NBER WORKING PAPER SERIES

# THE SPECIALNESS OF ZERO

Joshua S. Gans

Working Paper 26485
http://www.nber.org/papers/w26485

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2019

Thanks to Kevin Bryan, Len Goff, Shannon Liu, Barry Nalebuff, Fiona Scott Morton, Hal Varian and Glen Weyl for helpful discussions. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

The Specialness of Zero
Joshua S. Gans
NBER Working Paper No. 26485
November 2019
JEL No. L11,L41

## ABSTRACT

A model is provided whereby a monopolist firm chooses to price its product at zero. This outcome is shown to be driven by the assumption of 'free disposal' alongside selection markets (where prices impact on a firm's costs). Free disposal creates a mass point of consumers whose utility from the product is zero. When costs are negative, the paper shows that a zero price equilibrium can emerge. The paper shows that this outcome can be socially optimal and that, while a move from monopoly to competition can result in a negative price equilibrium, this can be welfare reducing. The conclusion is that zero can be a 'special zone' with respect to policy analysis such as in antitrust.

Joshua S. Gans
Rotman School of Management
University of Toronto
105 St. George Street
Toronto ON M5S 3E6
CANADA
and NBER
joshua.gans@gmail.com

# 1 Introduction

There appear to be an increasing number of instances where products are supplied to consumers at a zero price. For instance, in 2019, Charles Schwab and eTrade announced that commissions on stock trades would be eliminated.[1] This has also been common for advertising-based media models. This was the case with broadcast television over the past half century (3) and has been extended to Internet services such as search and social media. As these new services are now provided by firms with a large share of the market, antitrust authorities globally have started to pay attention to them. However, a common retort is that, as services are supplied to consumers for free, 'normal' antitrust concerns do not apply. Scholars have pointed out that the premise of this argument is flawed as there is nothing necessarily special about a zero price.

> Digital platforms are characterized by free services. "Free" is not a special zone where economics or antitrust do not apply. Rather, a free good is one where the seller has chosen to set a monetary price of zero and may set other, non-monetary, conditions or duties. It is possible that a digital market has an equilibrium price that is negative; in other words, because of the value of target advertising, the consumer's data is so valuable that the platform would pay for it. (16)

There is support in the literature for this view. Since the outset, the literature on platform competition has demonstrated that negative prices can emerge (5; 9).[2] Such payments raise participation on one-side of the platform allowing firms to earn more from the other (paying) side of the platform. From this perspective, the prevalence of a zero price in practice represents somewhat of a puzzle.

This has not prevented the theoretical literature from imposing a non-negative pricing constraint as an assumption based on that practice and then analysing situations where zero pricing is a corner solution. The reasons given, however, are informal. In some cases,

[1] The business model now involves earning revenue entirely from interest on account cash balances. For more on these types of business models see Anderson (2)

[2] In some cases, regulators have actually been concerned about negative prices (14). In addition, what appears to be zero pricing emerges in a number of settings; in particular, where products are sold as part of bundles of other goods.

justifications are based on transactions costs or that consumers are, in fact, paying a price in the form of a reduction in privacy (as their data is appropriated by the firm). However, the most common justifications allude to adverse selection and opportunistic behaviours (see (13; 7; 10)).

That being said, there appears to be no paper that provides a formal model generating a zero price as an equilibrium price for any of these reasons.[3] As a consequence, there has been no formal analysis of whether more competition in markets where zero prices exist would actually lead to improved social welfare outcomes once the conditions under which those prices emerge have been fully specified. The goal of this paper is to provide such a model and analysis. In so doing, it is demonstrated that the underlying conditions that generate zero pricing are relevant to welfare analysis and simple intuition based on unconstrained price theory may not apply.

A zero price is unlikely to emerge in equilibrium without some sort of anchor. The anchor analysed here focuses on the mass of consumers in many markets who derive no utility from consumption of a product. Usually, in industrial organisation research, a demand curve is given and, technically, passes through zero without any change in its properties (such as continuity or differentiability). What happens to demand with a zero price or below does not normally arise in equilibrium where costs are non-negative. Here, however, I draw from the standard assumption in the rest of microeconomics that there is free disposal.[4] Free disposal places a lower bound on people's utility from a product at 0. At that point, anyone in the economy is a potential consumer so the (inverse) market demand curve has a kink at zero. The paper demonstrates that it is this assumption that permits zero price as a potential equilibrium outcome even though this kink at 0 does not rule out a negative price – either as an equilibrium or a socially desirable outcome – if costs are negative.

The free disposal assumption is then paired with the literature on selection markets (no-

[3]I have searched, enquired of many and have yet to find a model in the literature. If one does exist, please let me know.

[4]Varian (17) notes that statements of linear demand often leave this lower-bound out but implicitly assume it.

tably (15)) where a consumers are not homogeneous with respect to the costs of supplying them. This is a natural way of modelling adverse selection in markets which, at least informally, was presented as a reason why negative prices may not arise. Importantly, in order to allow for an equilibrium with a negative price, the model here allows for the possibility that costs – including consumer-specific costs – are negative. As discussed below, this may represent potential earnings a firm might earn from consumers from another side of a two-sided platform (e.g., advertising).

The model here assumes that there is a mass point of consumers and that cost considerations may cause a monopolist to want to serve them or not. Importantly, the analysis to follow finds that a zero price could either attract that mass point (if there are no consumer-side (transaction) costs of engaging with the firm) or repel them (if consumers who are indifferent default to not purchasing a free product). As a result, it is shown that to understand the full welfare implications of a zero price, one must also observe whether it has enabled selling or deterring consumption by the mass point.

Significantly, it is demonstrated that when a monopolist chooses to price at zero, depending on consumers' default behaviour, it may not improve social welfare to charge a negative price. This is because when considering whether to serve the mass point or not, to both the monopolist and a social planner, no additional benefits will come other than a change in the costs (which may be negative) of production. The monopolist bears all of these social costs and hence, their decision coincides with the social planner.

Section 4 then considers what effect competition may have. It is found that, under conditions where a monopolist would charge a zero price, competition can lead to negative prices. However, it is demonstrated that this (competitive) outcome is socially undesirable; too much production occurs in a competitive equilibrium at too high a cost relative to the social optimum.

In what follows, I build a basic model with the elements described above and characterise the monopoly pricing problem. I then consider social welfare and the impact of competition.

Section 5 then considers whether have a means of managing costs can overturn these results. A final section concludes.

# 2 Model setup

There is a continuum of consumers in the economy on $i \in [0, 1]$. A consumer of type $i$ is characterised by three properties: their value for the product, their own costs incurred in purchasing or consuming the product and the costs they impose on the firm. Each is explained in turn. Each consumer can purchase at most one unit of the product of a firm where $p$ is the price per unit.

## 2.1 Intrinsic value

If a consumer uses a product they receive some value described by $u(i)$. Absent any other relevant factors in the product's purchase or consumption, the standard assumption in industrial organization is that a consumer's utility is quasi-linear and so their willingness to pay for a unit is $u(i)$. In most applications, analysis focuses on situations where, for each consumer, $u(i) > 0$. However, the standard assumption in microeconomics is to allow the possibility that, for some consumers $u(i) = 0$ and to assume that there is *free disposal* so that, as a consumer who receives a product or has access to a service can simply choose not to consume it, that it is not possible that $u(i) < 0$. To put it another way, a consumer must be compensated for being a potential consumer in the market.

To emphasise, the free disposal assumption is standard in economics. Indeed, Farrell and Gallini (13) argue that this is a reason for a non-negative price constraint.[5] However, as will be demonstrated, when a firm has an ancillary value on acquiring a consumer, this itself does not create an incentive to refrain from a negative price. Nonetheless, the free disposal assumption plays an important role here in that it implies that there is (potentially) a kink

[5] They write, "[a]t a negative price, people could take computers and use them for landfill." (p.679)

in consumer demand with a mass point of consumers for whom $u(i) = 0$. For expositional purposes, in what follows, a consumer with $u(i) > 0$ will be described as generic and those with $u(i) = 0$ as non-generic.

In what follows, a specific functional form will be relied upon for $u(i) = v - \mu i$ for $i < \frac{v}{\mu}$ and $u(i) = 0$ for $i \geq \frac{v}{\mu}$. Thus, the share of generic consumers is $\frac{v}{\mu}$ while the remainder are non-generic. The latter consumers will not use the product for any positive price.

## 2.2 Transaction costs

The second component of a consumer's type are the costs they personally incur in using the product, $t(i)$. These are assumed to be non-positive for all $i$. They are called *transaction costs* here because the costs are specific to a consumer type and, if they are strictly positive, a consumer who is otherwise indifferent between purchasing a product or not (i.e., an $i$ for whom $u(i) = p$) will choose not to purchase. Transaction costs may be literally the cost of transacting – e.g., the costs of filling out a form, travelling to a store, organizing payment – or they may be other costs that are incurred by the consumer – such as the costs associated with a loss of privacy, the annoyance of advertisements or incurred trying to mitigate these costs.

The set of consumers who purchase the product at a price of $p$ is $Q(p) \equiv \{i|u(i)-t(i) \geq p\}$ . Thus, using the earlier functional form for $u(i)$ and assuming that $t(i) = t$, demand, $q(p)$, for the product is as follows:

$$q(p) = \begin{cases} \frac{v-t-p}{\mu} & \text{if} \quad -t \leq p \\ 1 & \quad\quad -t > p \end{cases}$$

That is, quantity demanded is strictly decreasing in price until price falls below a threshold in which case the mass point of non-generic consumers for whom $u(i) = 0$ choose to purchase the product. Alternatively, if $t(i)$ is continuously differentiable in $i$ and $u(i) - t(i)$ is everywhere non-decreasing, there will remain a kink in demand at $i$ such that $u(i) = 0$ but there will be no mass point there.

To anticipate the results to follow, the existence of a mass point at $u(i) = 0$ will be a necessary (but not sufficient) condition for $p = 0$ to be an equilibrium price in any non-degenerate manner (i.e., as a chosen price not simply because of the specific coincidence of all of the model parameters but, instead, for a range of parameters). Thus, in what follows, it will be assumed that:

**(A1)** *There exists a share, $\alpha > 0$, of non-generic consumers for whom $t(i) = t \geq 0$.*

This assumption ensures there is a mass point in demand. The following approach captures this assumption in a tractable format alongside the functional form assumptions on intrinsic demand. Specifically, regardless of their value for the firm's product, there is some share of consumers in the population ($\alpha$) who have a transaction cost of $t(i) = t$ while the remainder have $t(i) = T > t$. Thus, who has significant transactions costs is independent of the value each consumer places on the product.[6]

At what price will that mass point purchase the product? For $t > 0$, that price will be $p = -t$. If $t = 0$, then if the price is $p = 0$, the mass point will purchase the product. That said, if $t > 0$, then, as $t \to 0$, the mass point will not purchase the product. Thus, depending upon whether (i) $t = 0$ or (ii) $t \to 0$, $p = 0$ will either add (i) or not add (ii) the mass point as purchasers. In what follows, it is demonstrated that whether cases (i) or (ii) hold have different implications for the rationale behind a zero price outcome (and its welfare properties). It is left to the specifics of particular applications whether (i) or (ii) is the more appropriate/reasonable assumption. The purpose here remains to characterize all of the cases that result in a zero pricing outcome.

## 2.3 Customer-specific costs

In terms of the costs supplying to consumers imposes, two assumptions are made. First, the firm faces a marginal cost, $c$, that is common to all consumers they supply. Significantly,

[6]Importantly, there is a share, $\alpha$, of non-generic consumers who have a transaction cost of $t$ while a share, $1 - \alpha$, have transaction cost of $T$.

$c$ need not be positive and, in fact, a necessary condition for a zero price to emerge in equilibrium is for it to be negative. We assume that $v > c$. Second, the firm also faces a cost, $ai$ specific to each consumer, $i$.

This assumption plays an important role and is worth elaborating on. The consumer-specific cost means that the firm is not indifferent as to who it supplies. This type of cost was introduced by Einav, Finkelstein, and Cullen (12) and Einav and Finkelstein (11), and further developed by Mahoney and Weyl (15) to model *selection markets*. In such markets, an increase in price may lead to higher or lower average costs depending upon whether they induce adverse or advantageous selection. For instance, if $a < 0$, then, by charging a higher price, $p$, consumers with a higher $i$ will not purchase the product and so the average cost serving a share, $q$, of consumers, $c + \frac{1}{2}aq$, will rise. Thus, a higher price deters "lower cost" consumers; that is, there is *adverse selection*. The reverse happens if $a > 0$ where a higher price lowers average costs; that is, there is *advantageous selection*.[7]

Selection markets have been useful for understanding insurance markets where a higher premium likely selects for consumers who have private information that their risk of an adverse event is higher (modelled as adverse selection or $a < 0$). They are also useful in the context of platforms whereby attracting a consumer to one side of the market allows the firm to earn revenue from the other side of the market. For instance, in a credit card network, those who may be willing to pay a high fee to avoid carrying cash may be those consumers who have large expenditures and are, therefore, of more value to merchants. This would be consistent with a model where $a > 0$. Similarly, a media outlet may attract readers of a certain type which can be better matched with advertisers or a social media network may be able to attract more reliable data from those who are active because they value the network. In this case, a lower price reduces the "quality" of consumers for advertisers and hence, the revenue a platform might earn. This is consistent with $a > 0$.

[7] Of course, it is possible that these selection costs do not operate in a monotone fashion as described here so there is adverse/advantageous selections over different consumer segments. It will be apparent that this possibility does not have a significant impact on the conclusions drawn in this paper.

Intuitively, the existence of selection effects complicates the choice of price by a firm. In their absence, price is chosen and driven by demand-side considerations (as captured by consumers' intrinsic value and transaction costs described above). When there are selection effects, the firm must take into account the 'quality' of the set of consumers a given price selects. This poses challenges for a firm that have monopoly power over price and for the outcomes of competitive markets. As will be shown, it is the firm's concern regarding the quality of consumers it attracts, in particular, from the mass point of consumers that will drive its incentives to set a zero price.

# 3 Zero pricing by a monopolist

One of the key motivations for this paper was the concern raised by some that dominant firms setting zero prices may raise antitrust concerns. To provide a basis for analysing this concern, an understanding of why a monopolist might set a zero price is required. The conditions for zero pricing by a monopolist will be established in this section. Then these conditions will be used in the following section to analyse the impact of competition being imposed.

## 3.1 Homogeneous transaction costs

To build intuition, the case where $\alpha = 1$ is analysed first. In this situation, all consumers face the same transaction cost, $t$. In doing this, we will analyse two cases: (i) where consumers default to purchase if they are otherwise indifferent; that is, where $t = 0$ and (ii) where consumers default to not purchase if they are otherwise indifferent; that is, where $t \to 0$. Given this, the $t$ notation will be dropped for the remainder of this section.

Before considering pricing, the monopolist's quantity decision is analysed. In so doing, it is not that the monopolist cannot choose to have a fraction of the non-generic consumers and either must supply all or none of them. The following proposition characterises this choice:

**Proposition 1** *Suppose that* $t \approx 0$ *and* $\alpha = 1$. *It is profit maximising to sell to the mass point of non-generic consumers if* $c < Min[-\frac{1}{2}a(1+\frac{v}{\mu}), -v(1+\frac{a}{\mu})]$. *It is profit maximising to sell to a share* $\frac{v}{\mu}$ *of consumers if* $c \in [-\frac{1}{2}a(1+\frac{v}{\mu}), -v(1+\frac{a}{\mu}))$.

**Proof**: Let $p(q)$ be the inverse demand for the product. First, note that marginal revenue, $MR(q)$, up to a market share of $\frac{v}{\mu}$ is $v - 2\mu q$ while $MC(q) = c + aq$. If at $q = \frac{v}{\mu}$, $MR \leq MC$ or $c \geq -v(1+\frac{a}{\mu})$, then the firm prefers to set price such that $q \leq \frac{v}{\mu}$, than to price so that mass point purchases its product. Second, profits at $q = \frac{v}{\mu}$ are $\pi(\frac{v}{\mu}) = (p(\frac{v}{\mu}) - c - \frac{1}{2}a\frac{v}{\mu})\frac{v}{\mu}$ while profits at $q = 1$ are $\pi(1) = p(1) - c - \frac{1}{2}a$. Suppose that $p(\frac{v}{\mu}) \approx p(1) \approx 0$ then the firm prefers to sell to the mass point rather than not if $\pi(\frac{v}{\mu}) \leq \pi(1) \implies c \leq -\frac{1}{2}a(1+\frac{v}{\mu})$ . Note, therefore, that if $c < -v(1+\frac{a}{\mu})$ and $c > -\frac{1}{2}a(1+\frac{v}{\mu})$ then profits are maximised at $q \approx \frac{v}{\mu}$. This is possible if $-\frac{1}{2}a(1+\frac{v}{\mu}) < -v(1+\frac{a}{\mu})$ or $v\mu < a(\frac{1}{2}\mu - v)$. Thus, necessary conditions for $q = \frac{v}{\mu}$ to maximize profits are that $a > 0$ and $\frac{v}{\mu} > \frac{1}{2}$. $\square$

The proposition provides sufficient conditions as to whether the firm wants to sell to the mass point of consumers or not. Note that if $AC(1) = c + \frac{1}{2}a \geq 0$, then the firm does not want to sell to them as the average quality would be too low and the firm, as it could not set a positive price, would be making a loss. Because there is a non-convexity in the firm's profit maximisation problem, the proposition also provides conditions under which the firm will want to sell to all consumers *but* those in the mass point; i.e., $q = \frac{v}{\mu}$. This is something that can occur even if $AC(1) > 0$.

These considerations are relevant because, depending on consumer defaults, each may be associated with a zero pricing choice. First, suppose that $t = 0$ and consumers default to purchase when indifferent. In setting price, demand is continuous if $p > 0$ but jumps to 1 for $p \leq 0$ implying that the monopolist cannot obtain a market share in the range $[\frac{v}{\mu}, 1]$. This means that there are two candidate prices that maximise profit, $(p - c - \frac{1}{2}aq(p))q(p)$: (1) the standard monopoly mark-up over costs and (2) a price of 0. Given this, Proposition 1 provides a characterisation of when the monopolist sets $p = 0$; precisely when it wants to supply to the mass point of consumers (i.e., set $q = 1$). Note that a clear sufficient condition for this to occur is $c + a \leq 0$; making all consumers valuable in terms of ancillary earnings for the firm. In this case, it is optimal to cover the market as marginal cost, $MC(q) = c + aq(p)$, is less than marginal revenue.

Second, suppose that $t \to 0$ and consumers default not to purchase when indifferent. In this case, to attract the mass point of consumers, the firm would have to set $p < 0$. Thus, a necessary condition for a zero price in this case is that the firm does not want to supply the mass point of consumers. Note, however, that if it wants to supply $q < \frac{v}{\mu}$, then the monopolist would choose $p > 0$. It will find it optimal to do this if $MR(\frac{v}{\mu}) < MC(\frac{v}{\mu})$. If $MR(\frac{v}{\mu}) = MC(\frac{v}{\mu})$, then it is optimal to set $p = 0$. However, this is non-degenerate as it only holds where $c = -\frac{1}{2}a(1 + \frac{v}{\mu})$. However, if $MR(\frac{v}{\mu}) > MC(\frac{v}{\mu})$, then it is possible that $q = \frac{v}{\mu}$ is the optimal quantity. By setting $p = 0$, this share of consumers purchases the product. Proposition 1 provides these conditions.

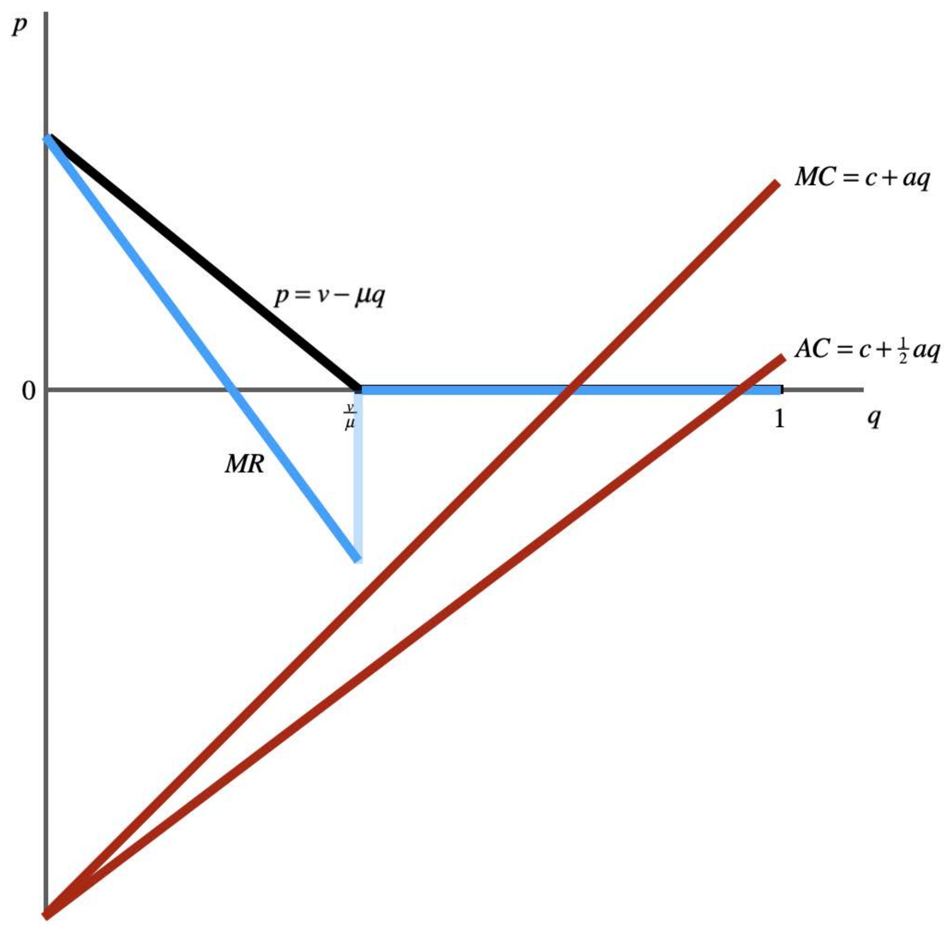
That said, it is perhaps useful to state some more interpretable sufficient conditions for this type of zero pricing outcome.

**Proposition 2** *Suppose that $\alpha = 1$. If (i) marginal cost at $q = \frac{v}{\mu}$ is less than marginal revenue (i.e., $-v > c + a\frac{v}{\mu}$); (ii) there is advantageous selection ($a \geq 0$) and (iii) the average cost supplying all consumers is positive (i.e., $c + \frac{1}{2}a > 0$), then, as transactions costs become arbitrarily small ($t \to 0$), the monopoly price is zero.*

This outcome is illustrated in Figure 1. Here, at a price of 0, $MR > MC$ while $AC(1) > 0$ (as per condition (i)). If, by contrast, $c > 0$, then marginal cost is always above 0 and standard monopoly pricing is optimal. More precisely, standard monopoly pricing may still be optimal if marginal revenue is less than marginal cost at the threshold quantity, $q = \frac{v}{\mu}$.

Note that, if $a < 0$ (violating (ii)), then apart from a degenerate case where the standard monopoly price is 0, it is optimal for the monopolist to cover the market. This could happen, for instance, if there are network effects on the other side of a platform. Similarly, if average costs associated with covering the entire market are negative (violating (iii)), then it is possible that the monopolist will choose cover the entire market.

**Figure 1: Monopoly with Zero Price**




## 3.2 Heterogeneous transaction costs

We now consider what happens when $\alpha < 1$. Recall that, in this case, a share of all consumers ($\alpha$) has a transaction cost of $t$ while the remainder have a higher transaction cost $T > t$. The distribution of transaction cost is independent of other consumer-specific factors. While this basic model captures the essential intuition that would drive a zero pricing outcome, it is useful to extend the model to incorporate significant transaction costs in some manner and examine whether a zero pricing outcome can be maintained. This is important as such consumer generated costs may not be caused by transactional issues but by a loss of utility from privacy if data is appropriated by the firm or from annoying advertising.

Under these assumptions, there exists a point whereby no consumer with both $u(i) < 0$ and a transaction cost of $T$ purchases the product while all consumers with a transaction cost of $t$ purchase the product. This point occurs at $p \approx 0$ with the specifics depending upon the consumers' default purchase rule in the case of indifference. Thus, the demand for the product is either:

$$\lim_{t \to 0} q(p,t) = \begin{cases} (1-\alpha)\frac{v-T-p}{\mu} + \alpha\frac{v-p}{\mu} & p \geq 0 \\ (1-\alpha)\frac{v-T-p}{\mu} + \alpha & \text{if } -T \leq p < 0 \\ 1 & p < -T \end{cases}$$

or

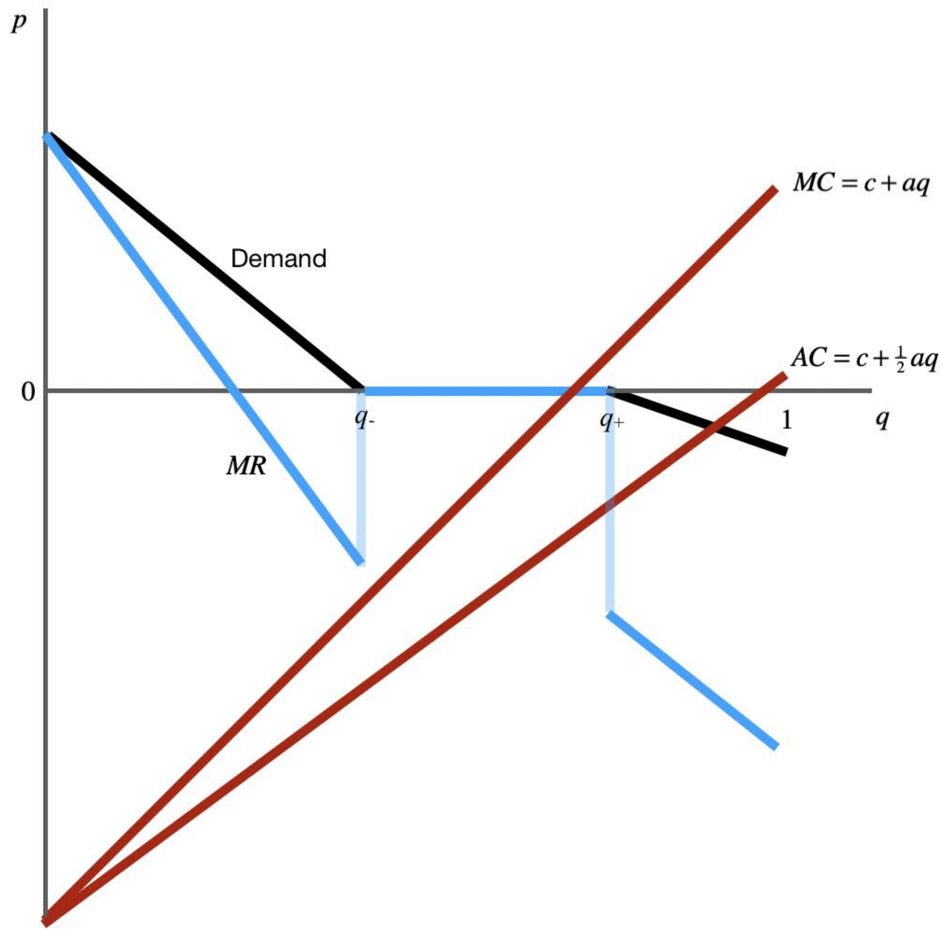
$$q(p,t)_{t=0} = \begin{cases} (1-\alpha)\frac{v-T-p}{\mu} + \alpha\frac{v-p}{\mu} & p > 0 \\ (1-\alpha)\frac{v-T-p}{\mu} + \alpha & \text{if } -T < p \leq 0 \\ 1 & p \leq -T \end{cases}$$

These are depicted in Figure 2. Importantly, there are now two kinks in demand. The first is the one explored earlier at $q_- \equiv (1-\alpha)\frac{v-T}{\mu} + \alpha\frac{v}{\mu}$ while the latter arises at $q_+ \equiv (1-\alpha)\frac{v-T}{\mu} + \alpha$. Thus, the critical factor driving whether the monopolist prices at zero is whether it wants to sell to the fraction $q_+ - q_- = (1-\alpha)\frac{v}{\mu}$ or not. As before, this will be driven by cost-side considerations. Specifically, it can be easily show that the firm will prefer to supply to the mass point than not if:

$$c + \frac{1}{2}a(q_- + q_+) < 0$$

Another way of putting this is that if the average of $MC(q_-)$ and $MC(q_+)$ is negative, then it is worthwhile for the firm to supply the mass point.

**Figure 2: Monopoly with Zero Price with Heterogeneous Transaction Costs**




The following proposition characterises the parameters that give rise to a zero equilibrium price with expensive disposal.

**Proposition 3** *A necessary condition for an equilibrium monopoly price of zero is:*

$$c \in \left[-(\mu + a(1-\alpha))\left(\frac{\alpha}{1-\alpha} + \frac{v-T}{\mu}\right), -(\mu + a)\frac{v-(1-\alpha)T}{\mu}\right].$$

*If, in addition, (i) $t \to 0$ and (ii) $c + \frac{1}{2}a(q_- + q_+) \geq 0$, then there exists a zero price equilibrium without the mass point (at $q_-$). If, in addition, (i) $t = 0$ and (ii) $c + \frac{1}{2}a(q_- + q_+) \leq 0$, then there exists a zero price equilibrium with the mass point (at $q_+$).*

**Proof**: First, note that an interior standard monopoly price for $q < q_-$ will be preferred to setting price such that $q = q_-$ if $MR(q_-) < MC(q_-) \implies c > \frac{((1-\alpha)T-v)(a+\mu)}{\mu}$. This gives the upper bound on $\alpha$ for the necessary condition in the proposition. Second, note that an interior standard monopoly price for $q > q_+$ will be preferred to setting price such that $q = q_+$ if $MC(q_+) < MR(q_+) \implies c < (-\mu - a(1-\alpha))\left(\frac{\alpha}{1-\alpha} + \frac{v-T}{\mu}\right)$. This gives the lower bound on $c$ completing the proof of the necessary condition whereby the firm prefers with $q = q_-$ or $q = q_+$.

The remaining sufficient conditions for zero pricing follow from a combination of a particular default case and the earlier derivation of whether the firm would prefer to supply the mass point, $q_+ - q_- = (1-\alpha)\frac{v}{\mu}$, or not. $\square$

Observe that when $\alpha = 0$ there is no range of $c$ for the necessary condition; that is, a non-degenerate zero pricing outcome is contingent on (A1). When $\alpha$ is high, this is both associated with high 'generic' demand and a larger share of 'non-generic' demand from consumers who choose to purchase as the price falls below zero. However, these two factors make it more likely a positive price will be chosen. By contrast, when $\alpha$ is low, there is both low 'generic' demand and a low share of 'non-generic' demand who purchase when the price becomes negative leading to an equilibrium price that is strictly negative. For intermediate levels of $\alpha$ a zero price equilibrium arises as depicted in Figure 2. Of course, whether this involves the mass point or not depends upon consumers' default choices when indifferent (condition (i) for each case in Proposition 3) and whether the firm prefers to sell to the mass point or not (condition (ii) for each).

# 4 Impact of competition

Having derived the conditions under which a monopolist would set a zero price, we now turn to the impact of competition and, specifically, whether its addition can improve welfare. In particular, recall that the motivation here is to examine, *conditional on a monopolist is observed to be pricing at zero*, whether welfare would be increased by the advent of competition. It is well-understood that when a monopolist charging a positive price, welfare can be improved by reducing prices. Here the focus is on whether it can be improved by moving from a zero to a negative price.

### 4.1 Welfare

To begin, it is useful to examine how the monopoly pricing outcome compares the socially optimal outcome. The welfare criterion used is the sum of consumer and producer surplus. Thus, in the absence of a kink in demand, the socially optimal outcome occurs where consumer willingness to pay is set equal to marginal cost. Here this is complicated somewhat by the fact that willingness to pay may be always higher than marginal cost; in which case, the socially optimal outcome is for all consumers to consume the product – something that could be achieved by setting $p = c + aq$. However, if marginal cost is above 0 for some range, then it may equal willingness to pay only where willingness to pay is 0. In this case, the optimal outcome could be achieved by rationing consumption to that point but this outcome could not be implemented using a simple linear price.

Given this, we can establish a remarkably clean coincidence between the monopolist's preferences and those of a social planner.

**Proposition 4** *For $t \approx 0$, the social planner's preference between supplying the product to the mass point or not coincides with that of the monopolist.*

In the mass point, $u(i) = 0$ and, by assumption, those consumers do not incur more than infinitesimal transaction costs. These are necessary conditions for zero pricing to be an equilibrium for the monopolist. Given this, the only welfare change is the difference, $AC(q_+)q_+ - AC(q_-)q_-$, which is the same as the change in monopoly profits from moving from $q_-$ to $q_+$. Intuitively, while a social planner would take into account consumer welfare, both it and the monopolist's decision are based purely on 'cost' side considerations as both consumer welfare and revenue do not change as we move from $q_-$ to $q_+$.

An implication of this is that if there are homogeneous transaction costs (that is, $\alpha = 1$), then, conditional on the monopolist's optimal quantity being $q_-$ or $q_+$, there will be no improvement in welfare from moving to a negative price. Either the mass point is not supplied, in which case, supplying it would raise social costs without any increase in consumer surplus or the mass point is supplied in which case a negative price will be a pure transfer from the producer to consumers.

Zero pricing is optimal when there are homogeneous transaction costs. When there are heterogeneous transaction costs, the social optimality of the monopolist's choice regarding a zero price depends on the default rule for consumers when they are indifferent. In particular, if consumers' default rule is not to purchase (that is, $t \to 0$), then a zero price by the monopolist is associated with an intention *not* to supply the mass point of non-generic consumers. A necessary condition for this to be an equilibrium choice for the monopolist is that $MC(q_+) > 0$. If this condition did not hold, then the monopolist would prefer to price slightly below zero. Under this condition, the marginal cost curve intersects the demand curve at a quantity below $q_+$. Thus, the social planner is necessarily considering between outputs $q_-$ and $q_+$ which, by Proposition 4, coincides with that of the monopolist.

By contrast, a monopolist's zero pricing choice may be socially sub-optimal if consumers' default to purchasing (that is, $t = 0$). In that case, the monopolist chooses a zero price if they want to supply the mass point of consumers up to $q_+$. If $MC(q_+) \geq 0$, this choice coincides with the social planner's preference by Proposition 4. However, if $MC(q_+) < 0$, the socially optimal quantity is greater than $q_+$ and can be achieved with a $p < 0$.

### 4.2 Competition

We now turn to consider (perfect) price competition with more than one firm in the market. As demonstrated by Mahoney and Weyl (15), in selection markets, competition drives price to average cost. Here, the kinked demand curve means that this is not necessarily the equilibrium outcome. In particular, if average costs are positive if the mass point of non-generic consumers are supplied (that is, if $c + \frac{1}{2}aq_+ > 0$) then competitive firms pricing so that this outcome occurs would not be profitable. In this case, they would set a price such that only $q_-$ consumers purchase in the market. At this point, any firm lowering price would end up making a loss. Note, however, that in this situation, if the monopolist were pricing so that quantity was $q_-$, then competition would not change this outcome and this outcome would also be socially optimal.

What if $c + \frac{1}{2}aq_+ < 0$? In this case, competitive firms would set a negative price and quantity would exceed $q_+$. How does this compare to the socially optimal outcome?

It can easily be seen that this outcome can be socially suboptimal. If $a > 0$, then $MC(q_+) > AC(q_+)$ and the social planner would prefer a smaller output. In other words, in a competitive market, price may be too low compared with a socially optimal and, indeed, a monopoly outcome. This mirrors Mahoney and Weyl's (15) result that when there is advantageous selection (where with $a > 0$), competition results in over-production relative to the social optimum. We have shown that this conclusion extends to the kinked demand curve case although their result regarding monopoly under-production does not hold under the conditions already discussed.

What does this imply about the impact competition will have on a market where a monopolist would set $p = 0$? First, if the consumers' default is not to purchase ($t \to 0$), then the monopolist's quantity is $q_-$ while it is possible that, if $c + \frac{1}{2}aq_+ < 0$, a competitive outcome will involve a negative price. However, by Proposition 4, this negative price is socially sub-optimal. That is, a competitive market leads to too many high cost consumers being selected.

Second, if the consumers' default is to purchase ($t = 0$), then a zero price by the monopolist will lead to them producing $q_+$. In this situation, if $c + \frac{1}{2}aq_+ < 0$, a competitive outcome will involve a negative price as will the social optimum. But does this mean that competition will lead to higher social welfare? It is easy to show that competition does not increase social welfare over a monopoly zero price outcome if:

$$q^* \equiv \frac{(1-\alpha)(v-T-c)+\alpha\mu}{(1-\alpha)a+\mu} > \frac{q_- + q_+}{2}$$

Intuitively, if $c$ is less negative (all other things remaining equal), a zero price by a monopolist will still be socially more desirable than a negative price under competition. The same is true as $a$ rises and there is a greater selection cost associated with lower pricing.

### 4.3 Cost changes

In platform markets, costs (specifically, negative costs) are generated by the firm's ability to earn revenue from the other side of the market. It is useful to remark that, if there is an effect of competition on that other side of the market, it is to reduce those earnings and to, therefore, raise costs.[8] In this situation, a full welfare analysis would require an examination of welfare on both sides of the market. However, in this case, both the socially optimal price and the competitive price are likely to be higher than those examined here. Moreover, in this situation, if the monopolist is pricing at zero, then an improvement in social welfare would lead to a positive rather than a negative price.

## 5 Cost management

Thusfar, the model assumes that the costs to the firm of various consumers are given and cannot be managed. In practice, firms that choose to charge a negative price often engage in 'cost management.' For example, for some years Microsoft has offered consumers valuable rewards for searches they conduct on its Bing search engine. Microsoft opts for what is essentially a negative price for each search because of the potential advertising revenue such activity could generate. However, it does not simply offer rewards verbatum. Instead, it engages on various practices to deter consumers who may be searching solely to earn rewards and not be engaging in advertisements (as well as bots designed to mimic them). Microsoft needs to do this otherwise the quality of consumers it delivers to advertisers might become

[8] See (6), (1) and (4)

too low and reduce the amount advertisers are willing to pay.[9]

While Microsoft has experimented with different policies, typically, consumers are only entitled to rewards if they actively opt in (by completing a tour that informs them how rewards work and how they can be redeemed). In addition, rewards are only generated if consumers conduct a certain number of searches and there are daily caps on the magnitude of rewards earned. Finally, Microsoft can observe searches and directly identify consumers who appear to be entering nonsense search terms just for rewards. Broadly, these practices are motivated by a desire to manage costs and to ensure that marginal costs are capped and genuine consumers are selected.

The interesting case that creates this issue is where the firm is motivated to avoid selling to the mass point of non-generic consumers by pricing at zero. Thus, we assume here that $t \to 0$ (consumers default to not purchasing) and $\alpha = 1$ (transaction costs are homogeneous). We want to see whether the ability to manage costs – particularly associated with the mass point – changes the conclusions regarding the social optimality of monopoly zero pricing in this case.

To explore this, suppose that, at a cost of $f > 0$, a firm can implement a cost management system that prevents any consumer for whom marginal cost is positive from purchasing the product.[10] What this means is that if $p < 0$, then the maximum quantity that can be sold is $q = -\frac{c}{a}$; which is less than 1 if we assume that marginal cost when the entire market is covered is positive. Thus, a firm setting a negative price will earn profits of $-p\frac{c}{a} + \frac{1}{2}\frac{c^2}{a} - f$. By contrast, a firm setting $p = 0$ will earn $-c\frac{v}{\mu} - \frac{1}{2}a(\frac{v}{\mu})^2$ (where it is assumed that $-\frac{c}{a} > \frac{v}{\mu}$. In this case, cost management will be preferred to a zero price if:

$$-p\frac{c}{a} + \frac{1}{2}\frac{c^2}{a} - f > -c\frac{v}{\mu} - \frac{1}{2}a(\frac{v}{\mu})^2$$

[9]Note that web searches would fall into the non-free disposal model because those searches, unless performed by bots, would be costly for consumers to perform. That said, the point here regarding the quality of consumers and how to manage that quality remains.

[10](18) explore these techniques in a more micro-founded environment.

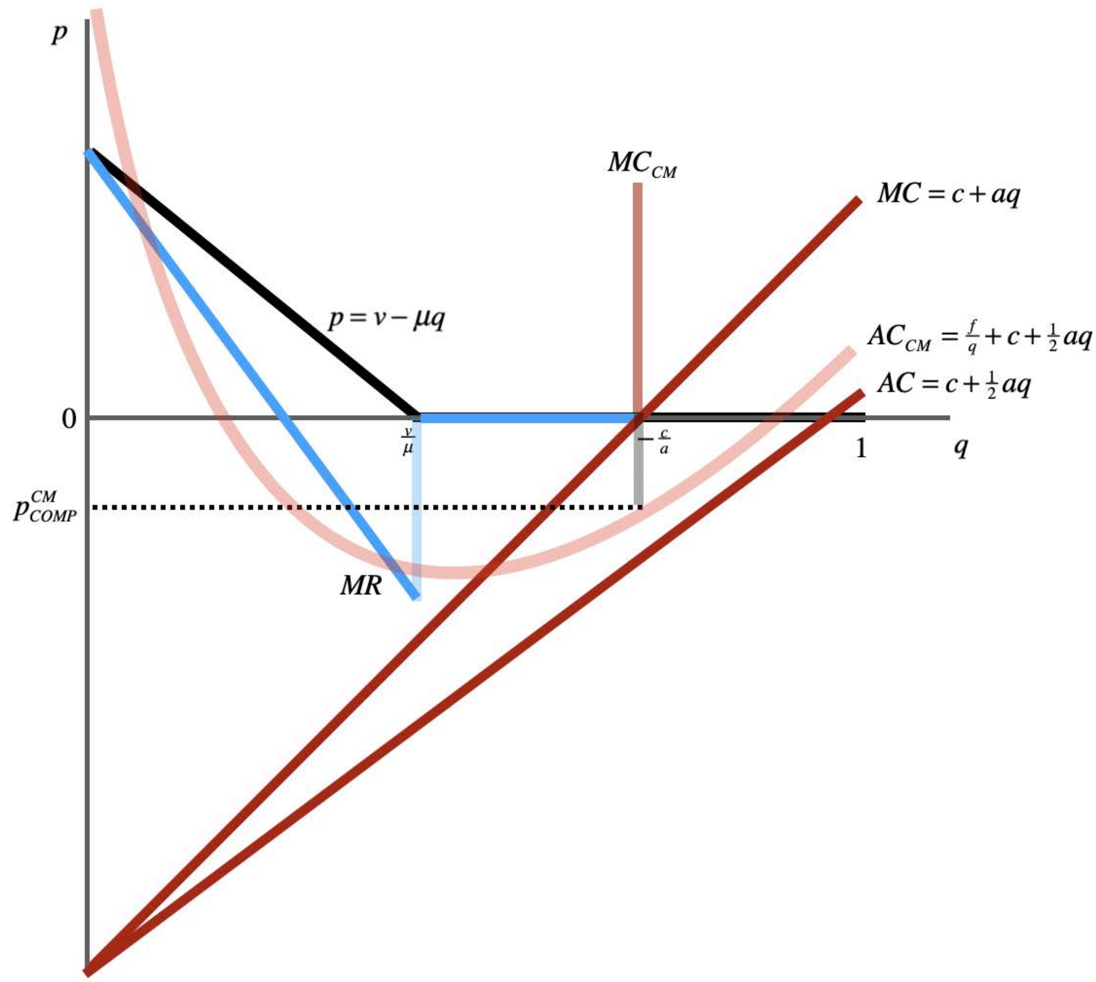
With cost management, the monopolist will set a price close to 0 as $t \to 0$. Thus, the monopolist will prefer cost management if $\frac{(av+c\mu)^2}{2a\mu^2} > f$.

By contrast, a competitive firm will set price equal to average cost or $p_{COMP}^{CM} = \frac{c^2}{2a} + f\frac{a}{c}$ under cost management where we assume, for simplicity, that there is just a single competitive firm that is price constrained in a perfectly contestable market. Note that, the average cost under cost management always lies above the average cost without cost management (that is, $AC_{CM} > AC$ in Figure 3). Thus, if there otherwise was a competitive equilibrium with a negative price, no firm would invest in cost management as their price would be undercut by their competitors. For this reason, to explore the incentives to invest in cost management, we assume that, in its absence, competitive firms are also setting a zero price. In this situation, one of those firms will have an incentive to adopt cost management if average cost with cost management are negative at $q = -\frac{c}{a}$; that is, $-f\frac{a}{c} + c - \frac{1}{2}a\frac{c}{a} < 0$ or $\frac{c^2}{2a} > f$. This outcome is depicted in Figure 3.[11]

This condition for a competitive equilibrium to involve cost management is more restrictive than the condition for a monopolist to do so; that is, $\frac{(av+c\mu)^2}{2a\mu^2} > \frac{c^2}{2a}$. Thus, cost management is less likely to occur under competition than under monopoly; specifically, if $\frac{(av+c\mu)^2}{2a\mu^2} > f > \frac{c^2}{2a}$ then cost management will be chosen under monopoly but not under competition.

[11]Observe that a competitive equilibrium under cost management does not result in a minimisation of average costs because there is a range of quantity, $q \in (\frac{v}{\mu}, -\frac{c}{a})$ that cannot be targeted by price alone.

**Figure 3: Equilibrium Outcomes with Cost Management**




This analysis shows that there is a greater incentive to engage in cost management under monopoly than under competition. We now turn to consider the welfare properties of these outcomes. Conditional on there being no cost management, if a monopolist chooses a zero price, this is socially optimal with quantity supplied at $q = \frac{v}{\mu}$. If there is cost management, the socially optimal quantity is $q = -\frac{c}{a}$ which is what arises in both the competitive and monopoly outcomes with cost management. Ignoring $f$, the increment to surplus moving between these two outcomes is $\frac{(av+c\mu)^2}{2a\mu^2}$. This is the same as the increment to the monopolist's (net) profit from adopting cost management. This follows the intuition of Proposition 2. However, it also demonstrates that pricing can remain inefficiently high under competition due to the diminished incentives to adopt cost management by competitive firms.

# 6 Conclusion

This paper has examined the conditions under which a zero price equilibrium may emerge in both monopoly and competitive markets. It identifies free-disposal plus advantageous selection with negative costs over a significant share of demand as key drivers of the choice of a zero price. Under the conditions that generate this outcome, it is demonstrated that a monopolist's pricing incentives can result in outcomes that are socially optimal while a negative price, should it occur in a competitive market, can be socially inefficient. This result is robust to the inclusion of cost management techniques that can limit high cost consumers from the market.

The results here suggest that there may be some credence to the notion that an observed zero pricing outcome in monopoly markets may require a specialised analysis of policy impacts – such as those that might arise from antitrust enforcement and regulation. This is because the conditions generating that outcome also lead to conditions under which competition (and lower pricing) may not improve welfare. This analysis would also apply to policy issues such as those advocating regulations to facilitate markets that pay consumers for data provided to firms (see (8)); requiring more careful exploration of the impact of such policies (including institutions that may overcome the limitations of relying on pricing for selection).

Nonetheless, it is important to note that the model here was designed to provide a general treatment and did not engage in a more micro-founded approach to the cost-side of the analysis. Such approaches have been typically found in the analysis of platforms and a critical issue for future work is an examination as to whether these would over-turn the qualitative results of this paper.

# References


[1] Ambrus, Attila, Emilio Calvano, and Markus Reisinger. "Either or both competition: A 'two-sided' theory of advertising with overlapping viewerships." *American Economic Journal: Microeconomics* 8.3 (2016): 189-222.

[2] Anderson, Chris, *Free: The Future of a Radical Price*, Hyperion: New York, 2009.

[3] Anderson, Simon P., and Stephen Coate. "Market provision of broadcasting: A welfare analysis." *Review of Economic Studies* 72.4 (2005): 947-972.

[4] Anderson, Simon P., Oystein Foros, and Hans Jarle Kind. "Competition for advertisers and for viewers in media markets." *Economic Journal* 128.608 (2017): 34-54.

[5] Armstrong, Mark "Competition in two-sided markets," *The Rand Journal of Economics*, 37 (3) (2006): 668-691.

[6] Athey, Susan, Emilio Calvano, and Joshua S. Gans. "The impact of consumer multi-homing on advertising markets and media competition." *Management Science* 64.4 (2016): 1574-1590.

[7] Amelio, Andrea and Bruno Jullien, "Tying and Freebie in Two-Sided Markets," *International Journal of Industrial Organization*, (2012): 436-446.

[8] Arrieta-Ibarra I, Goff L, Jiménez-Hernández D, Lanier J, Weyl EG. "Should We Treat Data as Labor? Moving beyond Free," *AEA Papers and Proceedings* Vol. 108, (2018): 38-42.

[9] Caillaud, Bernard and Bruno Jullien, "Chicken and Egg: Competition among Intermediation Service Providers," *Rand Journal of Economics*, Summer (2003): 309-328.

[10] Choi, Jay Pil and Doh-Shin Jeon, "A Leverage Theory of Tying in Two-Sided Markets with Non-Negative Price Constraints," *American Economic Journal: Microeconomics*, (2019), forthcoming.

[11] Einav, Liran, and Amy Finkelstein, "Selection in Insurance Markets: Theory and Empirics in Pictures," *Journal of Economic Perspectives* 25 (2011): 115–138.

[12] Einav, Liran, Amy Finkelstein, and Mark R. Cullen, "Estimating Welfare in Insurance Markets Using Variation in Prices," *Quarterly Journal of Economics* 125 (2010), 877–921.

[13] Farrell, Joseph and Nancy T. Gallini, "Second-Sourcing as a Commitment: Monopoly Incentives to Attract Competition," *Quarterly Journal of Economics*, (1988): 673- 694.

[14] Gans, Joshua S., "Are We Too Negative on Negative Fees for Payment Cardholders?" (April 14, 2018). Rotman School of Management Working Paper No. 3162627. Available at SSRN: https://ssrn.com/abstract=3162627.

[15] Mahoney, Neal and E. Glen Weyl, "Imperfect Competition in Selection Markets," *Review of Economics and Statistics*, 99(4), (2017): 637–651.

[16] Scott Morton, F., P. Bouvier, A.l Ezrachi, B. Jullien, R. Katz, G. Kimmelman, A. D. Melamed, and J. Morgenstern, *Committee for the Study of Digital Platforms Market Structure and Antitrust Subcommittee Report*, George J. Stigler Center for the Study of the Economy and the State, The University of Chicago Booth School of Business, (2019).

[17] Varian, Hal. "Price Discrimination and Social Welfare." *American Economic Review* 75:4, (1985): 870-75.

[18] Veiga, Andréi, and E. Glen Weyl. "Product design in selection markets." *Quarterly Journal of Economics* 131(2), (2016): 1007-1056.