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ABSTRACT

We explore a model in which banks strategically hold interconnected and opaque portfolios, despite increasing the likelihood they are subject to financial crises. In our framework, banks choose their degree of exposure to other banks to influence how investors can use their information. In equilibrium banks choose portfolios which are neither fully opaque, nor fully transparent. However, their portfolios are excessively interconnected to obfuscate investor information. Banks can create a degree of opacity that decreases welfare, and makes bank crises more likely. Our model is suggestive about the implications of asset securitization, as well as government bailouts.

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1 Introduction

Bank interconnectedness has increased dramatically in the past few decades both in the US and worldwide. A substantial fraction of connections are represented by overlapping portfolio exposures across financial institutions. For instance, at its peak asset securitization alone contributed $1.5 trillion to the increase in the interdependence of banks' portfolios. Such connections render the financial system inherently opaque. The lack of transparency plays a significant role in shaping market participants' beliefs, which have been a key ingredient in the unfolding of the global financial crisis. But how costly is it for banks to raise funds when investors are not able evaluate the web of interconnections between them?

In this paper we explore a model in which banks purposefully create opacity for their investors, by choosing interconnected portfolios. Even as they compete to attract investors, banks are able to raise funds at a negative premium for holding opaque portfolios. Moreover, banks decide on portfolio allocations which expose investors to uncertainty about banks' counterparty risk, and lead to an increase in the probability of banking crises. In other words, banks sow the seeds for crises in order to boost profits while they operate.

We consider a simple three-period model with two banks and two investors. Each bank has access to a risky investment project. A bank can decide to swap a fraction of its project with the other bank. If banks exchange a positive share of their projects, their portfolios are interdependent. Otherwise, their portfolios are independent.

To finance the projects, banks need to raise funds from investors. We consider an economy where banks compete to attract funds from investors by promising them a long-term debt contract that maximizes investors' expected payoff. Although banks offer competitive debt contracts, banks exercise market power when deciding their portfolio allocation. Thus, there exists a wedge between how banks manage their asset exposures and how their liabilities are set which allows banks to make positive profits in our model.

Each investor has the option to liquidate the contract prematurely against an outside redemption value. The outside redemption value represents the investors' reservation

value, and can be interpreted as a reduced form of secondary market liquidity in the tradition of Allen and Gale (1994), and more recently of Dewatripont and Tirole (2018). If the investor liquidates early, the bank receives zero. If the investor continues the contract, the bank repays him the face value of debt if the return of the portfolio is sufficiently high. Otherwise the bank defaults. The bank is the residual claimant on the return of the portfolio.

In this environment we consider two frictions. First, we assume that investors can use their funds to finance only one bank. The interpretation is that each bank needs to be monitored, as in Diamond (1984), and monitoring two banks is prohibitively expensive. This does not preclude the banks to offer competitive debt contracts, just as in a standard Bertrand-Nash setting studied by Matutes and Vives (1996) and estimated empirically by Egan, Hortacsu and Matvos (2018).

Second, we consider an information friction between banks and investors. We assume that each investor receives information that perfectly reveals the return of his bank's project, and it is completely uninformative about the other bank's project. Thus, if banks swap a fraction of their projects, the signal the investors receive is less informative about the success of their bank's overall portfolio. This can be, for instance, because banks specialize in lending to different industries and investors learn about their bank's industry, in the spirit of rational inattention. While empirical work on how much information investors decide to process is limited, evidence from international financial markets suggests that agents do indeed allocate more attention to countries whose assets make up a greater share of their portfolios (Mondria, Wu, and Zhang, 2010). Based on the information they receive, investors can decide to liquidate their debt contract before it matures, leading to a financial crisis. The degree of opacity of the financial system is then given by the degree of interconnectedness of banks' portfolios.

Our main focus is on whether banks choose opaque portfolios, whether opacity is an efficient outcome, and what it implies about the frequency of financial crises.

We obtain three sets of results. The first set of results characterizes the portfolio allocation that banks hold in equilibrium and the implications for welfare. Our

main finding is that banks often hold inefficiently opaque portfolios.[1] In particular, we show that in equilibrium, an intermediate level of opacity typically dominates both full transparency, as well complete opacity. In addition, we find that in equilibrium portfolio allocations are welfare reducing, unless investors' early redemption option is either too low or too high. We argue that unlike the traditional view on securitization, which suggests that banks sell securitized assets to benefit from an existing informational asymmetry they have relative to the investors, in our model banks exchange assets as a tool to *create* information asymmetry. By holding opaque portfolios banks can tilt the division of surplus in their own favor and capture rents, although this can impose net welfare losses.

The second set of results describes the implications for financial crises. Surprisingly, we find that banks choose a portfolio allocation that, at times, increases the likelihood of a financial crisis. Importantly, financial crises do not arise via the typical contagion mechanism. In fact, there is no contagion between banks in our model. Instead, financial crises arise as banks are liquidated early when investors terminate their debt contract prematurely. The mechanism that induces banks to choose a portfolio allocation which amplifies the likelihood of a financial crisis is solely due to opacity. When investors have incomplete information, the investors' liquidation decision is not precise. In particular, the investors can liquidate their bank in some states in which the bank would not default. The banks seek to exploit investors' information disadvantage to obtain a higher payoff albeit in fewer states of the world.

In the third set of results we emphasize the role of opacity by analyzing the benchmark in which investors have full information about both banks' project returns. Analyzing this case allows us to show that banks have interconnected portfolios beyond what they would have if they cannot obfuscate investor information, deliberately creating opacity for their investors. While even in the full information set-up, banks portfolio allocations can be inefficient, the implications for financial crisis are markedly differ-

[1] Jiang, Levine and Lin (2016) document that an intensification in competition reduces bank opacity. Note however that in our model we take as given that banks compete to attract funds and we do not provide predictions whether decreasing the level of competition would increase opacity.

ent. Since investors are perfectly informed, they choose to liquidate early only if the bank cannot repay the promised debt contract. As a result, banks cannot benefit from financial crises, thus they occur in equilibrium less often than it would be the case if portfolio allocation were optimal. This is in contrast with an economy with imperfect information where the probability of financial crisis can be higher in equilibrium.

Related Literature. This paper relates to several strands of literature. The most relevant studies are those on the opacity of banks, interconnections in the banking system and securitization.

It is well acknowledged that the banking system is opaque (see, e.g., Morgan, 2002; Flannery, Kwan and Nimalendran, 2013). The financial crisis of 2007-2008 emphasized the opacity of the financial system and prompted a line of research focused on the role of policies that improve transparency. Bouvard, Chaigneau, and de Motta (2015), Alvarez and Barlevy (2015), Goldstein and Leitner (2018), and Orlov, Zryumov, and Skrzypacz (2018) provide models that study the costs and benefits of disclosing bank-specific information. They show that increasing transparency is generally beneficial during financial crisis, but has ambiguous effects in normal economic times. A different perspective is provided by Dang, Gorton, Holmstrom and Ordonez (2017) who argue that banks need to be opaque to facilitate risk sharing. These papers take the degree of opacity in the banking system as given. In contrast, in our model the degree of opacity is endogenously determined by banks' portfolio choices. While we show that banks are indeed opaque in equilibrium, opacity is usually associated with welfare losses.

Recently, a few papers have explored the issue of endogenous information production in the context of financial markets. In Glode, Opp and Zhang (2018) and in Asriyan, Foarta and Vanasco (2018) agents who choose how much information to disclose forego informational rents to ensure that trade does not break down. Azarmsa and Cong (2018) explore how competition between financiers affects the incentives of an entrepreneur to produce information. These papers do not address the role of endogenous opacity in banking crises, which is one of our main interests of analysis.

In our model, opacity is a result of interconnections between banks. The literature

on bank linkages is rapidly growing, not surprisingly given the prominence of interconnectedness in the US financial system and its role during the last financial crisis. The papers that are more related to ours study interconnections that arise when banks have overlapping portfolio exposures. These papers are focused on understanding whether common exposures between banks' portfolios can amplify systemic risk. For instance, Shaffer (1994), Wagner (2010) and Ibragimov, Jaffee and Walden (2010) show that diversification is beneficial for each bank individually, but it can lead to greater systemic risk as banks' investments become more similar. Galeotti and Ghiglino (2019) study the role of portfolio linkages for excessive risk taking and volatility. Adrian and Brunnermeir (2016) propose a measure of systemic risk based on co-movement of financial institutions' assets and liabilities. However, with the exception of Allen, Babus and Carletti (2012), information is not central to the mechanism explored in these papers. Even in Allen et al. (2012) banks do not choose explicitly how opaque they want to be, albeit linkages between them are endogenous. In addition, in that model banks have market power relative to investors. Instead, we assume that investors do not just break even, but are able to extract rents from the banks.

Interconnections between banks in our set-up arise as banks exchange a fraction of their portfolio. While this can be interpreted as banks securitizing assets, our mechanism has differences with the one which securitization relies on. In particular, when securitizing, the originator signals the quality of the project by retaining a fraction on its balance-sheet (DeMarzo, 2004; Duffie, 2008; Chemla and Hennessy, 2014; Vanasco, 2017). This channel is absent from our model. Instead, our paper highlights an important adverse consequence of securitization generally overlooked in the literature: by securitizing assets, banks deliberately produce opaque portfolios to capture informational rents, although this can increase the probability of banking crises.

The rest of the paper is organized as follows. In section 2 we introduce the model set-up. Section 3 explains how we solve for equilibrium as well as for the optimal portfolio allocation. Our main results are presented in Section 4. In Section 5 we introduce the full information benchmark to highlight the role of opacity. Section 6 concludes.

2 Model Set-Up

Consider a three-period ($t = 0, 1, 2$) economy with two risk-neutral banks, denoted by $i = 1, 2$, and two risk-neutral investors, denoted by $I = 1, 2$. At date 0, each bank i has access to a risky investment project that returns R_i at date 2. For each bank i, the return R_i is an independent draw from a uniform probability distribution $G(\cdot)$, with support $[0, 1]$. Each investment project represents loans to a continuum of firms that have perfectly correlated risks. Each project requires initial investment c and is not scalable. To invest in the project, bank i raises funds from investor I.

Banks choose whether to exchange a fraction of their project. In particular, a bank i, chosen at random, proposes to exchange a fraction $(1 - \phi)$ of her project for a fraction $(1 - \phi)$ of bank j's project. Bank j can accept or reject. If she rejects, no exchange takes place. If she accepts, bank i's portfolio is

$$V_i(\phi) = \phi R_i + (1 - \phi) R_j,$$

while bank j's portfolio is

$$V_j(\phi) = \phi R_j + (1 - \phi) R_i.$$

In exchange for borrowing funds from the investor, each bank issues a debt contract with face value D that matures at date 2.[2] To capture that banks compete to attract funds from investors, we assume that the face value of debt is set to maximize the investor's expected surplus, given that he lends all his funds.

Although banks offer competitive debt contracts, banks nevertheless have market power when deciding their portfolio allocation $(\phi, 1 - \phi)$. In other words, bank i chooses the fraction ϕ of her project that she retains in her portfolio so to maximize her expected profit, as we describe in detail below. Indeed, in practice banks are often constrained to offer their investors returns that align with those offered by their competitors. However, banks have more flexibility in shaping their assets' risk exposure. This wedge between

[2]Banks are symmetric, thus debt contract each issues is the same ($D_i = D_j = D$).

how banks manage their asset exposures and how their liabilities are set is an important force which allows banks to make positive profits in our model. Exploring this wedge is also the rationale for assuming that at date 0 banks first decide on their portfolio allocation and then they issue the debt contract to their investors.

At date 1, a signal perfectly reveals the return R_i of the project of each bank i. Each investor I observes the return R_i, but not R_j. Each bank i observes both returns R_i and R_j whenever $\phi \in (0,1)$. It follows that, from the perspective of an investor, ϕ captures the degree of transparency of a bank's portfolio. When ϕ is high, then investor I's signal is more informative about the final realization of bank i's portfolio. Conversely, the lower ϕ is, the more opaque the bank's portfolio is, and thus the less informative investor's signal is.

After observing R_i, investor I decides whether to liquidate his investment, or continue and wait to receive D at date 2.[3] We represent investor I's decision through a function

$$s_I(R_i) = \begin{cases} 1 & \text{if investor } I \text{ continues bank } i \\ 0 & \text{if investor } I \text{ liquidates bank } i \end{cases}.$$

If an investor I chooses to liquidate, he receives an early redemption value $r < E(R_i)$. We set the amount of funds that an investor is endowed with $c < r$, to ensure that the investor's participation constraint is satisfied. Liquidation of bank i does not affect the state of the project of bank j. This is feasible since the projects that the banks invest in represent loans to a continuum of firms. Thus, liquidating a fraction ϕ of loans in region i and a fraction $(1-\phi)$ of loans in region j need not affect the success of the remaining firms in the either region.

In our model r is exogenous and stands for secondary market liquidity. The idea is that when the investor liquidates the bank early, projects are transferred to second best users and are worth less because of the misallocation mechanism proposed by Shleifer and Vishny (1992) and Kiyotaki and Moore (1997), and adopted by Lorenzoni

[3]With deposit insurance, investors' decision would be trivial. However, according to FDIC, only 53% of the dollar value of deposits was insured as of Q3 2019. Morevover, bigger banks have a larger fraction of uninsured deposits as Jiang, Matvos, Piskorski and Seru (2019) document.

(2008). If no information other than the bank early liquidation is available in a potential secondary market, then it is reasonable to assume that the redemption value at date 1 is constant, r. Each value of r can be seen as a fraction of the unconditional expected payoff of a bank's portfolio.

If an investor I chooses to continue, then he receives D at date 2, if $V_i \geq D$ and zero otherwise. The idea is that at date 2, when the projects mature and the bank cannot repay the investor, she enters costly bankruptcy. For tractability, we make the stark assumption that bankruptcy absorbs all the project payoff. In other words, the fraction that the investor receives of his bank's portfolio is 0.[4]

The bank i is the residual claimant and receives at date 2

$$\max\{V_i - D, 0\}.$$

Thus, the bank receives 0 both if it is liquidates at date 1 or if defaults at date 2.

In this set-up we use the following equilibrium concept.

Definition 1 *A symmetric equilibrium is given by a portfolio allocation $(\phi^*, 1-\phi^*)$, a face value of debt D^*, and continuation decision $s_I^*(R_i)$ of each investor I given signal R_i such that*

1. *the continuation decision maximizes each investor I's expected payoff at date* 1

$$\max_{s_I} \left\{ s_I(R_i) \cdot D \cdot \Pr(D \leq V_i(\phi) \,|R_i) + (1 - s_I(R_i)) \cdot r \right\};$$

2. *the face value of debt maximizes each investor I's expected payoff at date* 0

$$\max_{D} \mathbb{E}_{R_i} \left\{ s_I(R_i) \cdot D \cdot \Pr(D \leq V_i(\phi) \,|R_i) + (1 - s_I(R_i)) \cdot r \right\};$$

[4]The results are robust to the alternative assumption that the fraction that the investor receives of his bank's portfolio is a small positive α in the event of bank default at date 2. The robustness results are available upon request.

3. the portfolio allocation maximizes each bank i's expected payoff at date 0

$$\max_{\phi} \mathbb{E}_{R_i,R_j}\{s_I(R_i) \cdot \max[(V_i(\phi) - D), 0]\}.$$

Implicitly, the optimal continuation decision is a function of the face value of debt as well as the fraction of the project the bank retains in its portfolio, i.e. $s_I^*(R_i) = s_I^*(R_i; D, \phi)$. Similarly, the face value of debt is a function of the fraction of the project the bank retains in its portfolio, i.e. $D^* = D^*(\phi)$. In the exposition below, we take these dependencies as implicit so not to burden excessively the notation.

3 Model Solution

3.1 Equilibrium

We solve for the equilibrium in two steps. First we solve for the investors' optimal liquidation decision, and the optimal face value of debt, as described by condition (1) and (2) in Definition 1, given that each bank retains a fraction, ϕ, of her project. Second, we solve for the optimal portfolio allocation, ϕ, taking into account that each bank understands that investors behave optimally. We start with the case when $\phi \in (0, 1)$. We analyze the case of $\phi = 0$ and $\phi = 1$ separately, to highlight when banks have perfectly opaque or perfectly transparent portfolios.

In the first step, we start by analyzing investors' continuation decision. For each portfolio allocation, $\phi \in (0, 1)$, and each face value of debt, D, set at date 0, the optimal continuation decision at date 1 must satisfy

$$s_I^*(R_i) = \arg\max \{s_I(R_i) \cdot D \cdot \Pr(D \leq V_i(\phi) | R_i) + (1 - s_I(R_i)) \cdot r\}. \quad (1)$$

Thus, it is optimal for the investor I to continue funding bank i if the amount he expects to receive at date 2, given that his signal is R_i, $D \cdot \Pr(D \leq V_i(\phi) | R_i)$, is larger than the reservation value r, he obtains when he liquidates the bank at date 1. Investor's

decision to continue funding the bank depends thus on how high the face value of debt is, as well as on how high the probability that the investor gets repaid. Indeed, if the realization of R_i is sufficiently high that $D < \phi R_i$, then the investor gets repaid D with certainty. However, if $D \geq \phi R_i$, then the investor gets repaid D only if $R_j \geq \frac{D-\phi R_i}{1-\phi}$, which occurs with probability $\left(1 - G\left(\frac{D-\phi R_i}{1-\phi}\right)\right)$. Otherwise, the bank goes into default at date 2, and the depositor gets 0. Thus, everything else equal, the higher the face value of debt is, the lower the probability that the investor gets repaid when $D \geq \phi R_i$.

This implies that investor I finds it optimal to continue funding bank i when

$$\Pr\left(R_j \geq \max\left\{\frac{D-\phi R_i}{1-\phi}, 0\right\} \middle| R_i\right) \geq \frac{r}{D},$$

or when his signal is sufficiently large. In particular, investor's optimal continuation decision can be characterized by a threshold strategy as follows

$$s_I^*(R_i) = \begin{cases} 1 & \text{if } R_i \geq R^* \\ 0 & \text{if } R_i < R^* \end{cases}, \tag{2}$$

where

$$R^* = \max\left\{\min\left\{1, \frac{D}{\phi} - \frac{1-\phi}{\phi} G^{-1}\left(1 - \frac{r}{D}\right)\right\}, 0\right\}. \tag{3}$$

Thus, R^* can be interpreted as the probability that an investor liquidates his bank at date 1, under the assumption that the return of the projects is uniformly distributed. Equation (3) reveals that investor's continuation decision does not depend monotonically on D. Clearly, if the face value of debt is too low, the investor liquidates the bank, as he is better off cashing in the liquidation value r. However, the investor is also more likely to liquidate when the face value of debt is too high. While a high face value of debt benefits the investor, the investor expects he will be repaid with very low probability. For instance, in the extreme case when $D = 1$, the probability that the investor receives the face value of debt is zero. Hence, the investor finds it optimal to liquidate the bank and receive the liquidation value r. These forces play an important role in determining the optimal face value of debt, which we discuss next.

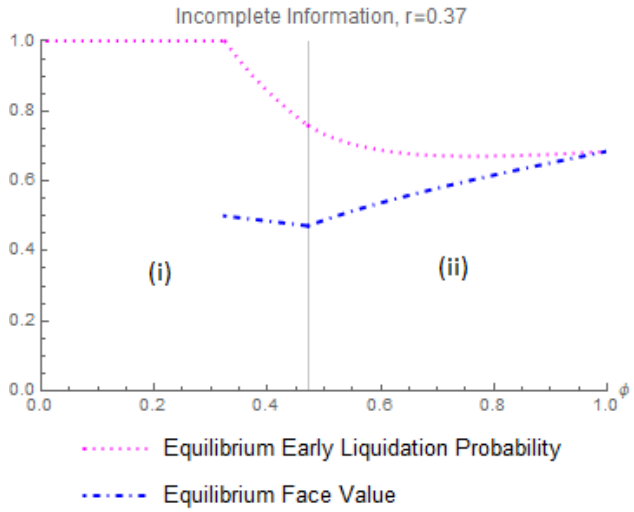

Figure 1: The figure illustrates the equilibrium liquidation probability, R^*, and the optimal face value of debt, D^*, as a function of the degree of transparency of a bank's portfolio, ϕ, for $r = 0.37$.

We turn to derive the optimal face value of debt taking as given a portfolio allocation ϕ, and taking into account that investors make the optimal continuation decision at date 1. Investor I's expected payoff is given by

$$W_I = \mathbb{E}_{R_i} \left\{ s_I^* \left(R_i \right) \cdot D \cdot \Pr(D \leq V_i \left(\phi \right) | R_i) + \left(1 - s_I^* \left(R_i \right) \right) \cdot r \right\}, \tag{4}$$

or

$$W_I = D \cdot \mathbb{E}_{R_i} \left(\Pr \left(R_j \geq \frac{D^* - \phi R_i}{1 - \phi} \right) \middle| R_i \geq R^* \right) \Pr \left(R_i \geq R^* \right) + r \cdot \Pr \left(R_i < R^* \right). \tag{5}$$

The first term on the right hand side of (4) and (5) represents investor I's expected payoff provided he continues funding bank i. The second term on the right hand side of (4) and (5) represents investor's expected payoff provided he liquidates the bank at date 1.

Thus, making use of (2), we obtain that the face value of debt satisfies the first

order condition

$$\frac{1}{D} = -\frac{\frac{\partial}{\partial D}\mathbb{E}_{R_i}\left(\Pr\left(R_j \geq \frac{D^* - \phi R_i}{1-\phi}\right)\middle| R_i \geq R^*\right)\Pr\left(R_i \geq R^*\right)}{\mathbb{E}_{R_i}\left(\Pr\left(R_j \geq \frac{D^* - \phi R_i}{1-\phi}\right)\middle| R_i \geq R^*\right)\Pr\left(R_i \geq R^*\right)}. \tag{6}$$

The left handside of Equation (6) represents the marginal benefit for an investor expressed as a percentage increase in the face value of debt. The right hand side of Equation (6) can be seen as the marginal cost for an investor represented as a percentage decrease in the expected probability of repayment associated with an increase in the face value of debt. An equivalent interpretation is that, in equilibrium, the elasticity of the expected probability of repayment with respect to the face value of debt is -1.

We distinguish two main cases when $R^* < 1$ and the investor is willing to continue his bank with positive probability: $D^* \leq \phi$ and $D^* > \phi$. When $D^* \leq \phi$, then there exists values of R_i such that bank i can repay the investor from the realization of its own project. Thus, the signal that investor I receives at date 1 can be sufficiently informative about whether he will be repaid at date 2. When $D^* > \phi$, it is necessary that bank j's project has a sufficiently good realization for bank i to be able to repay the debt to the investor at date 2. In this case, the signal that investor I receives at date 1 is less informative, as the investor is uncertain about whether he will be repaid or not even if R_i is high.

In the first case, when in equilibrium $D^* < \phi$, the first order condition (6) becomes

$$\int_{R^*}^{\frac{D}{\phi}} \left(1 - \frac{D - \phi R_i}{1-\phi}\right) dR_i + \int_{\frac{D}{\phi}}^{1} 1 dR_i + D\int_{R^*}^{\frac{D}{\phi}} \frac{\partial}{\partial D}\left(1 - \frac{D - \phi R_i}{1-\phi}\right) dR_i + D\int_{R^*}^{\frac{D}{\phi}} \frac{\partial}{\partial D} 1 dR_i = 0.$$

Integrating, we obtain that the face value of debt must satisfy the following equation

$$\left(D^* - R^*\phi\right)\left(3D^* - R^*\phi\right) = 2\phi\left(1-\phi\right)\left(1-R^*\right), \tag{7}$$

for any $\phi \in (0,1)$. We illustrate in Region (ii) in Figure 1 the solution to (7) as well the corresponding continuation decision of investor I, as a function of the degree of

transparency of his bank i, for a given reservation value r. The key observation is that as ϕ increases, investors' information improves. This leads to a more precise liquidation decision, and the liquidation threshold R^* and optimal face value D^* converge to each other. In fact, in the limit when investors are fully informed ($\phi \to 1$), the liquidation threshold is $R^* = D^*$.

In the second case, when in equilibrium $D^* > \phi$, the first order condition (6) becomes

$$\int_{R^*}^{1} \left(1 - \frac{D - \phi R_i}{1 - \phi}\right) dR_i + D \int_{R^*}^{1} \frac{\partial}{\partial D} \left(1 - \frac{D - \phi R_i}{1 - \phi}\right) dR_i = 0.$$

Integrating, we obtain that the face value of debt must satisfy the following equation

$$D^* = \frac{1}{4}\phi R^* - \frac{1}{4}\phi + \frac{1}{2}, \tag{8}$$

for any $\phi \in (0, 1)$. Similarly, the solution to (8) is illustrated in Region (i) in Figure 1. A change in the degree of transparency of a bank's portfolio, ϕ, implies that the probability of liquidation, R^*, and the face value of debt, D^*, move in the same direction in equilibrium. Figure 1 also shows that if the degree of transparency of bank i's portfolio is too low, then there exists no face value of debt for which the investors are willing to continue the bank, and $R^* = 1$.

A general characterization of the investor optimal decision, i.e. the probability R^* that a bank is liquidated and the face value of debt D^*, as a function of the investor's reservation value, r, and of the fraction that the bank retains of her own project, ϕ, is provided in Lemma A.1 in the Appendix. Figure 2 illustrates the results stated in the Lemma. When r is low and ϕ is not too high, as in Region (i) in Figure 2, the repayment an investor expects to receive from his bank at date 2 is sufficiently high that he always continues the bank. As r increases, the investor continues the bank with probability 1 only if he does not have sufficiently precise information about the bank's portfolio, or when ϕ is lower. Indeed, the threshold $\phi_{\text{i-ii}}(r)$ that separates Region (i) and Region (ii) is decreasing r. This is because when the investor is relatively uninformed

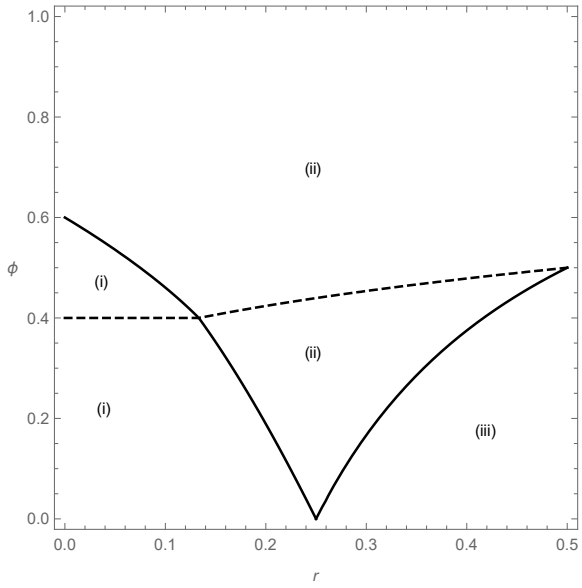

Figure 2: The figure illustrates how the liquidation probability R and the face value of debt D^* depend on the investor's reservation value, r, and of the fraction, ϕ. In region (i), $R = 0$. In region (ii), $R \in (0, 1)$. In region (iii), $R = 1$. The dashed lines in regions (i) and (ii) represent the thresholds above which the equilibrium face value of debt $D^* \leq \phi$, and below which $D^* > \phi$.

the face value of debt can be set to compensate him in expectation for not liquidating early, provided r is small. In other words, opacity makes investors passive when r is low. However, as the investors' reservation value, r, increases and banks portfolios remain relatively opaque, or ϕ is not too high, as in Region (iii) in Figure 2, investors no longer tolerate opacity. Thus, in Region (iii), there exists no level of the face value of debt for which an investor is willing to continue the bank at date 1. Thus opacity makes investors aggressive when r is high. This is reflected in the threshold $\phi_{\text{ii-iii}}(r)$ that separates Region (ii) and Region (iii) which is increasing r.

When ϕ is high and the banks' portfolios are relatively transparent, an investor continues his bank with probability $R^* \in (0, 1)$ for a large range of his reservation value, r, as shown in Region (ii) in Figure 2. This also holds for lower levels of ϕ, as long as r is not too high.

The dashed lines in Region (i) and Region (ii) in Figure 2 represent the thresholds above which the equilibrium face value of debt $D^* \leq \phi$, and below which $D^* > \phi$.

In the second step, we analyze banks' decision about what fraction, ϕ, of their loans to retain in their portfolio. When banks choose their optimal portfolio**,** they take into account that the investors make optimal decisions. That is, a bank i understands that each fraction $(1-\phi)$ of loans it exchanges with the other bank induces a continuation decision s_I, and a face value of debt D. Thus for each ϕ, and s_I^* and D^* associated with that ϕ, the bank i's expected payoff is

$$W_i = \mathbb{E}_{R_i}\left\{\mathbb{E}_{R_j}\{\max[(\phi R_i + (1-\phi)\, R_j - D^*), 0]\} \cdot s_I^*(R_i)\right\}$$

or

$$W_i = \mathbb{E}_{R_i}\left\{\mathbb{E}_{R_j}\left\{\left(\phi R_i + (1-\phi)\, R_j - D^*\right) \middle| R_j \geq \frac{D^* - \phi R_i}{(1-\phi)}\right\} \middle| R_i \geq R^*\right\} \Pr\left(R_i \geq R^*\right).$$

Isolating D^*, and substituting the investor's payoff, we can write bank i's expected payoff in (5) as

$$\begin{aligned} W_i \;=\; & \mathbb{E}_{R_i}\left\{\phi R_i + (1-\phi)\, \mathbb{E}_{R_j}\left\{R_j \middle| R_j \geq \frac{D^* - \phi R_i}{(1-\phi)}\right\} \middle| R_i \geq R^*\right\} \Pr\left(R_i \geq R^*\right) \quad (9) \\ & -D^* \cdot \mathbb{E}_{R_i}\left(\Pr\left(R_j \geq \frac{D^* - \phi R_i}{1-\phi}\right) \middle| R_i \geq R^*\right) \Pr\left(R_i \geq R^*\right). \quad (10) \end{aligned}$$

The first term in (9) represents the total surplus that is expected to be realized when investor I continues to fund bank i. The last term is simply the payoff that investor I's expects to receive if he continues the banks. Thus, in equilibrium, each bank i chooses the fraction, ϕ, of her project to retain in her portfolio such that the expected marginal total surplus equals the marginal investor's payoff.

The optimal degree of transparency, ϕ^* solves

$$\phi^* = \arg\max\left\{\int_{R^*}^{1}\int_{\max\left\{\frac{D^* - \phi R_i}{1-\phi}, 0\right\}}^{1} \left(\phi R_i + (1-\phi)\, R_j - D^*\right) dR_j dR_i\right\}.$$

The bank's objective is to choose a portfolio allocation $(\phi^*, 1-\phi^*)$ that would result in the lowest face value of debt, D, conditional on the investor continuing to fund the bank. The bank has a two-fold incentive to choose a portfolio allocation that minimizes D conditional on continuation. First, since the bank is the residual claimant on the realization of her portfolio, then the smaller D the bank needs to pay to the investor the more she is able to retain for herself in the states in which there is no default. Second, a smaller D lowers the probability of default as well. Note that although D is set competitively to maximize investors' expected payoff, banks' and investors' incentives are not perfectly aligned. The investor could benefit from a portfolio allocation that induces a higher D even though the bank defaults more often, as this could yield a higher expected payoff for him. This wedge is an expression of the market power that banks exercise when choosing ϕ.

In deciding a portfolio allocation, the bank weighs two forces: the ex-post probability of default at date 2 and the probability of liquidation at date 1. When possible, the bank chooses a portfolio allocation to decrease both the ex-post probability of default at date 2 and the probability of liquidation at date 1. This is the case when $D^* > \phi$ in Region (ii) in Figure 2. However, there are parameters for which the probability that the investor liquidates the bank is not monotonic in ϕ, as it is the case when $D^* \leq \phi$ in Region (ii) in Figure 2. Then, the bank faces a trade-off between the marginal change she induces in the ex-post probability of default relative to the marginal change she induces in the probability that the investors liquidate the bank. In this case, the optimal choice of the degree of transparency has interesting implications for welfare and the early liquidation of banks, as we discuss in Section 4.

3.2 Constrained Planner and Welfare

We start by introducing the social planner optimization problem. Our definition of welfare aggregates banks' and investors' expected payoffs as of date 0, and can be

written as

$$W^S = 2\times\mathbb{E}_{R_i}\left\{\left[\phi R_i \Pr\left(R_j \geq \frac{D^* - \phi R_i}{1-\phi}\right) + (1-\phi)\,\mathbb{E}\left(R_j | R_j \geq \frac{D^* - \phi R_i}{1-\phi}\right)\right]\cdot s_I^*(R_i) + r\cdot(1 - s_I^*(R_i))\right\}. \tag{11}$$

The social planner chooses a fraction of the project, ϕ^S, that each bank retains such that

$$\phi^S = \arg\max_{R^*}\int_{R^*}^{1}\left[\phi R_i\left(1 - G\left(\max\left\{\frac{D^* - \phi R_i}{1-\phi}, 0\right\}\right)\right) + (1-\phi)\int_{\max\left\{\frac{D^* - \phi R_i}{1-\phi}, 0\right\}}^{1} R_j dG(R_j)\right] dG(R_i) + rG(R^*),$$

given that the face value of debt satisfies (6) and investors take optimal continuation decisions that satisfy (1).

The social planner's and the banks' incentives are partially aligned, as both benefit when investors continue to fund the bank and when default is avoided. However, two wedges may arise between the social planner and the banks.

The first wedge arises because banks do not internalize that in case of early liquidation, investors nevertheless obtain their reservation value, r. In contrast, for the social planner, the early liquidation payoff represents a welfare gain even when a bank defaults. Moreover, as r increases, investors obtain a good payoff if they liquidate early. Thus, even if banks choose a portfolio allocation that decreases the probability of liquidation, it may be the case that the expected welfare gain from continuation increases by less. This force is most clearly identified by allowing investors to have full information about both banks' projects, a set-up which we characterize formally in the Section 5.

The second wedge arises because the social planner and the banks associate a different relative importance to being continued vs. defaulting. That is, the trade-off between the ex-post probability of default and the probability that the investors liquidate the bank resolves differently for the social planner. This is because, each bank, as a residual claimant on the return of her portfolio, may benefit from receiving a higher payoff albeit in fewer states of the world. Therefore, it may be optimal for the bank

to chose a portfolio allocation under which investor receive a lower face value of debt, even if this implies that investors are more likely to liquidate. In contrast, from the perspective of a social planner, the face value of debt is a transfer from the bank to the investor. Provided that investors are not too highly compensated for liquidating early, the social planner's objective is to chose a portfolio allocation $\left(\phi^S, 1-\phi^S\right)$ for each bank that yields a face value of debt, D^S, such that the surplus is realized in as many states of the world as possible. While the planner may also be concerned that the face value of debt is be too high, this is because the bank can become insolvent and default at date 2. This trade-off is present only when investors have incomplete information about their banks' portfolio.

3.3 Case Study: Full Opacity vs. Full Transparency

As a preamble to the full equilibrium characterization, it is instructive to consider the implications of our model if the only possible choices for the bank and the constrained planner were full opacity ($\phi = 0$) and full transparency ($\phi = 1$).

The case of $\phi = 1$, when each bank retains all of her project and the investors are perfectly informed, is straightforward. Investor I continues to fund bank i when the signal he receives R_i is larger than the face value of debt D, i.e. when he is certain that the bank does not default at date 2. In other words, his continuation strategy is given by (2) where $R^* = D$. At the same time, the face value of debt must solve the following maximization problem

$$\max_D \left\{ D \left[\int_{R^*}^{1} 1 dG\left(R_i\right) \right] + rG\left(R^*\right) \right\},$$

and the first order condition implies that

$$D^*\left(1\right) = \frac{1+r}{2}.$$

Thus, the investor continues the bank with probability $\frac{1-r}{2}$.

The case of $\phi = 0$, when each bank swaps the entire project with the other bank, is similarly straightforward. The signal that the investors receive at date 1 is not informative about the return of their bank portfolio. Thus, an investor continues the debt contract with the bank provided the amount he expects to be repaid at date 2, $D \cdot \Pr\left(D \leq V_i\left(0\right)\right)$, exceeds the reservation value he obtains when he liquidates the bank at date 1, r. Then, the face value of debt must solve the following maximization problem

$$\max_{D} \left\{ D \left[\int_{D}^{1} dG\left(R_j\right) \right] \right\},$$

and the first order condition implies that

$$D^*\left(0\right) = \frac{1}{2}.$$

Thus, if $r < \frac{1}{4}$ the investor continues the bank with probability 1 and receives $D^*\left(0\right)$ at date 2 with probability $\frac{1}{2}$. Otherwise, if $r \geq \frac{1}{4}$ there exists no face value of debt that can induce the investor to continue the bank.

Direct comparison of these two extreme cases reveal three key outcomes. First, from the perspective of the planner, it is always better to have fully transparent banks. In the absence of diversification benefits, full opacity only makes the investors' liquidation decision less precise, which leads to surplus being destroyed inefficiently. Thus, welfare under full transparency is higher than under full opacity.

Second, the banks prefer to be fully opaque if $r < \frac{1}{4}$, while they choose a fully transparent portfolio if $r \geq \frac{1}{4}$. When $r < \frac{1}{4}$, opacity makes investors passive, and they choose to continue their bank with probability 1. Banks favor this outcome, since they need to pay a lower face value of debt than when they have perfectly transparent portfolios. When $r \geq \frac{1}{4}$, opacity makes investors aggressive and they will always liquidate the bank at date 1. A bank can persuade the investor to continue the contract with positive probability if only and if she holds a perfectly transparent portfolio. In this case, the investor continues his banks with probability $\frac{1-r}{4}$. Therefore, the bank is too opaque relative to the constraint efficient allocation if and only if $r < \frac{1}{4}$. We show in

the next section how investors' and banks' incentives adjust to intermediate levels of opacity.

The third key implication is that the probability of early bank liquidation, or of a financial crisis, is always weakly lower in equilibrium compared to when the banks choose fully transparent portfolios, as in the constraint efficient outcome. In other words, banks always prefer to avoid early liquidation. However, as we argue in the next section, narrowing banks' choice to be either fully opaque or fully transparent misses an important consequence of opacity, namely bank strategic exposure to financial crises. We show that such behavior only emerges if banks can fine-tune how much information they provide to investors by choosing portfolios with intermediate level of opacity, $\phi \in (0, 1)$.

4 Equilibrium and Optimal Opacity

In this section we formally introduce our results. The first set of result characterizes the equilibrium portfolio allocation, and contrasts it with the portfolio allocation that maximizes welfare. The second set of results addresses the implications of the equilibrium portfolio allocation for financial crises. In our model, we define a *financial crisis* as the event that a bank is liquidated prematurely, at date 1.

We start by characterizing banks' equilibrium portfolio allocation in the following proposition.

Proposition 1 *There exists r_h such that for any $r < r_h$, banks hold in equilibrium interdependent and opaque portfolios, or $\phi^* \in (0, 1)$. If $r \geq r_h$, banks have independent and perfectly transparent portfolios, or $\phi^* = 1$.*

Figure 3 shows the equilibrium portfolio allocation, ϕ^*, as a function of the investors' reservation value, r. To gain intuition, we superimpose the equilibrium ϕ^* on Figure 2 which shows the various regions corresponding to the investor's optimal outcomes, i.e. the probability R^* that a bank is liquidated and the face value of debt D^*. In Region (i) in Figure 3 (2) r is small and investors' outside option is not that attractive. In this

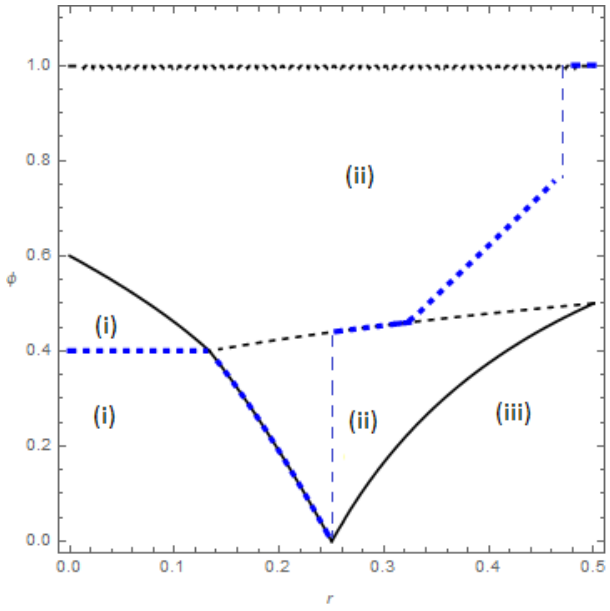

Figure 3: The figure illustrates the equilibrium degree of transparency, ϕ^*, depends on the investor's reservation value, r. We plot ϕ^* superimposed on Figure 2.

case, investors do not have an incentive to liquidate their bank early, particularly when ϕ is low and they are better informed. Thus $R^* = 0$. In this region, since the bank is not liquidated, she simply chooses the ϕ^* that minimizes the face value of debt D^* that it has to pay the investor.

As we transition to Region (ii) in Figure 3 (2), the probability R^* that a bank is liquidated is positive. This introduces complex trade-offs in how a bank chooses her portfolio allocation ϕ. In some cases, the bank prioritizes to be continued and chooses ϕ so to minimize the probability of liquidation, even though investors require a relatively high face value of debt, D^*, that can increase the default probability. Yet in other cases, the bank chooses ϕ to minimize D^* and thus the probability of default, even though this implies that it would be liquidated more often, as is the case when $D^* \leq \phi$.

Proposition 1 shows that when investors' payoff, r, from early liquidation is not too high, neither full transparency, nor full opacity are desirable for the banks. To see this, it is useful to first understand a bank's incentives at the two extremes. As we have discussed above, when $\phi = 1$ and the investor has perfect information, he liquidates the

bank if and only if the bank defaults at date 2. This allows the investor to extract a larger share of his bank's surplus, under the form of D^*. When $\phi = 0$, the investor is completely uninformed so he needs to be compensated more in expectation, in order to retain his investment in the bank. In addition, if banks either don't swap or fully swap their projects, there are no gains from diversification. In both cases, the bank is worse off than when she chooses an interior portfolio allocation. With an interior allocation, the bank's portfolio is opaque, and investor's liquidation decision is less precise. That is, the investor liquidates the bank even in some states in which the bank would not default and, conversely, the investor continues the bank even in some states in which the bank would default. The bank essentially seeks a degree of opacity ϕ to take most advantage of the two types of errors that the investor makes.

When the reservation value of the investor is too attractive, he will continue to fund the bank only when he is certain that he will receive a sufficiently high face value of debt, conditional on his information at date 1. In this case, the bank must retain all of his project, as the investor does not tolerate any degree of uncertainty about the portfolio return.

A direct implication of Proposition 1 is that banks incur a negative premium for holding opaque and interconnected portfolios, as long as $r \leq r_h$. This premium is defined as the difference between a bank's payoff when she holds a perfectly transparent portfolio ($\phi = 1$) and the payoff she obtains in equilibrium, conditional on being continued. The next proposition shows that the gain banks obtain from opacity is at the expense of the investors.

Proposition 2 *There exists r_l such that for any $r > r_l$, welfare is maximized when banks have independent and perfectly transparent portfolios, or $\phi^S = 1$. If $r < r_l$, welfare is maximized when banks have interdependent and opaque portfolios, and $\phi^S = \phi^* < 1$.*

The social planner finds it optimal that banks have independent portfolios if r is not too low. There are two reasons for this. First, even if the probability of liquidation may be higher when banks have independent portfolio than when they have interdependent portfolios, the welfare gain that investors obtain from their reservation value, r, may

compensate for the welfare loss when the bank is liquidated early. Second, investors have perfect information about their bank portfolio return when $\phi = 1$. As we have seen in the previous section, when $\phi = 1$, the face value of debt $D^* = R^*$. Thus investor I continues to fund bank i only when he is certain he will receive D, given he knows that the return of i's project is R_i at date 1. In other words, the investor liquidates the bank only when the bank would otherwise default at date 2. In this case, no surplus is destroyed inefficiently. In contrast, when $\phi < 1$ there can be a welfare loss because when bank i is liquidated the $(1 - \phi)$ share of project originated by bank j that i owns is liquidated as well, even though it could yield a high return.

A direct consequence of Proposition 2 is that equilibrium portfolio allocations decrease welfare for intermediate values of the investors' reservation value, r. In Figure 6 we plot the difference between the payoff that a bank obtains in equilibrium and the payoff that a bank obtains when she holds the efficient portfolio as a function of r. The figure illustrates that indeed banks gain from opacity, instead of incurring a cost, at the expense of the investors.

Formally, we show that $r_l < r_h$, such that the divergence between the social planner and the banks arises when $r \in (r_l, r_h)$. The next proposition introduces this result and identifies the main sources of the inefficiency.

Proposition 3 *For any $r \in (r_l, r_h)$ banks' portfolios are inefficiently opaque. Moreover, there exists $r_m \in (r_l, r_h)$ such that*

1. *If $r_l < r < r_m$, the investors continue their bank too frequently, or $R^*(\phi^*) < R^*(1)$.*
2. *If $r_m < r < r_h$, the investors liquidate their bank too frequently, or $R^*(\phi^*) > R^*(1)$.*

Proposition 3 essentially characterizes the probability of financial crises as a function of the banks' portfolio choices. Recall that we defined a financial crisis to be the event that a bank is liquidated at date 1. While financial crises may be efficient conditional on banks' choices, welfare losses arise nevertheless either because crises occur either too frequently or too infrequently. There are two channels that explain why inefficiencies

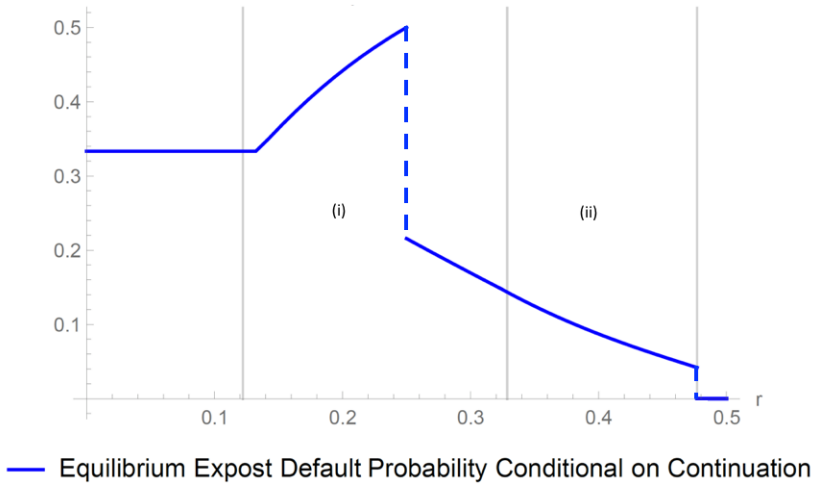

Figure 4: The figure illustrates $\mathbb{E}_{R_i}\big(\Pr(D > V_i | R_i > R^*)\big)$, the expected probability that a bank defaults at date 2, when at date 1 investors have learned the signal and decided to continue funding the bank, given the equilibrium degree of transparency, as well as the optimal degree of transparency. We distinguish two regions: Region (i), when $r \in (r_l, r_m)$ and Region (ii), when $r \in (r_m, r_h)$.

occur. First, the banks default too often at date 2 if they are not liquidated early. That is, there are cases when investors choose to forego r and not liquidate the bank, but the bank defaults at date 2 and investors receive zero. This is shown in the first part of Proposition 3. Second, banks are liquidated too often at date 1: there are cases when investors choose to liquidate the bank and only receive r. However, if the bank were not liquidated, it would be solvent and pay out $D > r$ to investors at $t = 2$. This is shown in the second part of Proposition 3.

The intuition is as follows. Given equilibrium D^* and ϕ^* set at $t = 0$, investor I has an ex-ante expectation of his payoff, in case he does not liquidate the bank. At $t = 1$, when investor I observes signal R_i, he updates the expectation of his payoff given his signal. When $r \in (r_l, r_m)$, banks choose relatively opaque portfolios. Thus investor I's signal is relatively uninformative. It follows that unless R_i is very low, the investor does not revise his expected payoff significantly downward, and he chooses to continue the bank. This leads the investor to continue funding the bank too often relative to the case when banks hold the socially optimal portfolio. Since a large part

of bank i portfolio depends on realization of R_j, ex-post the banks default frequently, as illustrated in Figure 4, Region (i).

When $r \in (r_m, r_h)$, the banks choose relatively transparent portfolios, since investors' reservation value, r, is high. As investor's interim signal at $t = 1$ is informative, he substantially updates his expectation about the payoff he obtains in case he does not liquidate the bank. Thus even for an intermediate realization of the signal, concern that his bank will default at date 2 prompts the investor to liquidate the bank. This leads to too frequent liquidation relative to the case when the bank holds the socially optimal portfolio. It also implies that the investor only continues to fund the bank when his signal is relatively high, which leads to a healthy bank balance sheet since R_i is a significant share of bank i's portfolio. As a result, conditional on continuation, the bank rarely defaults at $t = 2$, as illustrated in Figure 4, Region (ii). The bank favors this outcome as well since its payoff conditional on being continued is sufficiently high to compensate for the fact that she is liquidated more frequently.

Note that the liquidation probability is monotonically increasing in r, but the degree of opacity is non-monotonic, as illustrated in Figure 5. The first observation is intuitive: the liquidation value r represents investors outside option, and a higher outside option increases the probability of liquidating the bank. By the same logic, one would expect the equilibrium opacity to be monotonically decreasing in r. Indeed, higher bank transparency can increase investor's likelihood that he receives his date 2 payoff, attenuate the increase in probability of liquidation, and improve bank's payoff.

Although this logic holds once r is sufficiently high, it is not globally true. In fact for low levels of liquidation value, opacity is weakly increasing (transparency, ϕ, weakly decreasing) in r. The reason is that when r is not too high, banks recognize that by giving their investors a sufficiently risky payoff with a high upside, as represented by a high face value of debt, they can remove the liquidation threat all together. It follows that to induce a sufficiently high face value of debt, equilibrium opacity increases in r until it peaks at $r = \frac{1}{2}$, while R^* is zero, i.e. the bank is never liquidated. Above this threshold the investor's outside option is sufficiently high that he sometimes liquidates

the bank, regardless of the degree of opacity, as we explained above.

It is important to observe that the second part of Proposition 3 implies that the probability of a financial crisis is higher when banks have correlated portfolios than when they have independent portfolios. Counterintuitively, in our model banks choose a portfolio allocation that increases the likelihood of a banking crisis. At the same time, the face value of debt that compensates investors is lower than the one that prevails when the banks are less exposed to crises, as is the case when $\phi = 1$. A banking crisis can occur even when there is a positive probability that the bank's portfolio yields a positive return if continued. This suggests that government interventions and bailout policies can only increase the probability of banking crises by further distorting banks' incentives, and thus are inefficient.

Furthermore, the model also provides an interesting set-up to think about the timing of bailouts. Specifically, in the context of our model, the government can intervene at two distinct times. First, at $t = 1$ when investors receive a low signal and intend to liquidate the banks, the government can intervene and allow the banks to continue. Second, at $t = 2$, if a bank does not have enough resources, the government can intervene and repay the face value of debt to investors. Neither intervention involves a transfer of government resources to bank equity holders, but the two bailout policies have very different implications for the equilibrium degree of opacity, as well as for the banks' and investors' payoff. Thus, by explicitly modeling the strategic interactions between banks and investors, our framework allows for comparing the equilibrium implications of government intervention when banks are illiquid versus when they are insolvent.

Another interpretation of Proposition 3 is in terms of investors' use of their information. When banks have independent portfolios, investors are *relatively information insensitive*, in the sense that they only liquidate the bank if they anticipate a low return of their bank's project. Once the bank chooses to exchange projects, investors become *more information sensitive*, by taking actions based on the (partial) information that they receive about their bank portfolio.

While r in our set-up is an exogenous parameter, it can be thought as capturing

in a reduced form liquidity in the secondary market, as most recently modeled in Dewatripont and Tirole (2018). An improvement in the secondary market can be then equivalent to increasing r, which effectively acts as an outside option for investors. Then, an implication of our model is that more liquidity in the secondary market has an adverse effect: it provides incentives for investors to liquidate the bank early as they can guarantee themselves a safe return, and do not internalize the forgone residual claim which accrues to the bank if it is continued. Under this interpretation, improving secondary market liquidity makes the banking sector less stable, and destroys welfare through frequent banking crises.

5 Opacity vs. Full Information

A useful benchmark for our analysis is the full information case. In the full information case, we consider that at date 1 the two signals that perfectly reveal the return of banks' projects, R_i and R_j, are observed by both investors. This benchmark helps disentangle the sources of bank profits in the imperfect information case into two distinct constituents. The first component purely relates to banks' actions counteracting investors' bargaining position in setting the face value of debt. The second one exists since banks can obfuscate investor information. The full information benchmark isolates the first component. It also clarifies that the only reason banks choose to expose themselves to excessive early liquidation is to obfuscate investor information. If investors are fully informed, banks always choose a portfolio allocation that allows them to be continued as long as possible.

In the full information case, we consider that at date 1 the two signals that perfectly reveal the return of banks' projects, R_i and R_j, are observed by both investors. This case is informative of the extent to which opacity motives drive banks' decision to have interdependent portfolios, and the implications for financial crisis.

As in the main specification of our model, we start by analyzing investors' continuation decision. For each portfolio allocation, $\phi^{FI} \in (0, 1)$, and each face value of debt, D^{FI}, set at date 0 under full information, the optimal continuation decision that

investor I takes at date 1 must satisfy

$$s_I^{*/FI} = \arg\max \left\{ s_I^{FI} \cdot D^{FI} \cdot \Pr\left(D^{FI} \leq V_i\left(\phi^{FI}\right) | R_i, R_j \right) + \left(1 - s_I^{FI}\right) \cdot r \right\}. \quad (12)$$

Since at date 1 the investor is perfectly informed, he continues funding the bank when the return of his bank's portfolio is sufficiently high to repay the face value of debt D^{FI}. In other words, his continuation strategy is given by

$$s_I^{*/FI}\left(R_i, R_j\right) = \begin{cases} 1 & \text{if } V_i\left(\phi^{FI}\right) \geq D^{FI} \\ 0 & \text{if } V_i\left(\phi^{FI}\right) < D^{FI} \end{cases}. \quad (13)$$

To parallel the imperfect information case, we use $R^{*/FI} \equiv \Pr\left(V_i\left(\phi^{FI}\right) < D^{FI}\right)$ to denote the probability that the investor liquidates the bank at date 1. In contrast with the case when they have information only about their bank's project, with full information investors liquidation decision is perfectly accurate. That is, investors liquidate the bank only in the states in which the bank defaults at date 2, and, similarly, they continue the bank only in the states in which the banks is able to repay the face value of debt.

With full information, investor's I expected payoff at date 0, is given by

$$W_I^{FI} = \mathbb{E}_{R_i,R_j}\left\{ s_I^{*/FI}\left(R_i, R_j\right) \cdot D^{FI} + \left(1 - s_I^{*/FI}\left(R_i, R_j\right)\right) \cdot r \right\},$$

and the optimal face value of debt solves

$$D^{*/FI} = \arg\max \left\{ D^{FI} \cdot \Pr\left(D^{FI} \leq V_i\left(\phi^{FI}\right)\right) + r \cdot \left(1 - \Pr\left(D^{FI} \leq V_i\left(\phi^{FI}\right)\right)\right) \right\}. \quad (14)$$

The solution to (14) ensures that when the investor continues the bank, he will receive $D^{*/FI} > r$.

We then turn to analyze each bank's decision about what fraction, ϕ^{FI}, of their loans to retain in her portfolio. As in the imperfect information case, when banks choose their optimal portfolio, they take into account that the investors make optimal decisions. Thus for each ϕ^{FI}, and $s_I^{*/FI}$ and $D^{*/FI}$ associated with ϕ^{FI}, the bank i's

expected payoff is

$$W_i^{FI} = \mathbb{E}_{R_i,R_j}\{\max[(\phi R_i + (1-\phi) R_j - D^{*/FI}), 0] \cdot s_I^{*/FI}(R_i, R_j)\}.$$

Thus, the optimal portfolio allocation, $\phi^{*/FI}$ solves

$$\phi^{*/FI} = \arg\max \left\{ \mathbb{E}_{R_i,R_j} \left[\left(\phi R_i + (1-\phi) R_j - D^{*/FI} \right) | D^{*/FI} \leq V_i\left(\phi^{FI}\right) \right] \cdot \Pr\left(D^{*/FI} \leq V_i\left(\phi^{FI}\right) \right) \right\}.$$

The social planner optimization problem is similar to the incomplete information case. Total welfare aggregates banks' and investors' expected payoffs as of date 0, and can be written as

$$W^{S/FI} = \mathbb{E}_{R_i,R_j} \left[(\phi R_i + (1-\phi) R_j) | D^{*/FI} \leq V_i\left(\phi^{FI}\right) \right] \Pr\left(D^{*/FI} \leq V_i\left(\phi^{FI}\right) \right) + r\left(1 - \Pr\left(D^{*/FI} \leq V_i\left(\phi^{FI}\right) \right)\right). \tag{15}$$

The social planner chooses a fraction of the project, ϕ^S, that each bank retains to maximize (15), given that the face value of debt satisfies (14) and investors take optimal continuation decisions that satisfy (12).

Next we characterize the equilibrium portfolio allocation, as well as the portfolio allocation that maximizes welfare when investors are fully informed at date 1.

Proposition 4 *There exists r_H such that for any $r < r_H$, $\phi^{*/FI} = \max\left\{\frac{3-2r}{5}, \frac{1+2r}{3}\right\}$. If $r \geq r_H$, banks have independent portfolios in equilibrium and $\phi^{*/FI} = 1$.*

Note that for any portfolio allocation $(\phi, 1-\phi)$ that is an equilibrium, the portfolio allocation $(1-\phi, \phi)$ is also an equilibrium. This follows from the symmetry of the banks' portfolio return distribution in the full information case.

Proposition 4 implies that banks find it beneficial to exchange projects only as long as the reservation value of the investor, r, is not too high. With perfect information, banks have an incentive to exchange projects either to change the distribution of their portfolio to gain on the upside, or to benefit from diversification and default less often.

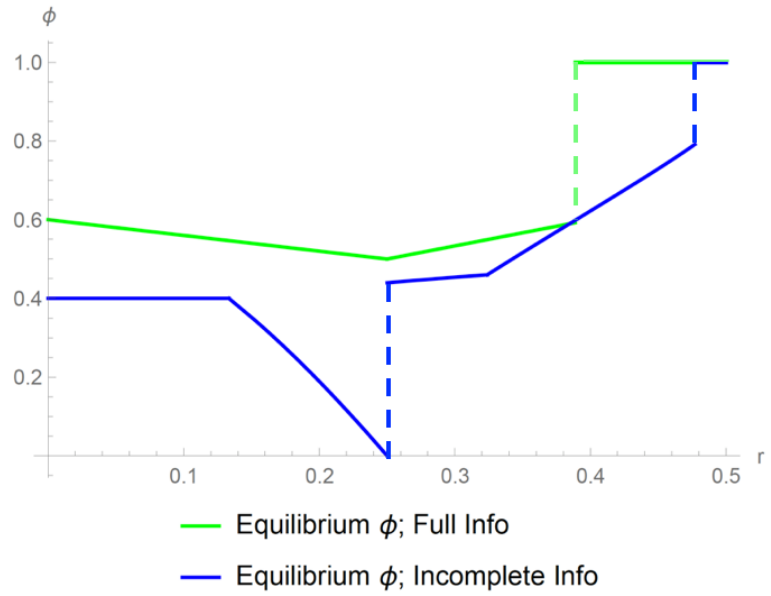

Figure 5: The figure illustrates the equilibrium portfolio allocation under full information, $\phi^{*/FI}$, as well as the equilibrium portfolio allocation under incomplete information, ϕ^*, as a function of the investors' reservation value r.

Under perfect information investors liquidate their bank at date 1 precisely when the bank will default at date 2. In this case, banks are able to retain a larger share of the aggregate surplus if they choose ϕ that both lowers the probability of default and face value of debt. Investors nevertheless could be better off with a portfolio allocation for which the bank defaults more often but that yields a higher face value of debt.

Exchanging projects brings diversification benefits as diversification lowers the probability of default. However, diversification also limits the payoff that a bank receives on the upside as well. Hence, perfect diversification is not optimal, in the sense that $\phi^{*/FI} \neq \frac{1}{2}$. This is not surprising. While models like Diamond (1984) inform our intuition that perfect diversification is desirable, this result tells us that including more independently risky projects equally-weighted in a portfolio strictly improves the default probability. However, even in the standard model of Diamond (1984) in which investors are offered a debt contract that allows them to break even, fixing the number of projects in the portfolio, a bank can be indifferent between various portfolio allocations.

To shed light on how opacity affects banks' decisions to have interdependent portfolio, it is then useful to compare the portfolio allocations that banks choose when they cannot obscure investor information, with the allocations that banks choose when they benefit from opacity as well. Figure 5 illustrates how the equilibrium allocation under full information, $\phi^{*/FI}$, compares with the equilibrium allocation under incomplete information, ϕ^*, as a function of the investors' reservation value r. It is immediate from the figure that bank incentive to obscure investor information leads them to hold excessively interconnected portfolios.

Proposition 5 *There exists r_L such that for any $r < r_L$, welfare is maximized when banks have correlated portfolios and $\phi^{S/FI} = \phi^{*/FI} = \frac{3-2r}{5}$. If $r \geq r_L$, welfare is maximized when banks have independent portfolio and $\phi^{S/FI} = 1$.*

Just as in the equilibrium result, note that for any portfolio allocation $(\phi, 1-\phi)$ that is welfare optimal, the portfolio allocation $(1-\phi, \phi)$ is also welfare optimal. This also follows from the symmetry of the banks' portfolio return distribution in the full information case.

Proposition 5 shows that an interior portfolio allocation is optimal only if the reservation value of the investor, r, is sufficiently low. Since $r_l < r_L < r_m$, even with full information, the equilibrium is inefficient if $r \in (r_L, r_m)$. The nature of inefficiency is similar to our main specification when investors have only partial information. When investors have access to full information about their bank's portfolio, banks choose interdependent portfolios not to dilute investor information, but to gain from diversification. However, the bank, just as in the partial information case, does not internalize that if it is not continued, the liquidation value r still accrues to investors. For the social planner on the other hand, the early liquidation payoff represents a welfare gain even when a bank defaults. This is the only wedge that arises between the social planner and the bank in the full information case.

As we discussed in Section 4, opacity introduces a second wedge that renders banks' portfolios inefficient. Banks significantly exploit the informational disadvantage of the investors to increase their profits relative to what they would obtain if they held the

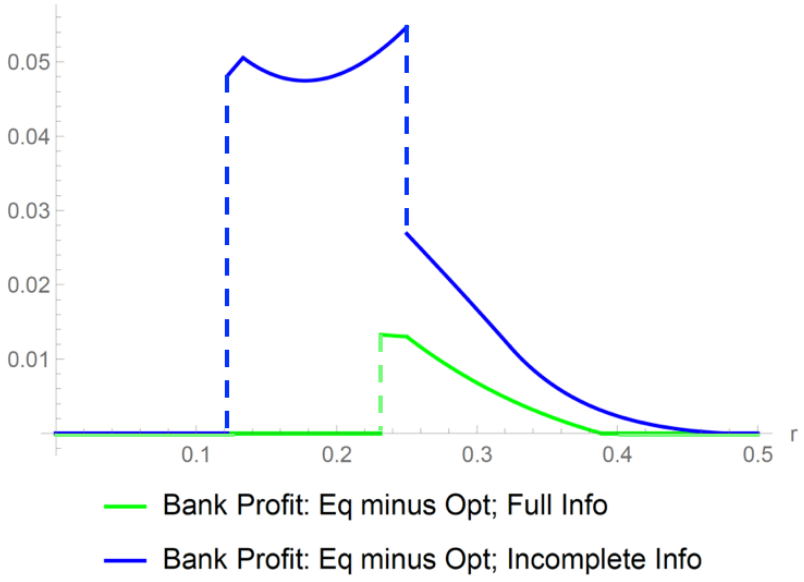

Figure 6: The figure illustrates the difference between the payoff that a bank obtains in equilibrium and the payoff that a banks obtains when she holds the efficient portfolio under full information, as well as under incomplete information, as a function of the investors' reservation value r.

optimal portfolio, as it is illustrated in Figure 6. Moreover, the region of inefficiency is wider in the partial information case than in the full information case, as the same figure shows.

The next proposition characterizes formally the main source of inefficiency when investors have perfect information, with implications for the probability of financial crises.

Proposition 6 *For any $r \in (r_L, r_H)$ banks portfolio allocations are inefficient. Moreover, the investors continue their bank too frequently, or $R^{*/FI}(\phi^{*/FI}) < R^{*/FI}(1)$.*

It is interesting to contrast the result in Proposition 6 with the result in Proposition 3. With full information about both project realizations, investors terminate their bank whenever the total ex-post return is below the face value they are promised to get. Perfectly informed investors are able to better tailor their joint $(D^{*/FI}, R^{*/FI})$ decision to minimize the loss in the event of bad realizations, while capturing as much surplus as possible. Thus, in the absence of opacity, banks cannot benefit from financial

crises. While banking crises occur in equilibrium, they are not as frequent as it would be the case if portfolio allocation were optimal. Only under opacity banks purposefully inflict crises in order to boost profits while they operate.

6 Conclusion

We explore a model in which banks strategically hold interconnected portfolios rendering the financial system opaque and more prone to financial crises. In our set-up, banks borrow funds from investors to finance risky projects, in exchange for risky long-term debt contracts. Before maturity investors observe a signal about their bank's project, and decide whether they liquidate their debt early against a fixed redemption value. Ex-ante, each bank can affect how investors use their information by exchanging a fraction of their project with another bank, thus creating opaque portfolios.

We show that generically, banks choose portfolios which are neither extremely opaque nor fully transparent. In other words, they provide investors with an optimal level of information. Furthermore, banks can choose a degree of opacity which makes equilibrium outcome constraint inefficient. However, rather counter-intuitively, the equilibrium degree of opacity does not deem the banks interim safe but ex-post in trouble. Instead, banks choose a degree of opacity that encourages crises: banks make themselves exposed to frequent termination if in return they end up paying extreme low returns to investors if they survive and become profitable. In other words, banks choose to expose themselves to financial crisis.

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A Appendix

A.1 Incomplete Information Derivations

Lemma A.1 *For any early redemption value $r \in \left(0, \frac{1}{2}\right)$ and any portfolio allocation $(\phi, (1-\phi))$ that banks hold, the probability, R^*, that an investor liquidates the bank at date 1, as well as the optimal face value of debt, D^*, with incomplete information are as follows.*

For any r,

1. *if $0 \leq \phi \leq \min\{\frac{2}{5}, \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\}$, then $D^* = \frac{2-\phi}{4}$ and $R^* = 0$.*
2. *if $\frac{2}{5} \leq \phi \leq \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$, then $D^* = \sqrt{\frac{2}{3}\phi(1-\phi)}$ and $R^* = 0$.*
3. *if $\max\{\frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right), 1 - \frac{1}{4r}\} < \phi \leq \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$ then $D^* = \frac{1}{6}\left(1 + \sqrt{1 + 12r(1-\phi)}\right)$ and $R^* = \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right)$, $0 < R^* < 1$.*
4. *if $\max\{\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right), \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)\} < \phi \leq 1$, then D^* is the largest root of equation*

$$-4D^3 + D^2(2r + \phi + 1) + r^2(\phi - 1) = 0,$$

and $R^* = \frac{1}{\phi}\left(D^* - (1-\phi)(1 - \frac{r}{D^*})\right)$, $0 < R^* < 1$.

5. *if $0 \leq \phi < 1 - \frac{1}{4r}$, then $R^* = 1$.*

Proof. Start from equations (3) and (6), letting $G \sim U[0,1]$. We will consider $R^* = 0$, $R^* = 1$ and $0 < R^* < 1$ separately. Moreover, we need to consider the following cases separately: $D^* < \phi$, and $D^* \geq \phi$. The distinction is that in the former case, lender I takes into account that even if the opaque part of portfolio of bank i, $(1-\phi)R_j$ returns zero, for sufficiently high realizations of R_i, $\frac{D^*}{\phi} \leq R_i \leq 1$, the lender will get paid if he chooses to continue the bank.

1. No early liquidation, $R^* = 0$.

1. $\frac{D^*}{\phi} > 1$. In this case, investor payoff simplifies to

$$W_I = D \int_0^1 \left(1 - \frac{D - \phi z}{1 - \phi}\right) dz.$$

The first order condition is

$$\frac{4D + \phi - 2}{2(\phi - 1)\phi} = 0,$$

which implies

$$D^* = \frac{1}{4}(2 - \phi).$$

The second order condition holds (SOC< 0), thus the above D^* is a maximum. $D^* > \phi$ requires $0 \leq \phi \leq \frac{2}{5}$, while $R^* = 0$ requires $\phi \leq \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$, which leads the first case.

2. $\frac{D^*}{\phi} < 1$. In this case, investor payoff simplifies to

$$W_I = \int_0^{\frac{D}{\phi}} \left(1 - \frac{D - \phi z}{1 - \phi}\right) dz + D(1 - \frac{D}{\phi})$$

The first order condition is

$$\frac{3D^2 + 2(\phi - 1)\phi}{2(\phi - 1)\phi} = 0,$$

which implies

$$D^* = \sqrt{\frac{2}{3}\phi(1 - \phi)}.$$

The second order condition holds (SOC< 0), thus the above D^* is a maximum. $D^* < \phi$ requires $\phi \geq \frac{2}{5}$, while $R^* = 0$ requires $\phi \leq \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$, which leads the second case.

2. Some early liquidation, $0 < R^* < 1$.

$$R^* = \frac{D}{\phi} - \frac{1 - \phi}{\phi}\left(1 - \frac{r}{D}\right)$$

We again separately consider two cases: $rac1-\phi\phi\left(1-\frac{r}{D}\right)$. We again separately consider two cases:

1. $\frac{D^*}{\phi}>1$. In this case, investor payoff simplifies to

$$W_I = D\int_{R^*}^{1}\left(1-\frac{D-\phi z}{1-\phi}\right)dz + rR^*$$

Substituting for R^* and taking first order condition implies

$$\frac{(D(1-3D)+r(1-\phi))\,(r(1-\phi)-D(1-D))}{2\phi(1-\phi)D^2}=0,$$

This is a quadratic equation with four roots: $D_1=\frac{1}{6}\left(1-\sqrt{1+12r(1-\phi)}\right)<0$. $D_{2,4}=\frac{1}{2}\left(1\pm\sqrt{4r\phi-4r+1}\right)$, and $R^*(D_2)=R^*(D_4)=1$. Thus, the only relevant face value is $D_3=\frac{1}{6}\left(1+\sqrt{1+12r(1-\phi)}\right)$. Note that $R^*(D)<1$ only if $D_2<D<D_4$, thus the optimal face value can be in this interval. Moreover, $D_2<D_3<D_4$.

Next, the second order condition is given by

$$\frac{\frac{r^2(1-\phi)^2}{D^3}+3D-2}{(1-\phi)\phi}$$

Letting SOC= 0 leads to a quadratic equation. Only two of the roots are in between D_2 and D_3, and $D_2<D_1^{soc}<D_3<D_2^{soc}<D_4$. Thus the second order condition changes sign twice on the interval $[D_2,D_4]$. Moreover, the second derivative evaluated at D_2 and D_4 is

$$\frac{1\pm\left(\sqrt{1-4r(1-\phi)}\right)(1-4r(1-\phi))}{2r\phi(1-\phi)^2}>0,$$

which implies that D_2 and D_4 are local minima, and that the second derivative is negative at D_3, thus D_3 is a (local) maximum. Since first order condition is positive between D_2 and D_3, and negative between D_3 and D_4, D_3 is the global

maximum in the interval $[D_2, D_4]$. Thus we have

$$D^* = \frac{1}{6}\left(1 + \sqrt{1 + 12r(1-\phi)}\right)$$
$$R^* = \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right).$$

Lastly, $D^* > \phi$ requires $\phi < \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right)$, $R^* < 1$ requires $\phi > 1 - \frac{1}{4r}$ and $R^* > 0$ requires $\phi > \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$. This leads the third case.

Moreover, R^* cannot exceed 1, thus $R^* = 1$ if $\phi \geq 1 - \frac{1}{4r}$, which leads the fifth case.

2. $\frac{D^*}{\phi} < 1$. Here we need to consider two sub-cases

   (a) $R^* < \frac{D^*}{\phi} \Leftrightarrow r < D^* < \phi$. In this case, investor payoff simplifies to

$$W_I = \int_{R^*}^{\frac{D}{\phi}} \left(1 - \frac{D - \phi z}{1 - \phi}\right) dz + D(1 - \frac{D}{\phi}) + rR^*,$$

   Substituting for R^* and taking the first order condition implies

$$\frac{-4D^3 + D^2(1 + \phi + 2r) - r^2(1-\phi)}{2D^2\phi} = 0.$$

   The numerator is a cubic function in D, with $\Delta = -432r^4(1-\phi)^2 + 4r^2(1-\phi)(1+\phi+2r)^3$, thus $\Delta < 0$ implies $(1+\phi+2r)^3 - 108r^2(1-\phi) < 0$. For any pair (r, ϕ) that satisfy $\Delta < 0$,

$$\phi < \tilde{\phi}(r) = \max\{\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right), \frac{1}{6}\left(-r + \sqrt{r(r+10)+1} + 1\right)\},$$

   and (r, ϕ) is covered by one of the first 3 cases. Thus when $\phi > \tilde{\phi}(r)$, $Delta > 0$ and the cubic first order condition has 3 distinct real roots, $D_1 < D_2 < D_3$. $D_1 < 0$, so it is not the solution. Moreover, note that the derivative of investors surplus approaches $-\infty$ as $D \to 0$ from above, and as

$D \to \infty$.

Next, the second order condition is given by

$$\frac{r^2(1-\phi) - 2D^3}{D^3\phi},$$

which has one root: $D^{soc} = \left(\frac{r^2(1-\phi)}{2}\right)^{\frac{1}{3}}$, and it is positive iff $D < D^{soc}$. Moreover, $D_2 < D^{soc} < D_3$, thus D_2 is a local minimum while D_3 is a local maximum. Thus either D_3 is the optimal face value, or the minimum feasible D, which in this case is $D = r$. Comparing the two values leads $W_I(\phi, r, D_3) > W_I(\phi, r, r)$, $\forall\ \phi > \tilde{\phi}(r)$. Thus we have

$$\begin{aligned} D^* &= D_3 \\ R^* &= \frac{1}{\phi}\left(D^* - (1-\phi)\left(1 - \frac{r}{D^*}\right)\right). \end{aligned}$$

Lastly, note that if $D^* = r$, first order condition implies that $\phi = r$. However, $r < \tilde{\phi}(r)$, which in turn implies that whenever $\frac{D^*}{\phi} < 1$, $D^* > r$ and thus the next case is never relevant.

(b) $R^* > \frac{D^*}{\phi} \Leftrightarrow D^* < \min\{r, \phi\}$. As argued above, this case does not arise in equilibrium.

Lastly, if $D^* = \phi$, first order condition implies $\phi = \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$, and $R^* > 0$ implies $\phi > \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$. This leads to the forth case.

Always liquidate.

$$\frac{D^*}{\phi} > 1$$

■

Proof of Proposition 1

We need to calculate the optimal ϕ for a bank i. That is, ϕ that maximizes

$$W_i = \int_{R^*}^{1} \int_{\max\left\{\frac{D^* - \phi R_i}{1-\phi}, 0\right\}}^{1} \left(\phi R_i + (1-\phi) R_j - D^*(\phi)\right) dR_j dR_i$$

As the optimal face value of debt changes depending on ϕ and r, we need to consider various cases, as defined in lemma A.1.

Case 1: $0 \leq \phi \leq \min\{\frac{2}{5}, \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\}$.

Here $D^* = \frac{2-\phi}{4}$ and $R^* = 0$. Substitute in the planner objective function to get

$$W_i = \frac{7\phi^2 + 12(1-\phi)}{96(1-\phi)}.$$

Since $R^* = 0$, the optimal face value and bank profit are independent of r. Observe that $\frac{dW_i}{d\phi} > 0$ and $\frac{d^2W_i}{d\phi^2} > 0$, thus the bank profit function is increasing and convex in this region. It follows that if the equilibrium level of opacity is in this region we will have

$$\phi^* = \min\left\{\frac{2}{5}, \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\right\},$$

or

$$\phi^* = \begin{cases} \frac{2}{5} & \text{if } 0 < r < \frac{2}{15} \\ \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right) & \text{if } \frac{2}{15} < r < \frac{1}{4} \end{cases}$$

Case 2: $\frac{2}{5} \leq \phi \leq \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$.

Here $D^* = \sqrt{\frac{2}{3}\phi(1-\phi)}$ and $R^* = 0$. Substitute in the bank objective function to get

$$W_i = \frac{1}{2} - \frac{8}{9}\sqrt{\frac{2}{3}(1-\phi)\phi}.$$

Since $R^* = 0$, again the optimal face value and bank profit are independent of r. Observe

that $\frac{dW_i}{d\phi} = 0$ at $\phi = \frac{1}{2}$ and $\frac{d^2W_i}{d\phi^2} > 0$, thus the objective function is convex, and the maximum is attained on one of the corners, i.e. $\phi^* = \frac{2}{5}$ or $\phi^* = \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$. Direct comparison of the bank profit on the two boundaries reveals that $W_i(\frac{2}{5}) \geq W_i\left(\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)\right)$ for $0 < r < \frac{2}{15}$. It follows that if the equilibrium level of opacity is in this region we will have

$$\phi^* = \frac{2}{5}.$$

Case 3: $\max\left\{\frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right), 1 - \frac{1}{4r}\right\} < \phi \leq \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$.

Here, $D^* = \frac{1}{6}\left(1 + \sqrt{1 + 12r(1-\phi)}\right)$ and $R^* = \frac{1}{3\phi}\left(-4 + 3\phi + \sqrt{1 + 12r(1-\phi)}\right)$, $0 < R^* < 1$. Substitute in the bank objective function to get

$$W_i = \frac{144r(1-\phi) - (42r(1-\phi) + 23)\sqrt{12r(1-\phi)+1} + 31}{162(1-\phi)\phi}.$$

It is more convenient to consider $r < \frac{1}{4}$ and $r > \frac{1}{4}$ separately. When $\frac{2}{15} < r < \frac{1}{4}$, $\frac{d^2W_i}{d\phi^2} > 0$. Since the bank objective is convex, maximum is attained at one of the two boundaries. Direct comparison reveals that $W_i\left(\frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\right) > W_i\left(\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)\right)$. On the other hand, when $\frac{1}{4} < r < \frac{1}{2}$, $\frac{dW_i}{d\phi} > 0$. Since the bank objective is increasing in ϕ, maximum bank profit is attained at maximum relevant ϕ, i.e. $\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$.

Thus if the equilibrium level of bank opacity is in this region, we have

$$\phi^* = \begin{cases} \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right) & \text{if } \frac{2}{15} < r < \frac{1}{4} \\ \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right) & \text{if } \frac{1}{4} < r < \frac{1}{2} \end{cases}$$

Case 4: $\max\{\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right), \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)\} < \phi \leq 1$.

Here, D^* is the largest root of equation

$$-4D^3 + D^2(2r + \phi + 1) + r^2(\phi - 1) = 0,$$

and $R^* = \frac{1}{\phi}\left(D^* - (1-\phi)(1-\frac{r}{D^*})\right)$, $0 < R^* < 1$. Bank profit is given by

$$W_i^{\text{case 4}} = W_i = \frac{3D^{*5} - 3D^{*4}(\phi+1) + D^{*3}\left(\phi^2+\phi+1\right) - r^3(1-\phi)^2}{6D^{*3}\phi}.$$

We will use $W_i^{\text{case 4}}$ for the objective function in this region since we use it to define the equilibrium thresholds.

We consider two cases separately, when $r < \frac{2}{15}$ and when $r > \frac{2}{15}$

- $\mathbf{r < \frac{2}{15}}$:

  In this region, $\frac{d^2 W_i^{\text{case 4}}}{d\phi^2} > 0, \forall\ \phi > \frac{3}{10}\left(1 - 2r + \sqrt{1-4r-6r^2}\right)$, thus the objective function is convex and the maximum is attained at one of the two boundaries. The upper boundary is $\phi = 1$ and the lower boundary is $\phi = \frac{3}{10}\left(1 - 2r + \sqrt{1-4r-6r^2}\right)$. The bank profit at the two boundaries is given by

  $$W_i\left(\frac{3}{10}\left(1 - 2r + \sqrt{1-4r-6r^2}\right)\right) = \frac{1}{2} - \frac{8}{45}\sqrt{2\left(\sqrt{1-2r(3r+2)}+1\right) + r\left(3r + 6\sqrt{1-2r(3r+2)}+2\right)},$$

  $$W_i(1) = \frac{(1-r)^2}{8}.$$

  Where the first expression uses continuity of bank objective function on the boundary $\frac{3}{10}\left(1 - 2r + \sqrt{1-4r-6r^2}\right)$, and case 2 above. Direct comparison of the two expressions reveals that the former expression is always larger than the latter. Thus in this range

  $$\phi^* = \frac{3}{10}\left(1 - 2r + \sqrt{1-4r-6r^2}\right)$$

- $\mathbf{r > \frac{2}{15}}$:

  Consider the first order condition

  $$\frac{dW_i^{\text{case 4}}(\phi, r)}{d\phi} = 0$$

The first order condition has either one or two solutions for $r \in (\frac{2}{15}, \frac{1}{2})$. Let ϕ_1 denote the larger solution. ϕ_1 exists for all $r \in (\frac{2}{15}, \frac{1}{2})$, $\phi_1 > \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$, and $\frac{d^2 W_i^{\text{case 4}}(\phi,r)}{d\phi^2} > 0$, i.e. ϕ_1 is a minimum.

Let ϕ_2 denote the smaller solution (if it exists). ϕ_2 exists only if $r > \hat{r} \in (\frac{2}{15}, \frac{1}{5})$, and $\frac{d^2 W_i^{\text{case 4}}(\phi,r)}{d\phi^2} < 0$, i.e. ϕ_1 is a maximum. However, ϕ_2 is not always larger than $\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$, thus it is not always a relevant solution. Moreover, $\frac{d\phi_2}{dr} > 0$.

Let $\phi^*_{FOC} = \phi_2$, and let r_z denote the level of $r \in (\frac{1}{4}, \frac{1}{2})$ such that $\phi^*_{FOC}(r) = \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$. Thus for $r > r_z$, ϕ^*_{FOC} is an interior (local) maximum.

Given the above argument, for $r \in (\frac{2}{15}, r_z)$, bank objective function is either decreasing or convex (or both) for $\phi \in \left(\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right), 1\right)$. Thus the maximum is attained at one of the two boundaries. The bank profit at the two boundaries is given by

$$W_i\left(\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)\right) =$$

$$\frac{46\sqrt{2r\left(r - \sqrt{r(r+10)+1} + 5\right)+1} + 2r\left(r - \sqrt{r(r+10)+1} + 5\right)\left(7\sqrt{2r\left(r - \sqrt{r(r+10)+1} + 5\right)+1} - 24\right)}{9\left(-r + \sqrt{r(r+10)+1} - 5\right)\left(-r + \sqrt{r(r+10)+1} + 1\right)}$$

and

$$W_i(1) = \frac{(1-r)^2}{8},$$

where the first expression uses continuity of bank objective function on the boundary $\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$, and case 3 above. Direct comparison of the two expressions reveal that the former expression is larger than the latter when $r \in (\frac{2}{15}, r_z)$. Thus in case 4, in this range

$$\phi^* = \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$$

Next, let r_h denote $r \in (r_z, 1)$ such that $W_i^{\text{case 4}}(\phi^*_{FOC}, r) = \frac{(1-r)^2}{8} = W_i(1)$. For any $r \in (r_z, r_h)$, bank surplus is first concave and then convex over the interval $\phi \in \left(\frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right), 1\right)$, with an interior (local) maximum and a larger interior (local) minimum. Thus the global maximum is obtained at either the local maximum, ϕ^*_{FOC}, or at the upper boundary $\phi = 1$. Direct comparison of the corresponding levels of objective functions reveals that $W_i^{\text{case 4}}(\phi^*_{FOC}(r), r) > \frac{(1-r)^2}{8} = W_i(1)$ for $r \in (r_z, r_h)$.

Finally, for $r > r_h$, the objective function is larger at the corner $\phi = 1$ compared to the interior local maximum, thus $\phi^* = 1$.

Putting the cases together,

$$\phi^* = \begin{cases} \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right) & \text{if } 0 < r < \frac{2}{15} \\ \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right) & \text{if } \frac{2}{15} < r < r_z \\ \phi^*_{FOC} & \text{if } r_z < r < r_h \\ 1 & \text{if } r_h < r < \frac{1}{2} \end{cases}$$

Case 5: $0 \leq \phi < 1 - \frac{1}{4r}$.
Here $R^* = 1$, thus $W_i = 0$.

Comparison across cases. Next for each r, we compare the optimum across cases. Again it is easiest to treat 3 ranges separately

1. $\mathbf{0 < r < \frac{2}{15}}$:

   Here we compare the maximum across cases 1, 2, and 4. Case 1 shows that when $0 < r < \frac{2}{15}$, maximum is attained at $\phi = \frac{2}{5}$. Case 2 shows that $\phi = \frac{2}{5}$ is also optimal in that range. Thus within cases 1 and 2, $\phi = \frac{2}{5}$ is optimal.

   Case 4 argues that if $r < \frac{2}{15}$, maximum bank profit is attained at $\phi = \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$. However, case 2 shows that $W_i(\frac{2}{5}) \geq W_i\left(\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)\right)$. Thus the maximum is attained at $\phi = \frac{2}{5}$.

2. $\frac{\mathbf{2}}{\mathbf{15}} < \mathbf{r} < \frac{\mathbf{1}}{\mathbf{4}}$:

Here we compare the maximum across cases 1, 3, and 4. Case 1 shows that when $\frac{2}{15} < r < \frac{1}{4}$, maximum is attained at $\phi = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$, which also maximizes bank profit over the region covered by case 3. The latter implies $W_i\left(\frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\right) \geq W_i(\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$. Since $\phi = \frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$ maximizes bank profit over the region covered by Case 4, the maximum is attained at $\phi = \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)$.

3. $\frac{\mathbf{1}}{\mathbf{4}} < \mathbf{r} < \frac{\mathbf{1}}{\mathbf{2}}$:

Here we compare the maximum across cases 3 and 4. Case 3 shows that $W_i\left(\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)\right) \geq W_i\left(1 - \frac{1}{4r}\right) = 0$. Comparing with case 4 in this region, and using continuity of the bank profit function at the boundary $\frac{1}{6}\left(1 - r + \sqrt{r(r+10)+1}\right)$ yields

$$\phi^* = \begin{cases} \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right) & \text{if } \frac{1}{4} < r < r_z \\ \phi^*_{FOC} & \text{if } r_z < r < r_h \\ 1 & \text{if } r_h < r < \frac{1}{2} \end{cases}$$

Putting all the regions together leads the final result.

$$\phi^* = \begin{cases} \frac{2}{5} & \text{if } 0 < r < \frac{2}{15} \\ \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right) & \text{if } \frac{2}{15} < r < \frac{1}{4} \\ \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right) & \text{if } \frac{1}{4} < r < r_z \\ \phi^*_{FOC} & \text{if } r_z < r < r_h \\ 1 & \text{if } r_h < r < \frac{1}{2} \end{cases} \tag{A.1}$$

where ϕ^*_{FOC} is the solution to $\frac{dW_i^{\text{case 4}}(\phi,r)}{d\phi} = 0$ with $\frac{d^2W_i^{\text{case 4}}(\phi,r)}{d\phi^2}|_{\phi^*_{FOC}} < 0$. r_z is the value of $r \in (\frac{1}{4}, \frac{1}{2})$ such that $\phi^*_{FOC}(r) = \frac{1}{6}\left(1 - r + \sqrt{r^2 + 10r + 1}\right)$, $r_z \approx 0.324$ and $r_h > r_z$ is the value of $r \in (\frac{1}{4}, \frac{1}{2})$ such that $W_i^{\text{case 4}}(\phi^*_{FOC}, r) = \frac{(1-r)^2}{8} = W_i(1)$, $r_h \approx 0.477$.

Proof of Proposition 2

We need to calculate the optimal ϕ for the social planner who faces the same friction as the bank facing the investors. That is, ϕ that maximizes

$$W^S = 2\times \int_{R^*}^{1} \left[\phi R_i \left(1 - \max\left\{ \frac{D^* - \phi R_i}{1-\phi}, 0 \right\} \right) + (1-\phi) \int_{\max\left\{ \frac{D^* - \phi R_i}{1-\phi}, 0 \right\}}^{1} R_j dR_j \right] dR_i + rR^*$$

As the optimal face value of debt changes depending on ϕ and r, we need to consider various cases, as defined in lemma A.1. Moreover, let

$$r_l = \frac{1}{3}\left(2\sqrt{\frac{7}{15}} - 1 \right), \tag{A.2}$$

and note that $r_l < \frac{2}{15}$. We will use r_l later to characterize the optimal ϕ^S.

Case 1: $0 \leq \phi \leq \min\{\frac{2}{5}, \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right)\}$.

Here $D^* = \frac{2-\phi}{4}$ and $R^* = 0$. Substitute in the planner objective function to get

$$W^S = \frac{13\phi^2 + 36(1-\phi)}{96(1-\phi)}$$

Since $R^* = 0$, the optimal face value and total surplus are independent of r. Observe that $\frac{dW^S}{d\phi} > 0$ and $\frac{d^2W^S}{d\phi^2} > 0$, thus the total surplus is increasing and convex in this region. It follows that if the socially optimal level of opacity is in this region we will have

$$\phi^S = \min\left\{ \frac{2}{5}, \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right) \right\},$$

or

$$\phi^S = \begin{cases} \frac{2}{5} & \text{if } 0 < r < \frac{2}{15} \\ \frac{2}{3}\left(2(1-2r) - \sqrt{(4r)^2 - 4r + 1}\right) & \text{if } \frac{2}{15} < r < \frac{1}{4} \end{cases}$$

Case 2: $\frac{2}{5} \leq \phi \leq \frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)$.

Here $D^* = \sqrt{\frac{2}{3}\phi(1=\phi)}$ and $R^* = 0$. Substitute in the planner objective function to get

$$W^S = \frac{1}{54}\left(27 - 4\sqrt{6(1-\phi)\phi}\right).$$

Since $R^* = 0$, again the optimal face value and total surplus are independent of r. Observe that $\frac{dW^S}{d\phi} = 0$ at $\phi = \frac{1}{2}$ and $\frac{d^2W^S}{d\phi^2} > 0$, thus the objective function is convex, and the maximum is attained on one of the corners, i.e. $\phi^S = \frac{2}{5}$ or $\phi^S = \frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)$. Direct comparison of the planner objective on the two boundaries reveals that $W^S(\frac{2}{5}) \geq W^S\left(\frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right)\right)$ for $0 < r < \frac{2}{15}$. It follows that if the socially optimal level of opacity is in this region we will have

$$\phi^S = \frac{2}{5}.$$

Case 3: $\max\{\frac{2}{3}\left(2(1-2r)-\sqrt{(4r)^2-4r+1}\right), 1-\frac{1}{4r}\} < \phi \leq \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$.

Here, $D^* = \frac{1}{6}\left(1+\sqrt{1+12r(1-\phi)}\right)$ and $R^* = \frac{1}{3\phi}\left(-4+3\phi+\sqrt{1+12r(1-\phi)}\right)$, $0 < R^* < 1$. Substitute in the planner objective function to get

$$W^S = \frac{6r(27\phi-12)(1-\phi) - (17-30r(1-\phi))\sqrt{12r(1-\phi)+1} + 37}{162(1-\phi)\phi}.$$

It is more convenient to consider $r < \frac{1}{4}$ and $r > \frac{1}{4}$ separately. When $\frac{2}{15} < r < \frac{1}{4}$, $\frac{d^2W^S}{d\phi^2} > 0$. Since the planner objective is convex, maximum is attained at one of the two boundaries. Direct comparison reveals that $W^S\left(\frac{2}{3}\left(2(1-2r)-\sqrt{(4r)^2-4r+1}\right)\right) < W^S\left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\right)$. On the other hand, when $\frac{1}{4} < r < \frac{1}{2}$, $\frac{dW^S}{d\phi} > 0$. Since the planner objective is increasing in ϕ, maximum welfare is attained at maximum relevant ϕ, i.e. $\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$.

Thus if socially optimal opacity is in this region, we have

$$\phi^S = \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$$

Case 4: $\max\{\frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right), \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\} < \phi \leq 1$.

Here, D^* is the largest root of equation

$$-4D^3 + D^2(2r + \phi + 1) + r^2(\phi - 1) = 0,$$

and $R^* = \frac{1}{\phi}\left(D^* - (1-\phi)(1 - \frac{r}{D^*})\right)$, $0 < R^* < 1$. The total welfare is given by

$$W^S = \frac{-3D^{*5} + 6D^{*4}r + D^{*3}\left(1 - 6r(1-\phi) + \phi(1+\phi)\right) + 3D^{*2}r^2(1-\phi) - r^3(1-\phi)^2}{6D^{*3}\phi}.$$

We consider two cases separately, when $r < \frac{2}{15}$ and when $r > \frac{2}{15}$

- $\mathbf{r < \frac{2}{15}}$:

  In this region, $\frac{d^2W^S}{d\phi^2} > 0$, $\forall\ \phi > \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$, thus the objective function is convex and the maximum is attained at one of the two boundaries. The upper boundary is $\phi = 1$ and the lower boundary is $\phi = \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$. The total surplus at the two boundaries is given by

  $$W^S\left(\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)\right) = \frac{1}{2} - \frac{2}{45}\sqrt{2\left(\sqrt{1 - 2r(3r+2)} + 1\right) + r\left(3r + 6\sqrt{1 - 2r(3r+2)} + 2\right)},$$

  $$W^S(1) = 1/8(3 + r(2 + 3r)).$$

  Where the first expression uses continuity of social planner objective function on the boundary $\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$, and case 2 above. Direct comparison of the two expressions reveal that there exists a threshold r_1, $r_1 < r_l < \frac{2}{15}$, such that the two curves cross only once at r_1, and for $r < \frac{2}{15}$, $W^S\left(\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)\right) > W^S(1)$ iff $r < r_1$. Thus in this range

  $$\phi^S = \begin{cases} \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right) & \text{if } 0 < r < r_1 \\ 1 & \text{if } r_1 < r < \frac{2}{15} \end{cases}$$

- $\mathbf{r > \frac{2}{15}}$:

In this region, there exists two thresholds, r_2 and r_3, such that: (1) $\frac{d^2W^S}{d\phi^2} > 0$, $\forall\ \phi > \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$ iff $\frac{2}{15} < r < r_2$, (2) $\frac{dW^S}{d\phi} > 0$, $\forall\ \phi > \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$ iff $r_3 < r < 1$, and (3) $r_3 < r_2$. It follows that for any r, $\frac{2}{15} < r < \frac{1}{2}$, either $\frac{d^2W^S}{d\phi^2} > 0$ or $\frac{dW^S}{d\phi} > 0\ \forall\ \phi > \frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$ (or both). Thus the maximum of the objective function is attained at one of the two boundaries.The total surplus at the two boundaries is given by

$$W^S\left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\right) = \frac{2\left(37-\frac{3}{2}r\left(5+r-\sqrt{r(r+10)+1}\right)\left(5+3r-3\sqrt{r(r+10)+1}\right)\right)}{9\left(5+r-\sqrt{r(r+10)+1}\right)\left(1-r+\sqrt{r(r+10)+1}\right)} + \frac{2\sqrt{2r\left(r-\sqrt{r(r+10)+1}+5\right)+1}\left(5r\left(r-\sqrt{r(r+10)+1}+5\right)-17\right)}{9\left(5+r-\sqrt{r(r+10)+1}\right)\left(1-r+\sqrt{r(r+10)+1}\right)},$$

$W^S(1) = 1/8(3+r(2+3r))$.

Where the first line uses continuity of social planner objective function on the boundary $\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)$, and case 3 above. Direct comparison of the two expressions reveal that $W^S(1) > W^S\left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\right)$ if $r > \frac{2}{15}$. Thus in this range

$$\phi^S = 1$$

Putting the two cases together,

$$\phi^S = \begin{cases} \frac{3}{10}\left(1-2r+\sqrt{1-4r-6r^2}\right) & \text{if } 0 < r < r_1 \\ 1 & \text{if } r_1 < r < \frac{1}{2} \end{cases}$$

Case 5: $0 \leq \phi < 1 - \frac{1}{4r}$.
Here $R^* = 1$, thus $W^S = r$.

Comparison across cases. Next for each r, we compare the optimum across cases. Again it is easiest to treat 3 ranges separately

1. $\mathbf{0 < r < \frac{2}{15}}$:

   Here we compare the maximum across cases 1, 2, and 4. Case 1 shows that when $0 < r < \frac{2}{15}$, maximum is attained at $\phi = \frac{2}{5}$. Case 2 shows that $\phi = \frac{2}{5}$ is also optimal in that range. Thus within cases 1 and 2, $\phi = \frac{2}{5}$ is optimal.

   In comparing with case 4, we consider two ranges separately, $r < r_1$ and $r > r_1$.

   If $r < r_1$, in case 4 $\phi = \frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)$ is optimal . Using case 2, $W^S(\frac{2}{5}) > W^S\left(\frac{3}{10}\left(1 - 2r + \sqrt{1 - 4r - 6r^2}\right)\right)$, thus $\phi^S = \frac{2}{5}$ in this range.

   If $r > r_1$, within the relevant range of case 4, $\phi = 1$ is optimal. We need to compare the welfare with $\phi = \frac{2}{5}$

$$W^S(\frac{2}{5}) = 37/90,$$
$$W^S(1) = \frac{1}{8}(r(3r + 2) + 3)$$

   Direct comparison shows that $W^S(\frac{2}{5}) > W^S(1)$ iff $r < r_l = \frac{1}{3}\left(2\sqrt{\frac{7}{15}} - 1\right)$. Thus in this range

$$\phi^S = \begin{cases} \frac{2}{5} & \text{if } 0 < r < r_l \\ 1 & \text{if } r_l < r < \frac{2}{15} \end{cases}$$

2. $\mathbf{\frac{2}{15} < r < \frac{1}{4}}$:

   Here we compare the maximum across cases 1, 3, and 4. Case 1 shows that when $\frac{2}{15} < r < \frac{1}{4}$, maximum is attained at $\phi = \frac{2}{3}\left(2(1 - 2r) - \sqrt{(4r)^2 - 4r + 1}\right)$. Case 3 shows that $W^S\left(\frac{1}{6}\left(1 - r + \sqrt{r(r + 10) + 1}\right)\right) \geq W^S\left(\frac{2}{3}\left(2(1 - 2r) - \sqrt{(4r)^2 - 4r + 1}\right)\right)$, and case 4 shows that $W^S(1) > W^S\left(\frac{1}{6}\left(1 - r + \sqrt{r(r + 10) + 1}\right)\right)$ in this region. Thus the maximum is attained at $\phi = 1$.

3. $\mathbf{\frac{1}{4} < r < \frac{1}{2}}$:

Here we compare the maximum across cases 3 and 4. Case 3 shows that $W^S\left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\right) \geq W^S\left(1-\frac{1}{4r}\right) = r$, and case 4 shows that $W^S(1) > W^S\left(\frac{1}{6}\left(1-r+\sqrt{r(r+10)+1}\right)\right)$ in this region. Thus the maximum is attained at $\phi = 1$.

Putting all the regions together leads the final result.

$$\phi^S = \begin{cases} \frac{2}{5} & \text{if } 0 < r < r_l \\ 1 & \text{if } r_l < r < \frac{1}{2} \end{cases}$$

where $r_l = \frac{1}{3}\left(2\sqrt{\frac{7}{15}} - 1\right) < \frac{2}{15}$

Proof of Proposition 3

Proposition 1 shows that for $r < r_h$, $\phi^* < 1$, while 2 shows that for $r > r_l$, $\phi = 1$. It follows that for $r \in (r_l, r_h)$, $\phi^* < \phi^S$, thus bank portfolios are inefficiently opaque.

Next, note that since for $r > r_l$, $\phi^S = 1$, the socially optimal R^* and D^* coincide,

$$R^*(1) = \max\{\min\{1, D^*(1)\}, 0\} = D^*(1) = \frac{1+r}{2},$$

as shown in the main text. Thus $R^*(1)$ is monotonically increasing in r. On the other hand, given the equilibrium ϕ^* in equation (A.1), $R^*(\phi^*)$ is given by

$$R^*(\phi^*) = \begin{cases} 0 & \text{if } r < \frac{1}{4} \\ \frac{-r+\sqrt{r(r+10)+1}+4\sqrt{2r\left(r-\sqrt{r(r+10)+1}+5\right)+1}-7}{-r+\sqrt{r(r+10)+1}+1} & \text{if } \frac{1}{4} < r \leq r_z \\ \frac{D^*(\phi^*_{FOC})}{\phi^*_{FOC}} + \frac{1-\phi^*_{FOC}}{\phi^*_{FOC}}\left(1 - \frac{r}{D^*(\phi^*_{FOC})}\right) & \text{if } r_z < r < r_h \\ \frac{1+r}{2} & \text{if } r_h < r < \frac{1}{2} \end{cases}$$

First, note that $r_l = \frac{1}{3}\left(2\sqrt{\frac{7}{15}} - 1\right) < \frac{1}{4}$, thus at r_l, $R^*(\phi^*(r_l)) < R^*(1) = \frac{1+r_l}{2}$. Second, evaluate R^* at r_z as defined in proposition 1 to get $R^*(\phi^*(r_z)) < R^*(1; r_z) = \frac{1+r_z}{2}$. Thus $R^*(\phi^*) \leq R^*(1)$ for $r \in (r_l, r_z)$. Third, evaluate R^* at r_h as defined in proposition 1 to get $R^*(\phi^*(r_h)) > R^*(1; r_h) = \frac{1+r_h}{2}$.

Since $R^*(1;r) = \frac{1+r}{2}$ it is continuous and increasing. Since ϕ^* is continuous for $r > \frac{1}{4}$, $R^*(\phi^*)$ is continuous for $r > \frac{1}{4}$ as well, including at r_z. Moreover, $\frac{dR^*(\phi^*(r))}{dr} = R^{*'}\left(\phi^*_{FOC}(r)\right)\phi^{*'}_{FOC}(r) > 0$. Along with $R^*(\phi^*(r_z)) < R^*(1;r_z)$ and $R^*(\phi^*(r_h)) > R^*(1;r_h)$, it follows that $R^*(\phi*)$ and $R^*(1)$ cross once in the interval (r_z, r_h), at $r = r_m$, such that $R^*(\phi^*) \leq R^*(1)$ for $r \leq r_m$ and $R^*(\phi^*) > R^*(1)$ for $r > r_m$. r_m is defined by $R^*(\phi^*(r_m)) = R^*(1;r_m)$, $r_m \approx 0.3287$.

A.2 Full Information Derivations

In this section we first introduce some useful notation and concepts, as well as a lemma with intermediate results. Then we proceed to the proofs.

Following the main text, with full information, the portfolio of a bank holding ϕ in her project and $(1-\phi)$ in the other bank's project has the following return cumulative distribution

$$H(z;\phi) = \begin{cases} \frac{z^2}{2\phi(1-\phi)} & \text{if } z < \phi \\ \frac{1}{1-\phi}(z - \frac{\phi}{2}) & \text{if } \phi \leq z \leq 1-\phi \\ 1 - \frac{(1-z)^2}{2\phi(1-\phi)} & \text{if } z > 1-\phi \end{cases} .$$

In this case, each investor I's expected payoff is given by

$$W_I^{FI} = D(1 - H(D,\phi)) + rH(D,\phi),$$

each bank i's expected payoff is given by

$$W_i^{FI} = \int_D^1 \left[(z-D)\frac{\partial H(z,\phi)}{\partial z}\right] dz,$$

while total welfare is given by

$$W^{S/FI} = \int_D^1 \left[z\frac{\partial H(z,\phi)}{\partial z}\right] dz + rH(D,\phi).$$

Note that $H(z;\phi)$ distribution is symmetric around $\phi = \frac{1}{2}$, so we only need to consider the case $0 \leq \phi \leq \frac{1}{2}$. The case $\frac{1}{2} \leq \phi \leq 1$ follows by symmetry. The following

lemma characterizes the probability, $R^{*/FI} = H(D, \phi)$, that an investor liquidates the bank at date 1, as well as the face value of debt, $D^{*/FI}$, as a function of the investor's reservation value, r, and of the fraction, ϕ, that the bank retains of her own project.

Lemma A.2 *For any early redemption value $r \in \left(0, \frac{1}{2}\right)$ and any portfolio allocation $(\phi, (1-\phi))$ that banks hold, the probability, $R^{*/FI}$, that an investor liquidates the bank at date 1, as well as the optimal face value of debt, $D^{*/FI}$, under full information are as follows:*

1. $r < \frac{1}{4}$.

   (a) $\phi \leq \frac{2r+2}{5}$, *then* $D^{*/FI} = \frac{1}{2}(r+1) - \frac{1}{4}\phi \in [\phi, (1-\phi)]$ *and* $R^{*/FI} = \frac{1}{1-\phi}\left(\frac{1}{2}(r+1) - \frac{3}{4}\phi\right)$.

   (b) $\phi > \frac{2r+2}{5}$, *then* $D^{*/FI} = \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} \in (r, \phi)$ *and* $R^{*/FI} = \frac{\left(\frac{1}{3}r + \frac{1}{3}\sqrt{r^2-6\phi^2+6\phi}\right)^2}{2\phi(1-\phi)}$.

2. $r \geq \frac{1}{4}$.

   (a) $\phi \leq \frac{2}{3}(1-r)$, *then* $D^{*/FI} = \frac{1}{2}(r+1) - \frac{1}{4}\phi \in [\phi, (1-\phi)]$ *and* $R^{*/FI} = \frac{1}{1-\phi}\left(\frac{1}{2}(r+1) - \frac{3}{4}\phi\right)$.

   (b) $\phi > \frac{2}{3}(1-r)$, *then* $D^{*/FI} = \frac{2}{3}r + \frac{1}{3} \in ((1-\phi), 1)$ *and* $R^{*/FI} = \left(1 - \frac{\left(\frac{2}{3} - \frac{2}{3}r\right)^2}{2\phi(1-\phi)}\right)$.

Proof. Since $\phi \leq \frac{1}{2}$, then $\phi \leq 1 - \phi$. Thus there are three relevant cases.

Case 1: $D < \phi$. The expected payoff of each investor is given by

$$W_I^{FI} = D\left(1 - \frac{D^2}{2\phi(1-\phi)}\right) + r\frac{D^2}{2\phi(1-\phi)}.$$

The first order condition with respect to D yields

$$\frac{1}{2\phi(\phi-1)}\left(3D^2 - 2rD - 2\phi(1-\phi)\right) = 0,$$

which has the following solutions

$$\begin{aligned} D_{1a} &= \frac{1}{3}r - \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} < 0 \\ D_{1b} &= \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} > 0 \end{aligned}$$

For D to be a feasible solutions, it must verify that $r < D < \phi$. We have that

$$\phi > \frac{2r+2}{5} \Rightarrow D_{1b} < \phi.$$

At the same time, $D_{1b} < \phi$ is sufficient to imply that $D_{1b} > r$, as

$$\phi > \frac{2r+2}{5} \Rightarrow \phi > \frac{1}{2} - \frac{1}{2}\sqrt{1-2r^2} \Rightarrow D > r$$

Case 2: $D > (1-\phi)$. The expected payoff of each investor is given by

$$W_I^{FI} = D\left(1 - \left(1 - \frac{(1-D)^2}{2\phi(1-\phi)}\right)\right) + r\left(1 - \frac{(1-D)^2}{2\phi(1-\phi)}\right).$$

The first order condition with respect to D yields

$$\frac{1}{2\phi\left(\phi-1\right)}\left(D-1\right)\left(2r-3D+1\right) = 0,$$

which admits the following solution

$$D_2 = \frac{2}{3}r + \frac{1}{3}.$$

For D to be a feasible solutions, it is sufficient that it verify that $D > (1-\phi)$, as $(1-\phi) > r$. We have that

$$D_2 > (1-\phi) \Longleftrightarrow \phi > \frac{2}{3}\left(1-r\right)$$

Case 3: $D \in [\phi, (1-\phi)]$. The expected payoff of each investor is given by

$$W_I^{FI} = D\left(1 - \frac{1}{1-\phi}\left(D - \frac{\phi}{2}\right)\right) + r\frac{1}{1-\phi}\left(D - \frac{\phi}{2}\right).$$

The first order condition with respect to D yields

$$\frac{1}{2(\phi-1)}(\phi - 2r + 4D - 2) = 0,$$

which admits the following solution

$$D_3 = \frac{1}{2}(r+1) - \frac{1}{4}\phi.$$

For D to be a feasible solutions, it is sufficient that it verify that $D_3 \leq (1-\phi)$, $D_3 \geq \phi$, and $D_3 > r$. We have that

$$D_3 \geq \phi \Leftrightarrow \frac{2}{5}(r+1) \geq \phi$$

and

$$D_3 \leq 1-\phi \Leftrightarrow \frac{2}{3}(1-r) \geq \phi.$$

Note that since $2(1-r) > \phi$, it follows that $D_3 > r$ as well.

■

Proof of Proposition 4

We need to calculate the optimal ϕ for a bank i. That is, ϕ that maximizes

$$W_i^{FI} = \int_D^1 \left[(z-D)\frac{\partial H(z,\phi)}{\partial z}\right] dz.$$

As the optimal face value of debt changes depending on ϕ and r, we need to consider various cases.

Case 1: $r < \frac{1}{4}$ & $\phi \leq \frac{2r+2}{5}$, and $r \geq \frac{1}{4}$ & $\phi \leq \frac{2}{3}(1-r)$. From Lemma A.2, we have that $D^{*/FI} = \frac{1}{2}(r+1) - \frac{1}{4}\phi$. Substituting into the objective function of bank i,

we obtain that

$$W_i^{FI} = \int_{D^{*/FI}}^{1-\phi} \left[(z - D^{*/FI}) \frac{\partial}{\partial z} \left(\frac{1}{1-\phi} (z - \frac{\phi}{2}) \right) \right] dz + \int_{1-\phi}^{1} \left[(z - D^{*/FI}) \frac{\partial}{\partial z} \left(1 - \frac{(1-z)^2}{2\phi(1-\phi)} \right) \right] dz$$

or

$$W_i^{FI} = \frac{1}{96\,(1-\phi)} \left(12\,(r-1)^2 + 12\phi\,(r-1) + 7\phi^2 \right)$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial \phi} W_i^{FI} = \frac{1}{96\,(\phi-1)^2} \left(12r^2 - 12r - 7\phi^2 + 14\phi \right).$$

Note that the second order condition with respect to ϕ yields

$$\frac{\partial^2}{\partial \phi^2} W_i^{FI} = \frac{1}{48\,(1-\phi)^3} \left(12r^2 - 12r + 7 \right) > 0$$

which implies that any feasible solution of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes bank i's objective we need to compare the value of W_i^{FI} evaluated at the corners. In particular, for $r < \frac{1}{4}$ we need to compare $W_i^{FI}\left(\frac{2r+2}{5}, D^{*/FI}, r\right)$ and $W_i^{FI}\left(0, D^{*/FI}, r\right)$, while for $r \geq \frac{1}{4}$ we need to compare $W_i^{FI}\left(\frac{2(1-r)}{3}, D^{*/FI}, r\right)$ and $W_i^{FI}\left(0, D^{*/FI}, r\right)$.

Consider first $r < \frac{1}{4}$. Then

$$W_i^{FI}\left(\frac{2r+2}{5}, D^{*/FI}, r\right) - W_i^{FI}\left(0, D^{*/FI}, r\right) = -\frac{1}{240r - 360}\left(30r^3 + 7r^2 - 16r + 7\right) > 0.$$

Thus

$$\phi^{*/FI} = \frac{2r+2}{5}.$$

Consider next $r \geq \frac{1}{4}$. Then

$$W_i^{FI}\left(\frac{2\,(1-r)}{3}, D^{*/FI}, r\right) - W_i^{FI}\left(0, D^{*/FI}, r\right) = -\frac{1}{72\,(2r+1)}\,(18r-7)\,(r-1)^2.$$

Thus, if $r < r_H = \frac{7}{18}$,

$$\phi^{*/FI} = \frac{2\,(1-r)}{3},$$

and, if $r > r_H$,

$$\phi^{*/FI} = 0.$$

Case 2: $r < \frac{1}{4}$ & $\phi > \frac{2r+2}{5}$. From Lemma A.2, we have that $D^{*/FI} = \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi}$. Substituting into the objective function of the social planner, we obtain that

$$\begin{aligned} W_i^{FI} &= \int_{D^{*/FI}}^{\phi} \left[\left(z - D^{*/FI}\right) \frac{\partial}{\partial z} \left(\frac{z^2}{2\phi(1-\phi)} \right) \right] dz + \int_{\phi}^{1-\phi} \left[\left(z - D^{*/FI}\right) \frac{\partial}{\partial z} \left(\frac{1}{1-\phi}(z - \frac{\phi}{2}) \right) \right] dz + \\ &\quad + \int_{1-\phi}^{1} \left[\left(z - D^{*/FI}\right) \frac{\partial}{\partial z} \left(1 - \frac{(1-z)^2}{2\phi(1-\phi)} \right) \right] dz \end{aligned}$$

or

$$W_i^{FI} = \frac{1}{2} - \left(\frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} \right) + \frac{1}{6} \frac{\left(\frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi} \right)^3}{\phi\,(1-\phi)}$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial \phi} W_i^{FI} = \frac{2\phi - 1}{\sqrt{r^2 - 6\phi^2 + 6\phi}} \left(1 + \frac{1}{162\phi^2} \frac{\left(r + \sqrt{r^2 - 6\phi^2 + 6\phi} \right)^2}{(\phi - 1)^2} \left(r\sqrt{r^2 - 6\phi^2 + 6\phi} + 3\phi^2 - 3\phi + r^2 \right) \right)$$

which has a unique solution of $\phi = \frac{1}{2}$. Note however that the second order condition with respect to ϕ evaluated at $\phi = \frac{1}{2}$ yields

$$\begin{aligned} \frac{\partial^2}{\partial \phi^2} W_i^{FI} \Big|_{\phi=\frac{1}{2}} &= \frac{2r^2 + 3}{(r^2 - 6\phi^2 + 6\phi)^{\frac{3}{2}}} \Big|_{\phi=\frac{1}{2}} + r \left(\frac{\partial}{\partial \phi} \frac{(2\phi - 1)}{162\phi^2} \frac{\left(r + \sqrt{r^2 - 6\phi^2 + 6\phi} \right)^2}{(\phi - 1)^2} \right) \Bigg|_{\phi=\frac{1}{2}} \\ &= \frac{2r^2 + 3}{(r^2 - 6\phi^2 + 6\phi)^{\frac{3}{2}}} \Big|_{\phi=\frac{1}{2}} + r \left(\frac{\left(r + \sqrt{r^2 - 6\phi^2 + 6\phi} \right)^2}{(1 - \phi)} \frac{1}{81\phi^3} \right) \Bigg|_{\phi=\frac{1}{2}} > 0 \end{aligned}$$

which implies that the solution $\phi = \frac{1}{2}$ of the first order condition would be a local mini-

mum. Thus, to find the value of ϕ that maximizes bank i's objective we need to compare the value of W_i^{FI} evaluated at the corners. In particular, we need $W_i^{FI}\left(\frac{2(1-r)}{3}, D^{*/FI}, r\right)$ and $W_i^{FI}\left(\frac{1}{2}, D^{*/FI}, r\right)$. We show that

$$W_i^{FI}\left(\frac{2\left(1-r\right)}{3}, D^{*/FI}, r\right) - W_i^{FI}\left(\frac{1}{2}, D^{*/FI}, r\right) > 0.$$

Thus,

$$\phi^{*/FI} = \frac{2\left(1-r\right)}{3},$$

Case 3: $r \geq \frac{1}{4}$ & $\phi > \frac{2}{3}\left(1-r\right)$. From Lemma A.2, we have that $D^{*/FI} = \frac{2}{3}r + \frac{1}{3}$. Substituting into the objective function of of bank i, we obtain that

$$W_i^{FI} = \int_{D^{*/FI}}^{1} \left(\left(z - D^{*/FI}\right) \frac{\partial}{\partial z}\left(1 - \frac{(1-z)^2}{2\phi(1-\phi)}\right)\right) dz$$

or

$$W_i^{FI} = \frac{1}{6\phi\left(\phi-1\right)}\left(\frac{2}{3}r - \frac{2}{3}\right)^3$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial \phi} W_i^{FI} = \frac{4}{81\phi^2\left(\phi-1\right)^2}\left(2\phi-1\right)\left(1-r\right)^3 \leq 0.$$

Note that the second order condition with respect to ϕ yields

$$\frac{\partial^2}{\partial \phi^2} W_i^{FI} = \frac{8}{81\phi^3\left(1-\phi\right)^3}\left(1-r\right)^3\left(3\phi^2 - 3\phi + 1\right) > 0$$

Thus, bank i's expected payoff is always decreasing in this case.

Thus, the derivations in Case 1 and Case 2 imply that for $r < \frac{1}{4}$ bank i's expected payoff is maximized at $\phi^{*/FI} = \min\left\{\frac{2r+2}{5}, \frac{2(1-r)}{3}\right\}$. Similarly, the derivations in Case 1 and Case 3 imply that for $r \geq \frac{1}{4}$ bank i's expected payoff is maximized at $\phi^{*/FI} = \min\left\{\frac{2r+2}{5}, \frac{2(1-r)}{3}\right\}$ if $r \leq r_H$ and at $\phi^{*/FI} = 0$ if $r > r_H$.

Note that for any portfolio allocation $(\phi, 1-\phi)$ that is an equilibrium, the portfolio allocation $(1-\phi, \phi)$ is also an equilibrium. This follows from the symmetry of

the banks' portfolio return distribution in the full information case. We will work with $(1-\phi, \phi)$ to make it comparable with the analysis with incomplete information. Thus, bank i's expected payoff is maximized at $\phi^{*/FI} = \max\left\{\frac{3-2r}{5}, \frac{21+2r}{3}\right\}$ if $r \leq r_H$ and at $\phi^{*/FI} = 1$ if $r > r_H$.

Proof of Proposition 5

We need to calculate the optimal ϕ for the social planner. That is, ϕ that maximizes

$$W^{S/FI} = \int_D^1 \left[z \frac{\partial H(z,\phi)}{\partial z} dz \right] + rH(D,\phi).$$

As the optimal face value of debt changes depending on ϕ and r, we need to consider various cases.

Case 1: $r < \frac{1}{4}$ & $\phi \leq \frac{2r+2}{5}$, and $r \geq \frac{1}{4}$ & $\phi \leq \frac{2}{3}(1-r)$. From Lemma A.2, we have that $D^{*/FI} = \frac{1}{2}(r+1) - \frac{1}{4}\phi$. Substituting into the objective function of the social planner, we obtain that

$$\begin{aligned} W^{S/FI} &= \int_{D^{*/FI}}^{1-\phi} \left[z \frac{\partial}{\partial z} \left(\frac{1}{1-\phi}(z - \frac{\phi}{2}) \right) \right] dz + \int_{1-\phi}^{1} \left[z \frac{\partial}{\partial z} \left(1 - \frac{(1-z)^2}{2\phi(1-\phi)} \right) \right] dz \\ &\quad + r \frac{1}{1-\phi} \left(D^{*/FI} - \frac{\phi}{2} \right) \end{aligned}$$

or

$$W^{S/FI} = \frac{1}{96(1-\phi)} \left(36r^2 - 60r\phi + 24r + 13\phi^2 - 36\phi + 36 \right).$$

Note that the second order condition with respect to ϕ yields

$$\frac{\partial^2}{\partial \phi^2} W^{S/FI} = \frac{1}{48(1-\phi)^3} \left(36r^2 - 36r + 13 \right) > 0,$$

which implies that any feasible solution of the first order condition would be a local minimum. Thus, to find the value of ϕ that maximizes welfare we need to compare the value of $W^{S/FI}$ evaluated at the corners. In particular, for $r < \frac{1}{4}$ we need to compare

$W^{S/FI}\left(\frac{2r+2}{5}, D^{*/FI}, r\right)$ and $W^{S/FI}\left(0, D^{*/FI}, r\right)$, while for $r \geq \frac{1}{4}$ we need to compare $W^{S/FI}\left(\frac{2(1-r)}{3}, D^{*/FI}, r\right)$ and $W^{S/FI}\left(0, D^{*/FI}, r\right)$.

Consider first $r < \frac{1}{4}$. Then

$$W^{S/FI}\left(\frac{2r+2}{5}, D^{*/FI}, r\right) - W^{S/FI}\left(0, D^{*/FI}, r\right) = -\frac{1}{240r - 360}\left(90r^3 + 13r^2 - 64r + 13\right)$$

Thus, if $r < r_L \simeq 0.23$,

$$\phi^{S/FI} = \left(\frac{2r+2}{5}\right),$$

and, if $r > r_L$,

$$\phi^{S/FI} = 0.$$

Consider next $r \geq \frac{1}{4}$

$$W^{S/FI}\left(\frac{2\left(1-r\right)}{3}, D^{*/FI}, r\right) - W^{S/FI}\left(0, D^{*/FI}, r\right) = -\frac{1}{72\left(2r+1\right)}\left(54r - 13\right)\left(r-1\right)^2 < 0$$

Thus

$$\phi^{S/FI} = 0.$$

Case 2: $r < \frac{1}{4}$ & $\phi > \frac{2r+2}{5}$. From Lemma A.2, we have that $D^{*/FI} = \frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi}$. Substituting into the objective function of the social planner, we obtain that

$$\begin{aligned} W^{S/FI} \;=\; & \int_{D^{*/FI}}^{\phi}\left[z\frac{\partial}{\partial z}\left(\frac{z^2}{2\phi(1-\phi)}\right)\right]dz + \int_{\phi}^{1-\phi}\left[z\frac{\partial}{\partial z}\left(\frac{1}{1-\phi}(z-\frac{\phi}{2})\right)\right]dz + \\ & + \int_{1-\phi}^{1}\left[z\frac{\partial}{\partial z}\left(1 - \frac{(1-z)^2}{2\phi(1-\phi)}\right)\right]dz + r\frac{D^{*/FI2}}{2\phi(1-\phi)} \end{aligned}$$

or

$$W^{S/FI} = \frac{1}{6\phi\left(1-\phi\right)}\left(-3\phi^2 + 3\phi - 2\left(\frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi}\right)^3 + 3r\left(\frac{1}{3}r + \frac{1}{3}\sqrt{r^2 - 6\phi^2 + 6\phi}\right)^2\right)$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial\phi}W^{S/FI}=\frac{(2\phi-1)}{162\phi^2}\frac{\left(r+\sqrt{r^2-6\phi^2+6\phi}\right)}{(\phi-1)^2\sqrt{r^2-6\phi^2+6\phi}}\left(\left(6\phi-6\phi^2+5r^2\right)\underbrace{\left(\sqrt{6\phi-6\phi^2+r^2}-r\right)}_{>0}+10r^3\right)\leq 0.$$

Note that the second order condition with respect to ϕ yields

$$\begin{aligned}\frac{\partial^2}{\partial\phi^2}W^{S/FI}=&\underbrace{\frac{1}{27\phi^2(\phi-1)^2\left(r^2+6\phi(1-\phi)\right)^{\frac{3}{2}}}}_{>0}\times\\&\underbrace{\left(\underbrace{\left(10\phi^2-10\phi+5\right)}_{>0}r^4+\underbrace{\left(216\phi^3-108\phi^4-153\phi^2+45\phi\right)}_{>0}r^2+\underbrace{\left(18\phi^4-36\phi^3+18\phi^2\right)}_{>0}\right)}_{>0}\\&+\underbrace{\frac{3(2\phi-1)^2 10r^4}{162\phi^2(\phi-1)^2}\frac{1}{\left(r^2-6\phi^2+6\phi\right)^{\frac{3}{2}}}+\frac{10}{81}\frac{r^4}{\phi^3(1-\phi)^3}\left(3\phi^2-3\phi+1\right)\frac{1}{\sqrt{r^2-6\phi^2+6\phi}}}_{>0}\\&+\underbrace{\frac{10}{81}\frac{r^3}{\phi^3(1-\phi)^3}\left(3\phi^2-3\phi+1\right)}_{>0}\end{aligned}$$

which implies that any feasible solution of the first order condition would be a local minimum. Thus, the total welfare is always decreasing in this case.

Case 3: $r\geq\frac{1}{4}$ & $\phi>\frac{2}{3}(1-r)$. From Lemma A.2, we have that $D^{*/FI}=\frac{2}{3}r+\frac{1}{3}$. Substituting into the objective function of the social planner, we obtain that

$$W^{S/FI}=\int_{D^{*/FI}}^{1}\left(z\frac{\partial}{\partial z}\left(1-\frac{(1-z)^2}{2\phi(1-\phi)}\right)\right)dz+r\left(1-\frac{(1-D^{*/FI})^2}{2\phi(1-\phi)}\right)$$

or

$$W^{S/FI}=\frac{1}{81\phi(\phi-1)}\left(10r^3-30r^2+81r\phi^2-81r\phi+30r-10\right).$$

Taking the first order condition with respect to ϕ yields

$$\frac{\partial}{\partial\phi}W^{S/FI}=\frac{10}{81\phi^2(\phi-1)^2}(1-2\phi)(r-1)^3\leq 0.$$

Note that the second order condition with respect to ϕ yields

$$\frac{\partial^2}{\partial\phi^2}W^{S/FI} = \frac{20}{81\phi^3\left(1-\phi\right)^3}\left(1-r\right)^3\left(3\phi^2-3\phi+1\right) \geq 0$$

Thus, the total welfare is always decreasing in this case.

Thus, the derivations in Case 1 and Case 2 imply that for $r < \frac{1}{4}$ social welfare is maximized at $\phi^{S/FI} = \frac{2r+2}{5}$ if $r \leq r_L$ and at $\phi^{S/FI} = 0$ if $r > r_L$. Similarly, the derivations in Case 1 and Case 3 imply that for $r \geq \frac{1}{4}$ social welfare is maximized at $\phi^{S/FI} = 0$.

Note that for any portfolio allocation $(\phi, 1-\phi)$ that is an optimum, the portfolio allocation $(1-\phi, \phi)$ is also an optimum. This follows from the symmetry of the banks' portfolio return distribution in the full information case. We will work with $(1-\phi, \phi)$ to make it comparable with the analysis with incomplete information. Thus,social welfare is maximized at $\phi^{*/FI} = \frac{3-2r}{5}$ if $r \leq r_L$ and at $\phi^{*/FI} = 1$ if $r > r_L$.

Proof of Proposition 6

The first of the part of the proposition simply follows as a corollary of Proposition 4 and of Proposition 5.

For the second part of the proposition, we need to show that for any $r \in (r_L, r_H)$, then $R^{*/FI}\left(\phi^{*/FI}\right) \leq R^{*/FI}\left(1\right)$.

Since $r < r_H$, we know from Proposition 4 that $\phi^{*/FI} = \min\left\{\frac{2r+2}{5}, \frac{2(1-r)}{3}\right\}$, and from Lemma A.2 we have that

$$R^{*/FI}\left(\phi^{*/FI}\right) = \begin{cases} \frac{1}{1-\frac{2r+2}{5}}\left(\frac{1}{2}\left(r+1\right) - \frac{3}{4}\times\frac{2r+2}{5}\right) & \text{if } r \in (r_L, \frac{1}{4}] \\ \frac{1}{1-\frac{2(1-r)}{3}}\left(\frac{1}{2}(r+1) - \frac{3}{4}\times\frac{2(1-r)}{3}\right) & \text{if } r \in \left(\frac{1}{4}, r_H\right) \end{cases}$$

Because of symmetry we also have that

$$R^{*/FI}\left(\phi^{*/FI}=1\right) = R^{*/FI}\left(\phi^{*/FI}=0\right)$$

or

$$R^{*/FI}(1) = \frac{1}{1-0}\left(\frac{1}{2}(r+1) - \frac{3}{4}\times 0\right) = \frac{1}{2}(r+1).$$

It is straightforward to show that

$$R^{*/FI}\left(\phi^{*/FI}\right) - R^{*/FI}(1) < 0.$$

Indeed

$$R^{*/FI}\left(\phi^{*/FI}\right) - R^{*/FI}(1) = \begin{cases} -\frac{1}{4r-6}\left(2r^2+r-1\right) < 0 & \text{if } r \in (r_L, \frac{1}{4}] \\ -\frac{1}{4r+2}\left(2r^2-3r+1\right) < 0 & \text{if } r \in \left(\frac{1}{4}, r_H\right) \end{cases}$$