NBER WORKING PAPER SERIES

ASSET PRICES AND TIME-VARYING RISK

Robert Flood

Working Paper No. 2780

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 1988

This research is part of NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the authors not those of the National Bureau of Economic Research.

NBER Working Paper #2780
December 1988

ASSET PRICES AND TIME-VARYING RISK

ABSTRACT

Observers have often characterized asset markets as being subject to periods of tranquility and periods of turbulence. Until recently, however, researchers were unable to produce closed-form asset pricing formulas in a model environment of time-varying risk. Some work by Abel provided us with the insights needed to produce such formulas. This paper gives an exposition of how to develop the formulas in an environment where the formulas may be obtained using a simple extension of standard tools.

While the paper is intended mainly as an exposition of new work, it also contains a report on the asset market effect of fiscal reform. It is found that entering a period of weak coordination between government spending and taxing (tax rate) policy is good for stock prices.

Robert Flood
Research Department
International Monetary Fund
Washington, DC 20431

I. Introduction

The purpose of this paper is to provide an exposition in a familiar setting of some new methods in asset pricing and to indicate some aspects of the extension of these methods to the open economy. While the paper is designed as an exposition of these new ideas, the fiscal policies studied in Section III may be applicable to discussions of the asset-market effects of recent U.S. fiscal policy efforts.

The paper builds on some work by Andrew Abel (1986) on obtaining closed-form asset pricing formulas for a model environment where agents understand correctly that the nature of risk is time-varying. 1/ In particular, Abel's work was the first that allowed time-varying dividend risk and obtained explicit closed-form pricing formulas for a representative-agent asset-pricing model where agent preferences display constant relative risk aversion.

Researchers have often described asset markets as being characterized by periods of turbulence and periods of tranquility. Before Abel's work, however, we were unable to study explicitly agents' reactions to new information about the riskiness of the asset pricing environment in a framework of a well-specified time-series model of tranquility and turbulence. Indeed, typical in asset pricing work is the examination of agents' reactions to inconceivable once-and-for-all shifts in higher moments. These shifts are inconceivable because the models are solved

---

1/ Others who have adopted and extended the Able model include Giovannini (1987) and Hodrick (1987) both of whom include money in their models and can therefore price assets in nominal terms. In Appendix II, some aspects of nominal asset pricing are discussed.

typically conditionally on the agents supposing that relevent higher moments are assuredly constant through time. Researchers then study the effects of risk changes by changing moments (apparently forever) in a pricing formula constructed under the assumption that such moments would never change. The asset market applicability of the study of such miraculous events is unclear.

If risk, in some sense, changes through time then one way to model explicitly intertemporal variations in the risk environment is by making risk the outcome of a stochastic process. The advantage to adopting an explicit stochastic process for risk is that a stochastic process is useful for separating shocks to the risk environment from predictable changes in the risk environment. Since it is the agents' reactions to their understanding of the time variation in risk that provides the link between the risk environment and asset prices we should model carefully the formation of agents' beliefs about the current and future riskiness of their environment.

Abel studies risk as a time-series process assuming representative agents with rational expectations, constant-relative-risk-aversion period utility functions and assuming a particular functional form for distribution functions generating the underlying shocks. This paper will study agents' reactions to shocks in the time-series process generating their stochastic environment without assuming a representative agent and without making particular functional form distributional assumptions. In fact, in this model the stochastic environment enters the rational expectations problem in a very familiar way and obtaining explicit closed-form solutions requires only a minor extension of the now well-know method of linear undetetermined coefficients.

In order to simplify the analysis some of Able's assumptions are changed. For clarity, the simplest assumption set has been chosen. In making the assumption substitutions, however, a possibly important aspect of decision making may be lost. In particular, it is assumed that time-separable quadratic preferences are capable of capturing adequately the consumption-saving decision facing agents. Given this approximation it is simple to obtain closed-form asset pricing functions for a wide range of distribution functions for the exogenous variables without assuming that individual agents are very similar aside from the functional form of preferences. While the quadratic preferences assumption buys a lot of simplicity it also costs something in terms of a possibly important dimension of risk aversion. With quadratic period utility, agents are risk averse with respect the risks involving the returns on their asset holding but they are risk neutral with respect to the riskiness of the underlying environment. Quadratic preferences induce linearity in the second moments (or variances) of returns. This linearity is the source of much of the simplicity of the present approach.

The paper is divided into four additional sections. In Section II the basic asset pricing model is developed for an infinitely lived representative agent inhabiting a world with time-varying risk. Two simple experi- ments ("warm-ups") are conducted in this section to familiarize the reader with the asset-price solution algorithm. The warm-ups are chosen to illustrate simply environments that might be encountered in practice. In the first example, dividend risk moves exogenously and is unconnected to any other aspect of market fundamentals. In the second example the dividend risk is modelled as a function of past disturbances to dividends. In Section IIII the model is subjected to two kinds of fiscal policy experiments. The first fiscal experiment studies how fiscal feedback policy (from the level of output) influences the asset market impact of a shift in the riskiness of the environment. The second fiscal experiment studies the asset market effect of entering a period of "fiscal reform". Section IV contains some concluding remarks. The text of the paper is followed by three appendices, the first listing some text-referenced coefficients, the second discussing some issues concerning nominal asset pricing and the third explaining a heterogenous-agent multi-country reinterpretation of the model.

II. A Real Asset Pricing Model

The model to be developed in this section is suitable for pricing assets in a hypothetical world that functions without money. Initially it is assumed that a family of representative agents inhabits a closed economy where output is exogenous, government is absent. Preferences are of the representative agent are represented by a quadratic period utility function with lifetime utility being the discounted sum of the period utilities. Some of these assumptions will be relaxed later. The agent's problem is:

$$\max E_t \sum_{i=0}^{\infty} u(c_{t+i})\beta^i, \quad 0 < \beta < 1$$

subject to:

$$c_{t+i} + q_{t+i}k_{t+i} = q_{t+i}k_{t+i-1} + d_{t+i}k_{t+i-1}, \quad i=0,1,2,\ldots,$$

where

E_t = the mathematical expectation operator conditional on complete time t information,

$u(\ )$ = the period utility function,

c_{t+i} = per capita consumption in period t+i,

q_{t+i} = price of equity in terms of consumption goods at time t+i,

k_{t+i} = number of equity shares held by the representative agent at time t+i,

d_{t+i} = dividend paid at the beginning of t+i to the holder of one equity share during t+i-1.

The agents' first order condition for maximizing expected lifetime utility at time t is:

$$(1) \quad u'(c_t)q_t = \beta E_t\{u'(c_{t+1})[q_{t+1} + d_{t+1}]\}$$

Define $z_t \equiv u'(c_t)q_t$; $a_t \equiv u'(c_t)d_t$. Equation (1) now becomes:

$$(2) \quad z_t = \beta E_t z_{t+1} + \beta E_t a_{t+1}.$$

To simplify, assume that a fixed capital stock is the only factor of production, that all output is paid to equity holders as dividends and that there is one equity share per representative agent. Therefore, $d_t = y_t$, where y_t is per capita output at time t. Output depreciates fully in one period so that storage is futile. In equilibrium, therefore, $c_t = y_t$.

The solution of (2) is:

$$z_t = \sum_{i=1}^{\infty} E_t a_{t+i}\beta^i.$$

The solution excludes dynamically-based inteterminacies and is the type of solution that will be dealt with throughout the paper.

Now, assume a particular functional form for period utility:

$$(3) \quad u(c_t) = \alpha c_t - (1/2)c_t^2$$

which is quadratic utility. 1/ Since choices are invariant to linear transformations of the period utility function, the constant has been suppressed a constant in (3) which has been written to highlight the fact that marginal period utility for quadratic preferences is a one parameter family of functions.

Using (3) and $c_t = y_t - d_t$, equation (2) becomes

$$(3a) \quad z_t = \beta E_t z_{t+1} + \beta E_t(\alpha - y_{t+1})y_{t+1},$$

which may also be written as

1/ Quadratic utility is objectionable to some--apparently on aesthetic grounds. It is sometimes mentioned that any fixed-parameter quadratic utility function will eventually be "out grown" by an economy growing without bound. Another way of phrasing this objection is to say that attention should be restricted to model environments where per-capita consumption can grow without bound. This seems to me to be a singularly unworldly restriction.

Another line of attack on quadratic utility is that other utility functions such as constant relative (or constant absolute) risk aversion seem a priori "more plausible". Such a point is, of course, unarguable. One can argue, however, that since empirical work on the stock market (e.g., Flood, Hodrick, and Kaplan (1987)) rejects many popular period utility functions including constant relative risk aversion, constant absolute risk aversion, risk neutrality and quadratic utility there is no way to choose among the popular forms on the basis of conformity with the data. Consequently, for studies like the current one it seems sensible to adopt the utility function which makes exposition as transparent as possible--quadratic utility.

One objection to quadratic period utility which should not be dismissed is that quadratic period utility implies risk neutrality with respect to the riskiness of the environment. As was mentioned in the text this is the technical source of this paper's simplicity.

$$(3b)\quad z_t = \beta E_t z_{t+1} + \beta[\alpha E_t y_{t+1} - (E_t y_{t+1})^2 - h_t],$$

where h_t is the conditional variance of y_{t+1},

$$h_t = E_t[(y_{t+1} - E_t y_{t+1})^2].$$

Like y_t, h_t is an exogenous variable and it will provide most of the "action" presently. In particular, the time series process for h_t will be the vehicle for modelling tranquil and turbulent periods. Notice that the t subscript on h_t refers to the period of the information set relevant to the h_t calculation.

Before moving on to the "warm-ups" notice that equation (3b) is a first-order linear stochastic difference equation in the variable z_t with the composite forcing process given by the right-hand-most term in (3a) or (3b). The experiments to be performed here all involve altering the composite forcing term. If we had simply adopted a linear time series process for the composite forcing term then we would solve (3b) directly by the methods of Hansen and Sargent (1981a, b). This would be an expedient method of solving for z_t, but it would not allow us to study the riskiness of the environment in isolation from other aspects of the forcing process.

Our choice of period utility function implies the moments of y_t that will appear in the forcing term in (3b). With quadratic utility the first two moments of y_t will appear. If a different period utility function had been adopted different moments would appear in the forcing process.

The variable z_t is the marginal utility attached to a unit of the equity. It is equal to period marginal utility of goods times the goods price of the equity. It turns out that it is convenient to work with z_t rather that q_t and so most of the discussion centers on z_t. Usually this is a harmless simplification but not always; in example 2 in this section we study a situation where the simplification is not harmless.

The exposition begins with some pedantic examples to make clear the aspects of the this type of problem and solution method that are familiar from previous work and those that are novel to this approach.

Example 1: Constant first moment, first order auto-regressive conditional variance

Assume the following time series processes for y_t and h_t:

(4a) $y_t = y + w_t$

(4b) $E_t h_{t+1} = \phi h_t + h, \ -1 < \phi\beta < 1,$

where w_t is mean zero and serially uncorrelated. The stochastic process for y_t and h_t are assumed to be such that both are always positive. Further, α is large enough compared to any possible value of y_t that period marginal utility of consumption is always positive, which requires

$y_t < \alpha$. Imposing an upper bound of α on y_t implies boundaries on h_t. 1/ Innovations in h_{t+1} can come from a variety of sources and maybe correlated with innovations in the y_t process or not. 2/

1/ One way to keep variables positive is to introduce stochastic processes of the form:

$$x_{t+1} = x + \nu_{t+1} \quad x_t \ , \ \nu_{t+1} \geq 0.$$

where ν_{t+1} is a positive stochastic coefficient with $E_t\nu_{t+1} = \nu$ and x_t is any variable in the model. If written in the standard linear form, the additive disturbance becomes proportional to x_t. It is convenient to think about many of the variance and covariance processes in the paper in these terms. Further, when stochastic processes are assumed for variables like y_t, one can think of the disturbances as being generated as above.

The requirement $z_t > 0$ implies that h_t is bounded above as well as below would be necessary,therefore, for the variance process, to impose conditions such as $0 < \nu < 1$ with a moving upper bound on ν_{t+1} such that h_{t+1} meets the variance upper bound. Phillipe Weil pointed out to me the upper bound implied by the condition $z_t > 0$.

In applied work one would work with noncentral higher moments (here the second moment) rather than with variances and covariances so that variance conditions could be ignored. On using this type of model in practice see footnote 4.

2/ What is appearing here is not a dynamically-based indeterminacy similar to "bubbles" or "sunspots." What is going on is the required modeling (and identification and estimation in applied work) of an unfamiliar (possibly nonlinear) forcing process. In fact, the nonlinear rational expectations model in this paper is even more tightly constrained than the typical linear rational expectations model in the sense that more exogenous variable equations need to be estimated along with the pricing equation than in the linear case for fully efficient estimation.

The choice of exogenous variable processes used in the examples was guided by the goal of exposition--not estimation--and may not be a good approach in applied work. For instance, to estimate parameters in example 3 (below in the text), I would estimate simultaneously: (i) equation (10a)--the reduced-form pricing function; (ii) equation (9a)--the y_t process; and (iii) a process on y_t^2. The y_t^2 process will place additional overidentifying restrictions on the model (compared to estimating only the reduced-form pricing function and a y_t process). From discussions with Robert Hodrick, and Adrian Pagan it seems to me that it might be a good idea in most applied work in this area to keep the estimated model in terms of y_t and y_t^2 (or their analogs) as in equation (3a) rather than invoking the decomposition allowing the transition to equation (3b). It would be simple in example 3 to reformulate the pricing function to exclude ht and put the agents'

An explicit example where y_t shocks drive h_t is given later. For now, we need not be concerned with how conditional variance shocks enter the model or how they might be correlated with shocks to other variables. Since $E_t y_{t+1}$ is constant, recognize from (3b) that the only variable entering the forcing process is h_t. This equation can be solved using the method of linear undetermined coefficients by "guessing" that the solution is linear in h_t, substituting the trial solution into the equilibrium condition and treating the resulting equation as an identity. In particular, consider the trial solution

$$(5) \qquad z_t = \lambda_{10} + \lambda_{11} h_t .$$

The standard undetermined-coefficients algorithm reveals that (5) is a solution only for

$$(5a) \qquad \lambda_{10} = \frac{\beta}{1 - \beta} \left\{ \theta - \frac{\beta h}{1 - \beta\phi} \right\},$$

$$(5b) \qquad \lambda_{11} = \frac{-\beta}{1-\beta\phi},$$

2/ (Cont'd from page 9) forecasting problem and the reduced-form pricing function in terms of a y_t^2 forcing process in addition to the y_t forcing process. The term h_t was used in the text for ease of exposition and could be reintroduced for experiments once one has obtained consistent estimates of the behavioral parameters.

In practice, h_t is difficult to model because the researcher has no direct observations on h_t and because constructing h_t requires that the researcher correctly model the agents' $E_t y_{t+1}$. This point was made by Pagan (1984) and is very closely related to Flavin's (1983) critique of some of the empirical stock-price variance bounds literature. At the time of Flavin's paper it was pointed out to me by Richard Porter that the variance bounds literature could be consistently recast in terms of first and second moments and that message seems to extend here.

where $\theta = (\alpha - y)y > 0$. Now divide each side of (5) by $(\alpha - y_t)$ to obtain:

$$(6) \quad q_t = \frac{\lambda_{10} + \lambda_{11}h_t}{\alpha - y_t}$$

In this example, two things are responsible for the stochastic behavior of the price of equity, q_t. First, real output disturbances will influence y_t and thereby $u'(y_t) = u'(c_t)$. The influence of current output shocks therefore shows up in the denominator of (6) with an increase in y_t (through an increase in w_t) increasing q_t. Second, new information altering agents' beliefs about the variance of <u>future</u> real output shocks shows up in h_t. Information that makes agents think that future output variance will be higher, will lower q_t at the rate $\lambda_{11}/u'(c_t)$. Notice that λ_{11} depends on ϕ, which is one of the parameters in the h_t time series process. If ϕ is large then a shock to h_t is relatively persistent and has a large effect on q_t as compared to a case where ϕ is small and shocks to h_t are transitory. Notice that the framework can handle very persistent shocks to h_t in that it can handle any ϕ such that $|\phi| < 1/\beta$.

<u>Example 2: ARCH</u>

The methods of Engle (1982) and Bollerslev and Engel (1986) suggest an alternative formulation of the behavior of the conditional variance, h_t. Retain equations (3b) and (4a), the equilibrium Euler equation and the y_t process respectively, but alter the structure of the conditional variance equation to:

$$(7) \quad h_t = h + \delta w_t^2.$$

This example differs from the previous one in that output shocks, w_t, now alter agents' beliefs about the distribution of future output shocks. If an output shock is large (in absolute value) the agents will raise their current beliefs about the variance of future output shocks. Here, make use of the fact $h_t = E_t w_{t+1}^2$ and recognize that (7) is structurally very similar to (4b). Consequently the trial solution for z_t retains its previous functional form but with δw_t^2 replacing ϕh_t. In particular:

$$(8) \quad z_t = \lambda_{20} + \lambda_{21} w_t^2,$$

where

$$(8a) \quad \lambda_{20} = \lambda_{10} \quad \text{(given in (5a))},$$

$$(8b) \quad \lambda_{21} = \frac{-\beta\delta}{1 - \beta\delta}$$

and

$$(8c) \quad q_t = \frac{\lambda_{20} - \lambda_{21} w_t^2}{\alpha - (y + w_t)}.$$

In this example, the effect of a positive output shock, w_t, on equity price, q_t (compared to $w_t = 0$) depends on specific parameter magnitudes since positive w_t both decreases current marginal utility of consumption acting toward increasing equity price and increases agents perception of future variance acting toward decreasing equity price. The net result will depend on specific parameters.

A negative w_t, on the other hand, lowers q_t (compared to $w_t = 0$). 1/ This example presents an interesting possible asymmetry in the reaction of equity price to positive versus negative real shocks.

III. Government Policy and Asset Prices

In this section the model developed above will be used to study two issues. First, how does government expenditure policy influence the way changes in an the economy's stochastic environment impact the asset market? Second, how does the asset market react to entering a period of likely fiscal reform? The concept of likely fiscal reform is characterized here by an upward shift in the covariance of innovations in government spending and innovations in the income tax rate. This definition of fiscal reform does not take a stand on the direction of movement of tax rates or government expenditure; it considers only the association between government spending changes and tax rate changes. While the questions studied in this section have some independent interest they were chosen to illustrate some aspects of the model. The first question is chosen to illustrate the asset pricing effect of the interaction of first moment and variance processes and the second is

---

1/ Examining equation (6), it is clear that while the present approach is helpful for examining the impact of underlying risk shifts on asset price, it is no big help in examining the effects of underlying volatility shifts on asset-price volatility. The problem is that to convert to asset price one must divide z_t by the marginal utility of consumption. Since this marginal utility will in equilibrium involve the level of output, the conversion to asset price from z_t involves dividing by a linear function of a stochastic disturbance. Asset price is therefore, in this set up, a nonlinear function of the underlying disturbances. Studying the volatility of asset price usually requires that the expectation of asset price be calculated. Such calculations for nonlinear functions are distribution-function specific and will be avoided here.

chosen to illustrate the adaptability of this kind of model to studying time series processes involving the covariance.

This section uses the closed-economy equity-pricing model introduced earlier. The closed economy is a useful fiction here since income tax rates need not be uniform across countries. Without uniform taxes agents need not face the same effective prices, making aggregation much more difficult than is the case presently.

1. Introducing the government

Introducing the government requires that the budget constraints and equilibrium conditions be modified. It is assumed that the government purchases goods in the goods market, collects a proportional income tax and collects a lump-sum tax. The government's budget constraint is:

$$(9) \quad g_t = \tau_t y_t + \theta_t$$

where g_t is per-capita government consumption τ_t is the income tax rate and θ_t is the per-capita lump-sum tax. The after-income-tax average individual budget constraint is now:

$$(9a) \quad c_t + q_t k_t + \theta_t = (q_t + (1 - \tau_t) d_t) k_{t-1}.$$

Goods market equilibrium requires:

$$(9b) \quad c_t + g_t = y_t$$

For the issues studied in this section it is assumed that government purchases do not influence the marginal utility of private consumption. The average Euler equation for equities, written for equilibrium is

$$\text{(9c)} \quad z_t = \beta E_t z_{t+1} + \beta E_t \left\{ (\alpha - y_{t+1} + g_{t+1})(1 - \tau_{t+1}) y_{t+1} \right\},$$

where z_t is now the product of the market price of equity and average marginal utility of consumption.

In writing (9c), equilibrium average consumption, $c_t = y_t - g_t$, has been used in place of c_t and private after-tax income becomes the pay-out to equity holders. Lump-sum taxes are present only as a government-budget balancing item.

2. Government feedback policy, volatility, and the equity market

The question to be investigated here is: "How does government expenditure policy influence the way in which volatility impacts the equity market?" To investigate this question we adopt the following settings.

$$\text{(10a)} \quad y_t = y + \rho_1 y_{t-1} + w_t \qquad |\rho_1| < 1$$

$$\text{(10b)} \quad E_t h_{t+1} = \phi h_t + h$$

$$\text{(10c)} \quad g_t = g + \rho_2 g_{t-1} + \eta y_t + \varepsilon_t, \qquad |\eta| < 1, |\rho_2| < 1.$$

Further, to focus on expenditure policy set $\tau_t = 0$ all t. The important difference between the set up currently and that used previously is that now a government expenditure policy rule has been adopted. According to this rule, equation (10c), government spending is determined by a constant, past government spending, the amount of current output and a white noise disturbance, ε_t.

The short cut of examining the influence of policy on z_t rather than on q_t is adopted. As before, the transformation between z_t and q_t can be made by dividing z_t by marginal utility. Keep in mind that by concentrating on z_t we are looking only into the future. This is reflected in equation (9c) by the fact that the entire right-hand-side of the z_t equation gives z_t as the current expectation of future events.

Following the methods of the previous section, substitute (10a), (10b) and (10c) into (9c) with $\tau_{t+1} = 0$ and find that the forcing process (for z_t) is linear in y_t, y_t^2, h_t, g_t, and $g_t y_t$. The following trial solution is therefore attempted:

$$(12) \quad z_t = \lambda_{40} + \lambda_{41} y_t + \lambda_{42} y_t^2 + \lambda_{43} h_t + \lambda_{44} g_t + \lambda_{45} g_t y_t,$$

which yields:

$$(12a) \quad \lambda_{42} = \beta\rho_1^2 (1 - \beta\rho_1^2)^{-1} [\eta\lambda_{45} + \eta - 1]$$

$$(12b) \quad \lambda_{43} = [\rho_1^2 (1 - \beta\phi)]^{-1} \lambda_{42}$$

$$(12c) \quad \lambda_{44} = y\beta\rho_2 [(1 - \rho_1\rho_2)(1 - \beta\rho_2^2)]^{-1}$$

$$(12d) \quad \lambda_{45} = \beta\rho_1\rho_2 (1 - \beta\rho_1\rho_2)^{-1}$$

The coefficients λ_{40} and λ_{41} are reported in the appendix.

The equity-market impact of h_t is given by λ_{43}, which is proportional to λ_{42}, the coefficient attached to y_t^2. This occurs because h_t enters the problem only through agents' forecasts of future squared output.

Government expenditure policy can influence the asset market impact of changes in risk. In particular, the asset market impact of risk is proportional to $[\eta\lambda_{45} + \eta - 1]$. The size and sign of this term depends on

η, which can assume any value between -1 and 1. 1/ When $\eta < 0$ government spending policy is counter-cyclical and the spending feedback exacerbates the effect of risk on the market. When n > 0 government spending is pro-cyclical and dampens the effect of risk on the market. What is going on is that counter-cyclical government spending policy, $\eta < 0$, induces a negative covariance of g_t and y_t. A larger absolute negative covariance (as would be induced by a positive ht shock) acts toward lowering the marginal utility of future dividends. This acts in conjunction with the standard effect of h_t to give larger variations in the equity market as a result of h_t disturbances. On the other hand, when $\eta > 0$ the above covariance is positive and its effect is to act toward offsetting h_t's standard effect.

The effect isolated here is conceptually distinct from the way the equity market reacts to an output shock. In particular, λ_{43} gives the partial derivative of z_t with respect to h_t. It is therefore answering a question about the equity market impact of a risk shock as distinct from an output shock.

3. <u>Fiscal reform and the equity market</u>

The question to be studied here is: "What is the equity market effect of entering a period of likely fiscal reform?" To confront the question with our tools, "fiscal reform" must be a <u>recurring event</u> with agents understanding the time series process of fiscal reforms. We can

---

1/ A condition that an increase in long-run output of one unit should bring about a less than unitary increase in long-run government spending, $\eta/(1 - \rho^1) < 1$, would ensure $\lambda_{43} < 0$. It is not clear, however, why in this model such a condition should be imposed since long-run output is not time-varying.

then come-to-grips with the narrower question: "What is the equity market effect of a disturbance to the "fiscal reform" time series process?" The following settings are adopted:

(13a) $y_t = y$,

(13b) $g_t = g + v_t$,

(13c) $\tau_t = \tau + x_t$,

(13d) $E_t x_{t+1} v_{t+1} = \delta_t$,

(13e) $E_t \delta_{t+1} = \phi\delta_t + \delta$.

In equation (13a), output has been set at a constant allowing us to concentrate attention on fiscal policy disturbances. In (13b) and (13c) government spending and the <u>tax rate</u> respectively are assumed to follow stochastic processes around fixed means. The disturbance terms in (13b) and (13c), v_t and x_t, are white noises. The conditional covariance of v_t and x_t is given by δ_{t-1} and the conditional expectation of the time-series process of this covariance is given by equation (13e). The conditional covariance, δ_t, is playing the same type of role played previously by h_t.

A period of "fiscal reform" is defined presently to be a period when δ_t is high. For an economy to move into a surprising period of fiscal reform is to have the economy receive a high δ_t. The fiscal reform model used here is cynical in that nothing ever happens to the means of per-capita government spending and the income tax rate. This is intended only as an analytical convenience allowing separation of the riskiness of a fiscal reform period from the type of reform expected.

Notice that the present idea of fiscal reform deals only with the coordination of tax and spending policies. It is eclectic with respect to the direction of change of both spending and the tax rate. A period of fiscal reform is therefore a period when the tax rate and per-capita spending are likely to move in the same direction.

For this issue we follow previous practice by substituting from equations (13a)-(13e) into equation (10c) and noticing that the forcing process (for z_t) is linear in δ_t. The trial solution therefore is:

(16) $$z_t = \lambda_{50} + \lambda_{51}\delta_t,$$

where

(16a) $$\lambda_{51} = \frac{-\beta y}{1 - \beta\phi},$$

The constant term, λ_{50} is given in the appendix. Since $\lambda_{51} < 0$, we find that entering a period of likely fiscal reform, i.e., entering a period of close coordination of tax and spending policy, decreases z_t. Conversely, entering a period of weak coordination of tax rates and per capita spending increases z_t.

The intuition for this result is that (roughly) things that are good for z_t are things that increase the expected future marginal utility of after tax dividends. The tax rate enters after tax dividends negatively while government spending enters marginal utility positively. The covariance of the tax rate and spending therefore enters negatively.

IV. Concluding Remarks

The purpose of this paper was to exposit and apply in a simple and familiar framework some ideas developed recently by Abel. This line of research allows us for the first time to ask in a consistent way questions concerning agents' reactions to predictable and unpredictable alterations in the riskiness of their environment. The framework seems to have wide potential applicability to problems such as options pricing and optimal government stabilization policy where the nature of and reactions to time-varying risk are at the center of the problem.

Text Coefficients

This appendix records several coefficients mentioned in the text.

$$(A1.1)\quad \lambda_{40} = (1-\beta)^{-1}\Big\{\beta[\lambda_{40}+\lambda_{41}y+\lambda_{42}y^2+\lambda_{43}(1-\phi)h+\alpha y-y^2+\eta y^2] + \lambda_{44}g+\lambda_{45}(\eta y^2+yg)\Big\}$$

$$(A1.2)\quad \lambda_{41} = \beta(1-\rho_1\beta)^{-1}\Big\{(\lambda_{42}2\rho_1+\lambda_{45}[\eta+\eta\rho_1]-2\rho_1+\eta+\rho_1\eta)y + \lambda_{44}\eta\rho_1+\alpha\rho_1+(\lambda_{55}\rho_1+\rho_1)g\Big\}$$

$$(A1.3)\quad \lambda_{50} = (1-\beta)^{-1}[\beta\lambda_{51}\,\delta+\beta y(\alpha-y+g)(1-\tau)]$$

Some Aspects of Nominal Asset Pricing

The model developed in the text was designed to allow pricing of assets in real terms in a hypothetical world that functions without money. One is interested, however, in nominal asset pricing since nominal units are the typical units of real-world asset pricing. While it is outside the direct realm of this paper, this appendix gives an example of asset pricing in a nominal environment where risk may be time-varying.

Money is introduced via the simplest mechanism available in the current literature, strong cash-in-advance. In this monetary introduction scheme, also known as a Clower constraint, it is assumed that agents must use money to buy goods. It is also assumed that when the money is acquired by agents, the agents know exactly the quantity of goods that they will purchase with the money. Later a somewhat looser version of a Clower-constrained model will be discussed.

Accommodating money in our model requires that we reset budget constraints. In particular, it is assumed that all money enters (leaves) the model via lump-sum transfers (taxes) from (to) the government. The government, which takes no role in this economy other than provider of money, has a budget constraint given by $P_t\theta_t = M_t - M_{t-1}$, where P_t is the money price of goods and M_t is the per-capita money stock.

The private individual now must obey a pair of constraints, the first constraining his purchases of asset and the second constraining his purchases of goods. This is natural in a cash-in-advance economy since only goods carry the special condition that they must be purchased with money. The constraints are:

(A2.1) $M_t + P_t q_t k_t + B_t = P_t\ (q_t + d_t)k_{t-1} + (1+i_{t-1})\ B_{t-1} + P_t T_t,$

(A2.2) $M_t = P_t c_t,$

with (A2.1) and (A2.2) constraining asset and goods purchases respectively. Previously defined symbols retain their meaning. B_t is per-capita nominal bonds and i_t is the nominal rate of interest. The Euler equation for bond holding is

(A2.3) $u'(c_t)/P_t = \beta(1 + i_t)E_t[u'(c_{t+1})/P_{t+1}].$

Imposing $c_t = y_t$ and $M_t = P_t y_t$ and rearranging obtain

(A2.4) $(1 + i)^{-1} = \beta E_t(g_{t+1} n_{t+1}),$

where $g_{t+1} = u'(y_{t+1})y_{t+1}/u'(y_t)y_t$ and $n_{t+1} = M_t/M_{t+1}$. Rewrite (A2.4) as

(A2.5) $(1 + i_t)^{-1} = \beta E_t g_{t+1} E_t n_{t+1} + \omega_t,$

where ω_t is the covariance of g_{t+1} and n_{t+1} conditional on time t information.

Leaving aside ωt for now, we see that the moments of y_{t+1} will enter $E_t g_{t+1}$ as directed by utility function choice. If quadratic utility is chosen, the first two moments of the y_{t+1} distribution will be important in determining the nominal interest rate. For other utility function choices other moments will enter the pricing function.

To the extent that money is exogenous to the private sector we have much less economic guidance when calculating $E_t n_{t+1}$. The problem is that we have no way of knowing offhand what mathematical function of money is being given a distribution function by the policy authorities. For example, suppose policy authorities choose a rule like $M_t = M + u_t$ where ut is white and has some known distribution. Since agents need to calculate $E_t n_{t+1}$ they must calculate $M_t E_t(1/M_{t+1})$ and if a distribution is given for u_{t+1} then aspects of that distribution function must be invoked in the calculation and, in general, the calculation will be a function of the entire shape of the distribution function.

For the above reason moments of the money supply rule other than those typically considered important for nominal interest rates might enter a reduced-form interest rate function. Of course there is no good reason for the authorities to adopt a distribution pertaining to the level of money. The monetary authority could make agents' calculations a lot easier if they were to adopt a rule on g_{t+1}, e.g., $g_{t+1} = g + w_{t+1}$, where w_{t+1} is white noise. For a monetary authority rule on g_{t+1} no moments of w_{t+1}'s distribution other than the mean, zero, would enter the nominal interest rate reduced form.

The reader may have noticed that the above methods for introducing unusual moments of the monetary disturbance into the interest rate reduced form are essentially unrelated to the asset pricing framework. These moments are being introduced by the model's requiring agents to form expectations of nonlinear functions of the policy variables. For the same reasons these moments could turn up in standard descriptive macro-models. Suppose, for example, that agents are following a log-linear macro-model. The solution of the model (e.g., for the interest rate) might require the model's agents to form expectations of future logs of money. If the policy authority is following money-supply a rule on the level of money with an additive shock then descriptions of the distribution of that shock (possibly including higher moments of the distribution) will appear in the reduced form expressions for the endogenous variables.

A much more interesting area of study is suggested by the term ωt in (A2.5). Through this term the covariance of a monetary variable, n_{t+1}, and a real variable, g_{t+1}, can influence the nominal rate of interest. Similar covariance terms have been an important topic in macroeconomics since its inception.

The model used in this appendix was adopted for ease of discussion. Essentially the same points apply in the cash-in-advance models of the type used by Hodrick (1987) and Giovannini (1987) where money must be gathered before the resolution of other aspects of the period's uncertainty. This alteration in the timing of the resolution of uncertainty simply alters the timing of variables entering the interest rate reduced form.

Heterogenous Agent, Multi-Country Reinterpretation

The analysis in the text was based on a representative-agent closed economy model to facilitate comparisons with other work in that setting. The purpose of this appendix is to show that the model we have used has a much broader interpretation than we have placed on it. Indeed we will find that once quadratic period utility has been assumed, aggregation in a world of heterogenous agents is straightforward.

In a world where agents from different countries are endowed with different wealth levels and have different period utility functions but where each agent has quadratic period utility, each agent faces the following problem:

$$\max E_t \sum_{i=0}^{\infty} (\alpha^j - [1/2](c^j_{t+i})^2)\beta^i, \; 0 < \beta < 1$$

subject to:

$$c^j_{t+i} + q_{t+i}k^j_{t+i-1} = q_{t+i}k^j_{t+i-1} + d_{t+i}k^j_{t+i-1}.$$

The symbols in the above problem retain most of their previous meaning, but now symbols with a j superscript apply only to individual j, where j = 1,2,3,...J.

It is not necessary to make any special assumptions about the country of location of the individuals for asset pricing. 1/ However, certain assumptions are required for simple aggregation. First, all agents use the same expectation operator; information is therefore homogenous across

---

1/ Of course locating agents by country would be crucial for country-bookeeping such as current account, capital accounts, etc.

agents. Second, the subjective discount rate, β, is identical for all agents. Third, all agents face identical prices for goods and assets.

The parameters in the period utility function and individual wealths can differ across individuals. As before, however, constant terms in the period utility functions have been ignored and each individual's period utility function has been normalized by dividing the function by 1/2 of the parameter attached to $(c_t^j)^2$. Individual period-utility-function differences are therefore contained entirely in the parameters α^j.

Each individual's optimization problem will imply an Euler equation like equation (1) in the text. Average these equation across individuals to obtain:

$$\text{(A3.1)} \quad \left(\sum_{i=1}^{J} [\alpha^j - c_t^j]/J\right)q_t = E_t\beta\left(\sum_{i=1}^{J} [\alpha^j - c_{t+1}^j]/J\right)(q_{t+1} + d_{t+1}).$$

Retaining the assumption of no government spending or taxation, equilibrium requires

$$(1/J)\sum_{j=1}^{J} c_t^j = y_t.$$

If all output is paid out as dividends we obtain an Euler equation very similar to (3a) in the text. The only difference between (A3.1) and (3a) is that in (A3.1), the Euler equation parameter derived from the period utility functions is the average across j of the constant terms in individual period marginal utility while α in (3a) is the representative agent's constant term in period marginal utility.

This implies that the results in the previous section are not very sensitive to the closed economy representative agent assumption. They would, however, be sensitive to different discount rates across agents or agents facing different effective prices as would happen in a multi-country setting with goods or asset market distortions.

References

Abel, Andrew B., "Stock Prices Under Time-Varying Dividend Risk: in Exact Solution in an Infinite Horizon General Equilibrium Model," mimeo, The Wharton School (University of Pennsylvania, Philadelphia, 1986).

Engle, Robert F., "Autoregressive Conditional Heteroscedascity With Estimates of the Variance of United Kingdom Inflations," Econometrica (July 1982).

_____, and Tim Bollerslev, "Modeling the Persistence of Conditional Variances," Econometric Reviews, Vol. 5 (1986), pp. 1-50.

Flavin, Marjorie, "Excess Volatility in the Financial Markets: A Reassessment of the Empirical Evidence," Journal of Political Economy, Vol. 91 (1983), pp. 929-56.

Flood, Robert, Robert Hodrick, and Paul Kaplan, "An Evaluation of Recent Evidence on Stock Market Bubbles," mimeo (Northwestern University, 1987).

Giovannini, Alberto, "Uncertainty and Liquidity," mimeo (Graduate School of Business, Columbia University, 1987).

Hansen, Lars, and Thomas Sargent (1981a), "Formulating and Estimating Dynamic Linear Rational Expectations Models," in Robert Lucas and Thomas Sargent (eds.), Rational Expectations and Econometric Practice (University of Minnesota Press, Minneapolis, 1981).

_____ (1981b), "Linear Rational Expectations Models for Dynamically Interrelated Variables," in Robert Lucas and Thomas Sargent (eds.), Rational Expectations and Econometric Practice (University of Minnesota Press, Minneapolis, 1981).

Hodrick, Robert J., "Risk, Uncertainty and Exchange Rates," mimeo (Kellogg Graduate School of Management, Northwestern University, 1987).

Pagan, Adrian, "Econometric Issues in the Analysis of Regressions with Generated Regressors," International Economic Review, Vol. 25 (1984), pp. 221-47.