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## ABSTRACT

Bequest constraints have played a major role in discussions of debt neutrality but their welfare implications were not sufficiently dealt with in the literature. In this paper we focus on the welfare implications of bequest constraints. We found that when institutional constraints to the transfer of resources from children to their parents exists the welfare of the parents' generation may be improved by an old age security scheme. Such a scheme is justified not by income redistribution consideration, as is typically the case, but rather on pure efficiency grounds. Due to its intergenerational transfer role the social security scheme is Pareto-improving with altruistic parents if, in addition, the real income effect which tends to raise children consumption is relatively strong.

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## 1. Introduction

Bequest constraints have played a major role in discussions of Barro's (1974) debt-neutrality theorem. In his famous article (1974), Barro showed that if finite-lived parents left their descendants positive bequests in the absence of government debt, the introduction of public debt would not affect parents' optimal consumption plans and would not create new opportunities to transfer resources from parents to children. Barro's proposition thus requires an interior solution or the absence of nonnegativity constraints on bequests. Subsequent developments of this subject have focused on the implications of such nonnegativity constraints and the possibility of boundary solutions; see Drazen (1978), Kimball (1987), Weil (1987), and Abel (1988). This literature focused mainly on the positive economics aspects of institutional constraints on bequests and their implications for government debt neutrality.

In this paper we focus on the welfare implications of bequest constraints within the framework of endogenous fertility developed in Nerlove, Razin and Sadka (1987). Gary Becker and Kevin Murphy (1988) pointed out the importance of bequest constraints when family decisions (such as fertility, investment in human capital, etc.) are endogenous, as a source of market failure, for public policies. In an early paper (1984) and subsequently in our book (Nerlove et al (1987, Chapter 8)) we dealt with a similar issue: the welfare implications of institutional constraints implying the inability of parents to force transfers among their children, who have different ability to make use of human capital.

When children have different abilities, so that investments in their human capital are not equally productive, and parents cannot enforce transfers among them, an egalitarian parental attitude may lead to inefficient investment in human and nonhuman capital. For example, the parents may be led to invest too much in the human capital of the low-ability children so that they will be equal (in the productivity of their human capital) to their more able siblings. The resulting market failure is shown to be alleviated by a tax on earned income and a subsidy to bequest.

In this paper, we continue this line of analysis dealing with the diseconomies associated with institutional constraints on the family by focusing on the implications of bequest constraints for the intergenerational distribution of welfare when no differences exist among children. We examine several potential welfare-improving policies designed to correct institutional constraints to negative bequests and the implications of such policies for population growth. Among such policies, we consider a social security scheme.

In his pioneering paper, Samuelson (1958) pointed out to the role of government debt (money) as a social contrivance for improving intertemporal allocations of resources within generations. A social security scheme by which each young generation pays a tax to finance a transfer to the old generation can, of course, play a similar role. This role is limited to intra-generational transfers, because in the original Samuelson's overlapping-generations model no intergenerational altruism exists: for each generation, life-time consumption is equal to own life-time income. In the context of intergenerational altruism, social security can also play a role of intergenerational transfers. Such a role is particularly important when there is a binding nonnegativity constraint on bequests. In this paper we highlight the role of social security in the context of intergenerational transfers in the form of parents' investment in the human capital of their children. Even though the social security tax is, like most taxes, distortionary, it nevertheless improves welfare. Other measures, such as a subsidy to human capital investment, are shown to be welfare-reducing because they introduce further distortions into the economy without playing any role for intergenerational transfers.

The paper is organized in the following manner. Section II introduces a stylized model used to demonstrate the existence of a potential market failure arising from the institutional constraints on bequests. Section III provides a Pareto-comparison between the laissez-faire allocation and the optimal allocation (from the altruistic parent's point of view). Section IV considers the effect of the institutional constraints on the rate of growth of population. In Section V we analyze alternative corrective policies for a bequest-constrained economy with endogenous population. Section VI contains concluding remarks. This section is followed by three technical appendices.

## II. Bequest-Constrained Equilibrium

To highlight the economic mechanism underlying the problem at hand we use a simple stylized model. Suppose there are only two periods, two generations and a single all-purpose composite good. The first generation consists of identical individuals (parents) who live for one period. Each parent is endowed with I units of the composite good. She consumes $c_1$ units of the composite good in the first period and bears n identical children who each possesses one unit of adult labor in the second period.

Investing e units of the single good in the first period in the education of a child augments her labor supply, as measured in efficiency units, to $g(e)$. We assume that the marginal product of human capital is positive but diminishing, i.e.:

(1) $g' > 0,\ g'' < 0.$

The child then earns $wg(e)$ in the second period where w is the wage rate per efficiency unit. Without loss of generality we henceforth set $w=1$. The parent can also bequeath b units of the composite good to each child in the first period. This bequest grows to be bR units in the second period, where $R - 1 > 0$ is the interest rate. For the sake of simplicity, R is assumed to be fixed.

The parent's budget constraint is thus

(2) $c_1 + n(e+b) = I.$

We assume that bequests cannot be negative. Accordingly:

(3) $b \geq 0.$

The consumption ($c_2$) of each child in the second period is constrained by:

(4) $c_2 \leq g(e) + bR.$

We assume that the nonnegativity constraint on bequest is only institutional. Namely, parents cannot obligate their children to repay debts that they (the parents) accumulate before they die. However, the economy lasts for two periods and is not constrained in its ability to transfer resources from the future back to the present. For instance, the economy (via the government) can borrow in the present from abroad at the rate of interest R-1 and repay the loan in the second period. Thus, constraint (3) is binding for the individual but not for the economy.1/ Therefore, there may be a market failure in the intertemporal allocation of resources.

Caring about the number and the well-being of her children, the parent in the first period choose $c_1$,n,e,b and a planned $c_2$ so as to maximize her utility function

(5) $u(c_1, n, c_2)$

subject to the resource constraints (2) and (4) and the institutional nonnegativity constraint on bequests (3). We call the solution to this optimization problem, $(c_1^*, n^*, c_2^*, b^*, e^*)$, the laissez-faire equilibrium. When the institutional constraint is not binding, namely the parent wishes to bequeath a positive amount to her children, it is straightforward to see that the parent invests in the human capital of each child up to the point where the marginal productivity of human capital is equal to the rate of interest:

(6) $g'(e^*) = R.$

The rate-of-return equalization ensures efficiency in the choice between human and nonhuman (physical) capital as means of transferring resources from the present to the future. Notice that this efficiency rule is obtained independently of the optimal choice of $c_1$,n,b and $c_2$. (Of

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course, (6) does not guarantee that the latter variables are optimally chosen.)

However, this efficiency rule no longer holds when the nonnegativity bequest constraint is binding. In the latter case $b^* = 0$ and (6) becomes:

(6a) $g'(e^*) > R$.

This implies by (1) that investment in human capital is insufficient. The parent would have liked to invest more in the human capital of her children if she could keep some of the return to this investment to herself by borrowing from her children. But this would have required b to be negative which is institutionally impossible. In this paper we focus on this bequest constrained equilibrium ($b^* = 0$).

There are several key factors which determine whether the bequest constraint is binding or not; see also Abel (1988). First, the magnitude of the parent's altruism towards her children, namely, the marginal rate of substitution of children's consumption for parent's consumption ($\partial c_2/\partial c_1$ along the parent's indifference curve), affects the size of the total transfer (i.e., e+b) to each child. The smaller the magnitude of parent's altruism, the smaller is the transfer and the larger is the likelihood that the nonnegativity bequest constraint be binding. Second, the magnitude of the marginal productivity of human capital investment (i.e., $g'$), relative to the return on physical capital (i.e., R), determines the composition of the total transfer b+e between b and e. The larger is $g'$, relative to R, the more that parent would like to invest in the human capital of their children, and even borrow for this purpose. In the latter case, the nonnegativity constraint becomes binding.

In contrast to the parent, the economy is not constrained by (3). The optimum for the economy is obtained by maximizing the utility function (5) subject to the resource constraints (2) and (4) only. The optimal allocation is denoted by $(c_1^{**}, n^{**}, c_2^{**}, e^{**}, b^{**})$. In this case $b^{**} < 0$ and investment in human capital is efficient:

(7) $g'(e^{**}) = R$

(hence $e^{**} > e^{*}$).

Obviously not only investment in human capital is distorted in the laissez-faire equilibrium due to the institutional constraint on bequests but also all other choice variables ($c_1$, $n$, and $c_2$) are distorted.

Notice that our concept of optimality here is considered from the parent's point of view, namely, the optimal allocation maximizes the parent's utility function (subject to the economy's resource constraints). Since the parent is altruistic towards her children, the optimal allocation may plausibly lead to a higher utility for children, compared to the laissez-faire allocation. In this case the optimal allocation Pareto dominates the laissez-faire allocation. However, this is not always true. It may well happen that the optimal allocation renders a lower utility (consumption) for children than the laissez-faire allocation. In this case the optimal allocation does not bring about a Pareto-improvement over the laissez-faire allocation.

## III. Optimum versus Laissez-Faire: A Pareto Comparison

As indicated in the preceding section, parent altruism toward children does not necessarily imply that the optimal allocation dominates in the Pareto sense the laissez-faire allocation. Namely, although the parent's utility is, by definition, greater at the optimum than at the laissez-faire, children utility (consumption) may be smaller. To see the factors at play in the movement from the laissez-faire to the optimal allocation and, in particular, their effect on children consumption, let us consider a simplified version of our model in which the number of children n is exogenous.

Notice that b is equal to zero at the laissez-faire allocation and becomes negative at the optimal allocation. Therefore, to analyze the difference between the laissez-faire and the optimal consumption per child, let us parameterize b and consider the effect on $c_2$ of lowering b from zero to its optimal level. That is, we first solve the problem

$$\max_{c_1, c_2, e} \quad u(c_1, n, c_2)$$

$$\text{s.t.:} \quad c_1 + n(e+b) \leq I$$

$$c_2 \leq g(e) + bR$$

where b is treated as a parameter. We then consider the effect of changing b on the solution for $c_2$, denoted by $\bar{c}_2$ (b).

A straightforward comparative statistic analysis yields:

$$\frac{d\bar{c}_2}{db} = \frac{1}{\Delta} [(R-g')n^2(u_{11} - u_{13}u_1/u_3)] + \frac{1}{\Delta} [R\ u_3 g'']$$

where

$$\Delta = n^2 u_{11} - 2ng'\ u_{13} + u_{33}(g')^2 + u_3 g''$$

is negative by the second-under conditions for utility maximization.

Notice that the expression for $d\bar{c}_2/db$ is composed of two terms. The first represents the real income (welfare) effect of relaxing the bequest constraint. Since the constraint is binding along the path from the laissez-faire to the optimum, then $g' > R$ along this path. Also, the expression $u_{11} - u_{13}\, u_1/u_3$ is negative when $c_2$ is a normal good, which is a plausible assumption. Hence, the first-term is negative, working in the direction of increasing $c_2$ as the bequest constraint is relaxed. That is, the real income effect works in the direction of increasing $c_2$ as one moves from the laissez-faire to the optimum. Thus, the real income effect, as by itself, enhances Pareto-improvement, when moving from the laissez-faire to the optimum.

The second term in the expression for $d\bar{c}_2/db$ is unambiguously positive. It stems from the fact that relaxing the bequest constraint allows the parent to provide less consumption for each one of her children. Thus, the second term tends, as by itself, to lower the utility of children in the optimal allocation, making the laissez-faire and the optimal allocations Pareto-noncomparable. If the income effect on $c_2$ is relatively strong, then it dominates the second effect and the children utility is higher in the optimal allocation than in the laissez-faire allocation: The optimal allocation Pareto-dominates the laissez-faire allocation. On the other hand, if the real income effect is relatively small, the two allocations are Pareto-noncomparable.

The above analysis was carried out under the assumption that the number of children is exogenous. However, it should be clear that the same factors are at work when n is endogenous. Indeed, the children utility may well be lower in the optimum than in the laissez-faire also when n is endogenous. Appendix B provides such an example.

## IV. The Effect of the Bequest Constraint on Population Growth

As was already mentioned earlier, a binding bequest constraint forces parents to underinvest in the human capital of their children. It may seem that this distortion will induce parents to bring an inadequate number of children (i.e., $n^* < n^{**}$), because they face a binding limit on the "financing" of children.

This may indeed be the case. The loss of utility due to the institutional constraint on bequest may lead to a smaller number of children in the laissez-faire solution relative to the optimum. To gain some insight into the effects determining the difference between the number of children in the two allocations, observe that the laissez-faire allocation with a binding bequest constraint can be represented as a solution to the following problems:

$$\max_{(c_1,n)} u^*(c_1,n) \tag{8}$$

$$\text{s.t.: } c_1 + ne^* = I,$$

where $u^*(c_1,u) = u(c_1,n,g(e^*))$.

The optimal allocation can be represented as a solution to the following problem:

$$\max u^{**}(c_1,n)$$

$$c_1 + n(e^{**} + b^{**}) = I, \tag{9}$$

where $u^{**}(c_1,n) = u(c_1,n,g(e^{**}) + b^{**}R)$.

In both (8) and (9) only $c_1$ and n are the choice variables, while investment in human capital (e) and bequest (b) are set at their predetermined solution levels ($e^*$ and $b^* = 0$ for the laissez-faire allocation; $e^{**}$ and $b^{**} < 0$ for the optimal allocation). A comparison between the above two consumer optimization problems suggests two differences. First, the objective (utility) function is different. Second, the "price" of n in the budget constraint is different. The laissez-faire solution is suboptimal because the transfer of resources from parents to children cannot be efficiently channeled via physical and human capital investments. If the removal of the constraint on the efficient allocation of investment between human and physical capital leads to a larger total yield (i.e., $c_2^* = g(e^*) < g(e^{**}) + b^{**}R = c_2^{**}$) with a smaller total investment (i.e., $e^* > e^{**} + b^{**}$), then it is plausible that the laissez-faire rate of population growth is too slow (i.e., $n^* < n^{**}$). To see this, refer to Figure 1. The laissez-faire budget constraint AB′ is steeper than the optimal budget constraint AB′, because the laissez-faire price of children is larger than the optimal price of children ($e^* > e^{**} + b^{**}$). If an increase in the quality of children (i.e., $c_2 = g(e) + bR$) changes preferences in $(c_1,n)$-space in favor of number of the children (which is plausible) then the indifference curves corresponding to $u^{**}$ are steeper than those corresponding to $u^*$. If furthermore $u^{**}$ implies that the number of children is not a Giffen good, then the optimal allocation must yield a larger number of children than the laissez-faire allocation.

However, this is not always true. In fact, the existence of the non-negativity constraint on bequest might have forced the parent to allow more consumption per child than otherwise desired. Hence, the removal of the constraint will, in this case, lower the consumption per child, i.e. $c_2^{**} = g(e^{**}) + b^{**}R < g(e^*) = c_2^*$ (see the preceeding section). Since investment is more efficiently channelled in the optimum, this will require a smaller total investment in the optimum: $e^{**} + b^{**} < e^{*}$. If, as before, the decline in the quality of children changes preferences in $(c_1, n)$-space against number of children, then the indifference curves corresponding to $u^{**}$ are flatter than those corresponding to $u^{*}$ (see Figure 2). In this case, the optimal allocation can be either to the right or to the left of point D, showing that the optimal number of children can be either smaller or larger than the laissez-faire number of children. Appendix B presents an example in which the optimal number of children is indeed smaller than the laissez-faire number of children.

## V. Corrective Policies

The source of the market failure in the presence of an institutional constraint on negative bequests is the inability of parents to transfer resources from their heirs to themselves. Such a transfer enables parents to have an efficient number of children, to invest efficiently in their human capital and makes parents and, possibly, each one of their children better off.[2]

A first-best remedy to this market failure may seem to be a per-parent subsidy in the first period debt-financed by a head tax on children in the second period. However, notice that when fertility is endogenous (as is the case here), a head tax on children is not neutral, because it affects the "price" of children, and thereby distorts parents' behavior. In the absence of genuine second-period lump-sum taxes we therefore examine second-best corrective policies. Such policies are aimed to raise the parent's utility beyond the laissez-faire level, but they cannot attain the first-best level.

Consider then a consumption tax at the rate $t_c$ (in the two periods), an education subsidy at the rate s', a child allowance of a', an income tax at the rate t' in the second period, and a lump-sum tax T' in the first period.[3] Observe that this menu of policy tools includes, as a special case, a social security scheme. Such a scheme obtains when $t'>0$ and $T'<0$, i.e. when children are taxed in order to provide a lump-sum transfer to parents. The taxes modify the budget constraints (2) and (4) faced by the parent to:

(10) $$(1+t_c)c_1+n(1-s')e-a'n \leq I-T'$$

and

(11) $$(1+t_c)c_2 \leq (1-t')g(e).$$

In what follows we examine the effects of sufficiently small taxes at the neighborhood of the laissez-faire allocation ($t_c{*}=s'{*}=a'{*}=t'{*}=T'{*}=0$). Since at this allocation, $b* = 0$, we ignore inheritance taxes. Dividing (10) and (11) by $(1+t_c)$, we see that the consumption tax $t_c$ is redundant. We therefore rewrite (10) and (11) as:

(10a) $$c_1 + n(1-s)e - an \leq I-T$$

and

(11a) $$c_2 \leq (1-t)g(e),$$

where

$1-s = (1-s')/(1+t_c)$,

$a = a'/(1+t_c)$,

$1-t = (1-t')/(1+t_c)$, and

$I-T = (I-T')/(1+t_c)$.

The parent maximizes the utility function (5), subject to the budget constraints (10a) and (11a). The first-order conditions are given in Appendix C. Denote the solution to this problem by:

$$\text{(12)} \qquad \begin{aligned} &\text{(a)} \quad c_1 = C_1(s,a,t,T) \\ &\text{(b)} \quad n = N(s,a,t,T) \\ &\text{(c)} \quad e = E(s,a,t,T) \\ &\text{(d)} \quad c_2 = C_2(s,a,t,T) \end{aligned}$$

and the indirect utility function by:

$$\text{(13)} \qquad v(s,a,t,T) = u(C_1(s,a,t,T),N(s,a,t,T),C_2(s,a,t,T)).$$

The government, which can lend and borrow also over periods in which parents and children do not overlap, faces the following present-value budget constraint:

$$\text{(14)} \qquad [aN(s,a,t,T) + sE(s,a,t,T)N(s,a,t,T) - T]R - tN(s,a,t,T)g[E(s,a,t,T)] \le 0.$$

As mentioned before, we examine small changes in s, a, t and T around the laissez-faire allocation ($s^*=a^*=t^*=T^*=0$). Totally differentiating v with respect to s, a, t and T yields:

$$\text{(15)} \qquad \begin{aligned} dv &= v_s ds + v_a da + v_t dt + v_T dT \\ &= p_1 NE ds + p_1 N da - p_2 g(E) dt - p_1 dT. \end{aligned}$$

(see Appendix C, equation (C6)). Totally differentiating the government's budget constraint (14) with respect to s, a, t and T yields (at $s^*=a^*=t^*=T^*=0$):

$$\text{(16)} \qquad (N da + NE ds - dT)R - Ng(E)dt = 0.$$

We now examine the welfare effects of changes in each one of the distortionary taxes s, a and t, offset by a change in the first period lump-sum tax T. These revenue-neutral welfare effects are derived from (15) and (16):

$$\text{(17)} \qquad dv/ds = v_s + v_T dT/ds$$
$$= p_1(NE - dT/ds) = 0;$$
$$\text{(18)} \qquad dv/da = p_1(N - dT/da) = 0;$$
$$\text{(19)} \qquad dv/dt = - p_2 g(E) - p_1 dT/dt$$
$$= - (p_2 R - p_1 N) g(E)/R$$
$$= p_3 g(E) R \geq 0.$$

Thus, (19) implies that a small income tax (in the second period) which is financed by a first-period lump-sum subsidy (a negative T) is welfare improving. This is a sort of an old age social security scheme. The rationale for this result is as follows. On the one hand, such a policy transfers resources from the children to the parent and thereby alleviates the nonnegativity constraint on bequests and thus raises welfare. On the other hand, an income tax is distortionary because it discourages investment in human capital. However, this distortionary effect is of a second-order magnitude, since in general a small tax has only a second-order effect on welfare around the laissez-faire equilibrium. Therefore, a second-period income tax with a first-period lump-sum subsidy increases welfare. Interestingly, an income tax is justified here on pure efficiency grounds rather than on the more conventional redistribution grounds. We also drew a similar conclusion in Nerlove et al. (1984, 1987, Chapter 8) in the context of differential ability of children.

As should be clear from the analysis of section III, since the parent is altruistic, a policy which enhances the parent's welfare may improve the children's welfare as well. Indeed, employing similar methods to those of section III and making use of (16), one can show that the effect of our social security scheme on the children utility (consumption) is :

$$\frac{dc_2}{dt} = \frac{1}{D} n^2 [g(u_{11} - \frac{u_1}{u_3} u_{13})(\frac{g'}{R} - 1)] + \frac{1}{D} u_3 [(g')^2 - gg'']$$

where

$D = n^2 u_{11} - 2ng' u_{13} + g'' u_3 + u_{33}(g')^2 < 0,$

by the second-order conditions for utility maximization.[4] As in section III, there are two conflicting effects on $c_2$: the income effect works in the direction of increasing $c_2$; but the second term reflects the desire of the parent to lower $c_2$, as the bequest constraint is effectively relaxed. When the income effect is relatively strong, the social security scheme is Pareto-improving.

Equations (17) and (18) imply that a small education subsidy and a child allowance have no first-order effects on welfare. This is obvius since they do not play any role in intertemporal redistributions (since they are financed by a lump-sum tax in the same period) and their distortionary effect on welfare is, as in general, only of a second-order magnitude. Furthermore, due to this lack of any intertemporal distribution role, one can show that a <u>finite</u> education subsidy/tax or a <u>finite</u> child allowance/tax reduce welfare. To see this, observe that the parent's choices of $c_1$, n, $c_2$, and e in the presence of s, a and T satisfy the budget constraints:

(20) $$C_1(s,a,0,T) + N(s,a,0,T)(1-s)E(s,a,0,T) - aN(s,a,0,T) \leq I - T$$

and

(21) $$C_2(s,a,0,T) \leq g[E(s,a,0,T)].$$

Since the government's budget constraint in this case is

(22) $$aN(s,a,0,T) + sE(s,a,0,T)N(s,a,0,T) - T = 0,$$

it follows from (20) and (22) that

(23) $$C_1(s,a,0,T) + N(s,a,0,T)E(s,a,0,T) \leq I.$$

Thus, the bundle chosen in the presence of s,a, and T is affordable under the laissez-faire budget constraints (compare (23) and (21) with (2) and (4), respectively). Therefore, an education tax or subsidy and a child allowance or tax, financed by a first-period lump-sum tax or subsidy cannot possibly increase welfare. Such taxes or subsidies will usually reduce welfare.

## V. Conclusions

When institutional constraints to the transfer of resources from children to their parents exists, welfare of the parents' generation may be improved by some sort of an old age social security scheme: a lump-sum subsidy to parents, financed by debt creation to be paid by the revenues from an income tax on children. Such an income tax is justified not by income redistribution considerations, as is typically the case, but rather on pure efficiency grounds. Due to its intergenerational-transfer role, the social security scheme is Pareto-improving with altruistic parents if, in addition, the real income effect which tends to raise children consumption is relatively strong. If the model is extended to an infinite overlapping generations model, the social security will not only tax children when they are young but will also subsidize them when they are old. In such a setup, social security is more likely to be Pareto-improving.

Other conceivably corrective measures such as subsidies to parents' expenditures on children's education or a tax on children are shown to be welfare reducing.

We also explore the implication of such a corrective policy for the children welfare and for the rate of population growth. We identified a feature which can make this policy to improve the welfare of children. This feature is a strong real income effect on children consumption. Intuition suggests that population growth will be greater when bequest constraints are eliminated, or ameliorated. However, this is not necessarily the case due to income effects and because of the trade-offs between numbers and welfare of children are altered.

## Appendix A

In this appendix we describe a simple closed economy with overlapping generations in which a parent is constrained in her ability to transfer resources from the future back to the present, while the society is not. Suppose an individual, whom we call Eve, lives for three periods. Eve is born in the first period in which she receives an education from her parents and inherits $b \geq 0$ from them at the end of this period. In the second period she works, consumes and saves for the third period. In the third and last period of her life Eve has children, invests in their human capital and bequeaths $b' \geq 0$ to each of them.

Now consider the third period of Eve's life. In this period also a person from another generation, Adam, who was born in the previous period, is alive. Adam is in the second period of his life in which he works and can save. Thus, the society can transfer resources in the third period of Eve's life from Adam to her. Next period, Eve's children will be in the second (working/saving) period of their lives and society can transfer resources from them back to Adam. In this way, society is able to transfer resources indirectly from Eve's children to herself (via Adam). Eve, herself, cannot make such a transfer due to the nonnegativity constraint on bequests.

## Appendix B

Let the utility function be

(B1) $$u(c_1,n,c_2) = c_1 + an - hn^2/2 + n\log c_2$$

and let g take the form

(B2) $$g(e) = 2e^{1/2}.$$

The optimal allocation is obtained by maximizing (B1) subject to (B2), (2) and (4). The solution to e is given by $g'(e) = R$, implying that

(B3) $$e^{**} = 1/R^2,\ g(e^{**}) = 2/R.$$

Upon substitution, the optimal allocation can be found by solving

(B4) $$\underset{(b,n)}{\mathrm{Max}}\ [I-(1/R^2+b)n+an-hn^2/2+n\log(2/R+bR)].$$

The first-order conditions are:

(B5) $$-1/R^2-b+a-hn+\log(2/R+bR) = 0.$$

and

(B6) $$-1+R/(2/R+bR) = 0.$$

From (B6) we can find the optimal bequest:

(B7) $$b^{**} = (R^2-2)/R^2.$$

Assuming that $R < 2^{1/2}$ ensures that $b^{**} < 0$. Substituting (B7) into (B5) we can solve for the optimal number of children:

(B8) $$n^{**} = [\log R + 1/R^2+a-1]/h.$$

Substituting (B7) and (B3) into (4) yields the optimal level of $c_2$:

(B9) $C_2^{**} = R.$

The *laissez-faire* allocation will have $b^* = 0$ in this case. This allocation can be found by solving the following optimization problem.

(B10) $$\underset{\{e,n\}}{\text{Max}}\ [I-en+an-hn^2/2+n\log 2e^{1/2}].$$

The first-order conditions are:

(B11) $$-e+a-hn+\log 2e^{1/2} = 0,$$

and

(B12) $$-1+1/2e = 0.$$

From (B12) we can solve for the *laissez-faire* quantity of investment in human capital:

(B13) $$e^* = 1/2.$$

Substituting (B13) into (B11) we can find the *laissez-faire* number of children:

(B14) $$n^* = (-1/2+a+\log 2^{1/2})/h.$$

Substituting (B13) into (4) yields the *laissez-faire* level of $c_2$:

(B15) $$c_2^* = 2^{1/2}.$$

Comparing (B8) with (B14), we can see that if R is sufficiently small so that $2(\log R+1/R^2) < 1+\log 2$, then $n^*$ will be larger than $n^{**}$. Thus, there is no presumption that a binding bequest constraint causes parents to bring a fewer than optimal number of children.

Comparing (B9) with (B14), we can see that since, by assumption, $R < 2^{1/2}$, then $c_2^* > c_2^{**}$. This result is in line with the analysis of section III which suggests that when the real income effect on $c_2$ is relatively small, then $c_2$ is lower in the optimum than in the laissez-faire. Indeed, in this example $c_2$ is a neutral good so that the real income effect is zero.

## Appendix C

The parent maximizes $u(c_1,n,c_2)$ subject to the following budget constraints:

(C1) $$c_1 + n(1-s)e + nb - an \leq I - T$$

(C2) $$c_2 \leq (1-t)g(e) + bR$$

(C3) $$b \geq 0.$$

The Langrangian is

(C4) $$L = u(c_1,n,c_2) + p_1[I - T - c_1 - n(1-s)e - nb + an] + p_2[(1-t)g(e) + bR - c_2] + p_3b,$$

where $p_1, p_2, p_3 \geq 0$.

The first-order conditions (when constraint (C3) is binding, i.e., b = 0) are:

(C5)
$$\begin{aligned}
&\text{(a)} \quad u_1 - p_1 = 0\\
&\text{(b)} \quad u_2 - p_1(1-s)e + p_1a = 0\\
&\text{(c)} \quad u_3 - p_2 = 0\\
&\text{(d)} \quad - p_1n(1-s) + p_2(1-t)g' = 0\\
&\text{(e)} \quad - p_1n + p_2R + p_3 = 0.
\end{aligned}$$

Notice that, at the *laissez-faire* allocation, (C5)(d) and (C5)(e) imply that

$$g' = R + p_3/p_2 \geq R$$

which is (6a).

Differentiating (C4) with respect to s,a,t and T, employing the envelope theorem, yields:

(a) $v_s = p_1NE$

(C6) (b) $v_a = p_1 N$

(c) $v_t = -p_2 g(E)$

(d) $v_T = -p_1$.

## Footnotes

1. A similar distinction between the intertemporal constraint faced by the individual and by the economy exists also in a closed-economy when generations overlap (see Appendix A).

2. As shown in Section III, maximizing the altruistic parent's utility will lead also to a higher children utility if the real income effect on $c_2$, arising from the effective relaxation of the bequest constraint, is relatively strong; see below.

3. Since first-period income is exogenous, then an income tax in the first period is essentially a non-distortionary lump-sum tax.

4. As in Section III, n is assumed exogenous in the derivation of $dc_2/dt$.

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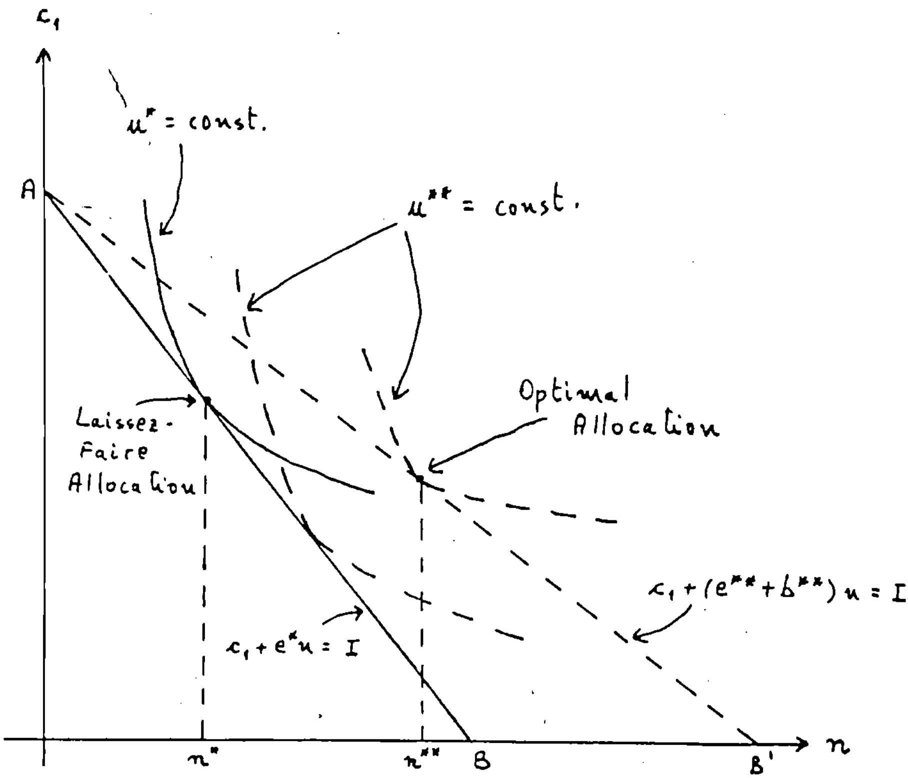
$c_1$
$u^* = \text{const.}$
A
$u^{**} = \text{const.}$
Optimal Allocation
Laissez-Faire Allocation
$c_1 + (e^{**} + b^{**})n = I$
$c_1 + e^* n = I$
$n^*$
$n^{**}$
B
B'
$n$

Figure 1

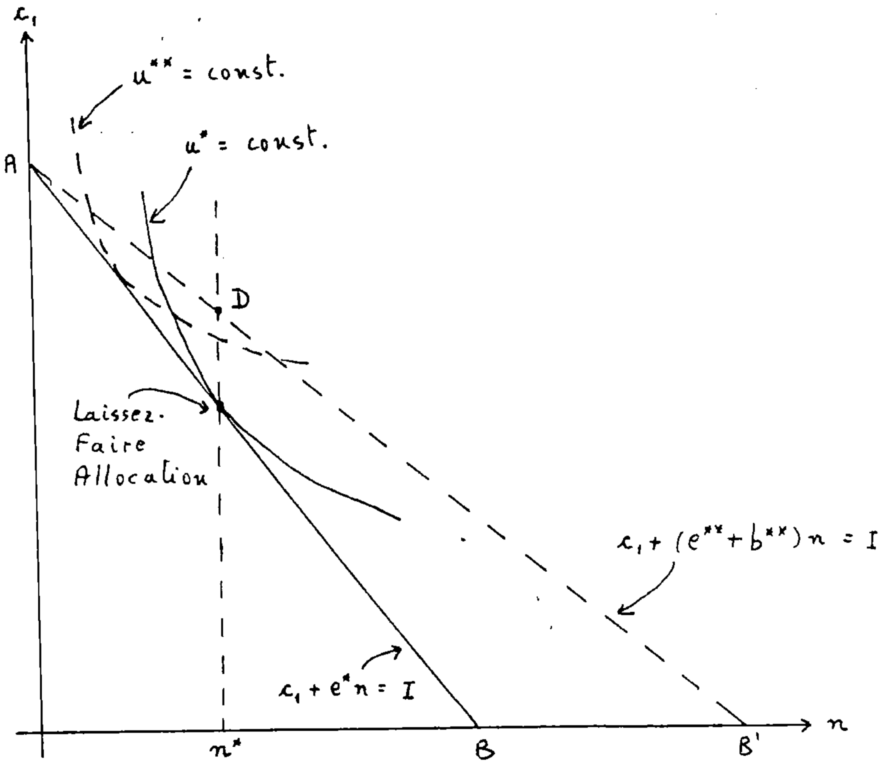
$c_1$
$u^{**}$ = const.
$u^{*}$ = const.
A
D
Laissez-
Faire
Allocation
$c_1 + (e^{**} + b^{**})n = I$
$c_1 + e^{*}n = I$
$n^{*}$
B
B'
$n$

Figure 2