\left(1 + \frac{\omega\bar{\epsilon}}{\theta\rho} \right)^2 (\eta-1)} \end{aligned} \quad (132)$$

However, now

$$\begin{aligned}
\mathbf{A}_0^{-1} \hat{\mathbf{B}}_1 \boldsymbol{\Gamma}_{t+1} &= \left[1 + \frac{\omega \bar{\epsilon}}{\theta \rho} \right]^{-1} \begin{bmatrix} \frac{-\beta}{\eta-1} & 0 & 0 & 0 \\ \frac{\omega \beta}{(\eta-1)\theta \rho} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \hat{\lambda}_{Ht+1}^C \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} -\beta \\ \frac{\omega \beta}{\theta \rho} \end{bmatrix} \frac{\Delta \hat{\lambda}_{Ht+1}^C}{\left(1 + \frac{\omega \bar{\epsilon}}{\theta \rho} \right) (\eta - 1)}
\end{aligned} \tag{133}$$

Focusing on the first entry of \mathbf{J}_t , consumer price inflation is given by:

$$\pi_{Ct} = \frac{(-\Delta \hat{\lambda}_{Ht+1}^C)}{\left(1 + \frac{\omega \bar{\epsilon}}{\theta \rho} \right) (\eta - 1)} * \left[\beta \left(1 - \left(\frac{\omega \bar{\epsilon}}{1 + \frac{\omega \bar{\epsilon}}{\theta \rho}} \right) \right) + \frac{\bar{\epsilon}(\omega - 1)}{\theta \rho (1 + \frac{\omega \bar{\epsilon}}{\theta \rho})} \right] > 0, \tag{134}$$

For case (ii), note that $\mathbf{J}_{t+1} = \mathbf{0}$, but also

$$\mathbf{A}_0^{-1} \hat{\mathbf{B}}_1 \boldsymbol{\Gamma}_{t+1} = \left[1 + \frac{\omega \bar{\epsilon}}{\theta \rho} \right]^{-1} \begin{bmatrix} \frac{-\beta}{\eta-1} & 0 & 0 & 0 \\ \frac{\omega \beta}{(\eta-1)\theta \rho} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \hat{\lambda}_{Ht+1}^M \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{135}$$

Therefore,

$$\mathbf{J}_t = \mathbf{A}_0^{-1} \mathbf{A}_1 \mathbf{J}_{t+1} + \mathbf{A}_0^{-1} \hat{\mathbf{B}}_1 \boldsymbol{\Gamma}_{t+1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{136}$$

which implies that $\pi_{Ct} = 0$.

D Model with Physical Capital

In this section, we provide an extension of the baseline model that introduces physical capital in production. We first describe the model, and then compute simulated inflation in the model. This serves to demonstrate the robustness of the key findings to the inclusion of physical capital in the model.

D.1 Model Equations

We make four modifications to the baseline model. First, we modify the production function to include capital as a primary factor. Second, we introduce a law of motion for the capital and specify the production function and market structure for investment. Specifically, investment is produced

competitively from domestic output, subject to costs of adjusting the capital stock. Third, we derive an optimal investment condition by which marginal costs of creating a unit of capital is equal to the shadow price of capital. Fourth, we impose an arbitrage condition that equates the expected rate of return to capital to the real interest rate in the economy.

Formally, the aggregate production function is now:

$$Y_t = Z_t(L_t^\varrho K_t^{1-\varrho})^{1-\alpha} M_t^\alpha. \quad (137)$$

The law of motion for capital is:

$$K_t = K_{t-1}(1 - \delta) + I_t [1 - \varphi(I_t/K_{t-1})], \quad (138)$$

where the function $\varphi(\cdot)$ captures the cost of adjustment the capital stock, and it satisfies the following properties: $\varphi(\delta) = 0$, $\varphi'(\delta) = 0$, and $\varphi''(\cdot) > 0$. We assume that investment is produced using home goods, so the market clearing condition for home goods now becomes:

$$Y_{Ht} = C_{Ht} + M_{Ht} + I_t(1 + \varphi(I_t/K_{t-1})) + X_t + \frac{\phi}{2} \left(\frac{P_{Ht}}{P_{H,t-1}} - 1 \right)^2 Y_t \quad (139)$$

Since the price of one unit of investment is P_{Ht} , optimal investment implies that the shadow price of a capital good, P_{Kt} , is equal to the marginal cost of creating it:

$$P_{Kt}(1 - \varphi(I_t/K_{t-1}) - \varphi'(I_t/K_{t-1})I_t/K_{t-1}) = P_{Ht} \quad (140)$$

Finally, arbitrage implies that the rate of return of capital satisfies:

$$E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{P_{Ct}}{P_{C,t+1}} (1 + i_t) \right] = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{P_{Ct} P_{H,t+1}}{P_{C,t+1} P_{Ht}} \frac{(1 - \varrho)(1 - \alpha) \frac{MC_{t+1} Y_{t+1}}{P_{H,t+1} K_{t+1}} + \frac{P_{K,t+1}}{P_{H,t+1}} (1 - \delta)}{\frac{P_{Kt}}{P_{Ht}}} \right] \quad (141)$$

The log-linearized model with capital is presented in Table 4. The notation generally follows the baseline model. We add four new variables related to the cost of capital and capital accumulation, where hats denote log deviations from steady state: \hat{i}_t is physical capital investment, \hat{k}_t is the capital stock, \widehat{rpk}_t is the relative price of capital (relative to home output), and \widehat{rd}_t is the real rental rate of capital. The equilibrium system has 17 endogenous variables, including prices $\{\widehat{rw}_t, \widehat{rd}_t, \hat{r}_t, \widehat{rpk}_t, \widehat{rmc}_t, \hat{q}_t, \hat{\pi}_{Ht}, \hat{\pi}_{Ct}\}$ and quantities $\{\hat{y}_t, \hat{m}_t, \hat{l}_t, \hat{m}_{Ht}, \hat{k}_t, \hat{l}_t, \hat{c}_{Ht}, \hat{c}_t, \hat{x}_t\}$ and 4 exogenous variables $\{\hat{\lambda}_{Ht}^C, \hat{\lambda}_{Ht}^M, \hat{c}_t^*, \hat{z}_t\}$.

Table 4: Log-Linearization of the Model with Capital

Consumption-Leisure	$\hat{l}_t = -\frac{\rho}{\psi} \hat{c}_t + \frac{1}{\psi} \widehat{r\bar{w}}_t - \frac{1}{\psi(\eta-1)} \hat{\lambda}_{Ht}^C$
Consumption Allocation	$\hat{c}_{Ht} = \frac{\eta}{\eta-1} \hat{\lambda}_{Ht}^C + \hat{c}_t$
Euler Equation	$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\rho} (\hat{r}_t - E_t \pi_{Ct+1})$ $\hat{l}_t = \widehat{r\bar{m}c}_t + \hat{y}_t - \widehat{r\bar{w}}_t$
Input Choices	$\hat{m}_t = \widehat{r\bar{m}c}_t + \hat{y}_t - \frac{1}{\eta-1} \hat{\lambda}_{Ht}^M$ $\hat{m}_{Ht} = \frac{\eta}{\eta-1} \hat{\lambda}_{Ht}^M + \hat{m}_t$ $\hat{k}_t = -(\widehat{ir\bar{d}}_t + \delta \widehat{r\bar{p}k}_t) / (\bar{i} + \delta) + \widehat{r\bar{m}c}_t + \hat{y}_t$
Production Function	$\hat{y}_t = \hat{z}_t + (1 - \alpha) \left((1 - \varrho) \hat{k}_t + \varrho \hat{l}_t \right) + \alpha \hat{m}_t$
Law of Motion for Capital	$\hat{k}_t = (1 - \delta) \hat{k}_{t-1} + \delta \hat{i}_{t-1}$
Optimal Investment	$\widehat{r\bar{p}k}_t = \overline{ac} * (\hat{i}_t - \hat{k}_{t-1})$
Domestic Price Inflation	$\pi_{Ht} = \left(\frac{\epsilon - 1}{\phi} \right) \widehat{r\bar{m}c}_t + \beta E_t (\pi_{Ht+1})$
Consumer Price Index	$\pi_{Ct} = \pi_{Ht} + \frac{1}{(\eta-1)} \left(\hat{\lambda}_{Ht}^C - \hat{\lambda}_{Ht-1}^C \right)$
Market Clearing	$\hat{y}_t = \left(\frac{C_{H0}}{Y_0} \right) \hat{c}_{Ht} + \left(\frac{M_{H0}}{Y_0} \right) \hat{m}_{Ht} + \frac{I}{Y} \hat{i}_t + \left(\frac{X_0}{Y_0} \right) \hat{x}_t$ $\hat{x}_t = \frac{\eta}{\eta-1} \hat{\lambda}_{Ht}^C + \eta \hat{q}_t + \hat{c}_t^*$ $\hat{c}_t = \hat{c}_t^* + \frac{1}{\rho} \hat{q}_t$
Arbitrage	$r_t + \widehat{r\bar{p}k}_t - E_t [\pi_{Ht+1}] = E_t \left[\frac{\bar{i} + \delta}{1 + i} \left(\widehat{r\bar{m}c}_{t+1} + \hat{y}_{t+1} - \hat{k}_{t+1} \right) + \frac{1 - \delta}{1 + i} \widehat{r\bar{p}k}_{t+1} \right]$
Monetary Policy Rule	$\hat{r}_t = \omega \pi_{Ct}$

D.2 Inflation Dynamics with Capital Accumulation

To simulate the model with capital, we need to calibrate two new parameters. We set the capital share in value added, $(1 - \varrho)$ to 0.4. Note that our baseline model is the limit of the model with capital when $(1 - \varrho)$ tends to 0. We set the adjustment cost parameter, \overline{ac} ,⁴¹ to 0.5, which implies a standard deviation for the price of capital that is half the standard deviation of investment. This is close to the relative standard deviation in U.S. data; At business cycle frequencies, it is around 1/3, but it increases to around 1/2 when medium-term frequencies are included in the analysis [Comin and Gertler (2006)].

⁴¹ \overline{ac} is equal to $\varphi''(I_0/K_0) * (I_0/K_0)^2$, where I_0/K_0 denotes the investment to output ratio in steady state.

Figure 11: Simulation of Inflation in the Model with Physical Capital

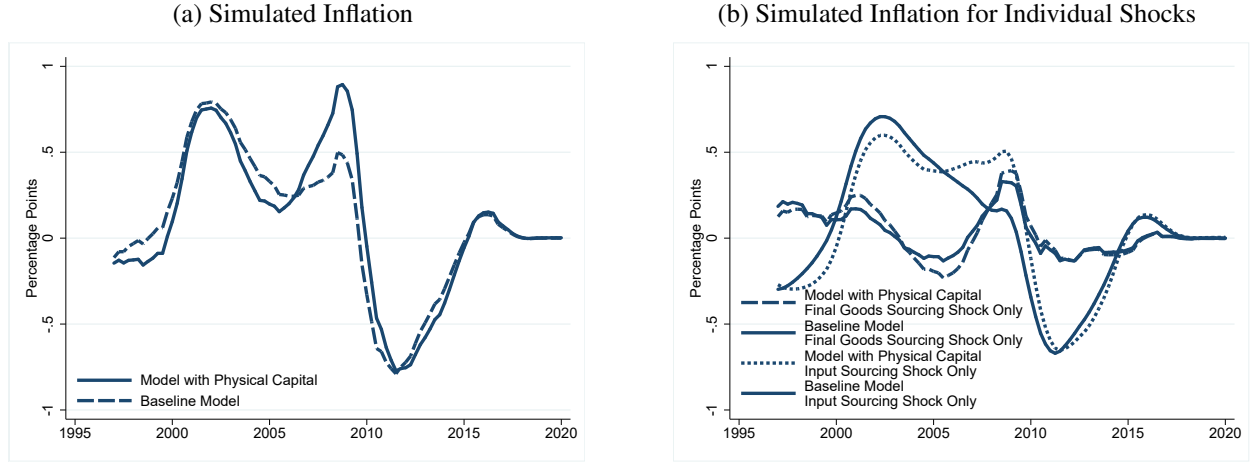


Figure 11a presents the simulated inflation series after feeding in the observed home sourcing shares in the model with capital. For comparison purposes we also plot the simulated inflation series in the baseline model. Figure 11b separates the inflation series resulting from changes in home sourcing shares for final consumption goods and for intermediate goods.

The main takeaway is that the dynamics of inflation in the model with capital are very similar to the baseline. Inflation increases in response to the decline in home sourcing during the first half of the simulation period, and then declines in the latter half as home sourcing of inputs reverts, much like the baseline model. The main difference vis-à-vis the baseline concerns dynamics around the Great Recession, where input sourcing shocks push inflation up more in the model with capital than the baseline model. This is a somewhat tangential feature of the data, on which we place relatively little weight.

E Three Equation Model with Capital Inflow Shocks

As described in the text, the model with capital inflow shocks makes one change to the baseline model (summarized in Tables 2-1). We drop the complete markets assumption, and we thus replace the risk sharing condition with a trade deficit equation given by:

$$TDY_t(P_{Ht}Y_t) = P_{Ft}\tau_{Ct}C_{Ft} + P_{Ft}\tau_{Mt}M_{Ft} - P_{Ht}X_t, \quad (142)$$

where $TDY_t = \frac{TD_t}{P_{Ht}Y_t}$ is the trade deficit as a share of gross output, which is treated as an additional exogenous variable (i.e., shock). This expression can be log-linearized as follows:

$$TD_0 \left(\widehat{tdy}_t + \hat{y}_t \right) = IM_0^C \left(\left(\frac{1 - \eta \lambda_{H0}^C}{(\eta - 1)(1 - \lambda_{H0}^C)} \right) \hat{\lambda}_{Ht}^C + \hat{c}_t \right) + IM_0^M \left(\left(\frac{1 - \lambda_{H0}^M}{(\eta - 1)(1 - \lambda_{H0}^M)} \right) \hat{\lambda}_{Ht}^M + \hat{m}_t \right) - EX_0 \hat{x}_t, \quad (143)$$

where we used $\hat{\lambda}_{Ft}^C = - \left(\frac{\lambda_{H0}^C}{1 - \lambda_{H0}^C} \right) \hat{\lambda}_{Ht}^C$, $\hat{\lambda}_{Ft}^M = - \left(\frac{\lambda_{H0}^M}{1 - \lambda_{H0}^M} \right) \hat{\lambda}_{Ht}^M$, $\hat{\lambda}_{Ht}^C = (1 - \eta)(\hat{p}_{Ht} - \hat{p}_{Ct})$, and $\hat{\lambda}_{Ht}^M = (1 - \eta)(\hat{p}_{Ht} - \hat{p}_{Mt})$ to rewrite the expression. Further, $TD_0 \equiv TDY_0(P_{H0}Y_0)$ is the trade deficit, $IM_0^C \equiv P_{F0}\tau_{C0}C_{F0}$ is the value of imports for final consumption, $IM_0^M \equiv P_{F0}\tau_{M0}M_{F0}$ is the value of imported inputs, and $EX_0 \equiv P_{H0}X_0$ is the value of exports, all evaluated at the date zero steady state.

We now proceed to use this trade deficit condition in place of the risk sharing condition to re-derive the Phillips and IS curves, as well as solve for the real natural rate of interest. These derivations support the discussion in main text.

E.1 Phillips Curve

Referring back to Appendix B.3, Step One and Step Two of the derivation are identical. Starting with Step Three, we need to solve for the real exchange rate gap. The goods market clearing condition is:

$$\hat{y}_t = s_C \left[\frac{\eta}{\eta - 1} \hat{\lambda}_{Ht}^C + \hat{c}_t \right] + s_M \left[\frac{\eta}{\eta - 1} \hat{\lambda}_{Ht}^M + \widehat{r\bar{m}c}_t + \hat{y}_t - \frac{1}{\eta - 1} \hat{\lambda}_{Ht}^M \right] + s_X \left[\frac{\eta}{\eta - 1} \hat{\lambda}_{Ht}^C + \eta \hat{q}_t + \hat{c}_t^* \right]. \quad (144)$$

Evaluating this expression in the flexible price equilibrium and taking differences gives us:

$$\hat{y}_t - \hat{y}_t^n = \left(\frac{s_C}{1 - s_M} \right) (\hat{c}_t - \hat{c}_t^n) + \left(\frac{s_M}{1 - s_M} \right) \widehat{r\bar{m}c}_t + \left(\frac{s_X \eta}{1 - s_M} \right) (\hat{q}_t - \hat{q}_t^n). \quad (145)$$

In Appendix B.3, we eliminated consumption using the risk sharing condition. Here we will use the trade deficit equation instead. Using Equation 143, we can substitute out for \hat{m}_t and \hat{x}_t , and then rearrange to get:

$$\begin{aligned} TD_0 \widehat{tdy}_t + (TD_0 - IM_0^M) \hat{y}_t &= \left[IM_0^C \left(\frac{1 - \eta \lambda_{H0}^C}{(\eta - 1)(1 - \lambda_{H0}^C)} \right) - EX_0 \left(\frac{\eta}{\eta - 1} \right) \right] \hat{\lambda}_{Ht}^C + IM_0^C \hat{c}_t \\ &+ IM_0^M \left(\frac{1 - \lambda_{H0}^M}{(\eta - 1)(1 - \lambda_{H0}^M)} - \frac{1}{\eta - 1} \right) \hat{\lambda}_{Ht}^M + IM_0^M \widehat{r\bar{m}c}_t - EX_0 \eta \hat{q}_t - EX_0 \hat{c}_t^*. \end{aligned} \quad (146)$$

Evaluate this expression at the flexible price equilibrium, where $\widehat{rmc}_t^n = 0$, and take differences to obtain:

$$(TD_0 - IM_0^M)(\hat{y}_t - \hat{y}_t^n) = IM_0^C(\hat{c}_t - \hat{c}_t^n) + IM_0^M \widehat{rmc}_t - EX_0\eta(\hat{q}_t - \hat{q}_t^n). \quad (147)$$

Note there is no direct effect of the trade deficit here, because it is assumed to be the same in the actual and flexible price equilibria. The steady state trade deficit matters for determination of the coefficients in this equation, but the main difference from the baseline model is that the real exchange rate gap is no longer linked to consumption via risk sharing.

Equations 145 and 147 allow us to solve for the real exchange rate gap in terms of the gross output gap and real marginal costs:

$$(\hat{q}_t - \hat{q}_t^n) = \left(\frac{IM_0^M + IM_0^C \left(\frac{1-s_M}{s_C} \right) - TD_0}{IM_0^C \left(\frac{s_X\eta}{s_C} \right) + EX_0\eta} \right) (\hat{y}_t - \hat{y}_t^n) + \left(\frac{IM_0^M - IM_0^C \left(\frac{s_M}{s_C} \right)}{IM_0^C \left(\frac{s_X\eta}{s_C} \right) + EX_0\eta} \right) \widehat{rmc}_t \quad (148)$$

Proceeding to Step Four, we need to link real wages and the output gap. Starting from Equation 59, we use the Equation 145 to replace the consumption gap:

$$\begin{aligned} \widehat{rw}_t - \widehat{rw}_t^n &= \left(\frac{\rho}{1 + \alpha\psi} \right) [\hat{c}_t - \hat{c}_t^n] + \left(\frac{\psi}{1 + \alpha\psi} \right) [\hat{y}_t - \hat{y}_t^n] \\ &= \left(\frac{\rho(1-s_M)}{(1 + \alpha\psi)s_C} + \frac{\psi}{1 + \alpha\psi} \right) (\hat{y}_t - \hat{y}_t^n) - \left(\frac{\rho}{1 + \alpha\psi} \frac{s_M}{s_C} \right) \widehat{rmc}_t - \frac{\rho\eta}{1 + \alpha\psi} \frac{s_X}{s_C} (\hat{q}_t - \hat{q}_t^n), \end{aligned} \quad (149)$$

where $\hat{c}_t - \hat{c}_t^n = \frac{1-s_M}{s_C} (\hat{y}_t - \hat{y}_t^n) - \frac{s_M}{s_C} \widehat{rmc}_t - \eta \frac{s_X}{s_C} (\hat{q}_t - \hat{q}_t^n)$. We then use Equation 57 to replace \widehat{rmc}_t , and rearrange to obtain:

$$(\widehat{rw}_t - \widehat{rw}_t^n) = \left(\frac{\rho(1-s_M) + \psi s_C}{(1 + \alpha\psi)s_C + \rho(1-\alpha)s_M} \right) (\hat{y}_t - \hat{y}_t^n) - \frac{\rho\eta s_X}{(1 + \alpha\psi)s_C + \rho(1-\alpha)s_M} (\hat{q}_t - \hat{q}_t^n) \quad (150)$$

Then we combine Equations 148, 150, and 57 to write the real wage gap as a function of the output

gap:

$$\widehat{rw}_t - \widehat{rw}_t^n = \bar{\chi} \dot{\chi} (\hat{y}_t - \hat{y}_t^n) \quad (151)$$

$$\text{with } \bar{\chi} \equiv \left[1 + \frac{\rho \eta s_X (1 - \alpha)}{(1 + \alpha \psi) s_C + \rho (1 - \alpha) s_M} \left(\frac{IM_0^M - IM_0^C \left(\frac{s_M}{s_C} \right)}{IM_0^C \left(\frac{s_X \eta}{s_C} \right) + EX_0 \eta} \right) \right]^{-1}$$

$$\text{and } \dot{\chi} \equiv \frac{1}{(1 + \alpha \psi) s_C + \rho (1 - \alpha) s_M} \left[\rho (1 - s_M) + \psi s_C - \rho \eta s_X \left(\frac{IM_0^M + IM_0^C \left(\frac{1 - s_M}{s_C} \right) - TD_0}{IM_0^C \left(\frac{s_X \eta}{s_C} \right) + EX_0 \eta} \right) \right].$$

In Step Five, we write real marginal costs as a function of the output gap:

$$\widehat{r\overline{mc}}_t = (1 - \alpha) \bar{\chi} \dot{\chi} [\hat{y}_t - \hat{y}_t^n]. \quad (152)$$

And finally, this gives us the domestic price Phillips curve:

$$\pi_{Ht} = \left(\frac{(\epsilon - 1)(1 - \alpha) \bar{\chi} \dot{\chi}}{\phi} \right) [\hat{y}_t - \hat{y}_t^n] + \beta E_t (\pi_{Ht+1}) \quad (153)$$

The end result of this analysis is that the domestic price Phillips curve is nearly identical to the baseline model with complete markets, but for a change in the slope of the curve. The consumer price Phillips curve then inherits this modest difference. Importantly, neither the domestic sourcing shocks, nor the capital inflow shocks shift the domestic price Phillips curve. As a result, only the domestic sourcing shocks for final goods will appear in the consumer price Phillips Curve, as in Equation 34.

E.2 IS Curve

Referring back to Appendix B.3, Step One of the derivation of the IS curve is identical. In Step Two, we link consumption and output gaps. Starting from Equation 145, we plug in for the real exchange rate gap using Equation 148, and then evaluate real marginal costs using Equation 152. Rearranging the result yields:

$$\hat{c}_t - \hat{c}_t^n = \bar{\theta} (\hat{y}_t - \hat{y}_t^n), \quad (154)$$

$$\text{with } \bar{\theta} \equiv \left(\frac{1 - s_M}{s_C} \right) \left[1 - \left[\left(\frac{s_M}{1 - s_M} \right) + \left(\frac{s_X \eta}{1 - s_M} \right) \left(\frac{IM_0^M - IM_0^C \left(\frac{s_M}{s_C} \right)}{IM_0^C \left(\frac{s_X \eta}{s_C} \right) + EX_0 \eta} \right) \right] (1 - \alpha) \bar{\chi} - \left(\frac{s_X \eta}{1 - s_M} \right) \left(\frac{IM_0^M + IM_0^C \left(\frac{1 - s_M}{s_C} \right) - TD_0}{IM_0^C \left(\frac{s_X \eta}{s_C} \right) + EX_0 \eta} \right) \right].$$

Via Step Three, the dynamic IS Curve follows:

$$\hat{y}_t - \hat{y}_t^n = E_t [\hat{y}_{t+1} - \hat{y}_{t+1}^n] - \frac{1}{\bar{\theta}\rho} (\hat{r}_t - \hat{r}_t^n). \quad (155)$$

As in the Phillips Curve, the immediate effect of relaxing the complete markets assumption is to change the slope parameter. The deeper impact of this change in the model is hidden from view, embedded in \hat{r}_t^n .

To derive the real natural rate, we refer back to Equation 83. To solve for consumption growth in the flexible price equilibrium, we first use the output market clearing and trade deficit equation to eliminate the real exchange rate and link output and consumption. From output market clearing, the real exchange rate is given by:

$$\hat{q}_t^n = \left(\frac{1-s_M}{s_X\eta} \right) \hat{y}_t^n - \left(\frac{1-s_M}{s_X(\eta-1)} \right) \hat{\lambda}_{Ht}^C - \left(\frac{s_C}{s_X\eta} \right) \hat{c}_t^n - \frac{1}{\eta} \hat{c}_t^* - \left(\frac{s_M}{s_X\eta} \right) \hat{\lambda}_{Ht}^M. \quad (156)$$

Then substitute this into the trade balance condition and rearrange to get:

$$\hat{y}_t^n = \Upsilon_C^y \hat{\lambda}_{Ht}^C + \hat{c}_t^n + \Upsilon_M^y \hat{\lambda}_{Ht}^M + \Upsilon_{tdy}^y \widehat{tdy}_t, \quad (157)$$

where the coefficients are given by:

$$\begin{aligned} \Upsilon_C^y &= \left[IM_0^C + EX_0 \left(\frac{s_C}{s_X} \right) \right]^{-1} \left[-IM_0^C \left(\frac{\eta\lambda_{H0}^C - 1}{(\eta-1)(1-\lambda_{H0}^C)} \right) + EX_0 \left(\frac{\eta}{\eta-1} \right) \left(\frac{s_C}{s_X} \right) \right] \\ \Upsilon_M^y &= \left[IM_0^C + EX_0 \left(\frac{s_C}{s_X} \right) \right]^{-1} EX_0 \left(\frac{s_M}{s_X} \right) > 0 \\ \Upsilon_{tdy}^y &= - \left[IM_0^C + EX_0 \left(\frac{s_C}{s_X} \right) \right]^{-1} TD_0 < 0 \quad \text{if } TD_0 > 0. \end{aligned}$$

Now we use the production function, labor supply, and real marginal cost equations to link output and consumption. The real marginal cost equation (recognizing that $\widehat{rmc}_t = 0$ in the flexible price equilibrium) and the labor supply condition yield:

$$\hat{l}_t^n = -\frac{\rho}{\psi} \hat{c}_t^n - \frac{\alpha}{\psi(\eta-1)(1-\alpha)} \hat{\lambda}_{Ht}^M - \frac{1}{\psi(\eta-1)} \hat{\lambda}_{Ht}^C + \frac{1}{\psi(1-\alpha)} \hat{z}_t. \quad (158)$$

Then we plug this in to the production function to get:

$$\hat{y}_t^n = -\frac{\rho}{\psi} \hat{c}_t^n + \left(1 + \frac{1}{\psi} \right) \left(\frac{1}{1-\alpha} \right) \hat{z}_t - \frac{\alpha}{(\eta-1)(1-\alpha)} \left(1 + \frac{1}{\psi} \right) \hat{\lambda}_{Ht}^M - \frac{1}{\psi(\eta-1)} \hat{\lambda}_{Ht}^C. \quad (159)$$

We then combine Equations 157 and 159 to solve for consumption:

$$\hat{c}_t^n = \Upsilon_z^c \hat{z}_t + \Upsilon_M^c \hat{\lambda}_{Ht}^M + \Upsilon_C^c \hat{\lambda}_{Ht}^C + \Upsilon_{tdy}^c \widehat{tdy}_t, \quad (160)$$

where the coefficients are given by:

$$\begin{aligned} \Upsilon_z^c &= \left(\frac{1+\psi}{\psi+\rho} \right) \left(\frac{1}{1-\alpha} \right) > 0 \\ \Upsilon_M^c &= -\frac{\psi}{\psi+\rho} \left[\frac{\alpha}{(\eta-1)(1-\alpha)} \left(\frac{1+\psi}{\psi} \right) + \Upsilon_M^y \right] < 0 \\ \Upsilon_C^c &= -\frac{\psi}{\psi+\rho} \left[\frac{1}{\psi(\eta-1)} + \Upsilon_C^y \right] \\ \Upsilon_\varepsilon^c &= -\frac{\psi}{\psi+\rho} \Upsilon_\varepsilon^y > 0. \end{aligned}$$

The final step is then to insert this solution for consumption into the Euler Equation, and solve for the natural real interest rate:

$$\hat{r}_t^n = \tilde{\Upsilon}_z E_t \Delta \ln z_{t+1} + \tilde{\Upsilon}_M E_t \Delta \ln \lambda_{Ht+1}^M + \tilde{\Upsilon}_C E_t \Delta \ln \lambda_{Ht+1}^C + \tilde{\Upsilon}_{tdy} E_t \Delta \ln tdy_{t+1}, \quad (161)$$

with $\tilde{\Upsilon}_z = \rho \Upsilon_z^c > 0$, $\tilde{\Upsilon}_M = \rho \Upsilon_M^c < 0$, $\tilde{\Upsilon}_C = \rho \Upsilon_C^c$, and $\tilde{\Upsilon}_{tdy} = \rho \Upsilon_{tdy}^c > 0$. And $\tilde{\Upsilon}_C < 0$ if $IM_0^C \frac{(\rho+1)(\eta\lambda_{H0}^C-1)}{(\eta-1)(1-\lambda_{H0}^C)} > \frac{X_0}{(\eta-1)} \frac{s_C}{s_X} (\rho\eta+1)$. The new result here is that an expected increase in the trade deficit ($E_t \Delta \ln tdy_{t+1}$) raises the natural rate of interest.

F Model with Variable Markups

Drawing on Section 3.2, we briefly describe new equilibrium conditions for the model with Kimball demand and dollar currency pricing. Consumers choose consumption of individual home and foreign varieties to minimize expenditure with the consumption aggregator given by Equation 41. In a symmetric firm equilibrium, this yields the following equilibrium conditions:

$$C_{Ht} = \nu \Psi \left(\frac{D_{Ct} P_{Ht}}{P_{Ct}} \right) C_t \quad (162)$$

$$C_{Ft} = (1-\nu) \Psi \left(\frac{D_{Ct} \tau_{Ct} P_{Ft}}{P_{Ct}} \right) C_t \quad (163)$$

$$\nu \Upsilon \left(\frac{C_{Ht}}{\nu C_t} \right) + (1-\nu) \Upsilon \left(\frac{C_{Ft}}{(1-\nu) C_t} \right) = 1 \quad (164)$$

$$P_{Ct} C_t = P_{Ht} C_{Ht} + \tau_{Ct} P_{Ft} C_{Ft}, \quad (165)$$

where $\Psi(x) \equiv \Upsilon'^{-1}(x)$. These replace first order conditions for the consumption allocation and the consumer price index in the baseline model.

On the production side, Home producers choose home and foreign input use to minimize costs given the input aggregator in Equation 42, and they set prices for sales to domestic buyers and export buyers separately. The new equilibrium conditions are:

$$M_{Ht} = \xi \Psi \left(\frac{D_{Mt} P_{Ht}}{P_{Mt}} \right) M_t \quad (166)$$

$$M_{Ft} = (1 - \xi) \Psi \left(\frac{D_{Mt} \tau_{Mt} P_{Ft}}{P_{Mt}} \right) M_t \quad (167)$$

$$\xi \Upsilon \left(\frac{M_{Ht}}{\xi M_t} \right) + (1 - \xi) \Upsilon \left(\frac{M_{Ft}}{(1 - \xi) M_t} \right) = 1 \quad (168)$$

$$P_{Mt} M_t = P_{Ht} M_{Ht} + \tau_{Mt} P_{Ft} M_{Ft}. \quad (169)$$

We assume that exporters face a constant elasticity demand curve in the foreign market, such that $X_t = \left(\frac{P_{Xt}}{E_t P_t^*} \right)^{-\epsilon_X} C_t^*$, similar to the baseline model. The (symmetric) firm's optimal prices then satisfy the following dynamic equation in the domestic market:

$$0 = 1 - \epsilon_{Ht} \left(1 - \frac{MC_t}{P_{Ht}} \right) - \phi \left(\frac{P_{Ht}}{P_{Ht-1}} - 1 \right) \left(\frac{P_{Ht}}{P_{Ht-1}} \right) + \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{P_{Ct}}{P_{C,t+1}} \phi \left(\frac{P_{Ht+1}}{P_{Ht}} - 1 \right) \frac{P_{H,t+1} Y_{H,t+1}}{P_{Ht} Y_{Ht}} \frac{P_{H,t+1}}{P_{Ht}} \right], \quad (170)$$

where $\epsilon_{Ht} = - \left[\Xi_\Psi \left(\frac{D_{Ct} P_{Ht}}{P_{Ct}} \right) \frac{C_{Ht}}{Y_{Ht}} + \Xi_\Psi \left(\frac{D_{Mt} P_{Ht}}{P_{Mt}} \right) \frac{M_{Ht}}{Y_{Ht}} \right]$ is the elasticity of demand at Home, with $\Xi_\Psi(x) \equiv \frac{\Psi'(x)}{\Psi(x)} x$ and $Y_{Ht} = C_{Ht} + M_{Ht}$. With the Klenow-Willis Υ -function, the elasticity of demand for Home goods by Home buyers is: $\epsilon_{Ht} = \frac{C_{Ht}}{Y_{Ht}} \epsilon_{Ht}^C + \frac{M_{Ht}}{Y_{Ht}} \epsilon_{Ht}^M$, with $\epsilon_{Ht}^C = \sigma \left(1 + \varepsilon \ln \frac{\sigma-1}{\sigma} - \varepsilon \ln \frac{D_{Ct} P_{Ht}}{P_{Ct}} \right)^{-1}$ and $\epsilon_{Ht}^M = \sigma \left(1 + \varepsilon \ln \frac{\sigma-1}{\sigma} - \varepsilon \ln \frac{D_{Mt} P_{Ht}}{P_{Mt}} \right)^{-1}$.

The firm's optimal prices in the export market are given by:

$$0 = 1 - \epsilon_X \left(1 - \frac{MC_t}{P_{Xt}} \right) - \phi \left(\frac{P_{Xt}}{P_{Xt-1}} - 1 \right) \left(\frac{P_{Xt}}{P_{Xt-1}} \right) + \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{P_{Ct}}{P_{C,t+1}} \phi \left(\frac{P_{Xt+1}}{P_{Xt}} - 1 \right) \frac{P_{X,t+1} X_{t+1}}{P_{Xt} X_t} \frac{P_{X,t+1}}{P_{Xt}} \right], \quad (171)$$

where ϵ_X is the constant elasticity of export demand.

On the import side, we assume foreign producers set import prices in dollars (exclusive of trade

costs), subject to adjustment costs. Their optimal pricing rule is analogous to the Home firms:

$$0 = 1 - \epsilon_{Ft} \left(1 - \frac{E_t M C_t^*}{P_{Ft}} \right) - \phi \left(\frac{P_{Ft}}{P_{Ft-1}} - 1 \right) \left(\frac{P_{Ft}}{P_{Ft-1}} \right) + \beta E_t \left[\left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\rho} \frac{E_t P_{Ct}^*}{E_{t+1} P_{Ct+1}^*} \phi \left(\frac{P_{Ft+1}}{P_{Ft}} - 1 \right) \frac{P_{Ft+1} Y_{Ft+1}}{P_{Ft} Y_{Ft}} \left(\frac{P_{Ft+1}}{P_{Ft}} \right) \right], \quad (172)$$

where $\epsilon_{Ft} = - \left[\Xi_{\Psi} \left(\frac{D_{Ct} \tau_{Ct} P_{Ft}}{P_{Ct}} \right) \frac{C_{Ft}}{Y_{Ft}} + \Xi_{\Psi} \left(\frac{D_{Mt} \tau_{Mt} P_{Ft}}{P_{Mt}} \right) \frac{M_{Ft}}{Y_{Ft}} \right]$ is the elasticity of import demand and $Y_{Ft} = \tau_{Ct} C_{Ft} + \tau_{Mt} M_{Ft}$.

Collecting and log-linearizing the model equilibrium conditions yields the system in Table 5. To reduce and simplify this system, we make the following observations.

First, we calibrate the model so that preference parameters ν and ζ match the domestic shares of final and input expenditure. With an appropriate choice of units, we have $P_{H0}/P_{C0} = \tau_{C0} P_{F0}/P_{C0} = 1$ and $P_{H0}/P_{M0} = \tau_{M0} P_{F0}/P_{M0} = 1$, so $C_{H0} = \nu C_0$, $C_{F0} = (1 - \nu) C_0$, $M_{H0} = \zeta M_0$, and $M_{F0} = (1 - \zeta) M_0$.

Second, with this result in hand, it is possible to show that $\hat{d}_{Ct} = 0$ and $\hat{d}_{Mt} = 0$ in any equilibrium. Working first with consumption, the final goods aggregator implies that aggregate consumption satisfies $\hat{c}_t = \frac{C_{H0}}{C_0} \hat{c}_{Ht} + \frac{C_{F0}}{C_0} \hat{c}_{Ft}$. Given the nominal spending identity $P_{Ct} C_t = P_{Ht} C_{Ht} + \tau_{Ct} P_{Ft} C_{Ft}$, then the price index can be expressed as $\hat{p}_{Ct} = \frac{C_{H0}}{C_0} \hat{p}_{Ht} + \frac{C_{F0}}{C_0} (\hat{\tau}_{Ct} + \hat{p}_{Ft})$, where $\frac{P_{H0}}{P_{C0}} = 1$ and $\frac{\tau_{C0} P_{F0}}{P_{C0}} = 1$. Then, plugging the first order conditions into the consumption aggregator and simplifying yields:

$$0 = \frac{C_{H0}}{C_0} \left[\left(\hat{d}_{Ct} + \hat{p}_{Ht} - \hat{p}_{Ct} \right) \right] + \frac{C_{F0}}{C_0} \left[\left(\hat{d}_{Ct} + \hat{\tau}_{Ct} + \hat{p}_{Ft} - \hat{p}_{Ct} \right) \right] = \hat{d}_{Ct},$$

where the second equality uses the prior result for \hat{p}_{Ct} . An identical procedure applied to inputs then returns $\hat{d}_{Mt} = 0$ as well.

Third, we can draw on the arguments in the text and these first two results to write relative prices as functions of changes in domestic sourcing shares:

$$\hat{p}_{Ht} - \hat{p}_{Ct} = -\frac{1}{\sigma - 1} \hat{\lambda}_{Ht}^C \quad (173)$$

$$\hat{\tau}_{Ct} + \hat{p}_{Ft} - \hat{p}_{Ct} = -\frac{1}{\sigma - 1} \hat{\lambda}_{Ft}^C = -\frac{1}{\sigma - 1} \frac{\lambda_{F0}}{\lambda_{H0}} \hat{\lambda}_{Ht}^C \quad (174)$$

$$\hat{p}_{Ht} - \hat{p}_{Mt} = -\frac{1}{\sigma - 1} \hat{\lambda}_{Ht}^M \quad (175)$$

$$\hat{\tau}_{Mt} + \hat{p}_{Ft} - \hat{p}_{Mt} = -\frac{1}{\sigma - 1} \hat{\lambda}_{Ft}^M = -\frac{1}{\sigma - 1} \frac{\lambda_{F0}}{\lambda_{H0}} \hat{\lambda}_{Ht}^M, \quad (176)$$

where we have used $\lambda_{Ht}^C + \lambda_{Ft}^C = 1$ and $\lambda_{Ht}^M + \lambda_{Ft}^M = 1$.

Fourth, given this rewriting, we can solve a subset of the equilibrium system to determine inflation. In particular, we can drop equations that pertain to demand for foreign final goods and inputs, and we can drop the dynamic pricing equation for imports and associated definitions of the elasticity of demand for imports.

Together, these four sets of results imply we can collapse down the equilibrium into the system presented in Table 6. In the table, $\widehat{r}\widehat{p}_{Xt} \equiv \widehat{p}_{Xt} - \widehat{p}_{Ht}$, and other variables match definitions in the baseline model.

A few additional words are helpful to interpret how we write the elasticity faced by domestic firms ($\widehat{\epsilon}_{Ht}$) in this table, and this discussion helps one interpret Equations 46-47 in the main text as well. In the steady state, $\epsilon_{H0}^C = \epsilon_{H0}^M = \sigma$. A sketch proof of this statement is as follows. Using the Klenow-Willis functional form, the first order condition for consumption is $\frac{C_{H0}}{vC_0} = \left(1 + \varepsilon \ln\left(\frac{\sigma-1}{\sigma}\right) - \varepsilon \ln\left(\frac{D_{C0}P_{H0}}{P_{C0}}\right)\right)^{\sigma/\varepsilon}$, where we have evaluated it in the steady state. Since $\frac{P_{H0}}{P_{C0}} = 1$ and $\frac{C_{H0}}{vC_0} = 1$ in the steady state, then $D_{C0} = \frac{\sigma-1}{\sigma}$. A parallel argument for inputs implies that $D_{M0} = \frac{\sigma-1}{\sigma}$ too. Then $\epsilon_{H0}^C = \sigma \left(1 + \varepsilon \ln\left(\frac{\sigma-1}{\sigma}\right) - \varepsilon \ln\left(\frac{D_{C0}P_{H0}}{P_{C0}}\right)\right)^{-1} = \sigma$ and $\epsilon_{H0}^M = \sigma \left(1 + \varepsilon \ln\left(\frac{\sigma-1}{\sigma}\right) - \varepsilon \ln\left(\frac{D_{M0}P_{H0}}{P_{M0}}\right)\right)^{-1} = \sigma$. Given this result, $\frac{C_{H0}}{Y_{H0}} \frac{\epsilon_{H0}^C}{\epsilon_{H0}} (\widehat{c}_{Ht} - \widehat{y}_{Ht}) + \frac{M_{H0}}{Y_{H0}} \frac{\epsilon_{H0}^M}{\epsilon_{H0}} (\widehat{m}_{Ht} - \widehat{y}_{Ht}) = 0$, since $\frac{C_{H0}}{Y_{H0}} \widehat{c}_{Ht} + \frac{M_{H0}}{Y_{H0}} \widehat{m}_{Ht} = \widehat{y}_{Ht}$. So, $\widehat{\epsilon}_{Ht} = \frac{C_{H0}}{Y_{H0}} \widehat{\epsilon}_{Ht}^C + \frac{M_{H0}}{Y_{H0}} \widehat{\epsilon}_{Ht}^M$, as in Equation 46. Further, the same results lead $\widehat{\epsilon}_{Ht}^C$ and $\widehat{\epsilon}_{Ht}^M$ to simplify as well, where only the parameter ε governs how relative prices influence deviations in elasticities from steady state.

G Multisector Model

The structure of the multisector model follows the one sector model closely, so we emphasize modifications in our discussion here.

Consumers Consumer preferences are given by Equation 18, with aggregate consumption defined as in Equations 49-50. The consumers intertemporal budget constraint is adjusted for the multisector structure:

$$\sum_s P_{Ht}(s) C_{Ht}(s) + \sum_s P_{Ft}(s) \tau_{Ct}(s) C_{Ft}(s) + E_t [Q_{t,t+1} D_{t+1}] \leq D_t + W_t L_t, \quad (177)$$

where the prices of the composite goods are $\{P_{Ht}(s), P_{Ft}(s)\}$. The parameter $\tau_{Ct}(s)$ is an iceberg trade cost, paid on imports.

Given $\{P_{Ht}(s), P_{Ft}(s), Q_{t,t+1}, W_t\}$ and initial asset holdings, the consumer's problem is to choose $\{C_t, C_{Ht}(s), C_{Ft}(s), L_t, D_{t+1}\}$ to maximize 18 given 49-50 and subject to 177 and the standard transversality condition.

Table 5: Log-Linearized Variable Markups Model Equilibrium

Consumption-Leisure	$-\frac{\rho}{\psi} \hat{c}_t + \frac{1}{\psi} [\hat{w}_t - \hat{p}_{Ht}] + \frac{1}{\psi} (\hat{p}_{Ht} - \hat{p}_{Ct}) = \hat{l}_t$
Consumption Allocation	$\hat{c}_{Ht} = -\sigma \left(\frac{C_{H0}}{vC_0} \right)^{-\varepsilon/\sigma} (\hat{d}_{Ct} + \hat{p}_{Ht} - \hat{p}_{Ct}) + \hat{c}_t$ $\hat{c}_{Ft} = -\sigma \left(\frac{C_{F0}}{(1-v)C_0} \right)^{-\varepsilon/\sigma} (\hat{d}_{Ct} + \hat{\tau}_{Ct} + \hat{p}_{Ft} - \hat{p}_{Ct}) + \hat{c}_t$ $0 = \exp \left(-\frac{1}{\varepsilon} \left(\frac{C_{H0}}{vC_0} \right)^{\varepsilon/\sigma} \right) \frac{C_{H0}}{C_0} [\hat{c}_{Ht} - \hat{c}_t] + \exp \left(-\frac{1}{\varepsilon} \left(\frac{C_{F0}}{(1-v)C_0} \right)^{\varepsilon/\sigma} \right) \frac{C_{F0}}{C_0} [\hat{c}_{Ft} - \hat{c}_t]$
Euler Equation	$\hat{c}_t = E_t (\hat{c}_{t+1} - (r_t - \pi_{Ct+1})/\rho)$ $[\hat{w}_t - \hat{p}_{Ht}] + \hat{l}_t = [\hat{m}c_t - \hat{p}_{Ht}] + \hat{y}_t$
Input Choices	$[\hat{p}_{Mt} - \hat{p}_{Ht}] + \hat{m}_t = [\hat{m}c_t - \hat{p}_{Ht}] + \hat{y}_t$ $\hat{m}_{Ht} = -\sigma \left(\frac{M_{H0}}{\zeta M_0} \right)^{-\varepsilon/\sigma} (\hat{d}_{Mt} + \hat{p}_{Ht} - \hat{p}_{Mt}) + \hat{m}_t$ $\hat{m}_{Ft} = -\sigma \left(\frac{M_{F0}}{(1-\zeta)M_0} \right)^{-\varepsilon/\sigma} (\hat{d}_{Mt} + \hat{\tau}_{Mt} + \hat{p}_{Ht} - \hat{p}_{Mt}) + \hat{m}_t$ $0 = \exp \left(-\frac{1}{\varepsilon} \left(\frac{M_{H0}}{\zeta M_0} \right)^{\varepsilon/\sigma} \right) \frac{M_{H0}}{M_0} [\hat{m}_{Ht} - \hat{m}_t] + \exp \left(-\frac{1}{\varepsilon} \left(\frac{M_{F0}}{(1-\zeta)M_0} \right)^{\varepsilon/\sigma} \right) \frac{M_{F0}}{M_0} [\hat{m}_{Ft} - \hat{m}_t]$
Marginal Cost	$\hat{m}c_t - \hat{p}_{Ht} = -\hat{z}_t + (1-\alpha)(\hat{w}_t - \hat{p}_{Ht}) + \alpha(\hat{p}_{Mt} - \hat{p}_{Ht})$
Domestic Price Setting	$\pi_{Ht} = -\frac{\hat{\epsilon}_{Ht}}{\phi} + \frac{(\epsilon_{H0}-1)}{\phi} (\hat{m}c_t - \hat{p}_{Ht}) + \beta E_t (\pi_{Ht+1})$
Import Price Setting	$\pi_{Ft} = -\frac{\hat{\epsilon}_{Ft}}{\phi} + \frac{(\epsilon_{F0}-1)}{\phi} \left[\hat{q}_t - (\hat{p}_{Ft} - \hat{p}_{Ct}) + (\hat{m}c_t^* - \hat{p}_{Ct}^*) \right] + \beta E_t (\pi_{Ft+1})$
Export Price Setting	$\pi_{Xt} = \frac{(\epsilon_{X0}-1)}{\phi} [(\hat{m}c_t - \hat{p}_{Ht}) - (\hat{p}_{Xt} - \hat{p}_{Ht})] + \beta E_t (\pi_{Xt+1})$
Auxiliary Inflation Definitions	$\pi_{Ht} = [(\hat{p}_{Ht} - \hat{p}_{Ct}) - (\hat{p}_{Ht-1} - \hat{p}_{Ct-1})] + \pi_{Ct}$ $\pi_{Xt} = [(\hat{p}_{Xt} - \hat{p}_{Ht}) - (\hat{p}_{Xt-1} - \hat{p}_{Ht-1})] + \pi_{Ht}$ $\pi_{Ft} = [(\hat{p}_{Ft} - \hat{p}_{Ct}) - (\hat{p}_{Ft-1} - \hat{p}_{Ct-1})] + \pi_{Ct}$ $\hat{\epsilon}_{Ht} = \left[\frac{C_{H0}}{Y_{H0}} \frac{\epsilon_{H0}^C}{\epsilon_{H0}} \hat{\epsilon}_{Ht}^C + \frac{M_{H0}}{Y_{H0}} \frac{\epsilon_{H0}^M}{\epsilon_{H0}} \hat{\epsilon}_{Ht}^M \right] + \frac{C_{H0}}{Y_{H0}} \frac{\epsilon_{H0}^C}{\epsilon_{H0}} (\hat{c}_{Ht} - \hat{y}_{Ht}) + \frac{M_{H0}}{Y_{H0}} \frac{\epsilon_{H0}^M}{\epsilon_{H0}} (\hat{m}_{Ht} - \hat{y}_{Ht})$ with $\hat{\epsilon}_{Ht}^C = -\frac{\varepsilon}{\sigma} \epsilon_{H0}^C (\hat{d}_{Ct} + \hat{p}_{Ht} - \hat{p}_{Ct})$ and $\hat{\epsilon}_{Ht}^M = -\frac{\varepsilon}{\sigma} \epsilon_{H0}^M (\hat{d}_{Mt} + \hat{p}_{Ht} - \hat{p}_{Mt})$
Elasticities	$\hat{\epsilon}_{Ft} = \left[\frac{C_{F0}}{Y_{F0}} \frac{\epsilon_{F0}^C}{\epsilon_{F0}} \hat{\epsilon}_{Ft}^C + \frac{M_{F0}}{Y_{F0}} \frac{\epsilon_{F0}^M}{\epsilon_{F0}} \hat{\epsilon}_{Ft}^M \right] + \frac{C_{F0}}{Y_{F0}} \frac{\epsilon_{F0}^C}{\epsilon_{F0}} (\hat{c}_{Ft} - \hat{y}_{Ft}) + \frac{M_{F0}}{Y_{F0}} \frac{\epsilon_{F0}^M}{\epsilon_{F0}} (\hat{m}_{Ft} - \hat{y}_{Ft})$ with $\hat{\epsilon}_{Ft}^C = -\frac{\varepsilon}{\sigma} \epsilon_{H0}^C (\hat{d}_{Ct} + \hat{\tau}_{Ct} + \hat{p}_{Ft} - \hat{p}_{Ct})$ and $\hat{\epsilon}_{Ft}^M = -\frac{\varepsilon}{\sigma} \epsilon_{H0}^M (\hat{d}_{Mt} + \hat{\tau}_{Mt} + \hat{p}_{Ft} - \hat{p}_{Mt})$
Price Indexes	$\hat{c}_t = \left(\frac{P_{H0}C_{H0}}{P_{C0}C_0} \right) (\hat{p}_{Ht} - \hat{p}_{Ct}) + \left(\frac{\tau_{C0}P_{F0}C_{F0}}{P_{C0}C_0} \right) (\hat{\tau}_{Ct} + \hat{p}_{Ft} - \hat{p}_{Ct}) + \left(\frac{P_{H0}C_{H0}}{P_{C0}C_0} \right) \hat{c}_{Ht} + \left(\frac{\tau_{C0}P_{F0}C_{F0}}{P_{C0}C_0} \right) \hat{c}_{Ft}$ $\hat{m}_t = \left(\frac{P_{H0}M_{H0}}{P_{M0}M_0} \right) (\hat{p}_{Ht} - \hat{p}_{Mt}) + \left(\frac{\tau_{M0}P_{F0}M_{F0}}{P_{M0}M_0} \right) (\hat{\tau}_{Mt} + \hat{p}_{Ft} - \hat{p}_{Mt}) + \left(\frac{P_{H0}M_{H0}}{P_{M0}M_0} \right) \hat{m}_{Ht} + \left(\frac{\tau_{M0}P_{F0}M_{F0}}{P_{M0}M_0} \right) \hat{m}_{Ft}$
Market Clearing	$\hat{y}_t = \frac{Y_{H0}}{Y_0} \hat{y}_{Ht} + \frac{X_0}{Y_0} \hat{x}_t$ $\hat{y}_{Ht} = \frac{C_{H0}}{Y_{H0}} \hat{c}_{Ht} + \frac{M_{H0}}{Y_{H0}} \hat{m}_{Ht}$ $\hat{y}_{Ft} = \frac{C_{F0}}{Y_{F0}} \hat{c}_{Ht} + \frac{M_{F0}}{Y_{F0}} \hat{m}_{Ft}$
Exports	$\hat{x}_t = -\epsilon_X ((\hat{p}_{Xt} - \hat{p}_{Ht}) + (\hat{p}_{Ht} - \hat{p}_{Ct}) - \hat{q}_t) + \hat{c}_t^*$
Complete Asset Markets	$\hat{c}_t = \hat{c}_t^* + \frac{1}{\rho} \hat{q}_t$
Monetary Policy Rule	$\hat{r}_t = \omega \pi_{Ct}$

Table 6: Simplified VM Model with Domestic Sourcing Shocks

Consumption-Leisure	$-\frac{\rho}{\psi}\hat{c}_t + \frac{1}{\psi}\widehat{r\bar{w}}_t - \frac{1}{\psi(\sigma-1)}\hat{\lambda}_{Ht}^C = \hat{l}_t$
Consumption Allocation	$\hat{c}_{Ht} = \frac{\sigma}{\sigma-1}\hat{\lambda}_{Ht}^C + \hat{c}_t$
Euler Equation	$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\rho}(\hat{r}_t - E_t\pi_{Ct+1})$
	$\hat{l}_t = \widehat{r\bar{m}c}_t + \hat{y}_t - \widehat{r\bar{w}}_t$
Input Choices	$\hat{m}_t = \widehat{r\bar{m}c}_t + \hat{y}_t - \frac{1}{\sigma-1}\hat{\lambda}_{Ht}^M$
	$\hat{m}_{Ht} = \frac{\sigma}{\sigma-1}\hat{\lambda}_{Ht}^M + \hat{m}_t$
Marginal Cost	$\widehat{r\bar{m}c}_t = (1-\alpha)\widehat{r\bar{w}}_t + \frac{\alpha}{\sigma-1}\hat{\lambda}_{Ht}^M - \hat{z}_t$
Domestic Price Inflation	$\pi_{Ht} = -\frac{1}{\phi}\hat{\epsilon}_{Ht} + \frac{(\epsilon_{H0}-1)}{\phi}\widehat{r\bar{m}c}_t + \beta E_t(\pi_{Ht+1})$
Export Price Inflation	$\pi_{Xt} = \left(\frac{\epsilon_X-1}{\phi}\right)(\widehat{r\bar{m}c}_t - \widehat{r\bar{p}}_{Xt}) + \beta E_t(\pi_{Xt+1})$
	with $\pi_{Xt} = [\widehat{r\bar{p}}_{Xt} - \widehat{r\bar{p}}_{Xt-1}] + \pi_{Ht}$
Consumer Price Inflation	$\pi_{Ct} = \pi_{Ht} + \frac{1}{\sigma-1}(\hat{\lambda}_{Ht}^C - \hat{\lambda}_{Ht-1}^C)$
	$\hat{\epsilon}_{Ht} = \frac{C_{H0}}{Y_{H0}}\hat{\epsilon}_{Ht}^C + \frac{M_{H0}}{Y_{H0}}\hat{\epsilon}_{Ht}^M$
Elasticities	with $\hat{\epsilon}_{Ht}^C = -\left(\frac{\epsilon}{\sigma-1}\right)\hat{\lambda}_{Ht}^C$
	and $\hat{\epsilon}_{Ht}^M = -\left(\frac{\epsilon}{\sigma-1}\right)\hat{\lambda}_{Ht}^M$
	$\hat{y}_t = \frac{Y_{H0}}{Y_0}\hat{y}_{Ht} + \frac{X_0}{Y_0}\hat{x}_t$
Market Clearing	$\hat{y}_{Ht} = \frac{C_{H0}}{Y_{H0}}\hat{c}_{Ht} + \frac{M_{H0}}{Y_{H0}}\hat{m}_{Ht}$
	$\hat{x}_t = -\epsilon_X\widehat{r\bar{p}}_{Xt} + \frac{\epsilon_X}{\sigma-1}\hat{\lambda}_{Ht}^C + \epsilon_X\hat{q}_t + \hat{c}_t^*$
	$\hat{c}_t = \hat{c}_t^* + \frac{1}{\rho}\hat{q}_t$
Monetary Policy Rule	$\hat{r}_t = \omega\pi_{Ct}$

Producers Similar to the baseline model, there is a unit continuum of varieties in each sector, which are produced under monopolistic competition. To simplify the notation, we assume that these varieties are aggregated into composite goods, which are then consumed at home and exported.⁴²

Varieties are aggregated by competitive intermediary firms into sector-level composites with the technology:

$$Y_t(s) = \left(\int_0^1 Y_t(s,i)^{(\epsilon(s)-1)/\epsilon_t(s)} dj \right)^{\epsilon(s)/(\epsilon(s)-1)}, \quad (178)$$

where $Y_t(s,i)$ is the quantity of variety i used to produce the composite Home good and $\epsilon(s)$ is the elasticity for sector s . Given prices $\{P_{Ht}(s,i)\}$ for individual varieties, cost minimization by the intermediaries yields these first order conditions and price indexes: $Y_t(s,i) = \left(\frac{P_t(s,i)}{P_{Ht}(s)} \right)^{-\epsilon(s)} Y_t(s)$ and $P_{Ht}(s) = \left[\int_0^1 P_{Ht}(s,i)^{1-\epsilon(s)} di \right]^{1/(1-\epsilon(s))}$.

The production function for individual varieties is given by Equations 52-54. Producers of differentiated output set the prices of their goods taking as given the demand and select the input mix to satisfy the implied demand. The firm chooses a sequence for $P_{Ht}(s,i)$ to maximize:

$$E \sum_{t=0}^{\infty} \beta^t \frac{C_t^{-\rho}}{C_0^{-\rho}} \frac{1}{P_{Ct}} \left[P_{Ht}(s,i) Y_t(s,i) - MC_t(s,i) Y_t(s,i) - \frac{\phi}{2} \left(\frac{P_{Ht}(s,i)}{P_{H,t-1}(s,i)} - 1 \right)^2 P_{Ht}(s) Y_t(s) \right], \quad (179)$$

where $MC_t(s,i)$ is the constant marginal costs of the firm (defined below). Further, firm i in sector s chooses $\{L_t(s,i), M_t(s,i), M_{Ht}(s,i), M_{Ft}(s,i)\}$ to minimize the cost of producing a given amount of output $Y_t(s,i)$. Like consumers, the firm must pay iceberg trade costs to import inputs, given by $\tau_{Mt}(s)$, where s denotes the source sector of the goods.

Closing the Model Output market clearing for composite goods is:

$$Y_t(s) = C_{Ht}(s) + \sum_{s'} \int_0^1 M_{Ht}(s,s',j) dj + X_t(s) + \int_0^1 \frac{\phi}{2} \left(\frac{P_{Ht}(s,i)}{P_{H,t-1}(s,i)} - 1 \right)^2 Y_t(s) di \quad (180)$$

where $X_t(s)$ is sector-level exports. We again assume that demand for exports has a CES structure: $X_t(s) = \left(\frac{P_{Ht}(s)}{S_t P_t^*(s)} \right)^{-\eta(s)} C_t^*(s)$ with $P_t^*(s) C_t^*(s) = \gamma(s) P_t^* C_t^*$ where the export demand elasticity is the same as that between Home and Foreign goods for domestic agents.⁴³

The labor market clearing condition is: $\sum_s \int_0^1 L_t(s,i) di = L_t$. The international risk sharing con-

⁴²In contrast, in the baseline model we define preferences and technologies over varieties directly. Here we move aggregation of varieties into a separate production sector to lighten the notation.

⁴³As in the baseline model, P_t^* and C_t^* are aggregate foreign prices and consumption. The model now accomodates sector-level shocks, via foreign sector-level prices $P_t^*(s)$.

dition applies, as in the baseline model. And we close the model via the same Taylor rule for domestic interest rates.

Equilibrium For reference, we collect equilibrium conditions in Table 7, without imposing a price normalization.

As in the baseline model, we work with the model equilibrium written in terms of domestic sourcing shares. $\Lambda_{Ct}(s) \equiv \frac{P_{Ht}(s)C_{Ht}(s)}{P_{Ct}(s)C_t(s)}$ and $\Lambda_{Mt}(s',s) = \frac{P_{Ht}(s')M_{Ht}(s',s)}{P_{Mt}(s',s)M_t(s',s)}$. Using first order conditions, we can related equilibrium prices to these shares as follows:

$$\frac{P_{Ht}(s)}{P_{Ct}(s)} = \left(\frac{\Lambda_{Ht}^C(s)}{v(s)} \right)^{1/(1-\eta(s))} \quad (181)$$

$$\frac{P_{Ht}(s')}{P_{Mt}(s',s)} = \left(\frac{\Lambda_{Ht}^M(s',s)}{\xi(s',s)} \right)^{1/(1-\eta(s'))} \quad (182)$$

Thus, we can swap out for $\frac{P_{Ht}(s)}{P_{Ct}(s)}$ and $\frac{P_{Ht}(s')}{P_{Mt}(s',s)}$ throughout the equilibrium system.

We collect log-linearized equilibrium conditions in Table 8. In the table, we define relative prices as follows: $\widehat{r\bar{w}}_t \equiv \widehat{w}_t - \widehat{p}_{Ct}$, $\widehat{r\bar{p}}_{Ct}(s) \equiv \widehat{p}_{Ct}(s) - \widehat{p}_{Ct}$, $\widehat{r\bar{m}\bar{c}}_t(s) \equiv \widehat{m\bar{c}}_t(s) - \widehat{p}_{Ht}(s)$, $\widehat{r\bar{p}\bar{m}}_t(s) \equiv \widehat{p}_{Mt}(s) - \widehat{p}_{Ct}$, and $\widehat{r\bar{p}}_t(s',s) \equiv \widehat{p}_t(s',s) - \widehat{p}_{Mt}(s)$. Given parameters, exogenous variables (foreign variables and domestic productivity), and domestic sourcing shares $\hat{\lambda}_{Ht}^C(s)$ and $\hat{\lambda}_{Ht}^M(s',s)$, an equilibrium is a path for prices $\{\widehat{r\bar{w}}_t, \widehat{r\bar{p}}_{Ct}(s), \widehat{r\bar{m}\bar{c}}_t(s), \widehat{r\bar{p}\bar{m}}_t(s), \widehat{r\bar{p}}_t(s',s), \hat{r}_t, \hat{q}_t, \pi_{Ht}(s), \pi_t\}$ and quantities $\{\hat{c}_t, \hat{l}_t, \hat{c}_t(s), \hat{c}_{Ht}(s), \hat{l}_t(s), \hat{y}_t(s), \hat{m}_t(s), \hat{m}_t(s',s), \hat{m}_{Ht}(s',s), \hat{x}_t(s)\}$ that solve the dynamic system in Table 8.

To simulate the model, we set parameters already defined in Table 3 to the same values, and we set $\eta(s) = \epsilon(s) = 3$, except where noted in the text. We set parameters that govern the multisector input-output structure to match data for 1996, with non-manufacturing sectors defined to include

agriculture, natural resources, and services. The parameters are given as follows:

$$\begin{aligned}
 \begin{bmatrix} \zeta(1) \\ \zeta(2) \end{bmatrix} &= \begin{bmatrix} 0.1854 \\ 0.8146 \end{bmatrix} \\
 \begin{bmatrix} \alpha(1) \\ \alpha(2) \end{bmatrix} &= \begin{bmatrix} 0.7896 \\ 0.4402 \end{bmatrix} \\
 \begin{bmatrix} \alpha(1,1) & \alpha(1,2) \\ \alpha(2,1) & \alpha(2,2) \end{bmatrix} &= \begin{bmatrix} 0.4868 & 0.0911 \\ 0.3028 & 0.3491 \end{bmatrix} \\
 \begin{bmatrix} \nu(1) \\ \nu(2) \end{bmatrix} &= \begin{bmatrix} 0.7755 \\ 0.9954 \end{bmatrix} \\
 \begin{bmatrix} \xi(1,1) & \xi(1,2) \\ \xi(2,1) & \xi(2,2) \end{bmatrix} &= \begin{bmatrix} 0.7221 & 0.7624 \\ 0.8876 & 0.9815 \end{bmatrix}.
 \end{aligned}$$

Table 7: Equilibrium Conditions for the Multisector Model

Consumption-Leisure	$C_t^{-\rho} \frac{W_t}{P_{Ct}} = \mu L_t^\psi$
Consumption Allocation	$C_t(s) = \zeta(s) \left(\frac{P_{Ct}(s)}{P_{Ct}} \right)^{-\vartheta} C_t$
	$C_{Ht}(s) = \nu(s) \left(\frac{P_{Ht}(s)}{P_{Ct}(s)} \right)^{-\eta(s)} C_t(s)$
	$C_{Ft}(s) = (1 - \nu(s)) \left(\frac{\tau_{Ct}(s) P_{Ft}(s)}{P_{Ct}(s)} \right)^{-\eta(s)} C_t(s)$
Euler Equation	$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{P_{Ct}}{P_{C,t+1}} (1 + i_{t+1}) \right]$
	$W_t L_t(s) = (1 - \alpha(s)) M C_t(s) Y_t(s)$
	$P_{Mt}(s) M_t(s) = \alpha(s) M C_t(s) Y_t(s)$
Input Choices	$M_t(s', s) = \frac{\alpha(s', s)}{\alpha(s)} \left(\frac{P_t(s', s)}{P_{Mt}(s)} \right)^{-\kappa} M_t(s)$
	$M_{Ht}(s', s) = \xi(s', s) \left(\frac{P_{Ht}(s')}{P_t(s', s)} \right)^{-\eta(s')} M_t(s', s)$
	$M_{Ft}(s', s) = (1 - \xi(s', s)) \left(\frac{\tau_{Mt}(s') P_{Ft}(s')}{P_t(s', s)} \right)^{-\eta(s')} M_t(s', s)$
Marginal Cost	$MC_t(s) = A \frac{W_t^{1-\alpha(s)} P_{Mt}(s)^{\alpha(s)}}{Z_t(s)} \quad \text{with} \quad A(s) \equiv \alpha(s)^{-\alpha(s)} (1 - \alpha(s))^{-(1-\alpha(s))}$
Input Prices	$P_{Mt}(s) = \left(\sum_{s'} \left(\frac{\alpha(s', s)}{\alpha(s)} \right) P_t(s', s)^{1-\kappa} \right)^{1/(1-\kappa)}$
	$P_t(s', s) = \left[\xi(s', s) P_{Ht}(s')^{1-\eta(s')} + (1 - \xi(s', s)) (\tau_{Mt}(s') P_{Ft}(s'))^{1-\eta(s')} \right]^{1/(1-\eta(s'))}$
Domestic Pricing	$(1 - \epsilon(s)) + \epsilon(s) \frac{MC_t(s)}{P_{Ht}(s)} - \phi \left(\frac{P_{Ht}(s)}{P_{H,t-1}(s)} - 1 \right) \frac{P_{Ht}(s)}{P_{H,t-1}(s)}$
	$+ E_t \left[\beta \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}} \frac{P_{Ct}}{P_{C,t+1}} \phi \left(\frac{P_{H,t+1}(s)}{P_{H,t}(s)} - 1 \right) \frac{P_{H,t+1}(s) Y_{t+1}(s)}{P_{Ht}(s) Y_t(s)} \frac{P_{H,t+1}(s)}{P_{Ht}(s)} \right] = 0$
	$P_{Ct} = \left(\sum_s \zeta(s) P_{Ct}(s)^{1-\vartheta} \right)^{1/(1-\vartheta)}$
Consumer Prices	$P_{Ct}(s) = \left(\nu(s) P_{Ht}(s)^{1-\eta(s)} + (1 - \nu(s)) (\tau_{Ct}(s) P_{Ft}(s)) \right)^{1/(1-\eta(s))}$
	$Y_t(s) = C_{Ht}(s) + \sum_{s'} M_{Ht}(s, s') + X_t(s) + \frac{\phi}{2} \left(\frac{P_{Ht}(s)}{P_{H,t-1}(s)} - 1 \right)^2 Y_t(s)$
Market Clearing	$X_t(s) = \left(\frac{P_{Ht}(s)}{P_{Ct} Q_t} \right)^{-\eta(s)} \frac{\gamma(s) P_t^* C_t^*}{P_t^*(s)}$
	$\left(\frac{C_t}{C_t^*} \right)^{-\rho} Q_t = \Xi$
	$\sum_s L_t(s) = L_t$
Monetary Policy Rule	$1 + i_t = (1 + i_0) \left(\frac{P_{Ct}}{P_{C,t-1}} \right)^\omega$

Table 8: Log-Linearized Equilibrium Conditions for the Multisector Model

Consumption-Leisure	$-\frac{\rho}{\psi} \hat{c}_t + \frac{1}{\psi} \widehat{r\bar{w}}_t = \hat{l}_t$
Consumption Allocation	$\hat{c}_t(s) = -\vartheta \widehat{r\bar{p}}_{Ct}(s) + \hat{c}_t$
Euler Equation	$\hat{c}_{Ht}(s) = -\frac{\eta(s)}{\eta(s)-1} \hat{\lambda}_{Ht}^C(s) + \hat{c}_t(s)$ $\hat{c}_t = E_t(\hat{c}_{t+1} - (r_t - \pi_{Ct+1})/\rho)$ $\widehat{r\bar{w}}_t + \hat{l}_t(s) = \widehat{r\bar{m}c}_t(s) - \frac{1}{\eta(s)-1} \hat{\lambda}_{Ht}^C(s) + \widehat{r\bar{p}}_{Ct}(s) + \hat{y}_t(s)$
Input Choices	$\widehat{r\bar{p}m}_t(s) + \hat{m}_t(s) = \widehat{r\bar{m}c}_t(s) - \frac{1}{\eta(s)-1} \hat{\lambda}_{Ht}^C(s) + \widehat{r\bar{p}}_{Ct}(s) + \hat{y}_t(s)$ $\hat{m}_t(s', s) = -\kappa \widehat{r\bar{p}}_t(s', s) + \hat{m}_t(s)$ $\hat{m}_{Ht}(s', s) = -\frac{\eta(s')}{\eta(s')-1} \hat{\lambda}_{Ht}^M(s', s) + \hat{m}_t(s', s)$
Real Marginal Cost	$\widehat{r\bar{m}c}_t(s) - \frac{1}{\eta(s)-1} \hat{\lambda}_{Ht}^C(s) + \widehat{r\bar{p}}_{Ct}(s) = (1 - \alpha(s)) \widehat{r\bar{w}}_t + \alpha(s) \widehat{r\bar{p}m}_t(s) - \hat{z}_t(s)$
Input Prices	$0 = \sum_{s'} \left(\frac{P_{M0}(s', s) M_0(s', s)}{P_{M0}(s) M_0(s)} \right) \widehat{r\bar{p}}_t(s', s)$ $\widehat{r\bar{p}}_t(s', s) = \left(\frac{1}{\eta(s')-1} \right) \hat{\lambda}_{Ht}^M(s', s) - \frac{1}{\eta(s)-1} \hat{\lambda}_{Ht}^C(s) - \widehat{r\bar{p}}_{Ct}(s)$
Domestic Pricing	$\pi_{Ht}(s) = \frac{(\epsilon(s)-1)}{\phi} \widehat{r\bar{m}c}_t(s) + \beta E_t[\pi_{Ht+1}(s)]$ $0 = \sum_s \left(\frac{P_{C0}(s) C_0(s)}{P_{C0} C_0} \right) \widehat{r\bar{p}}_{Ct}(s)$
Consumer Prices	$\pi_{Ht}(s) = -\frac{1}{\eta(s)-1} \left(\hat{\lambda}_{Ht}^C(s) - \hat{\lambda}_{Ht-1}^C(s) \right) + \widehat{r\bar{p}}_{Ct}(s) - \widehat{r\bar{p}}_{Ct-1}(s) + \pi_t$
Market Clearing	$\hat{y}_t(s) = \frac{C_{H0}(s)}{Y_0(s)} \hat{c}_{Ht}(s) + \sum_{s'} \frac{M_{H0}(s, s')}{Y_0(s)} \hat{m}_{Ht}(s, s') + \frac{X_0(s)}{Y_0(s)} \hat{x}_t(s)$ $\hat{x}_t(s) = \frac{\eta(s)}{\eta(s)-1} \hat{\lambda}_{Ht}^C(s) - \eta(s) \widehat{r\bar{p}}_{Ct}(s) + \eta(s) \hat{q}_t - (\hat{p}_t^*(s) - \hat{p}_{Ct}^*) + \hat{c}_t^*$ $\hat{c}_t = \hat{c}_t^* + \frac{1}{\rho} \hat{q}_t$ $\sum_s \frac{L_0(s)}{L_0} \hat{l}_t(s) = \hat{l}_t$
Monetary Policy Rule	$\hat{r}_t = \omega \pi_{Ct}$